Title: Holographic non-Fermi Liquids

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Abstract: We describe a class of non-Fermi liquid systems, using the AdS/CFT correspondence. The Fermi surfaces are studied by computing the response functions of fermionic operators. The scaling behavior near the Fermi surfaces is determined by conformal dimensions in an emergent IR CFT. The low-energy excitations near the Fermi momenta are not Landau quasiparticles. When the operator is marginal in the IR CFT, the full spectral function is precisely of the `marginal Fermi liquid' form, introduced as a phenomenological model of the `strange metal' phase of high temperature superconductors.

Non-Fermi liquids from holography

David Vegh
Simons Center for Geometry and Physics

STONY BROOK UNIVERSITY

Hong Liu, John McGreevy, DV arXiv:0903.2477
Thomas Faulkner, Hong Liu, John McGreevy, DV arXiv:0907.2694
Thomas Faulkner, Gary Horowitz, John McGreevy, Matthew Roberts, DV arXiv:0911.3402

(see also: Sung-Sik Lee, arXiv:0809.3402 ; Cubrovic, Zaanen, Schalm, arXiv:0904.1933)

Perimeter Institute – February 16, 2010

(Non-)Fermi liquids

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AdS/CFT background

- Review of AdS/CFT
- $AdS_4 - BH$ geometry
- Spinor Green's function

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Holographic non-Fermi liquids
- Fermi surfaces
- Near-horizon AdS_2 and emergent IR CFT
- Small ω expansion from matching

Fermions at finite density

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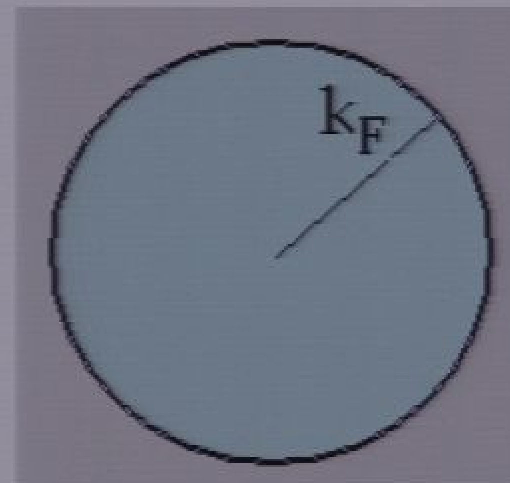
- Landau Fermi liquid theory

[Landau, 1957] [Abrikosov-Khalatnikov, 1963]



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- stable RG fixed point [Polchinski, Shankar]
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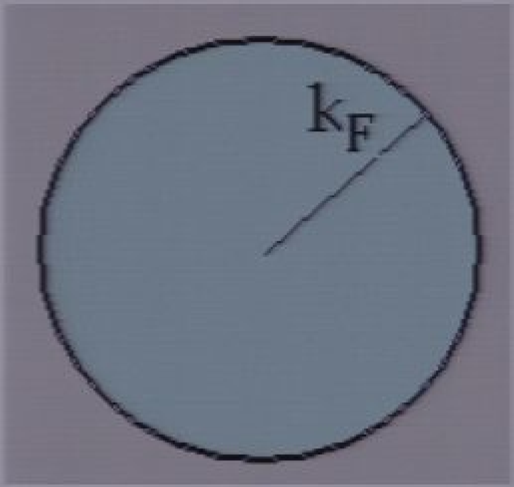
  $\rightarrow$ thermodynamics, transport properties
- appear as *poles* in the single-particle Green's function:



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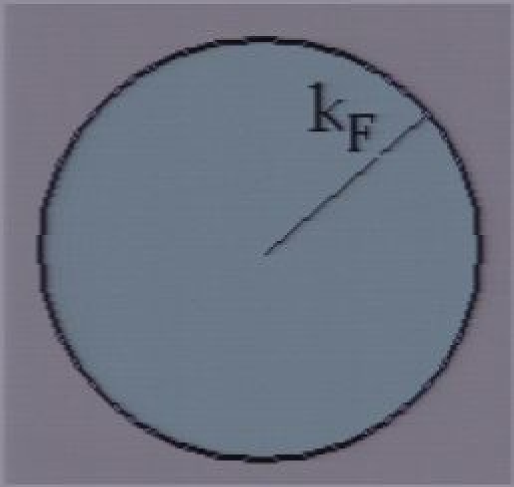

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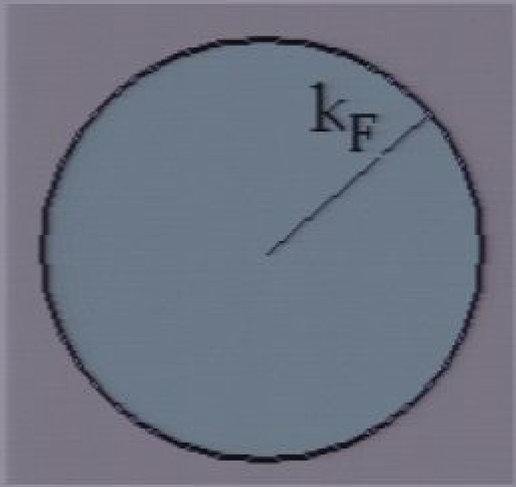

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$$A(\omega, \vec{k}) \equiv \mathrm{Im}\, G_R(\omega, \vec{k}) \overset{k_\perp \to 0}{\longrightarrow} Z\delta(\omega - v_F k_\perp) \quad \text{with Z finite}$$

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Organizing principle for non-Fermi liquids?

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a certain d-dimensional *conformal field theory*	$\Longleftrightarrow$	a string theory in (d+1)-dimensional *anti-de Sitter* spacetime

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finite chemical potential	$\Longleftrightarrow$	electrically charged black hole

$AdS_4 - BH$ geometry

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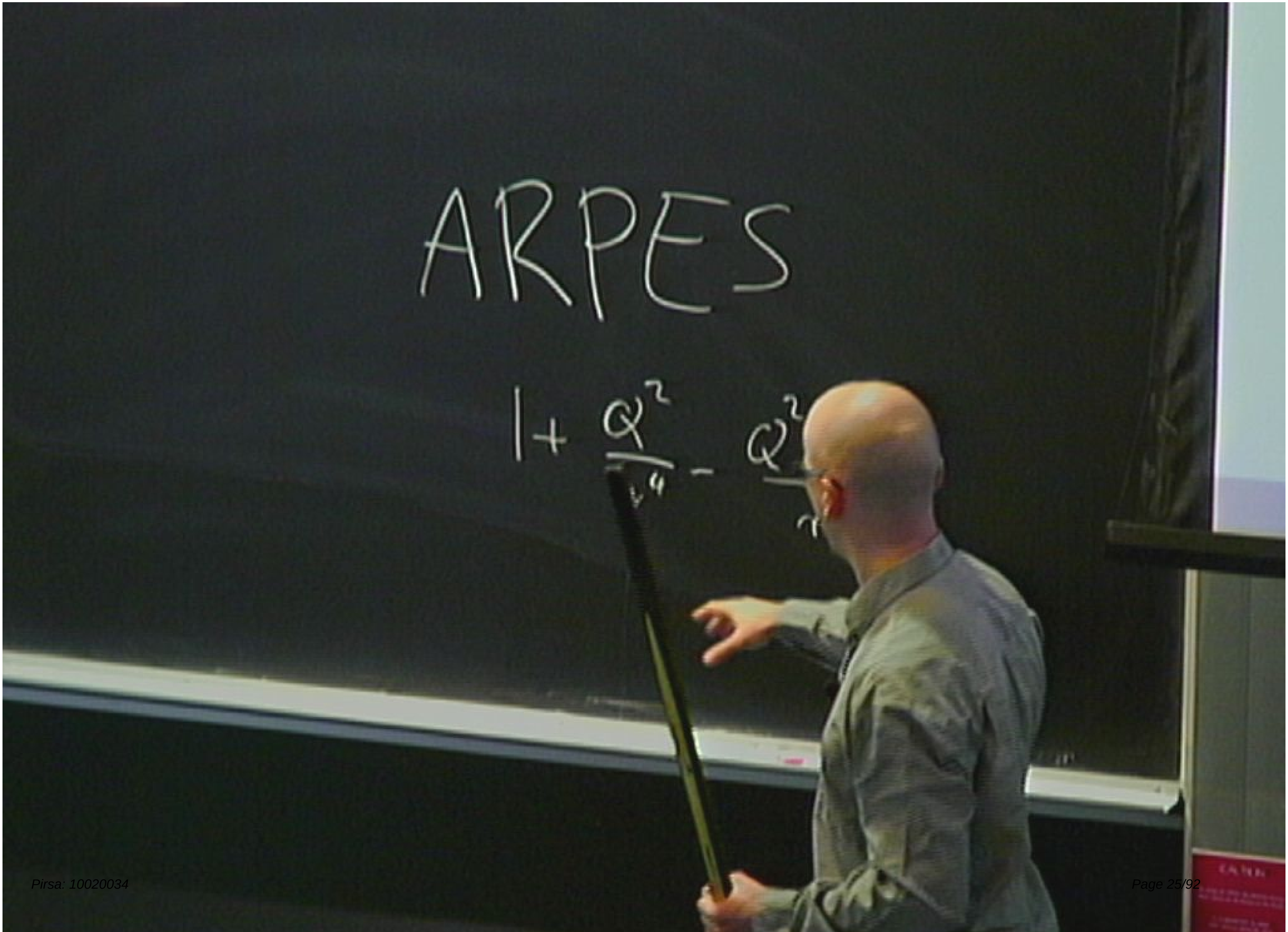
$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(\mathcal{R} + \frac{6}{R^2} - \frac{2\kappa^2}{g_F^2}F_{\mu\nu}F^{\mu\nu} + \dots\right)$$

