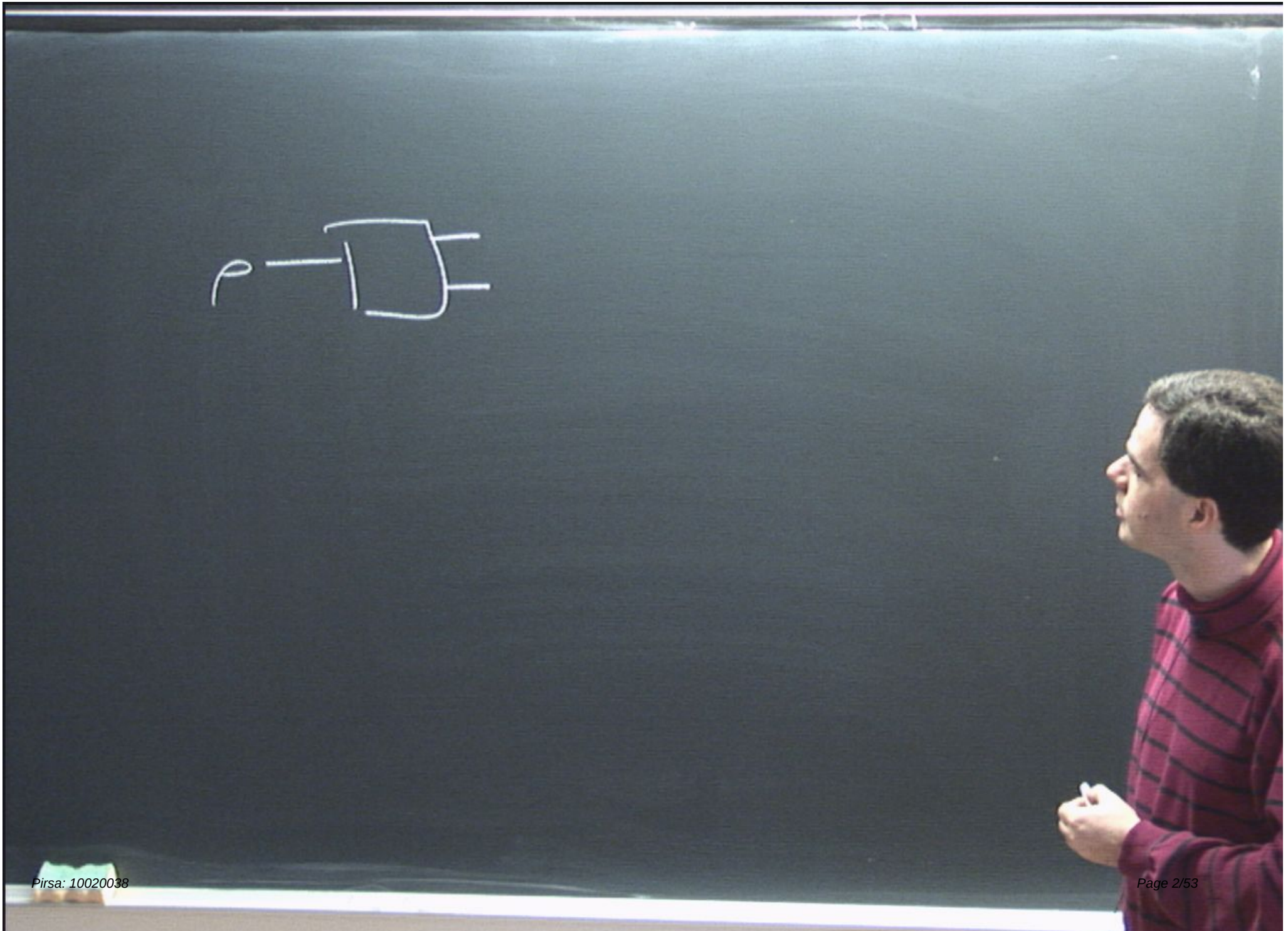
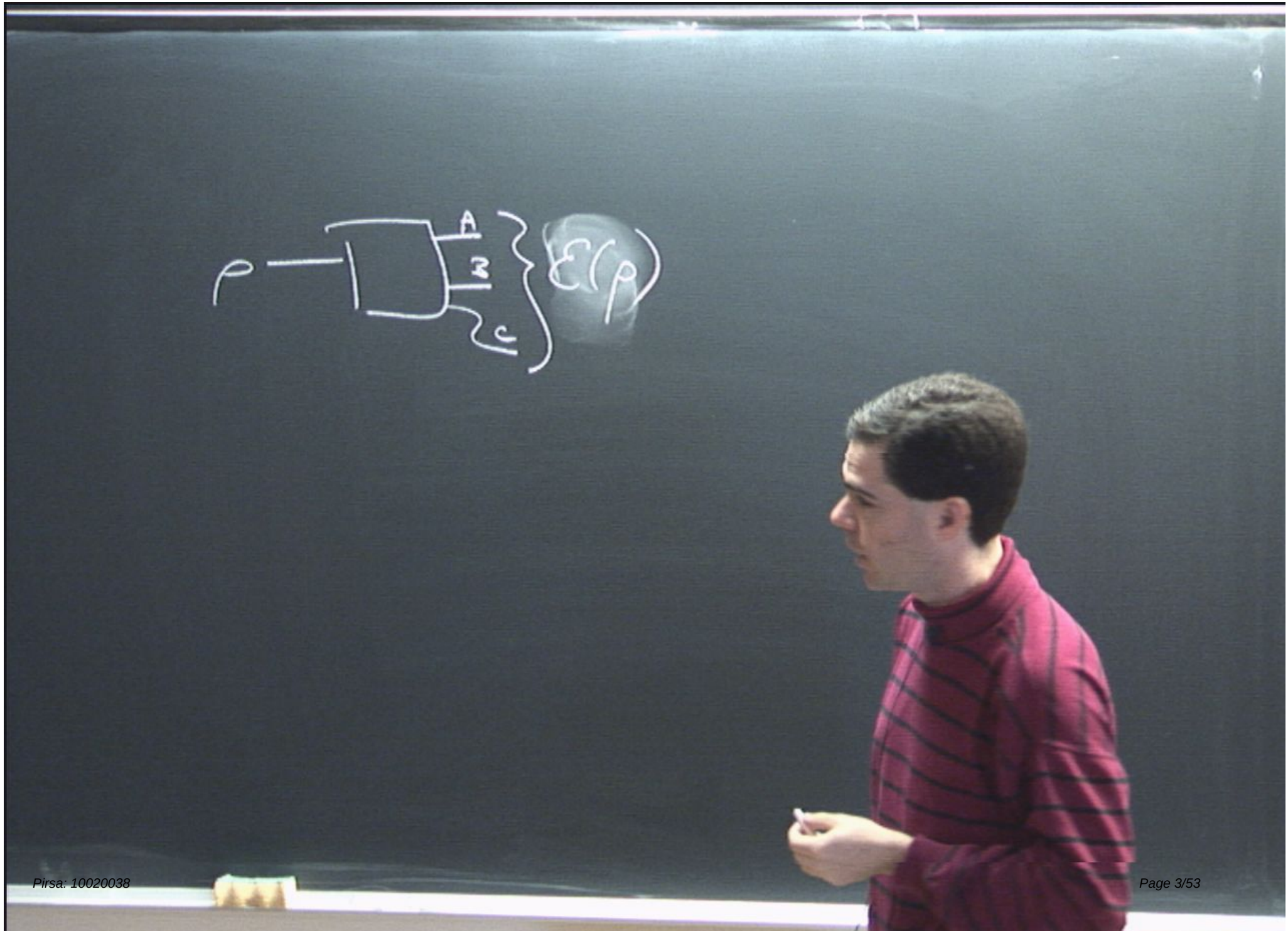
Title: Quantum Information - Review (PHYS 635) - Lecture 6

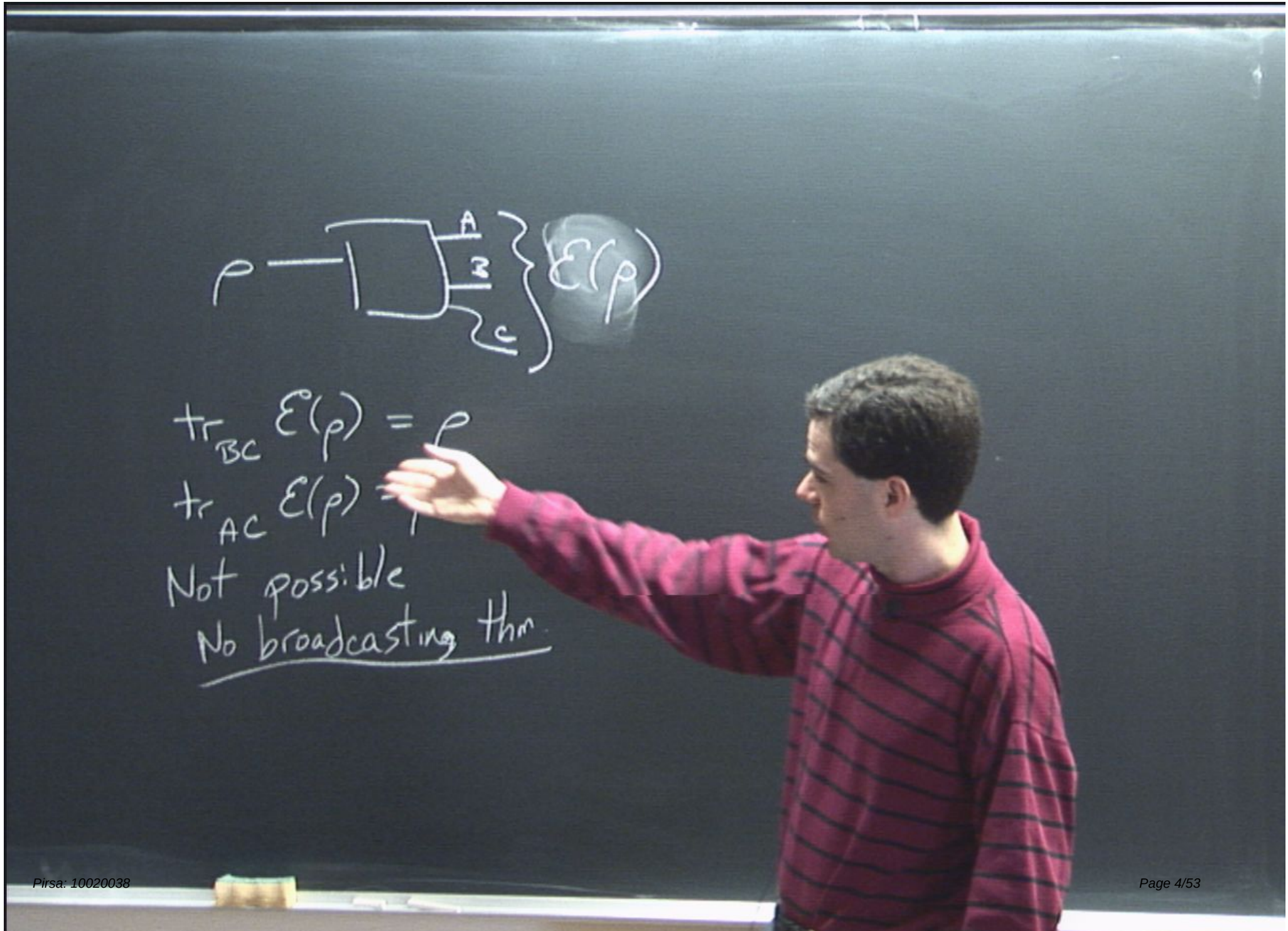
Date: Feb 01, 2010 09:00 AM

URL: http://pirsa.org/10020038

Abstract: Week 1: Basic topics (Qubits, quantum gates, quantum circuits, density matrices, quantum operations, entropy, entanglement) Week 2: Algorithms and complexity (Languages, complexity classes, oracles, RSA, Deutsch-Jozsa algorithm, Shor's algorithm, Grover's algorithm) Week 3: Information theory and implementations (Overview of implementations, quantum error correction, quantum cryptography, quantum information theory)




A
B
C
ρ
ε(ρ)


