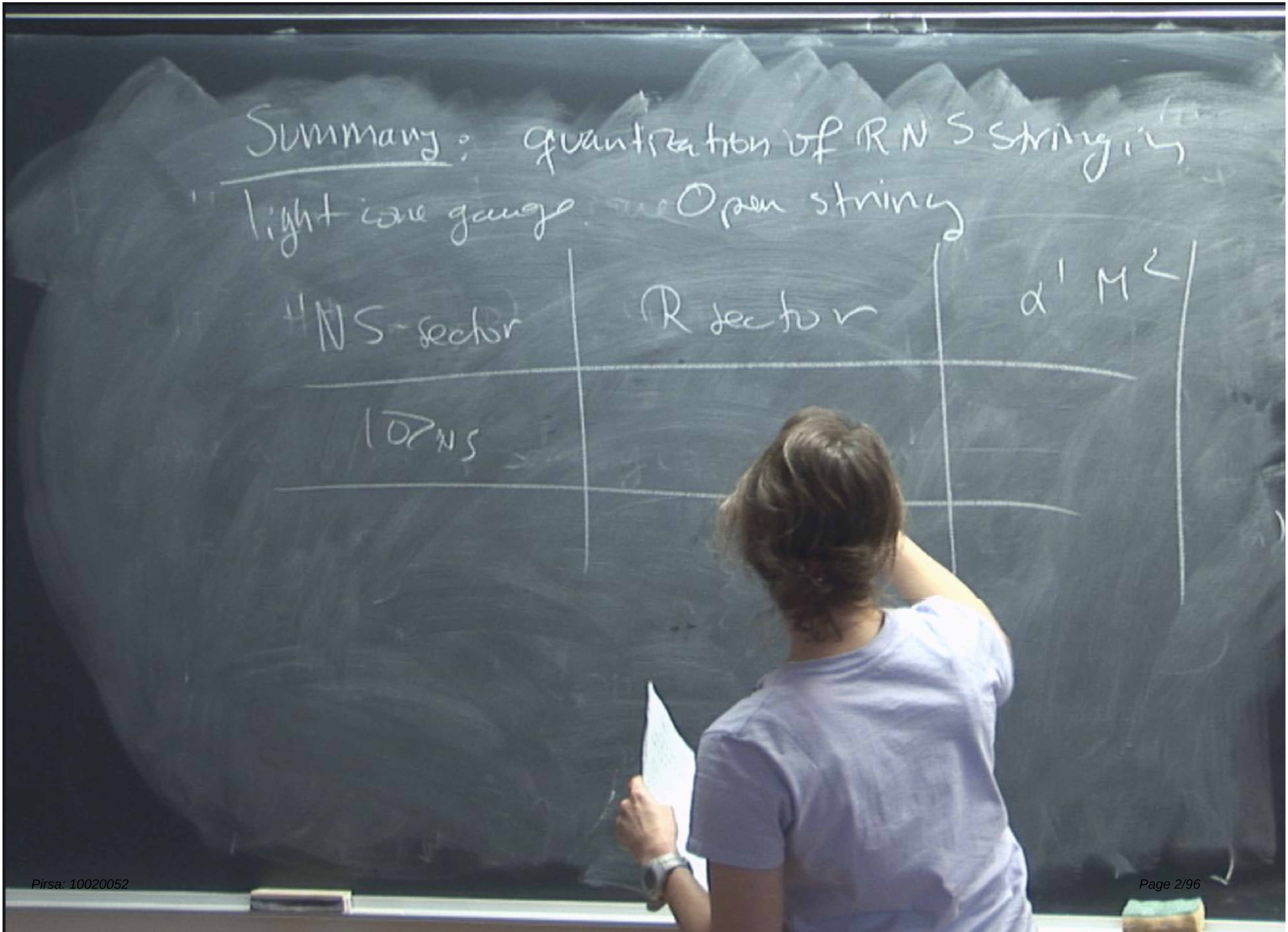
Title: String Theory - Review (PHYS 623) - Lecture 8

Date: Feb 03, 2010 11:20 AM

URL: http://pirsa.org/10020052

Abstract:

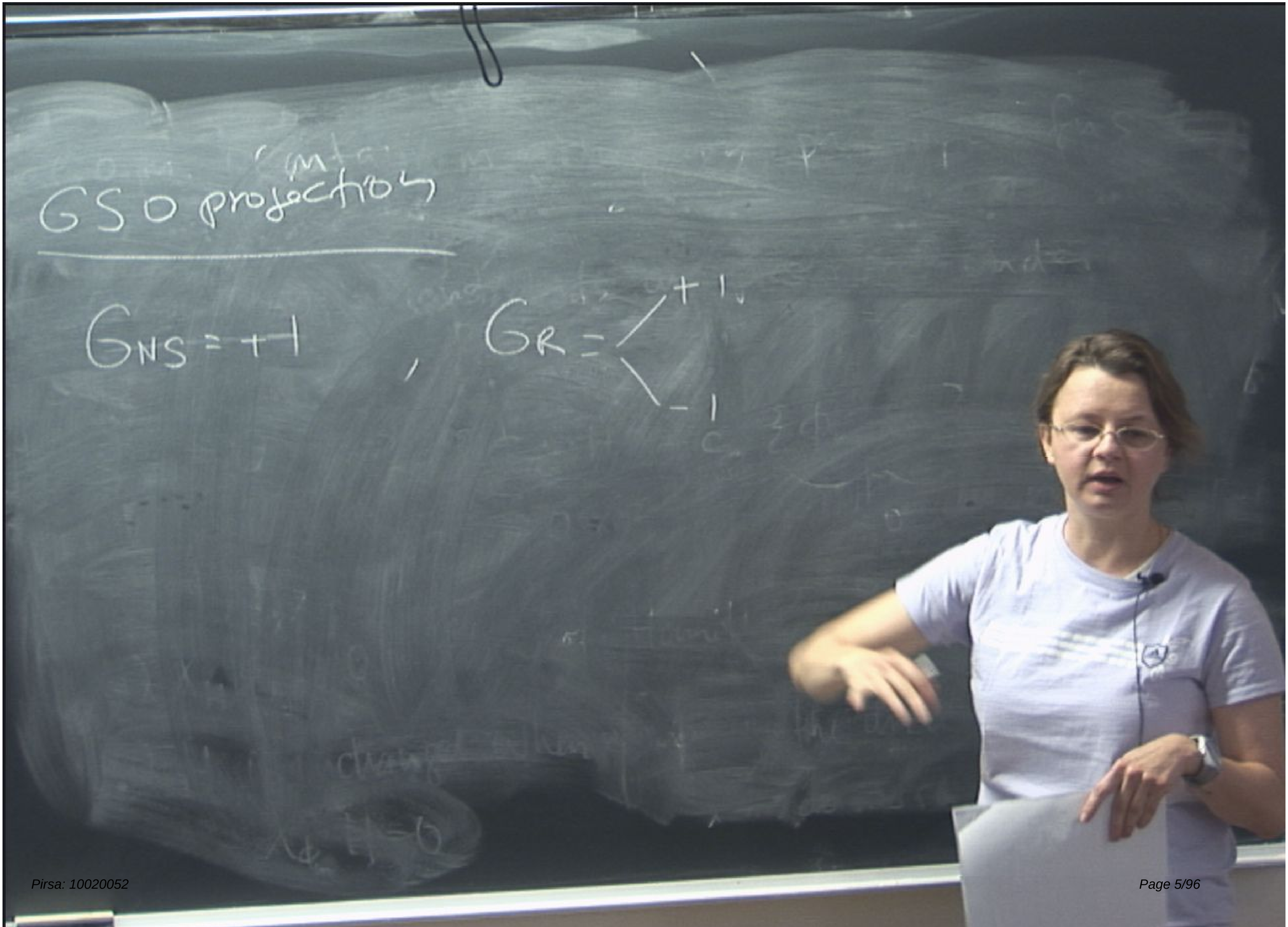

Summary: quantization of RNS string in
light cone gauge: Open string
NS-sector
R sector
α' M²
|0⟩NS

Summary: quantization of RNS string in light cone gauge: Open string

NS-sector	R sector	$\alpha' M^2$
$\|0\rangle_{NS}$		$-\frac{1}{2}$

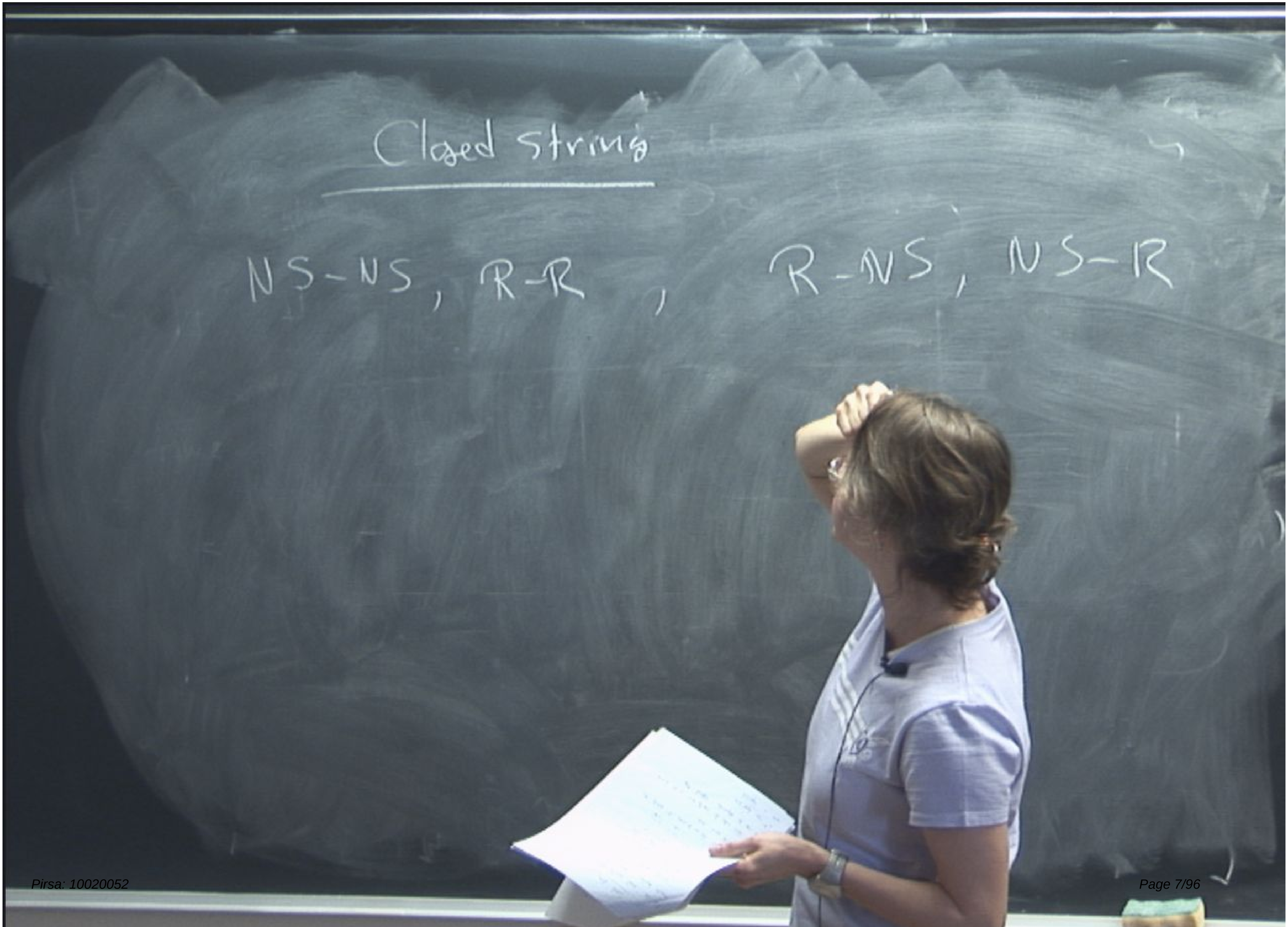
Summary: quantization of RNS string in light cone gauge. Open string

NS-sector	R sector	$\alpha' M^2$
$\|0\rangle_{NS}$		$-\frac{1}{2}$
$8_v \to \|i\rangle$	$8 \to \|a\rangle$	0


GSO projection
G_NS = +1 , G_R = +1
-1

GSO projection

$G_{NS} = +1$, $G_R = \begin{cases} +1 \\ -1 \end{cases}$


Closed string
NS-NS, R-R , R-NS, NS-R

Closed string

$\underbrace{\text{NS-NS}, \text{R-R}}_{\text{Space-time bosons}}$, $\underbrace{\text{R-NS}, \text{NS-R}}_{\text{fermions}}$

Closed string

NS-NS, R-R, R-NS, NS-R

$\underbrace{\text{NS-NS, R-R}}_{\text{space-time bosons}}$, $\underbrace{\text{R-NS, NS-R}}_{\text{fermions}}$

GSO projection

	G_{NS}	G_R	G_R
left movers	+1		
right movers	+1		

Closed string

NS-NS, R-R , R-NS, NS-R

$\underbrace{\text{NS-NS, R-R}}_{\text{space-time bosons}}$, $\underbrace{\text{R-NS, NS-R}}_{\text{fermions}}$

GSO projection

	G_{NS}	G_R	G_R
left movers	+1		
right movers	+1		

Closed string

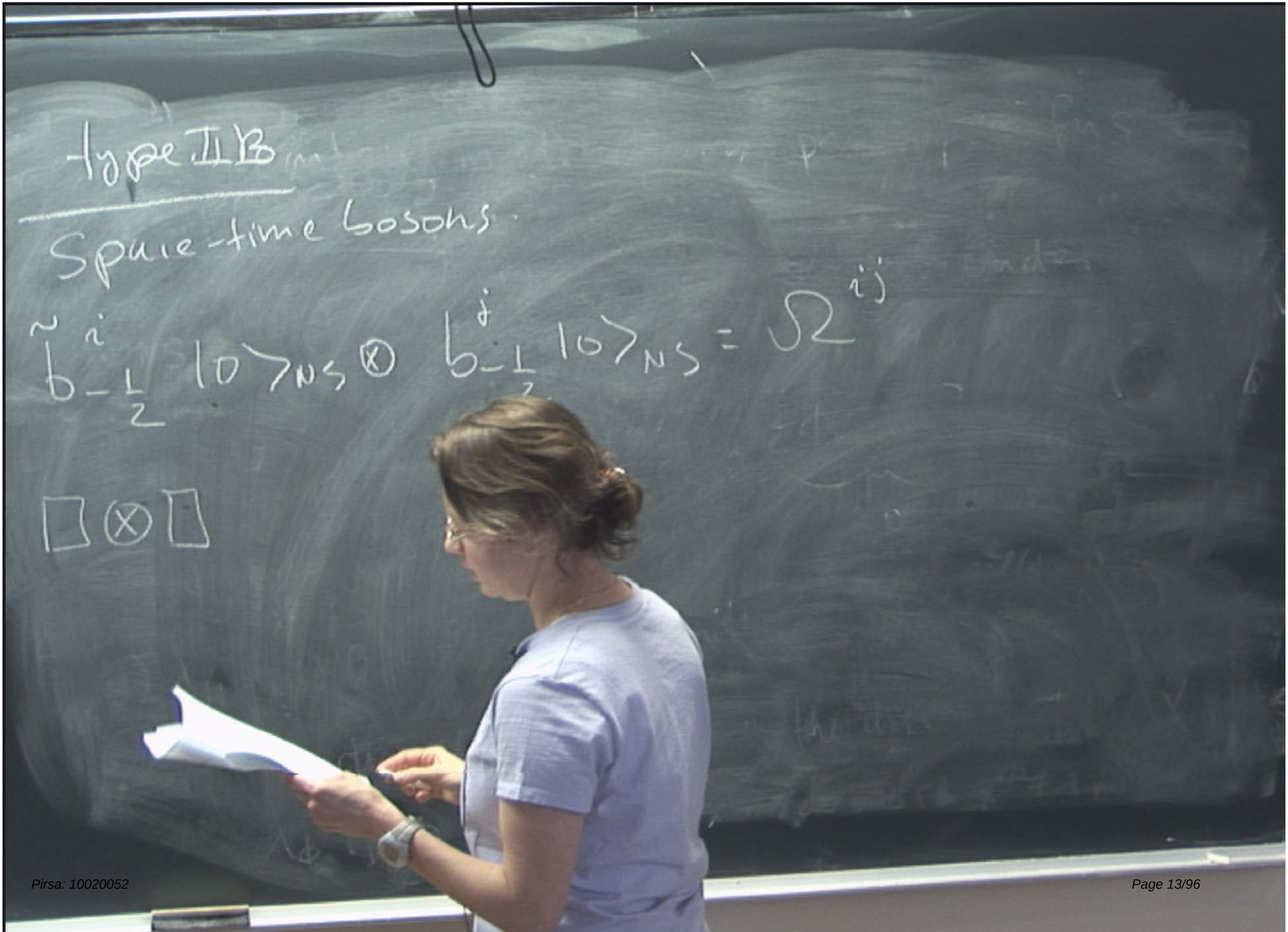
$$\underbrace{NS\text{-}NS,\ R\text{-}R}_{\text{space-time bosons}}\ ,\ \underbrace{R\text{-}NS,\ NS\text{-}R}_{\text{fermions}}$$

