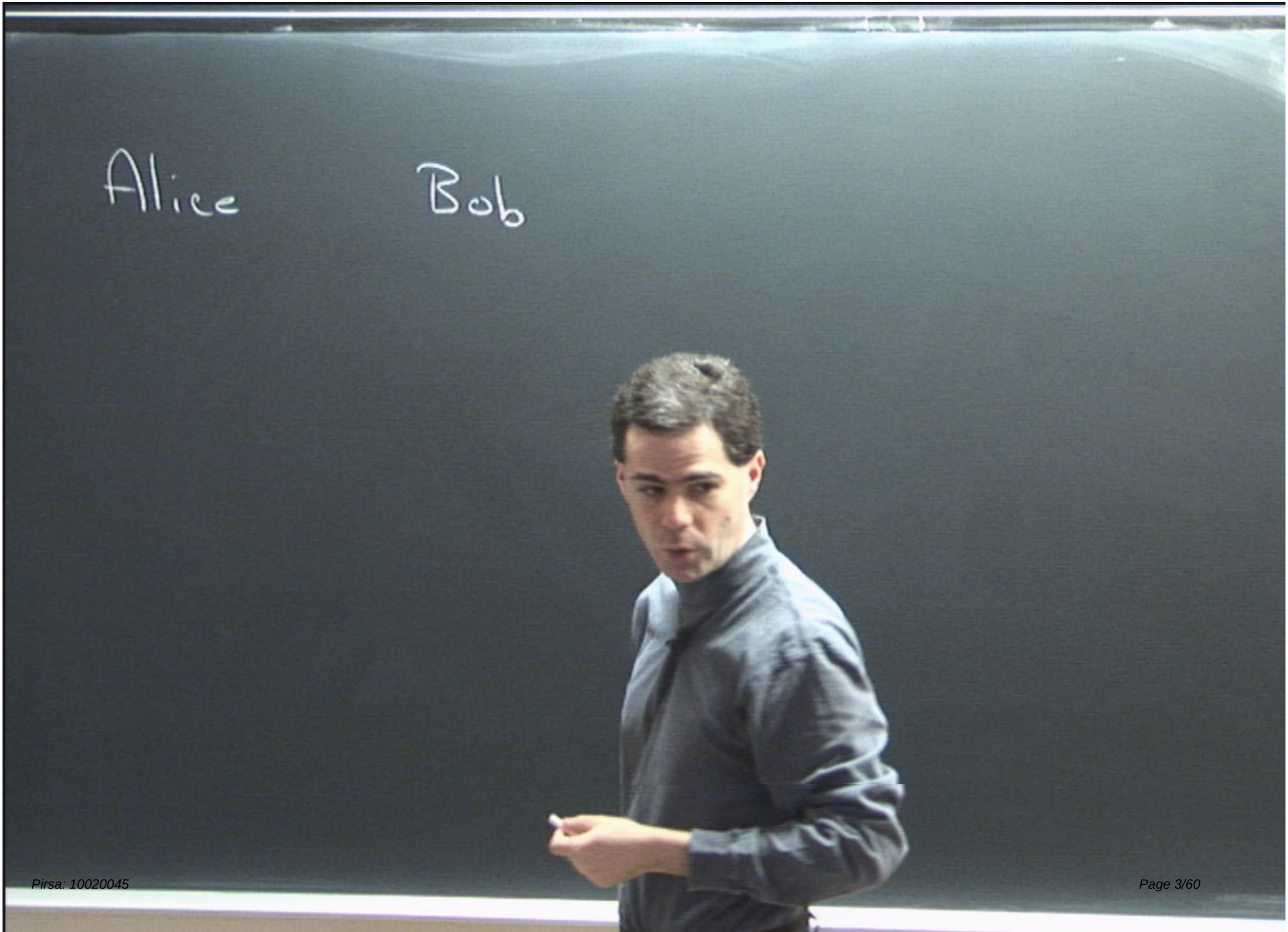
Title: Quantum Information - Review (PHYS 635) - Lecture 11

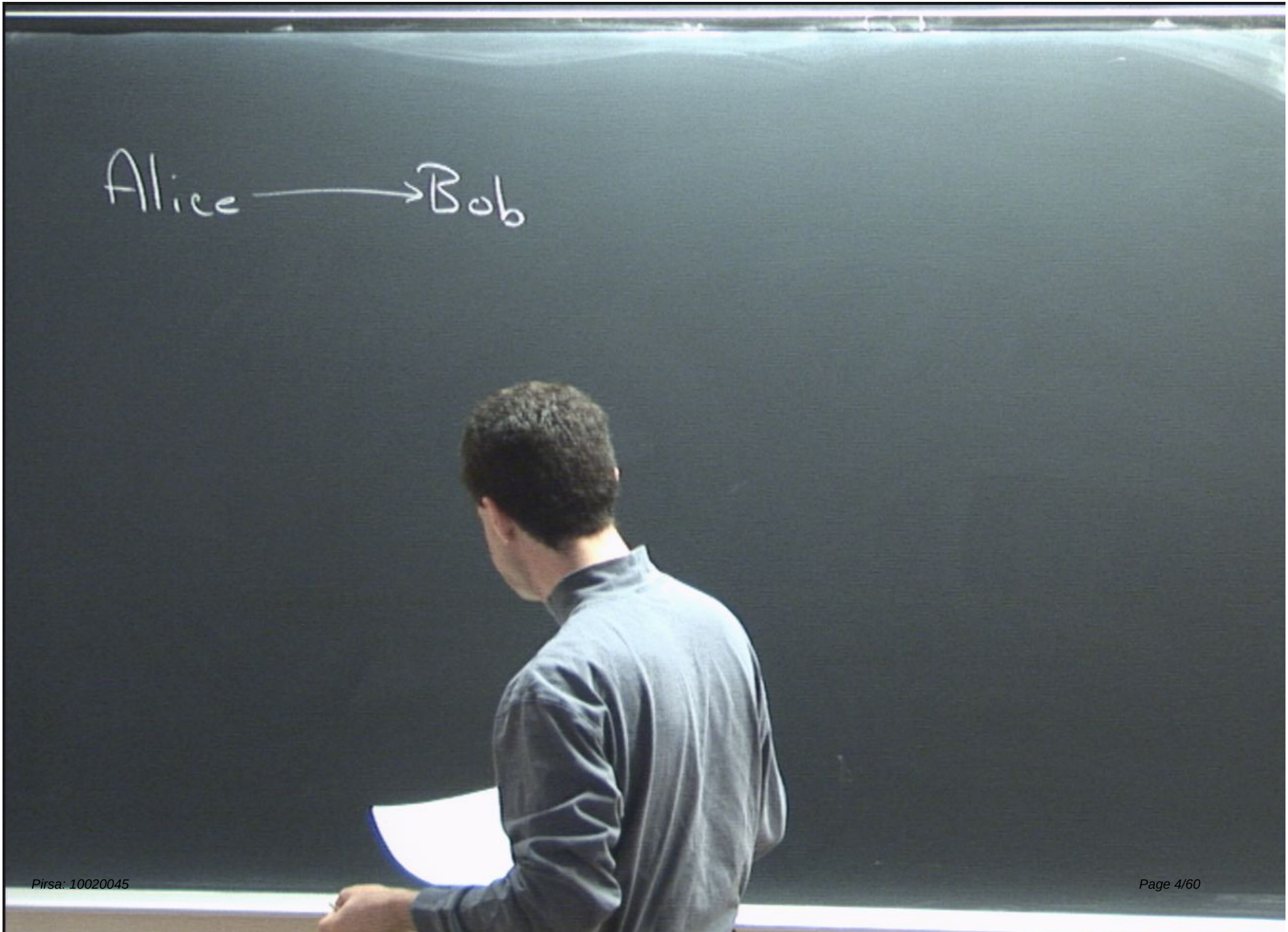
Date: Feb 08, 2010 09:00 AM

URL: http://pirsa.org/10020045

Abstract:


perimeter scholars
international


Alice
Bob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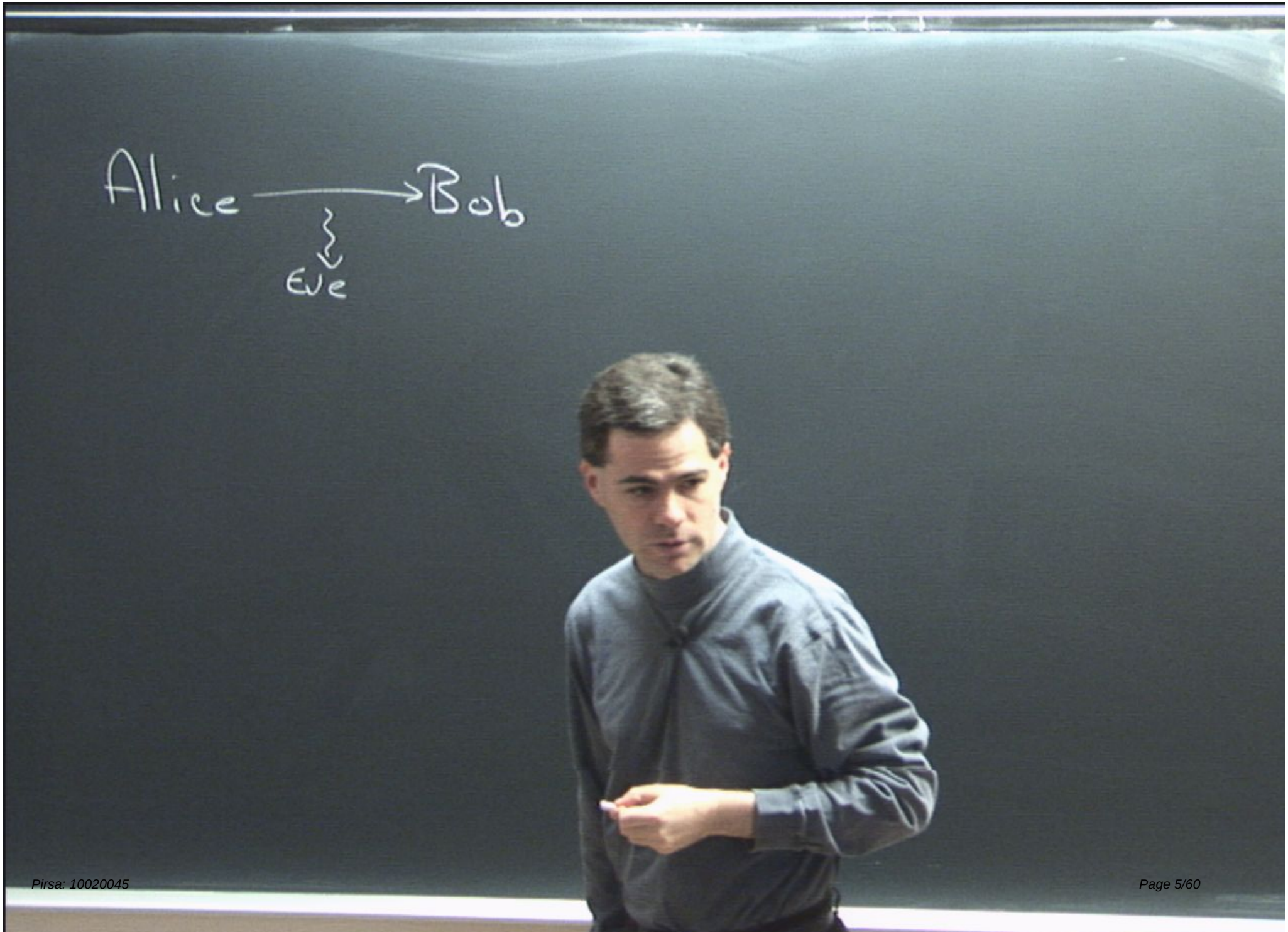
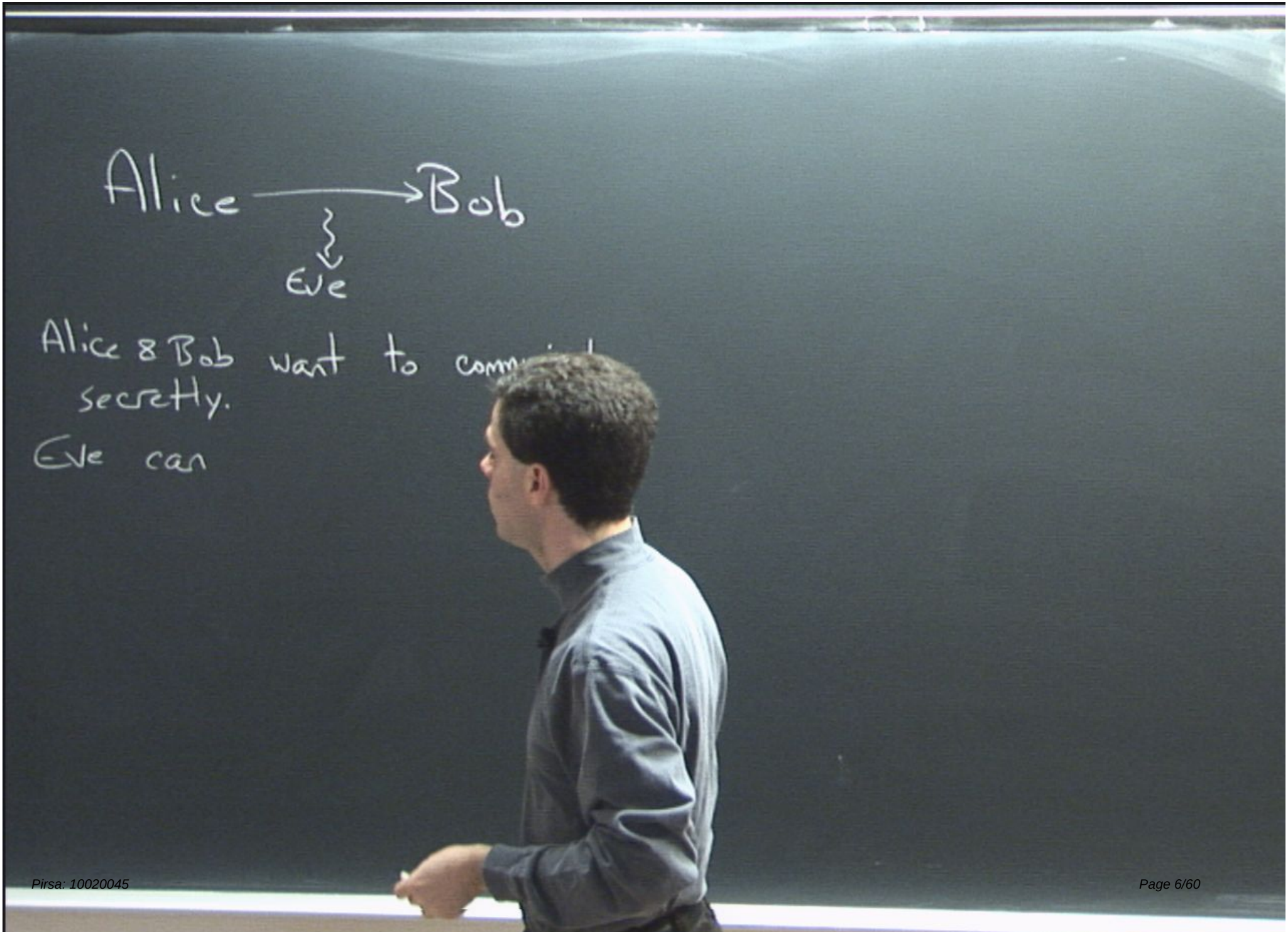
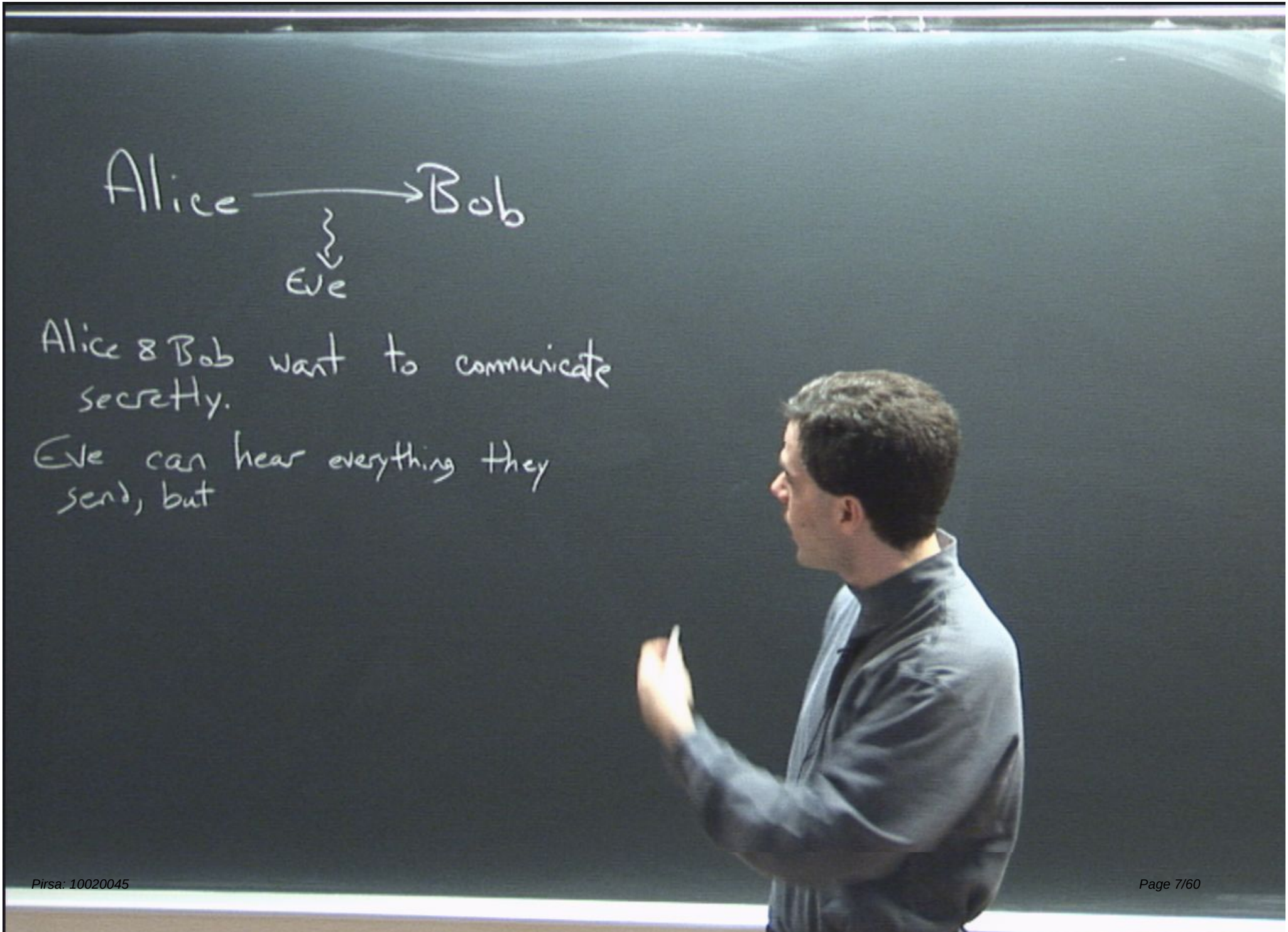

