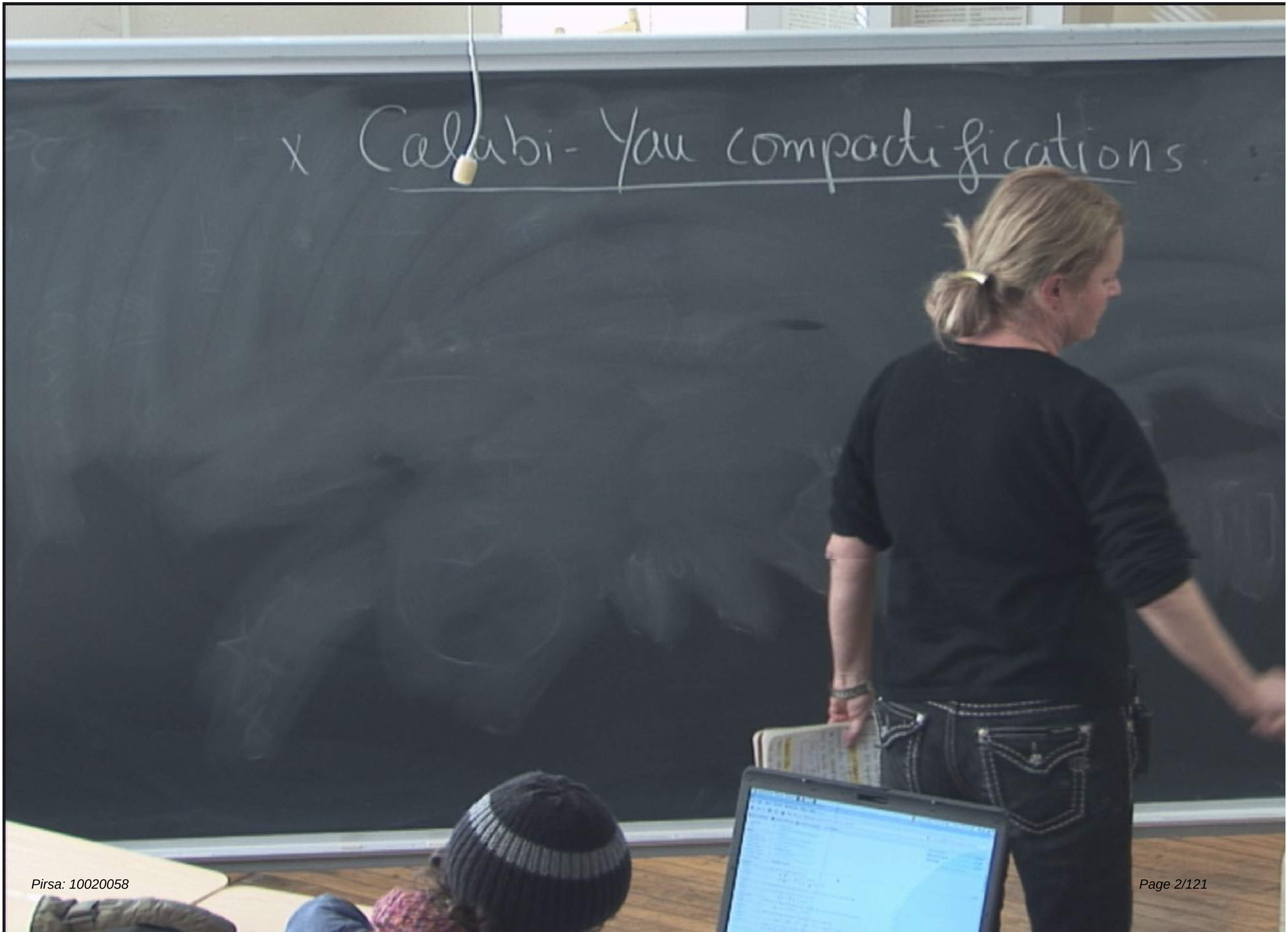
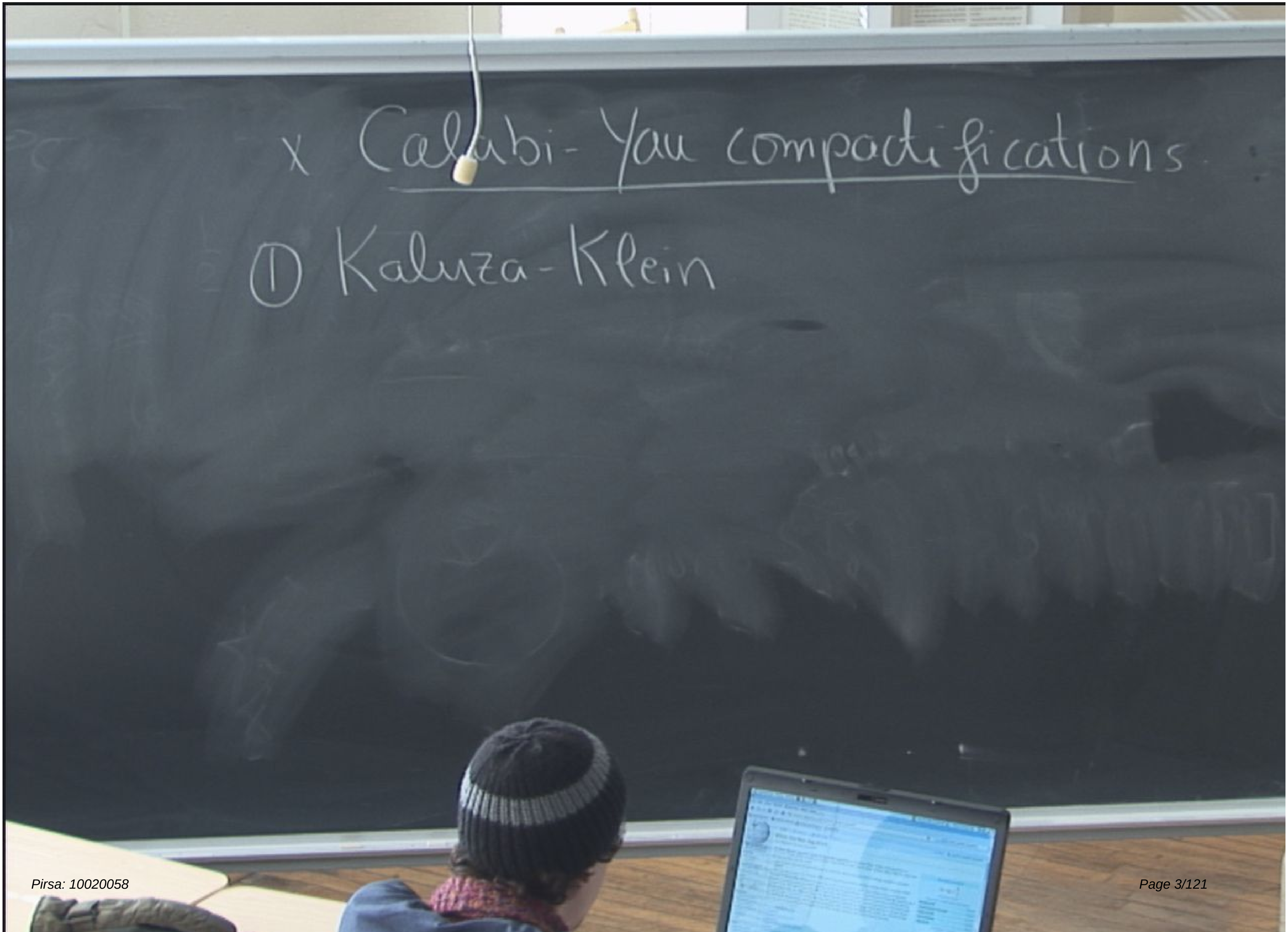
Title: String Theory - Review (PHYS 623) - Lecture 14

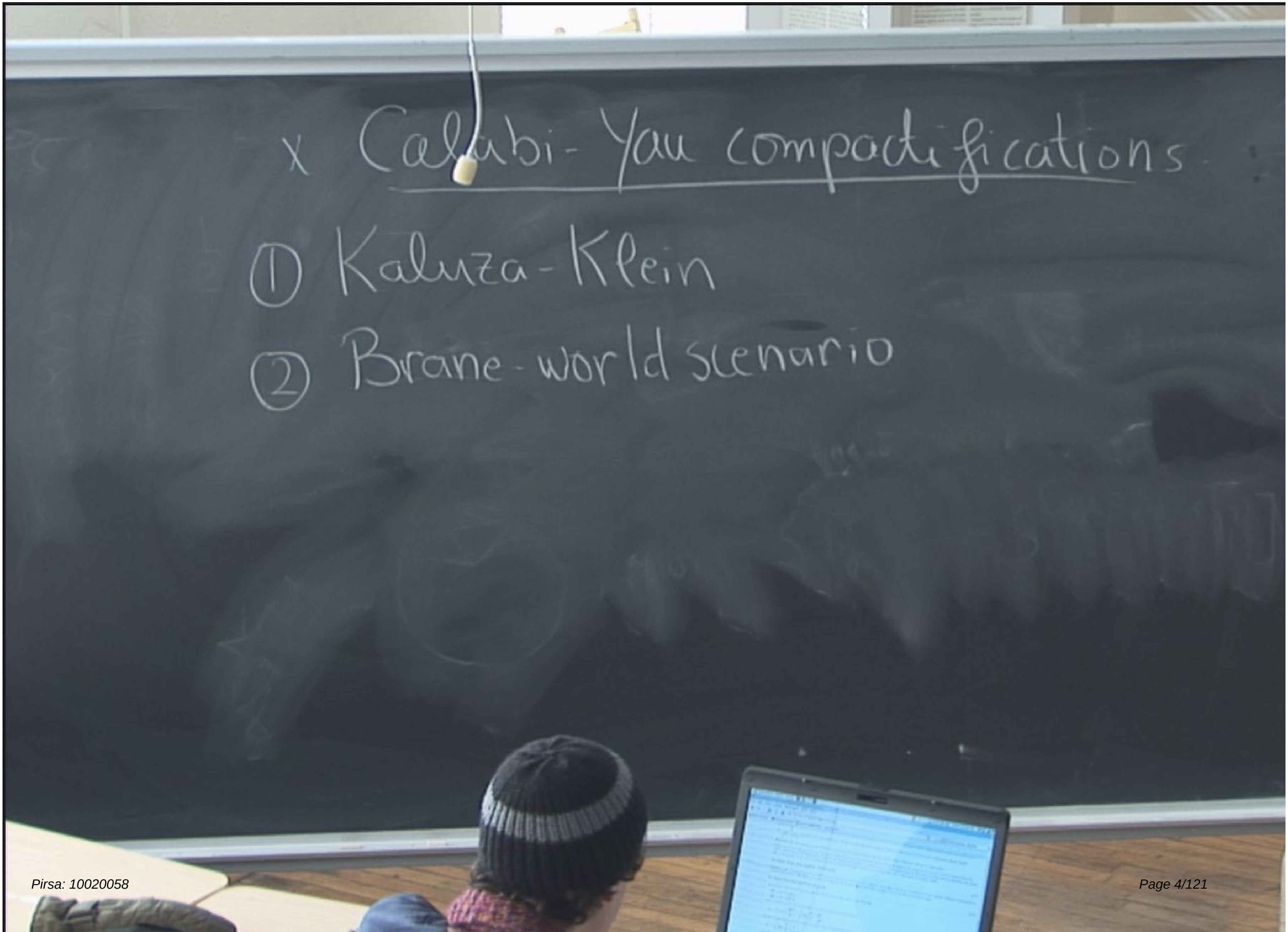
Date: Feb 11, 2010 11:20 AM

URL: http://pirsa.org/10020058

Abstract:


x Calabi-Yau compactifications


x Calabi-Yau compactifications
① Kaluza-Klein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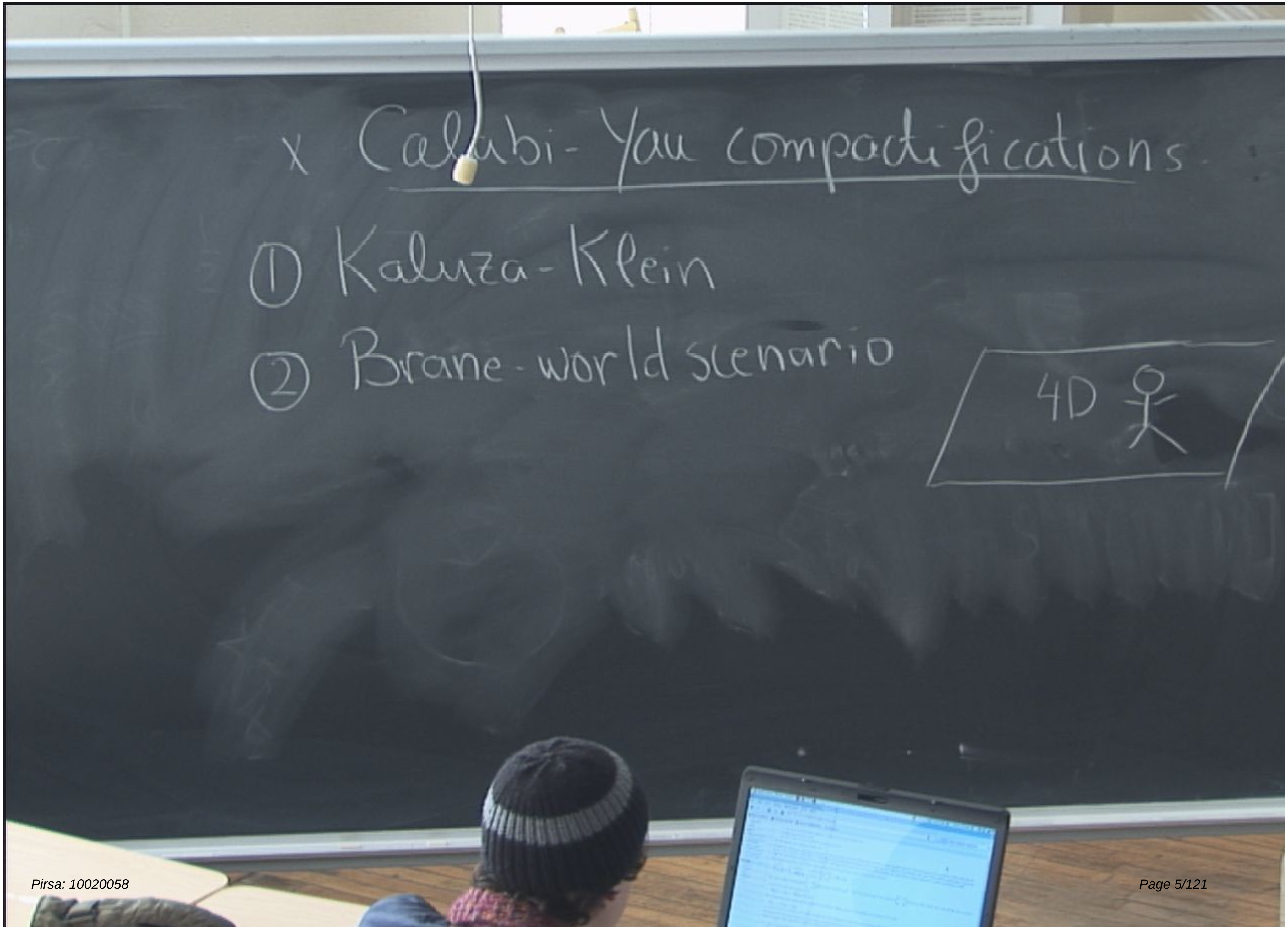
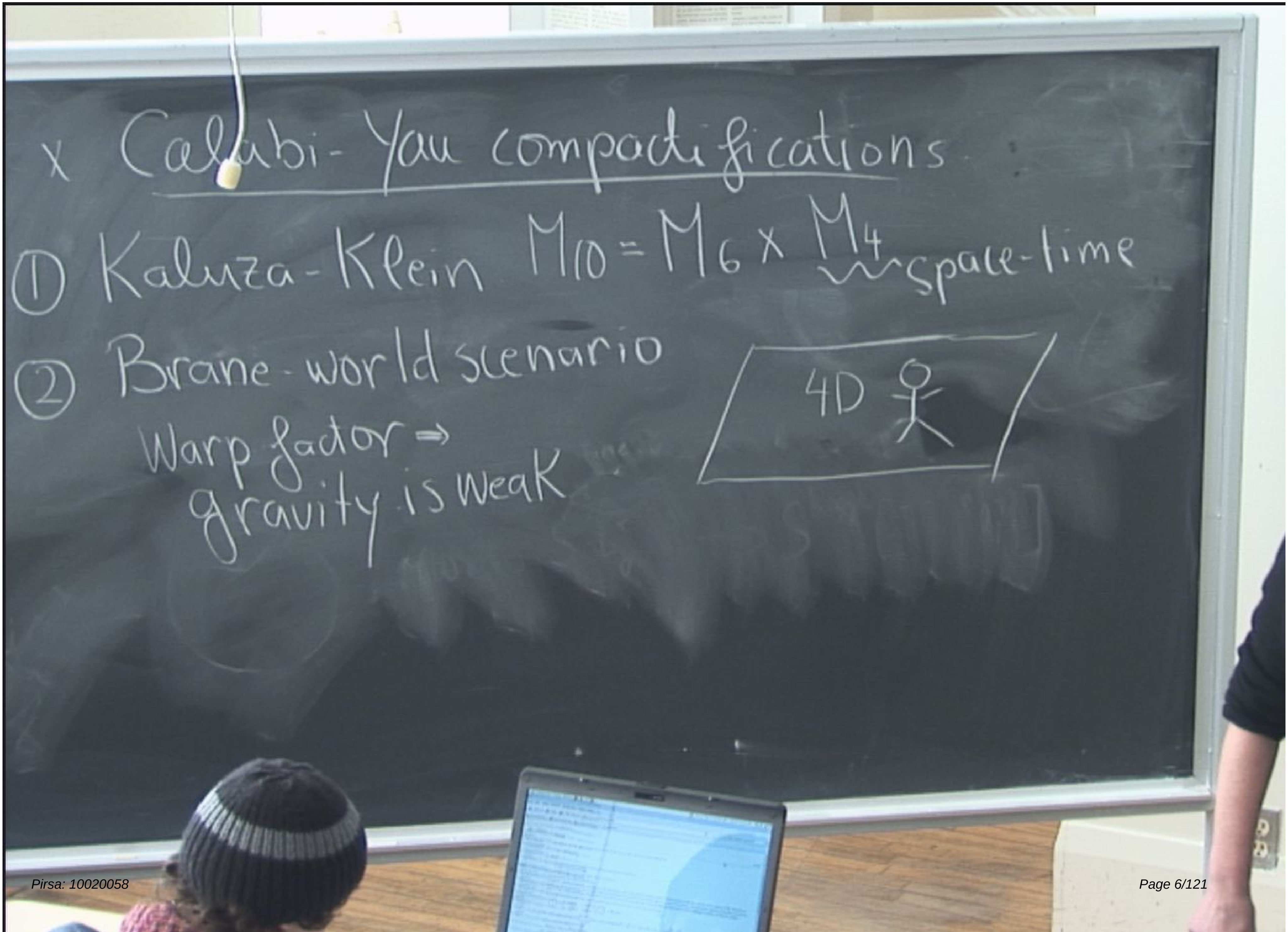
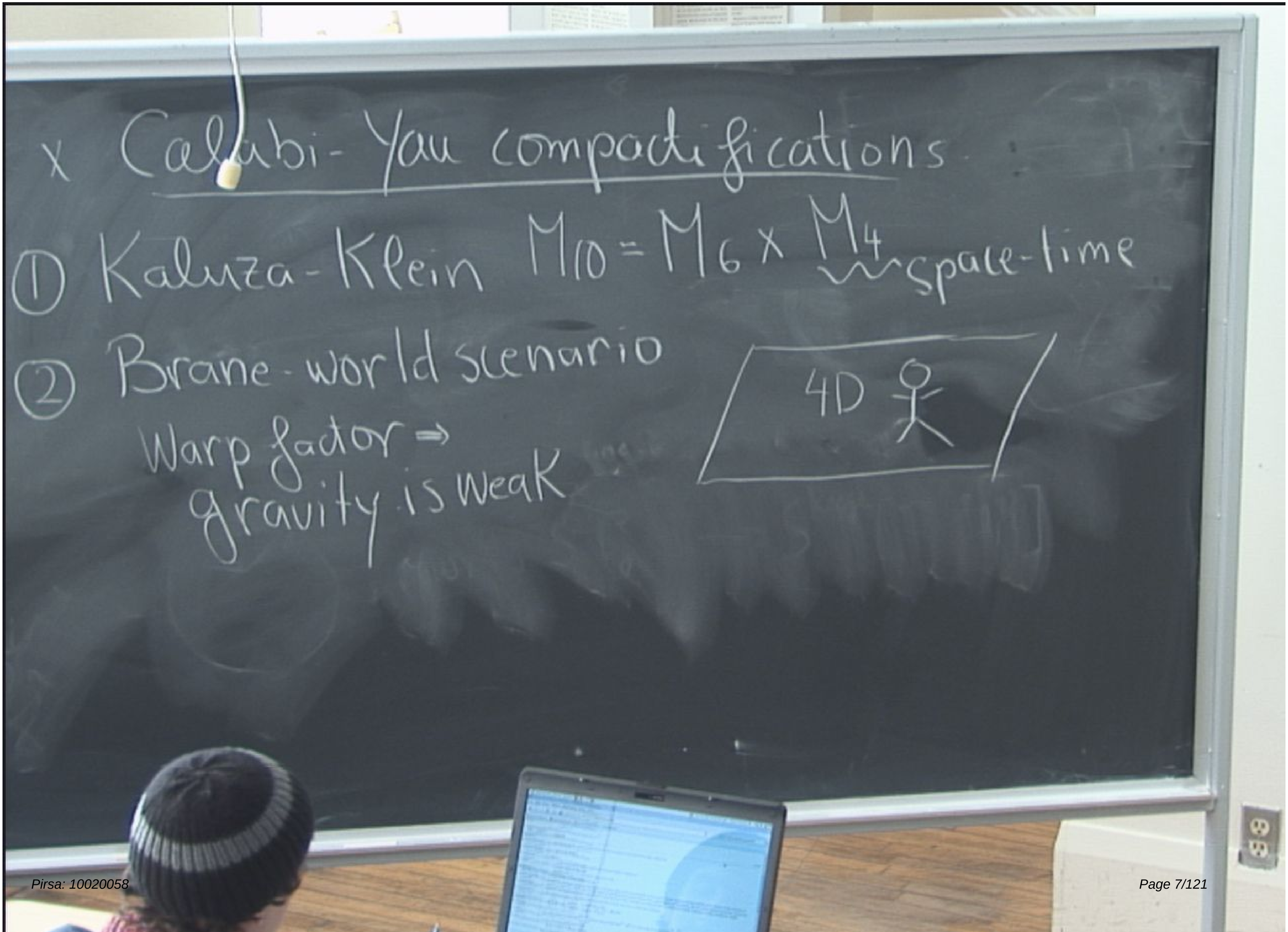
① Kaluza-Klein
② Brane-world scenario


x Calabi-Yau compactifications
① Kaluza-Klein
② Brane-world scenario
4D


x Calabi-Yau compactifications
① Kaluza-Klein $M_{10} = M_6 \times M_4$ space-time
② Brane-world scenario
Warp factor ⇒
gravity is weak
4D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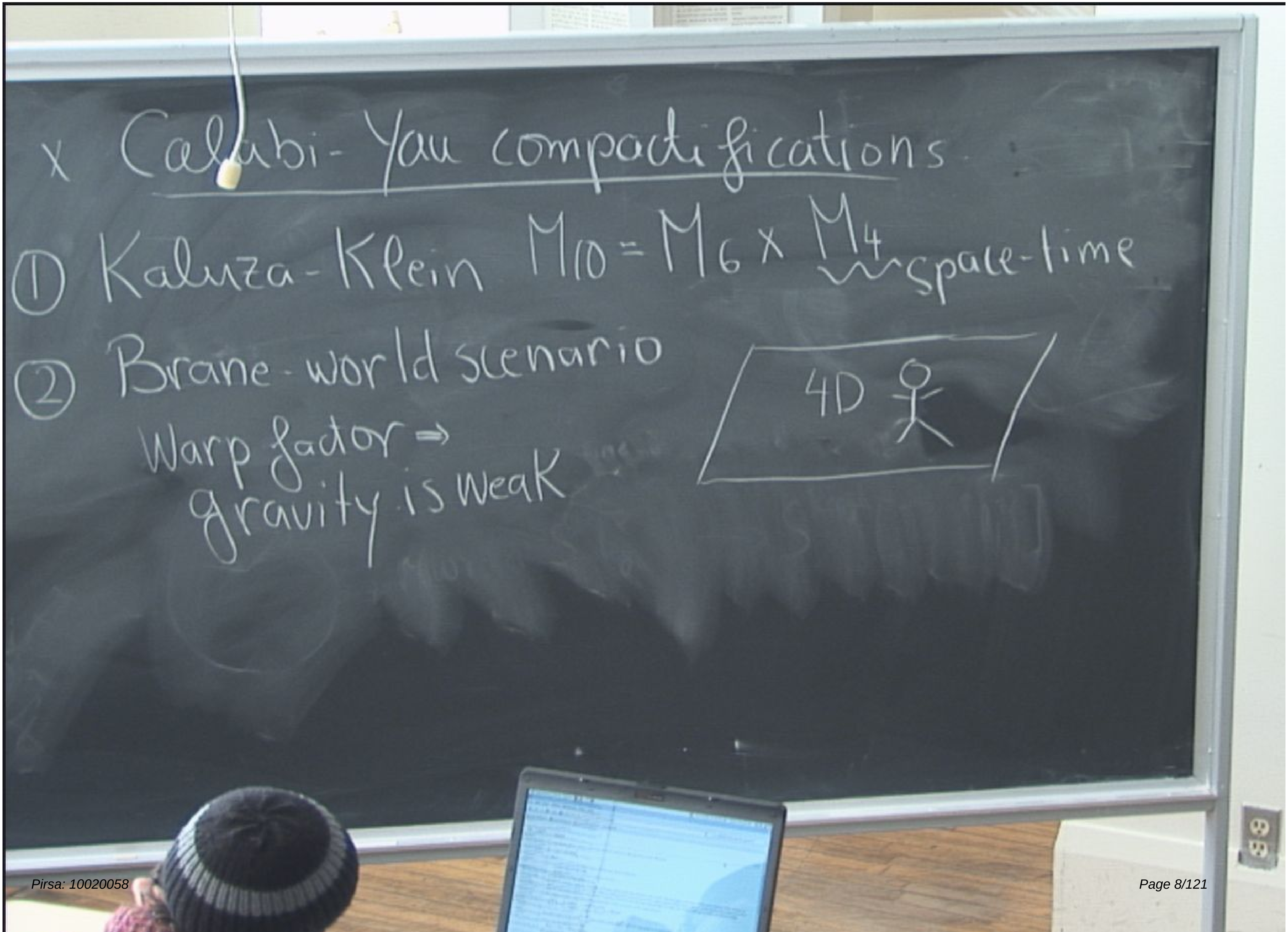
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① $M_{10} = M_6 \otimes M_4$

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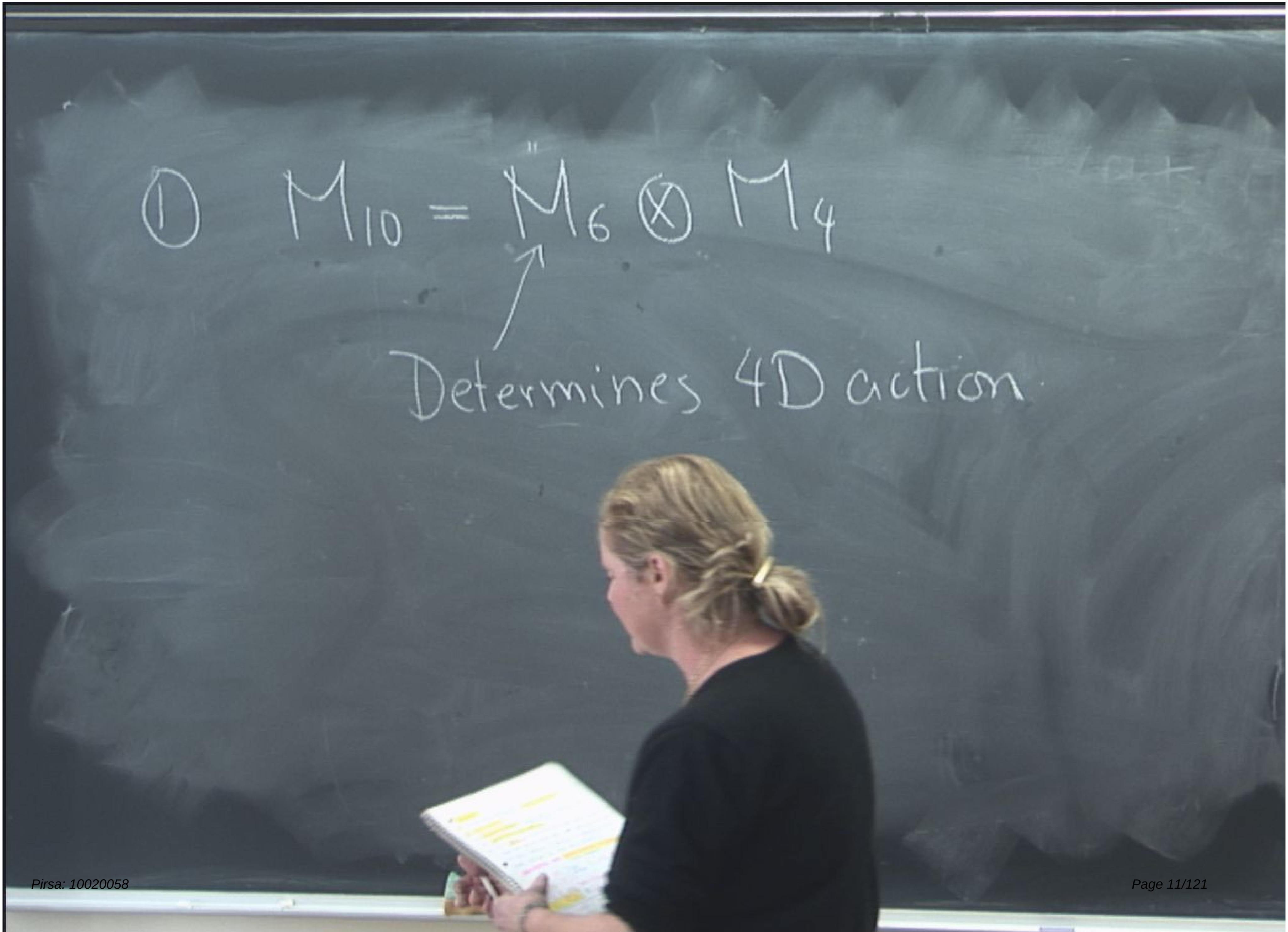
① $M_{10} = M_6 \otimes M_4$
Determines 4D action

