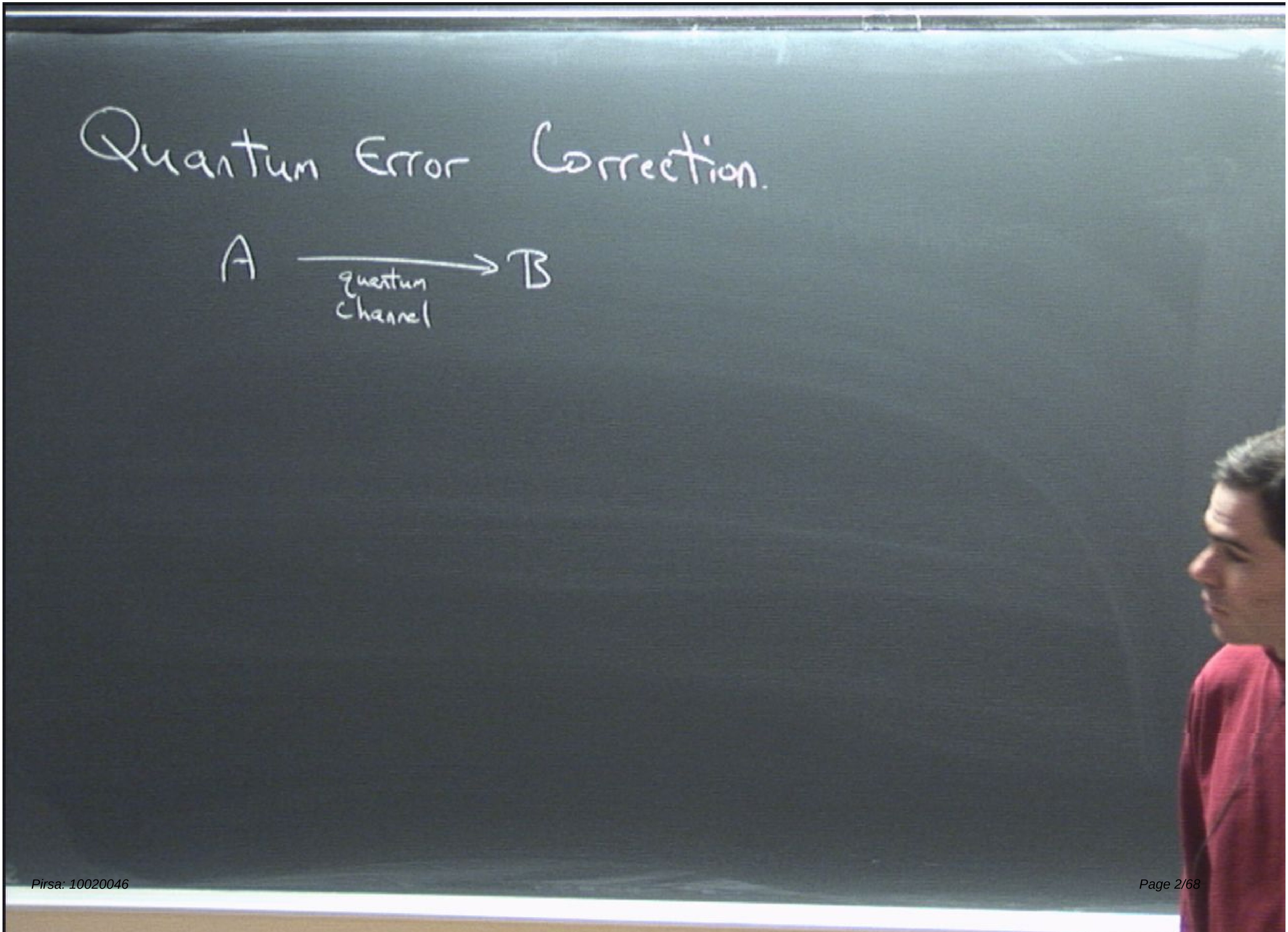
Title: Quantum Information - Review (PHYS 635) - Lecture 12

Date: Feb 09, 2010 09:00 AM

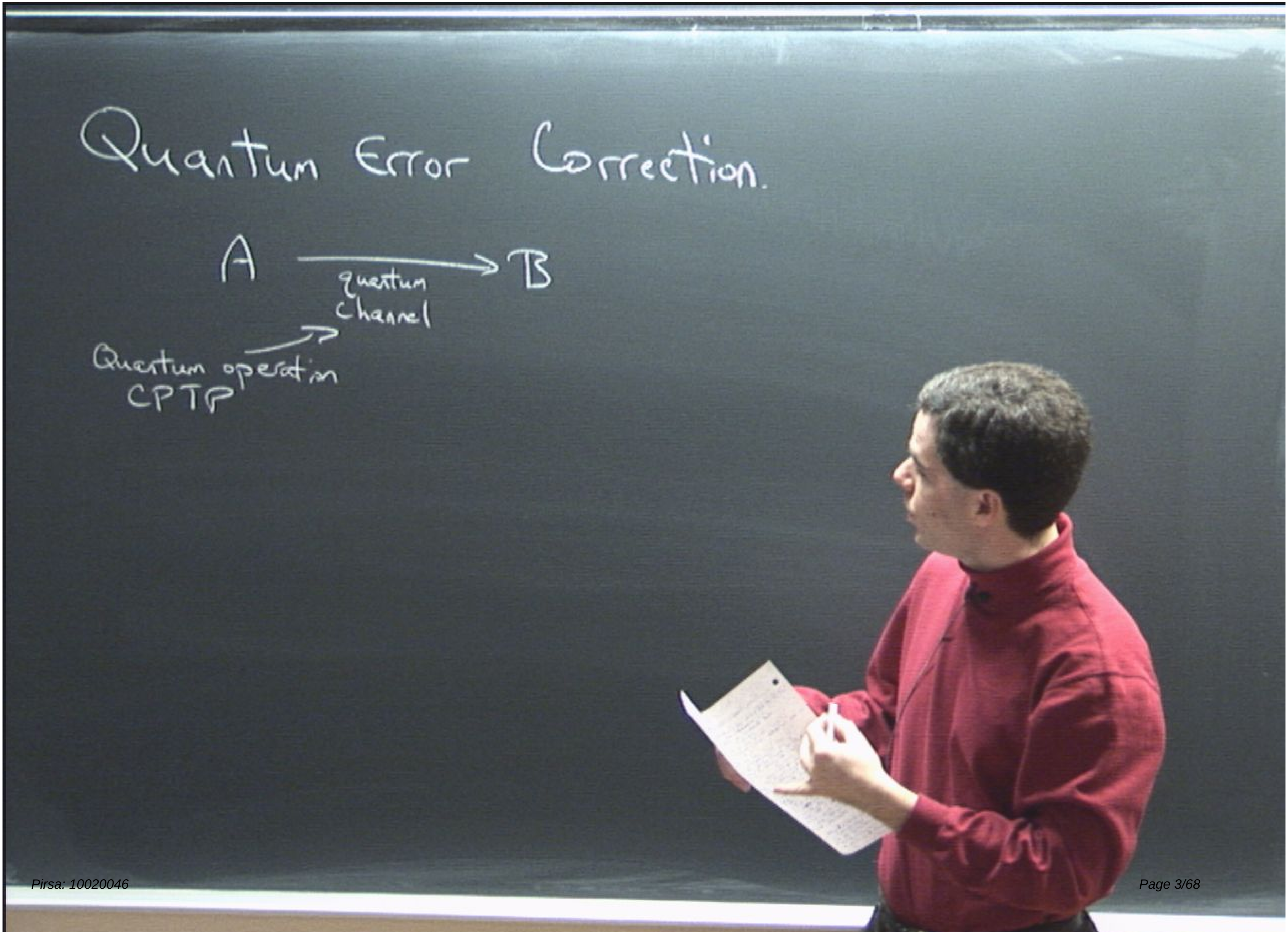
URL: http://pirsa.org/10020046

Abstract:



Quantum Error Correction.

$$A \xrightarrow[\text{quantum channel}]{} B$$



Quantum Error Correction.

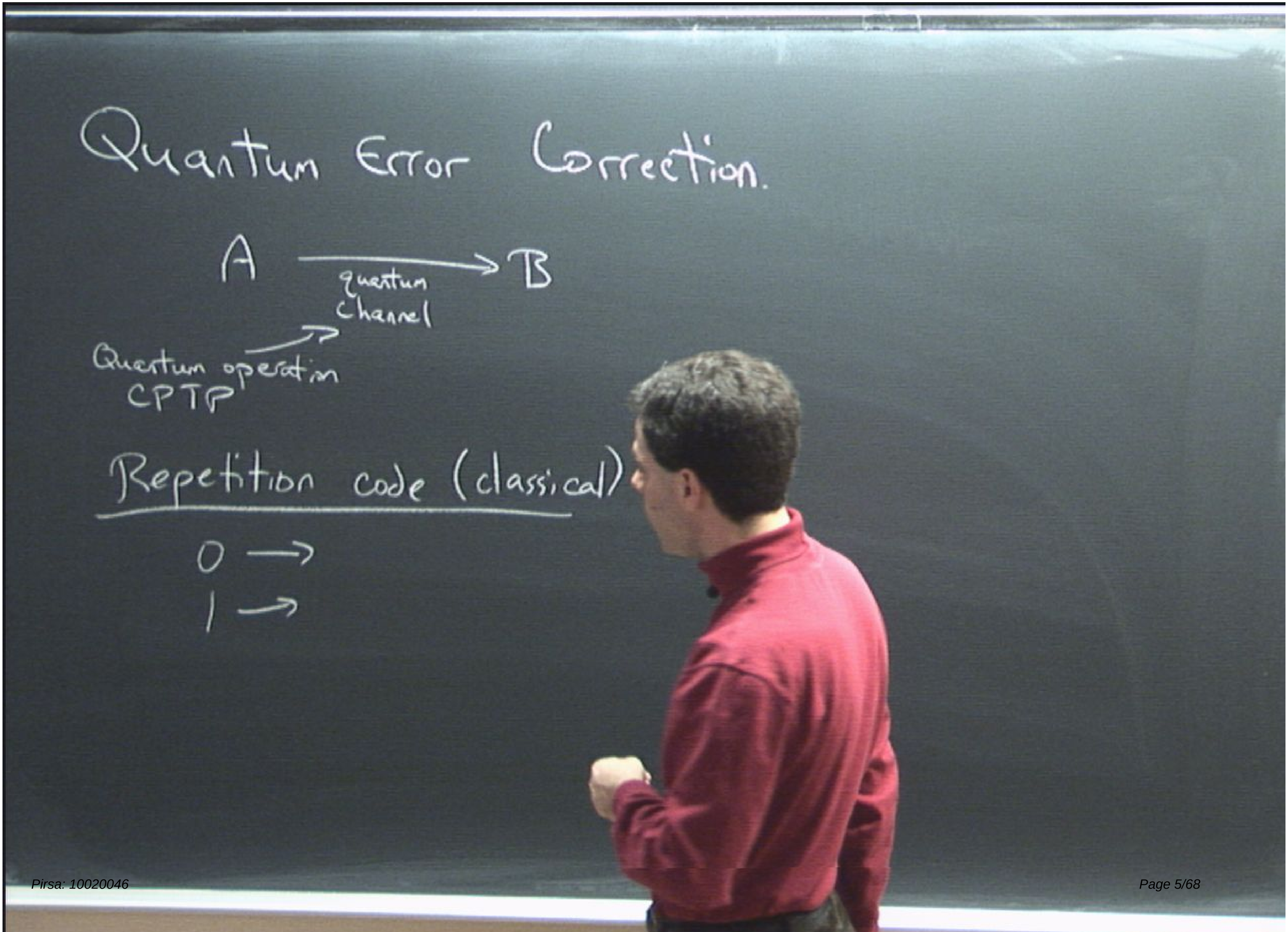
$$A \xrightarrow[\text{quantum channel}]{} B$$

Quantum operation
CPTP

Quantum Error Correction.

$$A \xrightarrow[\text{quantum channel}]{} B$$

Quantum operation
CPTP



Quantum Error Correction.

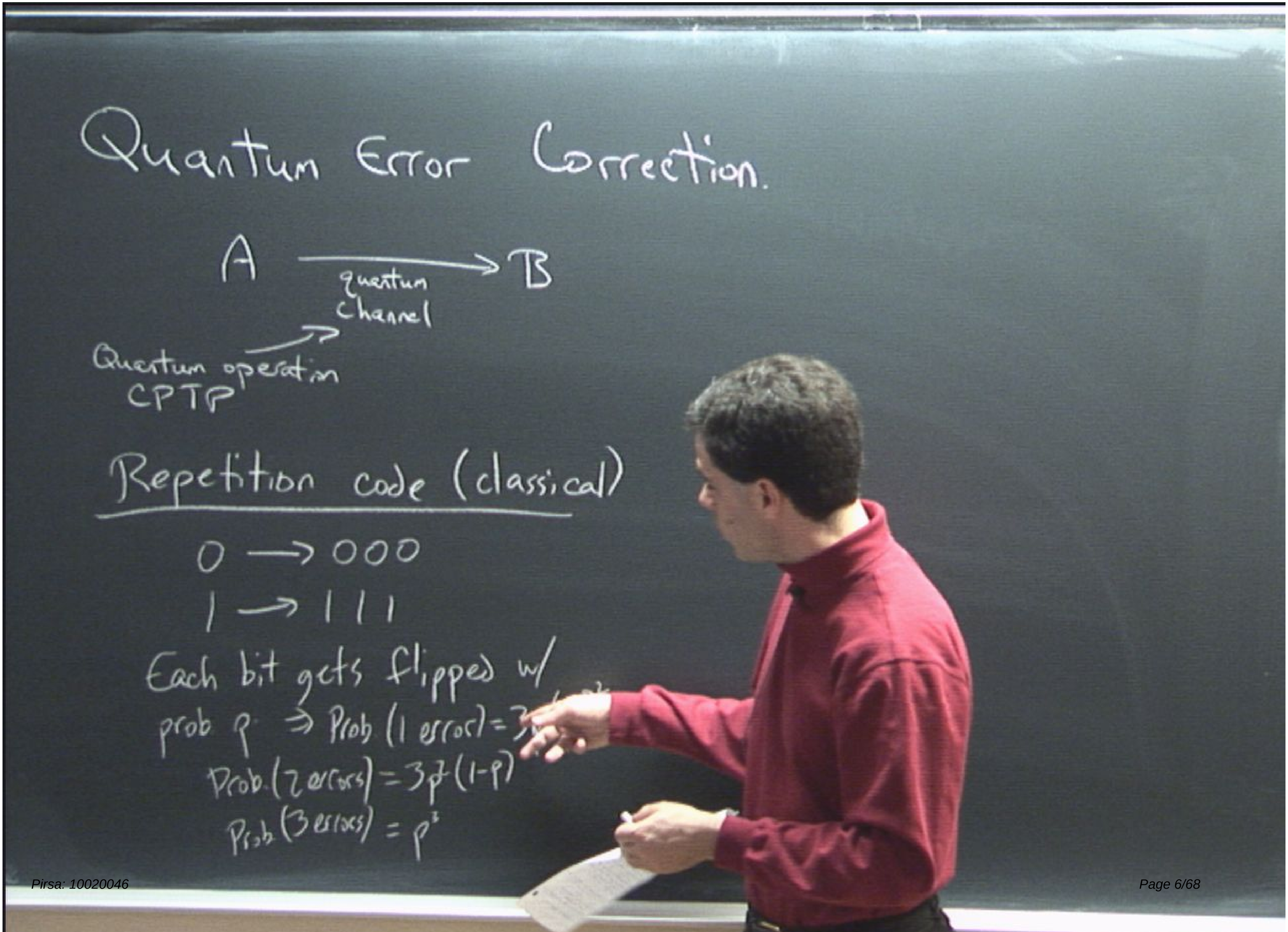
$$A \xrightarrow[\text{quantum channel}]{} B$$

Quantum operation
CPTP

Repetition code (classical)

$0 \longrightarrow$

$1 \longrightarrow$



Quantum Error Correction.

$$A \xrightarrow[\text{quantum channel}]{} B$$

Quantum operation
CPTP

Repetition code (classical)

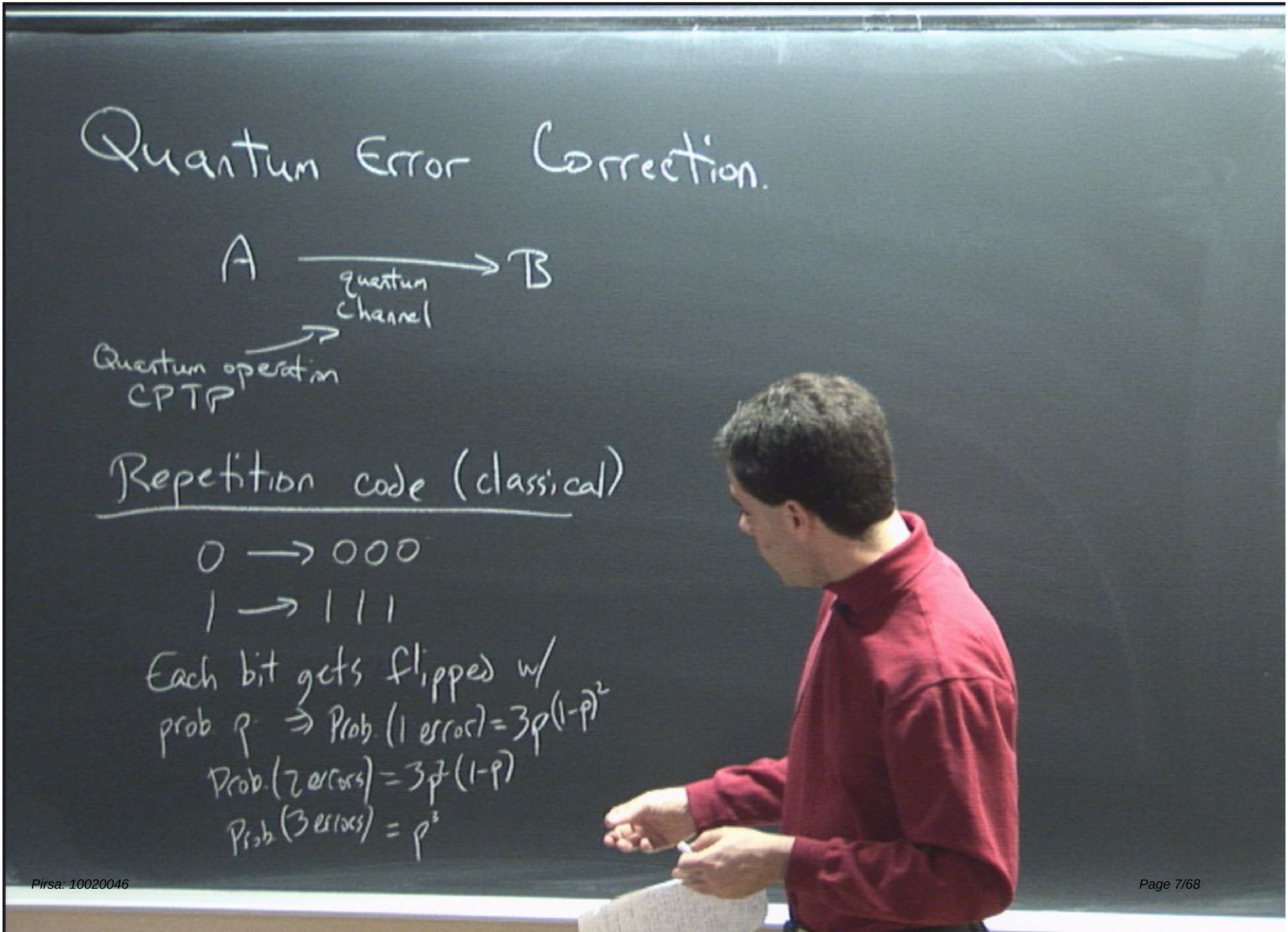
$0 \longrightarrow 000$

$1 \longrightarrow 111$

Each bit gets flipped w/ prob p $\Rightarrow$ Prob (1 error) $= 3$

Prob.(2 errors) $= 3p^2(1-p)$

Prob.(3 errors) $= p^3$



Quantum Error Correction.

$$A \xrightarrow[\text{quantum channel}]{} B$$

Quantum operation
CPTP

Repetition code (classical)

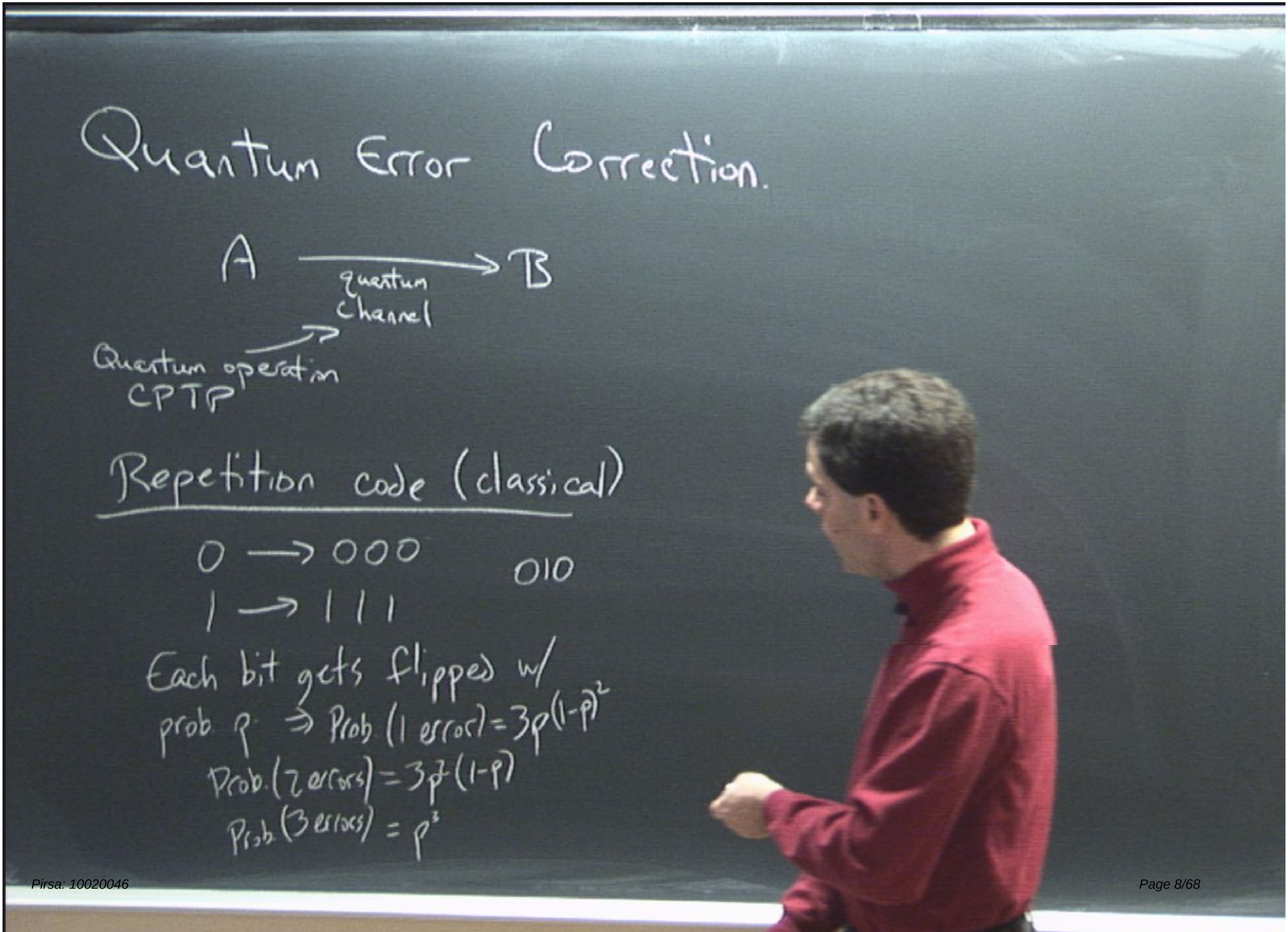
$$0 \longrightarrow 000$$

$$1 \longrightarrow 111$$

Each bit gets flipped w/ prob p $\Rightarrow$ Prob.(1 error) $= 3p(1-p)^2$

Prob.(2 errors) $= 3p^2(1-p)$

Prob.(3 errors) $= p^3$



Quantum Error Correction.

$$A \xrightarrow[\text{quantum channel}]{} B$$

Quantum operation
CPTP

Repetition code (classical)

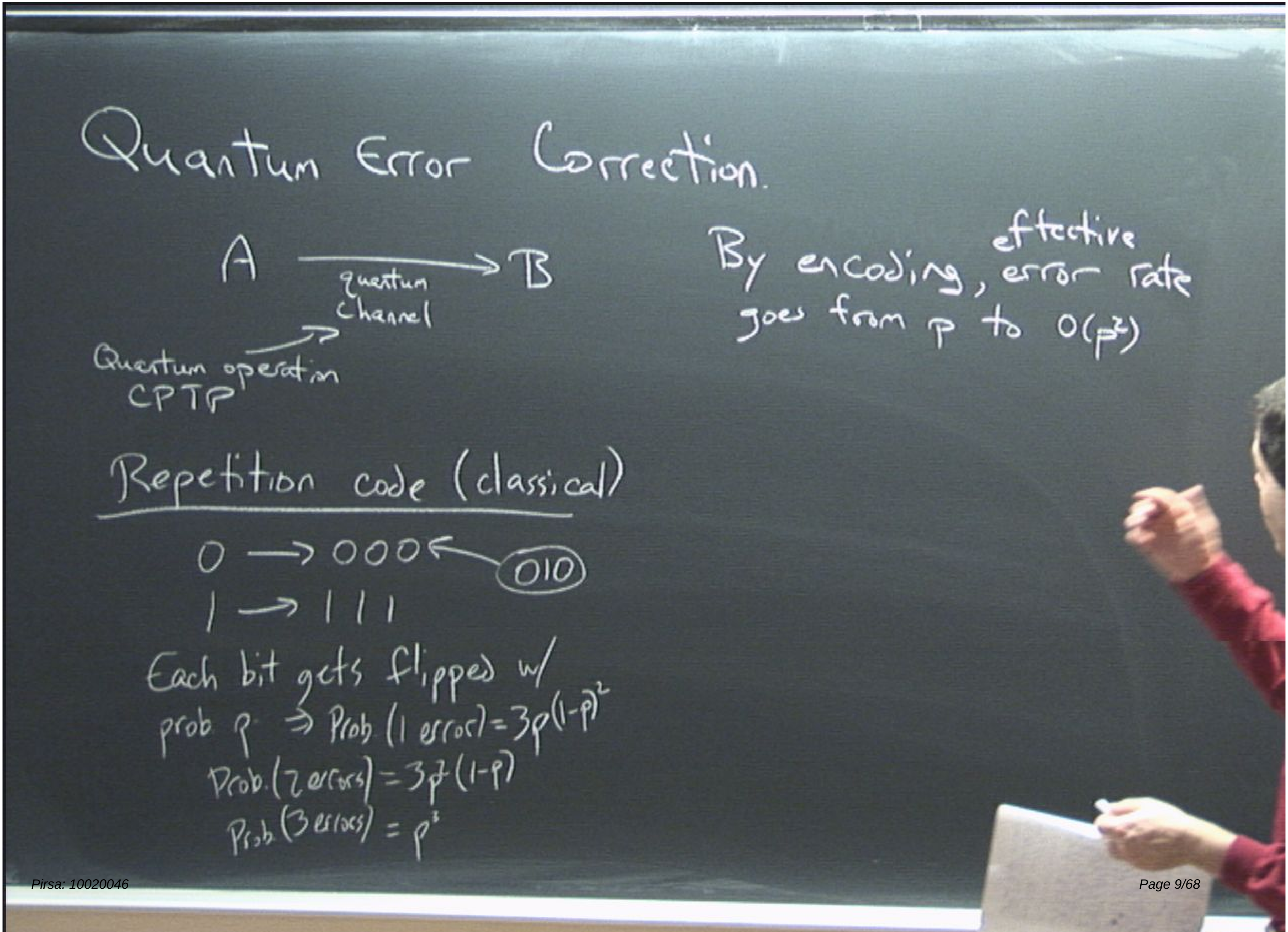
$0 \longrightarrow 000$ $\qquad 010$

$1 \longrightarrow 111$

Each bit gets flipped w/ prob. $p \Rightarrow$ Prob.$(1 \text{ error}) = 3p(1-p)^2$

Prob.$(2 \text{ errors}) = 3p^2(1-p)$

Prob.$(3 \text{ errors}) = p^3$



Quantum Error Correction.

$$A \xrightarrow[\text{quantum channel}]{} B$$

Quantum operation
CPTP

By encoding, effective error rate goes from p to $O(p^2)$

Repetition code (classical)

$$0 \longrightarrow 000 \longleftarrow \boxed{010}$$

$$1 \longrightarrow 111$$

Each bit gets flipped w/ prob p $\Rightarrow$ Prob.(1 error) $= 3p(1-p)^2$

Prob.(2 errors) $= 3p^2(1-p)$

Prob.(3 errors) $= p^3$

Quantum Error Correction.