Alice
Bob
Eve


Alice
Bob
Eve
Alice & Bob want to
secretly.
Eve can


Alice
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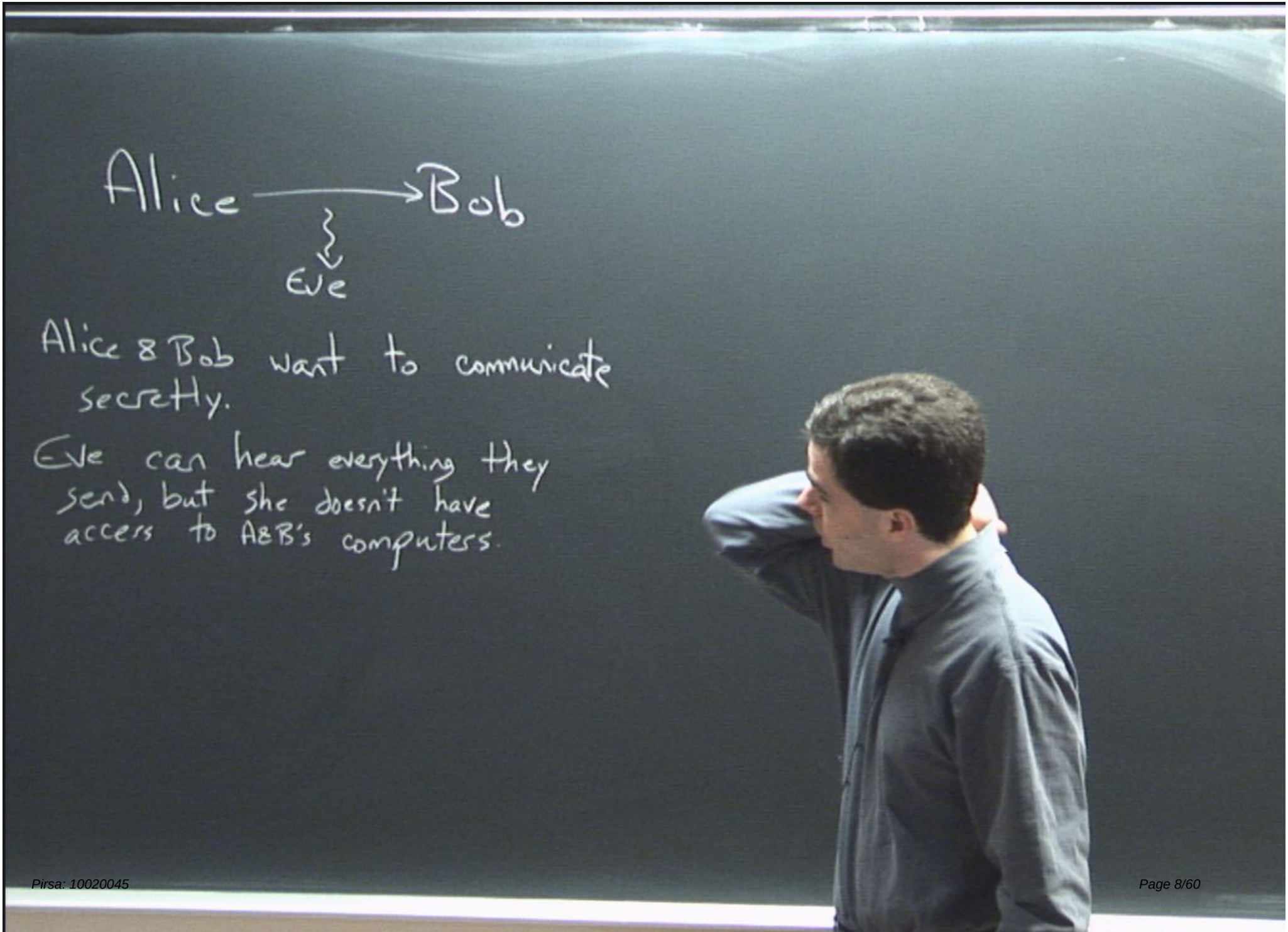
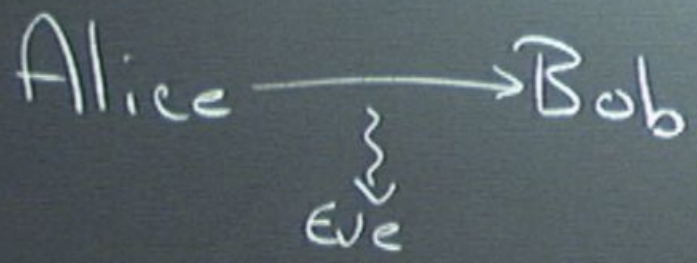
Alice & Bob want to communicate secretly.
Eve can hear everything they send, but


Alice
Bob
Eve
Alice & Bob want to communicate secretly.
Eve can hear everything they send, but she doesn't have access to A&B's computers.



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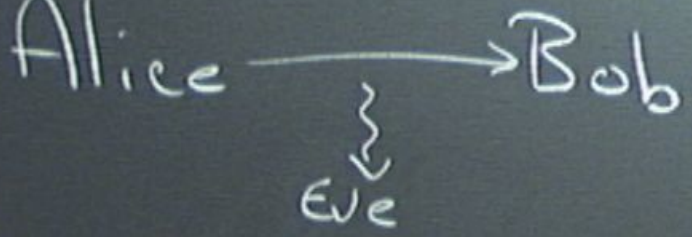
One-time pad:



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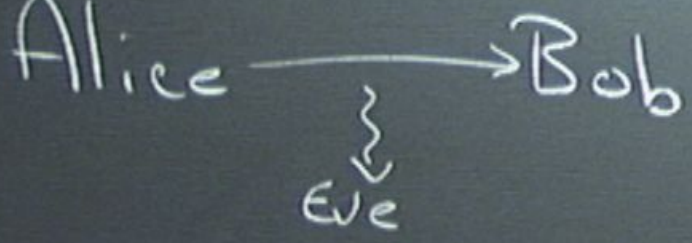
One-time pad:

Alice & Bob share a long private key k (a string of random bits)



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One-time pad:

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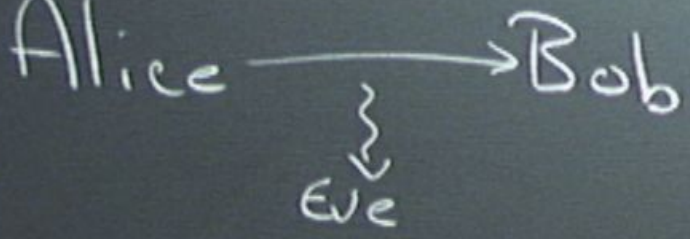
Alice converts her message to a string of bits



Alice & Bob want to communicate secretly.

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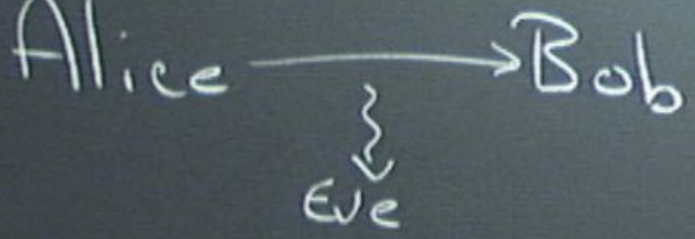
One-time pad:

Alice & Bob share a long private key k (a string of random bits)

Alice converts her message to a string of bits m

Sends $e = m \oplus k$ (bitwise XOR)



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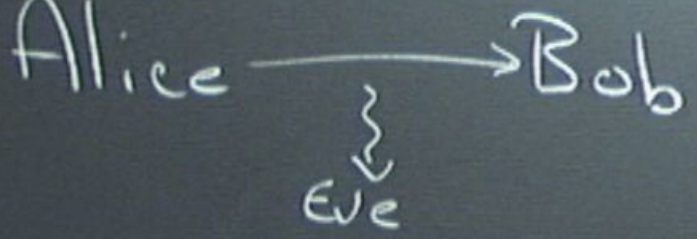
Bob decodes $e \oplus k = (m \oplus k) \oplus k = m$



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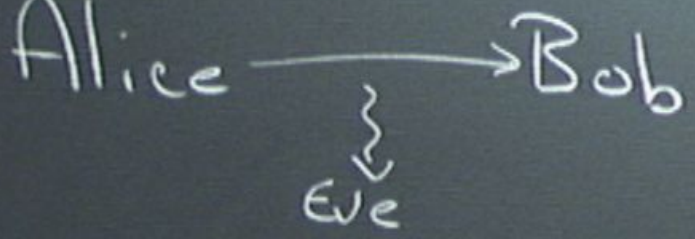
But Eve doesn't know k, so to her each bit looks completely random.



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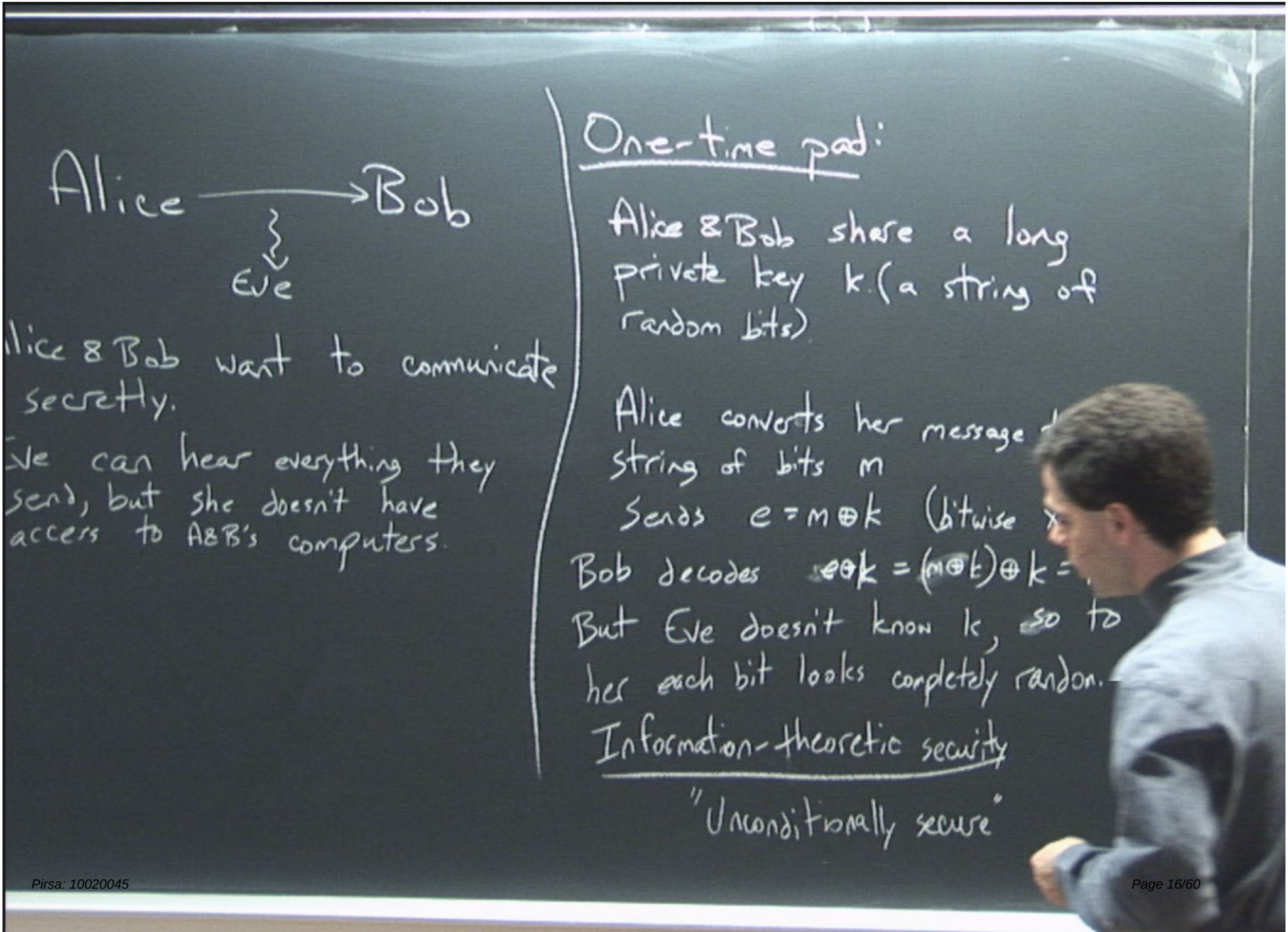
Information-theoretic security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One-time pad:
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Alice converts her message
string of bits m
Sends e = m ⊕ k (bitwise
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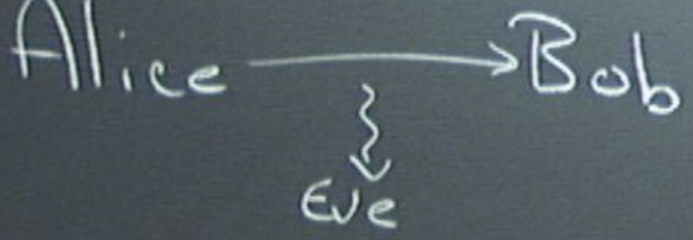
Information-theoretic security
"Unconditionally secure"



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One-time pad:

Alice & Bob share a long private key k (a string of random bits)

Alice converts her message to a string of bits m

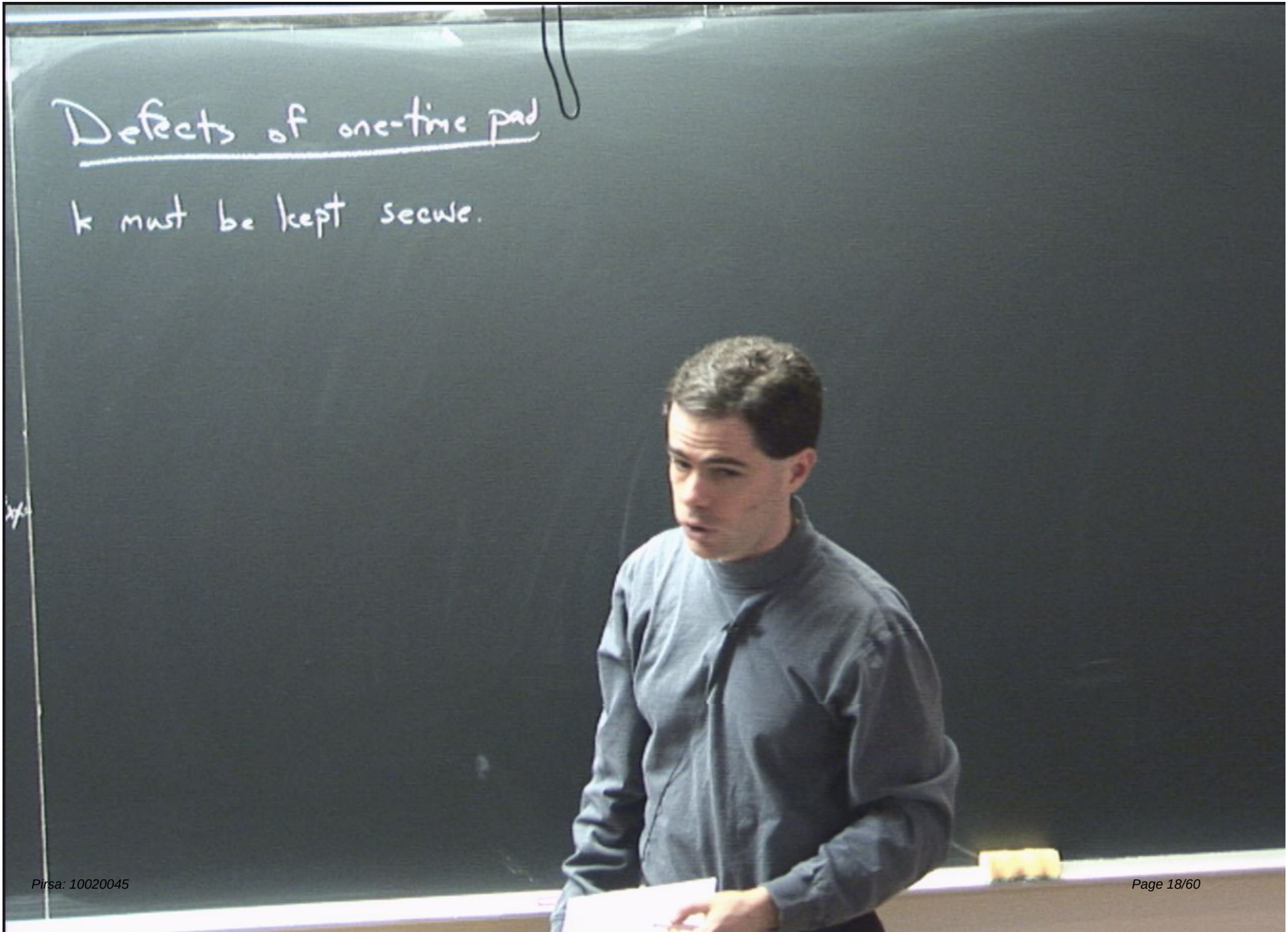
Sends $e = m \oplus k$ (bitwise XOR) "ciphertext"

