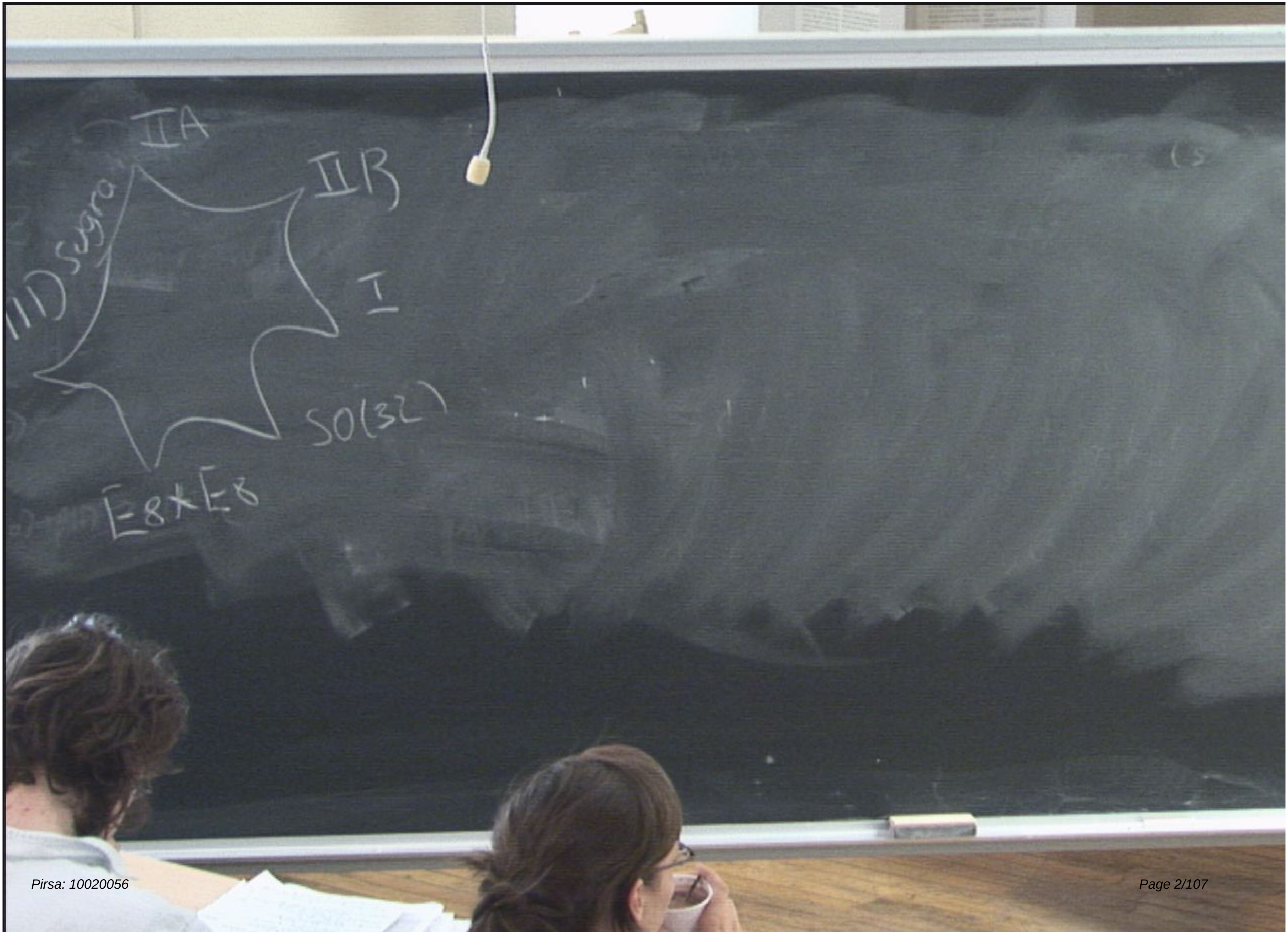
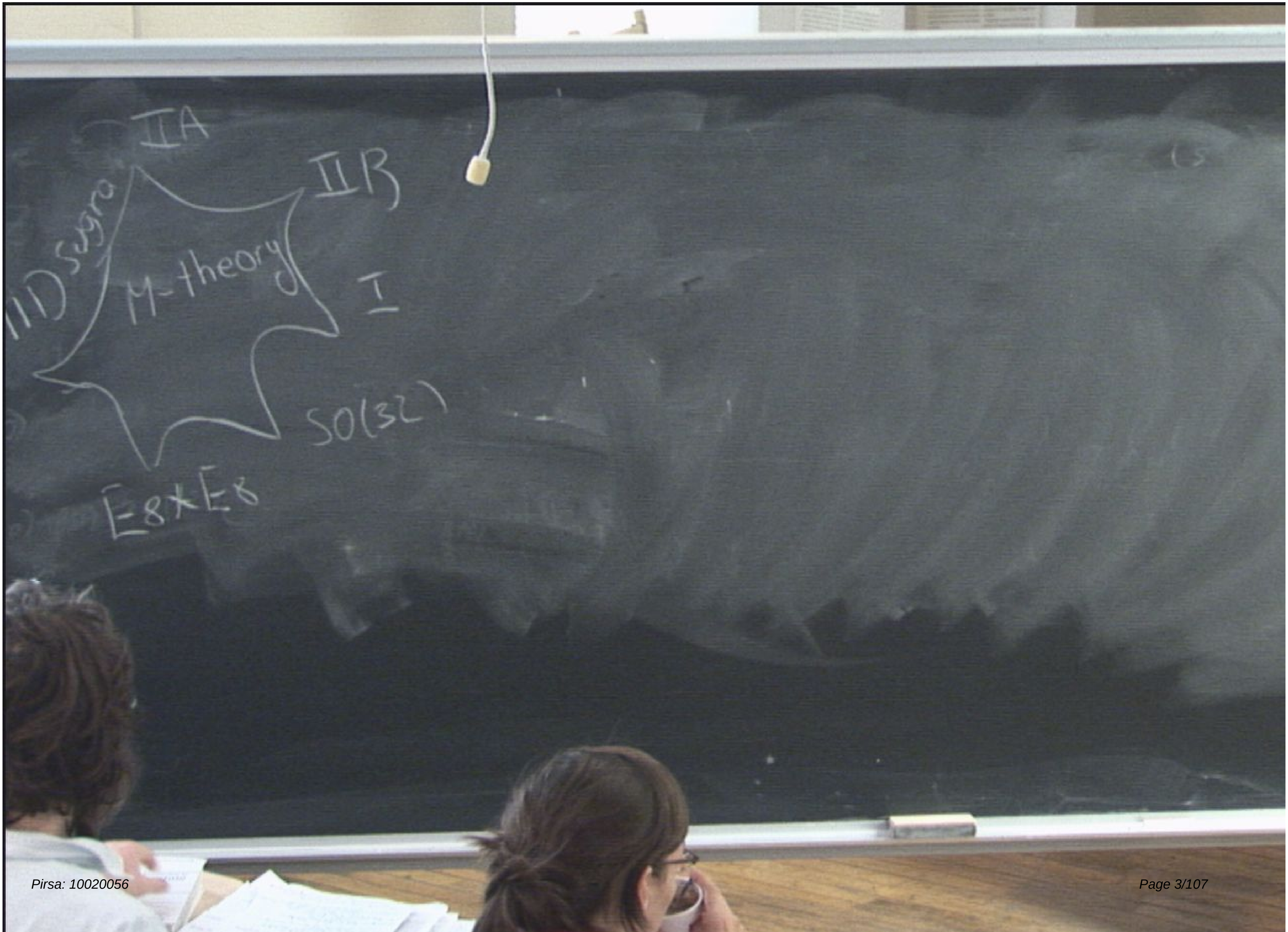
Title: String Theory - Review (PHYS 623) - Lecture 12

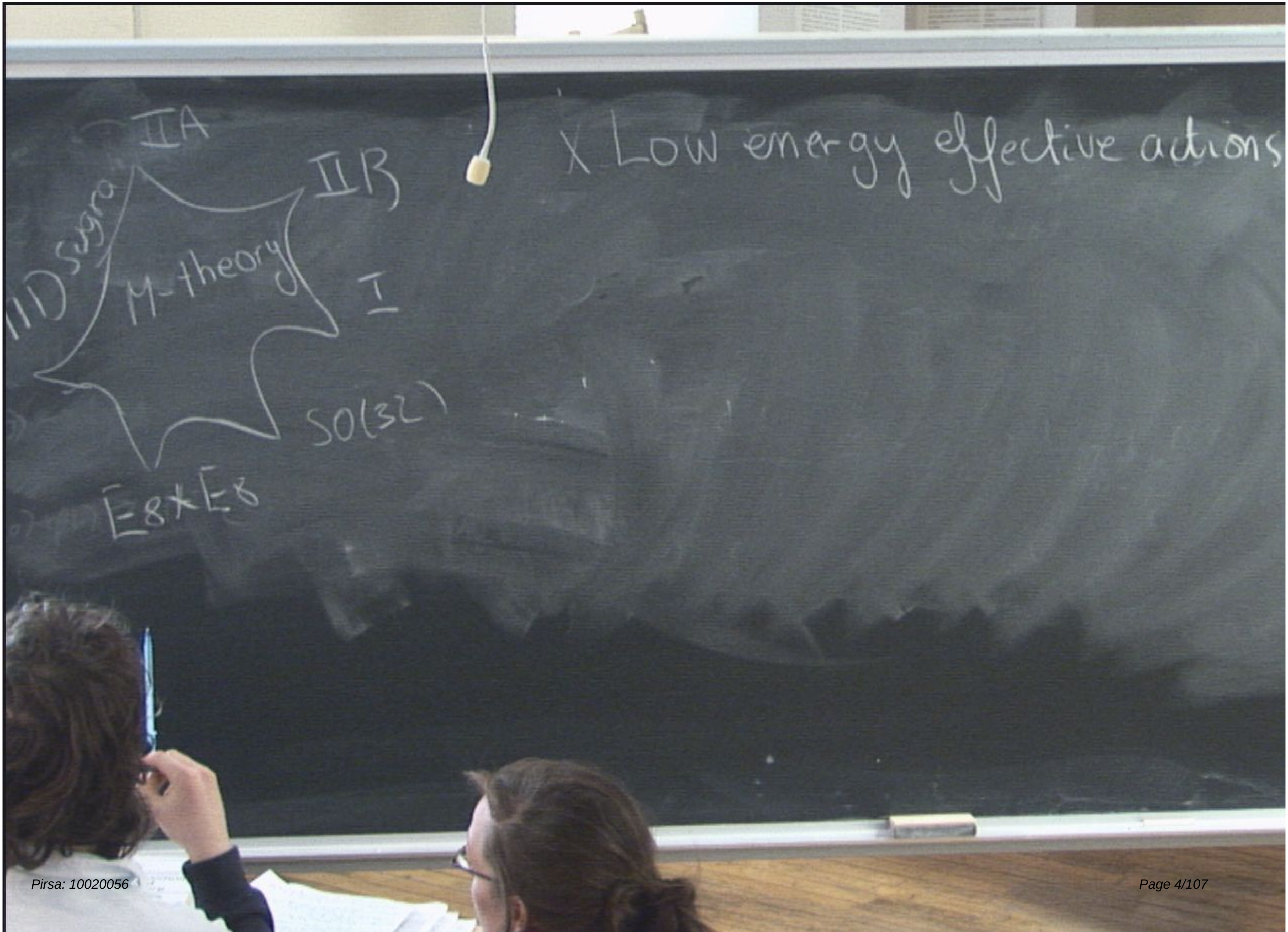
Date: Feb 09, 2010 11:20 AM

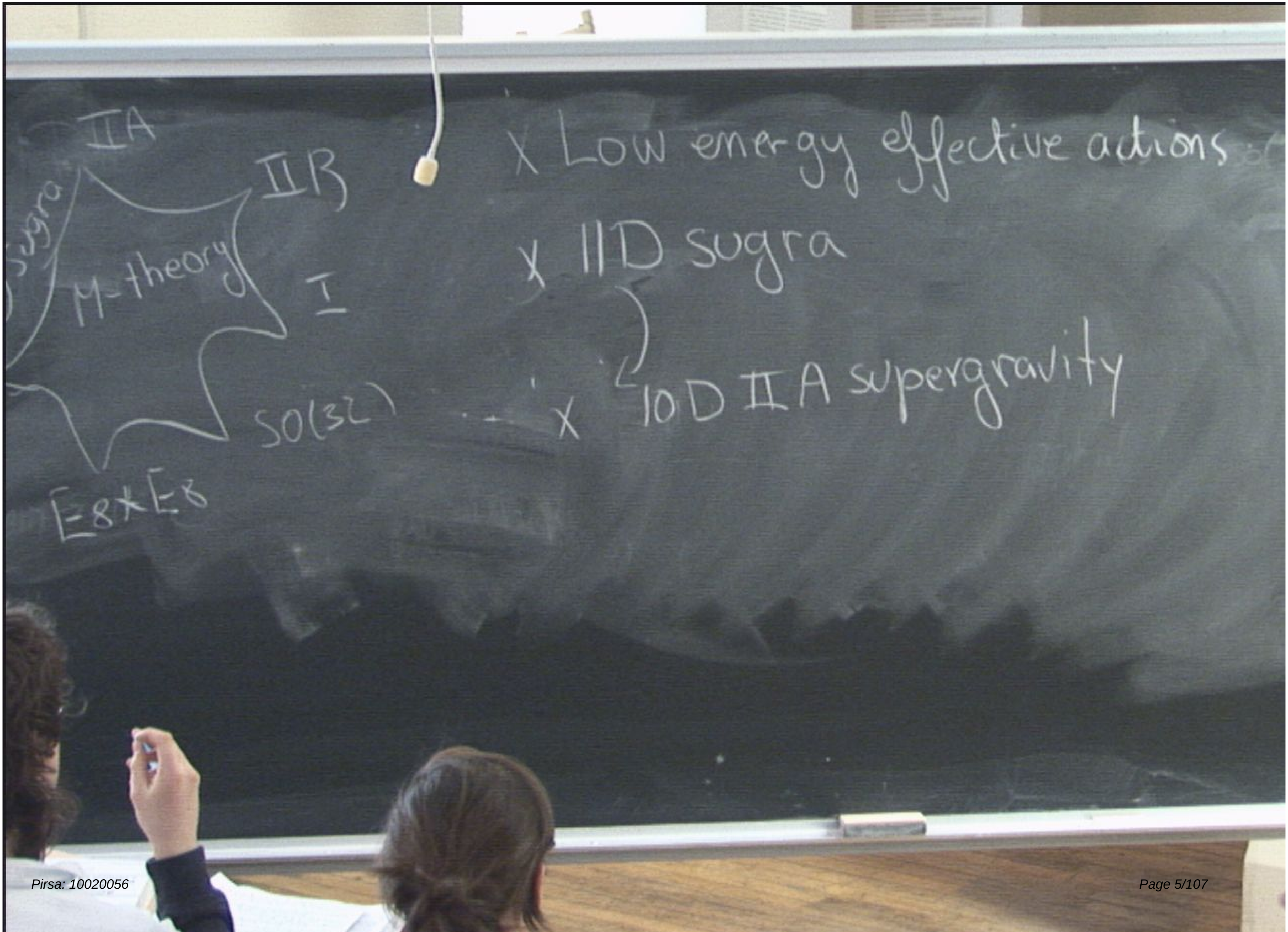
URL: http://pirsa.org/10020056

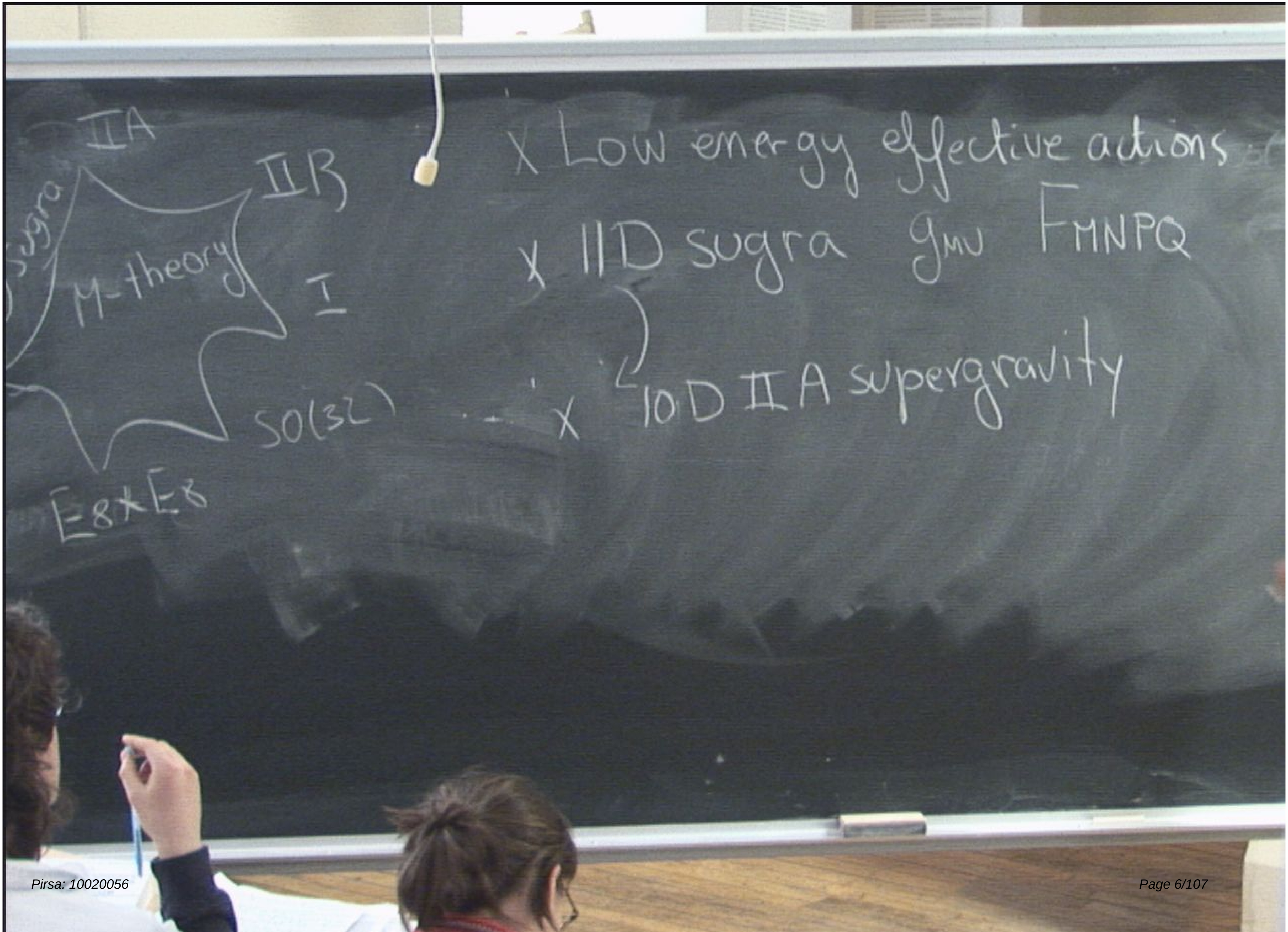
Abstract:

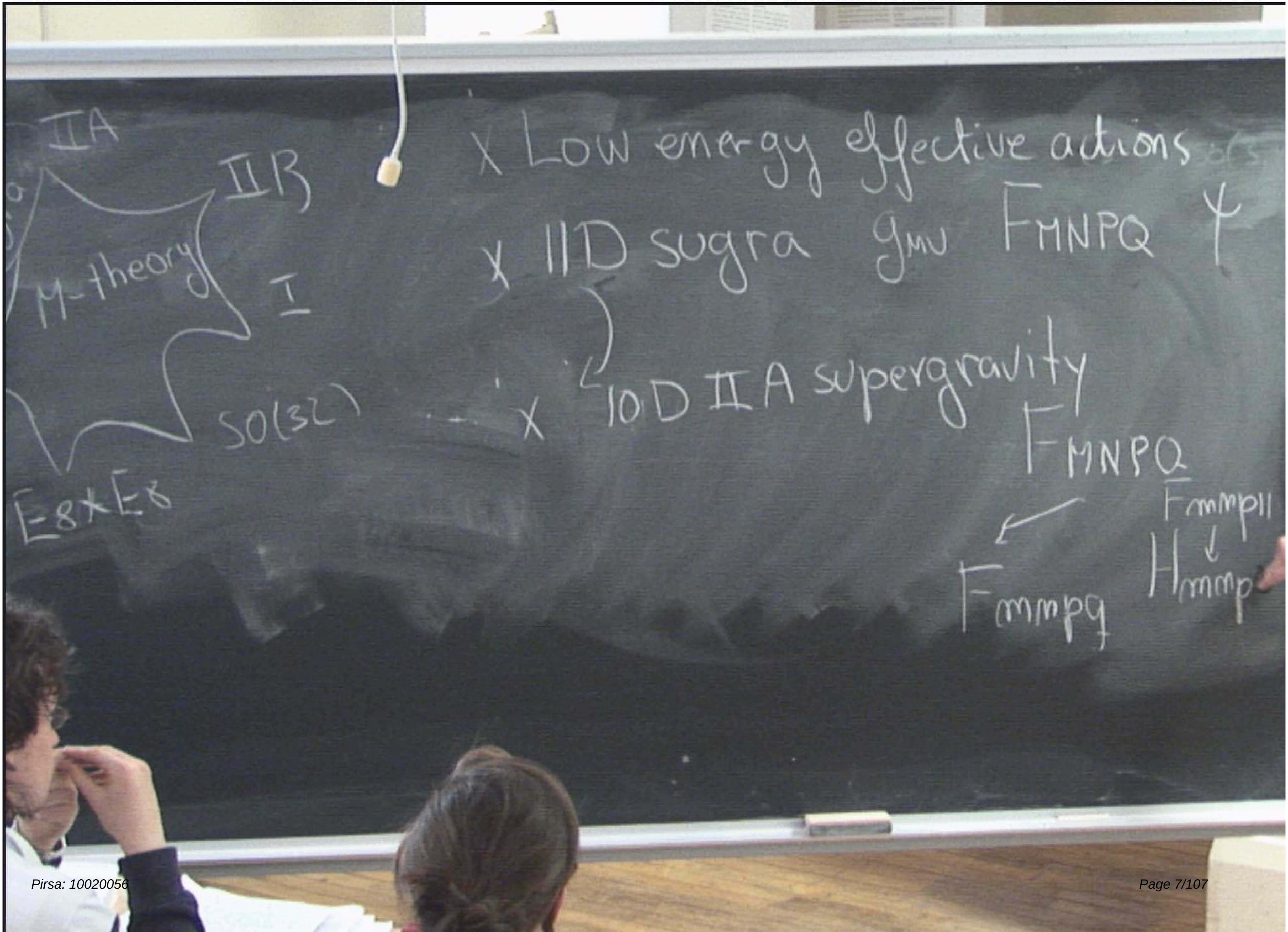

IIA
IIB
11D sugra
I
SO(32)
E8×E8

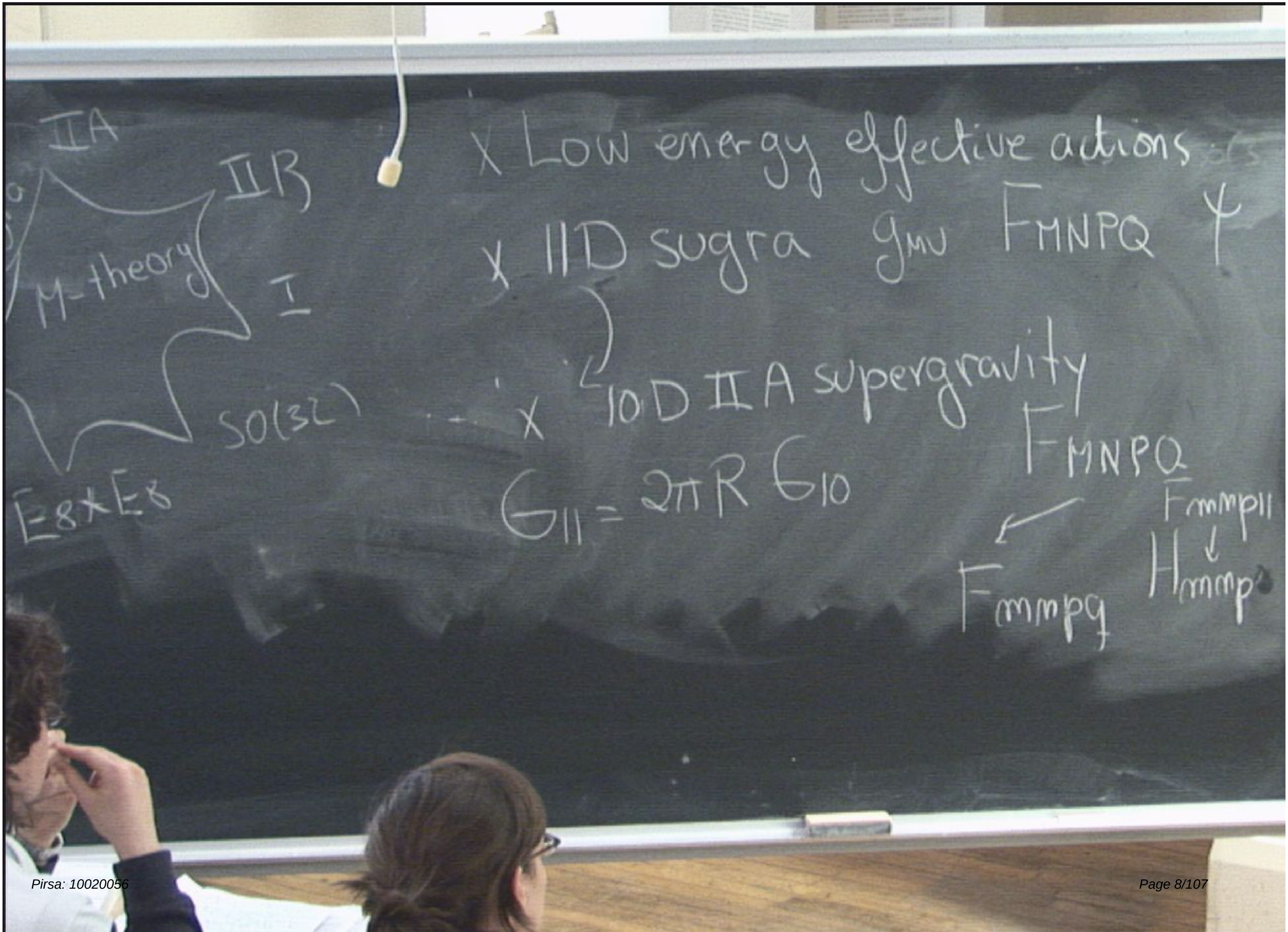

IIA
IIB
11D sugra
M-theory
I
SO(32)
E8×E8

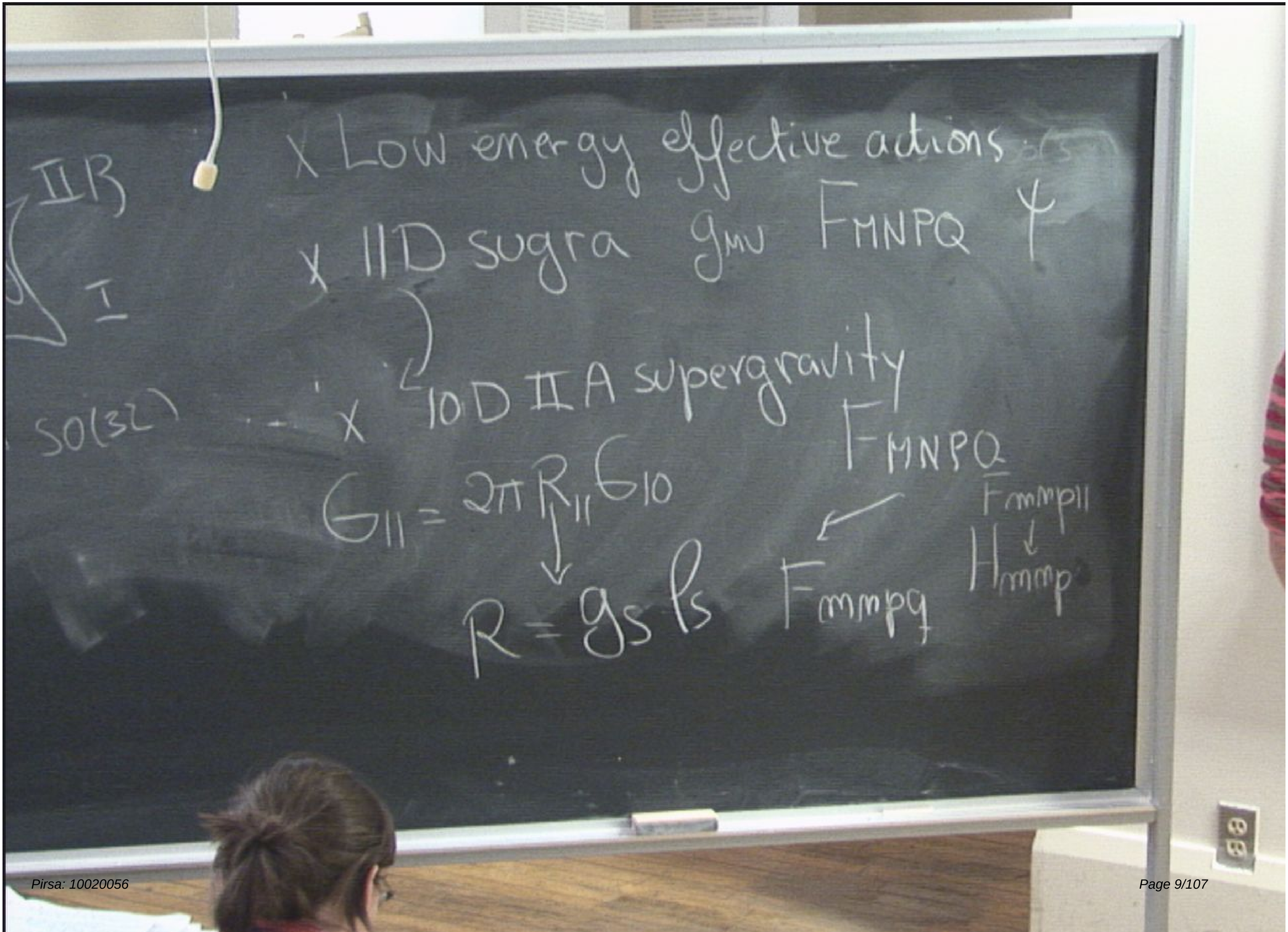

IIA
IIB
11D sugra
M-theory
I
SO(32)
E8×E8
× Low energy effective actions

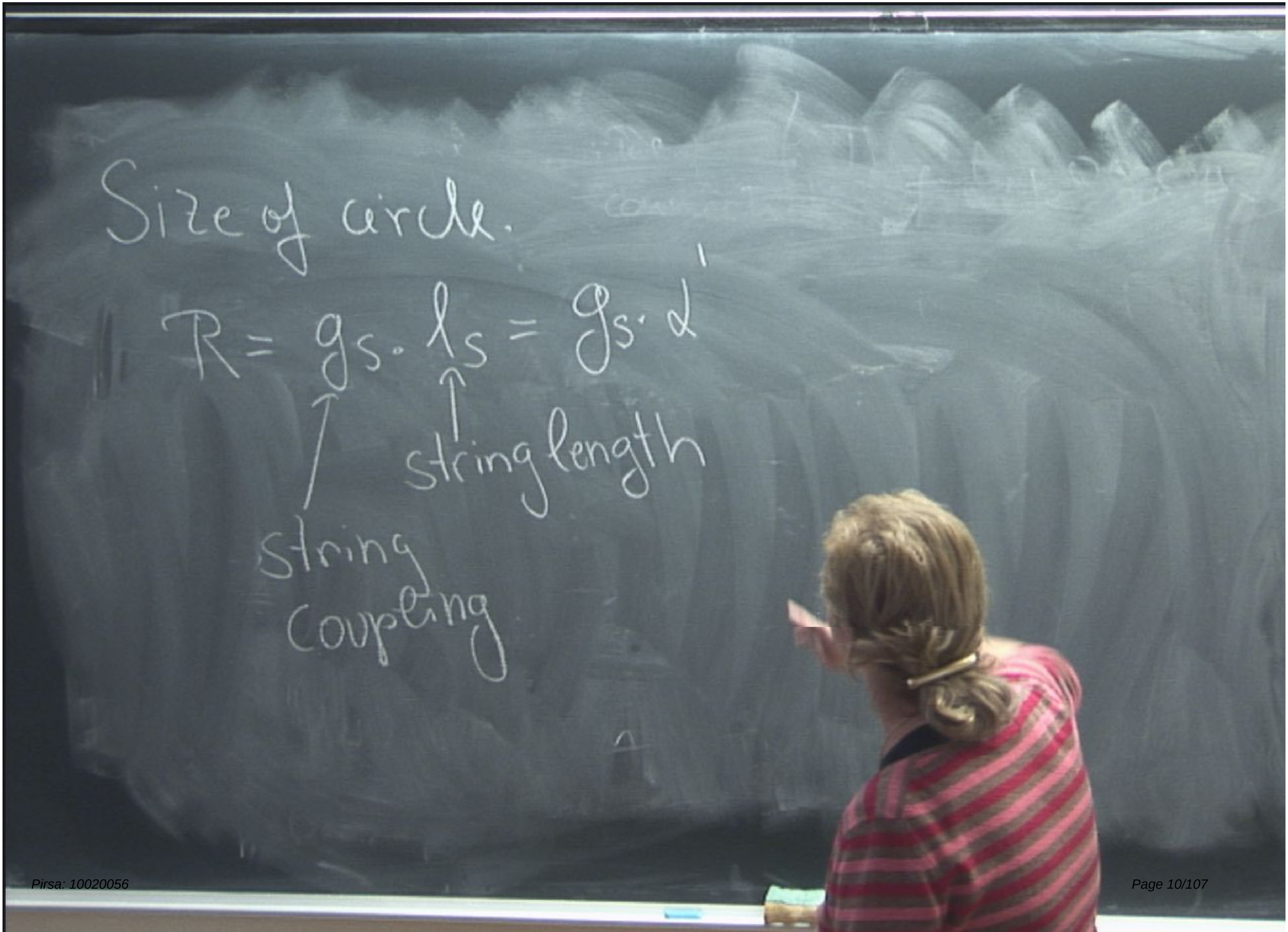

IIA
IIB
sugra
M-theory
I
SO(32)
E8×E8
x Low energy effective actions
x 11D sugra
x 10D IIA supergravity


IIA
IIB
I
SO(32)
E8×E8
sugra
M-theory
x Low energy effective actions
x 11D sugra g_{MU} F_{MNPQ}
x 10D IIA supergravity


IIA
IIB
M-theory
I
SO(32)
E8×E8
X Low energy effective actions
X 11D sugra g_{MN} F_{MNPQ} ψ
X 10D IIA supergravity
F_{MNPQ}
F_{mnp11}
F_{mnpq}
H_{mnp}


IIA
IIB
M-theory
I
SO(32)
E8×E8
× Low energy effective actions
× 11D sugra $g_{\mu\nu}$ F_{MNPQ} ψ
× 10D IIA supergravity
F_{MNPQ}
$G_{11} = 2\pi R G_{10}$
F_{mnp11}
H_{mnp}
F_{mnpq}


IIB
I
SO(32)
x Low energy effective actions
x 11D sugra g_{MN} F_{MNPQ} Ψ
x 10D IIA supergravity
F_{MNPQ}
$G_{11} = 2\pi R_{11} G_{10}$
F_{mnp11}
H_{mnp}
$R = g_s l_s$
F_{mnpq}


Size of circle.
$R = g_s \cdot l_s = g_s \cdot \alpha'$
string length
string coupling

Size of circle.

$$R = g_s \cdot l_s = g_s \sqrt{\alpha'}$$

string coupling

string length

$$g_s = e^{-\phi}$$

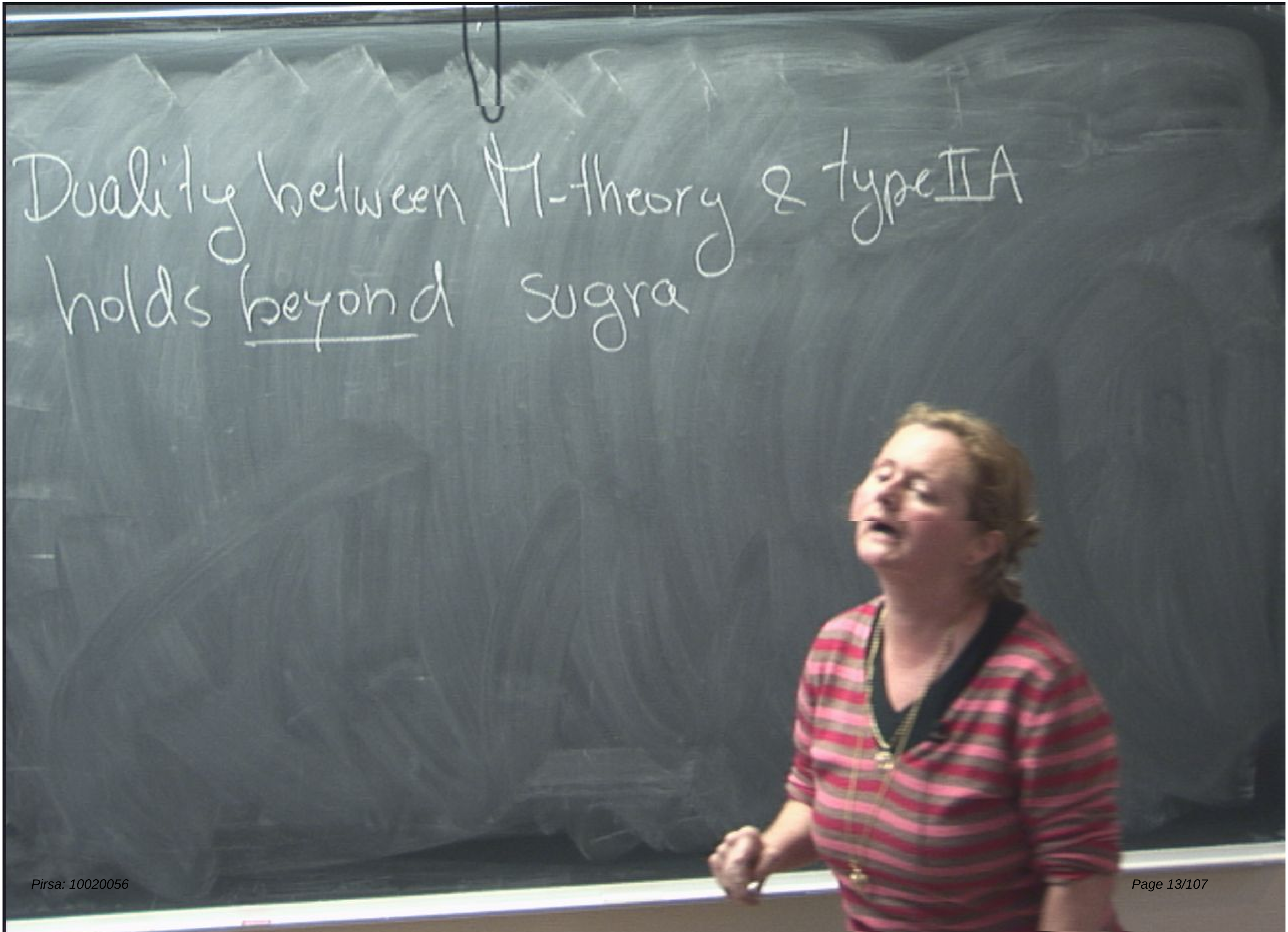
Size of circle.

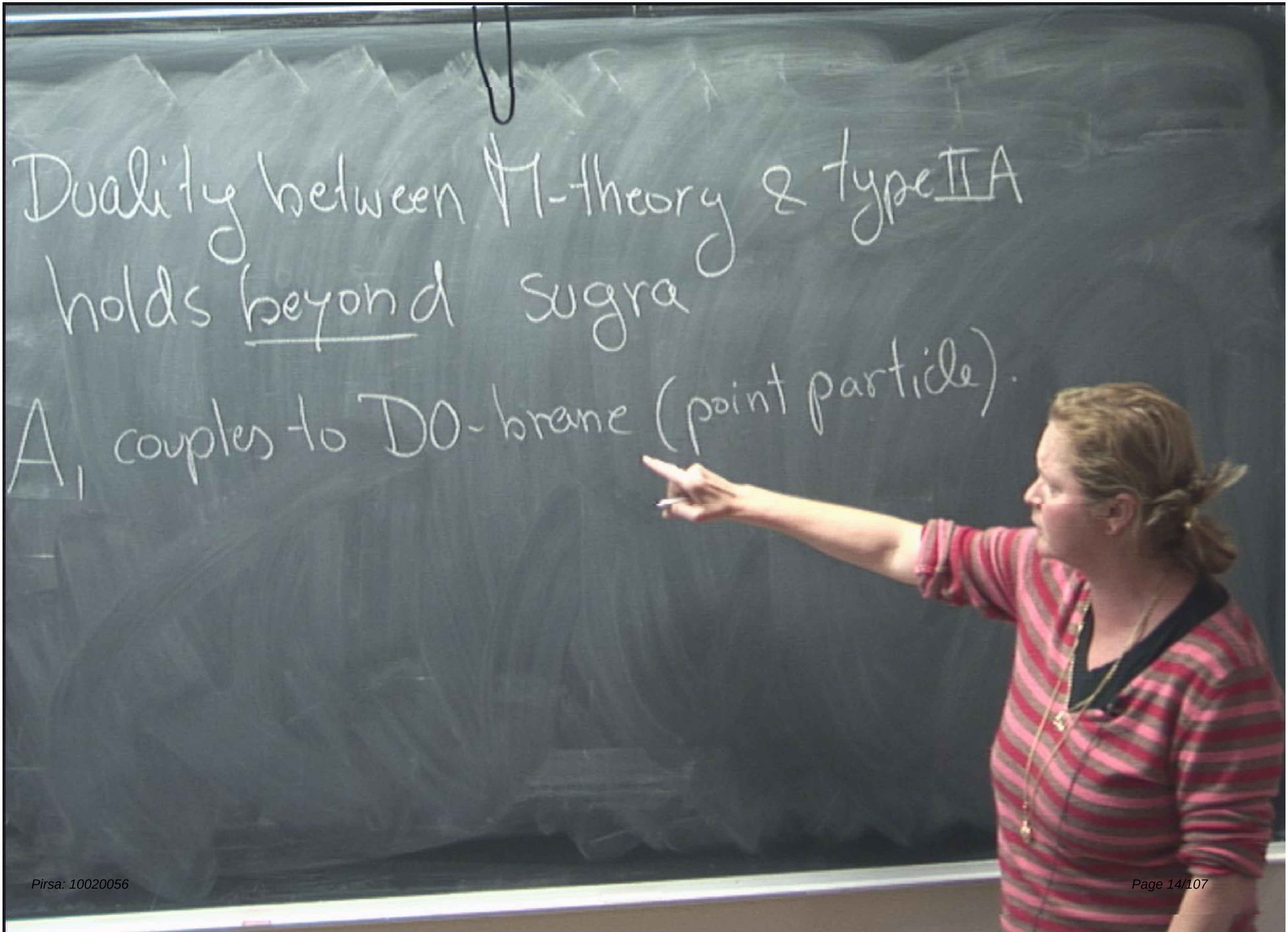
$$R = g_s \cdot l_s = g_s\sqrt{\alpha'}$$

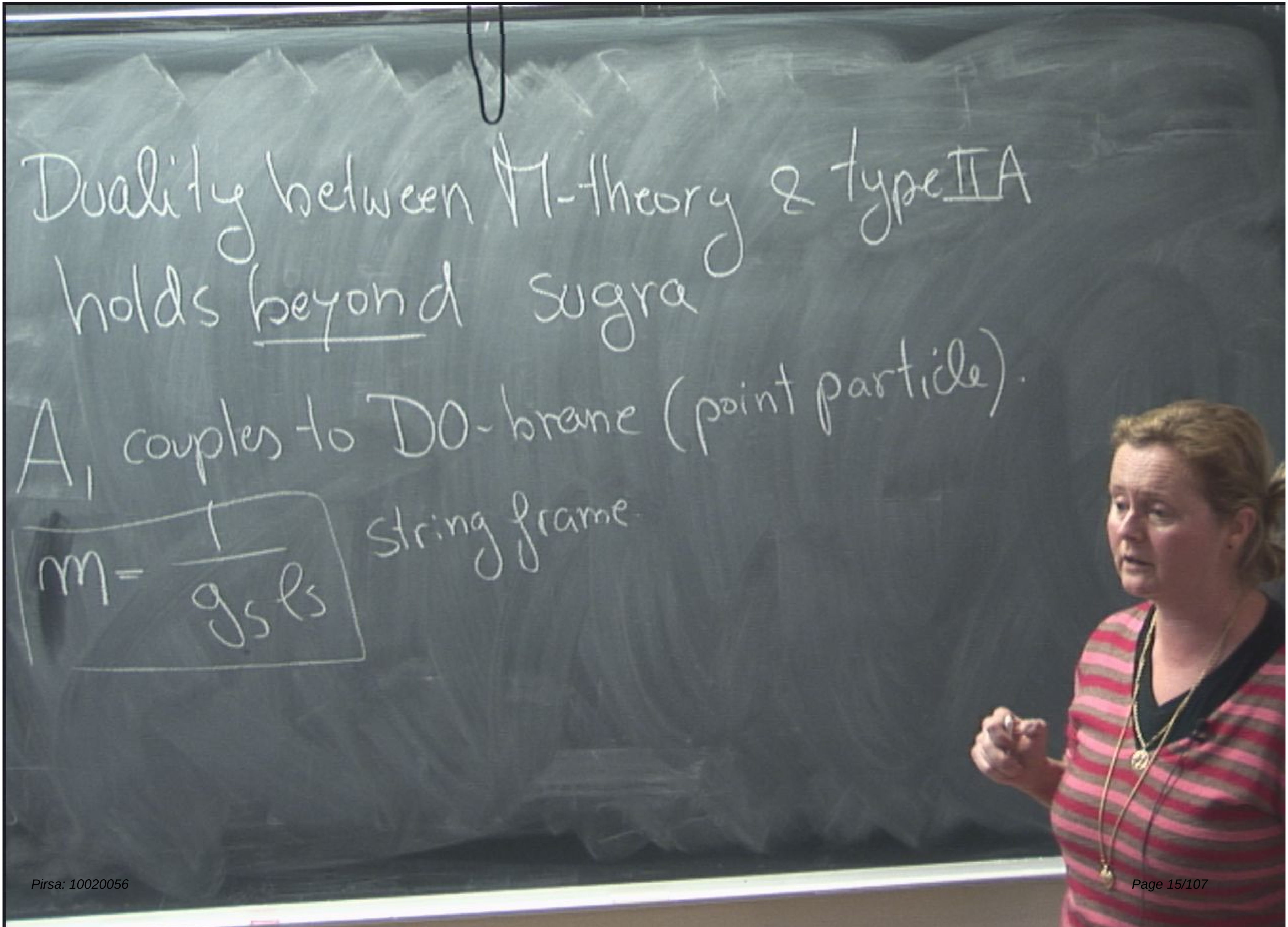
string coupling

string length

$$g_s = e^{-\phi}$$


Duality between M-theory & type IIA
holds beyond sugra


Duality between M-theory & type IIA
holds beyond sugra
A_1 couples to D0-brane (point particle).


Duality between M-theory & type IIA
holds beyond sugra
A_1 couples to D0-brane (point particle).
$\boxed{m = \frac{1}{g_s l_s}}$
string frame

Duality between M-theory & type IIA holds beyond sugra

A_1 couples to D0-brane (point particle).

$$\boxed{m = \frac{1}{g_s l_s}}$$ string frame

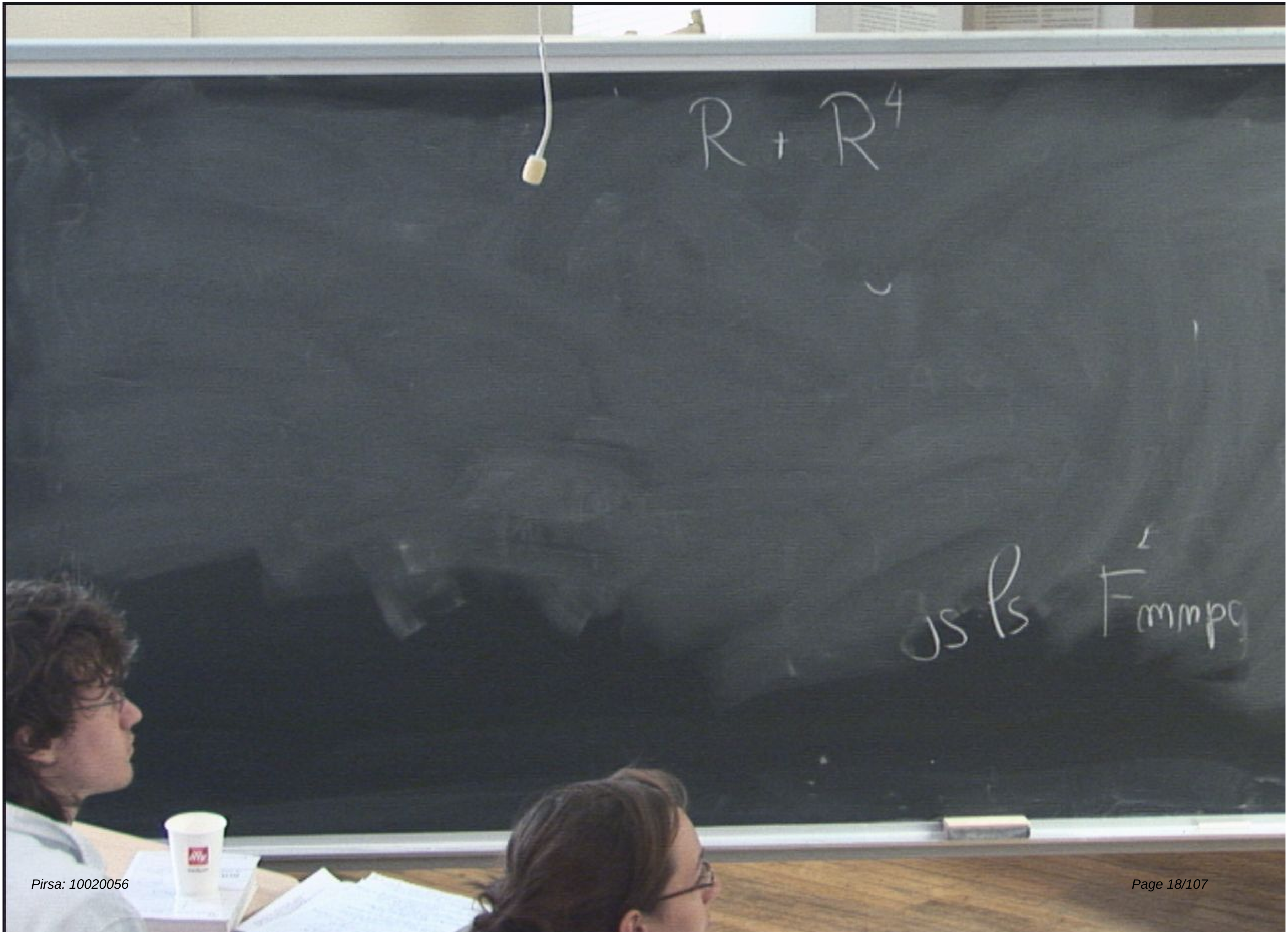
$g_s \to 0$ $\quad m \to \infty$ non-perturbative