$$A \xrightarrow[\text{quantum channel}]{} B$$

Quantum operation → CPTP

Repetition code (classical)

$$0 \longrightarrow 000 \longleftarrow \boxed{010}$$

$$1 \longrightarrow 111$$

Each bit gets flipped w/ prob p $\Rightarrow$ Prob (1 error) $= 3p(1-p)^2$

Prob. (2 errors) $= 3p^2(1-p)$

Prob. (3 errors) $= p^3$

By encoding, effective error rate goes from p to $O(p^2)$

Just assume 0 or 1 error, ignore the possibility of 2 errors

Quantum Error Correction.

$$A \xrightarrow[\text{quantum channel}]{} B$$

Quantum operation → channel
CPTP

Repetition code (classical)

$$0 \longrightarrow 000 \longleftarrow \boxed{010}$$

$$1 \longrightarrow 111$$

Each bit gets flipped w/ prob p $\Rightarrow$ $\text{Prob}(1 \text{ error}) = 3p(1-p)^2$

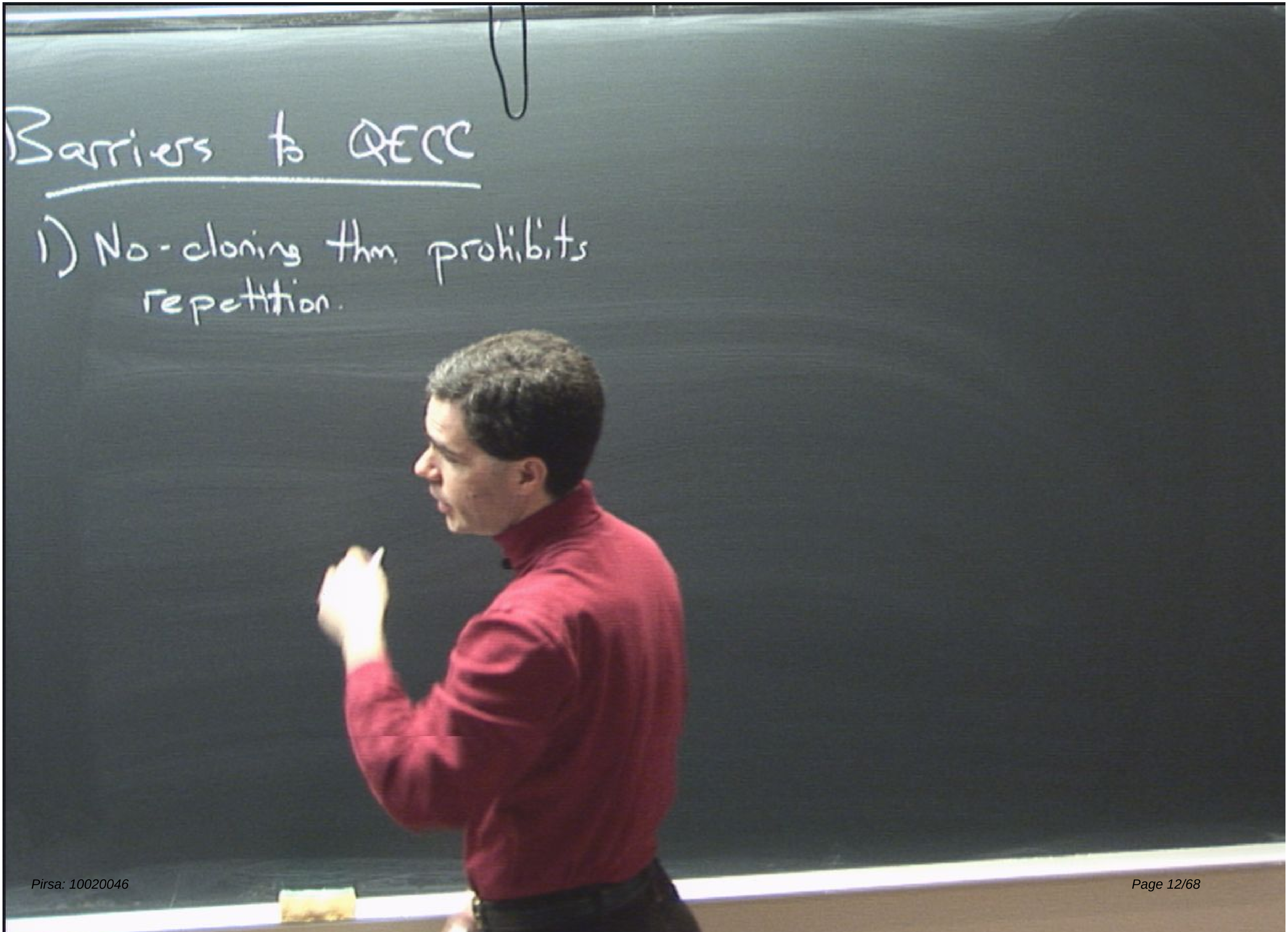
$\text{Prob.}(2 \text{ errors}) = 3p^2(1-p)$

$\text{Prob.}(3 \text{ errors}) = p^3$

By encoding, effective error rate goes from p to $O(p^2)$

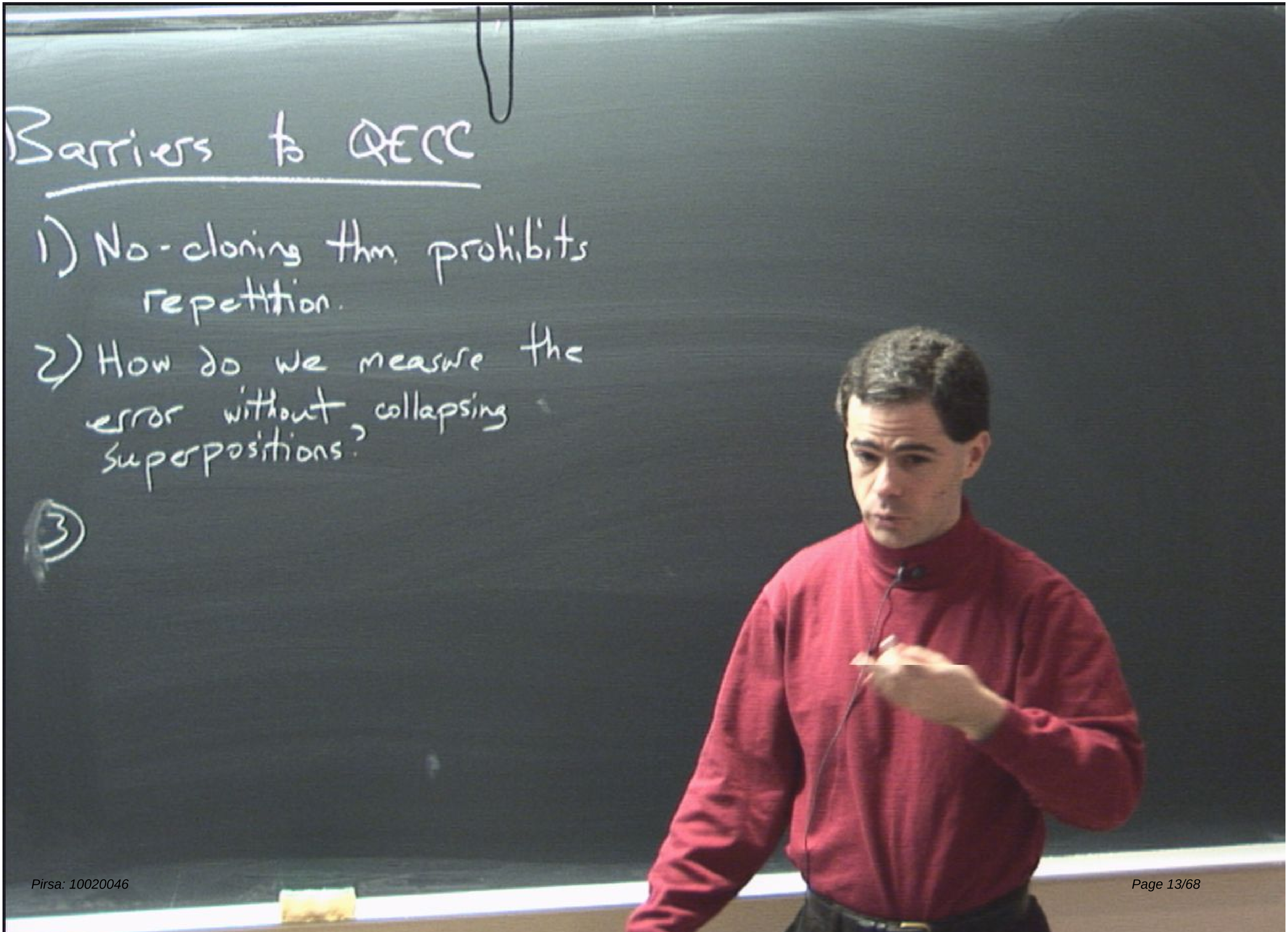
Just assume 0 or 1 error, ignore the possibility of 2 errors

For a code that corrects t errors, logical error prob. is $O(p^{t+1})$



Barriers to QECC

1) No-cloning thm. prohibits repetition.

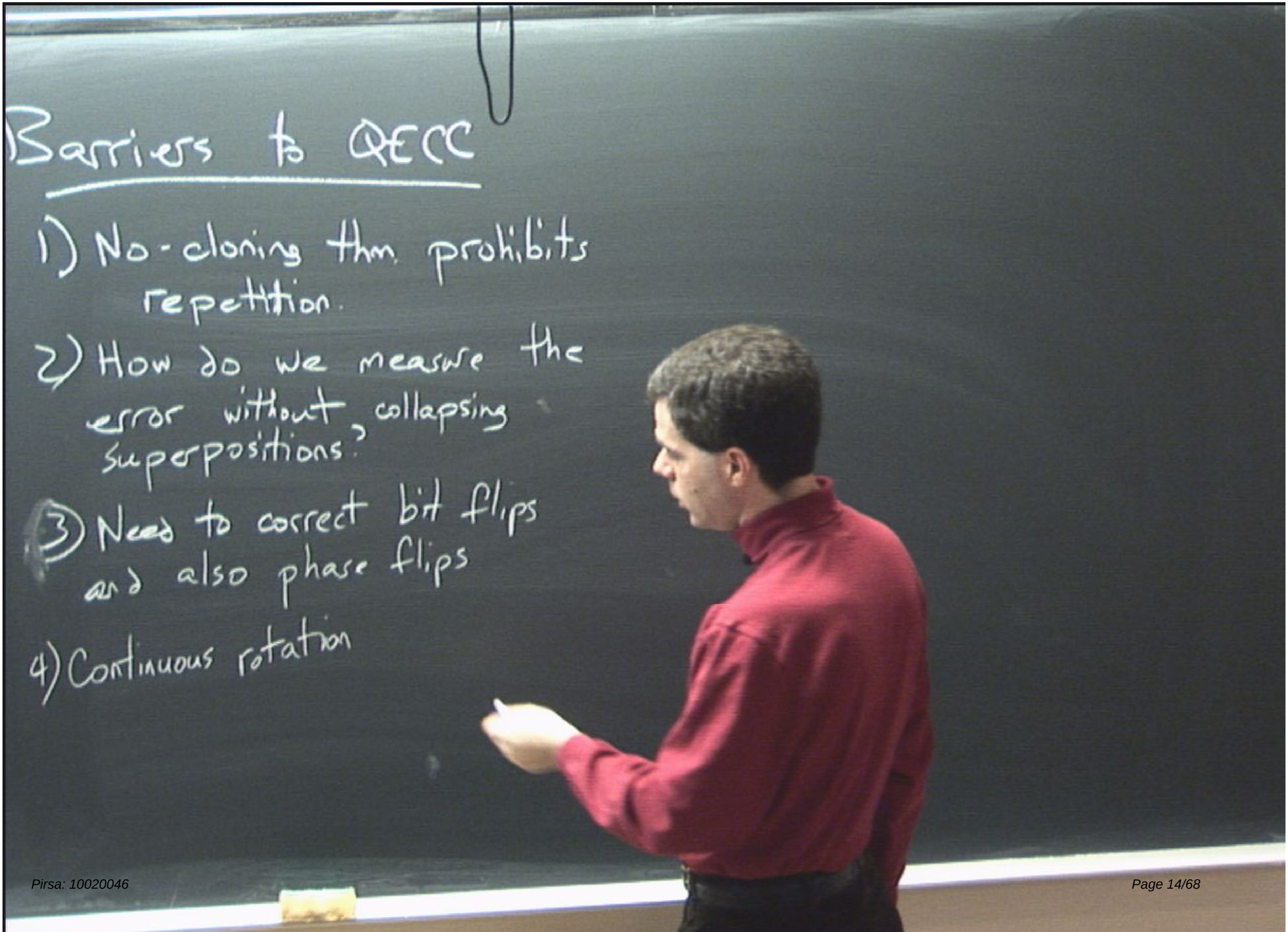


Barriers to QECC

1) No-cloning thm. prohibits repetition.

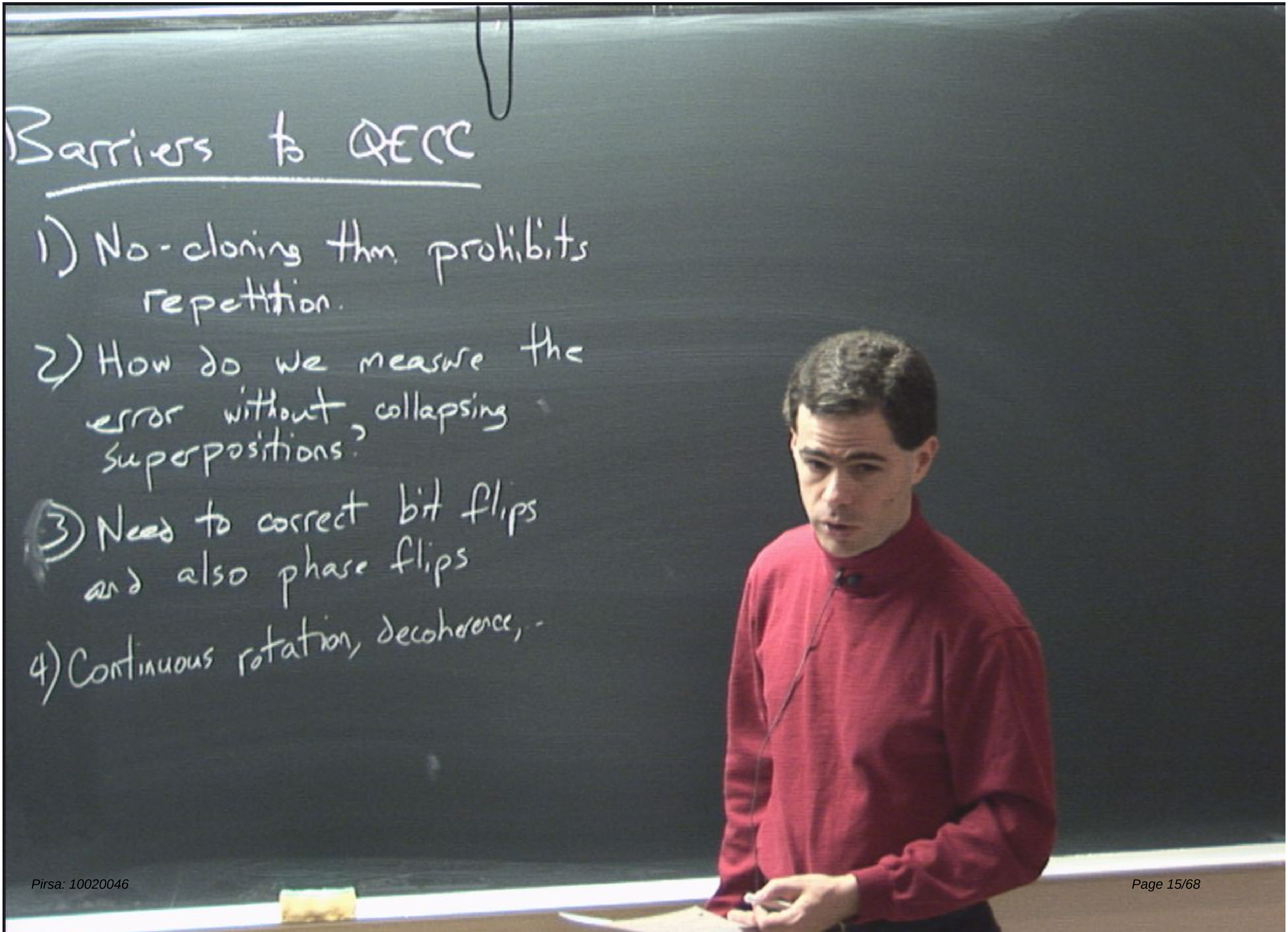
2) How do we measure the error without collapsing superpositions?

③



Barriers to QECC

1) No-cloning thm prohibits repetition.
2) How do we measure the error without collapsing superpositions?
3) Need to correct bit flips and also phase flips
4) Continuous rotation



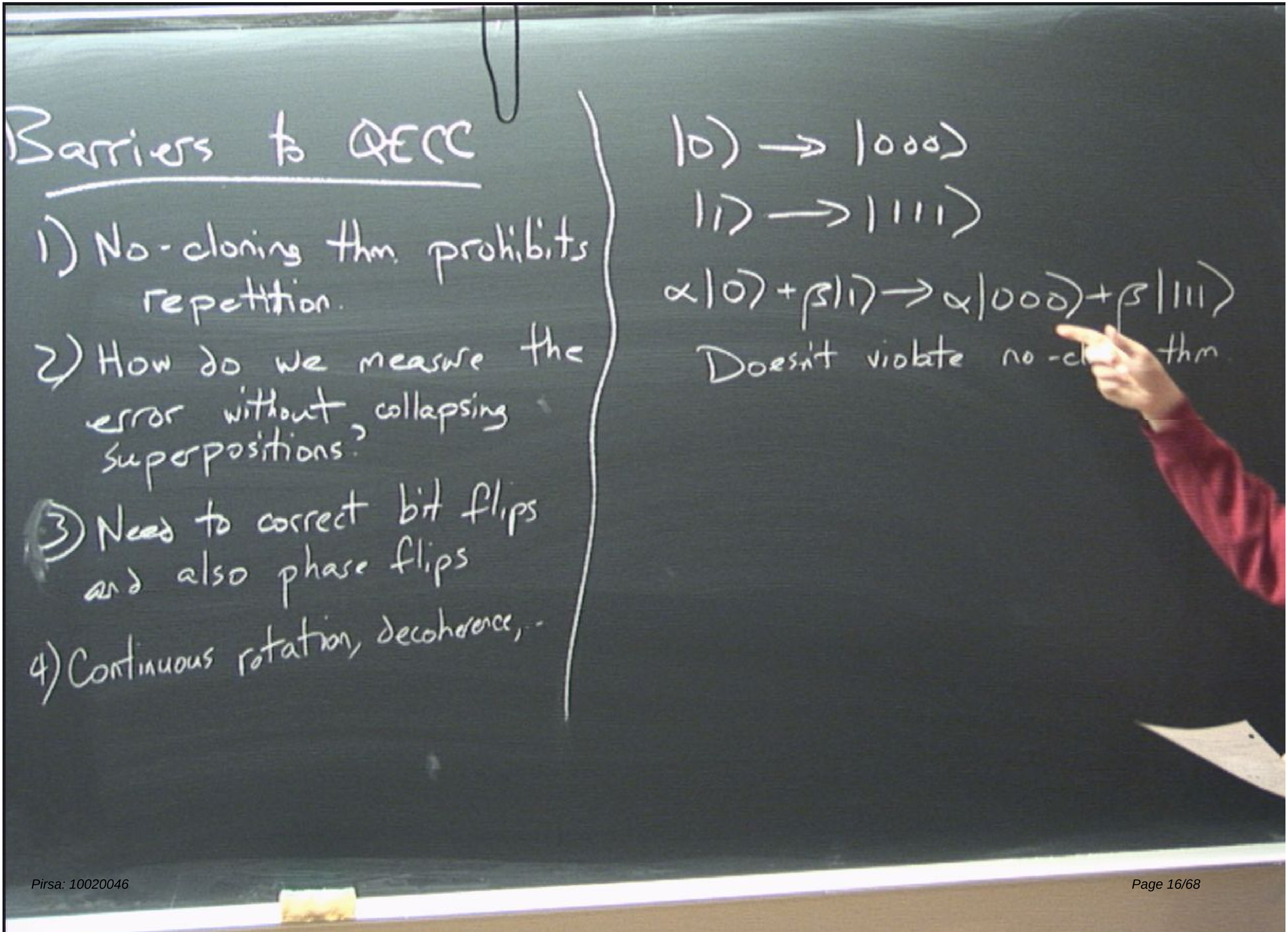
Barriers to QECC

1) No-cloning thm prohibits repetition.

2) How do we measure the error without collapsing superpositions?

3) Need to correct bit flips and also phase flips

4) Continuous rotation, decoherence, ..



Barriers to QECC

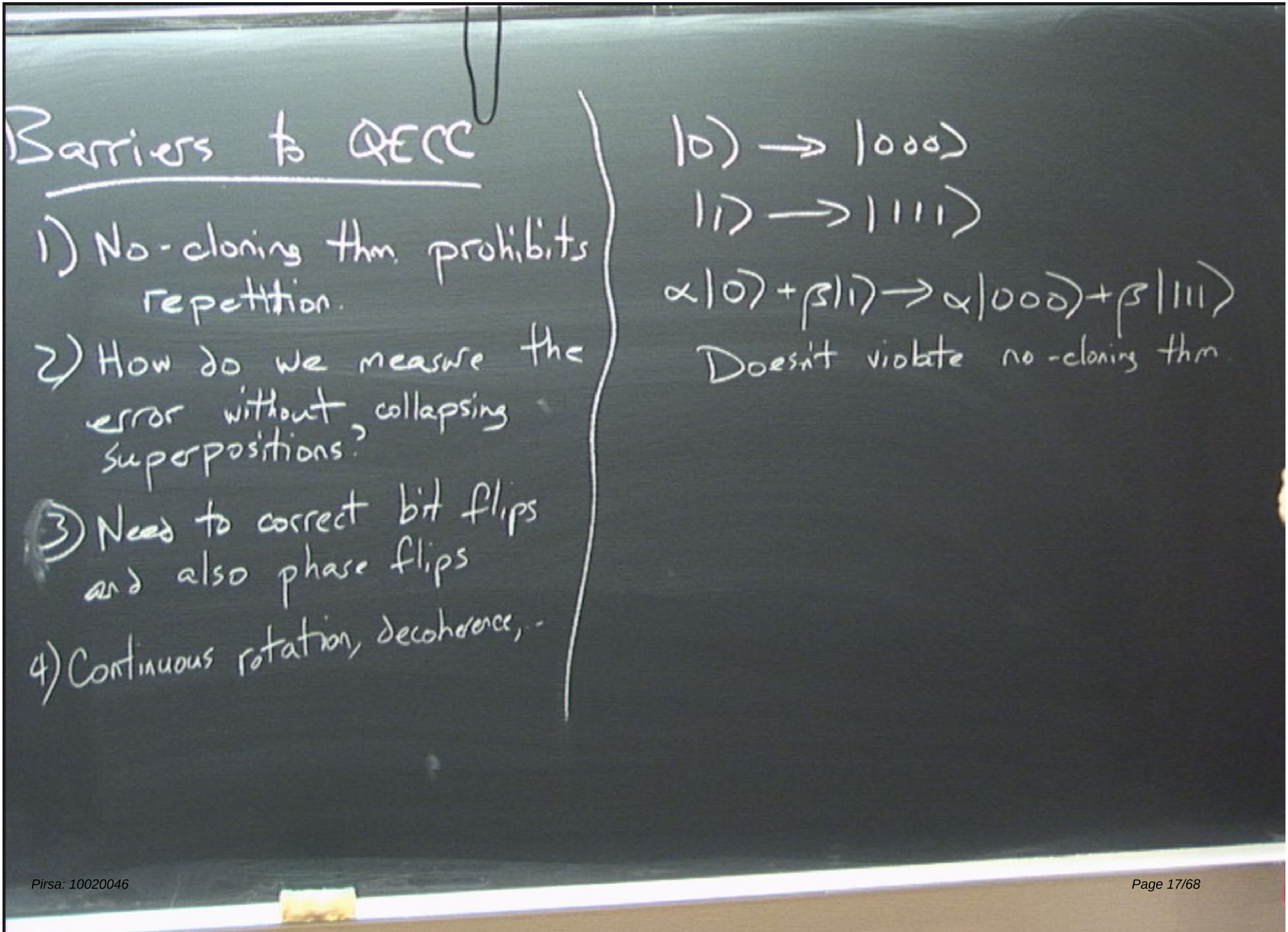
1) No-cloning thm prohibits repetition.
2) How do we measure the error without collapsing superpositions?
3) Need to correct bit flips and also phase flips
4) Continuous rotation, decoherence, ..

$$|0\rangle \longrightarrow |000\rangle$$

$$|1\rangle \longrightarrow |111\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \longrightarrow \alpha|000\rangle + \beta|111\rangle$$

Doesn't violate no-cl thm.

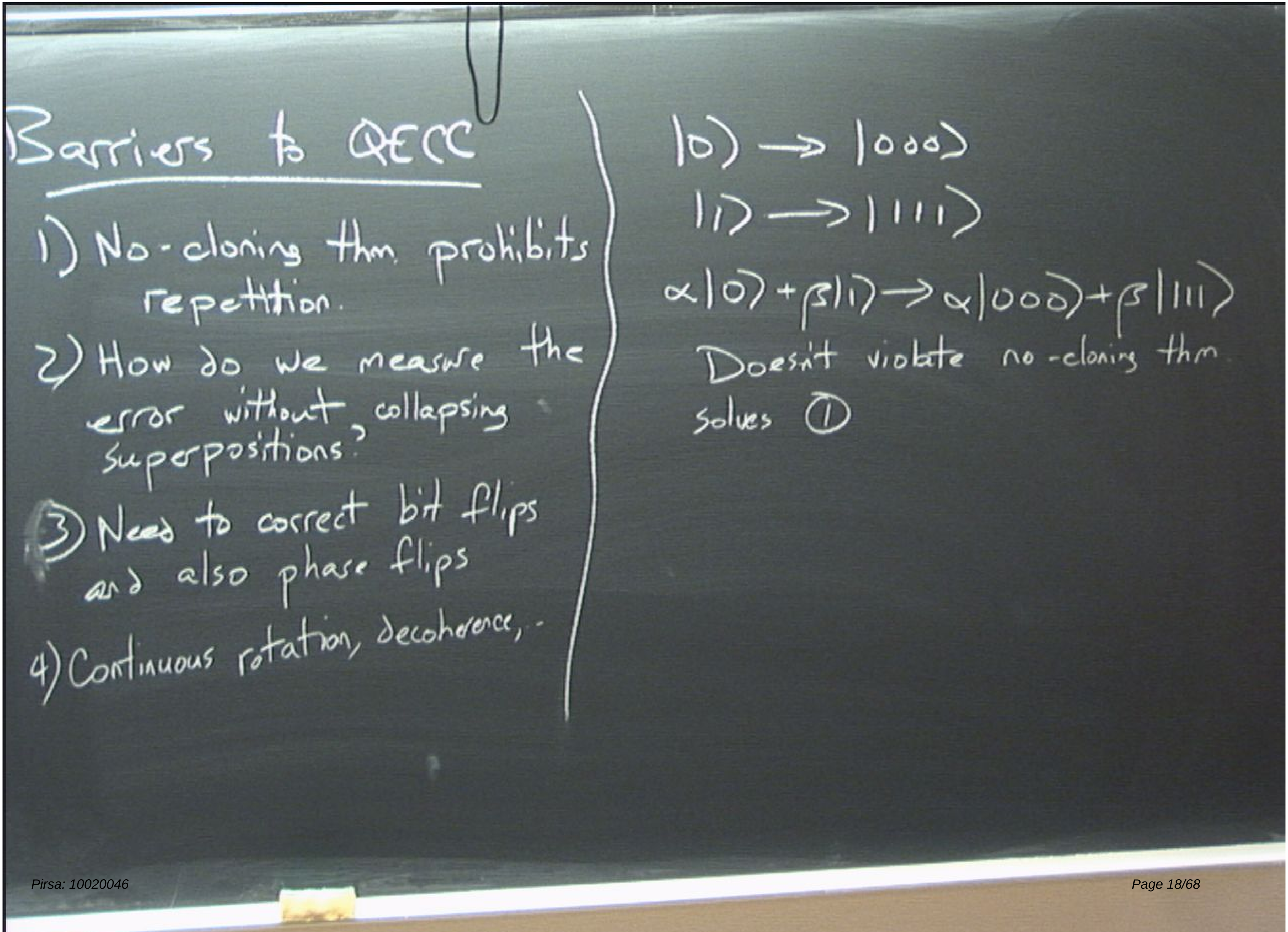


Barriers to QECC

1) No-cloning thm. prohibits repetition.
2) How do we measure the error without collapsing superpositions?
③) Need to correct bit flips and also phase flips
4) Continuous rotation, decoherence, ..

$$|0\rangle \longrightarrow |000\rangle$$
$$|1\rangle \longrightarrow |111\rangle$$
$$\alpha|0\rangle+\beta|1\rangle \longrightarrow \alpha|000\rangle+\beta|111\rangle$$

Doesn't violate no-cloning thm.