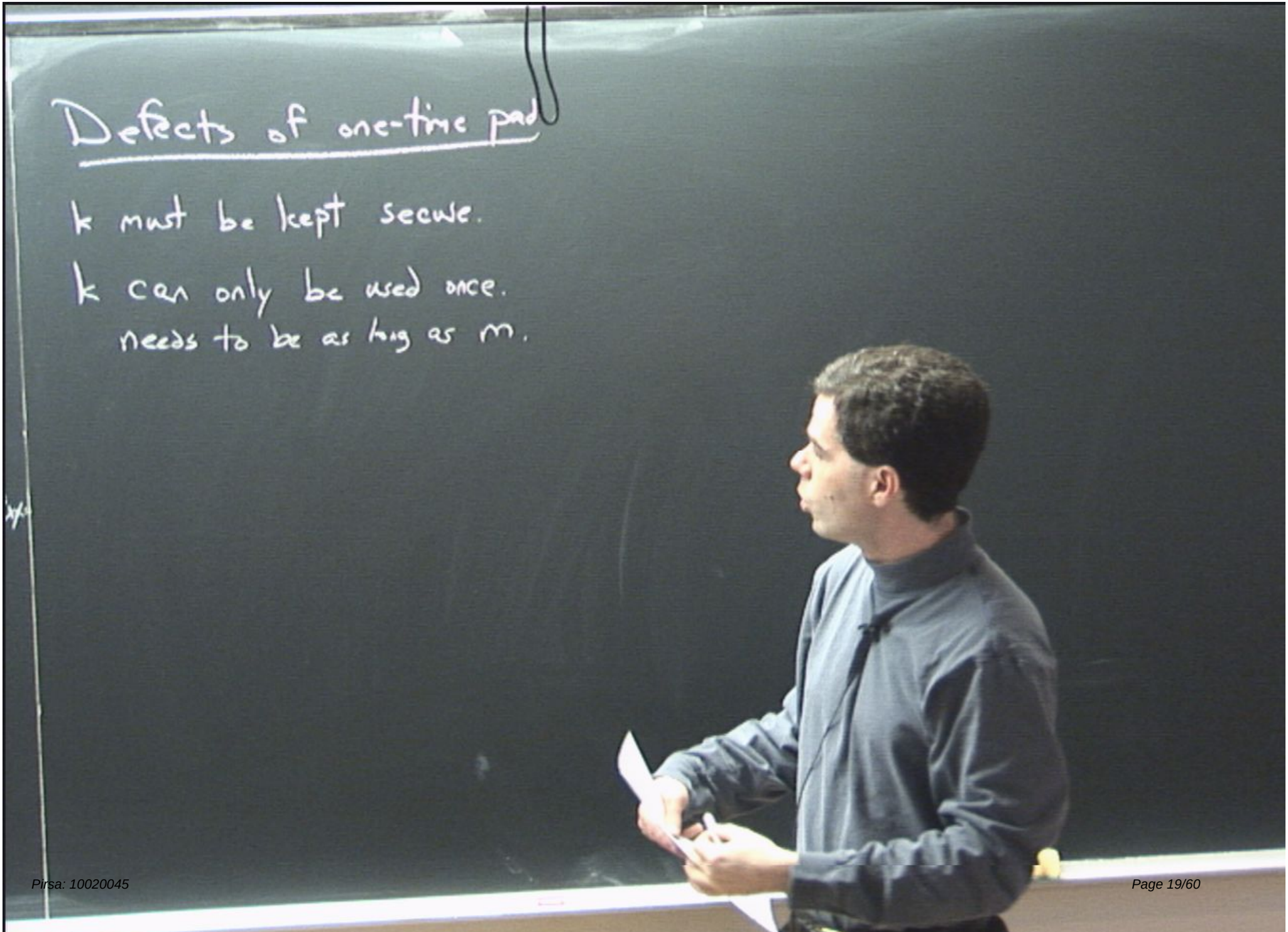
Bob decodes $e \oplus k = (m \oplus k) \oplus k = m$

But Eve doesn't know k, so to her each bit looks completely random.

Information-theoretic security

"Unconditionally secure"


Defects of one-time pad
k must be kept secure.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k can only be used once.
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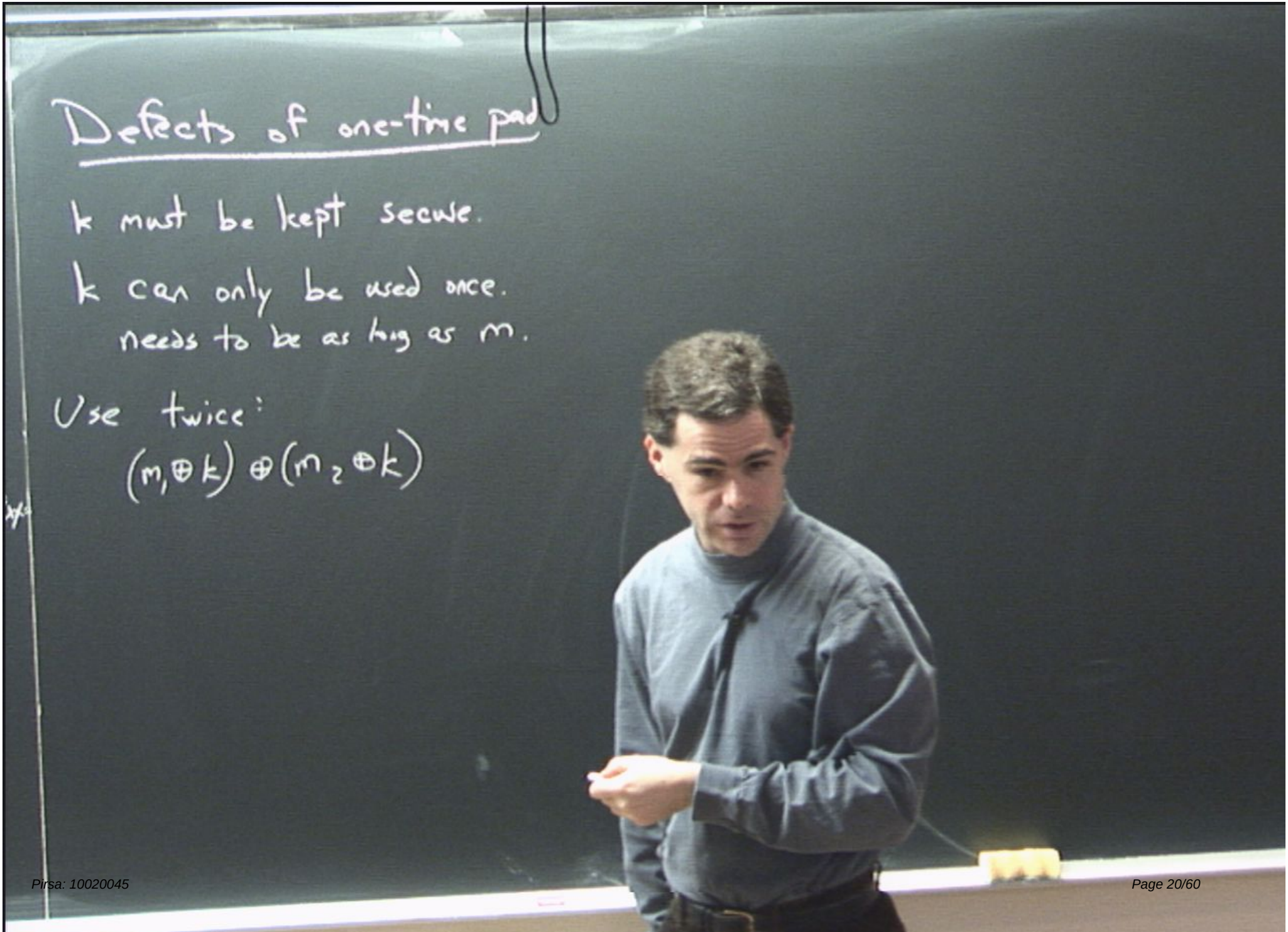
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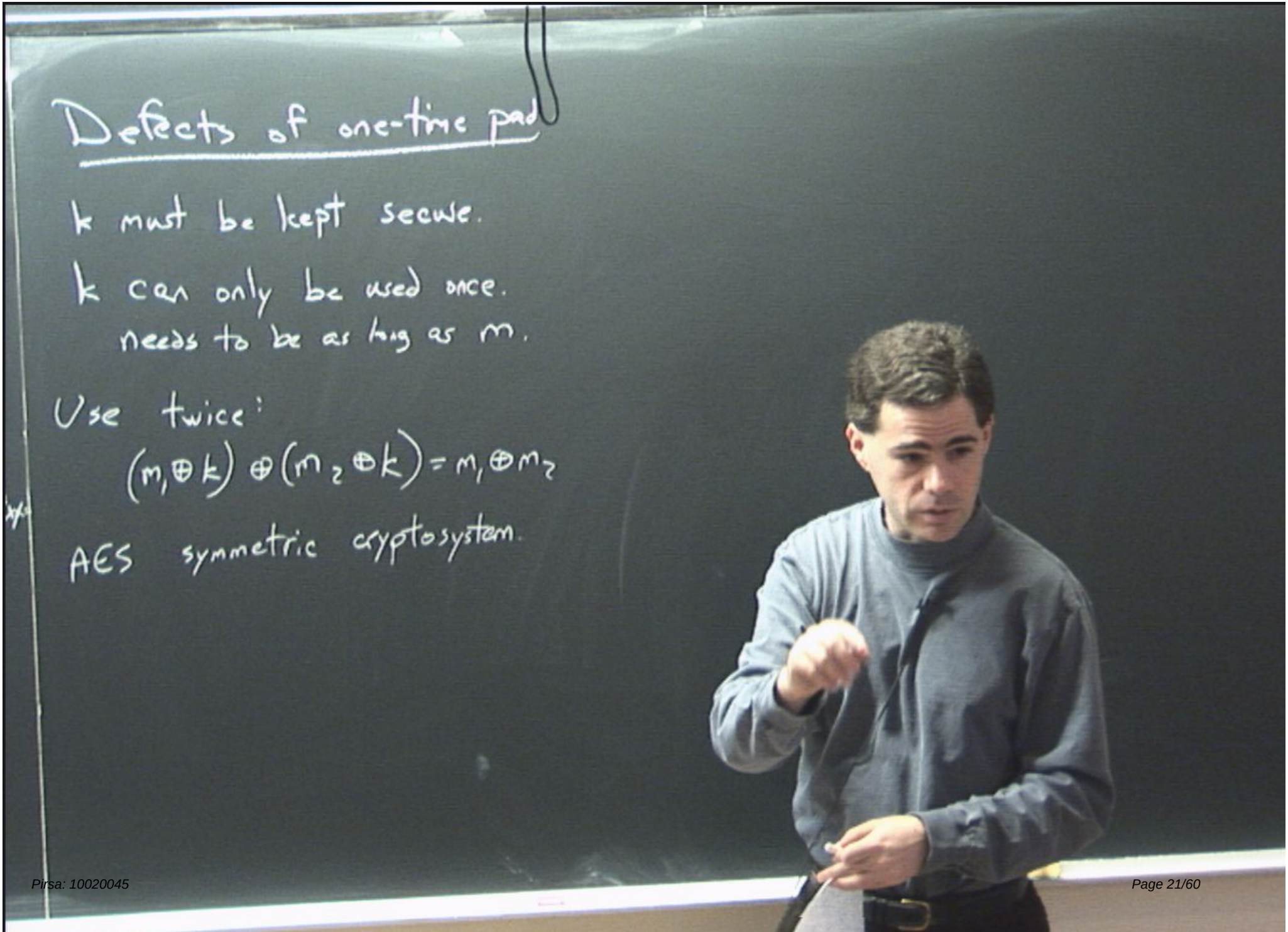
Use twice:
$(m_1 \oplus k) \oplus (m_2 \oplus k)$


Defects of one-time pad
k must be kept secure.
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Use twice:
$(m_1 \oplus k) \oplus (m_2 \oplus k) = m_1 \oplus m_2$
AES symmetric cryptosystem.

Defects of one-time pad

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Quantum key distribution (QKD)

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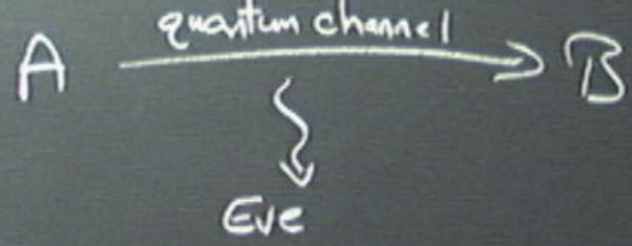

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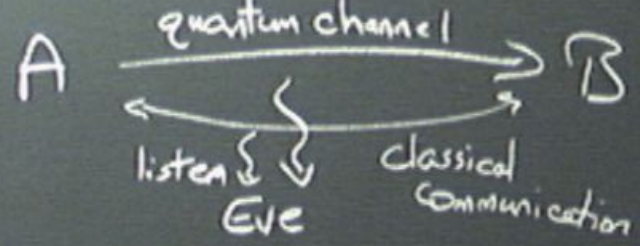

(1) Alice generate N pairs of random bits (a_i, r_i) $i = 1, \ldots, N$

Defects of one-time pad

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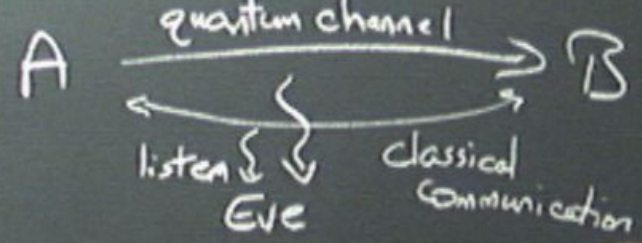

(1) Alice generate .f
random bits $(a_i, r_i$,N

a_i	r_i	
0	0	$\|0\rangle$
0	1	$\|1\rangle$

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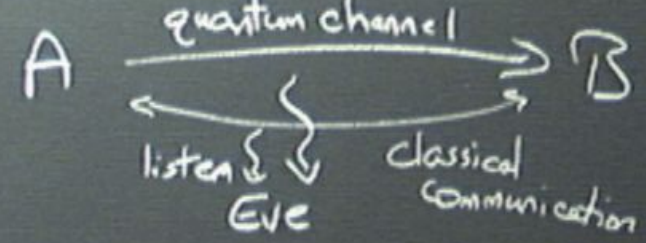

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0	1	$\|1\rangle$
1	0	$\|+\rangle = \|0\rangle + \|1\rangle$
1	1	$\|-\rangle = \|0\rangle - \|1\rangle$

(2)

Defects of one-time pad

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Use twice:

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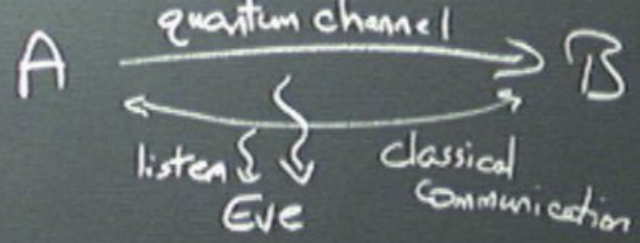
AES symmetric cryptosystem.

Quantum key distribution (QKD)



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(2) Alice sends the corresponding qubits to Bob for each i.

Defects of one-time pad

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(2) Alice sends the corresponding qubits to Bob for each i.

(3) Bob chooses a random basis for each qubit & measures it in that basis.