Charged black hole solution

$$ds^2 = \frac{r^2}{R^2}(-f(r)dt^2 + d\vec{x}^2) + R^2\frac{dr^2}{f(r)r^2}$$

$$f(r) = 1 + \frac{3}{r^4} - \frac{4}{r^3} \qquad A = \mu\left(1 - \frac{1}{r}\right)dt$$

where $\mu =$ chemical potential, horizon at $r = 1$.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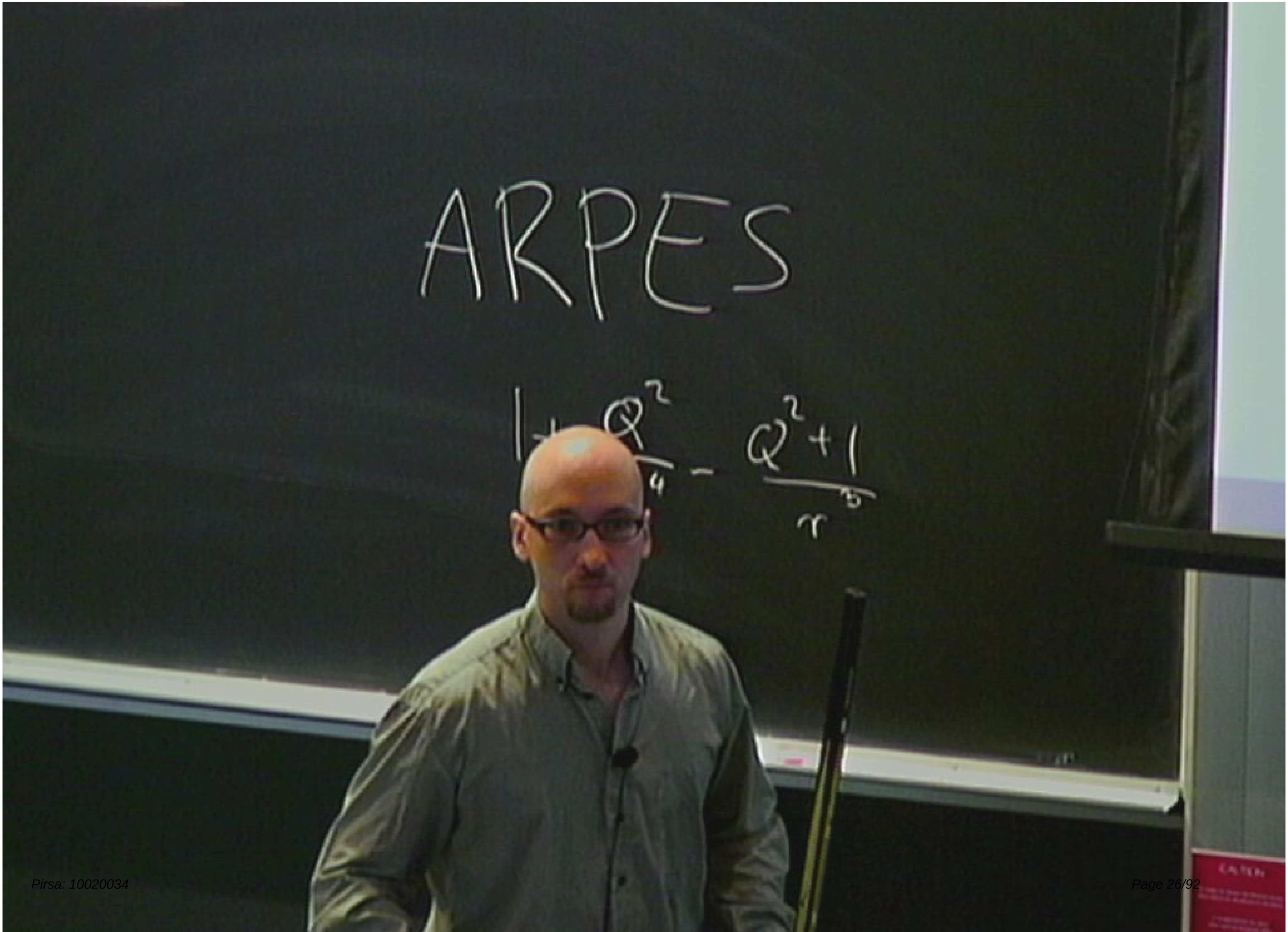
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Retarded spinor Green's function

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Introduce ψ spinor field in the $AdS - BH$ background

$$S_{probe} = \int d^4x \sqrt{-g} \left(\bar{\psi}(\not{D} - m)\psi + interactions \right)$$

with $D_\mu = \partial_\mu + \frac{1}{4}\omega_{ab\mu}\Gamma^{ab} - iqA_\mu$.

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Prescription [Henningson-Sfetsos] [Son-Starinets] [Iqbal-Liu]

- Solve the Dirac equation for the bulk spinor in $AdS - BH$
- Impose infalling boundary conditions at the horizon
- Expand the solution at the boundary

$$\psi = (-gg^{rr})^{-1/4} e^{-i\omega t + ikx}\Psi \qquad \Phi_\alpha = \frac{1}{2}(1 - (-1)^\alpha \Gamma^r \Gamma^t \Gamma^x)\Psi$$

$$\Phi_\alpha \overset{r\to\infty}{\approx} a_\alpha r^m \begin{pmatrix} 0 \\ 1 \end{pmatrix} + b_\alpha r^{-m} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad G_\alpha(\omega, k) = \frac{b_\alpha}{a_\alpha} \qquad \alpha = 1, 2$$

Fermi surfaces

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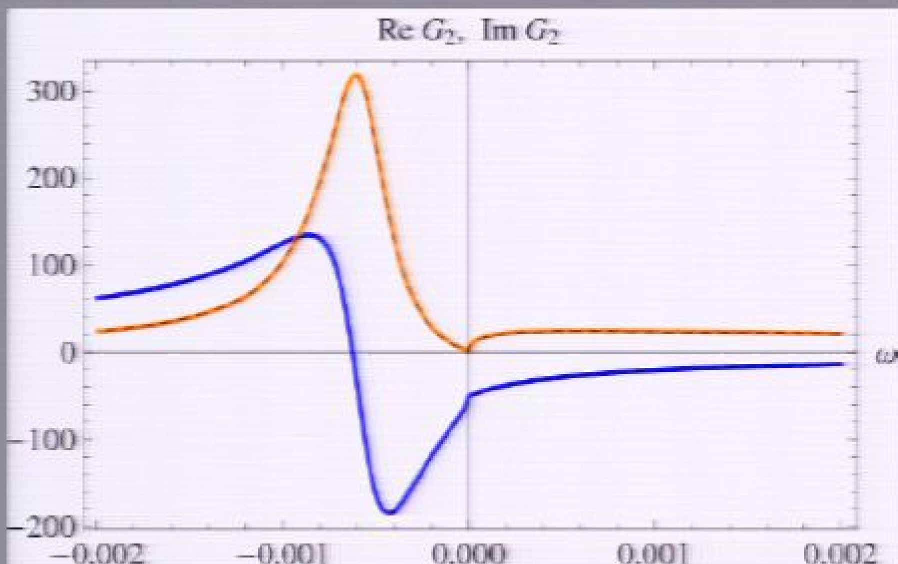

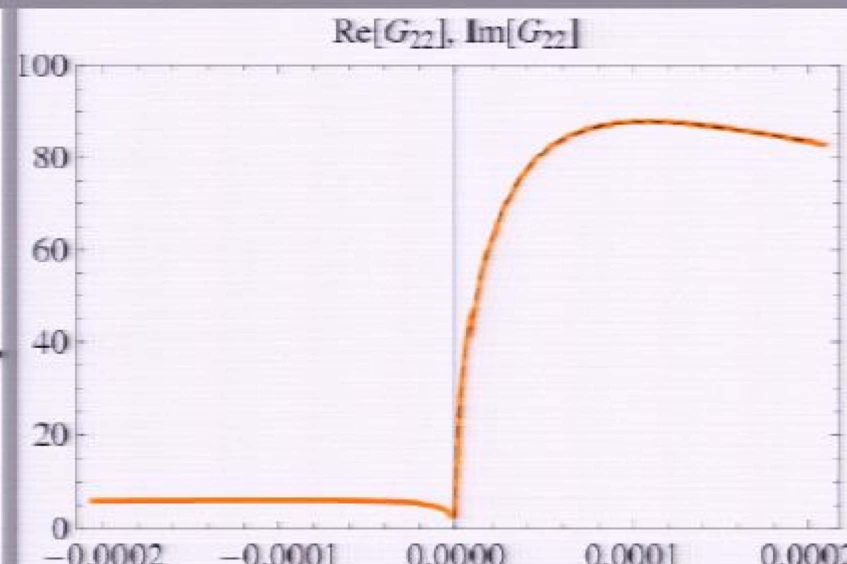

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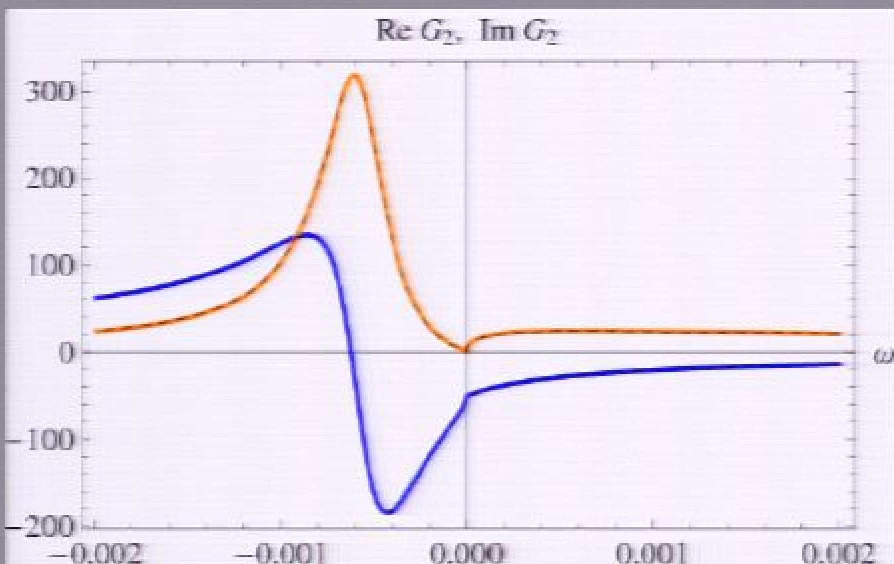