ρ
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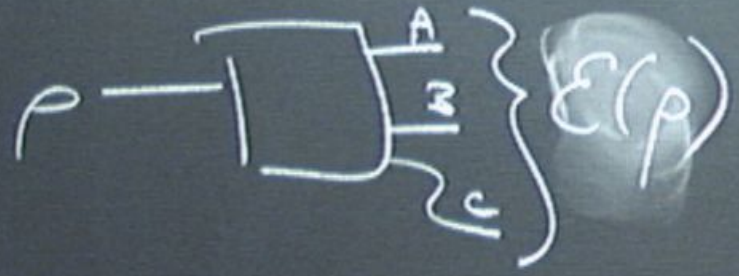
$\mathcal{E}(\rho)$
$\mathrm{tr}_{BC}\,\mathcal{E}(\rho) = \rho$
$\mathrm{tr}_{AC}\,\mathcal{E}(\rho) = \rho$
Not possible
No broadcasting thm



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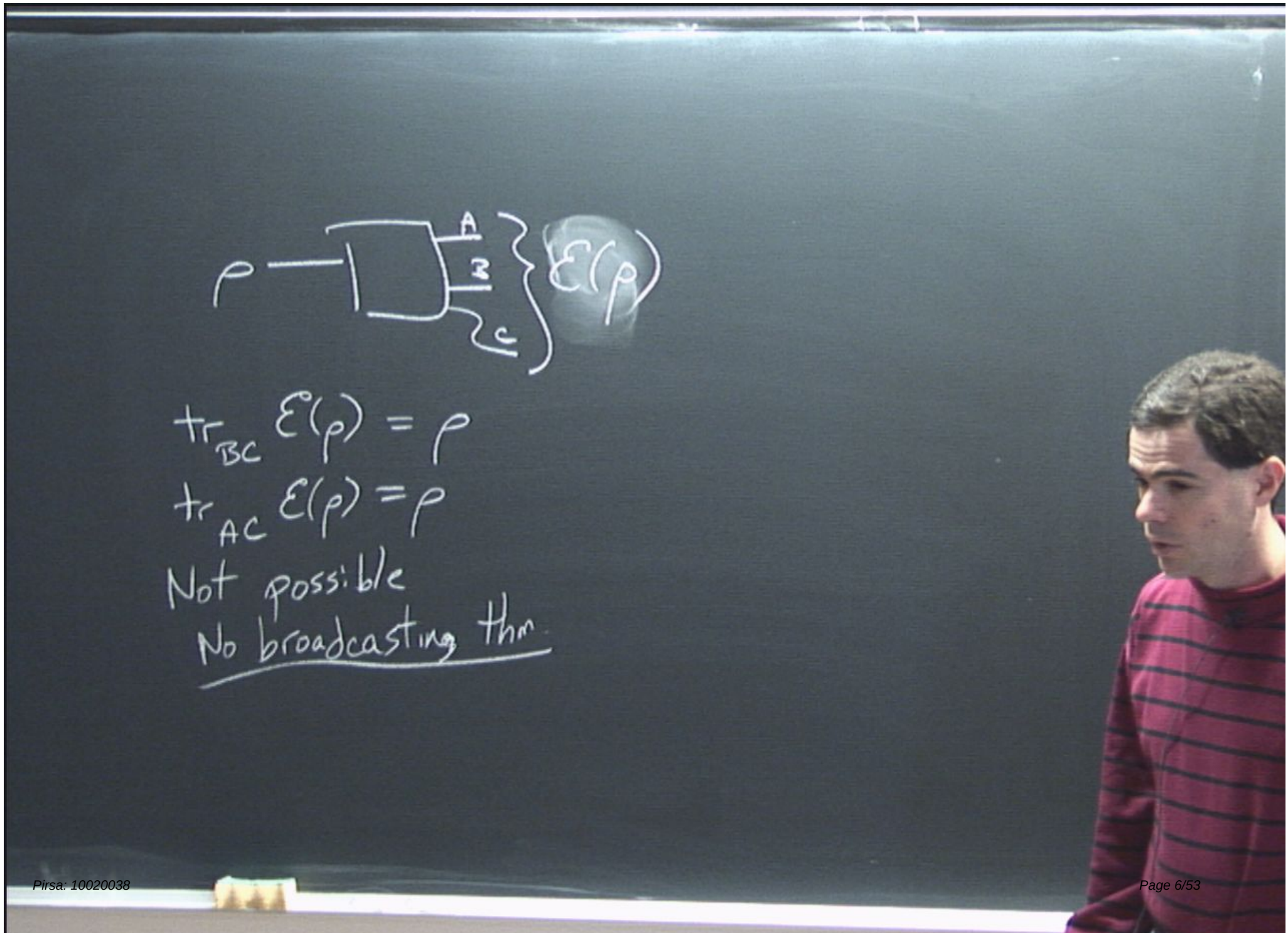
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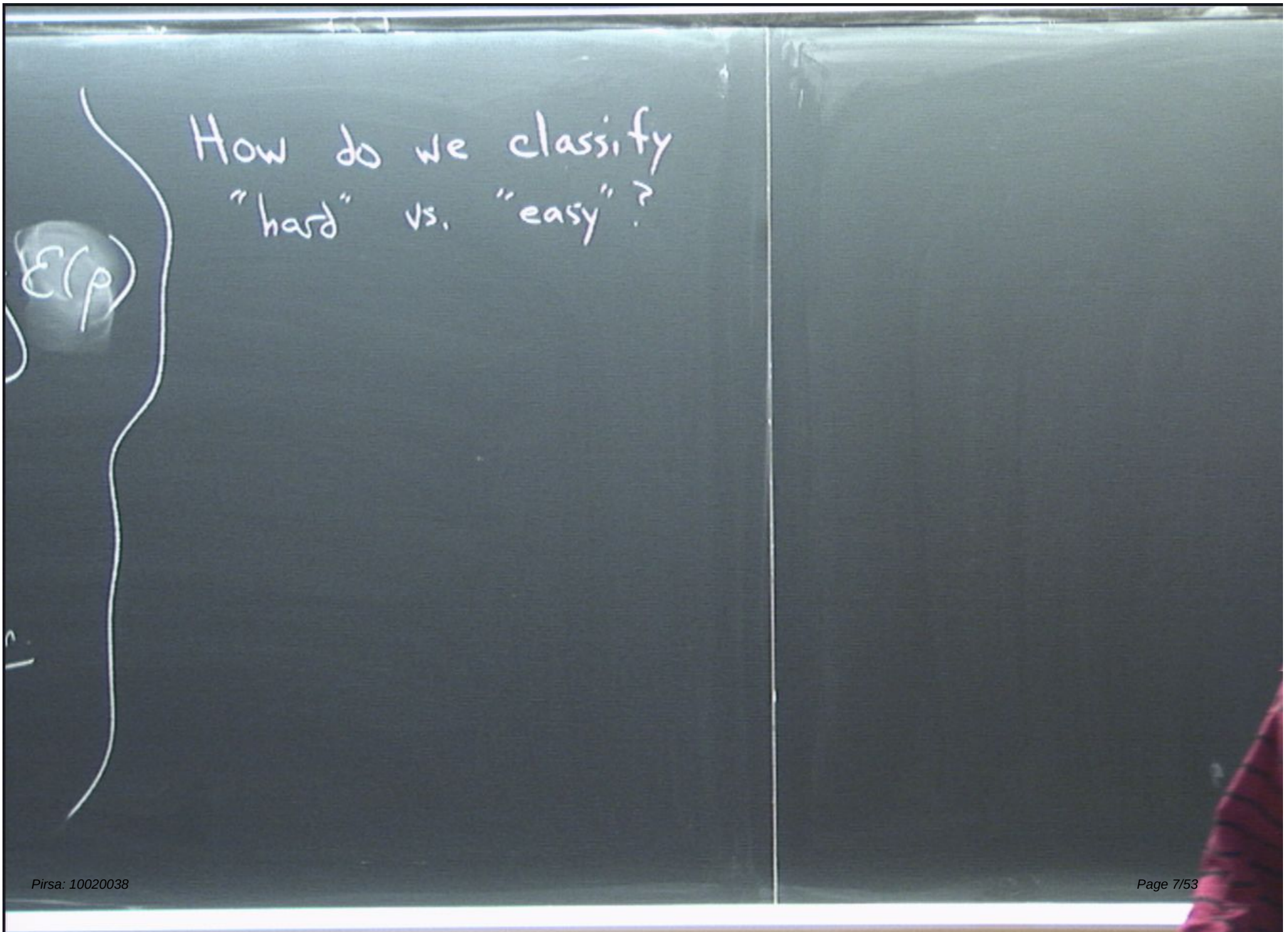
How do we classify
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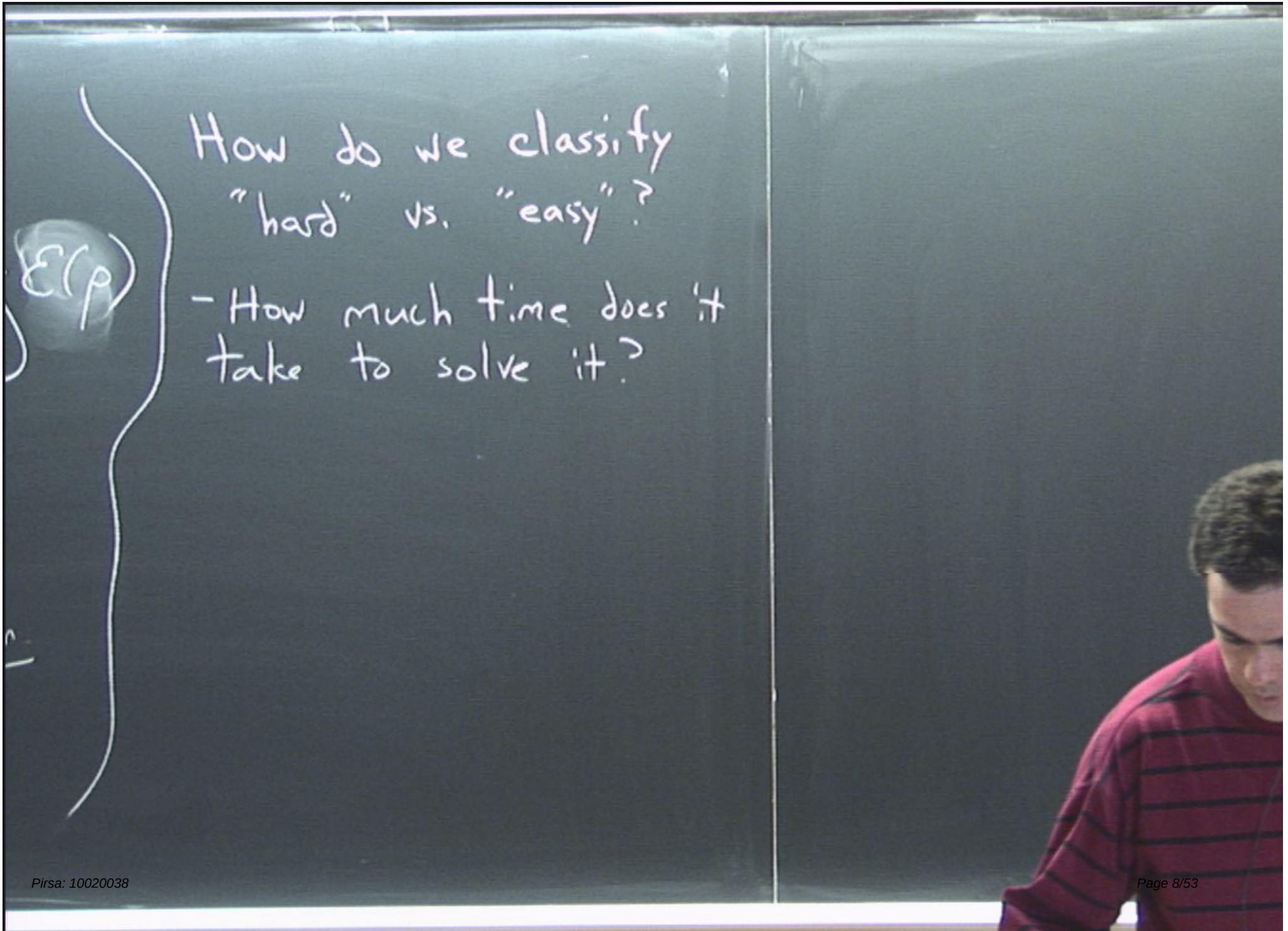
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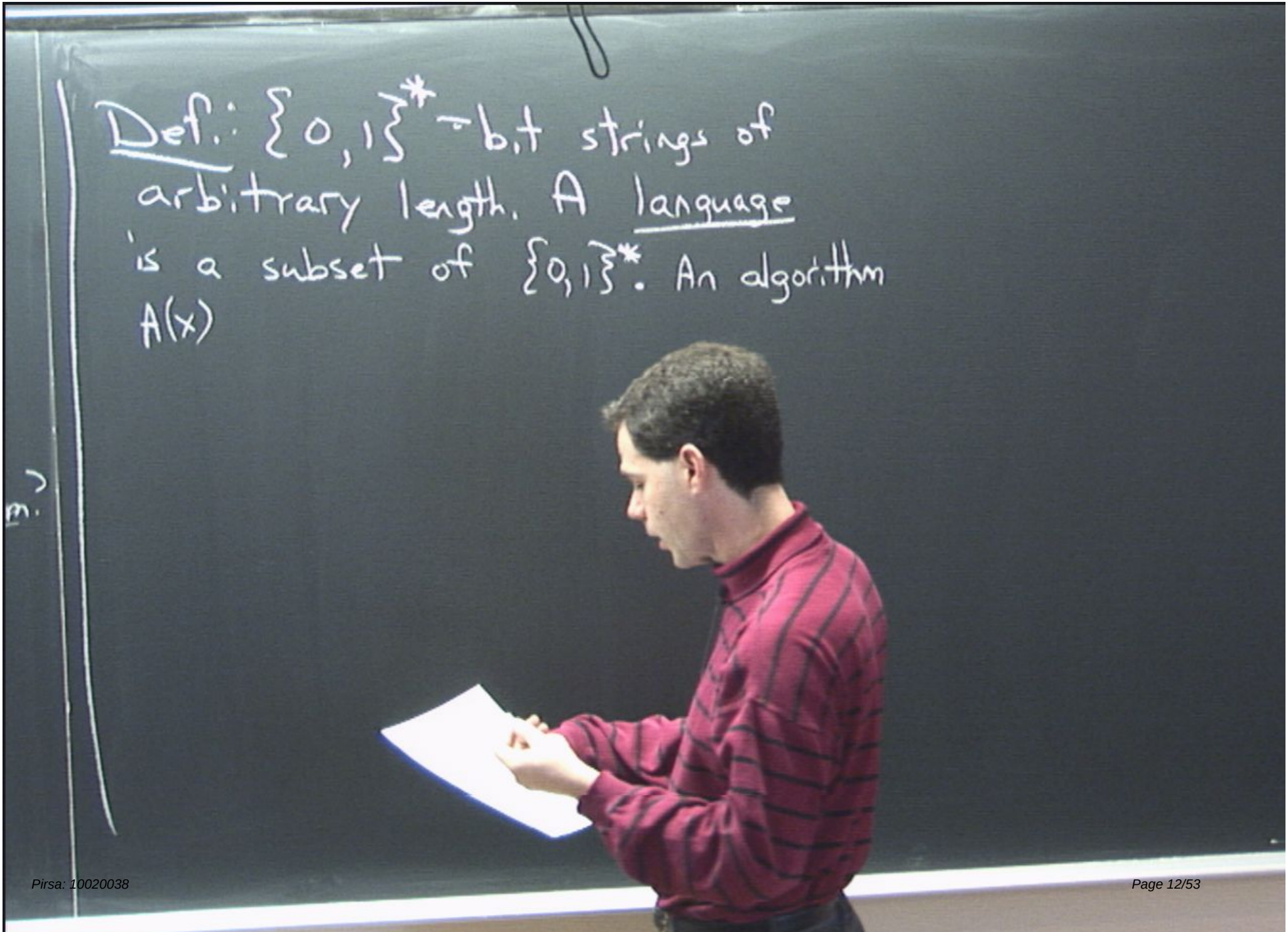
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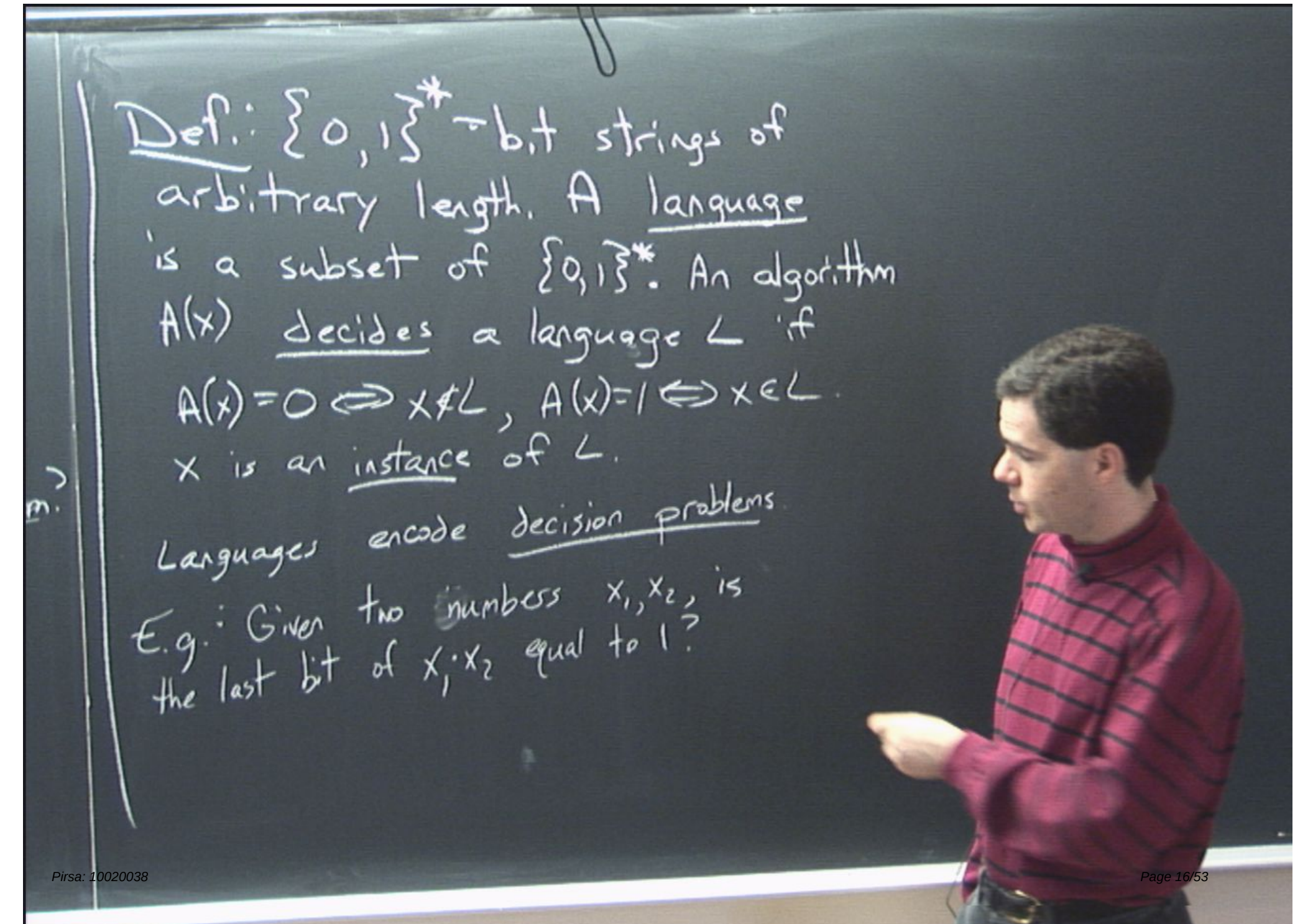