① Alice generate N pairs
random bits (a_i, r_i) $i = 1, \ldots$

a_i	r_i	
0	0	$\|0\rangle$
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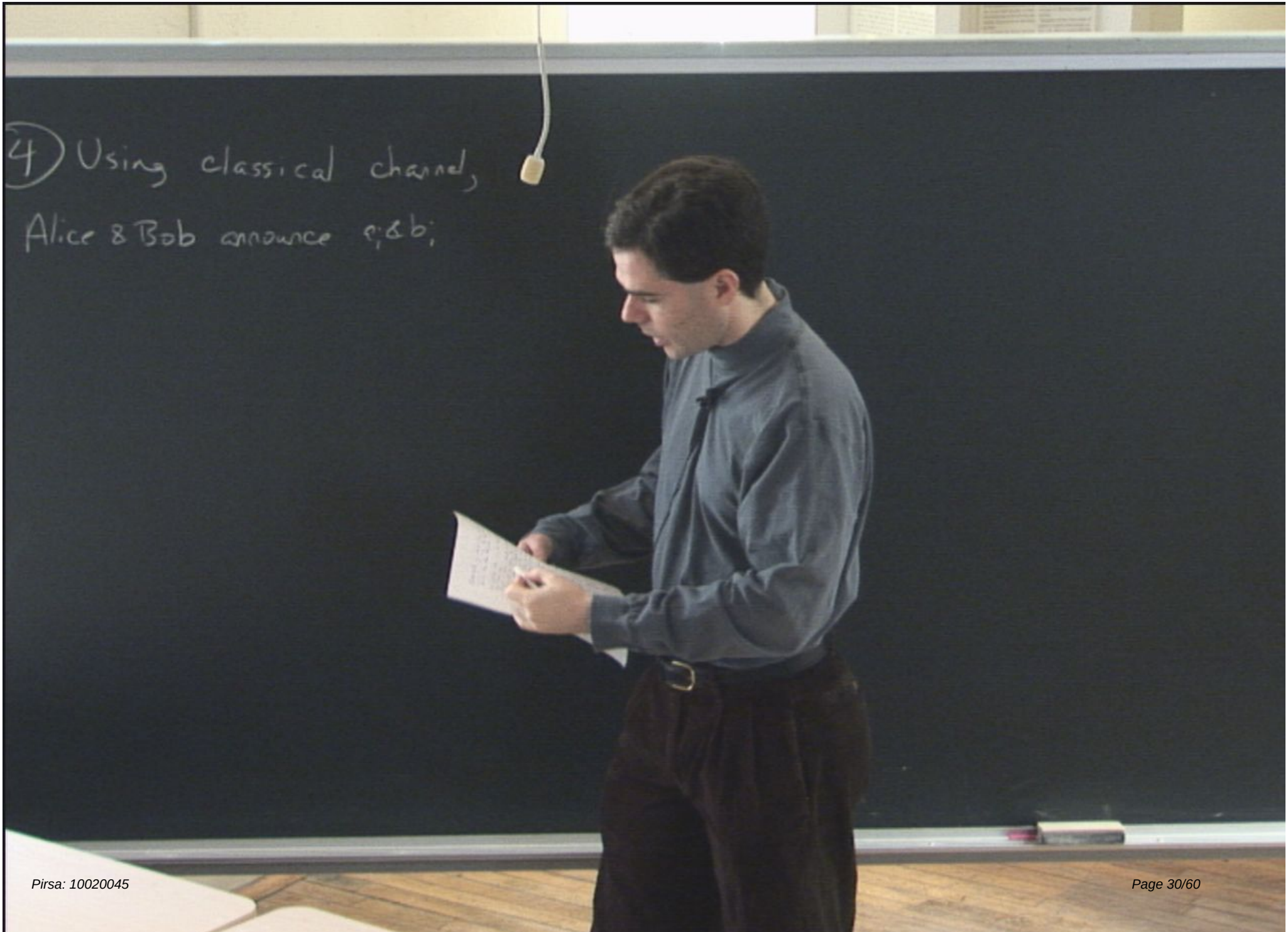
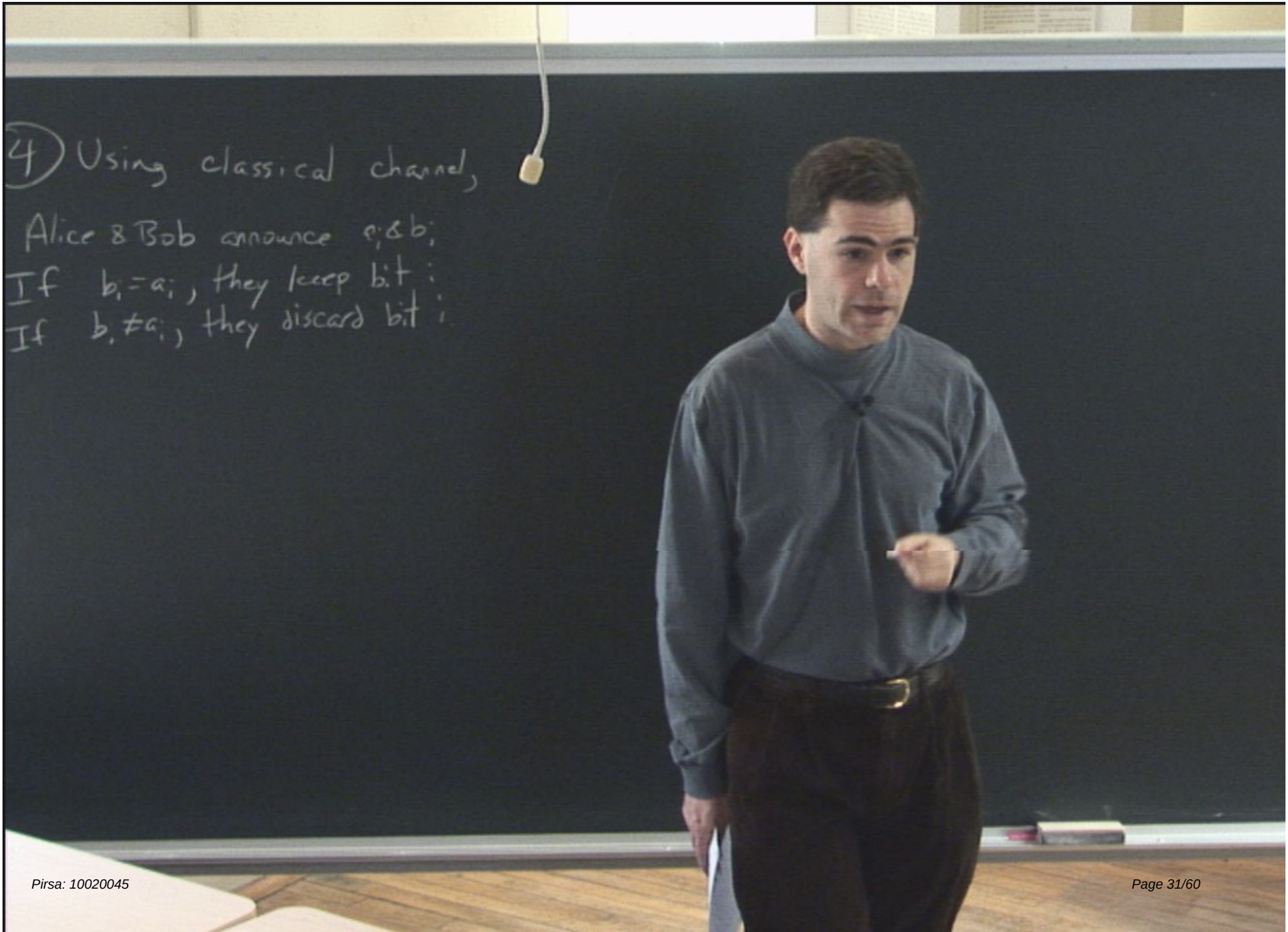

(4) Using classical channel,
Alice & Bob announce a; & b;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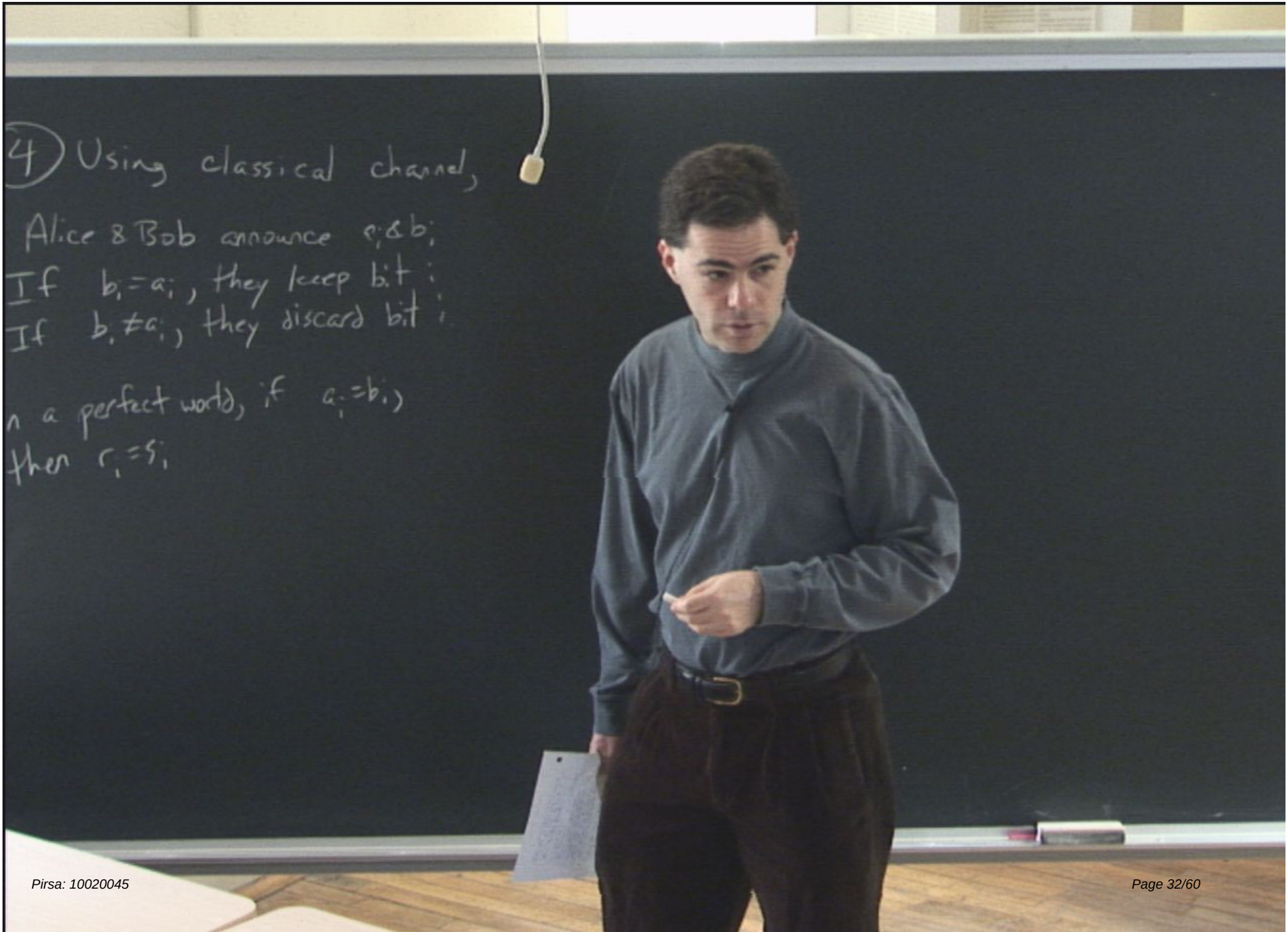
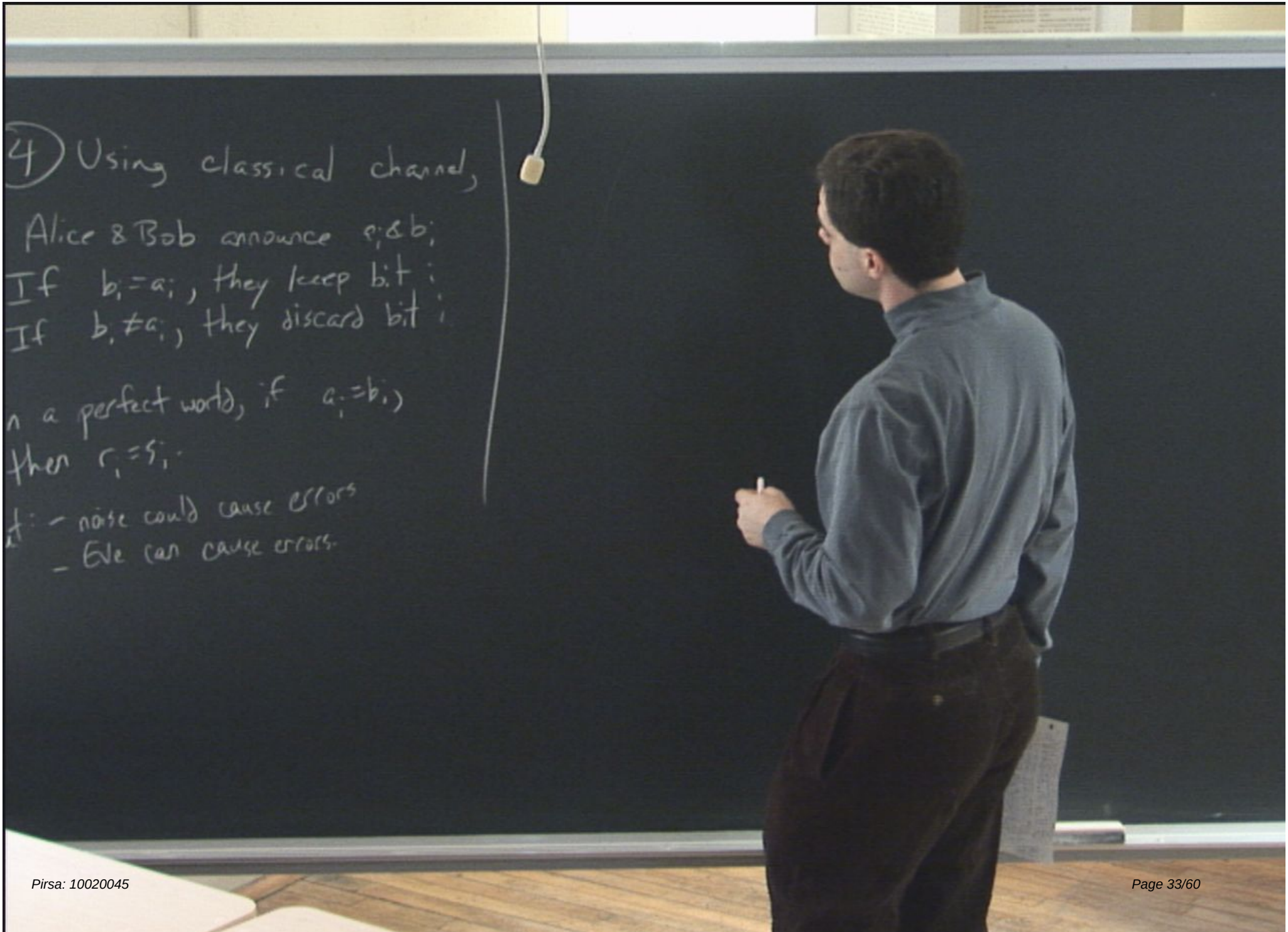
Alice & Bob announce a_i & b_i
If $b_i = a_i$, they keep bit i
If $b_i \neq a_i$, they discard bit i