Barriers to QECC

1) No-cloning thm prohibits repetition.
2) How do we measure the error without collapsing superpositions?
③ Need to correct bit flips and also phase flips
4) Continuous rotation, decoherence, ..

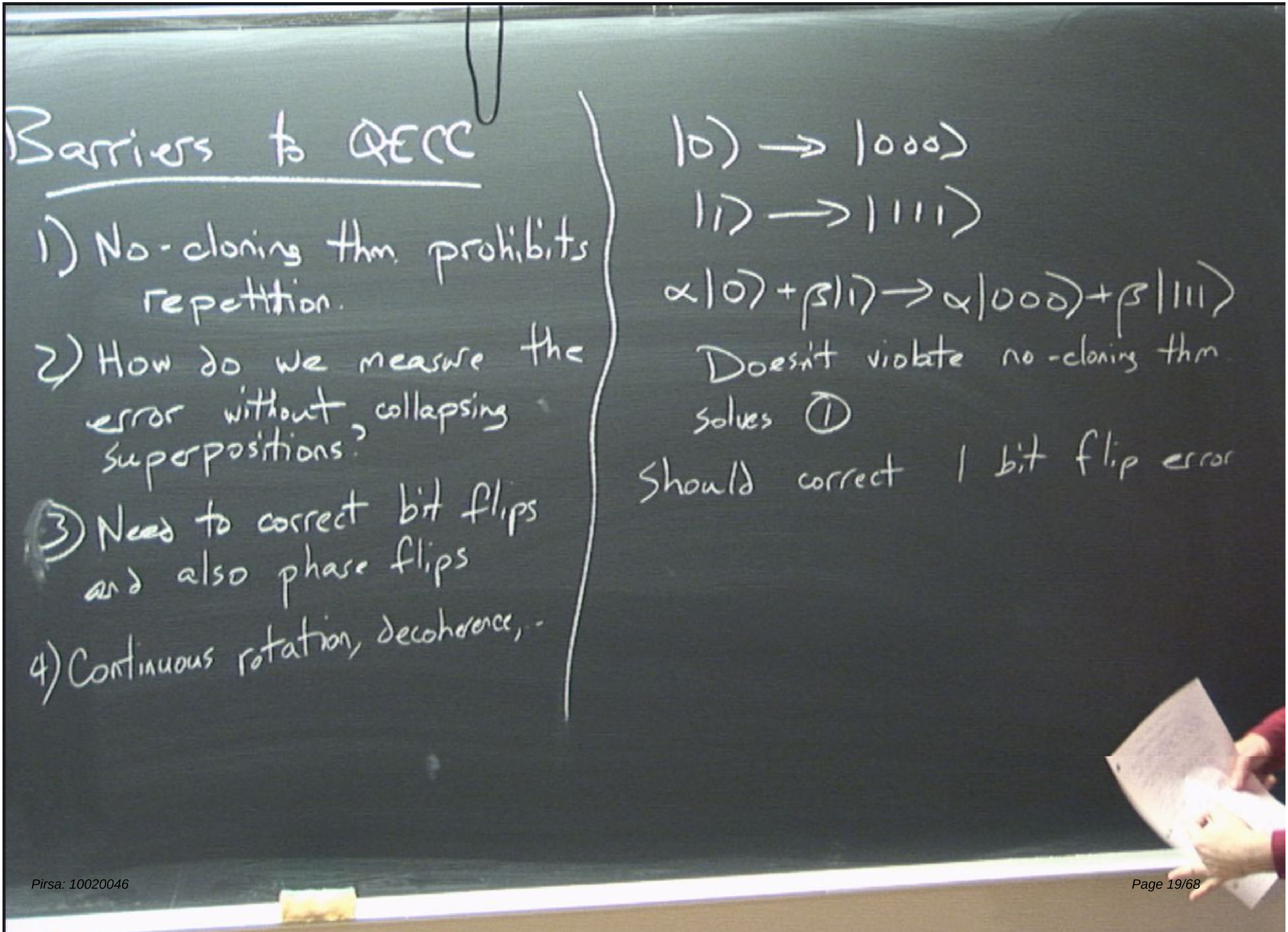
$$|0\rangle \longrightarrow |000\rangle$$

$$|1\rangle \longrightarrow |111\rangle$$

$$\alpha|0\rangle+\beta|1\rangle \longrightarrow \alpha|000\rangle+\beta|111\rangle$$

Doesn't violate no-cloning thm.

solves ①



Barriers to QECC

1) No-cloning thm. prohibits repetition.

2) How do we measure the error without collapsing superpositions?

③ Need to correct bit flips and also phase flips

4) Continuous rotation, decoherence, ..

$|0\rangle \rightarrow |000\rangle$

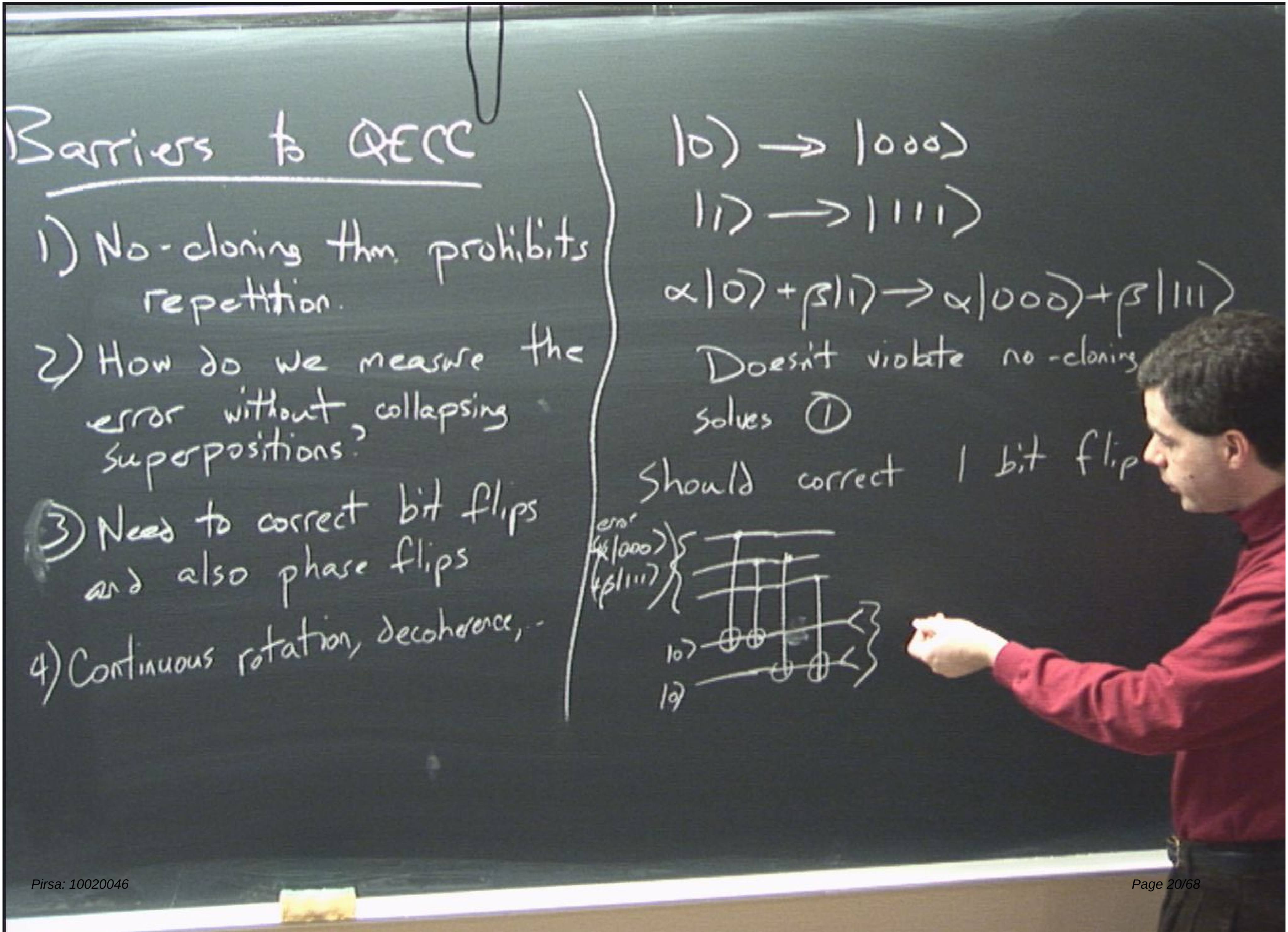
$|1\rangle \rightarrow |111\rangle$

$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$

Doesn't violate no-cloning thm.

solves ①

Should correct 1 bit flip error



Barriers to QECC

1) No-cloning thm prohibits repetition.

2) How do we measure the error without collapsing superpositions?

③ Need to correct bit flips and also phase flips

4) Continuous rotation, decoherence, ..

$|0\rangle \rightarrow |000\rangle$

$|1\rangle \rightarrow |111\rangle$

$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$

Doesn't violate no-cloning

solves ①

Should correct 1 bit flip

Barriers to QECC

1) No-cloning thm prohibits repetition.

2) How do we measure the error without collapsing superpositions?

3) Need to correct bit flips and also phase flips

4) Continuous rotation, decoherence, ..

$$|0\rangle \rightarrow |000\rangle$$

$$|1\rangle \rightarrow |111\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$$

Doesn't violate no-cloning thm.

solves ①

Should correct 1 bit flip error

error syndrome	error
00	None
10	1st bit
01	

riers to QECC

- o-cloning thm. prohibits repetition.
- ow do we measure the rror without collapsing uperpositions?
- eed to correct bit flips also phase flips
- tinuous rotation, decoherence, -

$|0\rangle \longrightarrow |000\rangle$

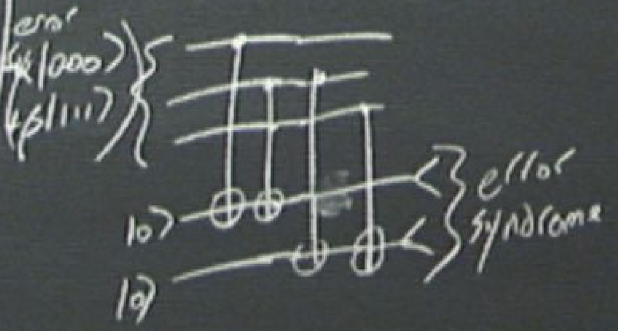
$|1\rangle \longrightarrow |111\rangle$

$\alpha|0\rangle + \beta|1\rangle \longrightarrow \alpha|000\rangle + \beta|111\rangle$

Doesn't violate no-cloning thm.

solves ①

Should correct 1 bit flip error



error syndrome	error
00	None
10	1st bit
01	3rd bit
11	2nd bit

iers to QECC

lo-cloning thm. prohibits repetition.

low do we measure the rror without collapsing uperpositions?

leed to correct bit flips d also phase flips

tinuous rotation, decoherence, ..

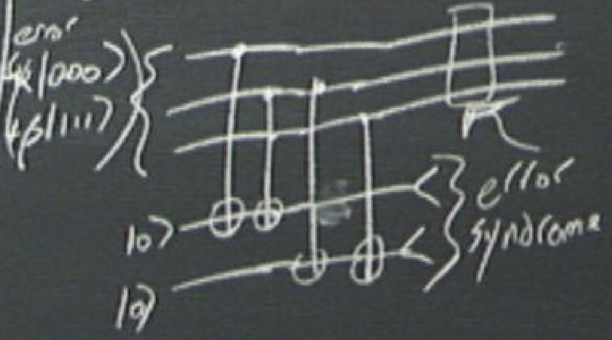
$$|0\rangle \rightarrow |000\rangle$$

$$|1\rangle \rightarrow |111\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$$

Doesn't violate no-cloning thm.

solves ①

Should correct 1 bit flip error

error syndrome	error
00	None
10	1st bit
01	3rd bit
11	2nd bit

iers to QECC

o-cloning thm prohibits repetition.

low do we measure the rror without collapsing uperpositions?

leed to correct bit flips d also phase flips

tinuous rotation, decoherence, ..

$$|0\rangle \rightarrow |000\rangle$$

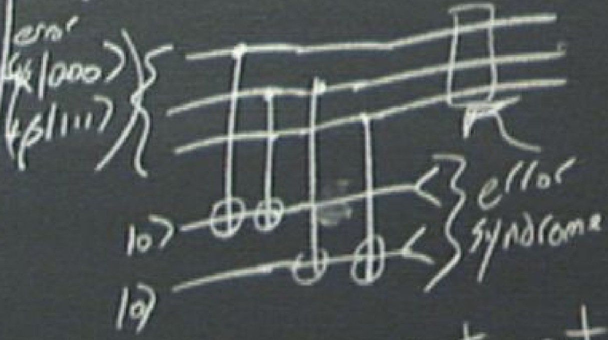
$$|1\rangle \rightarrow |111\rangle$$

$$\alpha|0\rangle+\beta|1\rangle \rightarrow \alpha|000\rangle+\beta|111\rangle$$

Doesn't violate no-cloning thm

solves ①

Should correct 1 bit flip error



error syndrome	error
00	None
10	1st bit
01	3rd bit
11	2nd bit

Measure error but not the encoded state solves ②

riers to QECC

- o-cloning thm prohibits repetition.
- low do we measure the rror without collapsing uperpositions?
- leed to correct bit flips also phase flips
- tinuous rotation, decoherence, ..

$|0\rangle \to |000\rangle$

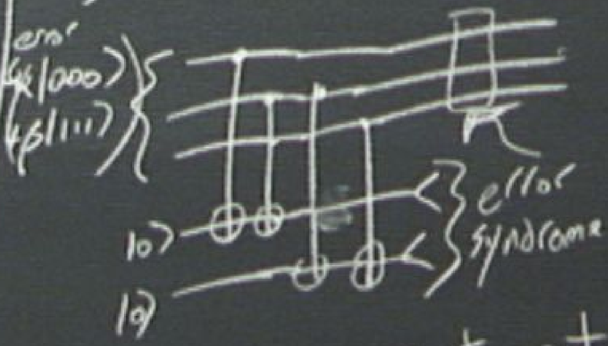
$|1\rangle \to |111\rangle$

$\alpha|0\rangle + \beta|1\rangle \to \alpha|000\rangle + \beta|111\rangle$

Doesn't violate no-cloning thm.

solves ①

Should correct 1 bit flip error



error syndrome	error
00	None
10	1st bit
01	3rd bit
11	2nd bit

Measure error but not the encoded state   solves ②

Barriers to QECC

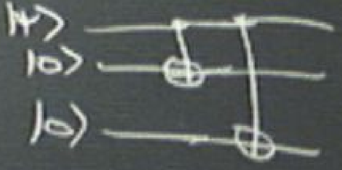
1) No-cloning thm prohibits repetition.

2) How do we measure the error without collapsing superpositions?

3) Need to correct bit flips and also phase flips

4) Continuous rotation, decoherence, ..

$|0\rangle \longrightarrow |000\rangle$



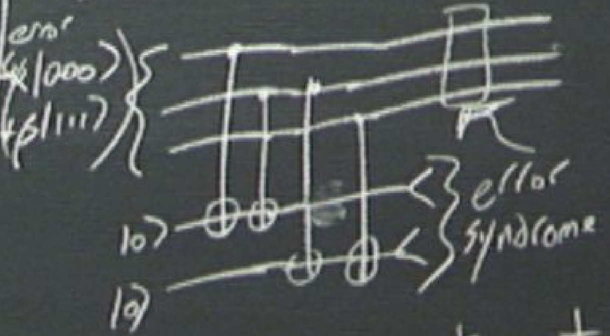
$|1\rangle \longrightarrow |111\rangle$

$\alpha|0\rangle + \beta|1\rangle \longrightarrow \alpha|000\rangle + \beta|111\rangle$

Doesn't violate no-cloning thm.

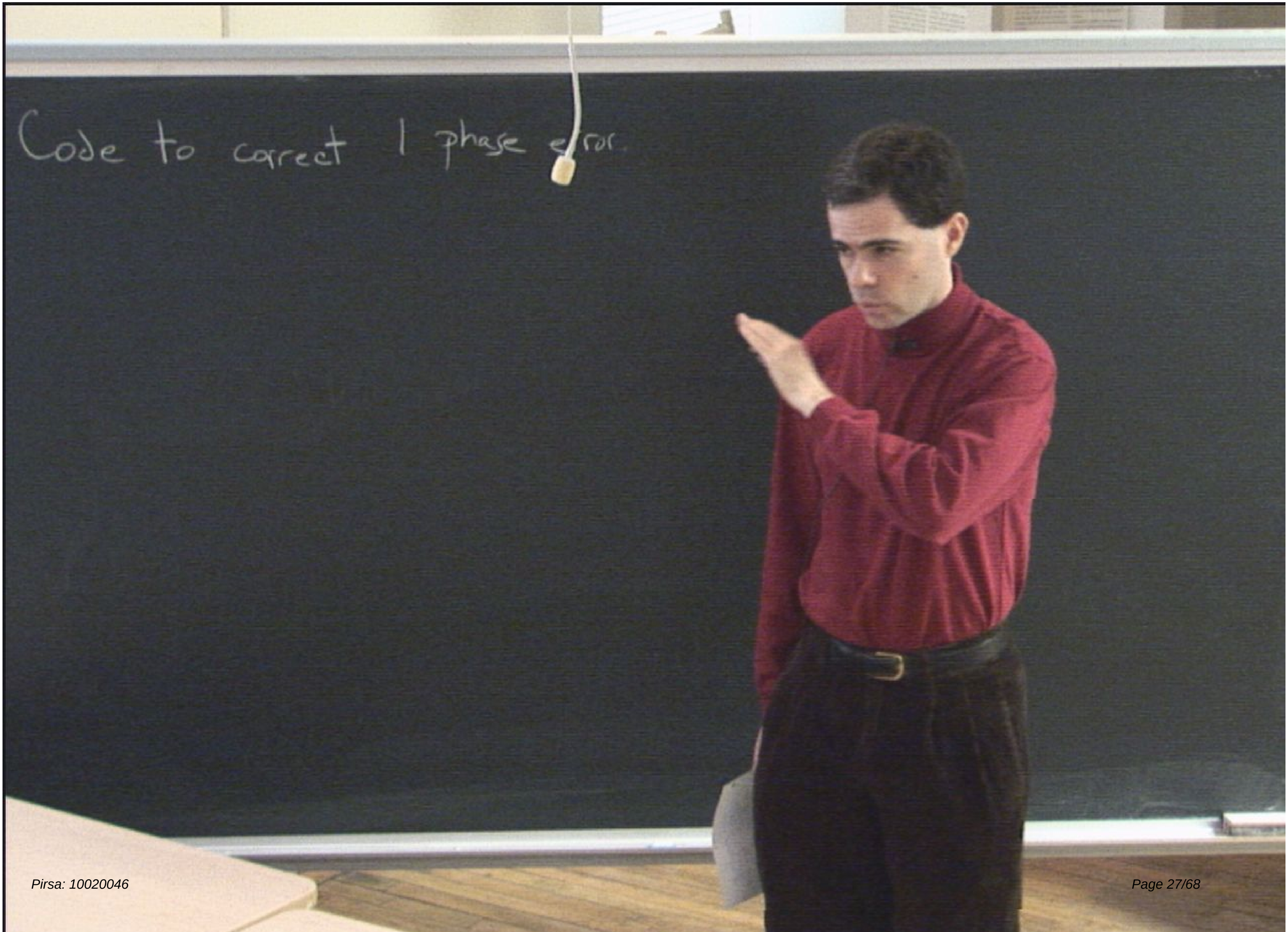
solves ①

Should correct 1 bit flip error

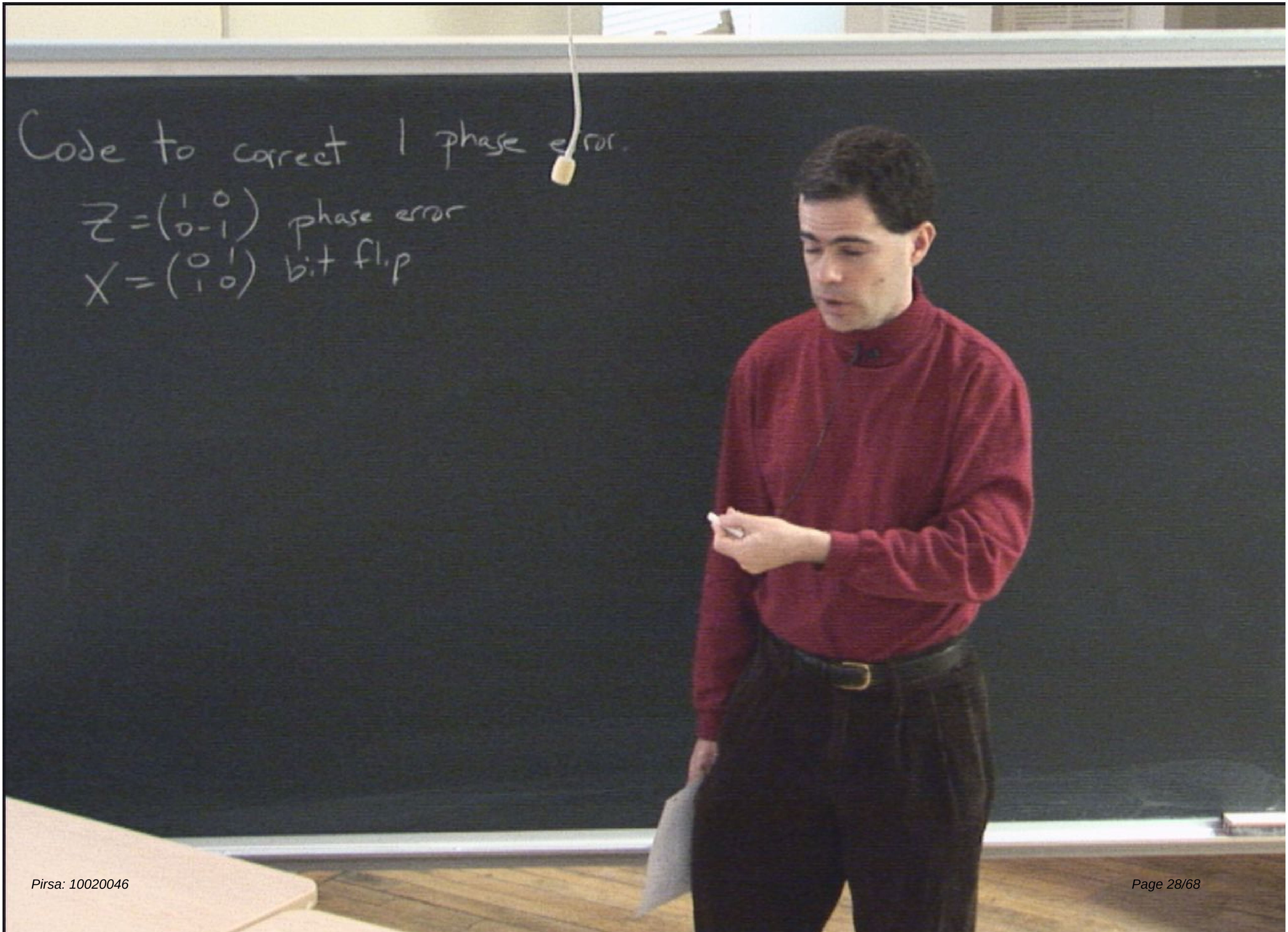


error syndrome	error
00	None
10	1st bit
01	3rd bit
11	2nd bit

Measure error but not the encoded state   solves ②



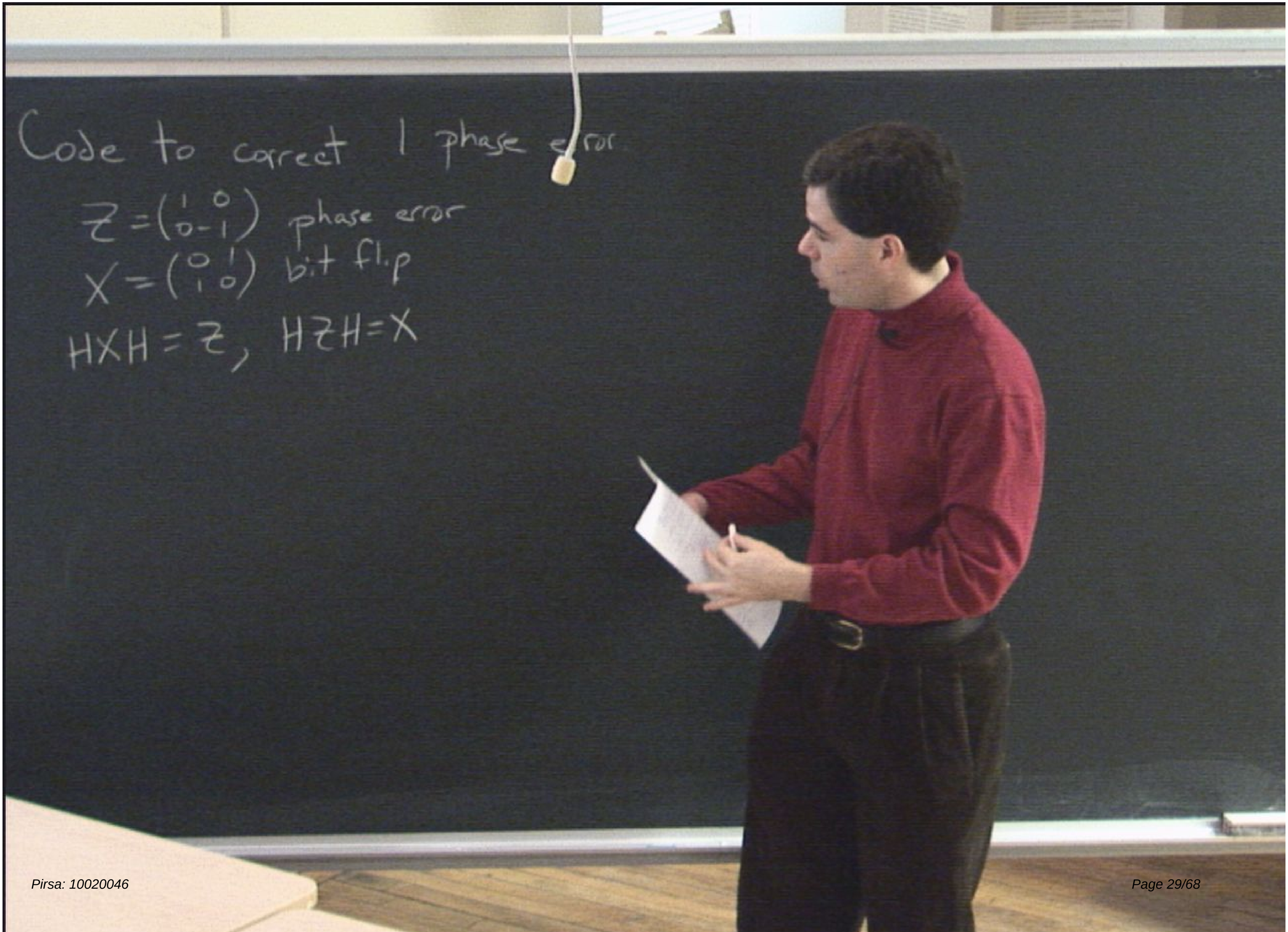
Code to correct 1 phase error.



Code to correct 1 phase error.