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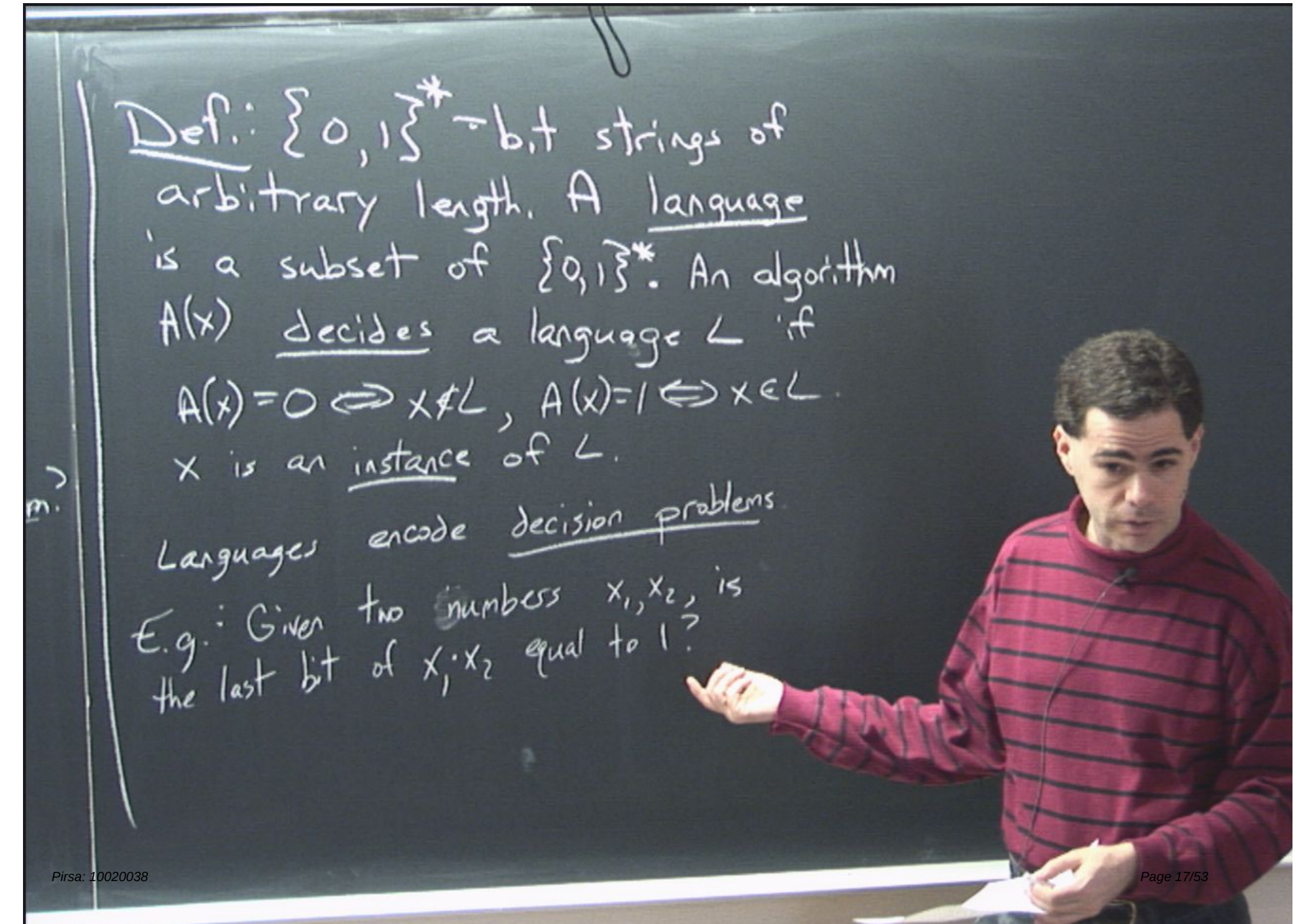