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1) $SU(3) \times SU(2) \times U(1)$

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2) $\mathcal{N}=1$ supersymmetry

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2) $\mathcal{N}=1$ supersymmetry

* hierarchy problem

*

① $M_{10} = M_6 \otimes M_4$

Determines 4D action

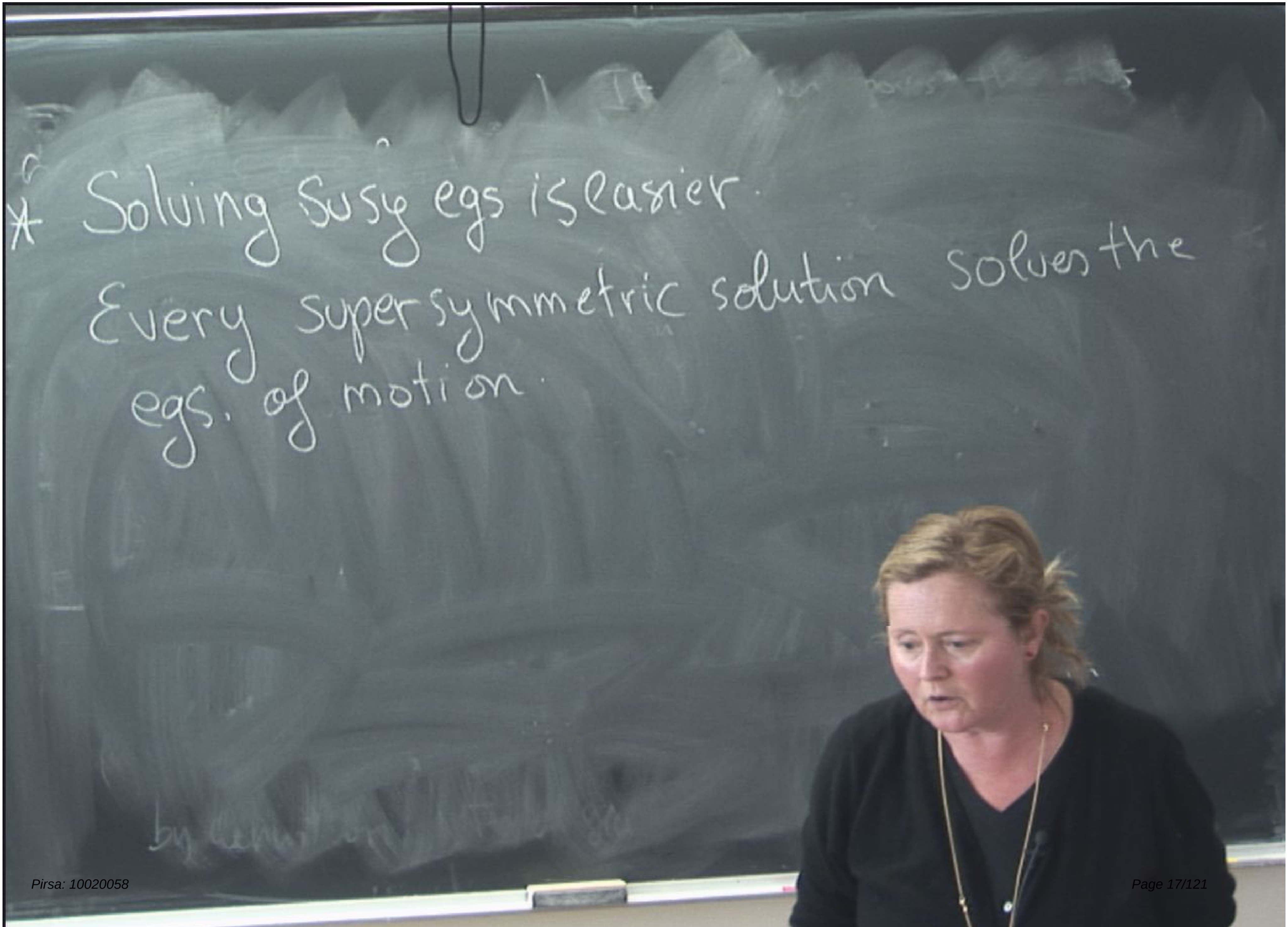
1) $SU(3) \times SU(2) \times U(1) = SU(5), SO(10)$

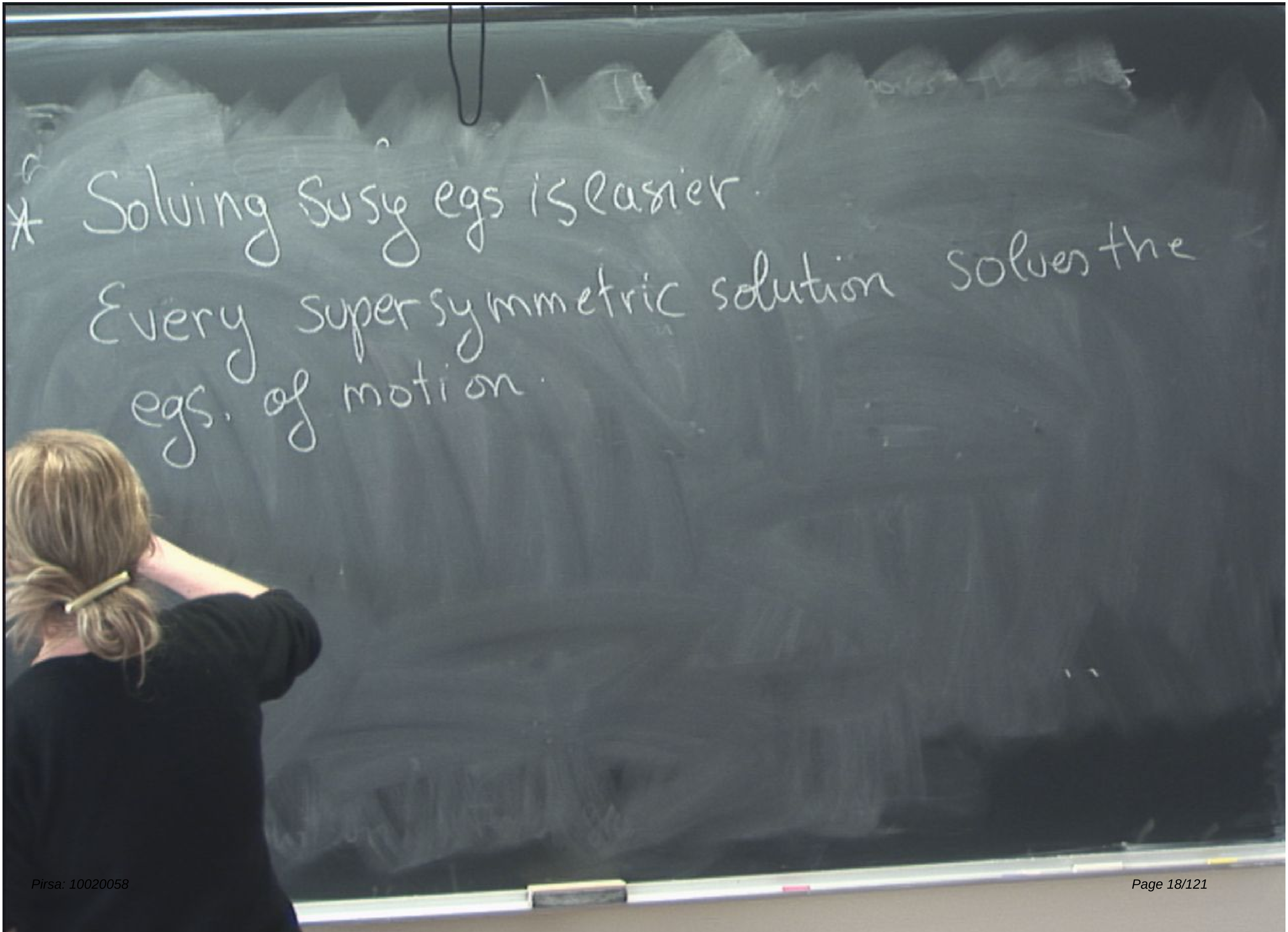
2) $\mathcal{N}=1$ supersymmetry

* hierarchy problem

* Unification of couplings

* Solving Susy eqs is easier.


* Solving Susy eqs is easier.
Every supersymmetric solution solves the eqs. of motion.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Every supersymmetric solution solves the eqs. of motion.

Global susy $H = Q^{\dagger} Q \leftarrow$ supercharge

$Q|B\rangle = |F\rangle$ $\quad Q|\psi$

$Q|F\rangle = |B\rangle$ $\quad$ Su

* Solving Susy eqs is easier.

Every supersymmetric solution solves the eqs. of motion.

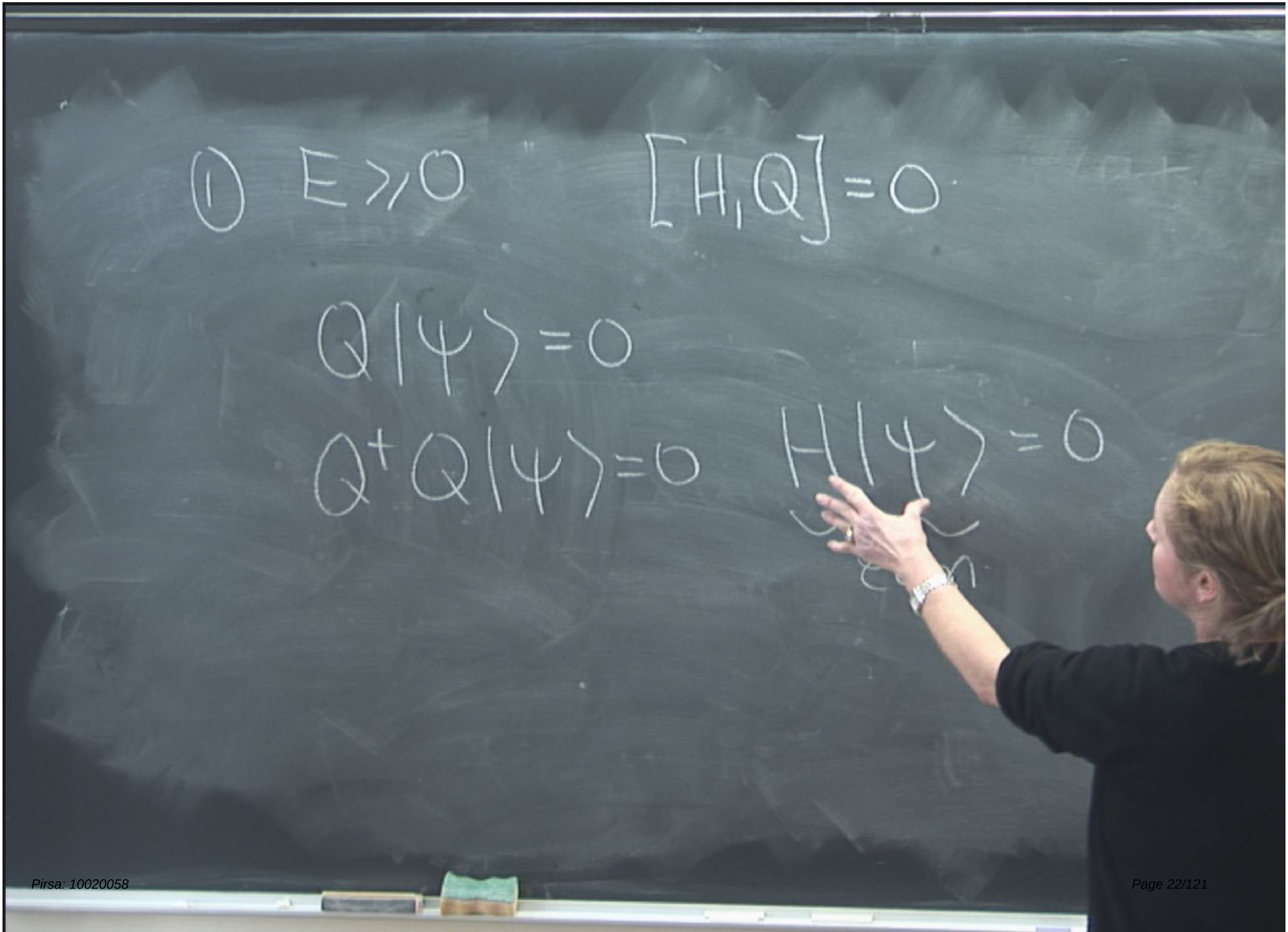
Global susy $H = Q^{+} Q \leftarrow$ supercharge

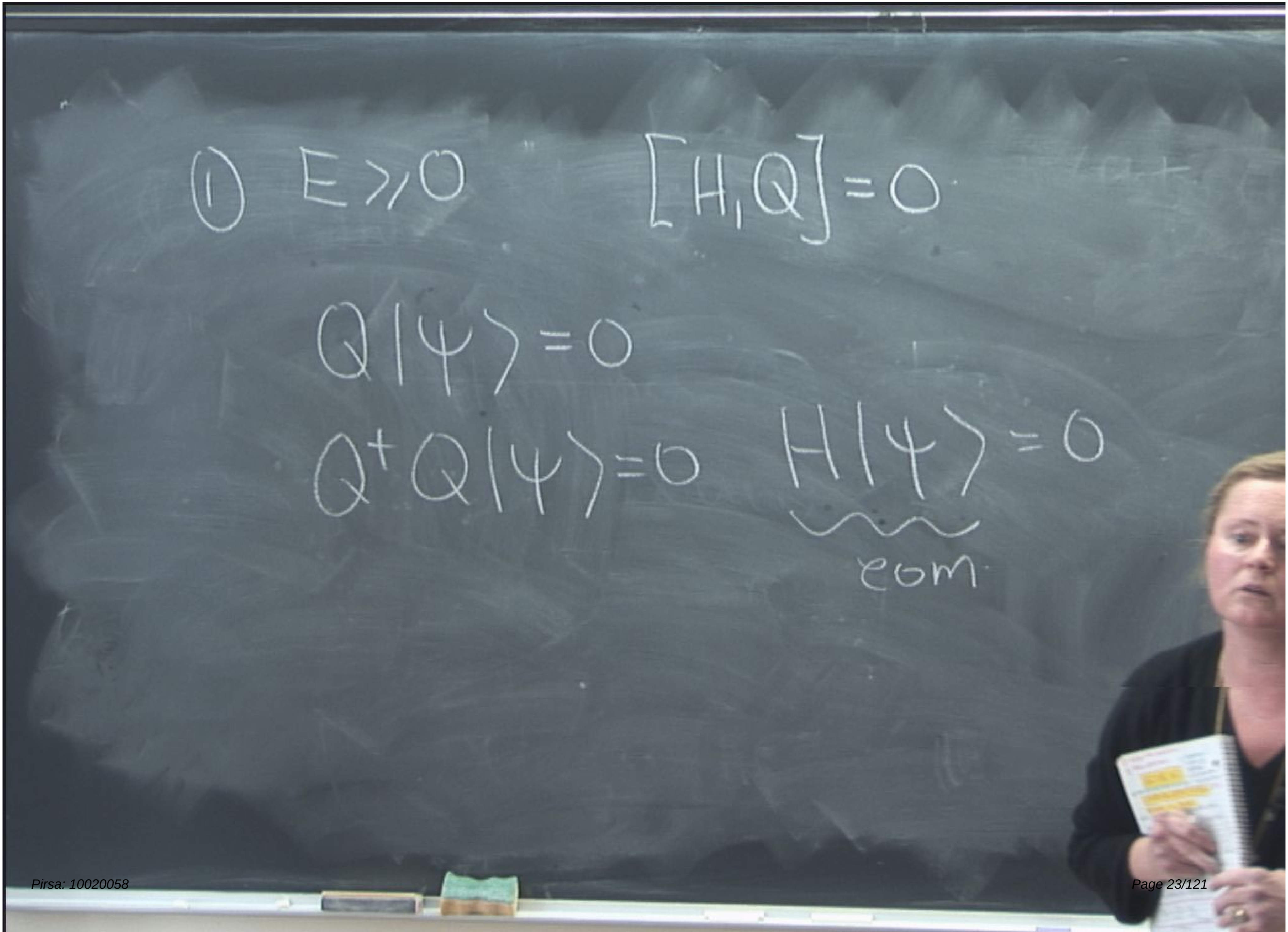
$Q|B\rangle = |F\rangle$ $\qquad Q|\psi\rangle = 0$

$Q|F\rangle = |B\rangle$ $\qquad$ "susic state"

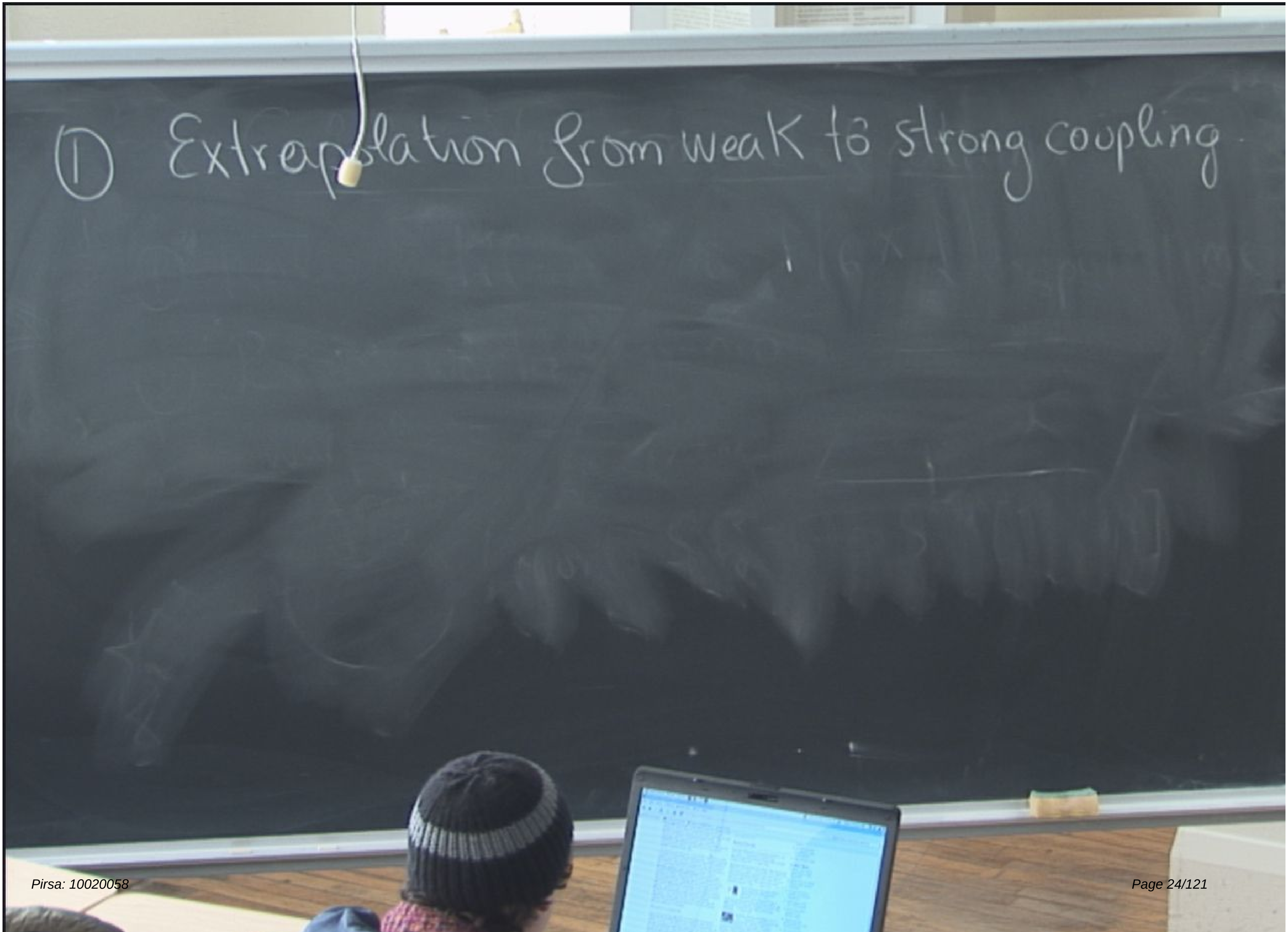
① $E \geq 0$

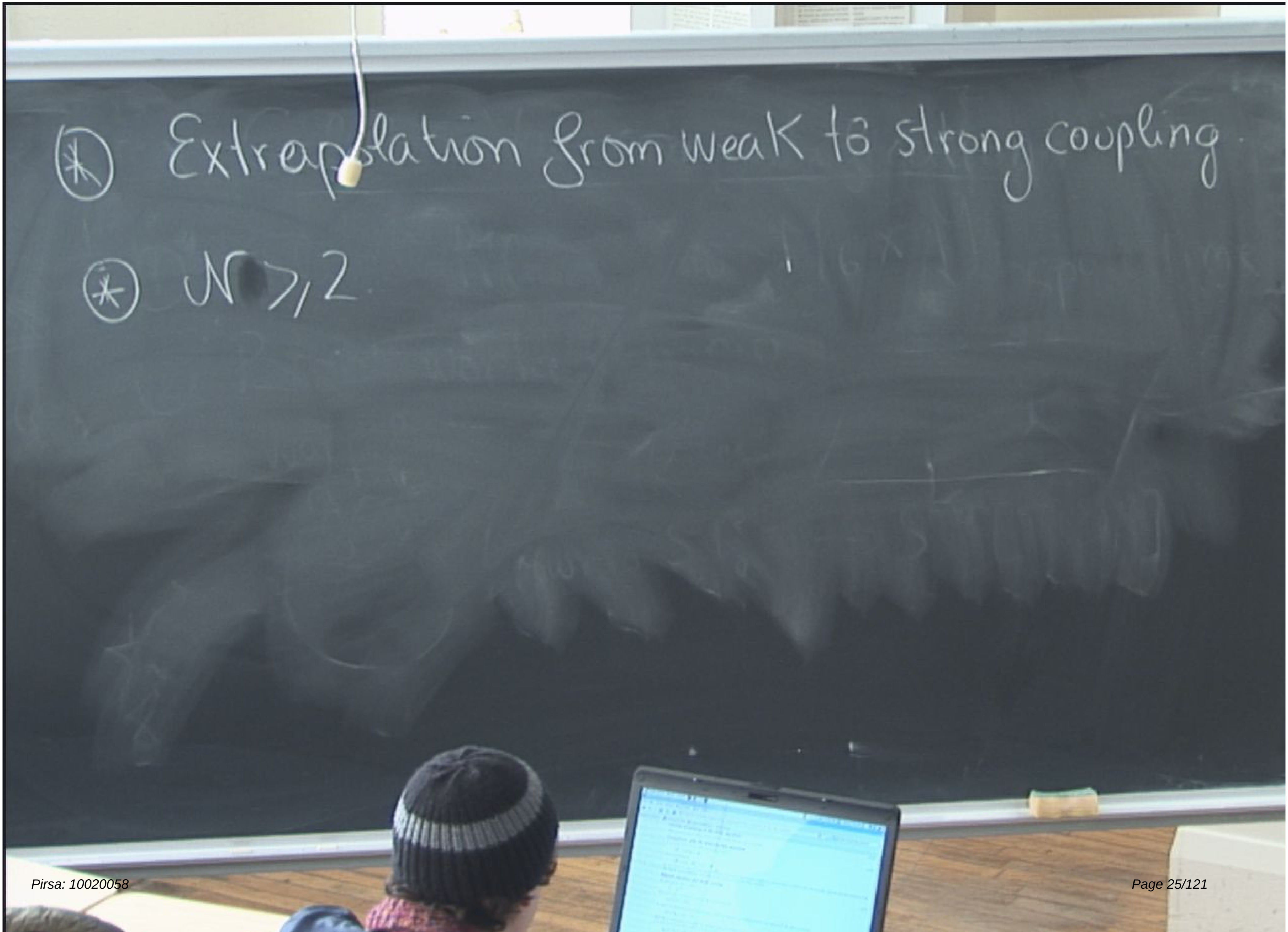
$[H, Q] = 0$

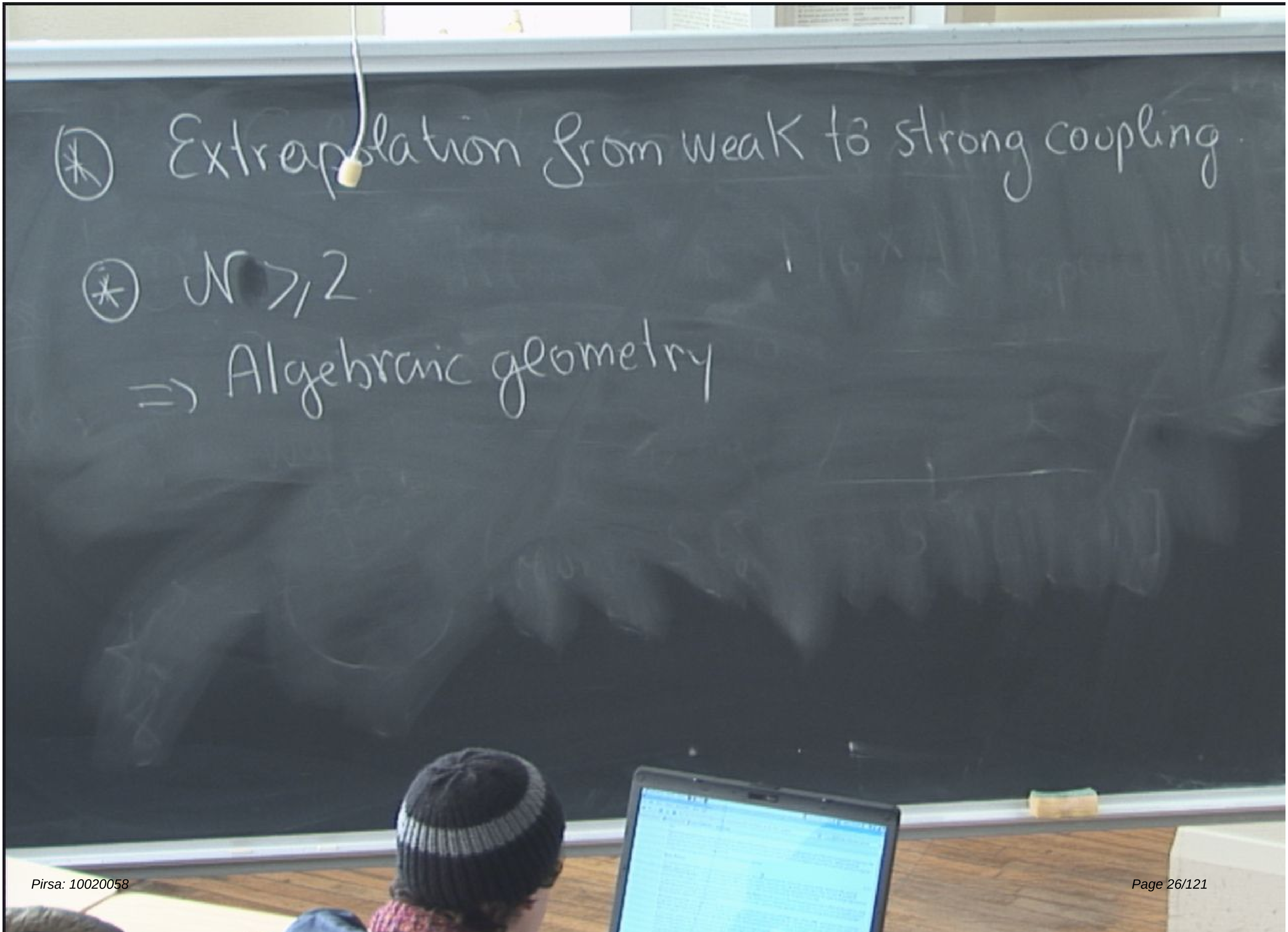

① E ≥ 0
[H, Q] = 0
Q|ψ⟩ = 0
Q†Q|ψ⟩ = 0
H|ψ⟩ = 0

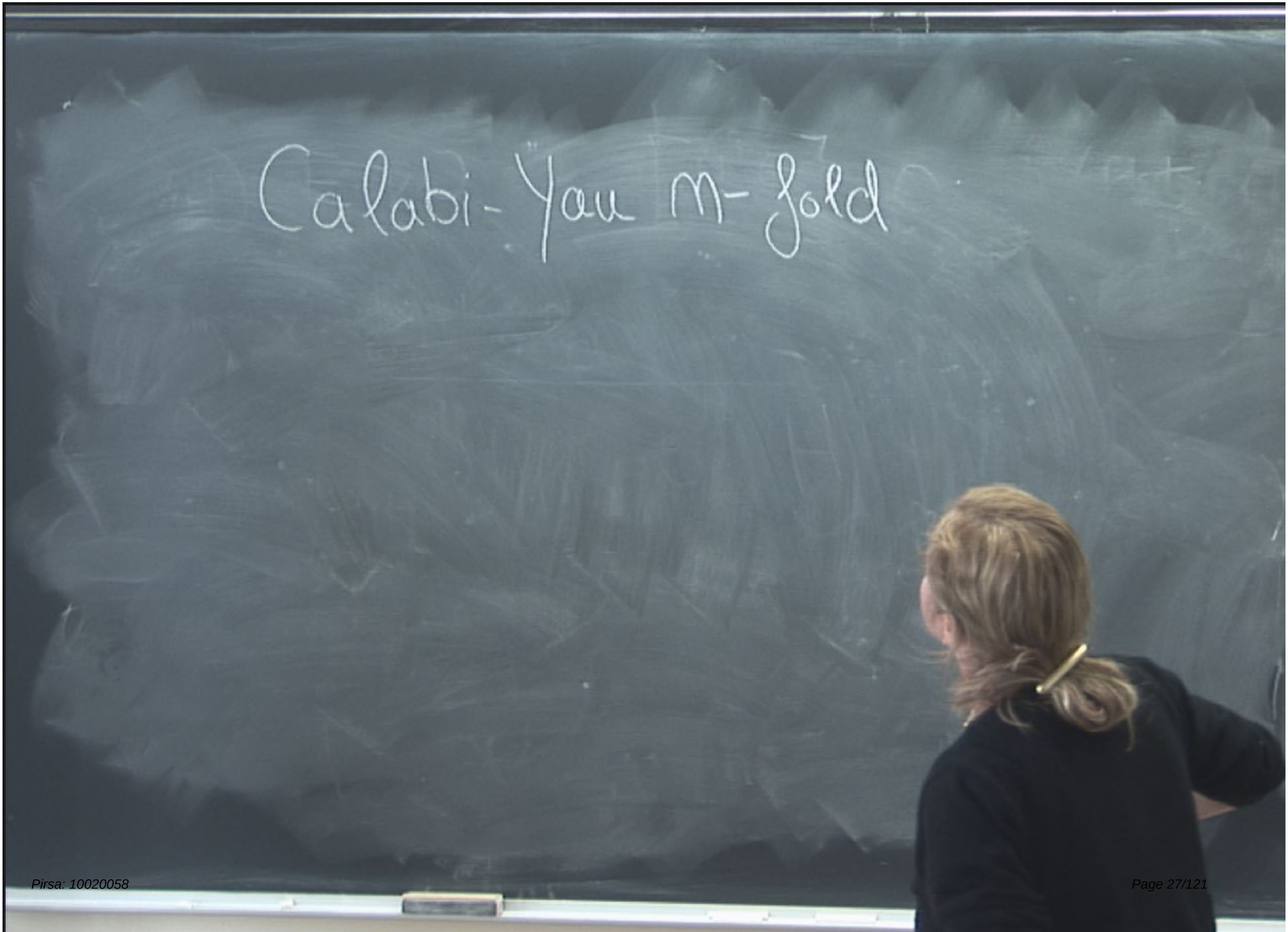


① $E \geq 0$
$[H, Q] = 0$
$Q|\psi\rangle = 0$
$Q^{\dagger} Q|\psi\rangle = 0$
$H|\psi\rangle = 0$
eom