(4) Using classical channel,
Alice & Bob announce r_i & b_i
If $b_i = a_i$, they keep bit i
If $b_i \neq a_i$, they discard bit i
n a perfect world, if $a_i = b_i$)
then $r_i = s_i$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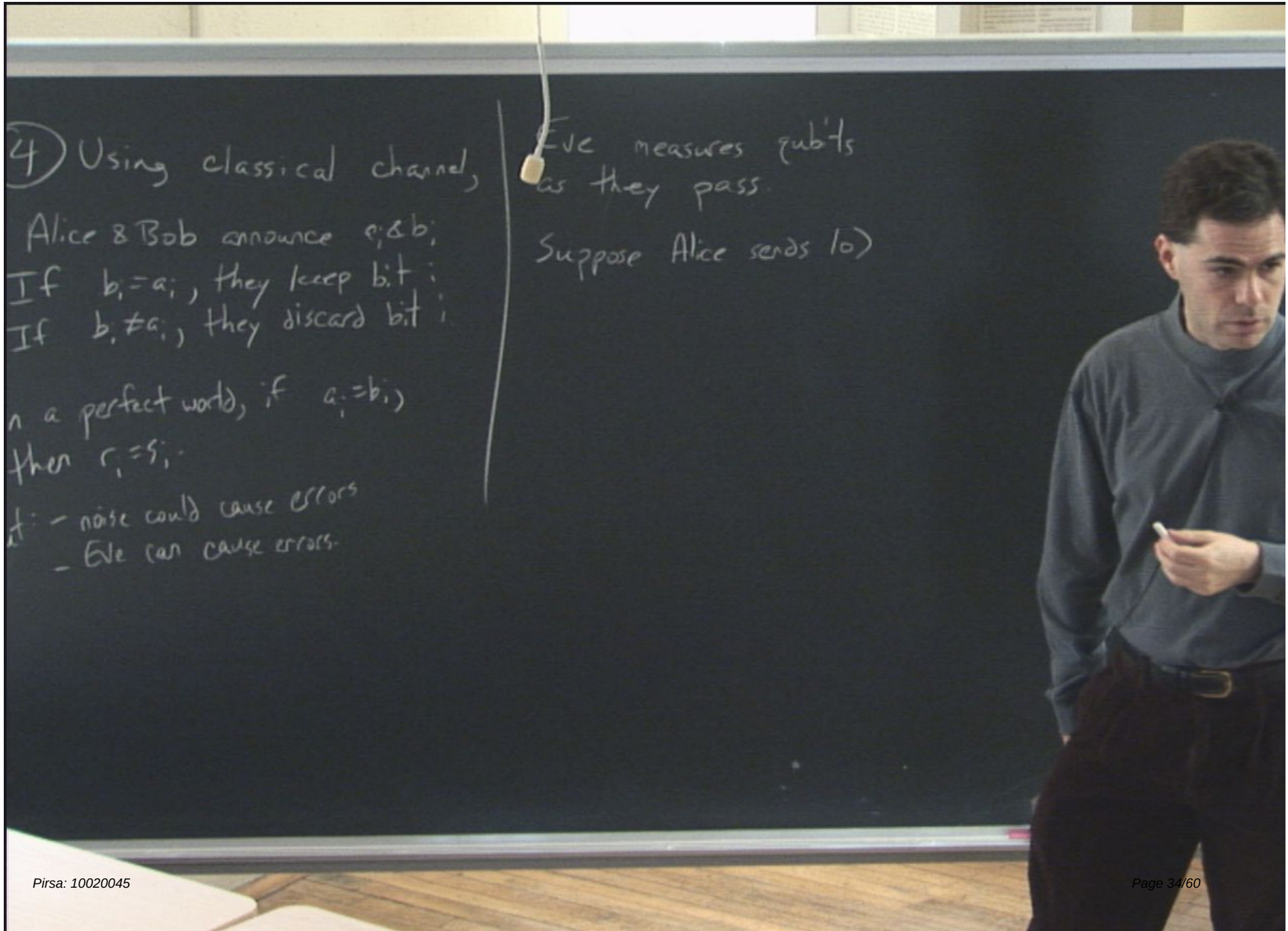
t: - noise could cause errors
- Eve can cause errors.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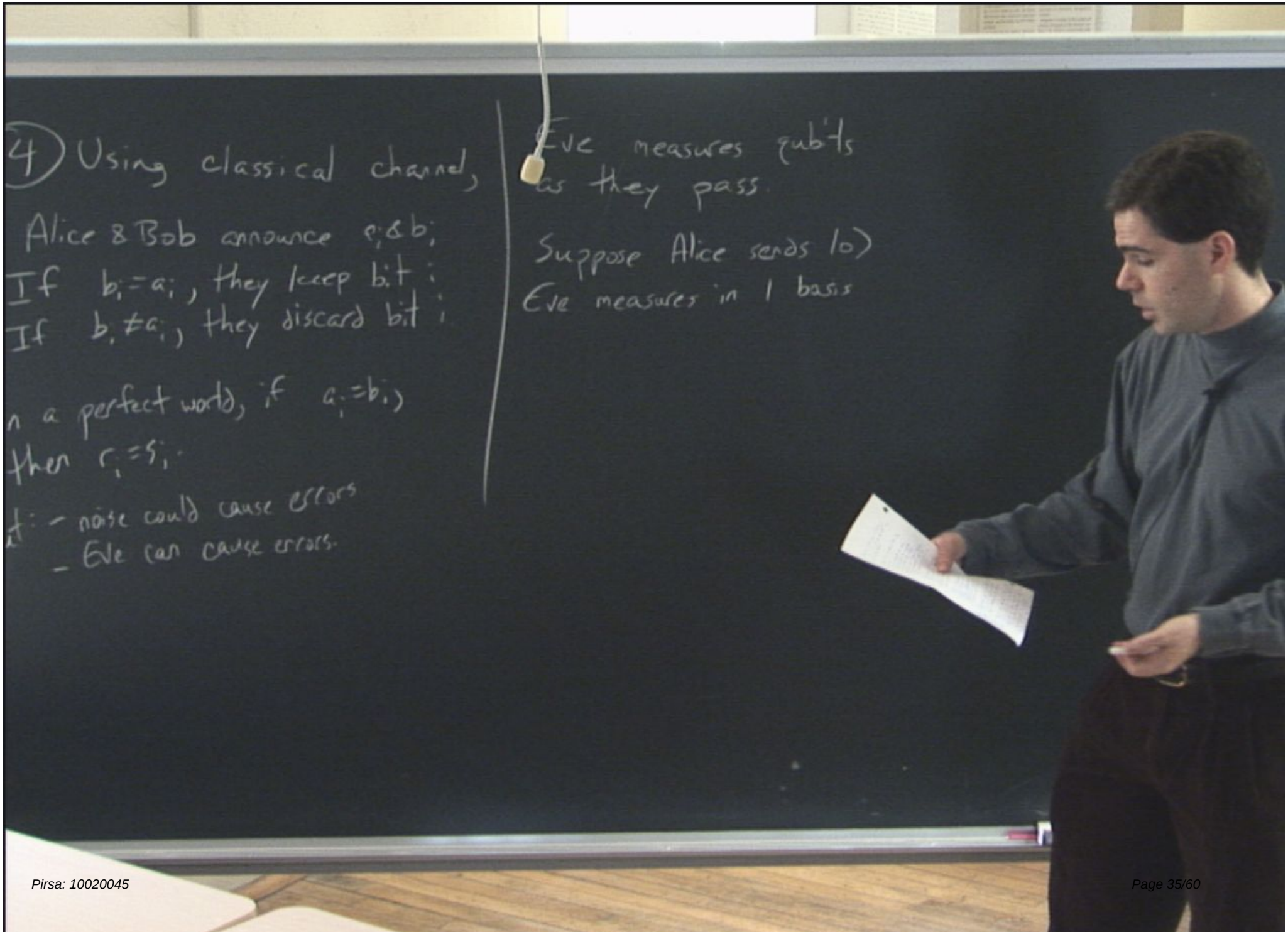
Eve measures qubits
as they pass.
Suppose Alice sends 1o)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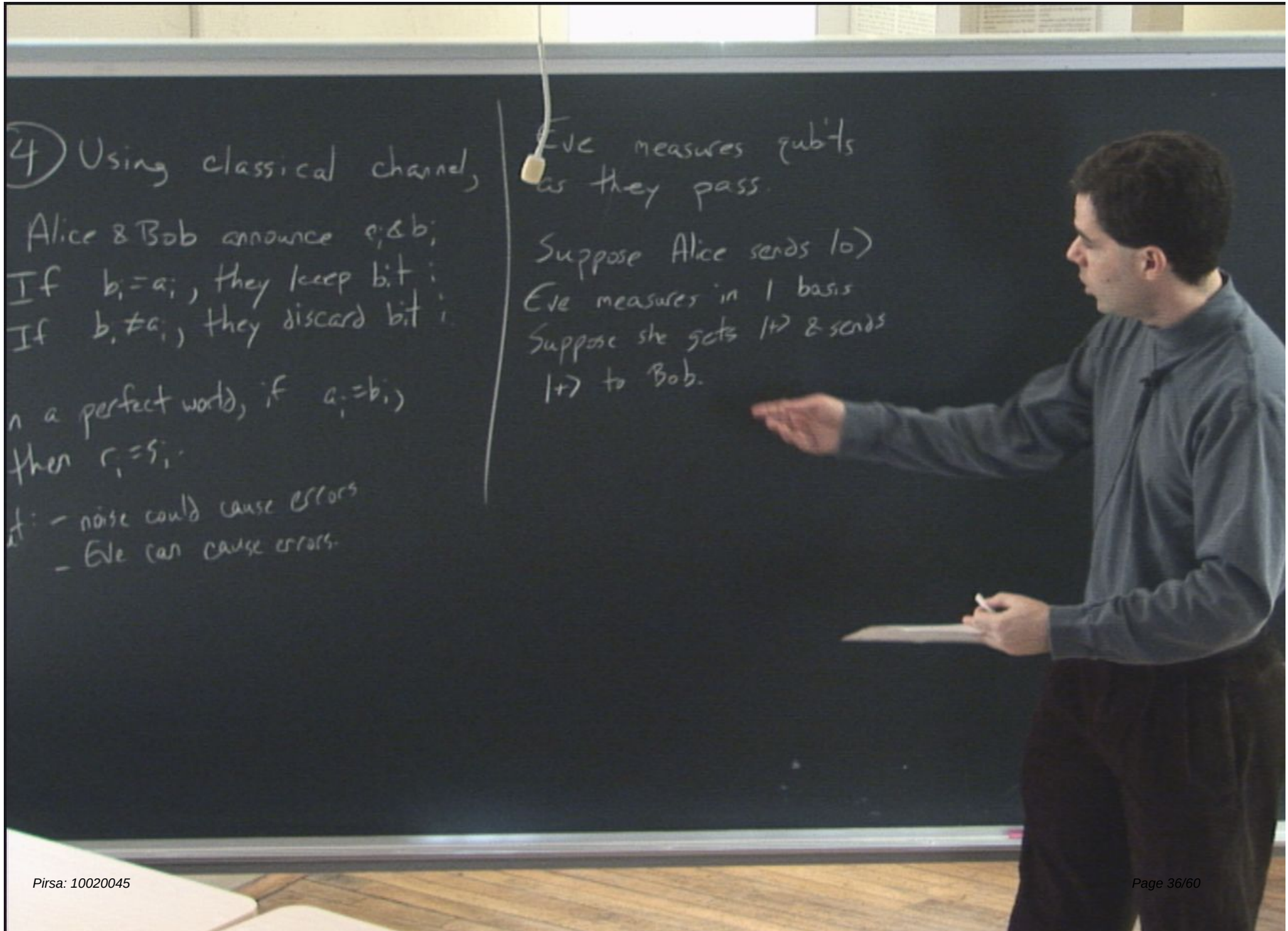
Suppose Alice sends |0⟩
Eve measures in I basis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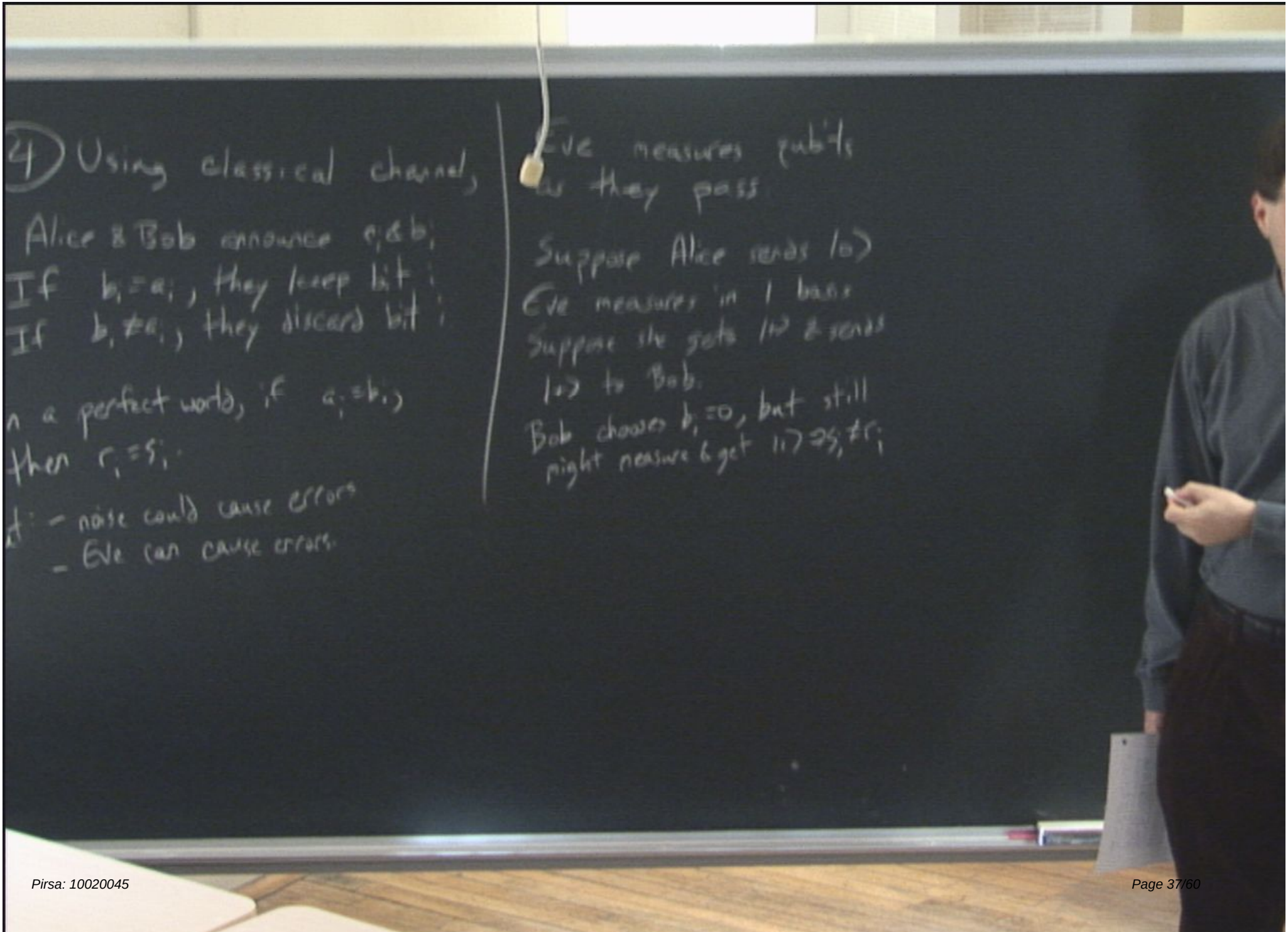
In a perfect world, if $a_i = b_i$
then $r_i = s_i$
But: - noise could cause errors
- Eve can cause errors.
Eve measures qubits
as they pass.
Suppose Alice sends $|0\rangle$
Eve measures in I basis
Suppose she gets $|+\rangle$ & sends
$|+\rangle$ to Bob.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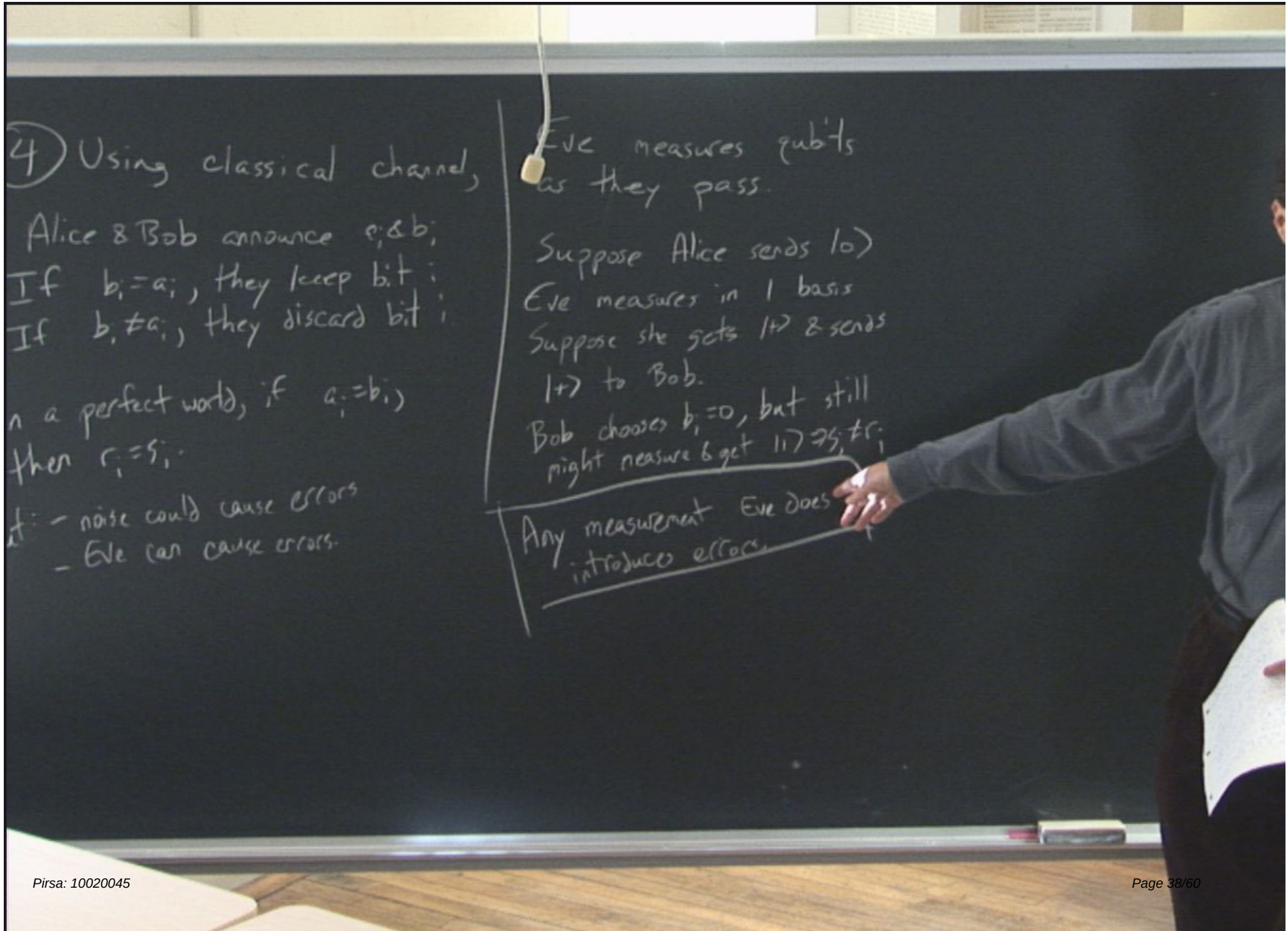
In a perfect world, if $a_i = b_i$,
then $r_i = s_i$;
- noise could cause errors
- Eve can cause errors.
Eve measures qubits
as they pass
Suppose Alice sends $|0\rangle$
Eve measures in I basis
Suppose she gets $|0\rangle$ & sends
$|0\rangle$ to Bob.
Bob chooses $b_i = 0$, but still
might measure 0 get $|1\rangle \Rightarrow s_i \neq r_i$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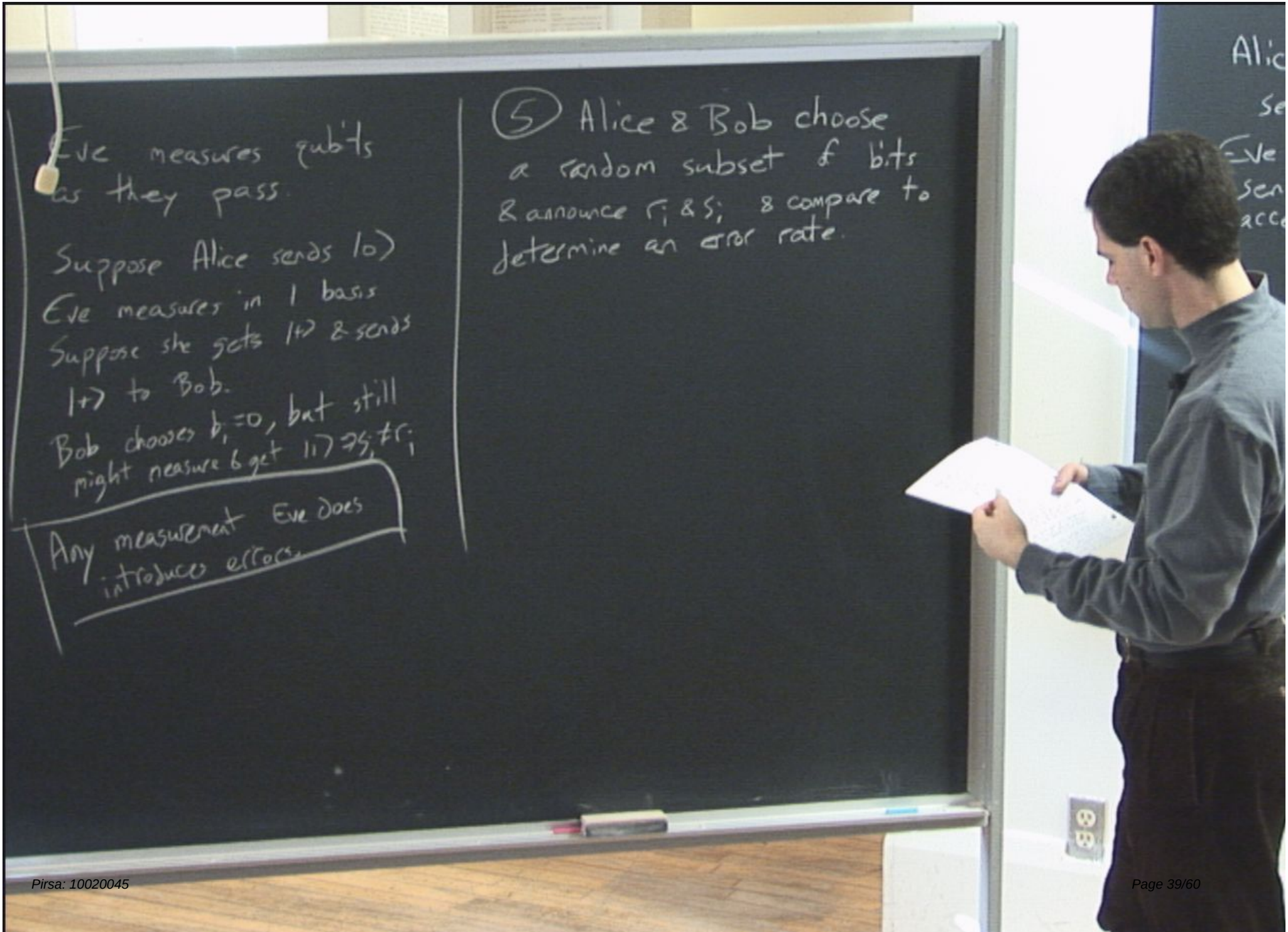
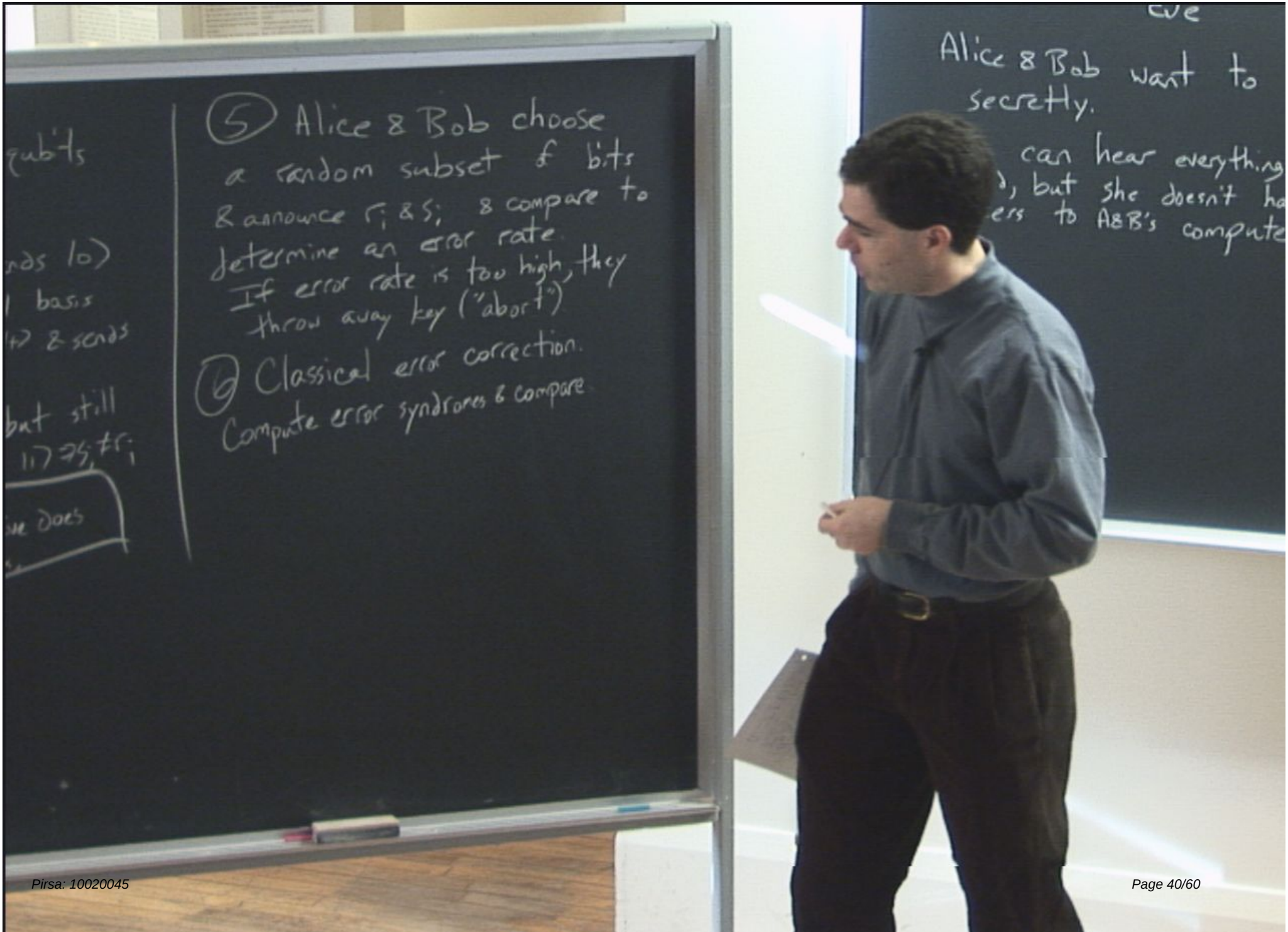
- Eve can cause errors.
Eve measures qubits as they pass.
Suppose Alice sends $|0\rangle$
Eve measures in I basis
Suppose she gets $|+\rangle$ & sends $|+\rangle$ to Bob.
Bob chooses $b_i = 0$, but still might measure & get $|-\rangle$, $s_i \neq r_i$
Any measurement Eve does introduces errors.