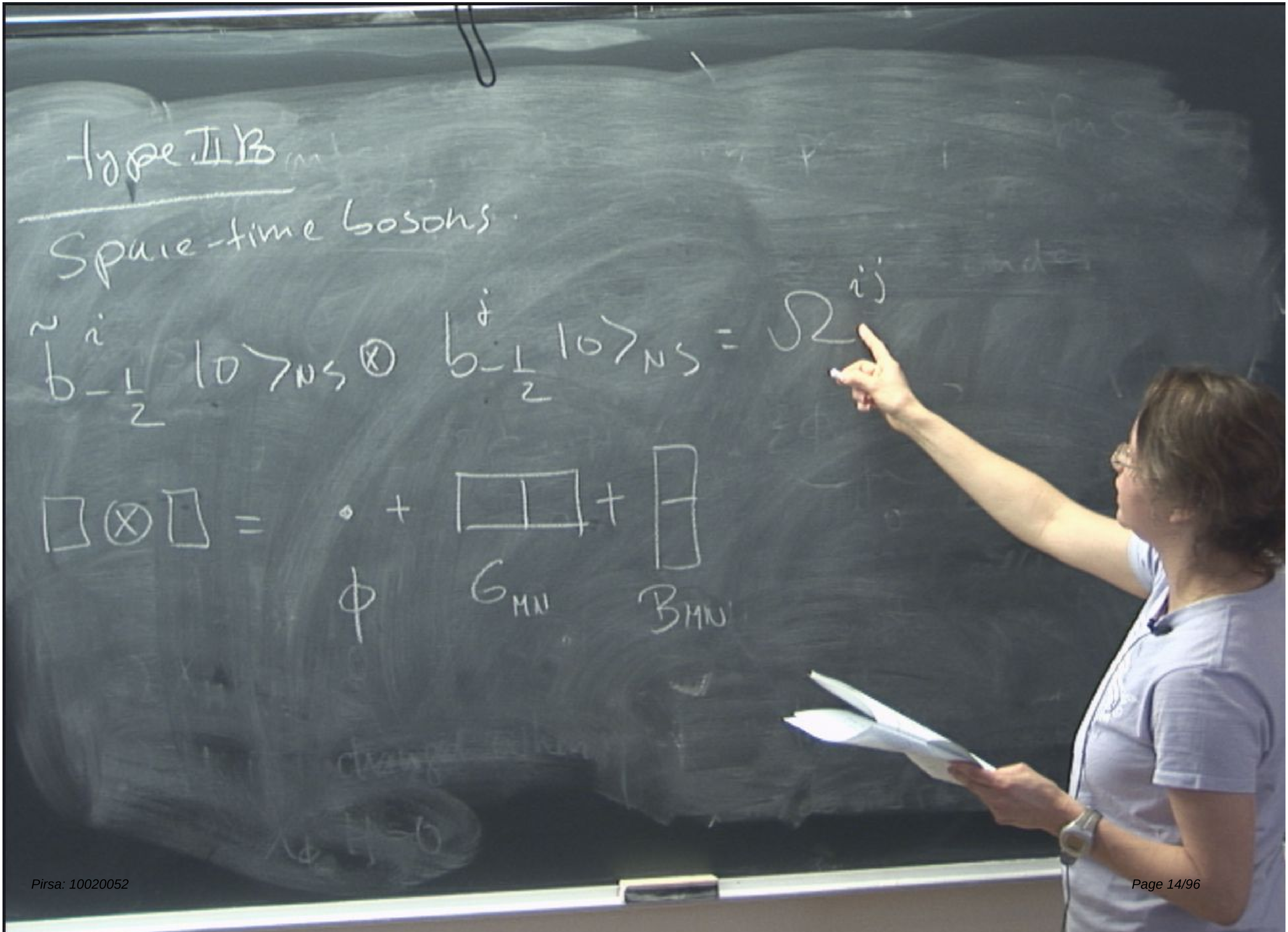
GSO projection

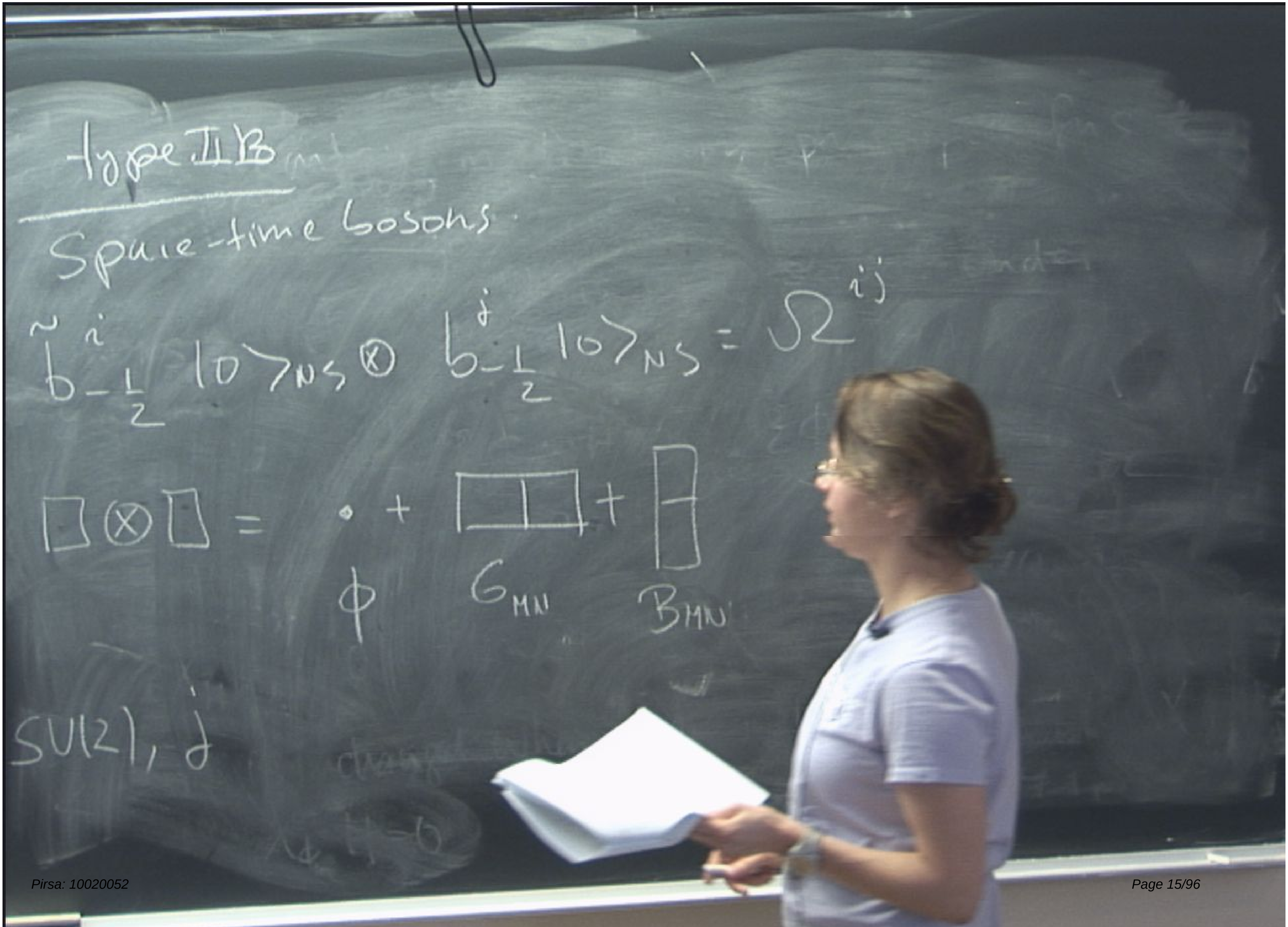
	G_{NS}	G_R	G_R
left movers	+1	+1	+1
right movers	+1	−1	+1
		type IIA	type IIB

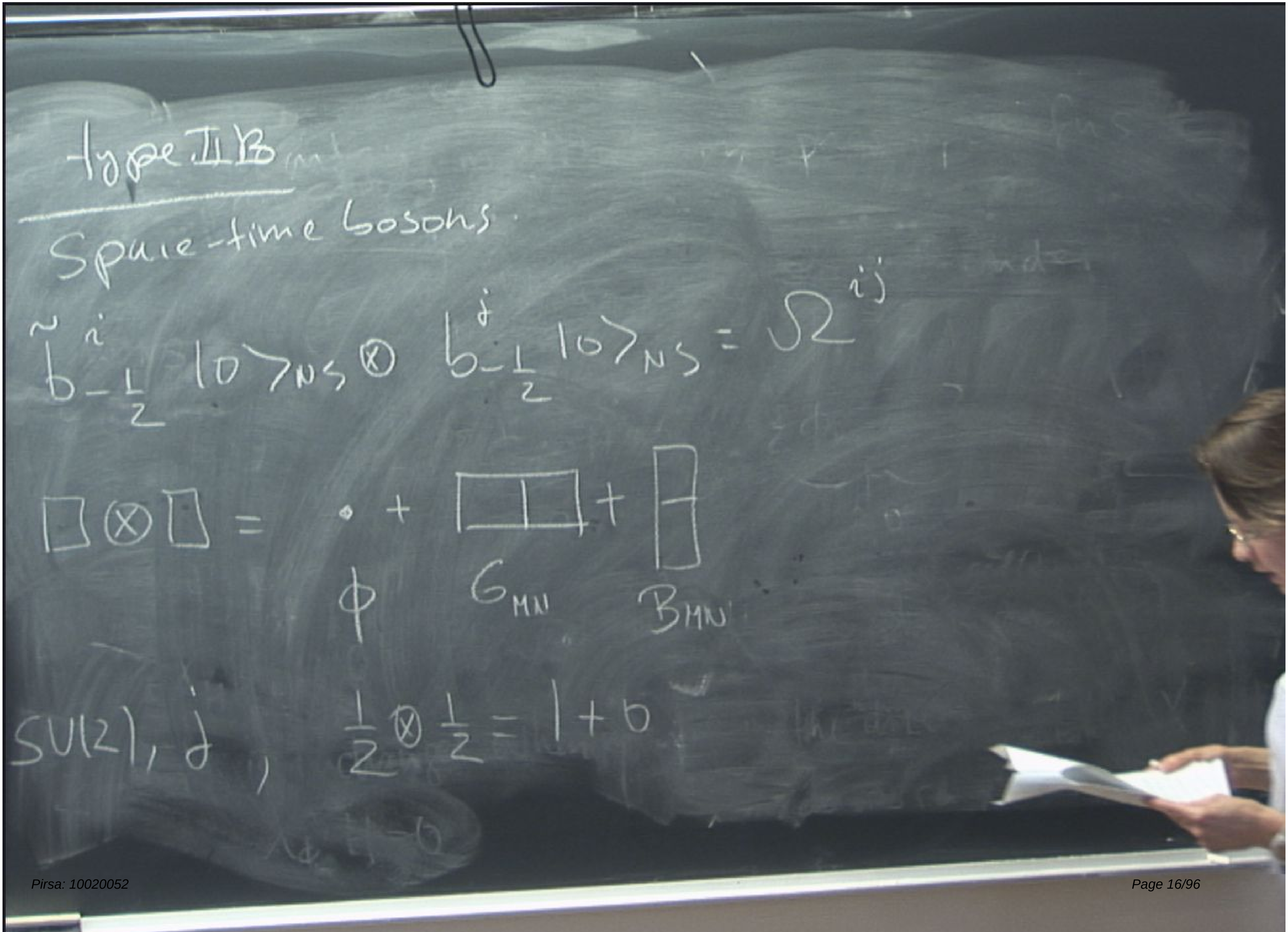
type IIB

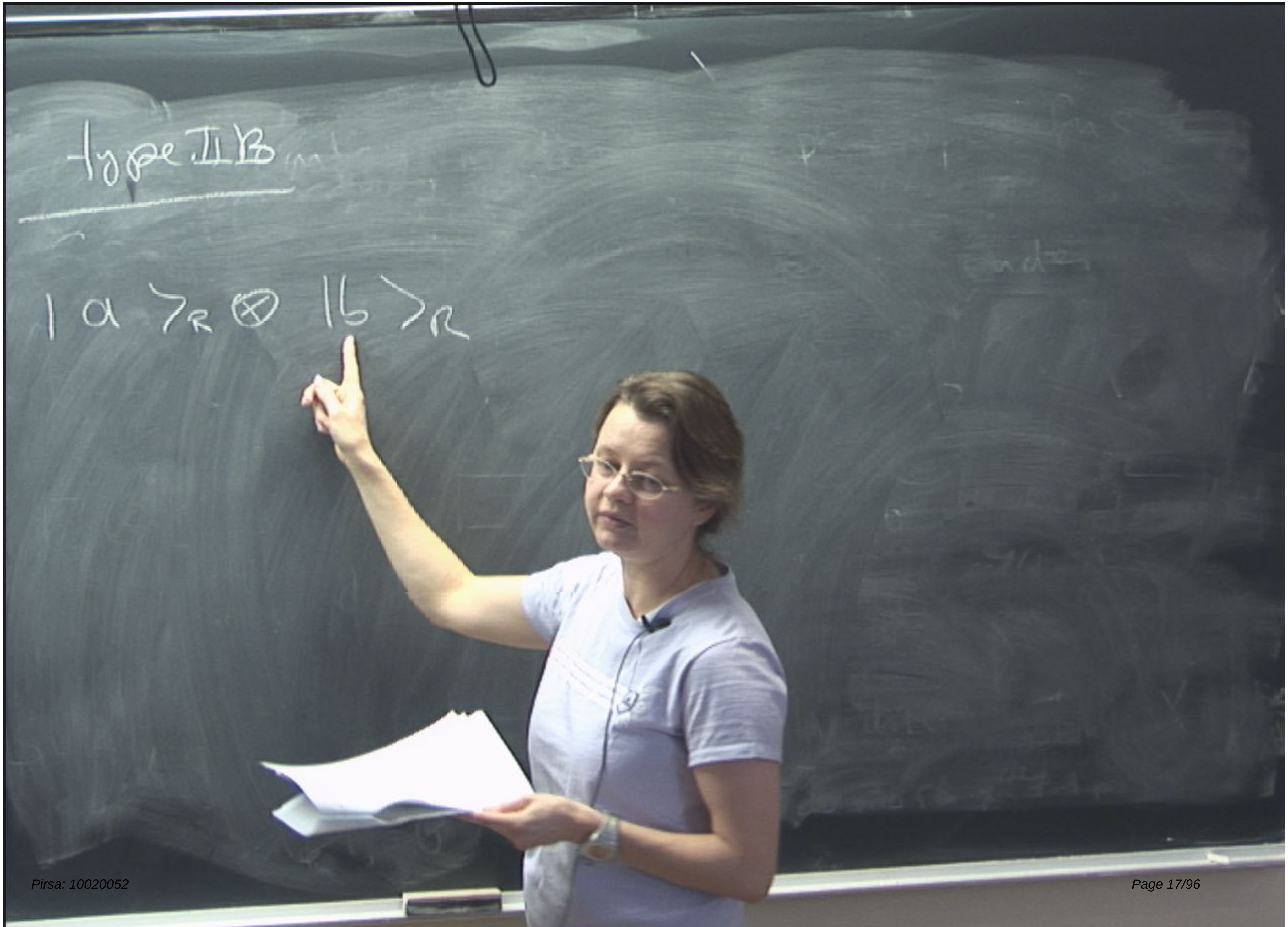
Space-time bosons

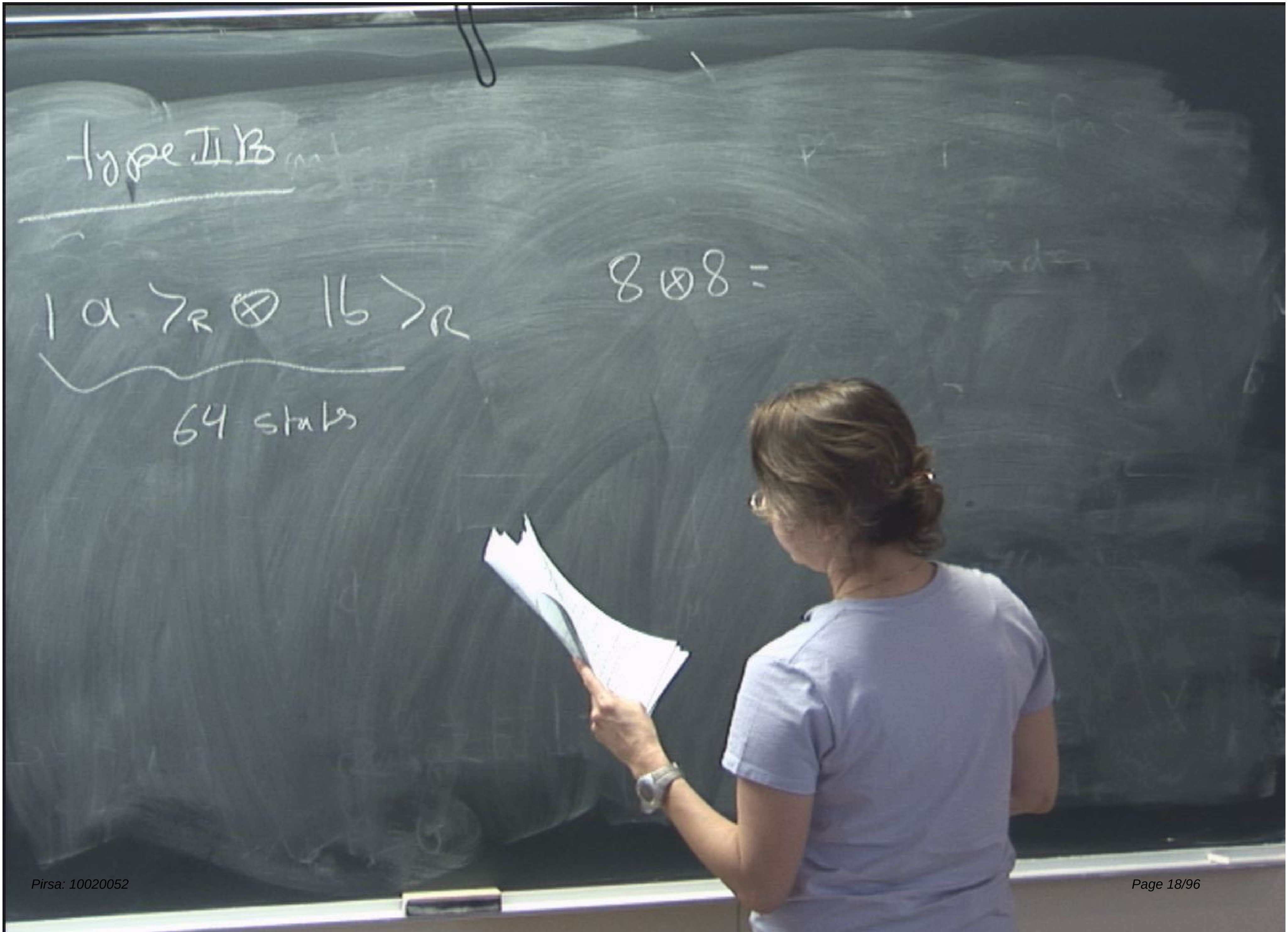

type IIB
Space-time bosons
$\tilde{b}^{i}_{-\frac{1}{2}} |0\rangle_{NS} \otimes b^{j}_{-\frac{1}{2}} |0\rangle_{NS} = \Omega^{ij}$
$\square \otimes \square$