① Extrapolation from weak to strong coupling


Extrapolation from weak to strong coupling.
$\mathcal{N} \geqslant 2$


⊛ Extrapolation from weak to strong coupling
⊛ $\mathcal{N} \geq 2$
$\Rightarrow$ Algebraic geometry


Calabi-Yau m-fold

Calabi-Yau m-fold $\quad CY_3 = M_6$

Calabi-Yau n-fold $\quad CY3 = M_6$

Is a (complex) Kähler manifold with $SU(n)$ holonomy

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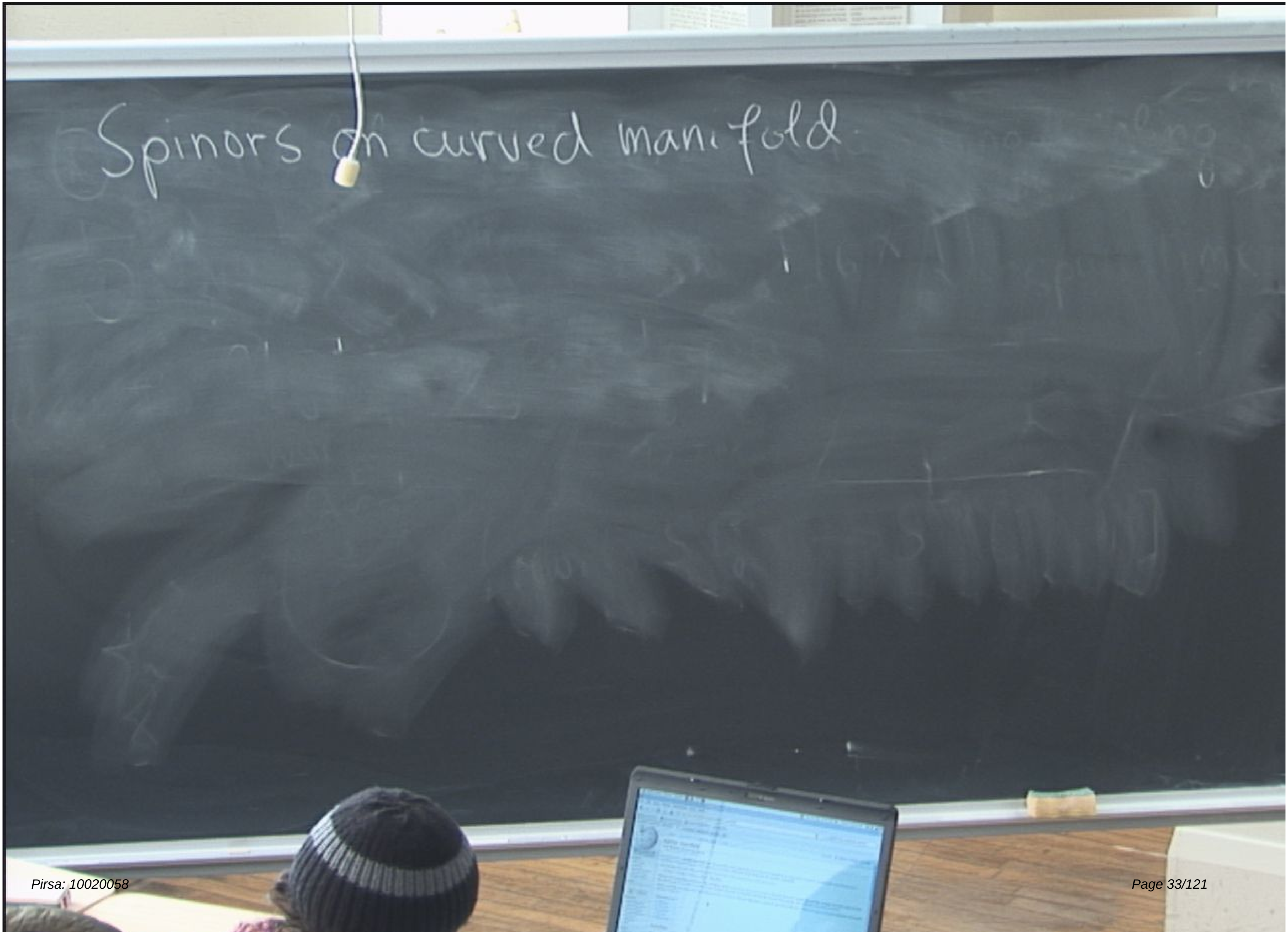
Is a (complex) Kähler manifold with $SU(n)$ holonomy $\triangleq$

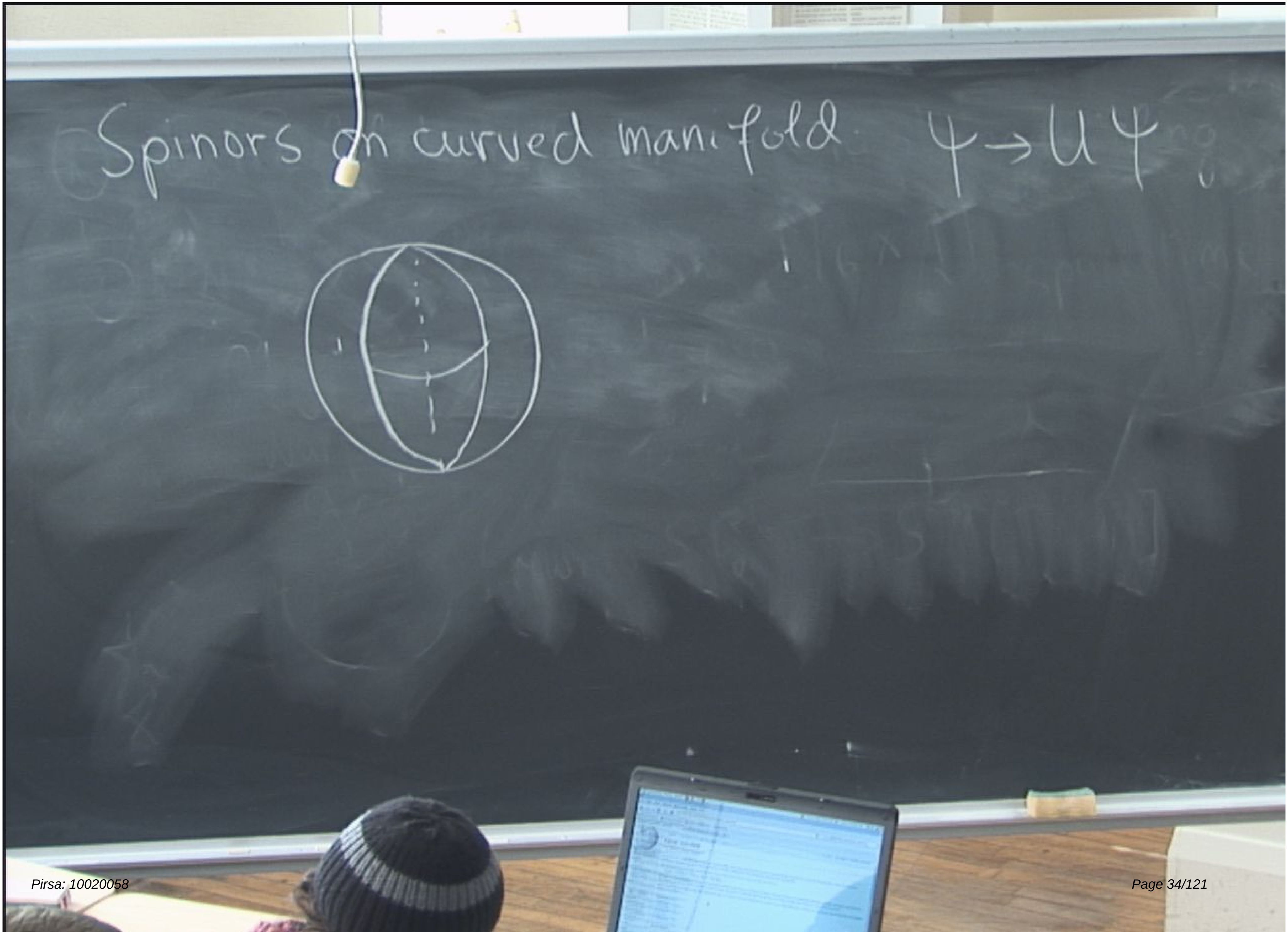
Kähler manifolds that are Ricci flat ($R_{ij}=0$)

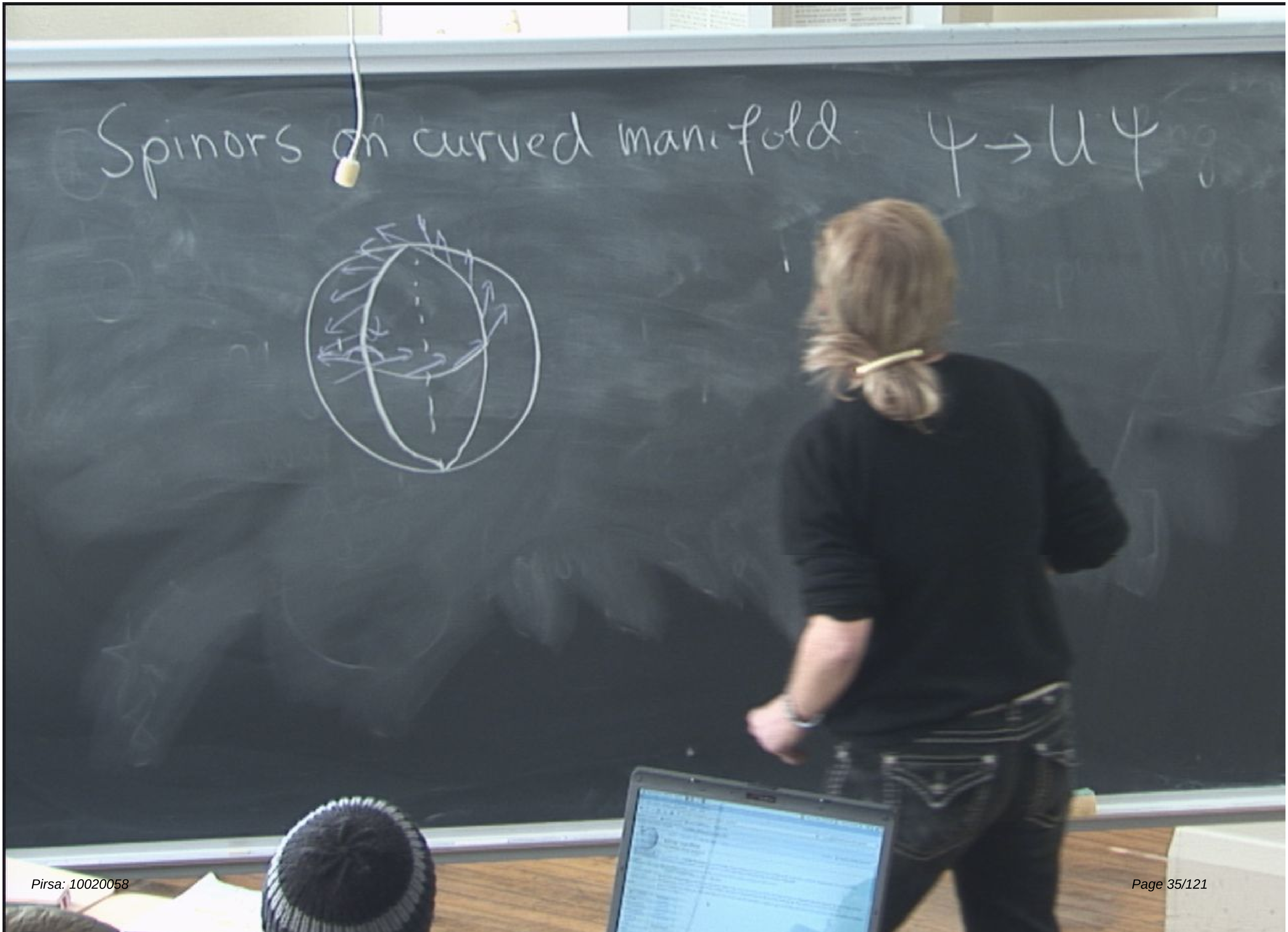
Calabi-Yau m-fold $CY3 = M_6$

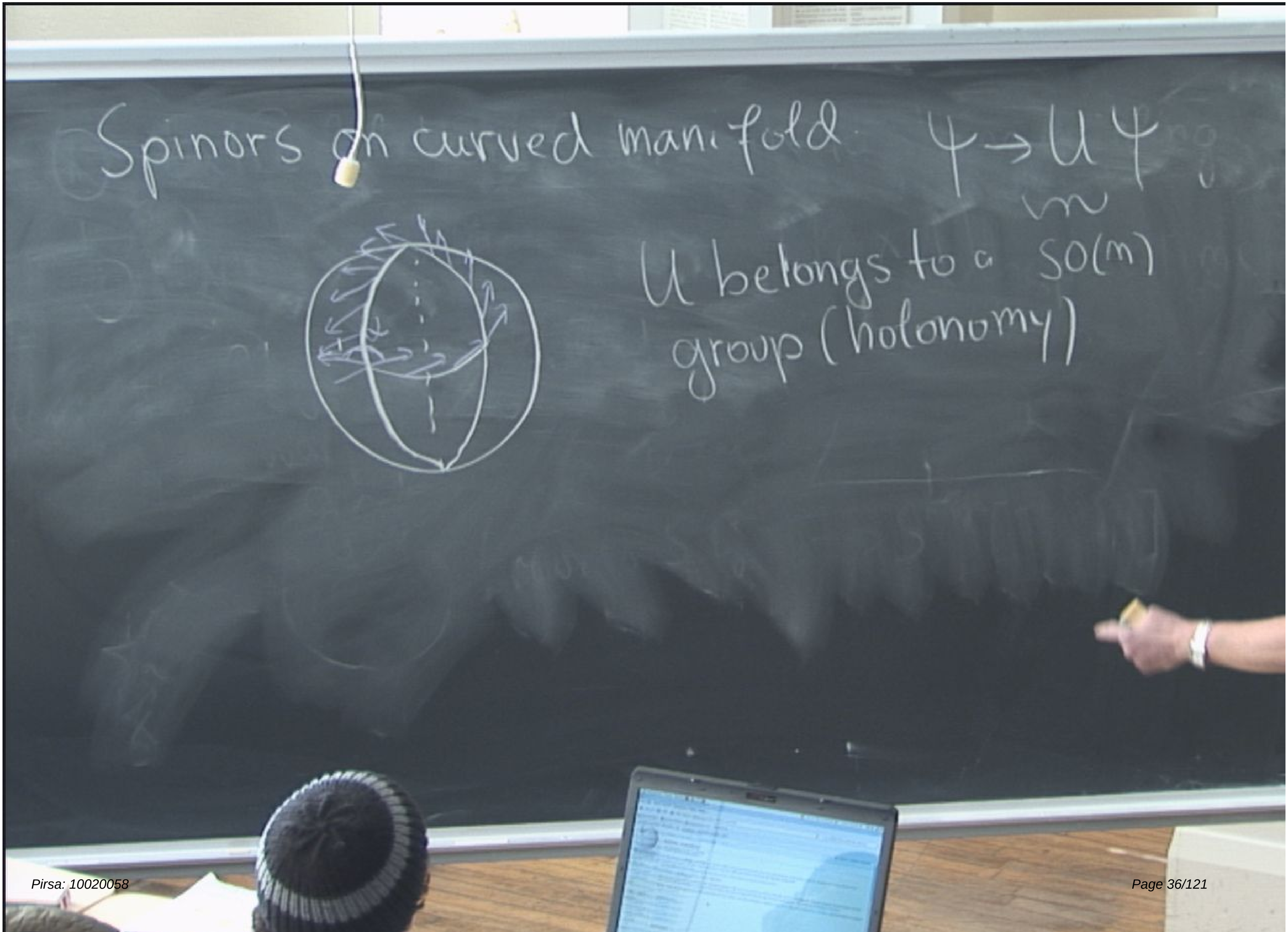
Is a (complex) Kähler manifold with $SU(m)$ holonomy $\triangleq$ Calabi-Conjecture Yau (1957)

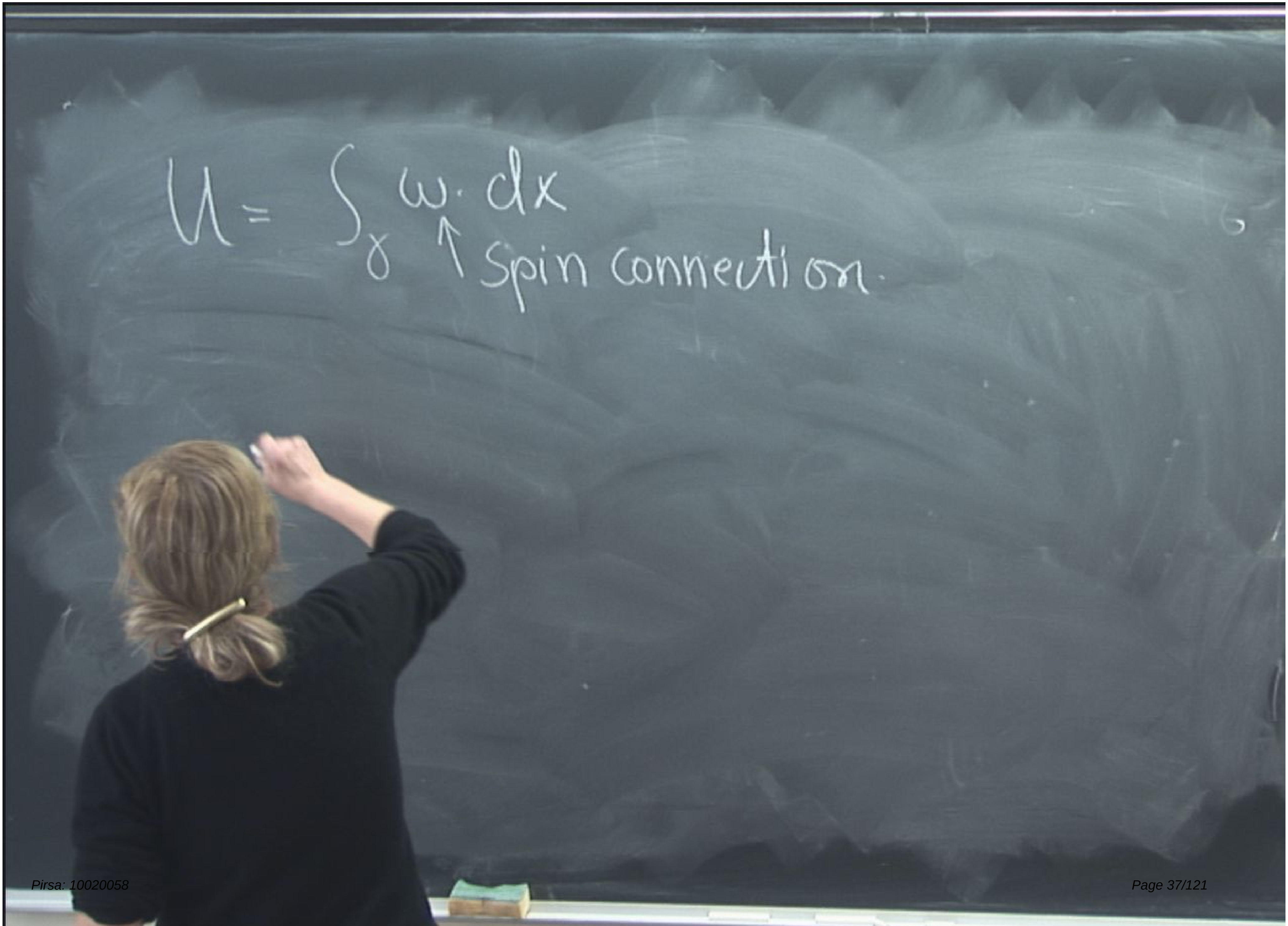
Kähler manifolds that are Ricci flat ($R_{ij} = 0$)


Spinors on curved manifold:


Spinors on curved manifold: ψ → Uψ


Spinors on curved manifold:
$\psi \to U\psi$


Spinors on curved manifold. $\psi \to U \psi$
U belongs to a SO(m)
group (holonomy)


$U = \int_\gamma \omega \cdot dx$
spin connection.

$$U = \int_\gamma \omega \cdot dx$$

$\uparrow$ spin connection.

$$U = \int_\gamma \omega \cdot dx$$

spin connection

$$\nabla_\mu \psi = \partial_\mu \psi + \frac{1}{4} \omega_\mu^{mn} \Gamma_{mn} \psi$$

spin connection

$$U = \int_\gamma \omega \cdot dx$$

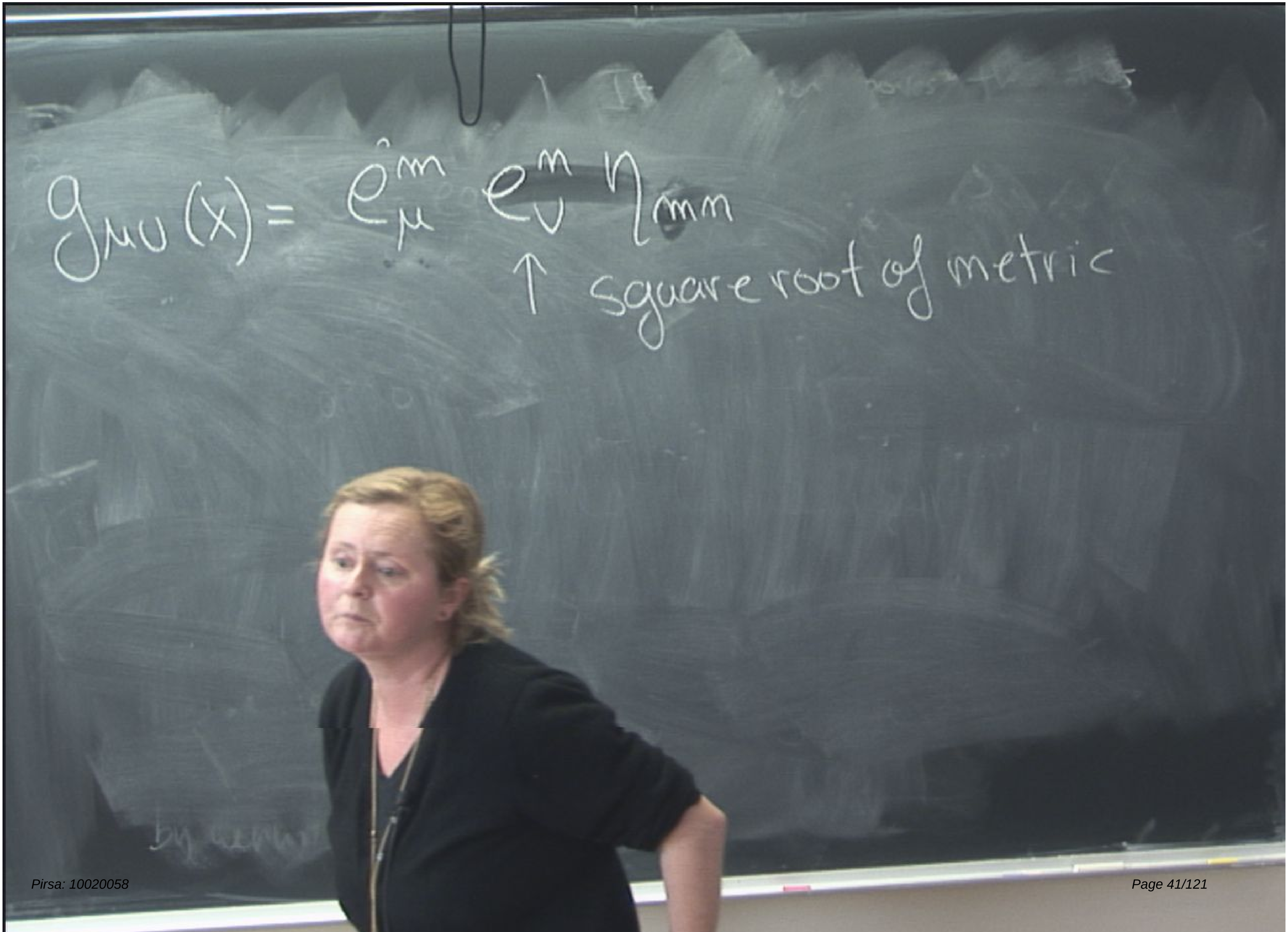
Spin connection.

