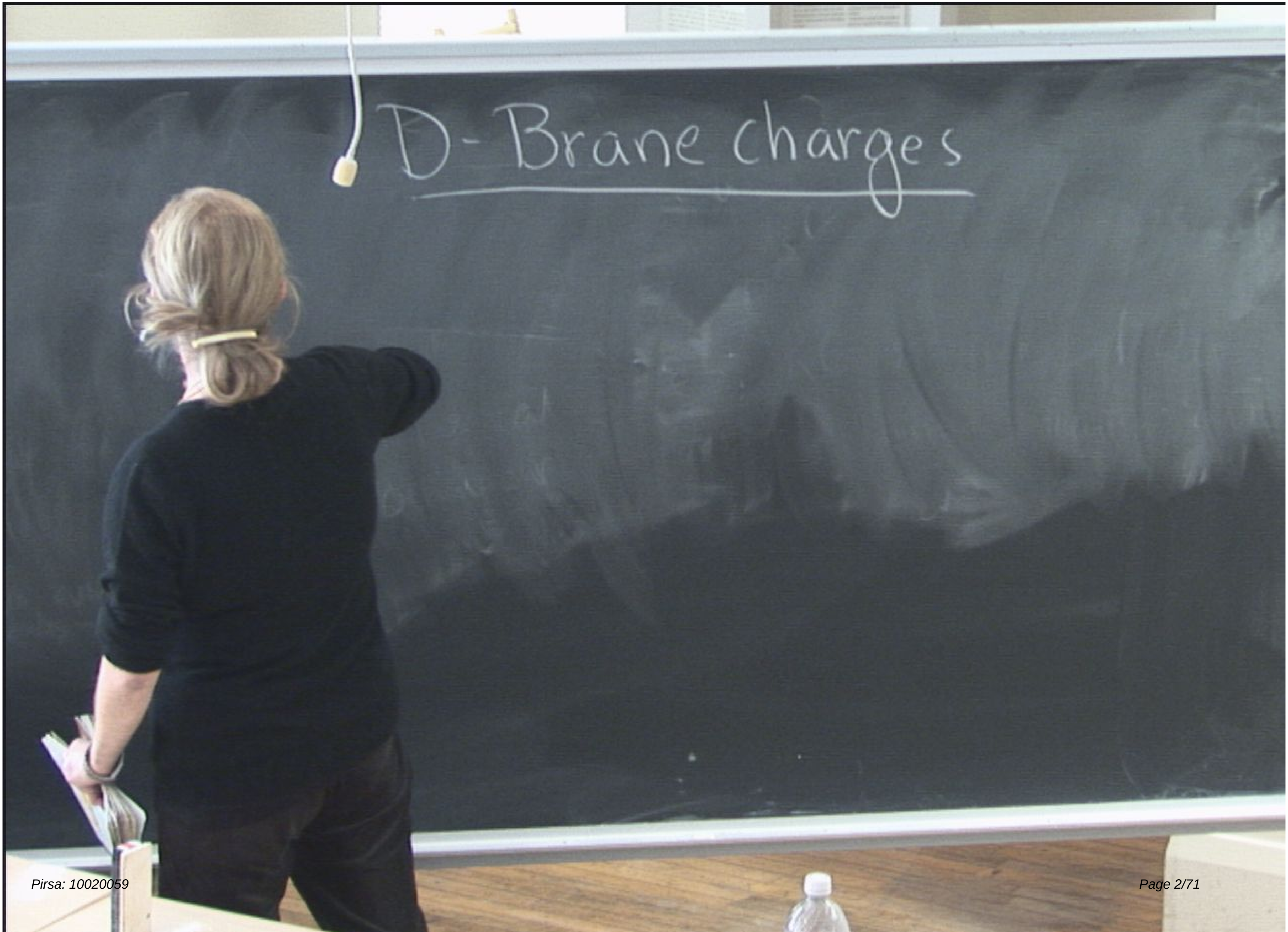
Title: String Theory - Review (PHYS 623) - Lecture 15

Date: Feb 12, 2010 11:20 AM

URL: http://pirsa.org/10020059

Abstract:


D-Brane charges

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n-form:

$$A_n = \frac{1}{n!} A_{\mu_1 \dots \mu_n} dX^{\mu_1} \wedge \dots \wedge dX^{\mu_n}$$

D-Brane charges

n-form:

$$A_n = \frac{1}{n!} A_{\mu_1 \ldots \mu_n} dX^{\mu_1} \wedge \ldots \wedge dX^{\mu_n}$$

$$F_{n+1} = d\underbrace{A_n}_{d\Lambda_{n-1}\ \text{gauge transform}}$$

Maxwell Theory $D=4$ A_μ.

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1) Maxwell eqs.

$$dF_2 = 0 \qquad d * F_2 = 0$$

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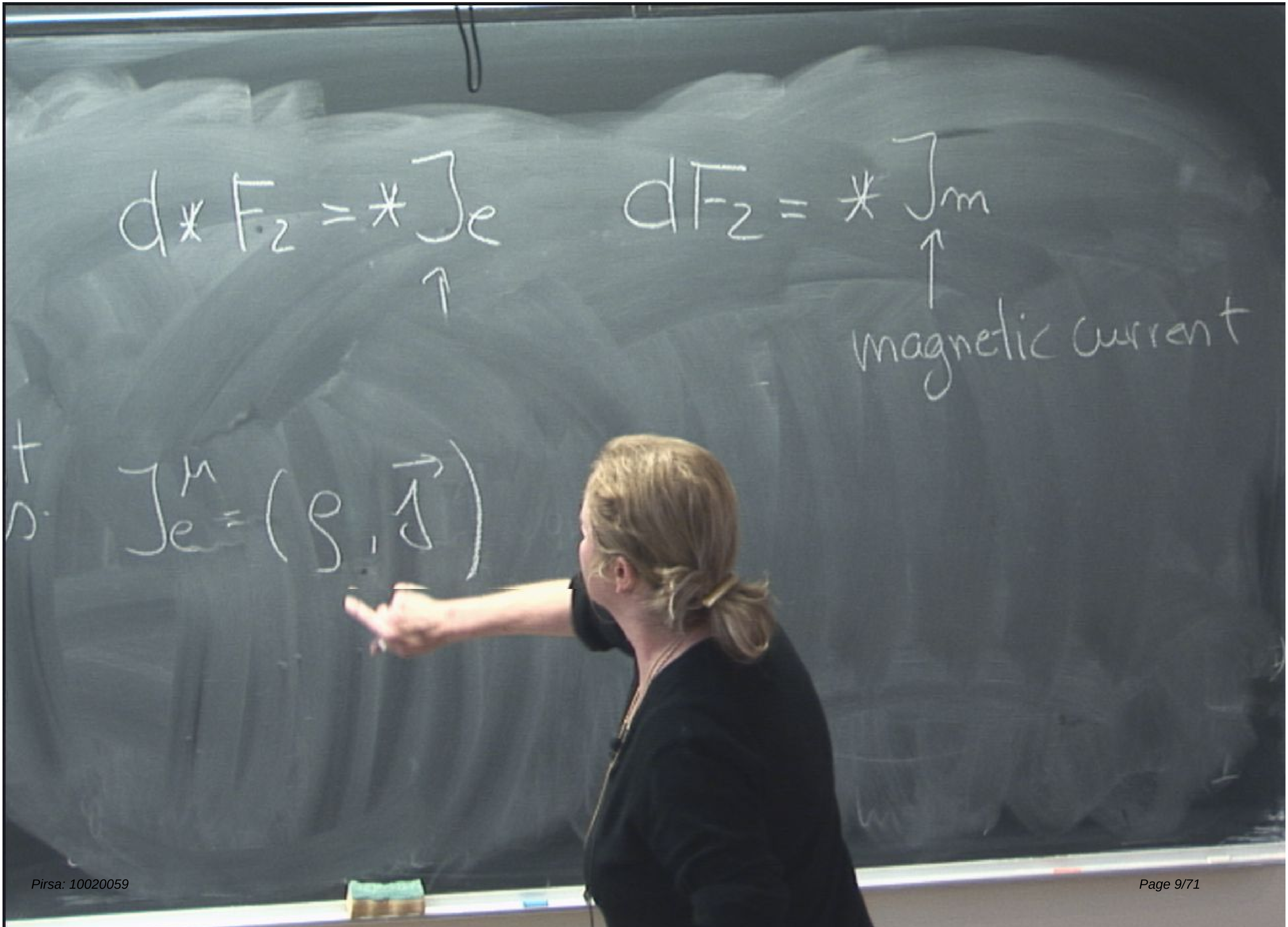
$$* F^{\mu_1 \mu_2 \cdots \mu_{D-2}} = \frac{\varepsilon^{\mu_1 \cdots \mu_D}}{2\sqrt{-g}} F_{\mu_{D-1} \mu_D}$$

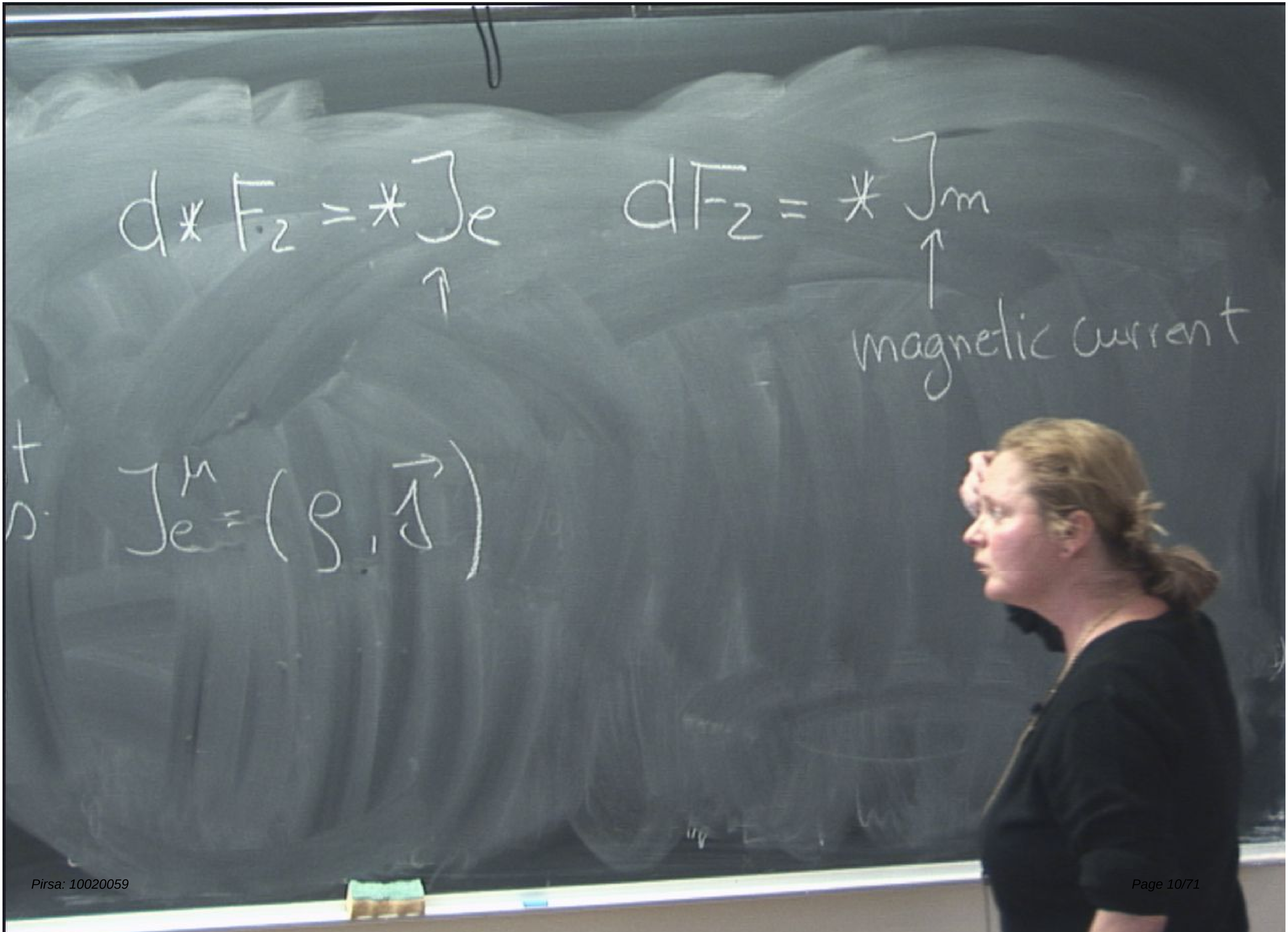
Maxwell Theory $D=4$ A_μ.