Type IIB
Space-time bosons.
$\tilde{b}^{i}_{-\frac{1}{2}} |0\rangle_{NS} \otimes b^{j}_{-\frac{1}{2}} |0\rangle_{NS} = \Omega^{ij}$
ϕ
G_{MN}
B_{MN}


type IIB
Space-time bosons
b̃^i_{-1/2} |0>_NS ⊗ b^j_{-1/2} |0>_NS = Ω^{ij}
φ
G_MN
B_MN
SU(2), j


type IIB
Space-time bosons
$\tilde{b}^i_{-\frac{1}{2}} |0\rangle_{NS} \otimes b^j_{-\frac{1}{2}} |0\rangle_{NS} = \Omega^{ij}$
ϕ
G_{MN}
B_{MN}
SU(2), j,
$\frac{1}{2} \otimes \frac{1}{2} = 1 + 0$


type IIB
$|a\rangle_R \otimes |b\rangle_R$


type IIB
|a>R ⊗ |b>R
64 states
8⊗8 =

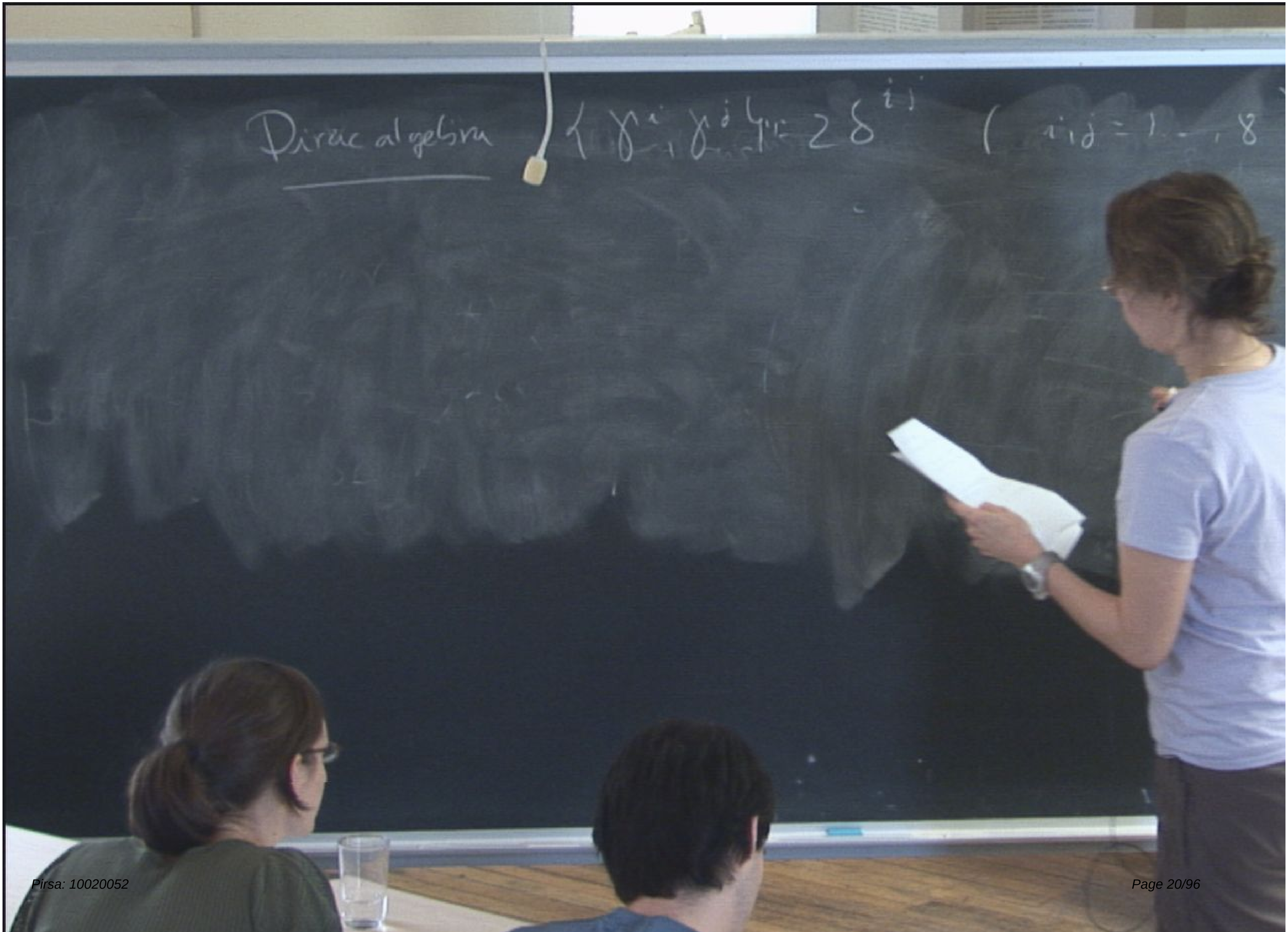
type IIB

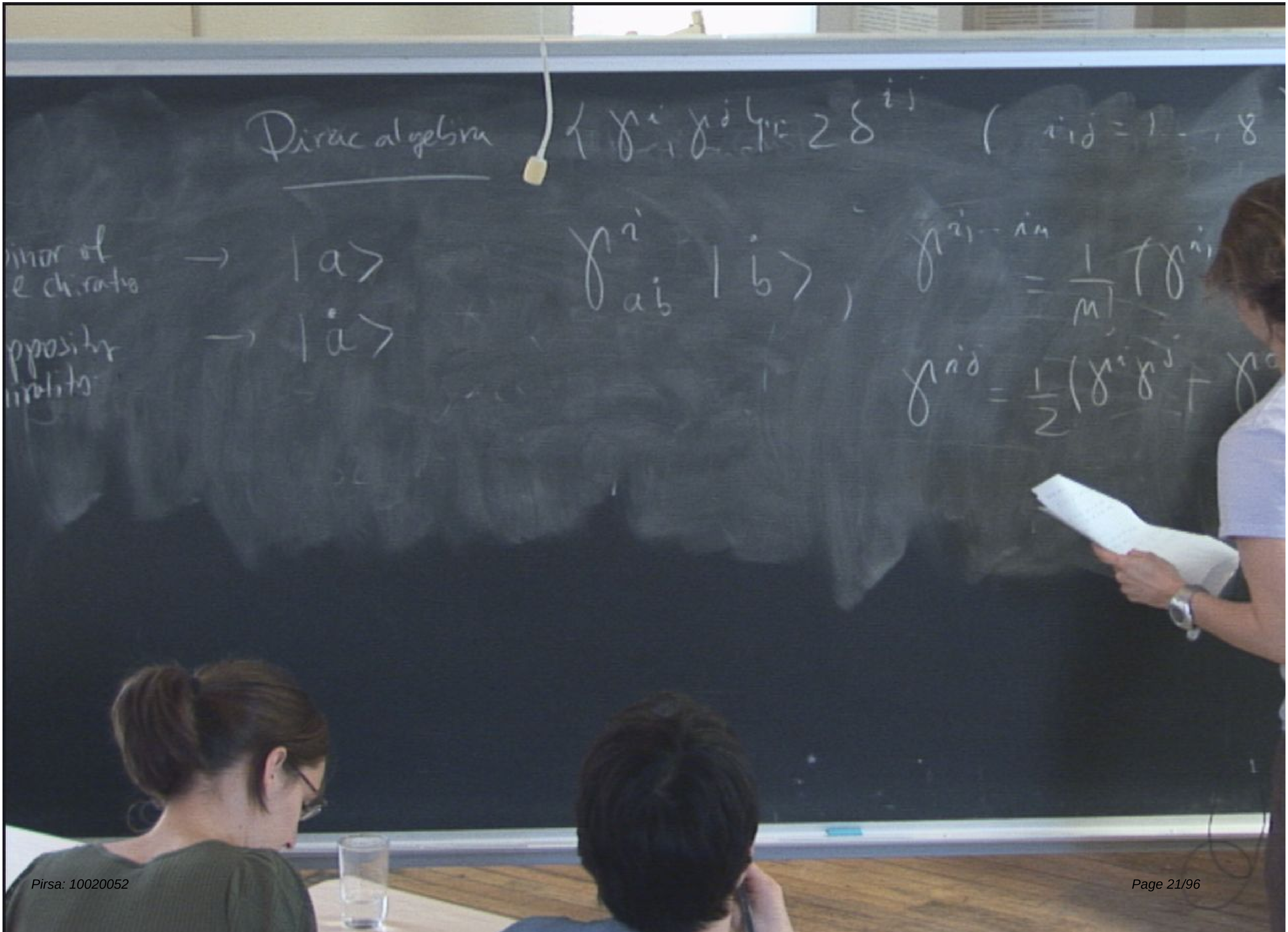
$$\underbrace{|a\rangle_R \otimes |b\rangle_R}_{\text{64 states}}$$

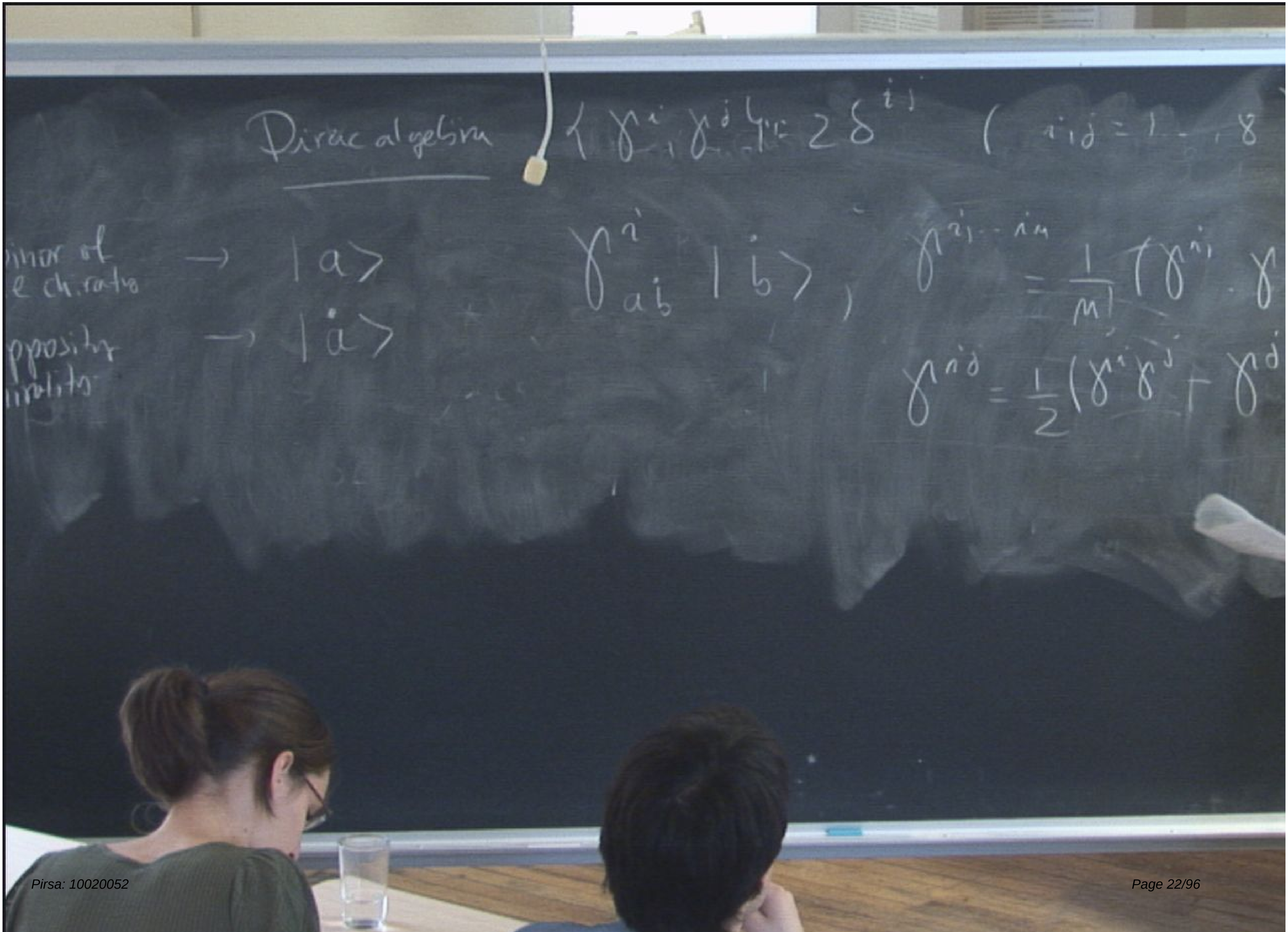
$$8 \otimes 8 = 28 + 35 + 1$$

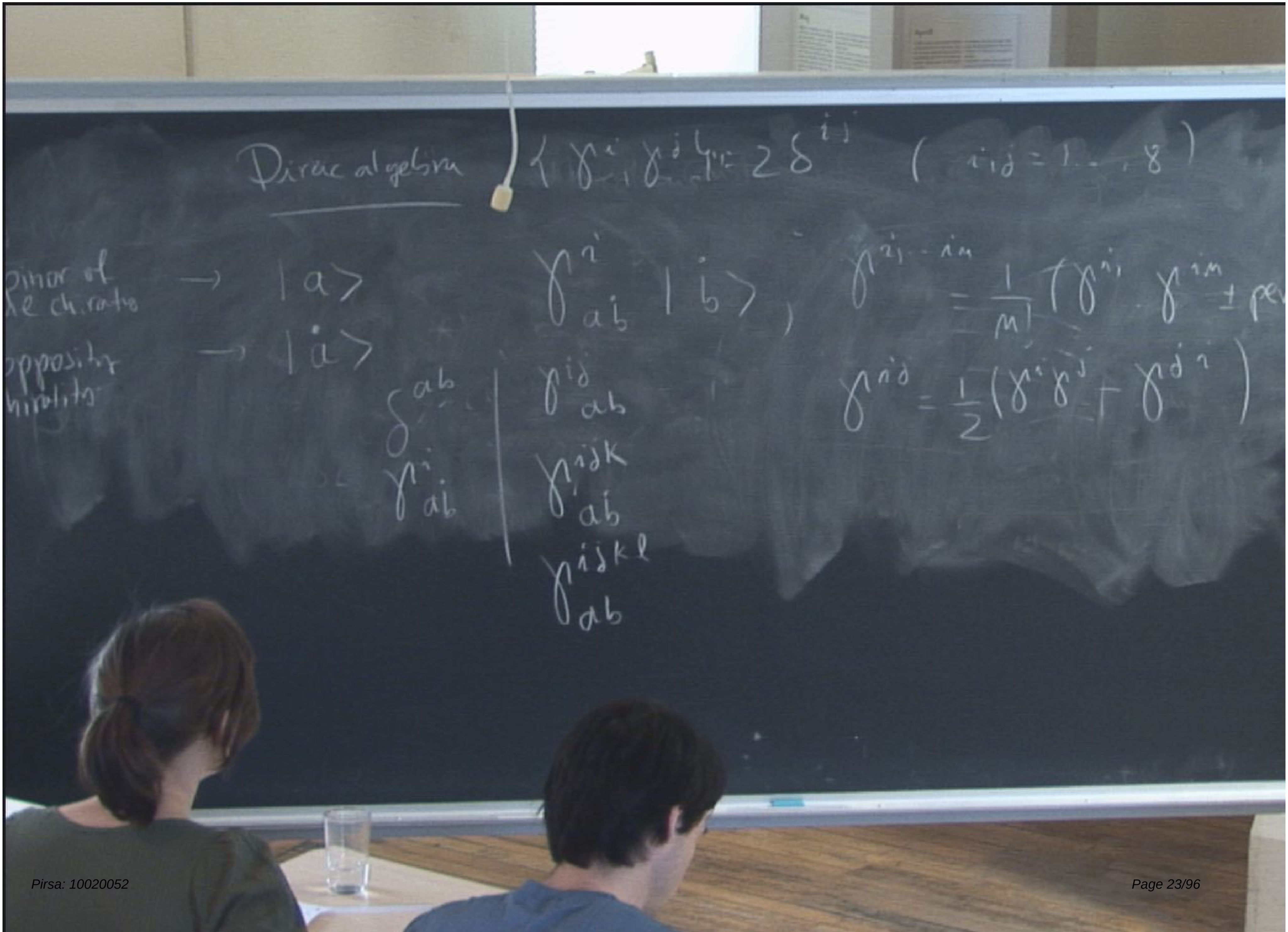
$$C^{(4)} \qquad C^{(2)} \qquad C^{(0)}$$

$$\delta^{ab}\, |a\rangle \otimes |b\rangle$$


Dirac algebra
$\{\gamma^i, \gamma^j\} = 2\delta^{ij}$ ($i,j = 1, \dots, 8$


Dirac algebra
{γ^i, γ^j} = 2δ^ij (i,j = 1,...,8
pinor of
e chirality → |a>
pposite
hirality → |ȧ>
γ^i_ab |b>,
γ^{i_1...i_n} = 1/n! (γ^i
γ^ij = 1/2 (γ^i γ^j + γ


Dirac algebra
{γ^i, γ^j} = 2δ^{ij}   (i, j = 1, ..., 8
|a>
γ^i_{ab} |b>
|ȧ>
γ^{ij} = 1/2 (γ^i γ^j − γ^j


Dirac algebra
$\{\gamma^i, \gamma^j\} = 2\delta^{ij}$ ($i, j = 1, \ldots, 8$)
pinor of he chiralit → $|a\rangle$
opposite chirality → $|\dot{a}\rangle$
$\gamma^i_{a\dot{b}} |\dot{b}\rangle$,
$\gamma^{i_1 \ldots i_n} = \frac{1}{n!}(\gamma^{i_1} \ldots \gamma^{i_n} \pm$ pe
$\gamma^{ij} = \frac{1}{2}(\gamma^i \gamma^j - \gamma^j \gamma^i)$
δ^{ab}
$\gamma^i_{a\dot{b}}$
γ^{ij}_{ab}
$\gamma^{ijk}_{a\dot{b}}$
γ^{ijkl}_{ab}