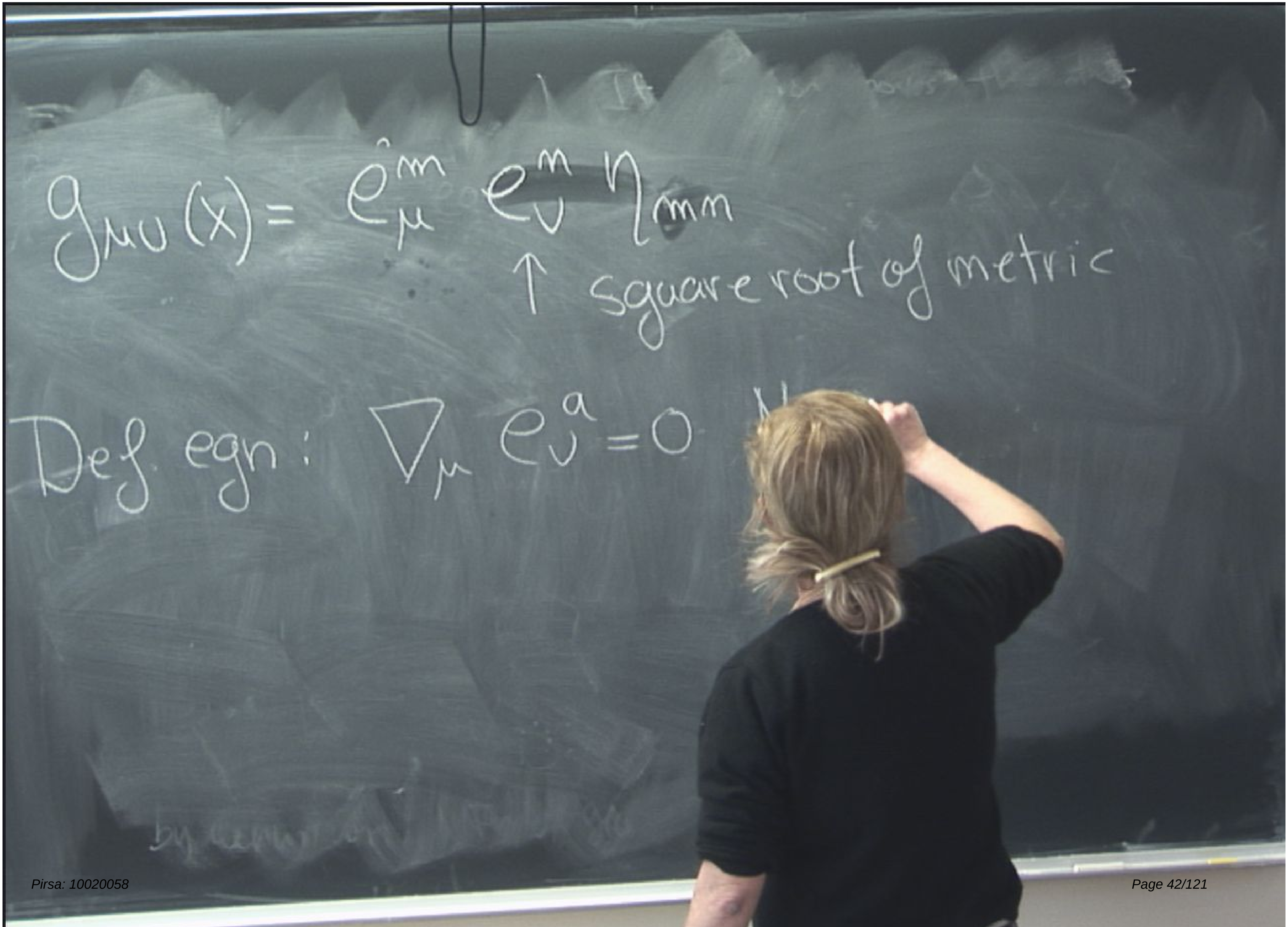
$$\nabla_\mu \psi = \partial_\mu \psi + \frac{1}{4} \omega_\mu^{mn} \Gamma_{mn} \psi$$

Labels group elements.

spin connection

$\mu = 0 \ldots 3$ Space-time index

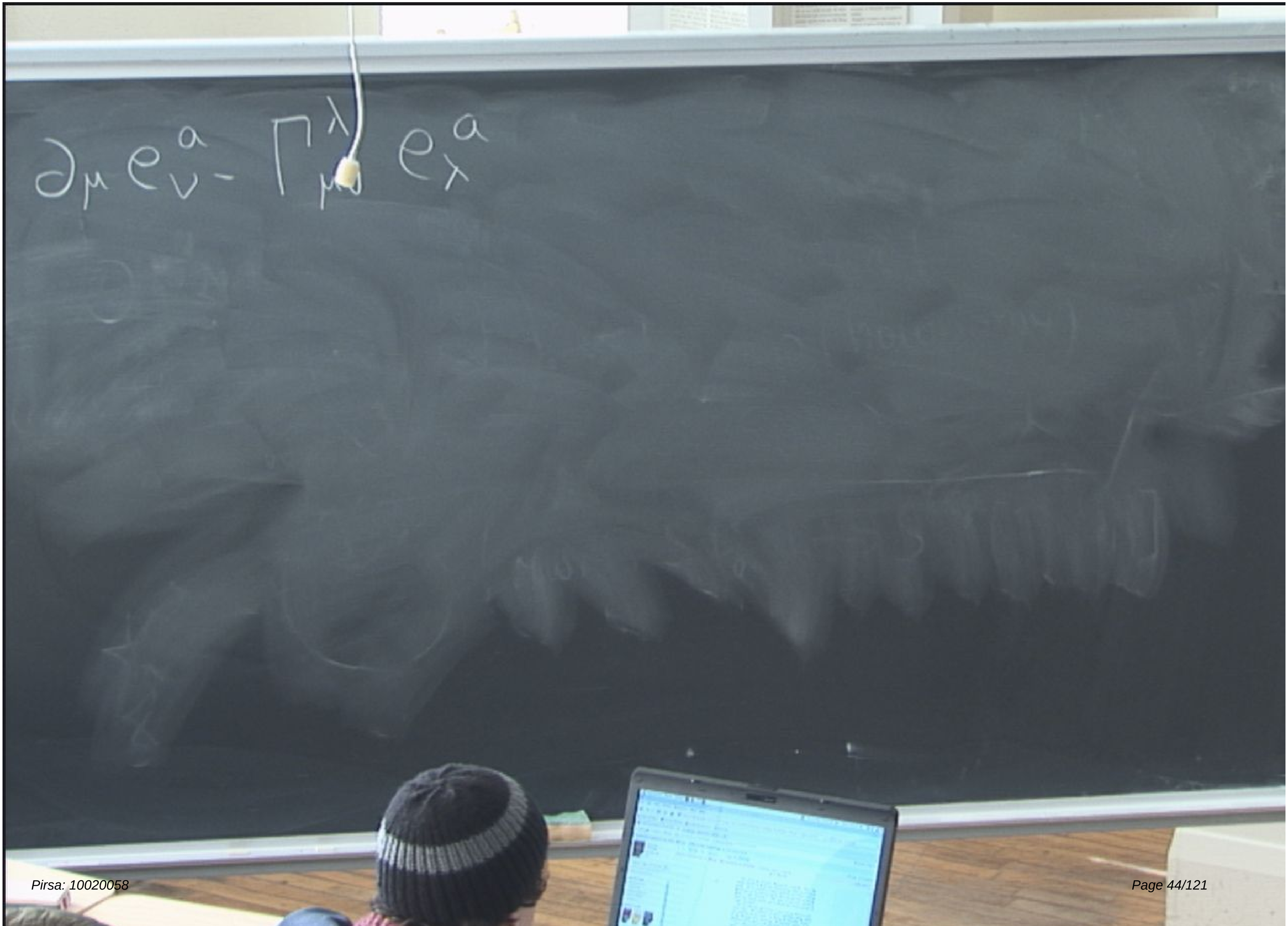

$g_{\mu\nu}(x) = e_\mu^m e_\nu^n \eta_{mn}$
square root of metric


$g_{\mu\nu}(x) = e^m_\mu e^n_\nu \eta_{mn}$
↑ square root of metric
Def. eqn: $\nabla_\mu e^a_\nu = 0$

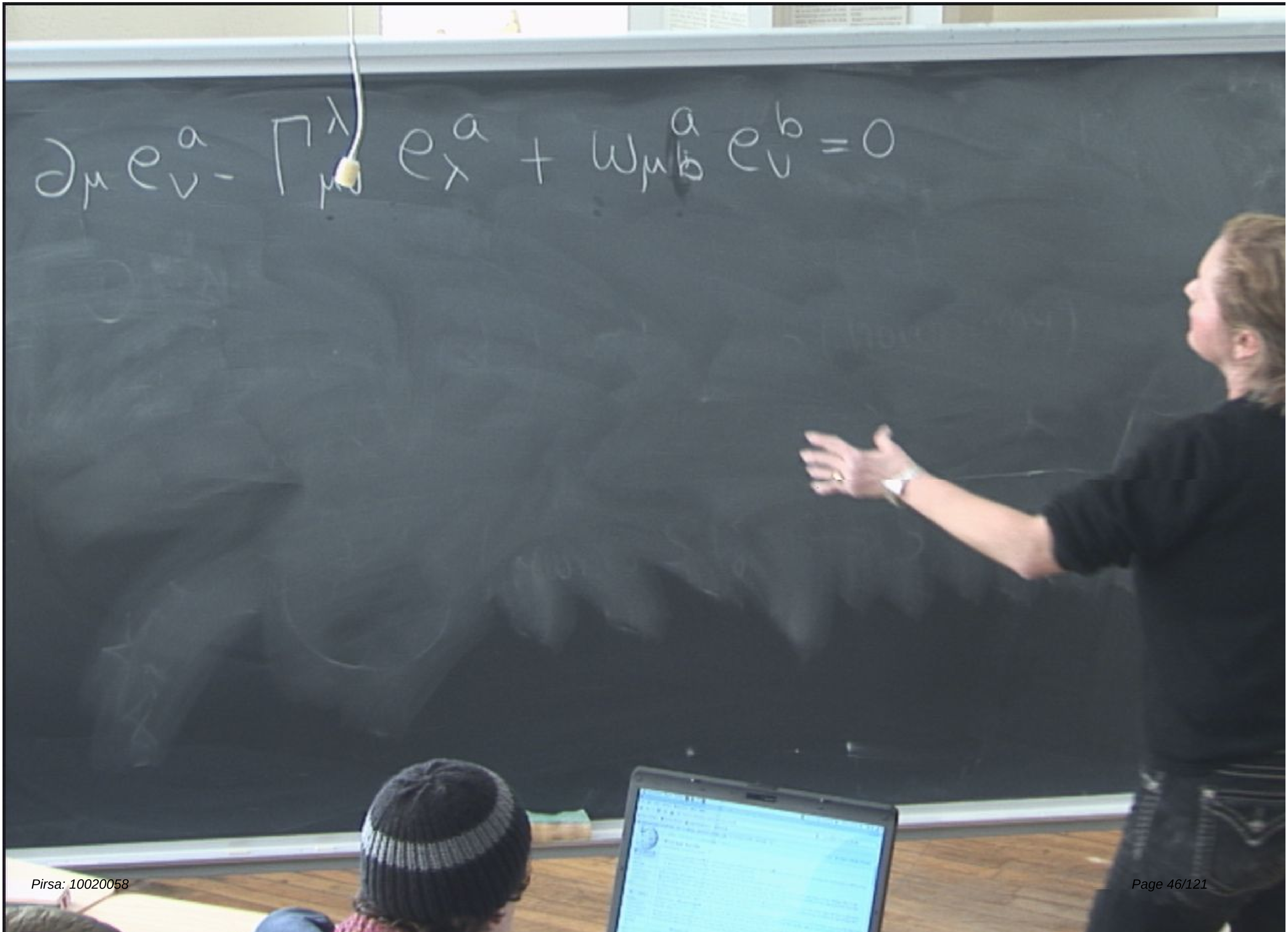
$$g_{\mu\nu}(x) = e_\mu^m \, e_\nu^n \, \eta_{mn}$$

↑ square root of metric

Def. eqn: $\nabla_\mu e_\nu^a = 0$ No analog in GR.


∂_μ e^a_ν - Γ^λ_μν e^a_λ

$$\partial_\mu e_\nu^a - \Gamma^\lambda_{\mu\nu} e_\lambda^a + \omega_\mu{}^a{}_b \, e_\nu^b = 0$$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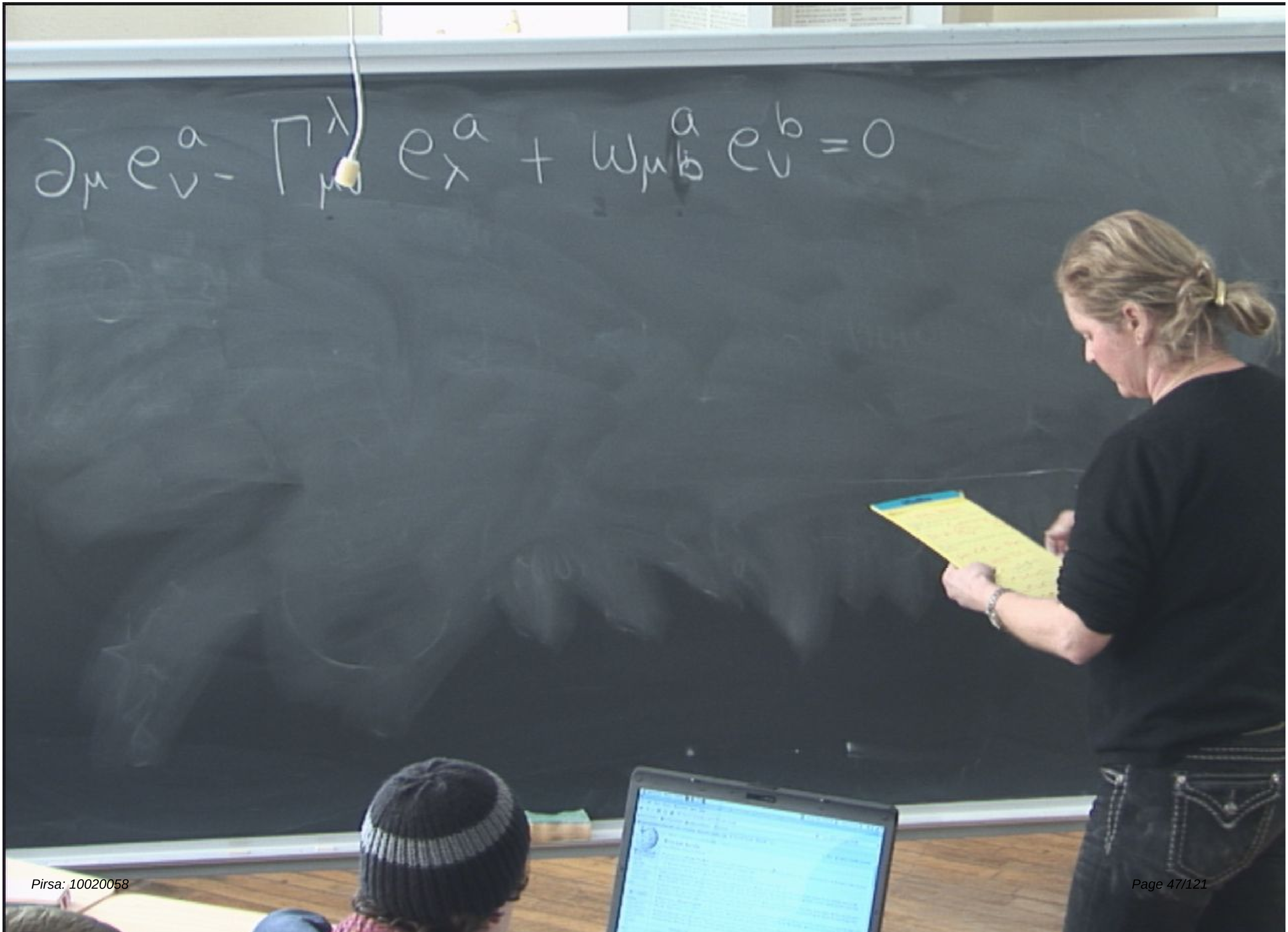
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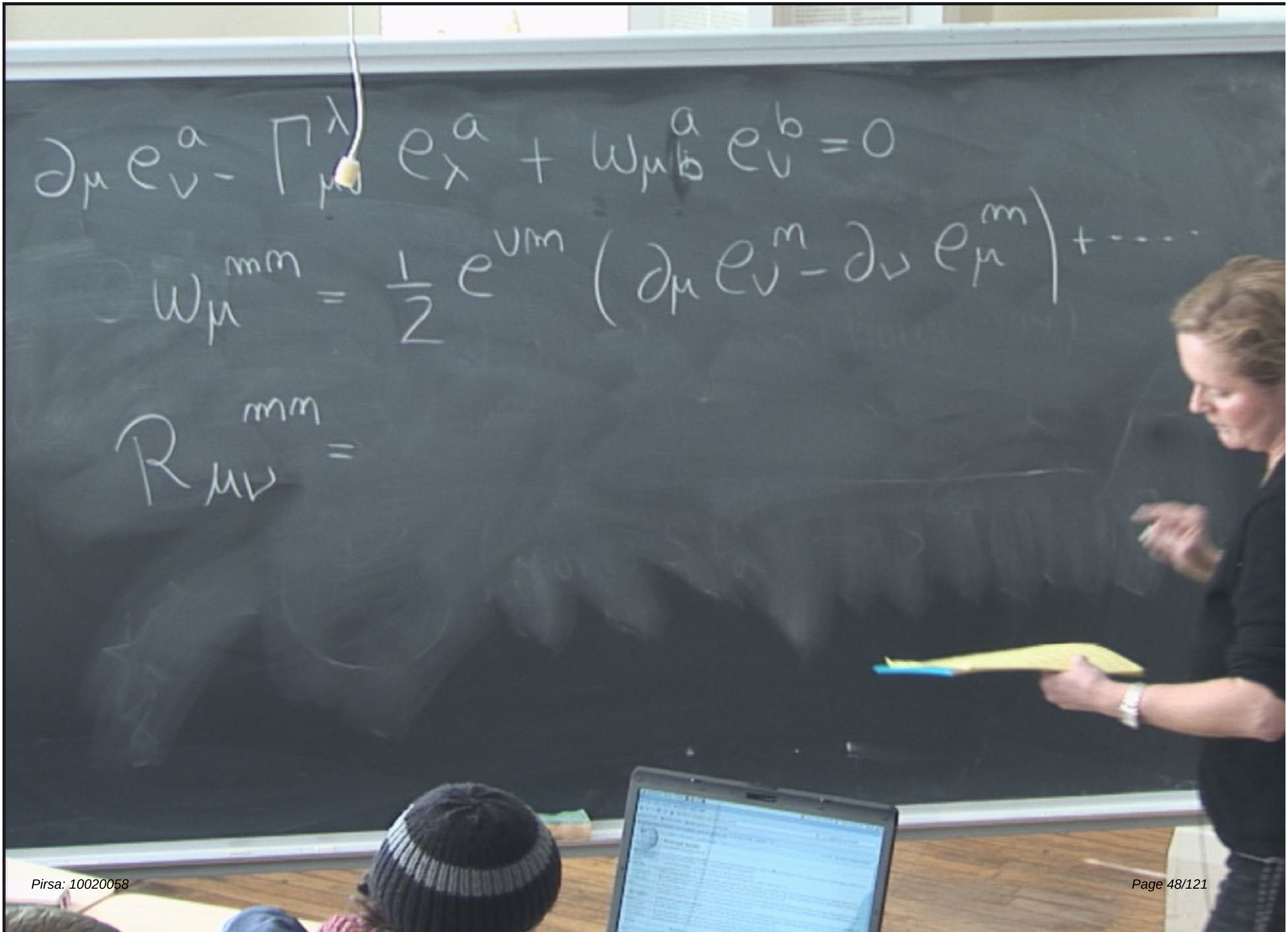
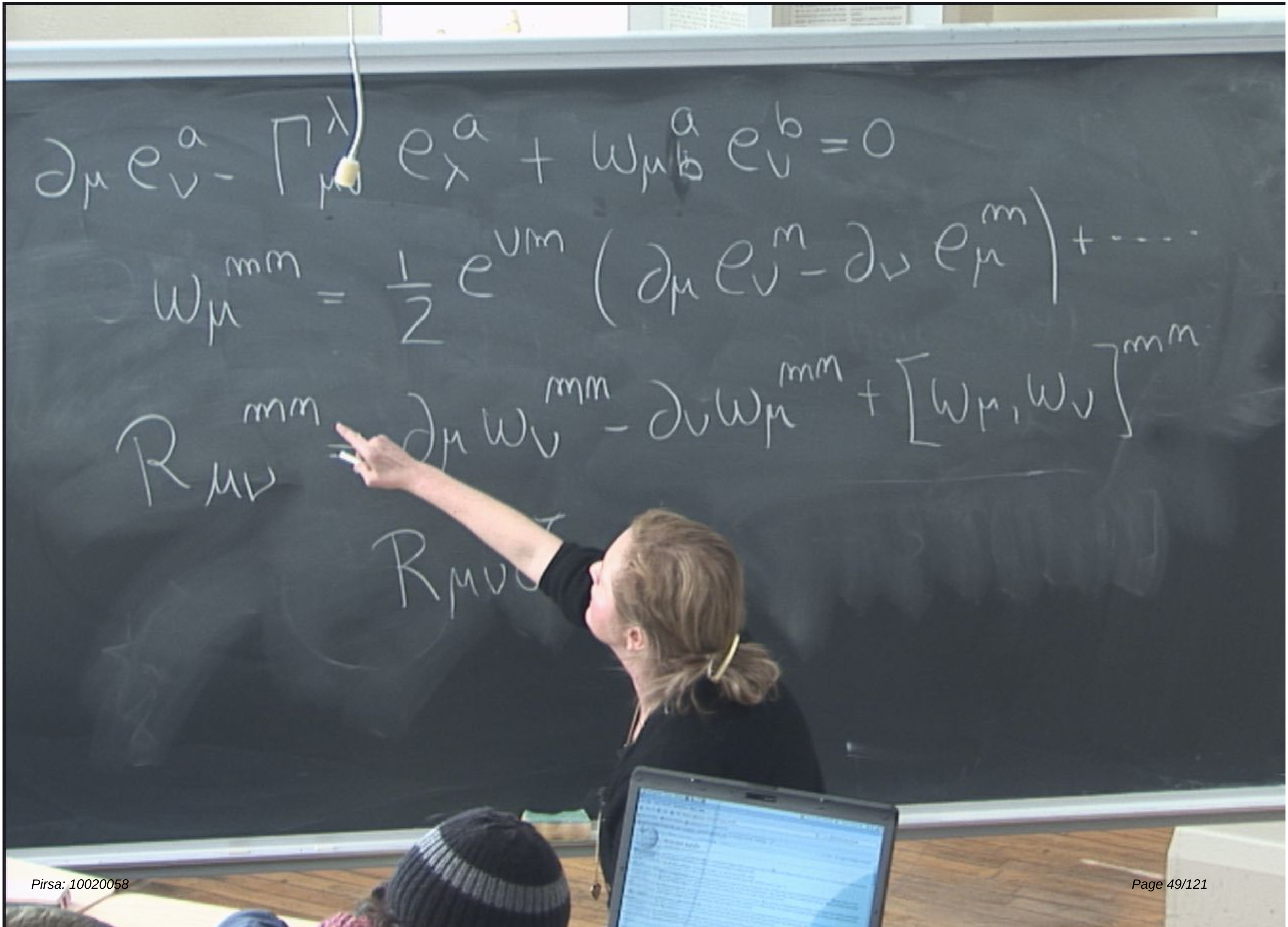


$\partial_\mu e_\nu^a - \Gamma^\lambda_{\mu\nu} e_\lambda^a + \omega_\mu{}^a{}_b e_\nu^b = 0$
$\omega_\mu{}^{mn} = \frac{1}{2} e^{\nu m} \left(\partial_\mu e_\nu^n - \partial_\nu e_\mu^n \right) + \cdots$
$R_{\mu\nu}{}^{mn} =$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$$R_{\mu\nu\sigma}{}^\tau = R_{\mu\nu}{}^m{}_n e_\sigma^n e_m^\tau$$

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Labels group elemen

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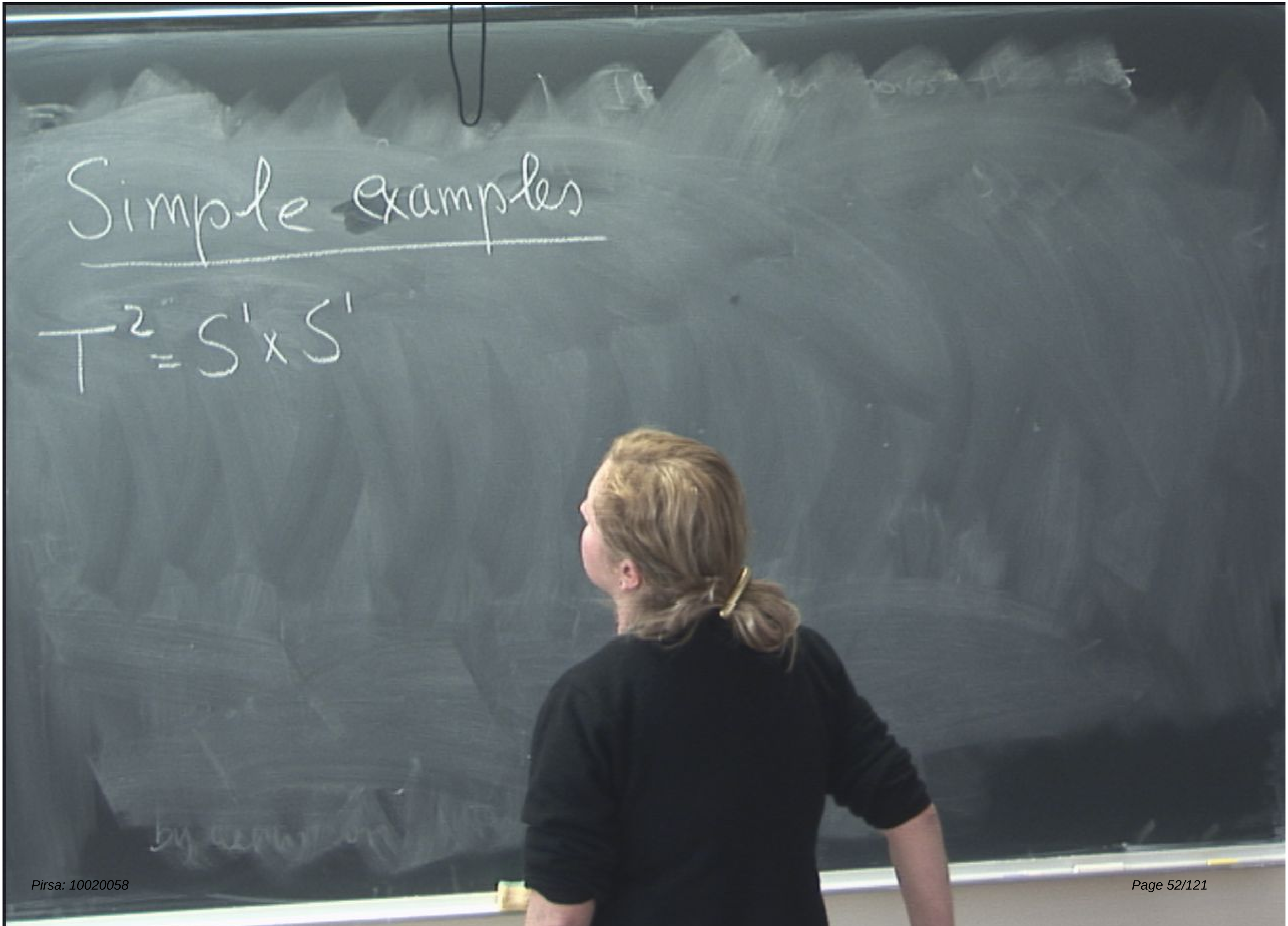
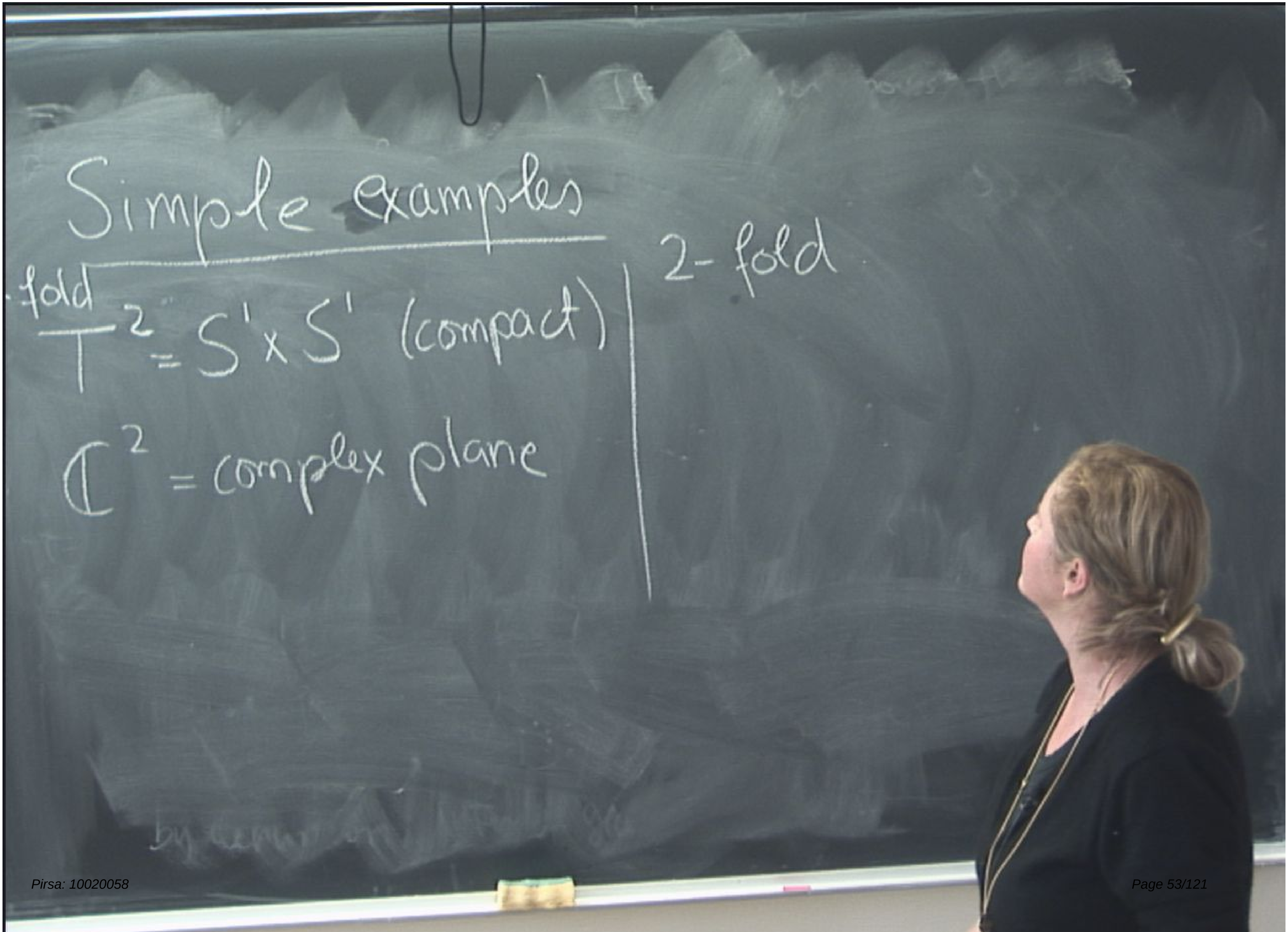
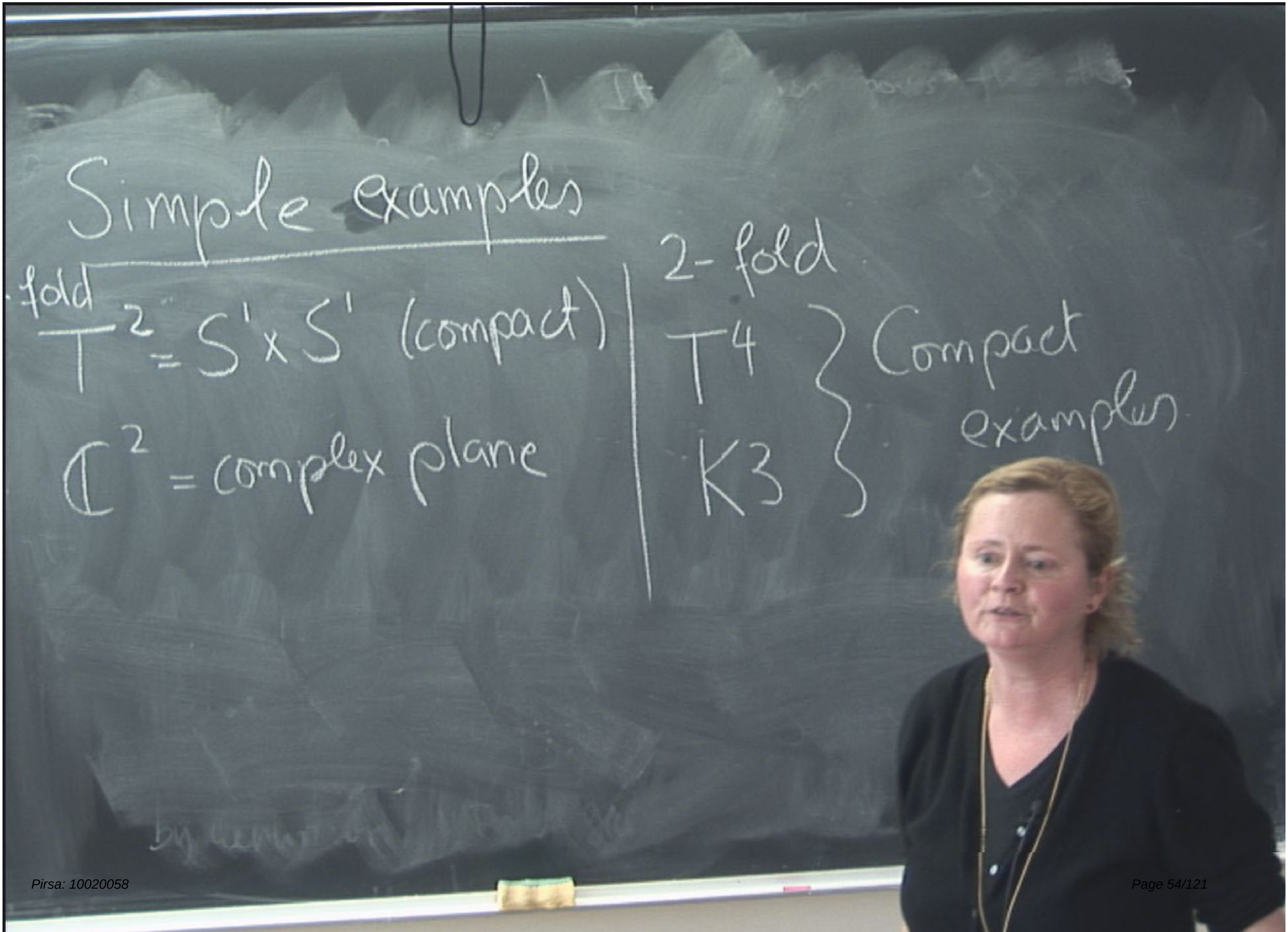

