Title: String Theory - Review (PHYS 623) - Lecture 10

Date: Feb 05, 2010 11:20 AM

URL: http://pirsa.org/10020054

Abstract:

Summary

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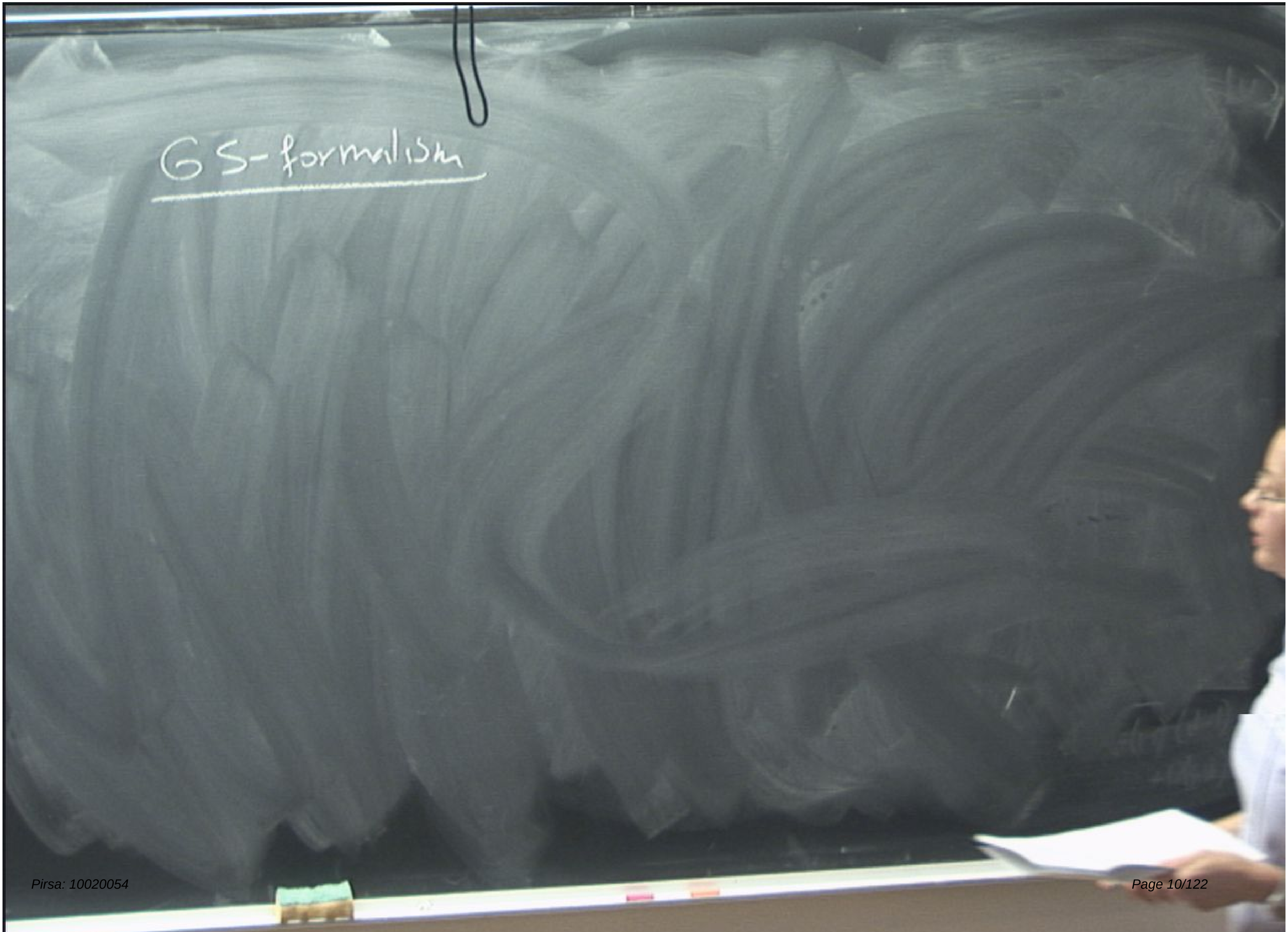
GS-formalism

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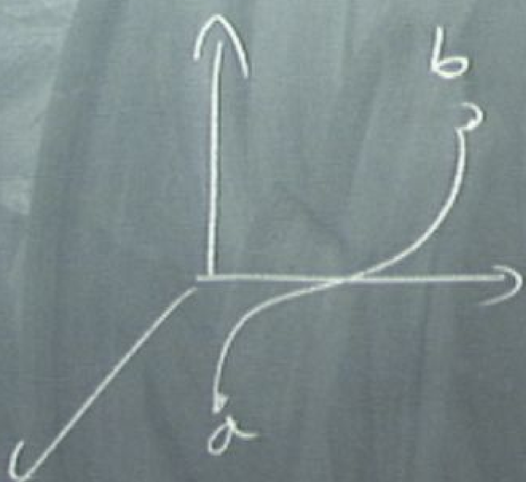

$$S = -m \cdot \int_a^b ds$$

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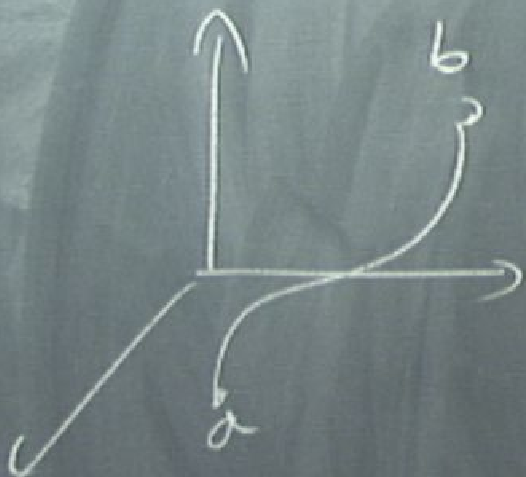
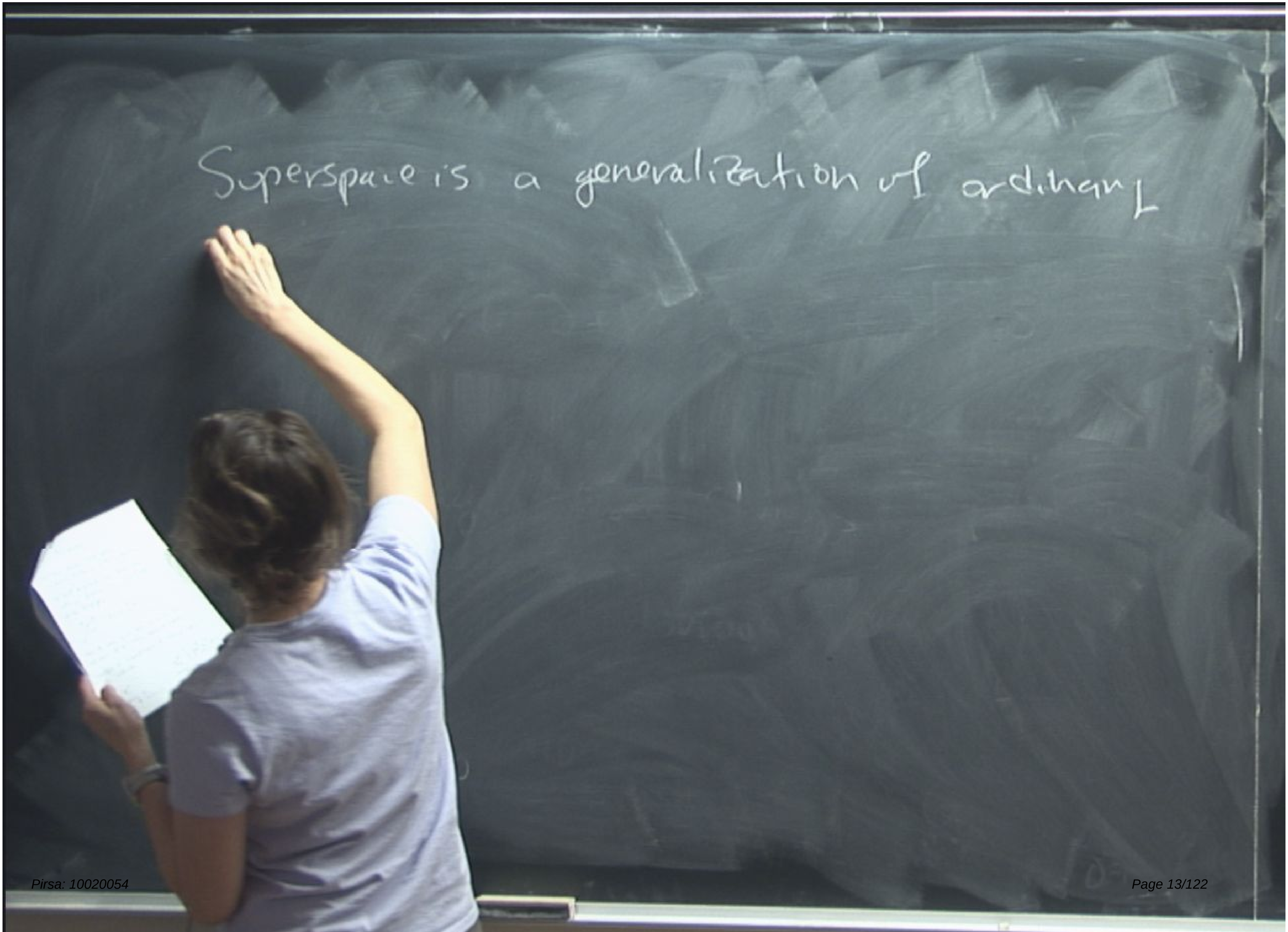


$$S = -m \cdot \int_a^b ds$$

→ invariant under Lorentz. trafo, rotations + translations (Poincaré group)!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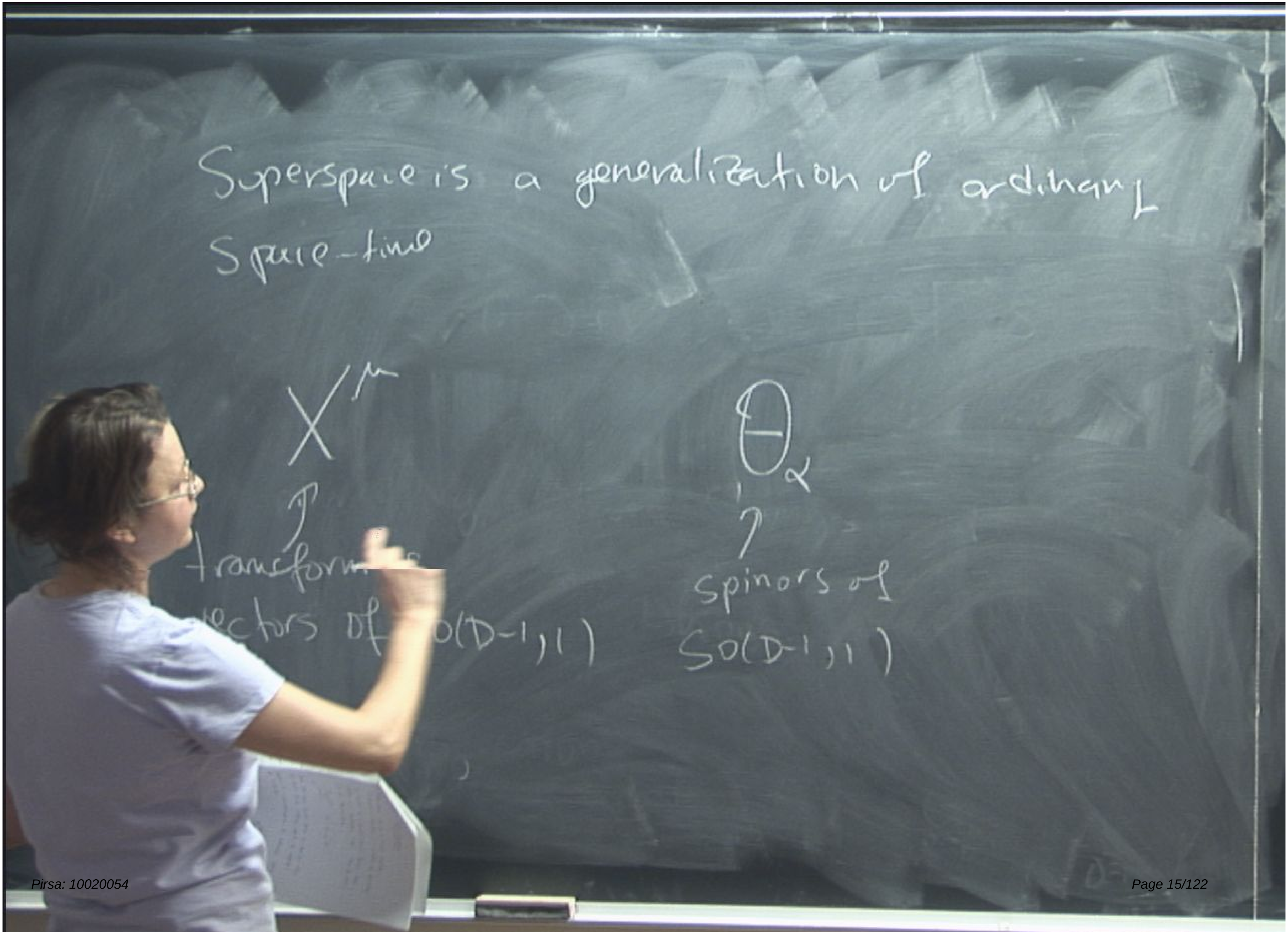
Space-time

X^μ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X^μ
θ_α
spinors of
SO(D-1,1)

Superspace is a generalization of ordinary
Space-time

X^μ

transform as
vectors of $SO(D-1,1)$

θ_α^A $A = 1, \dots, \mathcal{N}$

spinors of
$SO(D-1,1)$

Symmetries

$$x^\mu \to x^\mu + a^\mu$$

$$f(x^\mu + a^\mu) = f(x^\mu) + a^\mu \frac{\partial}{\partial x_\mu} f(x) + \mathcal{O}(a^2)$$

a is small!

$$f(x^\mu + a^\mu) = e^{i a_\mu P^\mu} \cdot f(x^\mu)$$

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$P_\mu = -i \frac{\partial}{\partial x_\mu}$ is the generator of translation translations form a non-compact abelian Lie group. The Lie algebra:

$$[P_\mu, P_\nu]$$

Rotations

Example 1

$$\vec{v}' = M \cdot \vec{v}$$

$$M = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

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$$= e^{i\phi S}$$

$SO(2)$ is abelian. $[S, S] = 0$

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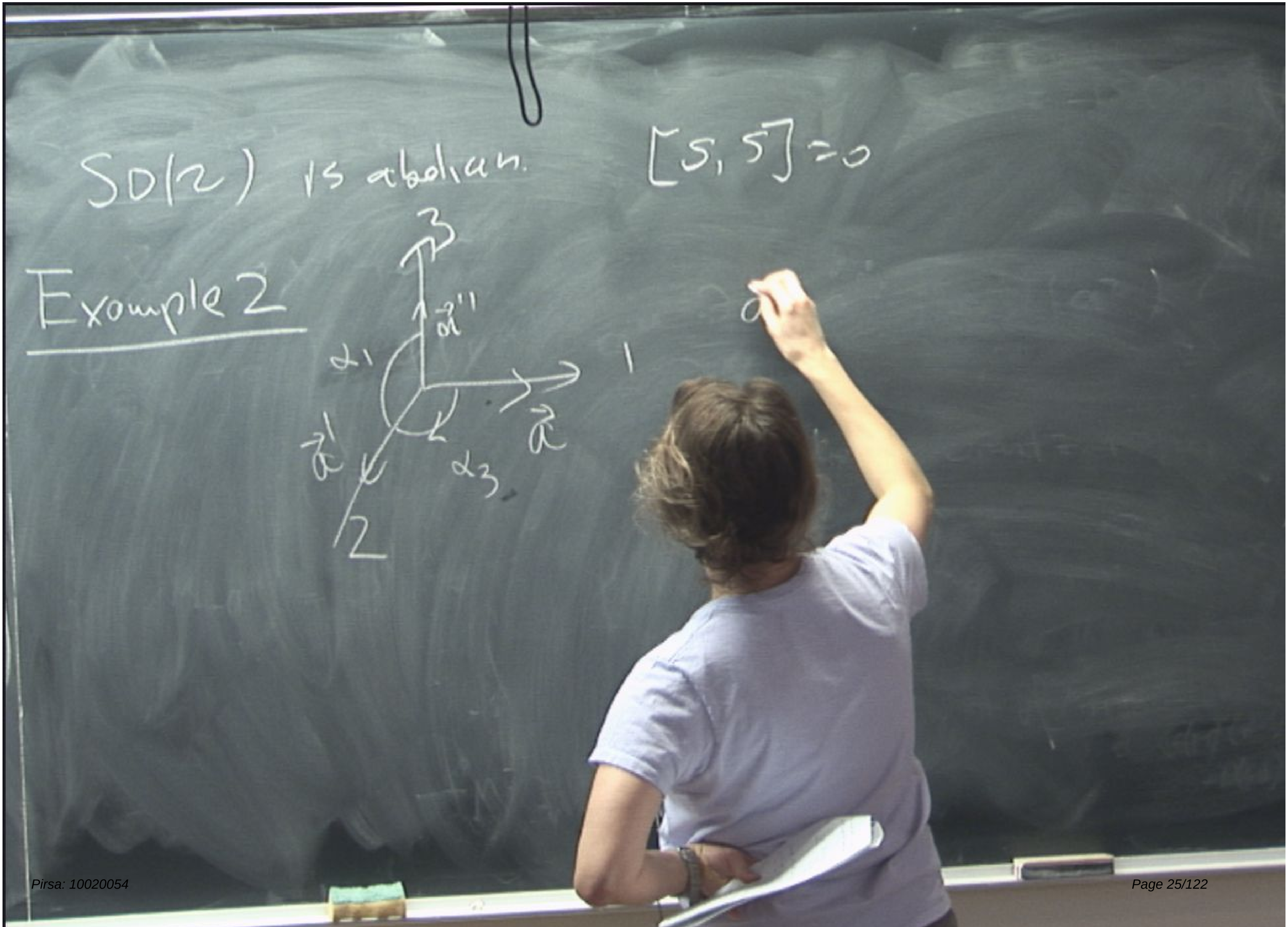
Example 2