$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ phase error

$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ bit flip

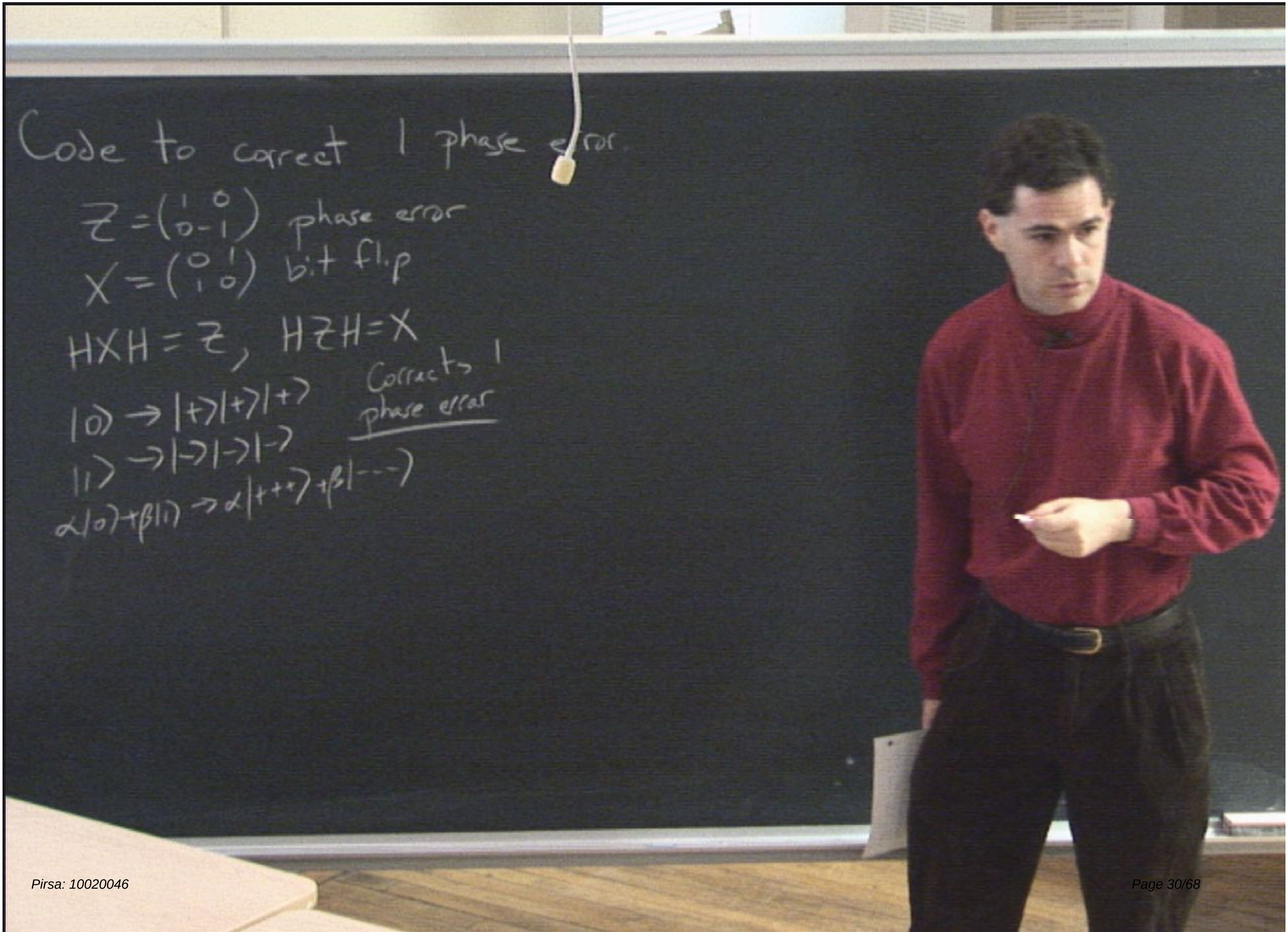


Code to correct 1 phase error

$Z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ phase error

$X = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ bit flip

$HXH = Z$, $HZH = X$



Code to correct 1 phase error.

$Z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ phase error

$X = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ bit flip

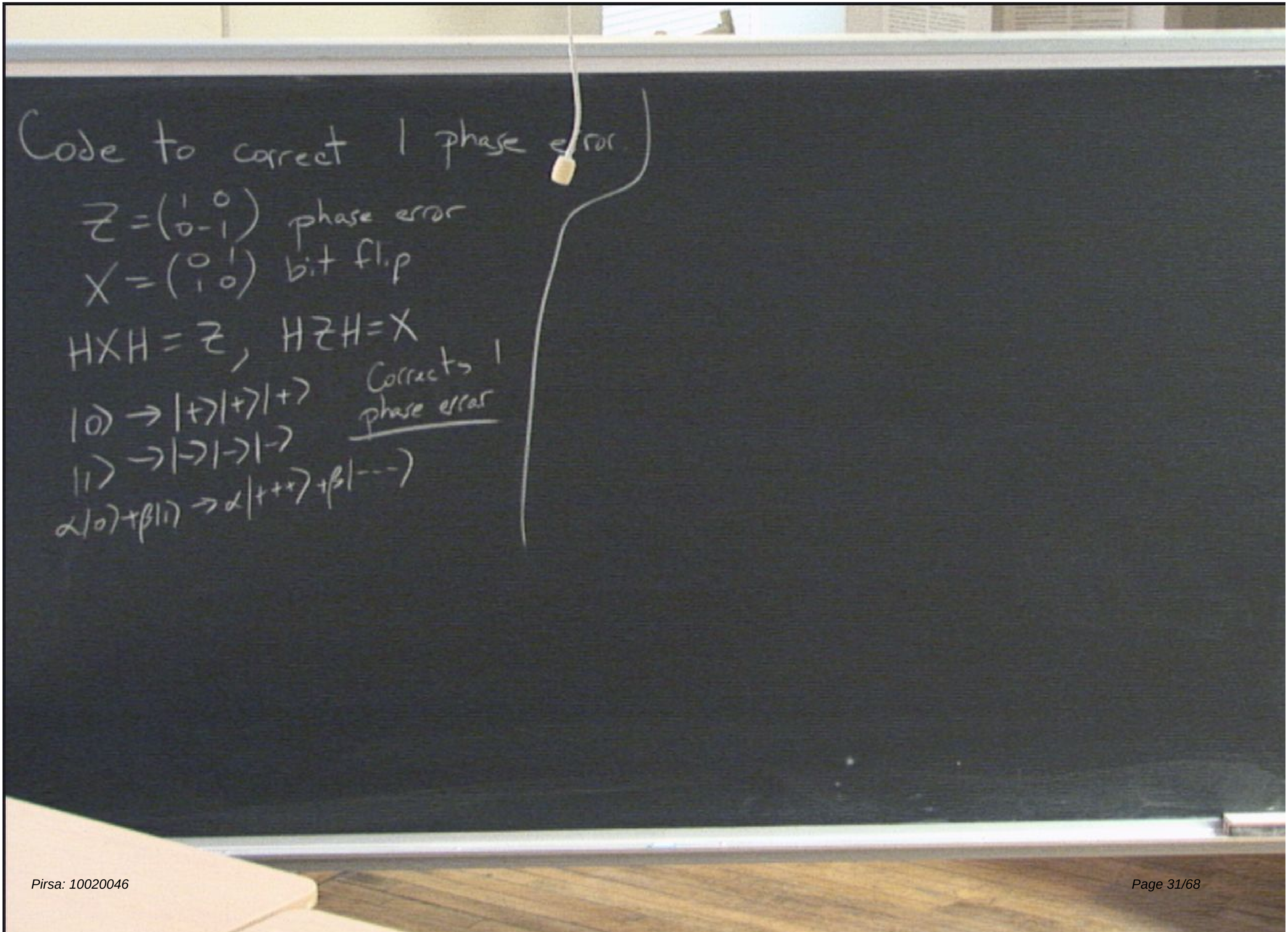
$HXH = Z, \quad HZH = X$

$|0\rangle \to |+\rangle|+\rangle|+\rangle$

Corrects 1 phase error

$|1\rangle \to |-\rangle|-\rangle|-\rangle$

$\alpha|0\rangle + \beta|1\rangle \to \alpha|+++\rangle + \beta|---\rangle$



Code to correct 1 phase error.

$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ phase error

$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ bit flip

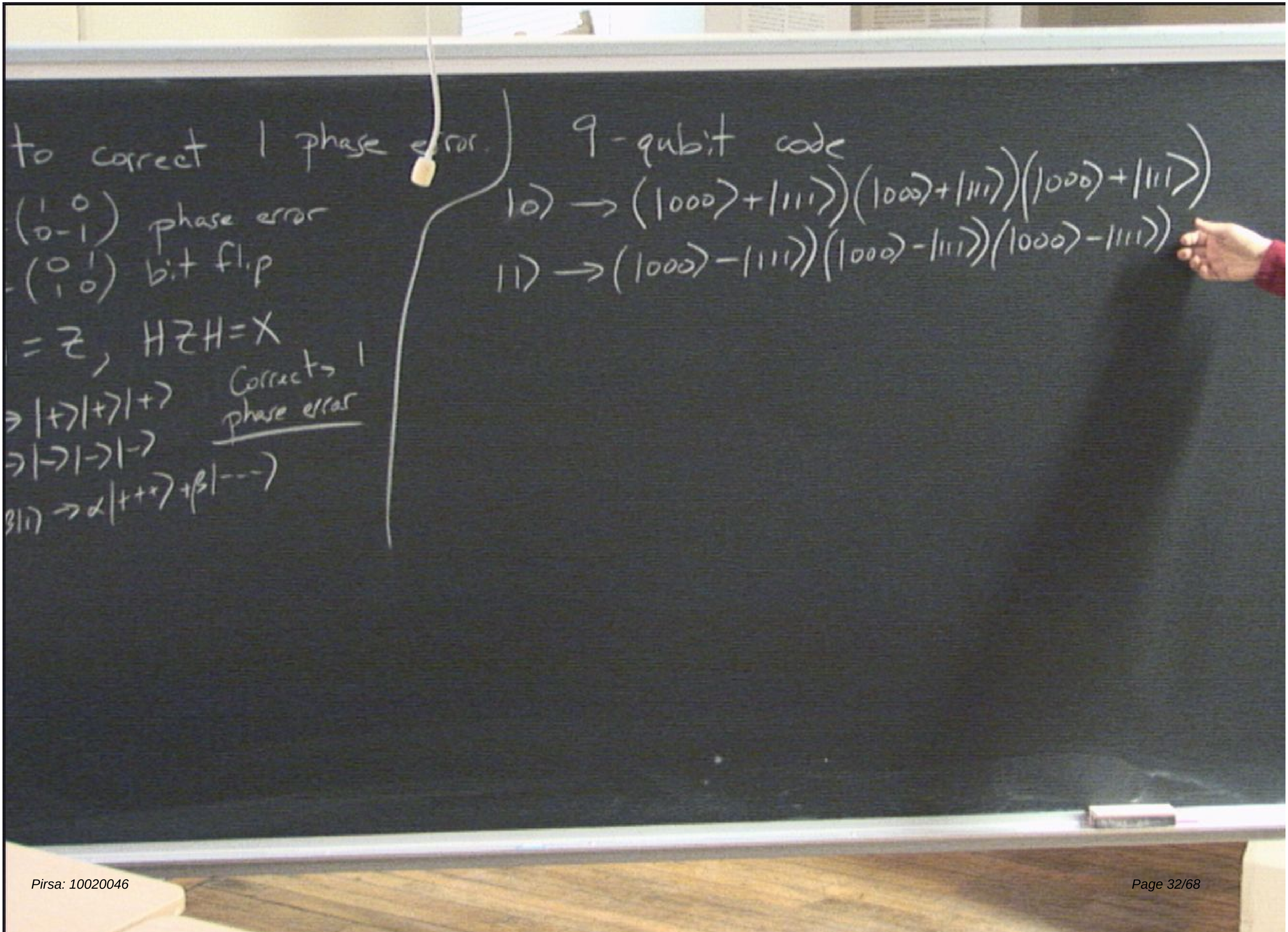
$HXH = Z, \quad HZH = X$

$|0\rangle \to |+\rangle|+\rangle|+\rangle$

$|1\rangle \to |-\rangle|-\rangle|-\rangle$

$\alpha|0\rangle + \beta|1\rangle \to \alpha|+++\rangle + \beta|---\rangle$

Corrects 1 phase error



to correct 1 phase error.

$\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ phase error

$\begin{pmatrix}0&1\\1&0\end{pmatrix}$ bit flip

$= Z,\ HZH = X$

$\rightarrow |+\rangle|+\rangle|+\rangle$

$\rightarrow |-\rangle|-\rangle|-\rangle$

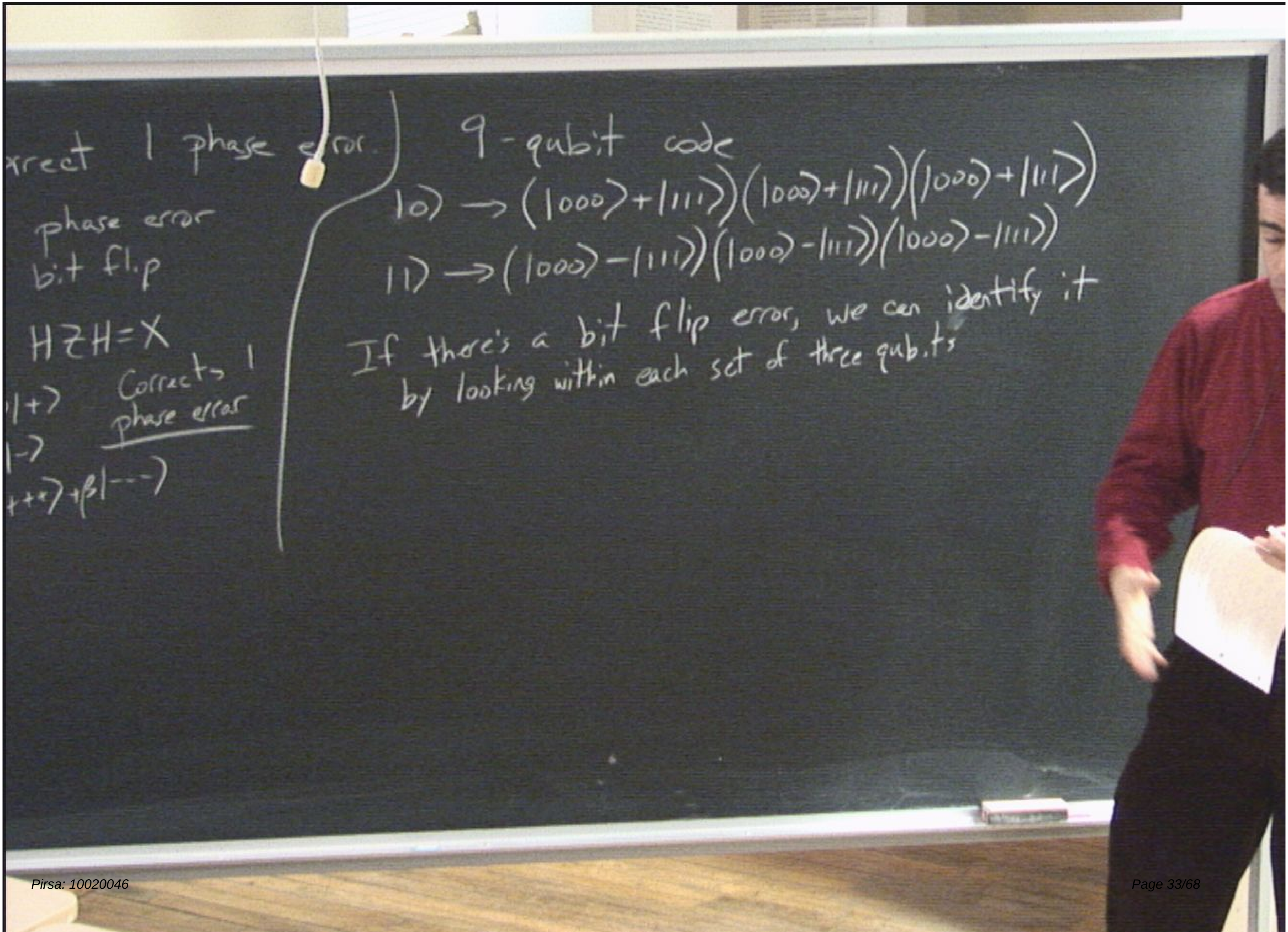
$\beta|1\rangle \rightarrow \alpha|+++\rangle + \beta|---\rangle$

Corrects 1 phase error

9-qubit code

$$|0\rangle \rightarrow (|000\rangle+|111\rangle)(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)$$

$$|1\rangle \rightarrow (|000\rangle-|111\rangle)(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)$$



rrect 1 phase error.

phase error

bit flip

$HZH=X$

$|+\rangle$ Corrects 1 phase error

$|-\rangle$

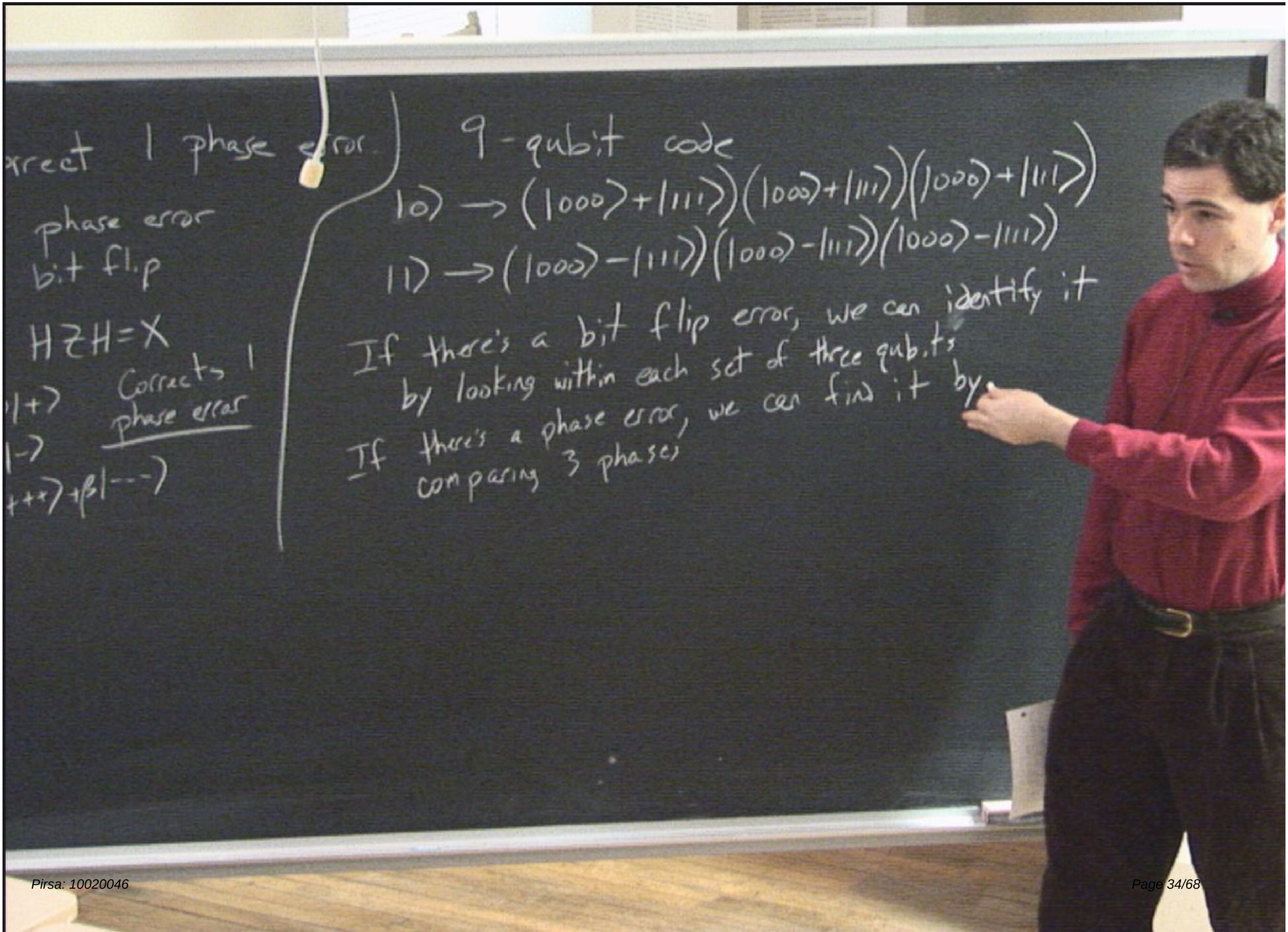
$++\rangle+\beta|---\rangle$

9-qubit code

$$|0\rangle \to (|000\rangle+|111\rangle)(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)$$

$$|1\rangle \to (|000\rangle-|111\rangle)(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)$$

If there's a bit flip error, we can identify it by looking within each set of three qubits



rrect 1 phase error.

phase error

bit flip

$HZH = X$

$|+\rangle$ Corrects 1 phase error

$|-\rangle$

$+++\rangle + \beta|---\rangle$

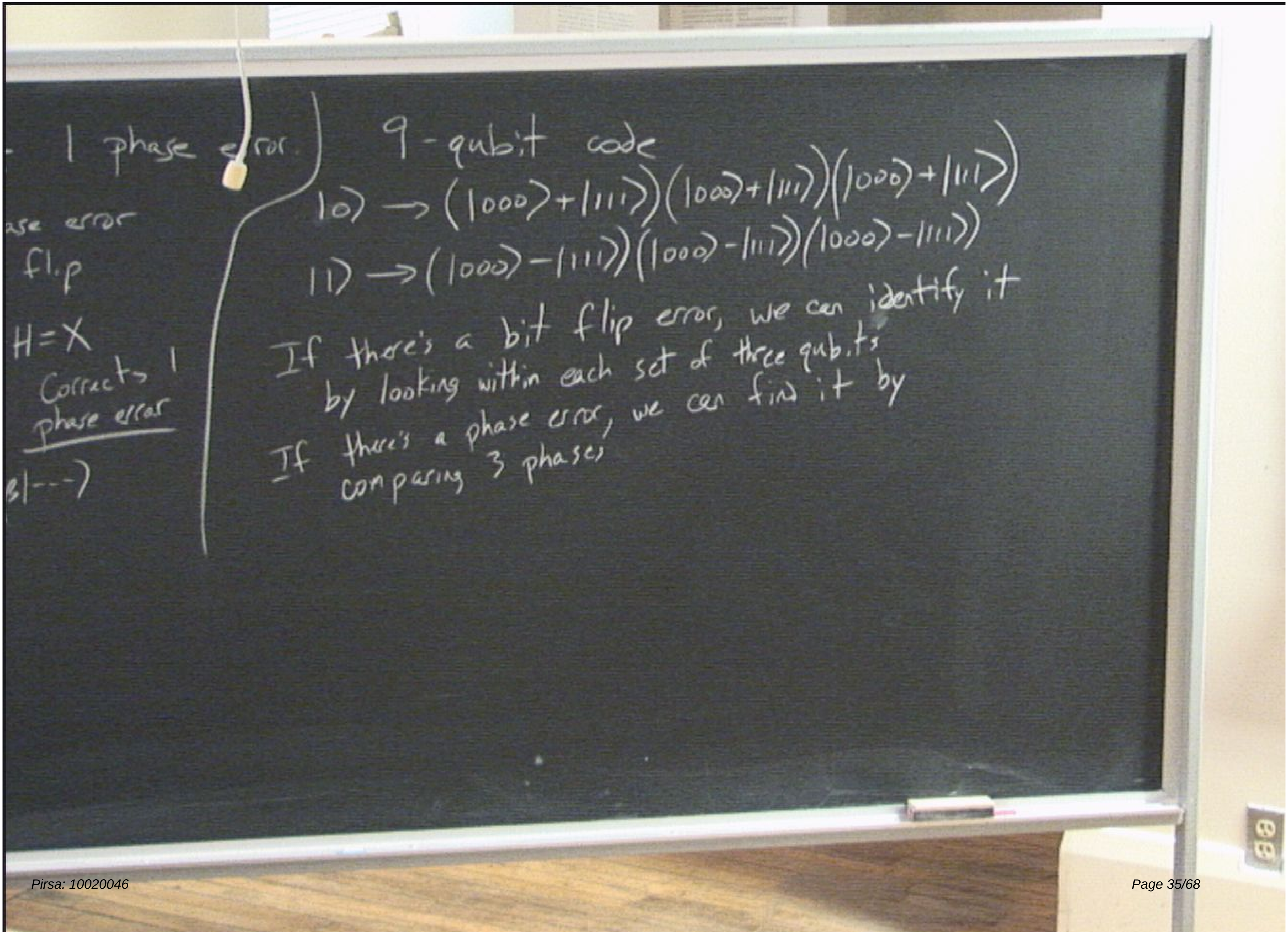
9-qubit code

$$|0\rangle \rightarrow (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$$

$$|1\rangle \rightarrow (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

If there's a bit flip error, we can identify it by looking within each set of three qubits

If there's a phase error, we can find it by comparing 3 phases



1 phase error.

ase error

flip

H=X

Corrects 1 phase error

$\beta|---\rangle$

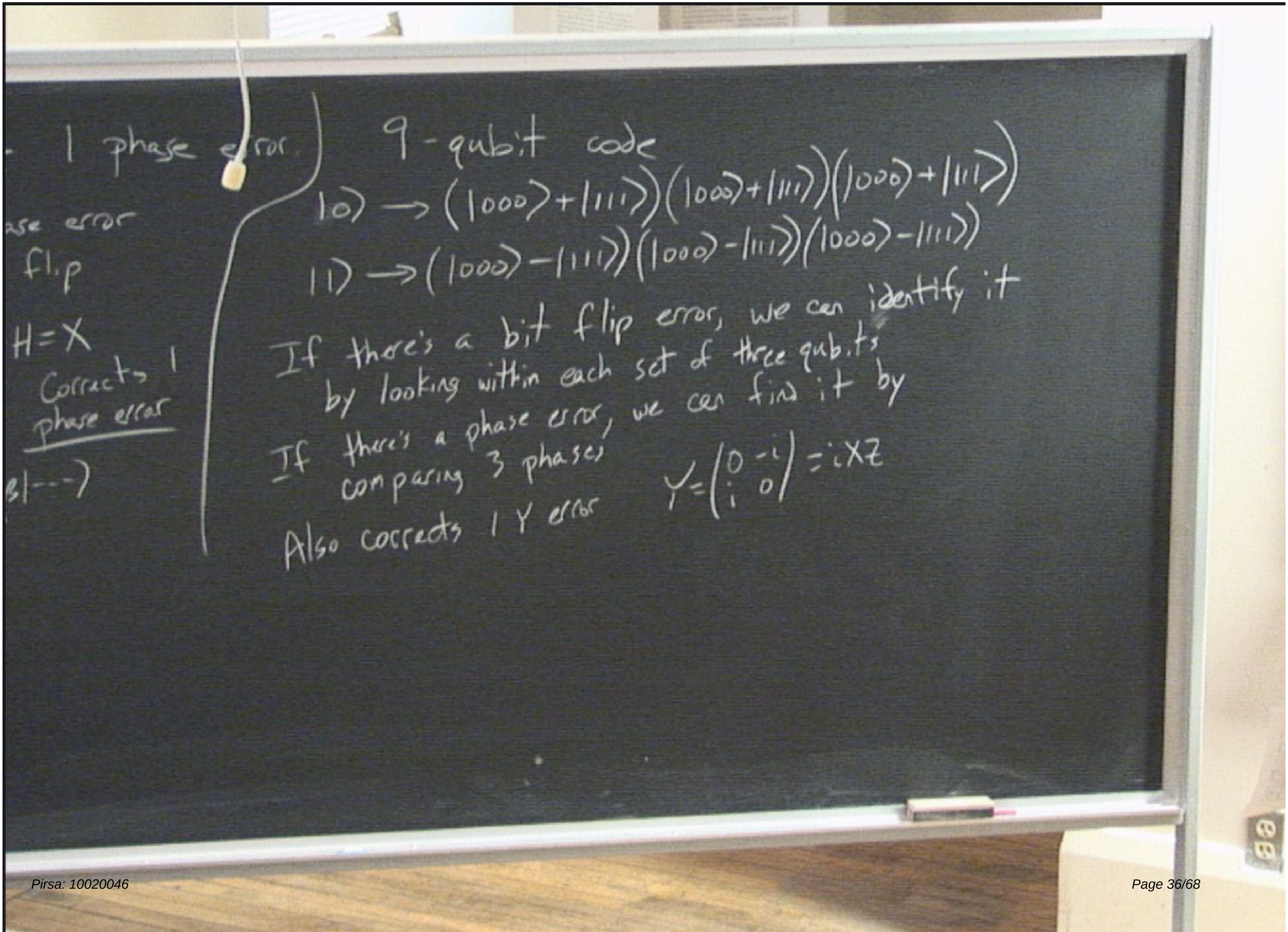
9-qubit code

$$|0\rangle \rightarrow (|000\rangle+|111\rangle)(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)$$

$$|1\rangle \rightarrow (|000\rangle-|111\rangle)(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)$$

If there's a bit flip error, we can identify it by looking within each set of three qubits

If there's a phase error, we can find it by comparing 3 phases



- 1 phase error.

ase error

flip

H=X

Corrects 1 phase error

$\beta|---\rangle$

9-qubit code

$$|0\rangle \to (|000\rangle+|111\rangle)(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)$$