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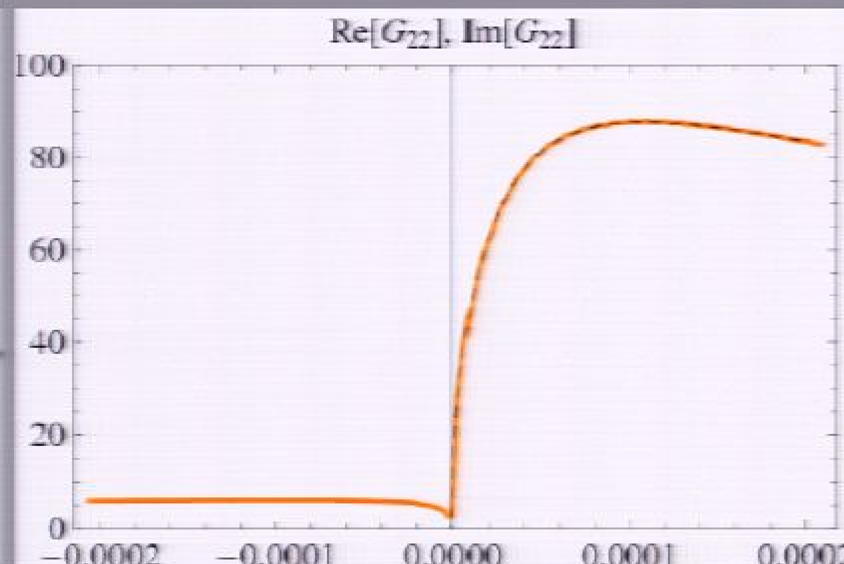

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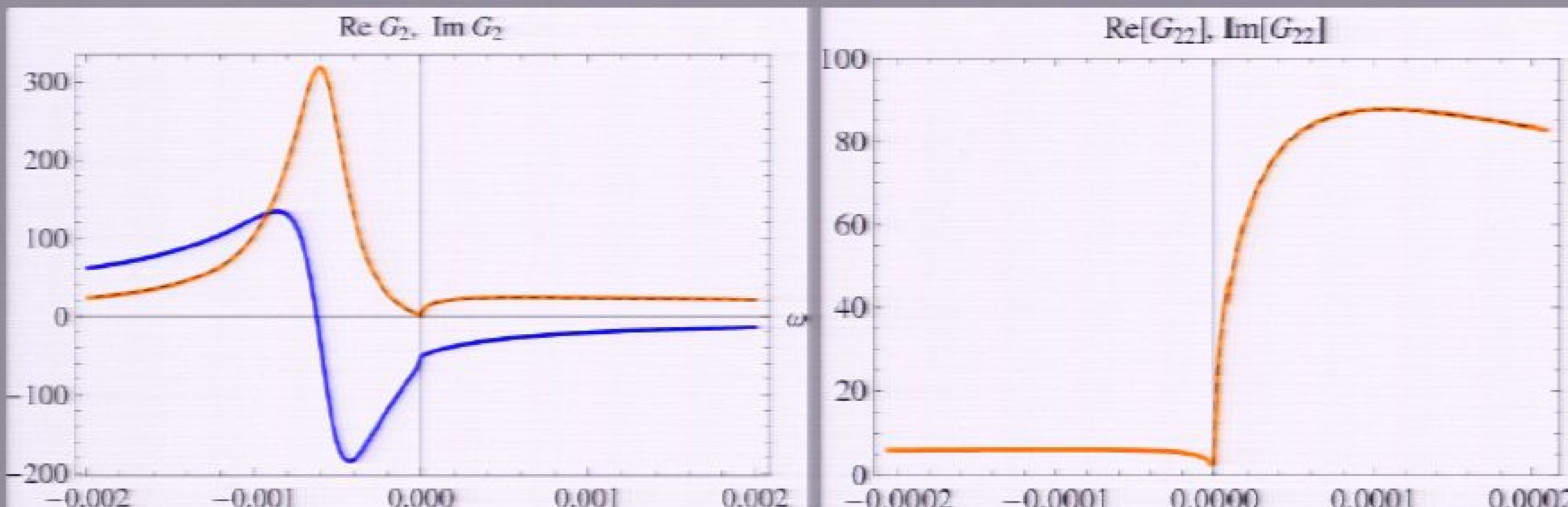

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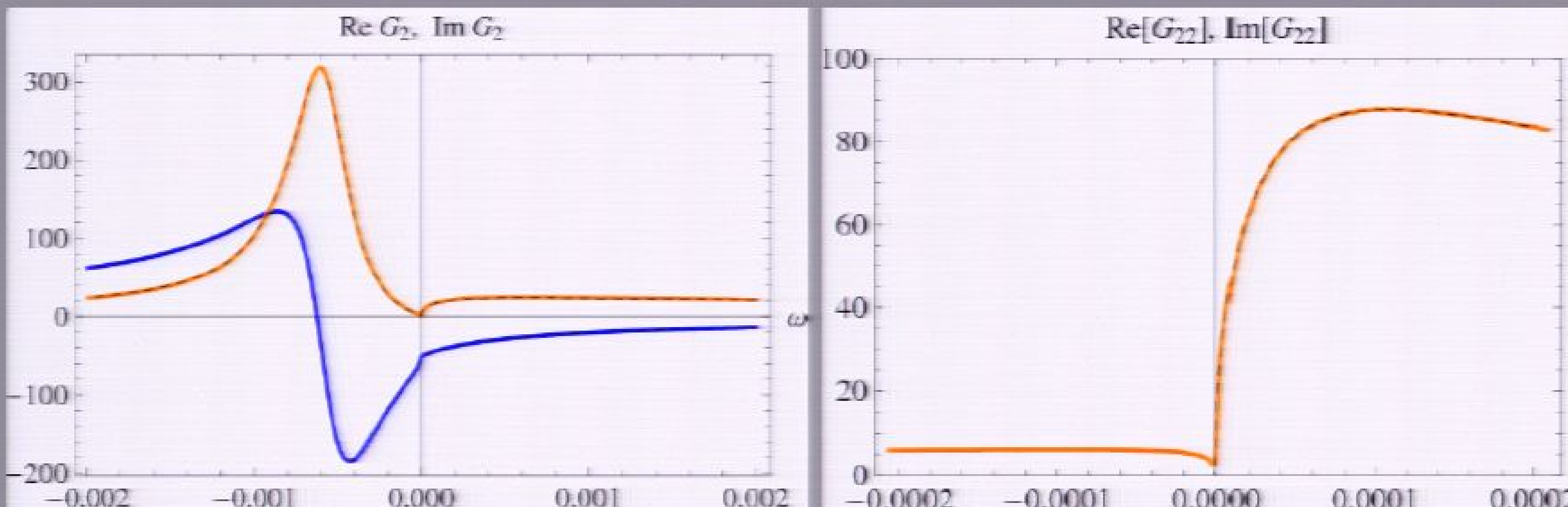

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Black hole geometry

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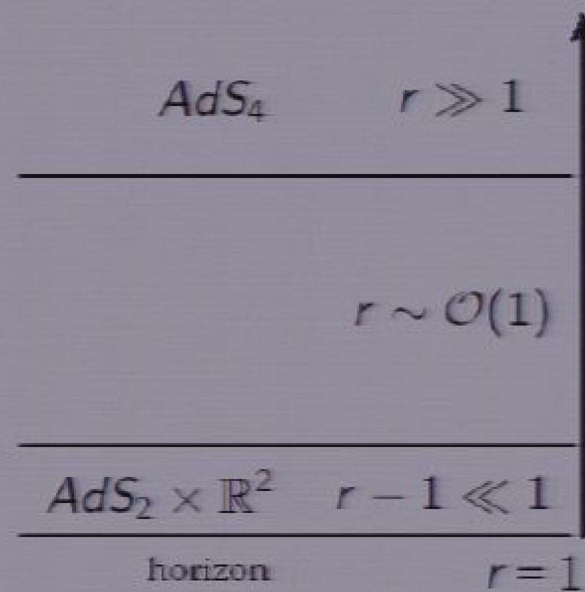

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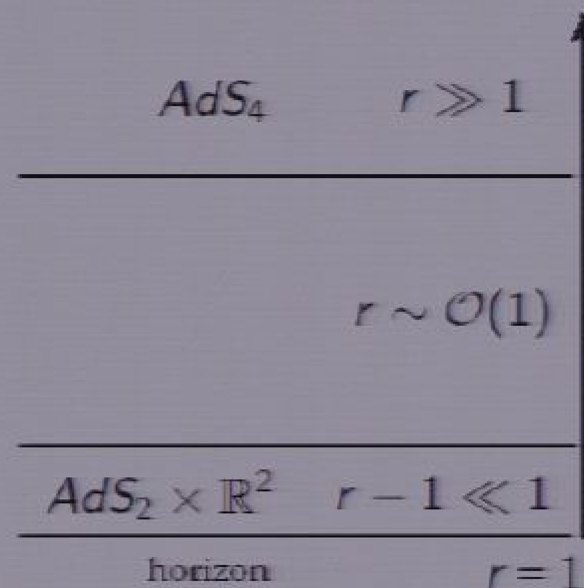