SO(2) is abelian.
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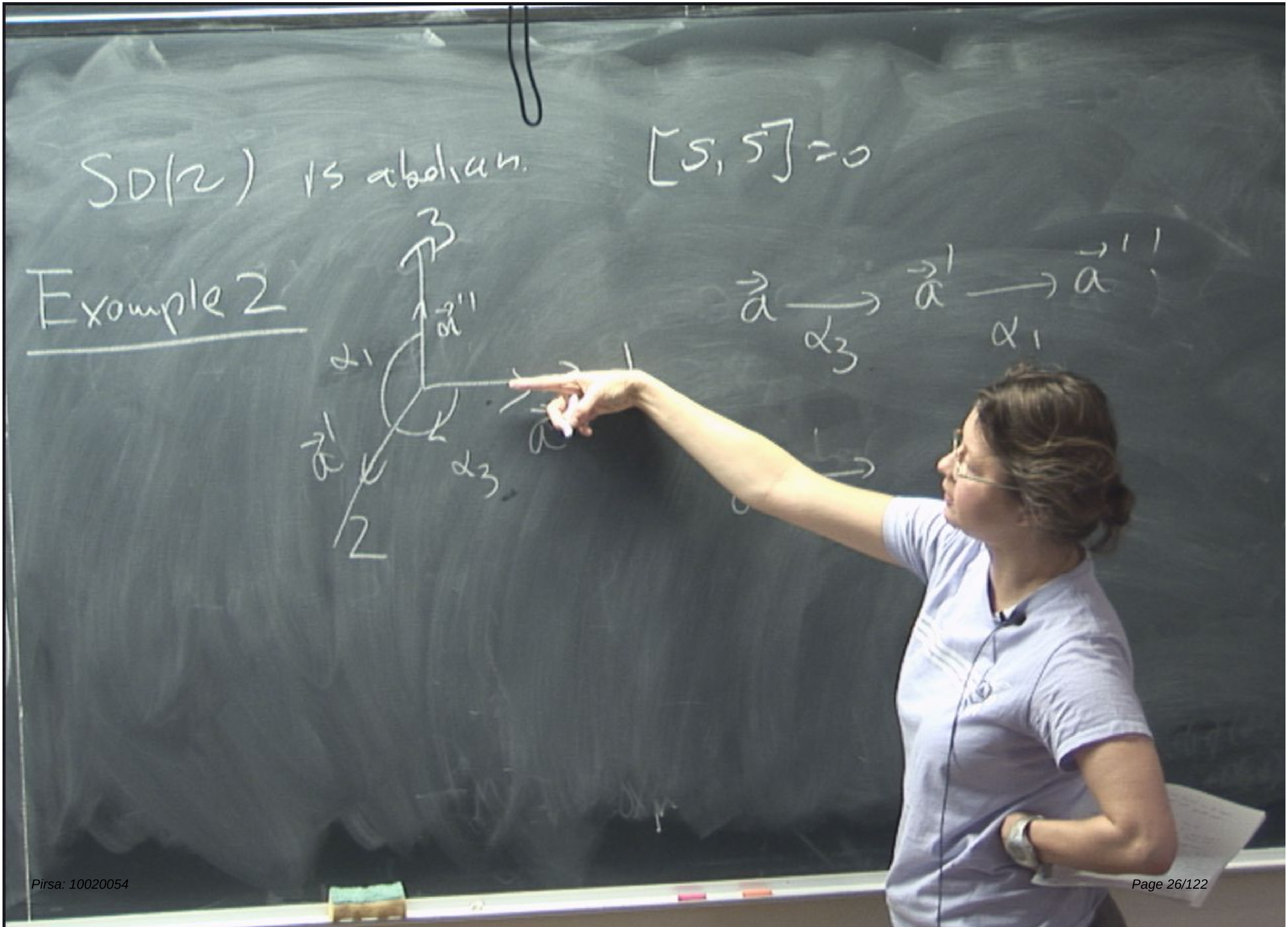
Example 2
3
$\vec{a}''$
α_1
1
$\vec{a}$
$\vec{a}'$
α_3
2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Example 2
$\vec{a} \xrightarrow[\alpha_3]{} \vec{a}' \xrightarrow[\alpha_1]{} \vec{a}''$
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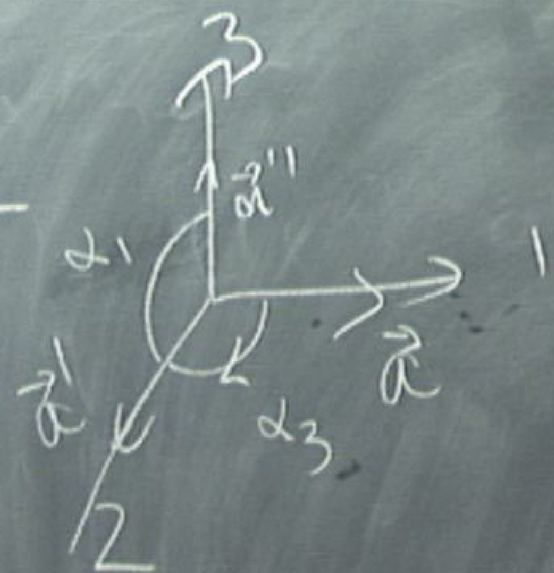

$$\vec{a} \xrightarrow[\alpha_3]{} \vec{a}' \xrightarrow[\alpha_1]{} \vec{a}''$$

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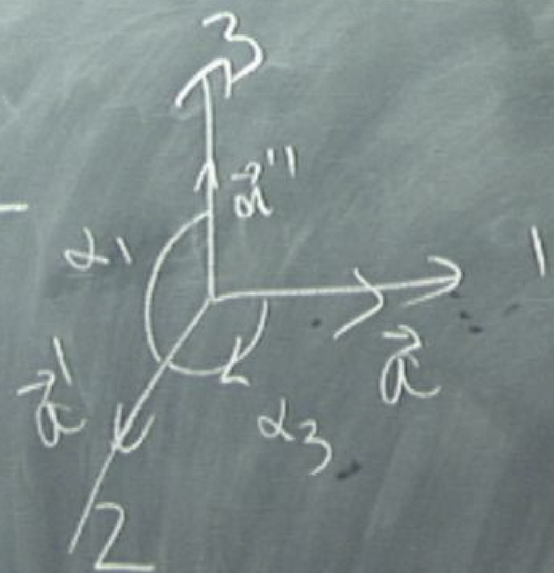

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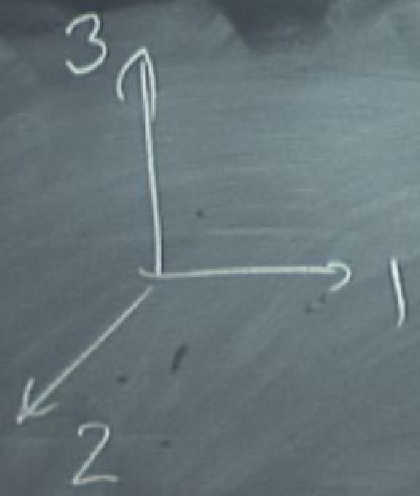
SO(3) is non-abelian



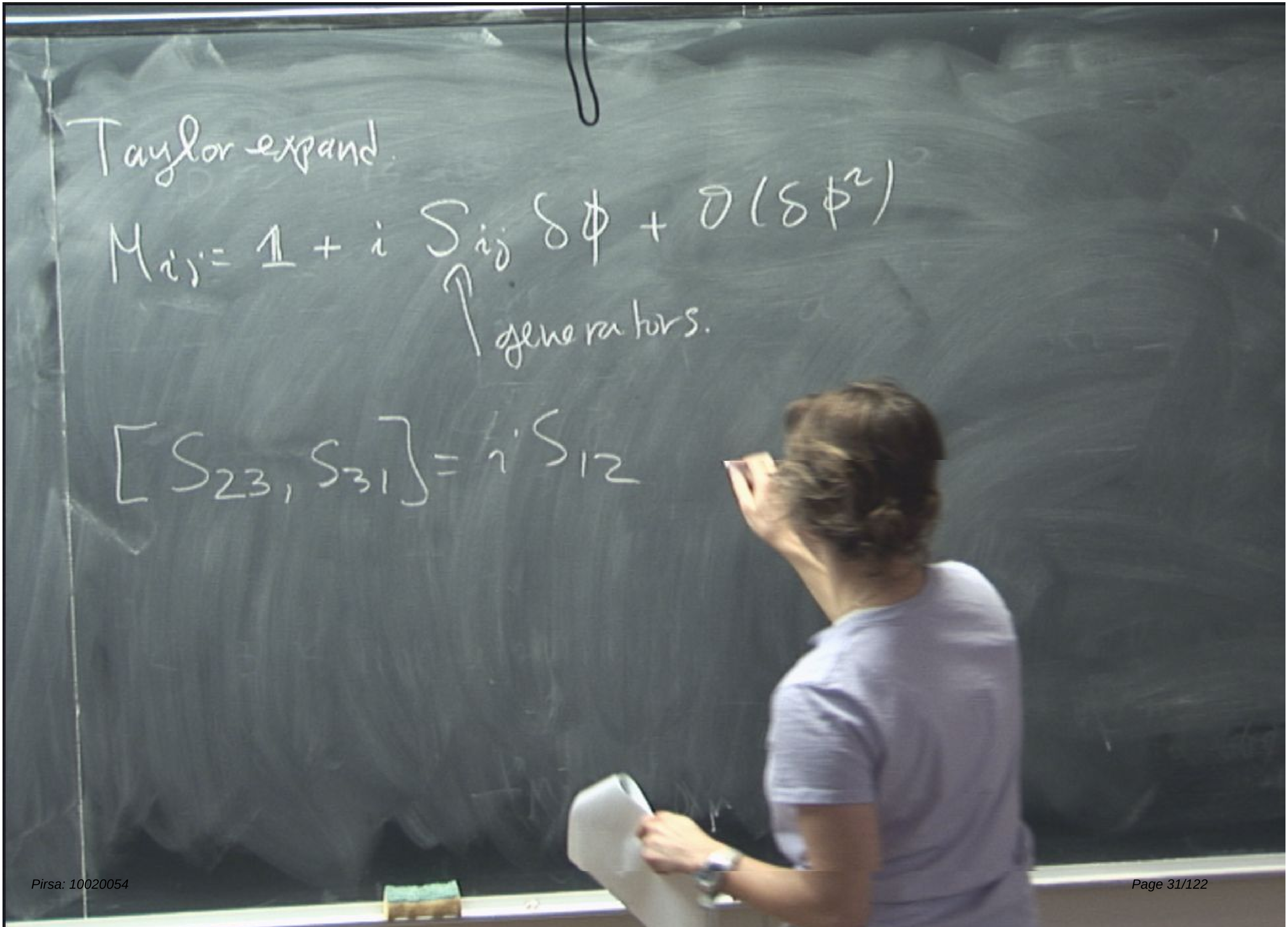
$$\vec{v}\,' = M_{ij}\,\vec{v}$$

$$M_{12} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{13} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$M_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{pmatrix}$$

Taylor expand.


Taylor expand.
$M_{ij} = \mathbb{1} + i\, S_{ij}\, \delta\phi + \mathcal{O}(\delta\phi^2)$
generators.
$[S_{23}, S_{31}] = i S_{12}$