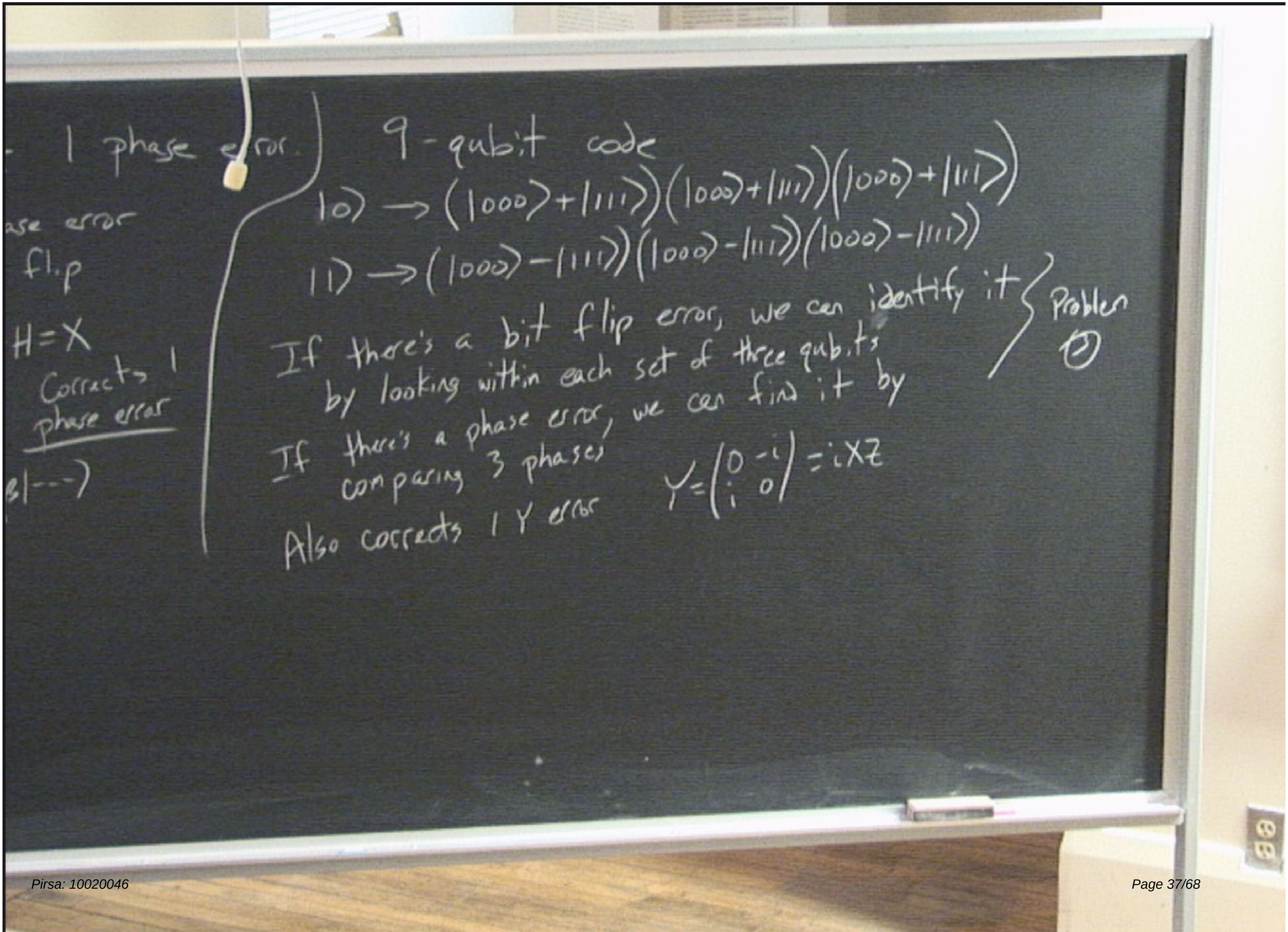
$$|1\rangle \to (|000\rangle-|111\rangle)(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)$$

If there's a bit flip error, we can identify it by looking within each set of three qubits

If there's a phase error, we can find it by comparing 3 phases

$$Y=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}=iXZ$$

Also corrects 1 Y error



1 phase error.

ase error

flip

$H=X$

Corrects 1 phase error

$\beta|---\rangle$

9-qubit code

$|0\rangle \rightarrow (|000\rangle+|111\rangle)(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)$

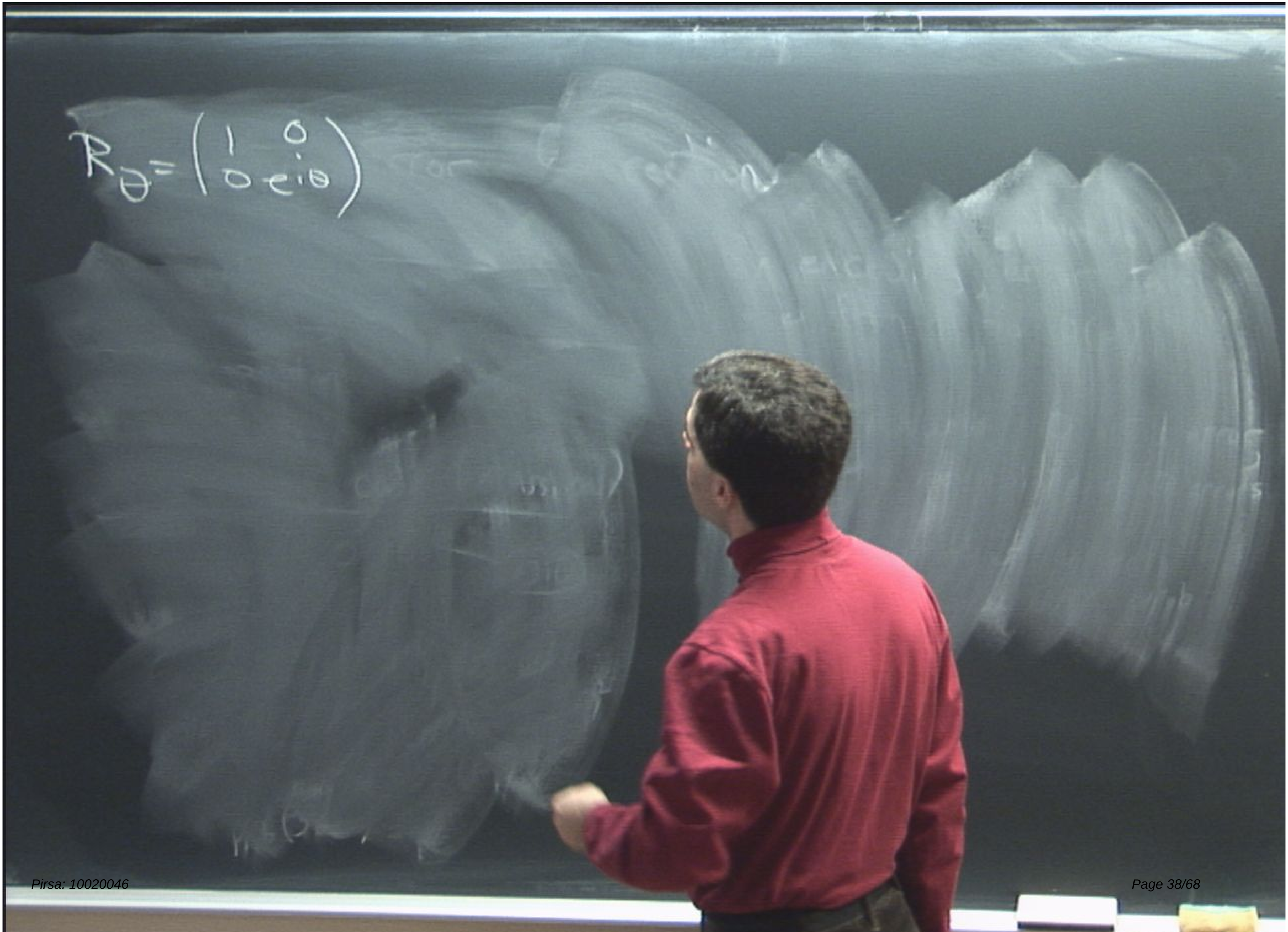
$|1\rangle \rightarrow (|000\rangle-|111\rangle)(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)$

If there's a bit flip error, we can identify it by looking within each set of three qubits

If there's a phase error, we can find it by comparing 3 phases

Problem ②

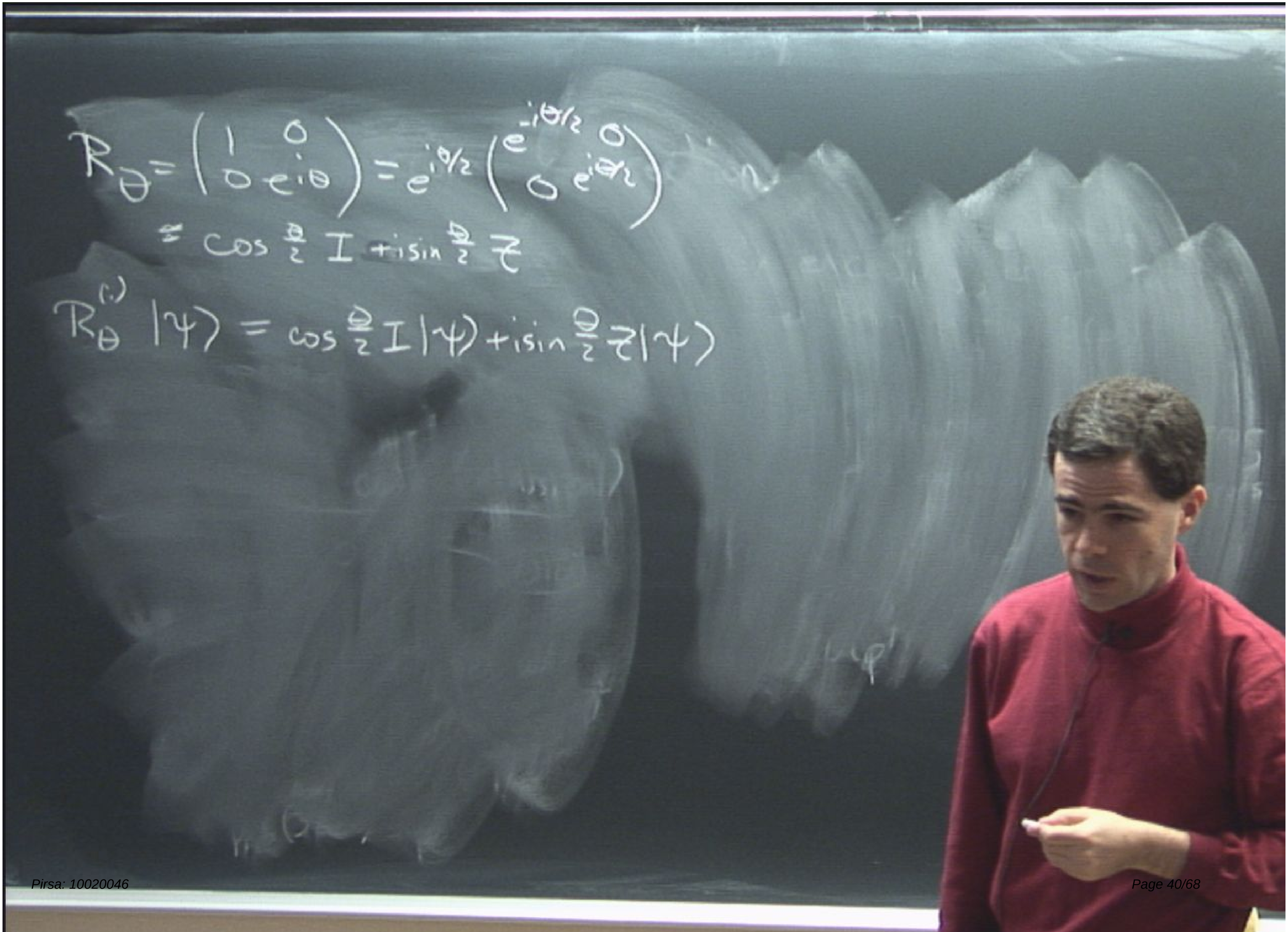
Also corrects 1 Y error $\quad Y=\begin{pmatrix}0&-i\\i&0\end{pmatrix}=iXZ$



$$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

$$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta/2} \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

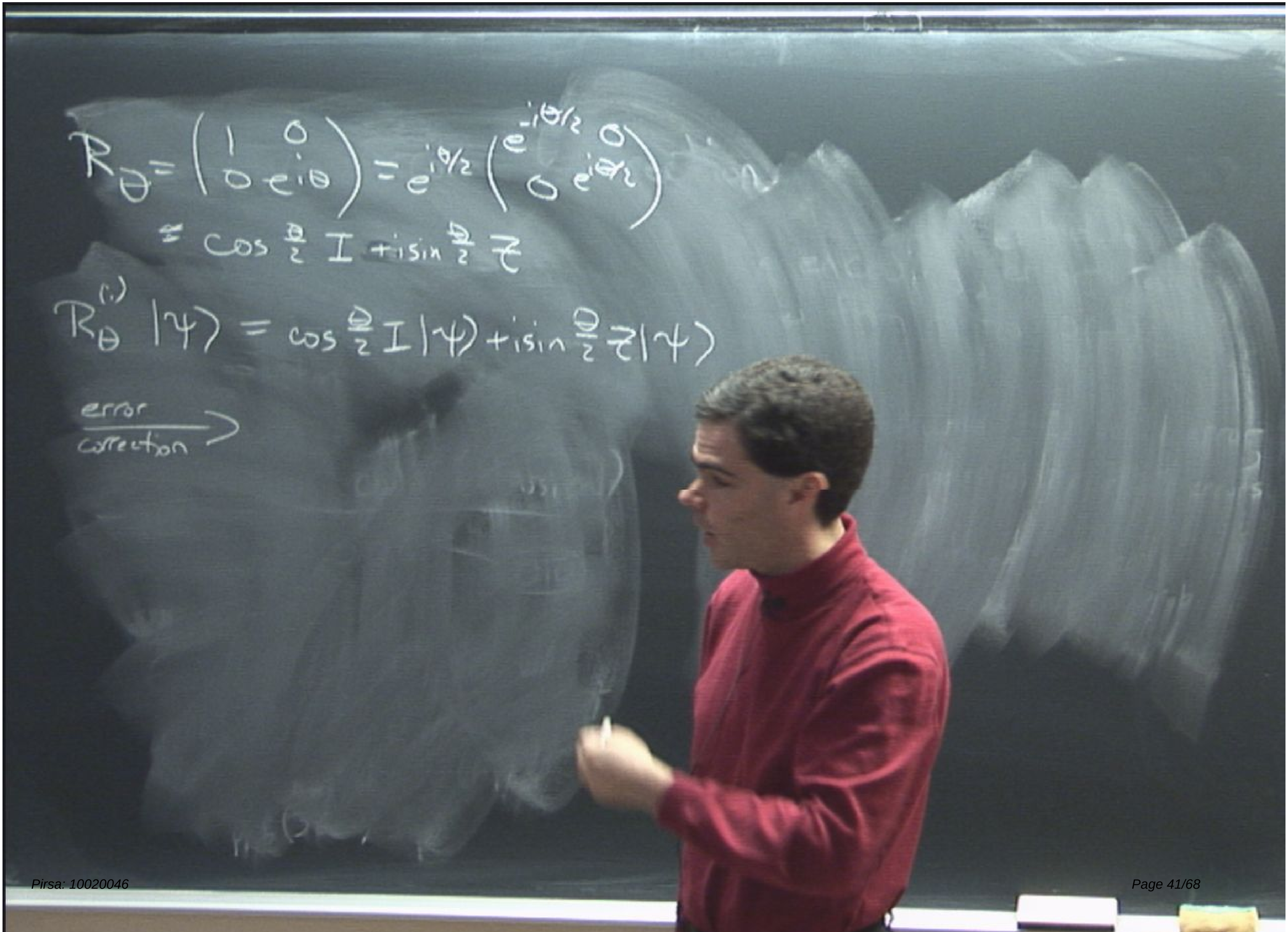
$$= \cos\frac{\theta}{2} I + i\sin\frac{\theta}{2} Z$$



$$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta/2} \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$$= \cos\frac{\theta}{2} I + i\sin\frac{\theta}{2} Z$$

$$R_\theta^{(i)} |\psi\rangle = \cos\frac{\theta}{2} I|\psi\rangle + i\sin\frac{\theta}{2} Z|\psi\rangle$$

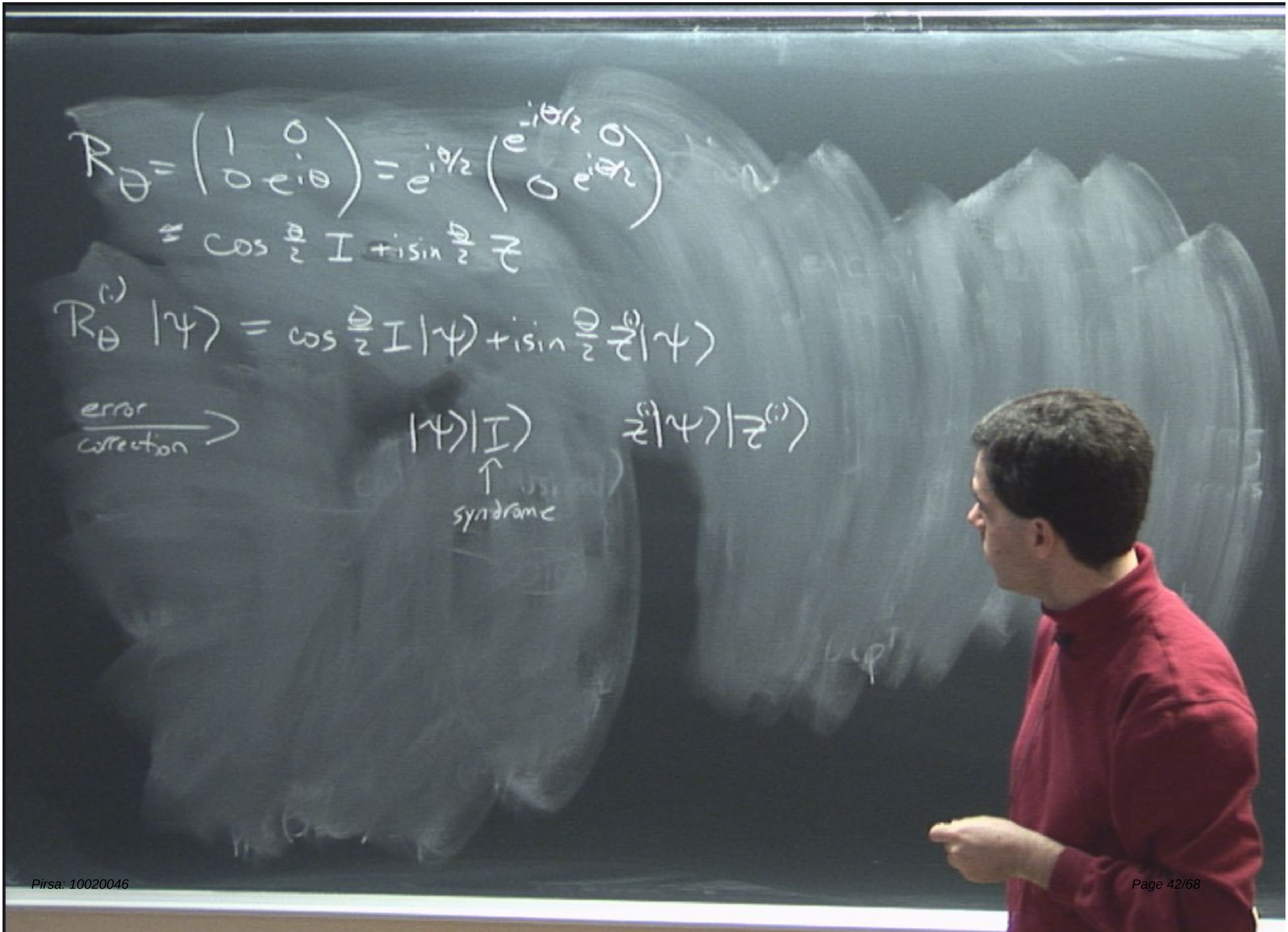


$$R_\theta = \begin{pmatrix}1 & 0\\ 0 & e^{i\theta}\end{pmatrix} = e^{i\theta/2}\begin{pmatrix}e^{-i\theta/2} & 0\\ 0 & e^{i\theta/2}\end{pmatrix}$$

$$\doteq \cos\frac{\theta}{2}\, I + i\sin\frac{\theta}{2}\, Z$$

$$R_\theta^{(i)}\,|\psi\rangle = \cos\frac{\theta}{2}\, I\,|\psi\rangle + i\sin\frac{\theta}{2}\, Z\,|\psi\rangle$$

$$\xrightarrow[\text{correction}]{\text{error}}$$

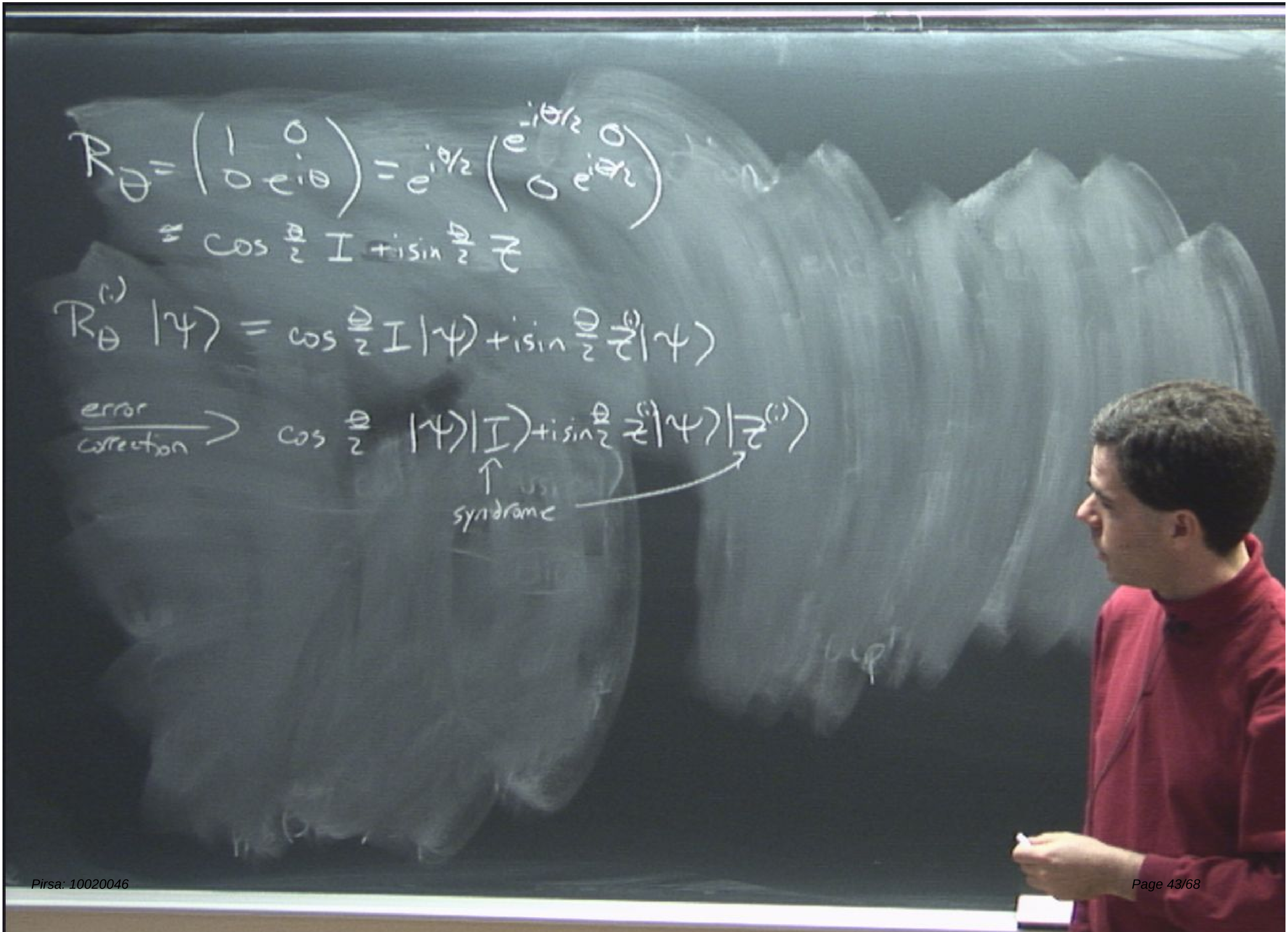


$$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta/2} \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$$= \cos\frac{\theta}{2} I + i \sin\frac{\theta}{2} Z$$

$$R_\theta^{(i)} |\psi\rangle = \cos\frac{\theta}{2} I |\psi\rangle + i \sin\frac{\theta}{2} Z^{(i)} |\psi\rangle$$

$$\xrightarrow[\text{correction}]{\text{error}} \quad |\psi\rangle \underset{\overset{\uparrow}{\text{syndrome}}}{|I\rangle} \qquad Z^{(i)} |\psi\rangle |Z^{(i)}\rangle$$

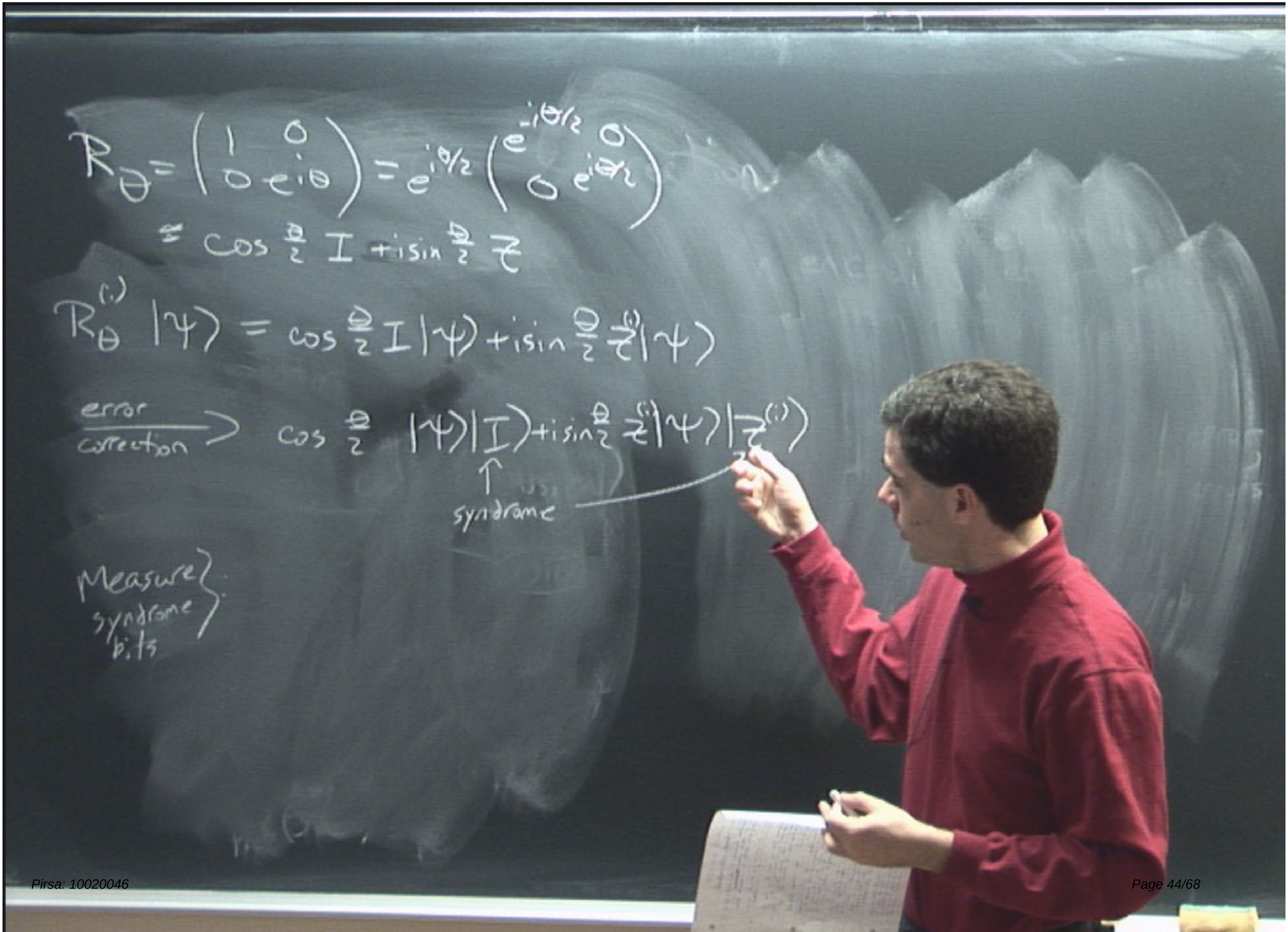


$$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta/2} \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$$= \cos\frac{\theta}{2} I + i\sin\frac{\theta}{2} Z$$

$$R_\theta^{(i)} |\psi\rangle = \cos\frac{\theta}{2} I |\psi\rangle + i\sin\frac{\theta}{2} Z^{(i)} |\psi\rangle$$

$$\xrightarrow[\text{correction}]{\text{error}} \cos\frac{\theta}{2} |\psi\rangle |\underset{\underset{\text{syndrome}}{\uparrow}}{I}\rangle + i\sin\frac{\theta}{2} Z^{(i)} |\psi\rangle |Z^{(i)}\rangle$$



$$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta/2} \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