type IIB

$|a\rangle_R \otimes |b\rangle_R$

$\underbrace{\qquad\qquad}_{\text{64 states}}$

$8 \otimes 8 = 28 + 35 + 1$

$C^{(4)} \quad C^{(2)} \quad C^{(0)}$

$\delta^{ab}\, |a\rangle \otimes |b\rangle$

$\gamma^{ij}_{ab}\, |a\rangle \otimes |b\rangle$

$\gamma^{ijkl}\, |a\rangle \otimes |b\rangle$

$\frac{8 \cdot 7}{2} = 28$

type IIB

$$|a\rangle_R \otimes |b\rangle_R$$

$\underbrace{\qquad\qquad}$ 64 states

$$8 \otimes 8 = 28 + 35 + 1$$

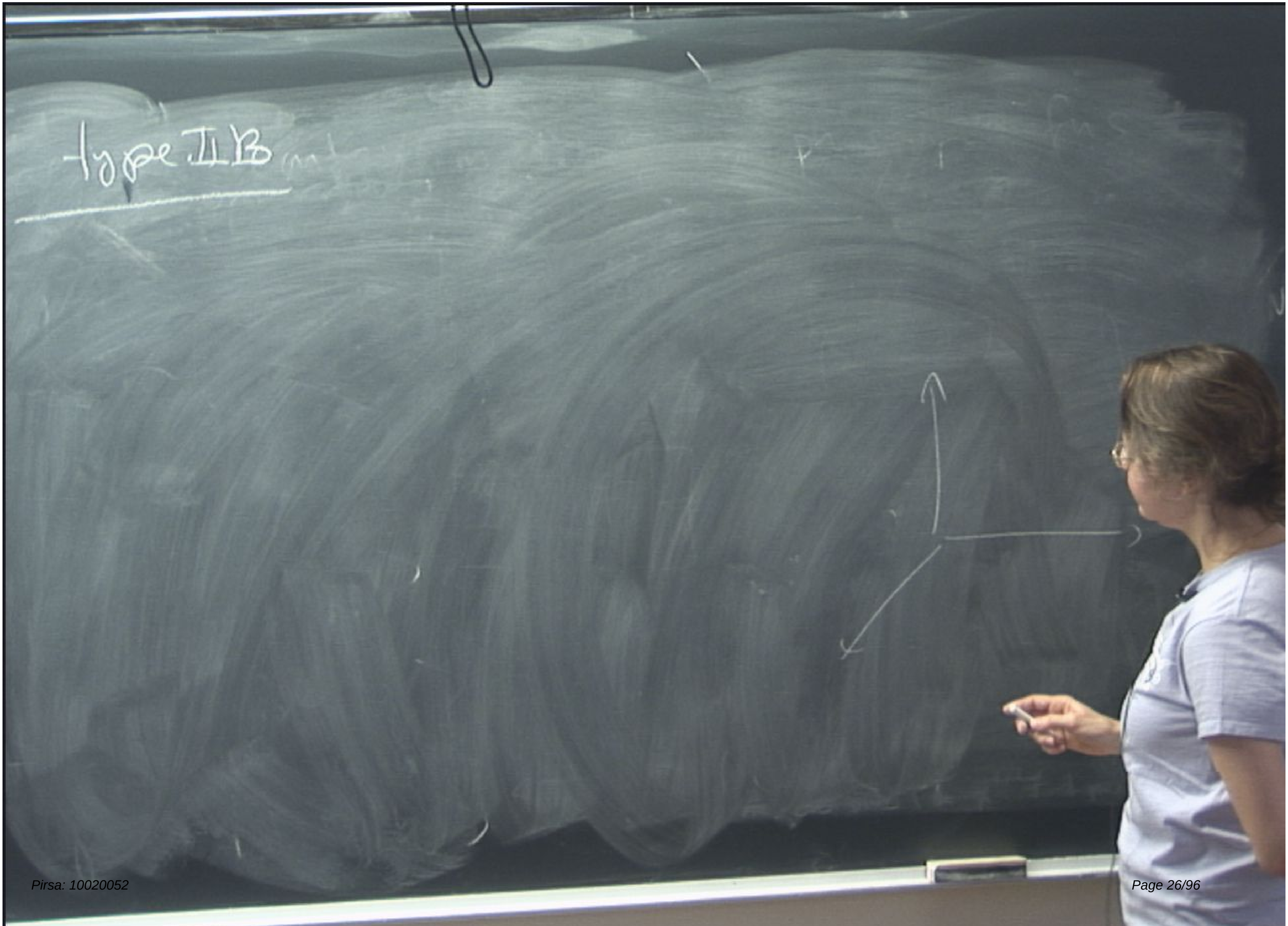
$C^{(4)}$ $C^{(2)}$ $C^{(0)}$

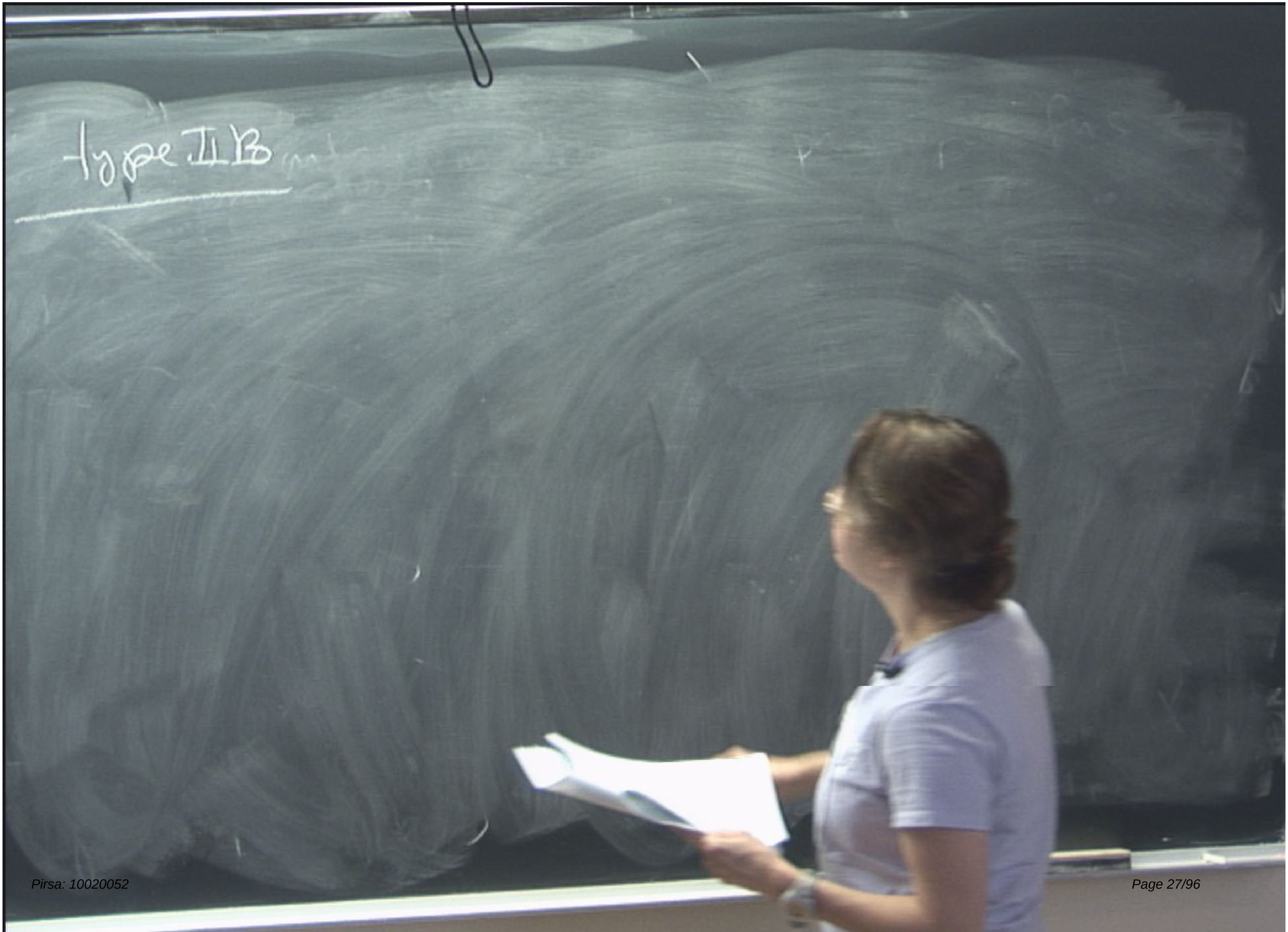
$\delta^{ab} |a\rangle \otimes |b\rangle \leftarrow 1$

$\frac{8 \cdot 7}{2} = 28$

$\gamma^{ij}_{ab} |a\rangle \otimes |b\rangle \leftarrow 28$

$\gamma^{ijkl} |a\rangle \otimes |b\rangle \leftarrow 35$ (check!)


type IIB


type IIB

type IIB

NS–NS $\quad \phi, \; G_{MN}, \; B_{MN}$

R–R $\quad C^{(0)}, \; C^{(2)}, \; C^{(4)}$

$A_M \leftarrow C^{(1)}$

type IIB

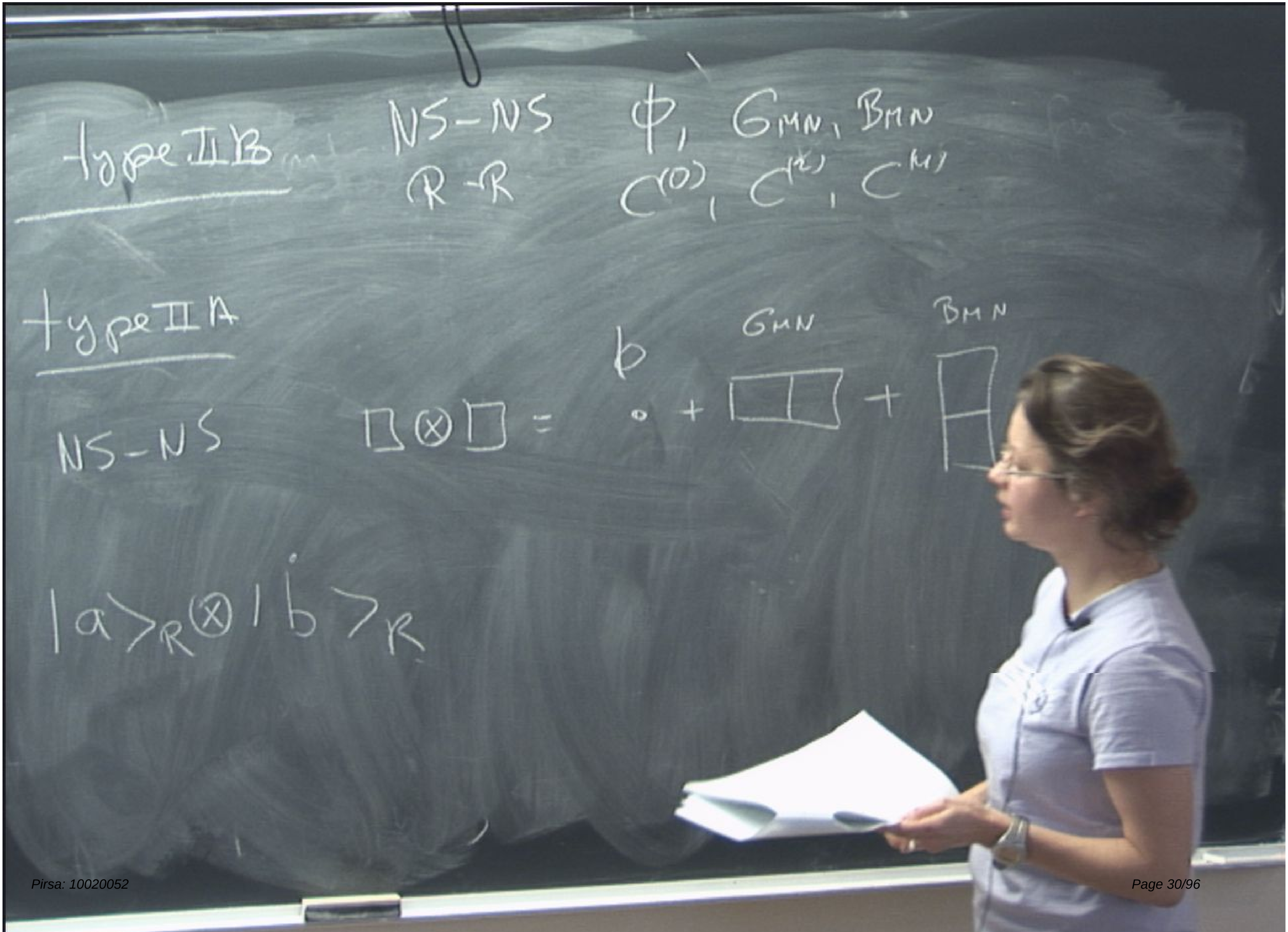
NS–NS $\quad \phi, \; G_{MN}, \; B_{MN}$

R–R $\quad C^{(0)}, \; C^{(2)}, \; C^{(4)}$

$A_M \leftarrow C^{(1)}$

$C^{(2)}_{MN}$

$C^{(4)}_{MNPQ}$


type II B
NS-NS φ, GMN, BMN
R-R C(0), C(2), C(4)
type II A
GMN
BMN
NS-NS
|a>R ⊗ |b>R

type IIB

NS–NS $\quad \phi,\ G_{MN},\ B_{MN}$

R–R $\quad C^{(0)},\ C^{(2)},\ C^{(4)}$

type IIA

$$\text{NS–NS} \qquad \square \otimes \square = \overset{b}{\bullet} + \overset{G_{MN}}{\begin{array}{|c|c|}\hline \ & \ \\ \hline\end{array}} + \overset{B_{MN}}{\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array}}$$

$$|a\rangle_R \otimes |b\rangle_R$$

$$\uparrow$$

$$8 \times 8 = 64$$

$$\gamma^{i}_{ab}\,|a\rangle \otimes |b\rangle \leftarrow C^{(1)}$$

$$\gamma^{ijk}_{ab}\,|a\rangle \otimes |b\rangle \leftarrow C^{(3)}$$

type IIB

NS–NS $\quad \phi,\ G_{MN},\ B_{MN}$

R–R $\quad C^{(0)},\ C^{(2)},\ C^{(4)}$

type II A

NS–NS

$$\square \otimes \square = \underset{\phi}{\cdot} + \underset{G_{MN}}{\square\square} + \underset{B_{MN}}{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}$$

$$|a\rangle_R \otimes |b\rangle_R$$

8×8

$$\gamma^{i}_{ab}\,|a\rangle \otimes |b\rangle \leftarrow C^{(1)} \leftarrow 8$$

$$\gamma^{ijk}_{ab}\,|a\rangle \otimes |b\rangle \leftarrow C^{(3)} \leftarrow$$

type IIB — NS-NS $\quad \phi,\ G_{MN},\ B_{MN}$

R-R $\quad C^{(0)},\ C^{(2)},\ C^{(4)}$

$$\frac{8\cdot 7\cdot 6}{3!}$$

type IIA

NS-NS

$$\square \otimes \square = \overset{\phi}{\cdot} + \overset{G_{MN}}{\square\square} + \overset{B_{MN}}{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}$$

$$|a\rangle_R \otimes |\dot{b}\rangle_R$$

$$8\times 8 = 64$$

$$8+56 = 64$$

$$\gamma^{i}_{a\dot{b}}\,|a\rangle\otimes|\dot{b}\rangle \leftarrow C^{(1)} \leftarrow 8$$

$$\gamma^{ijk}_{a\dot{b}}\,|a\rangle\otimes|\dot{b}\rangle \leftarrow C^{(3)} \leftarrow 56$$

type IIB   NS–NS $\phi,\ G_{MN},\ B_{MN}$
R–R $C^{(0)},\ C^{(2)},\ C^{(4)}$

$\frac{8\cdot 7\cdot 6}{3!}$

type IIA   NS–NS $\phi,\ G_{MN},\ B_{MN}$
R–R $C^{(1)},\ C^{(3)}$

$|a\rangle_R \otimes |\dot{b}\rangle_R$

$8\times 8 = 64$

$8+56 = 64$

$\gamma^{i}_{a\dot{b}}|a\rangle\otimes|\dot{b}\rangle \leftarrow C^{(1)} \leftarrow 8$

$\gamma^{ijk}_{a\dot{b}}|a\rangle\otimes|\dot{b}\rangle \leftarrow C^{(3)} \leftarrow 56$

type IIB   NS-NS $\phi,\ G_{MN},\ B_{MN}$

R-R $C^{(0)},\ C^{(2)},\ C^{(4)} \leftarrow$ even forms

type IIA   NS-NS $\phi,\ G_{MN},\ B_{MN}$

R-R $C^{(1)},\ C^{(3)} \leftarrow$ odd forms.

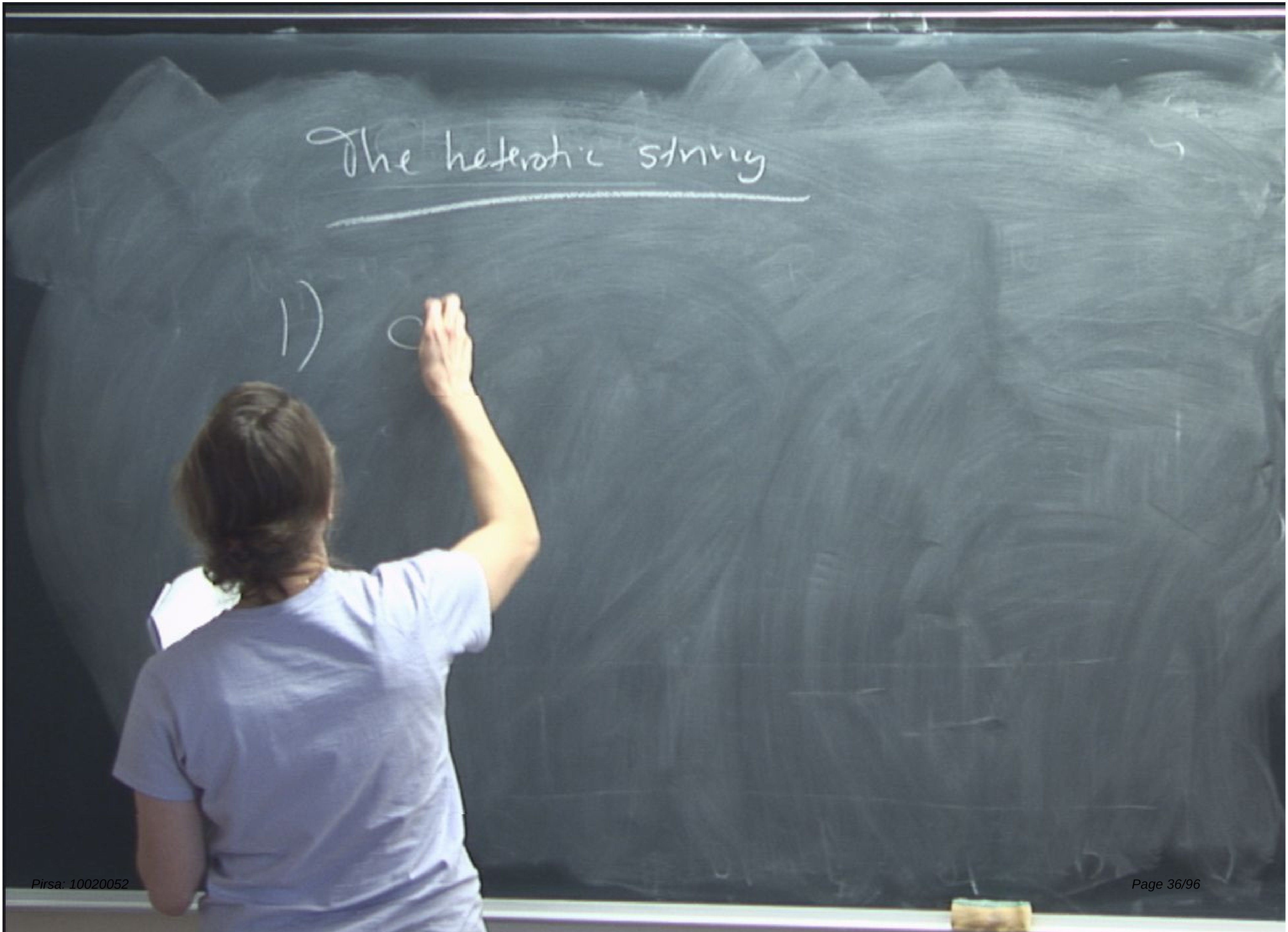
$|a\rangle_R \otimes |b\rangle_R$

$8 \times 8 = 64$

$8 + 56 = 64$

$\gamma^{i}_{ab}\,|a\rangle \otimes |b\rangle \leftarrow C^{(1)} \leftarrow 8$

$\gamma^{ijk}_{ab}\,|a\rangle \otimes |b\rangle \leftarrow C^{(3)} \leftarrow 5$


The heterotic string
1)

The heterotic string

1) closed bosonic strings

$$S=\int d^2\sigma\, \partial_\alpha X^\mu \partial^\alpha X_\mu$$

2) closed strings in RNS formalism

$$S=\int d^2\sigma\big(\partial_\alpha X_\mu \partial^\alpha X^\mu + \bar{\psi}^\mu \quad \psi$$

$$X^{\mu}(\sigma,\tau) = X_R^{\mu}(\tau-\sigma) + X_L^{\mu}(\tau+\sigma)$$

$$X^\mu(\sigma,\tau) = X_R^\mu(\tau-\sigma) + X_L^\mu(\tau+\sigma)$$

Left movers and right movers where decoupled.

$$X^{\mu}(\sigma,\tau) = X_R^{\mu}(\tau-\sigma) + X_L^{\mu}(\tau+\sigma)$$

Left movers and right movers where decoupled.

Take advantage of this independence.