1) Maxwell eqs.

$$dF_2 = 0 \qquad d * F_2 = 0$$ without sources.

$$*F^{\mu_1 \mu_2 \cdots \mu_{D-2}} = \frac{\varepsilon^{\mu_1 \cdots \mu_D}}{2\sqrt{-g}} F_{\mu_{D-1}\mu_D}$$


$d * F_2 = * J_e$
$dF_2 = * J_m$
magnetic current
$J_e^\mu = (\rho, \vec{J})$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$$\rho = e\, \delta^{(3)}(\vec{r})$$

$$d * F_2 = * J_e \qquad dF_2 = * J_m$$

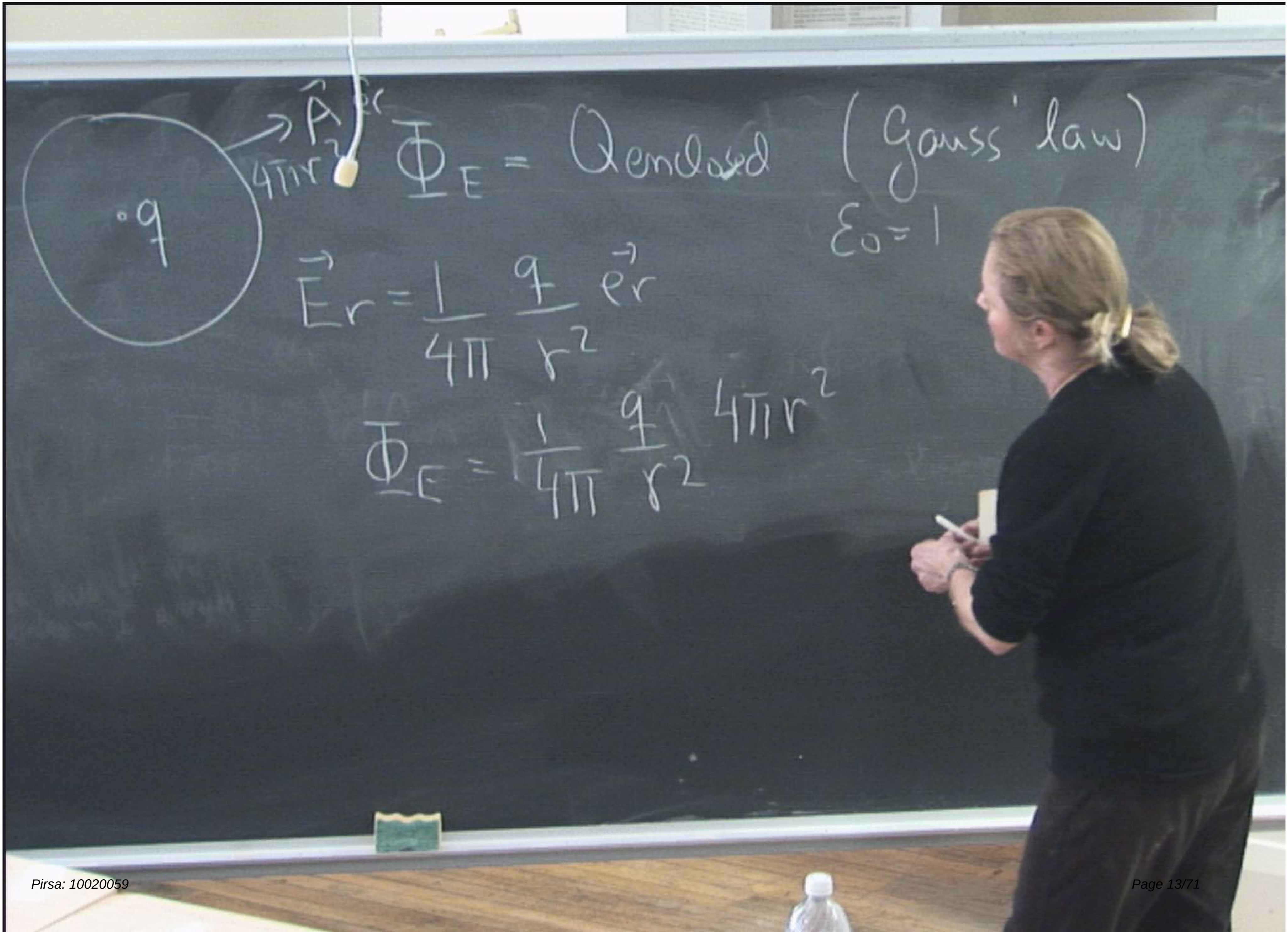
magnetic current

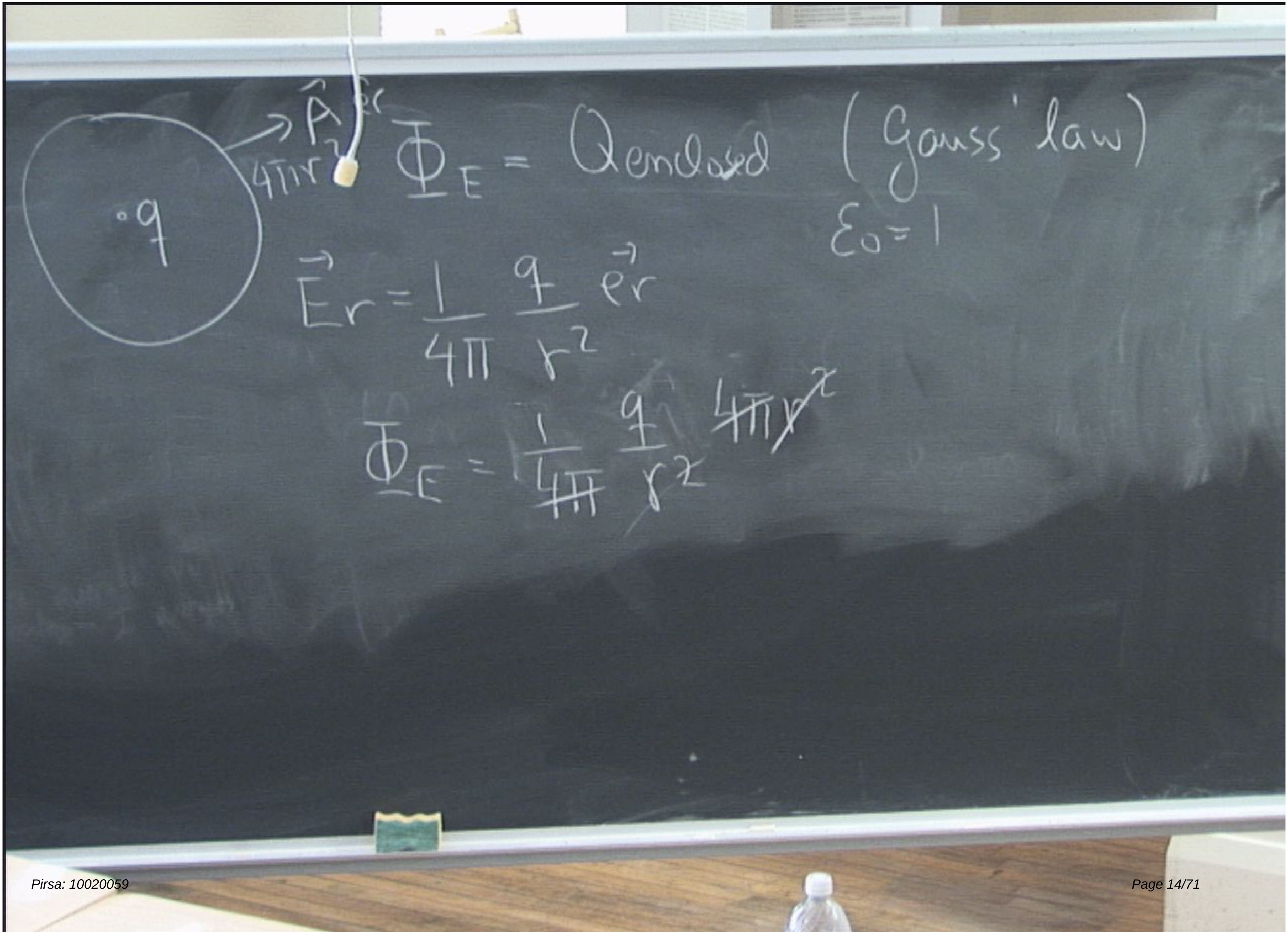
$$J_e^\mu = (\rho, \vec{J})$$

$$\rho = e \, \delta^{(3)}(\vec{r})$$

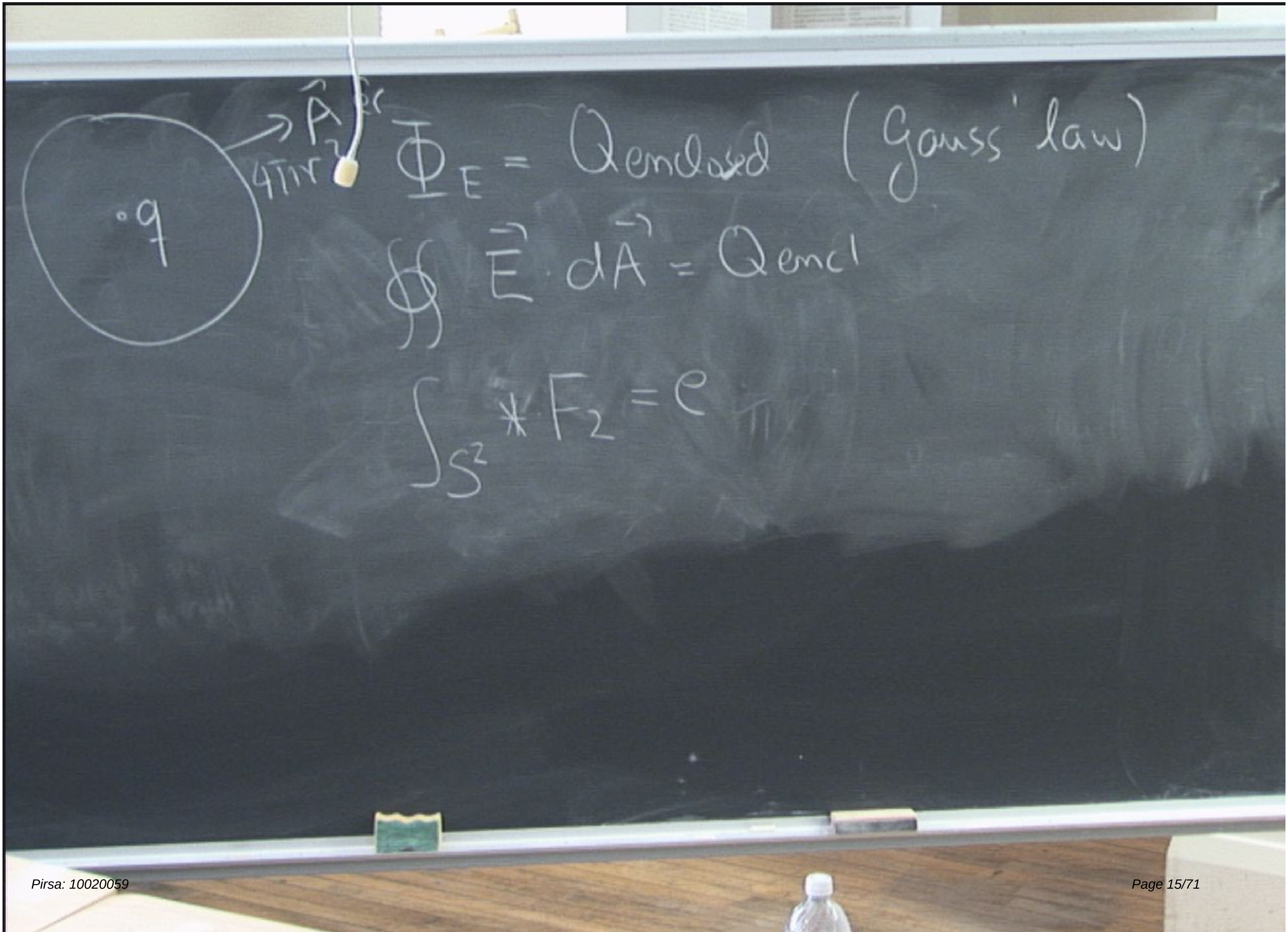
electric charge

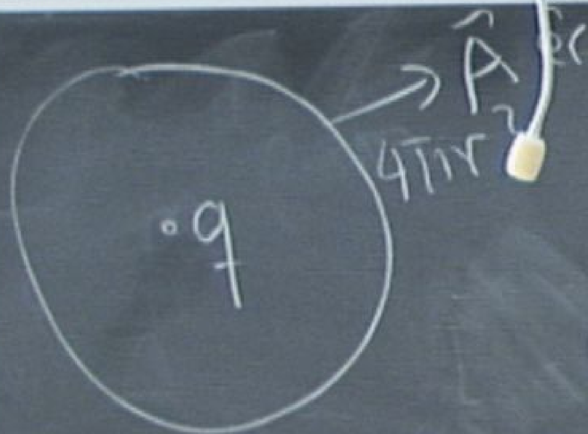
"g" magnetic charge.


q
$\hat{A}$ $\hat{e}_r$
$4\pi r^2$
$\Phi_E = Q_{enclosed}$ (Gauss' law)
$\varepsilon_0 = 1$
$\vec{E}_r = \frac{1}{4\pi} \frac{q}{r^2} \vec{e}_r$
$\Phi_E = \frac{1}{4\pi} \frac{q}{r^2} 4\pi r^2$



$\cdot q$
$\hat{A}$ $\hat{e}_r$
$4\pi r^2$
$\Phi_E = Q_{enclosed}$ (Gauss' law)
$\varepsilon_0 = 1$
$\vec{E}_r = \frac{1}{4\pi} \frac{q}{r^2} \vec{e}_r$
$\Phi_E = \frac{1}{\cancel{4\pi}} \frac{q}{r^2} \cancel{4\pi} \cancel{r^2}$