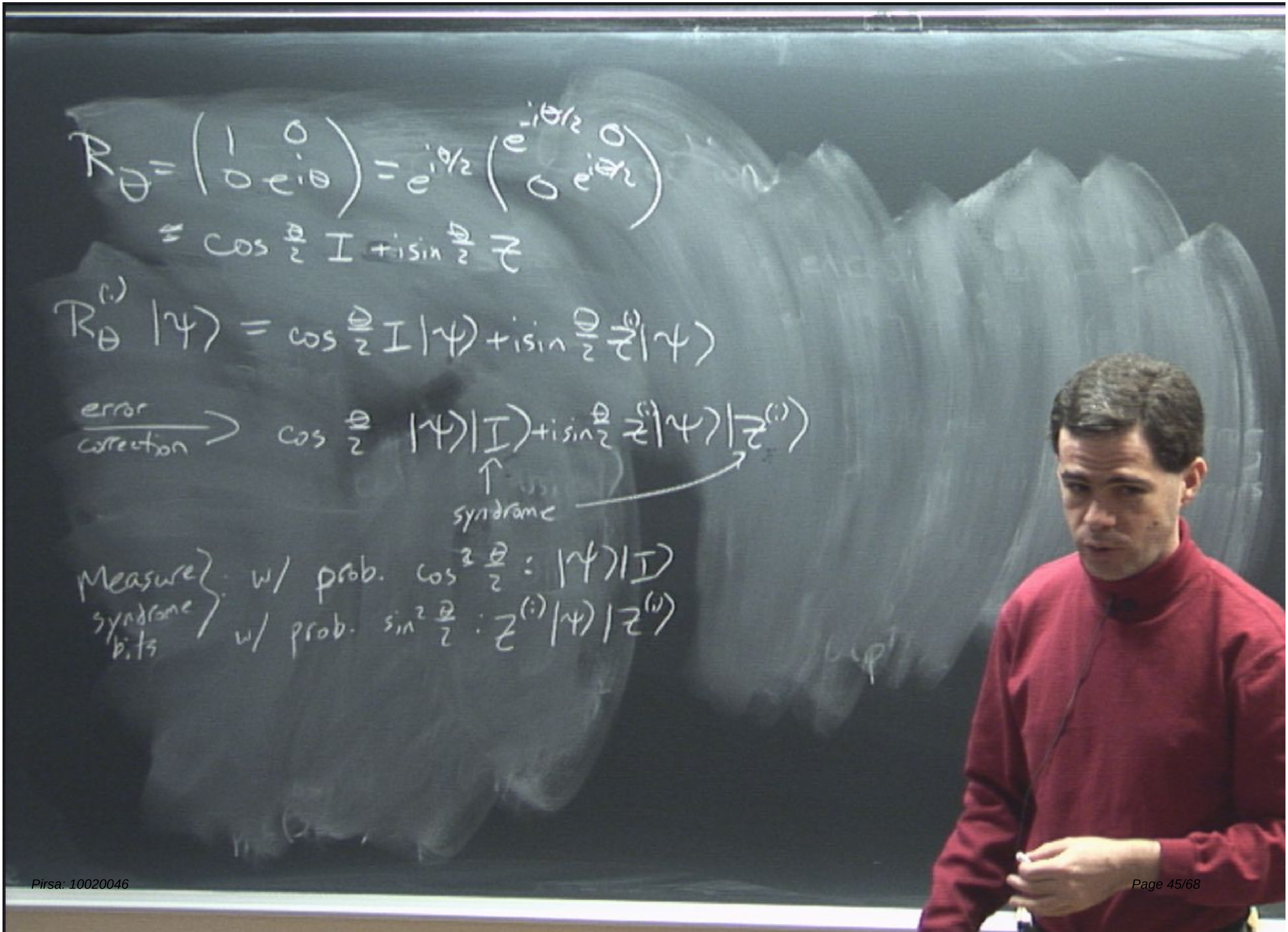
$$= \cos\frac{\theta}{2} I + i \sin\frac{\theta}{2} Z$$

$$R_\theta^{(i)} |\psi\rangle = \cos\frac{\theta}{2} I|\psi\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle$$

$$\xrightarrow[\text{correction}]{\text{error}} \cos\frac{\theta}{2} |\psi\rangle|I\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$$

syndrome

Measure syndrome bits }



$$R_\theta = \begin{pmatrix}1 & 0\\ 0 & e^{i\theta}\end{pmatrix} = e^{i\theta/2}\begin{pmatrix}e^{-i\theta/2} & 0\\ 0 & e^{i\theta/2}\end{pmatrix}$$

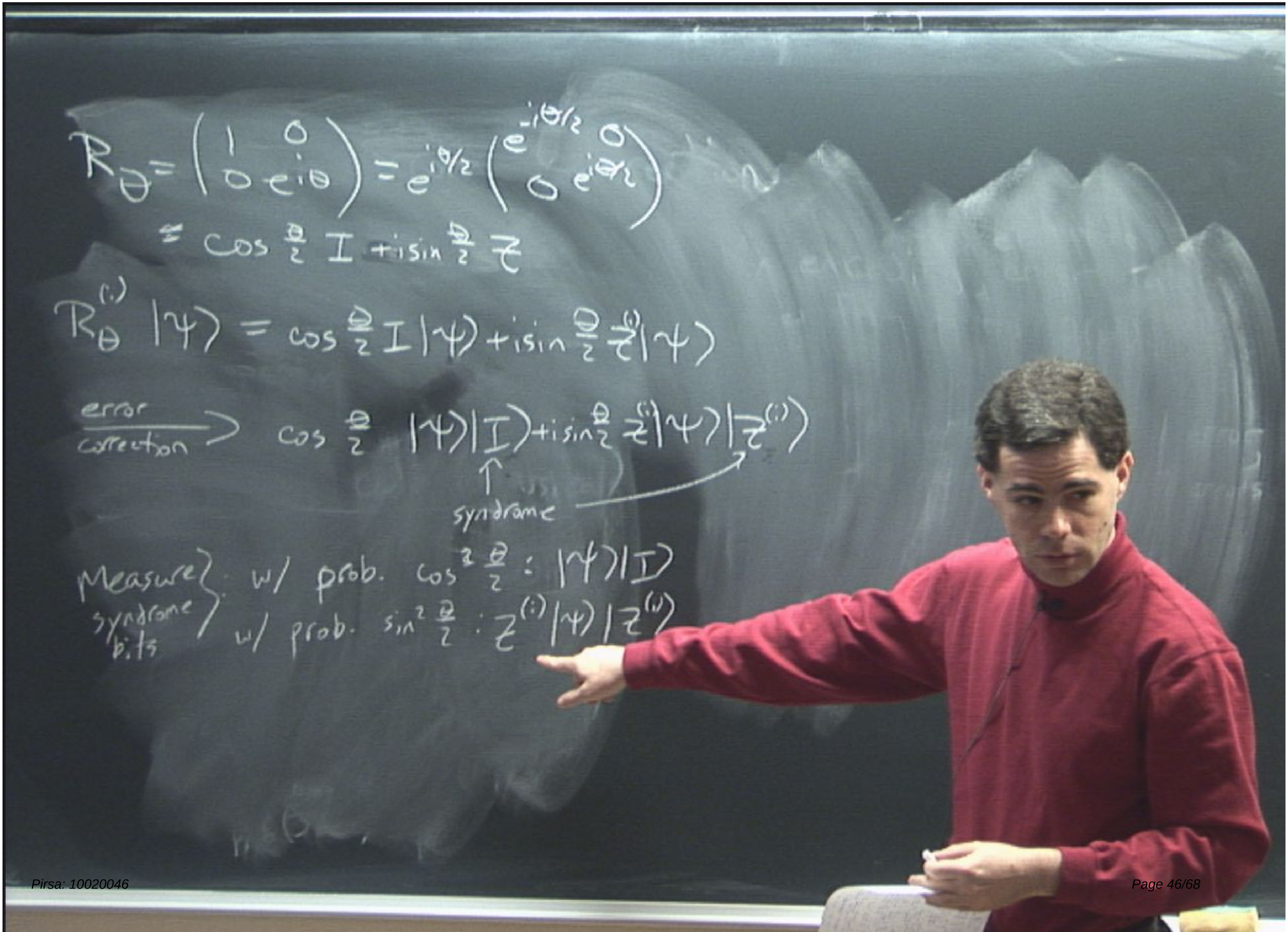
$$= \cos\frac{\theta}{2} I + i\sin\frac{\theta}{2} Z$$

$$R_\theta^{(i)}|\psi\rangle = \cos\frac{\theta}{2} I|\psi\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle$$

$$\xrightarrow[\text{correction}]{\text{error}} \cos\frac{\theta}{2}\,|\psi\rangle\underset{\uparrow}{|I\rangle} + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$$

syndrome

Measure syndrome bits $\left\}\begin{array}{l}\text{w/ prob. } \cos^2\frac{\theta}{2} : |\psi\rangle|I\rangle\\ \text{w/ prob. } \sin^2\frac{\theta}{2} : Z^{(i)}|\psi\rangle|Z^{(i)}\rangle\end{array}\right.$



$$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta/2}\begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$$= \cos\frac{\theta}{2} I + i\sin\frac{\theta}{2} Z$$

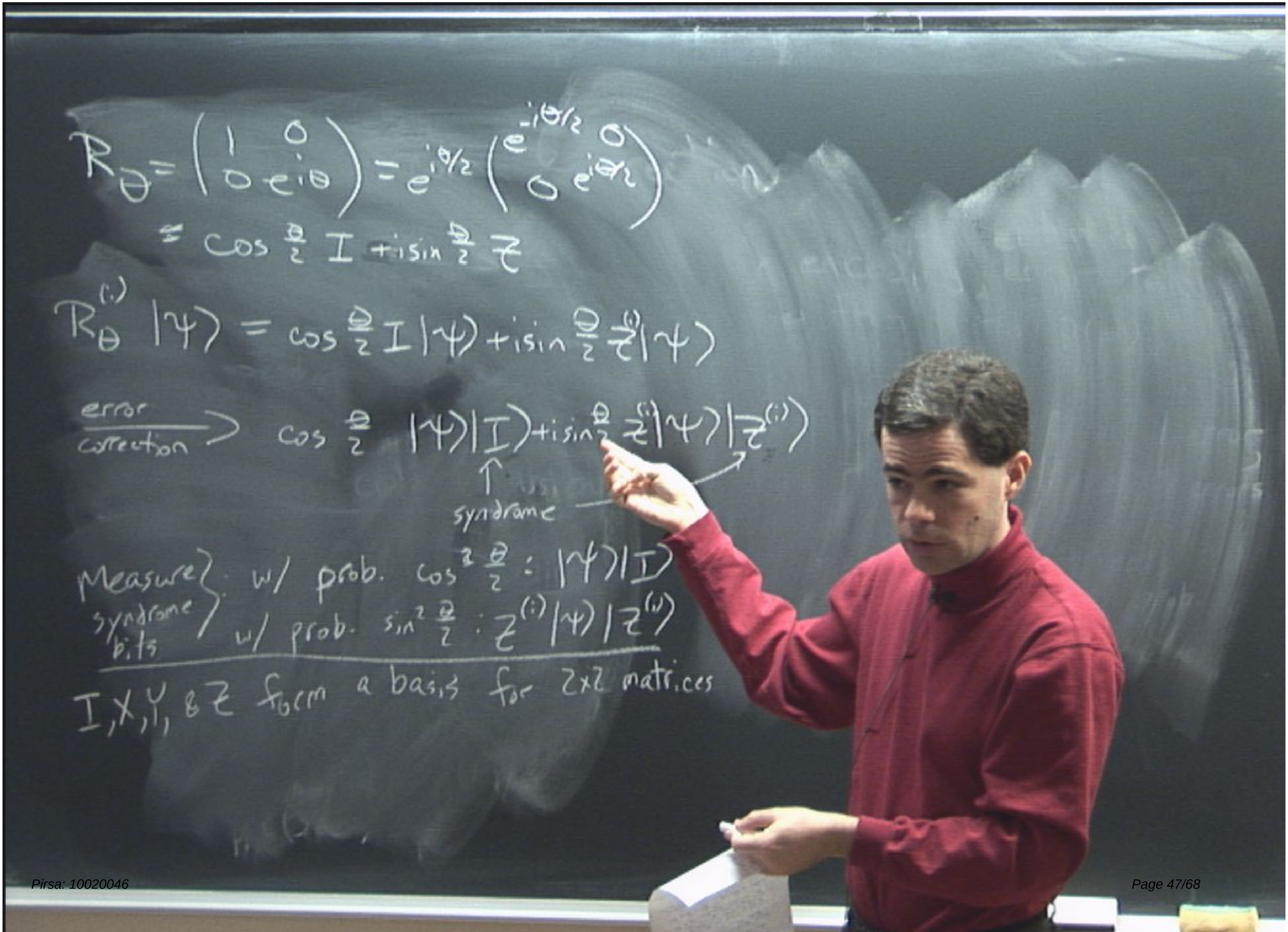
$$R_\theta^{(i)}|\psi\rangle = \cos\frac{\theta}{2} I|\psi\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle$$

$$\xrightarrow[\text{correction}]{\text{error}} \cos\frac{\theta}{2}\, |\psi\rangle|I\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$$

syndrome

Measure syndrome bits } w/ prob. $\cos^2\frac{\theta}{2}$: $|\psi\rangle|I\rangle$

w/ prob. $\sin^2\frac{\theta}{2}$: $Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$



$$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta/2} \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

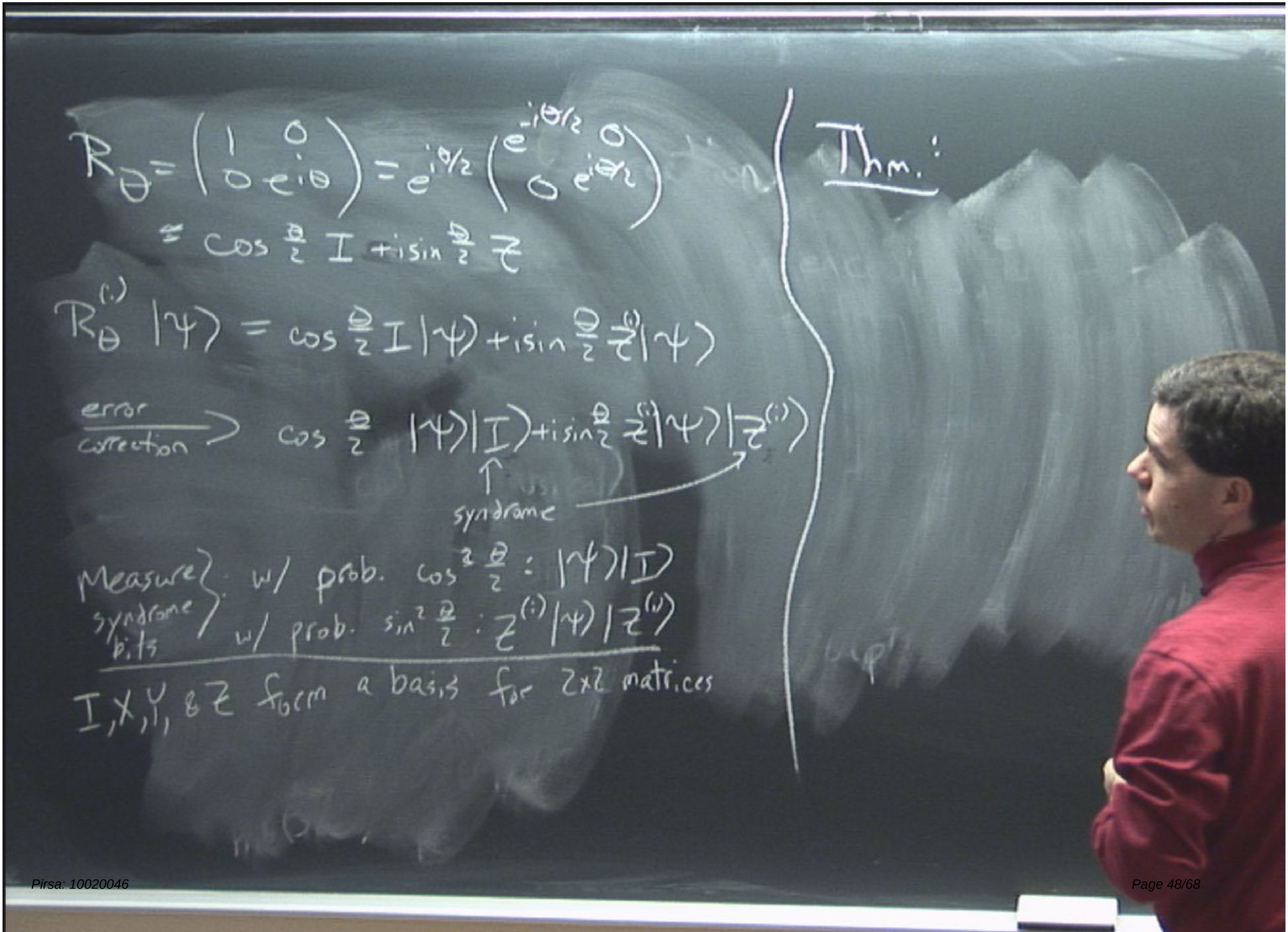
$$= \cos\frac{\theta}{2} I + i\sin\frac{\theta}{2} Z$$

$$R_\theta^{(i)} |\psi\rangle = \cos\frac{\theta}{2} I |\psi\rangle + i\sin\frac{\theta}{2} Z^{(i)} |\psi\rangle$$

$$\xrightarrow[\text{correction}]{\text{error}} \cos\frac{\theta}{2} \; |\psi\rangle \underset{\overset{\uparrow}{\text{syndrome}}}{|I\rangle} + i\sin\frac{\theta}{2} Z^{(i)} |\psi\rangle |Z^{(i)}\rangle$$

Measure syndrome bits $\left\}\begin{array}{l} \text{w/ prob. } \cos^2\frac{\theta}{2} : |\psi\rangle|I\rangle \\ \text{w/ prob. } \sin^2\frac{\theta}{2} : Z^{(i)}|\psi\rangle|Z^{(i)}\rangle \end{array}\right.$

$I, X, Y,$ & Z form a basis for 2×2 matrices



$$R_\theta = \begin{pmatrix}1 & 0\\ 0 & e^{i\theta}\end{pmatrix} = e^{i\theta/2}\begin{pmatrix}e^{-i\theta/2} & 0\\ 0 & e^{i\theta/2}\end{pmatrix}$$

$$= \cos\frac{\theta}{2} I + i\sin\frac{\theta}{2} Z$$

$$R_\theta^{(i)}|\psi\rangle = \cos\frac{\theta}{2} I|\psi\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle$$

$$\xrightarrow[\text{correction}]{\text{error}} \cos\frac{\theta}{2}\,|\psi\rangle|I\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$$

syndrome

Measure syndrome bits } w/ prob. $\cos^2\frac{\theta}{2}$: $|\psi\rangle|I\rangle$

w/ prob. $\sin^2\frac{\theta}{2}$: $Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$

I, X, Y, & Z form a basis for 2x2 matrices

Thm.:

$= \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta/2} \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$

$\cong \cos\frac{\theta}{2} I + i\sin\frac{\theta}{2} Z$

$|\psi\rangle = \cos\frac{\theta}{2} I|\psi\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle$

$\xrightarrow[\text{on}]{} \cos\frac{\theta}{2} \; |\psi\rangle|\underset{\uparrow}{I}\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$

syndrome

ure} w/ prob. $\cos^2\frac{\theta}{2}$: $|\psi\rangle|I\rangle$

ome} w/ prob. $\sin^2\frac{\theta}{2}$: $Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$

s

Y, & Z form a basis for 2x2 matrices

Thm.: Any QECC that corrects I, X, Y, Z single-qubit errors also corrects *any* single-qubit error.

Ba

1)

2)

3) N

an

4) Con

$$= \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta/2} \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$$= \cos\frac{\theta}{2} I + i\sin\frac{\theta}{2} Z$$

$$|\psi\rangle = \cos\frac{\theta}{2} I|\psi\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle$$

$$\xrightarrow[\text{on}]{} \cos\frac{\theta}{2}\, |\psi\rangle|\underset{\uparrow}{I}\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$$

syndrome

ure} w/ prob. $\cos^2\frac{\theta}{2}$: $|\psi\rangle|I\rangle$

ome} w/ prob. $\sin^2\frac{\theta}{2}$: $Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$

Y, & Z form a basis for 2x2 matrices

Thm.: Any QECC that corrects I, X, Y, Z single-qubit errors also corrects _any_ single-qubit error.

CPTP maps:

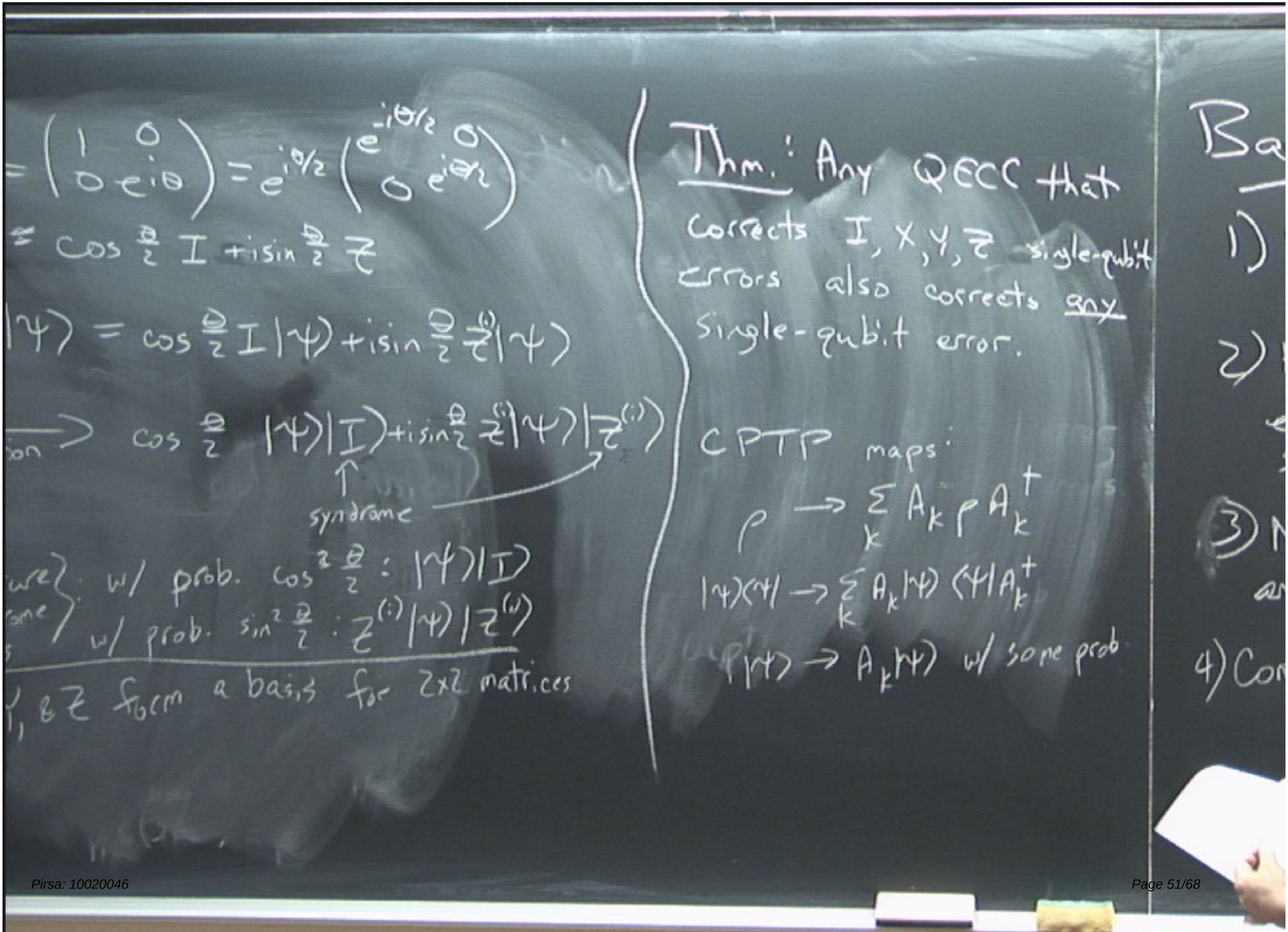
Ba

1)

2)

3) N

4) C



$= \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta/2} \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$

$= \cos\frac{\theta}{2} I + i\sin\frac{\theta}{2} Z$

$|\psi\rangle = \cos\frac{\theta}{2} I|\psi\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle$

$\xrightarrow[\text{on}]{} \cos\frac{\theta}{2} |\psi\rangle|\underset{\uparrow}{I}\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$

syndrome

ure} w/ prob. $\cos^2\frac{\theta}{2}$: $|\psi\rangle|I\rangle$

ome} w/ prob. $\sin^2\frac{\theta}{2}$: $Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$

s

Y, & Z form a basis for 2x2 matrices

Thm.: Any QECC that corrects I, X, Y, Z single-qubit errors also corrects any single-qubit error.

CPTP maps:

$\rho \longrightarrow \sum_k A_k \rho A_k^\dagger$

$|\psi\rangle\langle\psi| \longrightarrow \sum_k A_k|\psi\rangle\langle\psi|A_k^\dagger$

$|\psi\rangle \rightarrow A_k|\psi\rangle$ w/ some prob.

Ba

1)

2)

③ N

ar

4) Con

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta/2} \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$\cos\frac{\theta}{2} I + i\sin\frac{\theta}{2} Z$

$\psi\rangle = \cos\frac{\theta}{2} I|\psi\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle$

$\rightarrow \cos\frac{\theta}{2} \; |\psi\rangle|I\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$

$\uparrow$

syndrome

e} w/ prob. $\cos^2\frac{\theta}{2}$: $|\psi\rangle|I\rangle$

e} w/ prob. $\sin^2\frac{\theta}{2}$: $Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$

& Z form a basis for 2x2 matrices

Thm.: Any QECC that corrects I, X, Y, Z single-qubit errors also corrects any single-qubit error.

CPTP maps:

$$\rho \rightarrow \sum_k A_k \rho A_k^{\dagger}$$

$$|\psi\rangle\langle\psi| \rightarrow \sum_k A_k|\psi\rangle\langle\psi|A_k^{\dagger}$$

$|\psi\rangle \rightarrow \underbrace{A_k}|\psi\rangle$ w/ some prob

expand as sum of I, X, Y, Z

Barr

1) N

2) H

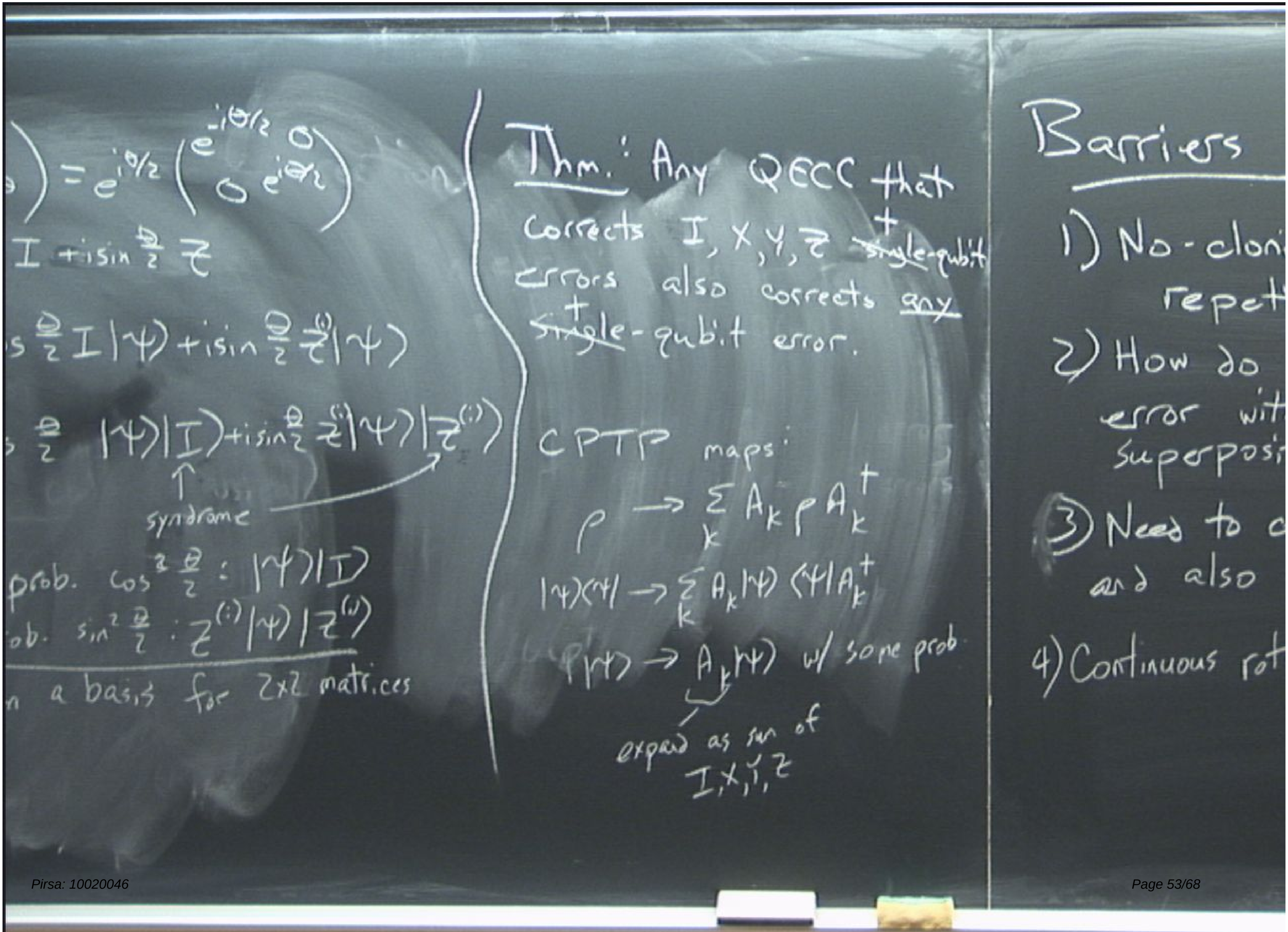
er

su

③ Ne

and

4) Conti



$) = e^{i\theta/2}\begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$

$I + i\sin\frac{\theta}{2} Z$

$s\frac{\theta}{2} I|\psi\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle$

$s\frac{\theta}{2} |\psi\rangle|I\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$

syndrome

prob. $\cos^2\frac{\theta}{2}$: $|\psi\rangle|I\rangle$

ob. $\sin^2\frac{\theta}{2}$: $Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$

n a basis for 2x2 matrices

Thm.: Any QECC that corrects I, X, Y, Z ~~single~~ t-qubit errors also corrects any ~~single~~ t-qubit error.