Simple examples
T^2 = S^1 x S^1


Simple examples
fold
$T^2 = S^1 \times S^1$ (compact)
$\mathbb{C}^2$ = complex plane
2-fold


Simple examples
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T^4
K3
Compact examples

Simple examples

fold

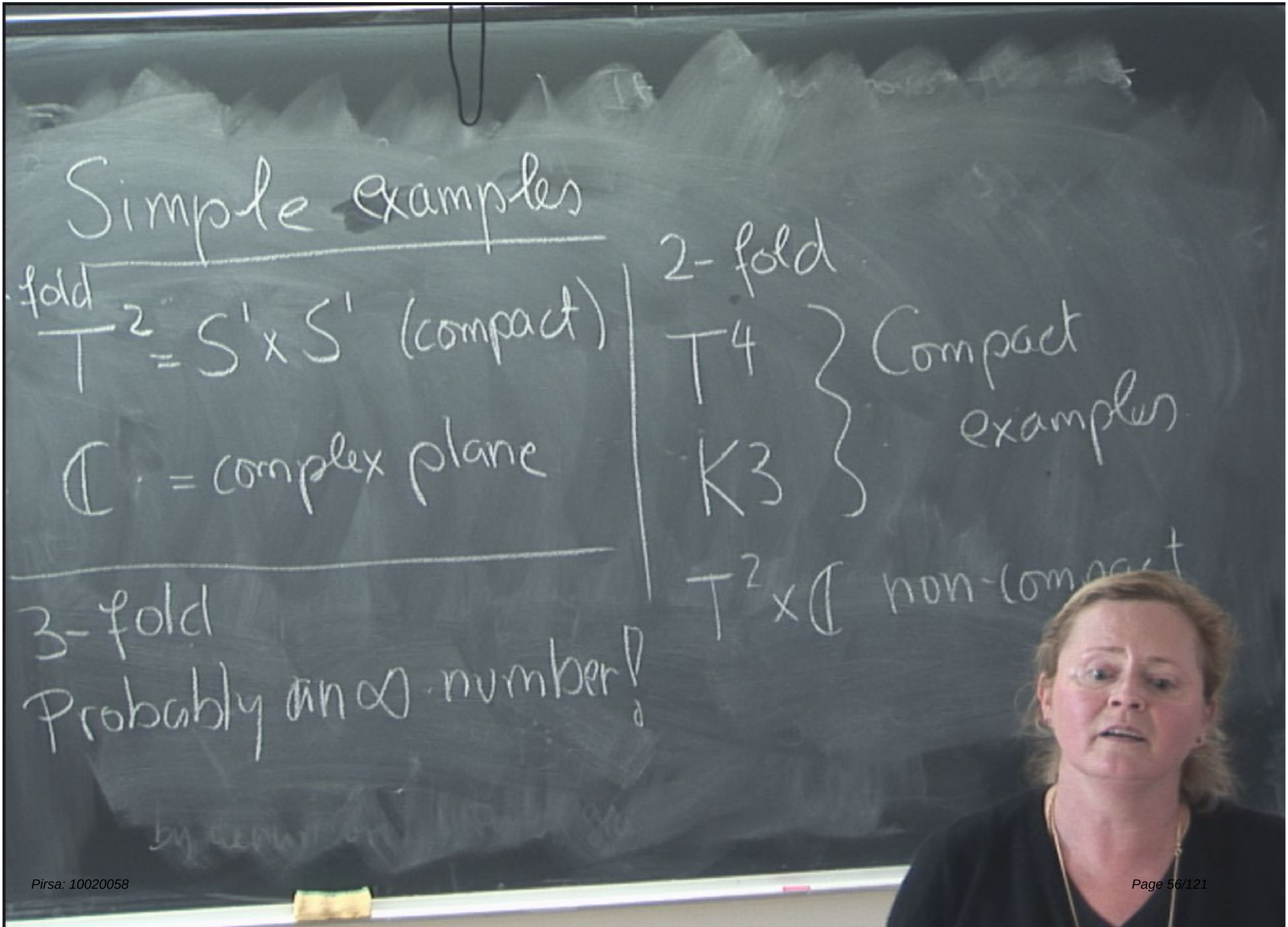
$T^2 = S^1 \times S^1$ (compact)

$\mathbb{C}$ = complex plane

2-fold

$\left.\begin{array}{l} T^4 \\ K3 \end{array}\right\}$ Compact examples

$T^2 \times \mathbb{C}$ non-compact


Simple examples
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$T^2 = S^1 \times S^1$ (compact)
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3-fold
Probably an ∞ number!
2-fold
T^4
K3
Compact examples
$T^2 \times \mathbb{C}$ non-com

Simple examples

fold

$T^2 = S^1 \times S^1$ (compact)

$\mathbb{C}$ = complex plane

3-fold

Probably an ∞ number!

2-fold

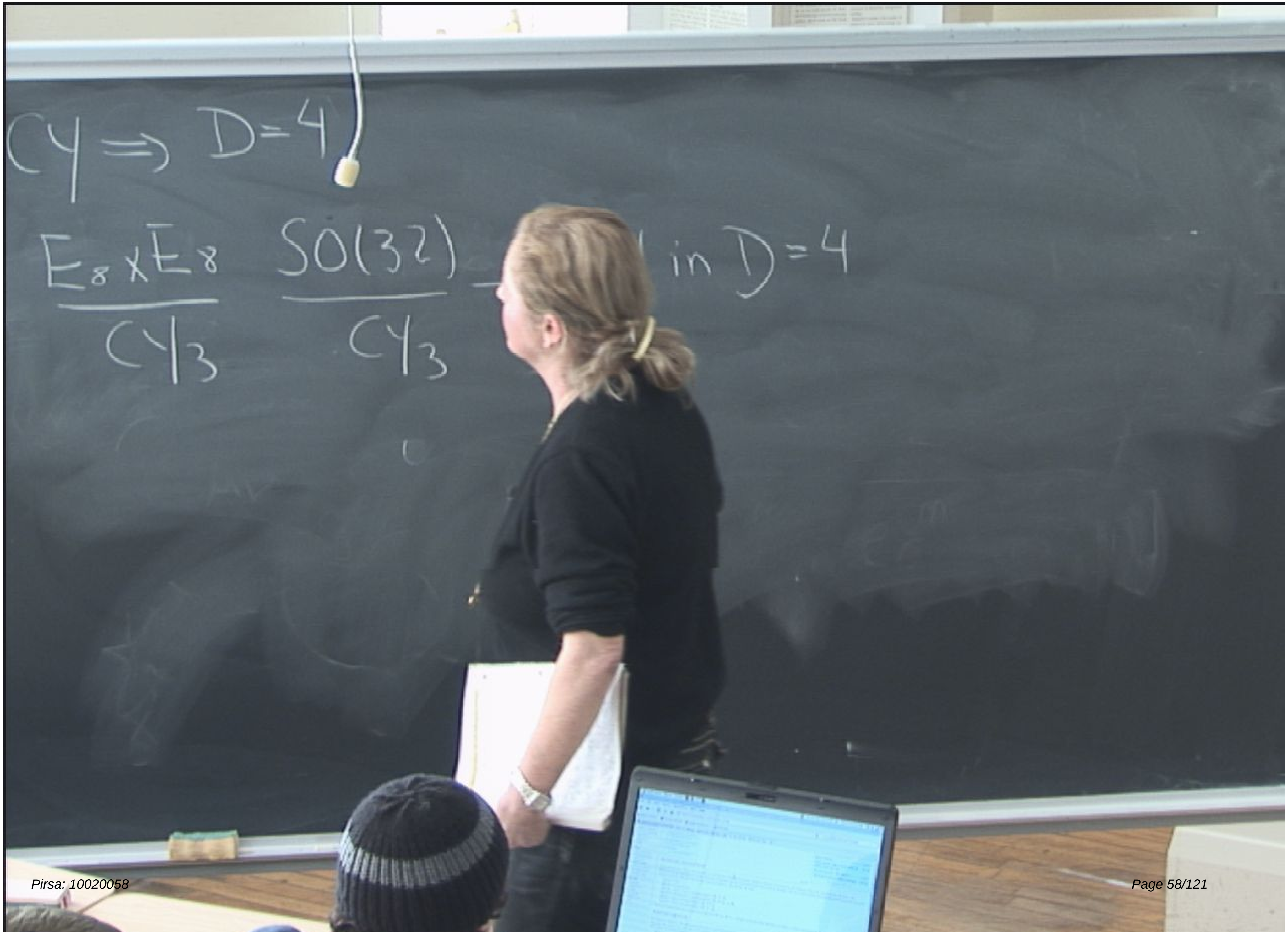
$K3 = T^4/\mathbb{Z}_2$

T^4

K3

Compact examples

$T^2 \times \mathbb{C}$ non-


CY ⟹ D=4
E8xE8 / CY3
SO(32) / CY3
in D=4

$$CY \Rightarrow D=4$$

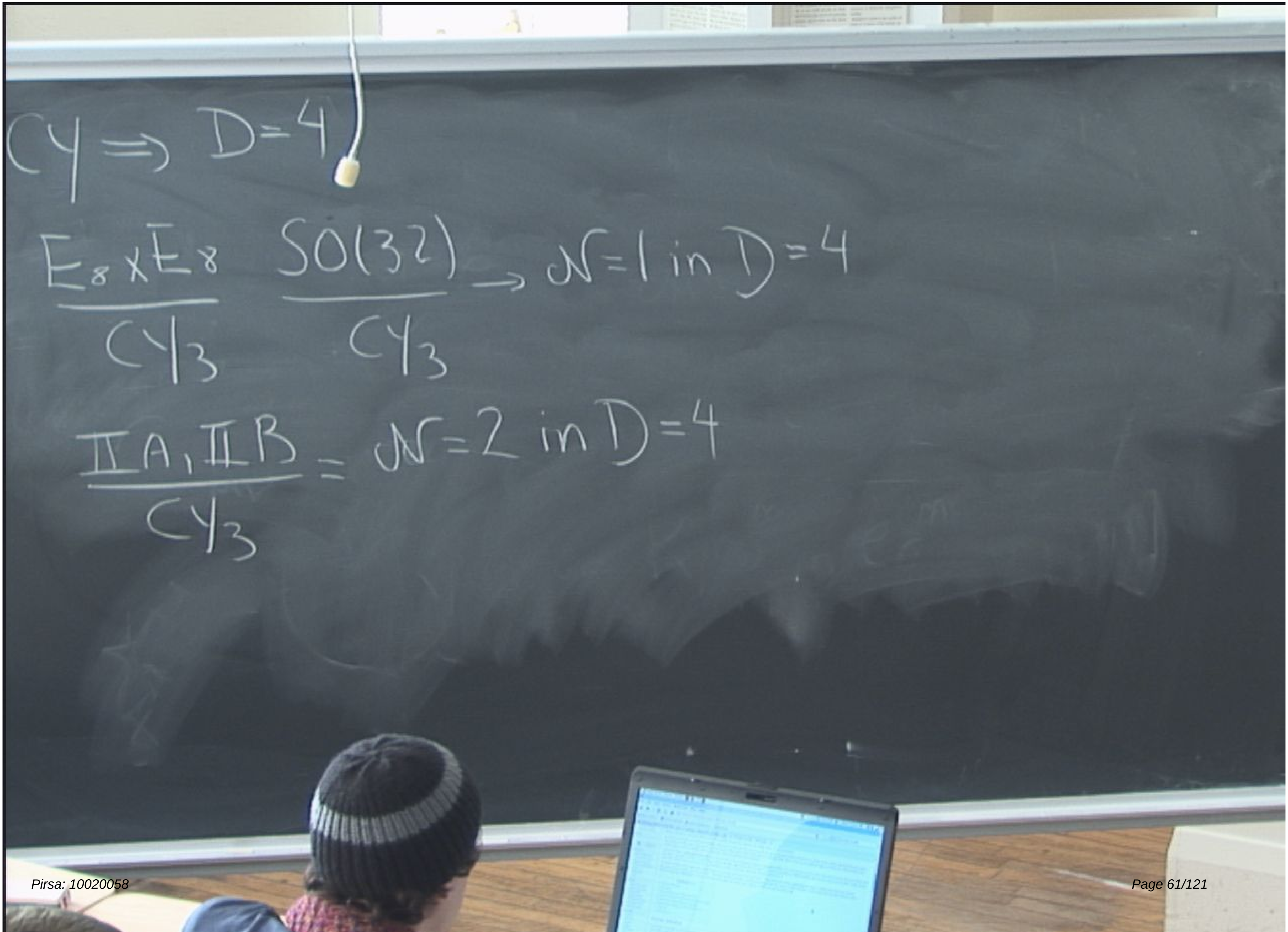
$$\frac{E_8 \times E_8}{CY_3} \quad \frac{SO(32)}{CY_3} \rightarrow \mathcal{N}=1 \text{ in } D=4$$

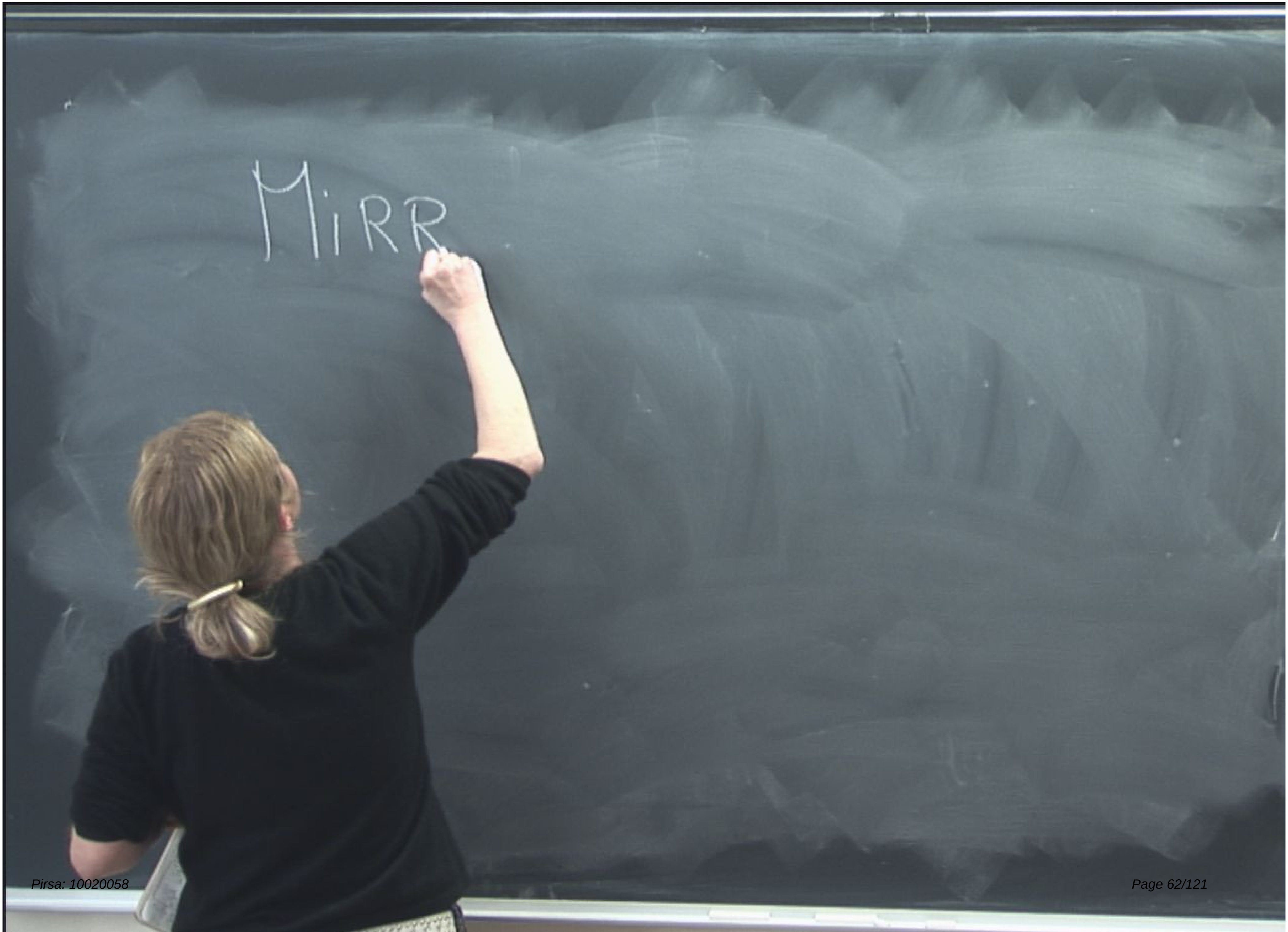
$$\frac{\mathrm{II}A, \mathrm{II}B}{CY_3} = \mathcal{N}=2 \text{ in } D=4$$

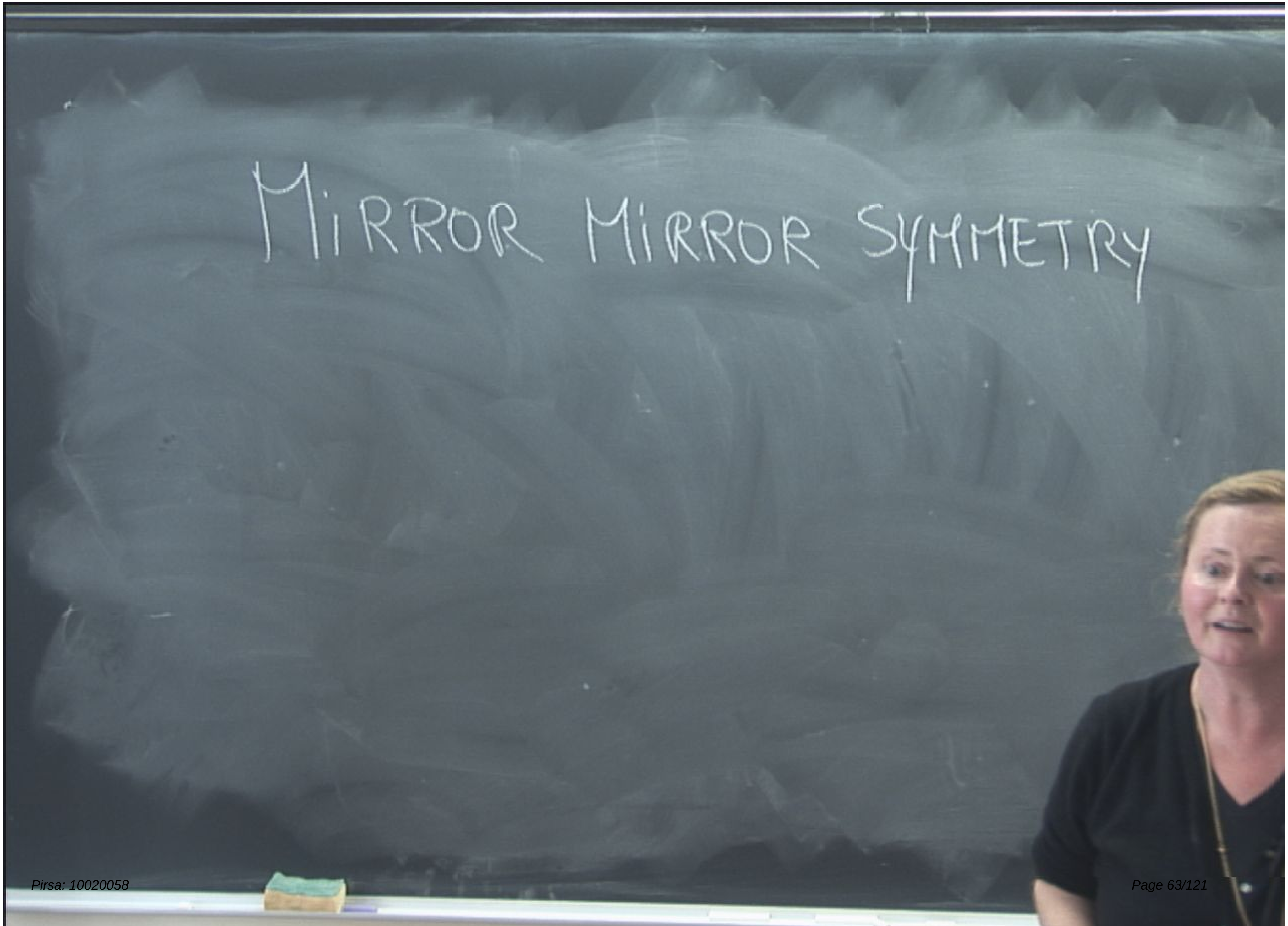
$CY \Longrightarrow D=4$

$\frac{E_8 \times E_8}{CY_3} \quad \frac{SO(32)}{CY_3} \longrightarrow \mathcal{N}=1$ in $D=4$

$\frac{\text{II}A, \text{II}B}{CY_3} = \mathcal{N}=2$ in $D=4$


$CY \Rightarrow D=4$
$\frac{E_8 \times E_8}{CY_3}$ $\frac{SO(32)}{CY_3} \rightarrow \mathcal{N}=1$ in $D=4$
$\frac{IIA, IIB}{CY_3} = \mathcal{N}=2$ in $D=4$


MiRR


MIRROR MIRROR SYMMETRY

MIRROR MIRROR SYMMETRY

MIRROR SYMMETRY

$$\frac{\text{Type IIA}}{R} \overset{\wedge}{=} \frac{\text{Type IIB}}{1/R} \quad \text{T-duality.}$$

MIRROR SYMMETRY

$$\frac{\text{Type II A}}{R} \hat{=} \frac{\text{Type II B}}{1/R} \quad \text{T-duality.}$$

$$\frac{\text{Type II A}}{M_6} \hat{=} \frac{\text{Type II B}}{\tilde{M}_6} \leftarrow \text{mirror of } M_6$$