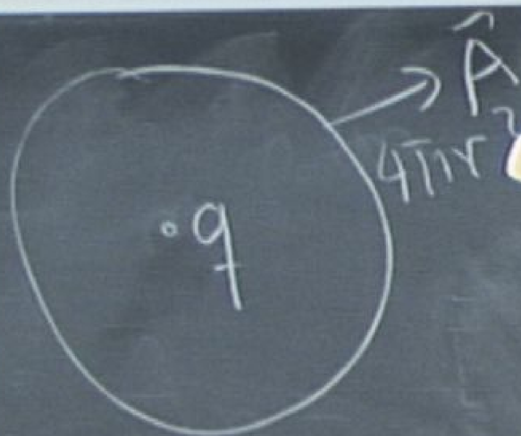

q
$\hat{A}\,\hat{r}$
$4\pi r^2$
$\Phi_E = Q_{enclosed}$ (Gauss' law)
$\oint\!\!\oint \vec{E} \cdot d\vec{A} = Q_{encl}$
$\int_{S^2} *F_2 = e$



$$\Phi_E = Q_{enclosed} \quad \text{(Gauss' law)}$$

$$\oiint \vec{E} \cdot d\vec{A} = Q_{encl}$$

$$\int_{S^2} *F_2 = e \qquad \int_{S^2} F_2 = g$$

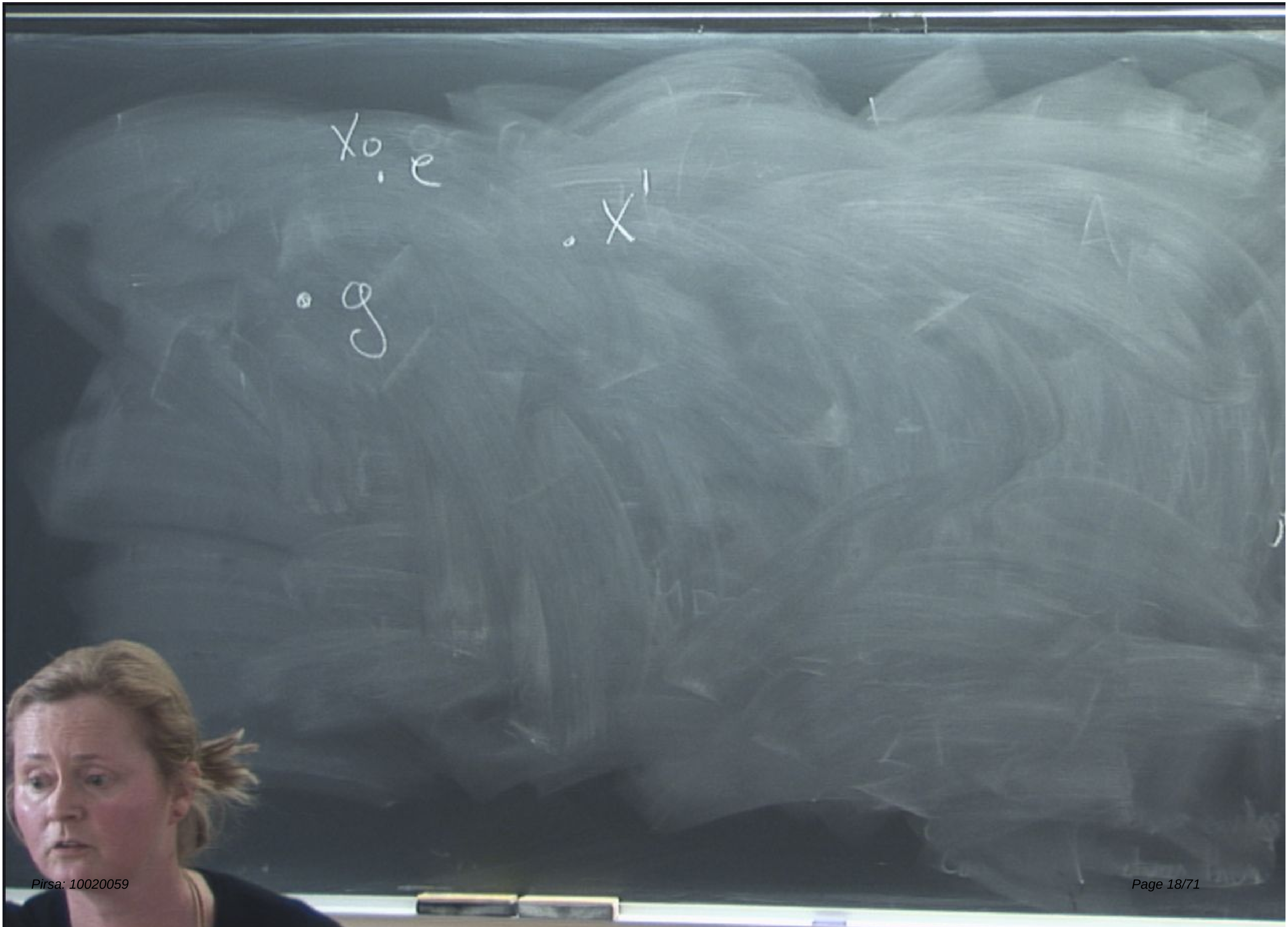


$$\Phi_E = Q_{enclosed} \quad \text{(Gauss' law)}$$

$$\oint \vec{E} \cdot d\vec{A} = Q_{encl}$$

$$\int_{S^2} *F_2 = e \qquad \int_{S^2} F_2 = g$$

<u>Dirac</u> $e \cdot g \in 2\pi\mathbb{Z}$


X0 . e
. X'
. g

$x_0 \cdot e$

$\cdot x'$

$\cdot g$

$$\psi \to \underbrace{U}_{\text{holonomy}} \psi$$

$$U = \exp \int \omega^\mu \cdot dx_\mu$$

$x_0 \cdot e$

$\cdot\, x'$ wave fct

$\cdot\, g$

$\Psi \rightarrow U \cdot \Psi$

$\uparrow$

$\exp i \int A_\mu dx^\mu$

$\psi \rightarrow \underbrace{U}_{\text{holonomy}} \psi$

$U = \exp \int \omega^\mu \cdot dx_\mu$

$x_0 \cdot e$

$\cdot\, x'$ wave fct

$\cdot\, g$

$\Psi \to U \cdot \Psi$

$$U \cdot \Psi = \exp i \underbrace{\int_{x_0}^{x'} A_\mu \frac{dx^\mu}{d\tau} d\tau}_{\text{holonomy}}$$