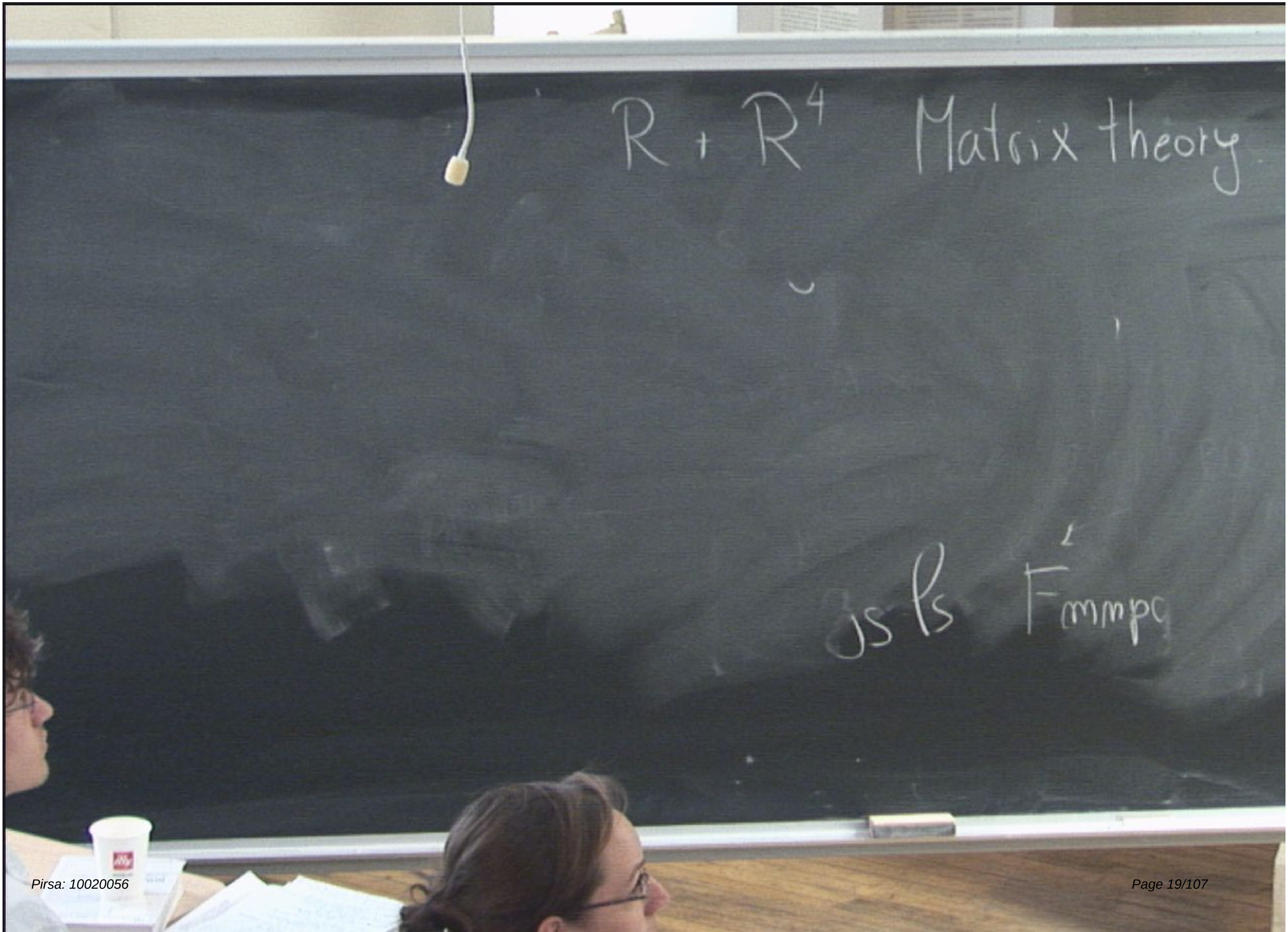
Duality between M-theory & type IIA
holds beyond sugra

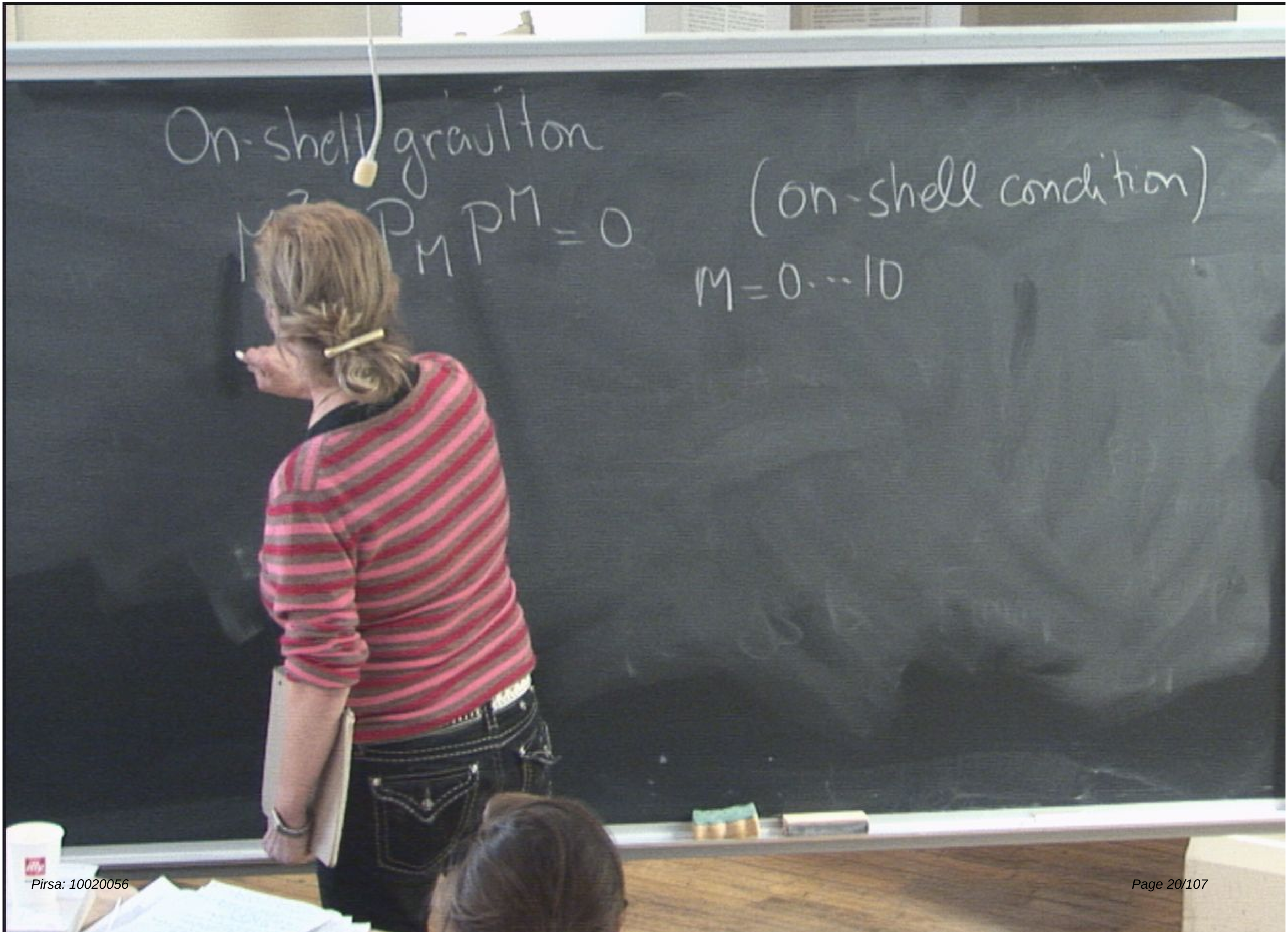
A_1 couples to D0-brane (point particle).

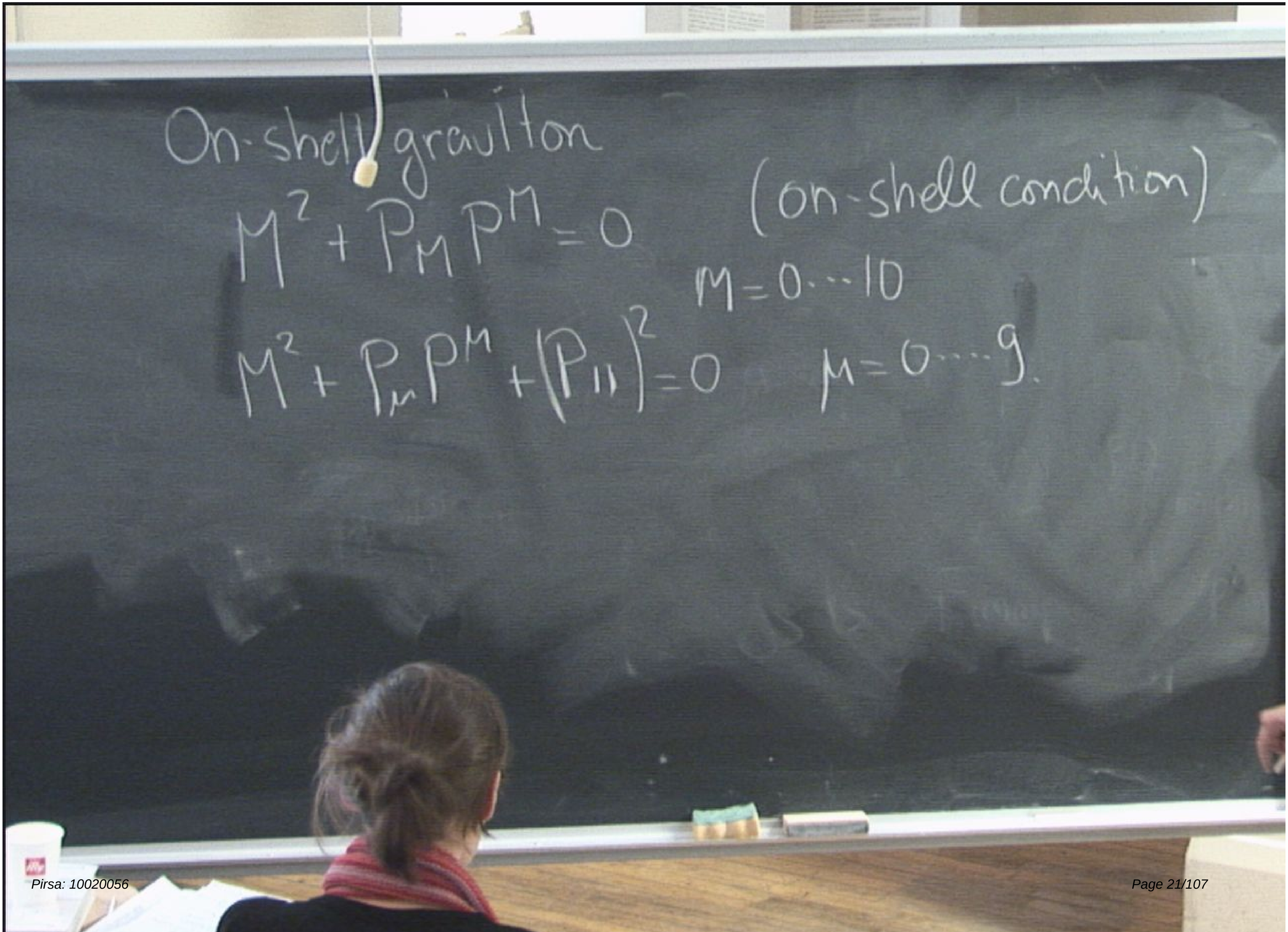
$$\boxed{m = \frac{1}{g_s l_s}}$$ string frame

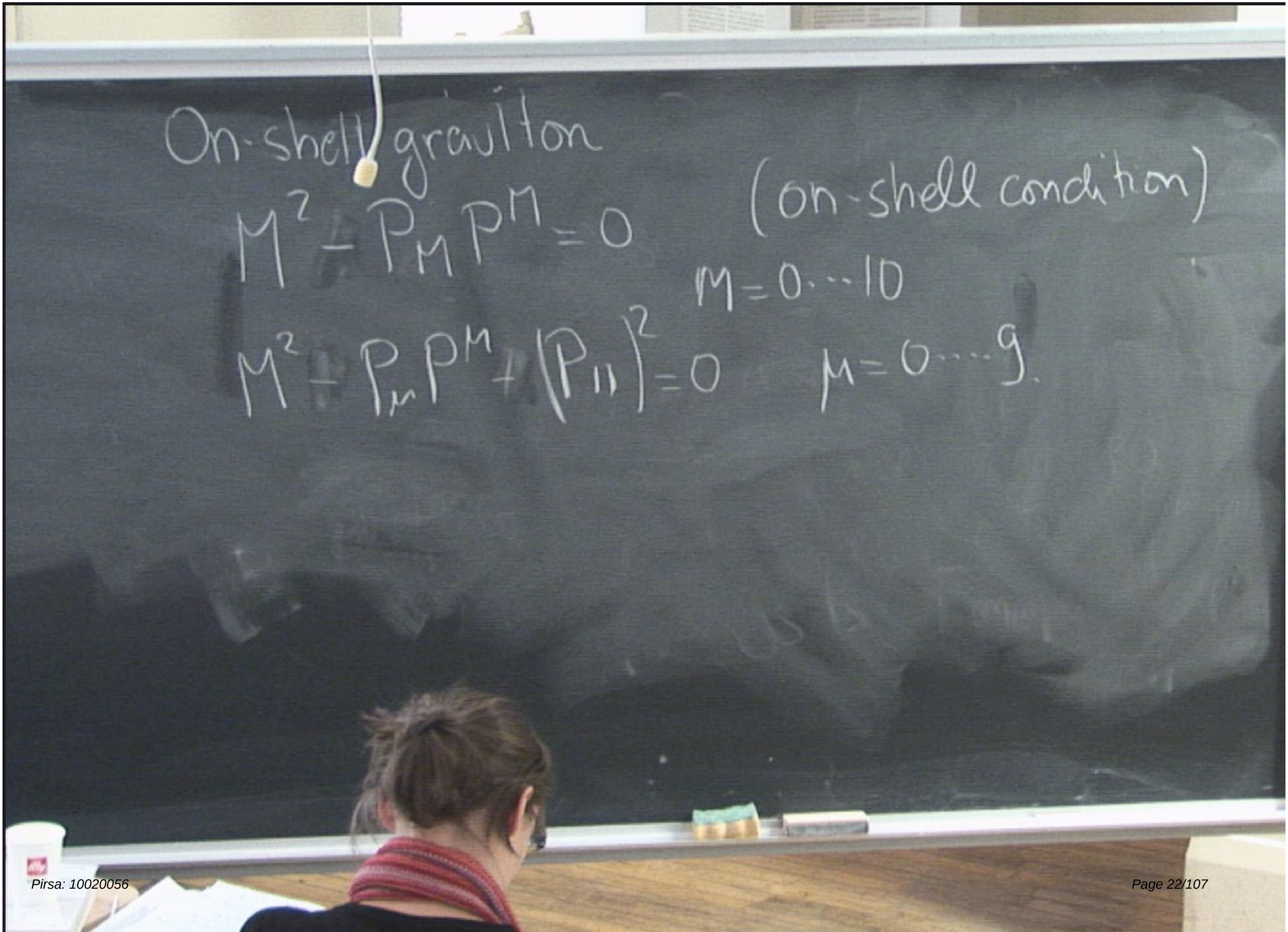
$g_s \to 0$ $m \to \infty$ non-perturbative

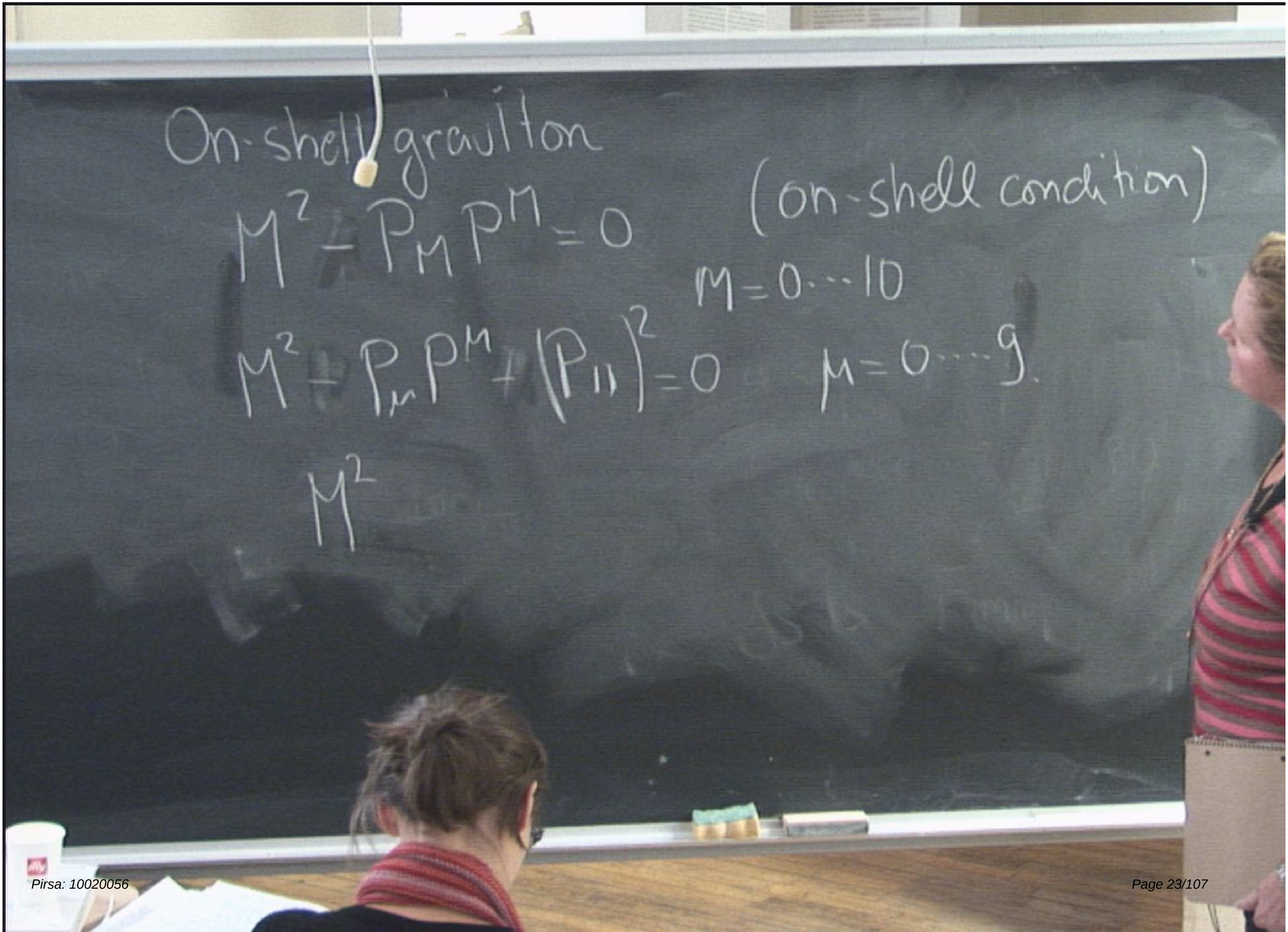

$R + R^4$
gs ls
F mmpq


$R + R^4$ Matrix theory
$g_s l_s$ F'_{mmpq}


On-shell graviton
$M^2 \; P_M P^M = 0$
(on-shell condition)
$M = 0 \cdots 10$


On-shell graviton
$M^2 + P_M P^M = 0$
(on-shell condition)
$M = 0 \cdots 10$
$M^2 + P_\mu P^\mu + (P_{11})^2 = 0$
$\mu = 0 \cdots 9$


On-shell graviton
$M^2 - P_M P^M = 0$ (on-shell condition)
$M = 0 \cdots 10$
$M^2 - P_\mu P^\mu - (P_{11})^2 = 0$ $\mu = 0 \cdots 9$


On-shell graviton
$M^2 - P_M P^M = 0$
(on-shell condition)
$M = 0 \cdots 10$
$M^2 - P_\mu P^\mu - (P_{11})^2 = 0$
$\mu = 0 \cdots 9$.
M^2

On-shell graviton

$$M^2 + P_M P^M = 0 \qquad \text{(on-shell condition)}$$

$$M = 0 \cdots 10$$

$$M^2 + P_\mu P^\mu + (P_{11})^2 = 0 \qquad \mu = 0 \cdots\cdots 9$$

$$M^2 \sim (P_{11})^2$$

On-shell graviton

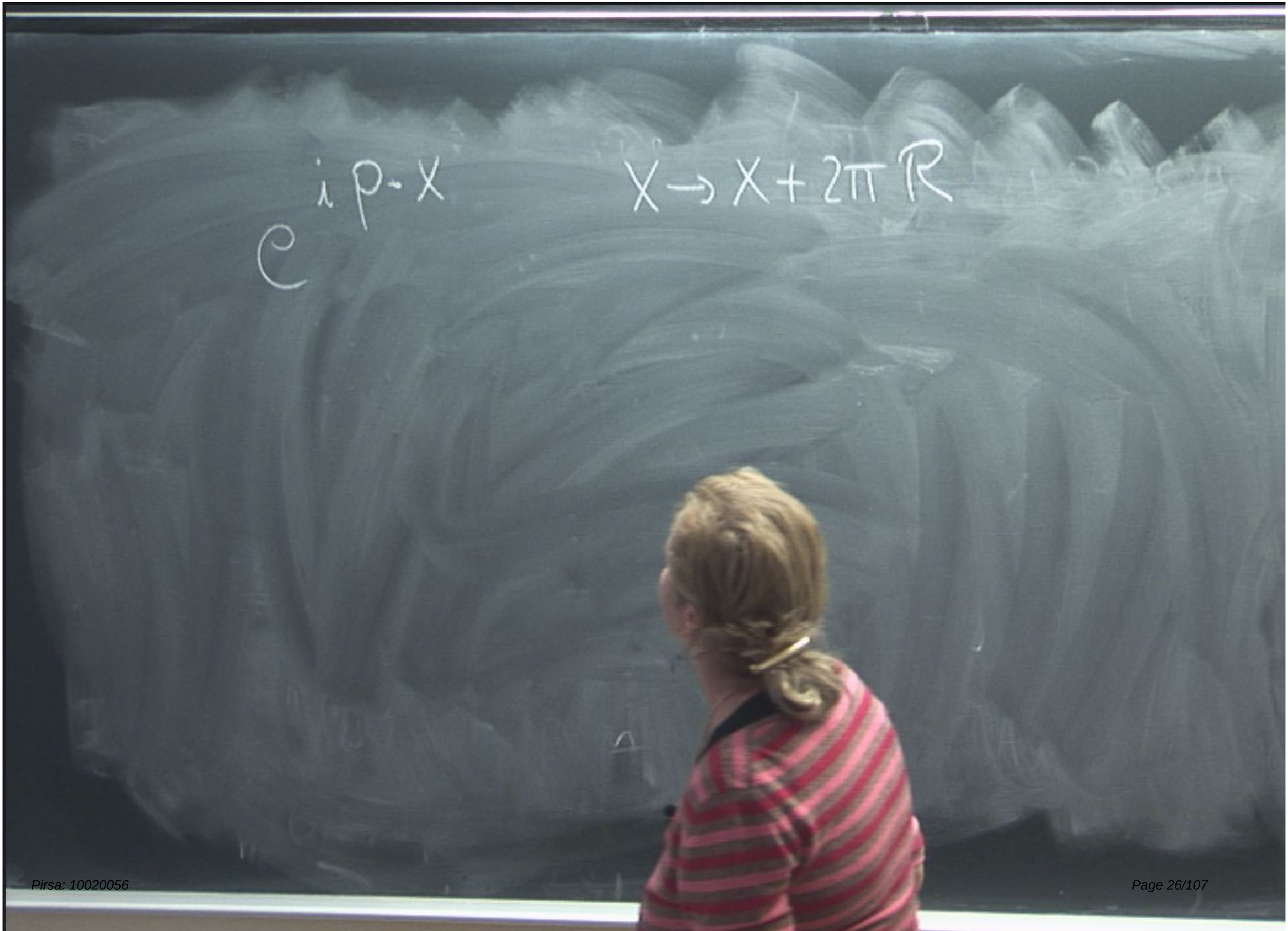
$M^2 + P_M P^M = 0$ (on-shell condition)

$M = 0 \cdots 10$

$M^2 + P_\mu P^\mu + (P_{11})^2 = 0$ $\mu = 0 \cdots 9$

$P_{11} = \frac{N}{R_{11}}$

$M^2 \sim (P_{11})^2 =$


$e^{ip\cdot x}$
$x \to x + 2\pi R$

$$e^{ip\cdot X} \qquad X \to X + 2\pi R$$

$$e^{iP(X+2\pi R)}$$

$e^{ip\cdot X}$ $\quad X \to X + 2\pi R$

$e^{iP(X+2\pi R)}$. $\quad ; R_{11} = g_s \cdot l_s$.