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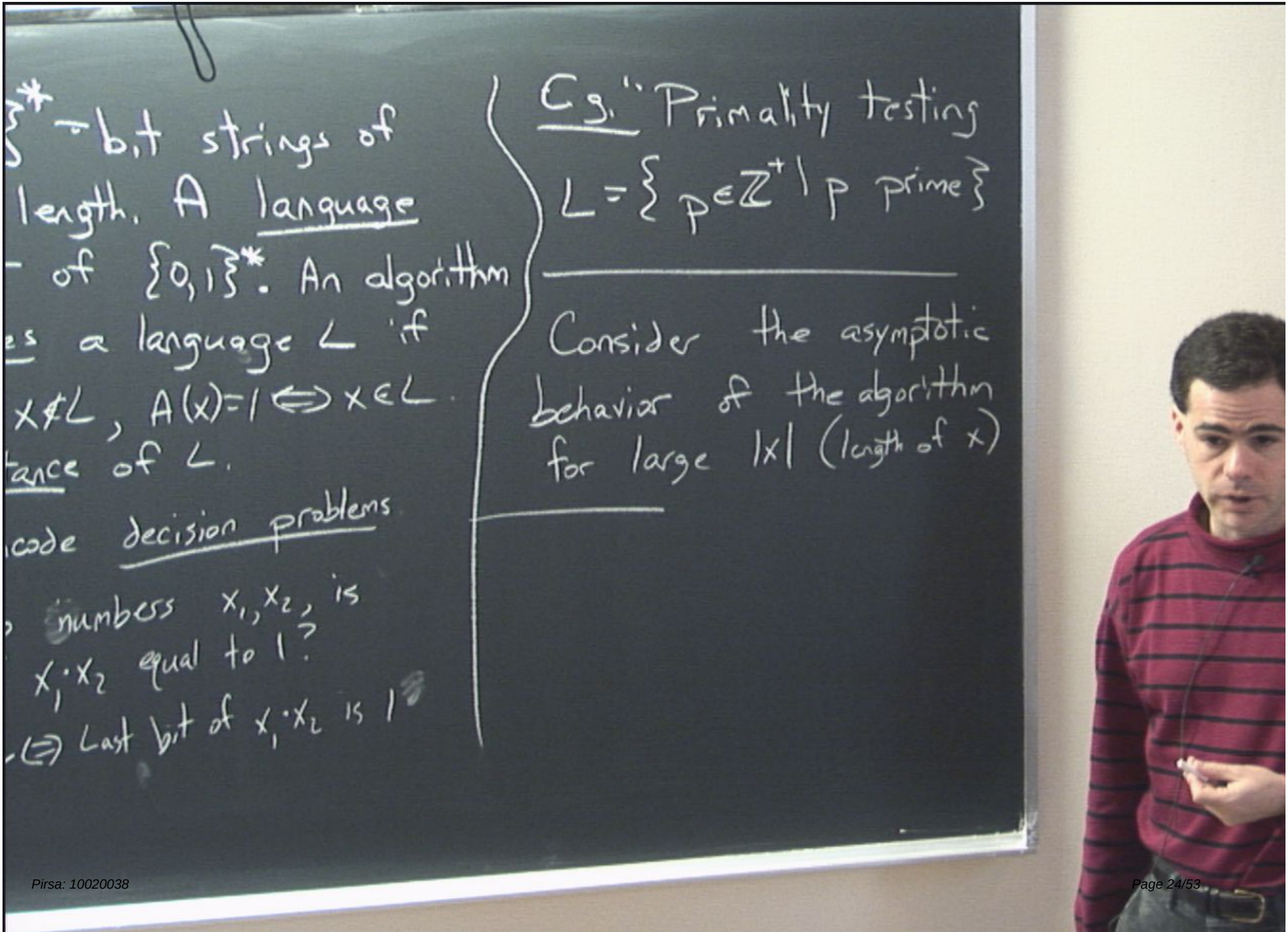
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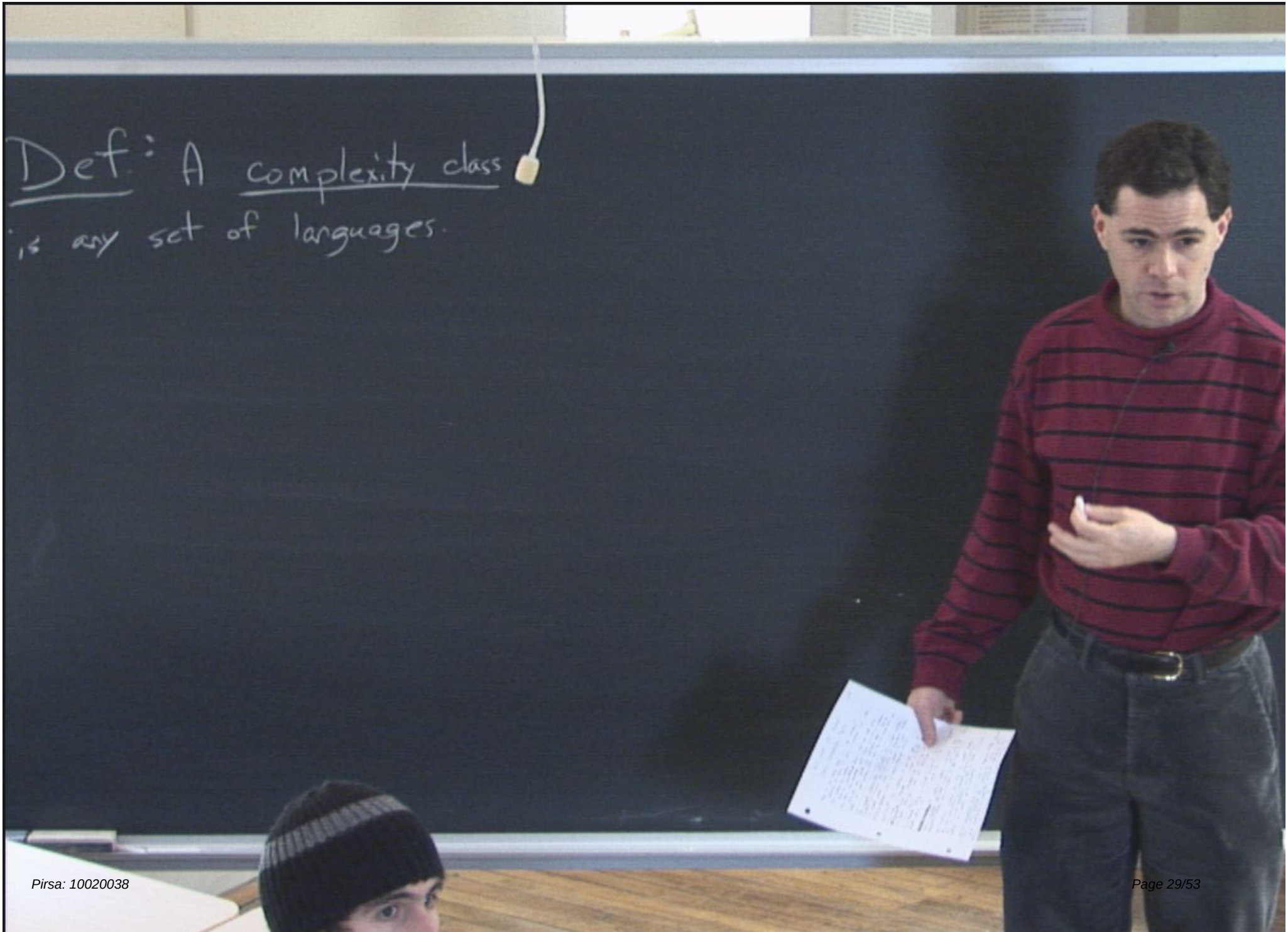
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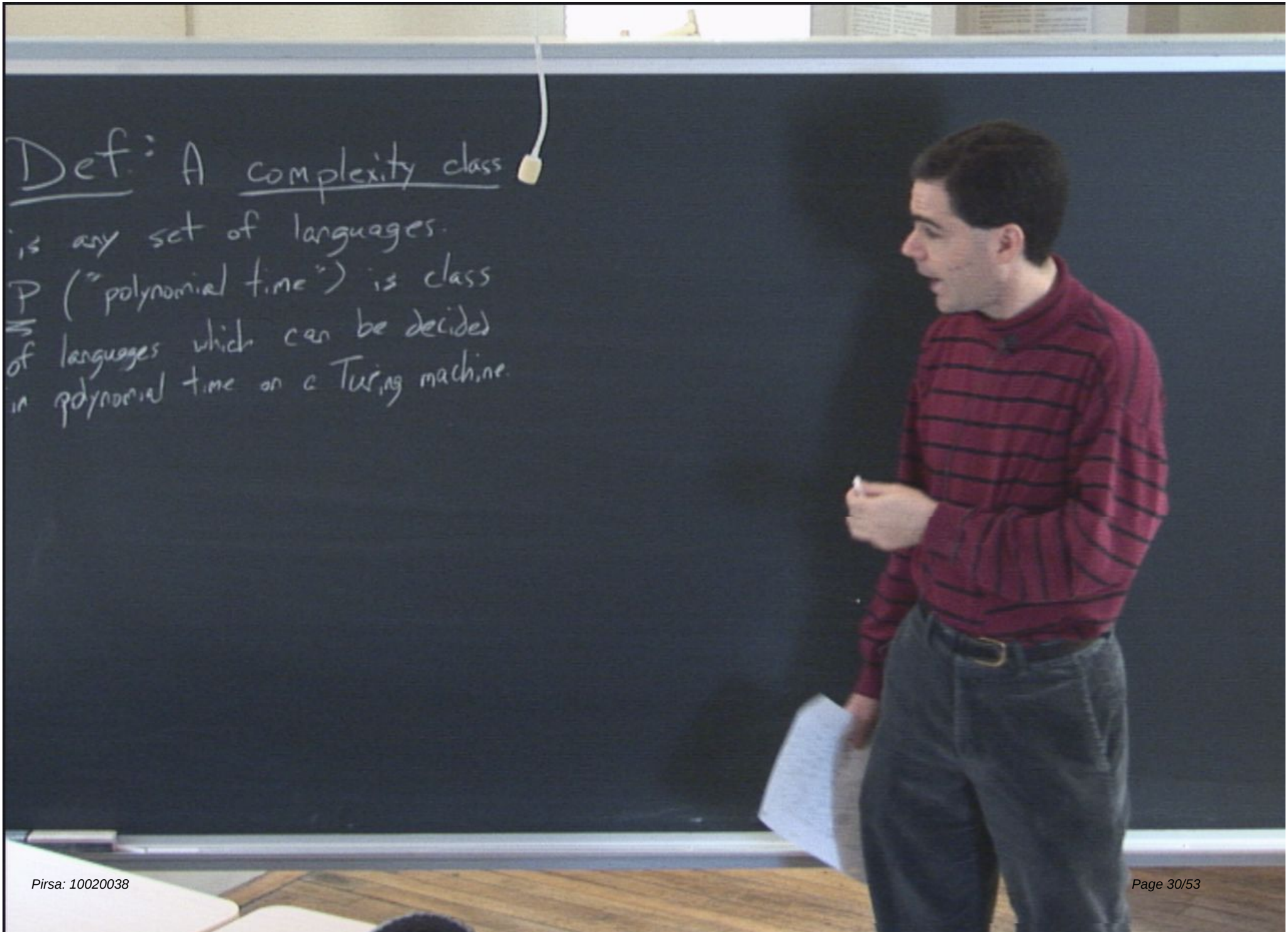
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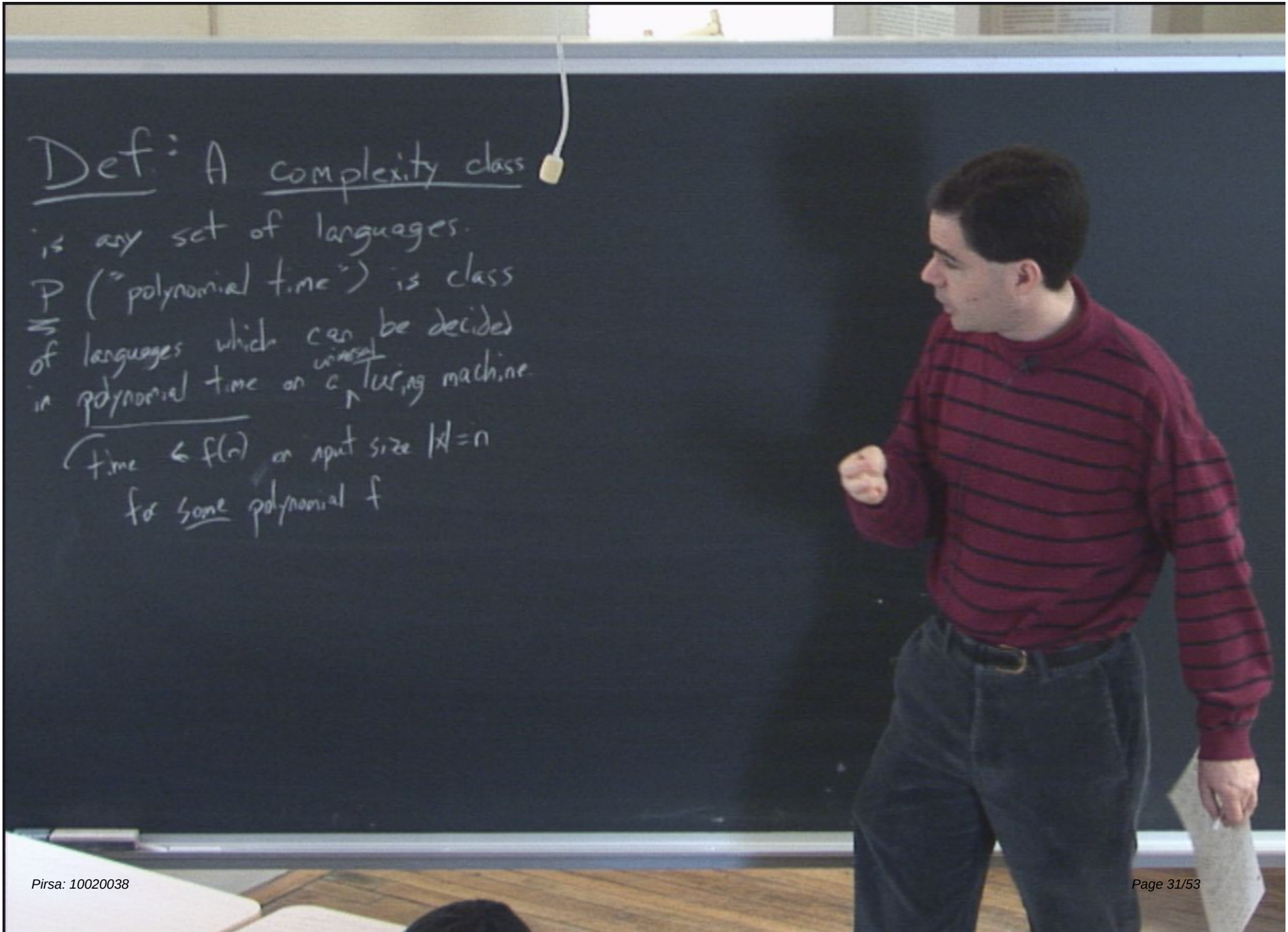