CPTP maps:

$$\rho \to \sum_k A_k \rho A_k^\dagger$$

$$|\psi\rangle\langle\psi| \to \sum_k A_k|\psi\rangle\langle\psi|A_k^\dagger$$

$$|\psi\rangle \to \underbrace{A_k}_{\text{expand as sum of } I,X,Y,Z}|\psi\rangle \text{ w/ some prob.}$$

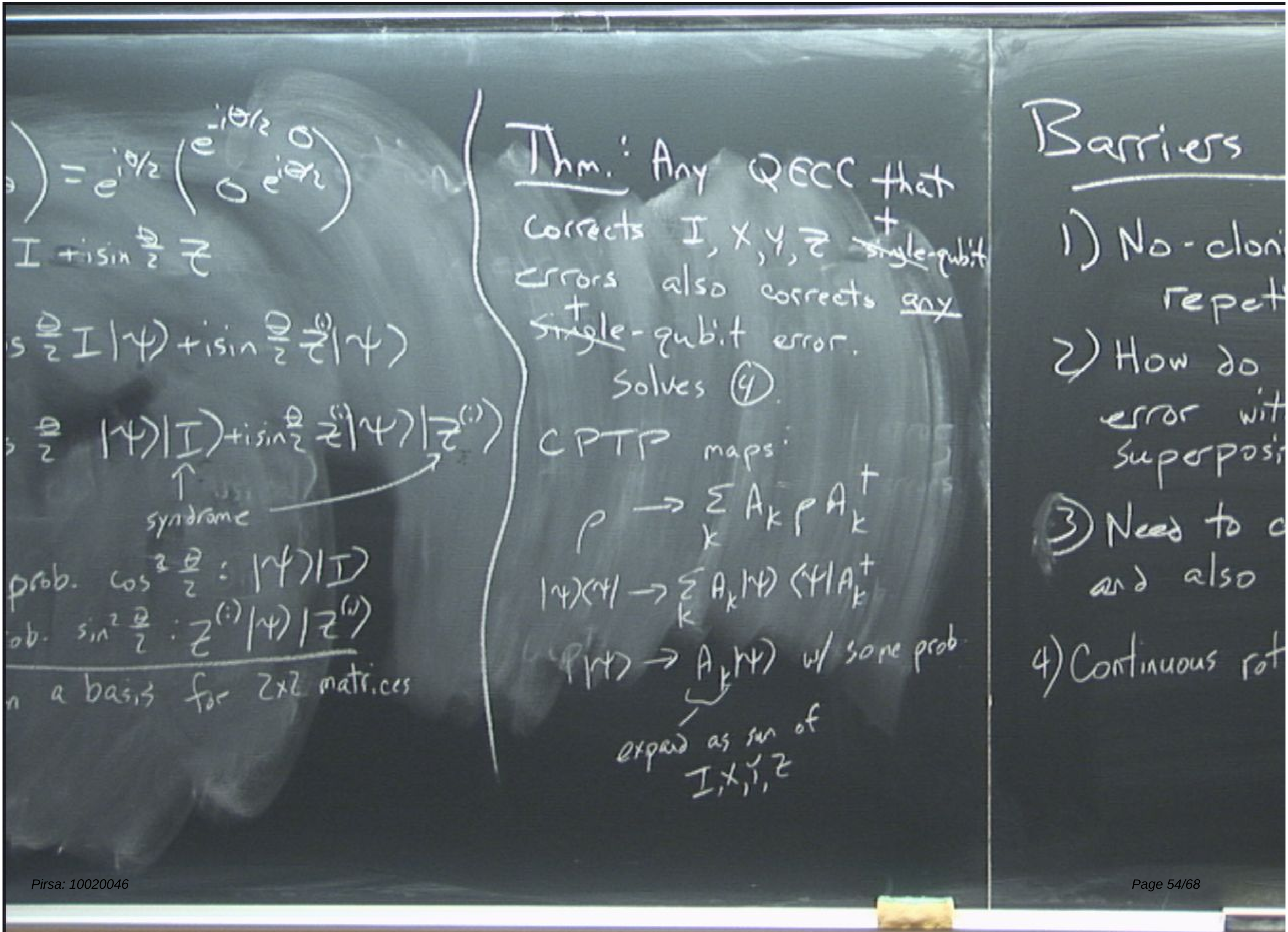
Barriers

1) No-cloni
repet

2) How do
error wit
superposi

3) Need to c
and also

4) Continuous rot



$) = e^{i\theta/2}\begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$

$I + i \sin\frac{\theta}{2} Z$

$s\frac{\theta}{2} I|\psi\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle$

$s\frac{\theta}{2} \; |\psi\rangle|\underset{\underset{\text{syndrome}}{\uparrow}}{I}\rangle + i\sin\frac{\theta}{2} Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$

prob. $\cos^2\frac{\theta}{2}$: $|\psi\rangle|I\rangle$

ob. $\sin^2\frac{\theta}{2}$: $Z^{(i)}|\psi\rangle|Z^{(i)}\rangle$

n a basis for 2x2 matrices

Thm.: Any QECC that corrects I, X, Y, Z t ~~single-qubit~~ errors also corrects any t ~~single~~-qubit error.

Solves ④.

CPTP maps:

$$\rho \to \sum_k A_k \rho A_k^\dagger$$

$$|\psi\rangle\langle\psi| \to \sum_k A_k|\psi\rangle\langle\psi|A_k^\dagger$$

$$|\psi\rangle \to \underbrace{A_k}_{\text{expand as sum of } I,X,Y,Z}|\psi\rangle \text{ w/ some prob.}$$

Barriers

1) No-cloni

repet

2) How do

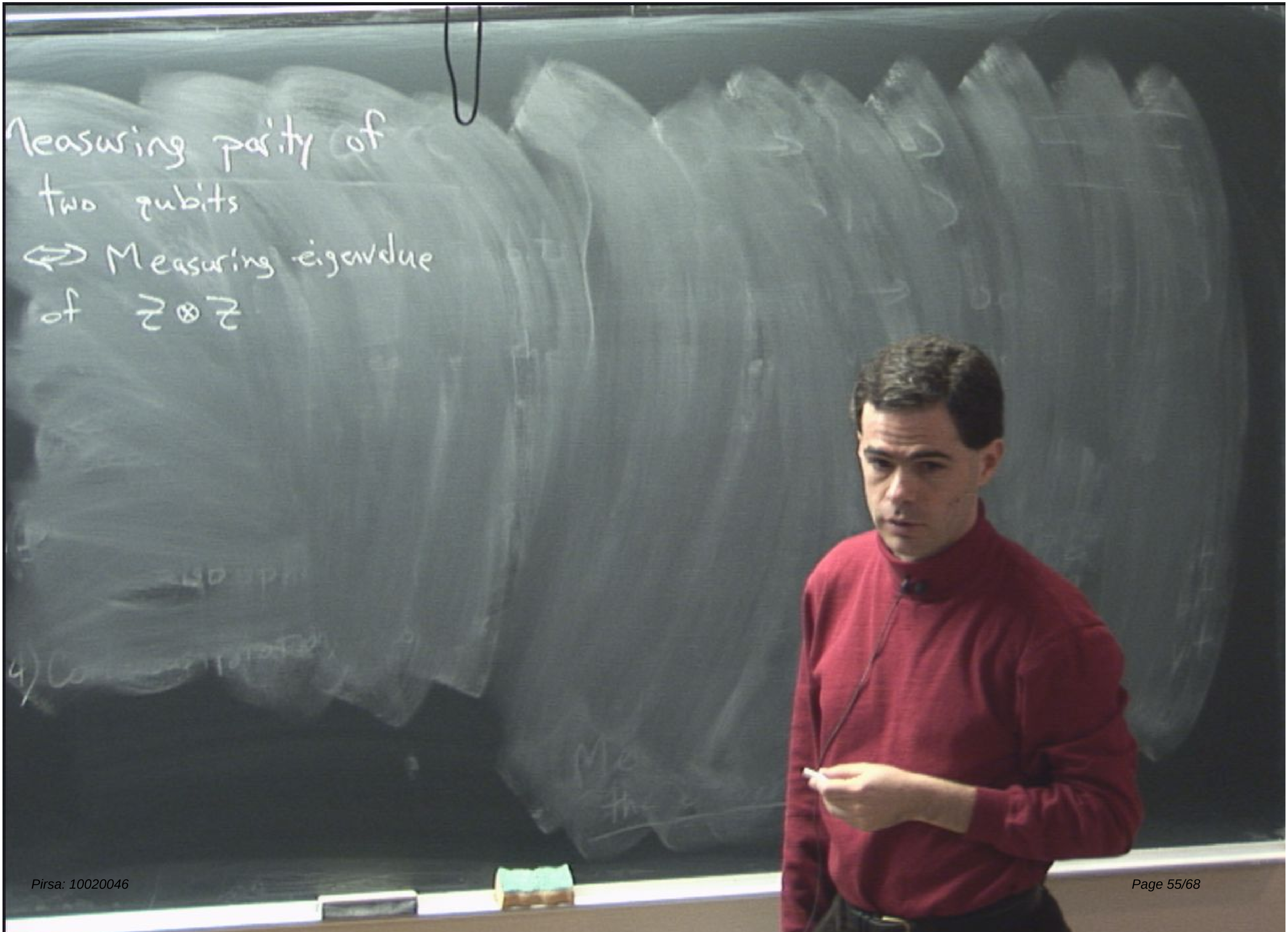
error wit

superposi

③ Need to c

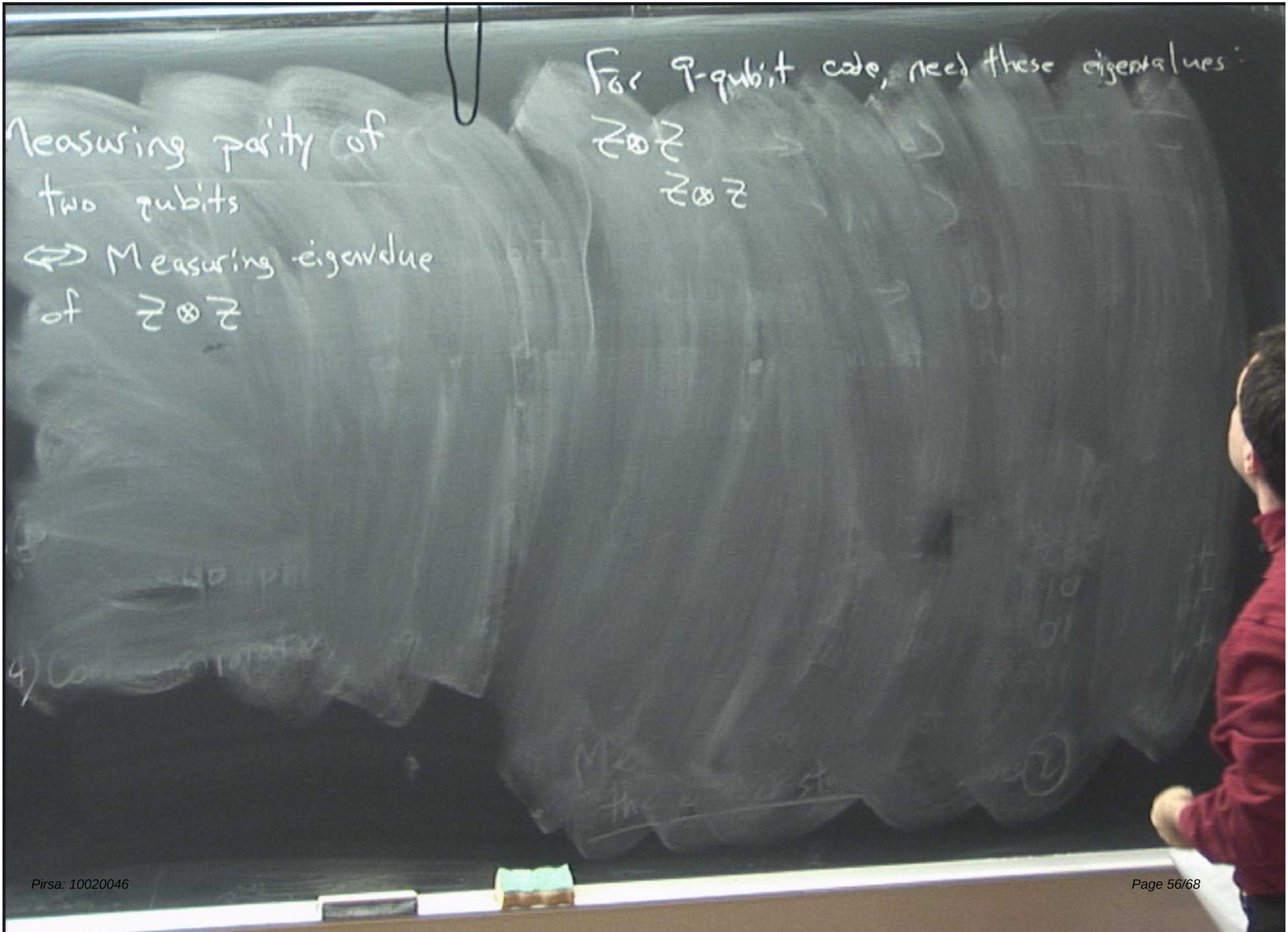
and also

4) Continuous rot



Measuring parity of two qubits

$\Leftrightarrow$ Measuring eigenvalue of $Z \otimes Z$



Measuring parity of two qubits

$\Leftrightarrow$ Measuring eigenvalue of $Z \otimes Z$

For 9-qubit code, need these eigenvalues:

$Z \otimes Z$

$\quad Z \otimes Z$

Measuring parity of two qubits

$\Leftrightarrow$ Measuring eigenvalue of $Z\otimes Z$

For 9-qubit code, need these eigenvalues:

$Z\otimes Z$

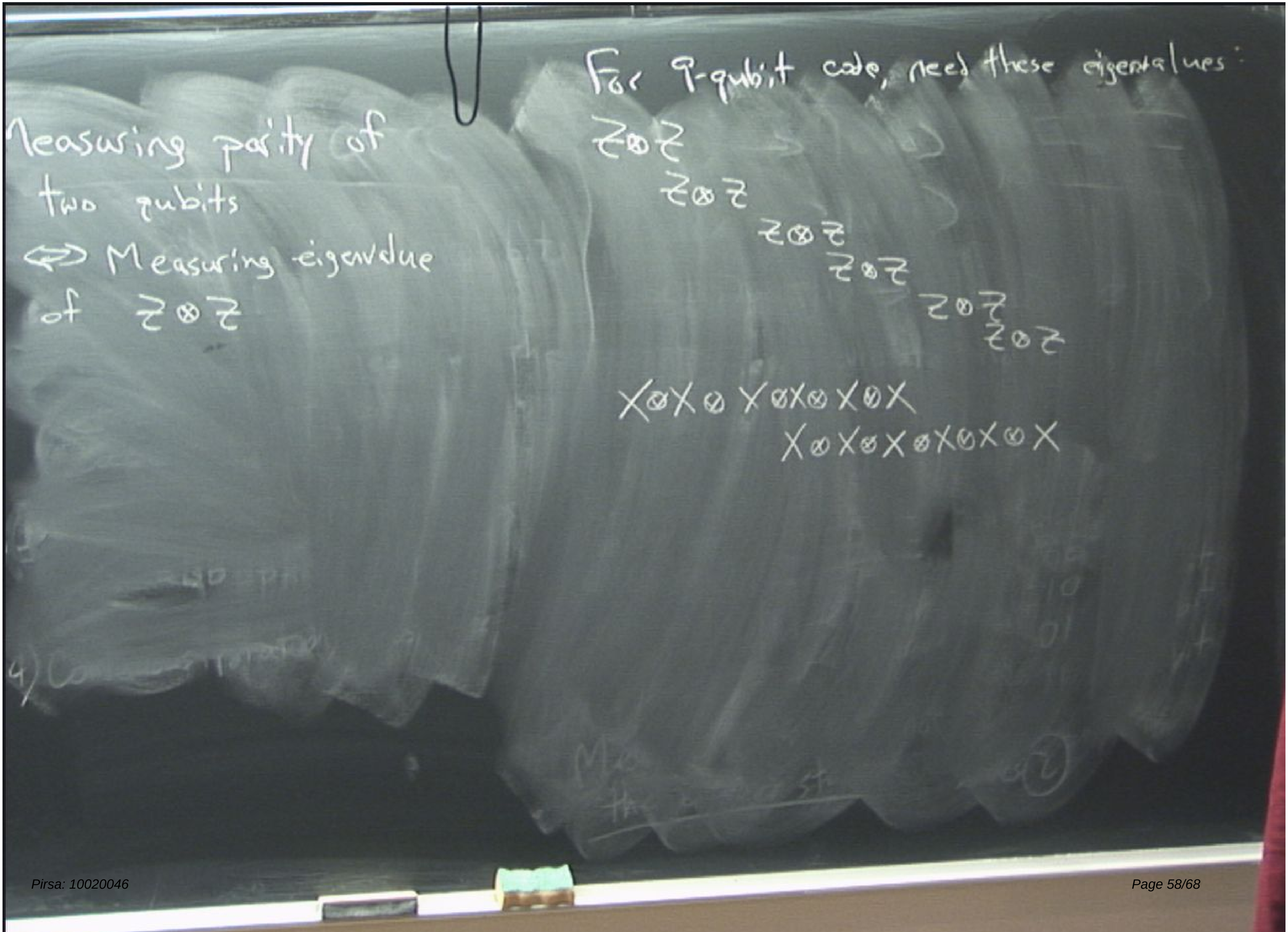
$Z\otimes Z$

$Z\otimes Z$

$Z\otimes Z$

$Z\otimes Z$

$Z\otimes Z$



Measuring parity of two qubits

$\Leftrightarrow$ Measuring eigenvalue of $Z \otimes Z$

For 9-qubit code, need these eigenvalues:

$$\begin{array}{lllllllll}
Z & Z & & & & & & & \\
 & Z & Z & & & & & & \\
 & & & Z & Z & & & & \\
 & & & & Z & Z & & & \\
 & & & & & & Z & Z & \\
 & & & & & & & Z & Z
\end{array}$$

$Z \otimes Z$

$\quad Z \otimes Z$

$\qquad\qquad Z \otimes Z$

$\qquad\qquad\quad Z \otimes Z$

$\qquad\qquad\qquad\qquad Z \otimes Z$

$\qquad\qquad\qquad\qquad\quad Z \otimes Z$

$X \otimes X \otimes X \otimes X \otimes X \otimes X$

$\qquad\qquad X \otimes X \otimes X \otimes X \otimes X \otimes X$

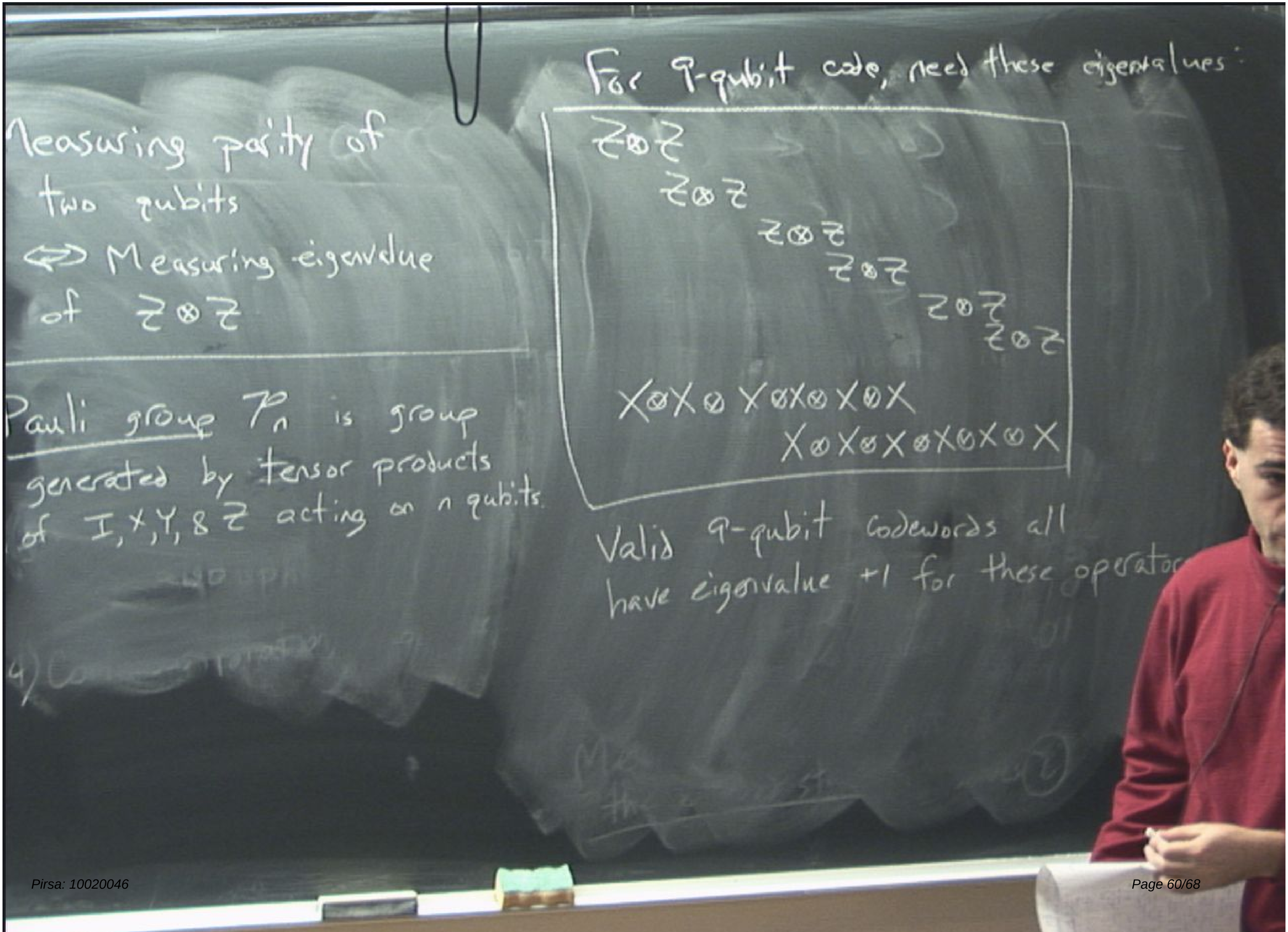
Measuring parity of two qubits

$\Leftrightarrow$ Measuring eigenvalue of $Z\otimes Z$

For 9-qubit code, need these eigenvalues:

$$\boxed{\begin{array}{l}
Z\otimes Z \\
\quad\;\; Z\otimes Z \\
\qquad\qquad\quad Z\otimes Z \\
\qquad\qquad\qquad\quad Z\otimes Z \\
\qquad\qquad\qquad\qquad\qquad Z\otimes Z \\
\qquad\qquad\qquad\qquad\qquad\qquad Z\otimes Z \\
X\otimes X\otimes X\otimes X\otimes X\otimes X \\
\qquad\qquad\quad X\otimes X\otimes X\otimes X\otimes X\otimes X
\end{array}}$$

Valid 9-qubit codewords all have eigenvalue +1 for these operators



Measuring parity of two qubits

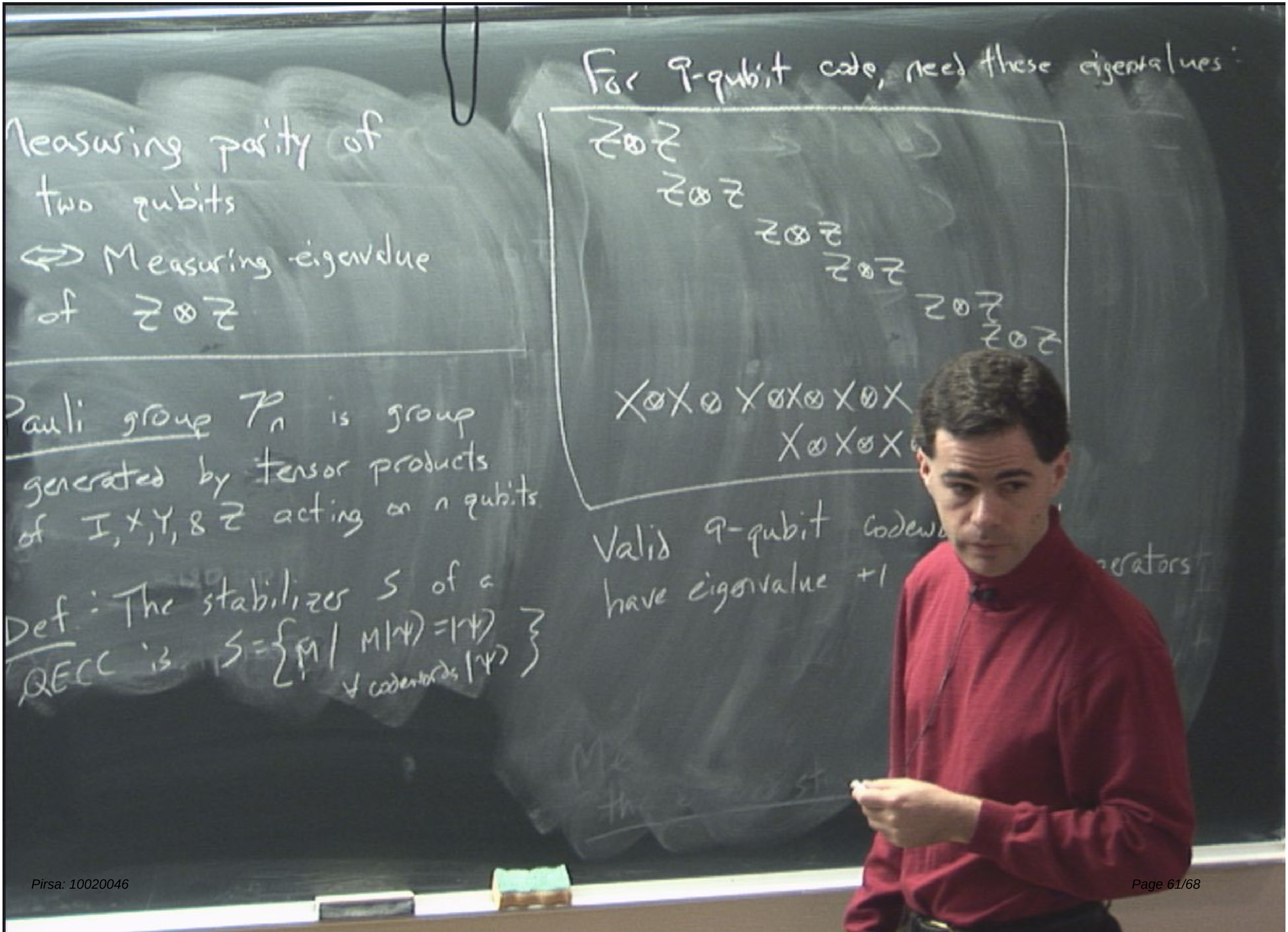
⟺ Measuring eigenvalue of $Z \otimes Z$

Pauli group $\mathcal{P}_n$ is group generated by tensor products of $I, X, Y, \& Z$ acting on n qubits.