$$\psi \to \underbrace{U}_{\text{holonomy}} \psi$$

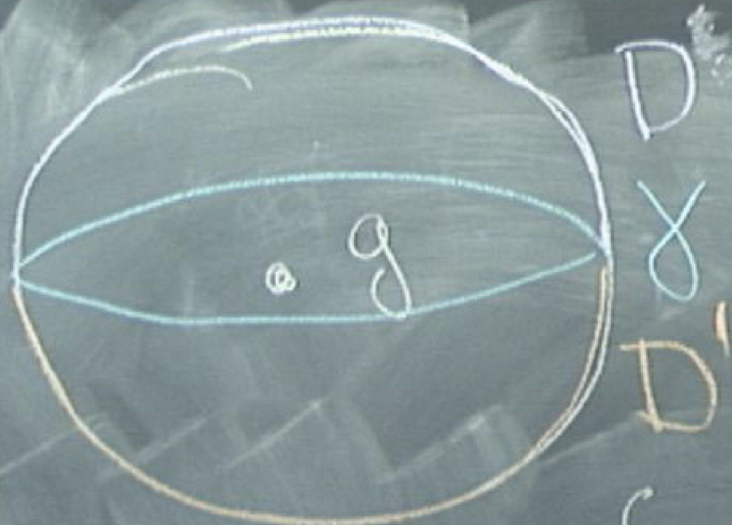
$$U = \exp \int \omega^\mu \cdot dx_\mu$$

$$F = dA$$

$$(= e^{i \oint_\gamma A^\mu dX_\mu}$$

Stokes

$$\oint_\gamma A \cdot dX = \iint_D F_2$$



$$F = dA$$

$$U = e^{\,i\oint_\gamma A^\mu dX_\mu}$$

Stokes

$$\oint_\gamma A \cdot dX = \iint_D F_2$$

$$\int_D F_2 - \int_{D'} F_2 = \int_{S^2} F_2 = g$$

$x_0 \cdot e$

$\cdot x'$ wave fct

$\cdot g$

$$\Psi \to U \cdot \Psi$$

$$\psi \to \underbrace{U}_{\text{holonomy}} \psi$$

$$U = \exp \int \omega^\mu \cdot dx_\mu$$

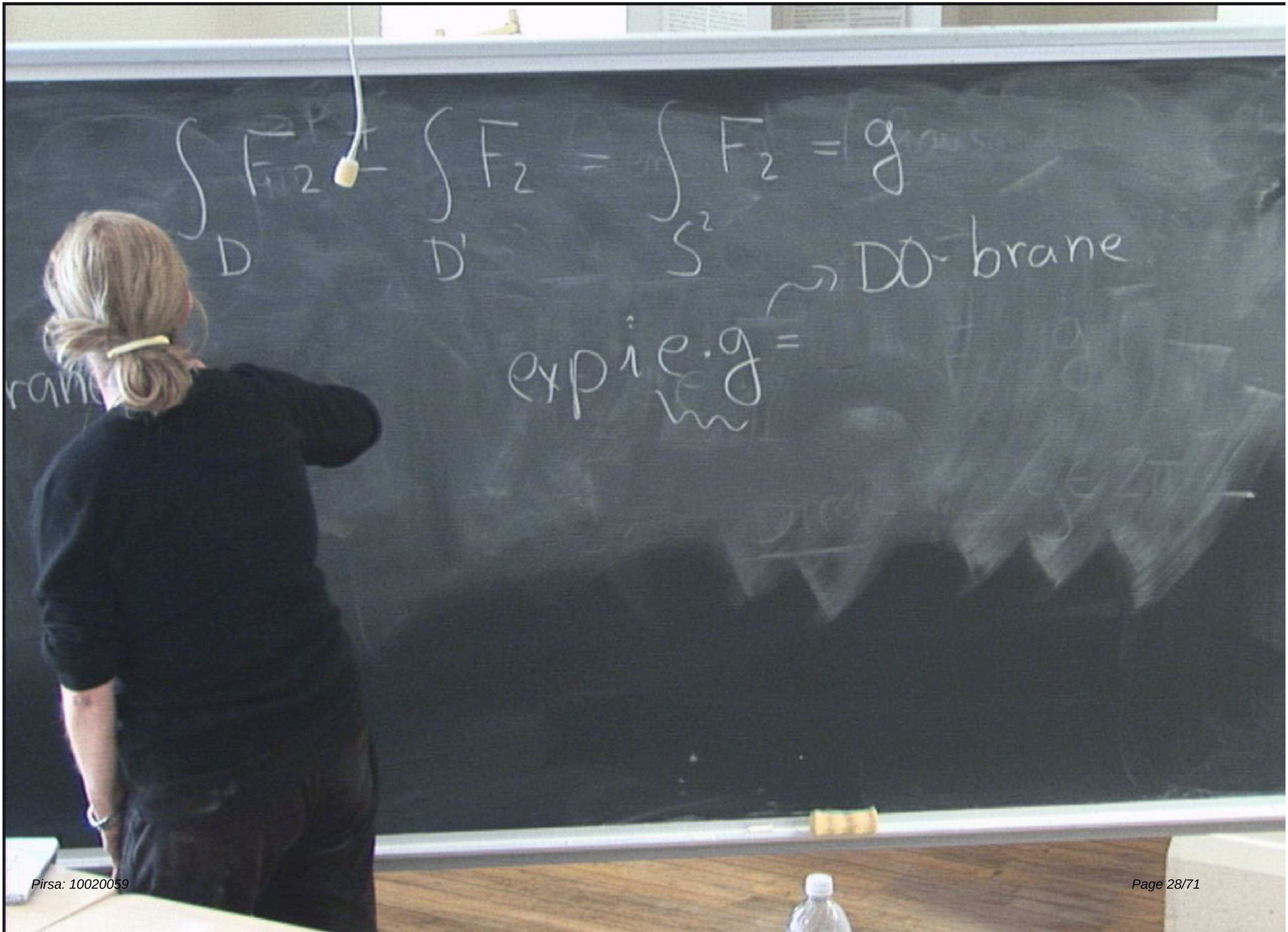
$$\underbrace{\exp ie \int_{x_0}^{x'} A_\mu \frac{dx^\mu}{d\tau} d\tau}_{\text{holonomy}}$$

$$\int_D F_2 - \int_{D'} F_2 = \int_{S^2} F_2 = g$$

$$\exp i \underbrace{e \cdot g}_{2\pi\mathbb{Z}} =$$

$$\int_D F_2 - \int_{D'} F_2 = \int_{S^2} F_2 = g \rightsquigarrow \text{D0-brane}$$

$$\exp i \underbrace{e \cdot g}_{2\pi\mathbb{Z}} =$$


$\int_D F_2 - \int_{D'} F_2 = \int_{S^2} F_2 = g$
D0-brane
$\exp i e \cdot g =$

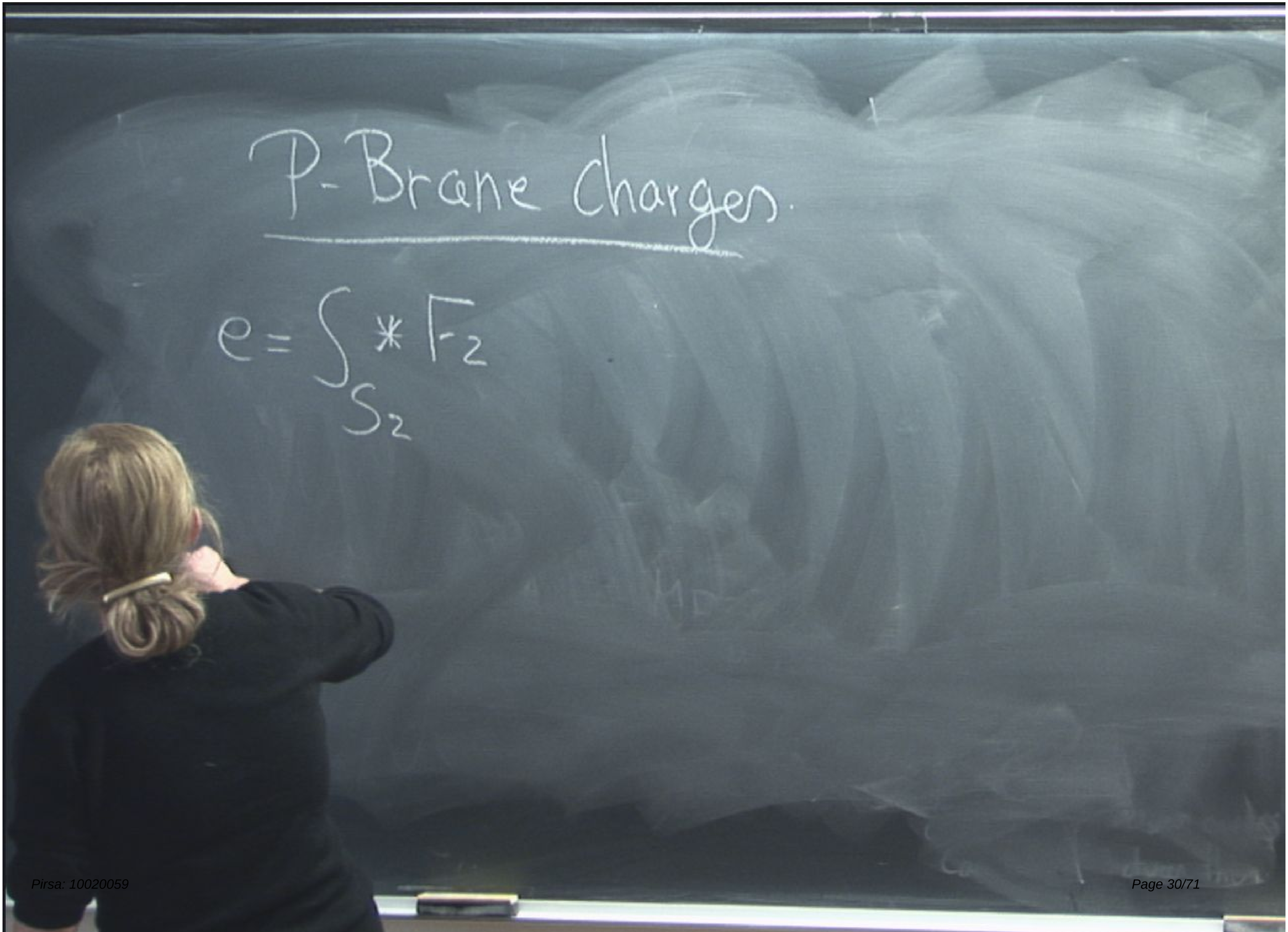
$$\int_D F_2 - \int_{D'} F_2 = \int_{S^2} F_2 = g$$

$\rightsquigarrow$ D0-brane

Branes:

$$\underbrace{\mu_p}_{\text{brane charge}} \underbrace{\mu_{6-p}}_{\text{charge of dual brane}} = 2\pi\mathbb{Z}$$

(Exercise 6.4)


P-Brane Charges.
$e = \int_{S_2} *F_2$

P-Brane Charges.

$$e = \int_{S_2} * F_2 \Rightarrow D = 4 \quad a\ S^2\ \text{surrounds } D0$$

1) Point particle in D-dimensions

P-Brane Charges.

$$e = \int_{S_2} * F_2 \Rightarrow D=4 \text{ a } S^2 \text{ surrounds D0}$$

1) Point particle in D-dimensions
is surrounded by D-2 sphere.