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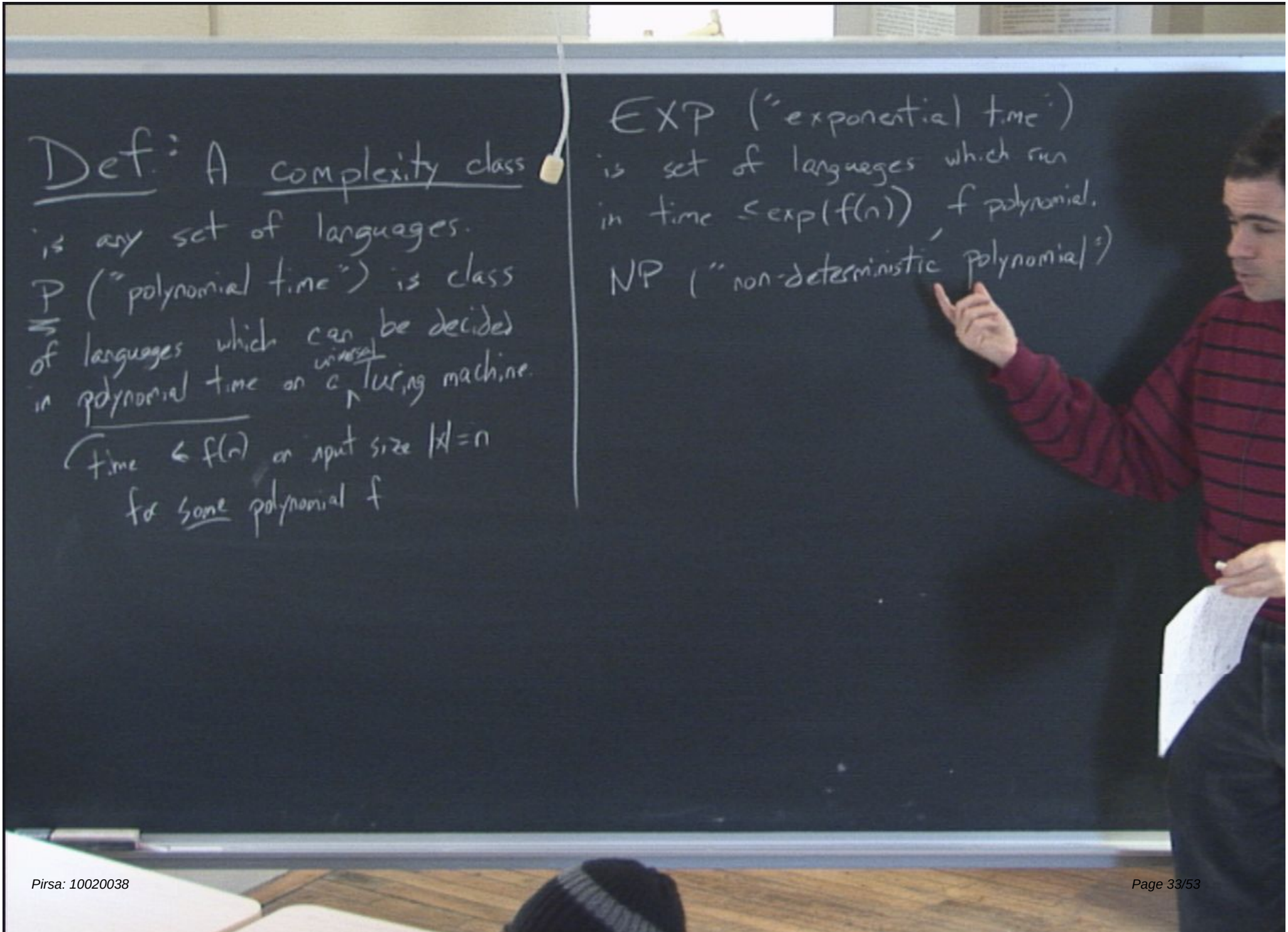
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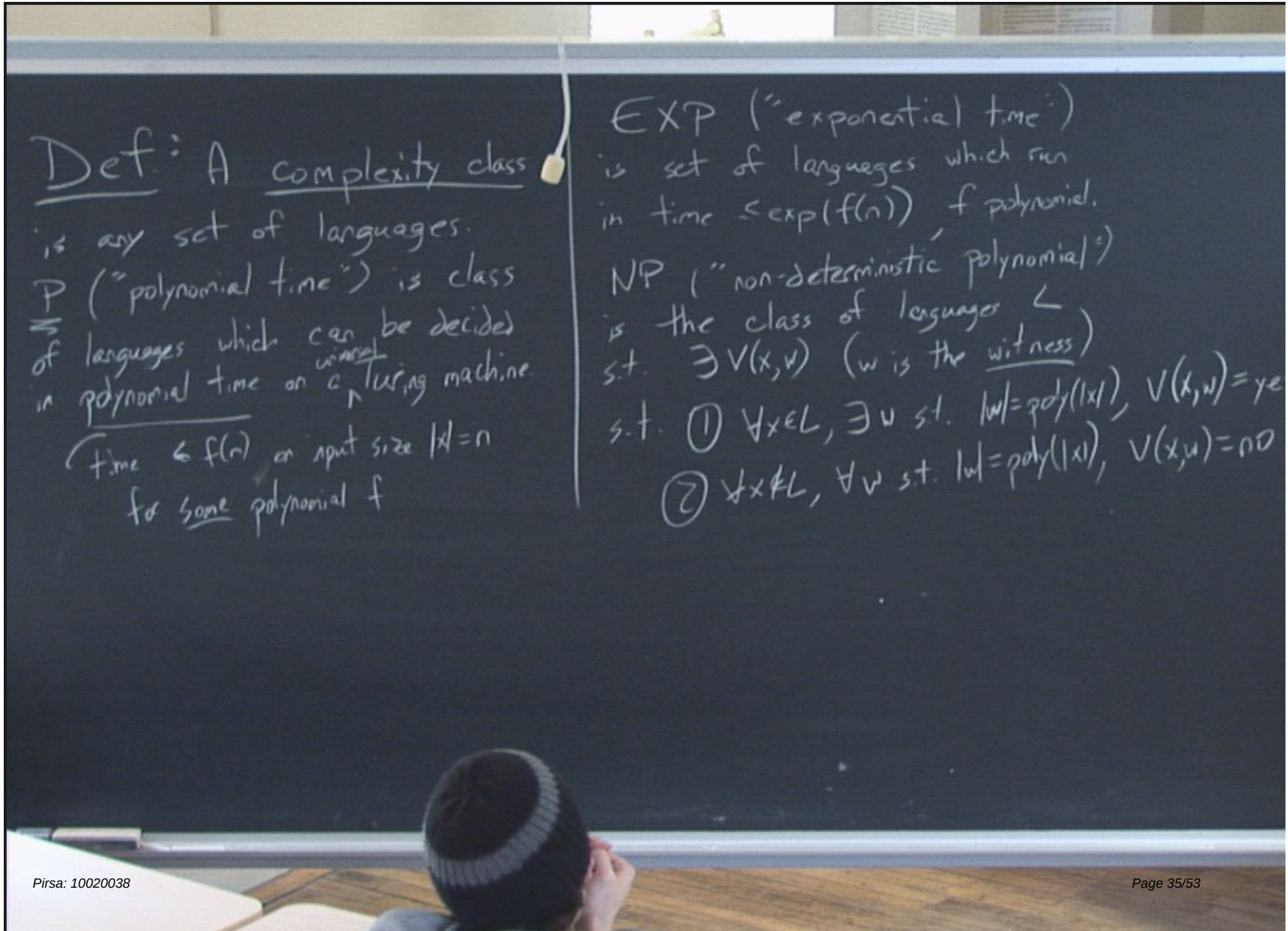
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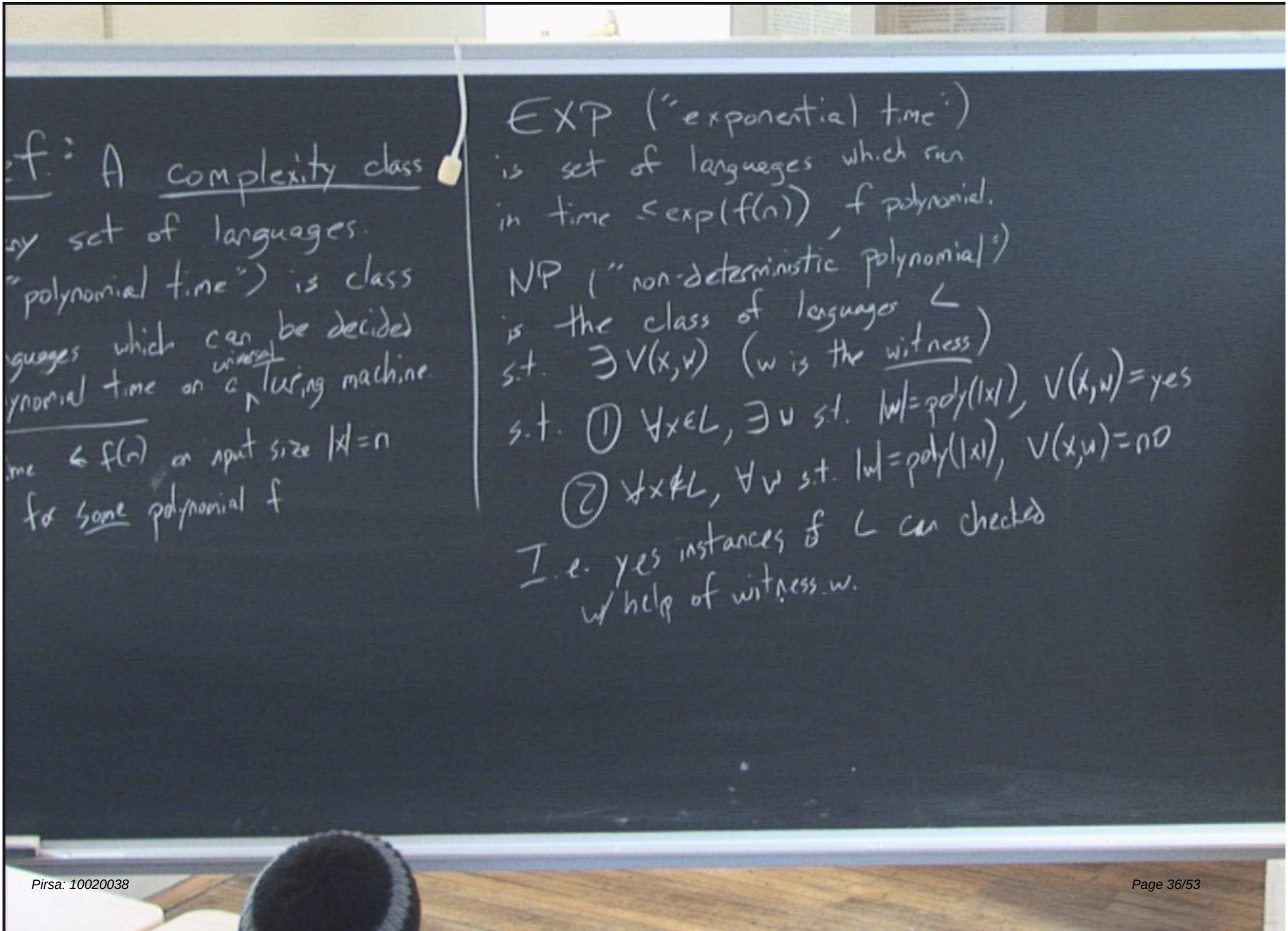
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s.t. ① $\forall x \in L$, $\exists w$ s.t. $|w| = \text{poly}(|x|)$, $V(x,w) = \text{yes}$

