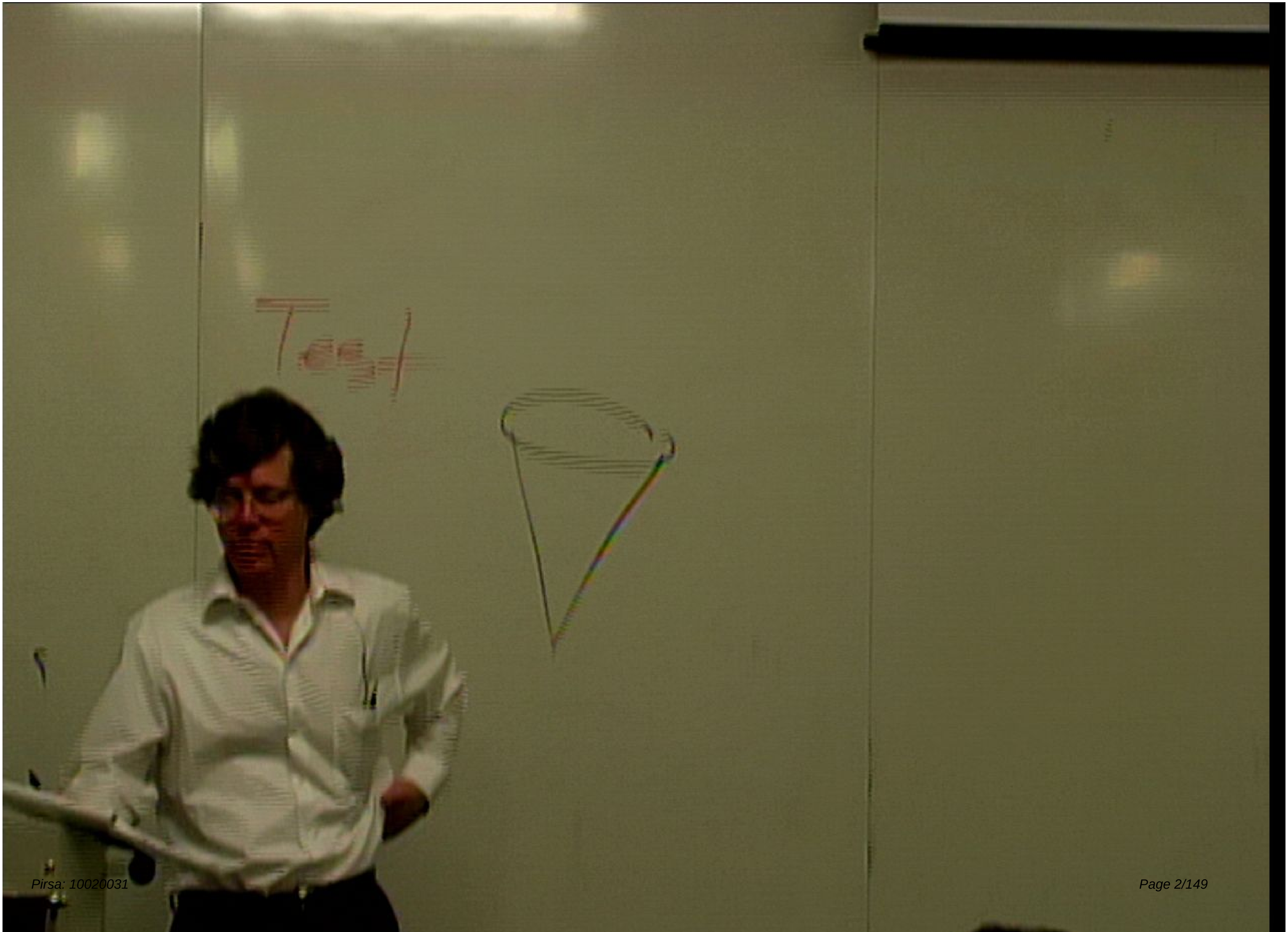
Title: Foundations and Interpretation of Quantum Theory - Lecture 11

Date: Feb 25, 2010 02:30 PM

URL: http://pirsa.org/10020031

Abstract: After a review of the axiomatic formulation of quantum theory, the generalized operational structure of the theory will be introduced (including POVM measurements, sequential measurements, and CP maps). There will be an introduction to the orthodox (sometimes called Copenhagen) interpretation of quantum mechanics and the historical problems/issues/debates regarding that interpretation, in particular, the measurement problem and the EPR paradox, and a discussion of contemporary views on these topics. The majority of the course lectures will consist of guest lectures from international experts covering the various approaches to the interpretation of quantum theory (in particular, many-worlds, de Broglie-Bohm, consistent/decoherent histories, and statistical/epistemic interpretations, as time permits) and fundamental properties and tests of quantum theory (such as entanglement and experimental tests of Bell inequalities, contextuality, macroscopic quantum phenomena, and the problem of quantum gravity, as time permits).



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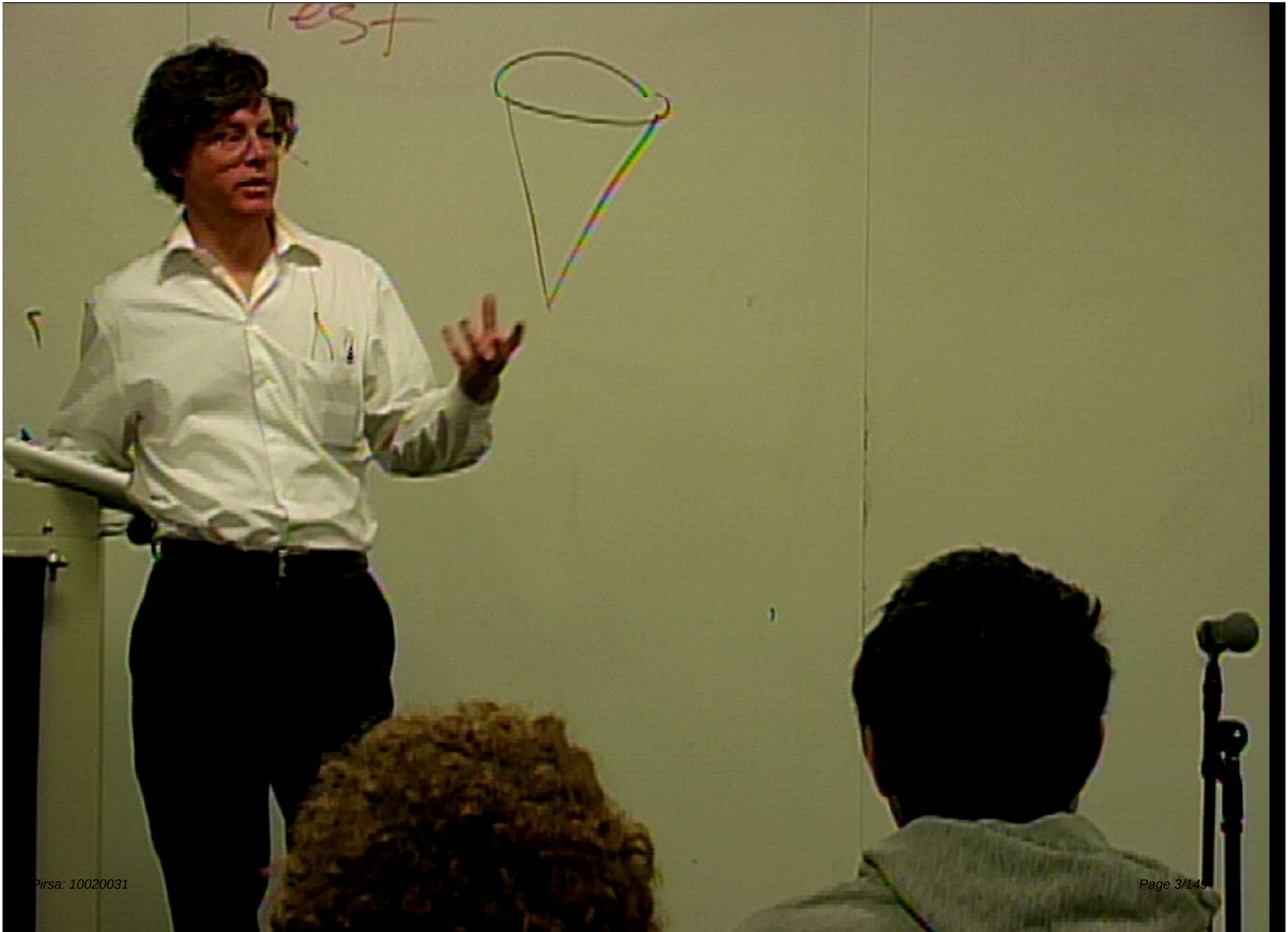
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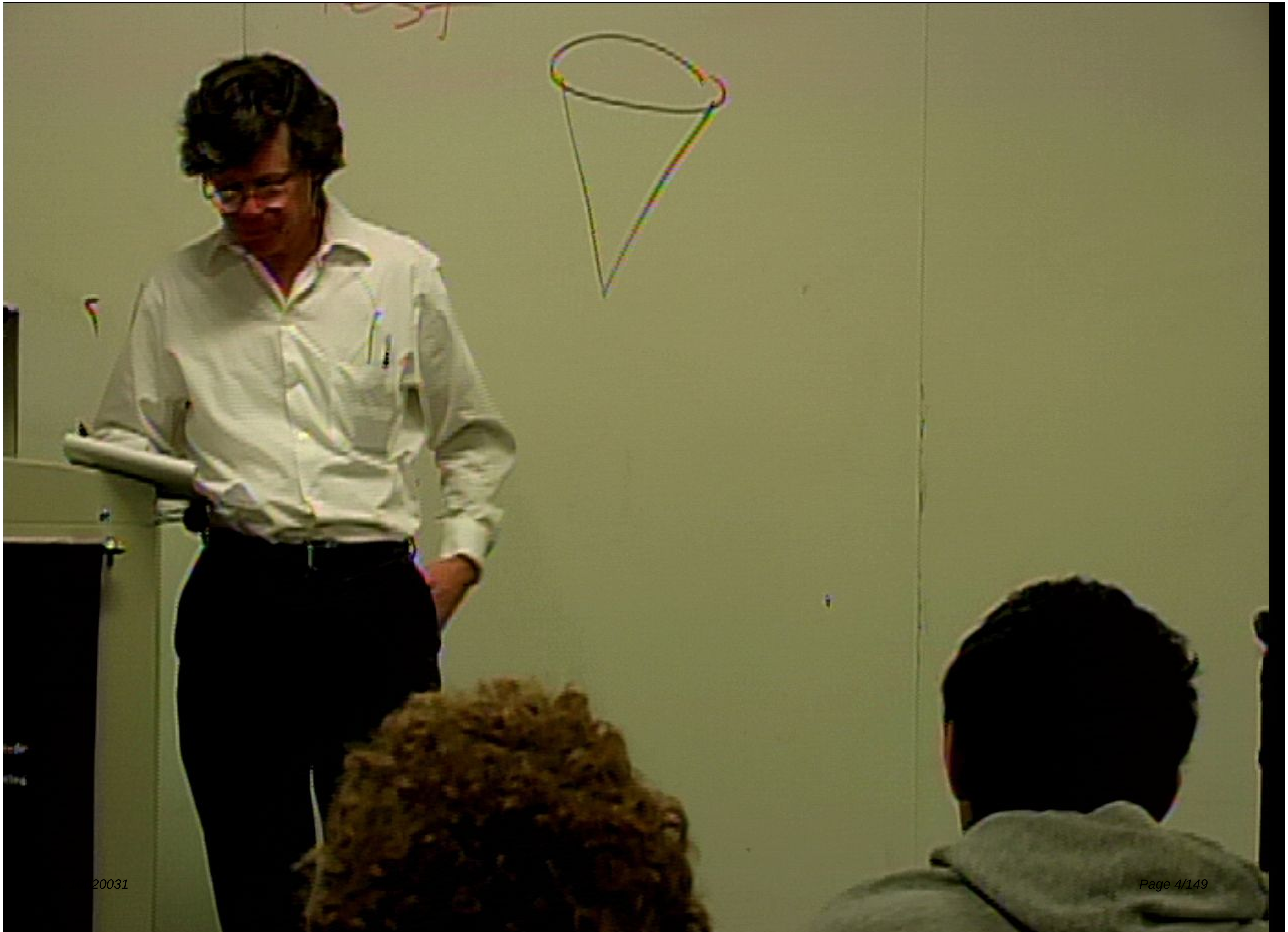