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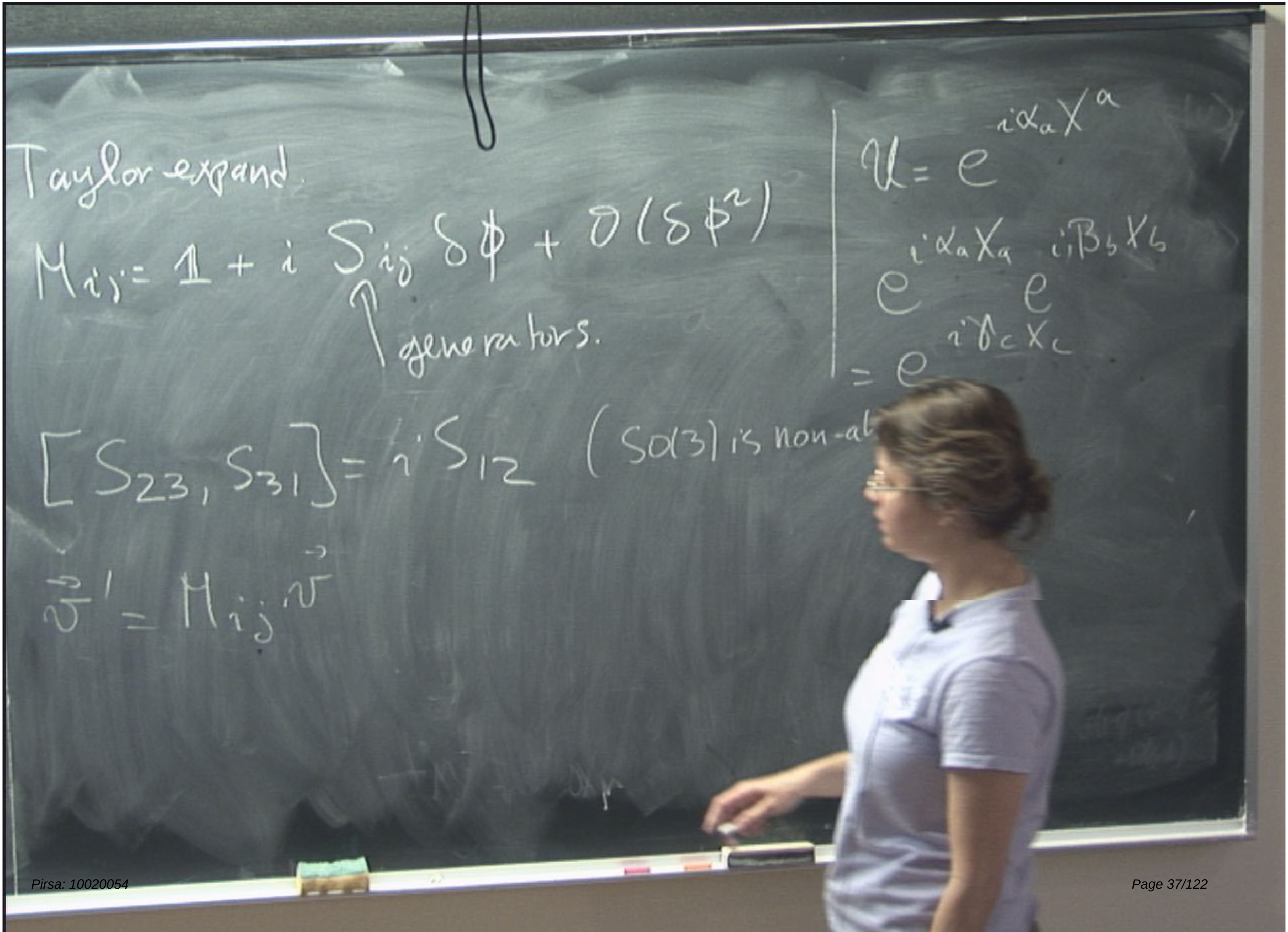
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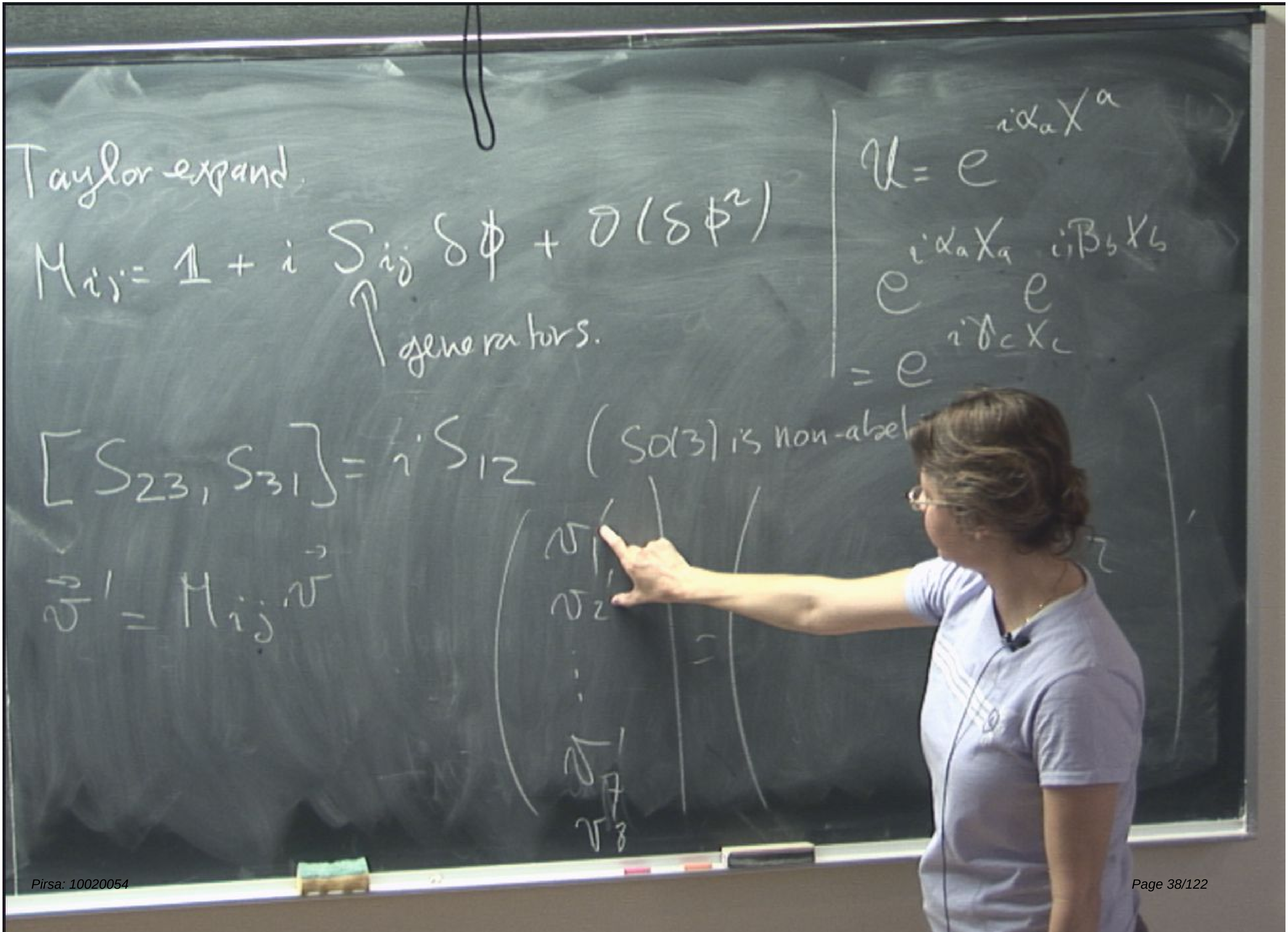
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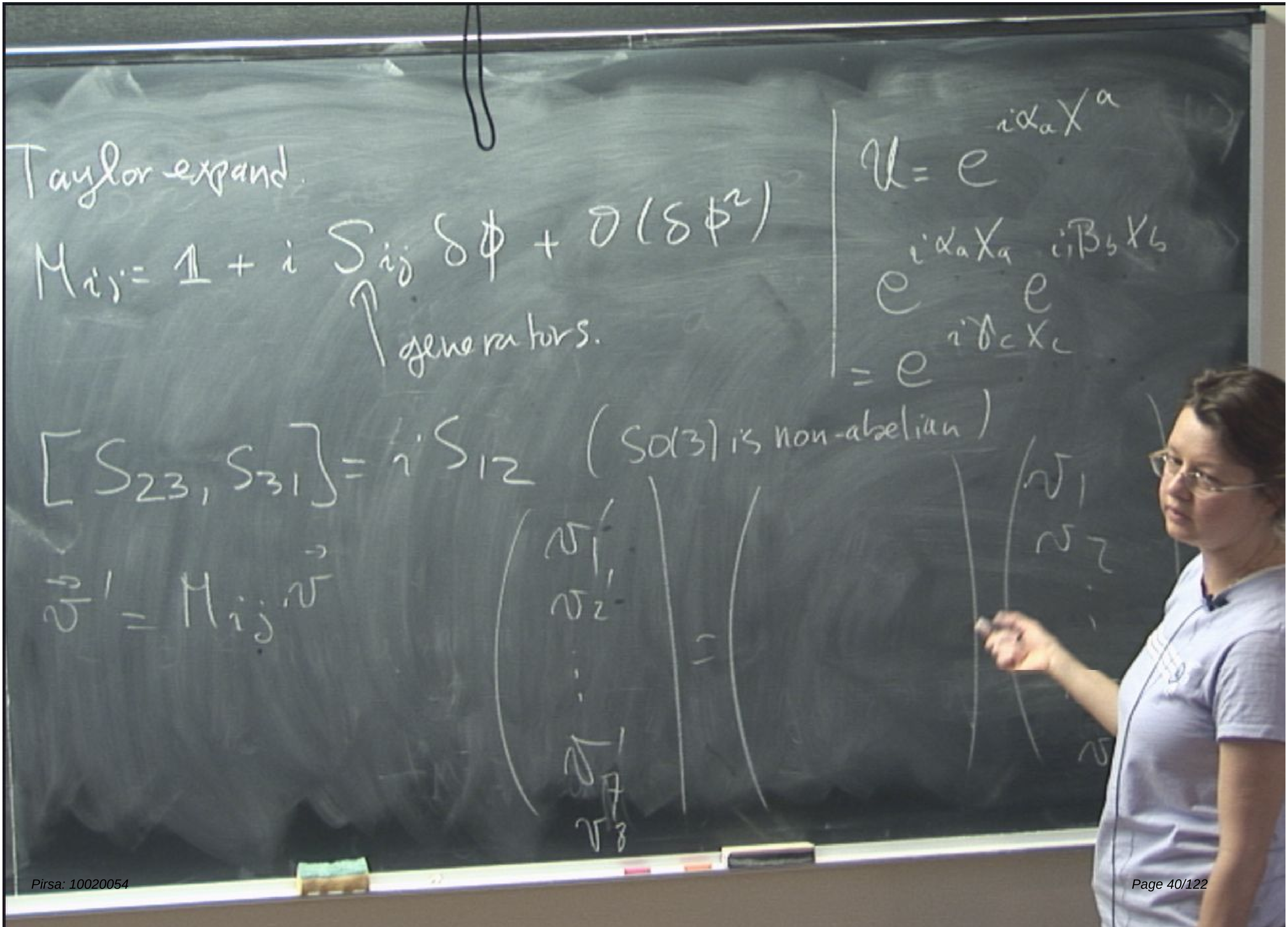
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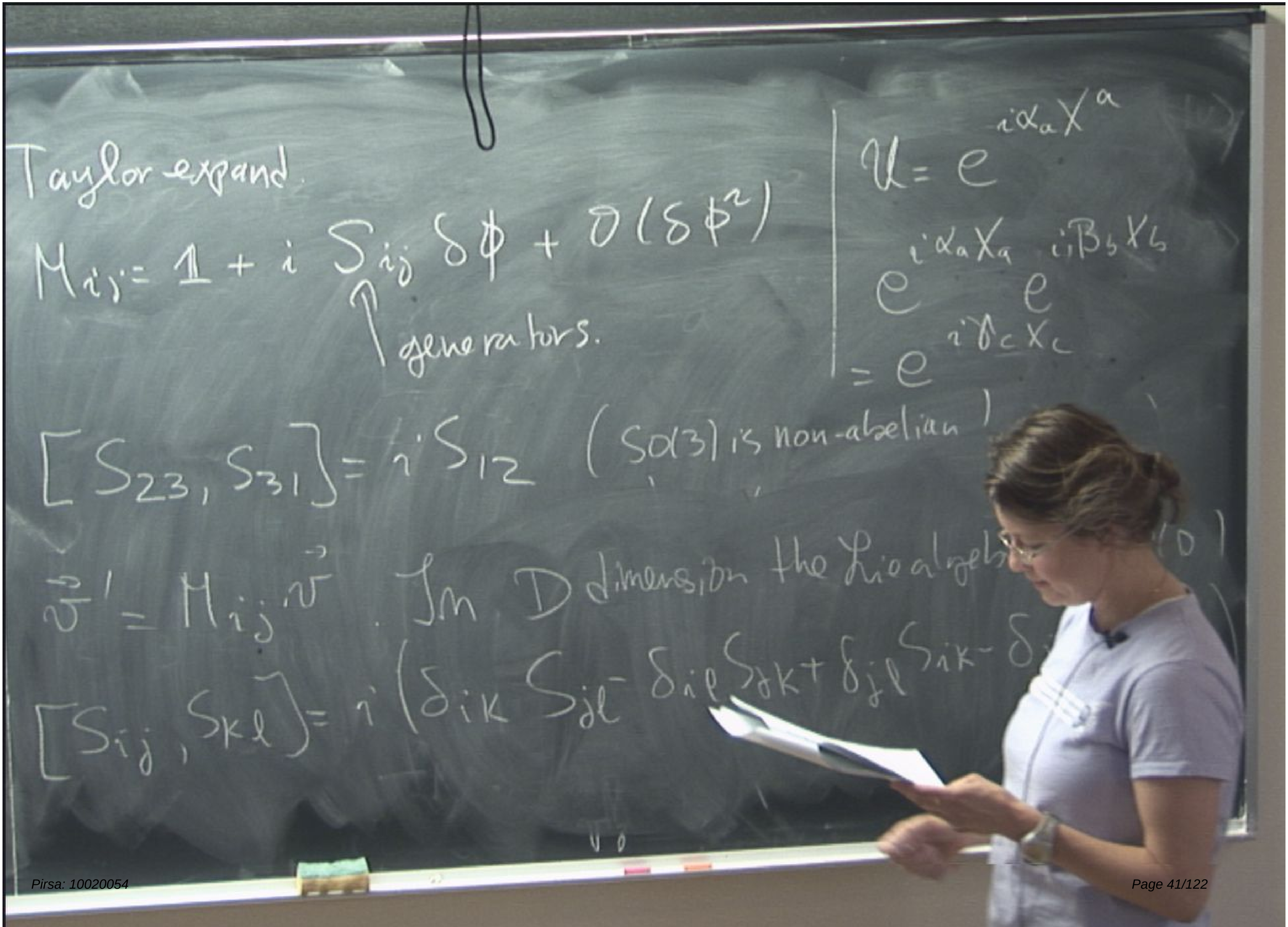
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In D dimension the Lie algebra
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In D dimension the Lie algebra SO(D)

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In D dimension the Lie algebra of SO(D)

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→ In the same way we can describe SO(D-1,1)

Translations in spinor coordinates:

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constant spinor

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constant spinor

$$X^\mu \longrightarrow X^\mu + \bar{\epsilon}_a \gamma^\mu_{ab} \theta_b$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

$$\left.\begin{aligned} \delta\theta_a &= \epsilon_a \\ \delta X^\mu &= \bar{\epsilon}\gamma^\mu\theta \end{aligned}\right\}$$ SUSY transformation.

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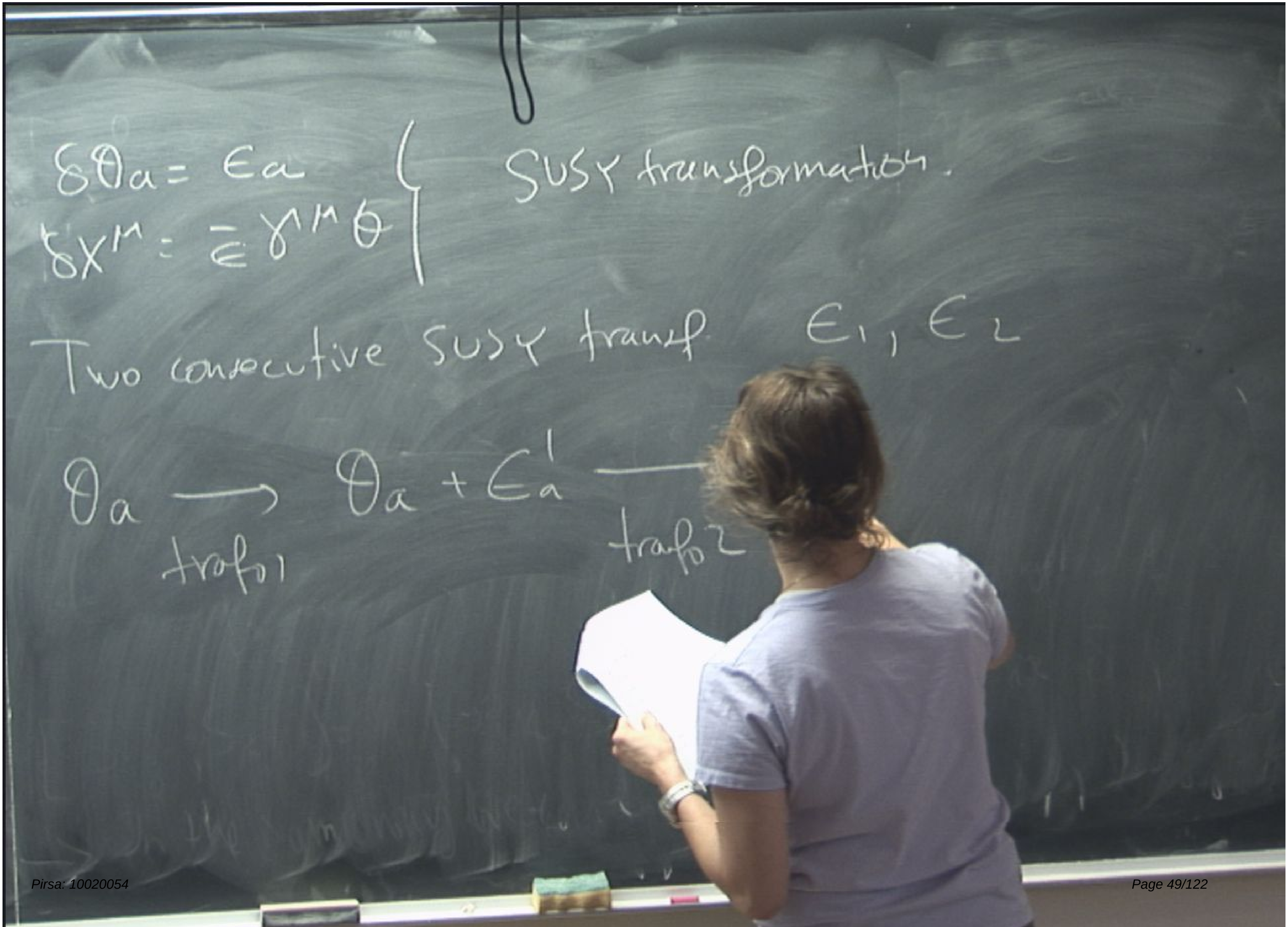
Two consecutive SUSY transf ϵ_1, ϵ_2

θ


$\delta\theta_a = \epsilon_a$
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SUSY transformation.
Two consecutive SUSY transf. ϵ_1, ϵ_2
$\theta_a \xrightarrow[\text{trafo 1}]{} \theta_a + \epsilon_a^1 \xrightarrow[\text{trafo 2}]{}$

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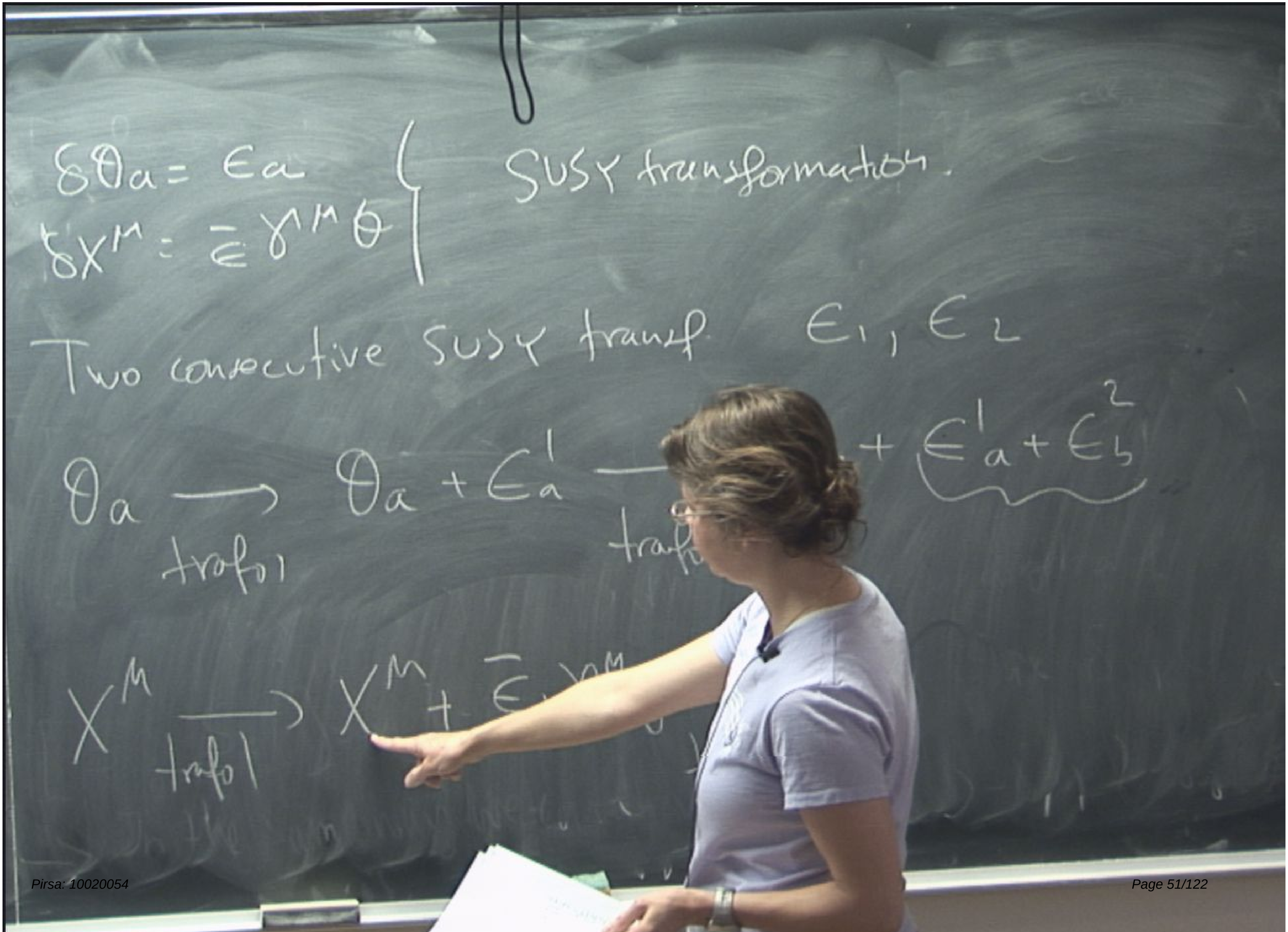
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$X^\mu \xrightarrow[\text{trafo 1}]{} X^\mu + \bar{\epsilon}$