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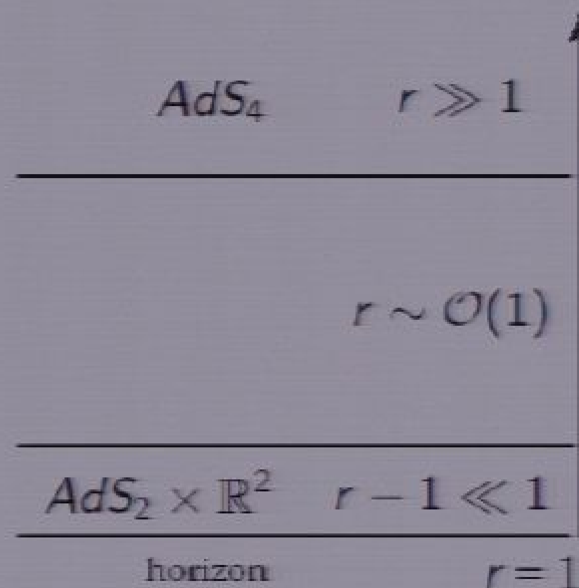
  Gravity in AdS_2 ⇔ (0+1)-d CFT



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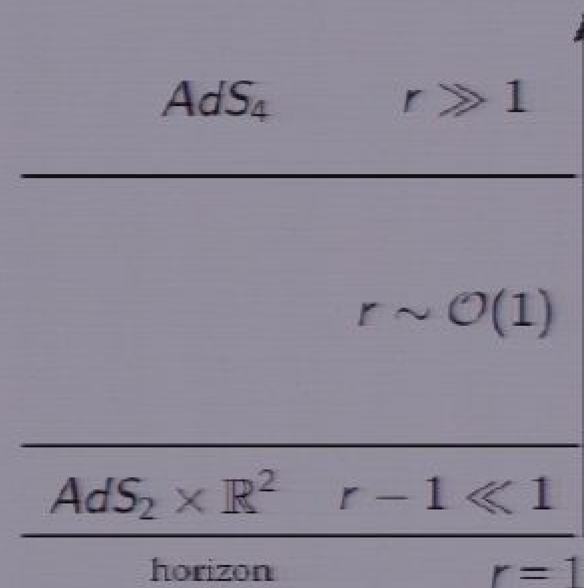

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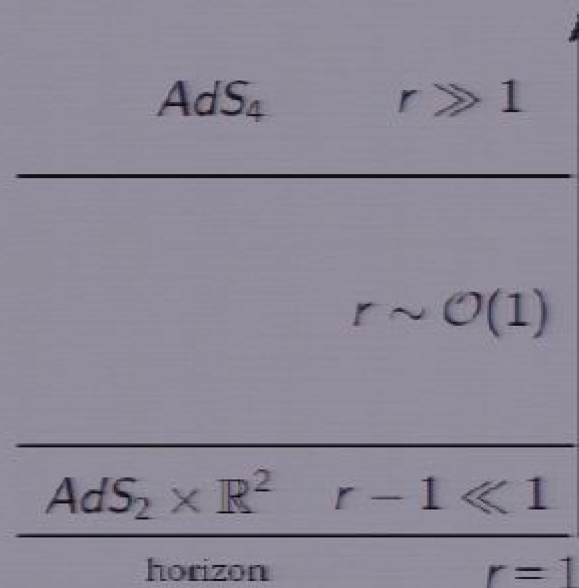

Small ω expansion

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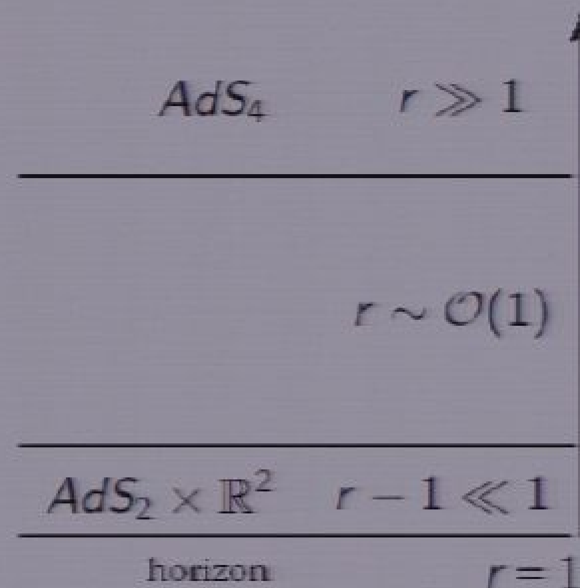
- Conformal dimensions in IR



$$\delta_{\vec{k}} = \frac{1}{2} + \nu_{\vec{k}} \qquad \nu_{\vec{k}} = \frac{1}{\sqrt{6}}\sqrt{\left(\Delta - \frac{3}{2}\right)^2 + k^2 - \frac{q^2}{2}} \qquad \mathcal{G}_k(\omega) = c(k)\omega^{2\nu_k}$$

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$$G_R(\omega, k) = \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2)\right)}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2)\right)}$$

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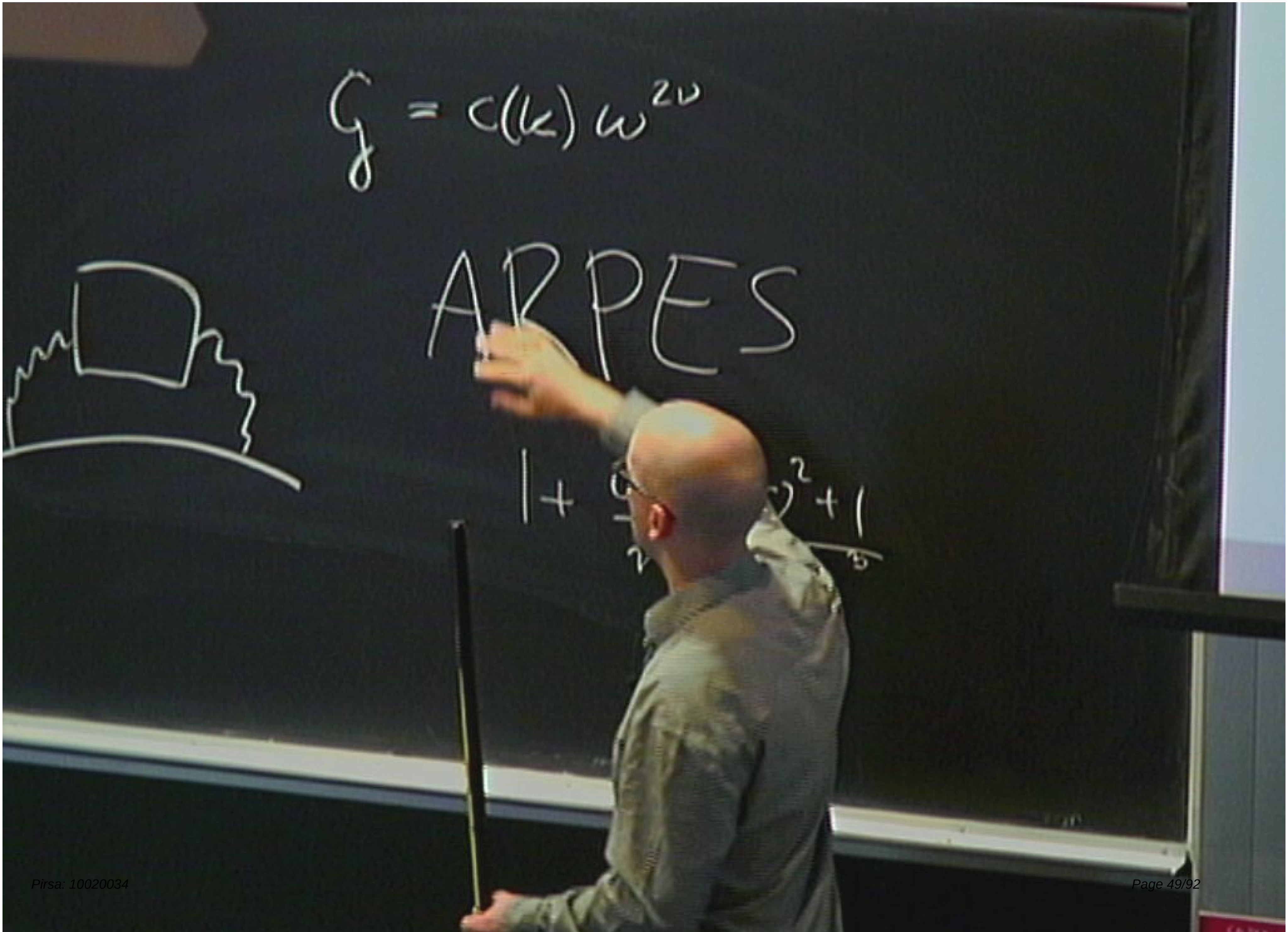
$\mathcal{G}_k(\omega) \in \mathbb{C}$ retarded correlator for $\mathcal{O}_{\vec{k}}$ in IR CFT