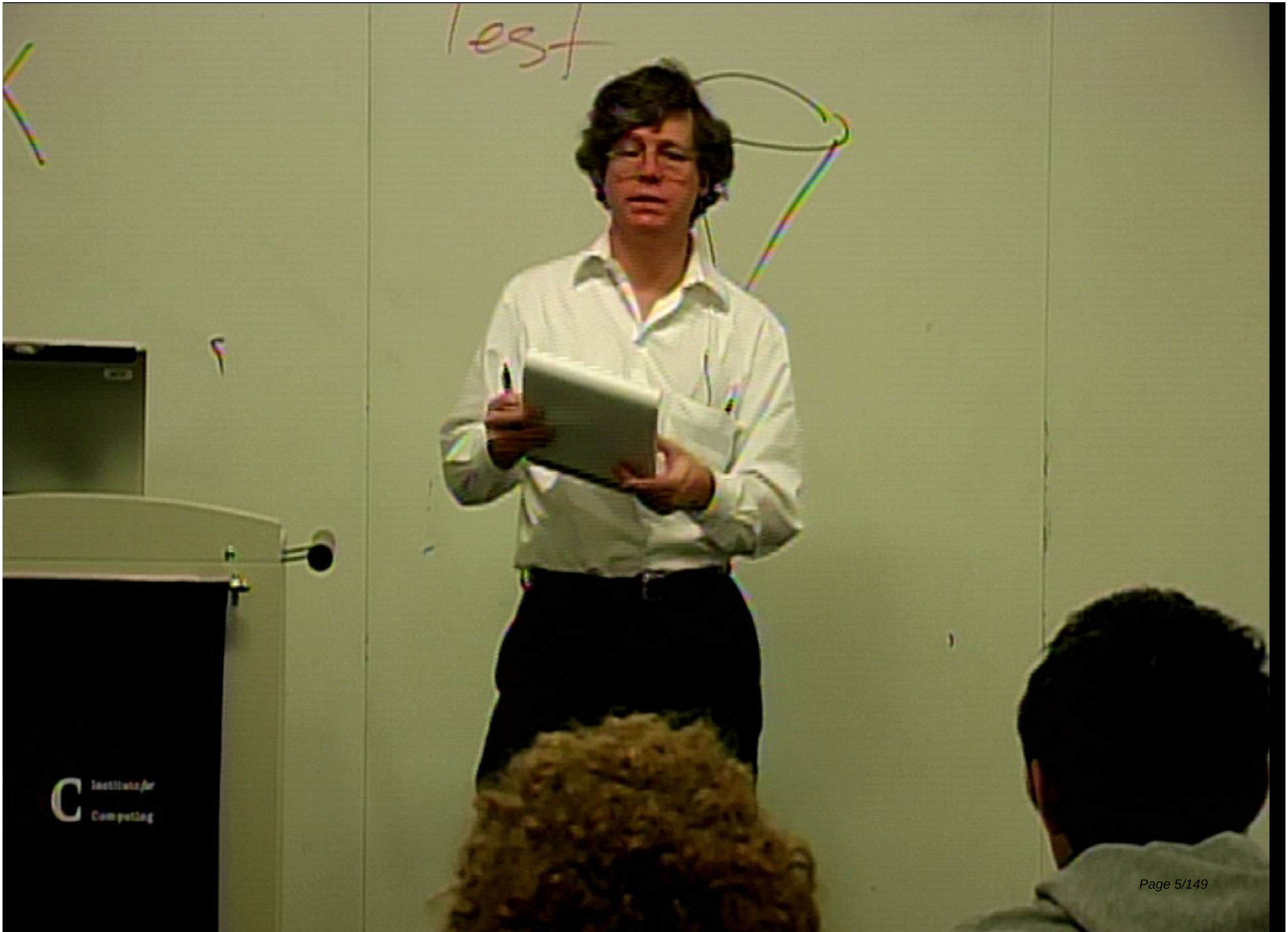
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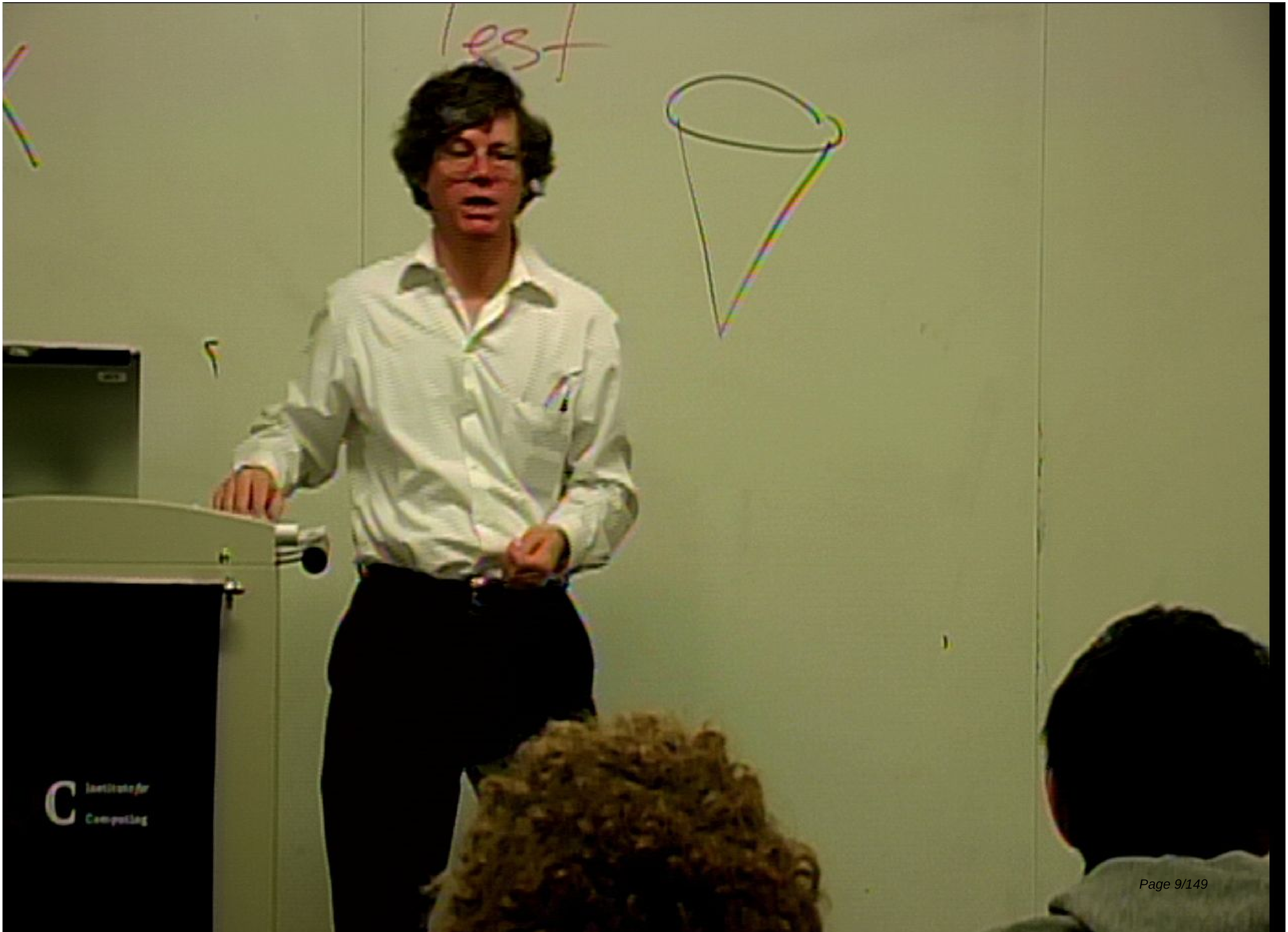
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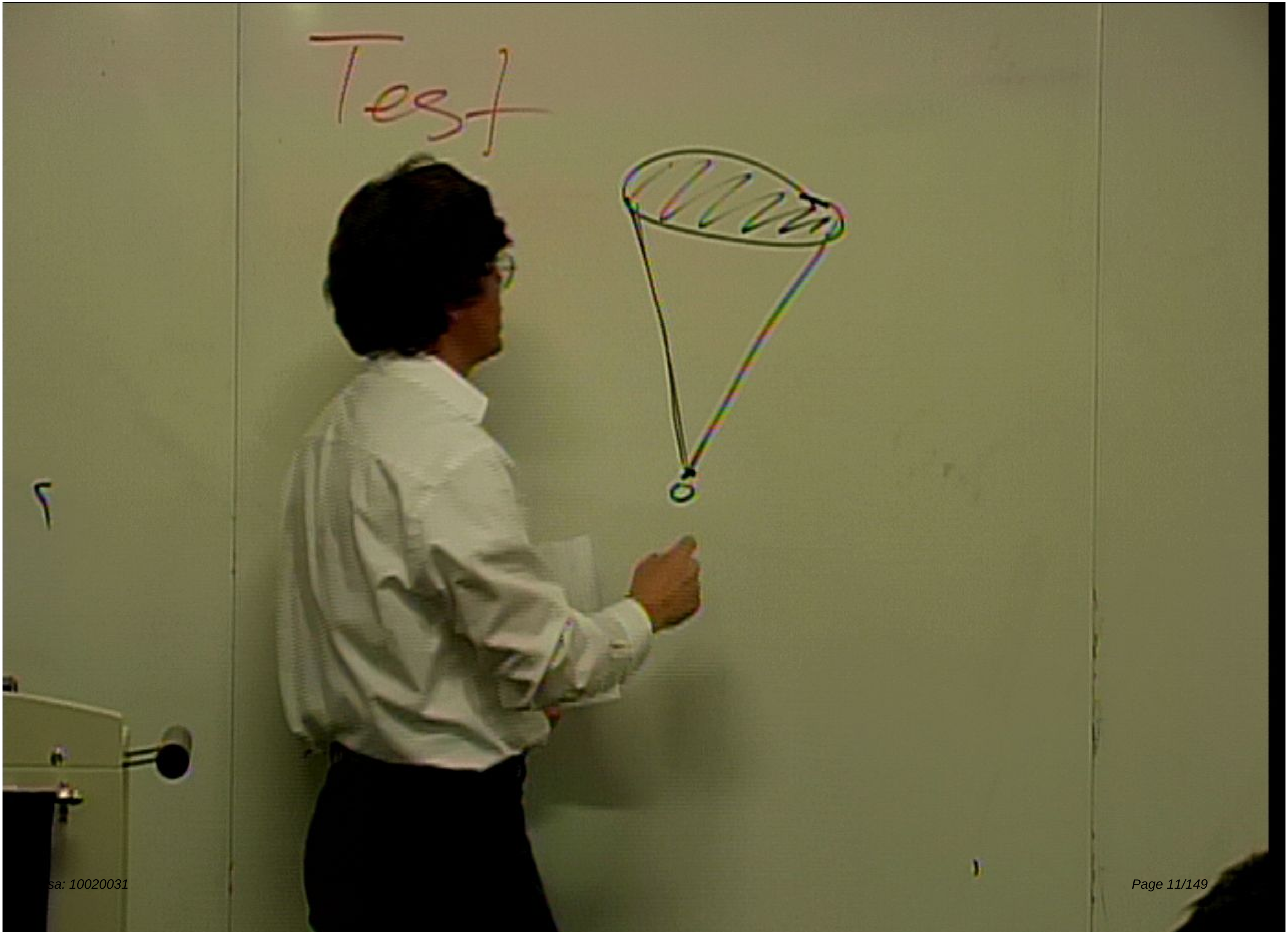
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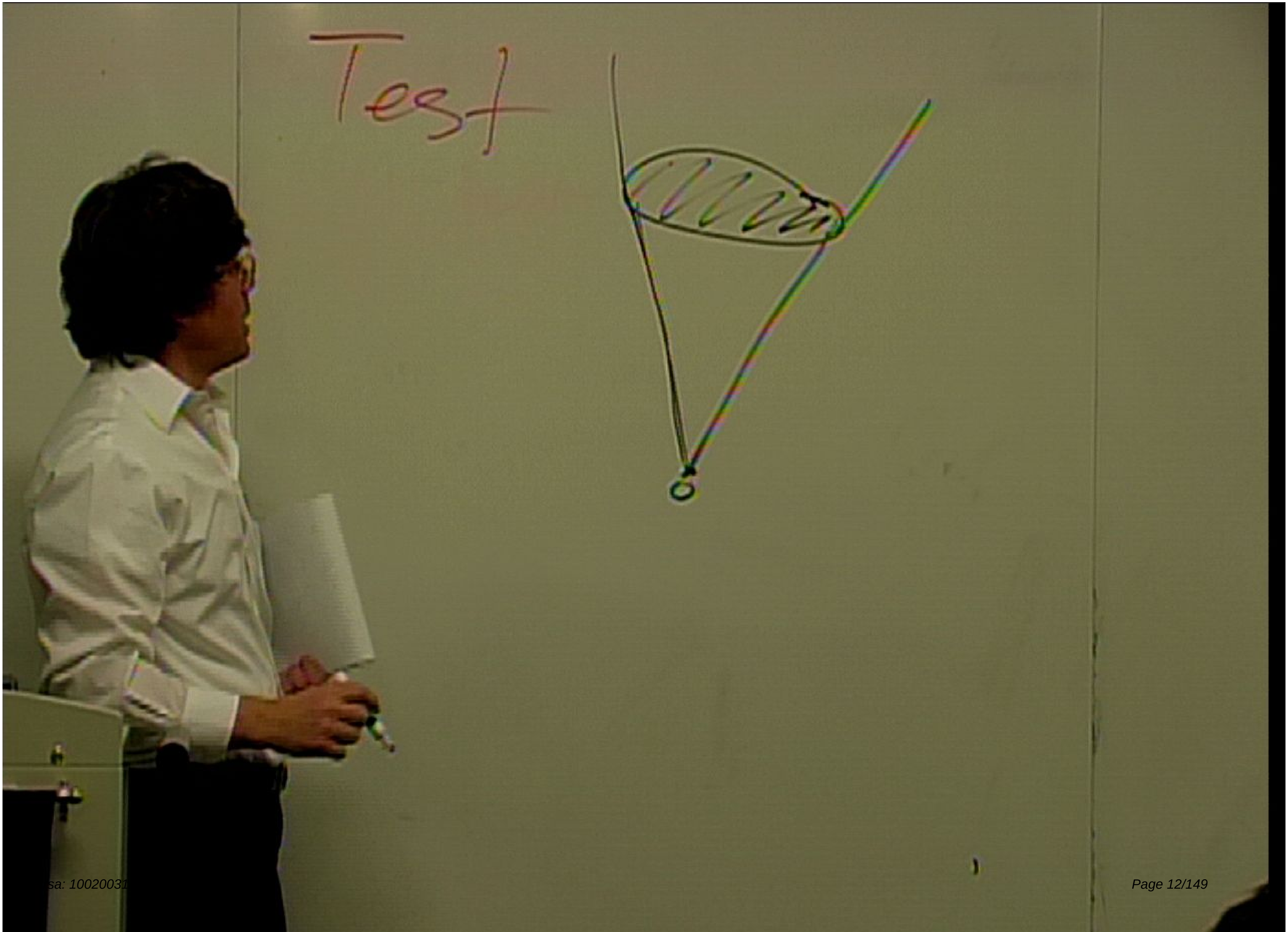
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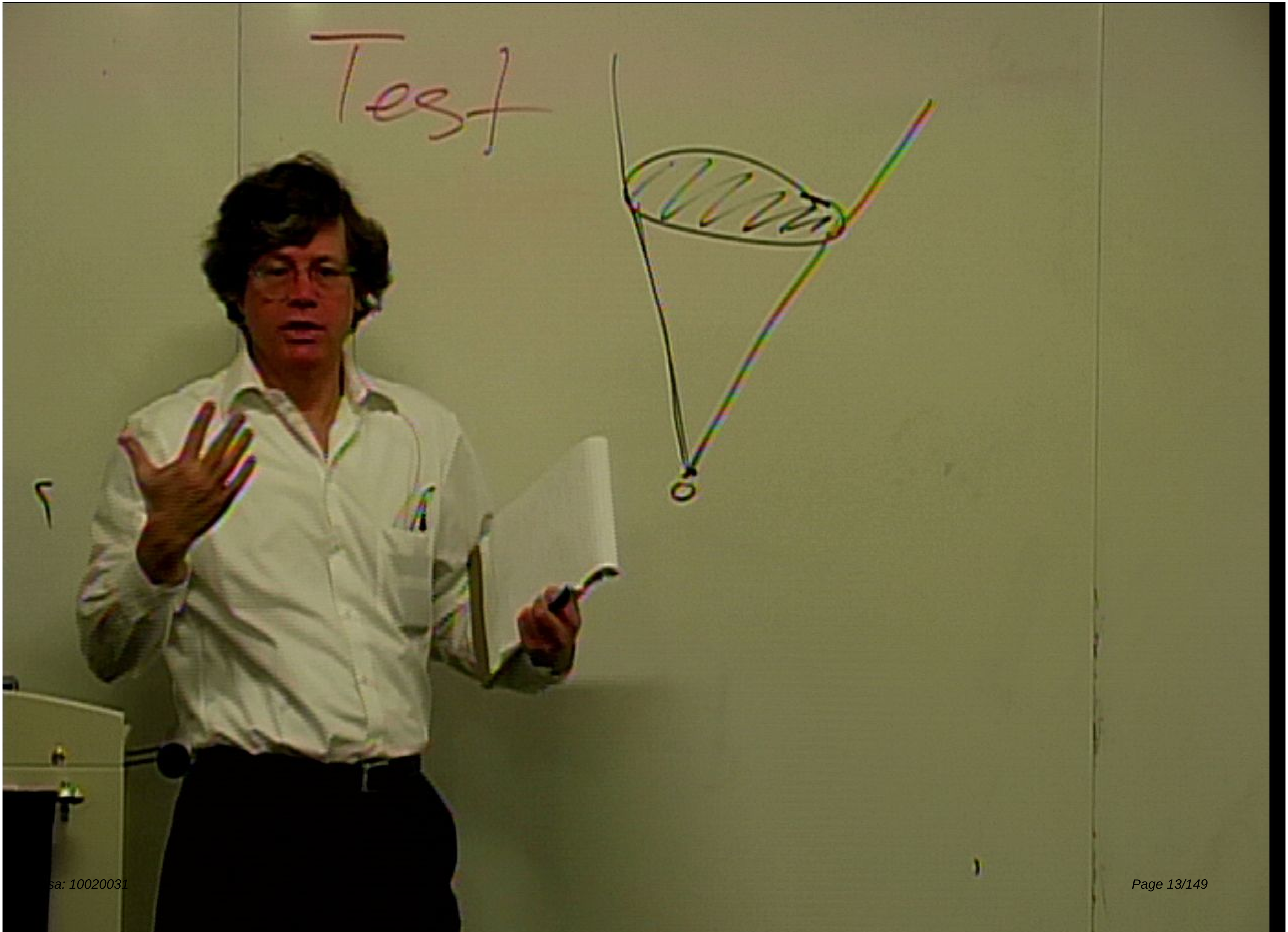
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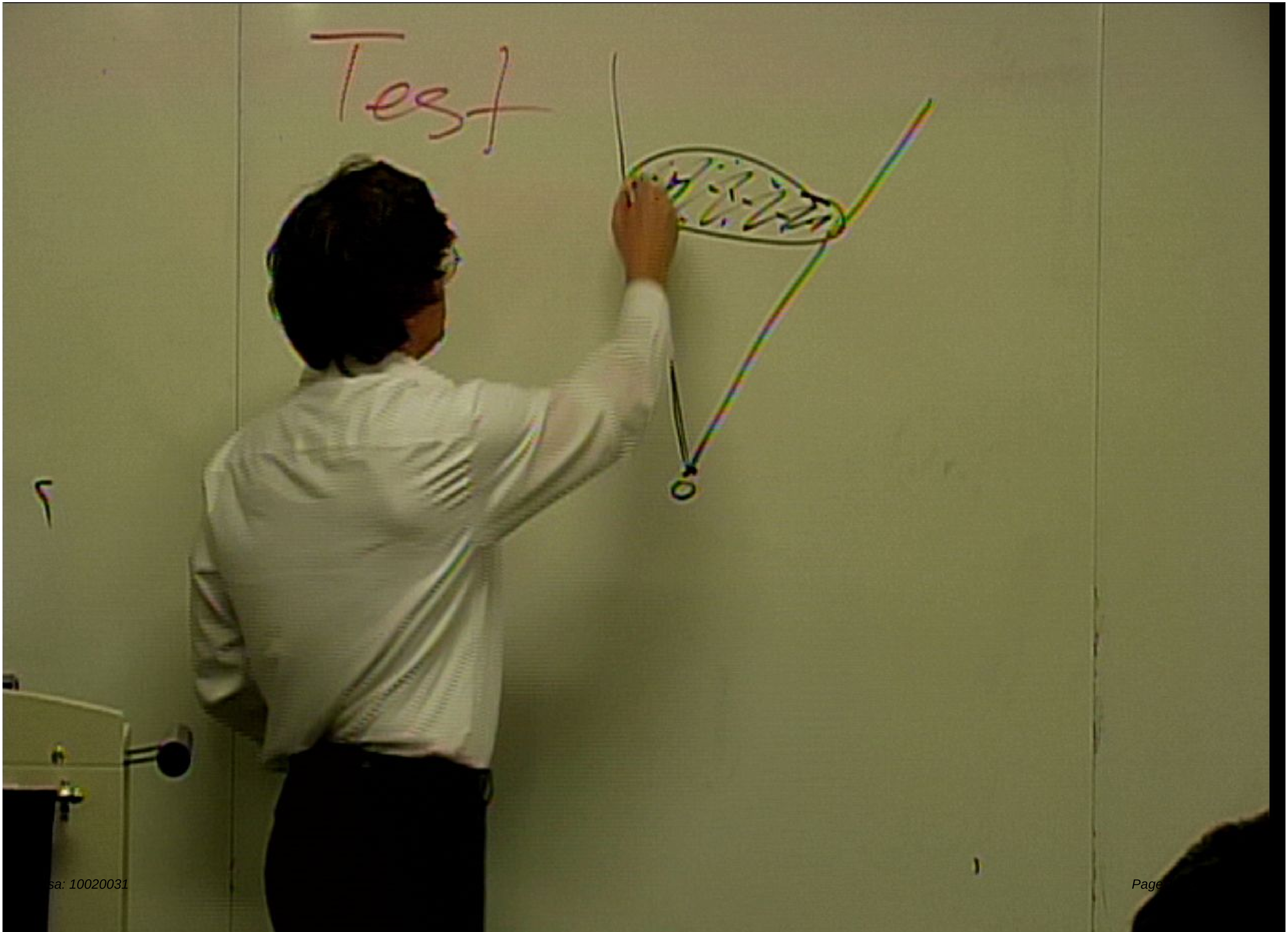
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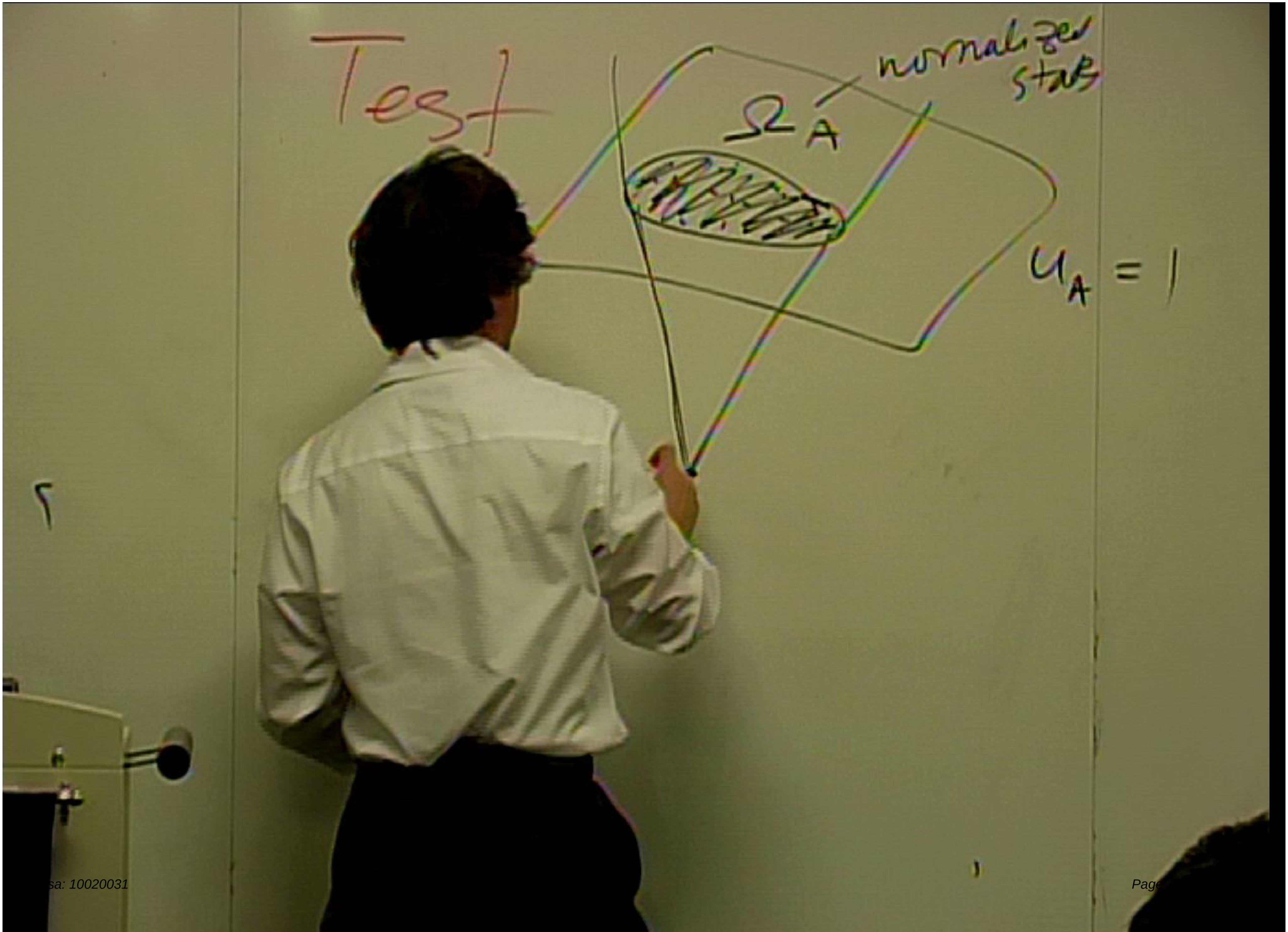
Test
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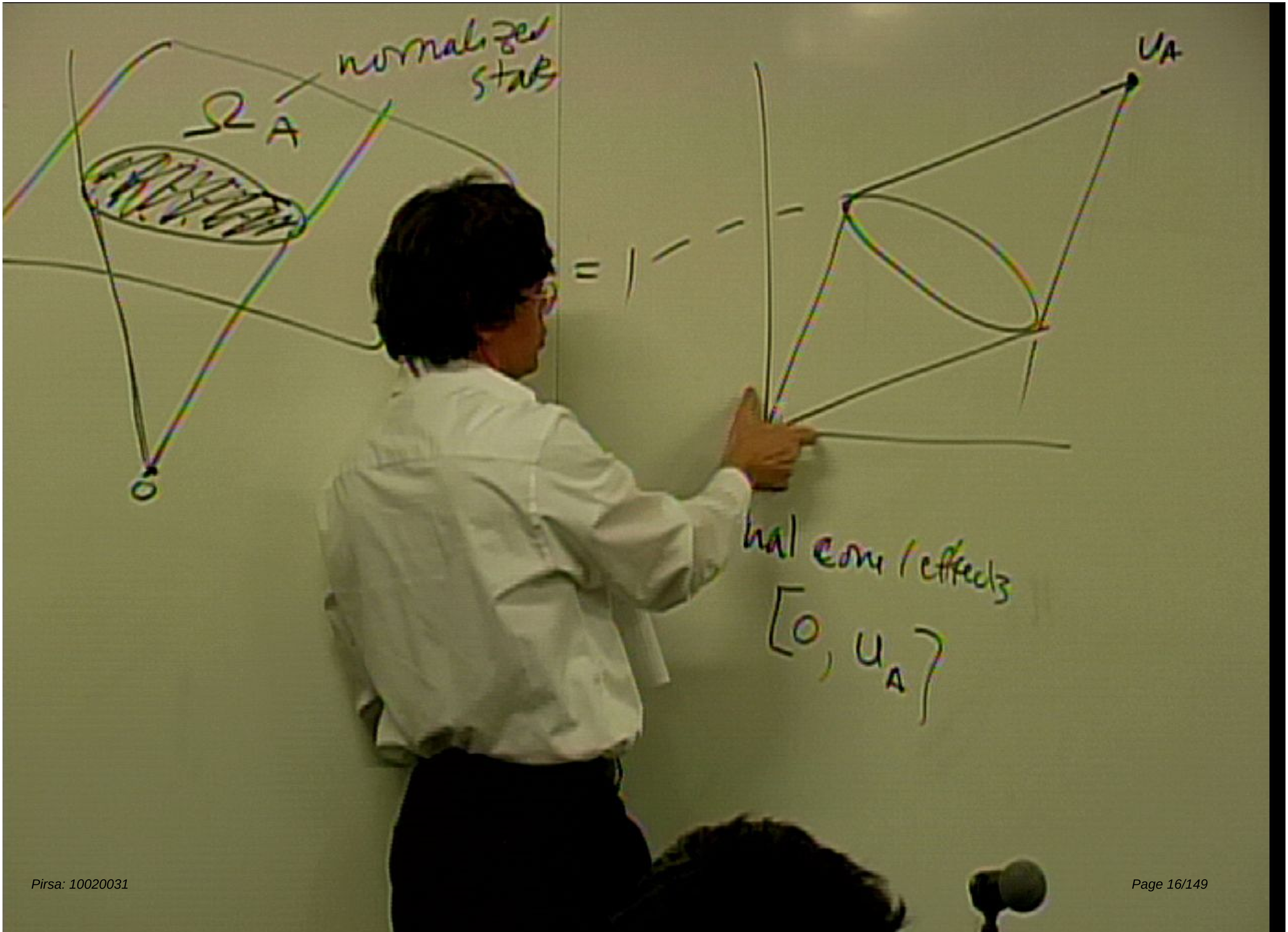
Ω_A
$u_A = 1$