D0 in type IIA by S^8:

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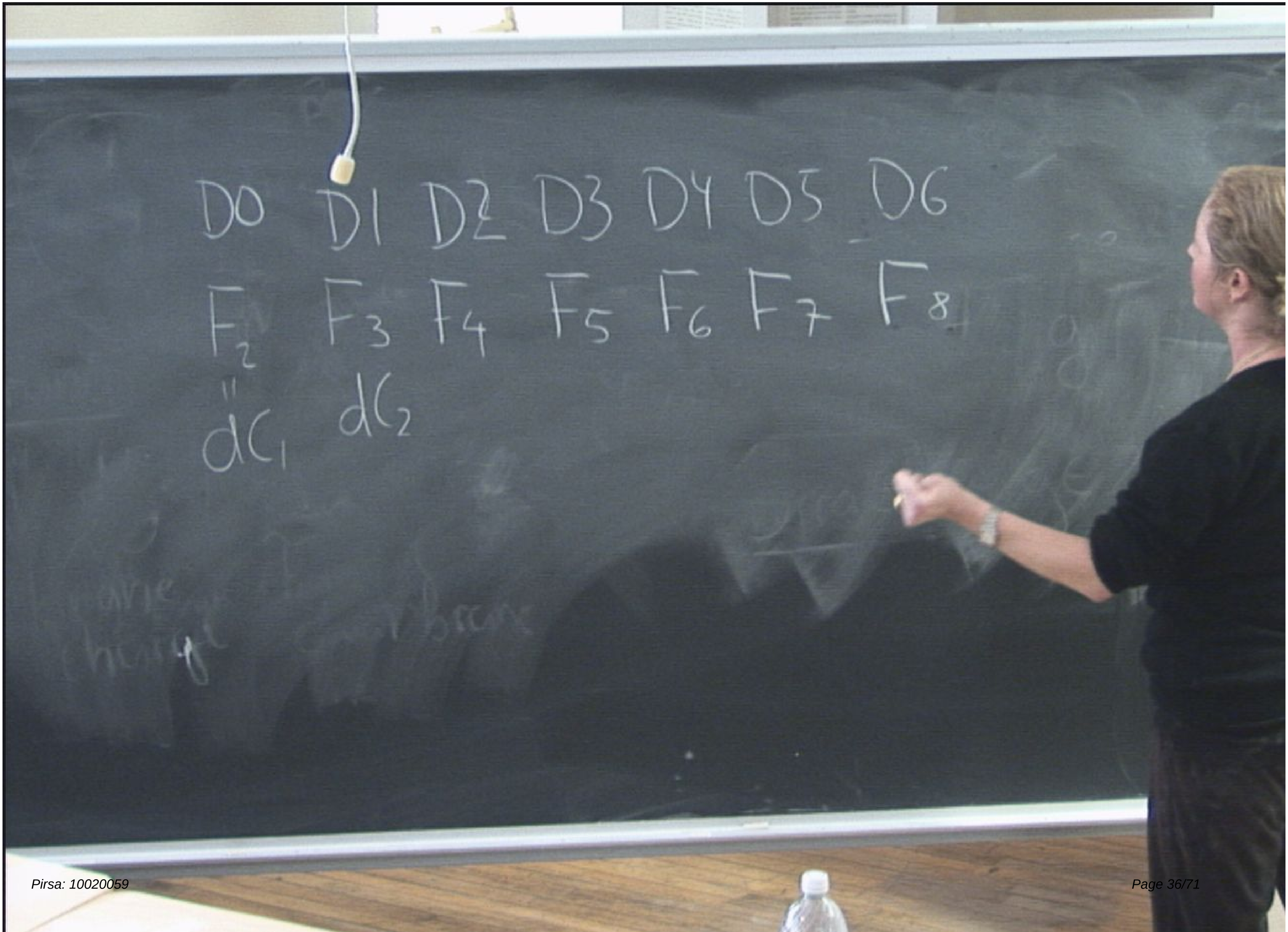
* In D space-time dimensions a 2-sphere

D-4 brane.

D0 in type IIA by S^8:

* In D space-time dimensions a 2-sphere
a D-4 brane.

D6-brane in D=10 is surrounded by S^2


D0 D1 D2 D3 D4 D5 D6
F2 F3 F4 F5 F6 F7 F8
dC1 dC2

D0 D1 D2 D3 D4 D5 D6

F_2 F_3 F_4 F_5 F_6 F_7 F_8

$\parallel$

dC_1 dC_2

$$S_{in} = \mu_p \cdot \int$$

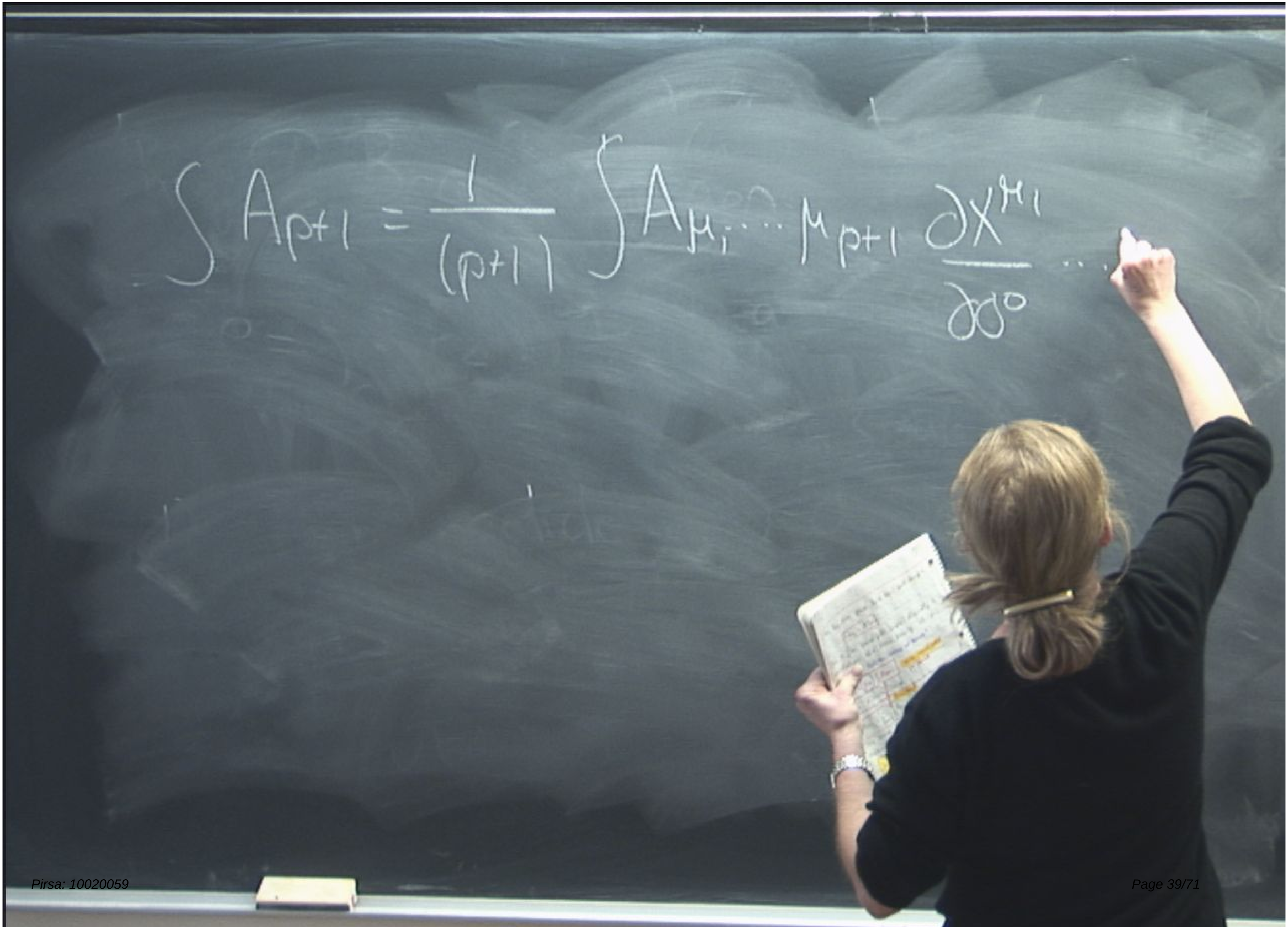
D0 D1 D2 D3 D4 D5 D6

F_2 F_3 F_4 F_5 F_6 F_7 F_8

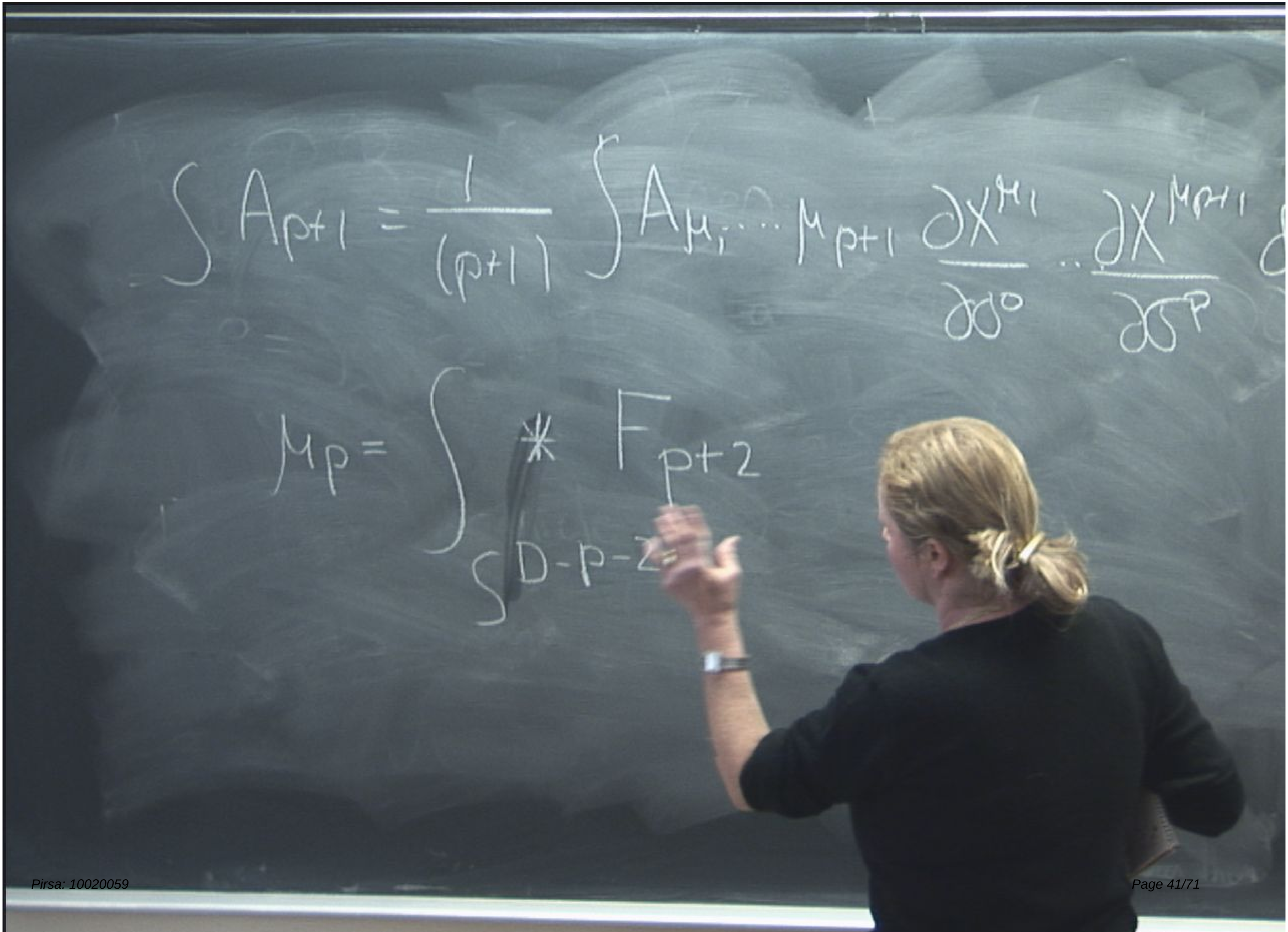
"