$M = \frac{N}{R_{11}}$ $\quad ; N = 1$ $\quad \boxed{M = \frac{1}{g_s l_s}}$

On-shell graviton

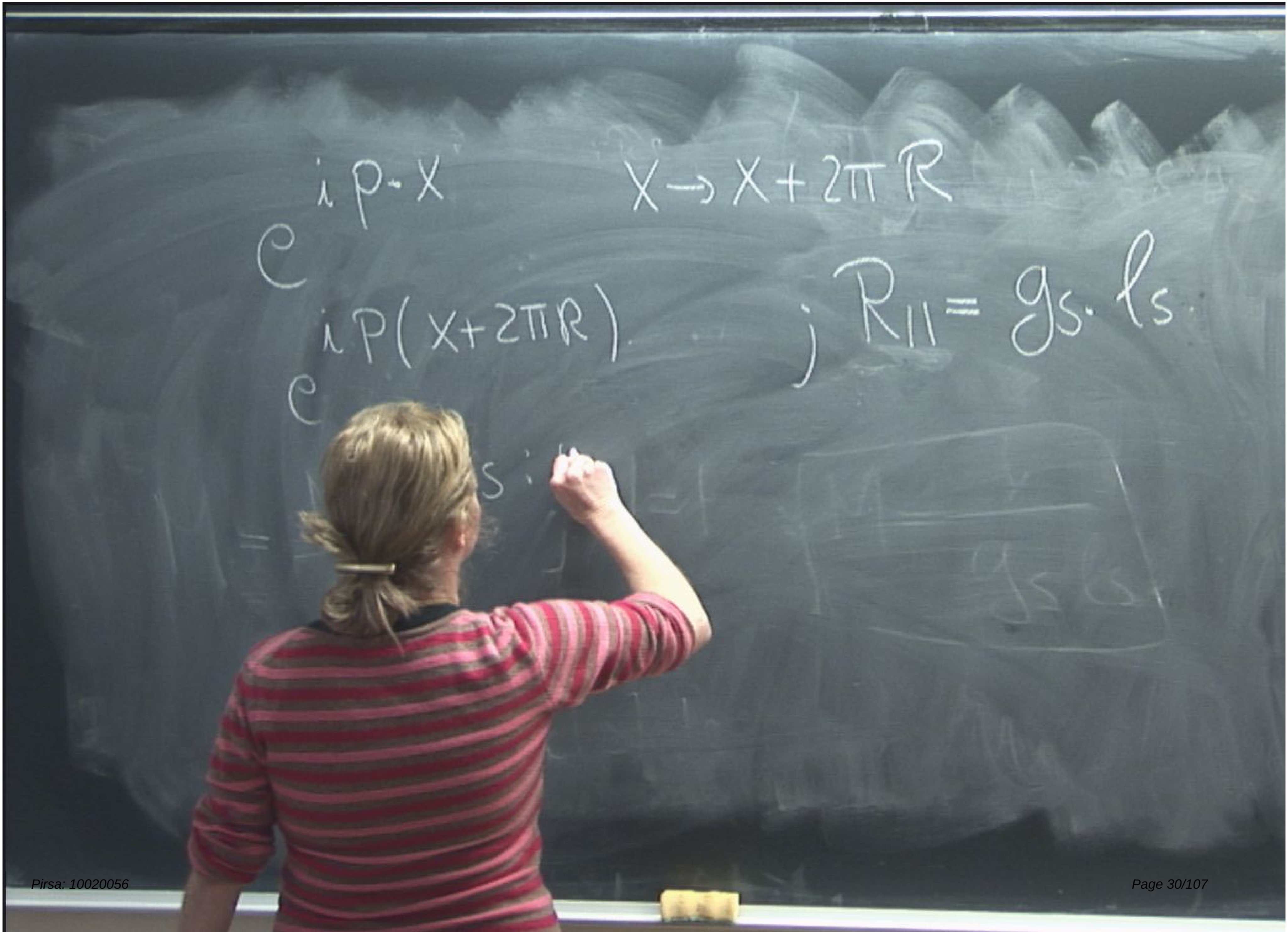
$$P_M P^M = 0$$ (on-shell condition)

$M = 0 \cdots 10$

$$P_\mu P^\mu - (P_{11})^2 = 0$$ $\mu = 0 \cdots 9$

$$M_{10}^2 \sim (P_{11})^2 =$$

$$\boxed{P_{11} = \frac{N}{R_{11}}}$$



e ip·x
x → x + 2π R
e iP(x+2πR)
; R11 = gs· ls·

$e^{ip\cdot X}$ $X \to X + 2\pi R$

$e^{ip(X+2\pi R)}$; $R_{11} = g_s \cdot l_s$.

g_s : weak coupling

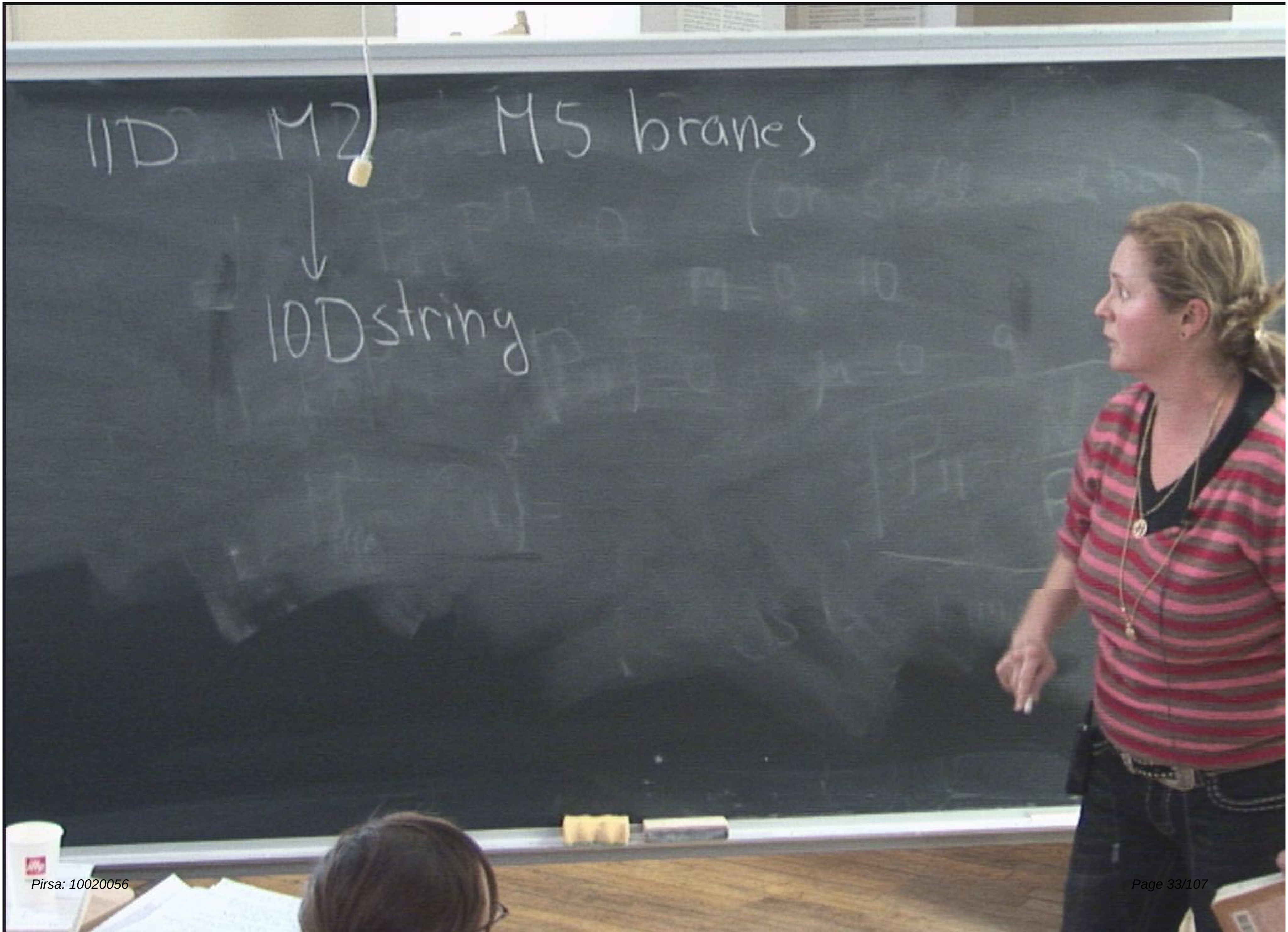
$e^{ip\cdot X}$ $\quad X \to X + 2\pi R$

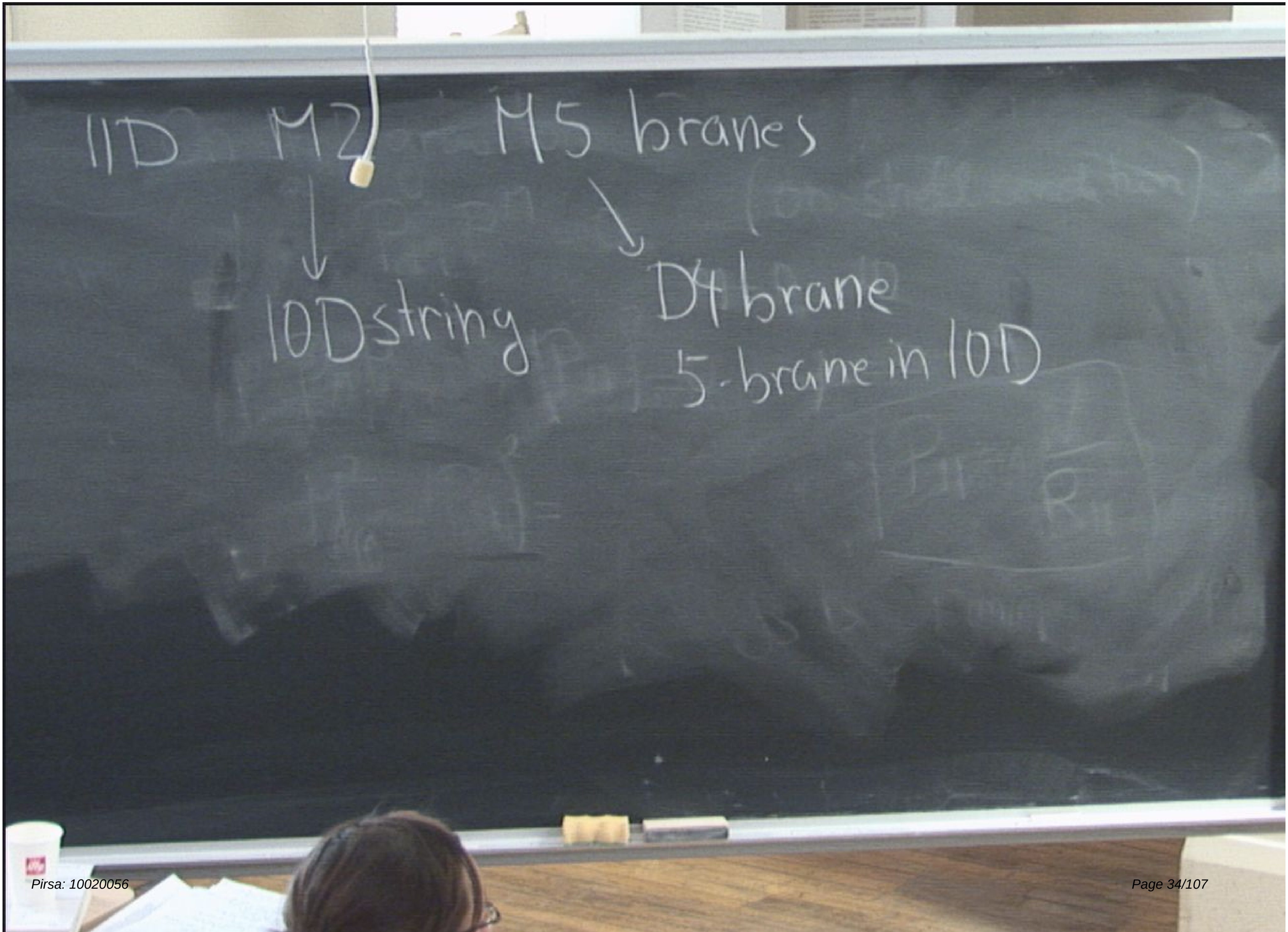
$e^{iP(X+2\pi R)}$; $R_{11} = g_s \cdot l_s$.

g_s : weak coupling Type IIA $R \to 0$

$g_s \to \infty$ $R \to \infty$ strong coupling

11D M-theory


11D M2 M5 branes
10D string

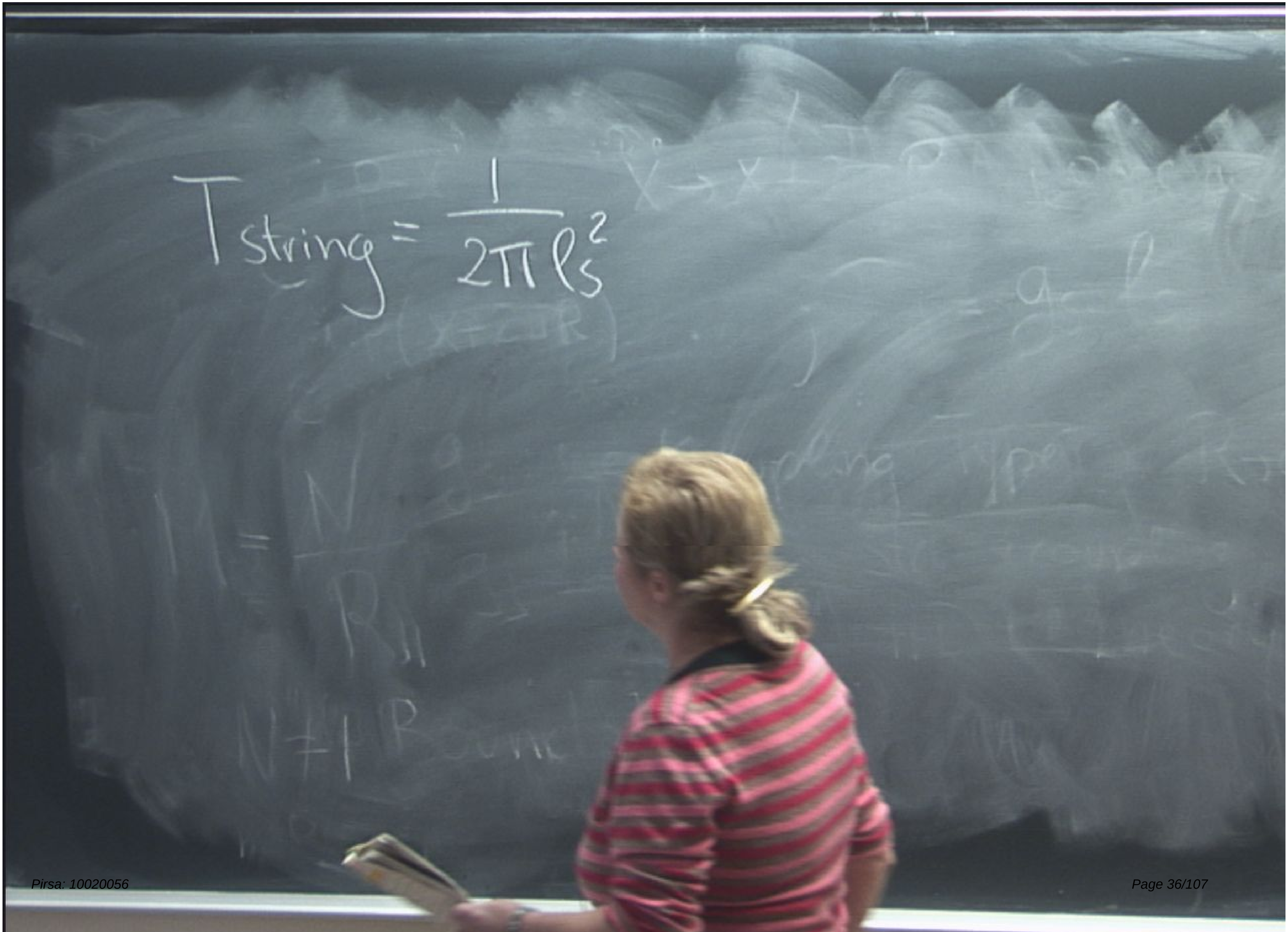

11D M2 M5 branes
10D string
D4 brane
5-brane in 10D

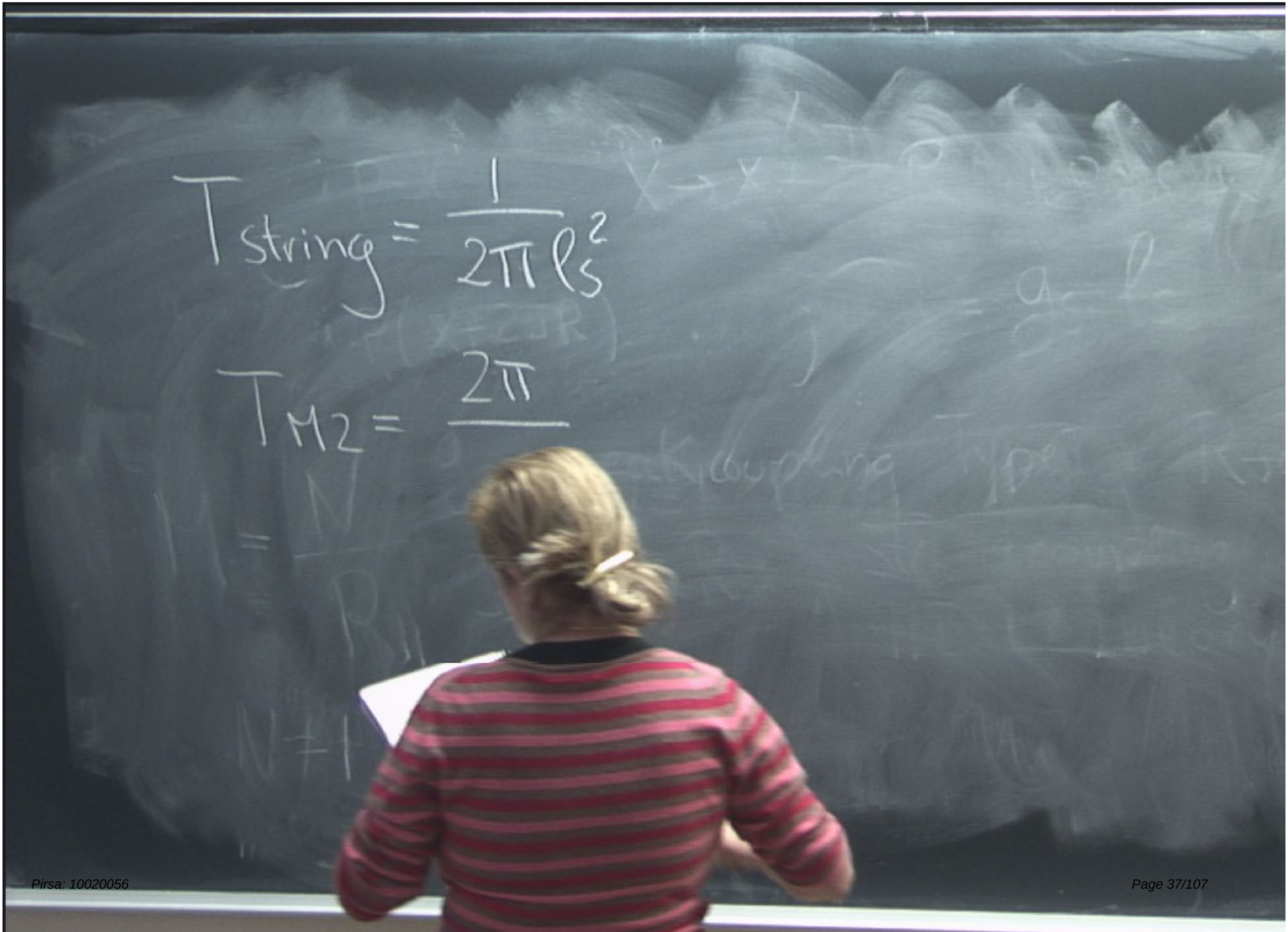
11D M2 M5 branes

10D string

D4 brane

5-brane in 10D


$T_{string} = \frac{1}{2\pi \ell_s^2}$

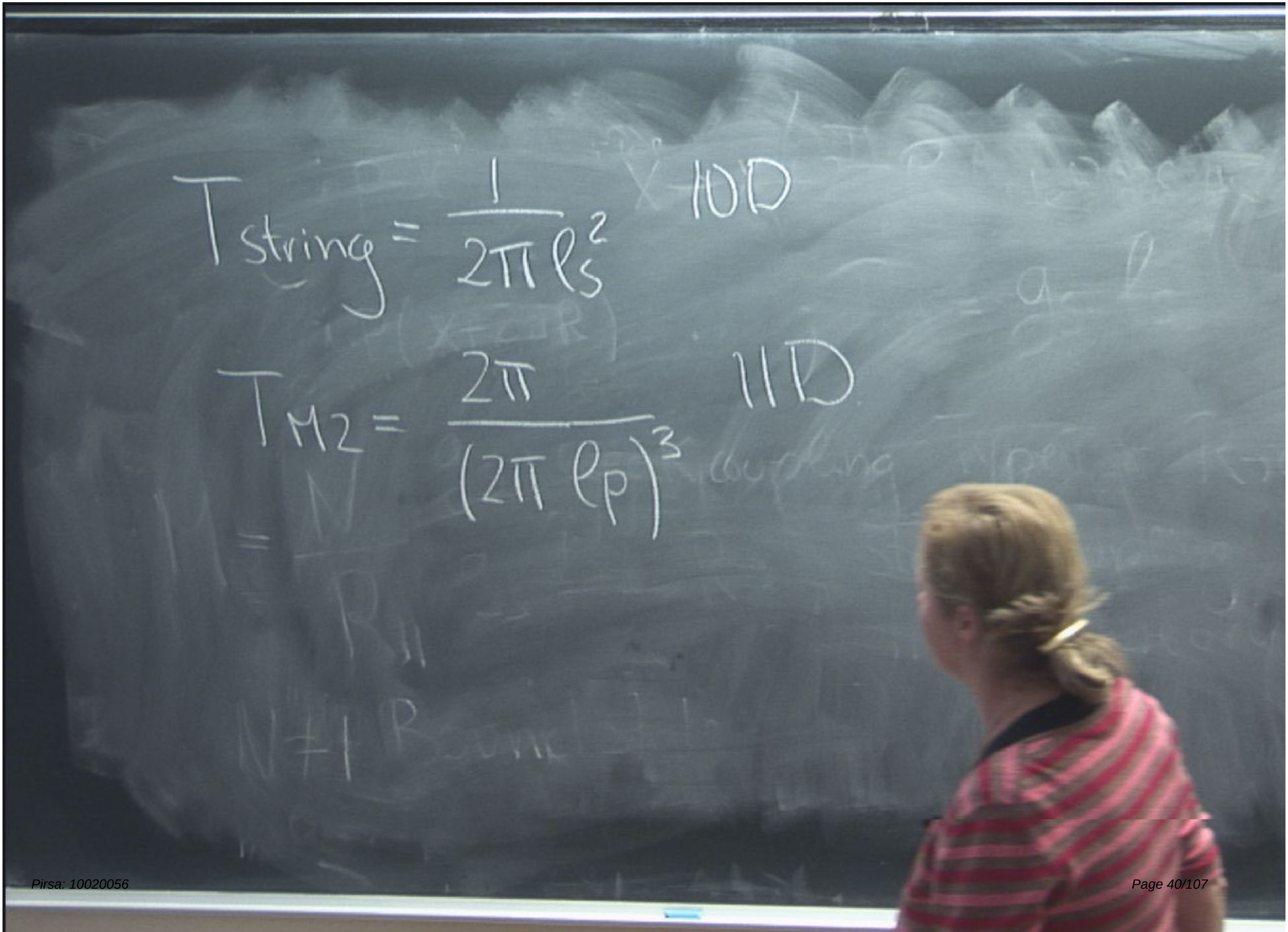

$T_{string} = \frac{1}{2\pi \ell_s^2}$
$T_{M2} = \frac{2\pi}{\quad}$

$$T_{string} = \frac{1}{2\pi \ell_s^2}$$

$$T_{M2} = \frac{2\pi}{(2\pi \ell_p)}$$

$$T_{string} = \frac{1}{2\pi \ell_s^2}$$

$$T_{M2} = \frac{2\pi}{(2\pi \ell_p)^3}$$



$T_{string} = \frac{1}{2\pi \ell_s^2}$ 10D
$T_{M2} = \frac{2\pi}{(2\pi \ell_p)^3}$ 11D

$$T_{string} = \frac{1}{2\pi \ell_s^2} \quad 10D$$

$$T_{M2} = \frac{2\pi}{(2\pi \ell_p)^3} \quad 11D$$

$$T_{string} \sim 2\pi R \cdot T_{M2}$$

$$T_{string} = \frac{1}{2\pi \ell_s^2} \quad 10D \qquad R_{11} = g_s \cdot \ell_s$$

$$T_{M2} = \frac{2\pi}{(2\pi \ell_p)^3} \quad 11D$$

$$T_{string} \sim 2\pi R_{11} T_{M2}$$

$$T_{string} = \frac{1}{2\pi \ell_s^2} \quad 10D \qquad R_{11} = g_s \cdot \ell_s$$

$$T_{M2} = \frac{2\pi}{(2\pi \ell_p)^3} \quad 11D \qquad \ell_p = g_s^{1/3} \cdot \ell_s$$

$$T_{string} \sim 2\pi R_{11} T_{M2}$$

$T_{string} = \frac{1}{2\pi \ell_s^2}$ 10D

$R_{11} = g_s \cdot \ell_s$

$T_{M2} = \frac{2\pi}{(2\pi \ell_p)^3}$ 11D

$\ell_p = \underbrace{g_s^{1/3}}_{} \cdot \ell_s$

$= T_{string} \sim 2\pi R_{11} T_{M2}$

K-K

ansatz

$$T_{string} = \frac{1}{2\pi \ell_s^2} \quad 10D \qquad R_{11} = g_s \cdot \ell_s$$

$$T_{M2} = \frac{2\pi}{(2\pi \ell_p)^3} \quad 11D \qquad \ell_p = g_s^{1/3} \cdot \ell_s$$

$$= \qquad T_{string} \sim 2\pi R_{11} T_{M2}$$

K-K ansatz

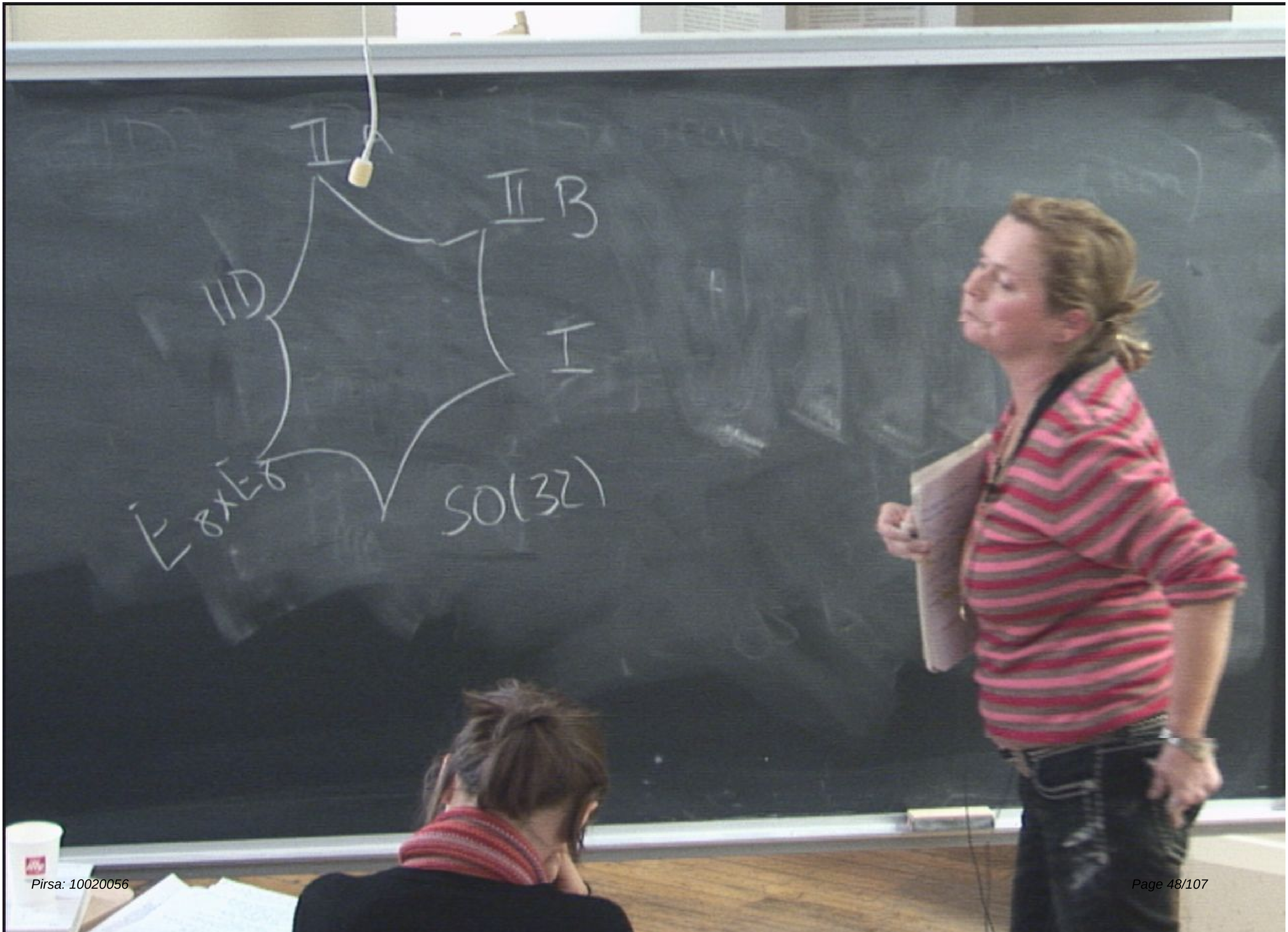
$$G_{MN} \cdot dx^M dx^N = e^{-2\phi/3} g_{\mu\nu} dx^\mu dx^\nu + (dx^{11} + A_\mu dx^\mu)^2$$