Eve measures qubits as they pass.
Suppose Alice sends |0⟩
Eve measures in I basis
Suppose she gets |+⟩ & sends |+⟩ to Bob.
Bob chooses $b_i = 0$, but still might measure 6 get $1/7 = s_i \neq r_i$
Any measurement Eve does introduces errors.
(5) Alice & Bob choose a random subset of bits & announce r_i & s_i & compare to determine an error rate.


qubits
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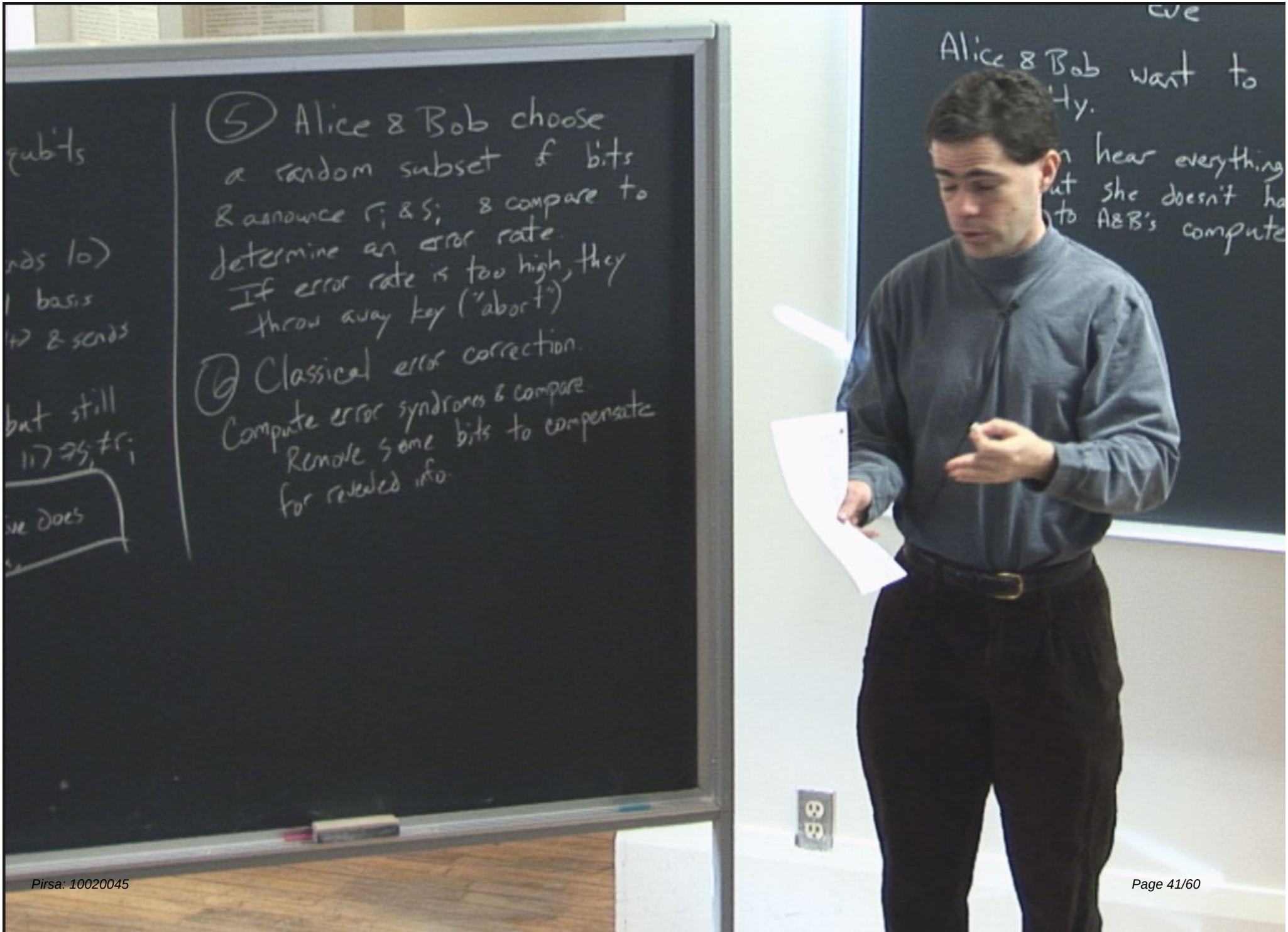
& sends
but still
Alice does
(5) Alice & Bob choose a random subset of bits & announce r_i & s_i & compare to determine an error rate. If error rate is too high, they throw away key ("abort")
(6) Classical error correction. Compute error syndromes & compare.
Alice & Bob want to secretly.
can hear everything
but she doesn't ha
ers to A&B's compute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Compute error syndromes & compare
Remove some bits to compensate
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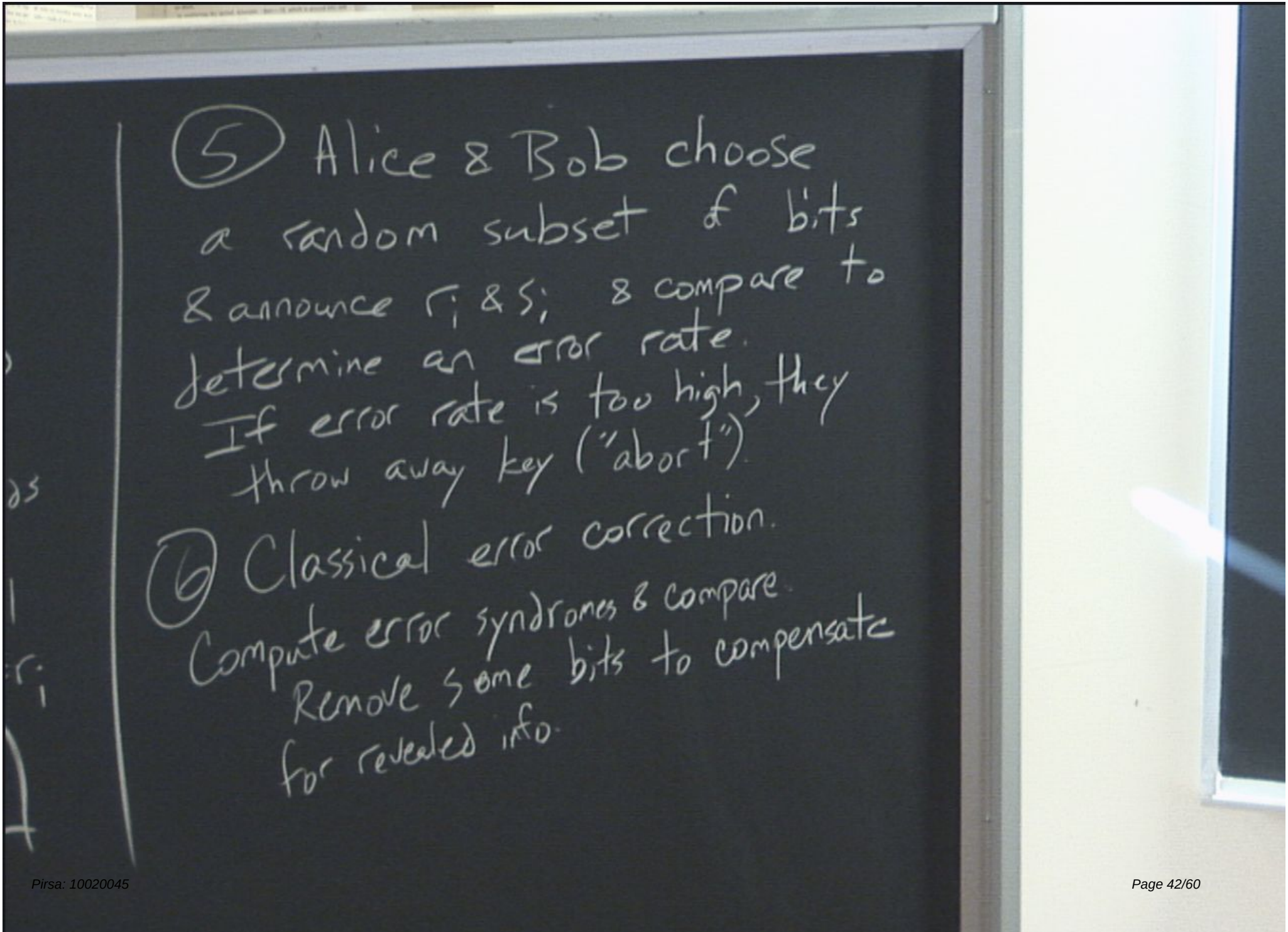
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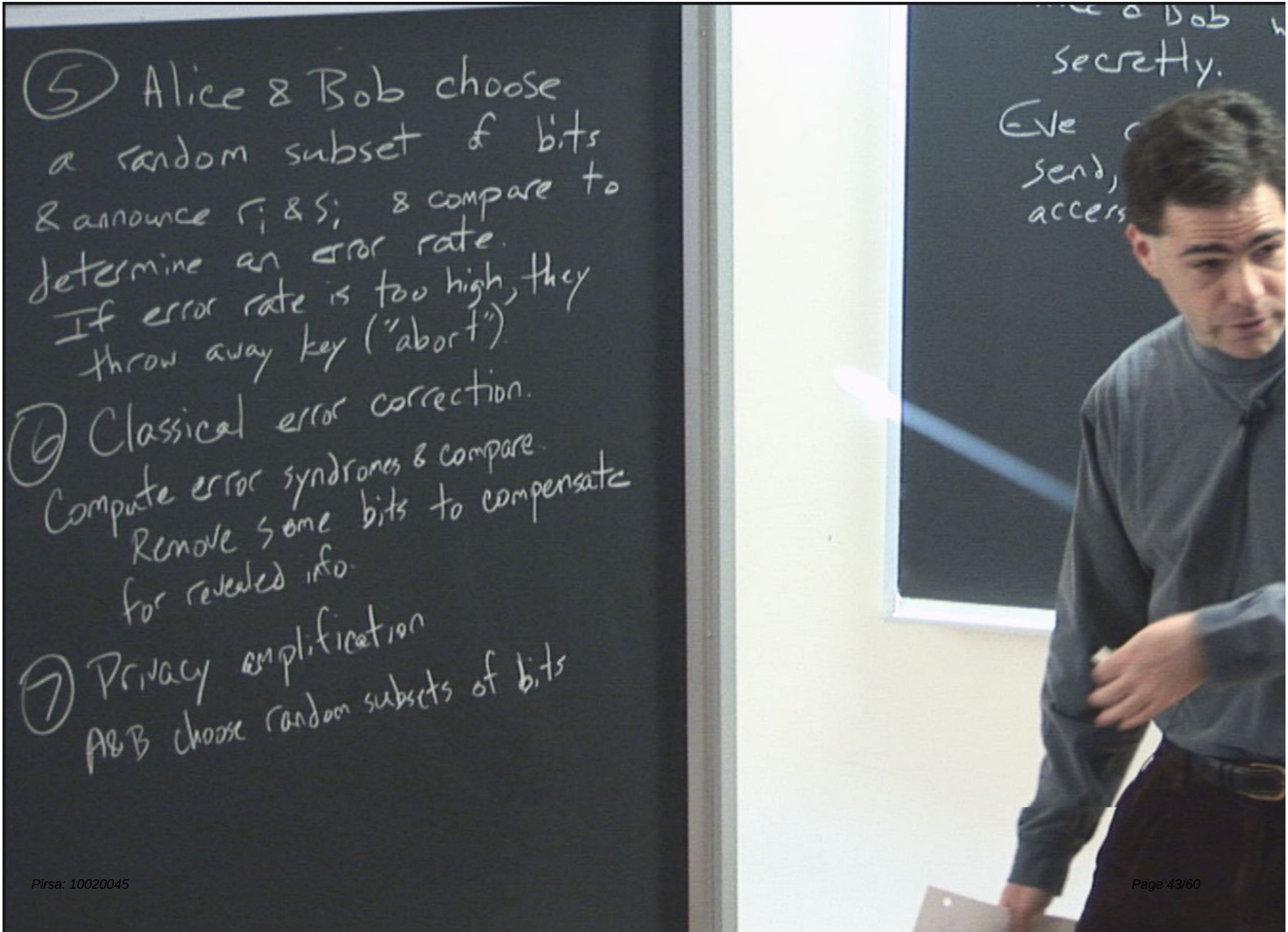
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Compute error syndromes & compare.
Remove some bits to compensate
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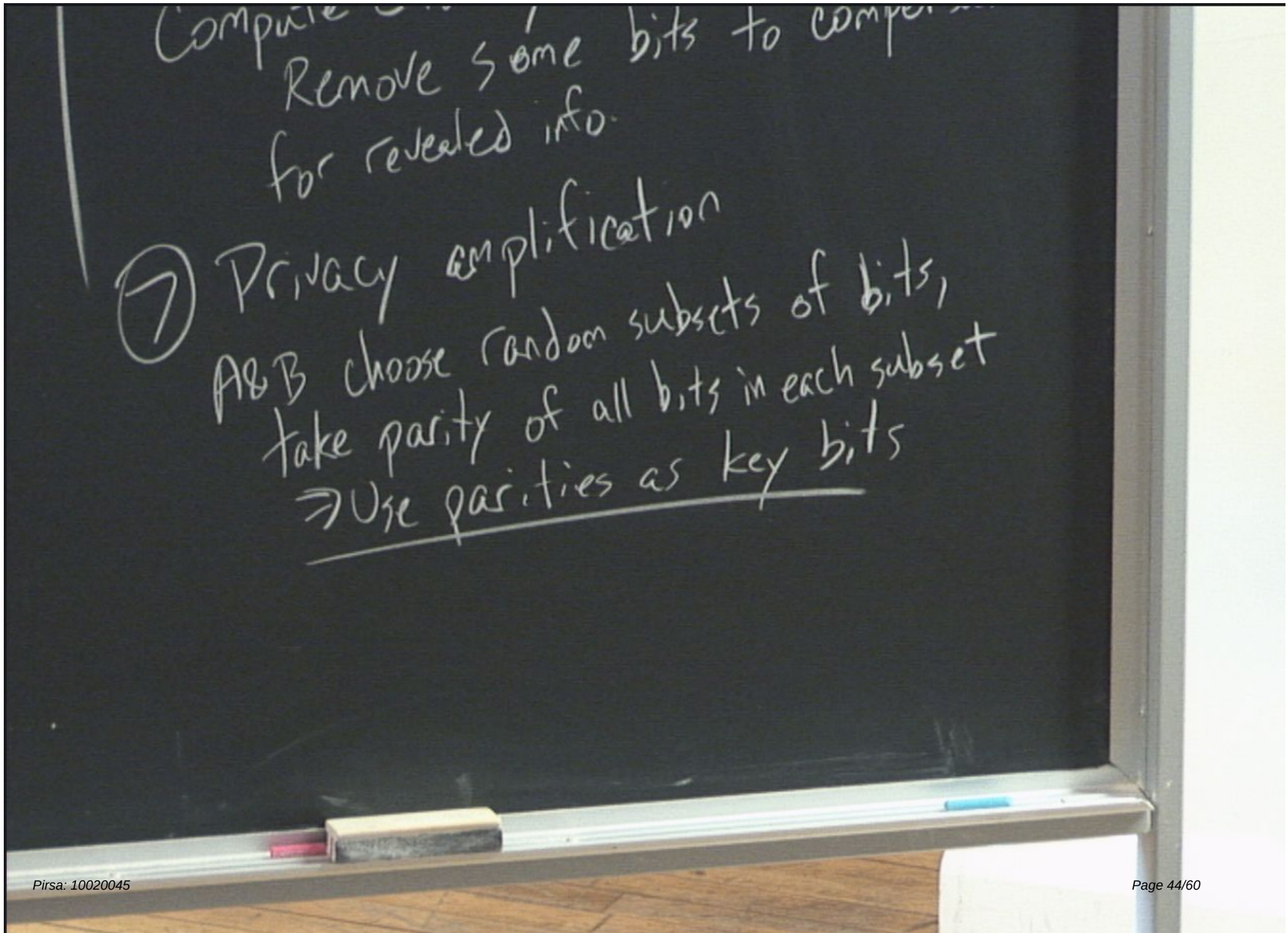
(7) Privacy amplification
A&B choose random subsets of bits
secretly.
Eve
send,
acces