normalized states
Ω_A
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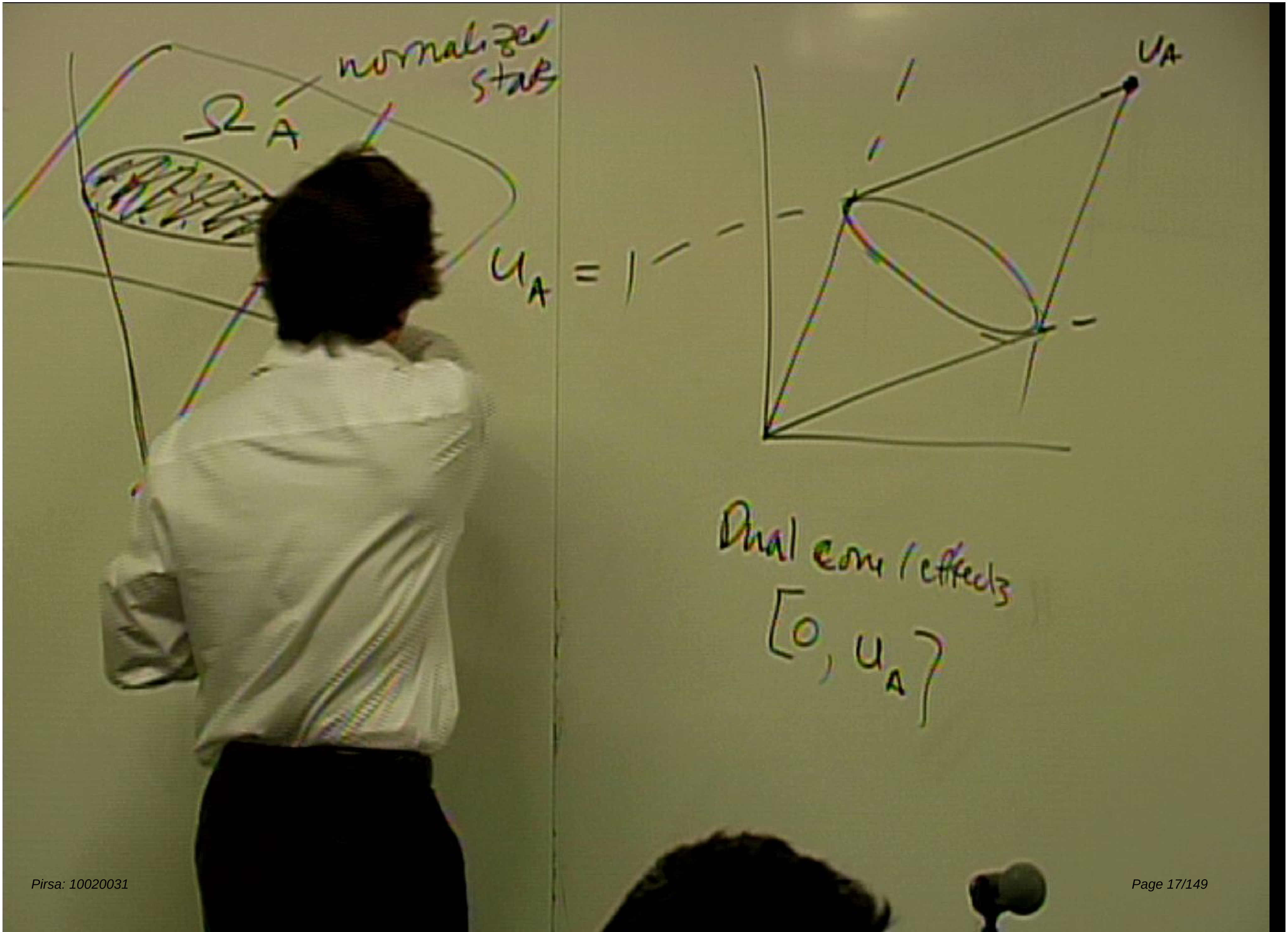
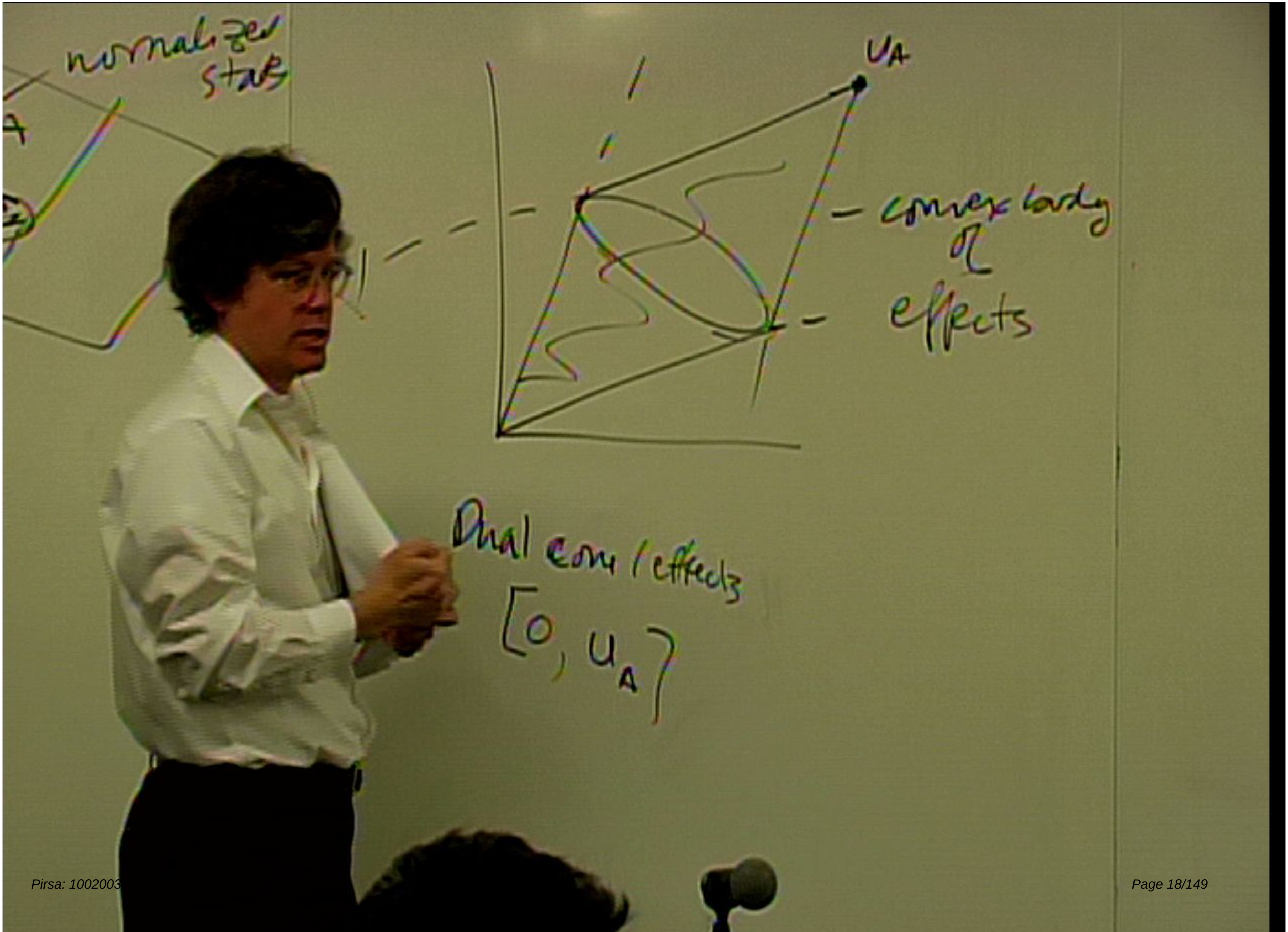
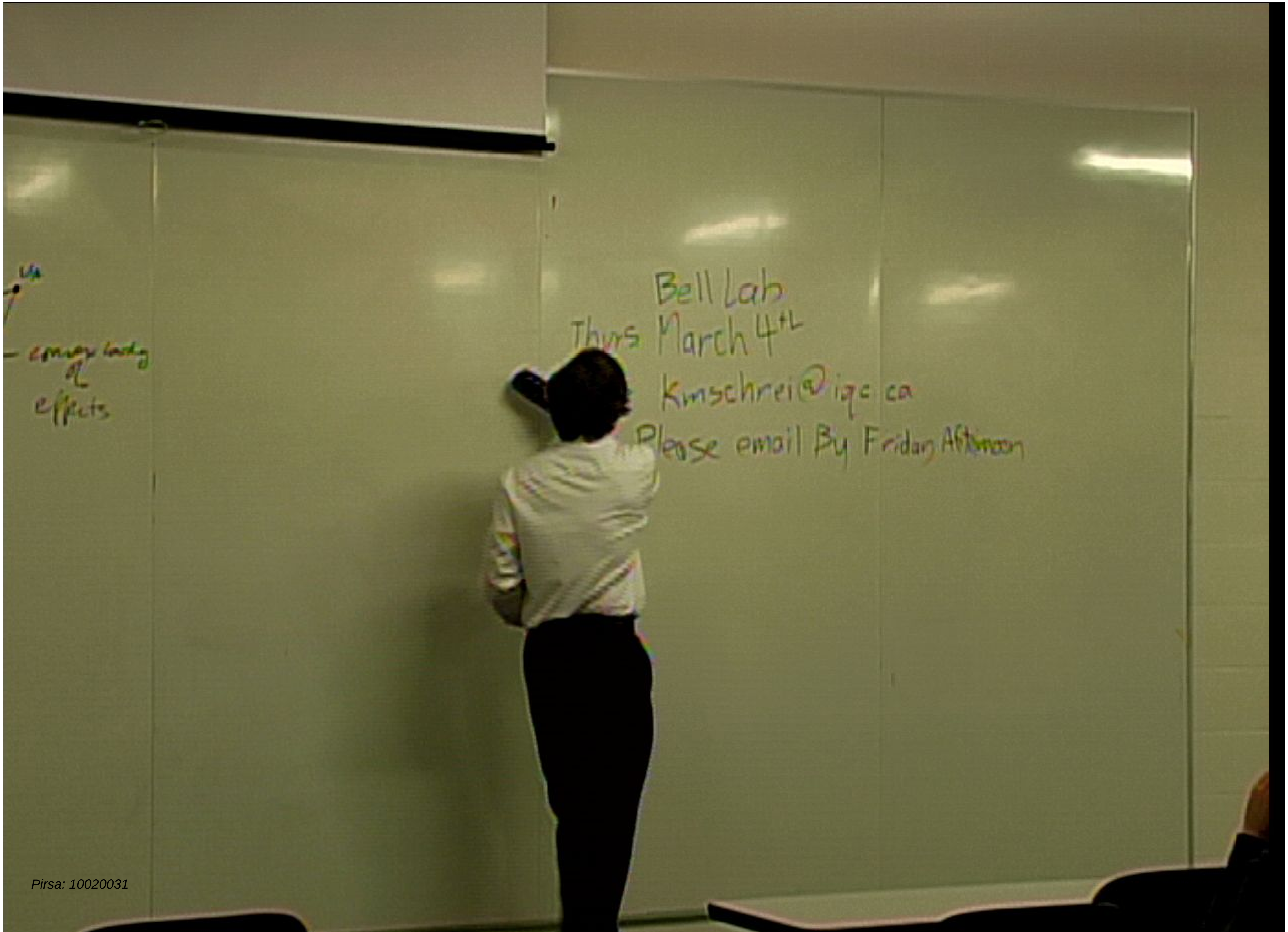
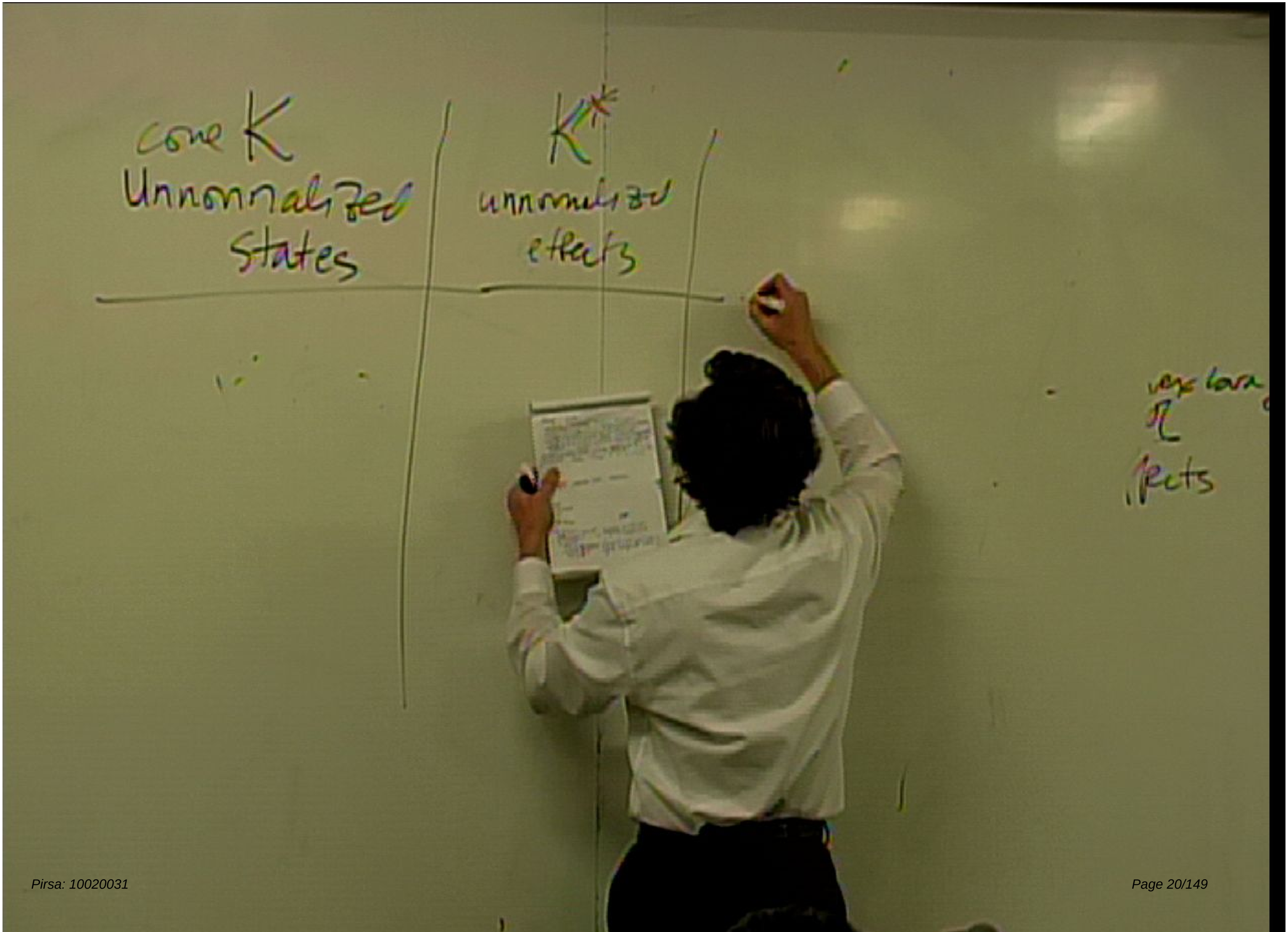
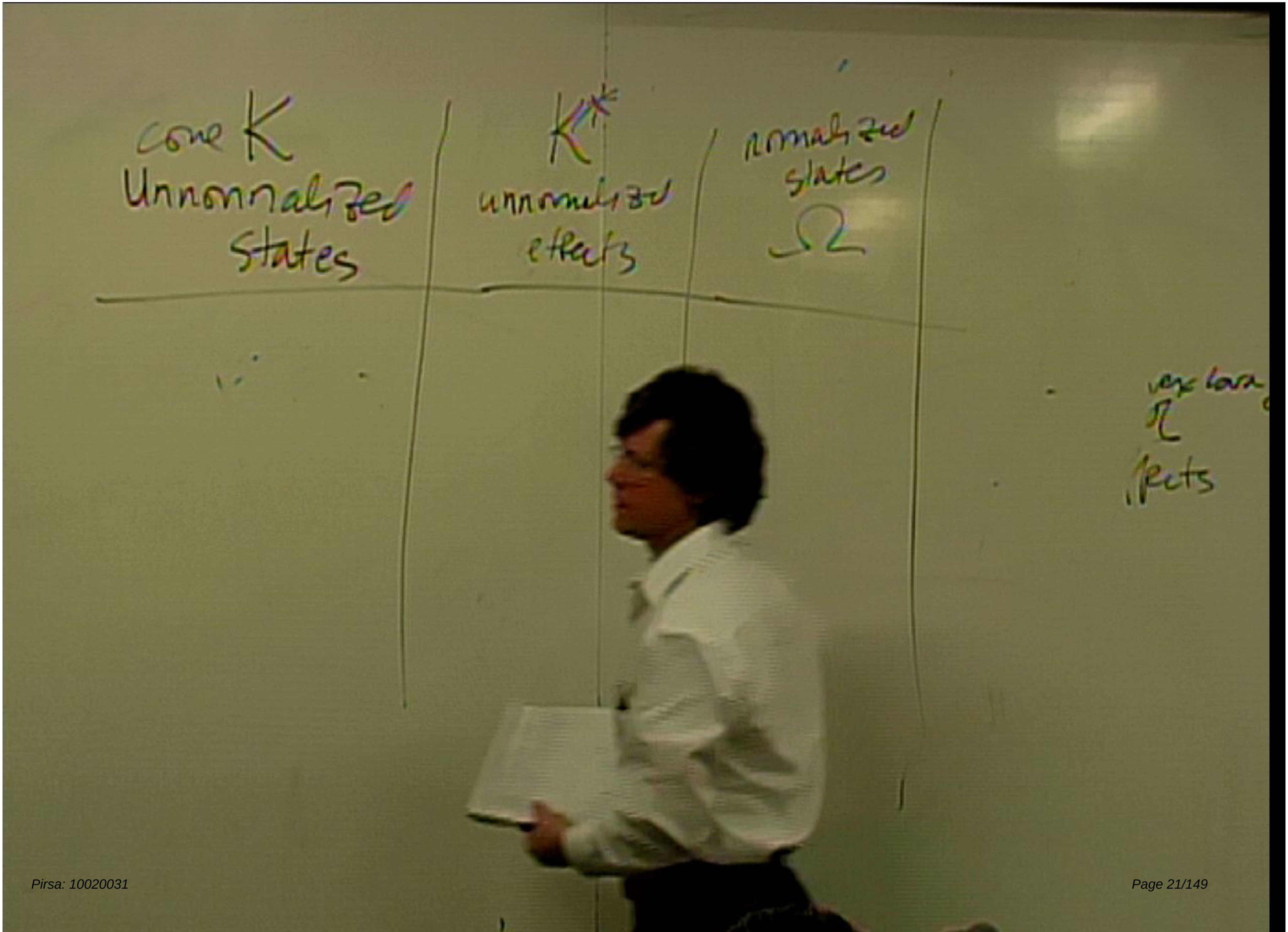
[0, u_A]


normalized states
Ω_A
$u_A = 1$
u_A
Dual cone / effects
$[0, u_A]$


normalized states
u_A
convex body of effects
Dual cone / effects
$[0, u_A]$


Bell Lab
Thurs March 4th
kmschrei@iqc.ca
Please email By Friday Afternoon
Va
effects