$$G_{MN} \cdot dx^M dx^N = e^{-2\phi/3} g_{\mu\nu} dx^\mu dx^\nu +$$

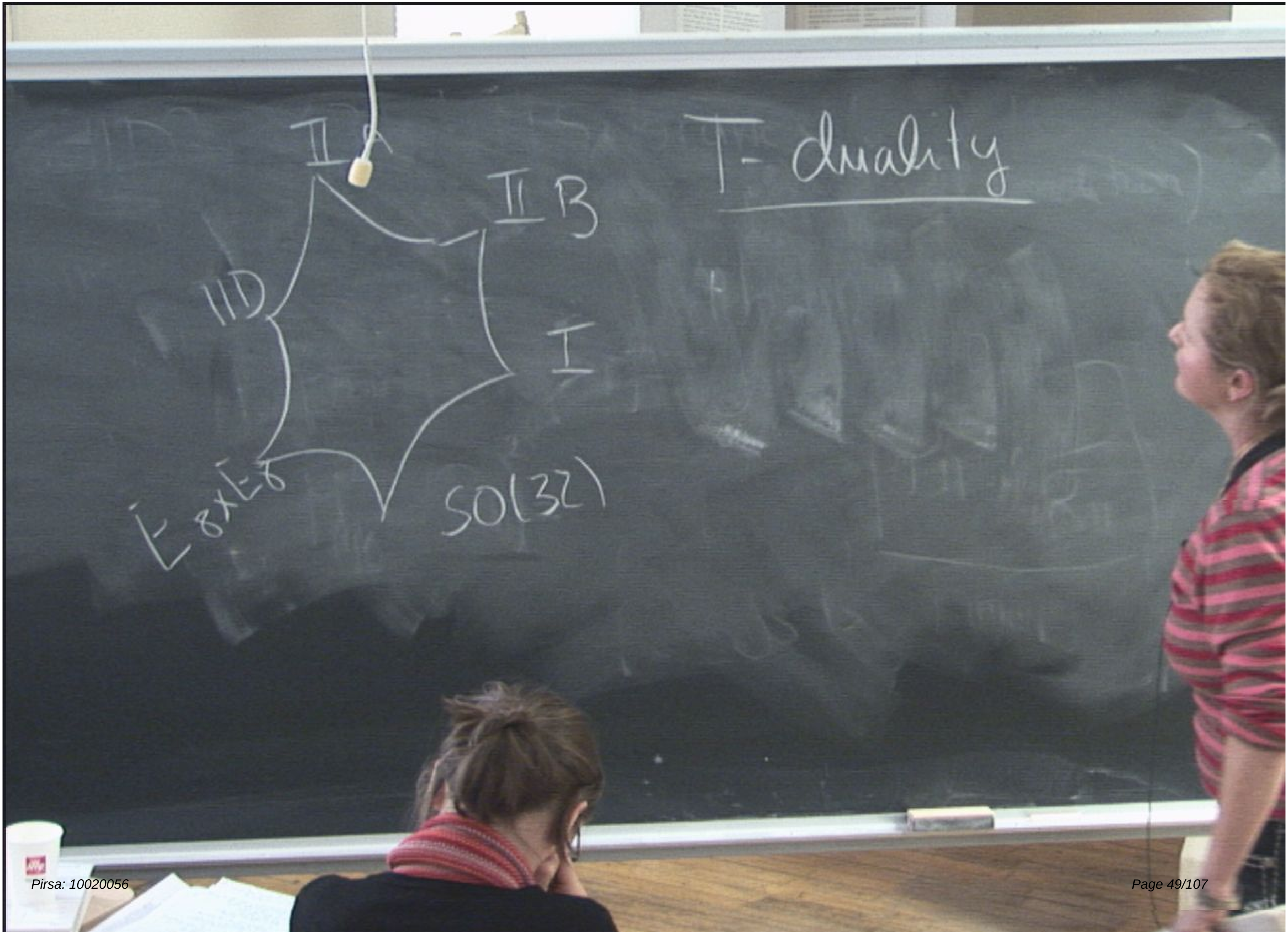
$$+ (dx^{11} + A_\mu dx^\mu)^2$$

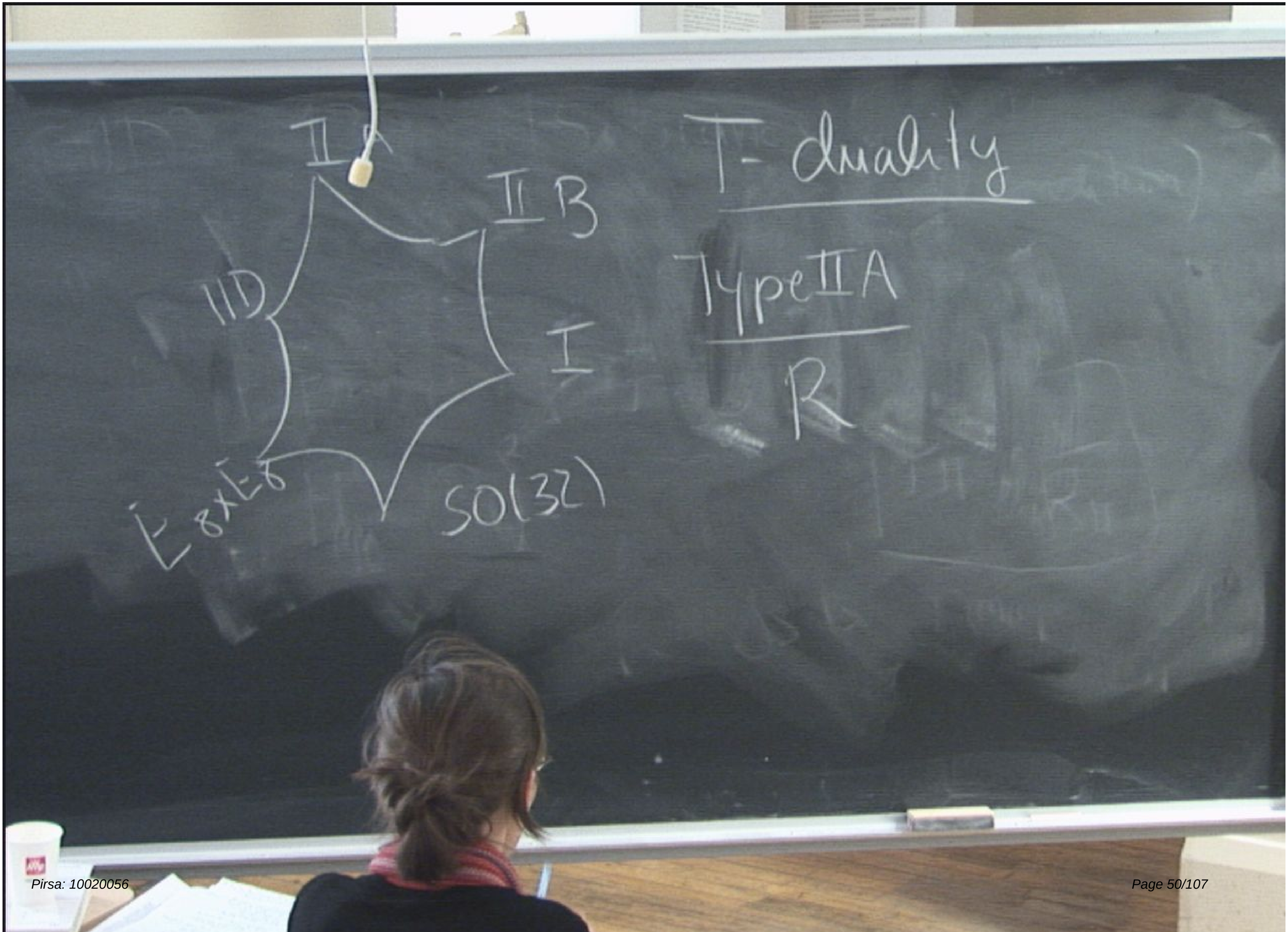
$g_s = e^{-\phi}$ di

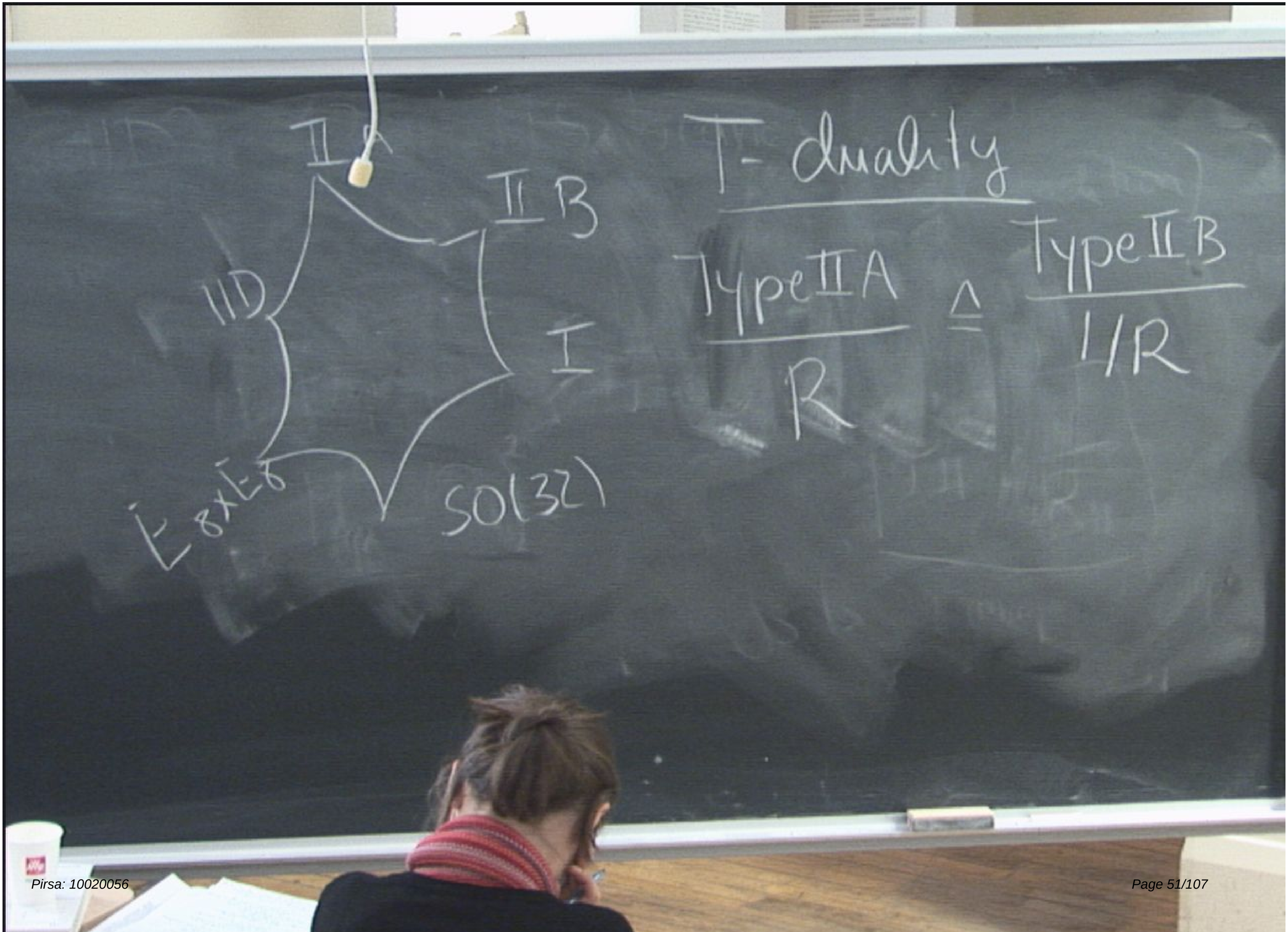
g_s^{IIB}

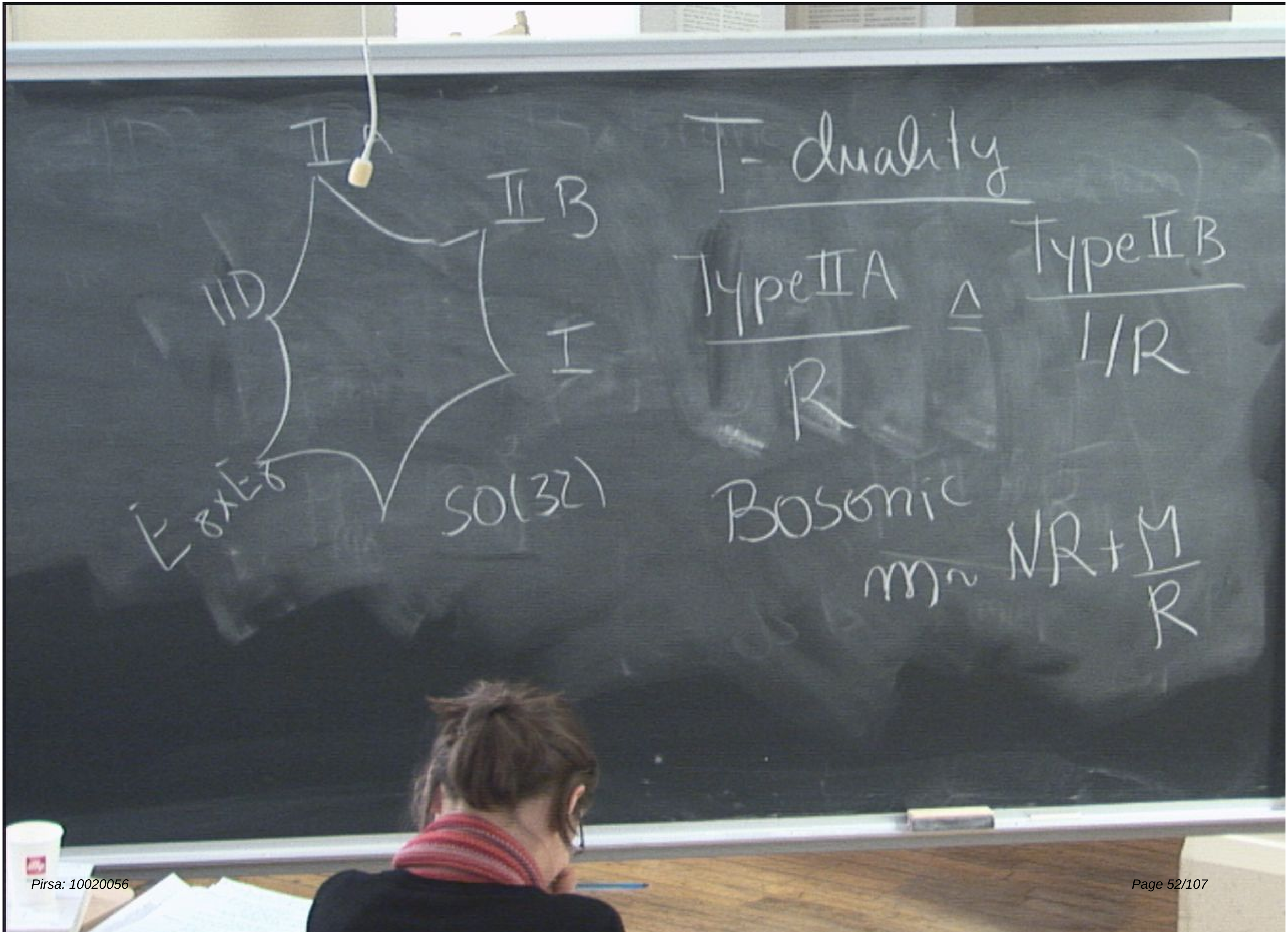


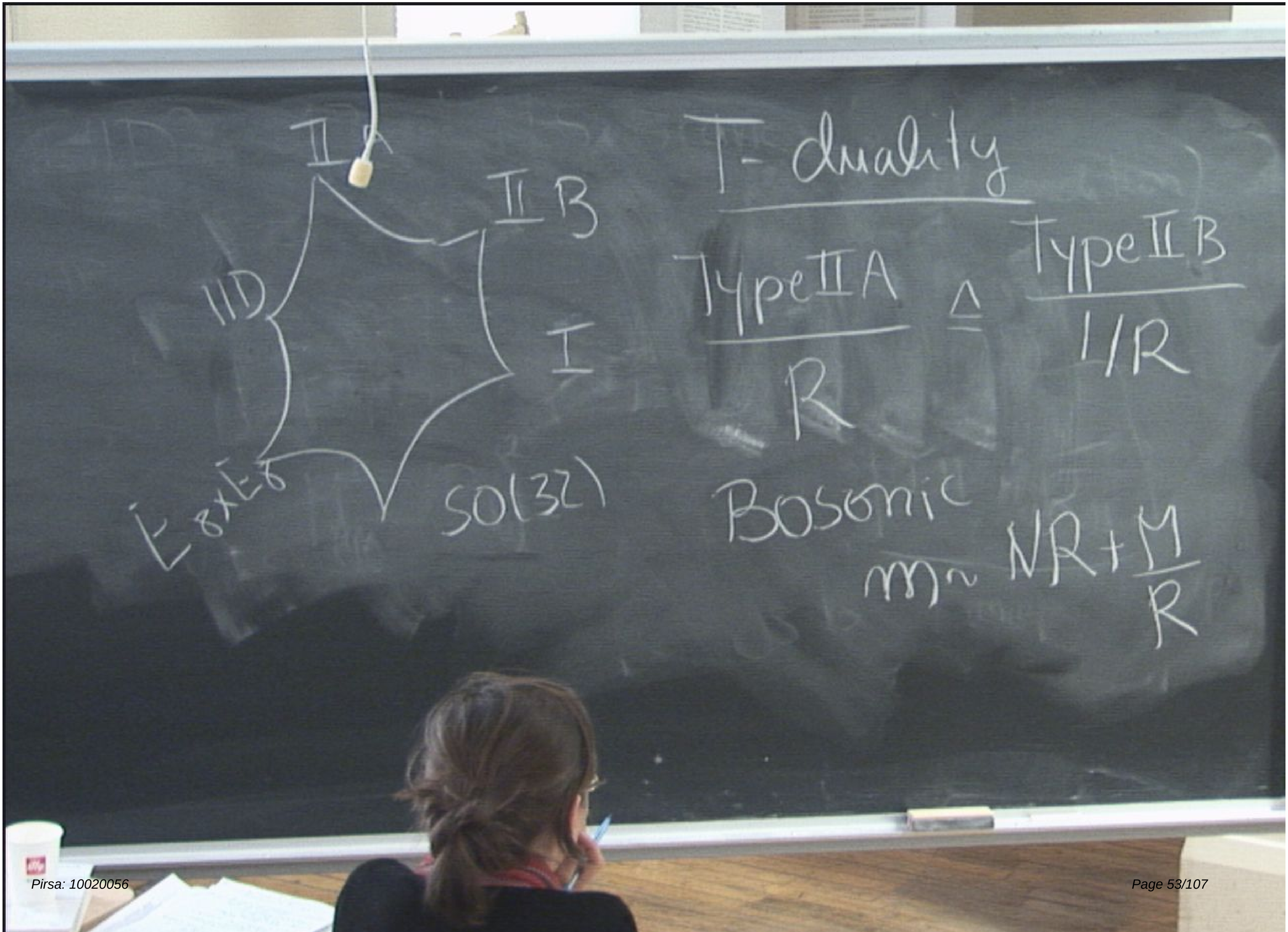
II A
II B
IID
I
E8xE8
SO(32)


T- duality
II A
II B
11D
I
E8xE8
SO(32)


IIA
IIB
11D
I
E8xE8
SO(32)
T- duality
Type IIA
R


II A
II B
11D
I
E8xE8
SO(32)
T- duality
Type II A
R
≙
Type II B
1/R


II A
II B
11D
I
E8xE8
SO(32)
T- duality
Type II A / R ≅ Type II B / 1/R
Bosonic
m ~ NR + M/R

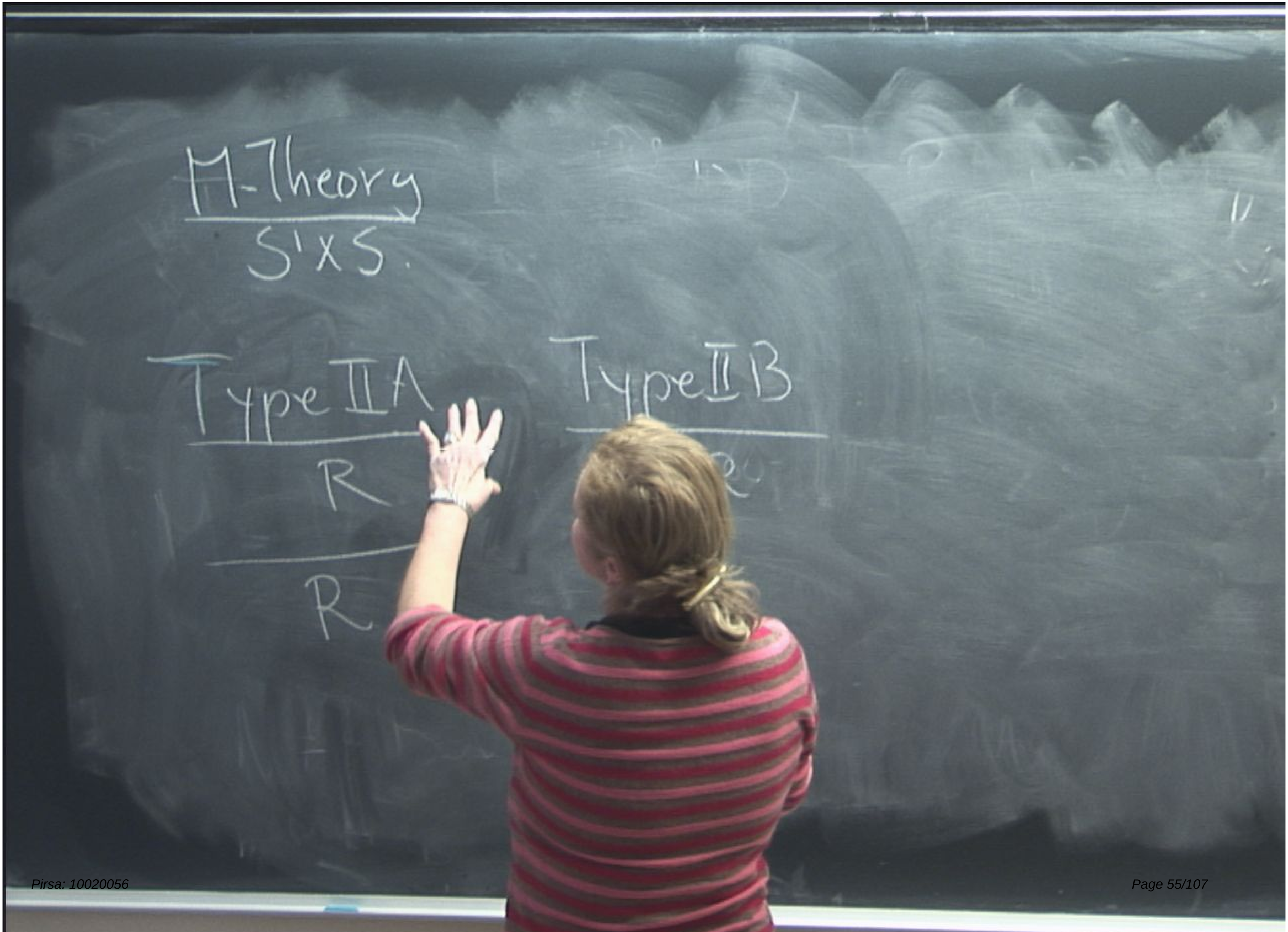

IIA
IIB
11D
I
E8xE8
SO(32)
T-duality
Type IIA / R ≅ Type IIB / 1/R
Bosonic
m ~ NR + M/R