Remove some bits to compen
for revealed info.
⑦ Privacy amplification
A & B choose random subsets of bits,
take parity of all bits in each subset
⇒ Use parities as key bits

Defects of one-time pad

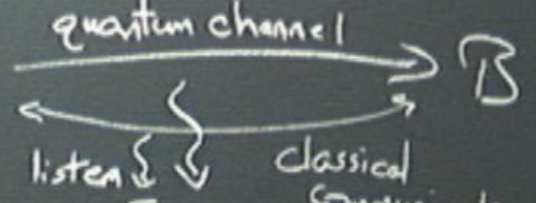
k must be kept secure.

k can only be used once.
needs to be as long as m.

se twice:
$(m_1 \oplus k) \oplus (m_2 \oplus k) = m_1 \oplus m_2$

AES symmetric cryptosystem.

Quantum key distribution (QKD)



BB84 " Bennett Brassard

(1) Alice generate N pairs of
random bits (a_i, r_i) $i = 1, \ldots$

a_i	r_i	
0	0	$\|0\rangle$
0	1	$\|1\rangle$
1	0	$\|+\rangle = \|0\rangle + \|1\rangle$
1	1	$\|-\rangle = \|0\rangle - \|1\rangle$

(2) Alice sends
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(3) Bob ch
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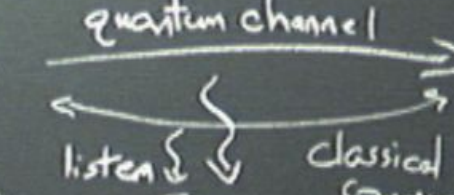

BB84 "Bennett Brassard 1984"

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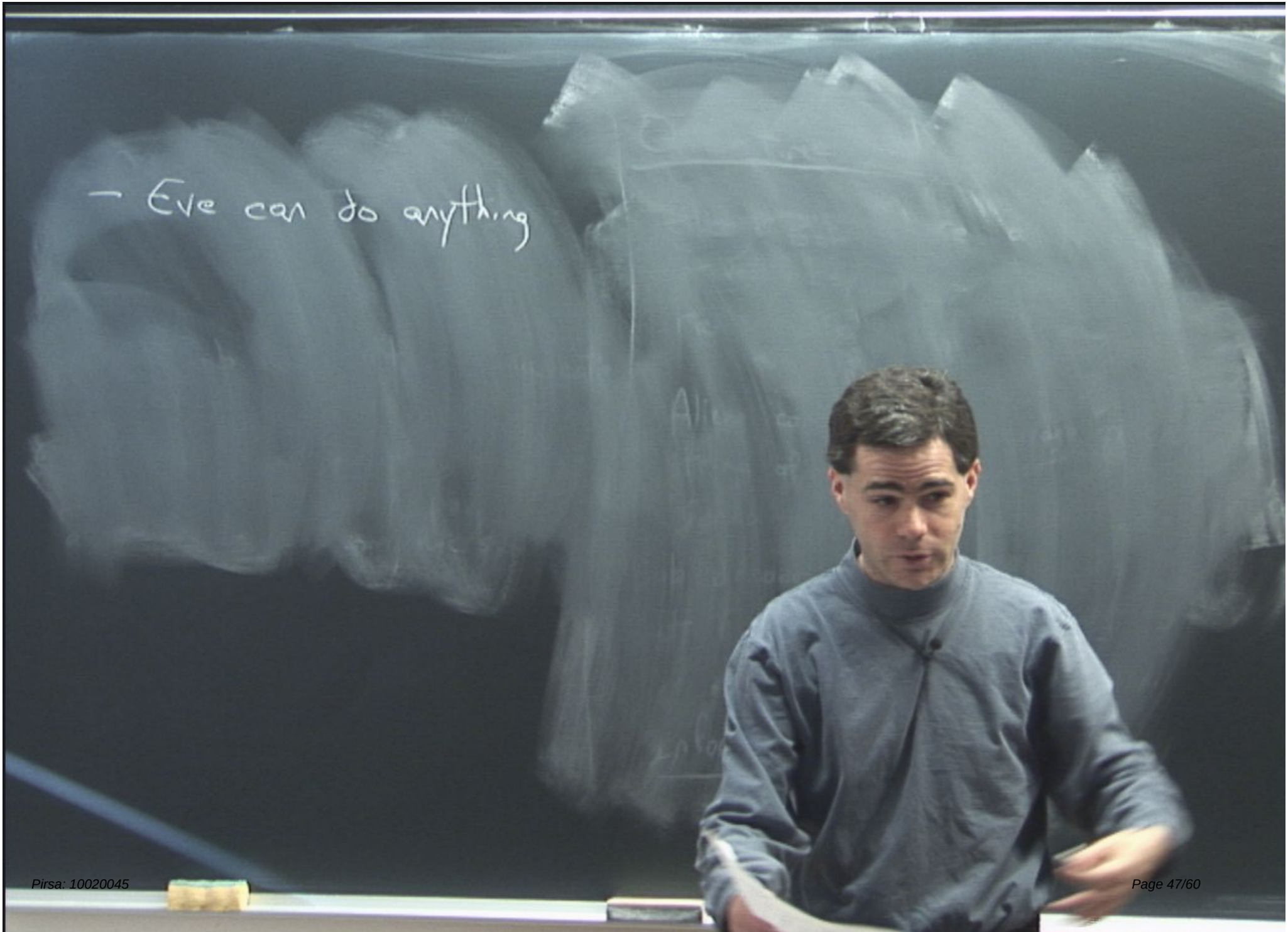
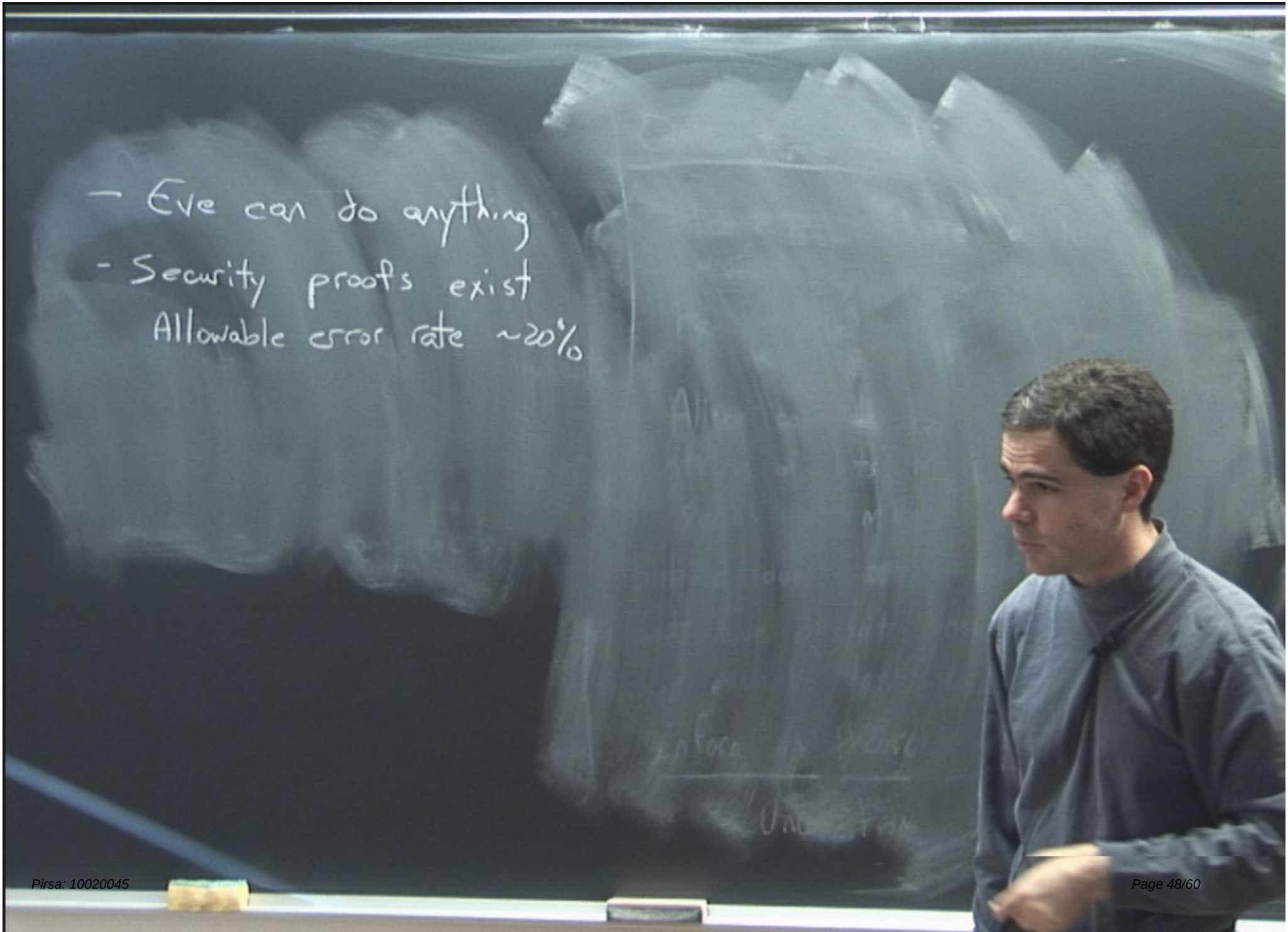
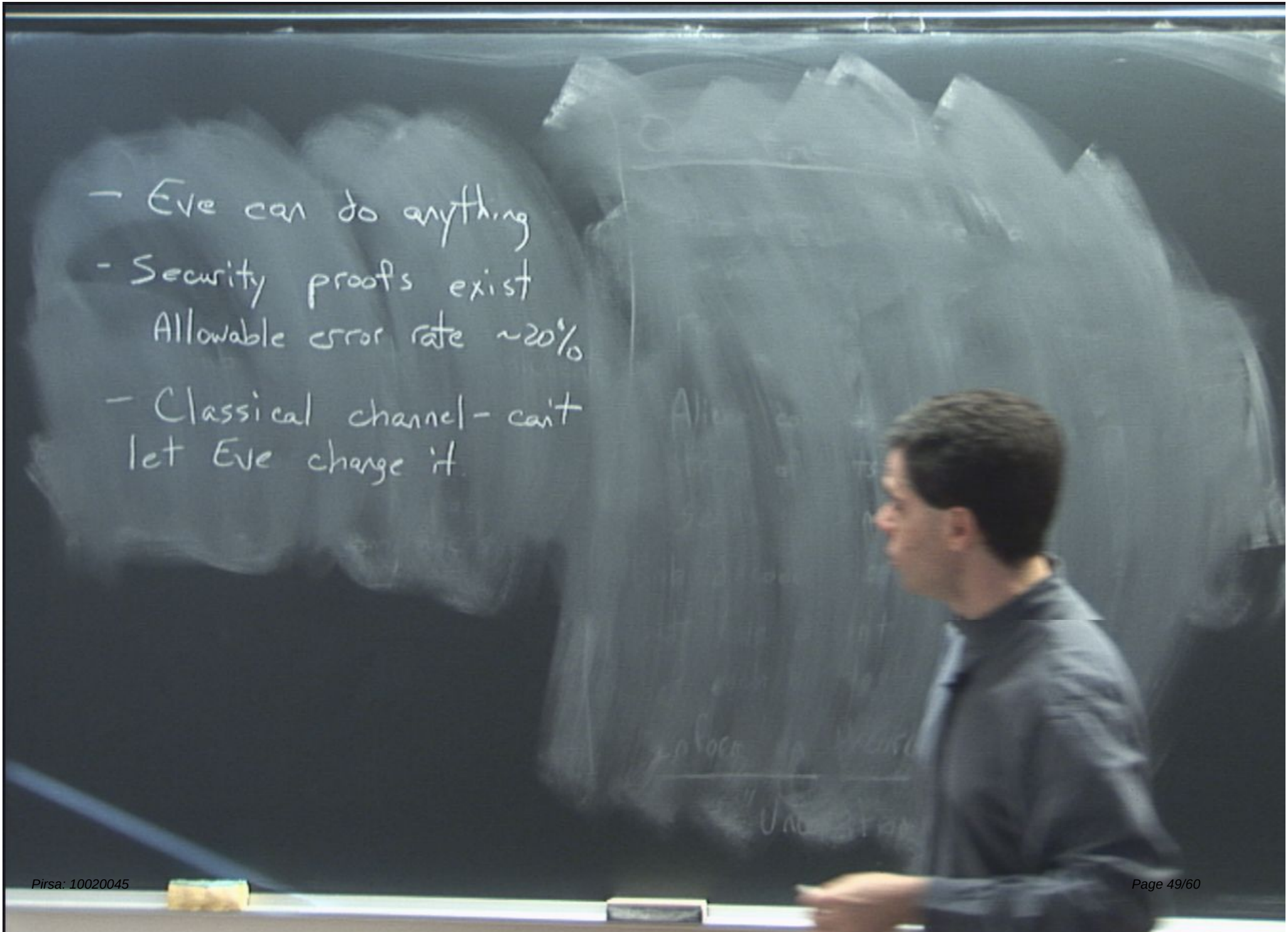
(2) Alice sends the corresponding qubits to Bob for each i.