cone K
Unnormalized states
K^*
unnormalized effects
very large
effects


cone K
Unnormalized states
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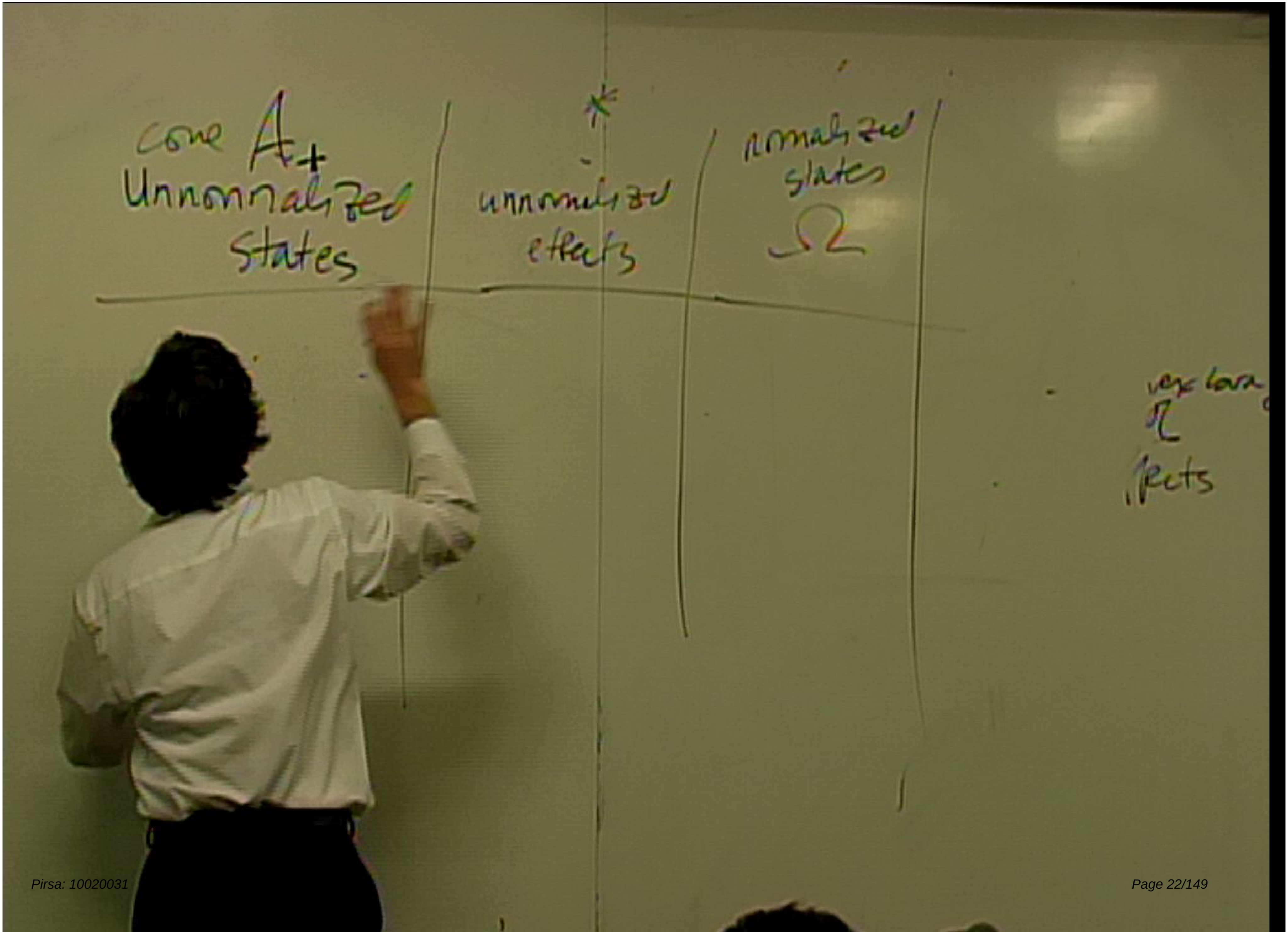
cone A_+
Unnormalized states
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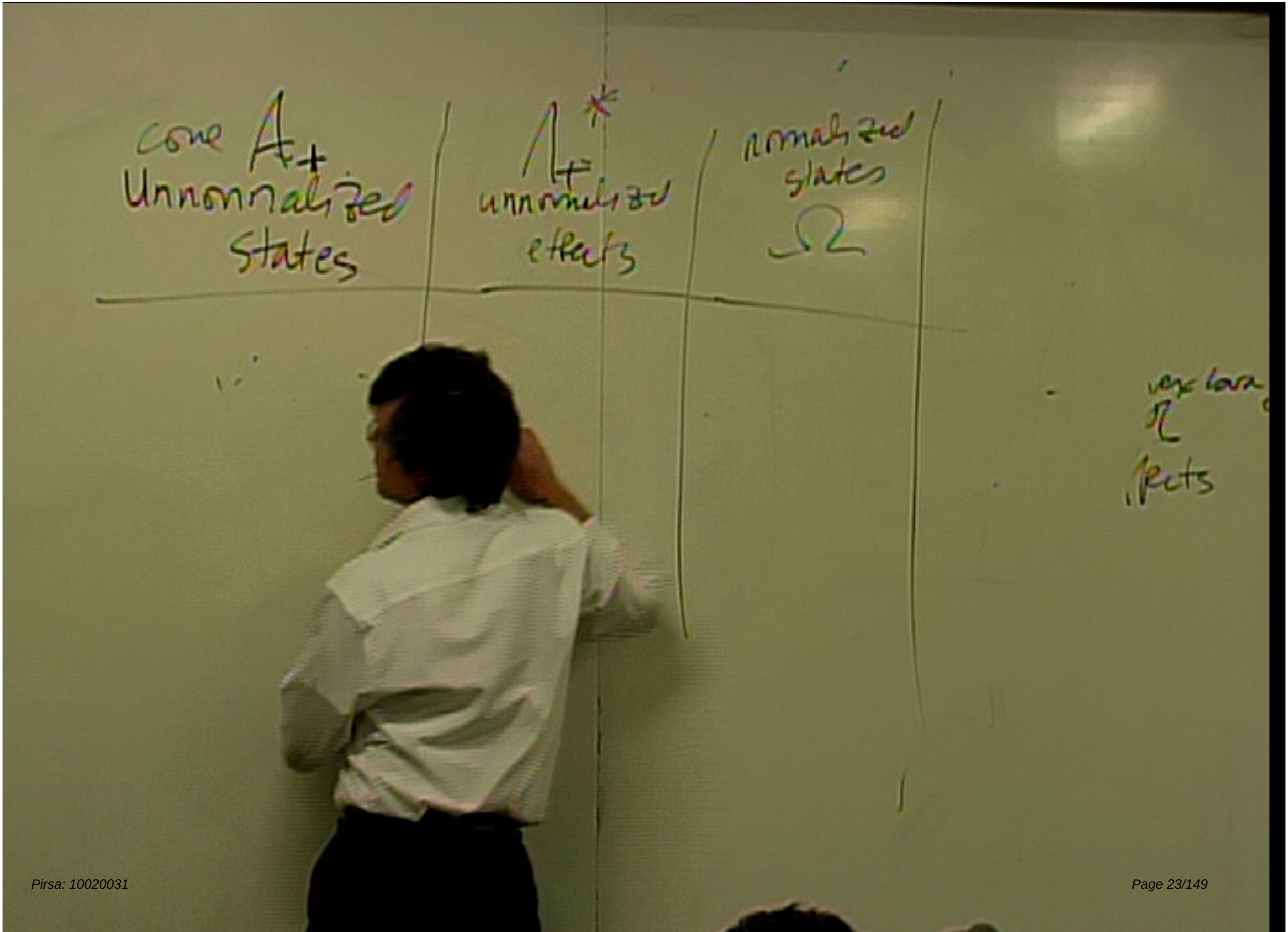
unnormalized effects
normalized states
Ω
effects



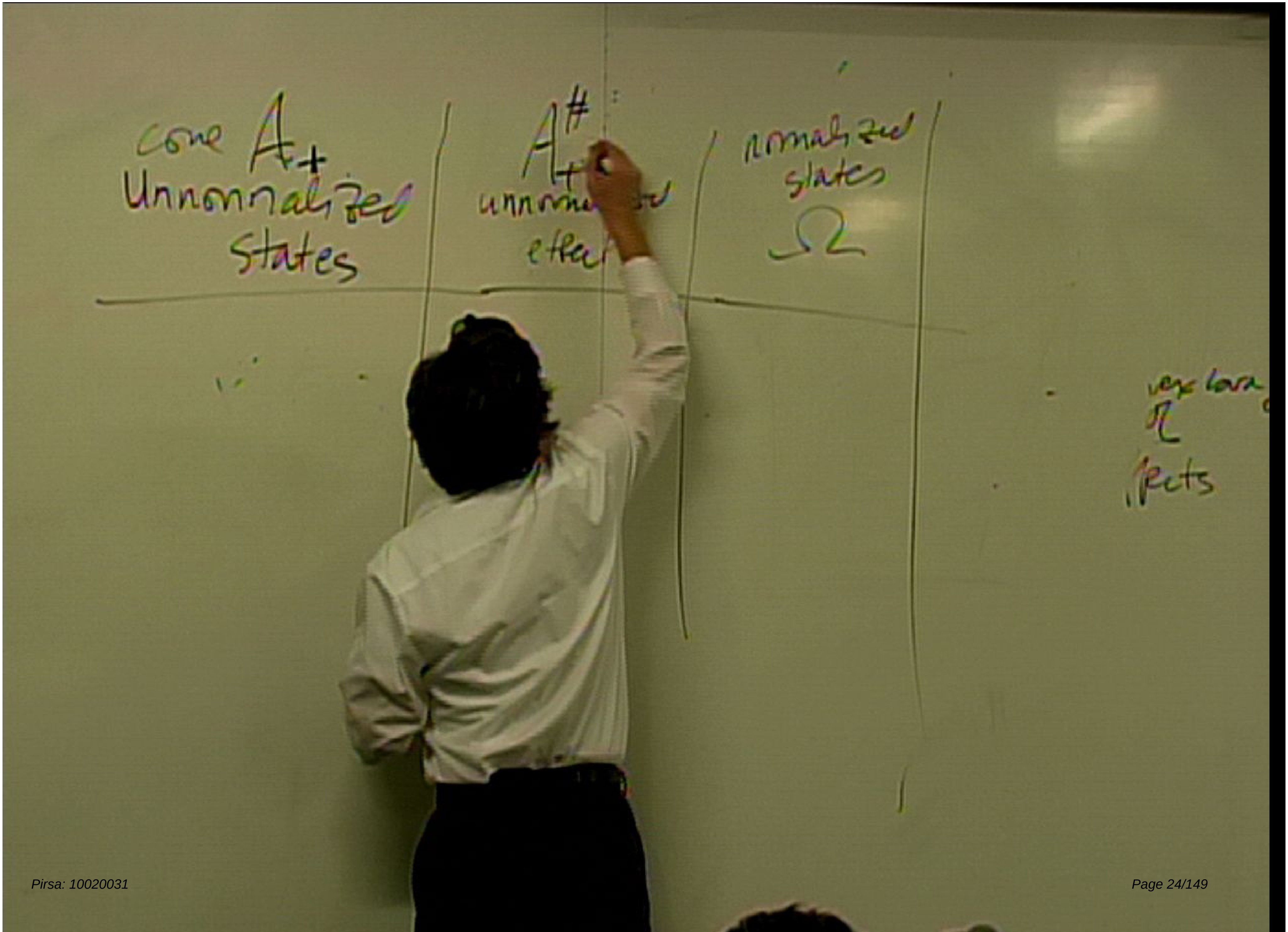
cone A_+
Unnormalized
states

A_+^*
unnormalized
effects

normalized
states
Ω

very large

fects


cone A_+
Unnormalized
states
$A_+^\#$
unnormalized
effects
normalized
states
Ω
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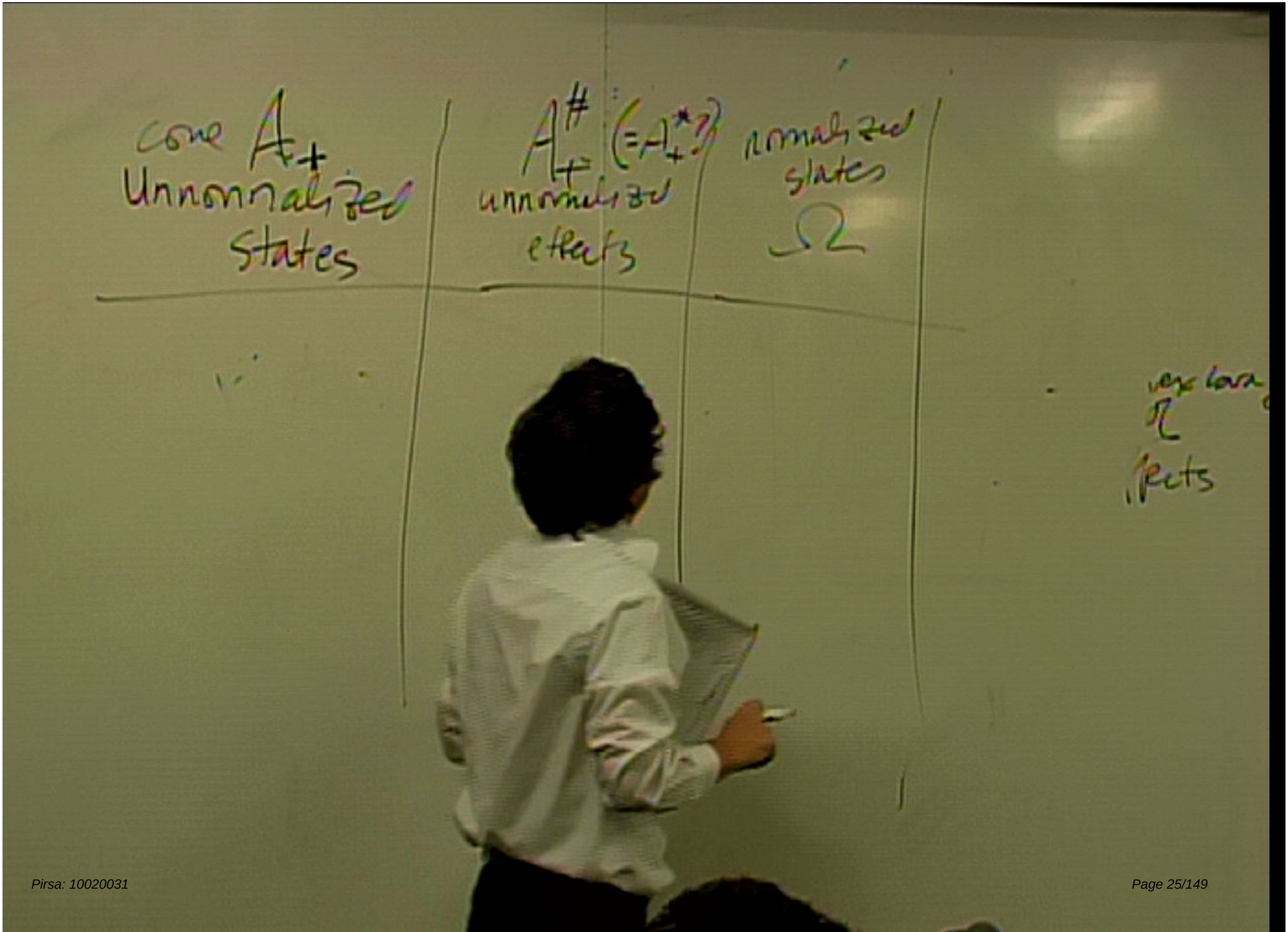
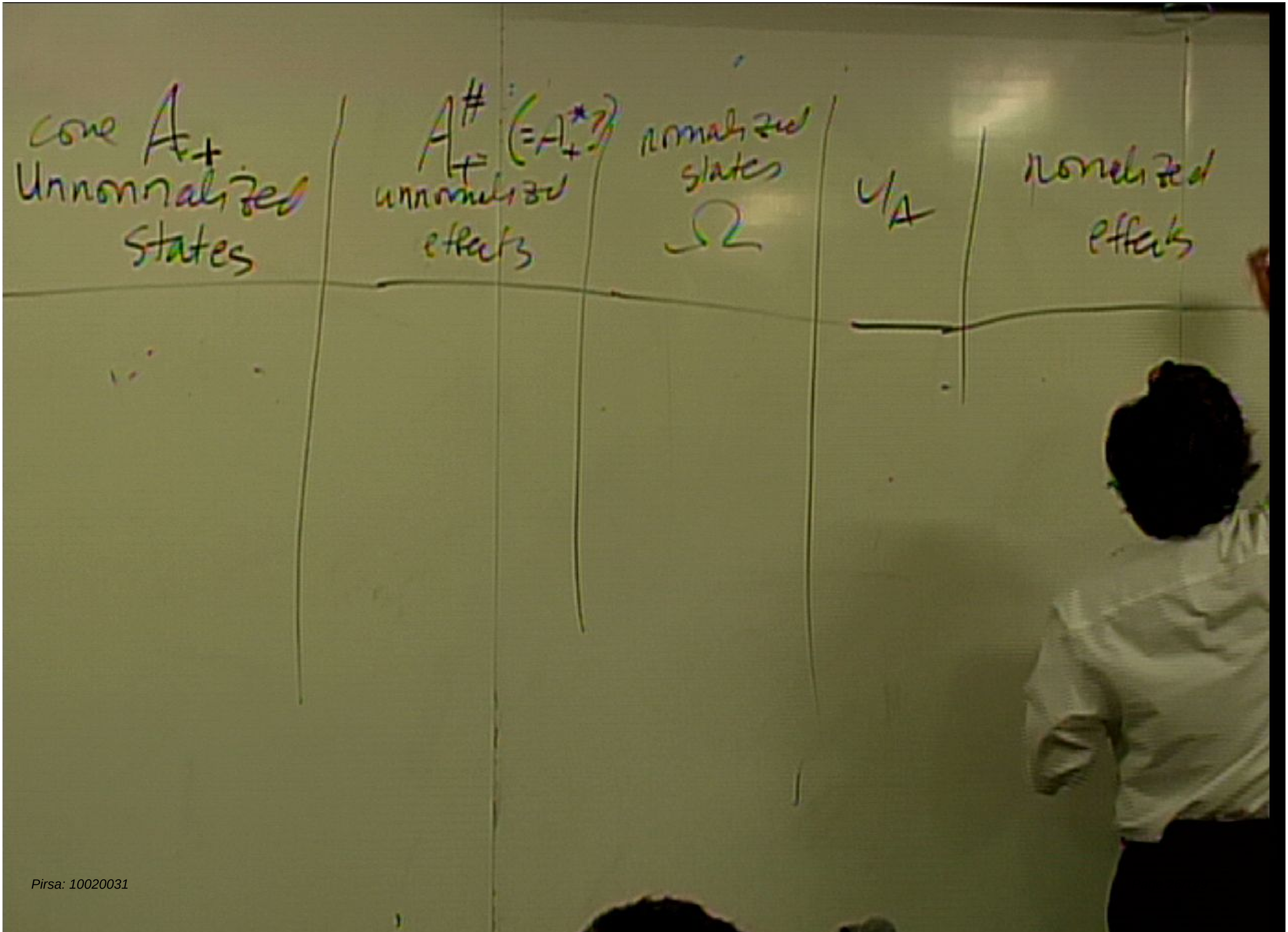
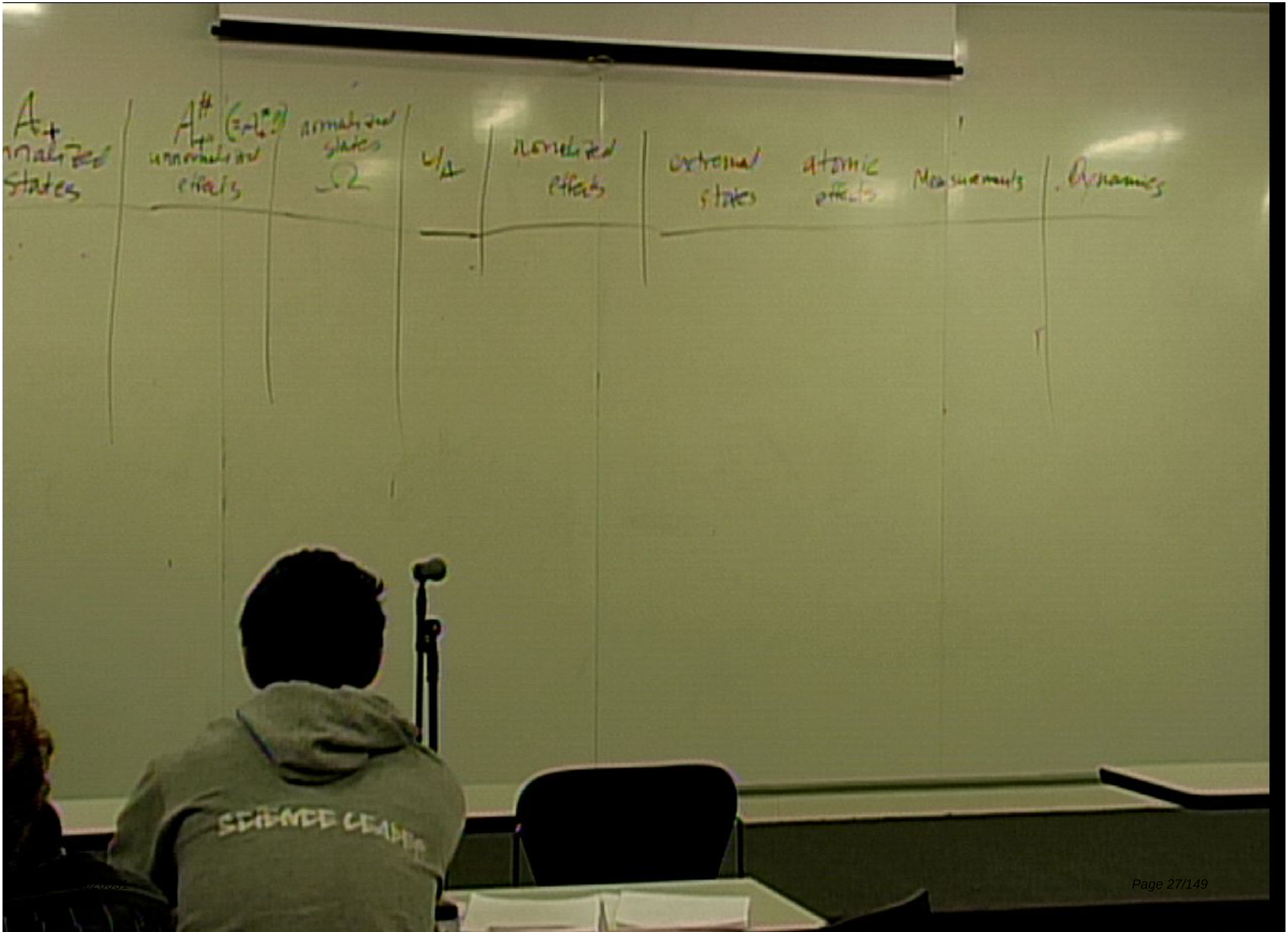

cone A_+
Unnormalized states
$A_+^{\#}$ (=A_+^*?)
unnormalized effects
normalized states
Ω
very long
Ω
ffects



cone A_+ Unnormalized states	$A_+^{\#}$ $(=A_+^{*}?)$ unnormalized effects	normalized states Ω	u_A	normalized effects