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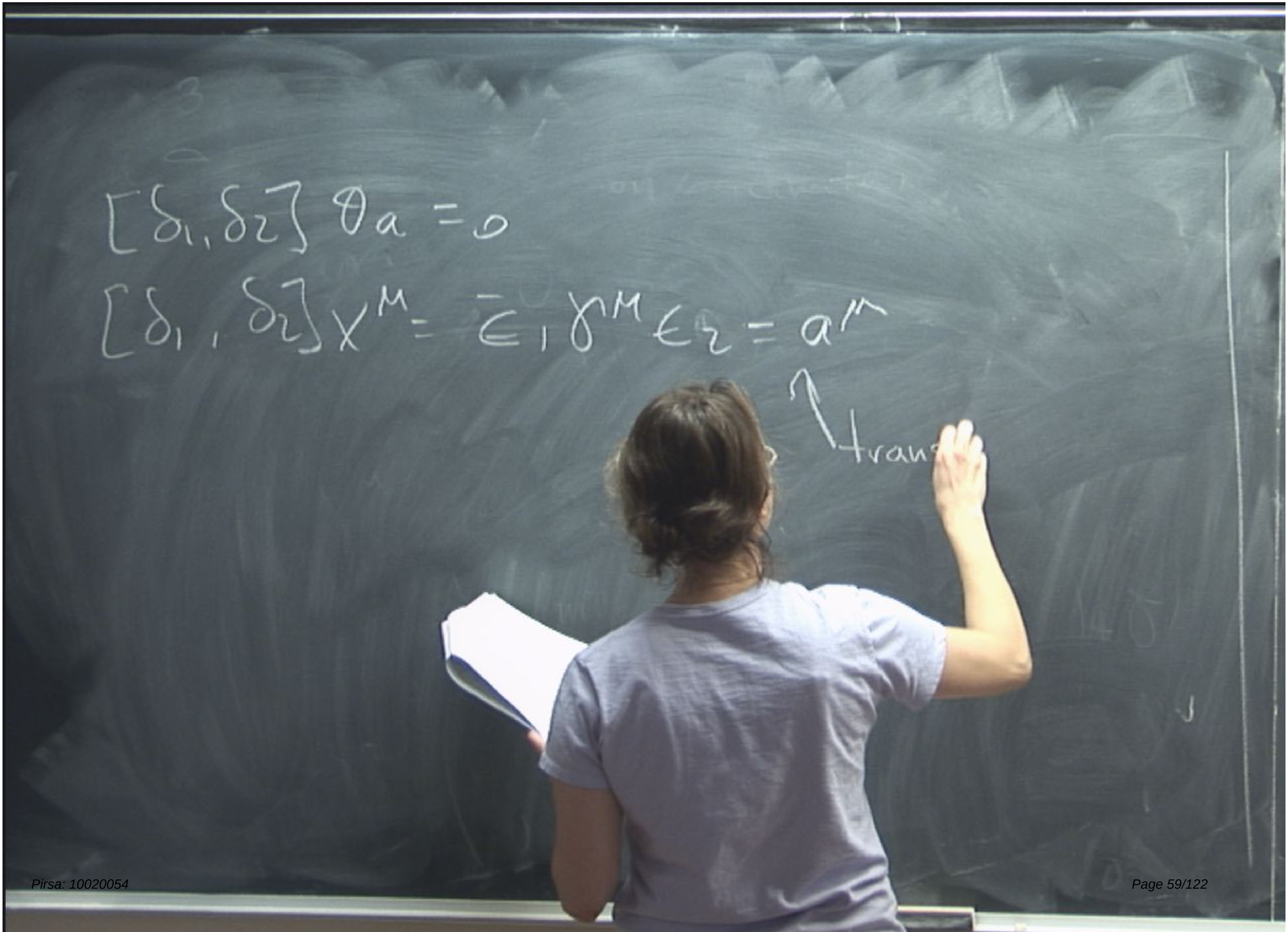
$$[\delta_1, \delta_2]\, \theta_a = 0$$

$$[\delta_1, \delta_2]\, X^M = \bar{\epsilon}_1 \gamma^M \epsilon_2 = a^M$$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tran

$$[\delta_1, \delta_2]\,\theta_a = 0$$

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translation

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translation

SUSY is a generalization of symmetries of space-time

$$[\delta_1, \delta_2]\, \theta_a = 0$$

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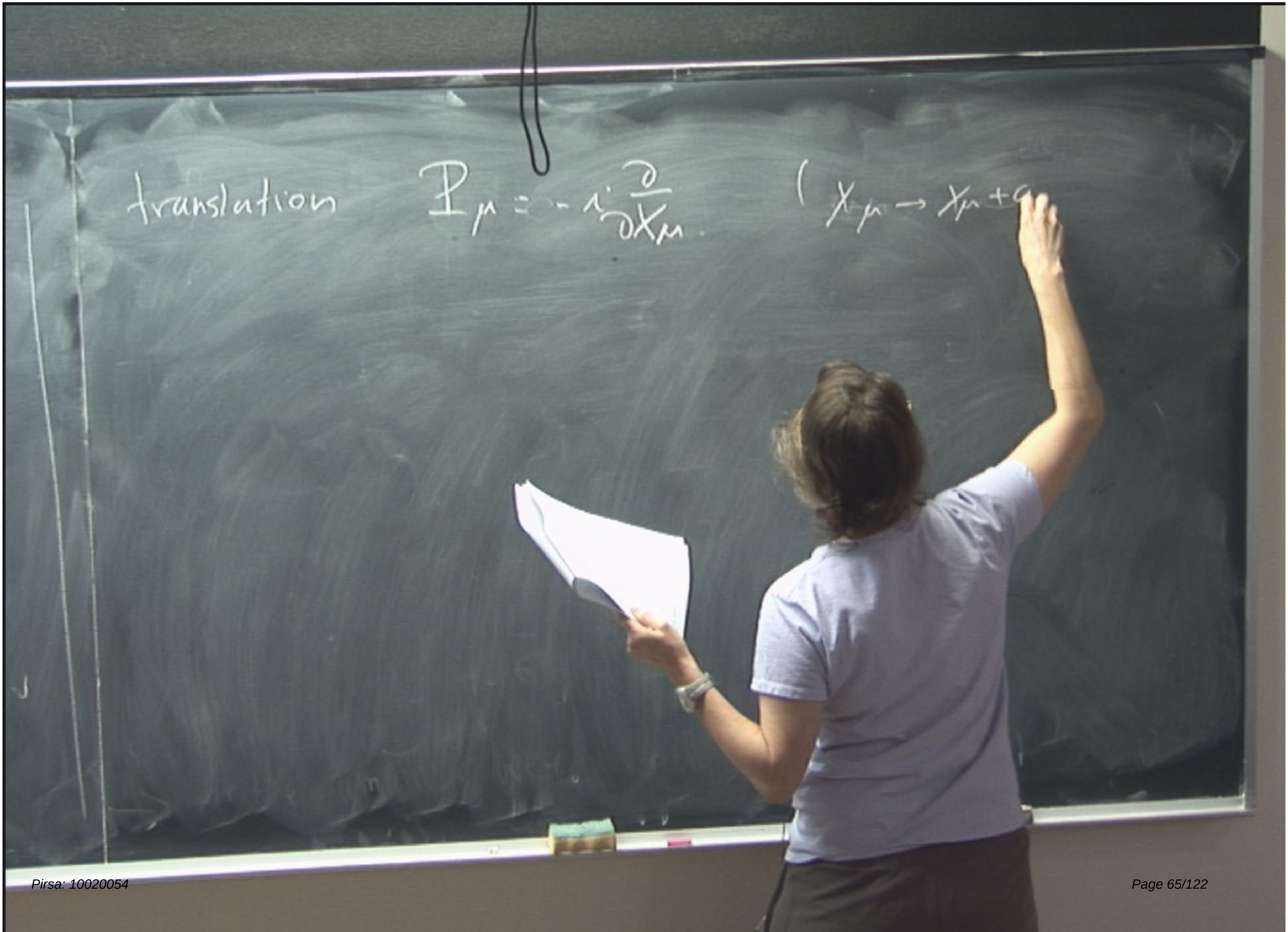
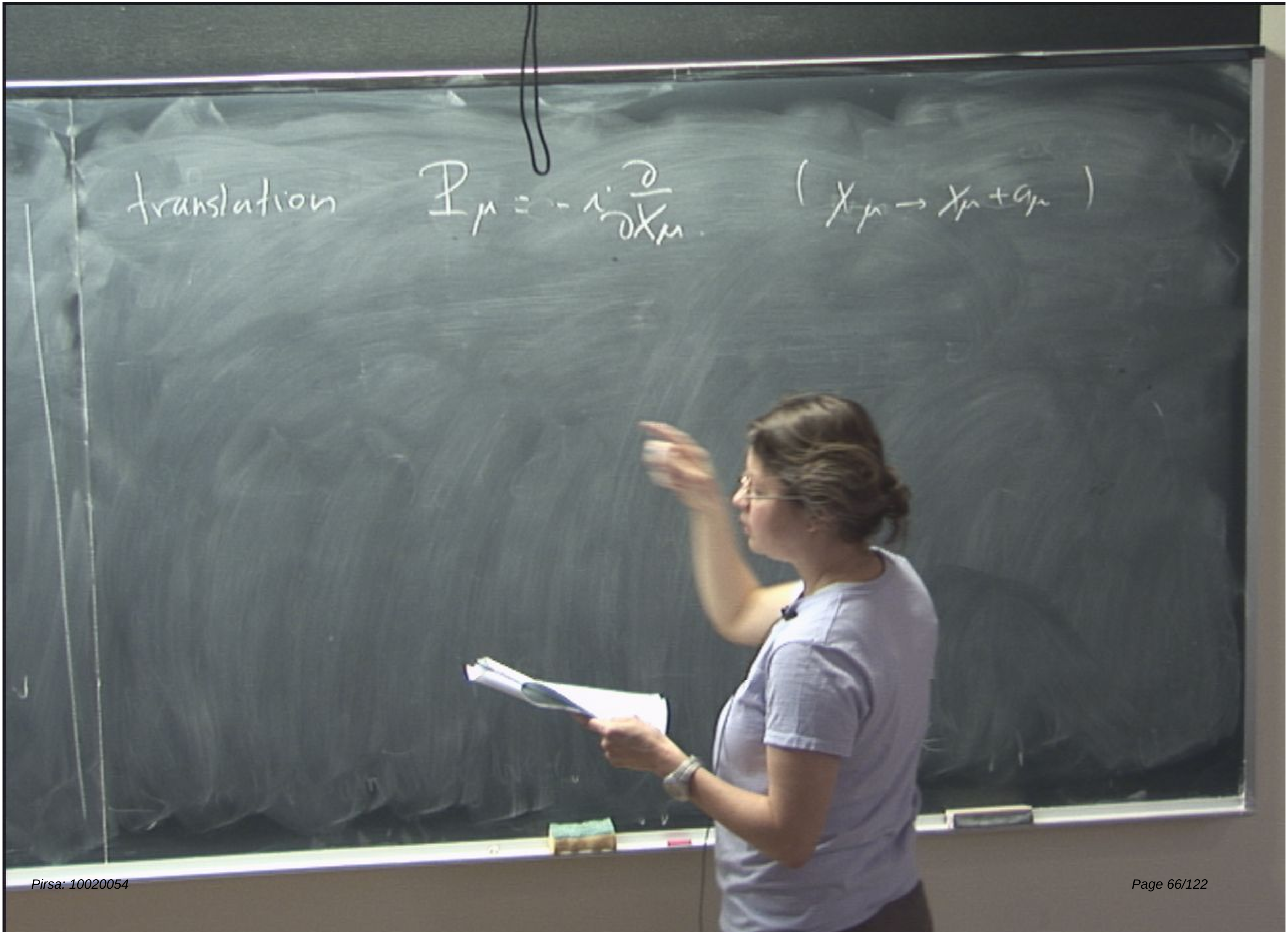
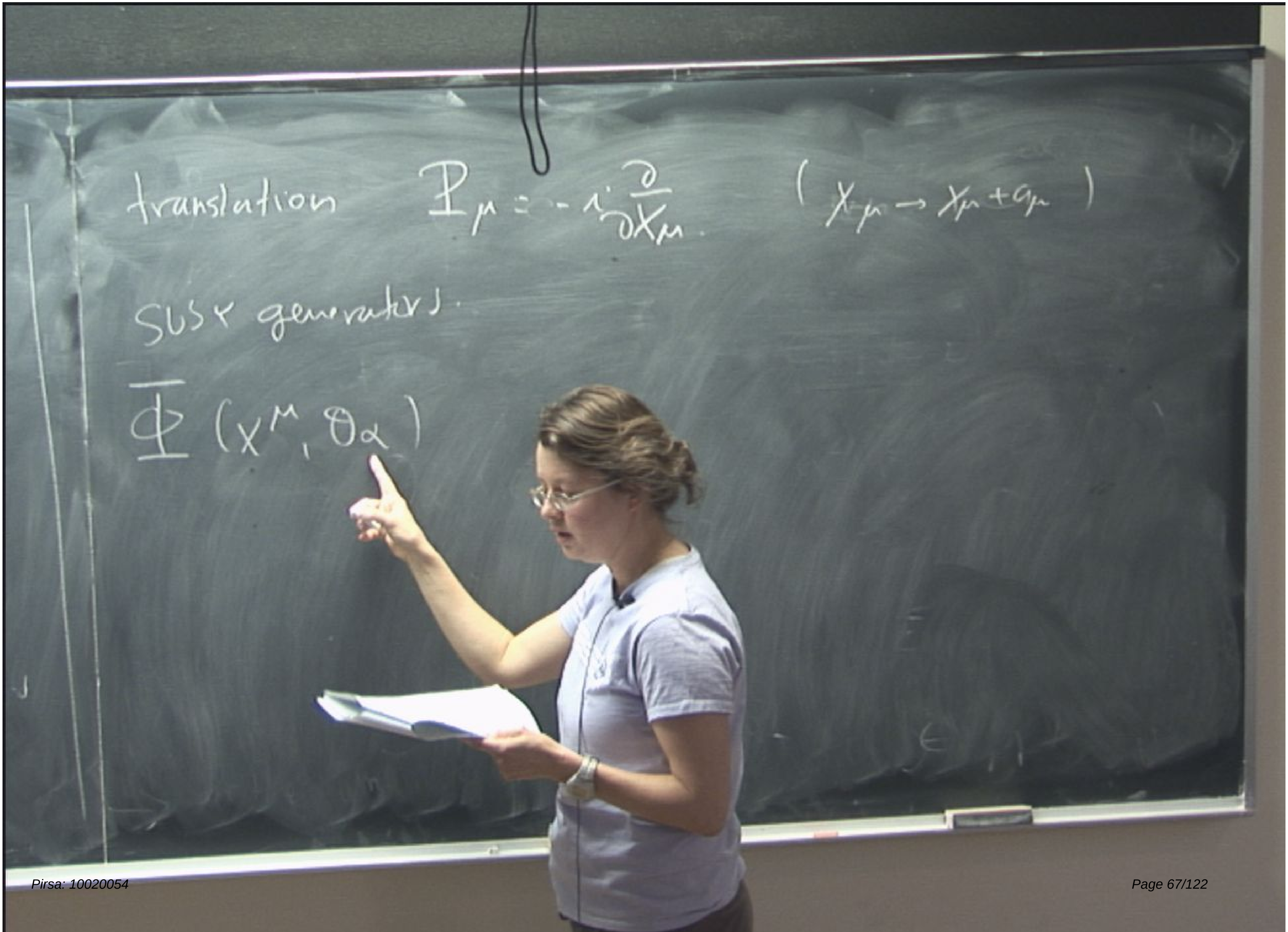
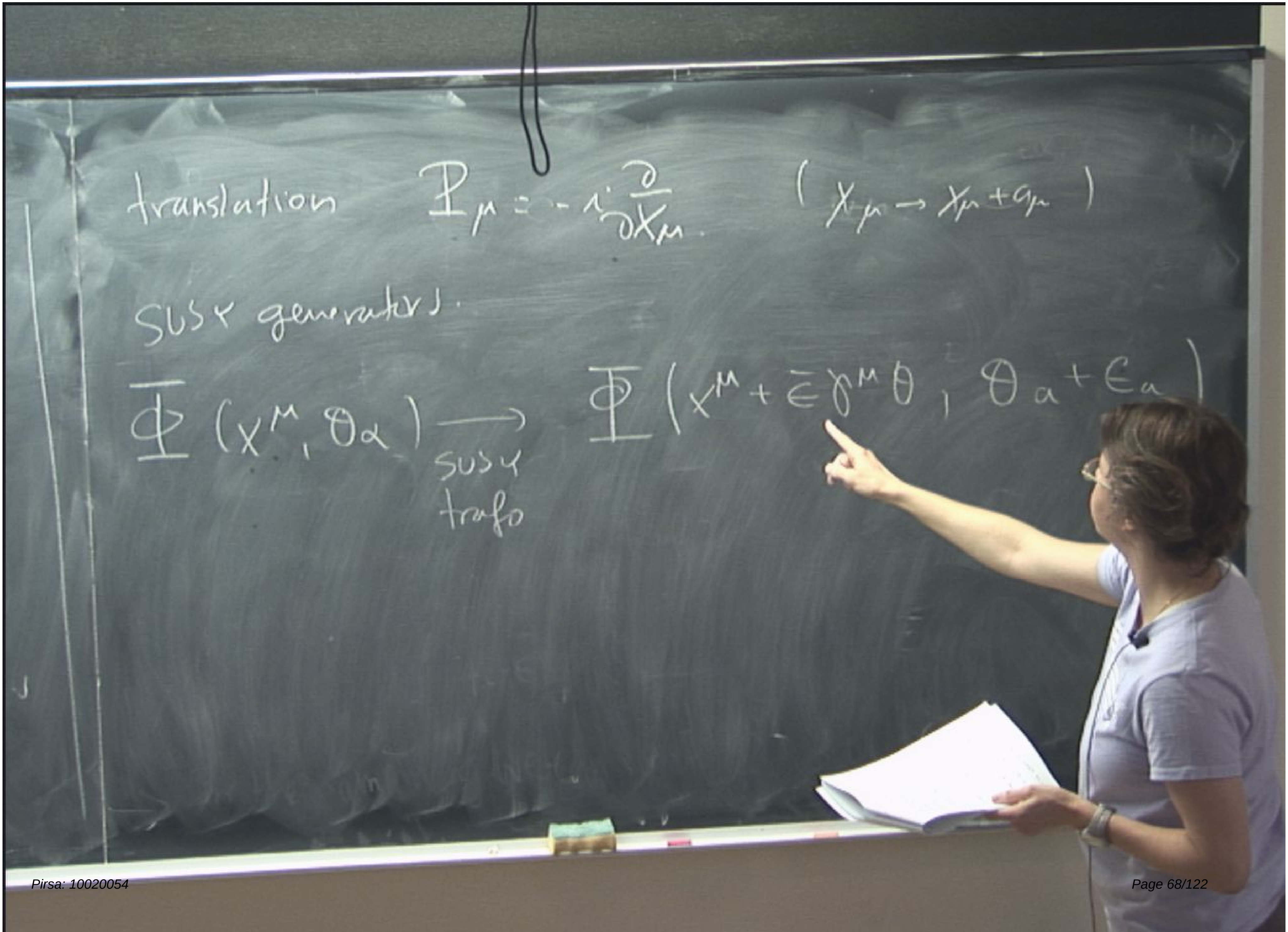
$$X^\mu \xrightarrow[\text{trafo1}]{} X^\mu + \bar{\epsilon}_1\gamma^\mu\theta \xrightarrow[\text{trafo2}]{} X^\mu + \underbrace{(\bar{\epsilon}_1 + \bar{\epsilon}_2)}_{\epsilon}\gamma^\mu\theta + \underbrace{\bar{\epsilon}_1\gamma^\mu\epsilon_2}_{a^\mu}$$


translation
$P_\mu = -i \frac{\partial}{\partial x_\mu}$
$(x_\mu \to x_\mu + a$


translation $P_\mu = -i \frac{\partial}{\partial x_\mu}$ $(x_\mu \to x_\mu + a_\mu)$


translation $P_\mu = -i \frac{\partial}{\partial x_\mu}$ ($x_\mu \to x_\mu + a_\mu$)
SUSY generators.
$\Phi(x^\mu, \theta_\alpha)$


translation $P_\mu = -i \frac{\partial}{\partial x_\mu}$ $(x_\mu \to x_\mu + a_\mu)$
SUSY generators.
$\Phi(x^\mu, \theta_\alpha) \xrightarrow[\text{trafo}]{\text{SUSY}} \Phi(x^\mu + \bar{\epsilon}\gamma^\mu\theta, \theta_\alpha + \epsilon_\alpha)$

translation $P_\mu = -i \frac{\partial}{\partial x_\mu}$ $\left(x_\mu \to x_\mu + a_\mu \right)$