② $\forall x \notin L$, $\forall w$ s.t. $|w| = \text{poly}(|x|)$, $V(x,w) = \text{no}$

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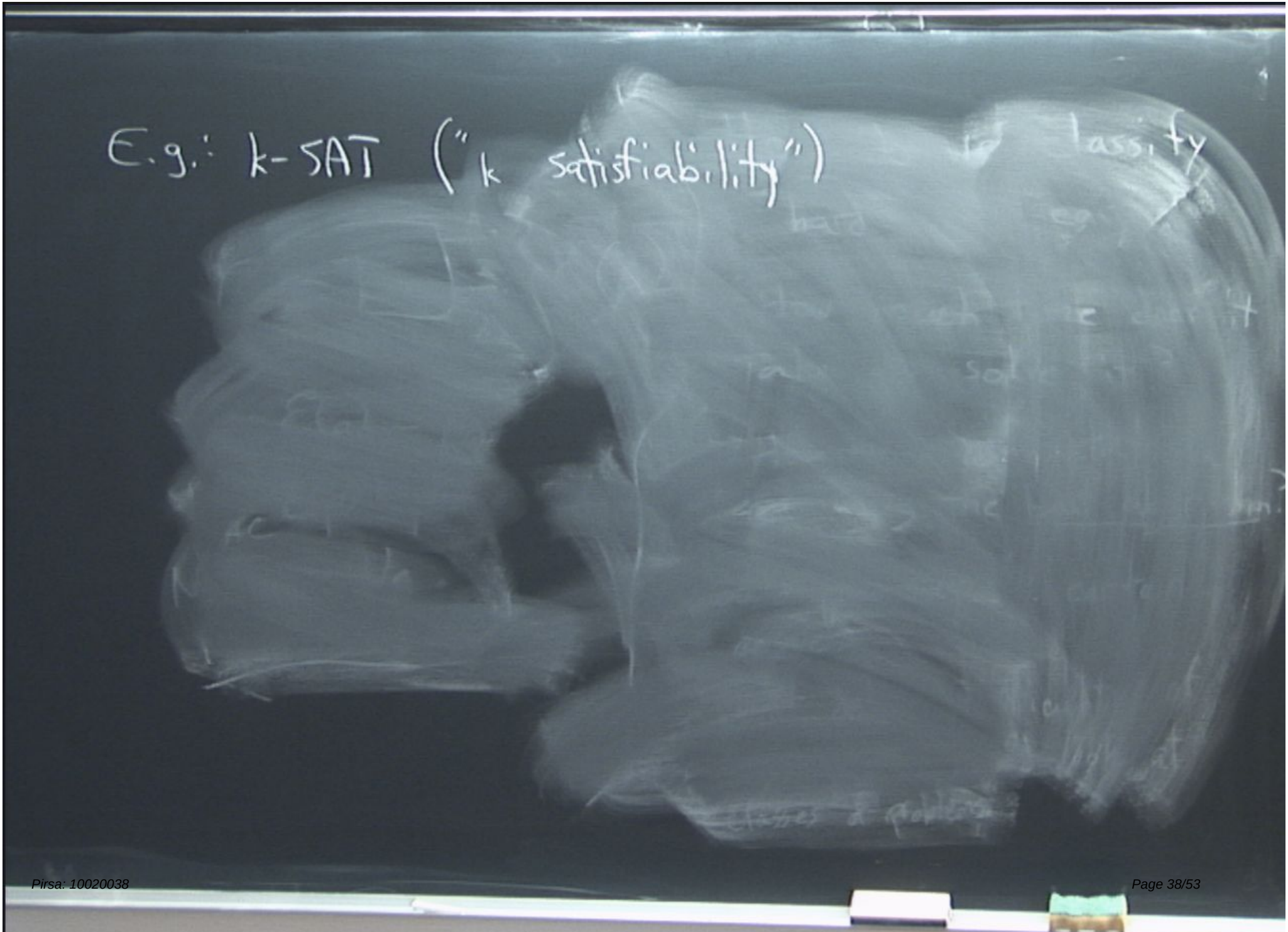
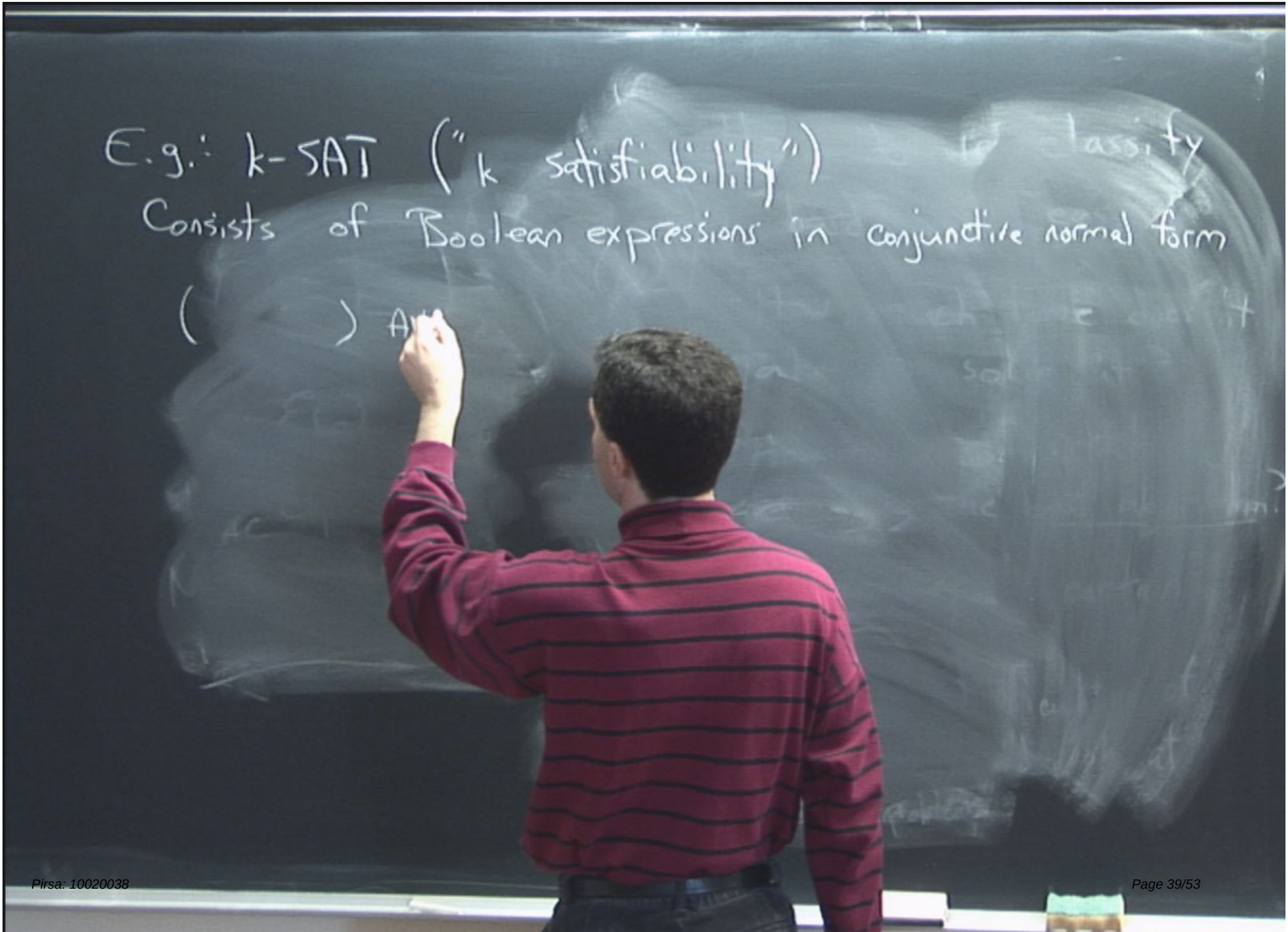

E.g.: k-SAT ("k satisfiability")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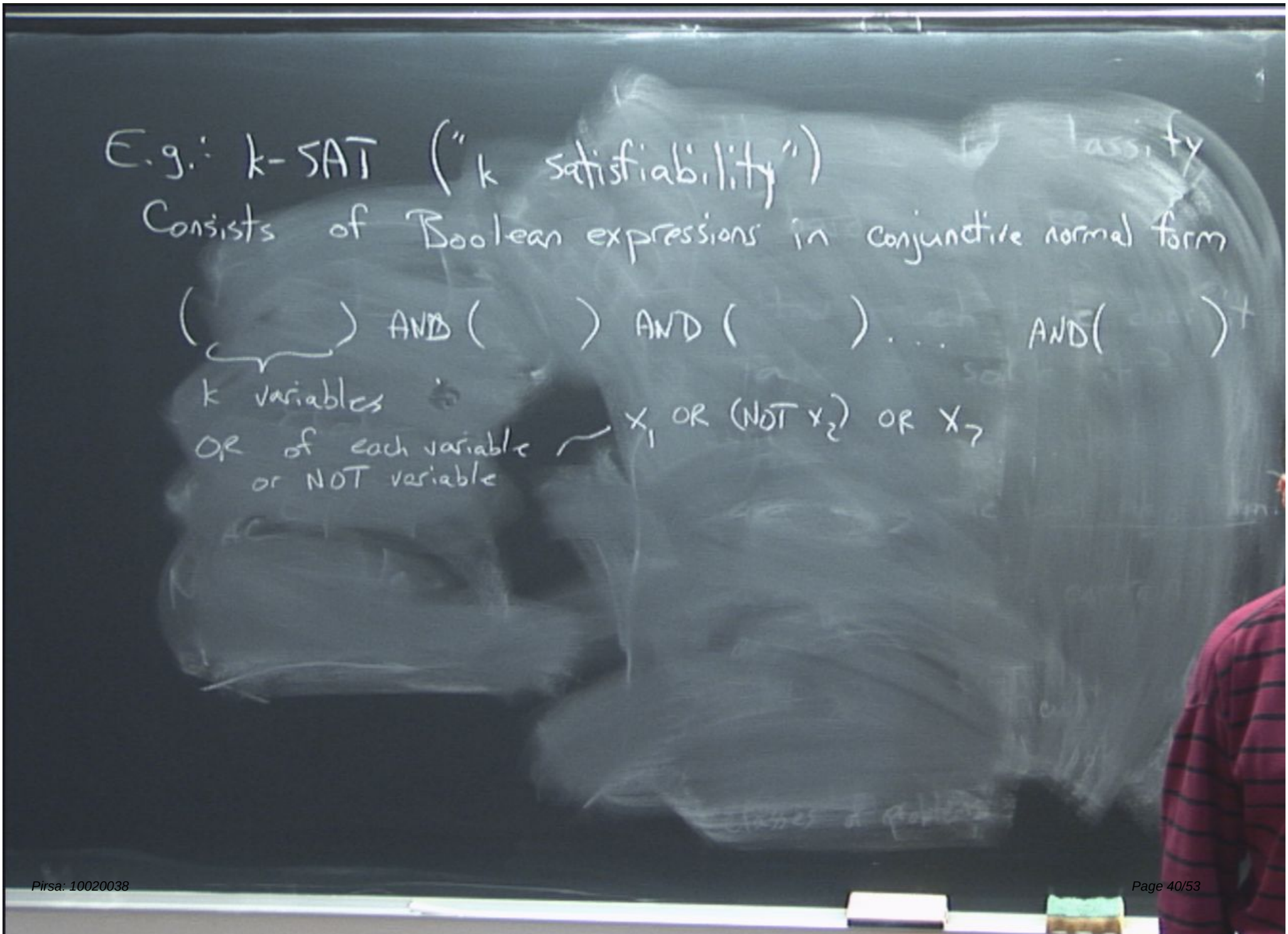
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k variables
OR of each variable or NOT variable
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A formula is in k-SAT (yes instance) if