A_+
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$A^{\#}_{+}$ (=A_+^*?)
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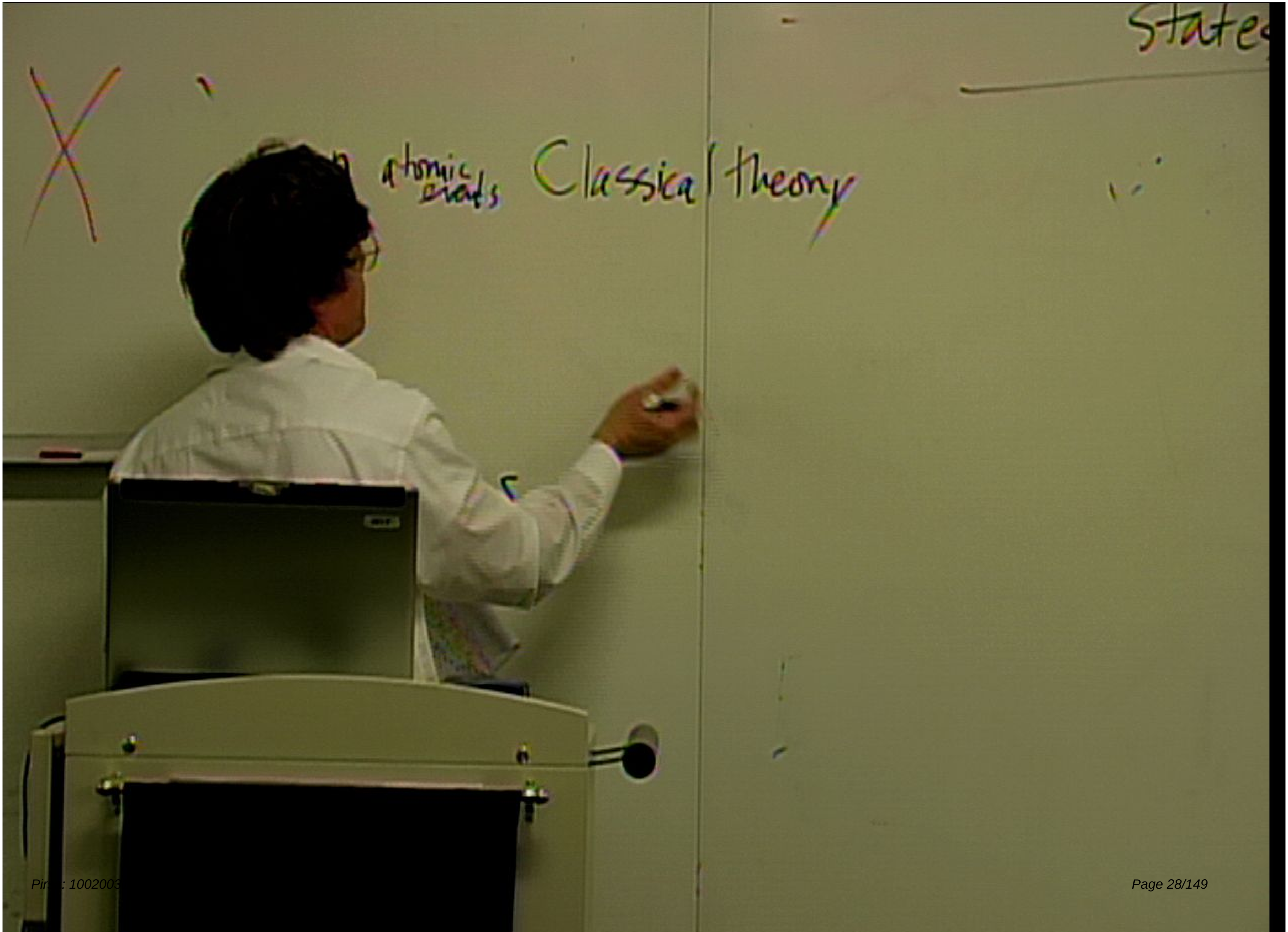
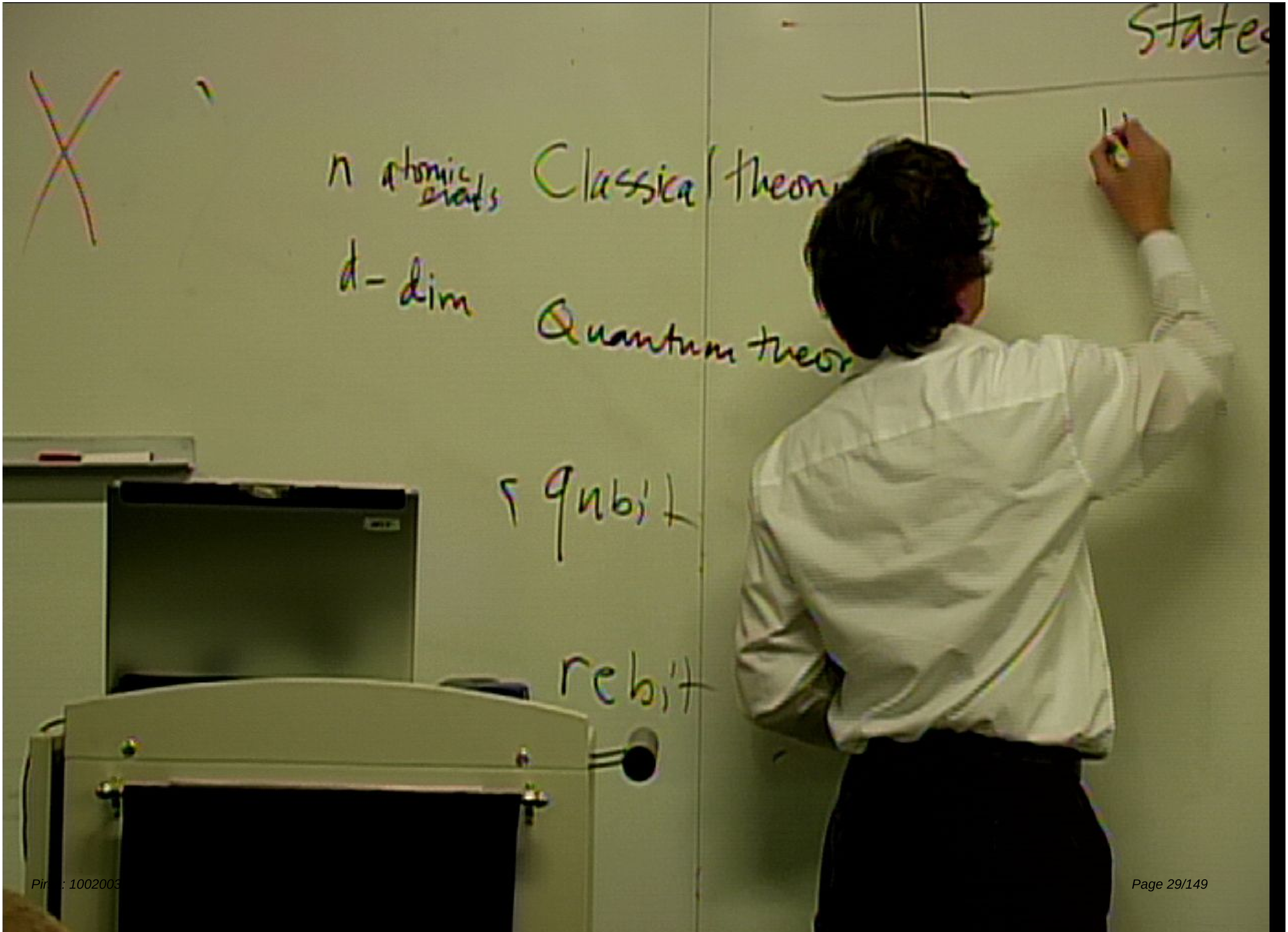
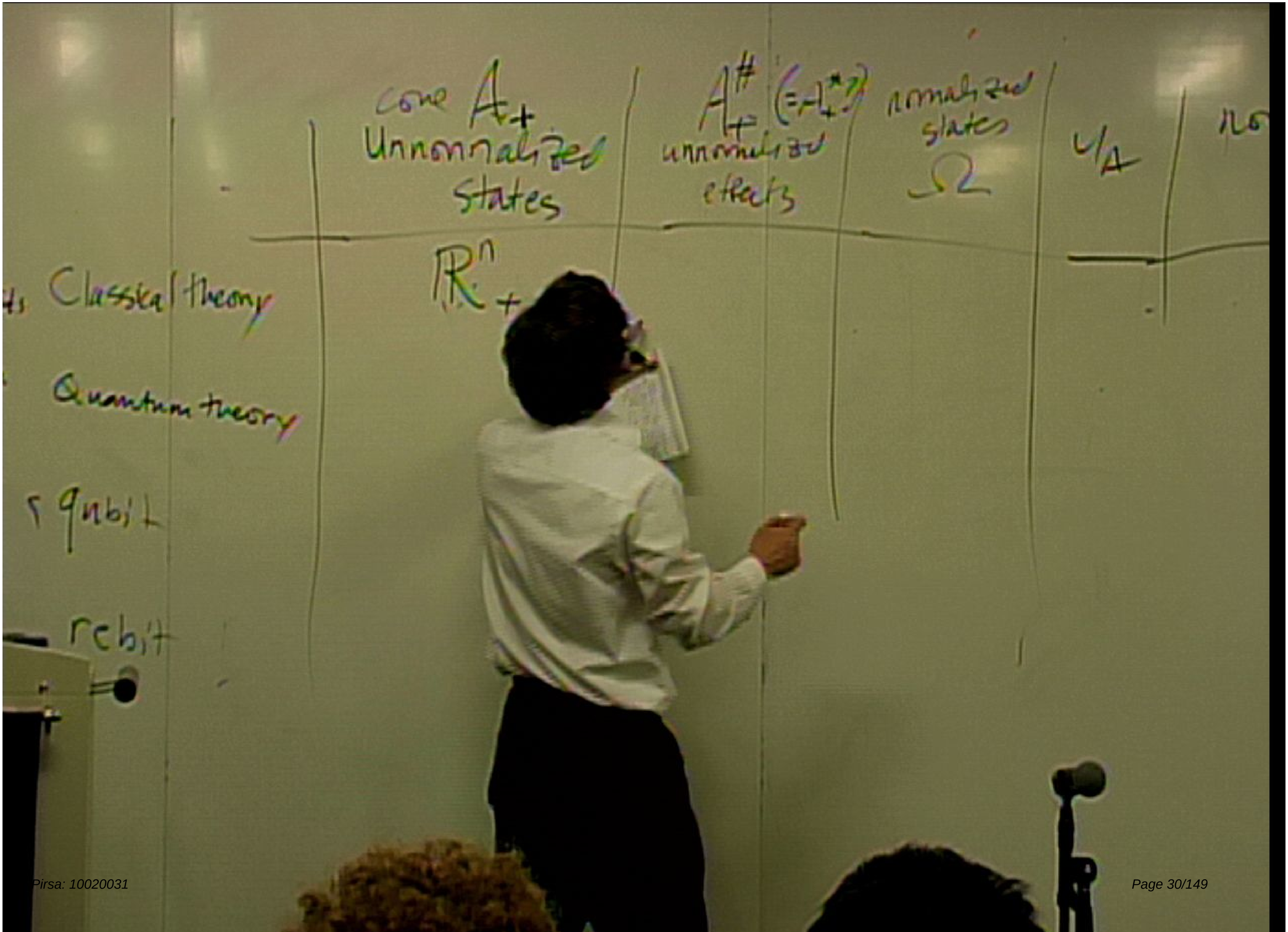
extremal states
atomic effects
Measurements
Dynamics

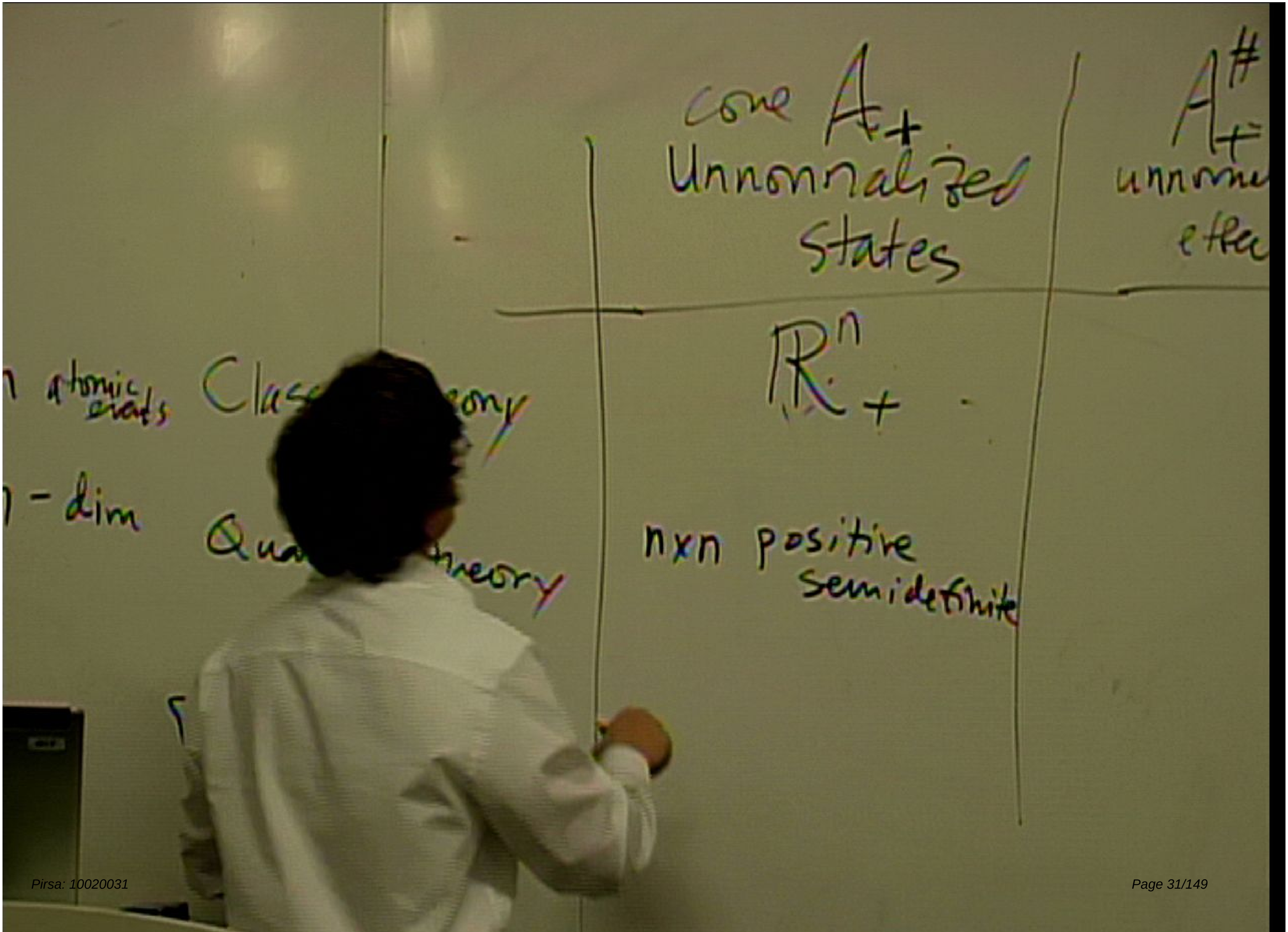


States
atomic events
Classical theory

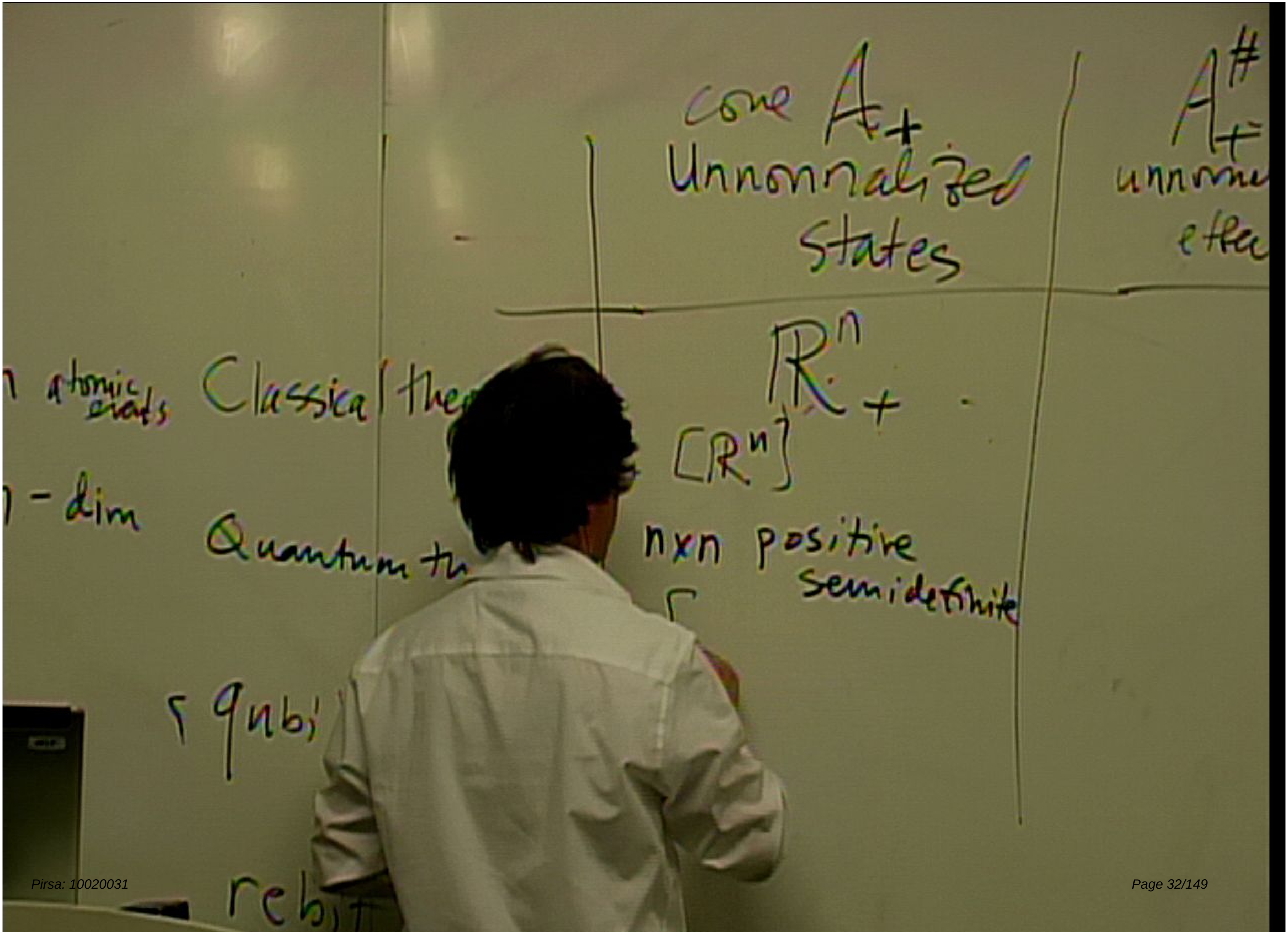

States
n atomic events
Classical theory
d-dim
Quantum theor
qubit
rebit



	cone A_+ Unnormalized states	$A_+^{\#}$ $(=A_+^{*}?)$ unnormalized effects	normalized states Ω	u/A	no
Classical theory	$\mathbb{R}^n_+$				
Quantum theory					
qubit					
rebit					