SUSY generators.

$$\Phi(x^\mu, \theta_a) \xrightarrow[\text{trafo}]{\text{SUSY}} \Phi\left(x^\mu + \bar{\epsilon}\gamma^\mu\theta,\ \theta_a + \epsilon_a\right)$$

$$\underset{\substack{\uparrow \\ \epsilon_a \text{ is} \\ \text{small}}}{=} \Phi(x^\mu, \theta_a) + \bar{\epsilon}\gamma^\mu\theta \frac{\partial \Phi}{\partial x^\mu}(x, \theta) + \epsilon_a \frac{\partial \Phi(x^\mu, \theta_a)}{\partial \theta^a} + \dots$$

$$\Phi(x^\mu, \theta_a) \longrightarrow \Phi(x^\mu, \theta^a) + \epsilon_a$$

$$\Phi(x^\mu, \theta_a) \longrightarrow \Phi(x^\mu, \theta^a) + \epsilon_a \left(\frac{\partial}{\partial \theta_a} + \right.$$

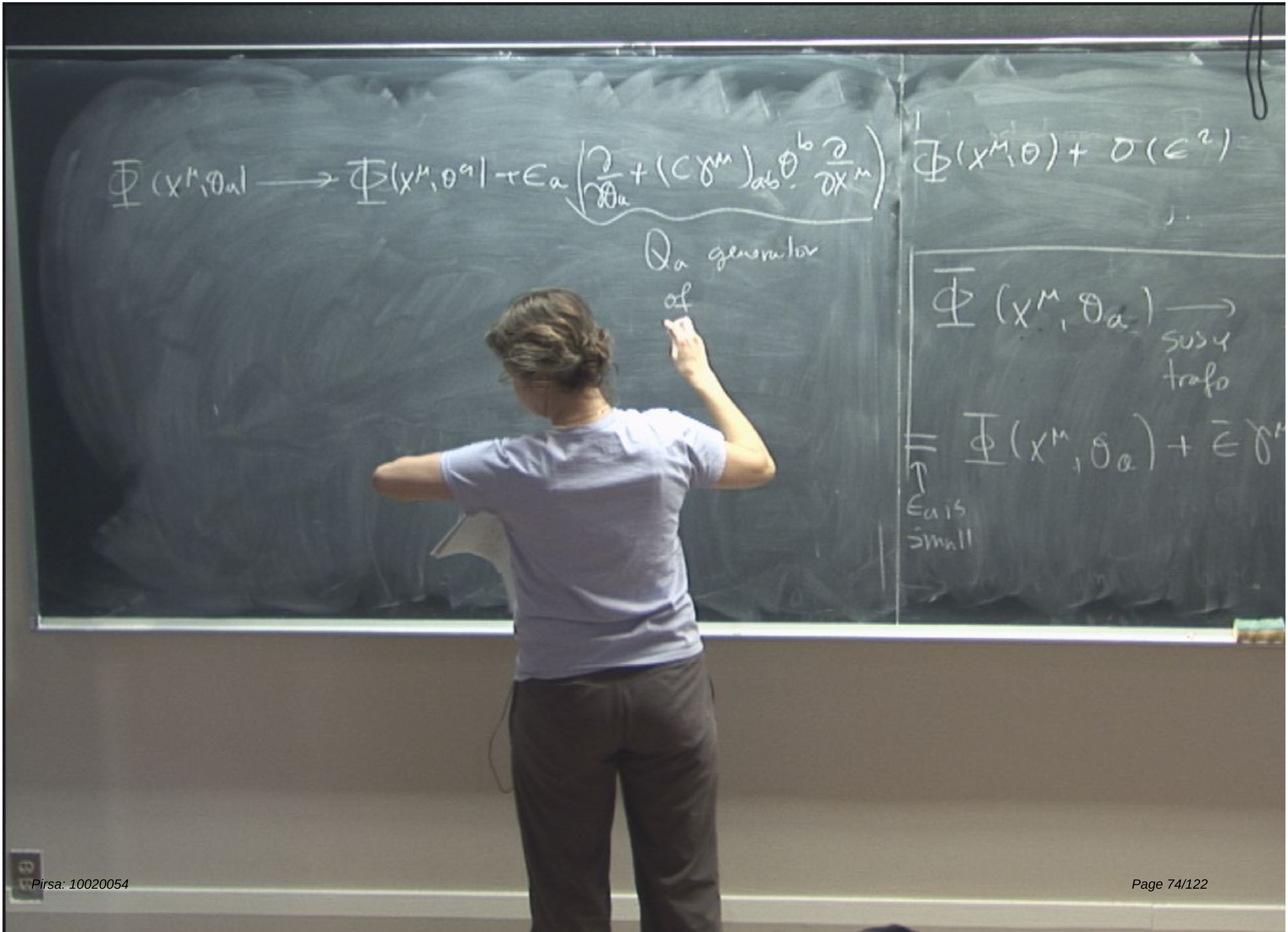
$$\Phi(x^\mu, \theta_a) \longrightarrow \Phi(x^\mu, \theta^a) + \epsilon_a \left(\frac{\partial}{\partial \theta_a} + (C\gamma^\mu)_{ab} \theta^b \frac{\partial}{\partial x^\mu} \right)$$

$$\theta_a) \longrightarrow \Phi(x^\mu, \theta^a) + \epsilon_a \left(\frac{\partial}{\partial \theta_a} + (C\gamma^\mu)_{ab} \theta^b \frac{\partial}{\partial x^\mu} \right) \Phi(x^\mu, \theta) + O(\epsilon^2)$$

$$\Phi(x^\mu, \theta_a) \xrightarrow[\text{susy trafo}]{}$$

$$= \Phi(x^\mu, \theta_a) + \bar{\epsilon}\,\theta$$

ϵ_a is small


$\Phi(x^\mu,\theta_a) \longrightarrow \Phi(x^\mu,\theta^a) + \epsilon_a \left(\frac{\partial}{\partial\theta_a} + (C\gamma^\mu)_{ab}\theta^b \frac{\partial}{\partial x^\mu}\right) \Phi(x^\mu,\theta) + O(\epsilon^2)$
Q_a generator of
$\Phi(x^\mu,\theta_a) \longrightarrow$ susy trafo
$= \Phi(x^\mu,\theta_a) + \bar{\epsilon}\gamma^\mu$
ϵ_a is small

$$\Phi(x^\mu, \theta_a) \longrightarrow \Phi(x^\mu, \theta^a) + \epsilon_a \underbrace{\left(\frac{\partial}{\partial \theta_a} + (C\gamma^\mu)_{ab}\,\theta^b \frac{\partial}{\partial x^\mu}\right)}_{Q_a \text{ generator of SUSY}} \Phi(x^\mu, \theta) + O(\epsilon^2)$$

$$Q_a = \frac{\partial}{\partial \theta_a} + \theta^b (C\gamma^\mu)_{ab} \frac{\partial}{\partial x^\mu}$$

$$\Phi(x^\mu, \theta_a) \xrightarrow[\text{trafo}]{\text{SUSY}}$$

$$\underset{\substack{\uparrow \\ \epsilon_a \text{ is small}}}{=} \Phi(x^\mu, \theta_a) + \bar{\epsilon}\gamma^\mu$$

$$\Phi(x^\mu,\theta) + O(\epsilon^2)$$

$$\theta_1\theta_2 + \theta_2\theta_1 = 0$$

$$\frac{\partial}{\partial\theta_1}(\theta_1\theta_2) = \theta_2$$

$$\frac{\partial}{\partial\theta_2}(\theta_1\theta_2) = -\theta_1$$

$$\Phi(x^\mu,\theta_a) \xrightarrow[\text{trafo}]{\text{susy}} \Phi(x^\mu + \bar\epsilon\gamma^\mu\theta,\ \theta_a + \epsilon_a)$$

$$= \Phi(x^\mu,\theta_a) + \bar\epsilon\gamma^\mu\theta\frac{\partial\Phi}{\partial x^\mu}(x,\theta) + \epsilon_a\frac{\partial\Phi(x^\mu,\theta_a)}{\partial\theta^a} + \ldots$$

ϵ_a is small

$$\Phi(x^\mu, \theta) + O(\epsilon^2)$$

$$\theta_1\theta_2 + \theta_2\theta_1 = 0$$

$$\frac{\partial}{\partial\theta_1}(\theta_1\theta_2) = \theta_2$$

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$$\Phi(x^\mu, \theta_a) \xrightarrow[\text{trafo}]{\text{susy}} \Phi(x^\mu + \bar\epsilon\gamma^\mu\theta, \theta_a + \epsilon_a)$$

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ϵ_a is small

$$\Phi(x^\mu, \theta_a) \longrightarrow \Phi(x^\mu, \theta^a) + \epsilon_a \underbrace{\left(\frac{\partial}{\partial \theta_a} + (C\gamma^\mu)_{ab} \theta^b \frac{\partial}{\partial x^\mu} \right)}$$

Q_a generator of SUSY

$$Q_a = \frac{\partial}{\partial \theta_a} + \theta^b (C\gamma^\mu)_{ab} \frac{\partial}{\partial x^\mu}$$

$$\{Q_a, Q_b\} = 2 (C\gamma^\mu)_{ab} P_\mu$$

$\theta_1\theta_2 + \theta_2\theta_1 = 0$

$\frac{\partial}{\partial\theta_1}(\theta_1\theta_2) = \theta_2$

$\frac{\partial}{\partial\theta_2}(\theta_1\theta_2) = -\theta_1$

$\Phi(x^\mu,\theta) + O(\epsilon^2)$

$\Phi(x^\mu,\theta_a) \xrightarrow[\text{trafo}]{\text{susy}} \Phi(x^\mu + \bar\epsilon\gamma^\mu\theta,\ \theta_a + \epsilon_a)$

$= \Phi(x^\mu,\theta_a) + \bar\epsilon\gamma^\mu\theta\frac{\partial\Phi}{\partial x^\mu}(x,\theta) + \epsilon_a\frac{\partial\Phi(x^\mu,\theta_a)}{\partial\theta^a} + \ldots$

ϵ_a is small

$\bar\epsilon = \epsilon^+ C$

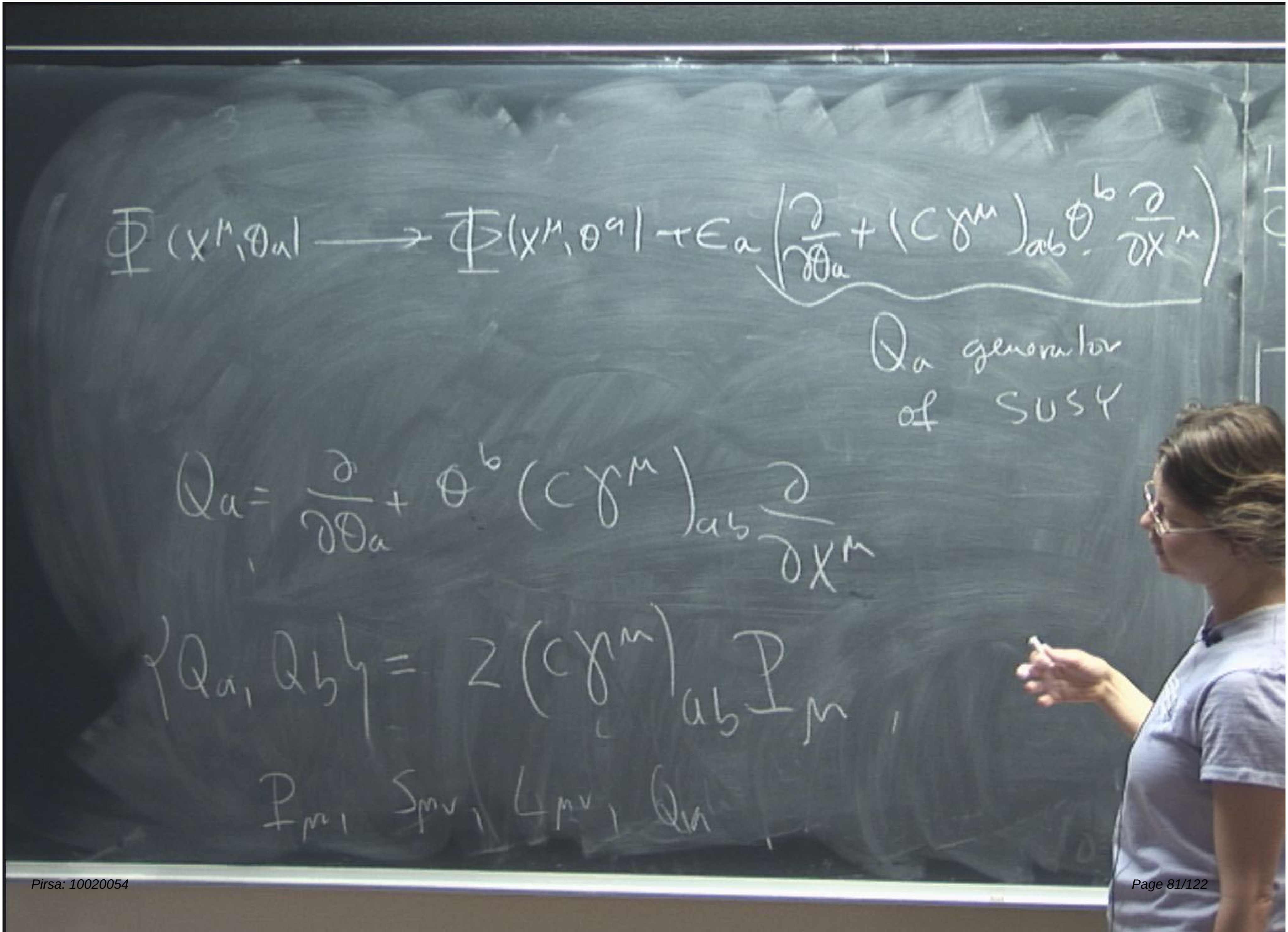
$$\Phi(x^\mu, \theta_a) \longrightarrow \Phi(x^\mu, \theta^a) + \epsilon_a \underbrace{\left(\frac{\partial}{\partial\theta_a} + (C\gamma^\mu)_{ab}\theta^b \frac{\partial}{\partial x^\mu}\right)}$$

Q_a generator of SUSY

$$Q_a = \frac{\partial}{\partial\theta_a} \ldots (C\gamma^\mu)_{ab} \frac{\partial}{\partial x^\mu}$$

$$\{Q_a, Q_b\} \ldots \gamma^\mu)_{ab} P_\mu ,$$

$P_\mu,$ ...


$\Phi(x^\mu, \theta_a) \longrightarrow \Phi(x^\mu, \theta^a) + \epsilon_a \underbrace{\left(\frac{\partial}{\partial\theta_a} + (C\gamma^\mu)_{ab}\theta^b \frac{\partial}{\partial x^\mu}\right)}$
Q_a generator of SUSY
$Q_a = \frac{\partial}{\partial\theta_a} + \theta^b (C\gamma^\mu)_{ab}\frac{\partial}{\partial x^\mu}$
$\{Q_a, Q_b\} = 2(C\gamma^\mu)_{ab}P_\mu$
$P_\mu, S_{\mu\nu}, L_{\mu\nu}, Q_a$

$\Phi(x^\mu, \theta) + O(\epsilon^2)$

$\theta_1\theta_2 + \theta_2\theta_1 = 0$

$\frac{\partial}{\partial\theta_1}(\theta_1\theta_2) = \theta_2$

$\frac{\partial}{\partial\theta_2}(\theta_1\theta_2) = -\theta_1$

Poincaré algebra → Superpoincaré which

is extension

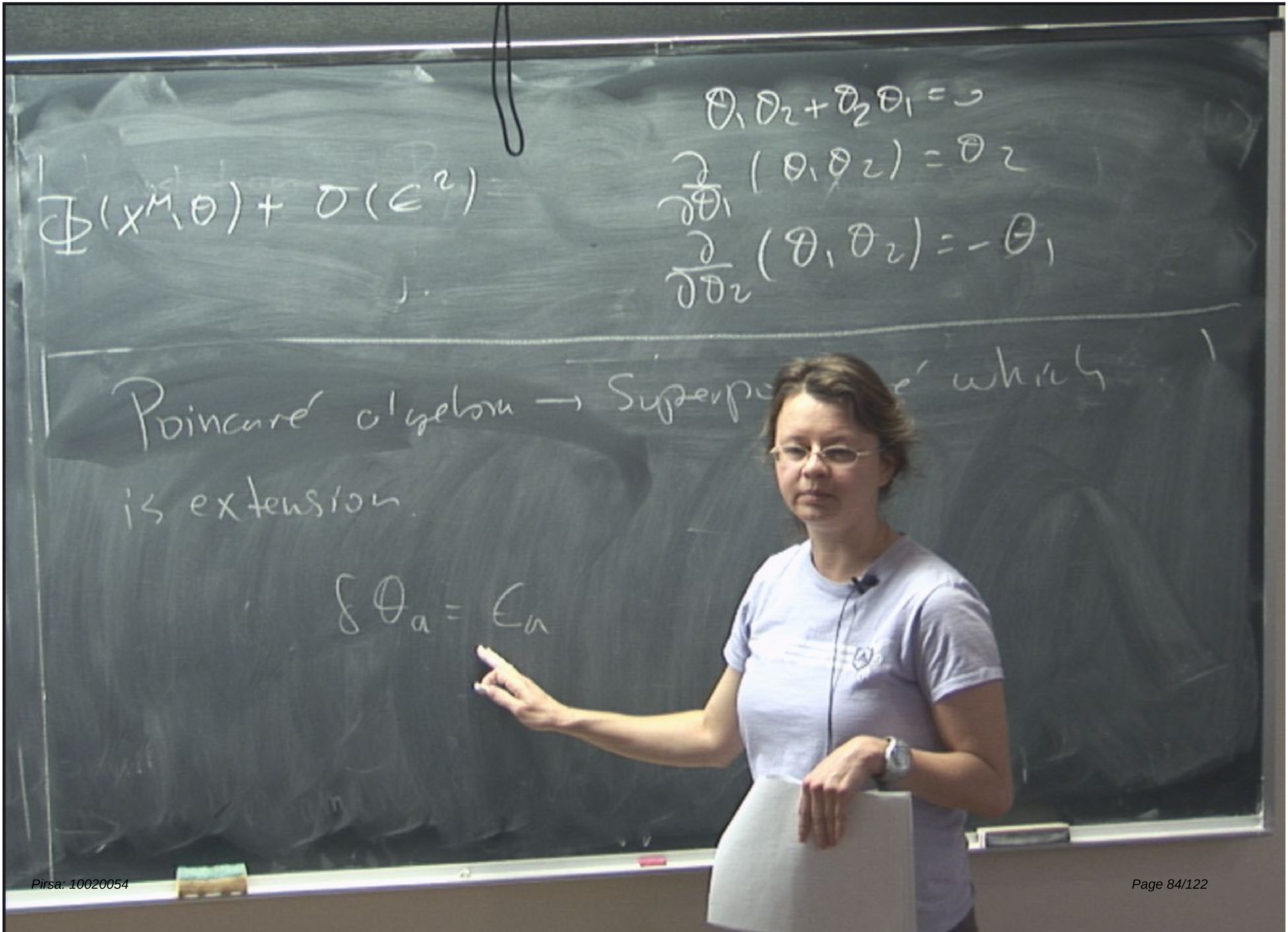
$\Phi(x^\mu, \theta) + O(\epsilon^2)$

$\theta_1 \theta_2 + \theta_2 \theta_1 = 0$

$\frac{\partial}{\partial \theta_1}(\theta_1 \theta_2) = \theta_2$

$\frac{\partial}{\partial \theta_2}(\theta_1 \theta_2) = -\theta_1$

Poincaré algebra → Superpoincaré which is extension.