For 9-qubit code, need these eigenvalues:

$$\boxed{\begin{array}{l}
Z\otimes Z \\
\quad Z\otimes Z \\
\qquad Z\otimes Z \\
\qquad\quad Z\otimes Z \\
\qquad\qquad Z\otimes Z \\
\qquad\qquad\quad Z\otimes Z \\
X\otimes X\otimes X\otimes X\otimes X\otimes X \\
\qquad\quad X\otimes X\otimes X\otimes X\otimes X\otimes X
\end{array}}$$

Valid 9-qubit codewords all have eigenvalue +1 for these operato



Measuring parity of two qubits

$\Leftrightarrow$ Measuring eigenvalue of $Z \otimes Z$

Pauli group $\mathcal{P}_n$ is group generated by tensor products of $I, X, Y,$ & Z acting on n qubits

Def: The stabilizer S of a QECC is $S = \{ M \mid M|\psi\rangle = |\psi\rangle \ \forall \text{ codewords } |\psi\rangle \}$

For 9-qubit code, need these eigenvalues:

$Z \otimes Z$

$Z \otimes Z$

$Z \otimes Z$

$Z \otimes Z$

$Z \otimes Z$

$Z \otimes Z$

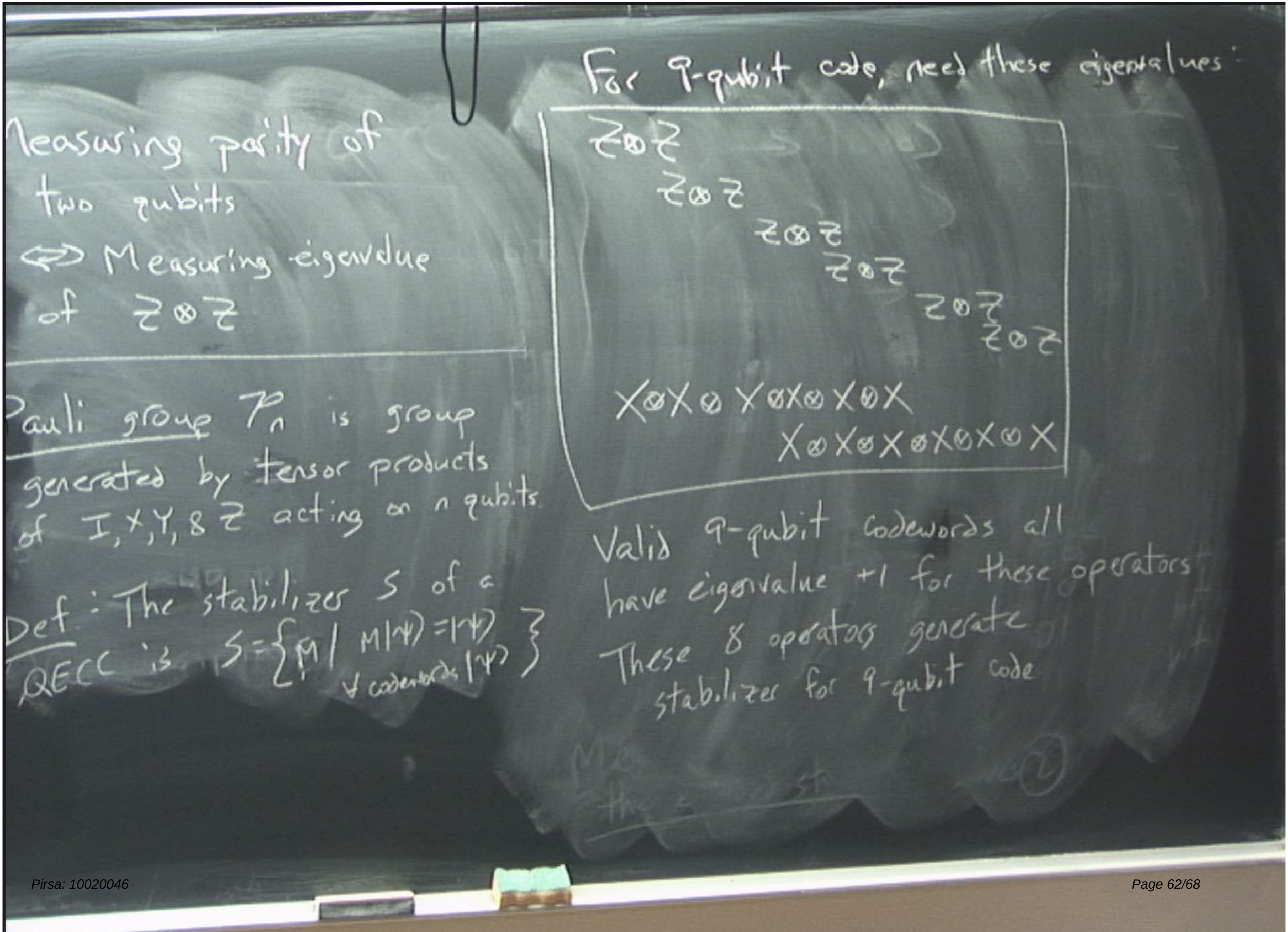
$X \otimes X \otimes X \otimes X \otimes X \otimes X$

$X \otimes X \otimes X$

Valid 9-qubit codew

erators

have eigenvalue +1



Measuring parity of two qubits $\Leftrightarrow$ Measuring eigenvalue of $Z \otimes Z$

Pauli group $\mathcal{P}_n$ is group generated by tensor products of $I, X, Y, \& Z$ acting on n qubits

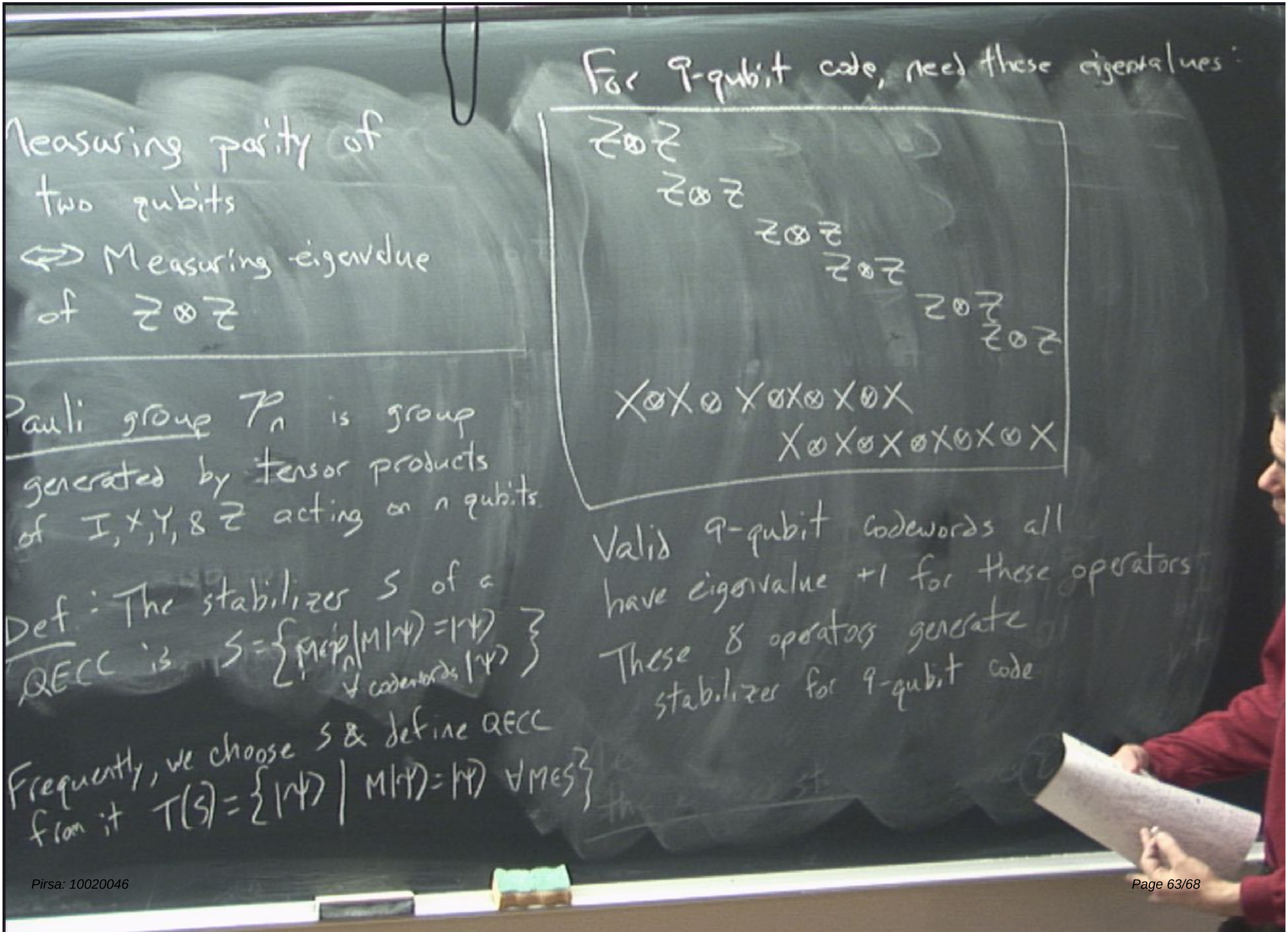
Def: The stabilizer S of a QECC is $S = \{M \mid M|\psi\rangle = |\psi\rangle \ \forall \text{ codewords } |\psi\rangle\}$

For 9-qubit code, need these eigenvalues:

$$\boxed{\begin{array}{l}
Z \otimes Z \\
\quad Z \otimes Z \\
\qquad Z \otimes Z \\
\qquad\quad Z \otimes Z \\
\qquad\qquad Z \otimes Z \\
\qquad\qquad\quad Z \otimes Z \\
X \otimes X \otimes X \otimes X \otimes X \otimes X \\
\qquad\qquad X \otimes X \otimes X \otimes X \otimes X \otimes X
\end{array}}$$

Valid 9-qubit codewords all have eigenvalue +1 for these operators

These 8 operators generate stabilizer for 9-qubit code



Measuring parity of two qubits

$\Leftrightarrow$ Measuring eigenvalue of $Z \otimes Z$

Pauli group $\mathcal{P}_n$ is group generated by tensor products of $I, X, Y, \& Z$ acting on n qubits

Def: The stabilizer S of a QECC is $S = \{ M \in \mathcal{P}_n \mid M|\psi\rangle = |\psi\rangle \ \forall \text{ codewords } |\psi\rangle \}$

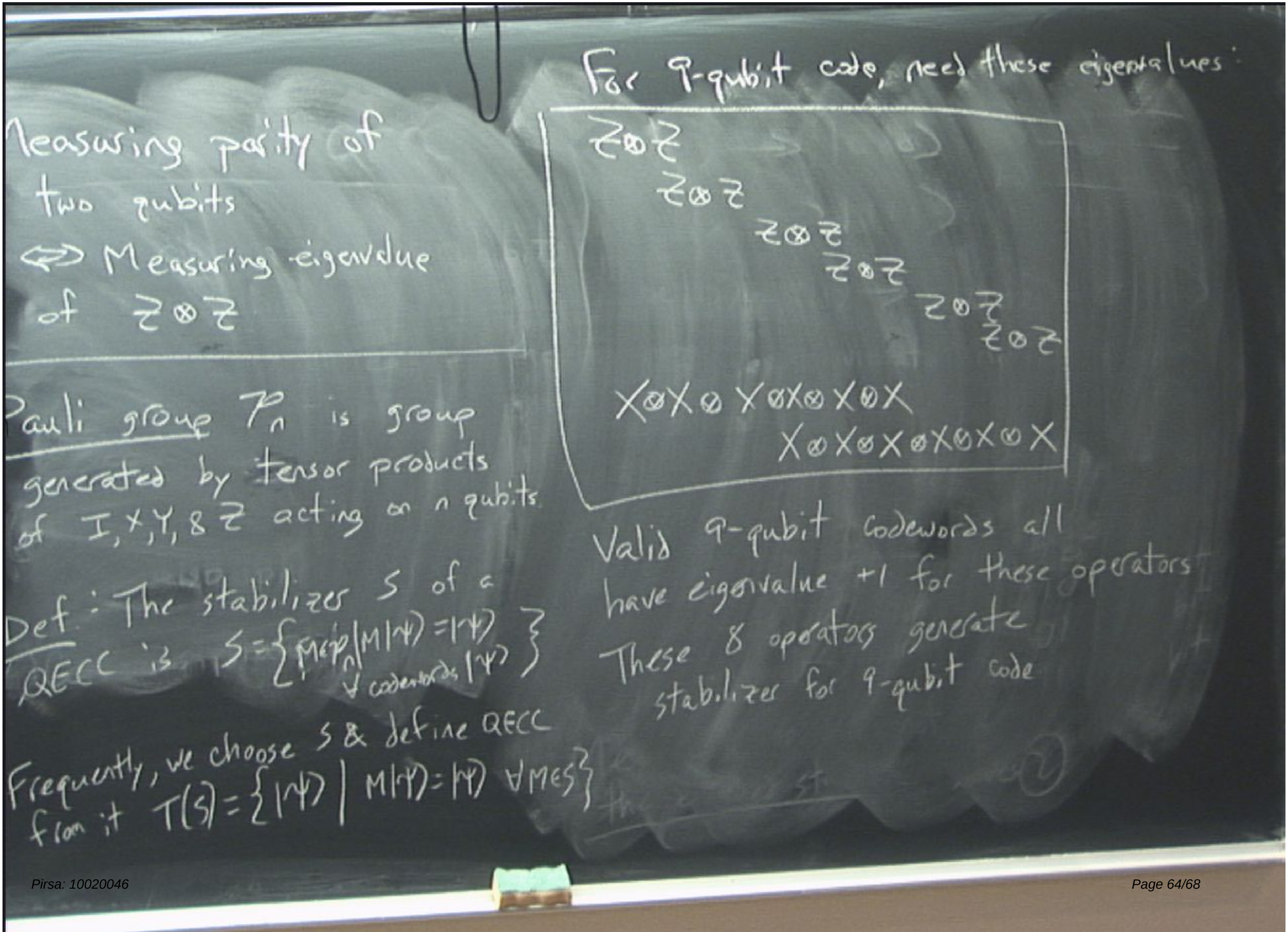
Frequently, we choose S & define QECC from it $T(S) = \{ |\psi\rangle \mid M|\psi\rangle = |\psi\rangle \ \forall M \in S \}$

For 9-qubit code, need these eigenvalues:

$$\boxed{\begin{array}{ccccccccc}
Z \otimes & Z & & & & & & & \\
& Z \otimes & Z & & & & & & \\
& & & Z \otimes & Z & & & & \\
& & & & Z \otimes & Z & & & \\
& & & & & & Z \otimes & Z & \\
& & & & & & & Z \otimes & Z \\
X \otimes & X \otimes & X \otimes & X \otimes & X \otimes & X & & & \\
& & & X \otimes & X \otimes & X \otimes & X \otimes & X \otimes & X
\end{array}}$$

Valid 9-qubit codewords all have eigenvalue +1 for these operators

These 8 operators generate stabilizer for 9-qubit code



Measuring parity of two qubits

⟺ Measuring eigenvalue of $Z \otimes Z$

Pauli group $\mathcal{P}_n$ is group generated by tensor products of $I, X, Y, \& Z$ acting on n qubits

Def: The stabilizer S of a QECC is $S = \{ M \in \mathcal{P}_n \mid M|\psi\rangle = |\psi\rangle \ \forall \text{ codewords } |\psi\rangle \}$

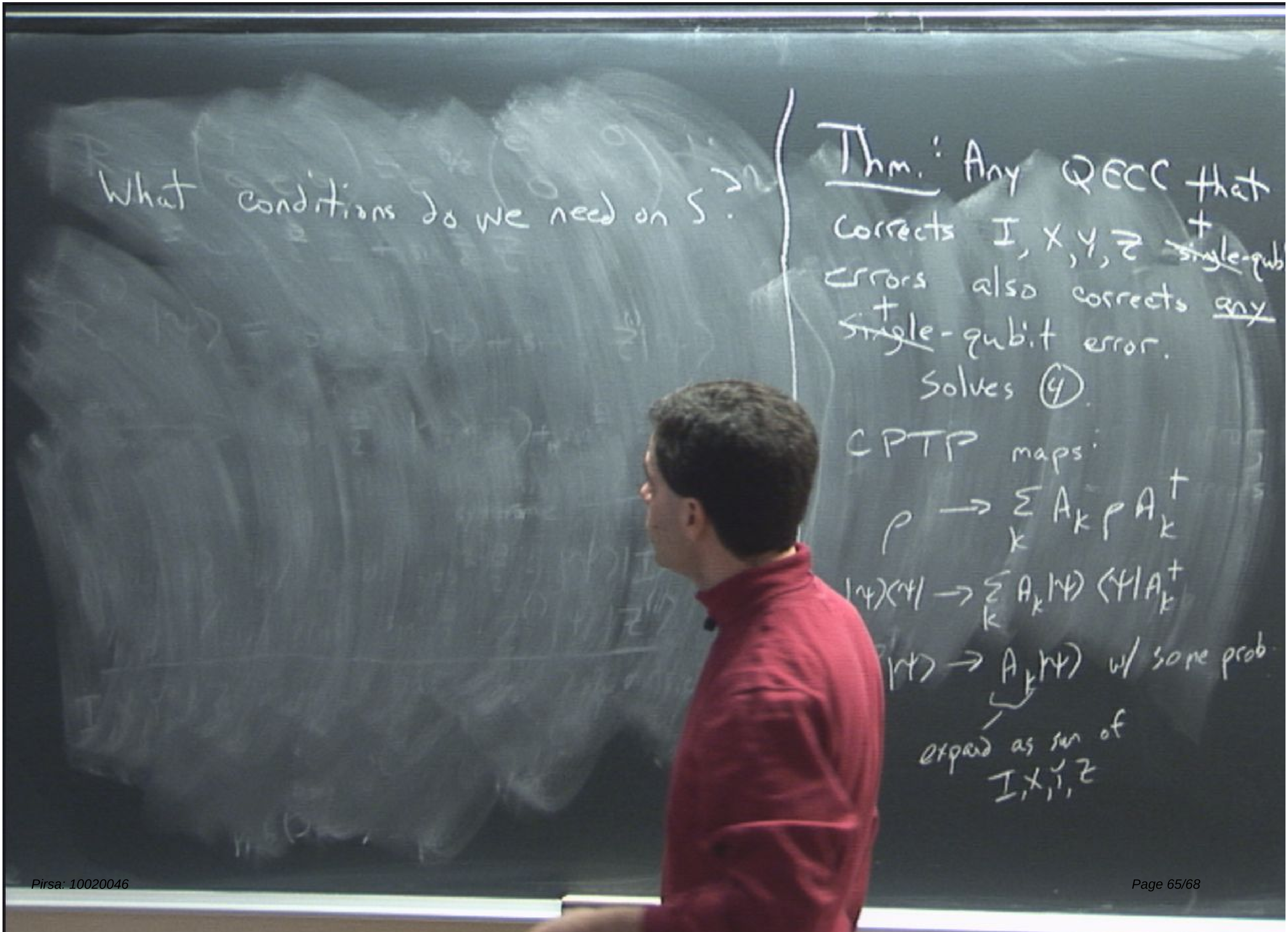
Frequently, we choose S & define QECC from it $T(S) = \{ |\psi\rangle \mid M|\psi\rangle = |\psi\rangle \ \forall M \in S \}$

For 9-qubit code, need these eigenvalues:

$$\boxed{\begin{array}{l} Z\otimes Z \\ \quad Z\otimes Z \\ \qquad\quad Z\otimes Z \\ \qquad\qquad Z\otimes Z \\ \qquad\qquad\qquad Z\otimes Z \\ \qquad\qquad\qquad\quad Z\otimes Z \\ X\otimes X\otimes X\otimes X\otimes X\otimes X \\ \qquad\qquad X\otimes X\otimes X\otimes X\otimes X\otimes X \end{array}}$$

Valid 9-qubit codewords all have eigenvalue +1 for these operators

These 8 operators generate stabilizer for 9-qubit code



What conditions do we need on S?

Thm.: Any QECC that corrects I, X, Y, Z ~~single~~ t-qub errors also corrects any ~~single~~ t-qubit error.

Solves ④.

CPTP maps:

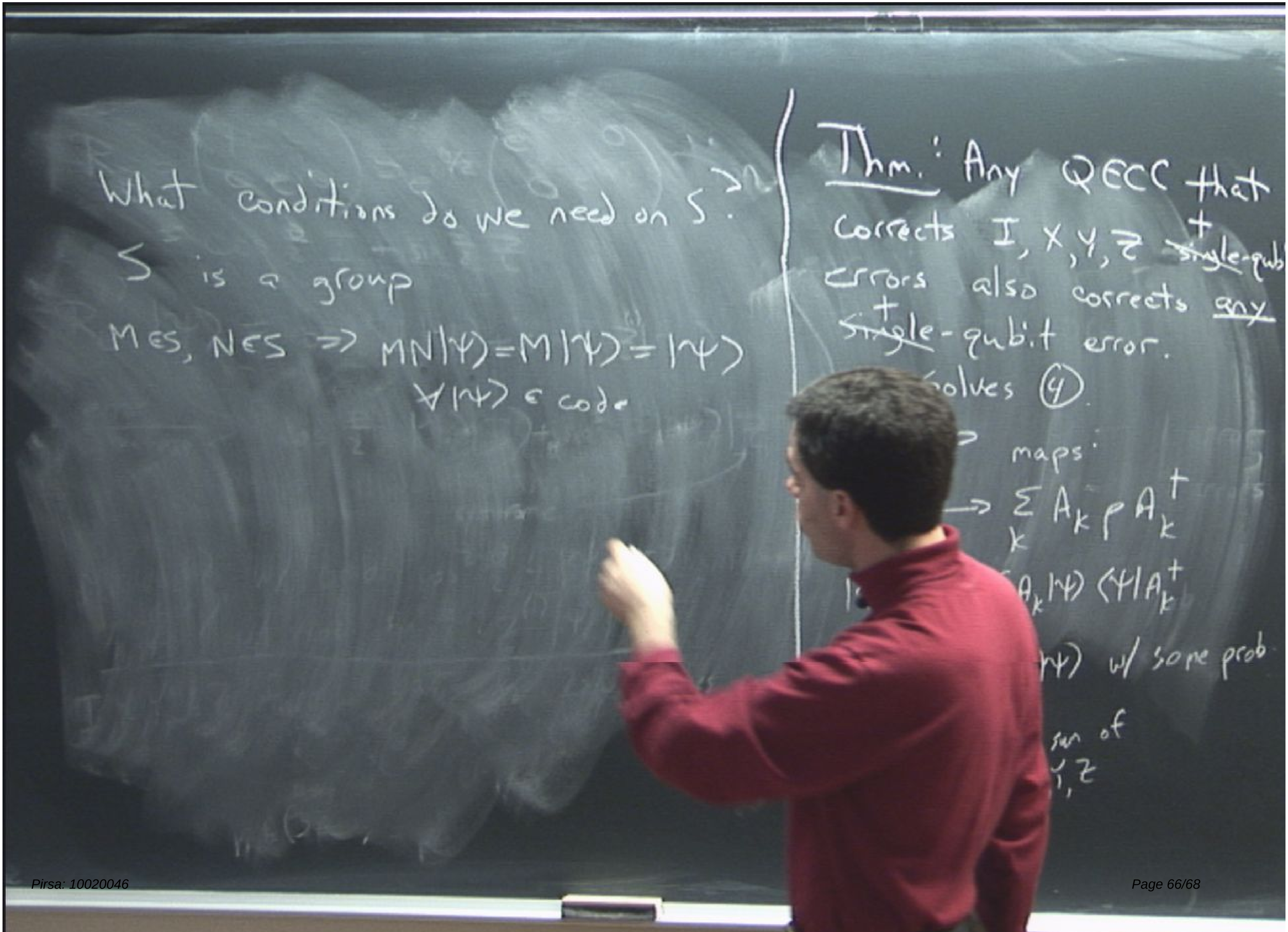
$$\rho \longrightarrow \sum_k A_k \rho A_k^\dagger$$

$$|\psi\rangle\langle\psi| \longrightarrow \sum_k A_k |\psi\rangle\langle\psi| A_k^\dagger$$

$$|\psi\rangle \rightarrow \underbrace{A_k}|\psi\rangle \text{ w/ some prob.}$$

expand as sum of

I, X, Y, Z



What conditions do we need on S?

S is a group

$M \in S,\ N \in S \Rightarrow MN|\psi\rangle = M|\psi\rangle = |\psi\rangle$

$\forall |\psi\rangle \in$ code

Thm.: Any QECC that corrects I, X, Y, Z ~~single~~ t-qubit errors also corrects any ~~single~~ t-qubit error.

solves ④.

maps:

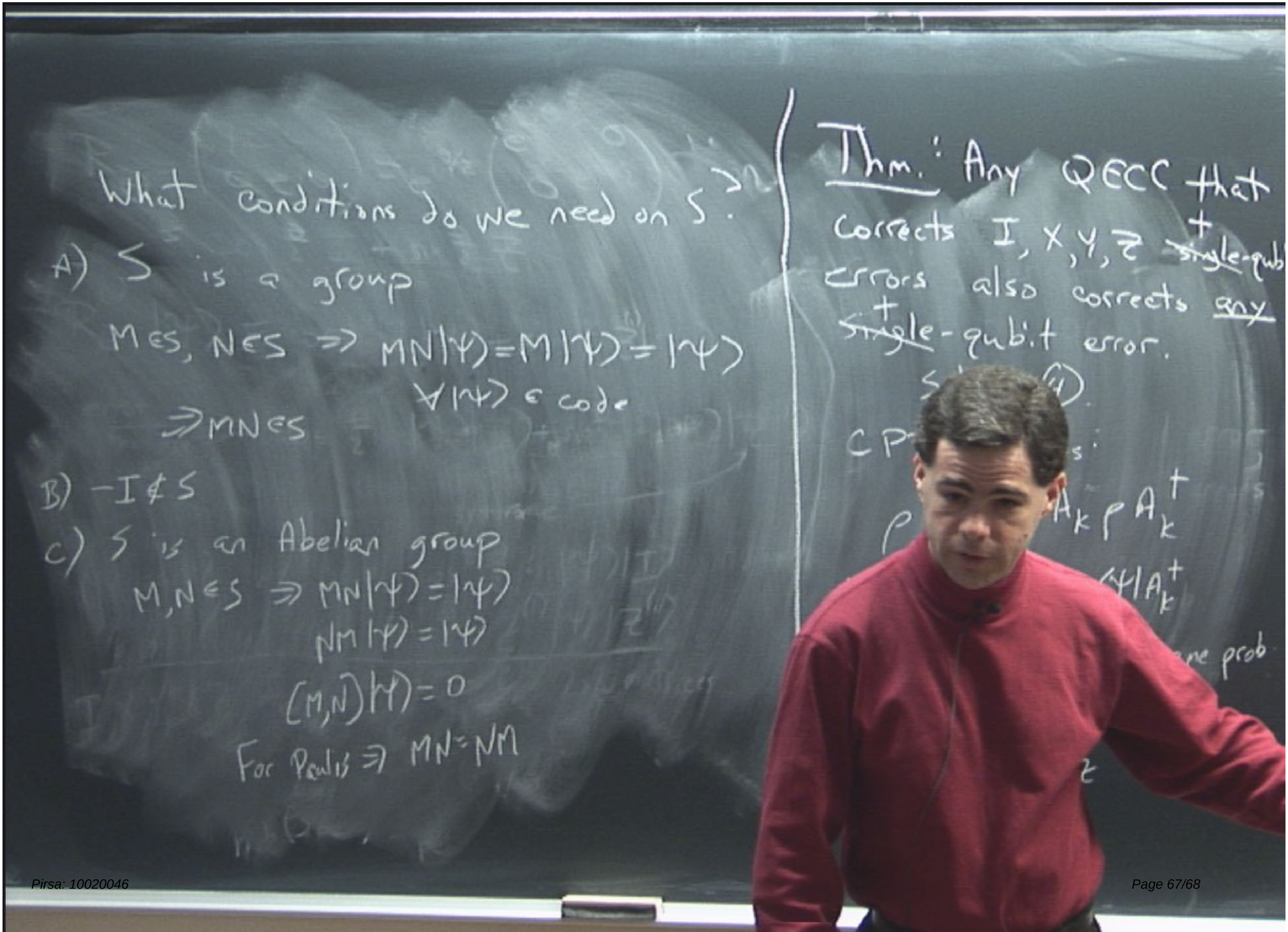
$\rightarrow \sum_k A_k \rho A_k^\dagger$

$A_k|\psi\rangle\langle\psi|A_k^\dagger$

$|\psi\rangle$ w/ some prob

sum of

Y, Z



What conditions do we need on S?

A) S is a group

$M \in S,\ N \in S \Rightarrow MN|\psi\rangle = M|\psi\rangle = |\psi\rangle$

$\forall |\psi\rangle \in \text{code}$

$\Rightarrow MN \in S$

B) $-I \notin S$

c) S is an Abelian group

$M, N \in S \Rightarrow MN|\psi\rangle = |\psi\rangle$

$NM|\psi\rangle = |\psi\rangle$

$[M,N]|\psi\rangle = 0$

For Paulis $\Rightarrow MN = NM$

Thm.: Any QECC that corrects I, X, Y, Z ~~single-qubit~~ t errors also corrects any ~~single~~ t-qubit error.

S (A).

CP s:

ρ $A_k \rho A_k^\dagger$

$\langle\psi| A_k^\dagger$

ne prob

t

What conditions do we need on S?

A) S is a group

$M \in S,\ N \in S \Rightarrow MN|\psi\rangle = M|\psi\rangle = |\psi\rangle$

$\forall |\psi\rangle \in$ code

$\Rightarrow MN \in S$

B) $-I \notin S$

C) S is an Abelian group

$M, N \in S \Rightarrow MN|\psi\rangle = |\psi\rangle$

$NM|\psi\rangle = |\psi\rangle$

$(M,N)|\psi\rangle = 0$

For Paulis $\Rightarrow MN = NM$

Thm.: Any QECC that corrects I, X, Y, Z t ~~single~~-qub errors also corrects any t ~~single~~-qubit error.

Solves ④.

CPTP maps:

$$\rho \longrightarrow \sum_k A_k \rho A_k^\dagger$$

$$|\psi\rangle\langle\psi| \longrightarrow \sum_k A_k|\psi\rangle\langle\psi|A_k^\dagger$$

$$|\psi\rangle \rightarrow \underbrace{A_k}_{\text{expand as sum of } I,X,Y,Z}|\psi\rangle \text{ w/ some prob.}$$