Title: Quantum Information - Review (PHYS 635) - Lecture 7

Date: Feb 02, 2010 09:00 AM

URL: http://pirsa.org/10020039

Abstract: Week 1: Basic topics (Qubits, quantum gates, quantum circuits, density matrices, quantum operations, entropy, entanglement) Week 2: Algorithms and complexity (Languages, complexity classes, oracles, RSA, Deutsch-Jozsa algorithm, Shor's algorithm, Grover's algorithm) Week 3: Information theory and implementations (Overview of implementations, quantum error correction, quantum cryptography, quantum information theory)

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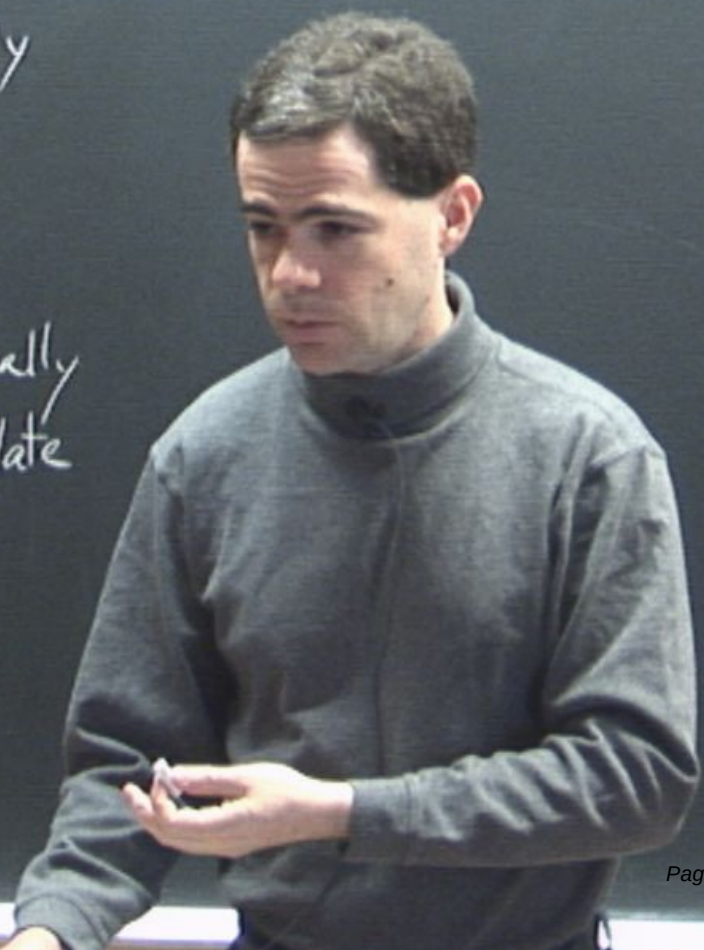
Strong Church-Turing thesis: All physically realistic models of computation can simulate each other using polynomial resources.



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<u>Def.</u>: BQP ("bounded quantum polynomial") is class of languages L s.t. $\exists$ poly-time quantum algorithm $A(x)$ s.t. a) if $x \in L$, $A(x) = \text{yes}$ w/ prob. $\geq \frac{2}{3}$, b) if $x \notin L$, $A(x) = \text{no}$ w/ prob. $\geq \frac{2}{3}$

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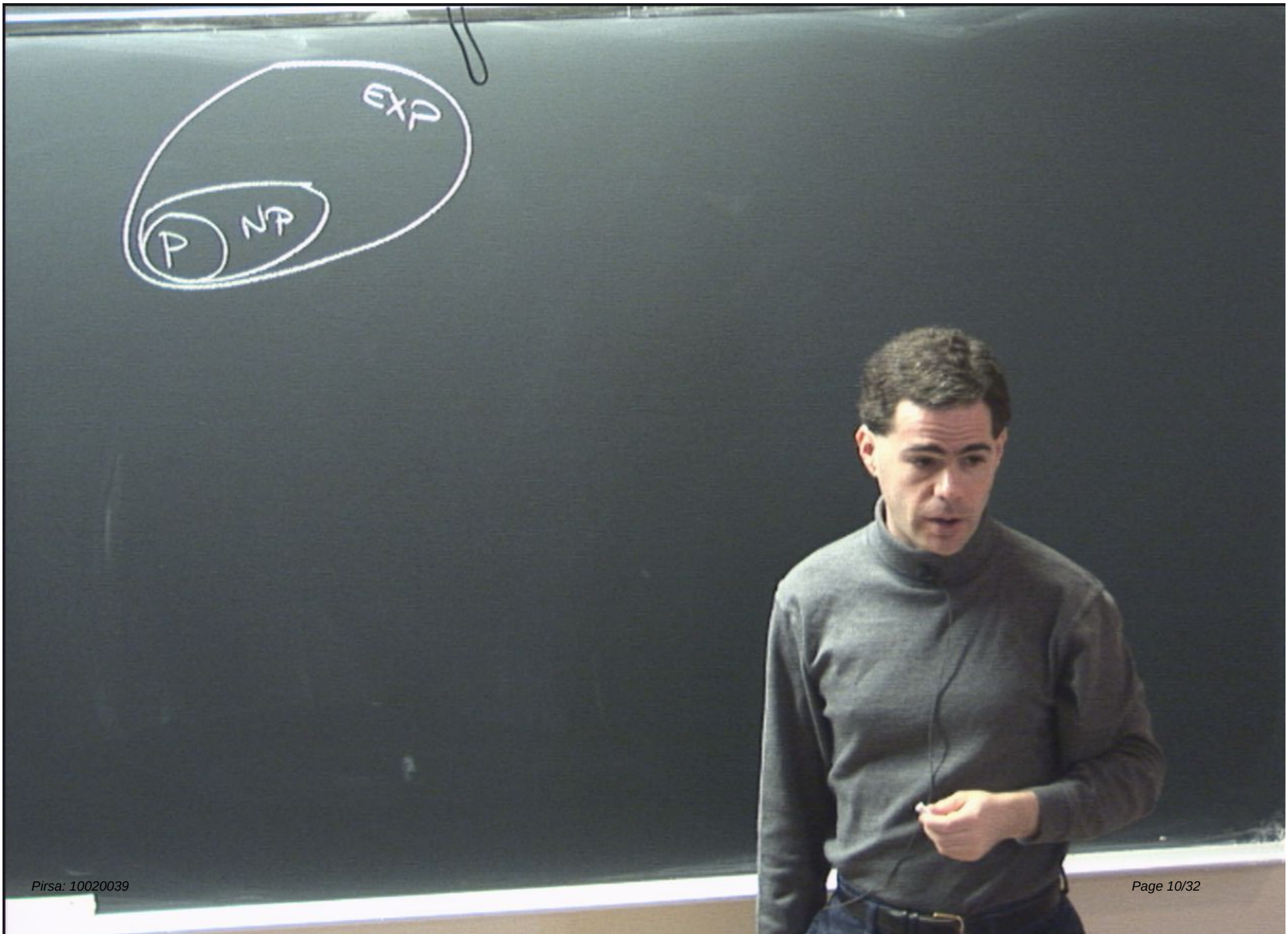
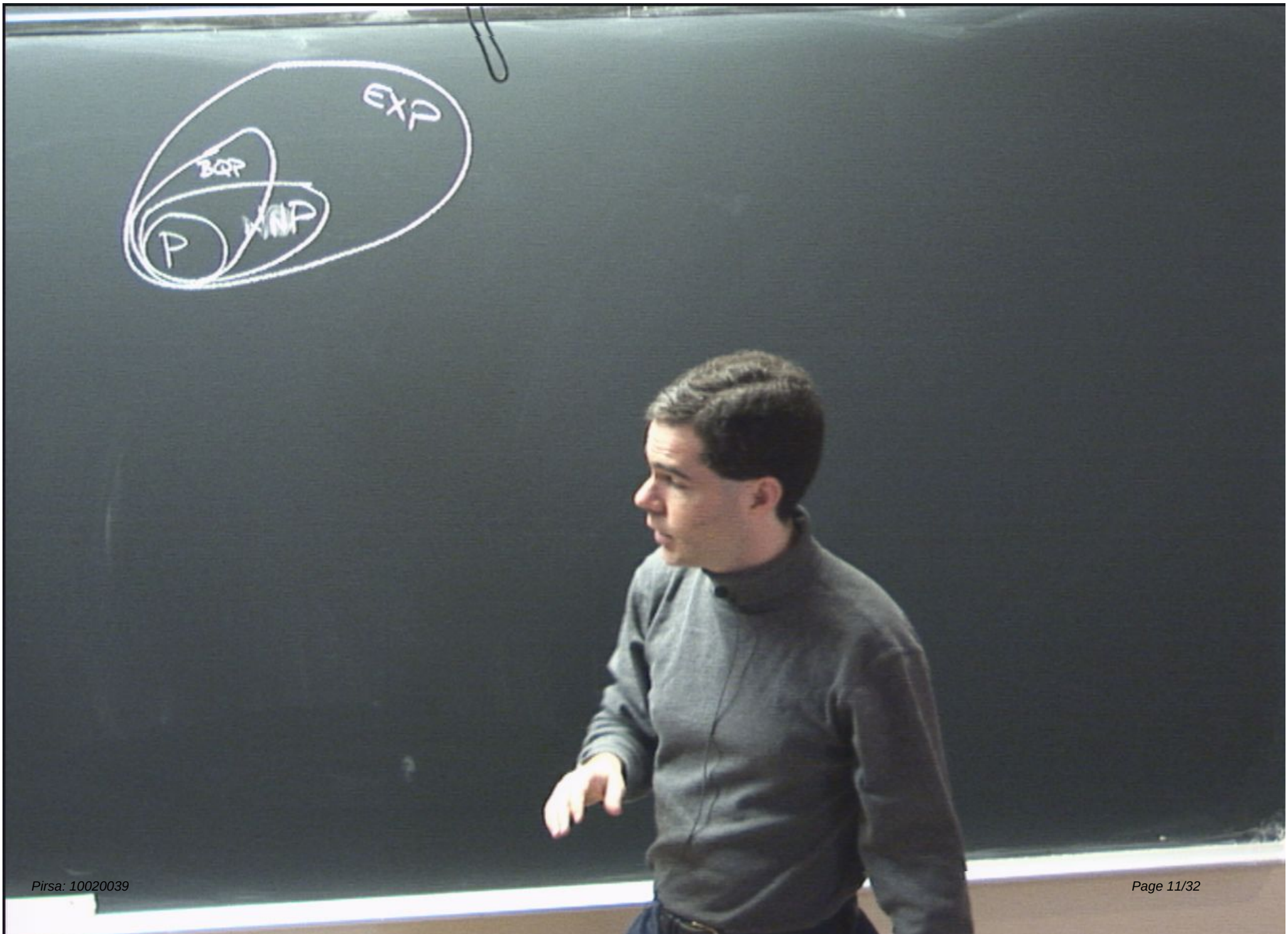