Type IIA

R

Type IIB

1/R


M-Theory
S¹xS.
Type IIA
R
R
Type IIB

M-Theory

$S^1 \times S^1$ ||

Type IIA

R

R

Type IIB

$1/R$

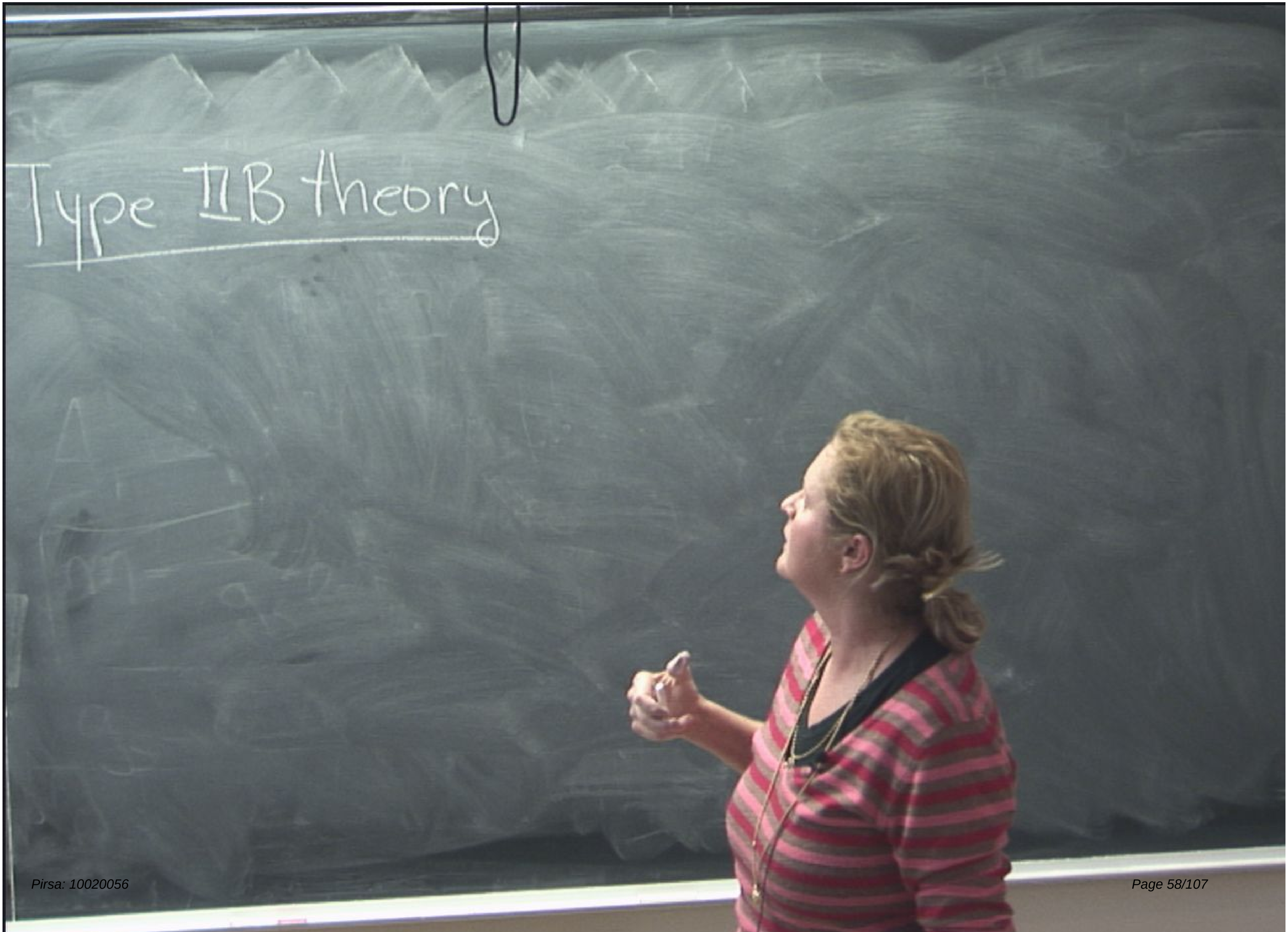
$$\frac{\text{M-Theory}}{S^1 \times S^1} \quad \|$$

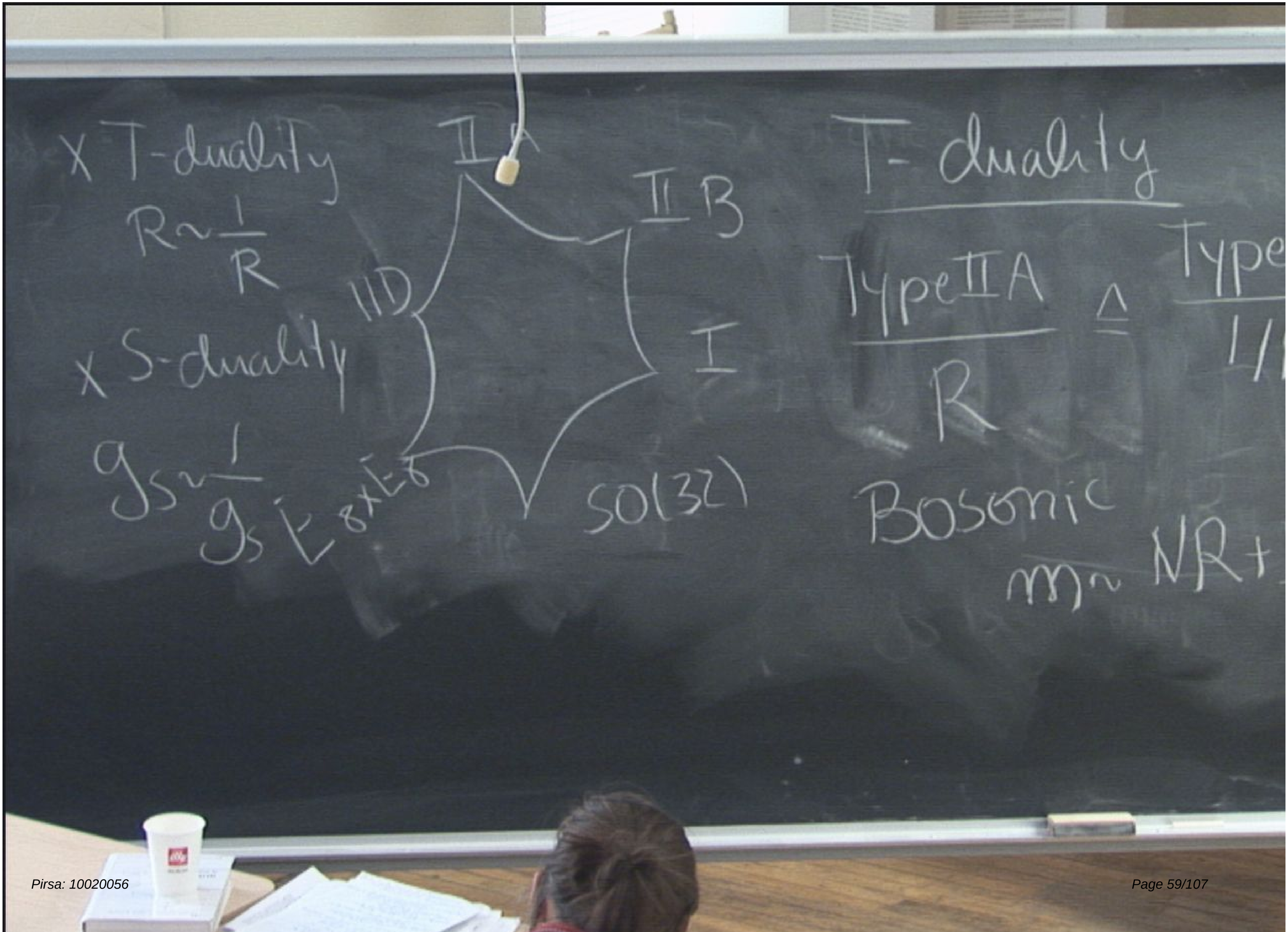
$$\frac{\text{Type IIA}}{R}$$

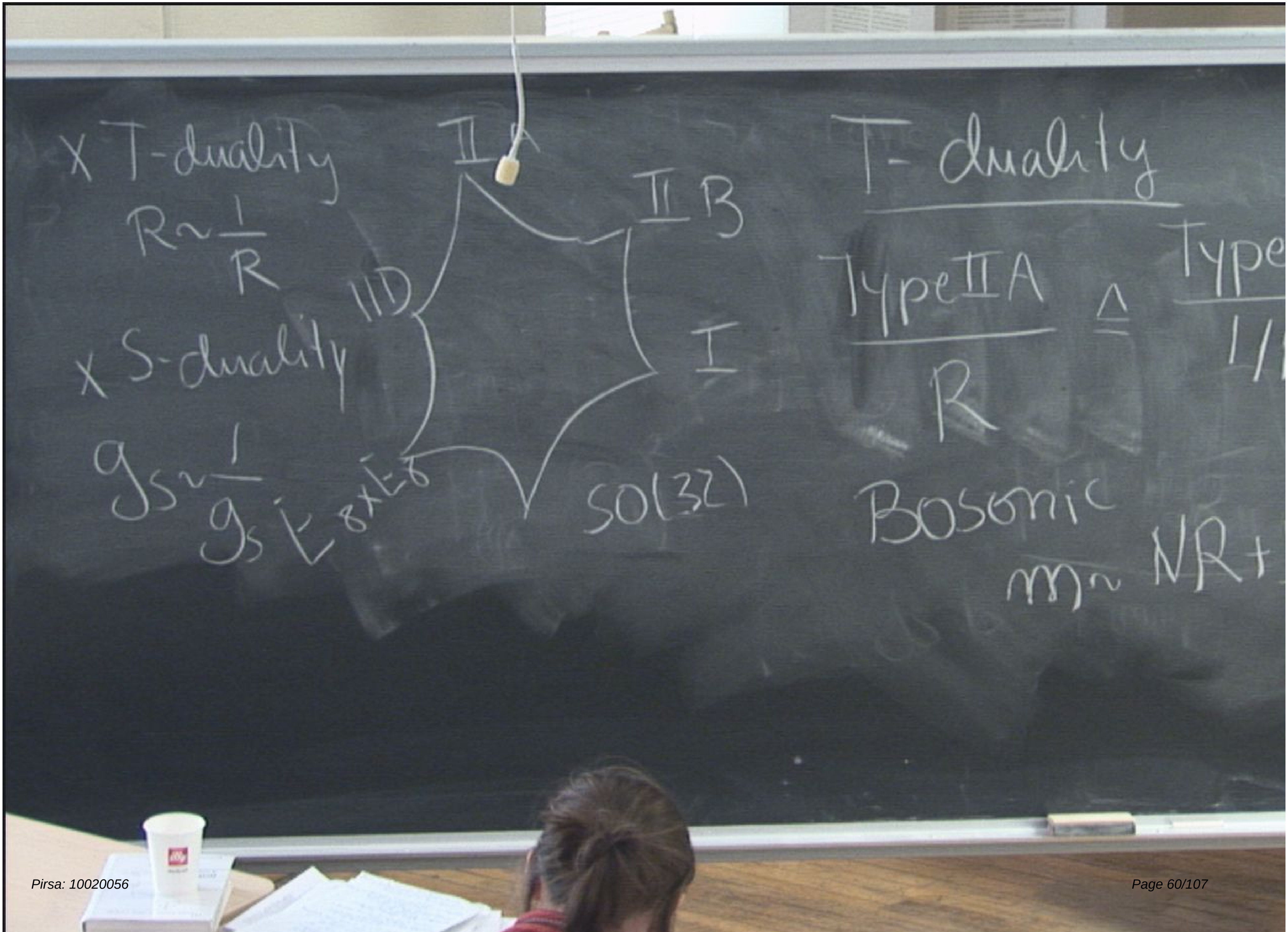
$$\frac{}{R}$$

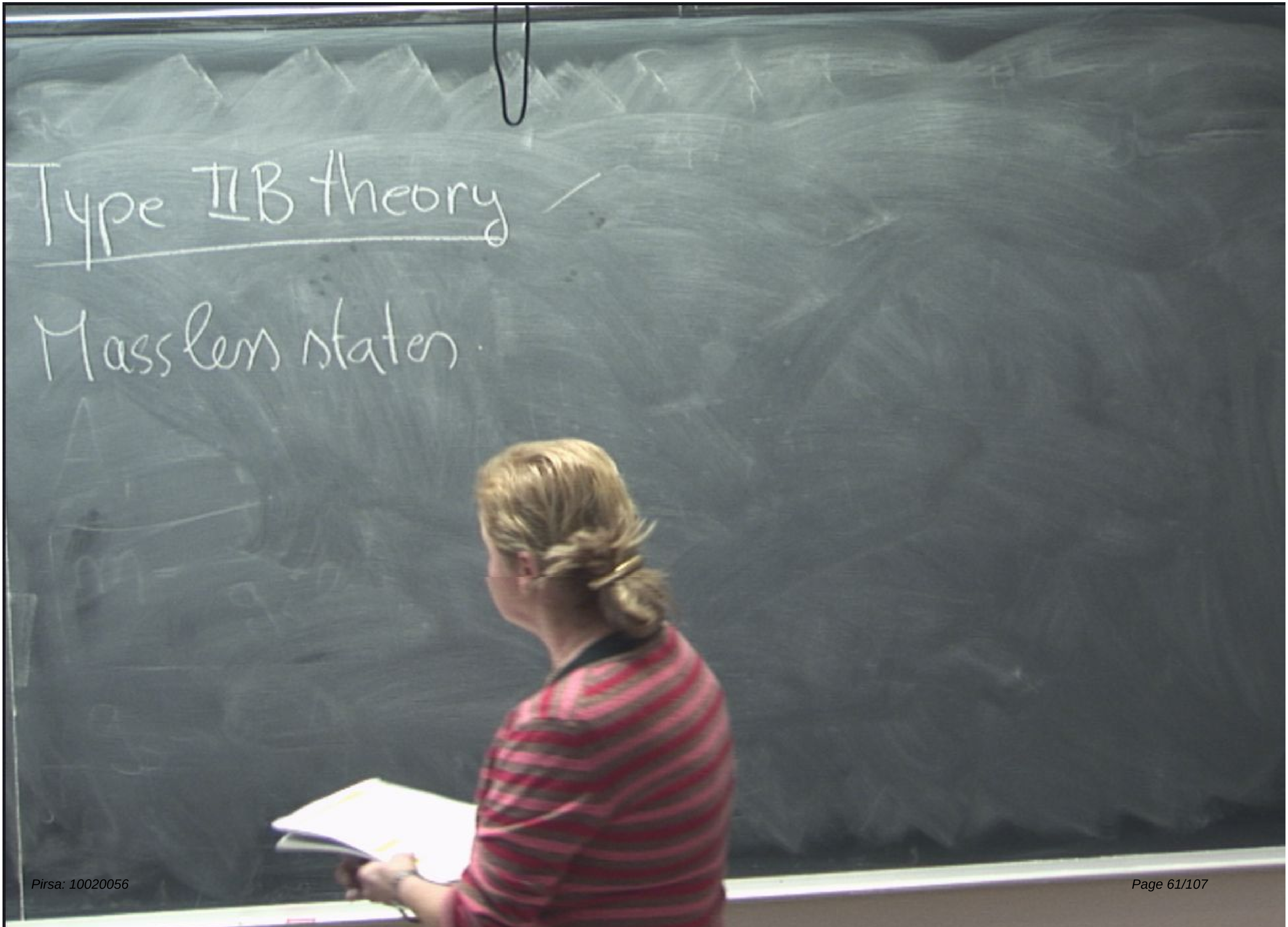
$$\frac{\text{Type IIB}}{1/R}$$

$$\frac{\text{M-theory}}{T^2} \simeq \frac{\text{IIB}}{R}$$


Type IIB theory


x T-duality
R ~ 1/R
x S-duality
g_s ~ 1/g_s
IIA
IIB
11D
I
SO(32)
E8xE8
T- duality
Type IIA / R
Bosonic
m ~ NR +


x T-duality
R ~ 1/R
x S-duality
gs ~ 1/gs
IIA
IIB
11D
I
SO(32)
E8xE8
T-duality
TypeIIA / R
Type
Bosonic
m ~ NR +

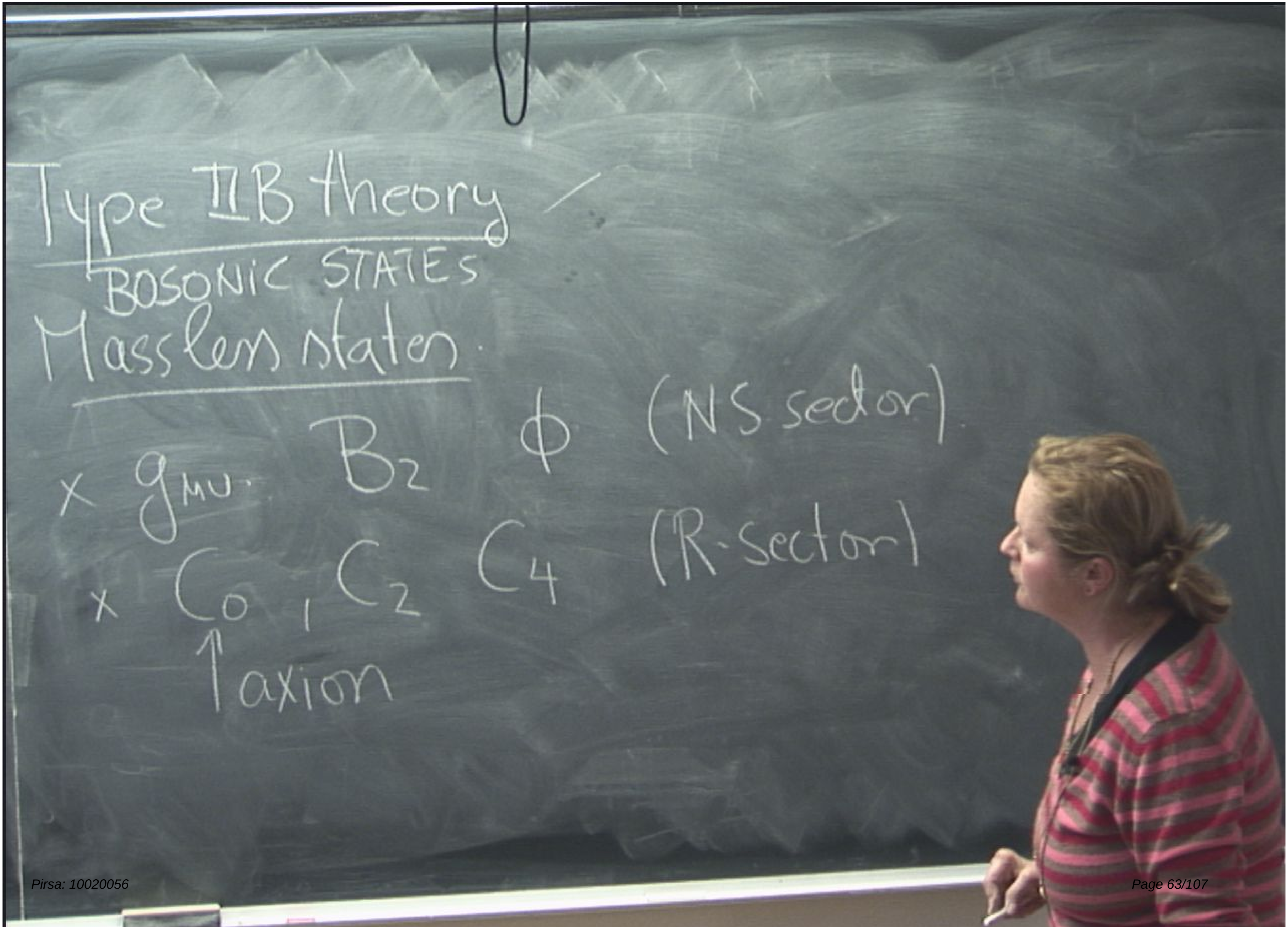

Type IIB theory
Massless states.

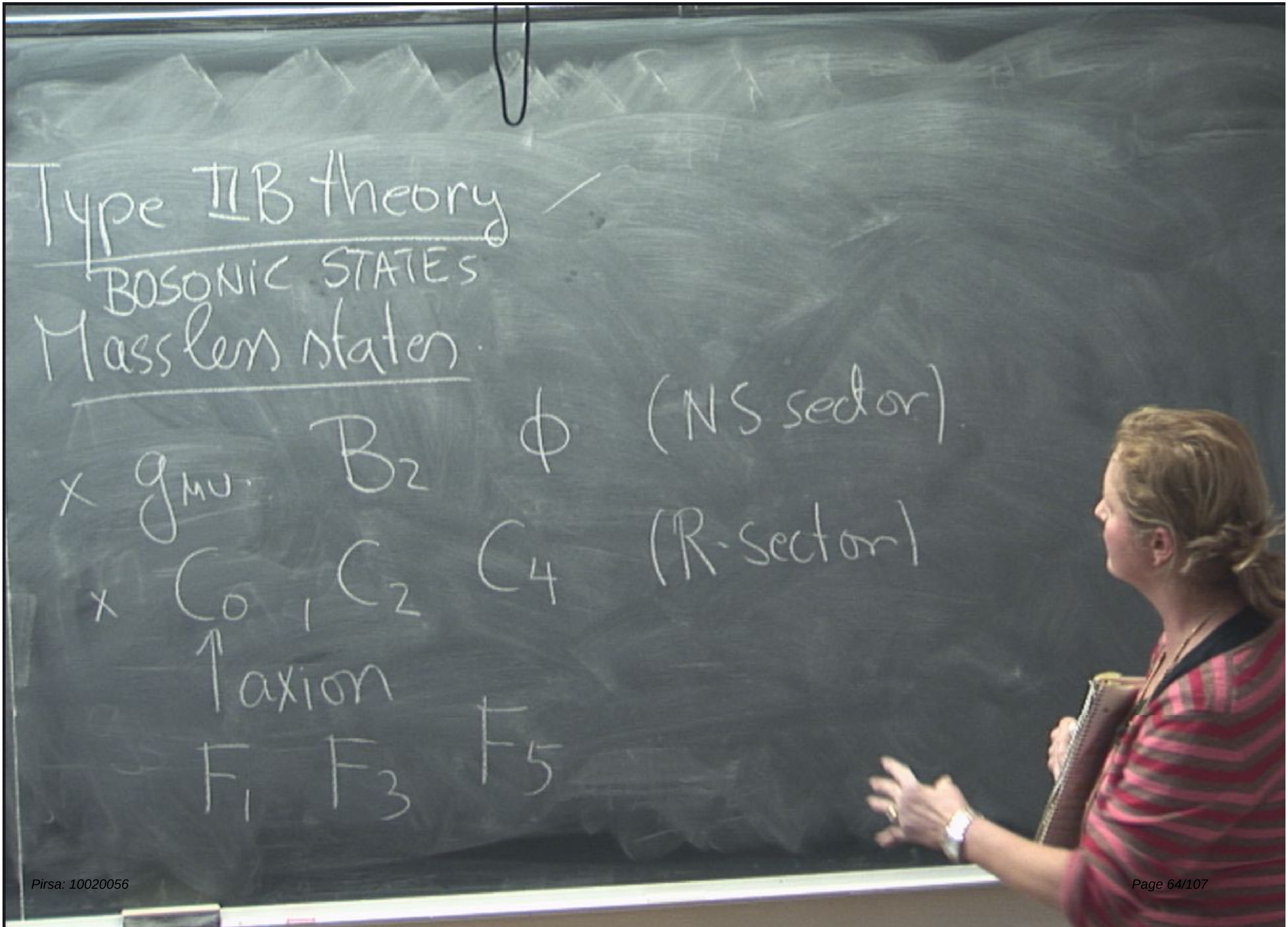
Type IIB theory

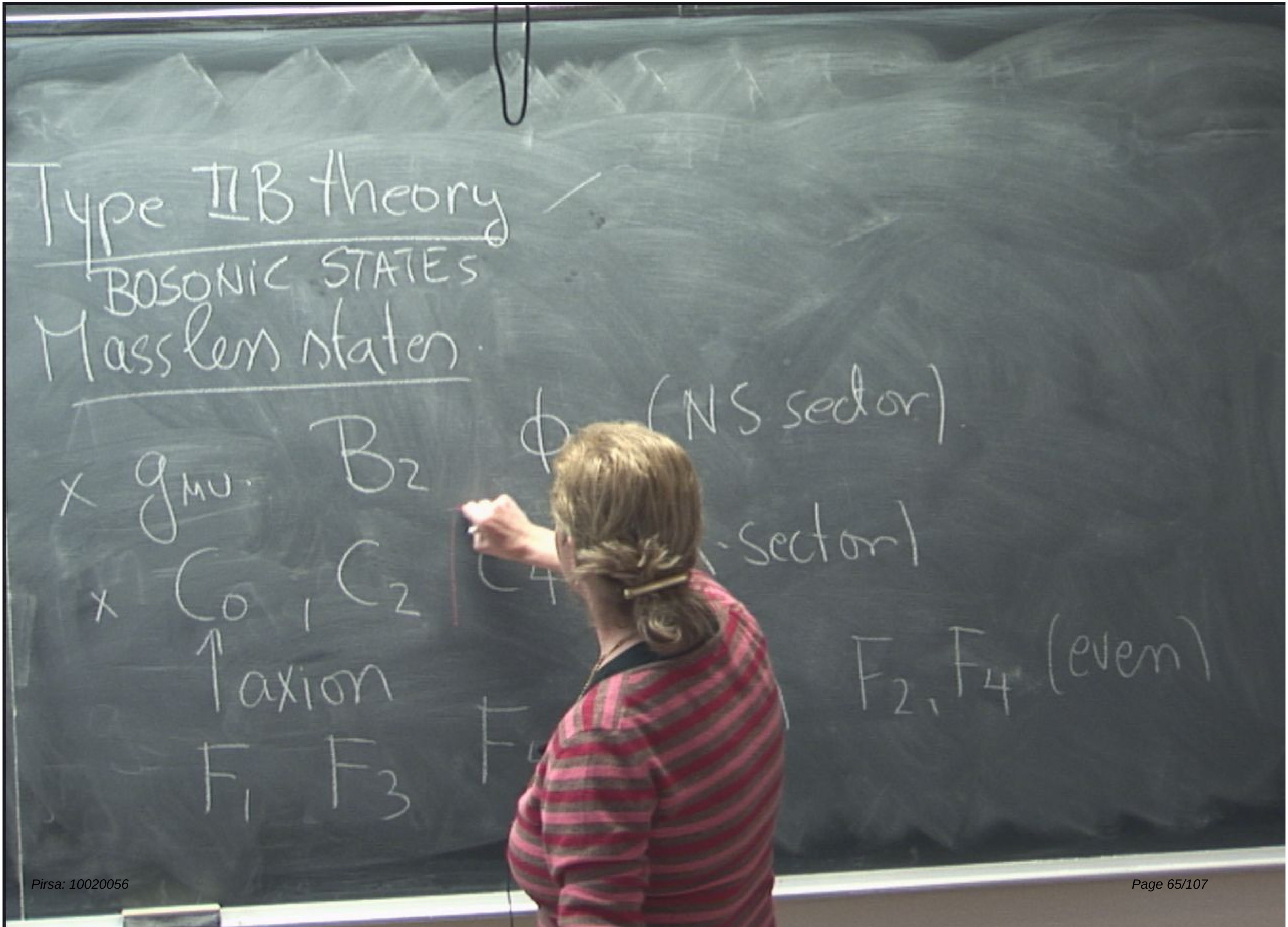
BOSONIC STATES

Massless states

met


Type IIB theory
BOSONIC STATES
Massless states.
x g_MU. B_2 φ (NS sector).
x C_0 , C_2 C_4 (R-sector)
↑ axion