G = c(k) ω^2ν
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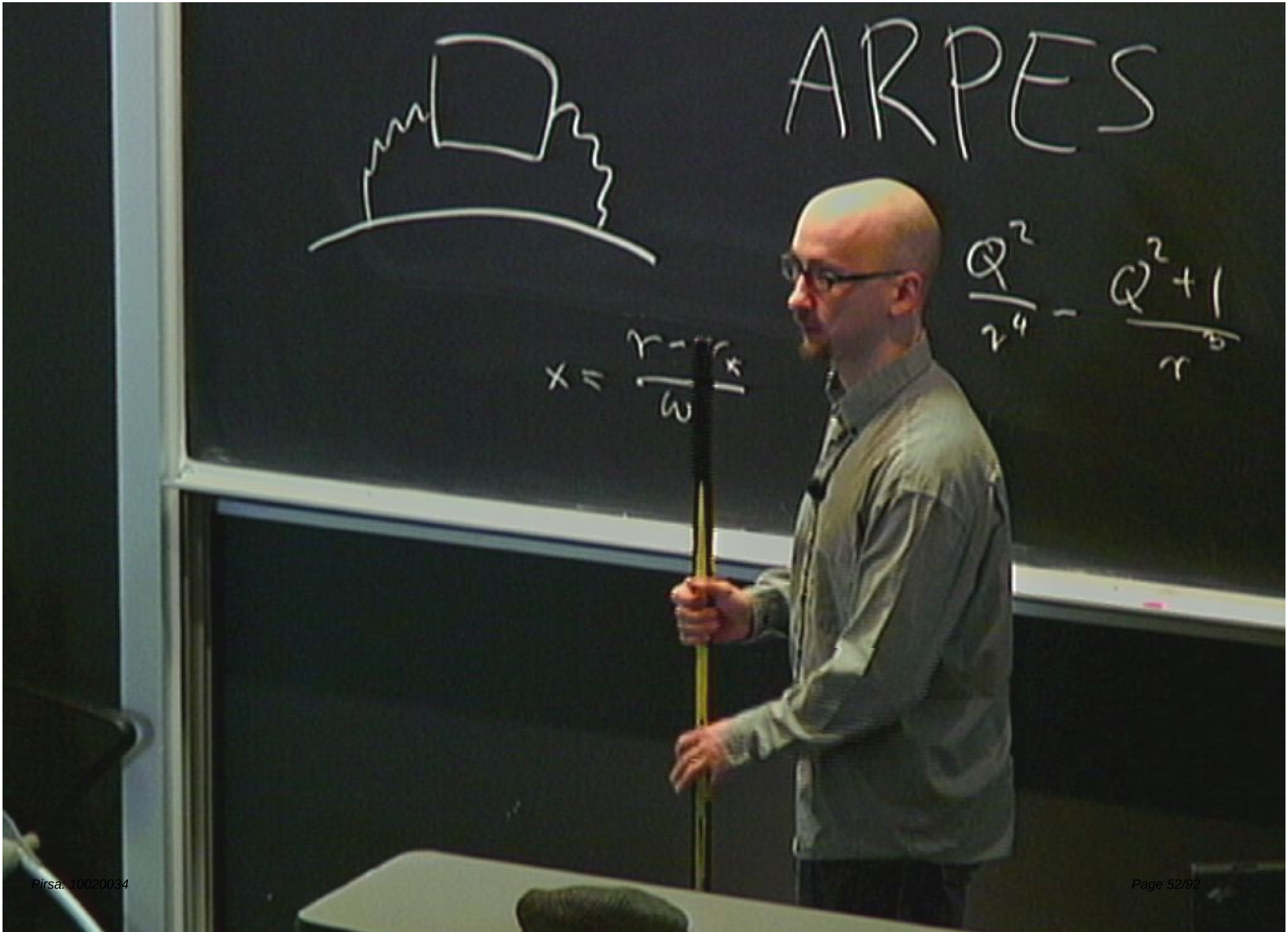
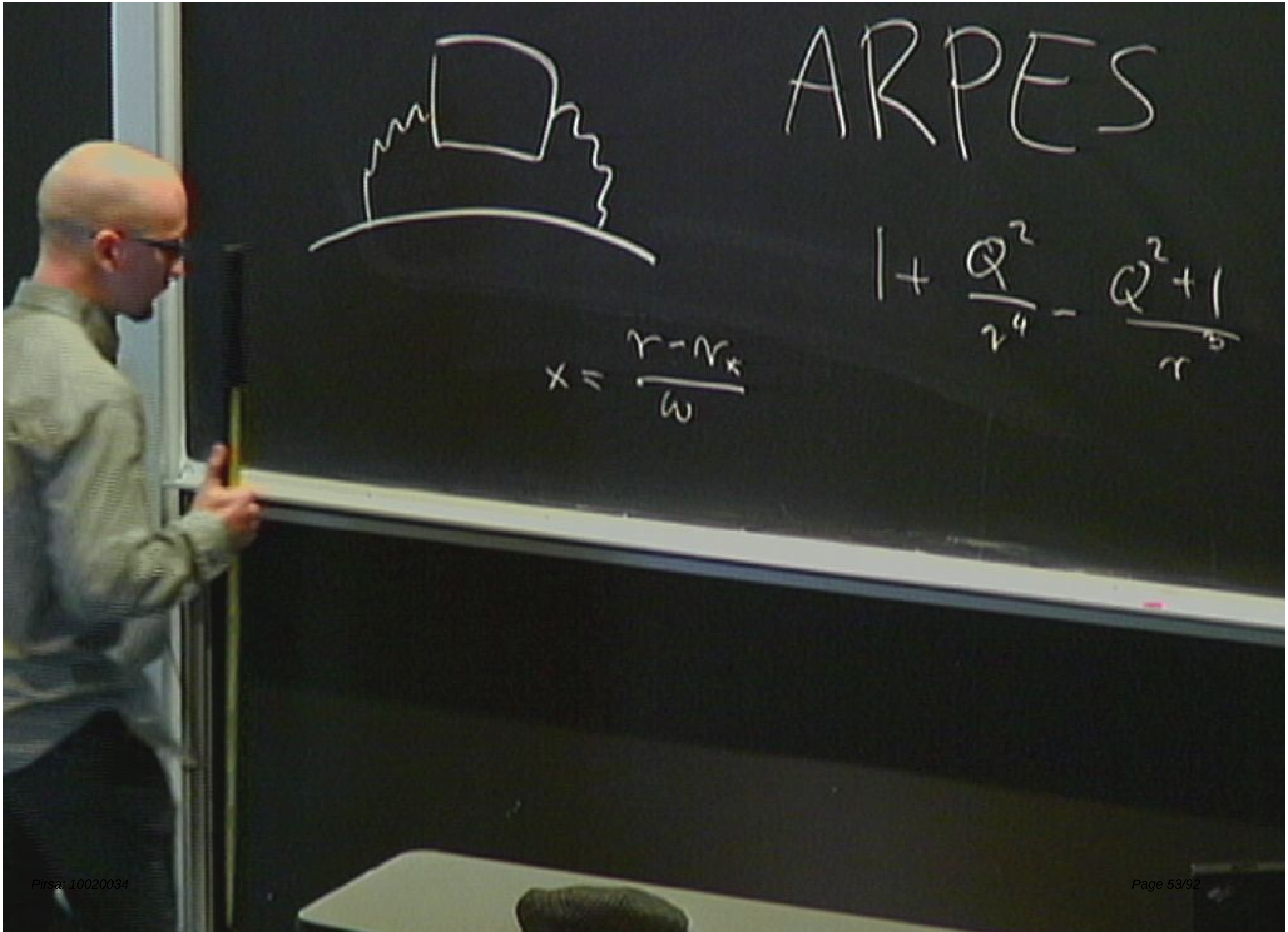
$a_\pm^{(0)}, a_\pm^{(1)}, b_\pm^{(0)}, b_\pm^{(1)} \in \mathbb{R}$ are k-dependent functions (from UV region)


ARPES
$\frac{Q^2}{r^4} - \frac{Q^2+1}{r^5}$
$x = \frac{r - r_*}{\omega}$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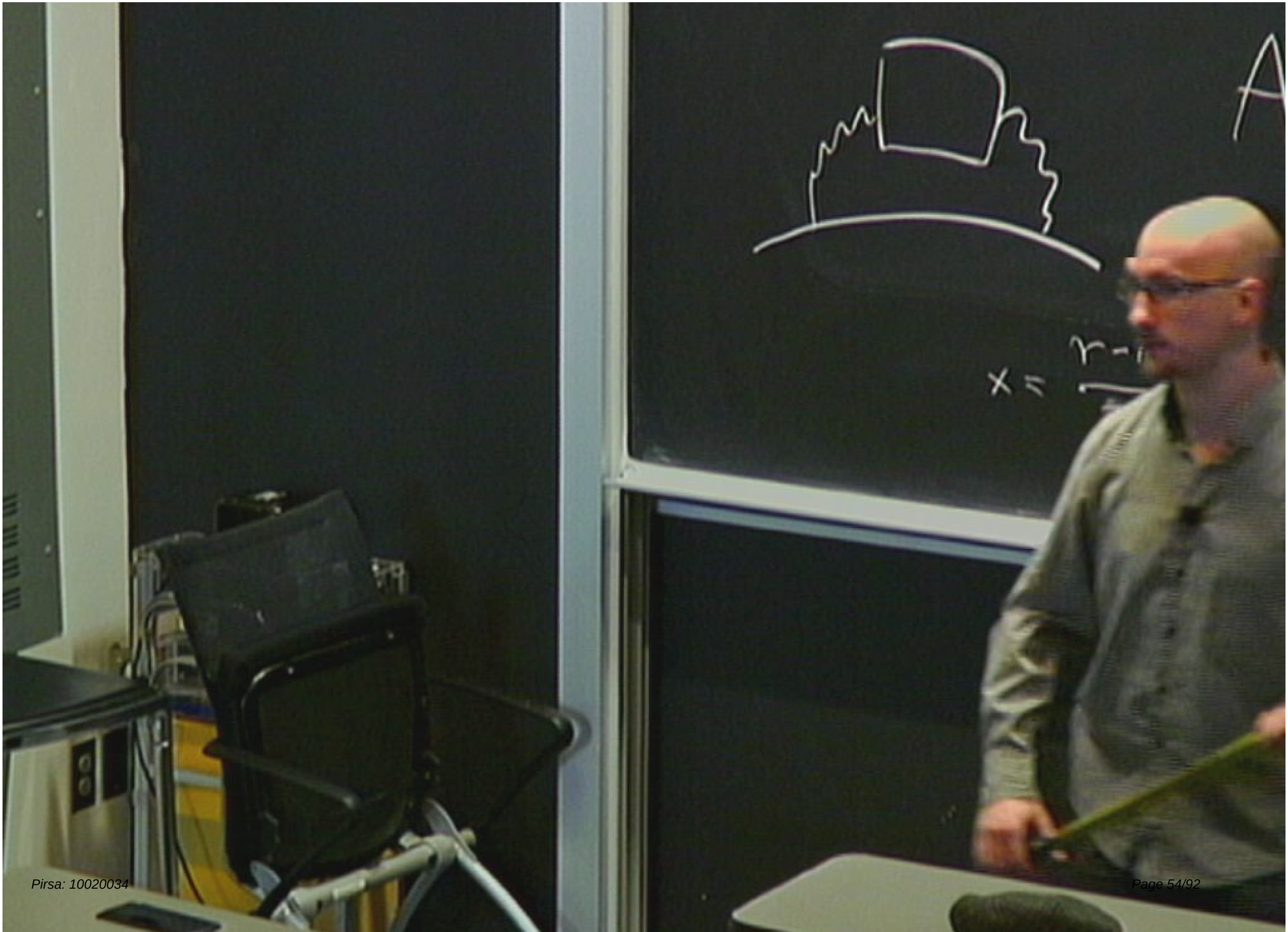

Small ω expansion

- To understand scaling around FS, study low-ω behavior of correlators.
- For extremal black hole, this is not straightforward, because ω-dependent terms in the Dirac equation always become singular near the horizon.