EXP
NP
P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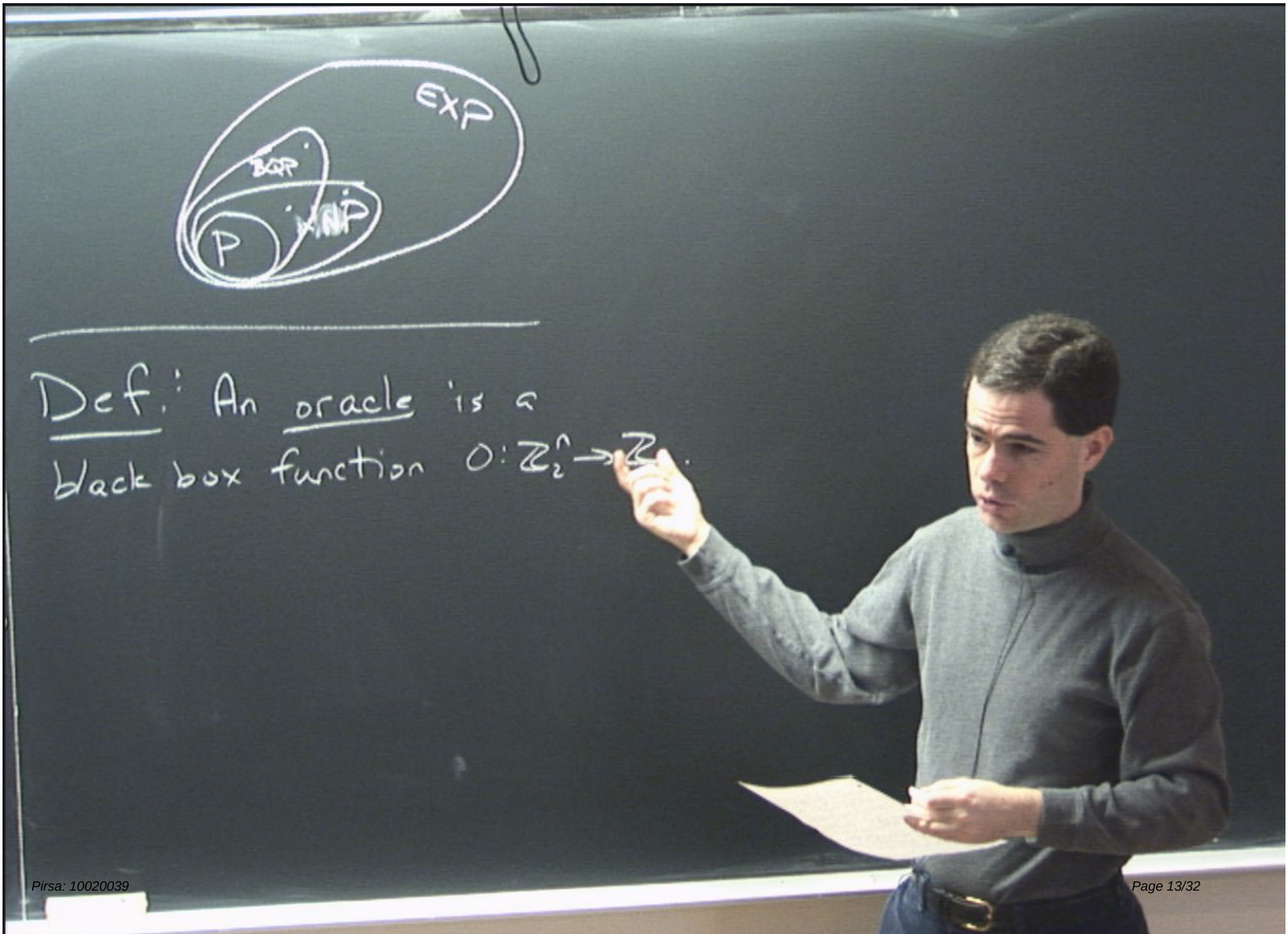
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Def.: An oracle is a