(3) Bob chooses a random basis for each qubit & measures it in that basis — gets s_i


– Eve can do anything


- Eve can do anything
- Security proofs exist
Allowable error rate ~20%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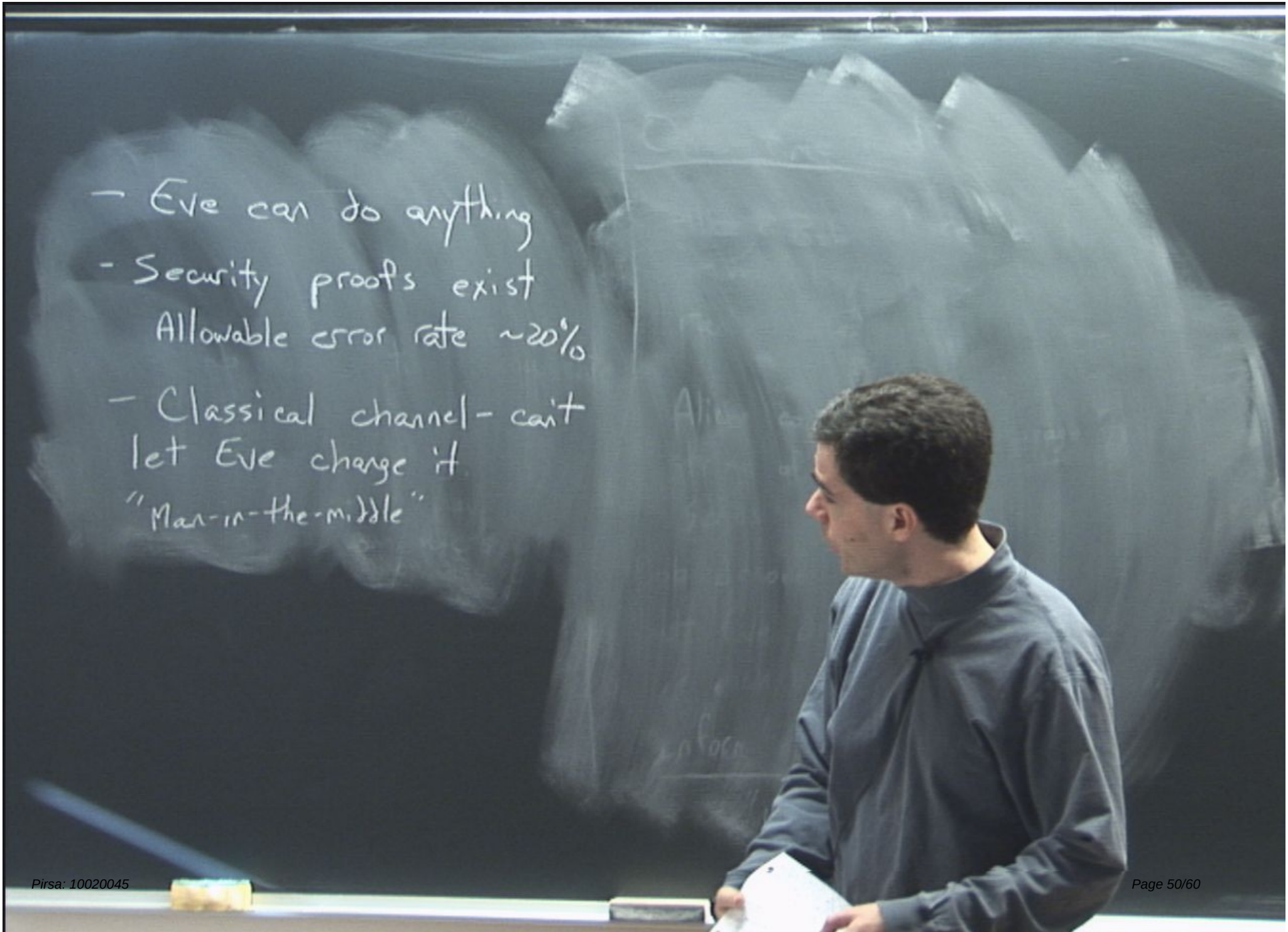
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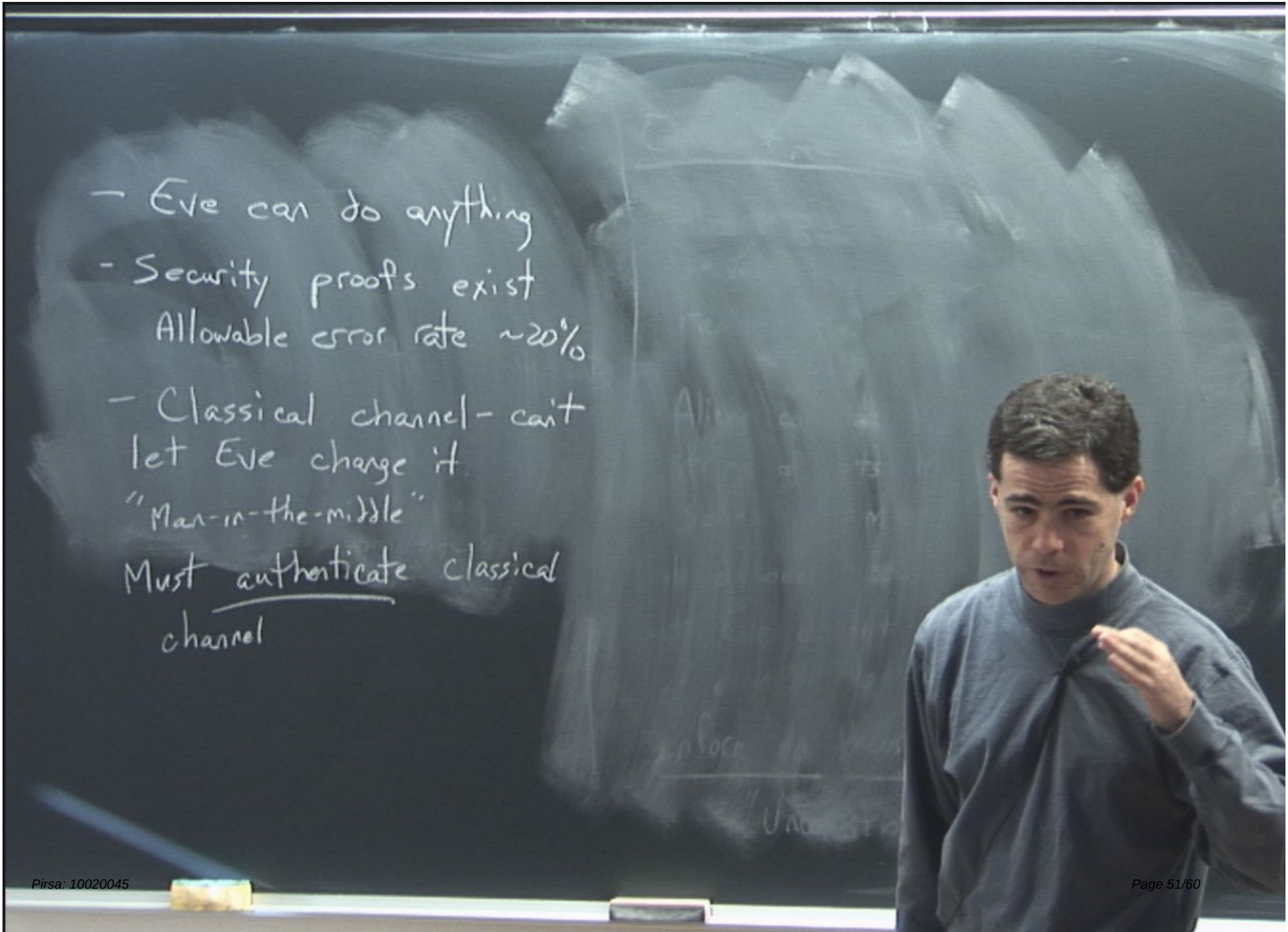
"Man-in-the-middle"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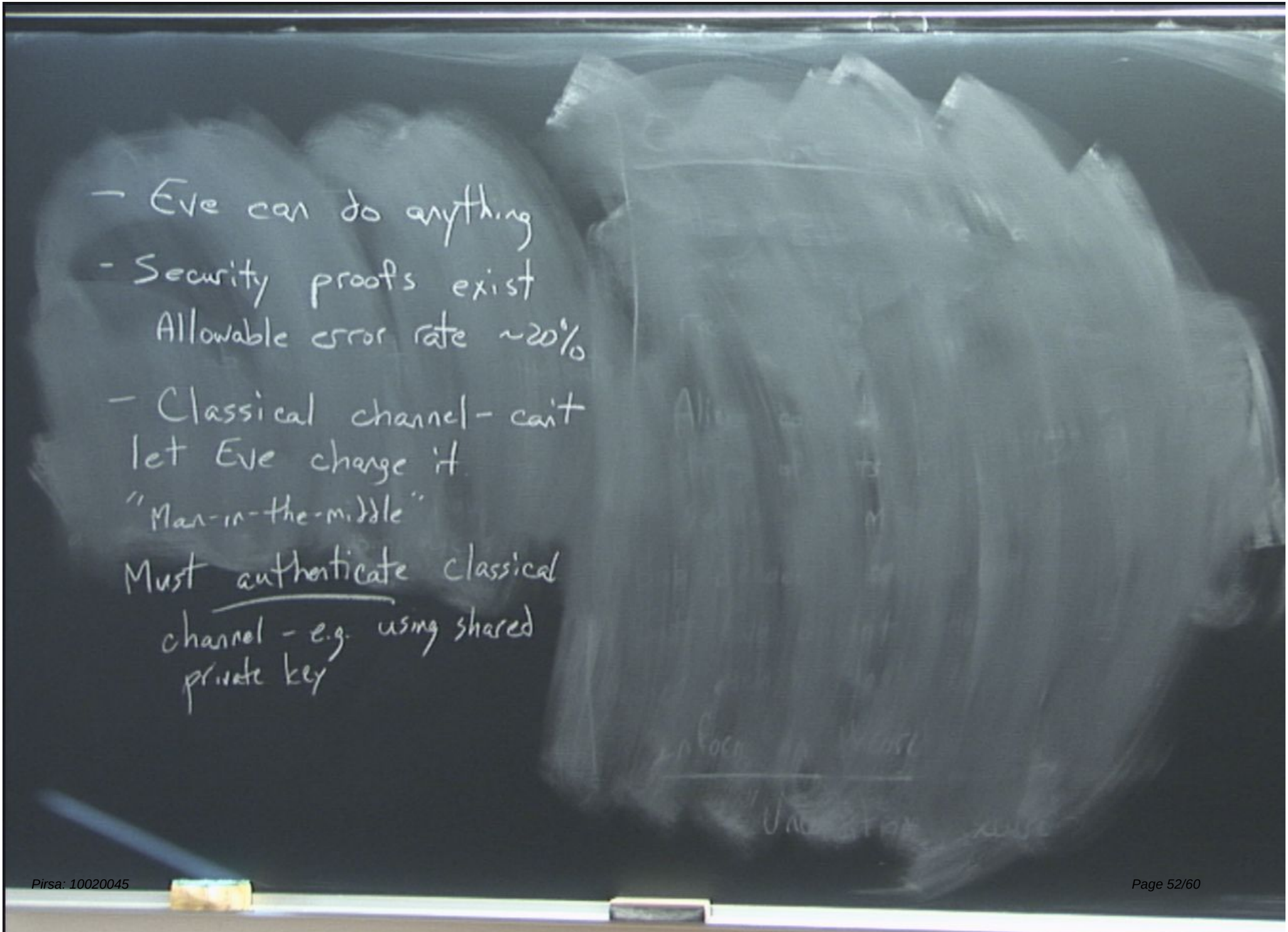
Must authenticate classical channel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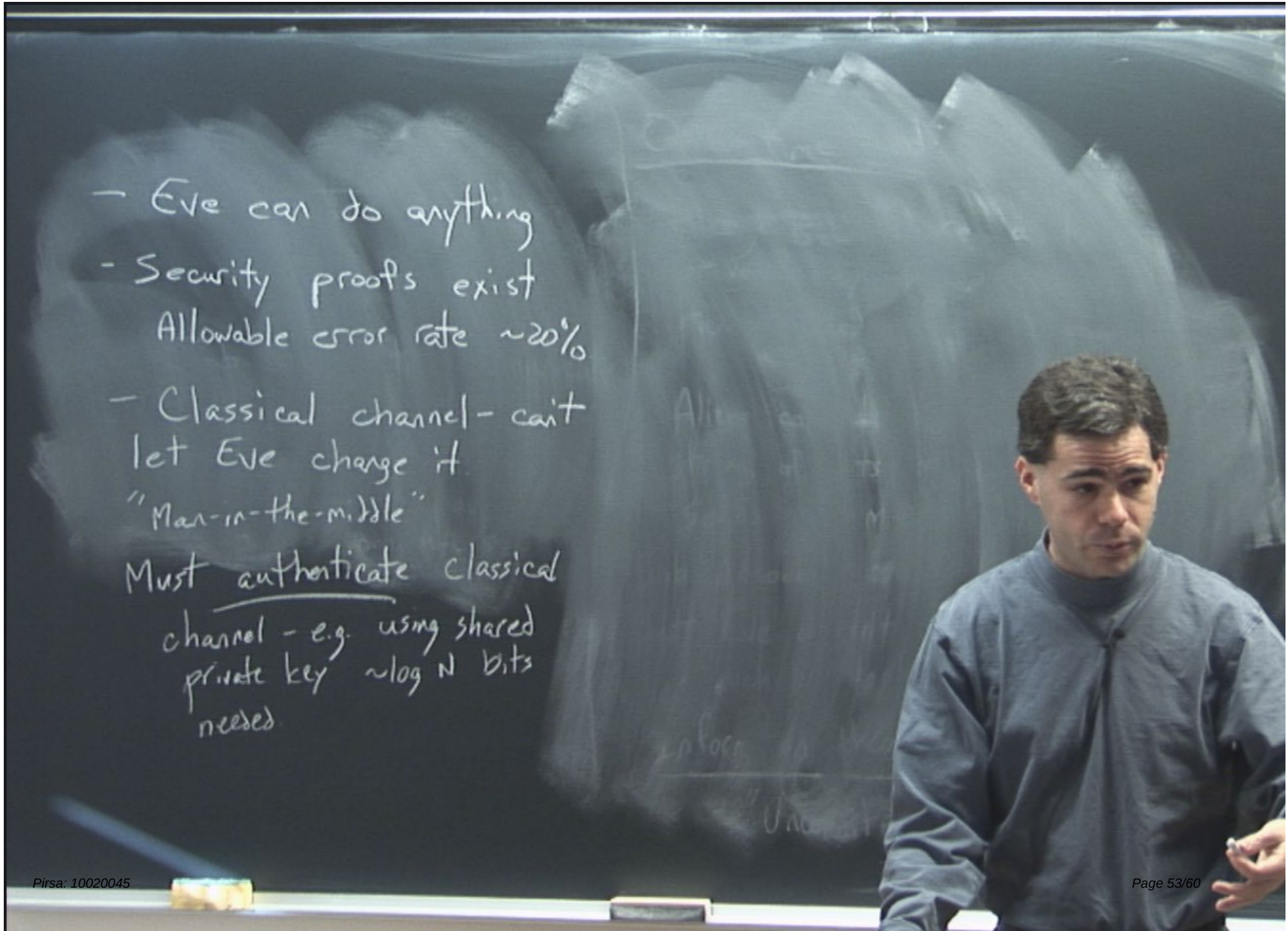
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QKD experimentally
photons through optical fibers

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