Strategy

- Separate spacetime into UV region and the $AdS_2 \times \mathbb{R}^2$ IR region.
- Perform small ω expansions separately.
- Match them at the overlapping region.

Result

$$G_R(\omega, k) = \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2)\right)}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2)\right)}$$

$\mathcal{G}_k(\omega) \in \mathbb{C}$ retarded correlator for $\mathcal{O}_{\vec{k}}$ in IR CFT
$a_\pm^{(0)}, a_\pm^{(1)}, b_\pm^{(0)}, b_\pm^{(1)} \in \mathbb{R}$ are k-dependent functions (from UV region)

Fermi surfaces

$$G_R(\omega, k) = \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2)\right)}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2)\right)}$$

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Suppose that for some k_F: $a_+^{(0)}(k_F) = 0$
Then, at small $\omega, k_\perp$ we have

$$G_R(k, \omega) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2 \mathcal{G}_{k_F}(\omega)} + \dots$$

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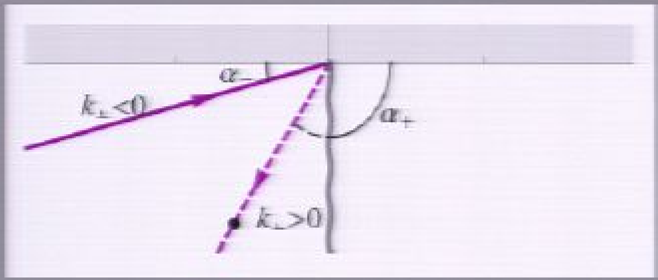
$$\mathcal{G}_k(\omega) = c(k)\omega^{2\nu_k} \qquad \nu_{\vec{k}} = \frac{1}{\sqrt{6}}\sqrt{m^2 + k^2 - \frac{q^2}{2}} \qquad h_1, h_2, v_F \in \mathbb{R}$$

This is the quasiparticle peak we saw earlier.

Fermi surfaces: singular and non-singular

Suppose $\nu_k < \frac{1}{2}$

$$G_R(k,\omega) = \frac{h_1}{k_\perp - h_2 \mathcal{G}_{k_F}(\omega)} + \dots$$



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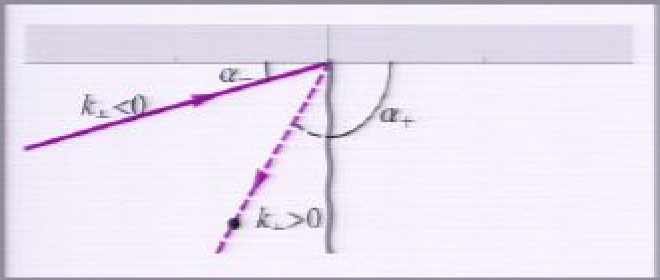

$$G_R(k,\omega) = \frac{h_1}{k_\perp - h_2 \mathcal{G}_{k_F}(\omega)} + \ldots$$

$$\omega_*(k) \sim k_\perp^z \quad z = \frac{1}{2\nu_{k_F}} > 1 \quad \frac{\Gamma(k)}{\omega_*(k)} = \text{const} \quad Z \sim k_\perp^{\frac{1-2\nu_{k_F}}{2\nu_{k_F}}} \to 0, k_\perp \to 0$$

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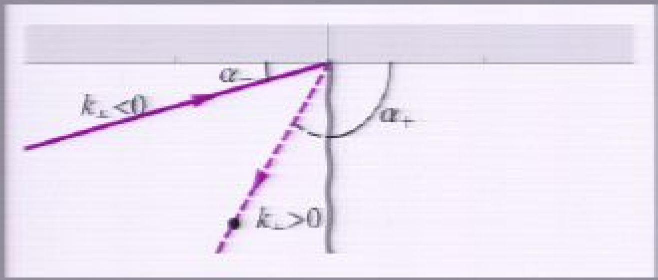

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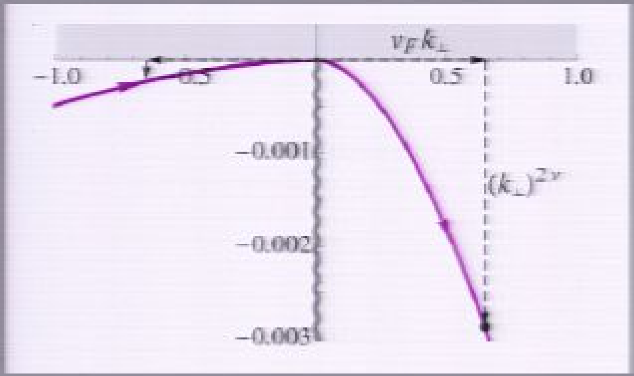

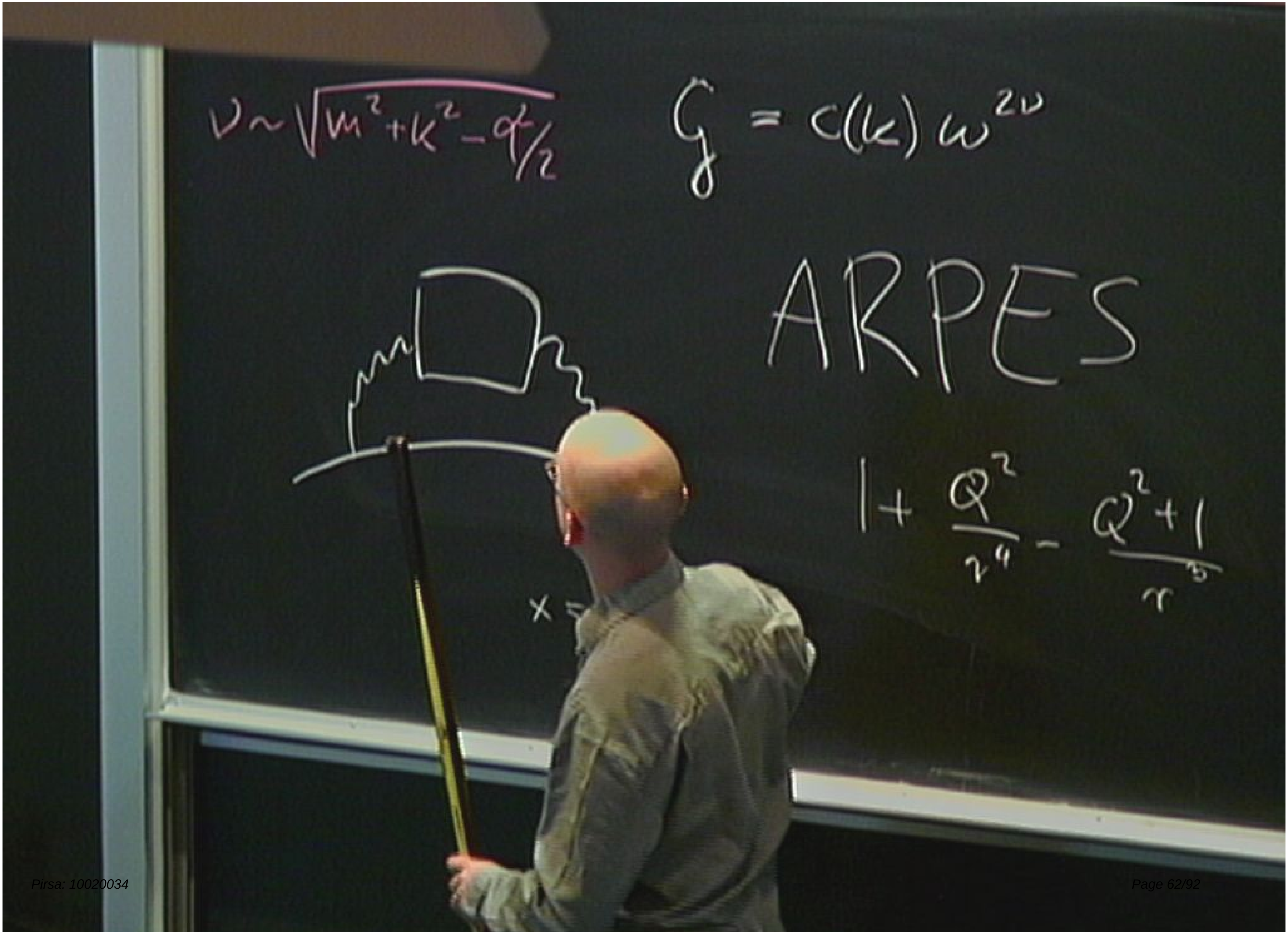
$\nu \sim \sqrt{m^2 + k^2 - q/2}$
$G = c(k)\,\omega^{2\nu}$
ARPES
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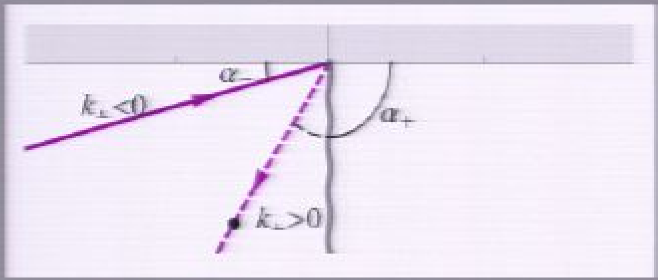