$\theta_1\theta_2+\theta_2\theta_1=0$
$\frac{\partial}{\partial\theta_1}(\theta_1\theta_2)=\theta_2$
$\frac{\partial}{\partial\theta_2}(\theta_1\theta_2)=-\theta_1$
$\Phi(x^\mu,\theta)+O(\epsilon^2)$
Poincaré algebra → Superp
which
is extension
$\delta\theta_a=\epsilon_a$

$$\theta_1\theta_2 + \theta_2\theta_1 = 0$$

$$\Phi(x^\mu,\theta) + O(\epsilon^2)$$

$$\frac{\partial}{\partial\theta_1}(\theta_1\theta_2) = \theta_2$$

$$\frac{\partial}{\partial\theta_2}(\theta_1\theta_2) = -\theta_1$$

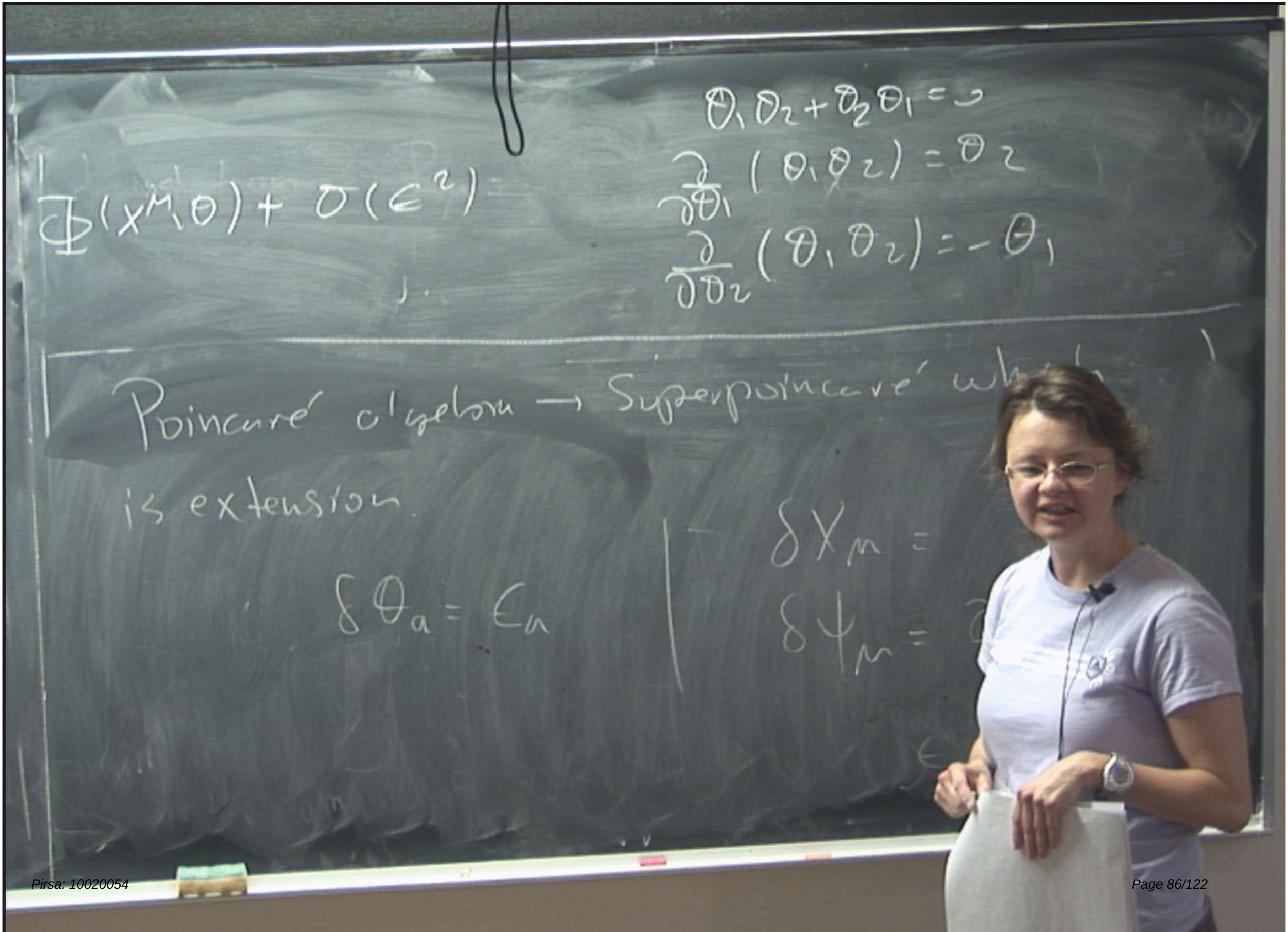
Poincaré algebra → Superpoincaré wh...

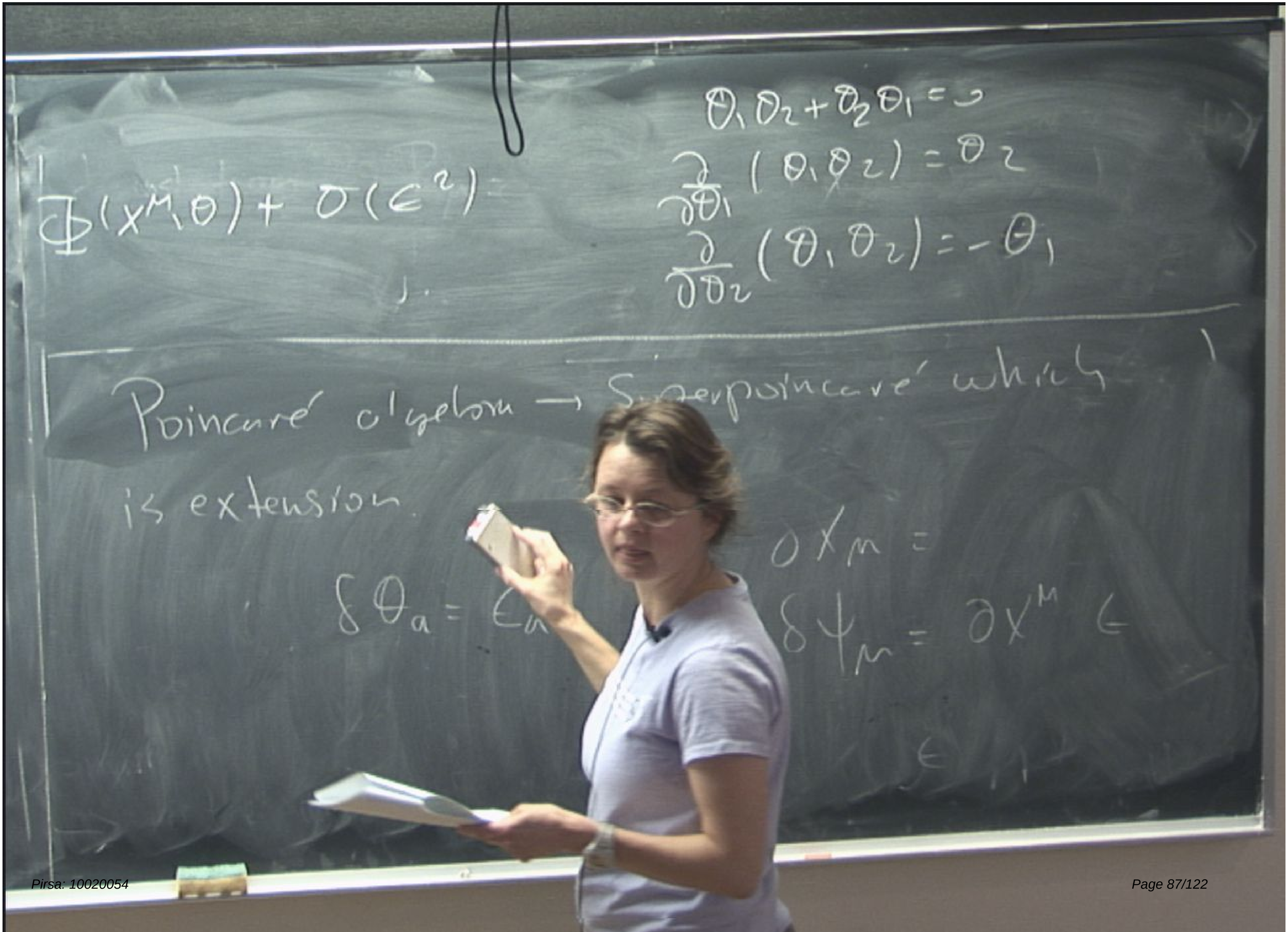
is extension

$$\delta\theta_a = \epsilon_a$$

$$\delta X_\mu =$$

$$\delta\psi_\mu =$$


$\theta_1\theta_2 + \theta_2\theta_1 = 0$
$\frac{\partial}{\partial\theta_1}(\theta_1\theta_2) = \theta_2$
$\frac{\partial}{\partial\theta_2}(\theta_1\theta_2) = -\theta_1$
$\Phi(x^\mu, \theta) + O(\epsilon^2)$
Poincaré algebra → Superpoincaré
is extension
$\delta\theta_a = \epsilon_a$
$\delta X_\mu =$
$\delta\psi_\mu =$


θ₁θ₂ + θ₂θ₁ = 0
∂/∂θ₁ (θ₁θ₂) = θ₂
∂/∂θ₂ (θ₁θ₂) = −θ₁
Φ(x^μ, θ) + O(ε²)
Poincaré algebra → Superpoincaré which
is extension
δθ_a = ε_a
δx_μ =
δψ_μ = ∂x^μ ε

$$\Phi(x^\mu,\theta) + O(\epsilon^2)$$

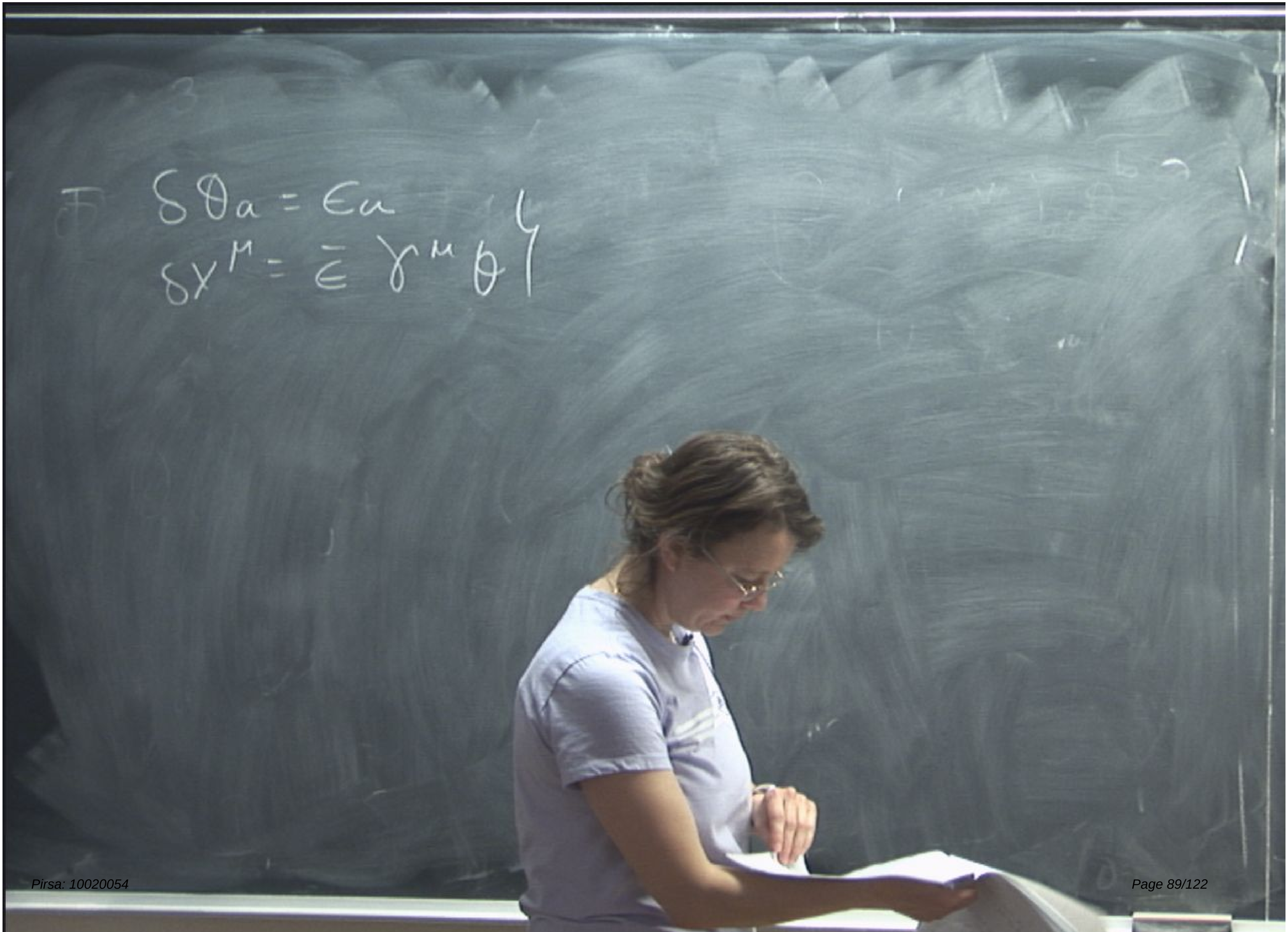
$$\theta_1\theta_2 + \theta_2\theta_1 = 0$$

$$\frac{\partial}{\partial\theta_1}(\theta_1\theta_2) = \theta_2$$

$$\frac{\partial}{\partial\theta_2}(\theta_1\theta_2) = -\theta_1$$

Poincaré algebra → Superpoincaré which is extension.

$$S = -m\int ds$$


δθa = εa
δx^μ = ε̄ γ^μ θ

$$\left.\begin{aligned}\delta\theta_a &= \epsilon_a \\ \delta x^\mu &= \bar{\epsilon}\,\gamma^\mu\,\theta\end{aligned}\right\}$$