Dp-brane

Chern-Simons term $\quad S = \mu_p \int A_{p+1}$


$\int A_{p+1} = \frac{1}{(p+1)} \int A_{\mu_1 \dots \mu_{p+1}} \frac{\partial X^{\mu_1}}{\partial \sigma^0} \dots$

$$\int A_{p+1} = \frac{1}{(p+1)} \int A_{\mu_1 \cdots \mu_{p+1}} \frac{\partial X^{\mu_1}}{\partial \sigma^0} \cdots \frac{\partial X^{\mu_{p+1}}}{\partial \sigma^p}$$


$\int A_{p+1} = \frac{1}{(p+1)} \int A_{\mu_1 \dots \mu_{p+1}} \frac{\partial X^{\mu_1}}{\partial \sigma^0} \dots \frac{\partial X^{\mu_{p+1}}}{\partial \sigma^p}$
$\mu_p = \int_{S^{D-p-2}} * F_{p+2}$

$$\int A_{p+1} = \frac{1}{(p+1)} \int A_{\mu_1 \cdots \mu_{p+1}} \frac{\partial X^{\mu_1}}{\partial \sigma^0} \cdots \frac{\partial X^{\mu_{p+1}}}{\partial \sigma^p}$$

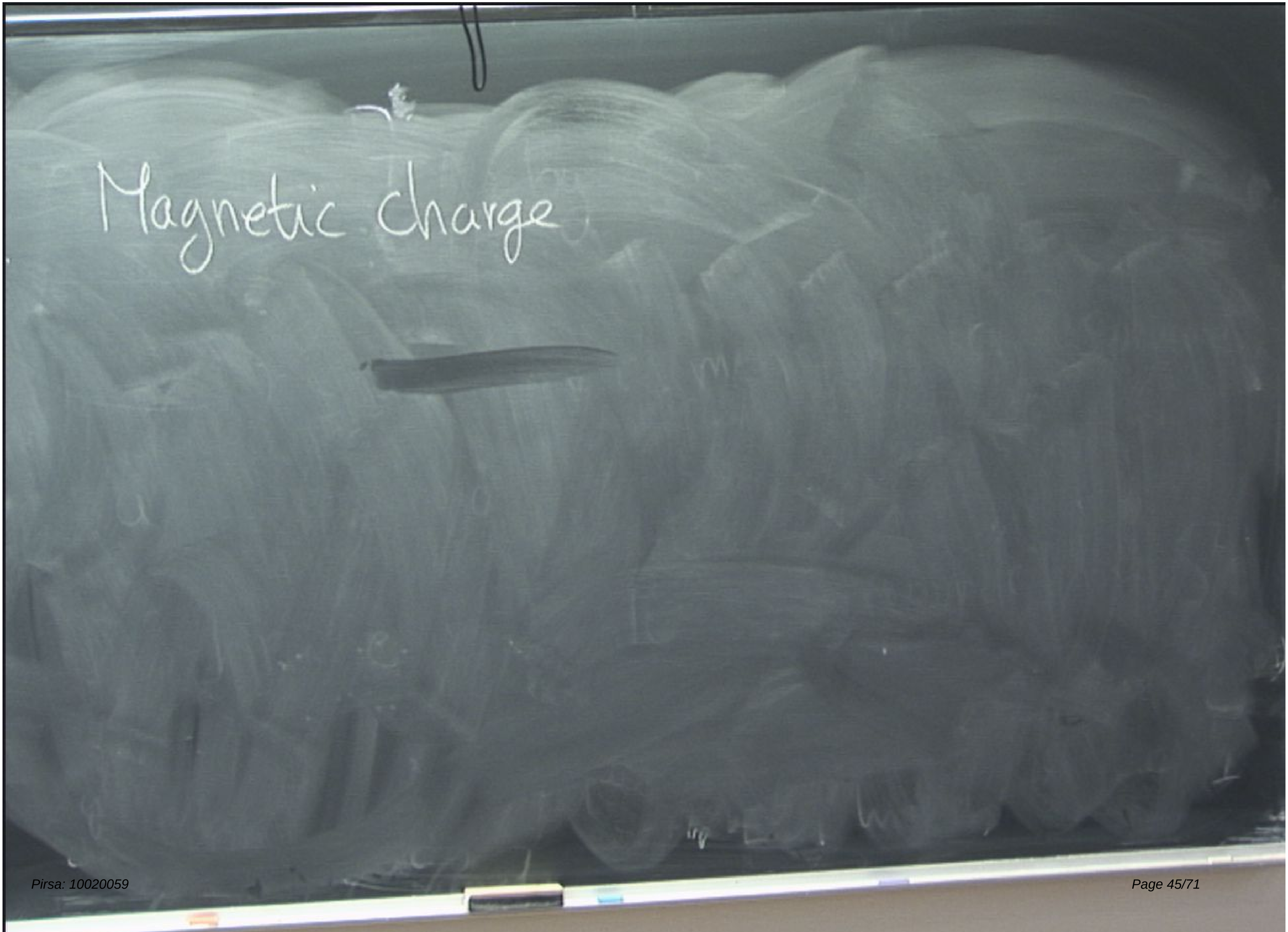
$$\mu_p = \int_{S^{D-p-2}} * F_{p+2}$$

$$\int A_{p+1} = \frac{1}{(p+1)!} \int A_{\mu_1 \ldots \mu_{p+1}} \frac{\partial X^{\mu_1}}{\partial \sigma^0} \cdots \frac{\partial X^{\mu_{p+1}}}{\partial \sigma^p}$$

$$\mu_p = \int_{S^{D-p-2}} {}^*F_{p+2} \leftarrow \text{p-brane charge}$$

$$\int A_{p+1} = \frac{1}{(p+1)!} \int A_{\mu_1 \cdots \mu_{p+1}} \frac{\partial X^{\mu_1}}{\partial \sigma^0} \cdots \frac{\partial X^{\mu_{p+1}}}{\partial \sigma^p} d\sigma.$$

$$\mu_p = \int_{S^{D-p-2}} * F_{p+2} \leftarrow \text{p-brane charge}$$


Magnetic charge

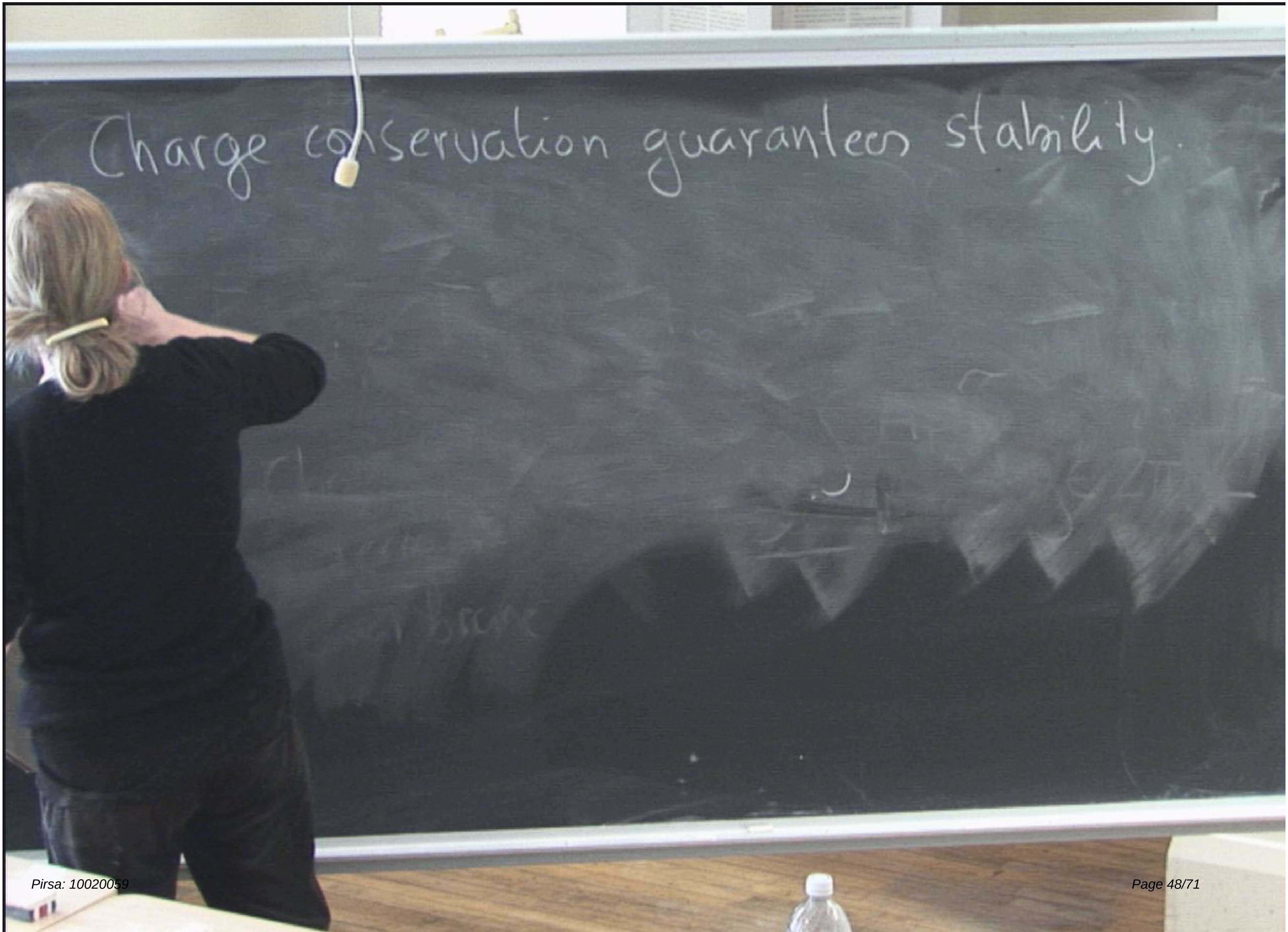
Magnetic charge

$$\mu_{D-p-4} = \int_{S^{p+2}} F_{p+2}$$

Magnetic charge

$$\mu_{D-p-4} = \int_{S^{p+2}} F_{p+2}$$

$$\mu_p \, \mu_{6-p} \in 2\pi \mathbb{Z}$$


Charge conservation guarantees stability.