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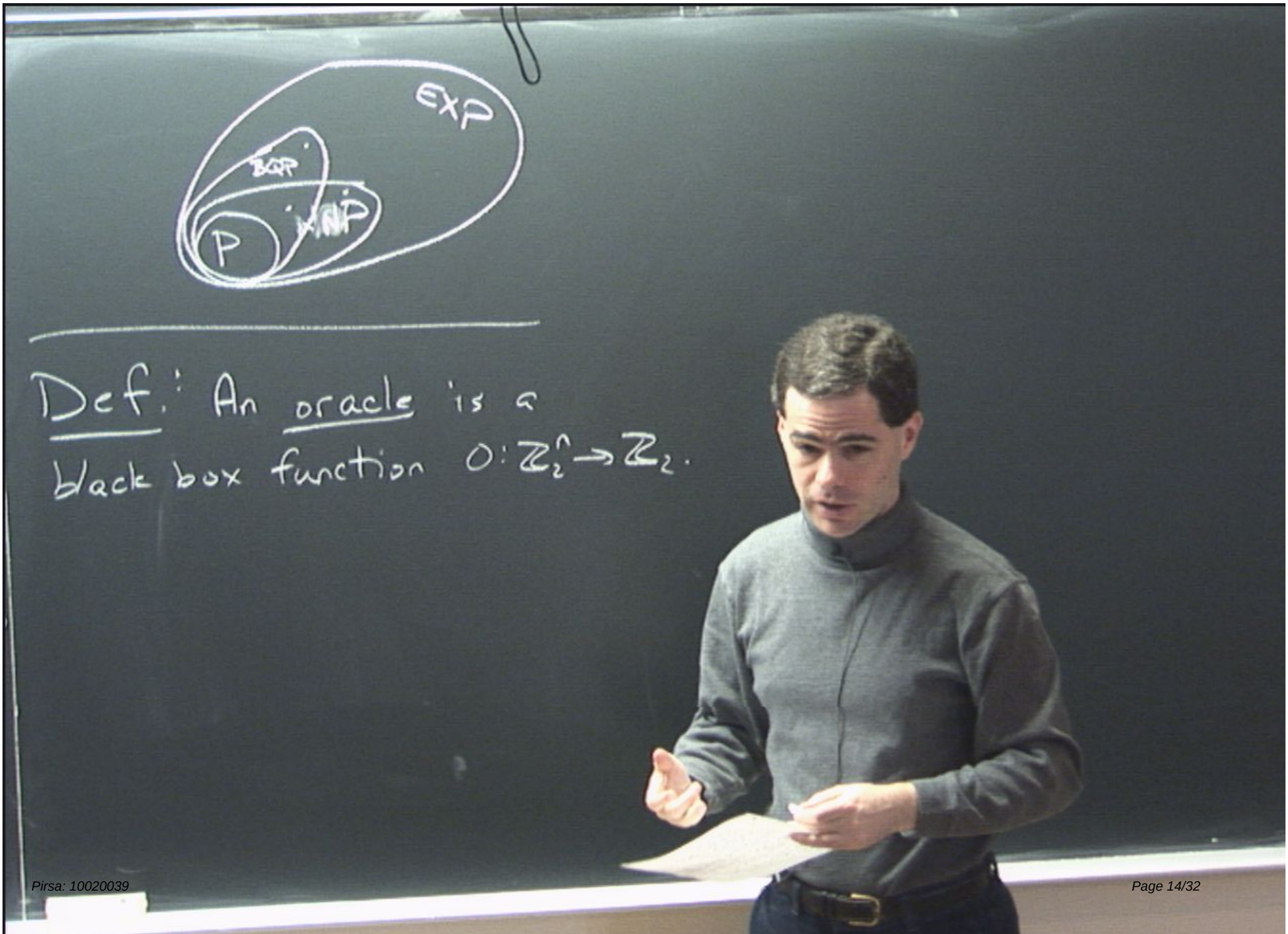
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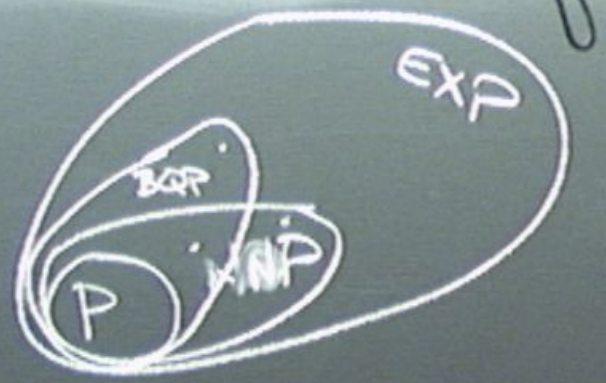
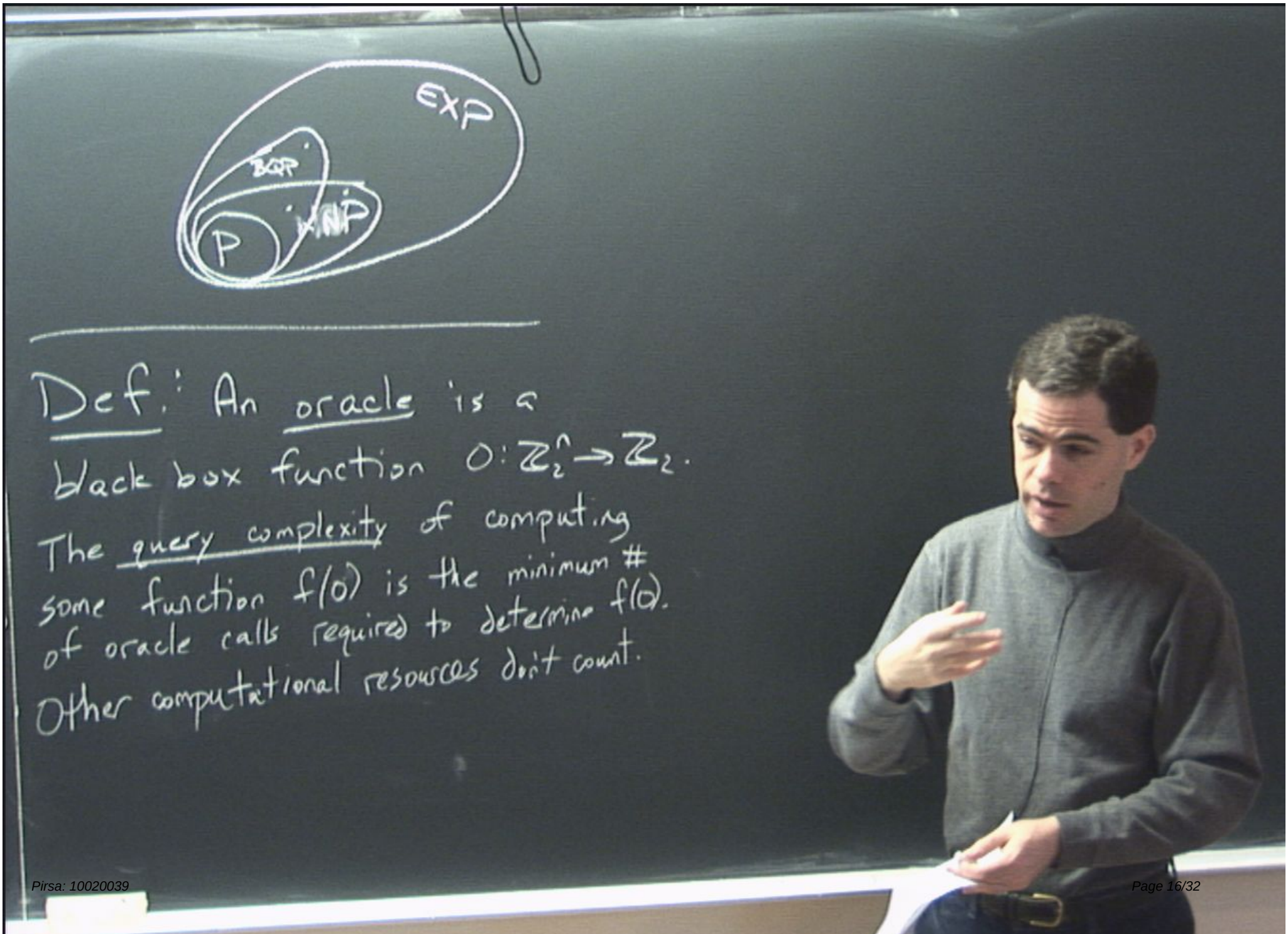