		cone A_+ Unnormalized states	$A^{\#}_{+}$ unnorma effe
n atomic events	Clas eory	$\mathbb{R}^n_+$	
n - dim	Qu heory	n×n positive semidefinite	


cone A_+
Unnormalized states
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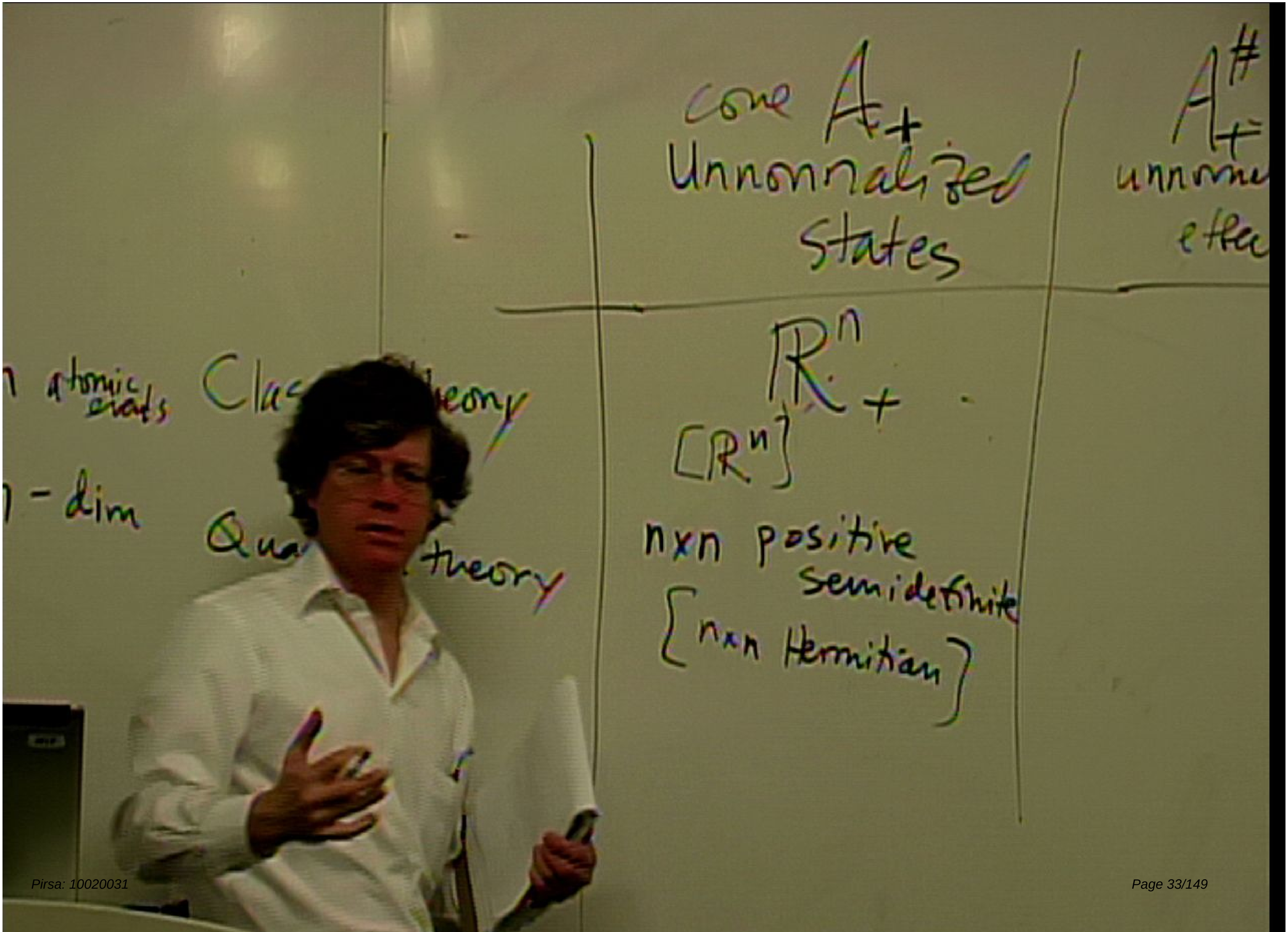
cone A_+
Unnormalized states
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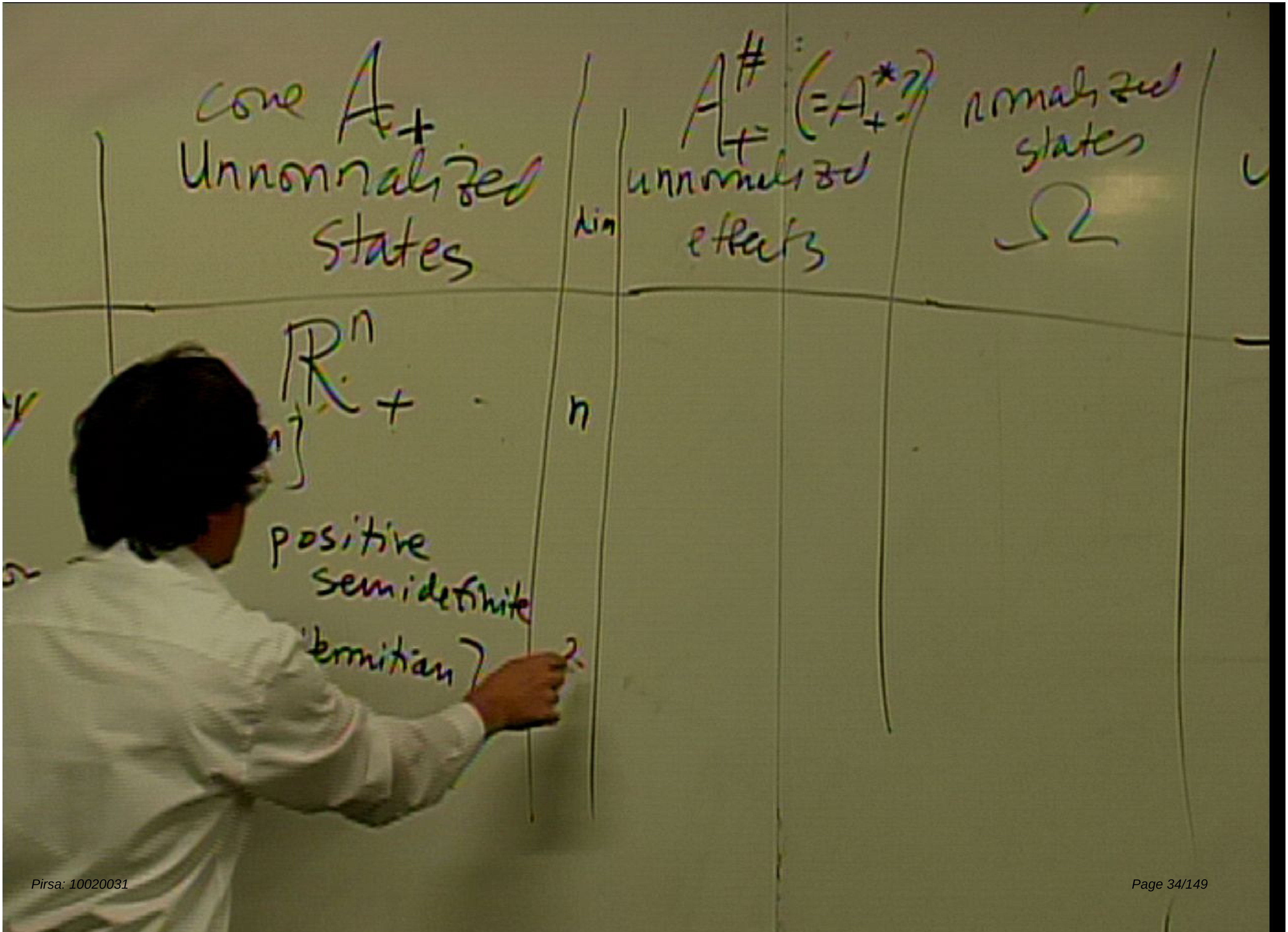
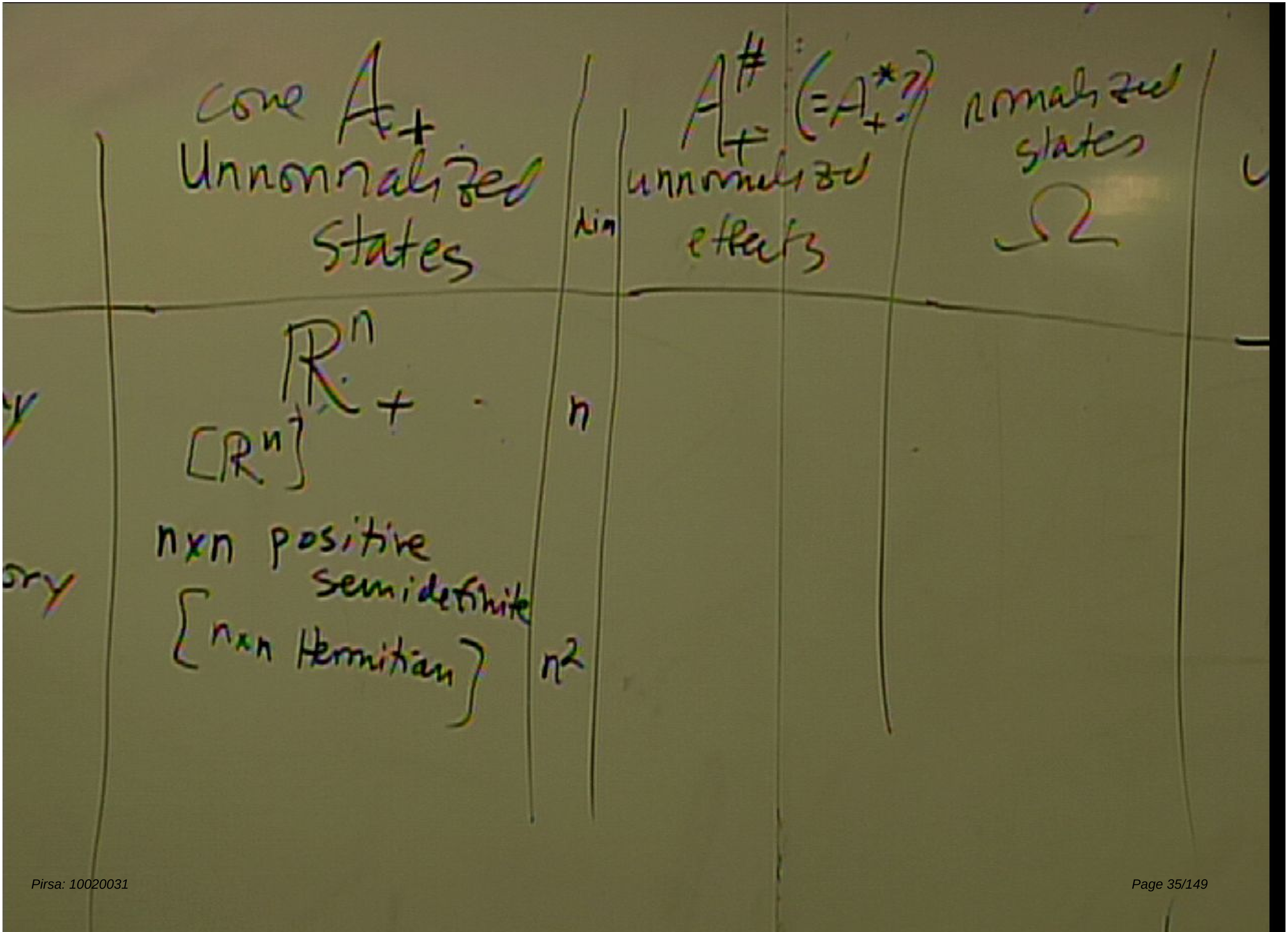
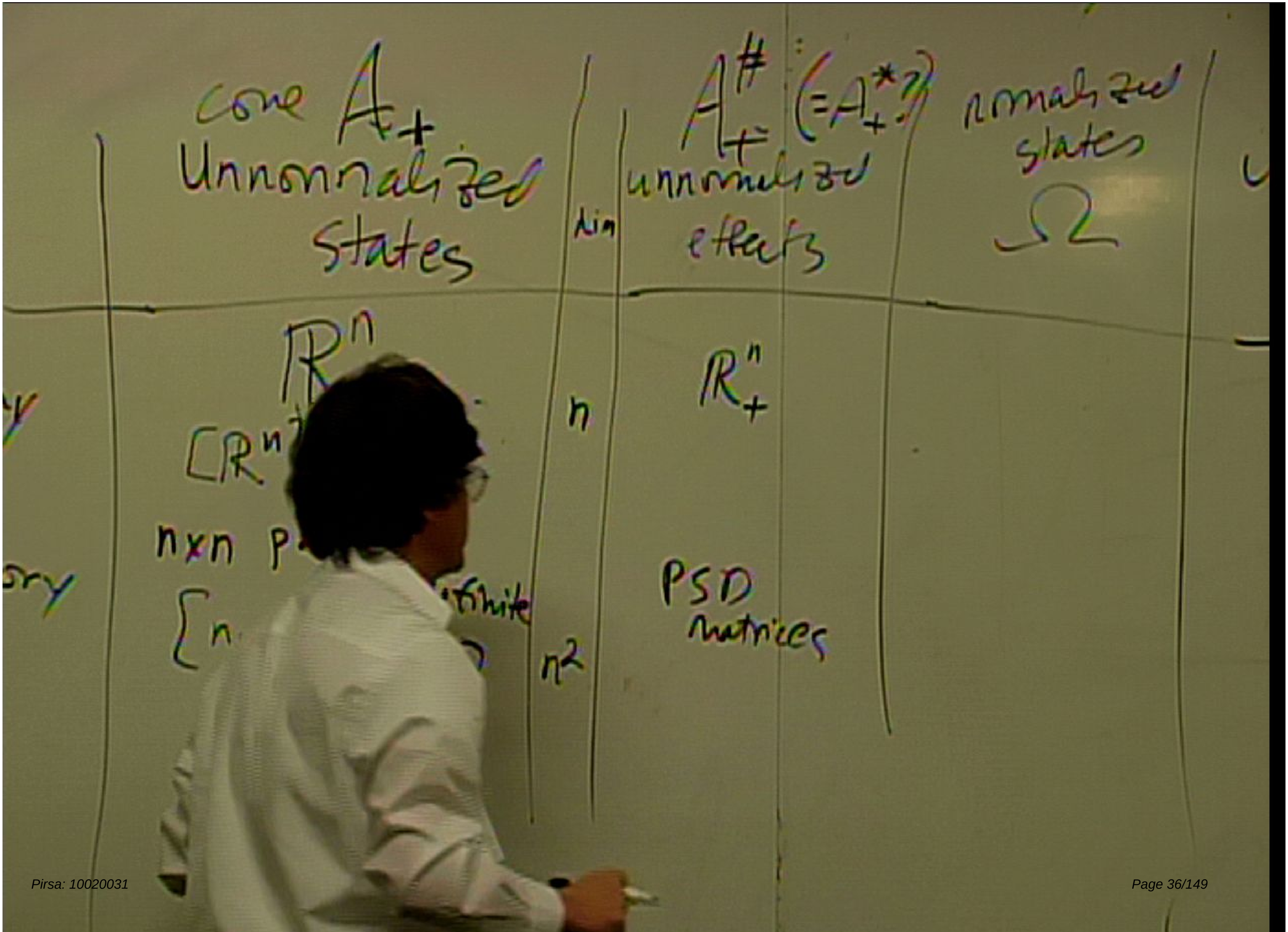
n×n positive semidefinite
{n×n Hermitian}



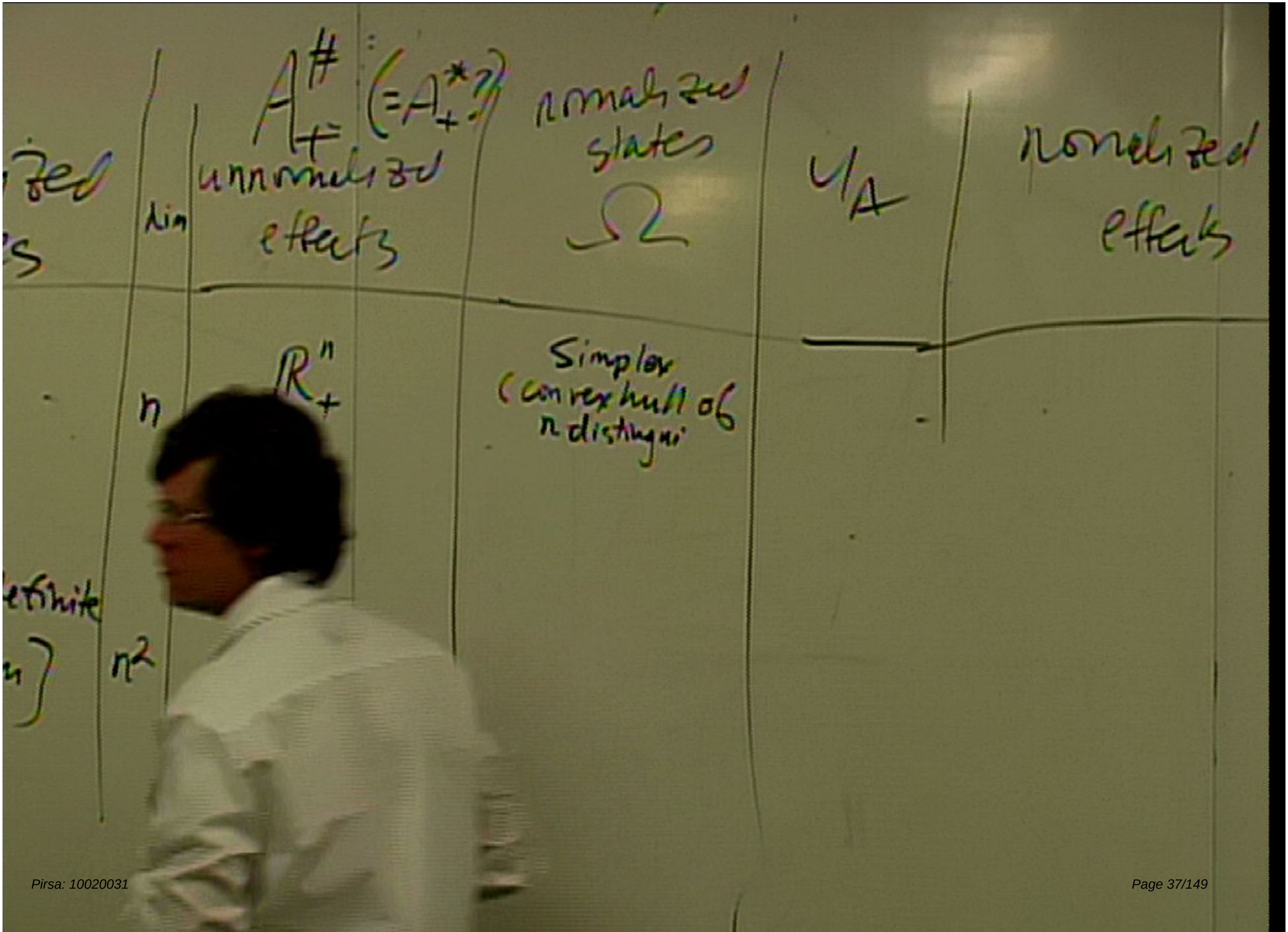
cone A_+ Unnormalized states	dim	$A_+^{\#}$: $(=A_+^{*}?)$ unnormalized effects	normalized states Ω
$\mathbb{R}^n_+$	n		
positive semidefinite ermitian	?		