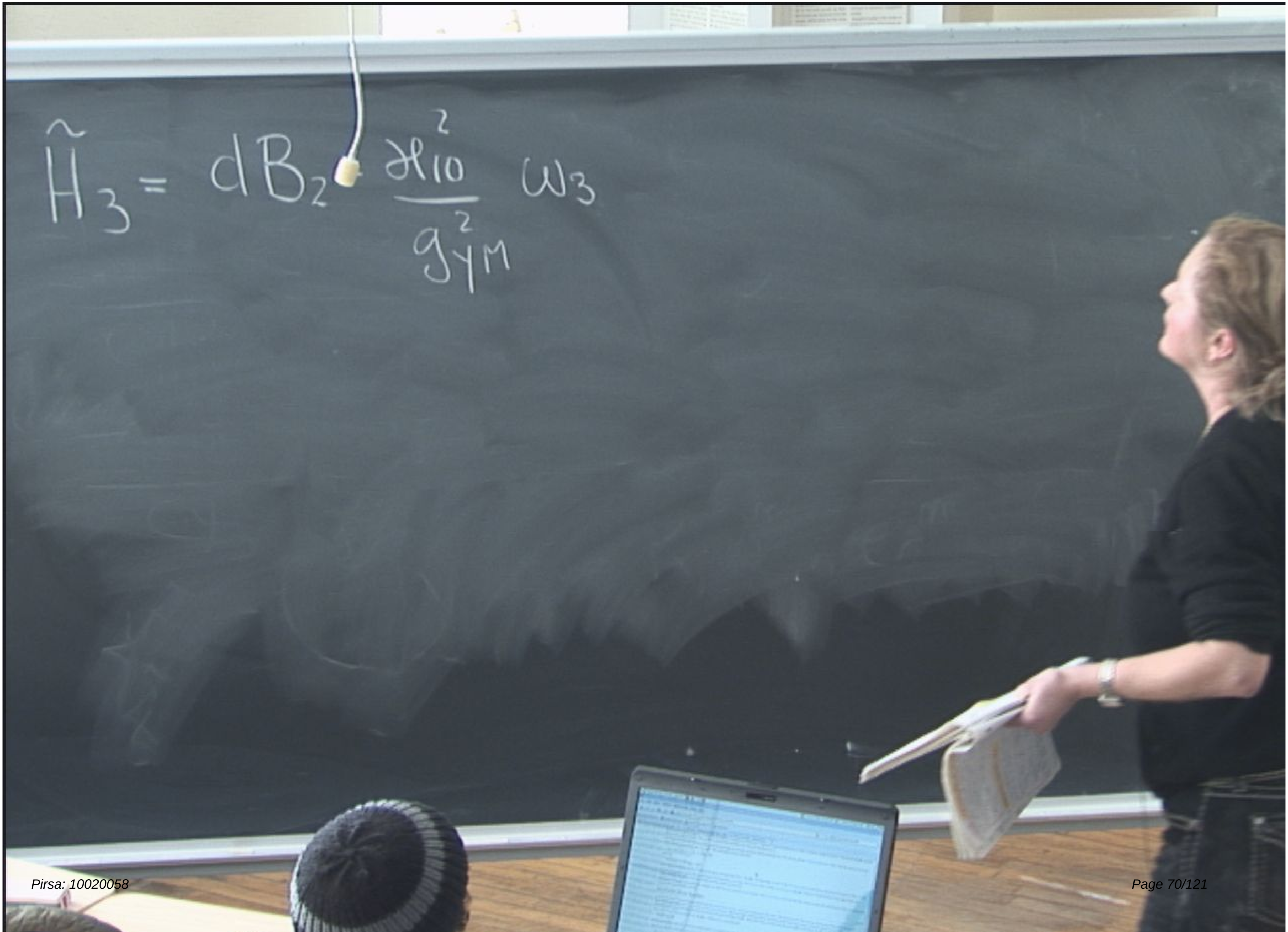

10D effective action for heterotic
S het =

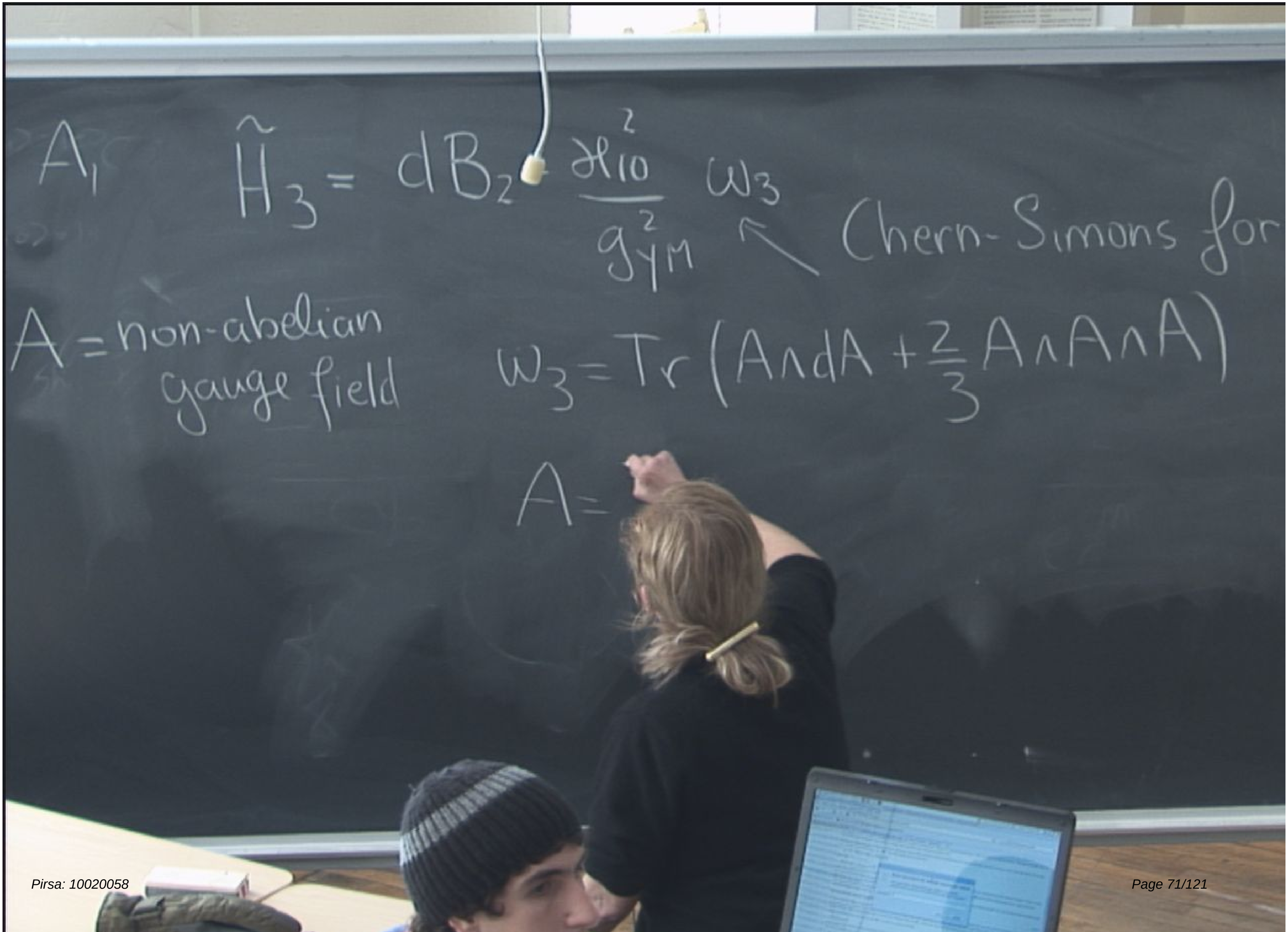
10D effective action for heterotic

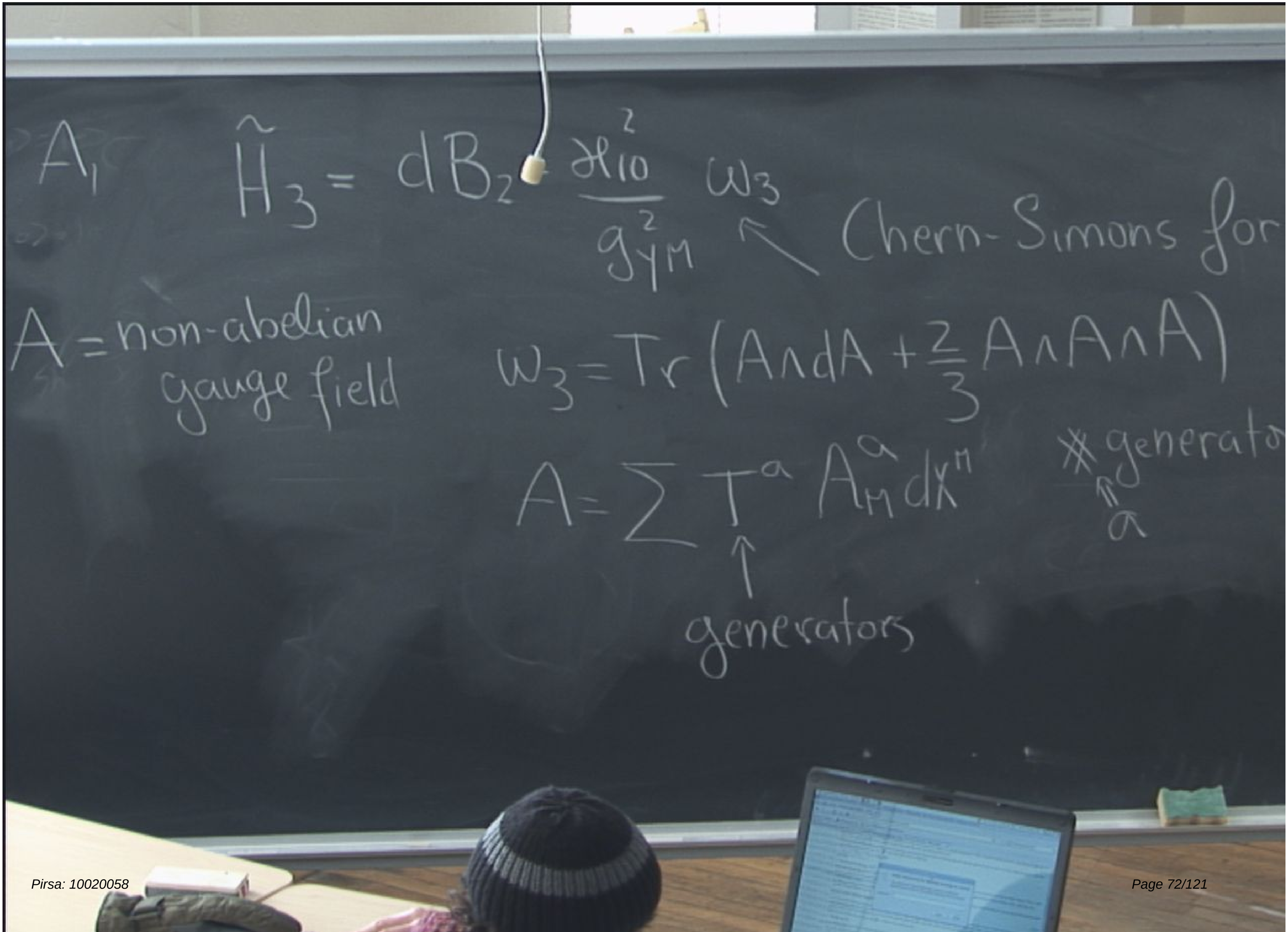
$$S_{het} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g}\, e^{-2\Phi} \left(R + 4(\partial\Phi)^2 - \frac{1}{2}\tilde{H}_3^2 \right.$$

10D effective action for heterotic

$$S_{het} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g}\, e^{-2\Phi} \left(R + 4(\partial\Phi)^2 - \frac{1}{2}\tilde{H}_3^2 + \frac{\kappa_{10}^2}{g_{YM}^2} \operatorname{tr} F^2 \right)$$


$\tilde{H}_3 = dB_2 \frac{\mathcal{H}_{10}^2}{g_{YM}^2} \omega_3$


A_1
$\tilde{H}_3 = dB_2 + \frac{\varkappa_{10}^2}{g_{YM}^2} \omega_3$
Chern-Simons for
A = non-abelian gauge field
$\omega_3 = \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$
A =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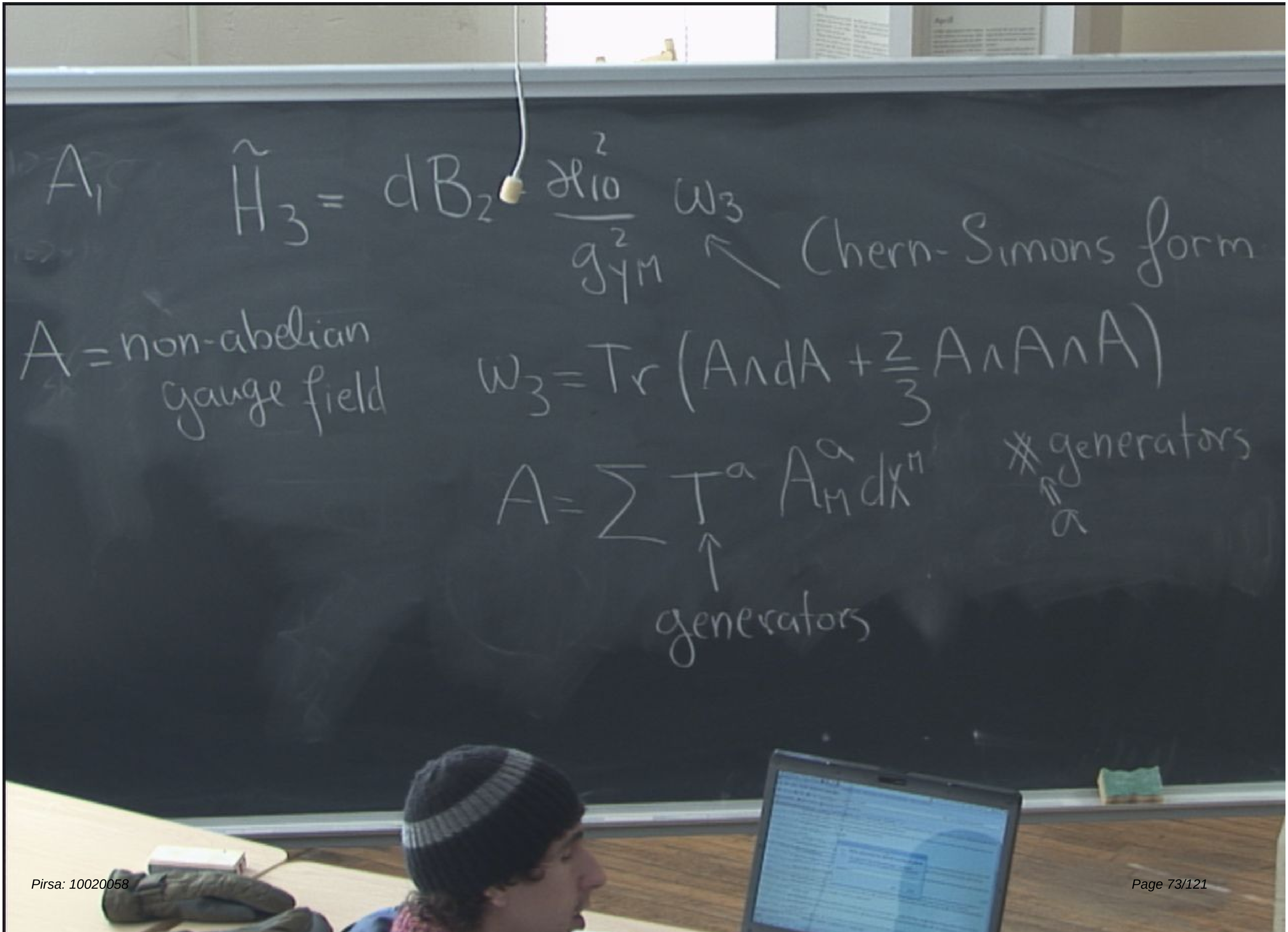
$\omega_3 = \mathrm{Tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A)$
$A = \sum T^a A^a_\mu dx^\mu$
generator
a
generators


A_1
$\tilde{H}_3 = dB_2 + \frac{\kappa_{10}^2}{g_{YM}^2}\omega_3$
Chern-Simons form
A = non-abelian gauge field
$\omega_3 = Tr\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$
$A = \sum T^a A_M^a dx^M$
generators
generators
a

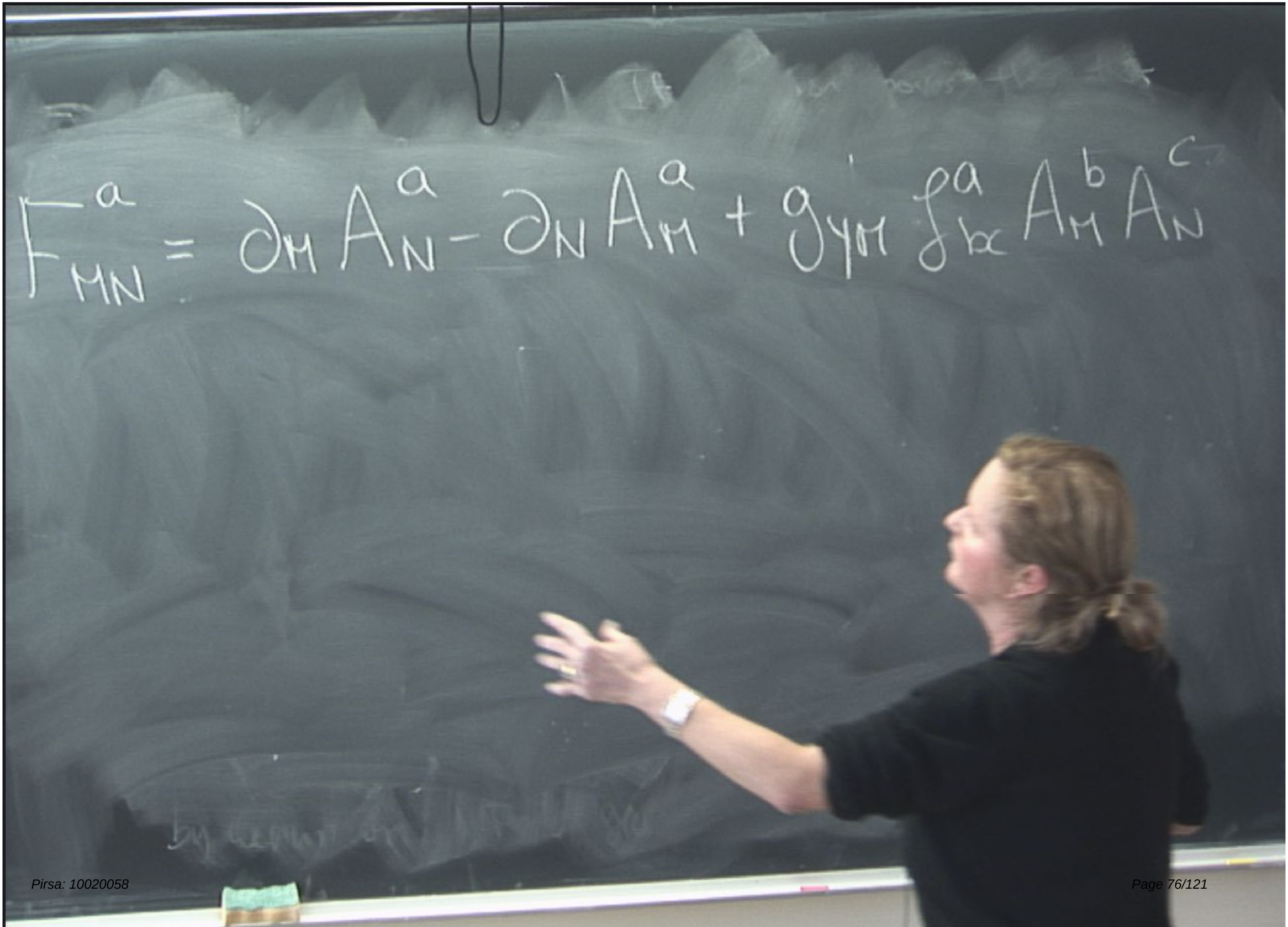
$$F = \sum_a T^a F^a$$

$$F^a = dA^a + g_{YM} f^a{}_{bc} A^b \wedge A^c$$

$$F = \sum_a T^a F^a$$

$$F^a = dA^a + g_{YM} f^a{}_{bc} A^b \wedge A^c$$

Structure constants.


$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + g_{YM} f^a_{bc} A^b_M A^c_N$

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$$\mathrm{tr} F^2 = \sum_a F^a_{MN} F^{MNa}$$

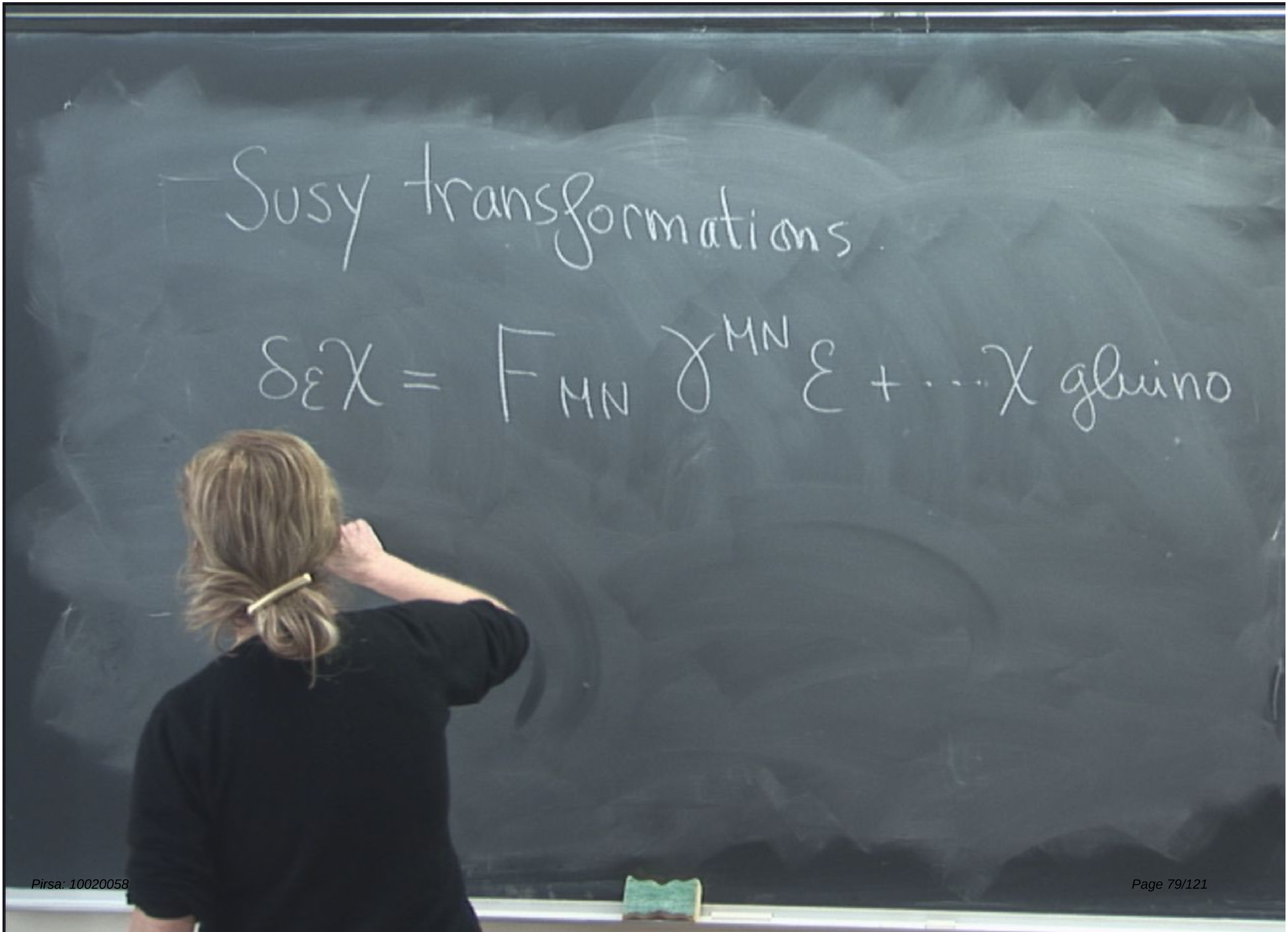
$$\tilde{H}_3 = dB_2 \; \frac{\kappa_{10}^2}{g_{YM}^2}\,\omega_3$$

Chern-Simons form . . .

-abelian
gauge field

$$\omega_3 = \mathrm{Tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)$$

$$\omega_3 = \sum_a A^a\wedge dA^a + \frac{2}{3}\, g_{YM}\, f_{abc}\, A^a\wedge A^b\wedge A^c$$


Susy transformations
$\delta_\varepsilon \chi = F_{MN}\, \gamma^{MN} \varepsilon + \cdots$ χ gluino

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$$\delta_\varepsilon \chi = F_{MN} \gamma^{MN} \varepsilon + \cdots \quad \chi \text{ gluino}$$

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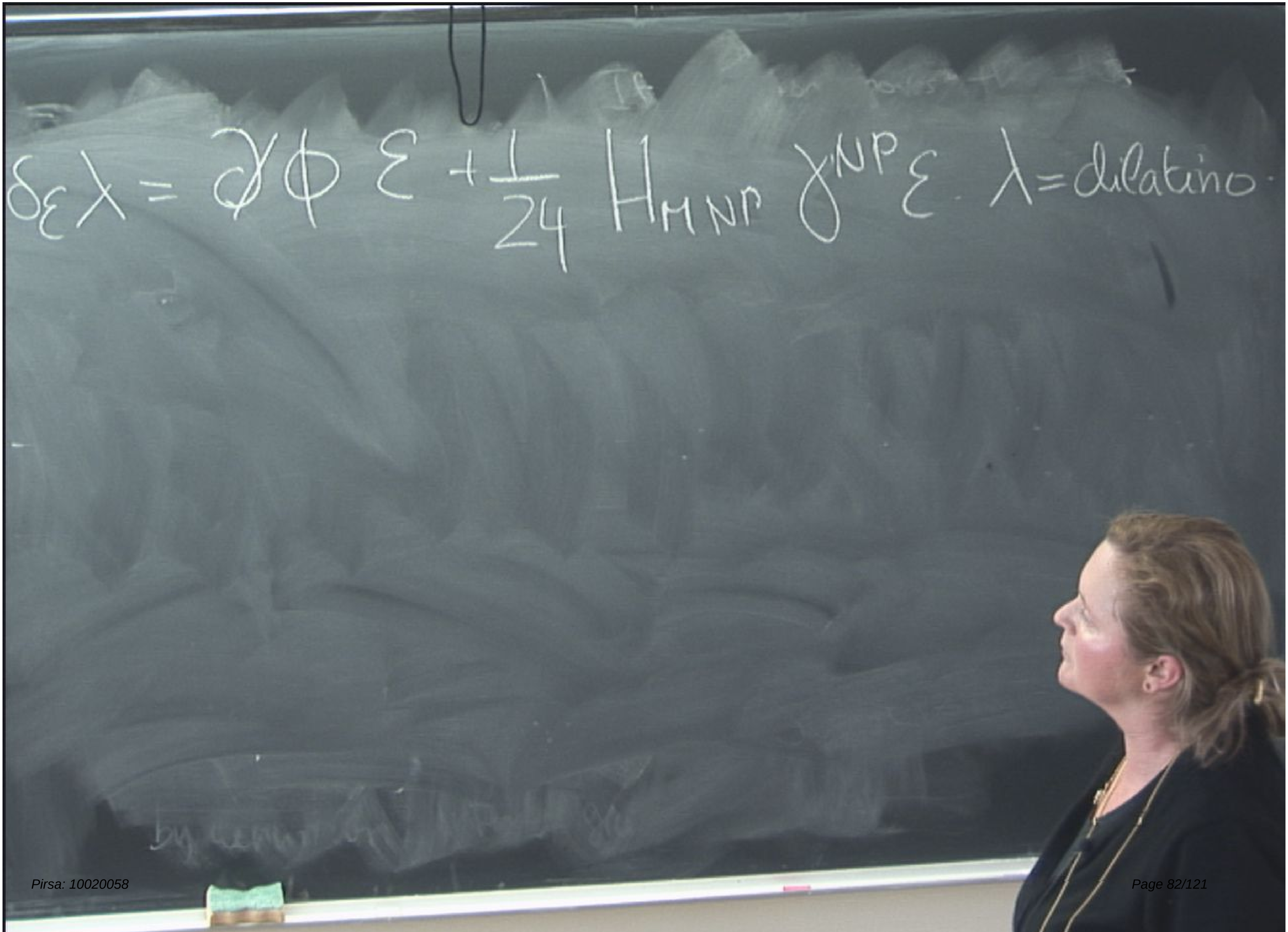
$\delta_\varepsilon \lambda = \partial\!\!\!/ \phi \, \varepsilon + \frac{1}{24} H_{MNP} \gamma^{NP} \varepsilon$. λ = dilatino

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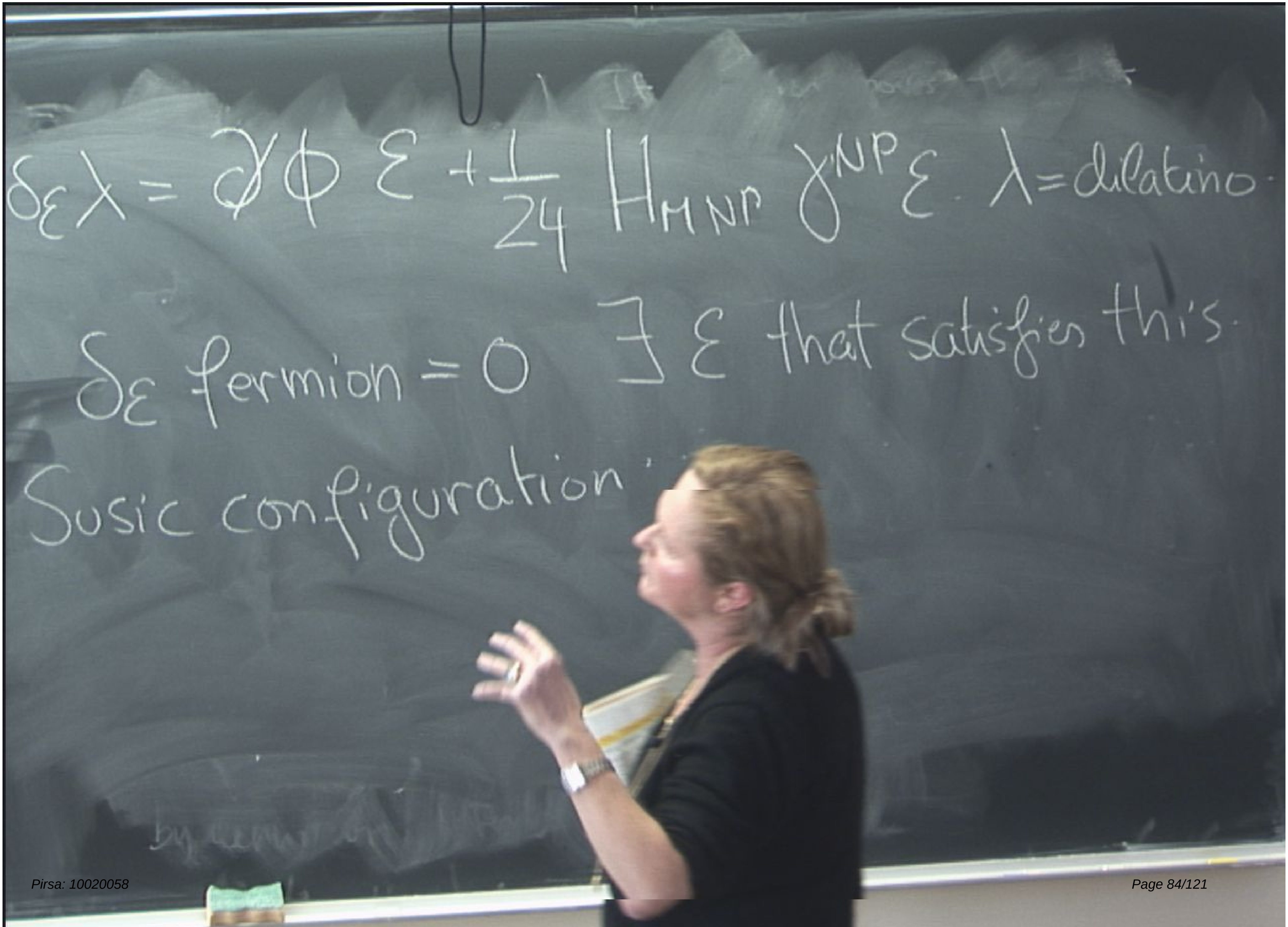
ε : 10D spinor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δ_ε fermion $= 0$ $\exists\, \varepsilon$ that satisfies this.
Susic configuration:

Susy transformations

$$\delta_\varepsilon \chi = F_{MN}\,\gamma^{MN}\,\varepsilon \doteq 0$$

gravitino

$$\delta_\varepsilon \psi_M = \nabla_M \varepsilon - \frac{1}{4} H_{MAB}\,\gamma^{AB}\,\varepsilon = 0$$

ε : 10D spinor

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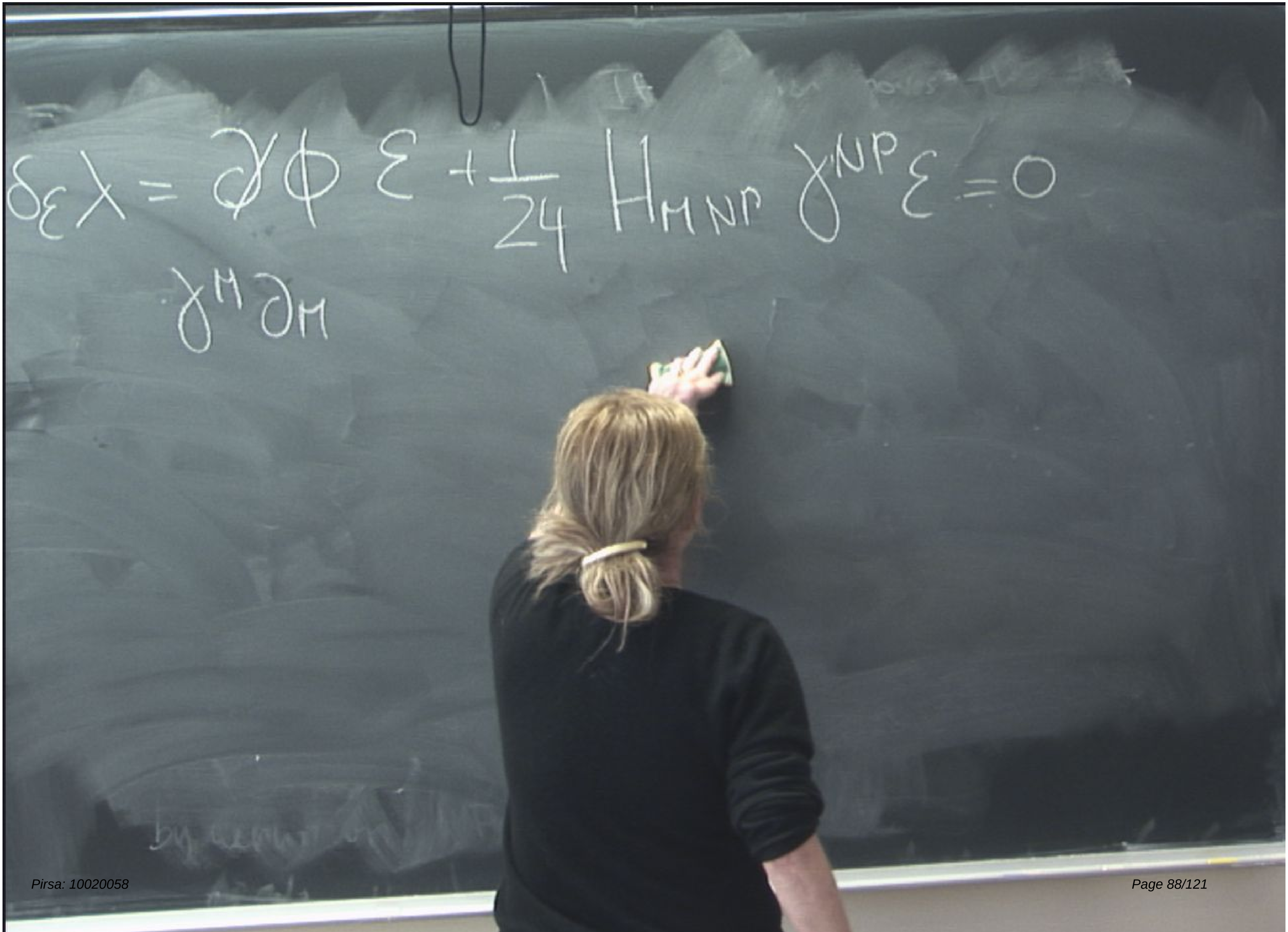
Susic configuration:

δ boson $=$ $\quad \chi, \psi_M, \lambda$

$$\delta_{\varepsilon}\lambda = \partial\!\!\!/\phi\,\varepsilon + \frac{1}{24} H_{MNP}\,\gamma^{MNP}\varepsilon = 0$$

Susic cor

δ boson =


$\delta_\varepsilon \lambda = \partial\!\!\!/ \phi \, \varepsilon + \frac{1}{24} H_{MNP} \gamma^{NP} \varepsilon = 0$
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Susy transformations (1985)

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Susy transformations (1985)

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ε : 10D spinor

$H = 0$

$\phi = \mathrm{Const}$

δ_ε

Simp

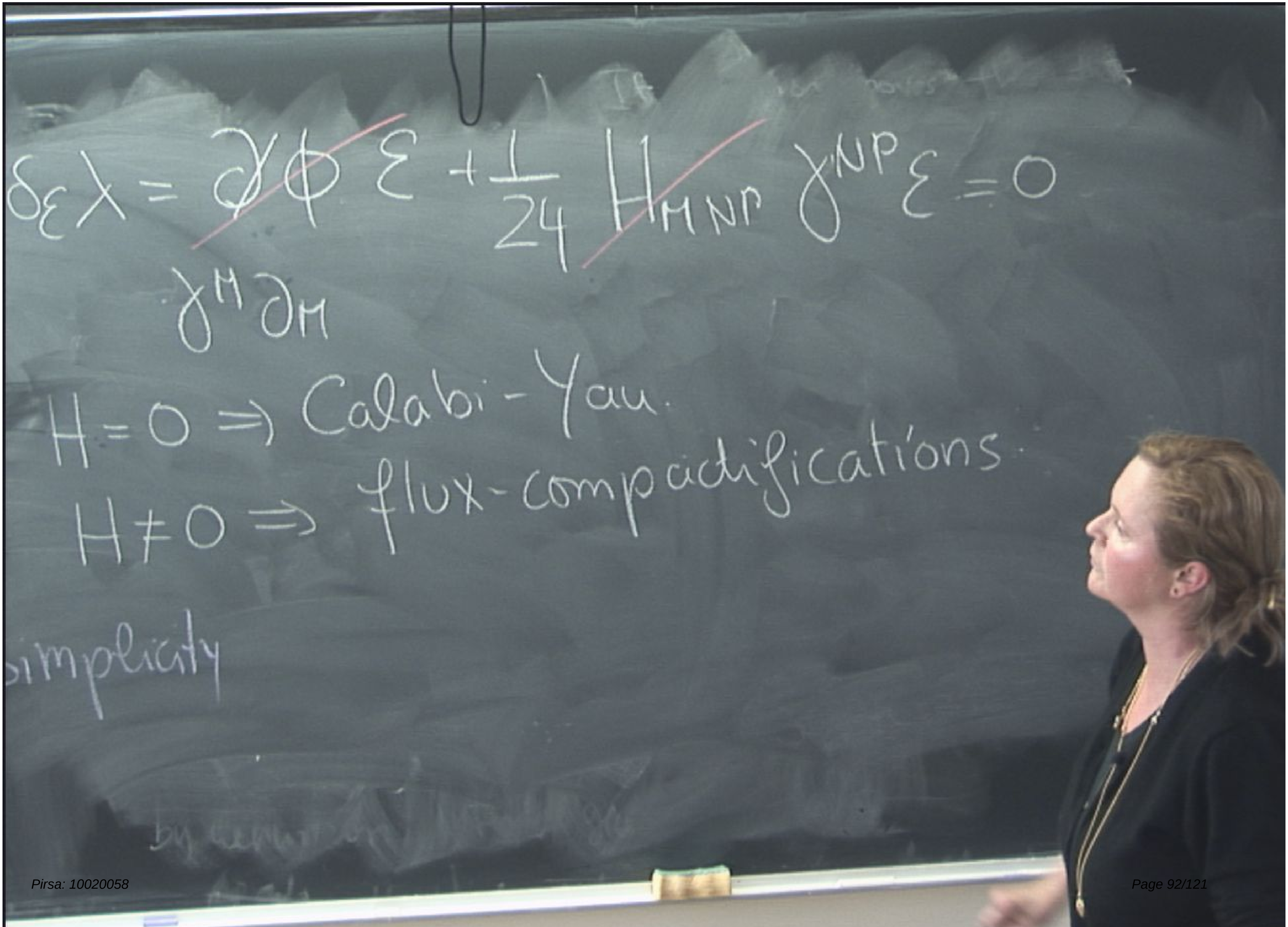
$$\delta_\varepsilon \lambda = \not{\partial} \phi \, \varepsilon + \frac{1}{24} H_{MNP} \gamma^{NP} \varepsilon = 0$$

$$\gamma^M \partial_M$$

$H = 0 \Rightarrow$ Calabi-Yau.

$H \neq 0 \Rightarrow$ flux-compactifications.

simplicity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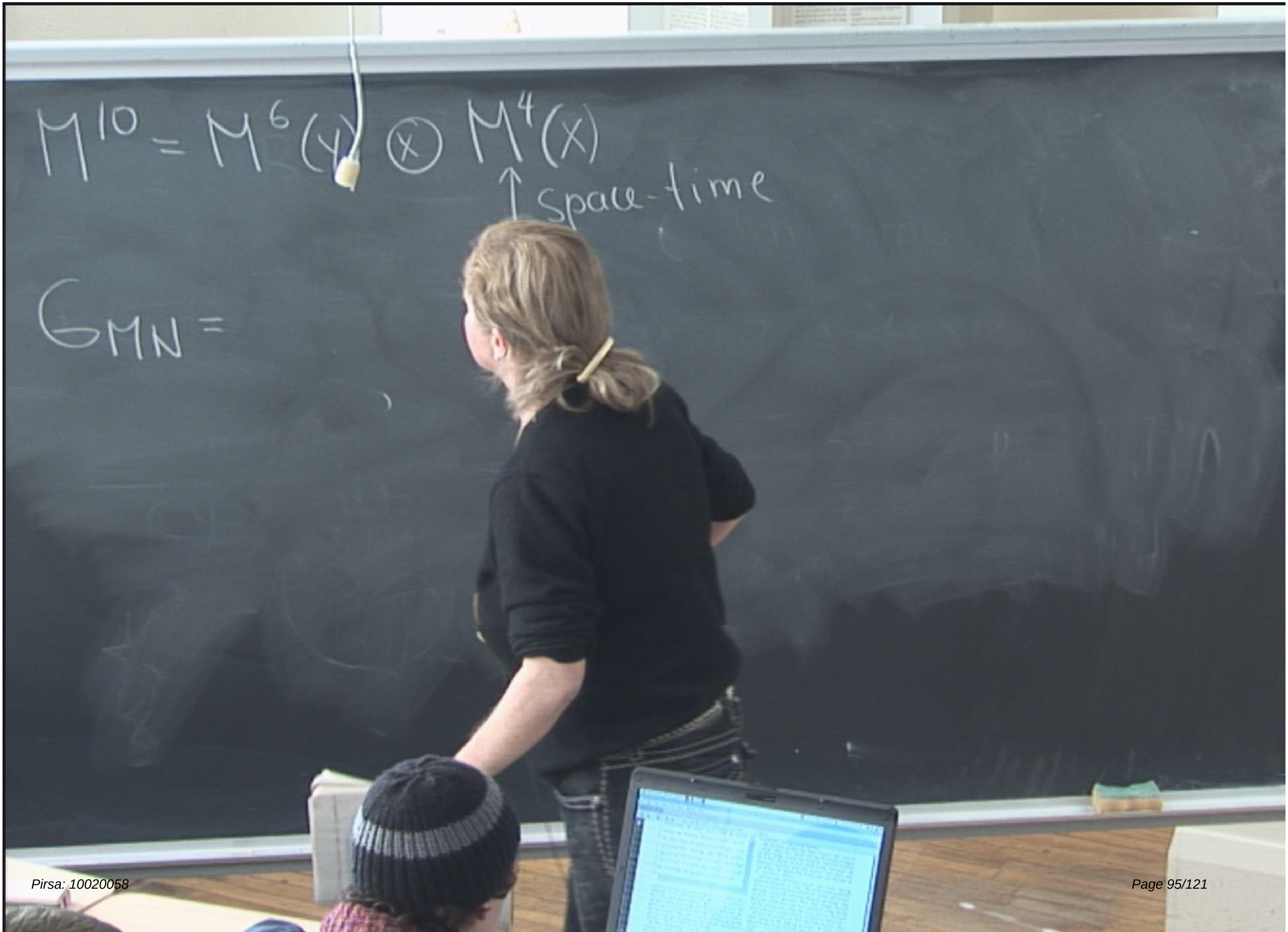
(1) $F_{MN}\gamma^{MN}\varepsilon = 0$

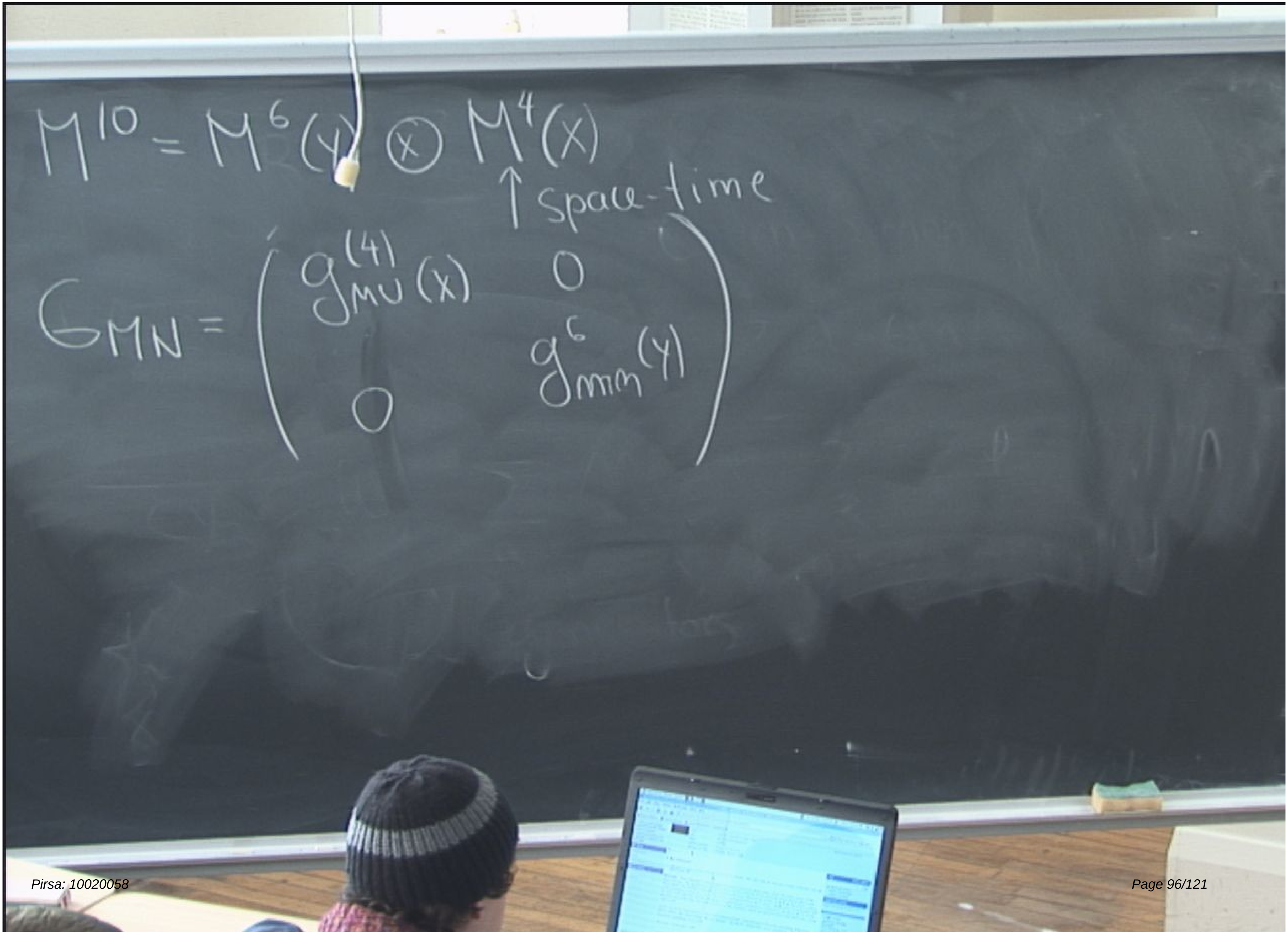
(2) $\nabla_M \varepsilon = 0$

1) $F_{MN}\gamma^{MN}\varepsilon = 0$

2) $\nabla_M \varepsilon^{(10)} = 0$

$\uparrow$ M = 10D indices


$M^{10} = M^6(y) \otimes M^4(x)$
space-time
$G_{MN} =$


$M^{10} = M^6(y) \otimes M^4(x)$
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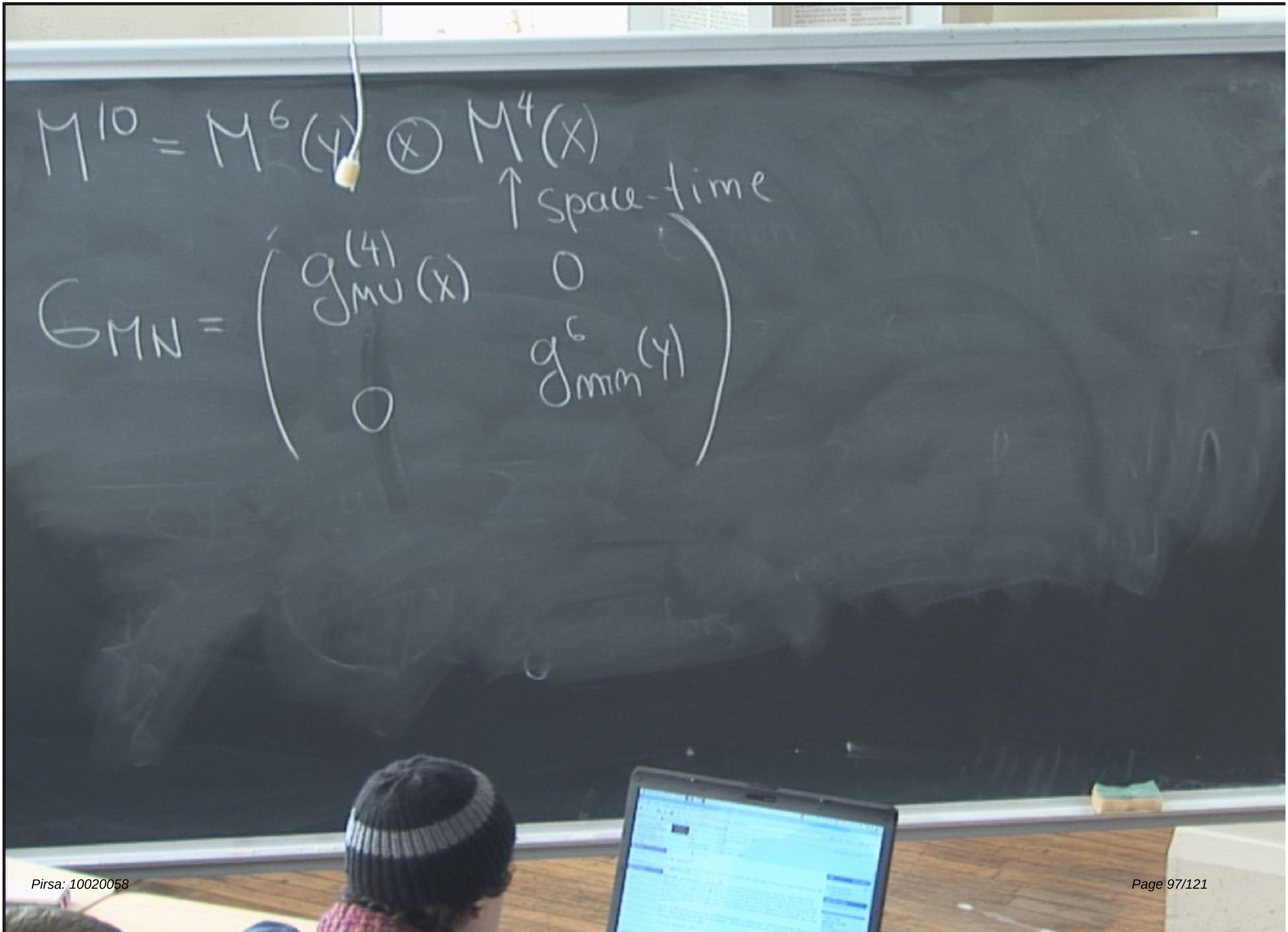
$G_{MN} = \begin{pmatrix} g^{(4)}_{\mu\nu}(x) & 0 \\ 0 & g^6_{mn}(y) \end{pmatrix}$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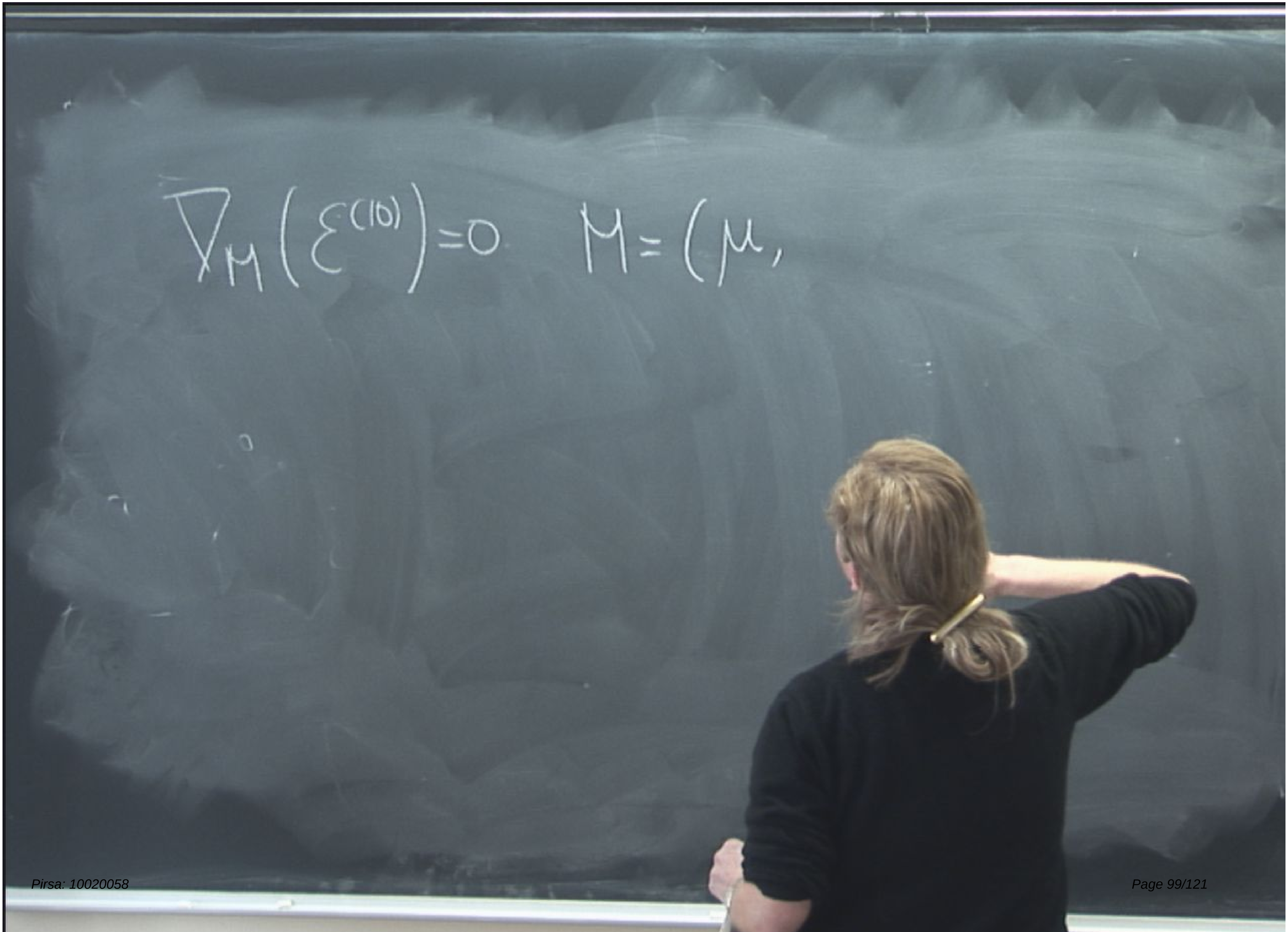
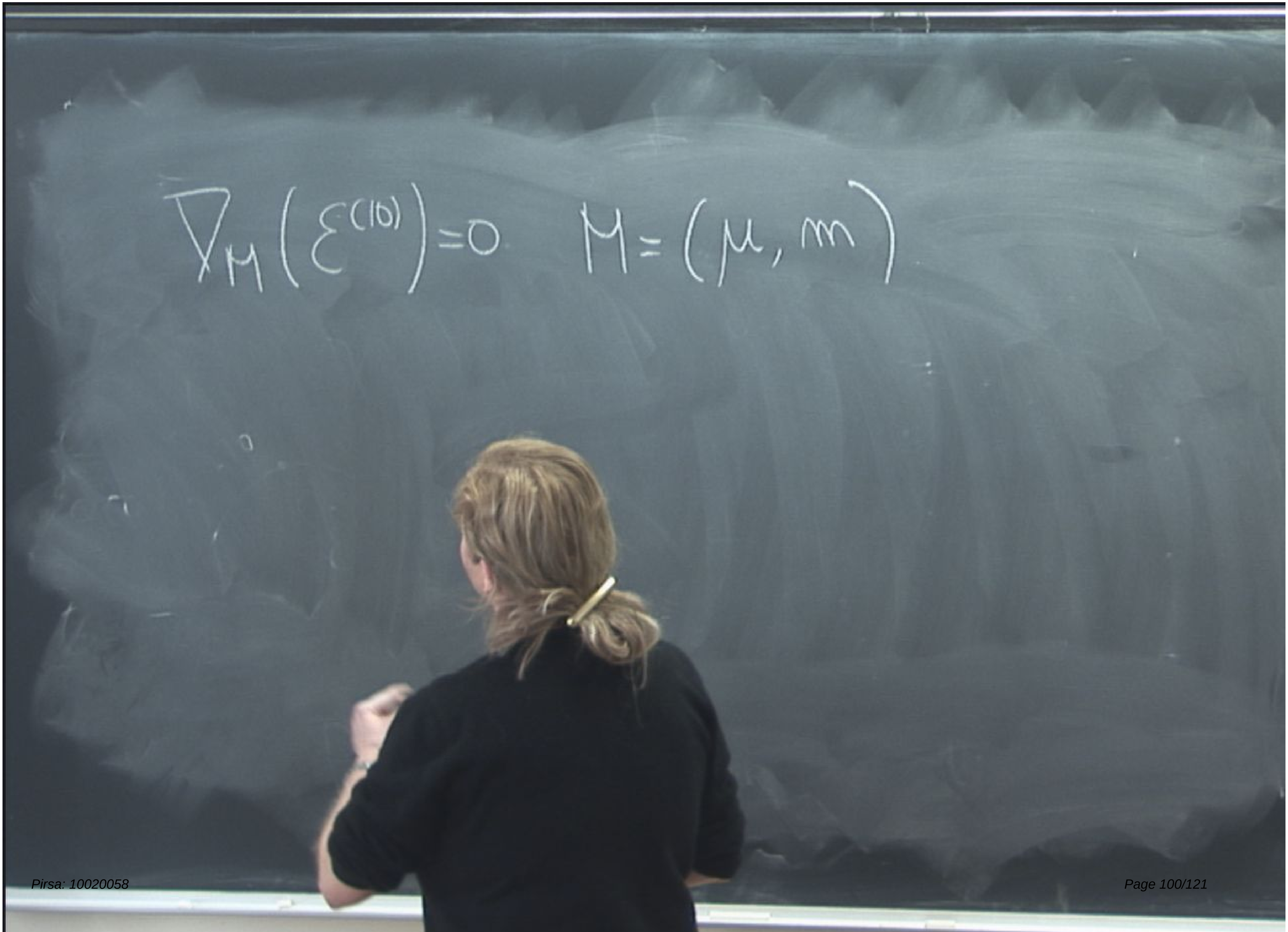
$$ {}^{10} = M^6(y) \otimes \underset{\uparrow \text{ space-time}}{M^4(x)} $$

$$ \varepsilon^{(10)} = \underbrace{\varepsilon^{(4)}}_{\text{4D spinor}} \otimes \underbrace{\varepsilon^{(6)}}_{\text{6D spinors}} + cc $$

$$ {}_{MN} = \begin{pmatrix} g^{(4)}_{\mu\nu}(x) & 0 \\ 0 & g^{6}_{mn}(y) \end{pmatrix} $$