Def.: An <u>oracle</u> is a black box function $O: \mathbb{Z}_2^n \to \mathbb{Z}_2$.

The <u>query complexity</u> of computing some function $f(O)$ is the minimum # of oracle calls required to determine $f(O)$.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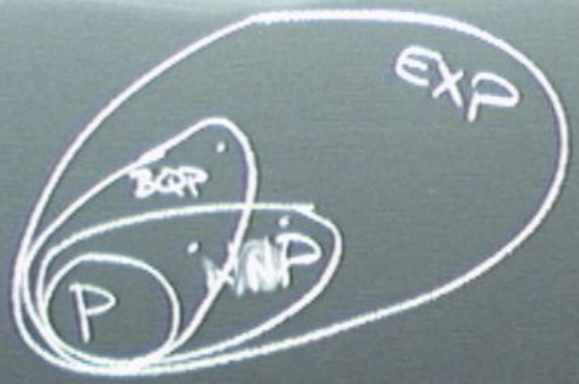
Other computational resources don't count.



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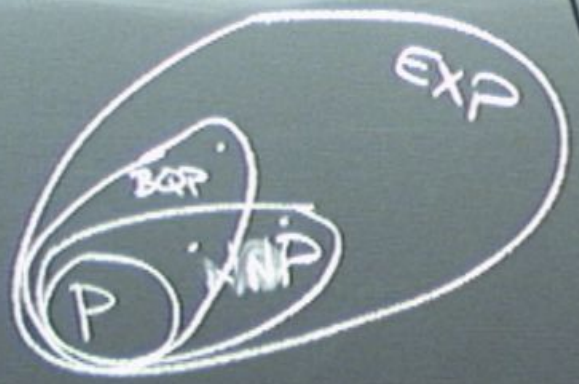

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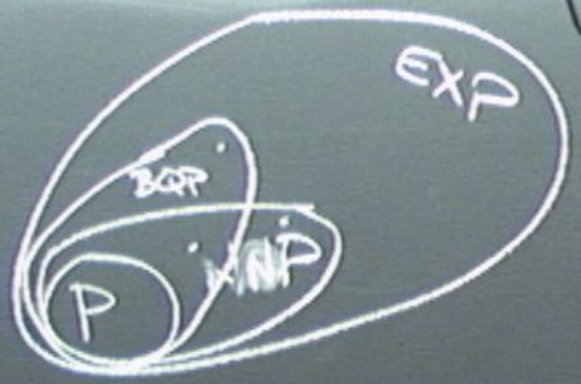

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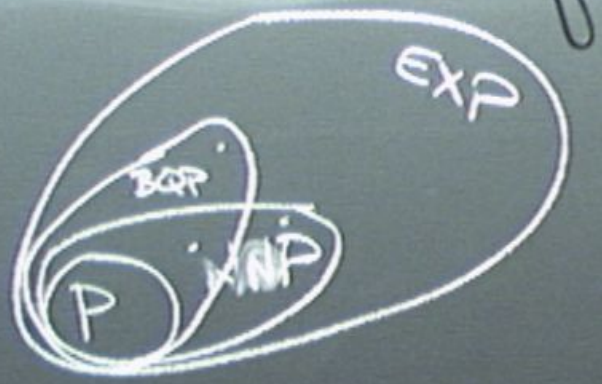