$S =$

$$\delta\theta_a = \epsilon_a$$
$$\delta x^\mu = \bar{\epsilon}\gamma^\mu\theta$$

$$S = -m\int ds = -m\int\sqrt{\dot{x}^\mu\cdot\dot{x}_\mu}\,d\tau$$

$$\delta\theta_a = \epsilon_a$$
$$\delta x^\mu = \bar{\epsilon}\gamma^\mu\theta$$

$$S = -m\int ds = -m\int\sqrt{\dot{x}^\mu\cdot\dot{x}_\mu}\,d\tau$$

$$\Pi_0^\mu = \dot{X}^\mu - \bar{\theta}\Gamma^\mu\dot{\theta}$$

$$\left.\begin{aligned} \delta\theta_a &= \epsilon_a \\ \delta x^\mu &= \bar{\epsilon}\gamma^\mu\theta \end{aligned}\right\}$$

$$S = -m\int ds = -m\int\sqrt{\dot{x}^\mu\cdot\dot{x}_\mu}\,d\tau$$

$$\Pi_0^\mu = \dot{x}^\mu - \bar{\theta}\Gamma^\mu\dot{\theta}$$

$$\Pi_0^\mu + \cancel{\bar{\epsilon}\gamma^\mu\dot{\theta}} - \cancel{\bar{\epsilon}\Gamma^\mu\dot{\theta}}$$

$$\left.\begin{aligned} \delta\theta_a &= \epsilon_a \\ \delta x^\mu &= \bar{\epsilon}\gamma^\mu\theta \end{aligned}\right\}$$

$$S = -m\int ds = -m\int\sqrt{\dot{x}^\mu\cdot\dot{x}_\mu}\,d\tau$$

$$\Pi_0^\mu = \dot{X}^\mu - \bar{\theta}\Gamma^\mu\dot{\theta}$$

$$\to \Pi_0^\mu + \cancel{\bar{\epsilon}\gamma^\mu\dot{\theta}} - \cancel{\bar{\epsilon}\Gamma^\mu\dot{\theta}}$$

invariant under SUSY trafo)

$$\left.\begin{aligned}\delta\theta_a &= \epsilon_a \\ \delta x^\mu &= \bar{\epsilon}\gamma^\mu\theta\end{aligned}\right\}$$

$$S = -m\int ds = -m\int\sqrt{\dot{x}^\mu\cdot\dot{x}_\mu}\,d\tau$$

$$\Pi_0^\mu = \dot{X}^\mu - \bar{\theta}\Gamma^\mu\dot{\theta}$$

$$\longrightarrow \Pi_0^\mu + \cancel{\bar{\epsilon}\gamma^\mu\dot{\theta}} - \cancel{\bar{\epsilon}\Gamma^\mu\dot{\theta}}$$

invariant under SUSY trafo)

$$\delta\theta_a = \epsilon_a \qquad \delta x^\mu = \bar{\epsilon}\gamma^\mu\theta$$

$$S = -m\int ds = -m\int\sqrt{\dot{x}^\mu\cdot\dot{x}_\mu}\,d\tau$$

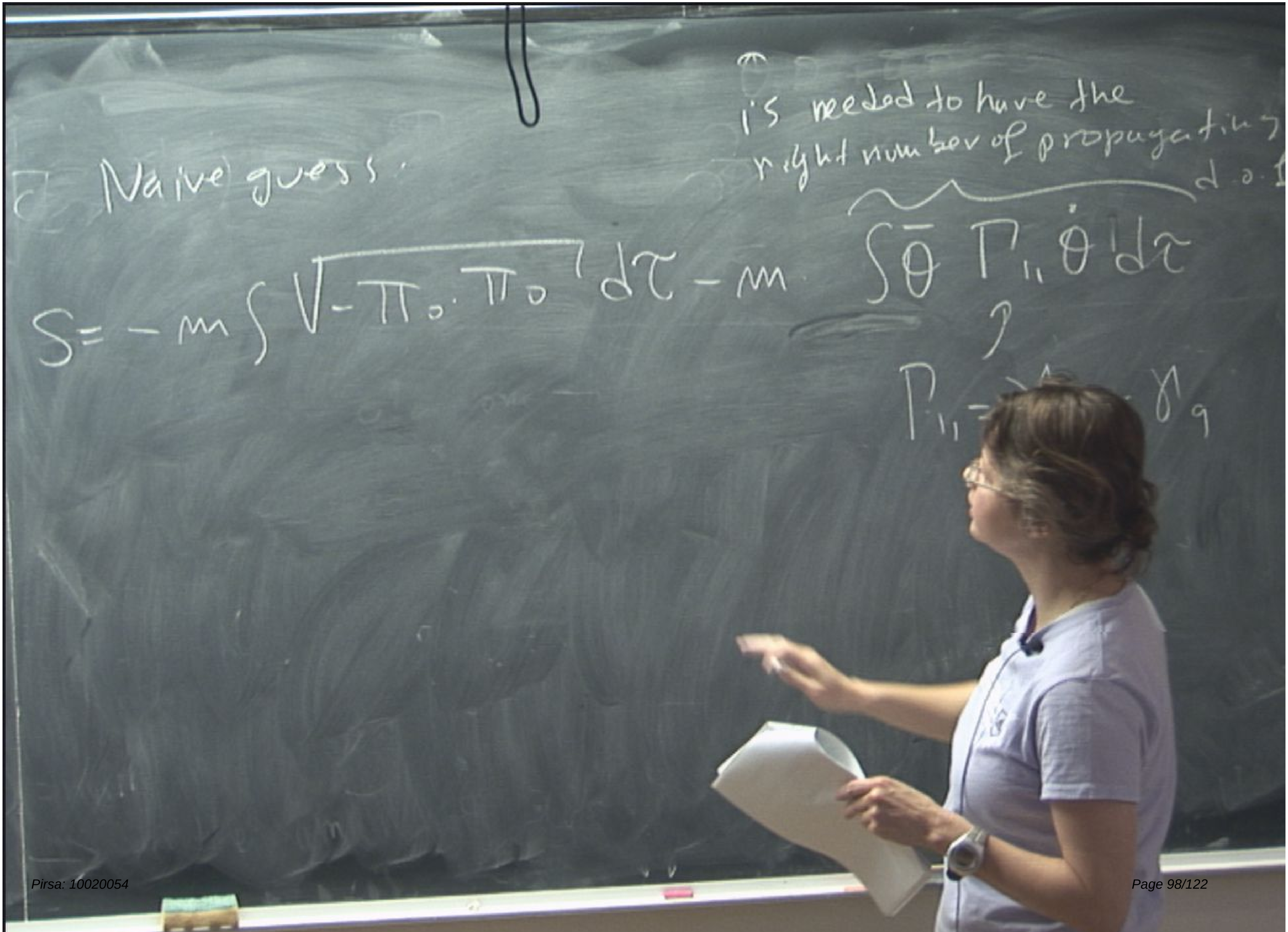
$$\Pi_0^\mu = \dot{X}^\mu - \bar{\theta}\gamma^\mu\dot{\theta}$$

$$\rightarrow \Pi_0^\mu + \cancel{\bar{\epsilon}\gamma^\mu\dot{\theta}} - \cancel{\bar{\epsilon}\gamma^\mu\dot{\theta}}$$

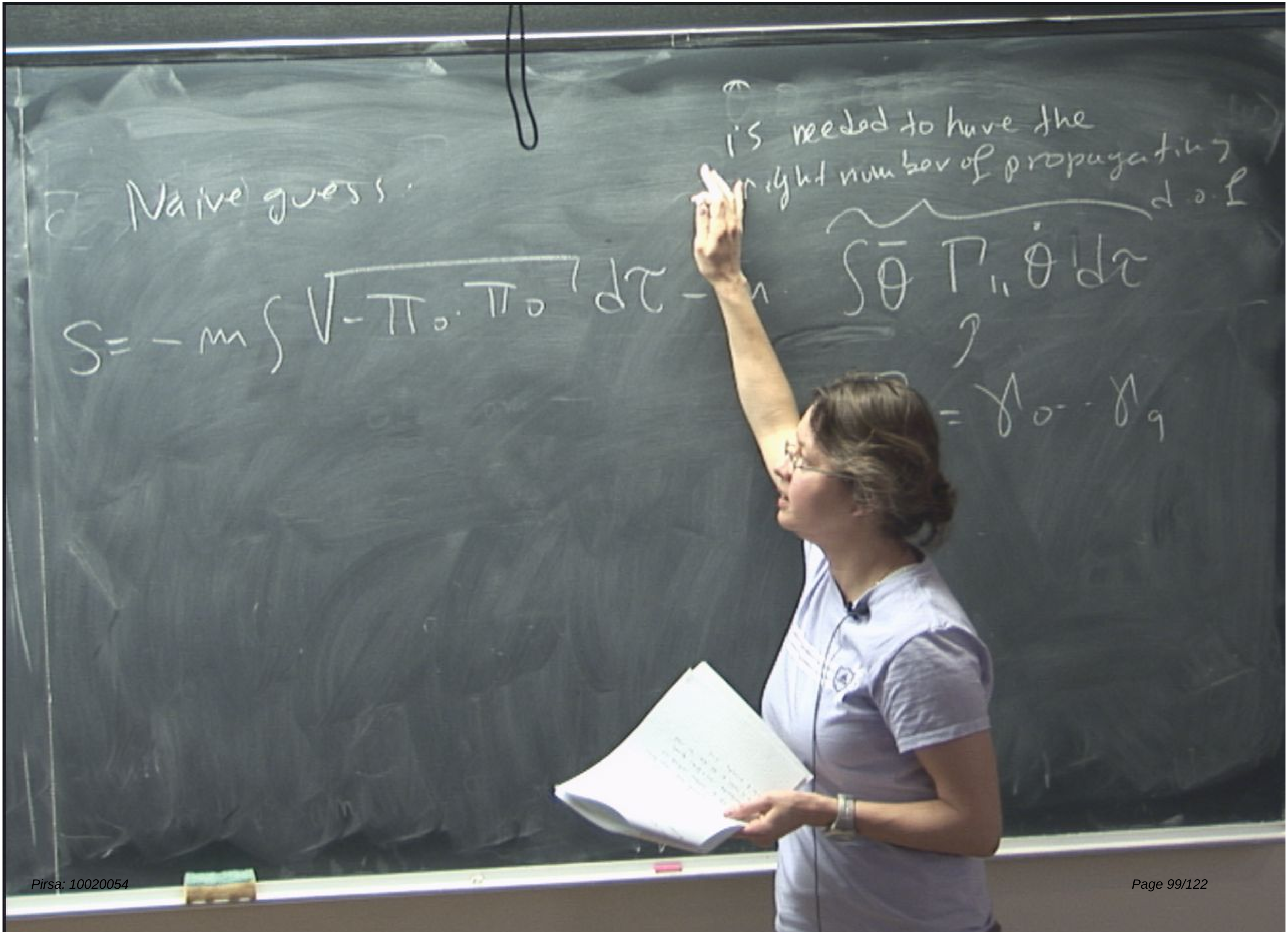
invariant under SUSY trafo)

Naive guess:

$$S = -m \int \sqrt{-\Pi_0 \cdot \Pi_0}\, d\tau$$



Naive guess:
$S = -m \int \sqrt{-\Pi_0 \cdot \Pi_0}\, d\tau - m \int \bar{\theta}\, \Gamma_{11}\, \dot{\theta}\, d\tau$
is needed to have the right number of propagating d.o.f.
$\Gamma_{11} = \gamma_0 \ldots \gamma_9$


Naive guess.
is needed to have the right number of propagating d.o.f
S = − m ∫ √(−Π₀·Π₀) dτ −
∫ θ̄ Γ₁₁ θ̇ dτ
= γ₀ ·· γ₉

Naive guess.

$$S = -m\int\sqrt{-\Pi_0\cdot\Pi_0}\,d\tau - m\underbrace{\int\bar\theta\,\Gamma_{11}\,\dot\theta\,d\tau}$$

is needed to have the right number of propagating d.o.f

$$\Gamma_{11} = \gamma_0\cdots\gamma_9$$

is needed to have the right number of propagating d.o.f

Naive guess:

$$S = -m \int \sqrt{-\Pi_0 \cdot \Pi_0}\, d\tau - m \underbrace{\int \bar{\theta}\, \Gamma_{11}\, \dot{\theta}\, d\tau}$$

$$\Gamma_{11} = \gamma_0 \cdots \gamma_9$$

D0-brane (point particle)

$$S = -m \int \sqrt{-\Pi_0 \cdot \Pi_0}\, d\tau - m \underbrace{\int \bar{\theta}\, \Gamma_{11} \dot{\theta}\, d\tau}$$

is needed to have the right number of propagating d.o.f.

$$\Gamma_{11} = \gamma_0 \cdots \gamma_9$$

D0-brane (point particle)

$$S = -m\int\sqrt{-\Pi_0\cdot\Pi_0}\,d\tau - m\underbrace{\int\bar{\theta}\,\Gamma_{11}\,\dot{\theta}\,d\tau}$$

is needed to have the right number of propagating d.o.f

$$\Gamma_{11} = \gamma_0\cdots\gamma_9$$

String

$$S = \int d^2\sigma\sqrt{-\det\partial_\alpha X^\mu\cdot\partial_\beta X_\mu}$$

D0-brane (point particle)

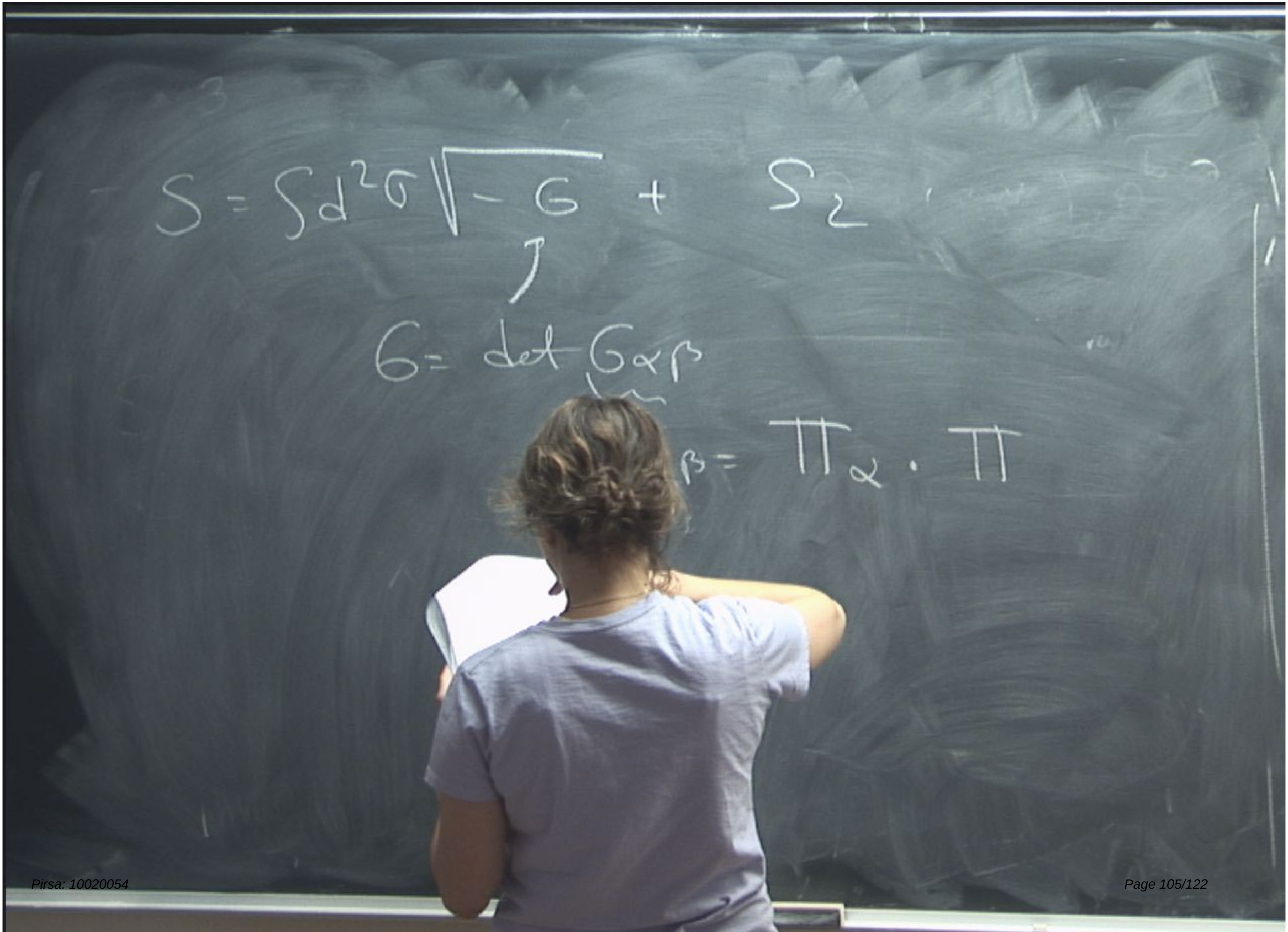
$$S = -m\int\sqrt{-\Pi_0\cdot\Pi_0}\,d\tau - m\underbrace{\int\bar{\theta}\,\Gamma_{11}\,\dot{\theta}\,d\tau}$$

is needed to have the right number of propagating d.o.f

$$\Gamma_{11} = \gamma_0\cdots\gamma_9$$

String

$$S = \int d^2\sigma\sqrt{-\det\partial_\alpha X^\mu\cdot\partial_\beta X_\mu}$$

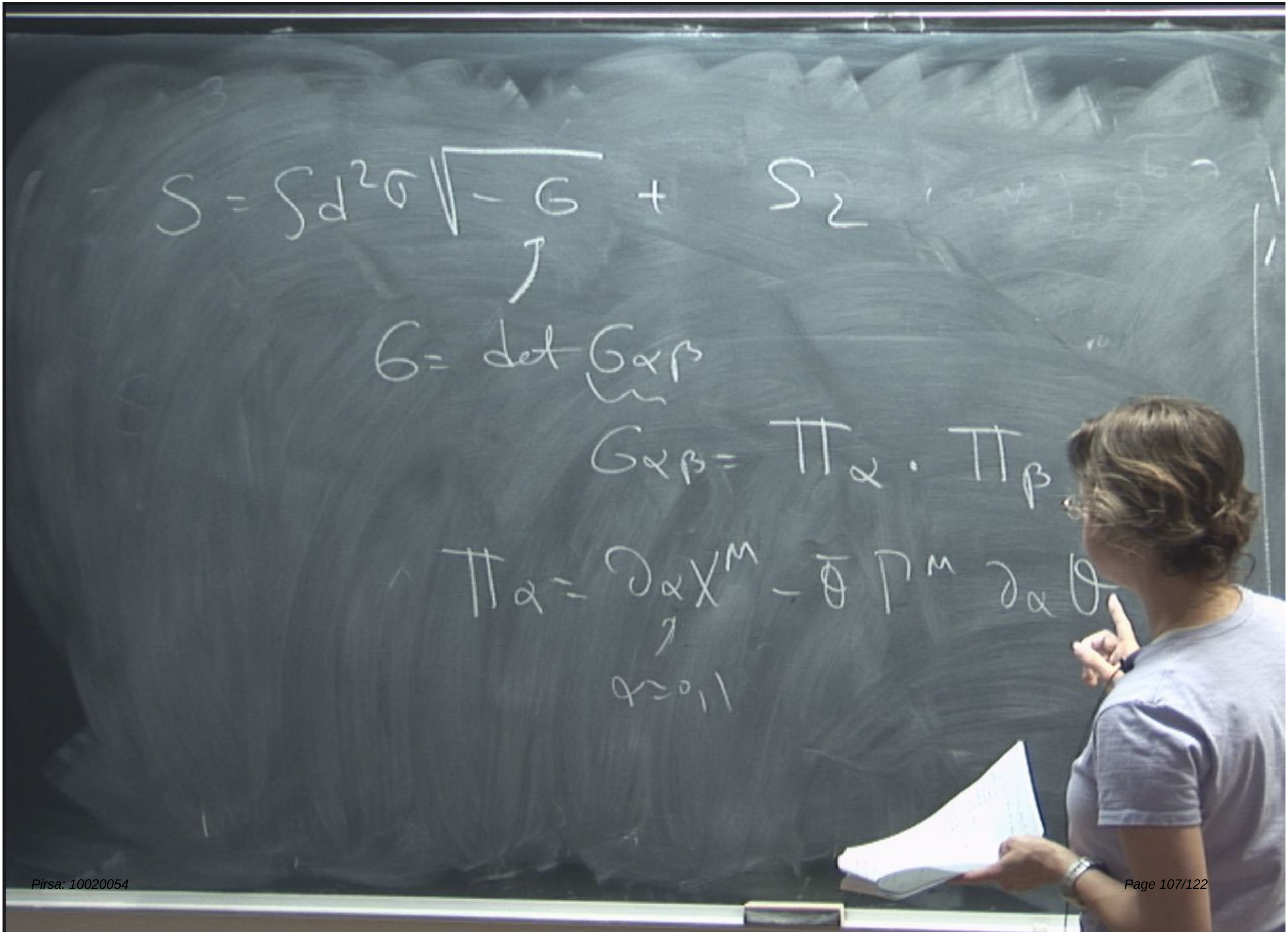

S = ∫d²σ √(−G) + S₂
G = det G_αβ
β = Π_α · Π

$$S = \int d^2\sigma \sqrt{-G} + S_2$$

$$G = \det G_{\alpha\beta}$$

$$G_{\alpha\beta} = \Pi_\alpha \cdot \Pi_\beta$$

$$\Pi$$


$S = \int d^2\sigma \sqrt{-G} + S_2$
$G = \det G_{\alpha\beta}$
$G_{\alpha\beta} = \Pi_\alpha \cdot \Pi_\beta$
$\Pi_\alpha = \partial_\alpha X^M - \bar{\theta}\Gamma^M \partial_\alpha \theta$
$\alpha = 0, 1$

$$S = \int d^2\sigma \sqrt{-G} + S_2$$

← right number of propagating d o f

$$G = \det G_{\alpha\beta}$$

$$G_{\alpha\beta} = \Pi_\alpha \cdot \Pi_\beta$$

$$\Pi_\alpha = \partial_\alpha X^M - \bar{\theta} \Gamma^M \partial_\alpha \theta$$