The heterotic string

1) Start:

$$S = \int d^2\sigma \,(\partial_+ X_\mu \partial_- X^\mu + \psi_- \partial_+ \psi_- + \psi_+ \partial_- \psi_+)$$

The heterotic string

1) Start:

$$S = \int d^2\sigma \, (\underbrace{\partial_+ X_\mu \partial_- X^\mu + \psi_- \partial_+ \psi_-} + \cancel{\psi_+ \partial_- \psi_+})$$

L	R
$X_L^\mu(\tau+\sigma)$	$X_R^\mu(\tau-\sigma)$
	$\psi^\mu(\tau-\sigma)$
$D=26$	$D=10$

The heterotic string

1) Start

$$S = \int d^2\sigma \, (\underbrace{\partial_+ X_\mu \partial_- X^\mu + \psi_- \partial_+ \psi_-}_{} + \cancel{\psi_+ \partial_- \psi_+})$$

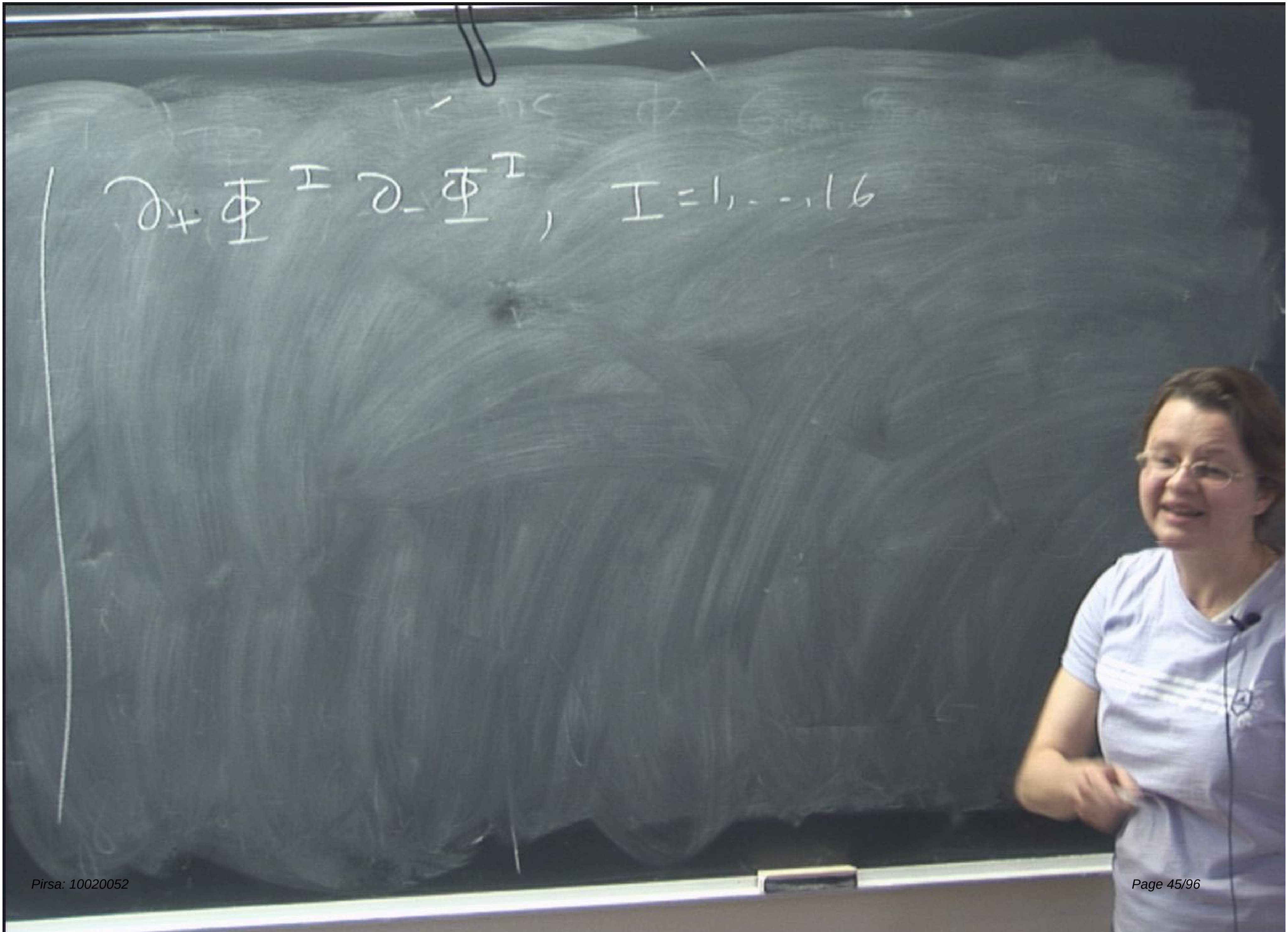
L	R
$X_L^\mu(\tau+\sigma)$	$X_R^\mu(\tau-\sigma)$
	$\psi^\mu(\tau-\sigma)$
$D = 26 = 10 + 16$	$D = 10$

The heterotic string

1) start:

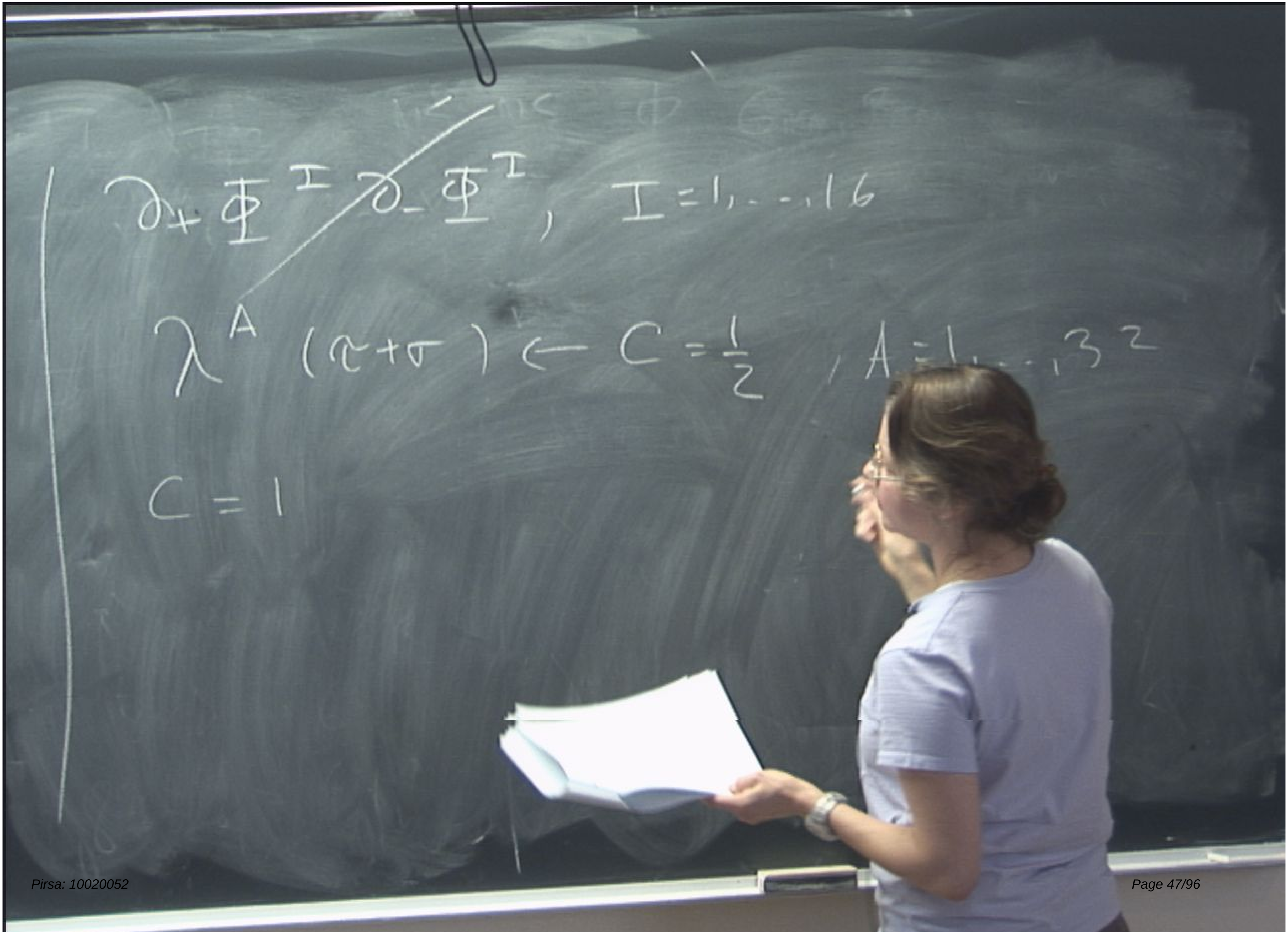
$$S = \int d^2\sigma \left(\partial_+ X_\mu \partial_- X^\mu + \psi_- \partial_+ \psi_- + \cancel{\psi_+ \partial_- \psi_+} \right)$$

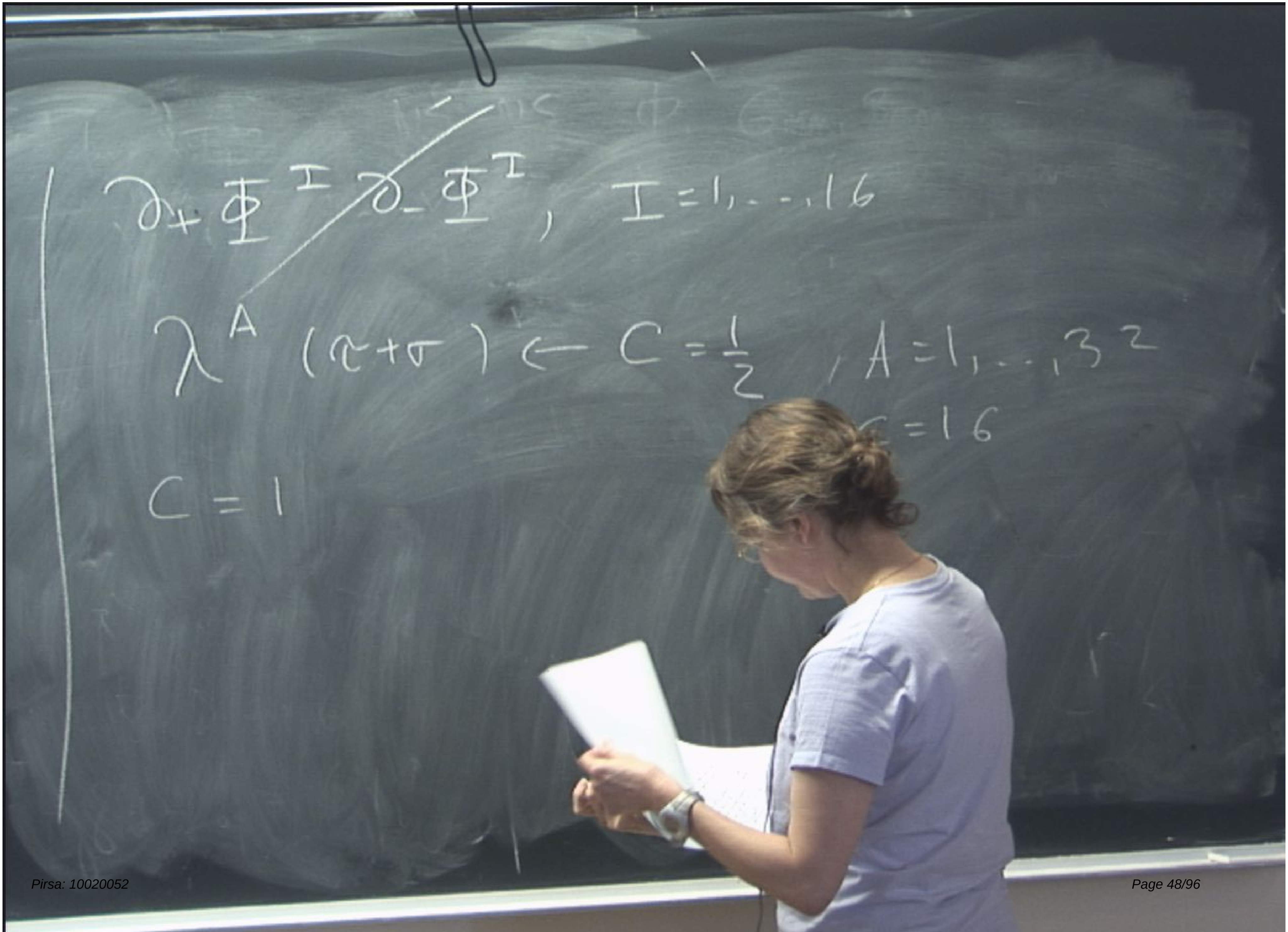
L	R
$X_L^\mu(\tau+\sigma)$	$X_R^\mu(\tau-\sigma)$
	$\psi^\mu(\tau-\sigma)$
$D = 26 = 10 + 16$	$D = 10$


$\partial_+ \Phi^I \partial_- \Phi^I , \quad I = 1, \ldots, 16$

$$\partial_+ \Phi^I \partial_- \Phi^I, \quad I = 1, \ldots, 16$$

$$\lambda^A (\tau + \sigma)$$


$\cancel{\partial_+ \Phi^I \partial_- \Phi^I}$, $I = 1, \ldots, 16$
$\lambda^A (\tau + \sigma) \leftarrow C = \frac{1}{2}$, $A = 1, \ldots, 32$
$C = 1$


$\partial_+ \Phi^I \partial_- \Phi^I$, $I = 1, \dots, 16$
$\lambda^A (\tau + \sigma) \leftarrow c = \frac{1}{2}$, $A = 1, \dots, 32$
$c = 16$
$c = 1$

$$\partial_+ \Phi^I \partial_- \Phi^I , \quad I = 1, \dots, 16$$

$$\lambda^A (\tau + \sigma) \leftarrow c = \frac{1}{2} , \quad A = 1, \dots, 32$$

$$c = 16$$

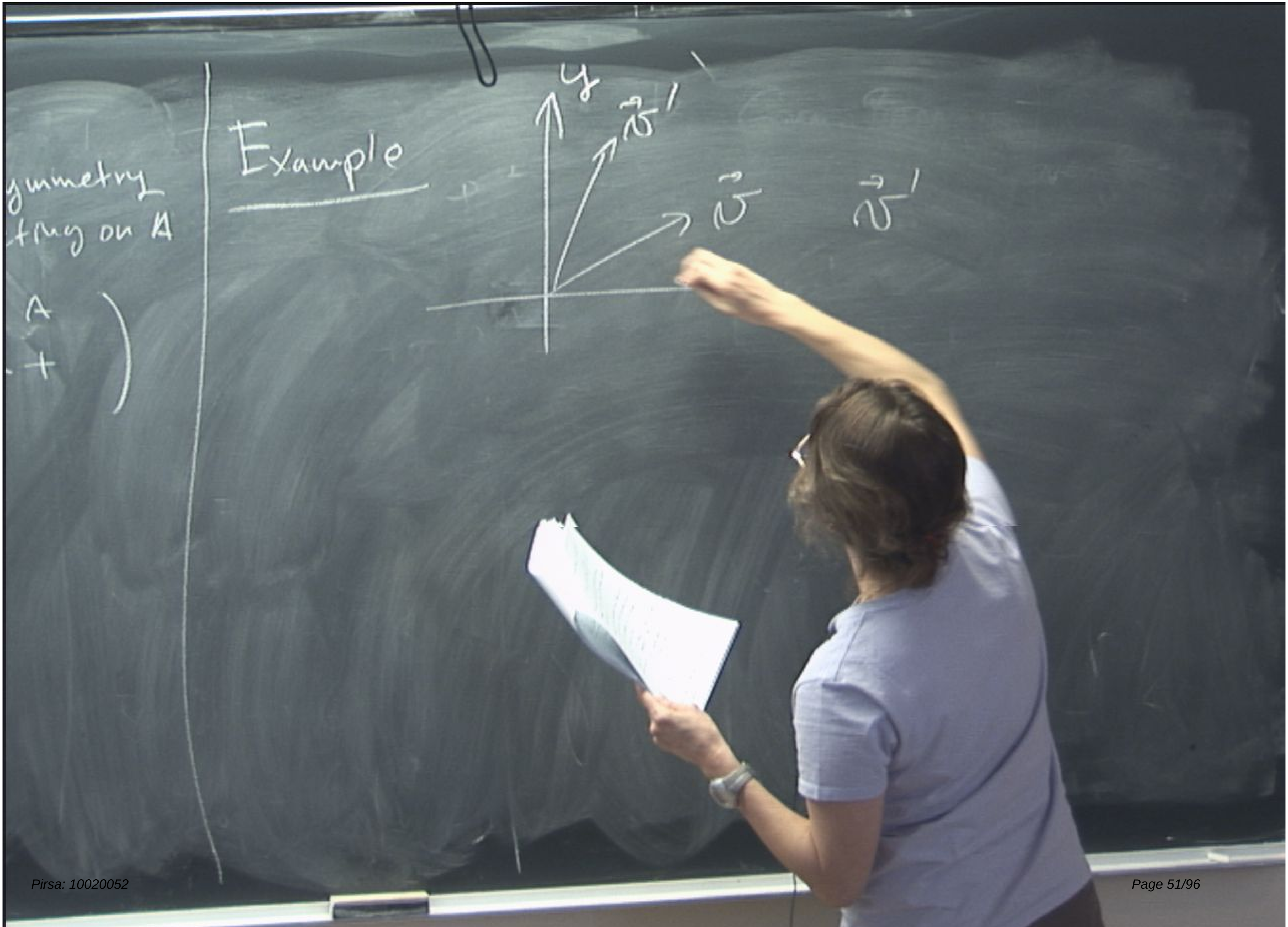
$$c = 1$$

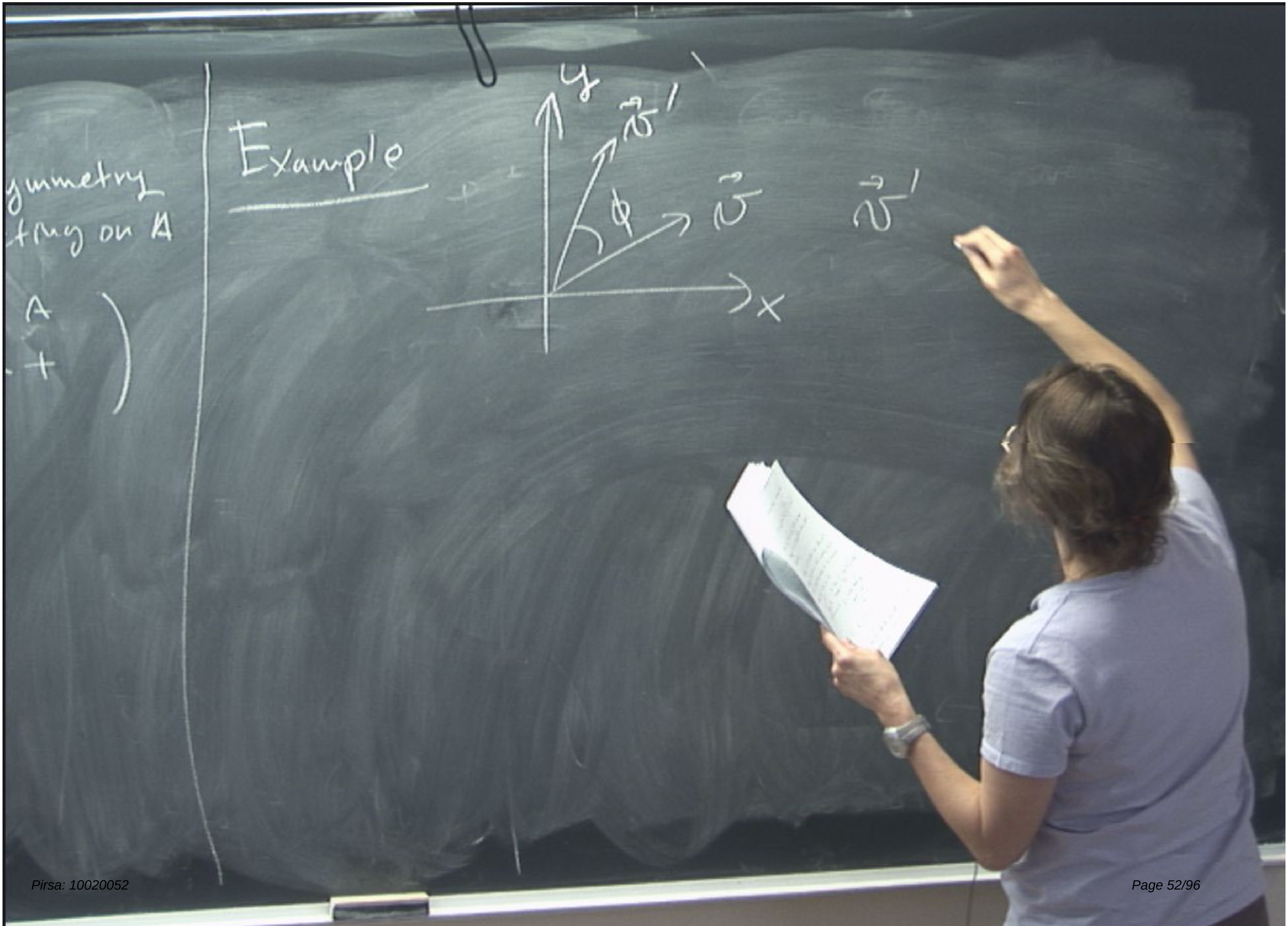
The heterotic string

1) start:

SO(32) symmetry acting on

$$S = \int d^2\sigma \left(\partial_+ X_\mu \partial_- X^\mu + \psi_- \partial_+ \psi_- + \sum_{i=1}^{32} \bar{\lambda}^A_+ \partial_- \lambda^A_+ \right)$$

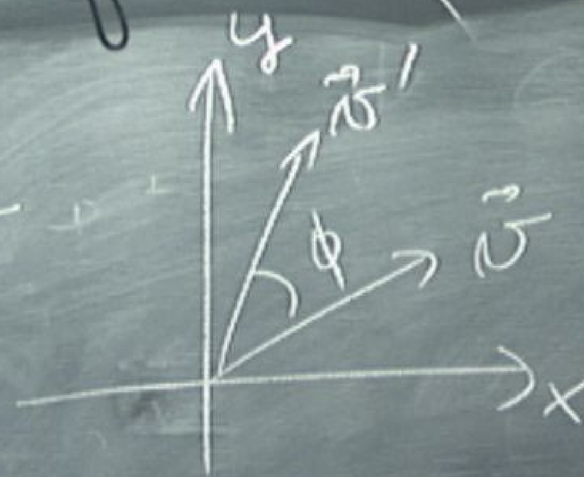

ymmetry
ting on A
Example
y
$\vec{v}'$
$\vec{v}$
$\vec{v}'$


Example
y
x
ϕ
$\vec{v}\,'$
$\vec{v}$
$\vec{v}\,'$

ymmetry
cting on A

A
+

Example



$$\vec{v}' = \underbrace{\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}}_{M} \vec{v}$$

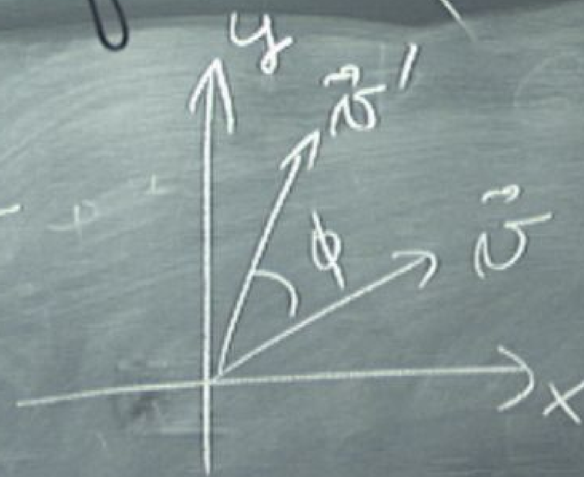
$$\vec{v}' = M \cdot \vec{v}$$

$$\left. \begin{array}{l} M \cdot M^T = \mathbb{1} \\ \det M = \mathbb{1} \end{array} \right\} M \in SO(2)$$

ymmetry
cting on A

A
+)

Example



$$\vec{v}' = \underbrace{\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}}_{M} \vec{v}$$

$$\vec{v}' = M \cdot \vec{v}$$

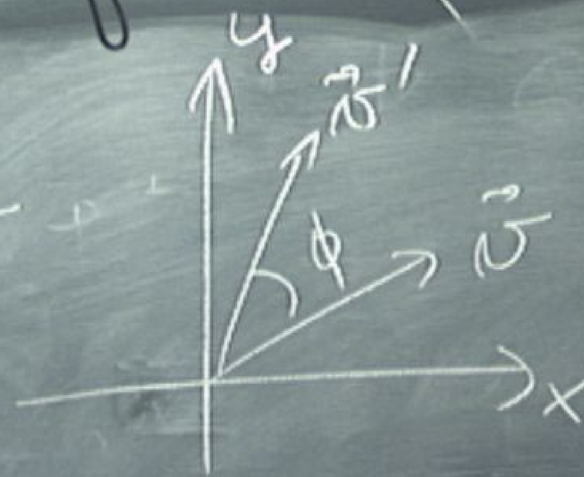
$$\left.\begin{array}{l} M \cdot M^T = \mathbb{1} \\ \det M = \mathbb{1} \end{array}\right\} M \in SO(2)$$

$$v_i' \cdot v_i' = v_i v_i$$

ymmetry
cting on A

A
+

Example



$$\vec{v}' = \underbrace{\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}}_{M} \vec{v}$$

$$\vec{v}' = M \cdot \vec{v}$$

$$\left.\begin{array}{l} M \cdot M^T = \mathbb{1} \\ \det M = \mathbb{1} \end{array}\right\} M \in SO(2)$$

$$v_i' \cdot v_i' = v_i v_i$$

The heterotic string

1) Start

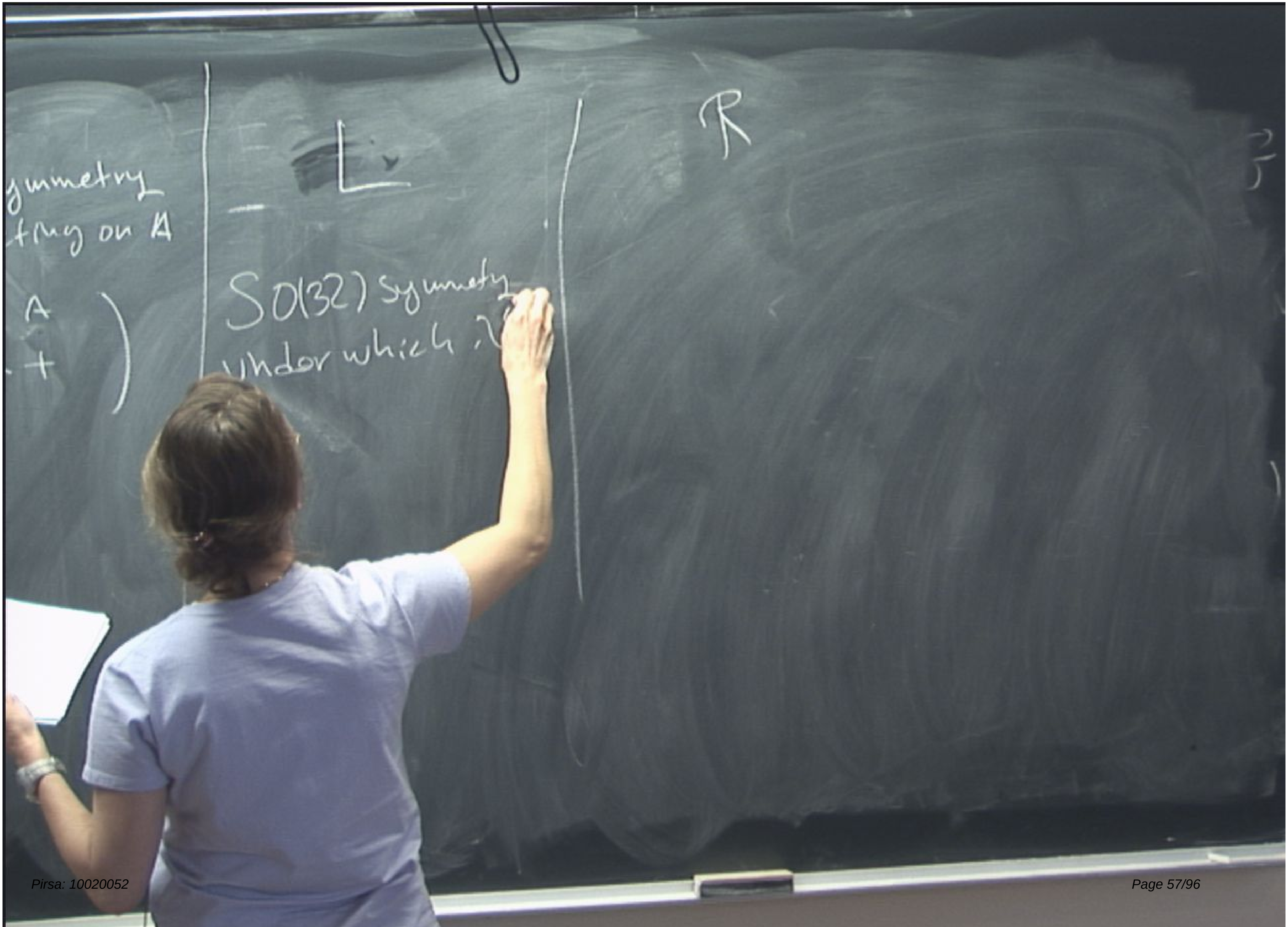
$$S = \int d^2\sigma \left(\partial_+ X_\mu \partial_- X^\mu + \psi_- \partial_+ \psi_- + \sum_{i=1}^{32} \bar{\lambda}^A \partial_+ \partial \ldots \right.$$

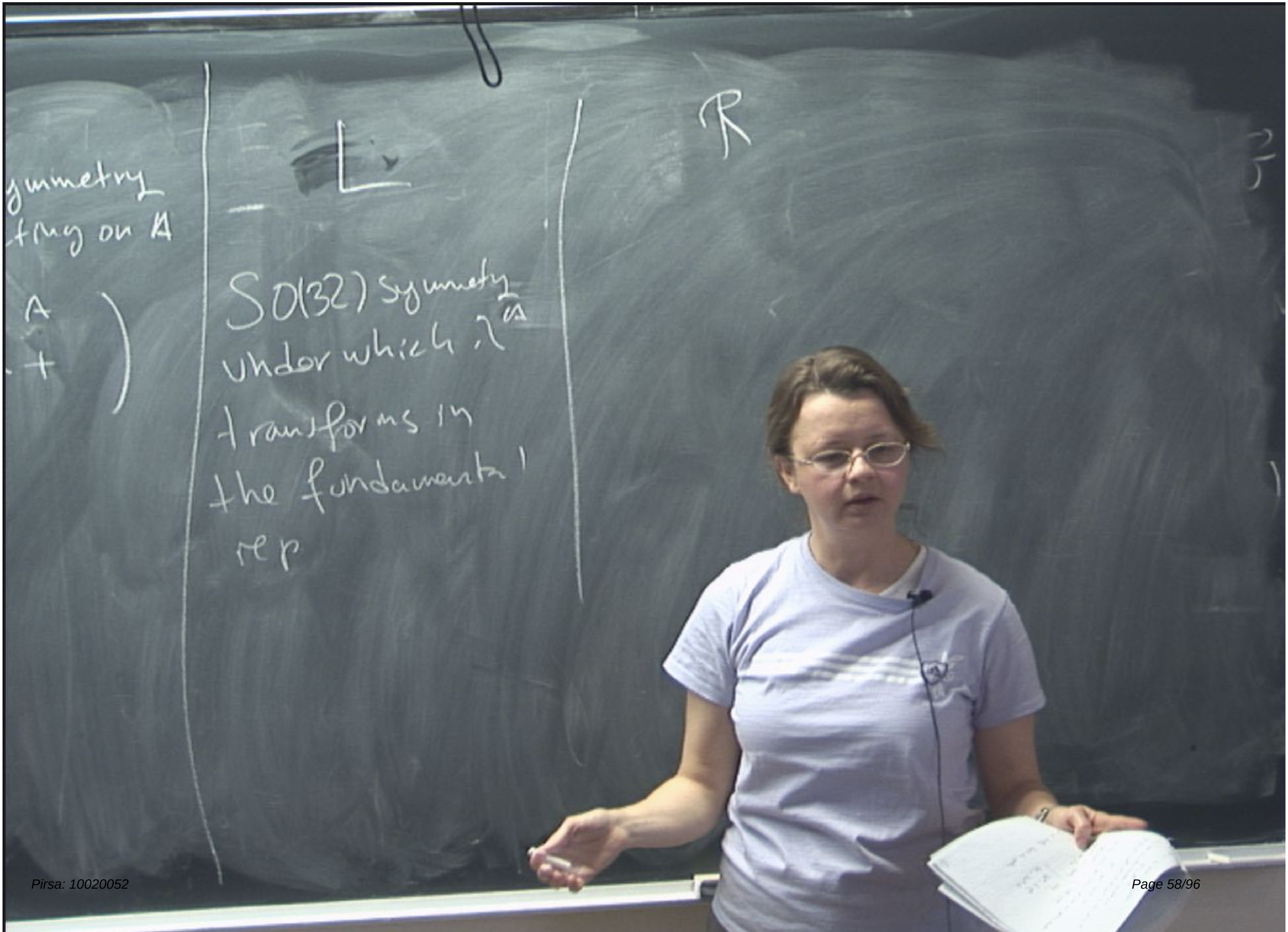
So(3…

$$\lambda^A \to M^{AB} \lambda^B$$

$$\bar{\lambda}^A \rho^\alpha \partial_\alpha \lambda^A \to \underbrace{M^{AB} M^{AC}}_{\delta^{AB}} \bar{\lambda}^B \rho^\alpha \partial_\alpha \lambda^C$$

since $M \cdot M^T = 1$


L
R
SO(32) symmetry
under which


ymmetry
cting on A
L
R
SO(32) symmetry
under which λ^A
transforms in
the fundamental
rep

ymmetry
ting on A

A
+

L

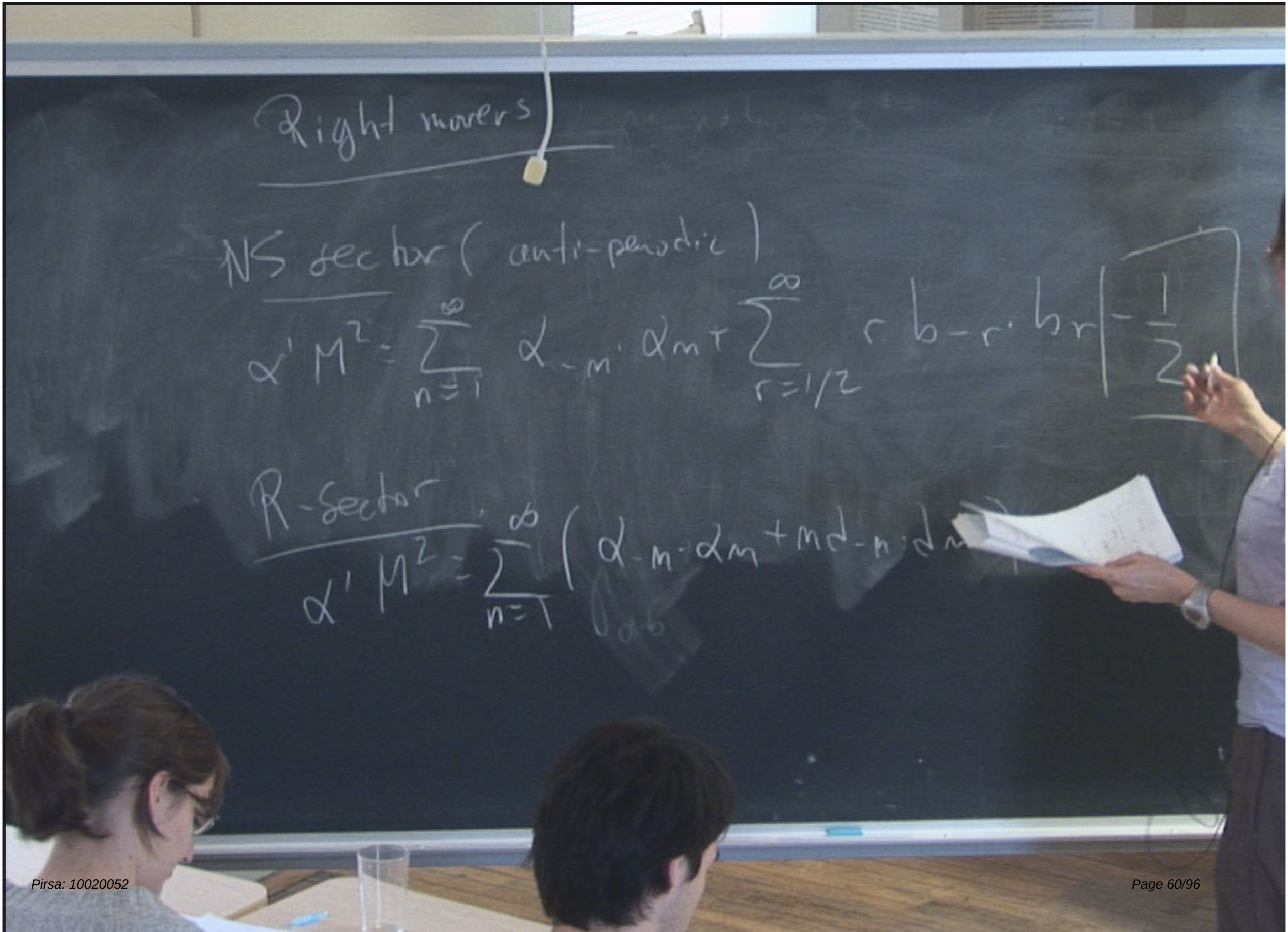
SO(32) symmetry
under which λ^A
transforms in
the fundamental
rep

R

SUSY

$\delta X^\mu = \bar{\epsilon}\,\psi_-^\mu$

$\delta\psi_- = \partial_- X^\mu \cdot \epsilon$


Right movers
NS sector (anti-periodic)
$\alpha' M^2 = \sum_{n=1}^{\infty} \alpha_{-m} \cdot \alpha_m + \sum_{r=1/2}^{\infty} r\, b_{-r} \cdot b_r - \frac{1}{2}$
R-sector
$\alpha' M^2 = \sum_{n=1}^{\infty} (\alpha_{-m} \cdot \alpha_m + m d_{-n} \cdot d_n$