	cone A_+ Unnormalized states	dim	$A_+^{\#}$ $(=A_+^*?)$ unnormalized effects	normalized states Ω	u
y	$\mathbb{R}^n_+$ $[\mathbb{R}^n]$	n			
ory	$n \times n$ positive semidefinite $\{n \times n$ Hermitian$\}$	n^2			

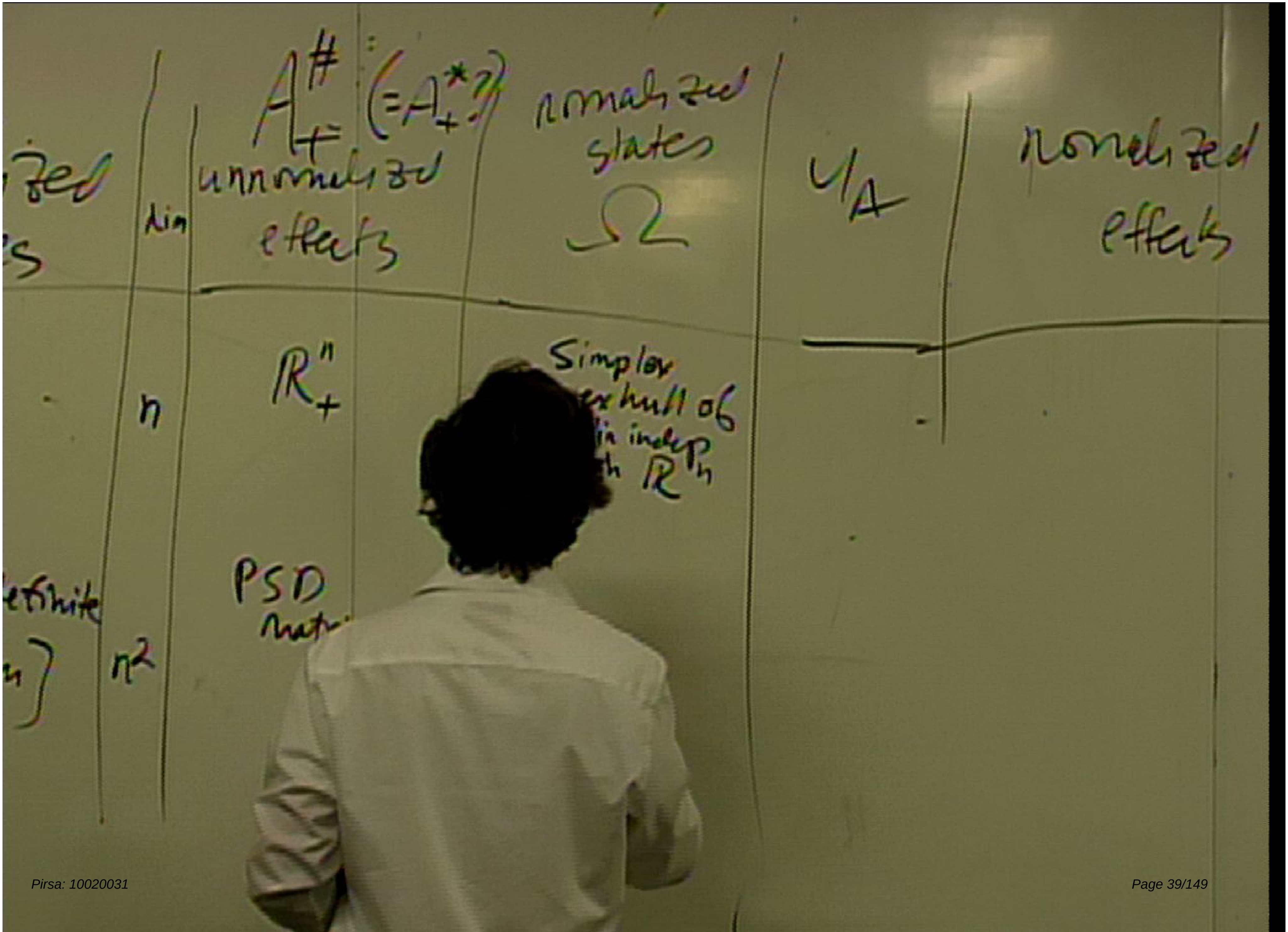


	cone A_+ Unnormalized states	dim	$A_+^\#$ ($=A_+^*$?) unnormalized effects	normalized states Ω	u
y	$\mathbb{R}^n$ $[\mathbb{R}^n$	n	$\mathbb{R}^n_+$		
ory	n×n p finite $\{n$	n^2	PSD matrices		

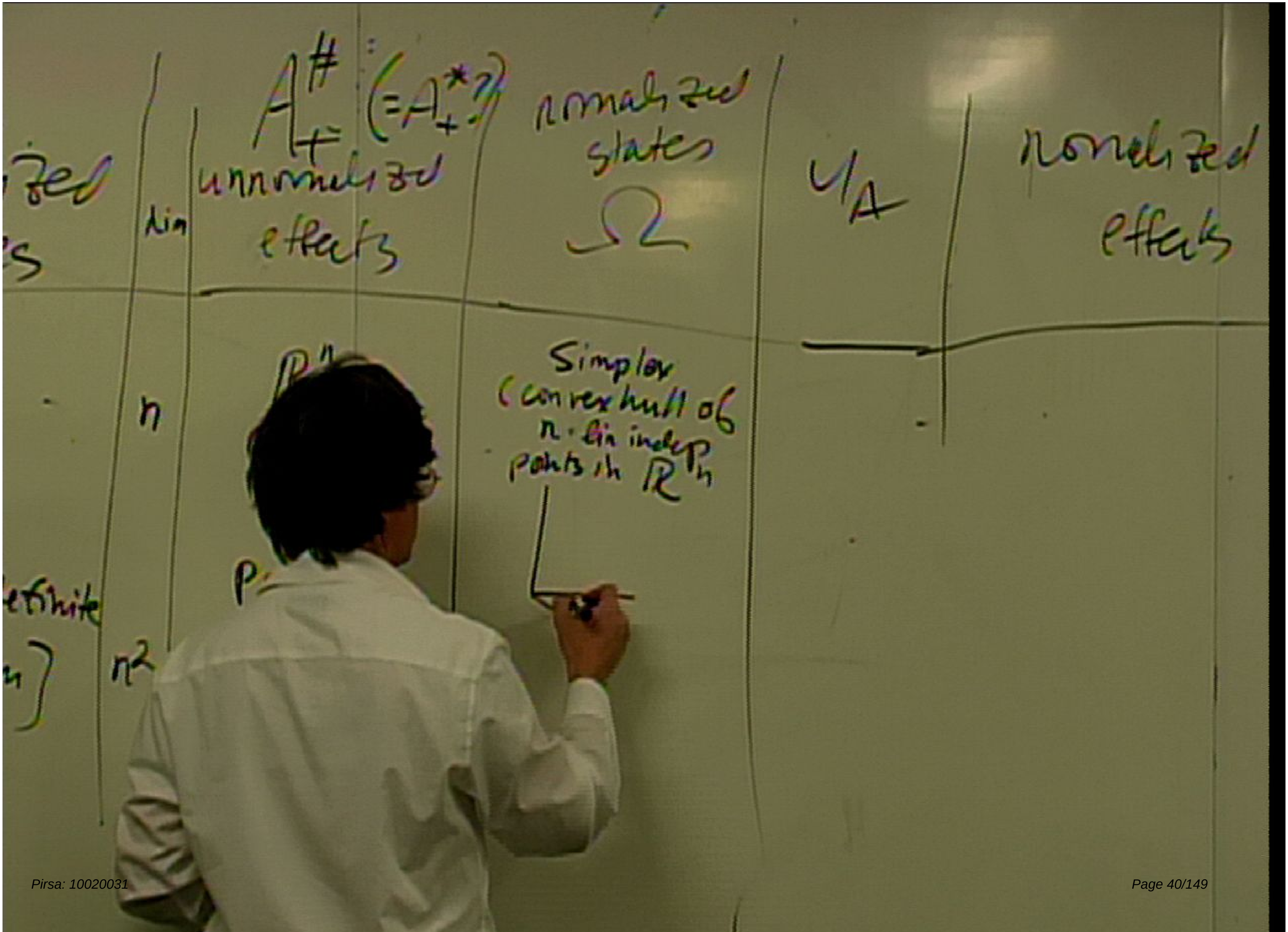


ized s	dim	$A_+^\#$ $(=A_+^*?)$ unnormalized effects	normalized states Ω	u_A	normalized effects
	n	$\mathbb{R}^n_+$	Simplex (convex hull of n distingui		
efinite n}	n^2				

ized s	dim	$A_+^\#$: (=A_+^*?) unnormalized effects	normalized states Ω	u_A	normalized effects
	n	$\mathbb{R}_+^n$	Simplex (convex hull of n·		
finite	n^2	PSD matrices			



zed s	dim	$A^{\#}_{+}$ (=A^{*}_{+}?) unnormalized effects	normalized states Ω	u_A	normalized effects
-	n	$\mathbb{R}^n_+$	Simplex ex hull of lin indep h $\mathbb{R}^n$		
efinite n}	n^2	PSD matr			



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dim
$A_+^\#$ (=A_+^*?)
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$\mathbb{R}^n$
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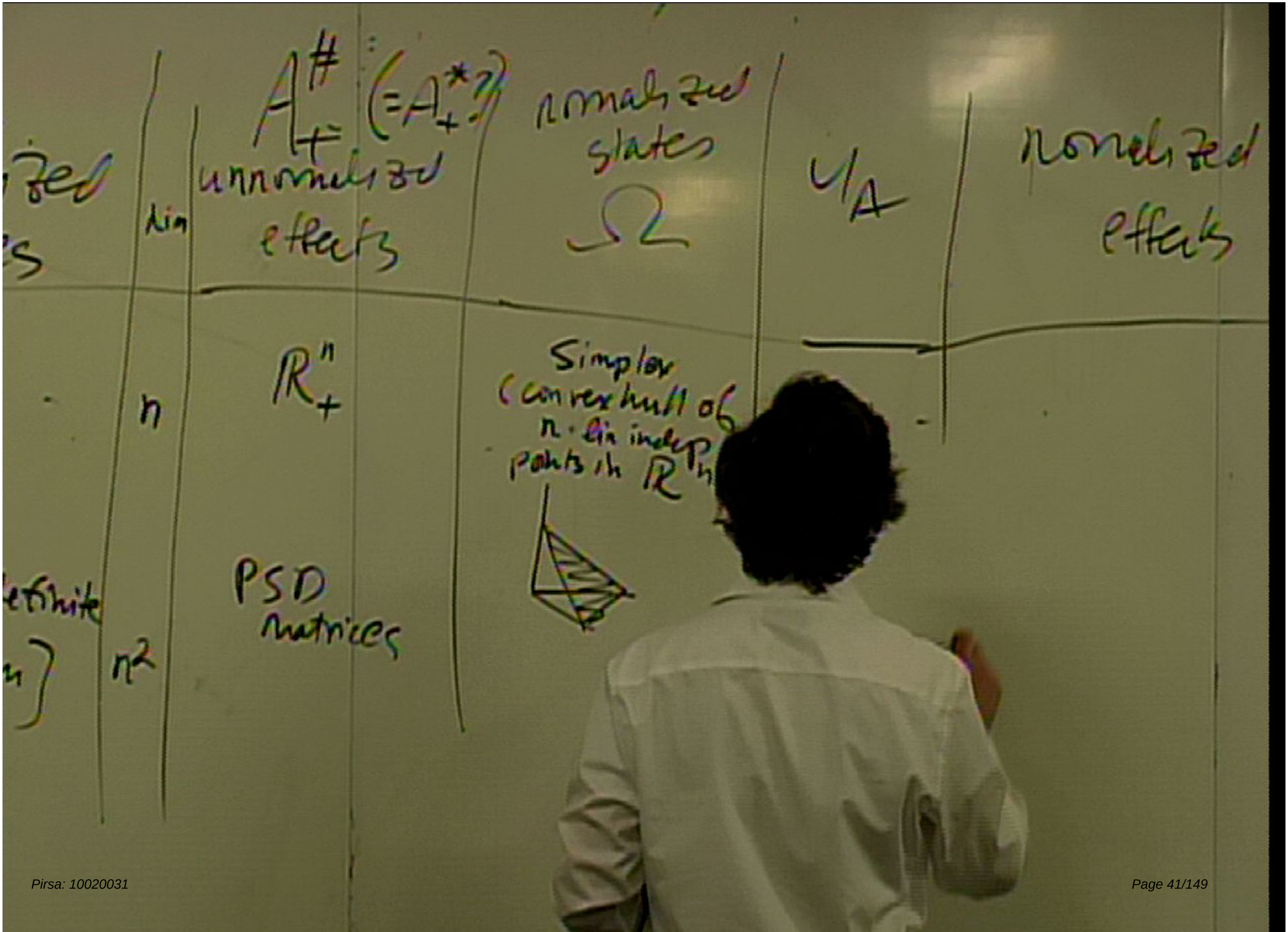
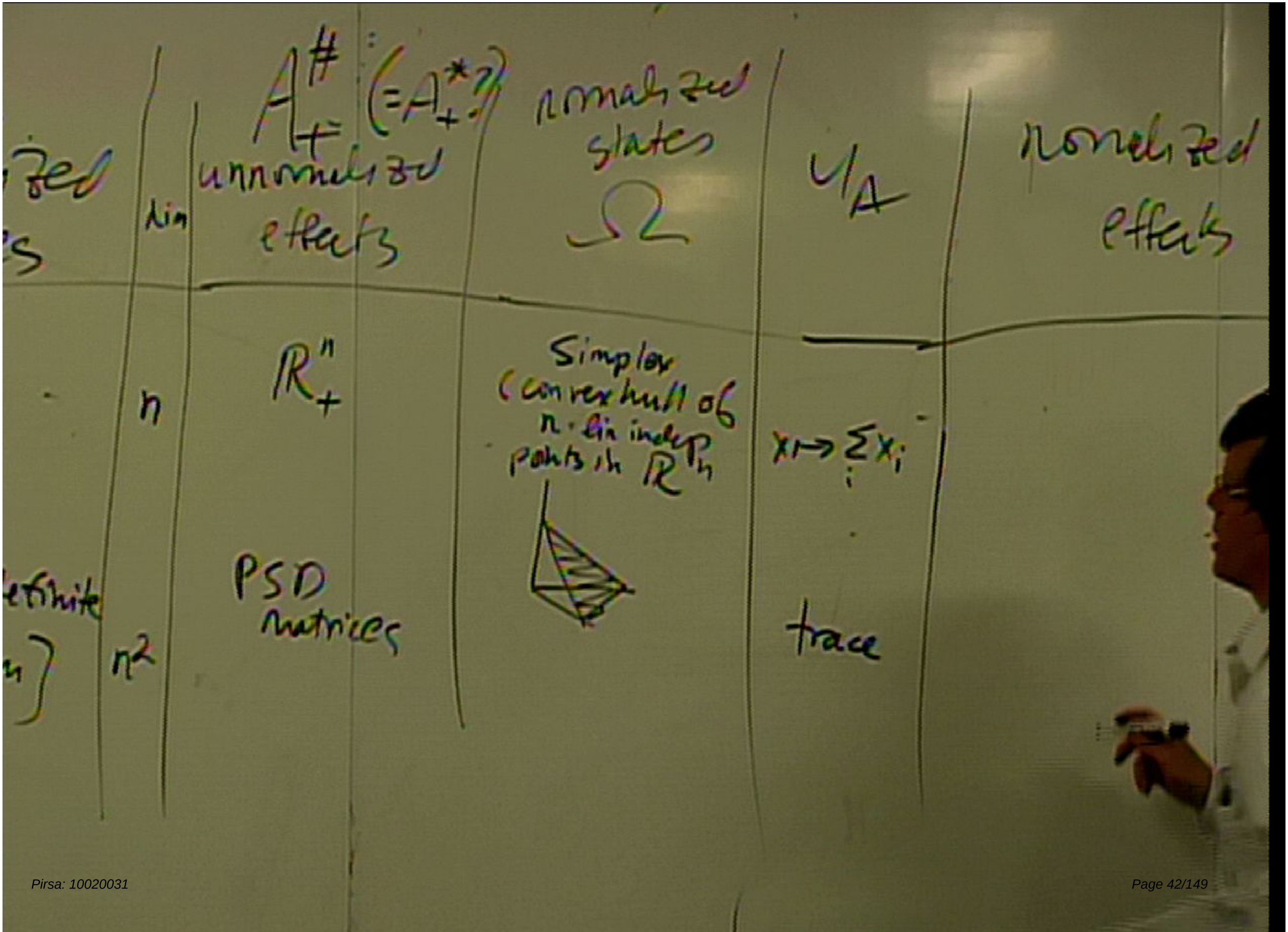
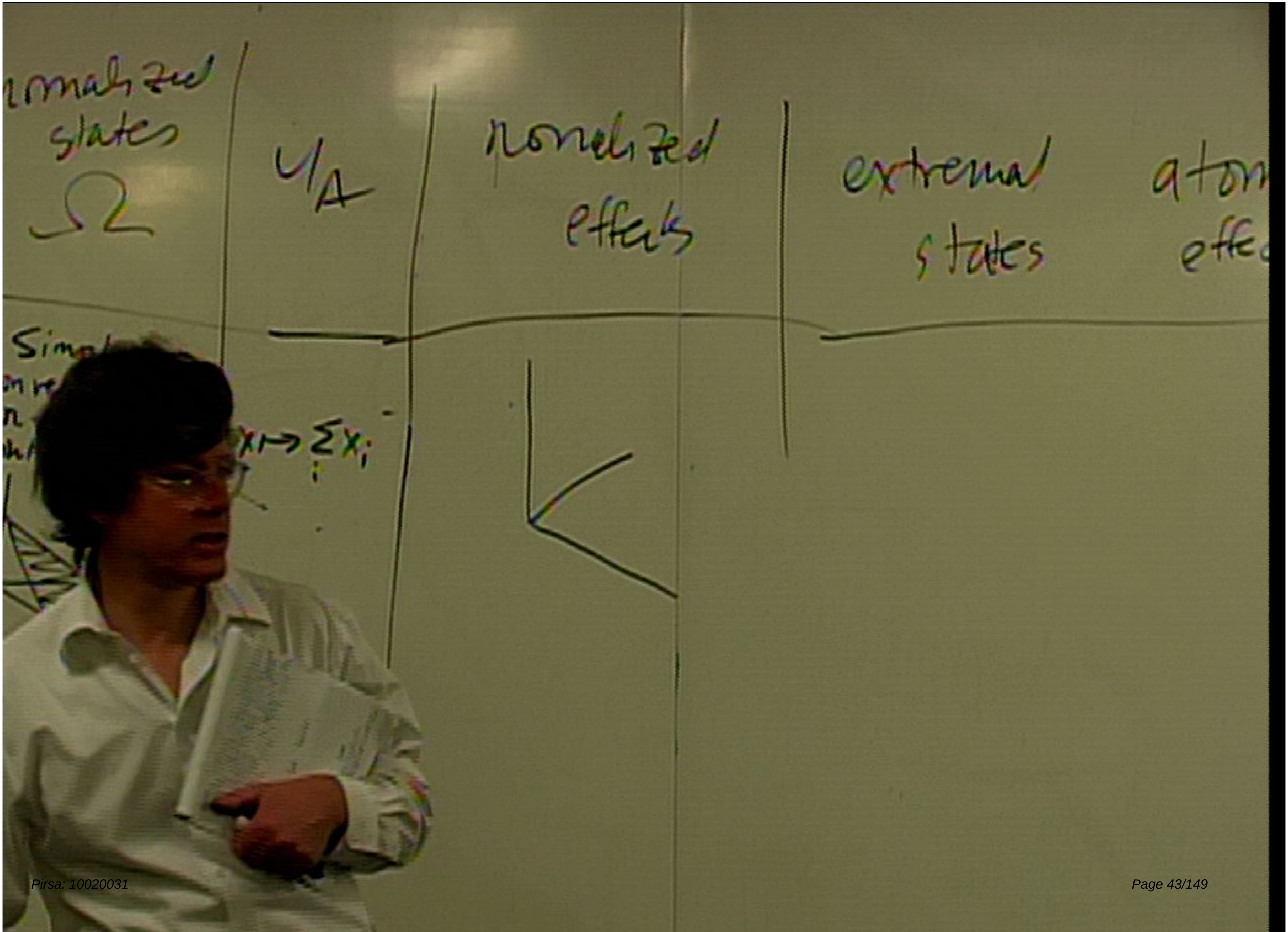