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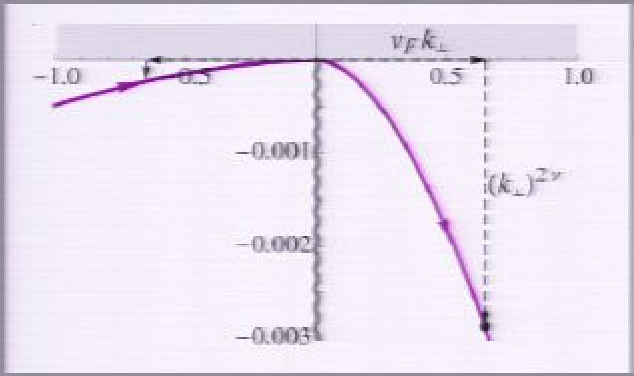

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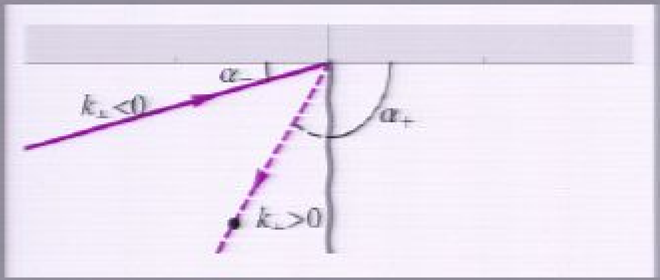

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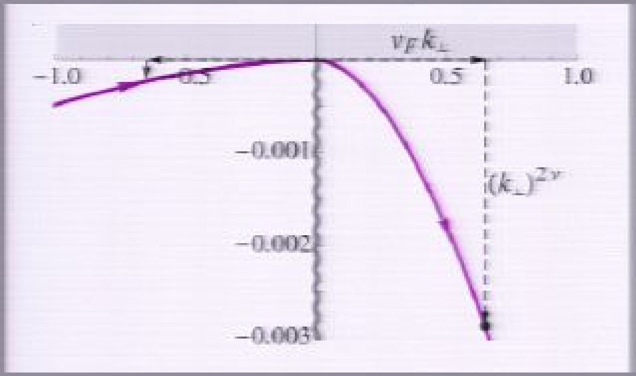

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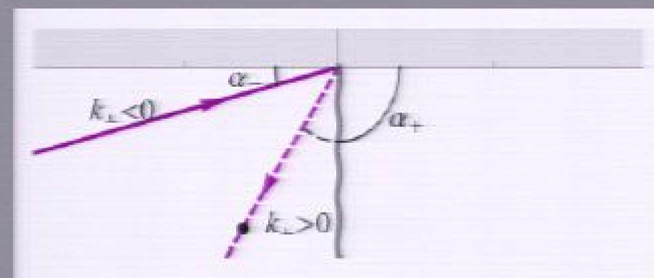

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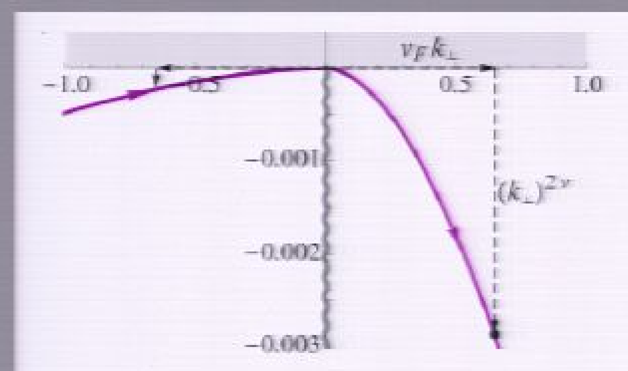

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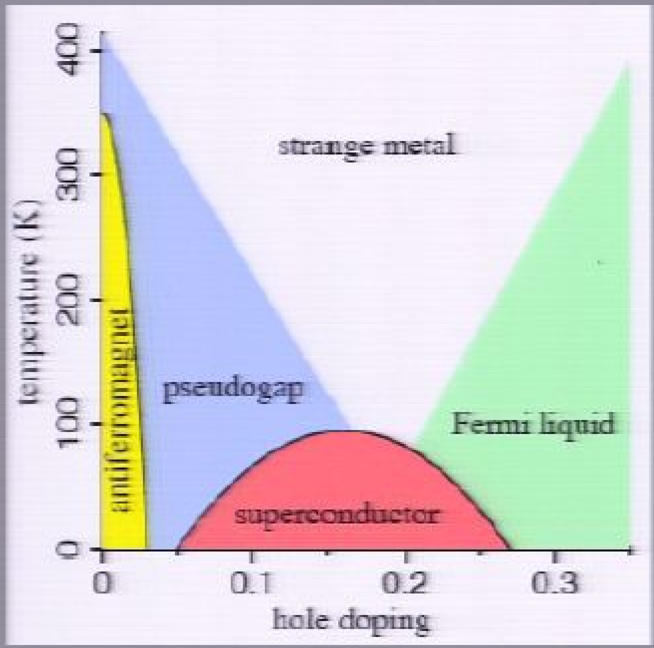
$$G_R(k,\omega) = \frac{h_1}{k_\perp + \tilde{c}_1 \omega \log \omega + c_1 \omega} + \dots$$

where $\tilde{c}_1 \in \mathbb{C}$, $c_1 \in \mathbb{R}$.

This is the Marginal Fermi liquid Green's function [Varma, 1989]