∃ assignment of variables ($x_1 = ?$, $x_2 = ?$, $x_3 = ?$,)

s.t. formula evaluates to TRUE

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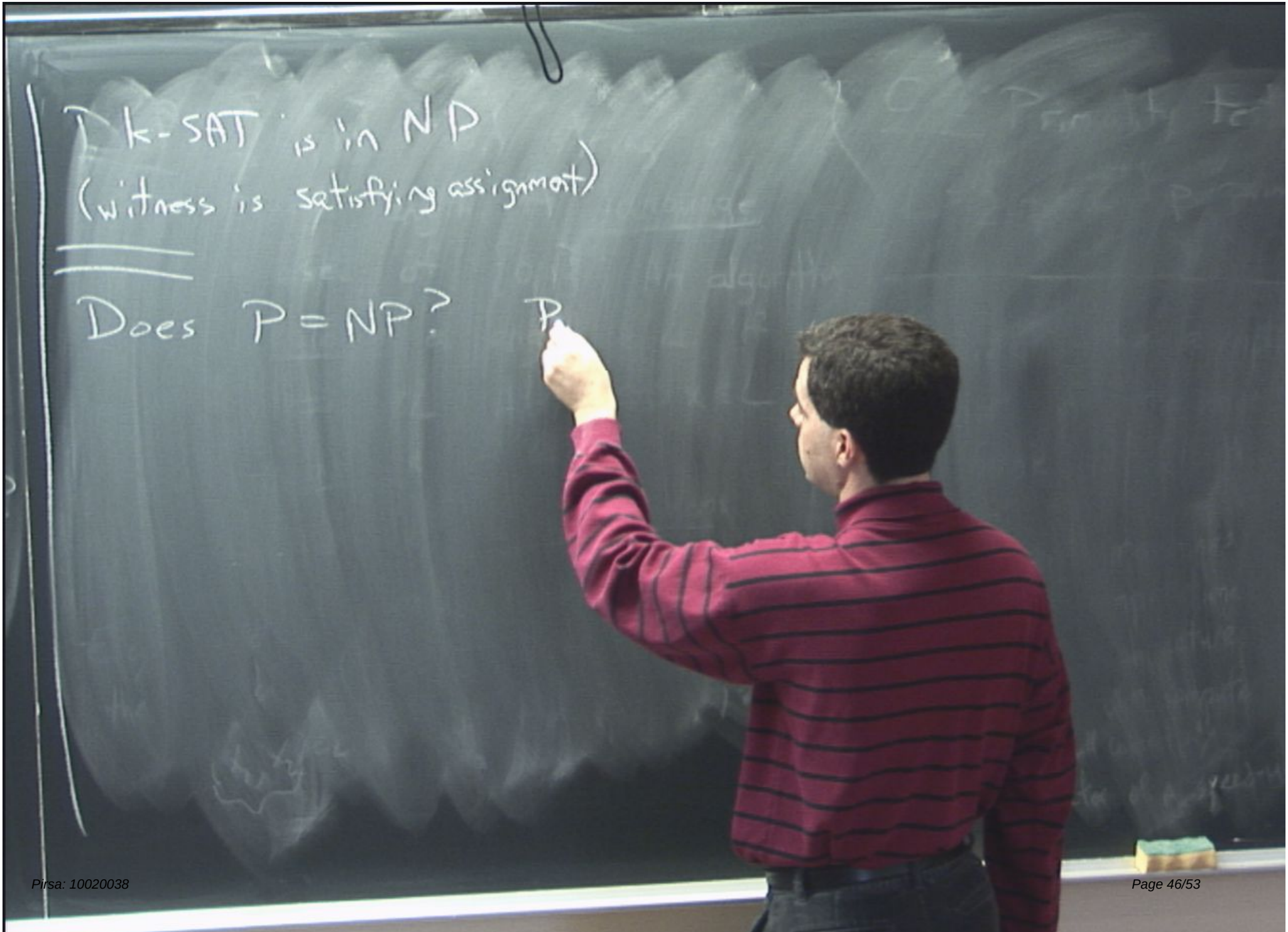
x_1 = TRUE, x_2 = TRUE, x_3 = FALSE — witness

k-SAT is in NP

(witness is satisfying assignment)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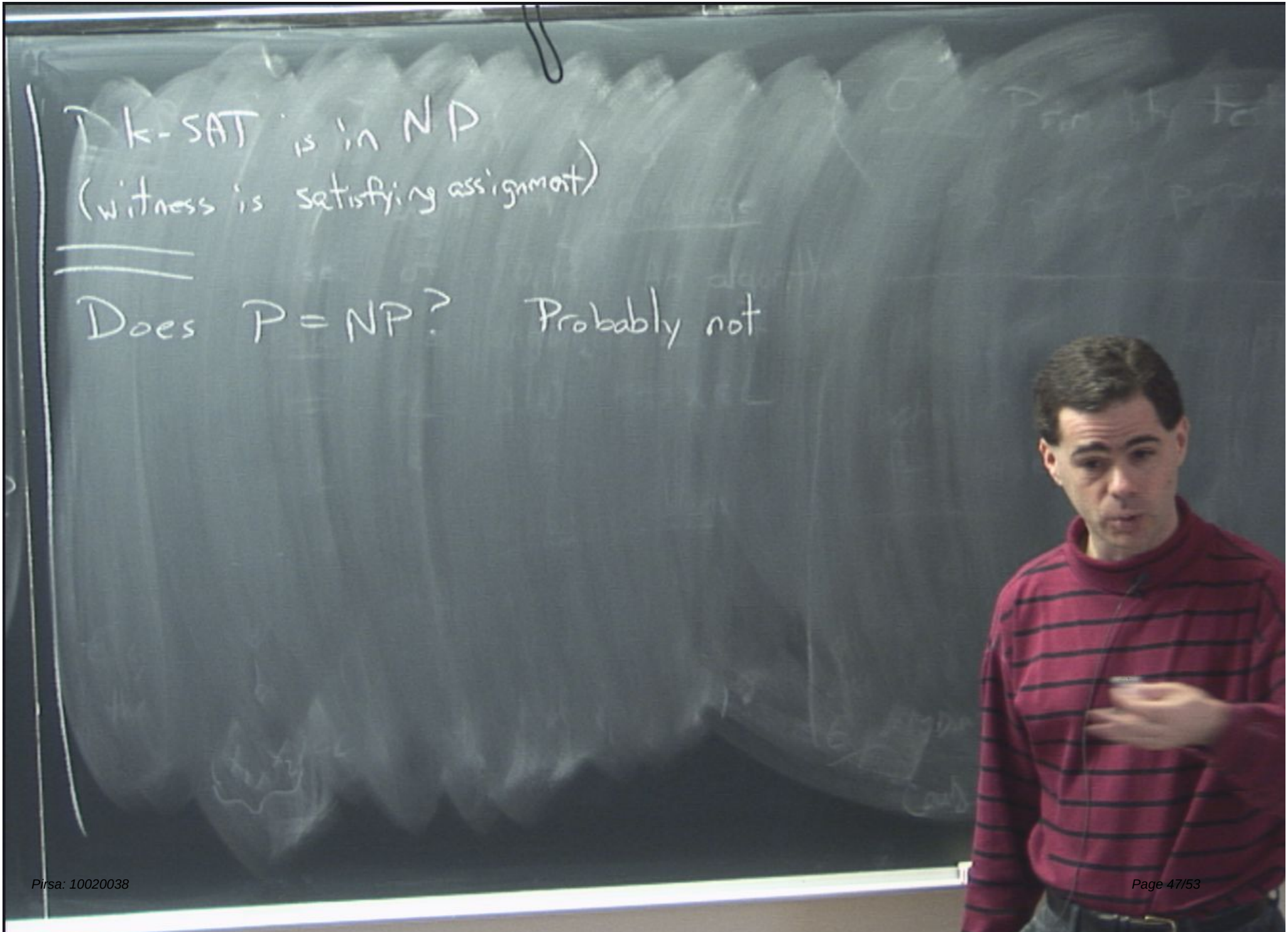
Does P=NP? P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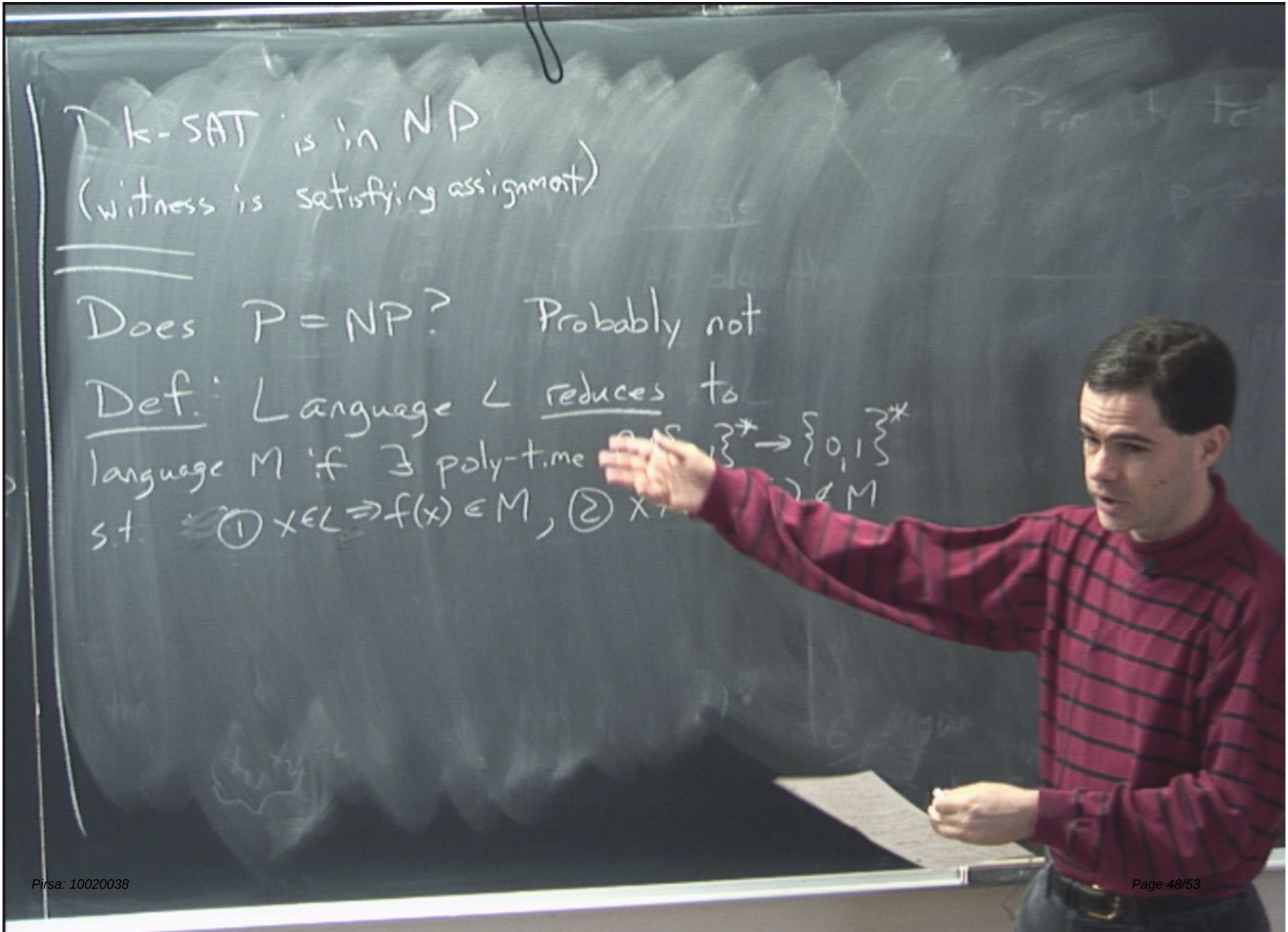
Does P = NP? Probably not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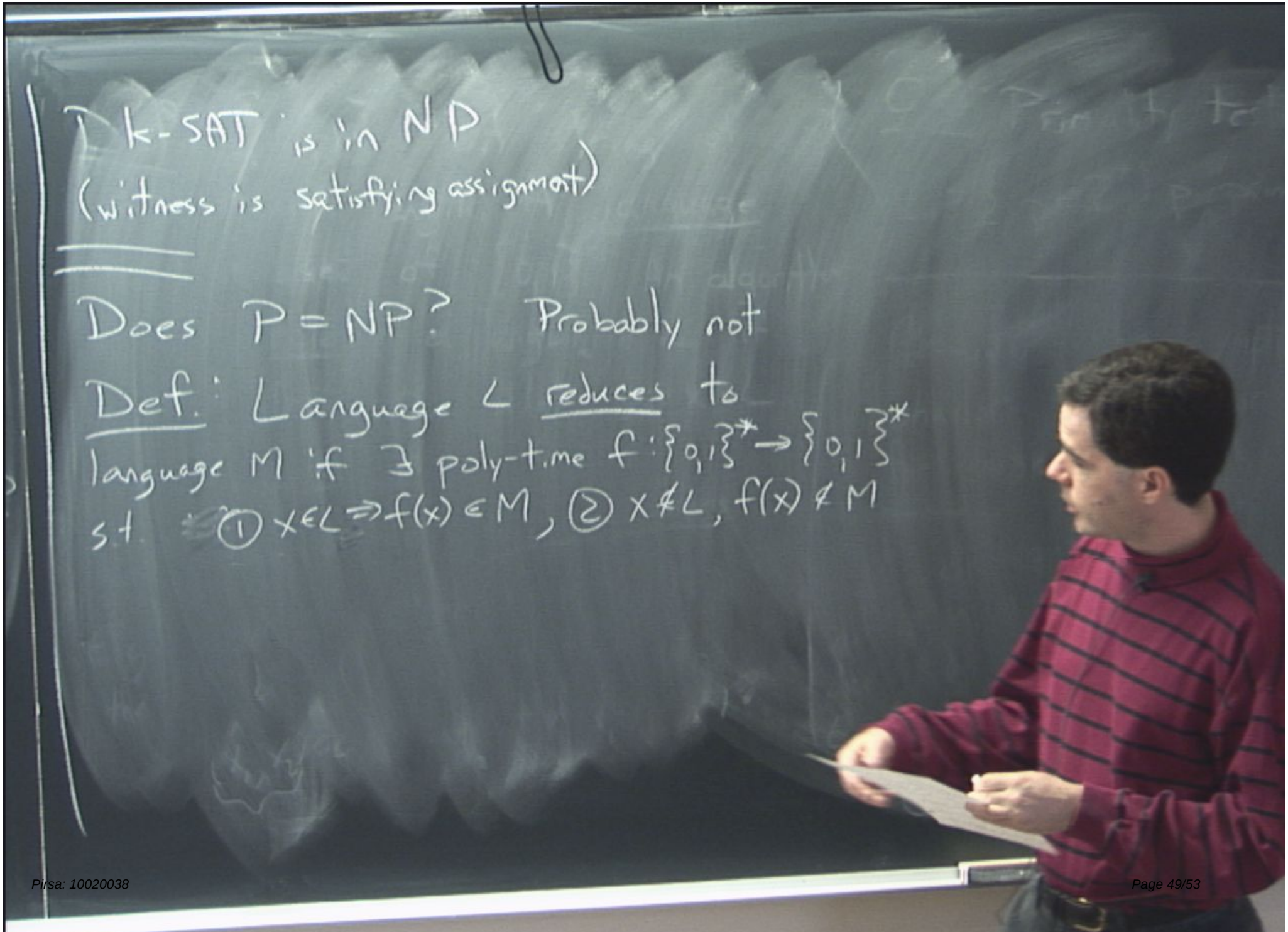
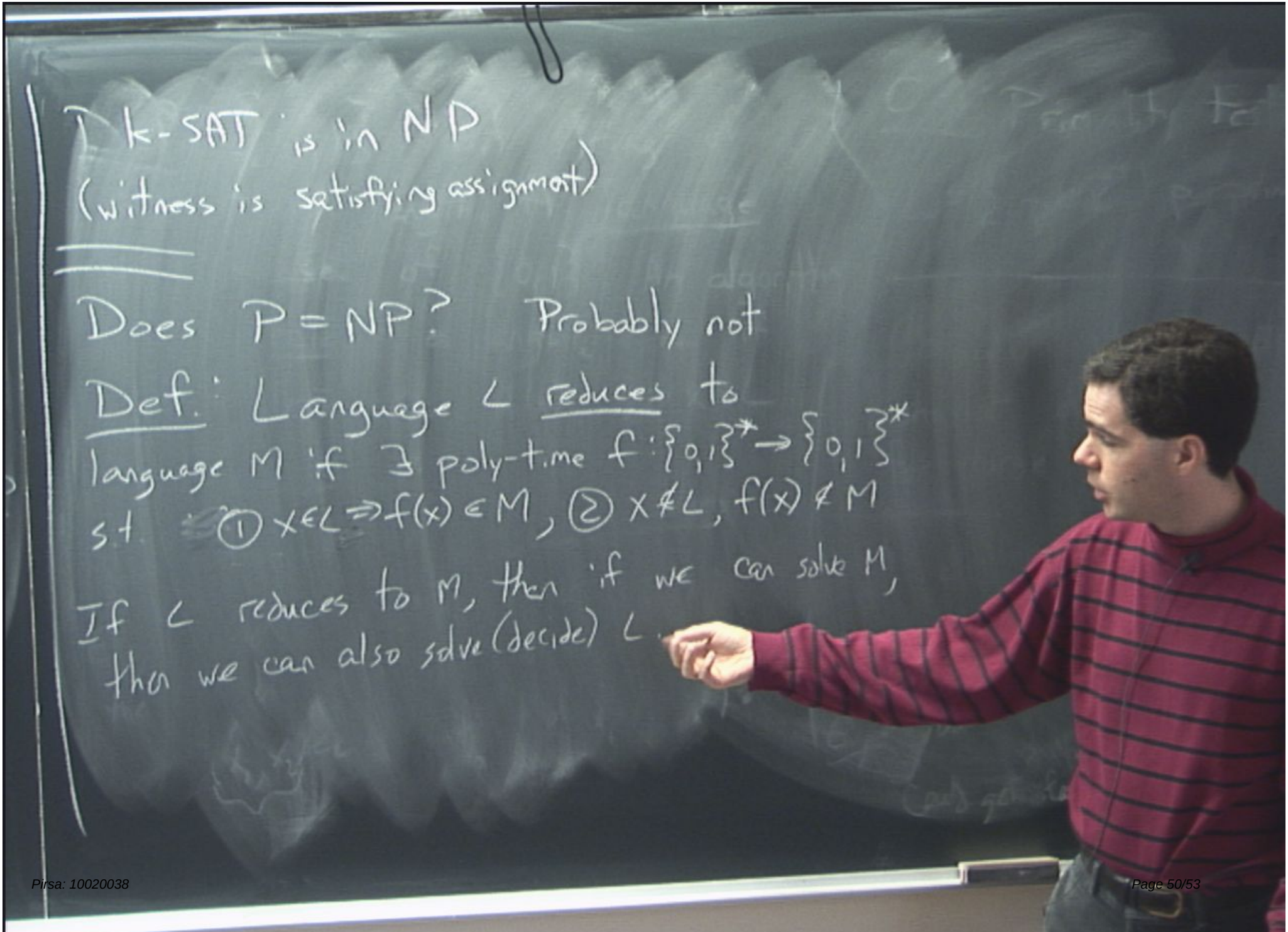
Def: Language L reduces to
language M if ∃ poly-time
s.t. ① x∈L ⇒ f(x) ∈ M, ②