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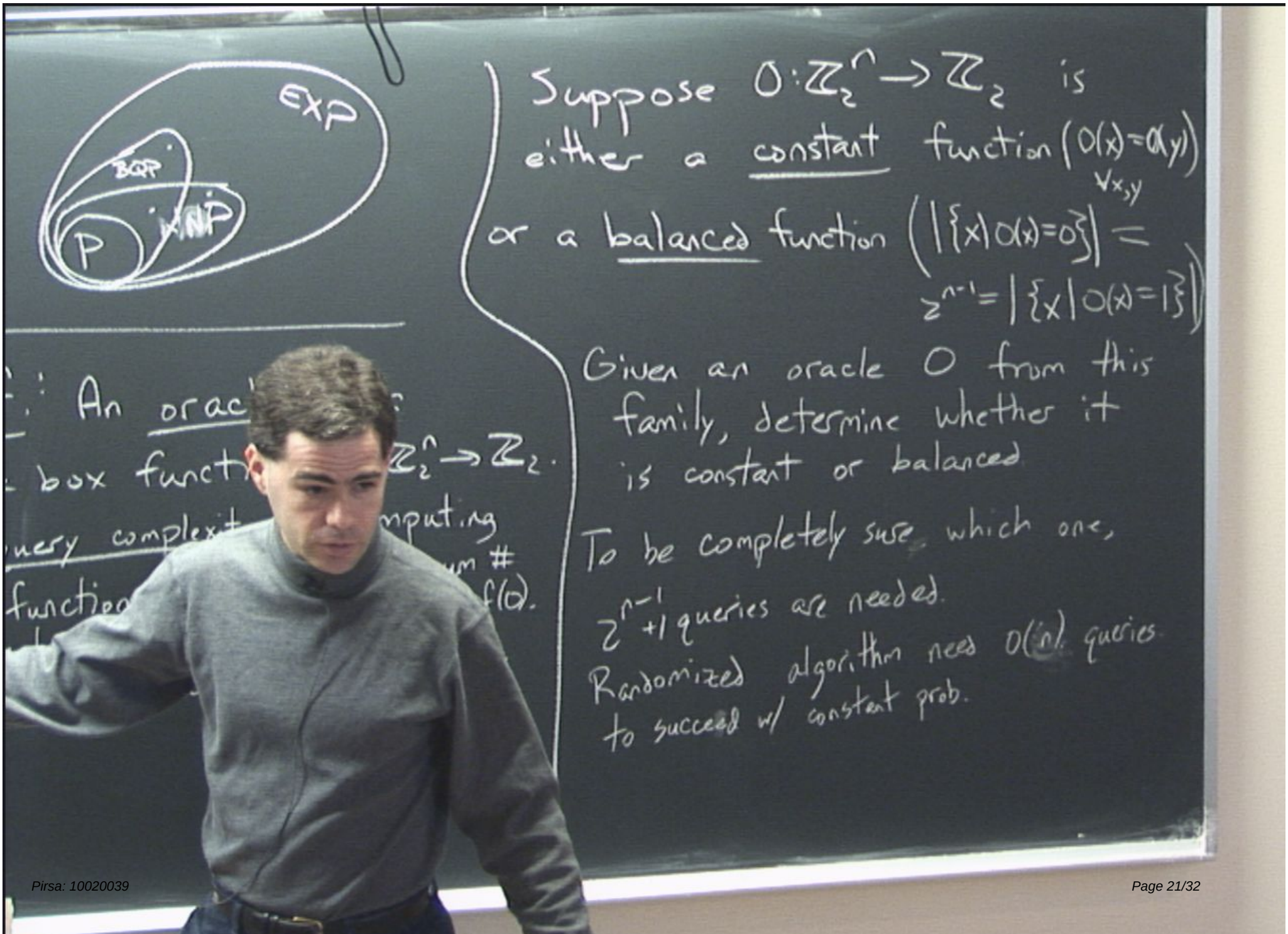
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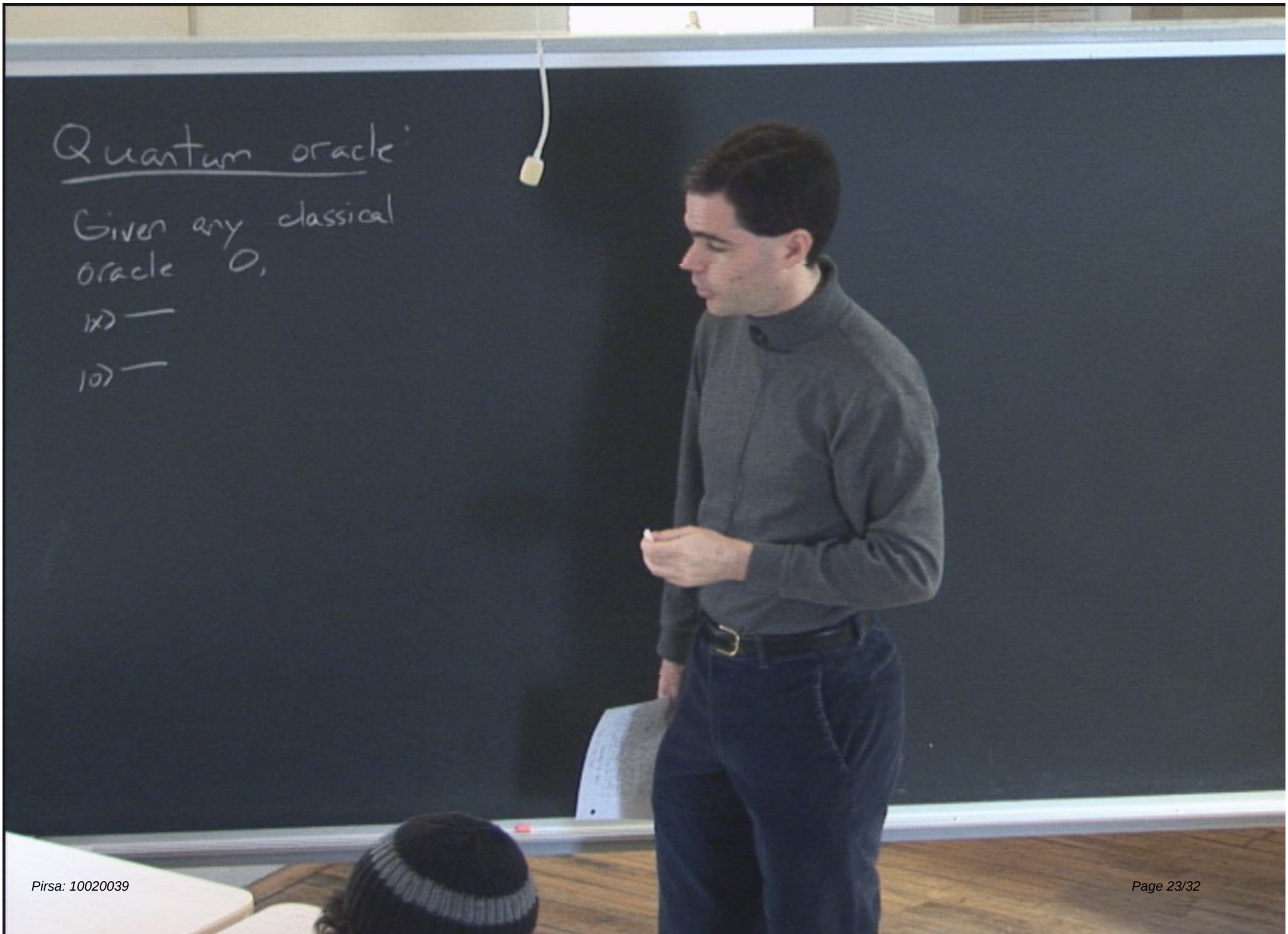
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|x⟩
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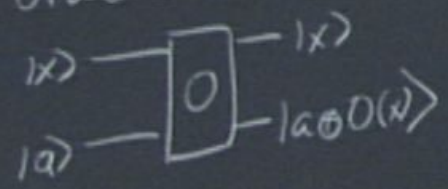

Here's a quantum algorithm for the constant/balanced oracle problem

Deutsch-Jozsa algorithm

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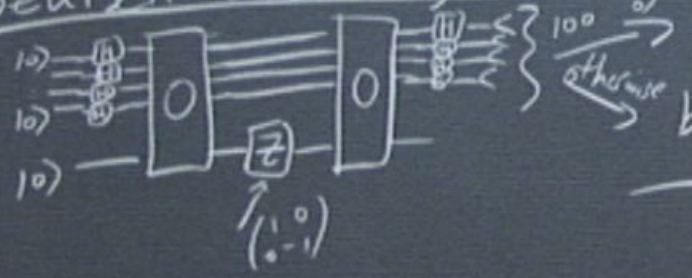

Constant:

$|0\ldots0\rangle|0\rangle \rightarrow$

oracle:

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$O(x)\rangle$

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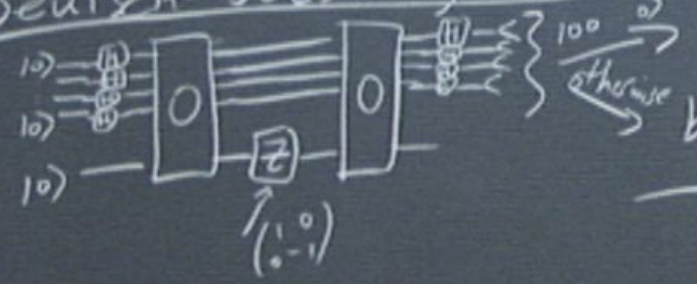
Deutsch-Jozsa algorithm



Constant:

$|0\rangle \quad 0\rangle|0\rangle \to \left(\sum_x |x\rangle\right) \otimes |0\rangle$

$\to \left(\sum_x |x\rangle \otimes |O(x)\rangle\right) = \left(\sum_x |x\rangle\right) \otimes |O(x)\rangle$