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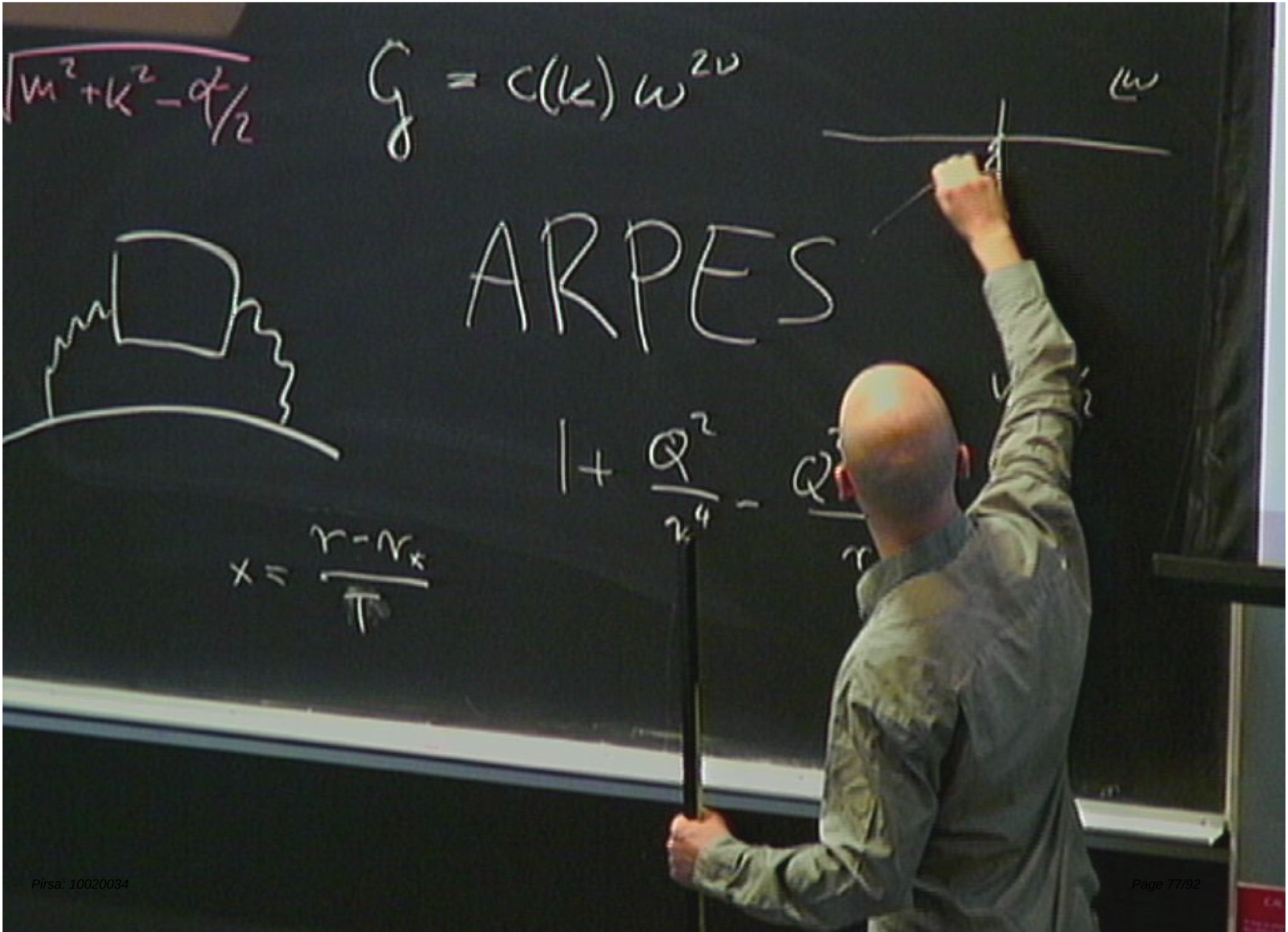
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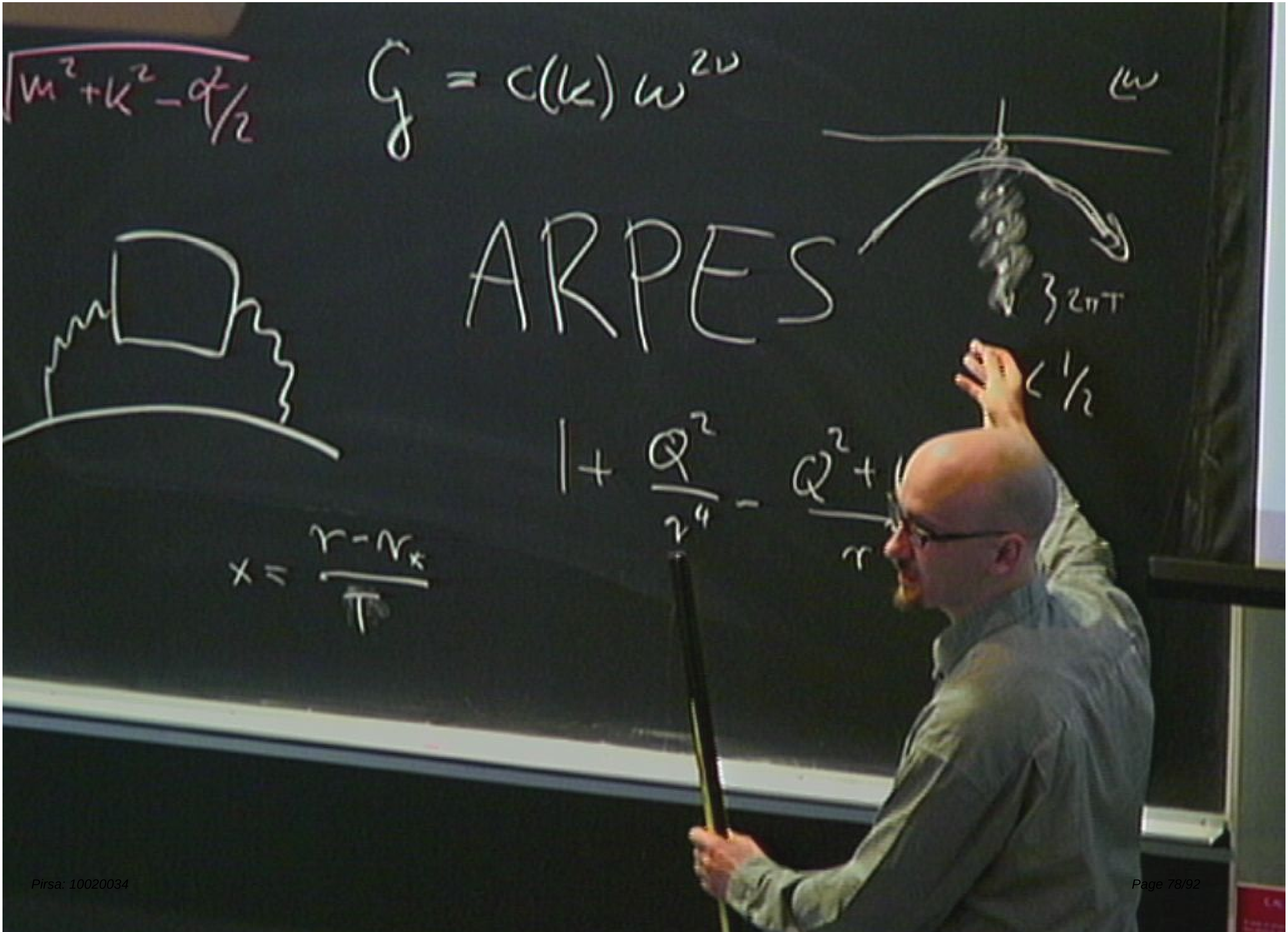
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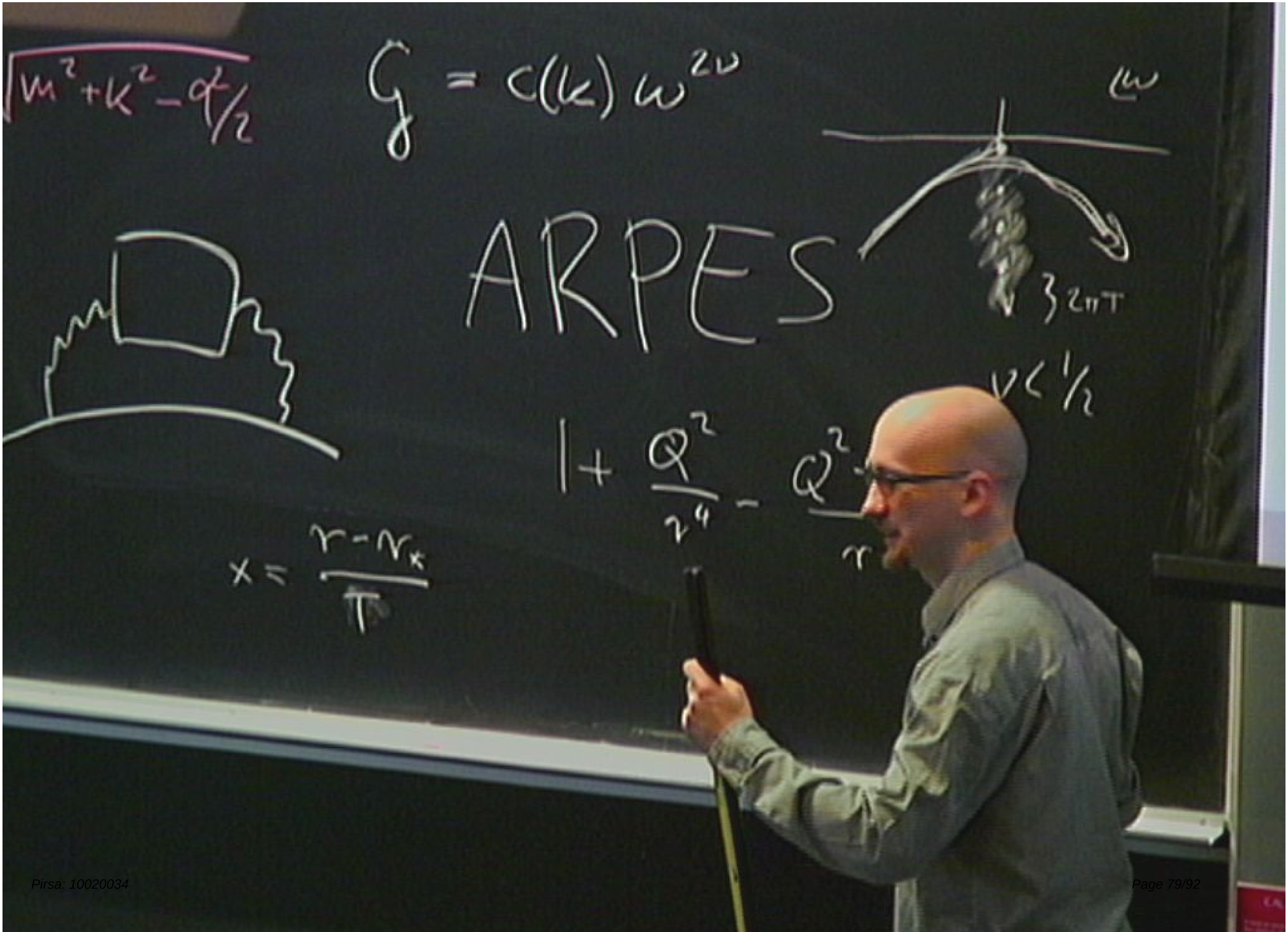

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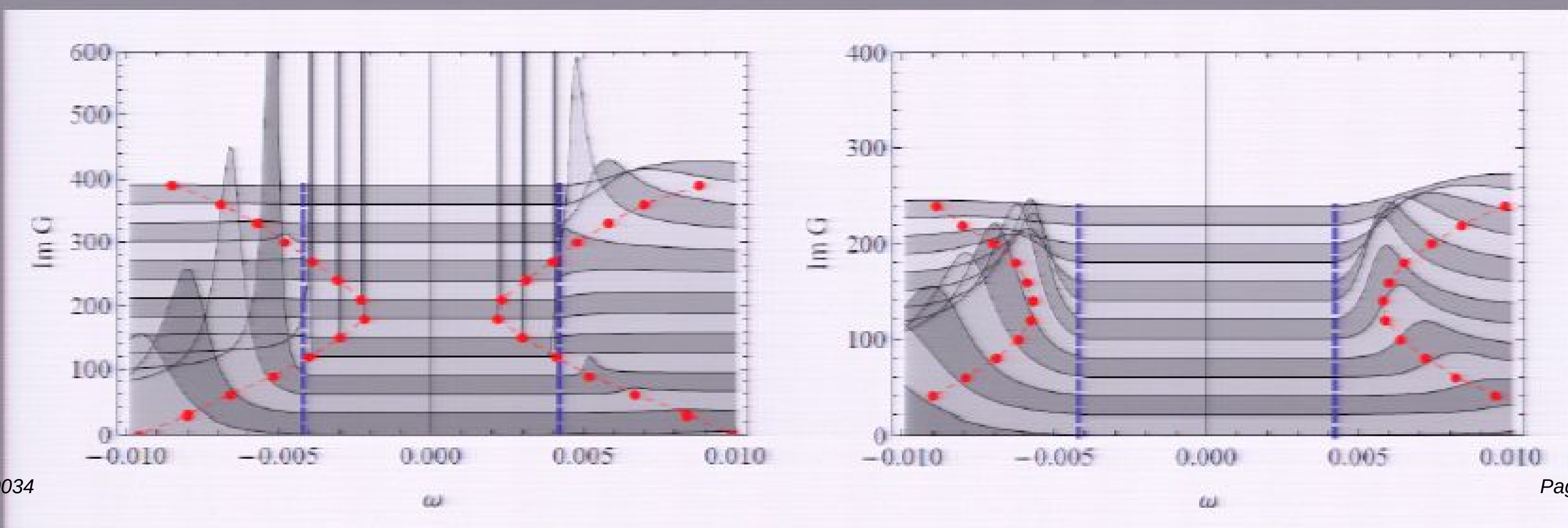

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