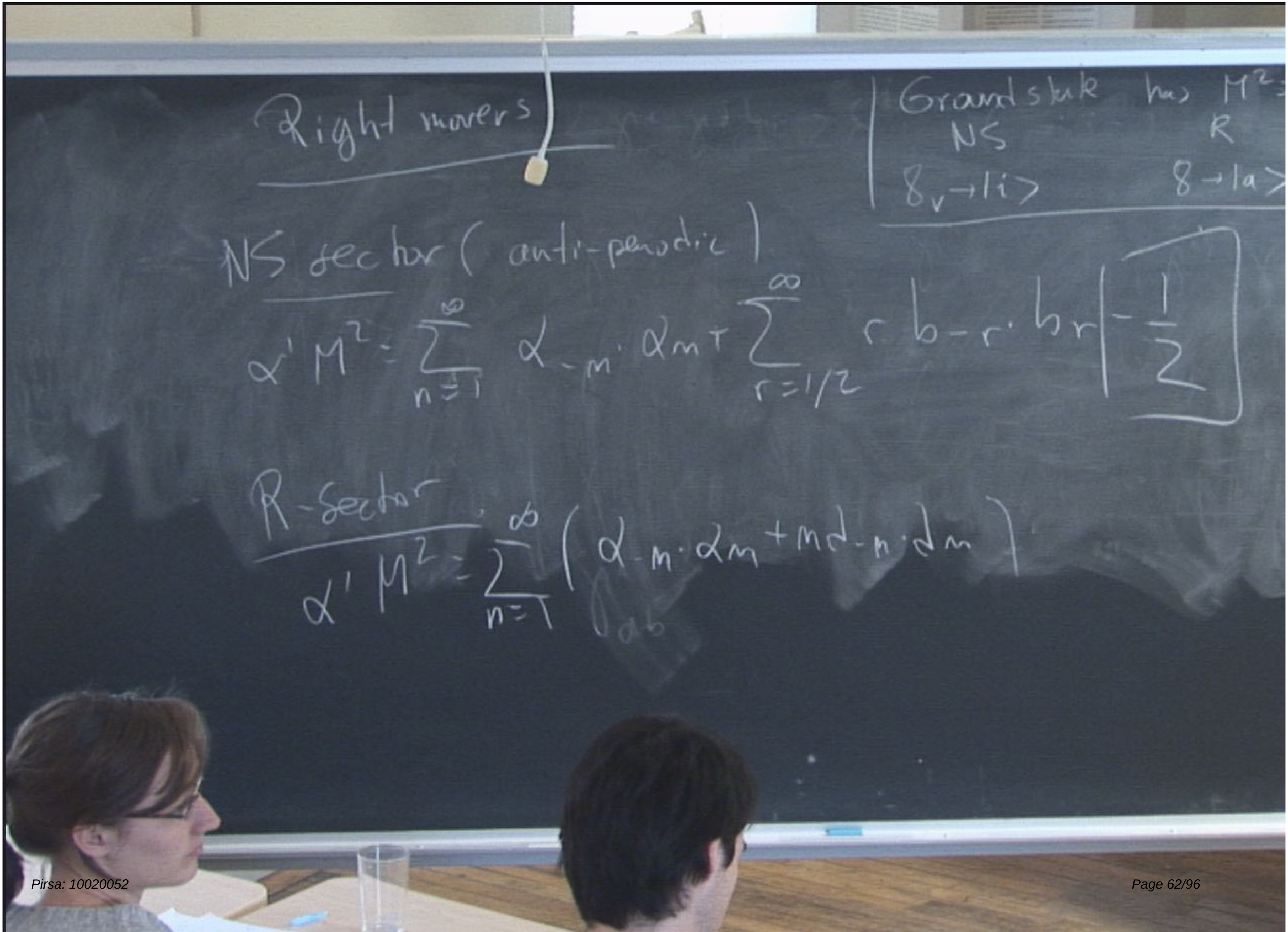
Right movers

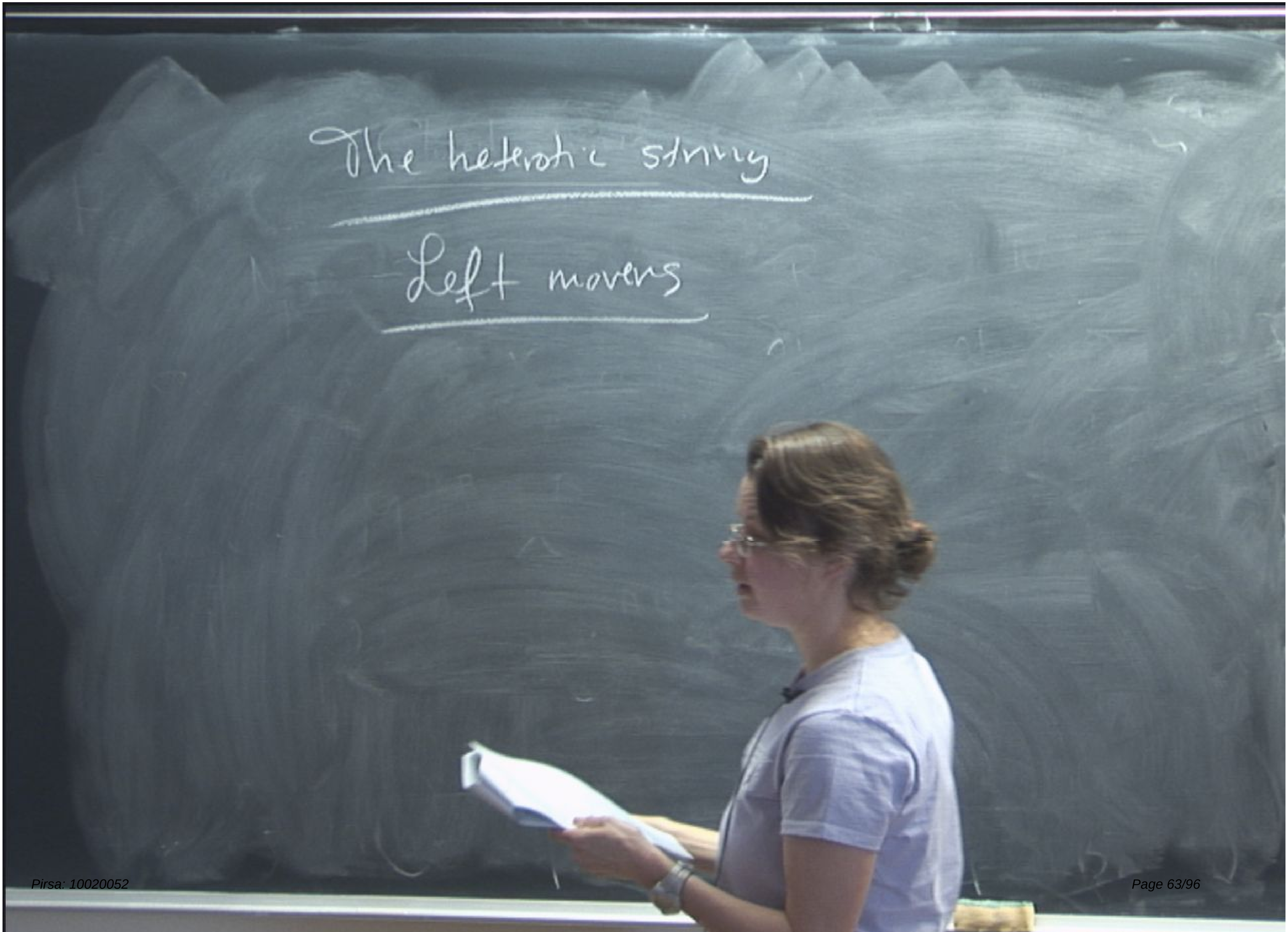
NS sector (anti-periodic)

$$\alpha' M^2 = \sum_{n=1}^{\infty} \alpha_{-m} \cdot \alpha_m + \sum_{r=1/2}^{\infty} r\, b_{-r} \cdot b_r \boxed{- \frac{1}{2}}$$

R-sector

$$\alpha' M^2 = \sum_{n=1}^{\infty} \left(\alpha_{-n} \cdot \alpha_n + n\, d_{-n} \cdot d_n \right)$$


Right movers
Grandstate has M²=
NS R
8ᵥ → |i> 8 → |a>
NS sector (anti-periodic)
α' M² = Σ_{n=1}^{∞} α₋ₙ · αₙ + Σ_{r=1/2}^{∞} r b₋ᵣ · bᵣ − 1/2
R-sector
α' M² = Σ_{n=1}^{∞} (α₋ₙ · αₙ + n d₋ₙ · dₙ)


The heterotic string
Left movers

The heterotic string

Left movers

$$\lambda^A(\sigma) = \underset{A}{\overset{P}{\pm}} \lambda^A(\sigma+\pi)$$

A-boundary conditions

$$\lambda^A(\tau+\sigma) = \sum_{r\in\mathbb{Z}+\frac{1}{2}} \lambda^A_r e^{-2ir(\tau+\sigma)}$$

$$\{\lambda^A_r, \lambda^B_s\} = \delta^{AB}\delta_{r+s,0}$$

The heterotic string

Left movers

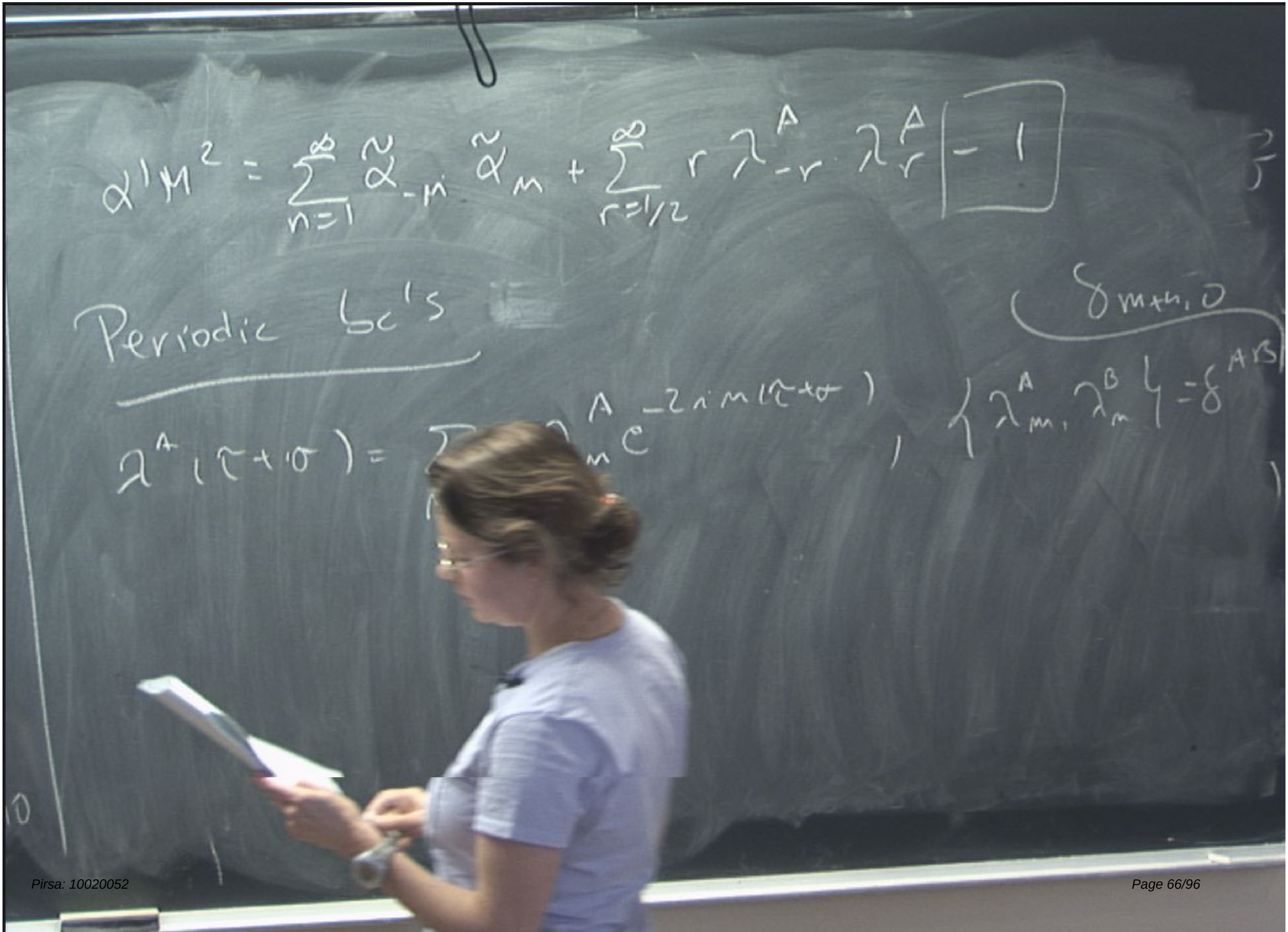
$$\lambda^A(\sigma) = \pm \lambda^A(\sigma+\pi)$$

P (for +), A (for −)

A - boundary conditions

$$\lambda^A(\tau+\sigma) = \sum_{r \in \mathbb{Z}+\frac{1}{2}} \lambda^A_r e^{-2ir(\tau+\sigma)}$$

$$\{\lambda^A_r, \lambda^B_s\} = \delta^{AB}\delta_{r+s,0}$$



$\alpha' M^2 = \sum_{n=1}^{\infty} \tilde{\alpha}_{-n} \tilde{\alpha}_n + \sum_{r=1/2}^{\infty} r \lambda^A_{-r} \lambda^A_r \boxed{-1}$
Periodic bc's
$\lambda^A(\tau+\sigma) = \sum \lambda^A_m e^{-2im(\tau+\sigma)}$,
$\{\lambda^A_m, \lambda^B_n\} = \delta^{AB} \delta_{m+n,0}$

$$\alpha' M^2 = \sum_{n=1}^{\infty} \tilde{\alpha}_{-n} \tilde{\alpha}_{n} + \sum_{r=1/2}^{\infty} r \lambda^{A}_{-r} \lambda^{A}_{r} \boxed{-1}$$

Periodic bc's

$$\lambda^{A}(\tau+\sigma) = \sum_{m \in \mathbb{Z}} \lambda^{A}_{m} e^{-2im(\tau+\sigma)}, \quad \{\lambda^{A}_{m}, \lambda^{B}_{n}\} = \delta^{AB}\delta_{m+n,0}$$

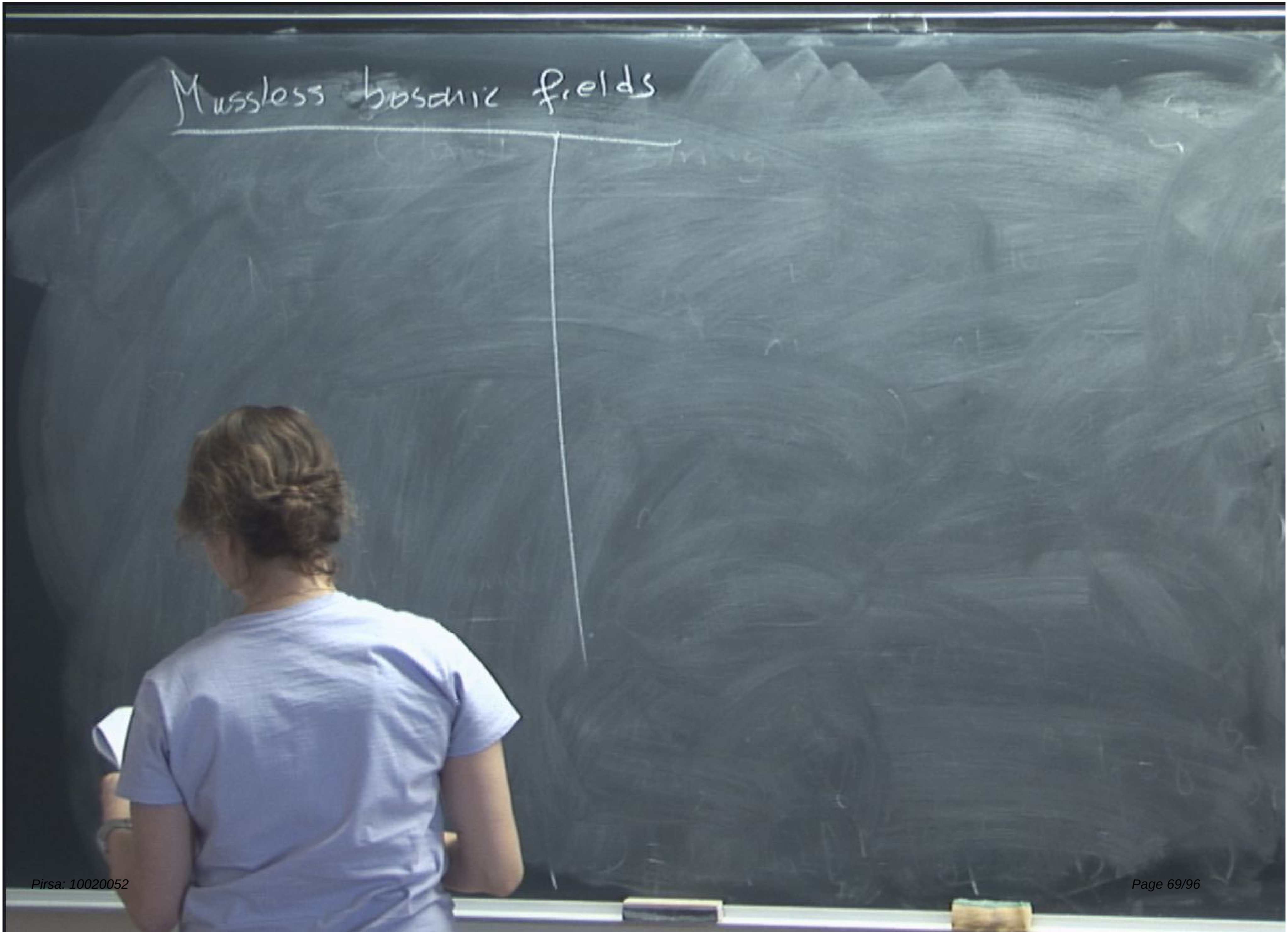
$$\alpha' M^2 = \sum_{n=1}^{\infty} \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n + \sum_{r=1/2}^{\infty} r \lambda^A_{-r} \lambda^A_r \boxed{-1}$$

Periodic bc's

$$\lambda^A(\tau+\sigma) = \sum_{m \in \mathbb{Z}} \lambda^A_m e^{-2im(\tau+\sigma)}, \quad \{\lambda^A_m, \lambda^B_n\} = \delta^{AB} \delta_{m+n,0}$$

$$\alpha' M^2 = \sum_{n=1}^{\infty} \left(\tilde{\alpha}_{-m} \cdot \tilde{\alpha}_m + m \lambda^A_{-m} \lambda^A_m \right) \boxed{+1}$$

normal ordering const.