Charge conservation guarantees stability.

gauge

IIA A_1 A_3 A_5 A_7

D0 D2 D4 D6

IIB A_0 A_2 A_4 A_6

D1 D5 D7

~~instanton~~

Worldvolume action for D-branes

Worldvolume action for D-branes.

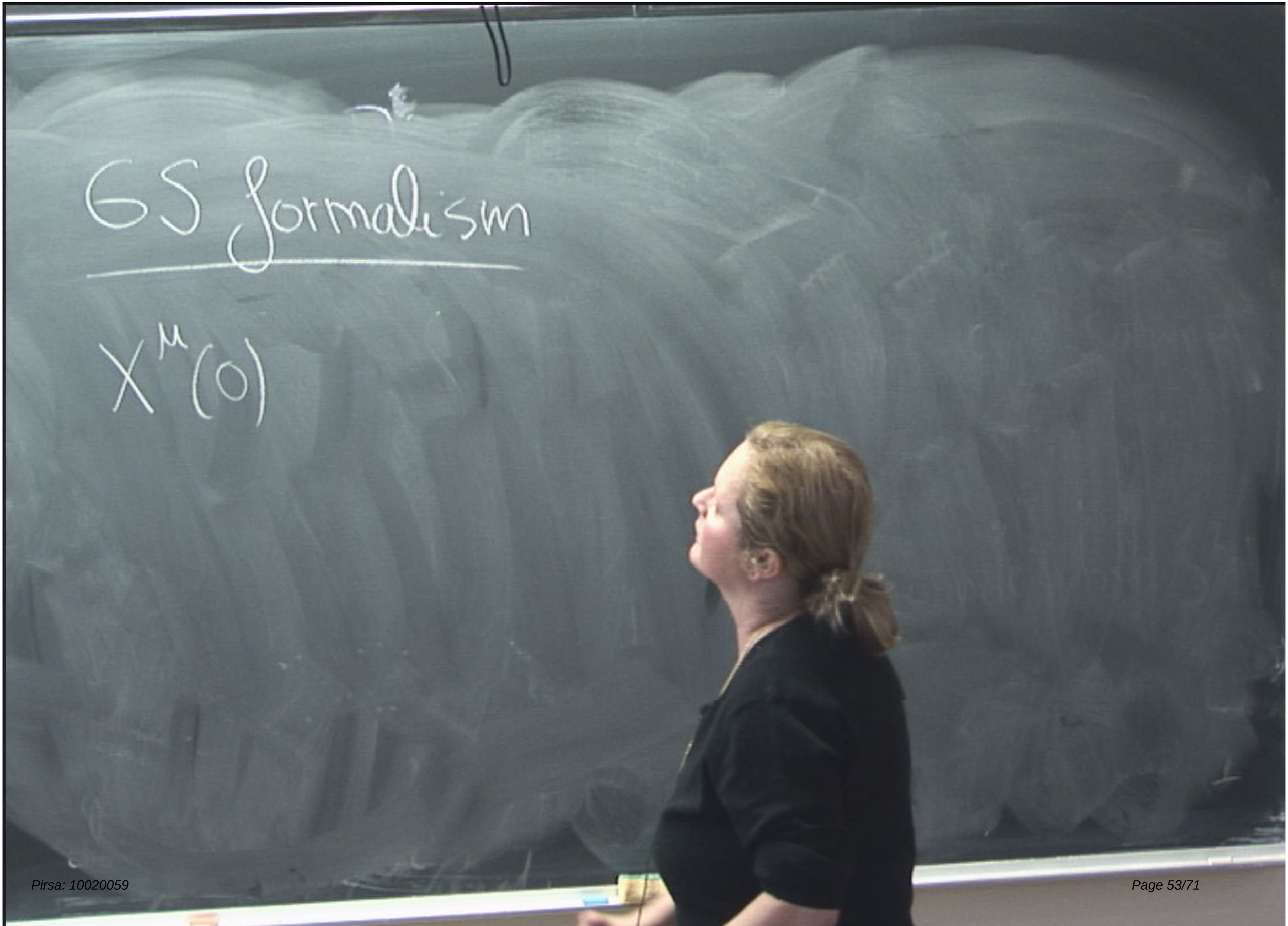
Dp-brane $\longleftrightarrow$ (p+1)-dimensional.

Worldvolume action for D-branes

Dp-brane $\longleftrightarrow$ $(p+1)$-dimensional.

$\Longrightarrow$ $(p+1)$-dim. field theory

(massless fields of open string)


GS formalism
$X^{\mu}(0)$

GS formalism

$X^\mu(0)$

$\theta^{1a}\ \theta^{2a}$

New ingredient

$A_\mu \Rightarrow U(1)$

1. D-brane in flat

DBI Dirac-Born-Infeld

$$S = -T_{Dp} \int d^{p+1} \sqrt{-\det\left(G_{\alpha\beta} + K F_{\alpha\beta}\right)}$$

T_{Dp}: tension

$K = 2\pi\alpha'$

DBI Dirac - Born - Infeld

$$S = -T_{Dp} \int d^{p+1} \sqrt{-\det\left(G_{\alpha\beta} + K\, \mathcal{F}_{\alpha\beta}\right)}$$

T_{Dp}: tension

K: $2\pi\alpha'$

$$G_{\alpha\beta} = \eta_{\mu\nu}\, \Pi_\alpha^{\ \mu}\, \Pi_\beta^{\ \nu}$$

$$\Pi_\alpha^\mu = \partial_\alpha X^\mu - \bar{\Theta}^A \Gamma^\mu \partial_\alpha \Theta^A$$

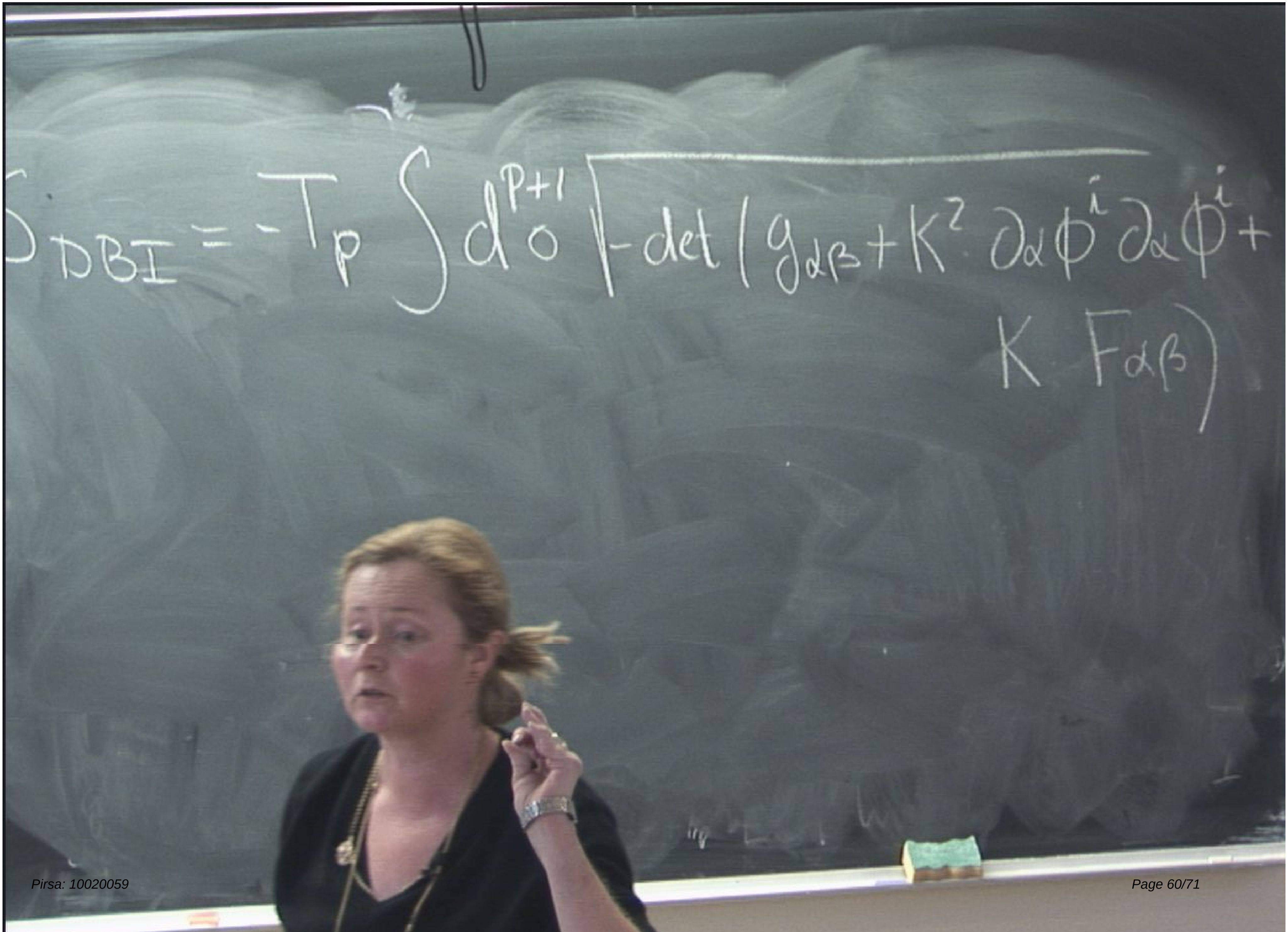
$$\Pi_\alpha^\mu = \partial_\alpha X^\mu - \bar{\Theta}^A \Gamma^\mu \partial_\alpha \Theta^A$$

$$\mathcal{F}_{\alpha\beta} = \underset{\uparrow\, dA}{F_{\alpha\beta}} + \underbrace{b_{\alpha\beta}}_{\Theta}$$