$\alpha = 0, 1$

$$S = \int d^2\sigma \sqrt{-G} + S_2$$

right number of propagating d.o.f.

$$G = \det \underbrace{G_{\alpha\beta}}$$

$$G_{\alpha\beta} = \Pi_\alpha \cdot \Pi_\beta$$

$$\Pi_\alpha = \partial_\alpha X^M - \bar\theta \Gamma^M \partial_\alpha \theta$$

$$\alpha = 0, 1$$

$$S_1 = \int d^2\sigma \sqrt{h}\, h^{\alpha\beta}\, \Pi_\alpha \Pi_\beta + S_2$$

$$S = \int d^2\sigma \sqrt{-G} + S_2$$

← right number of propagating d.o.f.

$$G = \det G_{\alpha\beta}$$

$$G_{\alpha\beta} = \Pi_\alpha \cdot \Pi_\beta$$

$$\Pi_\alpha = \partial_\alpha X^M - \bar{\theta}\, \Gamma^M \partial_\alpha \theta$$

$$\alpha = 0, 1$$

$$S_1 = \int d^2\sigma \sqrt{h}\, h^{\alpha\beta}\, \Pi_\alpha \cdot \Pi_\beta + S_2$$

$$S = \int d^2\sigma \sqrt{-G} + S_2$$

right number of propagating d o f

$$G = \det G_{\alpha\beta}$$

$$G_{\alpha\beta} = \Pi_\alpha \cdot \Pi_\beta$$

$$\Pi_\alpha = \partial_\alpha X^M - \bar{\theta}\,\Gamma^M \partial_\alpha \theta$$

$\alpha = 0,1$

$$S_1 = \int d^2\sigma \sqrt{h}\, h^{\alpha\beta}\, \Pi_\alpha \cdot \Pi_\beta + S_2$$

$$S = \int d^2\sigma \sqrt{-G} + S_2$$

← right number of propagating d o f

$$G = \det G_{\alpha\beta}$$

$$G_{\alpha\beta} = \Pi_\alpha \cdot \Pi_\beta$$

$$\Pi_\alpha = \partial_\alpha X^M - \bar{\theta}\Gamma^M \partial_\alpha \theta$$

$\alpha = 0, 1$

$$S_1 = \int d^2\sigma \sqrt{h}\, h^{\alpha\beta}\, \Pi_\alpha \cdot \Pi_\beta + S_2$$

D0-brane (point particle)

is needed to have the right number of propagating d.o.f

$$S = -m\int\sqrt{-\Pi_0\cdot\Pi_0}\,d\tau - m\underbrace{\int\bar{\theta}\,\Gamma_{11}\,\dot{\theta}\,d\tau}$$

$$\Gamma_{11} = \gamma_0\cdots\gamma_9$$

Additional symmetry →

κ-symmetry

in addition to Superpoincaré

D0-brane (point particle) $\underbrace{\text{is needed to have the right number of propagating}}$ d.o.f

$$S = -m\int\sqrt{-\Pi_0 \cdot \Pi_0}\, d\tau - m \int \bar{\theta}\,\Gamma_{11}\,\dot{\theta}\, d\tau$$

$$\Gamma_{11} = \gamma_0 \cdots \gamma_9$$

Additional symmetry →

κ-symmetry

in addition to invariance under Superpoincaré transfo

to quantize : choose again light-cone gauge.

$\Gamma_{11} = \gamma_0 \cdots \gamma_9$

Additional symmetry →

$\mathcal{H}$-symmetry

in addition to invariance under superpoincaré trafo to quantize : choose again light cone gauge.

→ more details in book!

$$\Gamma_{11} = \gamma_0 \cdots \gamma_9$$

Additional symmetry →

$\mathcal{H}$-symmetry

in addition to invariance under superpoincaré trafo

to quantize : choose again light-cone gauge.

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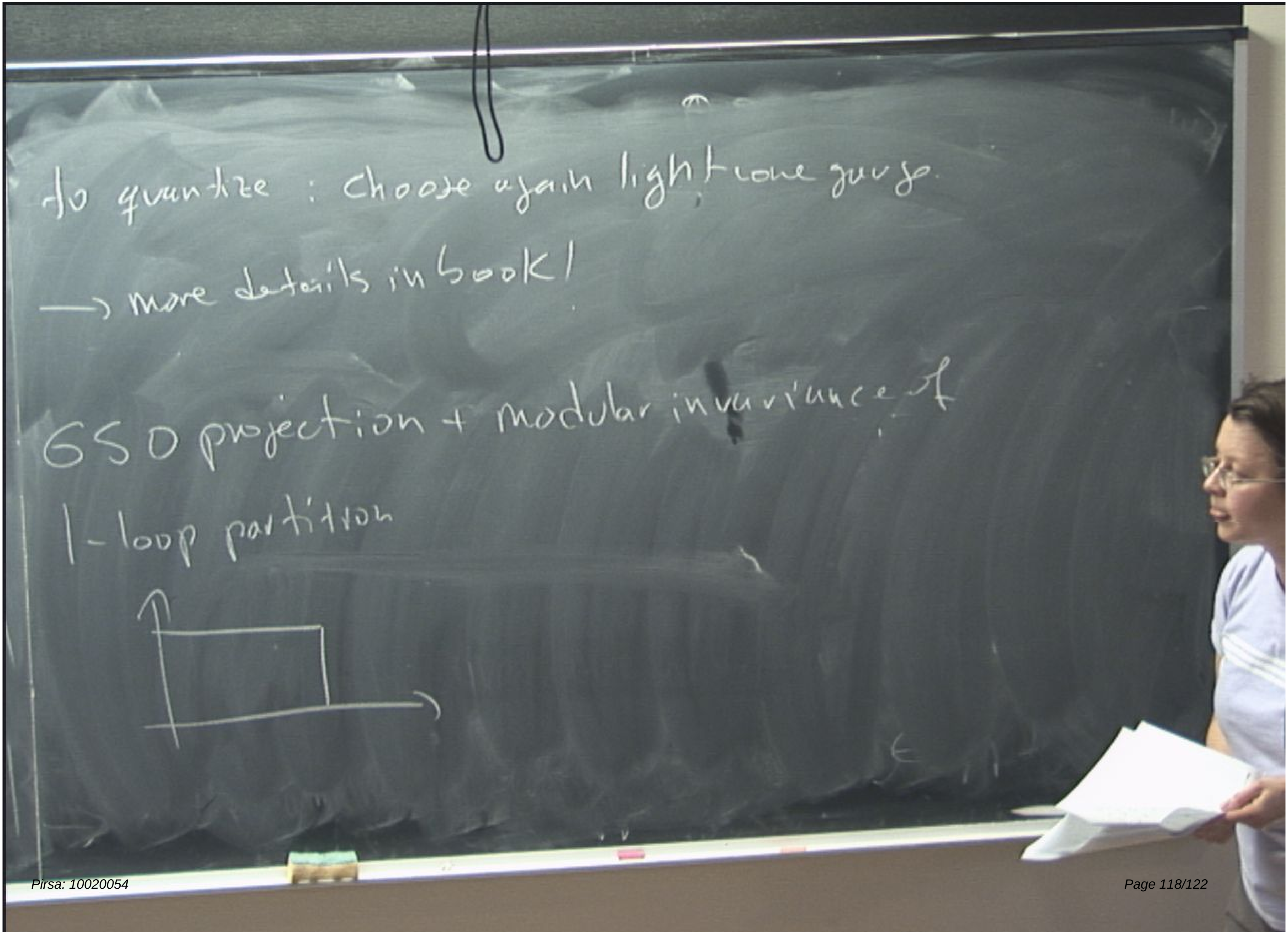
GSO projection + modular invariance of

1-loop partition

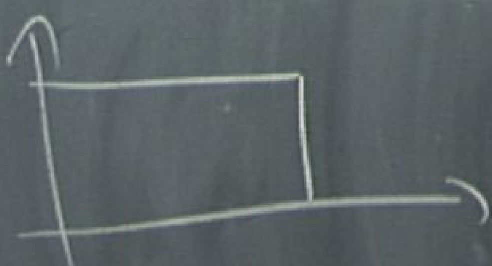

to quantize : choose again light cone gauge.
→ more details in book!
GSO projection + modular invariance of
1-loop partition

to quantize: choose again light-cone gauge.

→ more details in book!

GSO projection + modular invariance of

1-loop partition (section 9.4. GSW II)

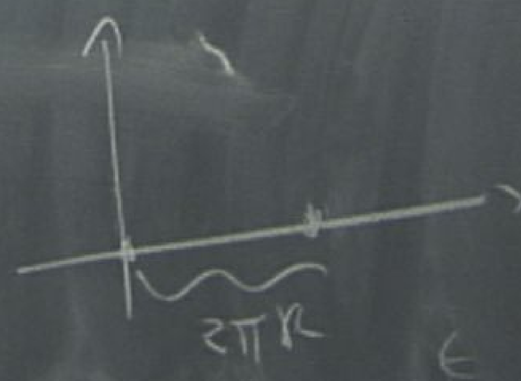


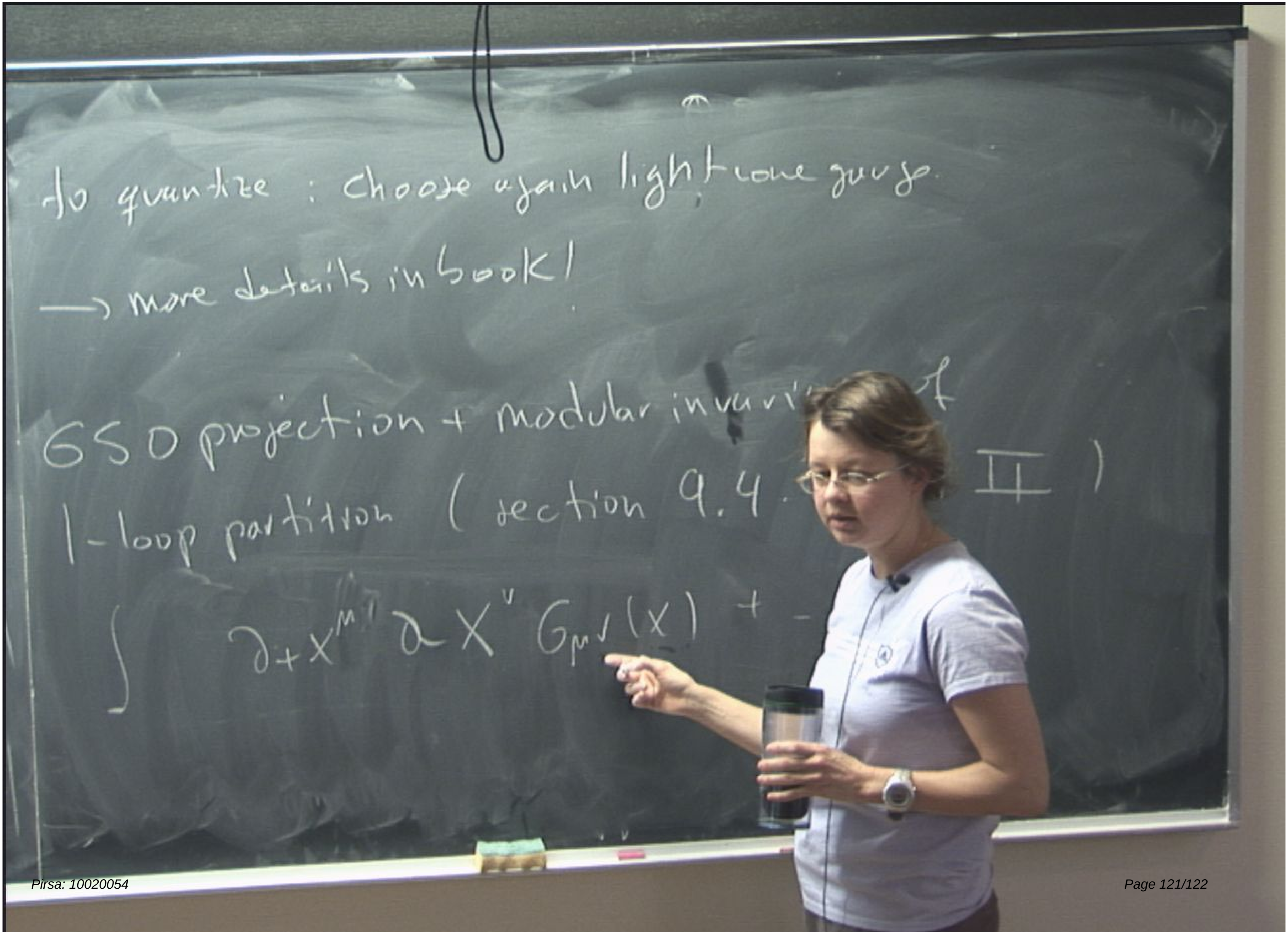
to quantize : choose again light-cone gauge.

→ more details in book!

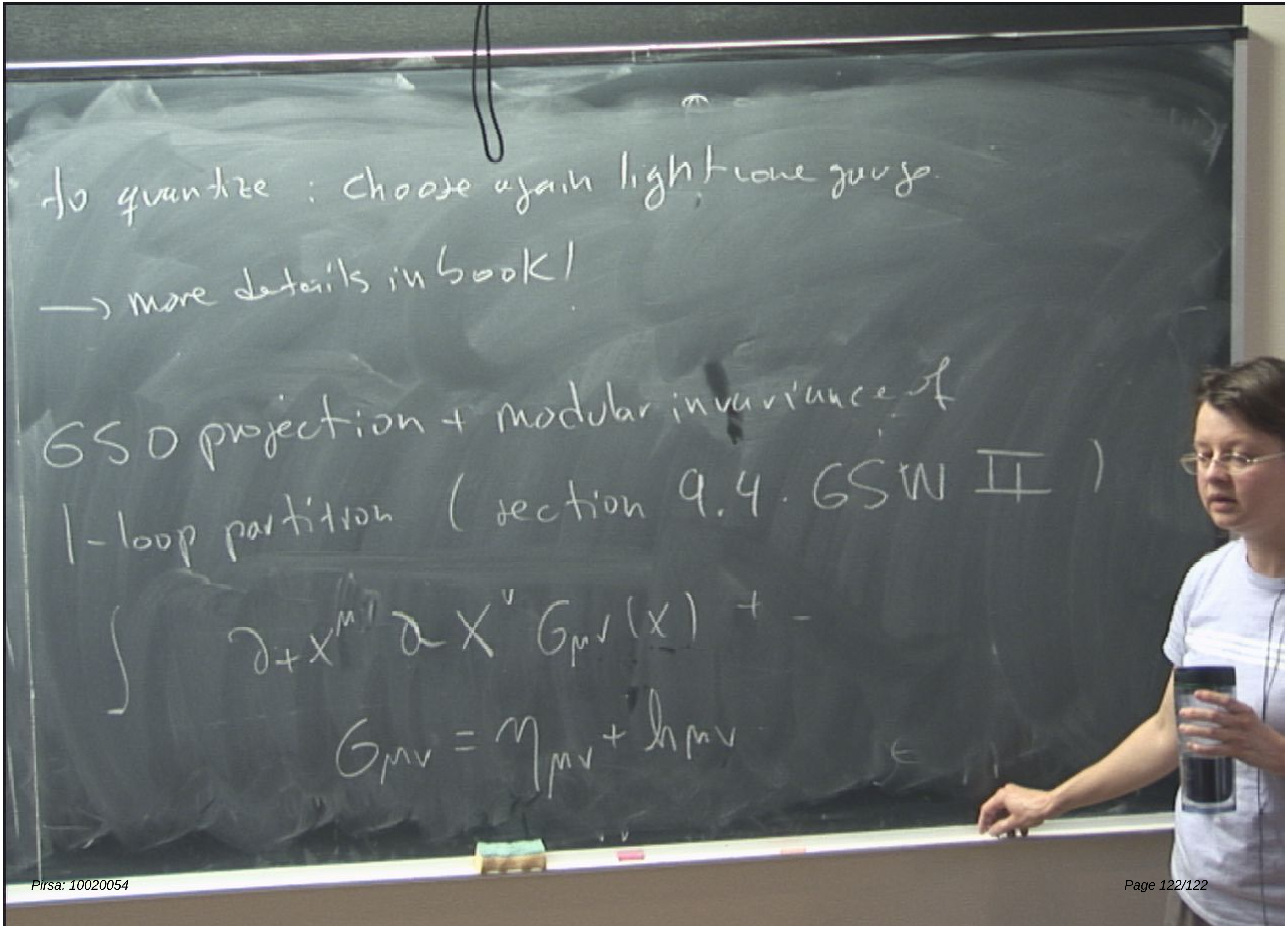
GSO projection + modular invariance of

1-loop partition (section 9.4. GSW II)





to quantize : choose again light cone gauge.
→ more details in book!
GSO projection + modular invarian of
1-loop partition (section 9.4.
II)
$\int \partial_+ X^{\mu} \partial_- X^{\nu} G_{\mu\nu}(X) +$



to quantize : choose again light cone gauge.
→ more details in book!
GSO projection + modular invariance of
1-loop partition (section 9.4. GSW II)
$\int \partial_+ X^\mu \partial_- X^\nu G_{\mu\nu}(X) + \ldots$
$G_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$