Massless bosonic fields

Massless bosonic fields

Left movers	Right movers
8_v	
$\tilde{\alpha}^j_{-1}\|0\rangle_L = \|j\rangle$	$\|i\rangle, \ \|a\rangle$
$\lambda^A_{-\frac{1}{2}}\lambda^B_{-\frac{1}{2}}\|0\rangle_L = \|\Omega^{AB}\rangle$	

Massless bosonic fields

Left movers	Right movers
$\to \tilde{\alpha}^{j}_{-1}\, \lvert 0\rangle_L = \lvert j\rangle$	$\underbrace{\lvert i\rangle}_{8_v},\ \underbrace{\lvert a\rangle}_{8}$
$\lambda^{A}_{-\frac{1}{2}}\, \lambda^{B}_{-\frac{1}{2}}\, \lvert 0\rangle_L = \lvert \Omega^{AB}\rangle$	

$$|j\rangle \otimes |i\rangle = 8_v \otimes 8_v \leftarrow \phi, G_{MN}, B_{MN}$$

$$|j\rangle \otimes |i\rangle = 8_v \otimes 8_v \longleftarrow \phi, G_{MN}, B_{MN}$$

($N=1$ SUGRA mplet)

$$|\tilde{j}\rangle \otimes |\tilde{i}\rangle = 8_v \otimes 8_v \leftarrow \phi,\ G_{MN},\ B_{MN}$$

(N=1 SUGRA mplet)

$$\lambda^{A}_{-\frac{1}{2}} \cdot \lambda^{B}_{-\frac{1}{2}} |0\rangle_L \otimes |i\rangle = A^{i}_{AB}$$

$$|j\rangle \otimes |\tilde{i}\rangle = 8_v \otimes 8_v \longleftarrow \phi,\ G_{MN},\ B_{MN}$$

(N=1 SUGRA multiplet)

$$\underbrace{\lambda^{A}_{-\frac{1}{2}} \cdot \lambda^{B}_{-\frac{1}{2}}}_{\substack{\text{transforms in adjoint} \\ \text{rep of } SO(32)}} |0\rangle_L \otimes |\tilde{i}\rangle = A^{\tilde{i}}_{AB}\ (8_v)$$

$$|j\rangle \otimes |i\rangle = 8_v \otimes 8_v \longleftarrow \phi,\ G_{MN},\ B_{MN}$$

($N=1$ SUGRA multiplet)

$$\underbrace{\lambda^{A}_{-\frac{1}{2}} \cdot \lambda^{B}_{-\frac{1}{2}}}\ |0\rangle_L \otimes |i\rangle = A^{i}_{AB}\ (8_v)$$

transforms in adjoint rep of SO(32) | (non-abelian SO(32) gauge field.

$$|\tilde{j}\rangle \otimes |\tilde{i}\rangle = 8_v \otimes 8_v \longleftarrow \phi,\ G_{MN},\ B_{MN}$$

(N=1 SUGRA multiplet)

$$\underbrace{\lambda^{A}_{-\frac{1}{2}} \cdot \lambda^{B}_{-\frac{1}{2}}}_{\text{transforms in adjoint rep of } SO(32)} |0\rangle_L \otimes |\tilde{i}\rangle = A^{\tilde{i}}_{AB} \; (8_v)$$

transforms in adjoint rep of SO(32) (non-abelian SO(32) gauge field.)

Massless bosonic fields

$$\vec{v}\,' = U \cdot \vec{v}$$

$$U \cdot U^T = \mathbb{1}$$
$$\det U = 1$$

$SO(3)$

ϕ is very small

$$\mathcal{U} = \mathbb{1} + i\alpha_a X_a + \ldots$$

parametrize
different
rotations

ϕ is very small

$$\mathcal{U} = \mathbb{1} + i\alpha_a X_a + \dots$$

parametrize different rotations

generators

ϕ is very small.

$$\mathcal{U} = \mathbb{1} + i\alpha_a X_a + \dots$$

parametrize different rotations

generators

$$\rightarrow \quad [X_a, X_b] = i \underbrace{f_{abc}}_{\text{structure constants}} X_c \quad (\text{Lie al}$$

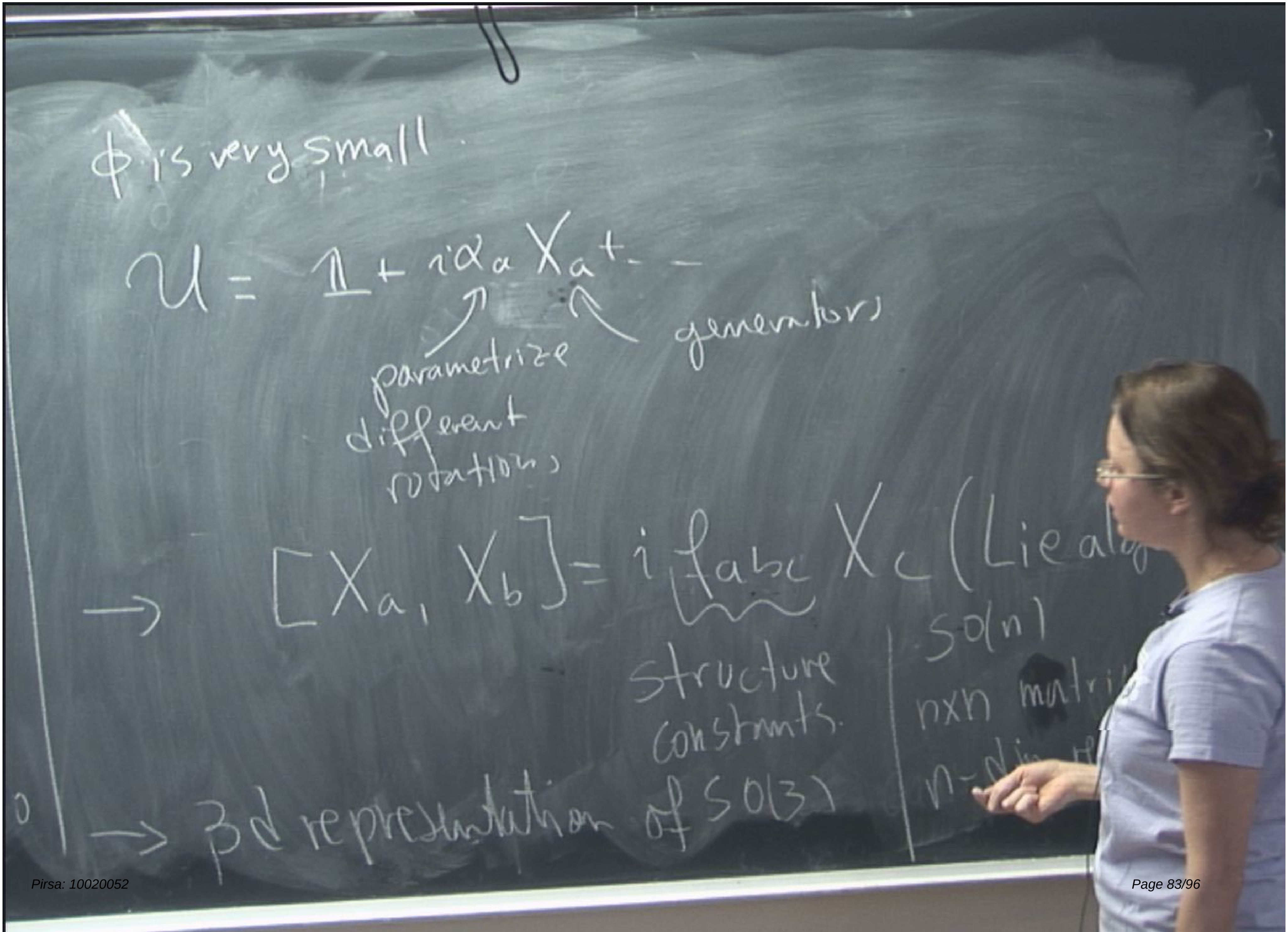
ϕ is very small

$$\mathcal{U} = \mathbb{1} + i\alpha_a X_a + \ldots$$

parametrize different rotations

generators

$$\rightarrow \quad [X_a, X_b] = i \underbrace{f_{abc}}_{\text{structure constants}} X_c \quad \text{(Lie algebra)}$$



φ is very small
U = 1 + iα_a X_a + ...
parametrize different rotations
generators
→ [X_a, X_b] = i f_abc X_c (Lie al
structure constants
SO(n)
nxn
→ 3d representation of SO(3)

ϕ is very small.

$$\mathcal{U} = \mathbb{1} + i\alpha_a X_a + \ldots$$

parametrize different rotations

generators

$$\rightarrow \quad [X_a, X_b] = i \underbrace{f_{abc}}_{\text{structure constants}} X_c$$

(Lie algebra

SO(n)

nxn matrices

n-dim

$\rightarrow$ 3d representation of SO(3)

ϕ is very small.

$\mathcal{U} = \mathbb{1} + i\alpha_a X_a + \cdots$

parametrize different rotations

generators

$\rightarrow [X_a, X_b] = i f_{abc} X_c$ (Lie algebra)

structure constants.

$SO(n)$

$n \times n$ matrices

n-dim rep.

$\rightarrow$ 3d representation of $SO(3)$

Massless bosonic fields

adjoint rep $(X_a)_{bc} = i f_{abc}$

what values do the indices b, c take

Massless bosonic fields

adjoint rep $\quad (X_a)_{bc} = i f_{abc}$

what values do the indices b, c take $\rightarrow$ number of generators

Massless bosonic fields

adjoint rep $$(X_a)_{bc} = i f_{abc}$$

what values do the indices b, c take $\rightarrow$ number of generators

rotations in planes

1-2
1-3
2-3

Massless bosonic fields

adjoint rep $\quad (X_a)_{bc} = i f_{abc}$

what values do the indices

b, c take → number of genera[tors]

rotations in planes
1-2
1-3
2-3

rotations in

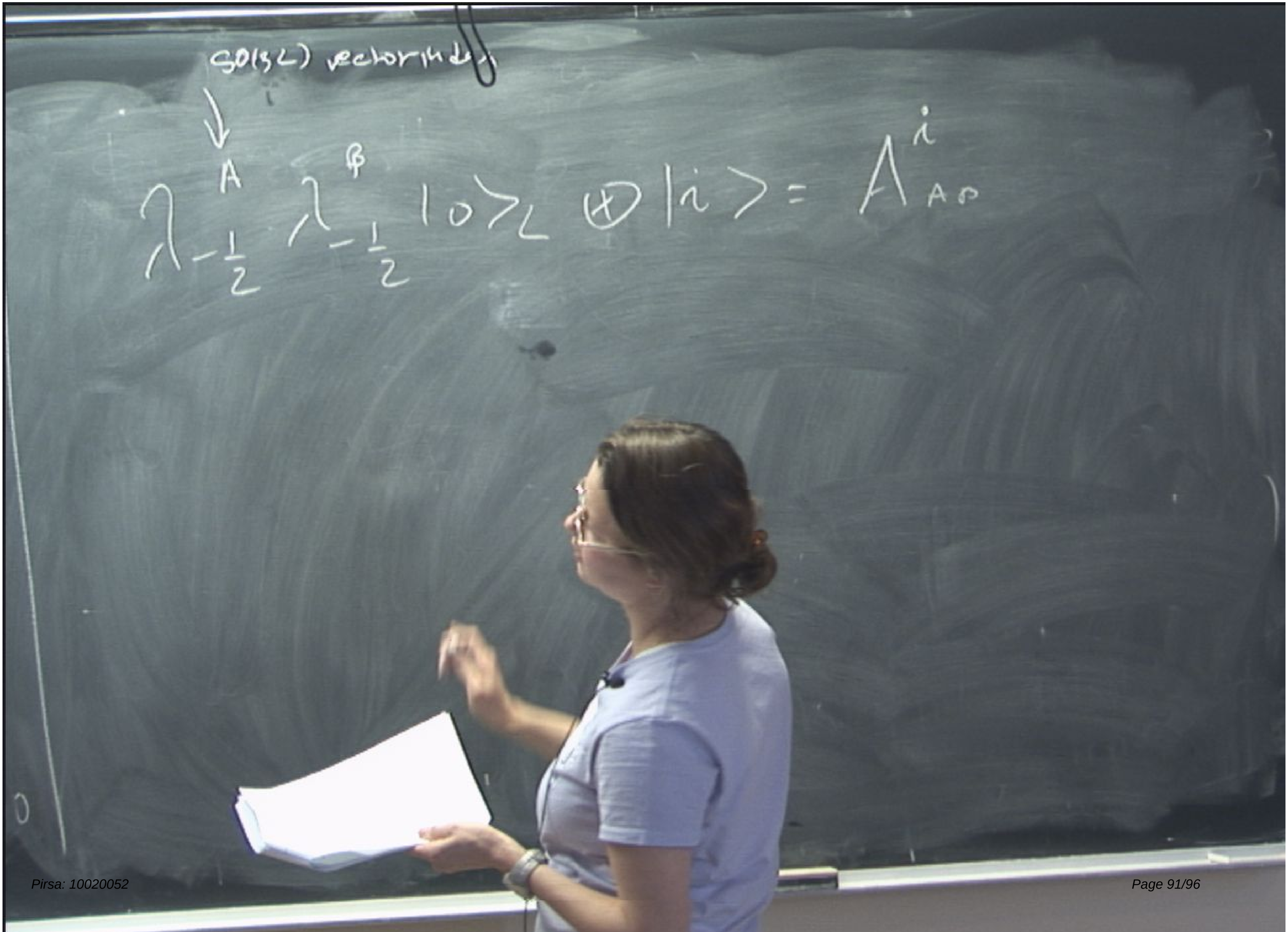
n dimensio[ns]

Massless bosonic fields

adjoint rep $\quad (X_a)_{bc} = i f_{abc}$

what values do the indices b, c take $\rightarrow$ number of generators

rotations in planes	rotations in n dimensions
1-2	$\underbrace{i-j}$
1-3	$\frac{n(n-1)}{2}$
2-3	


SO(8,L) vector index
$\lambda^{A}_{-\frac{1}{2}} \lambda^{B}_{-\frac{1}{2}} |0\rangle_L \otimes |i\rangle = A^{i}_{AB}$

SO(32) vector index

$$\lambda^{A}_{-\frac{1}{2}}\,\lambda^{B}_{-\frac{1}{2}}\,|0\rangle_L \otimes |i\rangle = A^{i}_{AB}$$

$$\frac{32\cdot 31}{2} = \frac{M(M-1)}{2} \leftarrow$$ adjoint rep. of SO(32)

SO(32) vector index

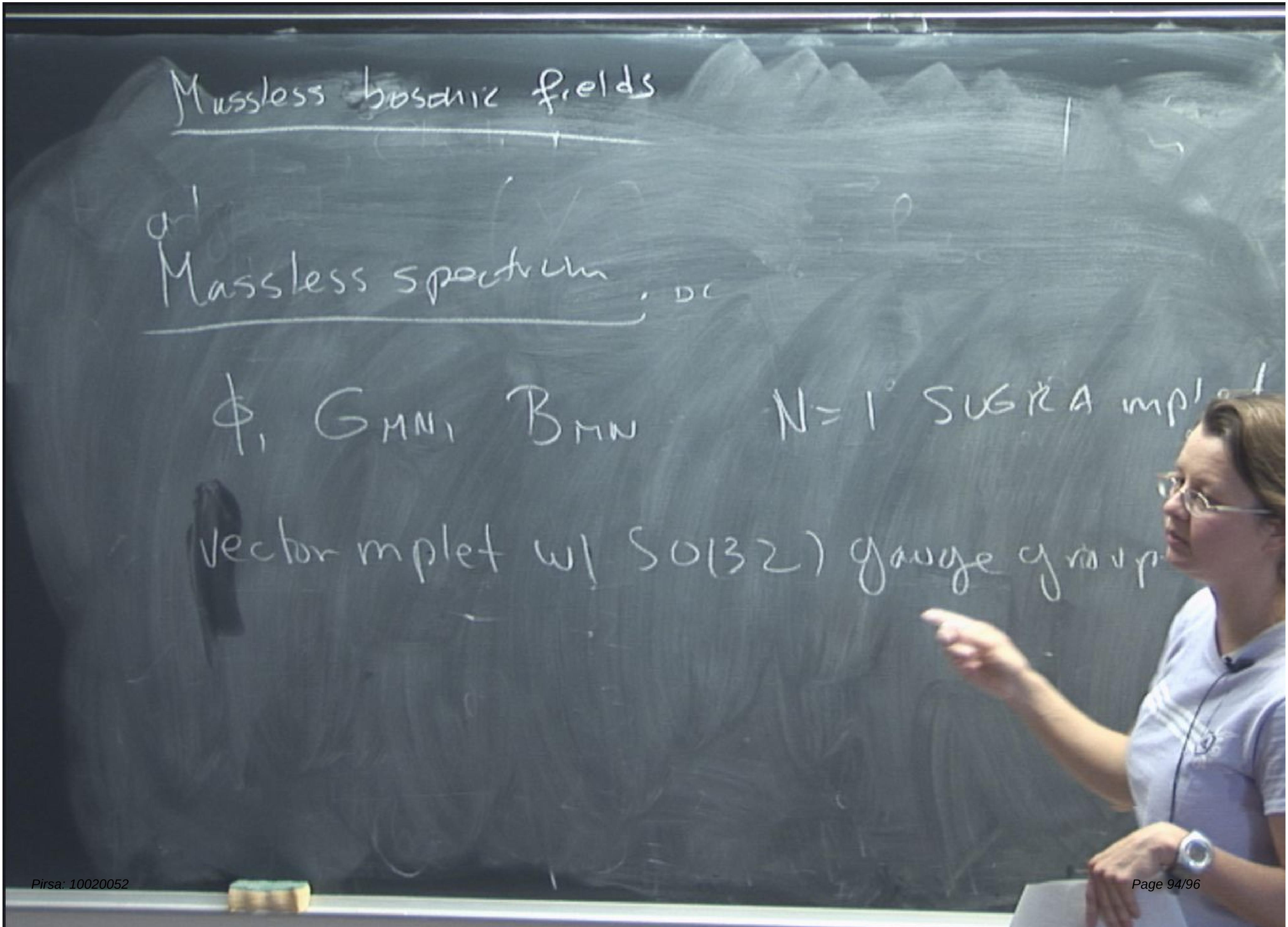
$$\lambda^{A}_{-\frac{1}{2}} \lambda^{B}_{-\frac{1}{2}} |0\rangle_L \otimes |i\rangle = A^{i}_{AB}$$

$$\frac{32 \cdot 31}{2} = \frac{M(M-1)}{2} \leftarrow \text{adjoint rep. of } SO(32)$$

rep. needed for

A^{i}_{AB} to be SO(32)

gauge field in space-time


Massless bosonic fields
and
Massless spectrum
φ, G_MN, B_MN
N=1 SUGRA mplet
vector mplet w/ SO(32) gauge group

Massless bosonic fields

Massless spectrum

$\phi,\ G_{MN},\ B_{MN}$ N=1 SUGRA mplet

vector mplet w/ $\underbrace{SO(32)}$ gauge group

→ one additional consisten superstring theor

$SO(32) \to E_8 \times E_8$

Massless bosonic fields

Massless spectrum

ϕ, G_{MN}, B_{MN} N=1 SUGRA mplet.

vector mplet w/ SO(32) gauge group

→ one additional consisten superstring theor

$SO(32) \to E_8 \times E_8$