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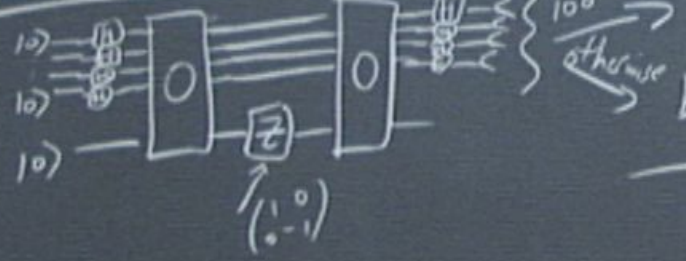

Constant:

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$$\to \left(\sum_x |x\rangle \otimes |O(x)\rangle\right) = \left(\sum_x |x\rangle\right)\otimes|O(0)\rangle$$

$$\to (-1)^{O(0)}\left(\sum_x |x\rangle\right)\otimes|O(0)\rangle$$

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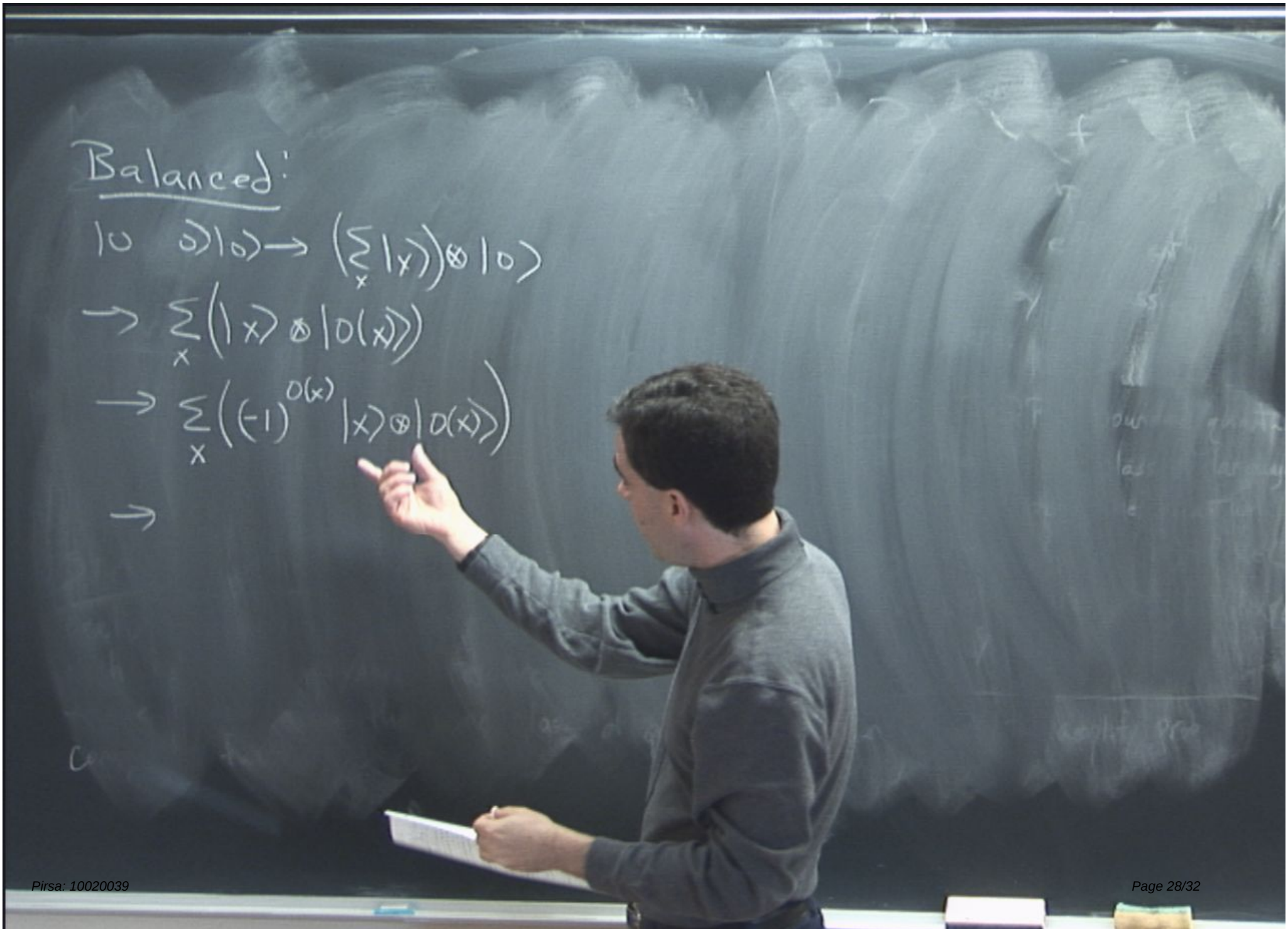
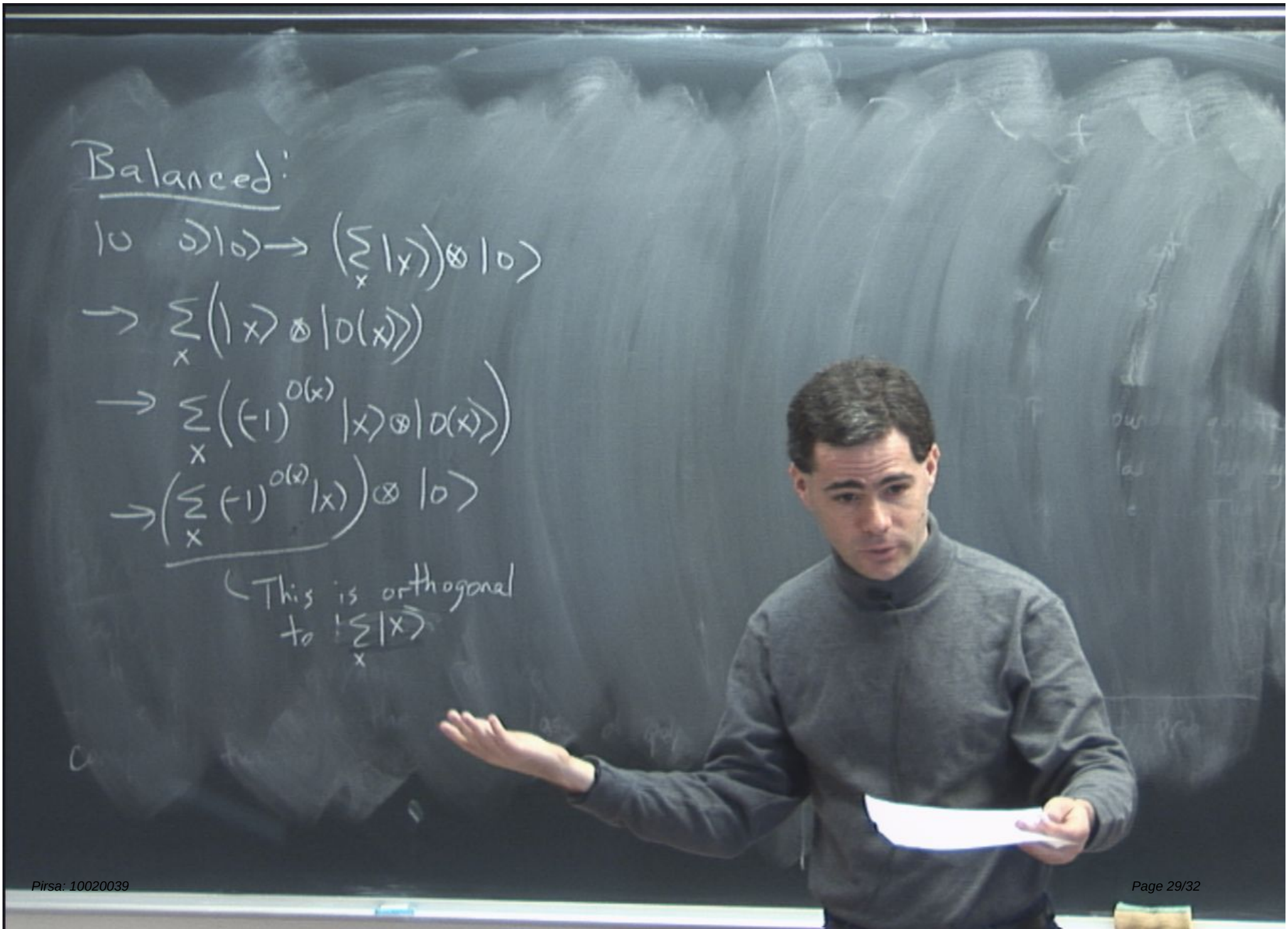
$$\to (-1)^{O(0)}|0\ldots0\rangle\otimes|0\rangle$$