Type IIB theory
BOSONIC STATES
Massless states.
× $g_{\mu\nu}$. B_2 ϕ (NS sector)
× C_0, C_2 C_4 (R-sector)
axion
F_1 F_3 F_5


Type IIB theory
BOSONIC STATES
Massless states
x gMU. B2 ϕ (NS sector).
x C0, C2 C4 sector)
axion
F2, F4 (even)
F1 F3

Type IIB theory

BOSONIC STATES

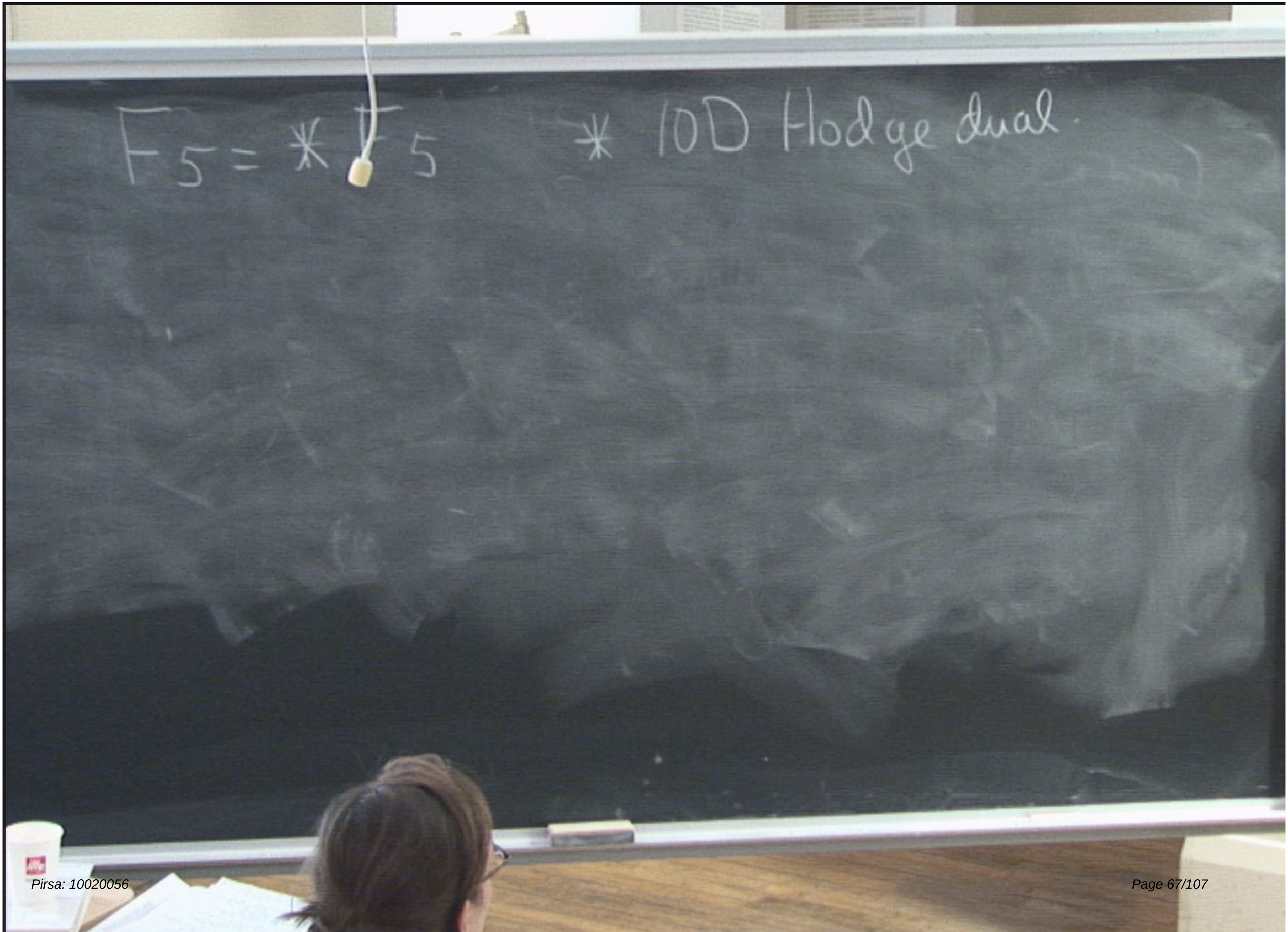
Massless states

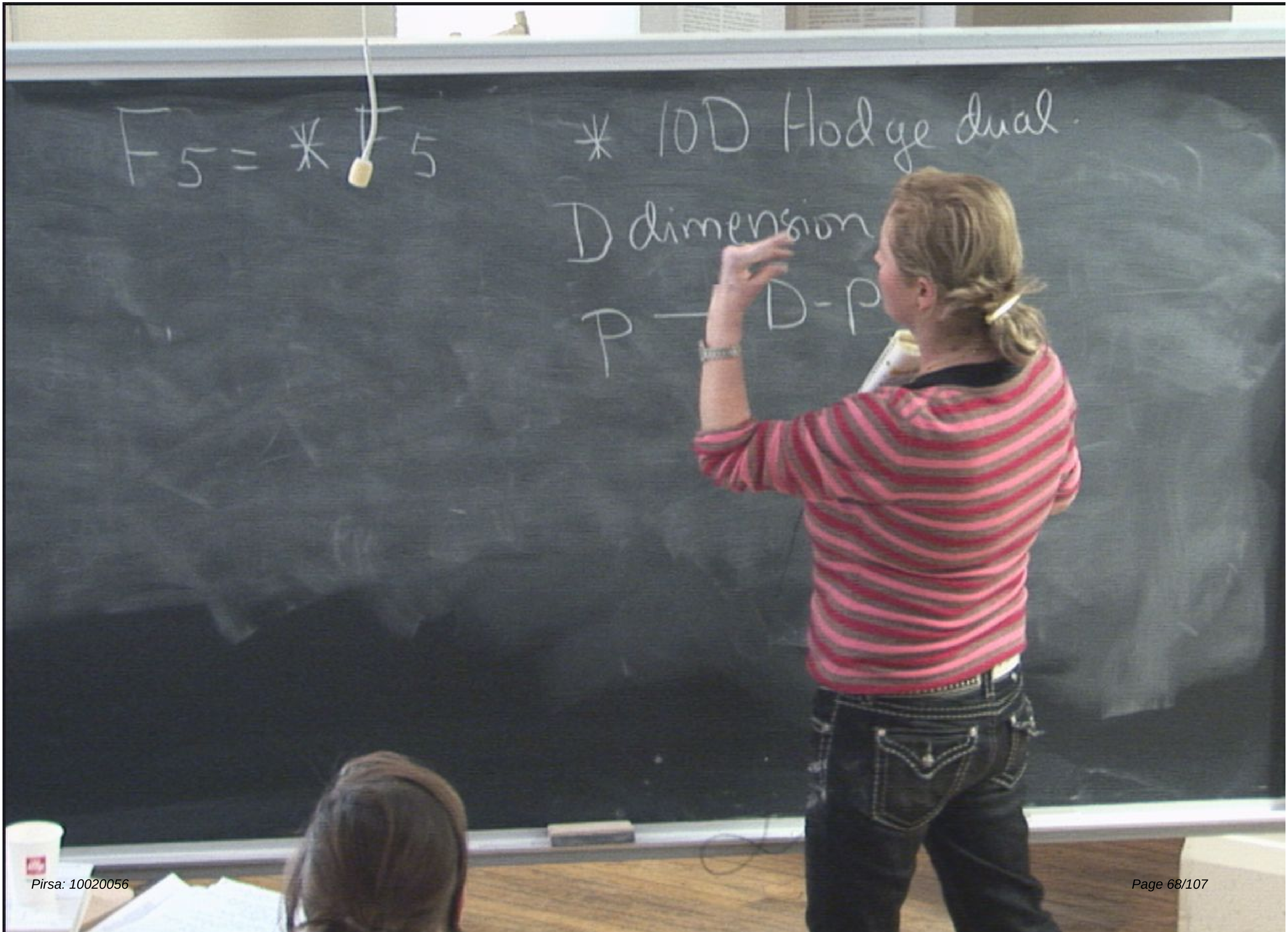
$\times\ g_{\mu\nu}$. B_2 ϕ (NS sector)

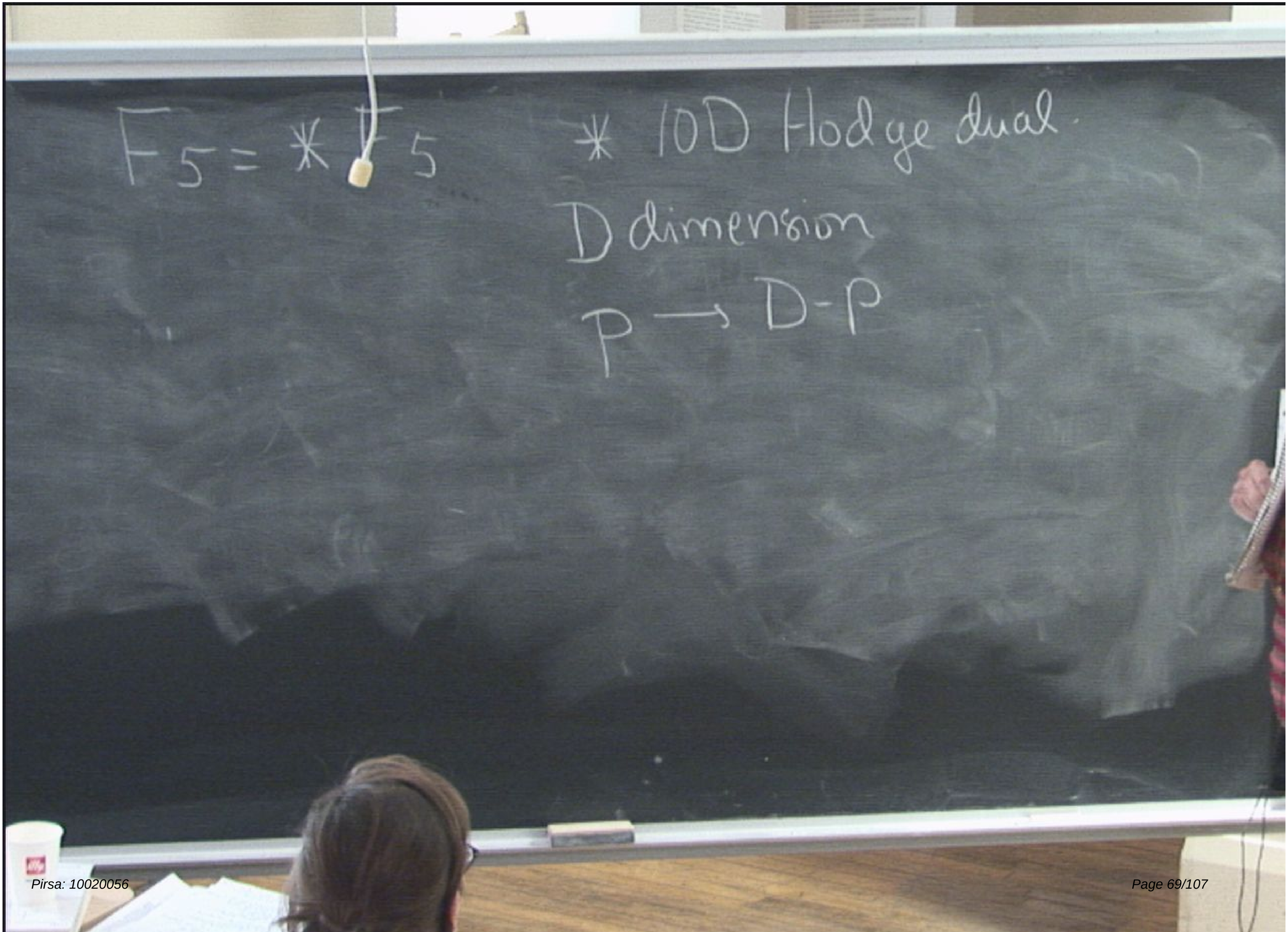
$\times\ C_0$, C_2 $\boxed{C_4}$ (R-sector)

↑ axion

F_1 F_3 F_5 IIA F_2, F_4 (even)


$F_5 = *F_5$
$*$ 10D Hodge dual.


F5 = * F5
* 10D Hodge dual.
D dimension
p — D-p


$F_5 = * F_5$
* 10D Hodge dual.
D dimension
$p \longrightarrow D-p$

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$\int d^{10}x \, |F_5|^2$

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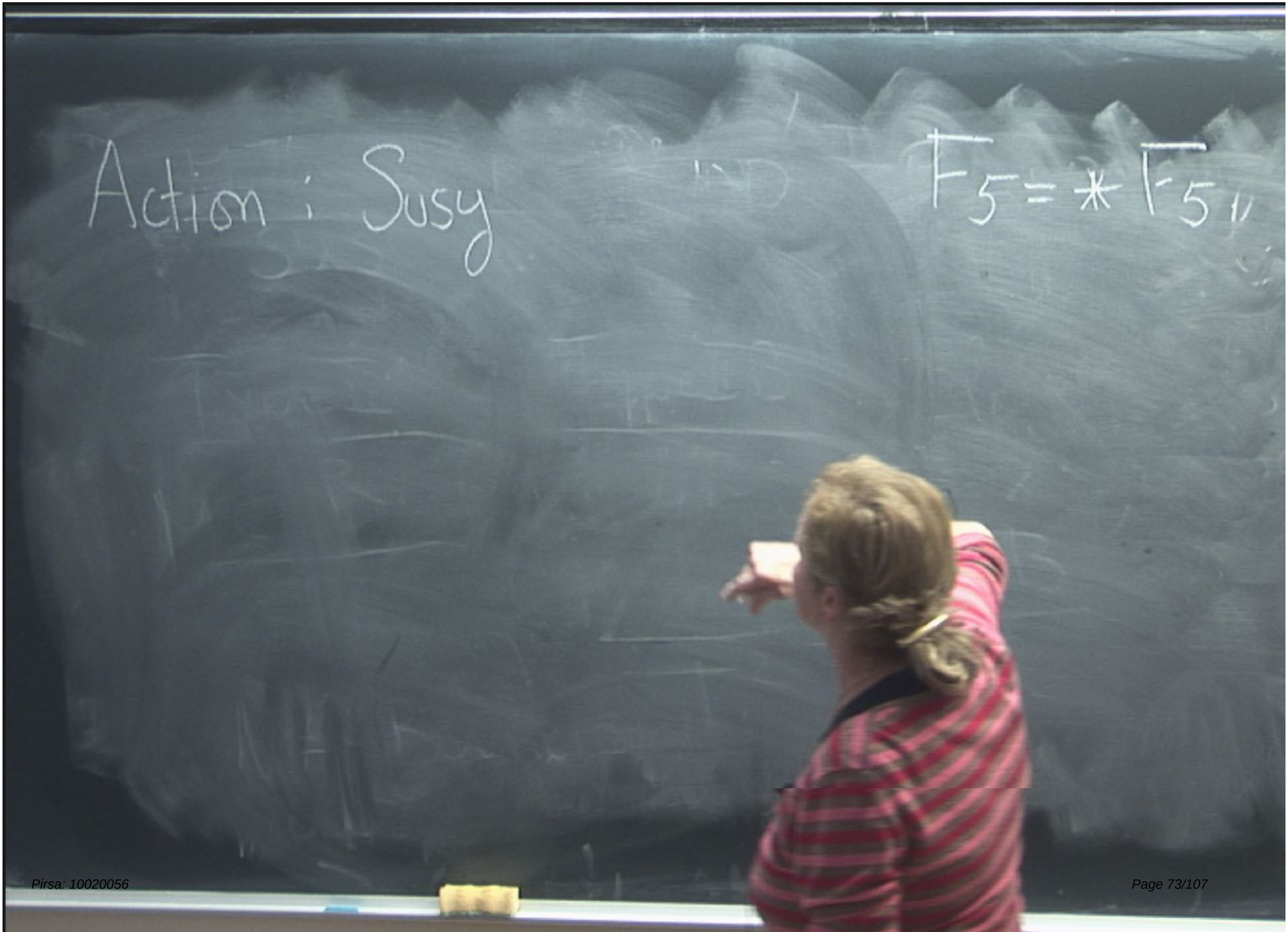
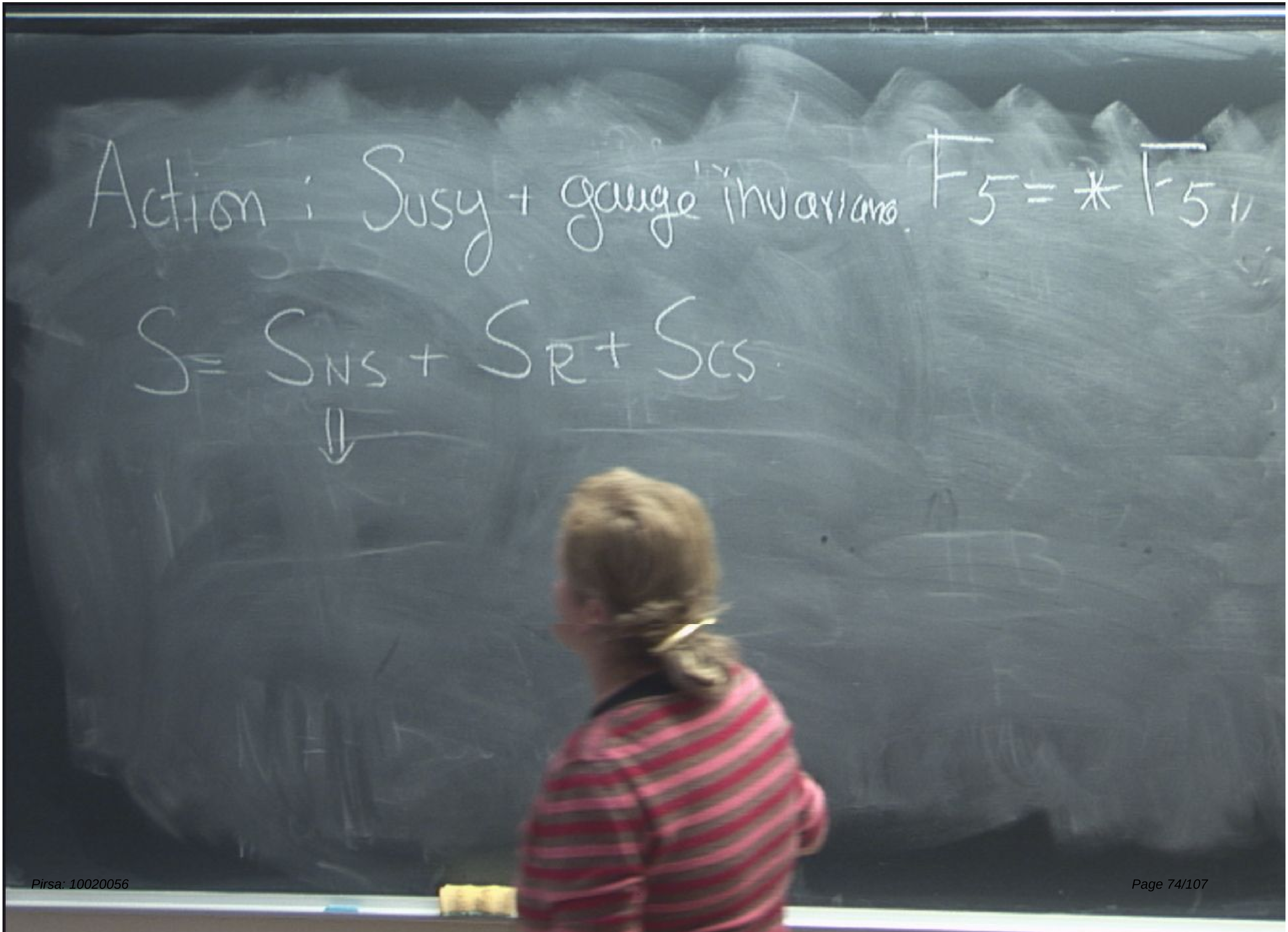

Action : Susy
$F_5 = * F_5$


Action : Susy + gauge invariance. $F_5 = *F_5$
$S = S_{NS} + S_R + S_{CS}$

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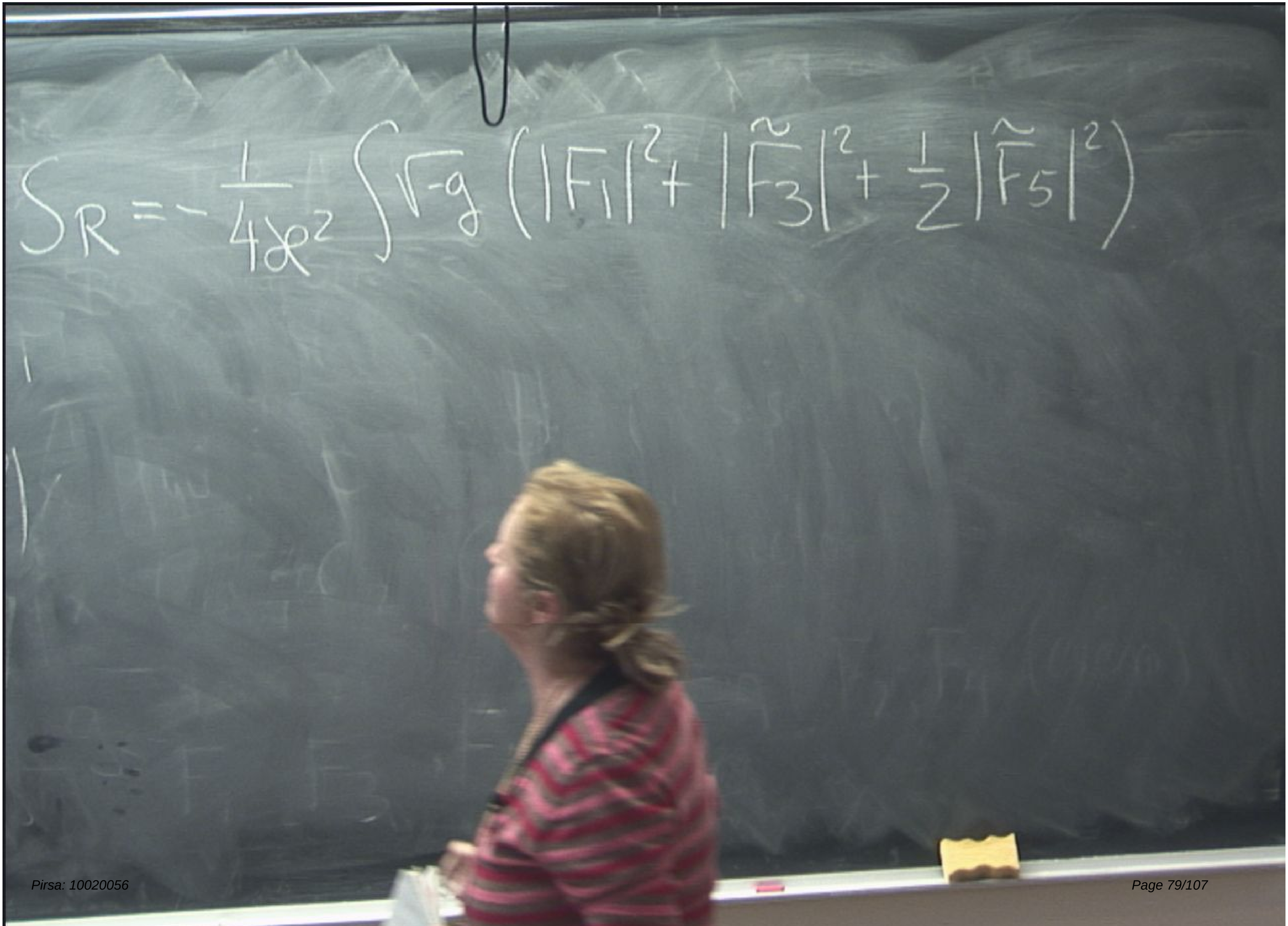
$$S_{NS} = \frac{-1}{4\kappa^2} \int d^{10}x \sqrt{-g}\, e^{-2\phi} \left(R + 4(\partial\phi)^2 - \frac{1}{2}|H_3|^2\right)$$


$S_R = -\frac{1}{4\kappa^2}\int\sqrt{-g}\left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right)$