Static gauge: $X^\mu = \sigma^\alpha$

$\uparrow$ $(p+1)$-coordinates gauged fixed

Φ^i remaining X^μ


$S_{DBI} = -T_p \int d^{p+1}\sigma \sqrt{-\det\left(g_{\alpha\beta} + K^2\, \partial_\alpha \phi^i \partial_\alpha \phi^i + K\, F_{\alpha\beta}\right)}$

$$S_{DBI} = -T_p \int d^{p+1}\sigma \sqrt{-\det\left(g_{\alpha\beta} + K^2\, \partial_\alpha \phi^i \partial^\alpha \phi^i + K\, F_{\alpha\beta}\right)}$$

$$S_{DBI} = -T_p \int d^{p+1}\sigma \sqrt{-\det\left(g_{\alpha\beta} + K^2\, \partial_\alpha \phi^i \partial_\beta \phi^i + K\, F_{\alpha\beta}\right)}$$

Φ, $g_{\alpha\beta}$, $B_{\alpha\beta}$

$$S_{DBI} = -T_p \int d^{p+1}\sigma \sqrt{-\det\left(g_{\alpha\beta} + K^2\, \partial_\alpha \phi^i \partial_\beta \phi^i + K\, F_{\alpha\beta}\right)}$$

Φ, $g_{\alpha\beta}$, $B_{\alpha\beta}$ NS of type II

$$S = -T_p \int d^{p+1}\, e^{-\phi} \sqrt{-\det\left(g_{\alpha\beta} + B_{\alpha\beta}\right.}$$

$$S = -T_p \int d^{p+1}\, e^{-\phi} \sqrt{-\det\left(g_{\alpha\beta} + B_{\alpha\beta} + K^2 \partial_\alpha \phi^i \partial_\beta \phi^i + K F_{\alpha\beta}\right)}$$

$\uparrow$ tension

$$S = - T_p \int d^{p+1} e^{-\phi} \sqrt{-\det\left(g_{\alpha\beta} + B_{\alpha\beta} + K^2 \partial_\alpha \phi^i \partial_\beta \phi^i + K F_{\alpha\beta}\right)}$$

tension

From RR: $S_{cs} =$

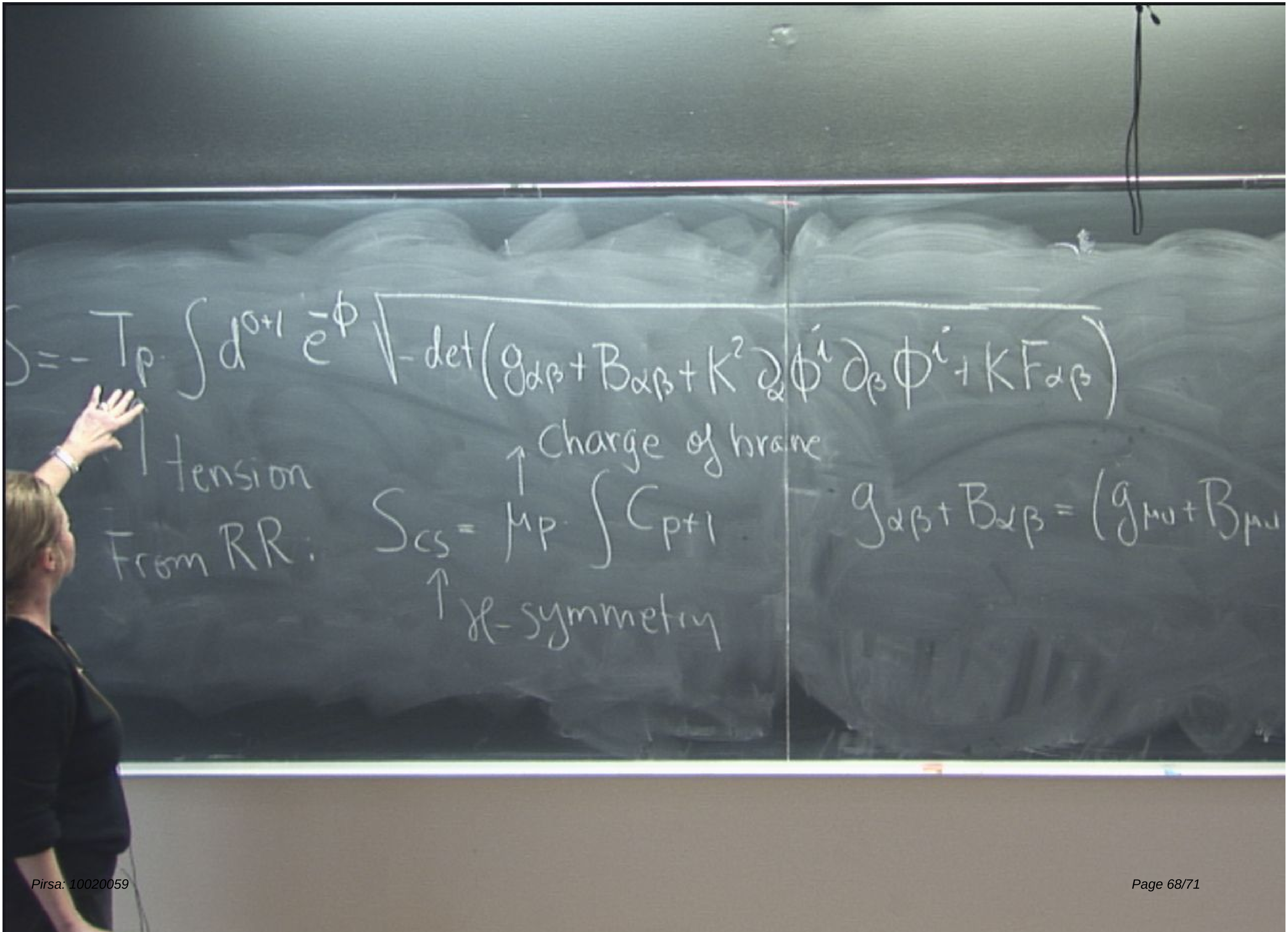
$$S = -T_p \int d^{p+1} e^{-\phi} \sqrt{-\det\left(g_{\alpha\beta} + B_{\alpha\beta} + K^2 \partial_\alpha \phi^i \partial_\beta \phi^i + K F_{\alpha\beta}\right)}$$

$\uparrow$ tension

$\uparrow$ Charge of brane

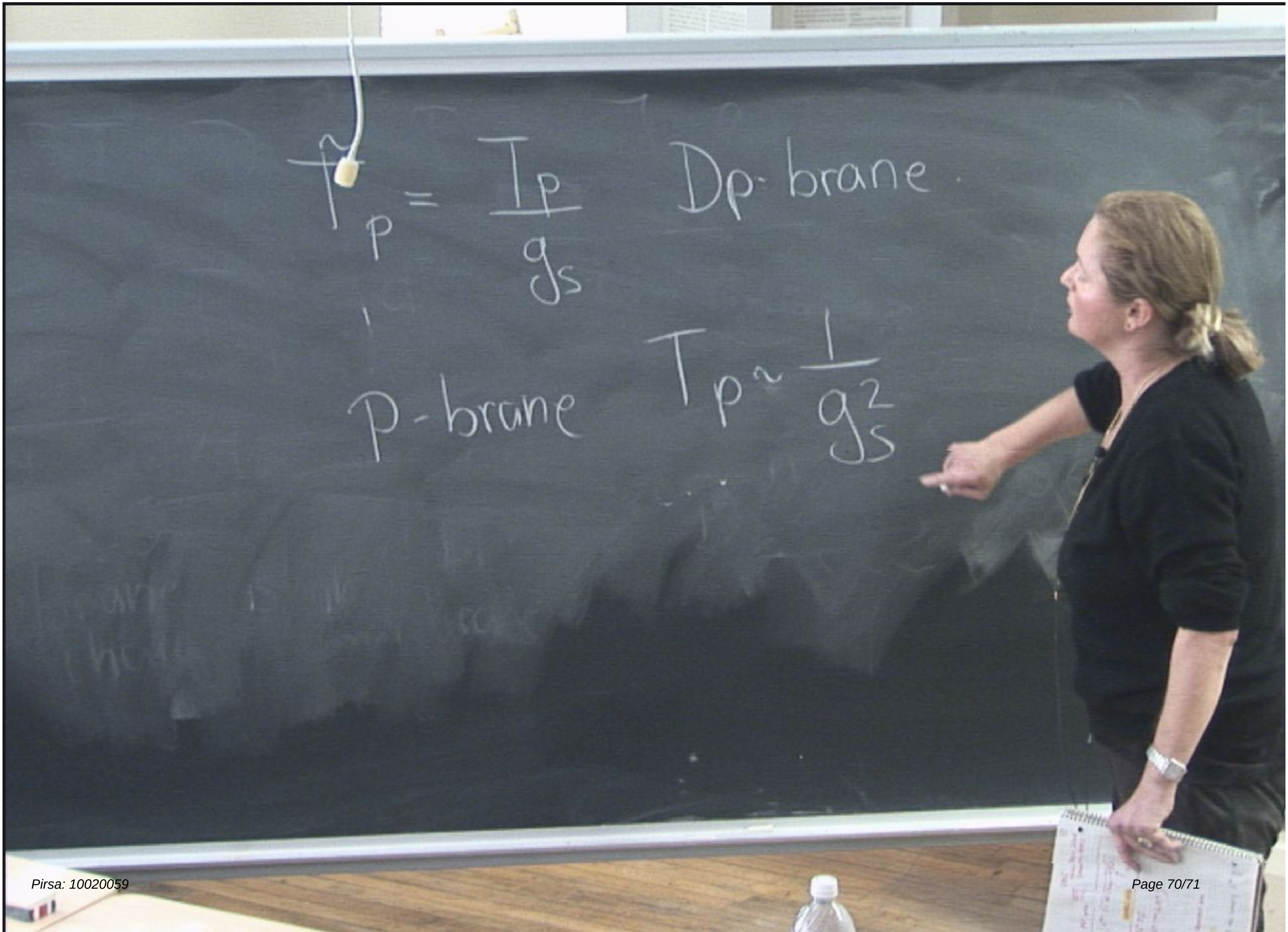
From RR: $S_{CS} = \mu_p \int C_{p+1}$

$\uparrow$ κ-symmetry


$S = -T_p \int d^{p+1} e^{-\phi} \sqrt{-\det(g_{\alpha\beta} + B_{\alpha\beta} + K^2 \partial_\alpha \phi^i \partial_\beta \phi^i + K F_{\alpha\beta})}$
tension
Charge of brane
From RR: $S_{CS} = \mu_p \int C_{p+1}$
κ-symmetry
$g_{\alpha\beta} + B_{\alpha\beta} = (g_{\mu\nu} + B_{\mu\nu}$

$$\tilde{T}_p = \frac{T_p}{g_s} \quad \text{Dp-brane.}$$

$$\text{p-brane} \quad T_p \sim \frac{1}{g_s^2}$$


$\tilde{T}_p = \frac{T_p}{g_s}$ Dp-brane
p-brane $T_p \sim \frac{1}{g_s^2}$

$\tilde{T}_p = \frac{T_p}{g_s}$ Dp-brane.

$g_s \to 0$

p-brane $T_p \sim \frac{1}{g_s^2} \to$ very large