Balanced:
$|0\ \ 0\rangle|0\rangle \rightarrow \left(\sum_x |x\rangle\right) \otimes |0\rangle$
$\rightarrow \sum_x \left(|x\rangle \otimes |0(x)\rangle\right)$
$\rightarrow \sum_x \left((-1)^{0(x)} |x\rangle \otimes |0(x)\rangle\right)$
$\rightarrow$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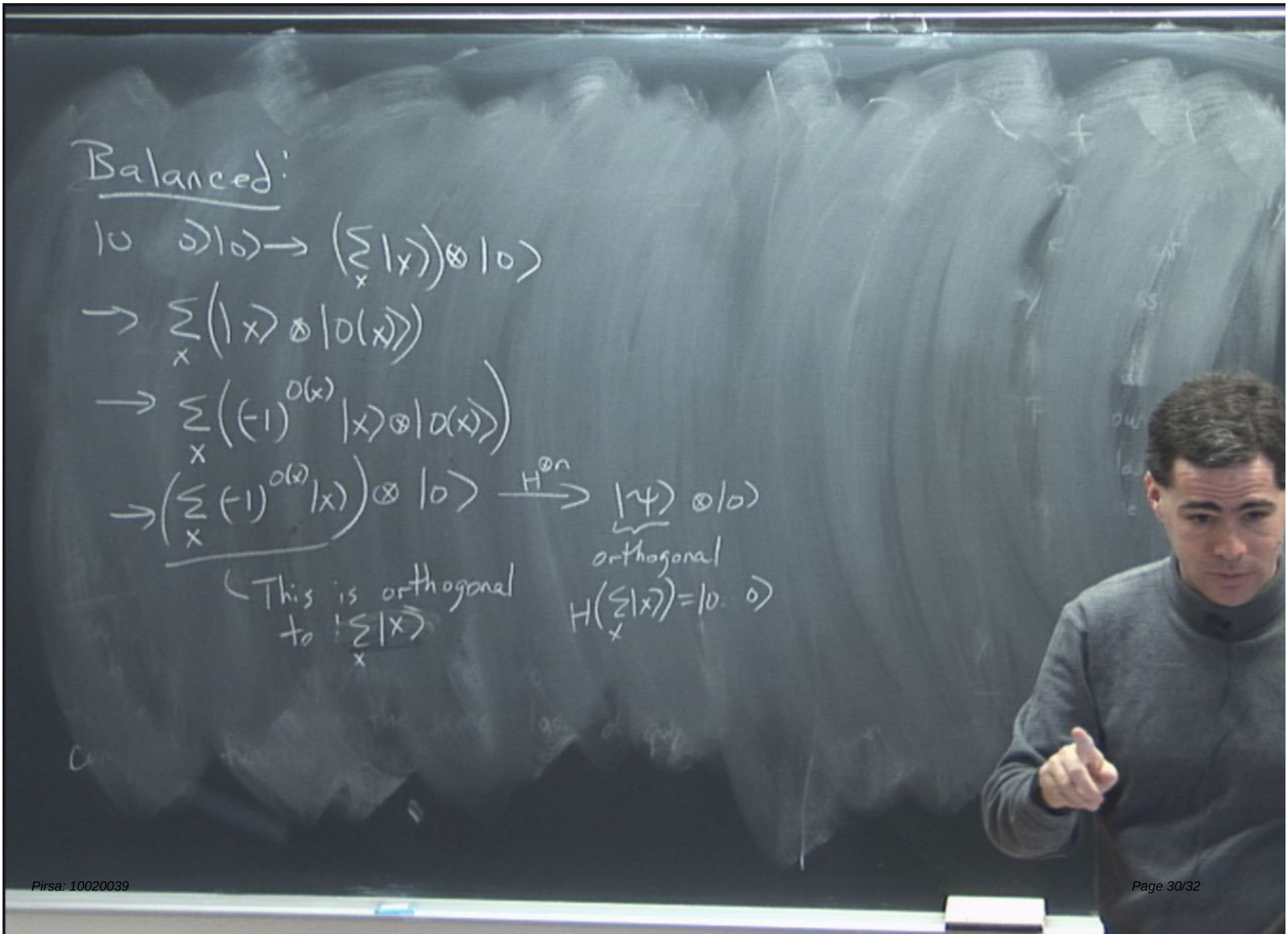
This is orthogonal to $\sum_x |x\rangle$