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Check gauge invariance

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$\Downarrow$

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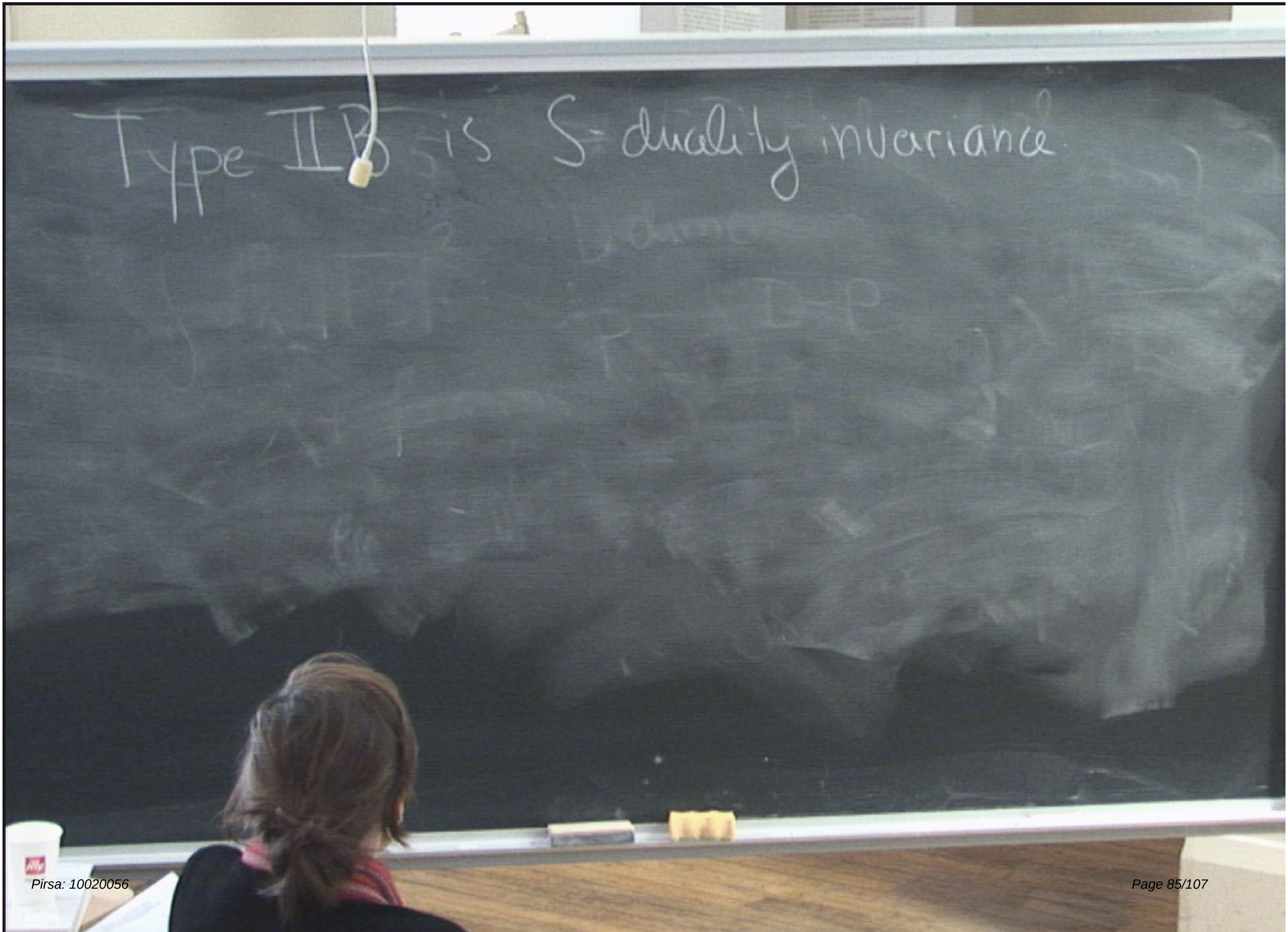
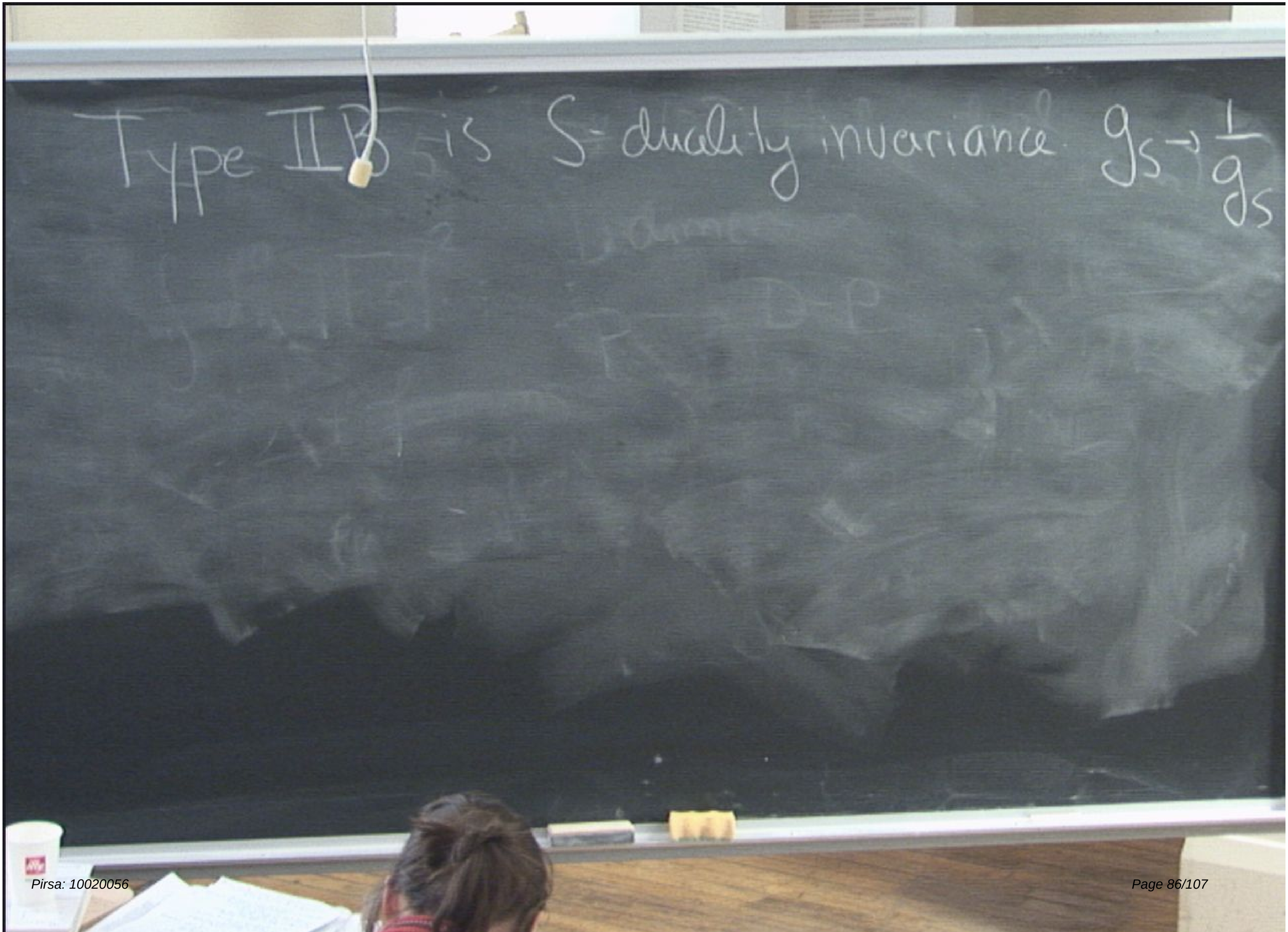
$\tilde{F}_4 = dA_3 + \cdots$

$A_3 \to d\Lambda_2$


Type IIB is S-duality invariance


Type IIB is S-duality invariance $g_s \to \frac{1}{g_s}$

Type IIB sugra is invariant under $SL(2,\mathbb{R})$

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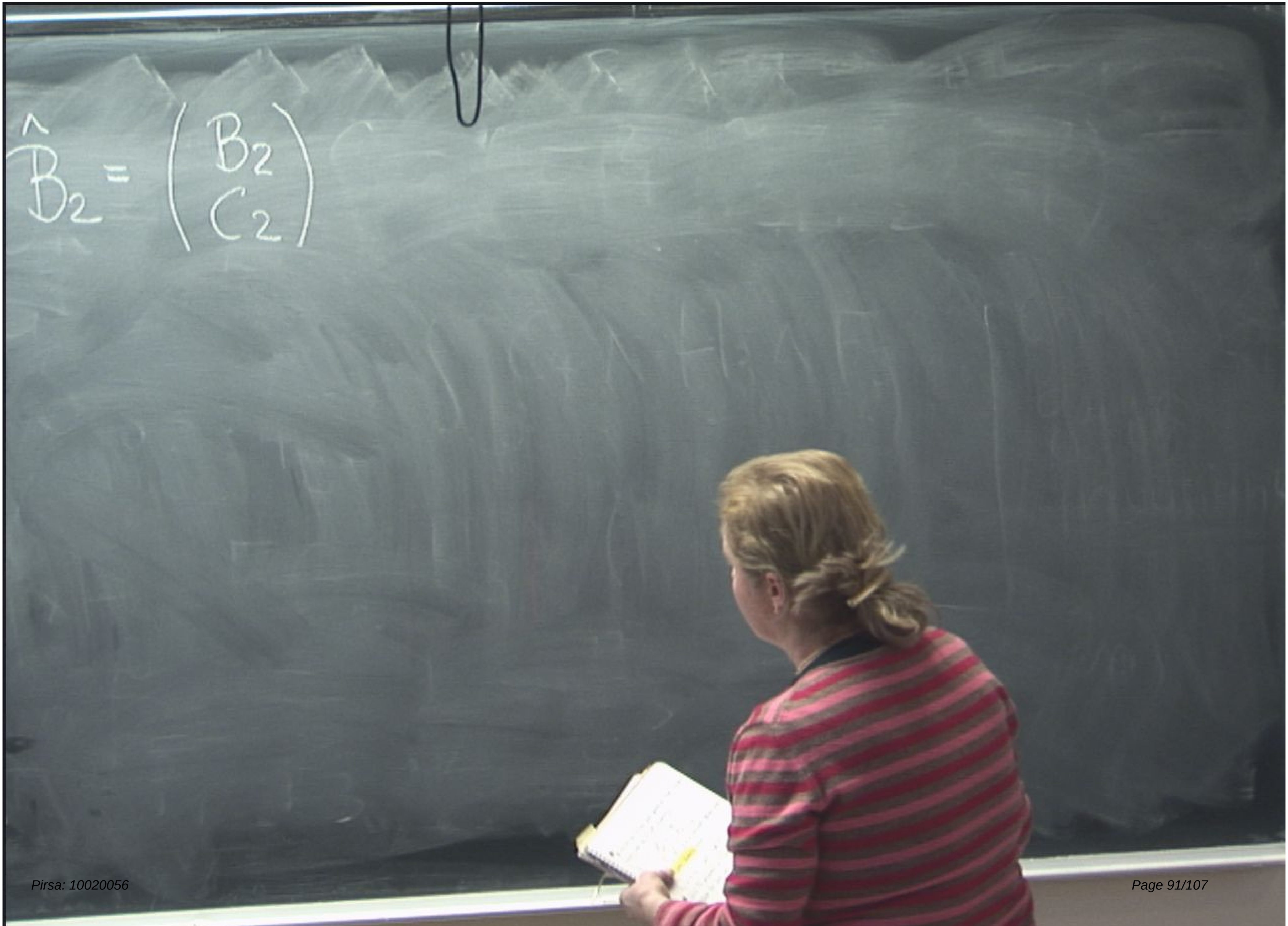
$\Rightarrow SL(2,\mathbb{Z})$ full string theory.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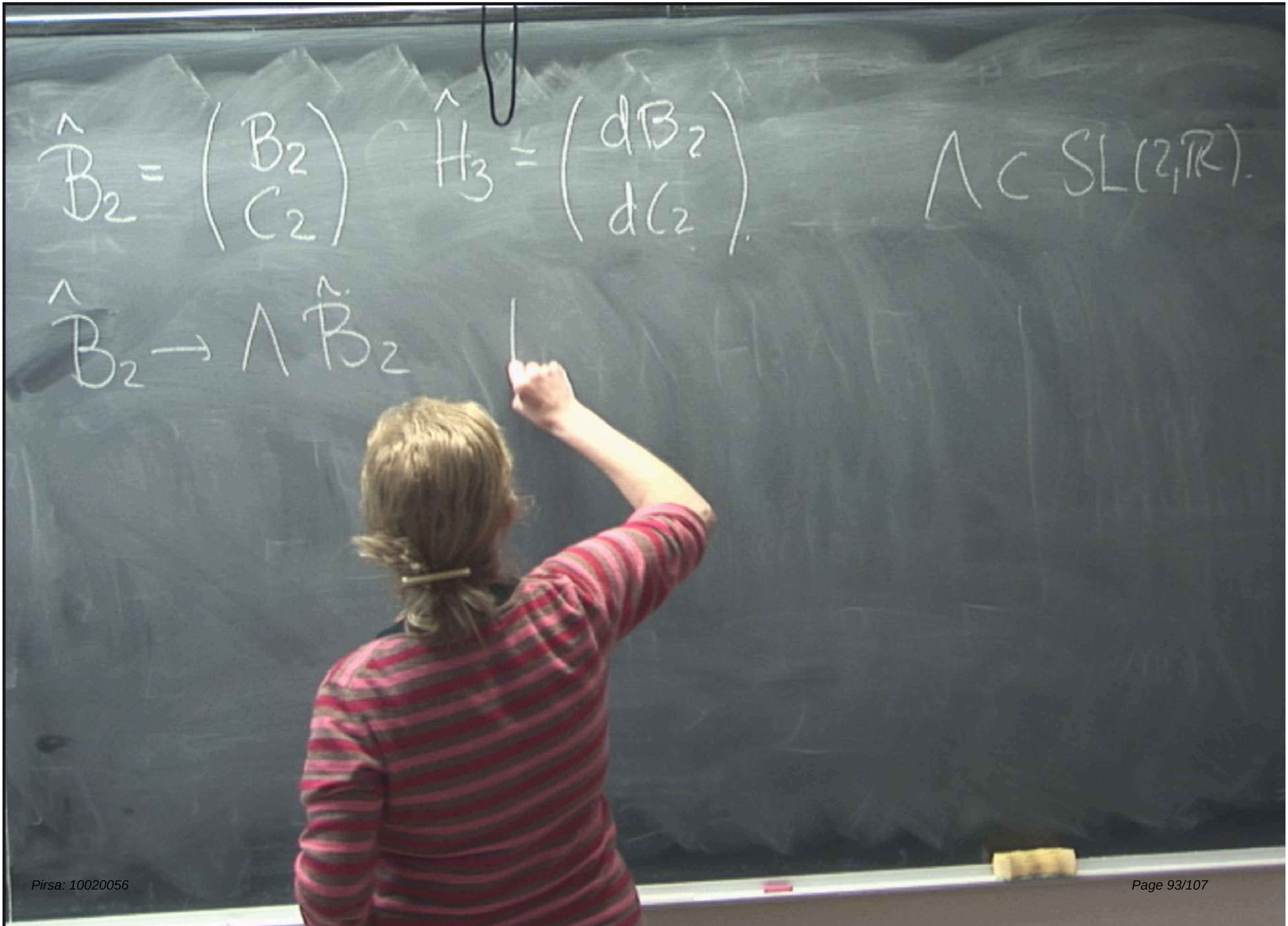
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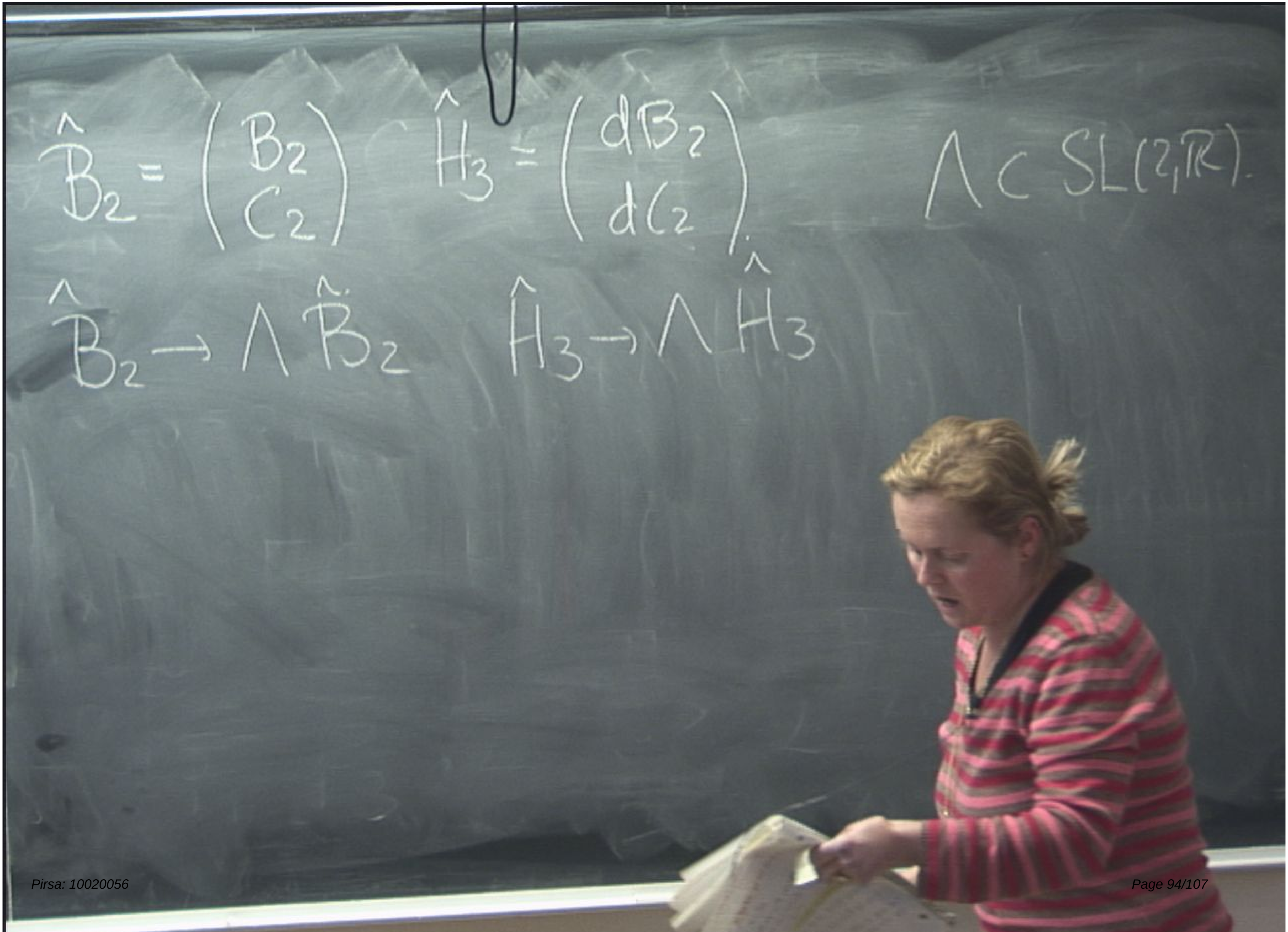

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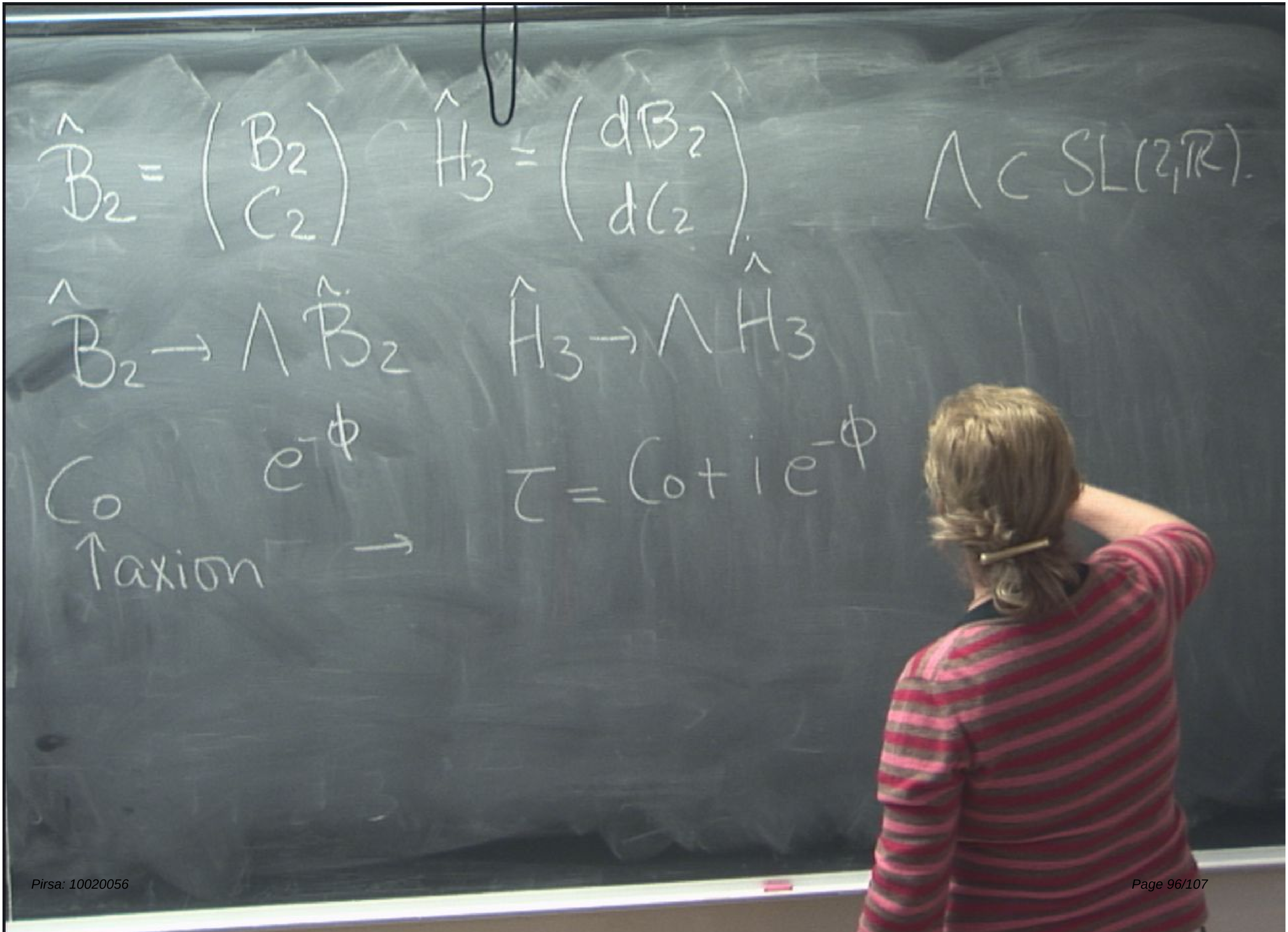
$\uparrow$ axion $\longrightarrow$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↑axion →

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$\uparrow$ axion $\longrightarrow$ $a=0$ $b=1$ $d=0$ $c=-1$

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d} \quad \tau \to -\frac{1}{\tau}$$

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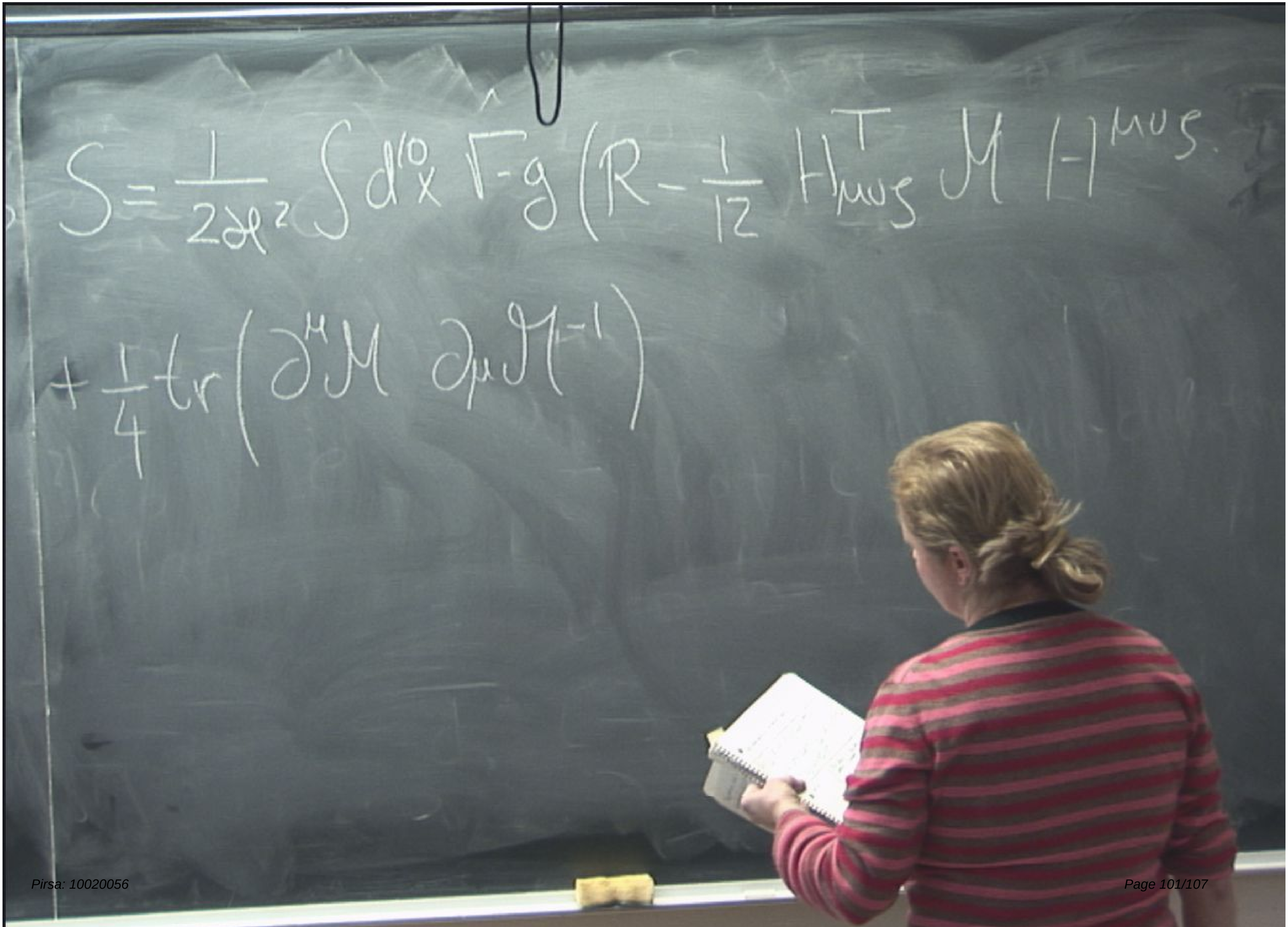
$a = 0 \quad b = 1 \quad d = 0 \quad c = -1$

$$\tau \to \frac{a\tau + b}{c\tau + d} \qquad \tau \to -\frac{1}{\tau} \qquad g_s \to \frac{1}{g_s}$$ S-duality

$$\mathcal{M} = e^{\phi}\begin{pmatrix} |\tau|^2 & -C_0 \\ -C_0 & 1 \end{pmatrix}$$

$$\mathcal{M}^{-1} = e^{\phi}\begin{pmatrix} 1 & C_0 \\ C_0 & |\tau|^2 \end{pmatrix}$$

$$\mathcal{M} \to (\Lambda^{-1})^T \mathcal{M} \Lambda^{-1}$$


$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left(R - \frac{1}{12} H^{T}_{\mu\nu\rho} \mathcal{M} H^{\mu\nu\rho} \right.$
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$$+ \int \varepsilon^{ij}\, C_4 \wedge H_3^{(i)} \wedge H_3^{(j)}$$

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$$\Lambda = e^{\phi}\begin{pmatrix} |\tau|^2 & -C_0 \\ -C_0 & 1 \end{pmatrix}$$

① Eqeuivalence of actions.

$$\mathcal{M}^{-1} = e^{\phi}\begin{pmatrix} 1 & C_0 \\ C_0 & |\tau|^2 \end{pmatrix}$$

② Invariance under $SL(2,\mathbb{R})$

$$\mathcal{M} \to (\Lambda^{-1})^T \mathcal{M} \Lambda^{-1} \quad \tau$$