k-SAT is in NP
(witness is satisfying assignment)
Does P = NP? Probably not
Def: Language L reduces to
language M if ∃ poly-time f: {0,1}* → {0,1}*
s.t. ① x∈L ⇒ f(x)∈M, ② x∉L, f(x)∉M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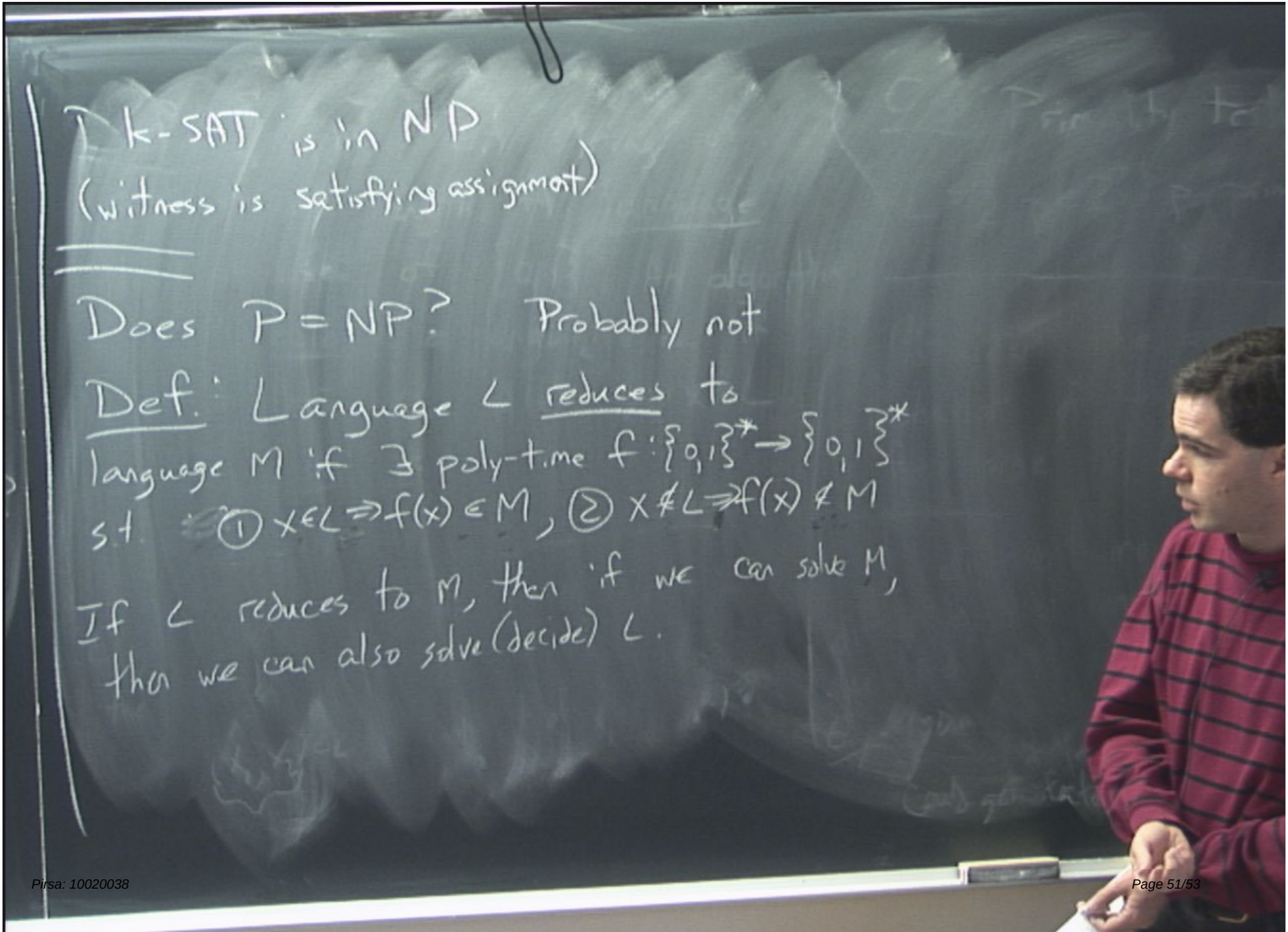
If L reduces to M, then if we can solve M,
then we can also solve (decide) L.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... Language L reduces to

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① $x \in L \Rightarrow f(x) \in M$, ② $x \notin L \Rightarrow f(x) \notin M$

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Def: A language M is complete for complexity class C if $\forall L \in C$

...T is in NP
... is satisfying assignment)

P = NP? Probably not

: Language L reduces to
... M if $\exists$ poly-time $f: \{0,1\}^* \to \{0,1\}^*$
... ① $x \in L \Rightarrow f(x) \in M$, ② $x \notin L \Rightarrow f(x) \notin M$

... reduces to M, then if we can solve M,
... we can also solve (decide) L.

Def: A language M is complete for complexity class C if $\forall L \in C$, L reduces to M.