$\nabla_M(\epsilon^{(10)})=0 \quad M=(\mu,$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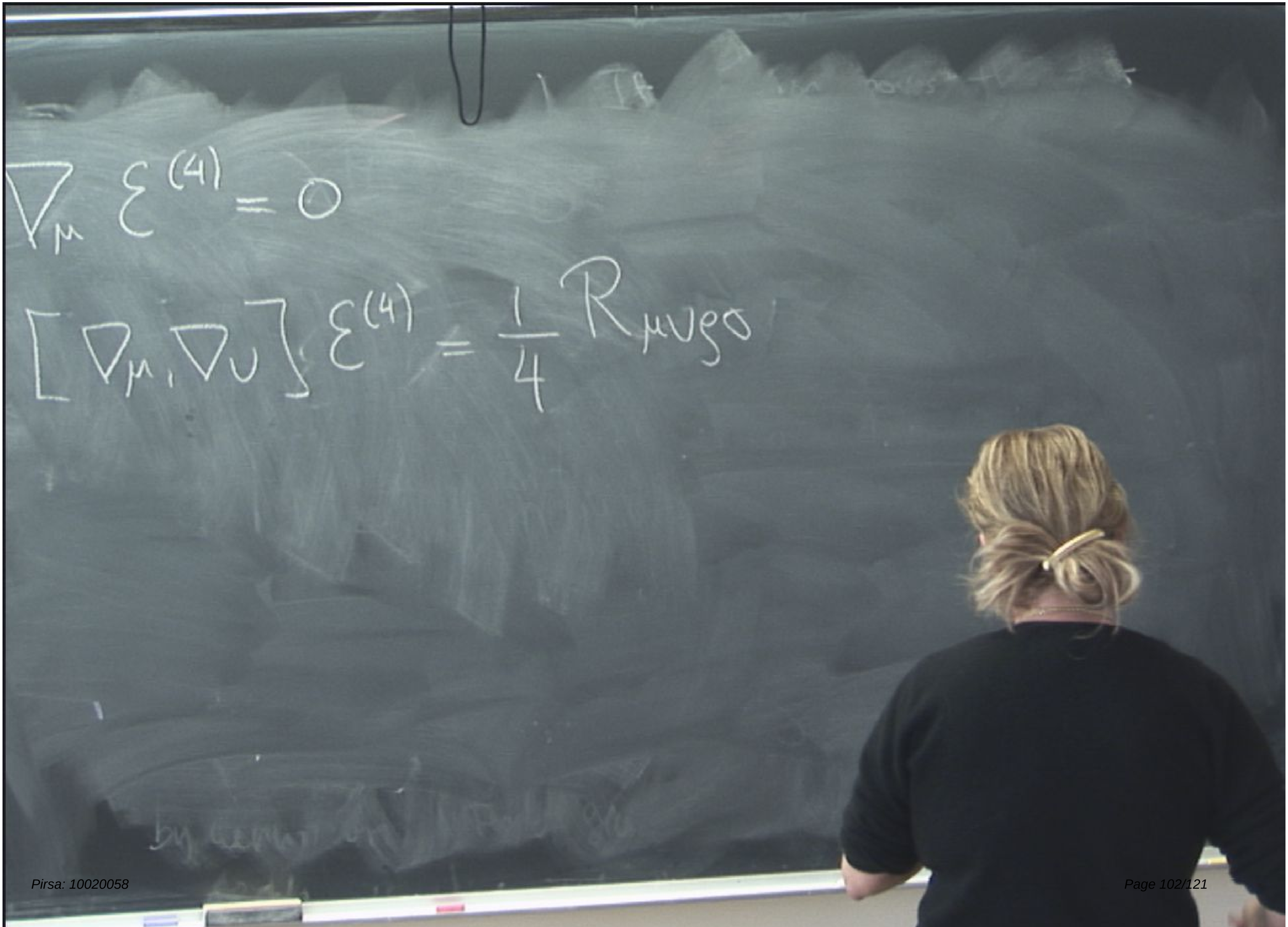
$$② \quad \nabla_m \mathcal{E}^{(6)} = 0$$


$\nabla_\mu \mathcal{E}^{(4)} = 0$
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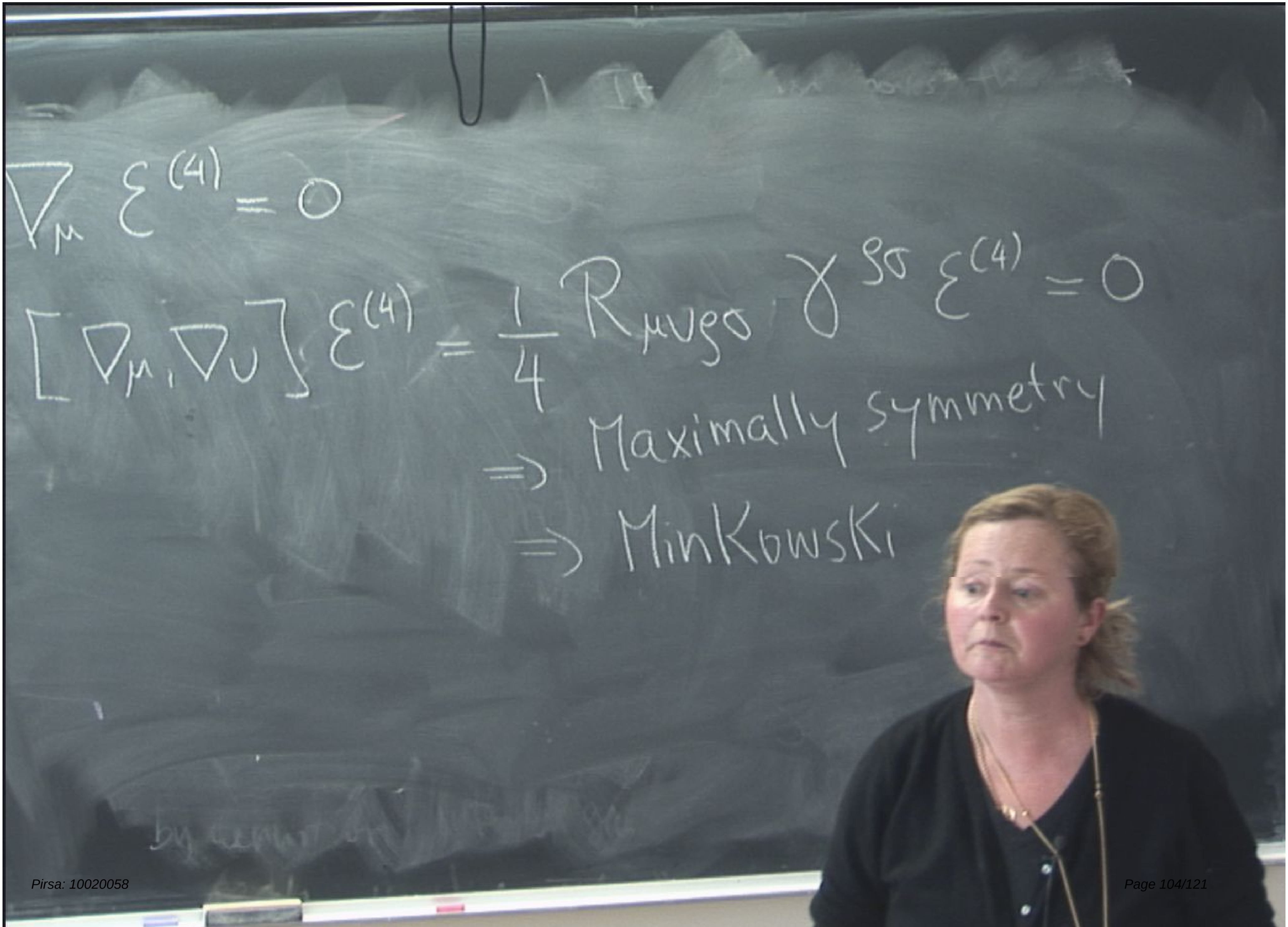
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$\Rightarrow$ Maximally symmetry
$\Rightarrow$ Minkowski

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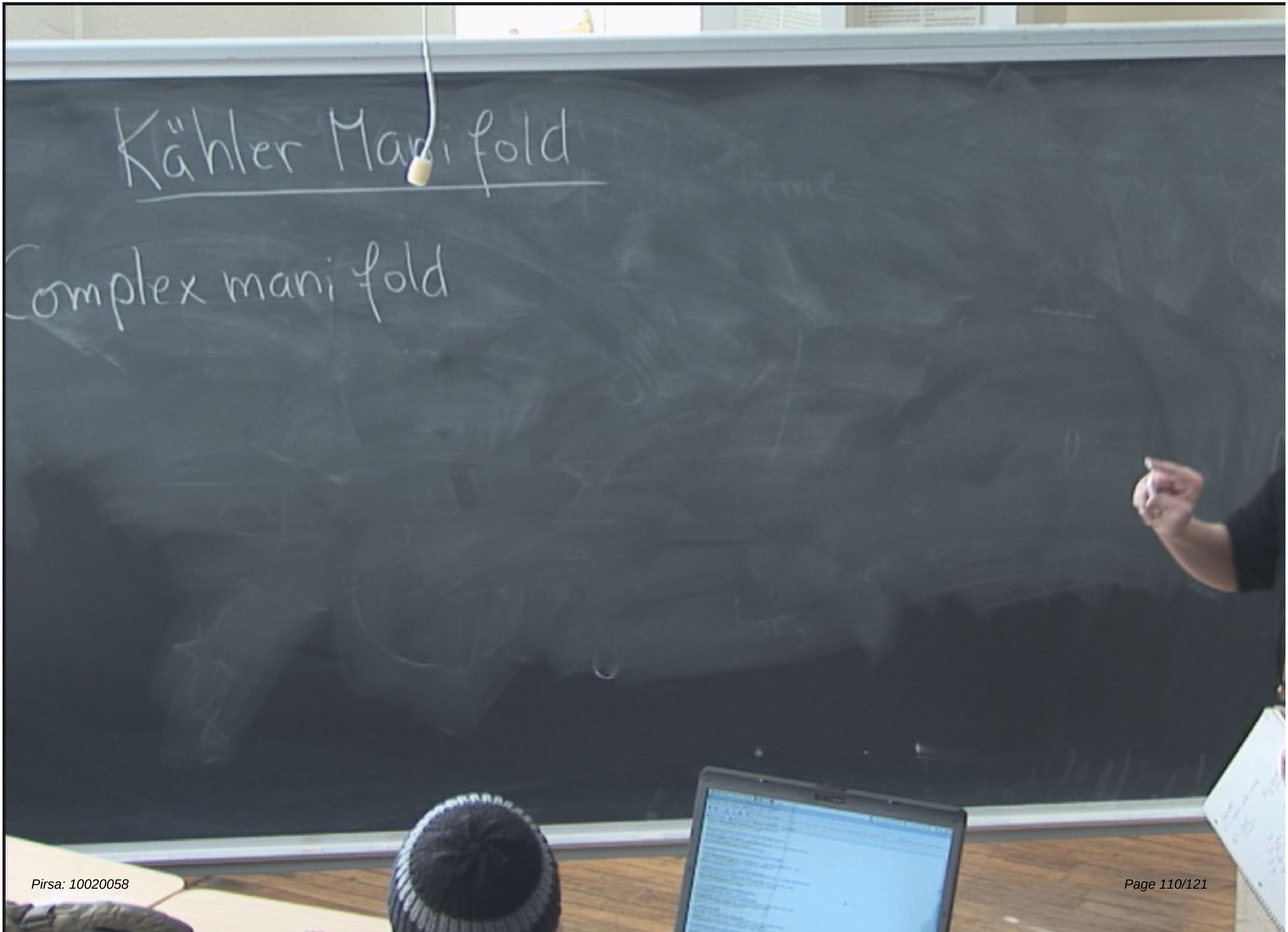
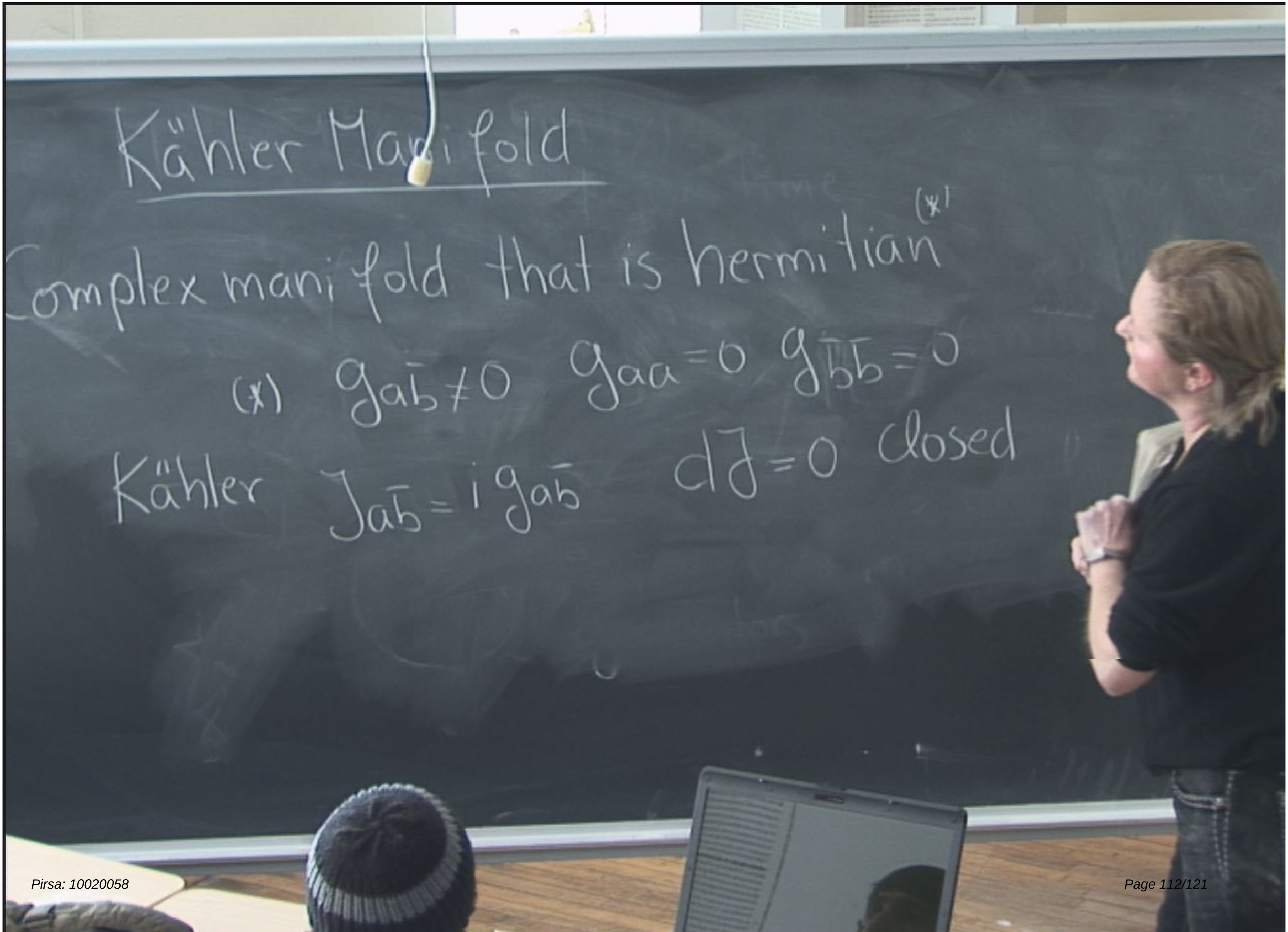
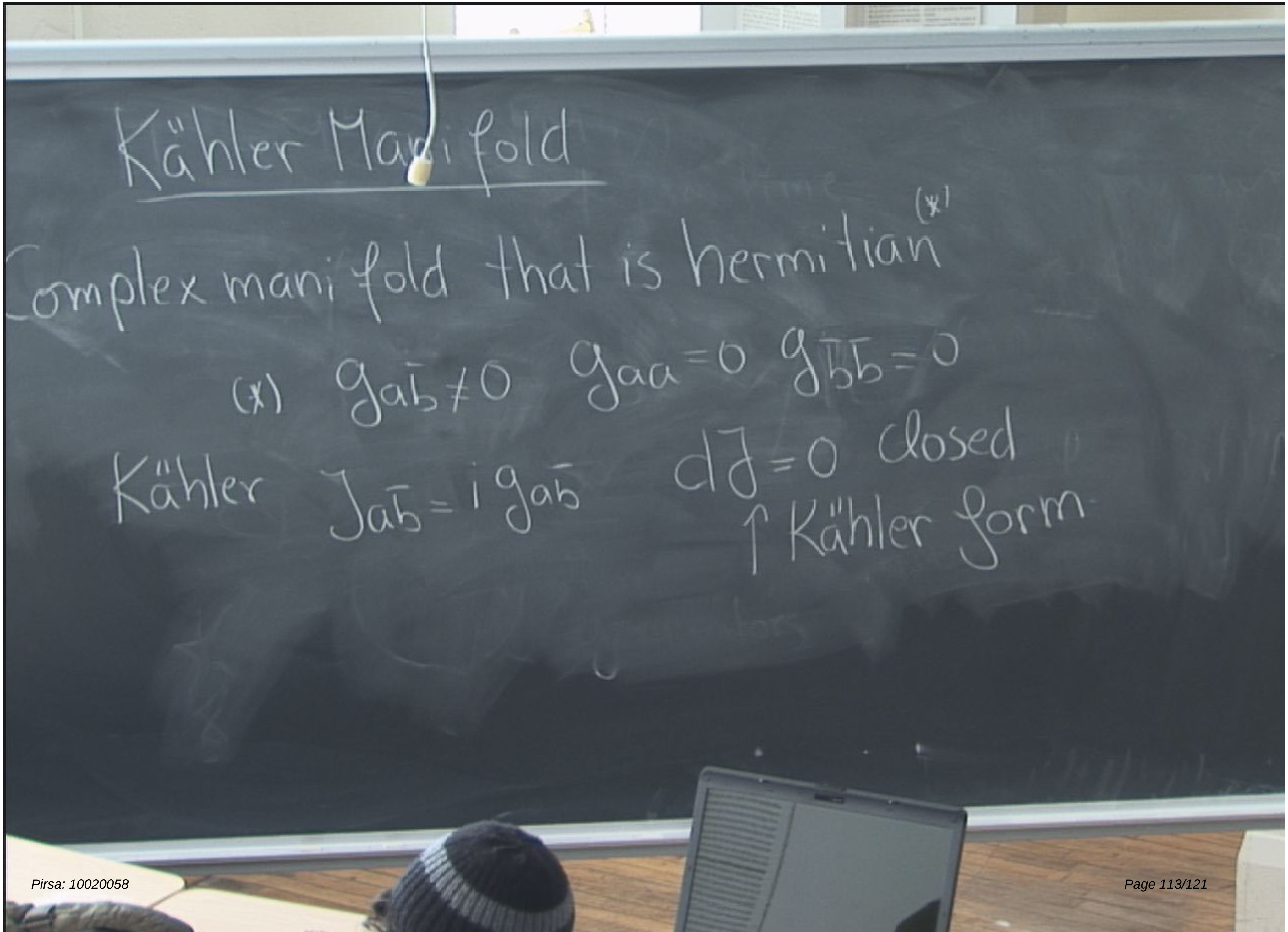

Kähler Manifold
Complex manifold

Kähler Manifold

Complex manifold that is hermitian (*)

(*) $g_{a\bar{b}}=0$ $g_{aa}=0$ $g_{\bar{a}\bar{a}}=0$


Kähler Manifold
Complex manifold that is hermitian (*)
(*) $g_{a\bar{b}} \neq 0$ $g_{aa} = 0$ $g_{\bar{b}\bar{b}} = 0$
Kähler $J_{a\bar{b}} = i g_{a\bar{b}}$ $dJ = 0$ closed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↑ Kähler form

S^2 is Kähler

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$$J = J_{a\bar{b}}\, dz^a \wedge d\bar{z}^{\bar{b}}$$

$$J_{a\bar{b}} = i\, g_{a\bar{b}}$$

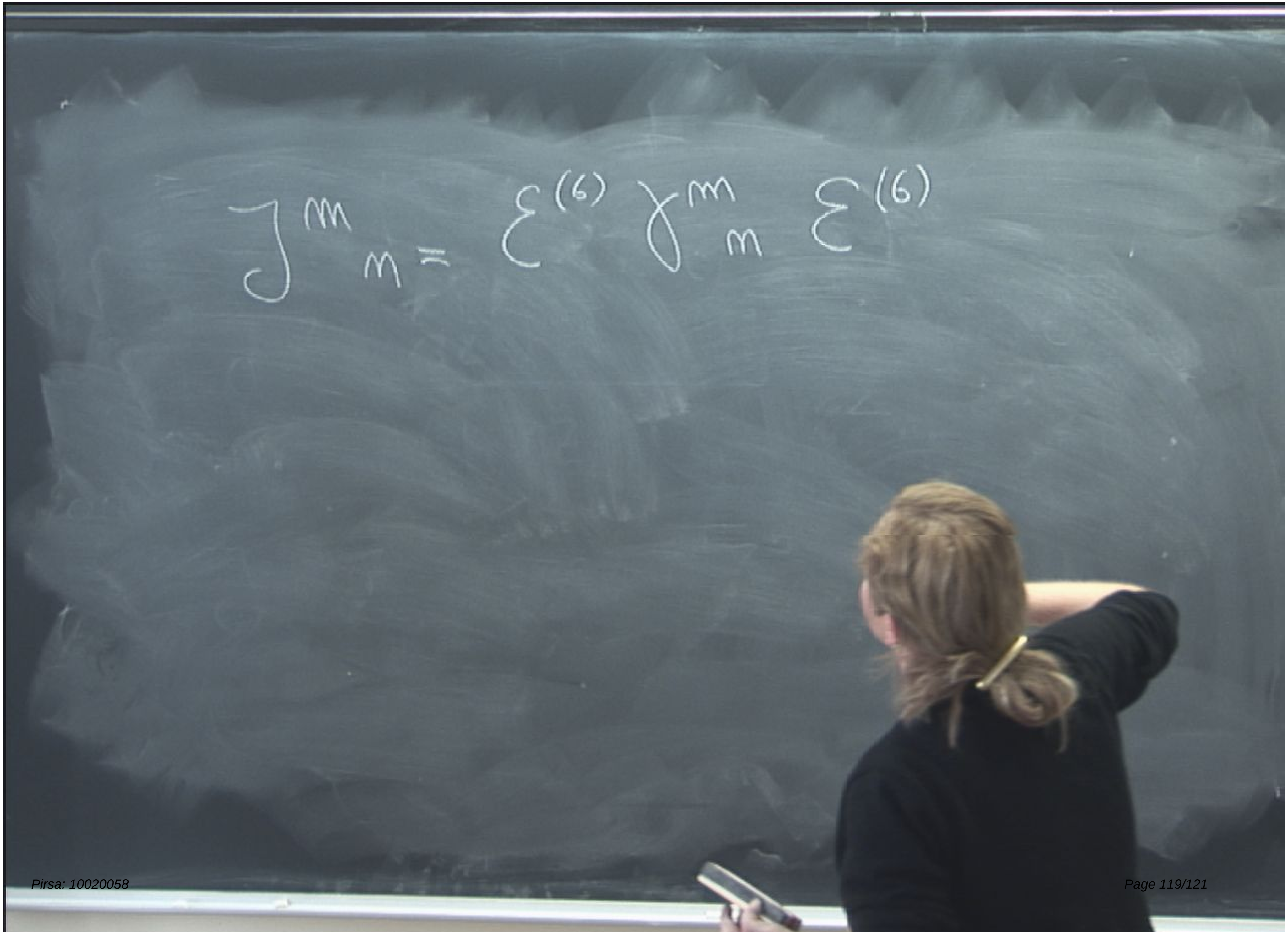
$$\nabla_\mu \varepsilon^{(4)} = 0$$

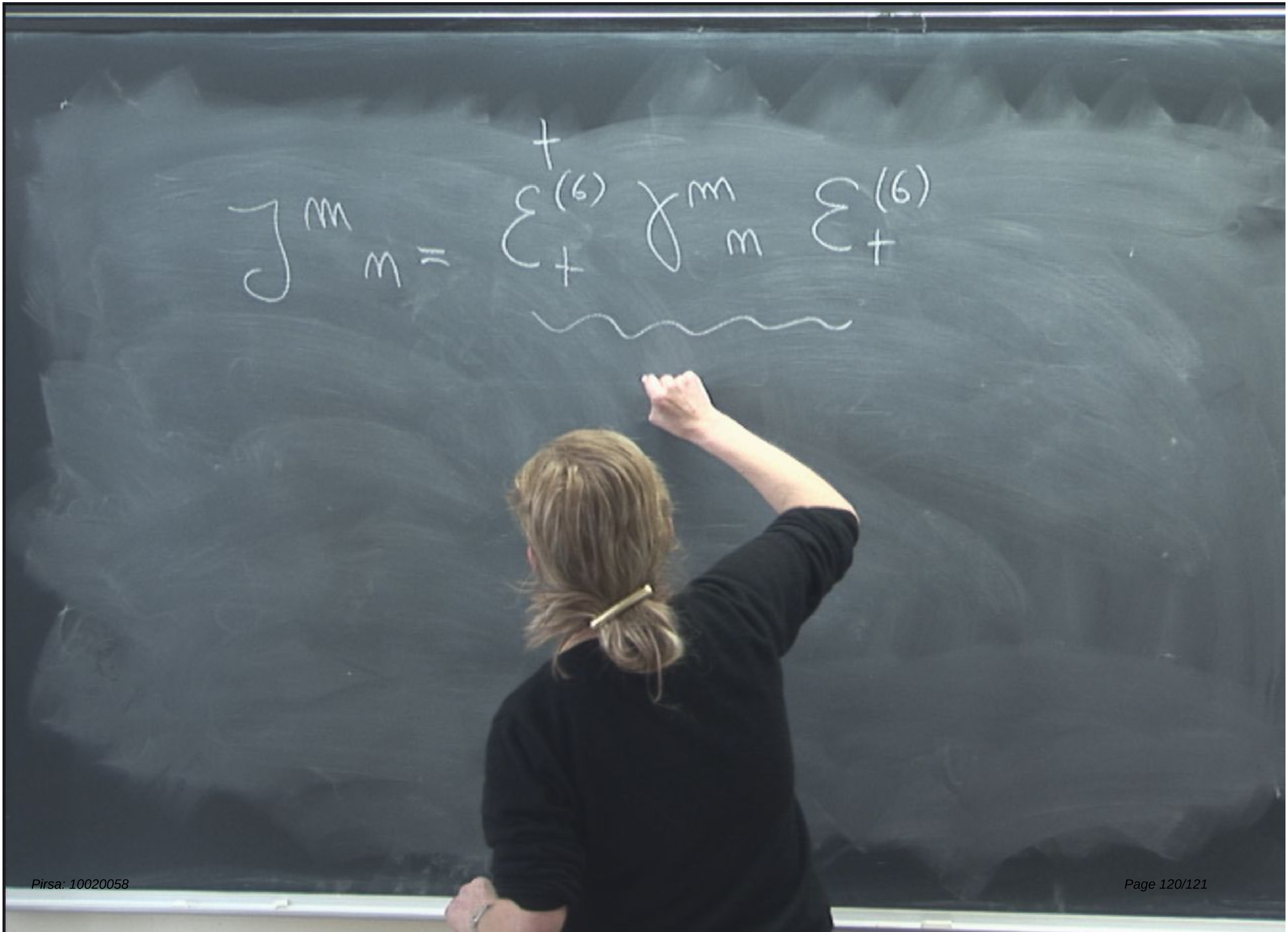
$$[\nabla_\mu, \nabla_\nu]\, \varepsilon^{(4)} = \frac{1}{4} R_{\mu\nu\rho\sigma}\, \gamma^{\rho\sigma}\, \varepsilon^{(4)} = 0$$

$$[\nabla_m, \nabla_n]\, \varepsilon^{(6)} = \frac{1}{4} R_{mnpq}\, \gamma^{pq}\, \varepsilon^{(6)} = 0$$

$\nabla_m \varepsilon^{(6)} = 0$ Killing spinor eqn

Covariantly constant spinor


$J^{m}{}_{m} = \varepsilon^{(6)} \gamma^{m}{}_{m} \varepsilon^{(6)}$


$J^{m}{}_{m} = \underline{\mathcal{E}_{+}^{(6)\dagger} \gamma^{m}{}_{m} \mathcal{E}_{+}^{(6)}}$

$$J^{m}{}_{n} = \underbrace{\varepsilon_{+}^{(6)\dagger}\, \gamma^{m}{}_{n}\, \varepsilon_{+}^{(6)}}_{\text{Kähler form}}$$