Balanced:
$|0\ldots0\rangle|0\rangle \to \left(\sum_x |x\rangle\right)\otimes|0\rangle$
$\to \sum_x \left(|x\rangle \otimes |O(x)\rangle\right)$
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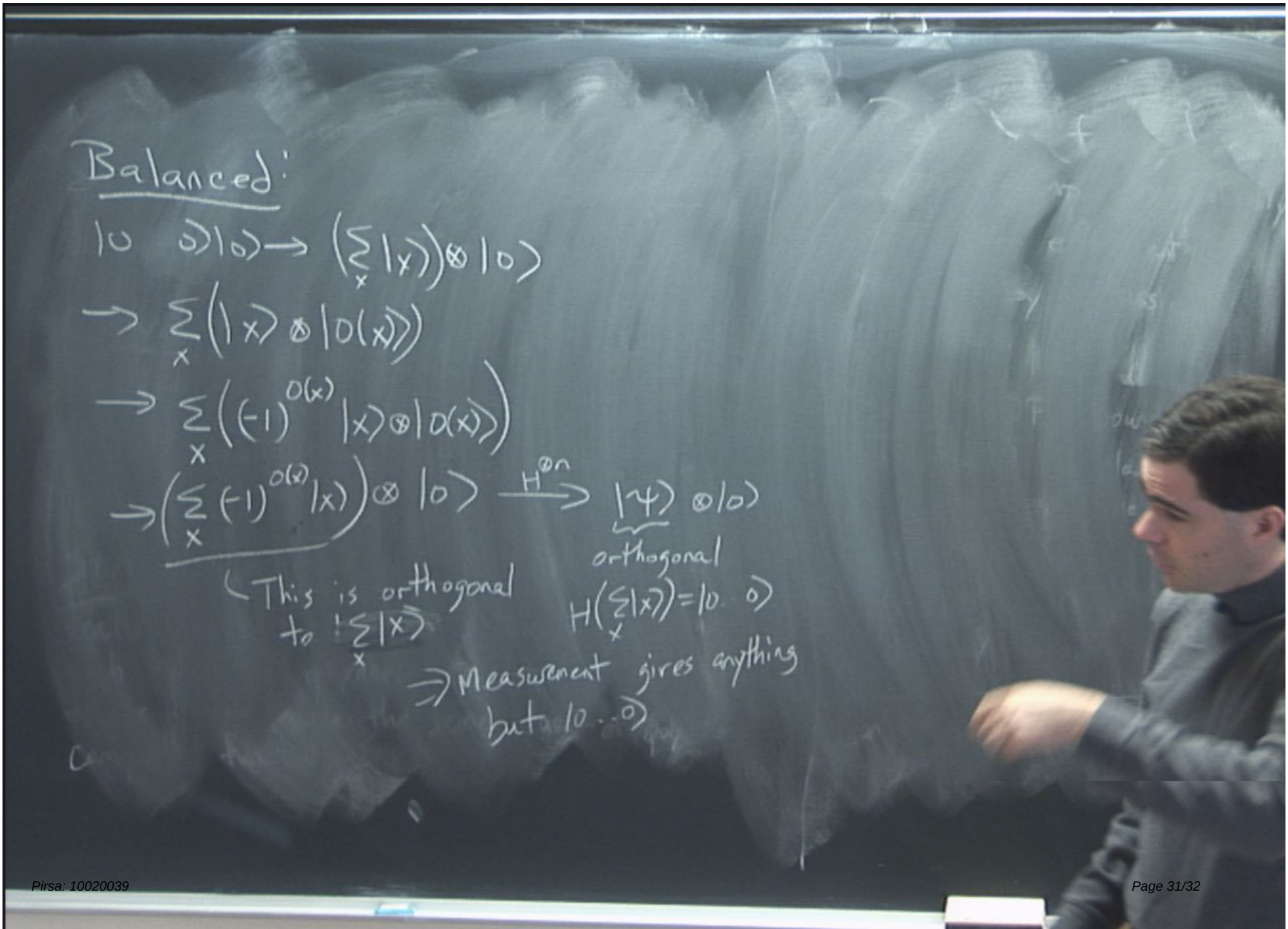
orthogonal
This is orthogonal to $\sum_x |x\rangle$
$H\left(\sum_x |x\rangle\right) = |0\ldots0\rangle$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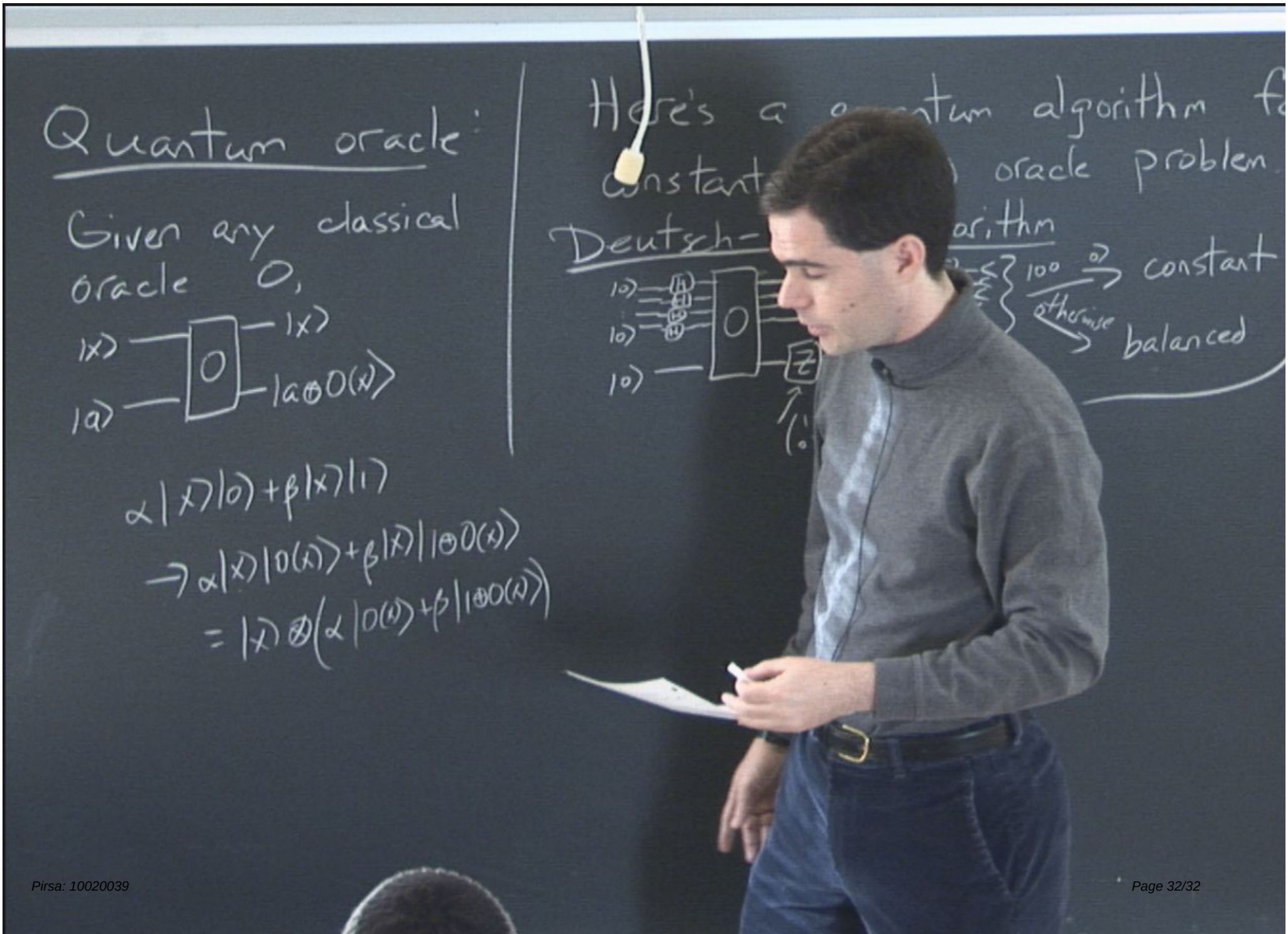
orthogonal
This is orthogonal to $\sum_x |x\rangle$
$H\left(\sum_x |x\rangle\right) = |0\ \ldots\ 0\rangle$
$\Rightarrow$ Measurement gives anything but $|0\ldots 0\rangle$


Quantum oracle:
Given any classical oracle O,
|x⟩
|a⟩
O
|x⟩
|a⊕O(x)⟩
α|x⟩|0⟩+β|x⟩|1⟩
→ α|x⟩|O(x)⟩+β|x⟩|1⊕O(x)⟩
= |x⟩⊗(α|O(x)⟩+β|1⊕O(x)⟩)
Here's a quantum algorithm
constant
oracle problem
Deutsch-
orithm
|0⟩
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|0⟩
O
Z
100
constant
otherwise
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