ized s	dim	$A^{\#}_{+}$ $(=A^{*}_{+}?)$ unnormalized effects	normalized states Ω	u_A	normalized effects
.	n	$\mathbb{R}^n_+$	Simplex (convex hull of n · lin indep points in $\mathbb{R}^n$	-	
efinite n}	n^2	PSD matrices			




normalized states
Ω
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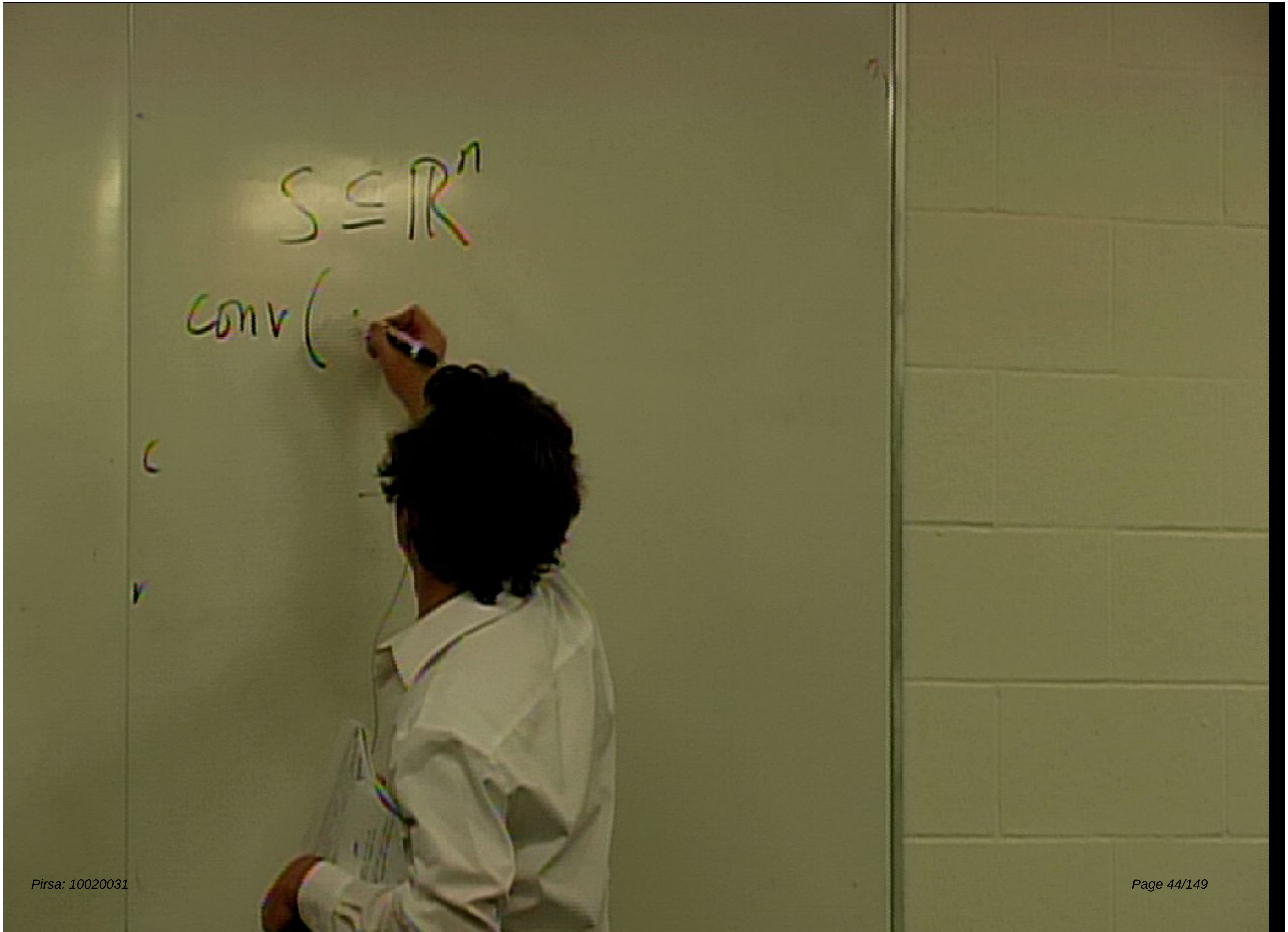
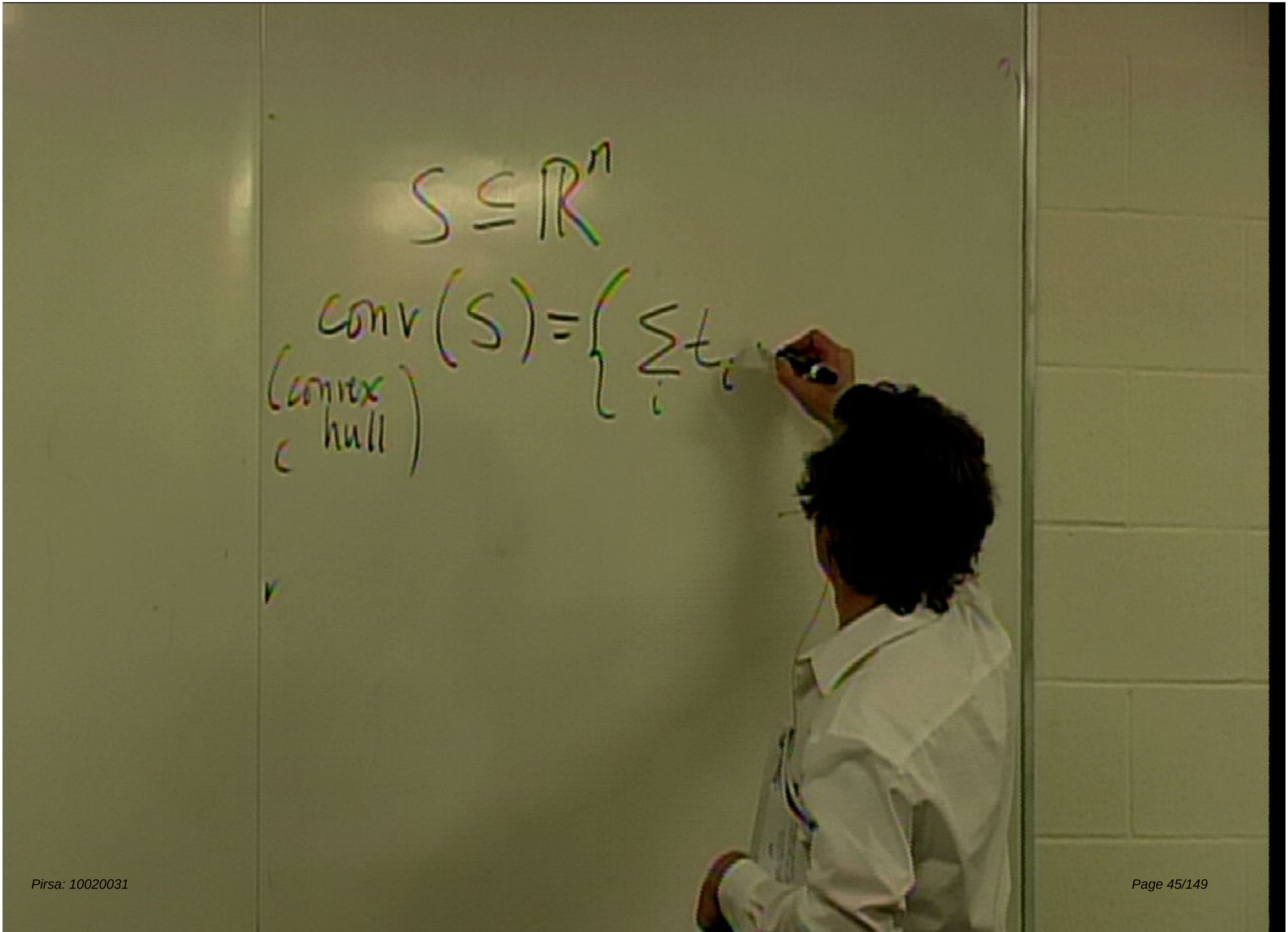
normalized effects
extremal states
atom effe
Sim
$x \mapsto \sum_i x_i$



$S \subseteq \mathbb{R}^n$

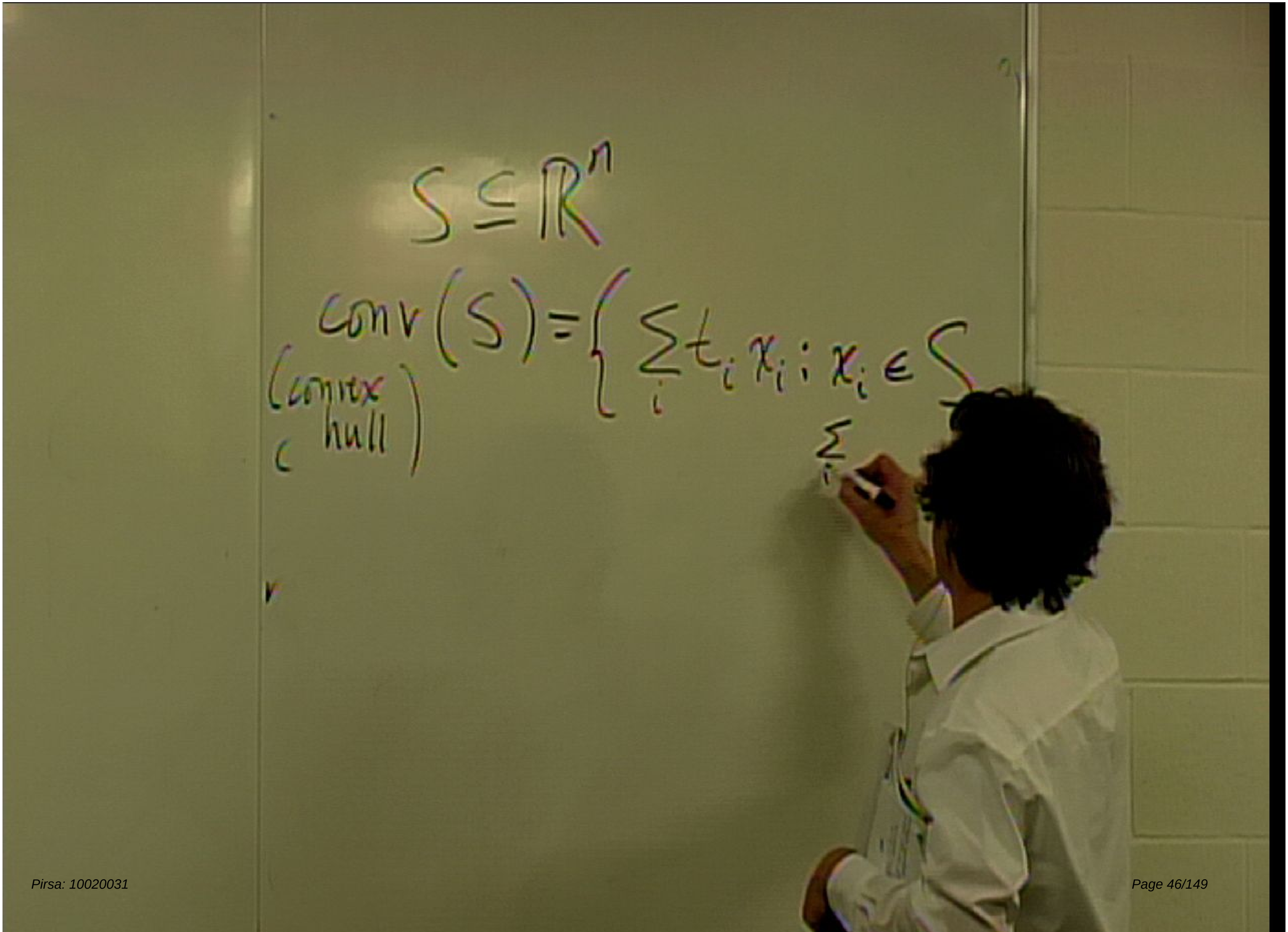
conv(



$S \subseteq \mathbb{R}^n$

$\mathrm{conv}(S) = \{ \sum_i t_i$

(convex hull)

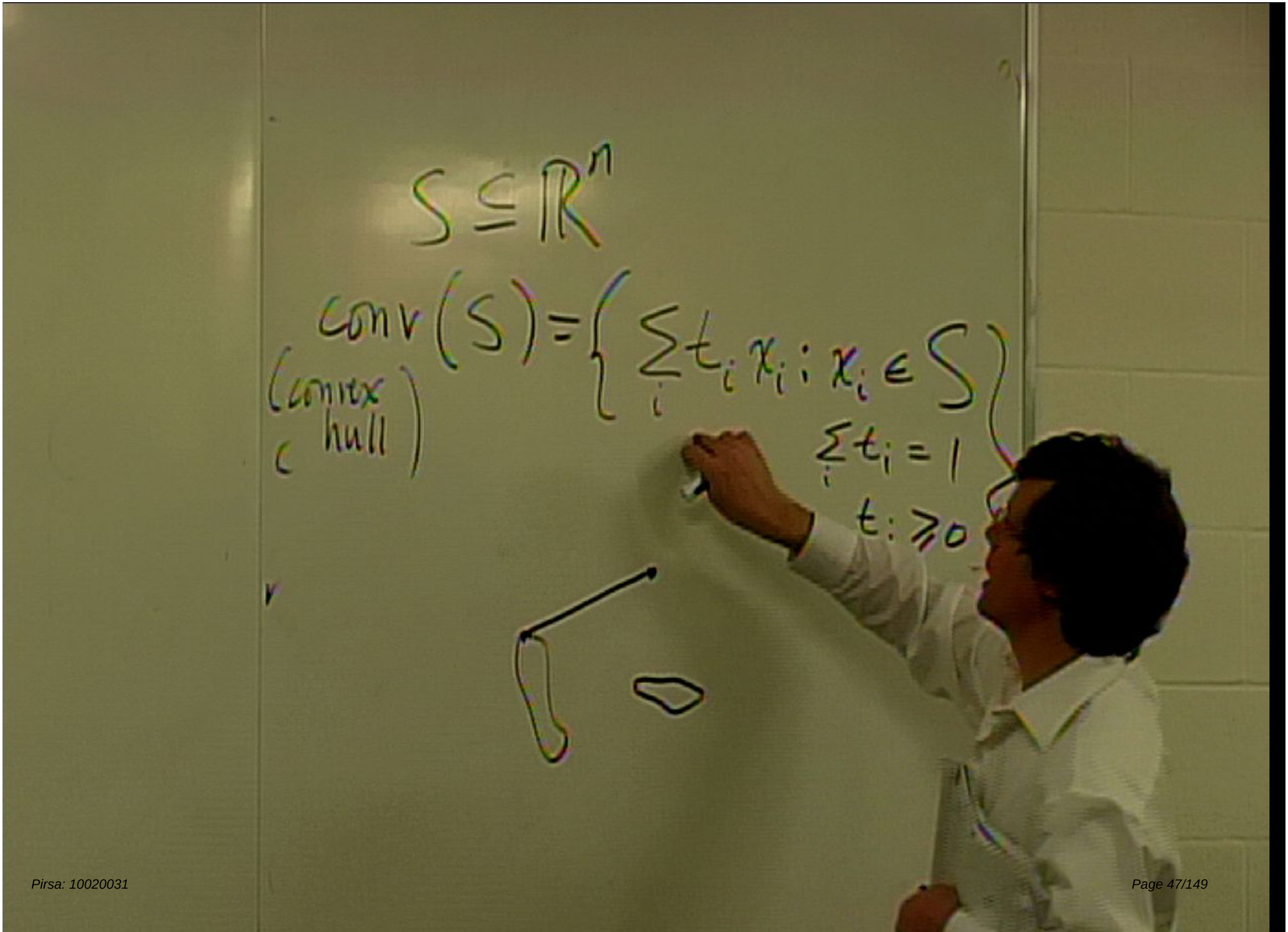


$S \subseteq \mathbb{R}^n$

$\mathrm{conv}(S) = \{ \sum_i t_i x_i : x_i \in S$

$\sum_i$

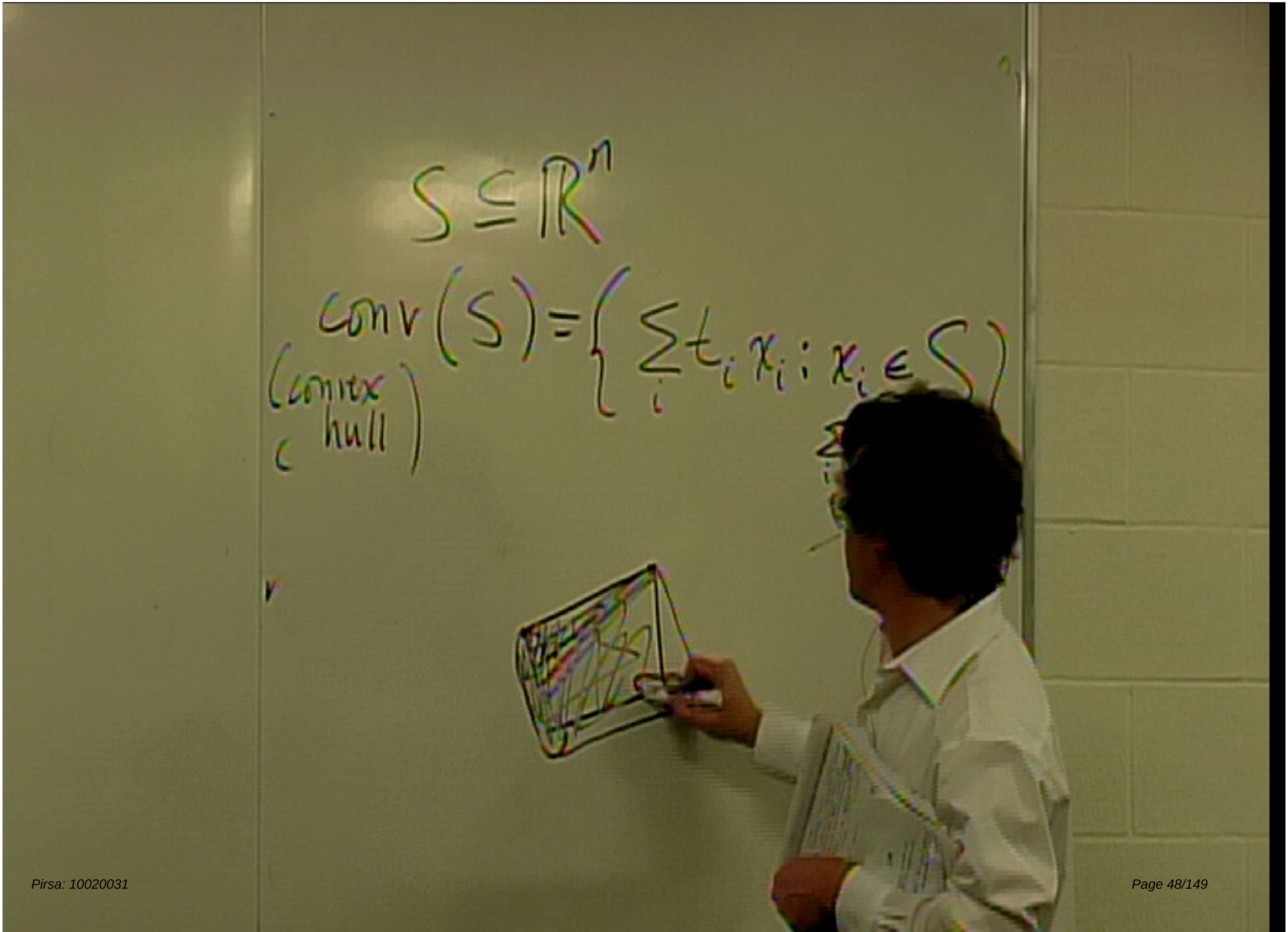
(convex hull)



$$S \subseteq \mathbb{R}^n$$

$$\operatorname{conv}(S) = \left\{ \sum_i t_i x_i : x_i \in S, \ \sum_i t_i = 1, \ t_i \geq 0 \right\}$$

(convex hull)



$S \subseteq \mathbb{R}^n$

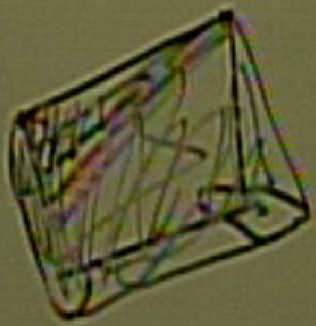
$\mathrm{conv}(S) = \left\{ \sum_i t_i x_i : x_i \in S \right.$

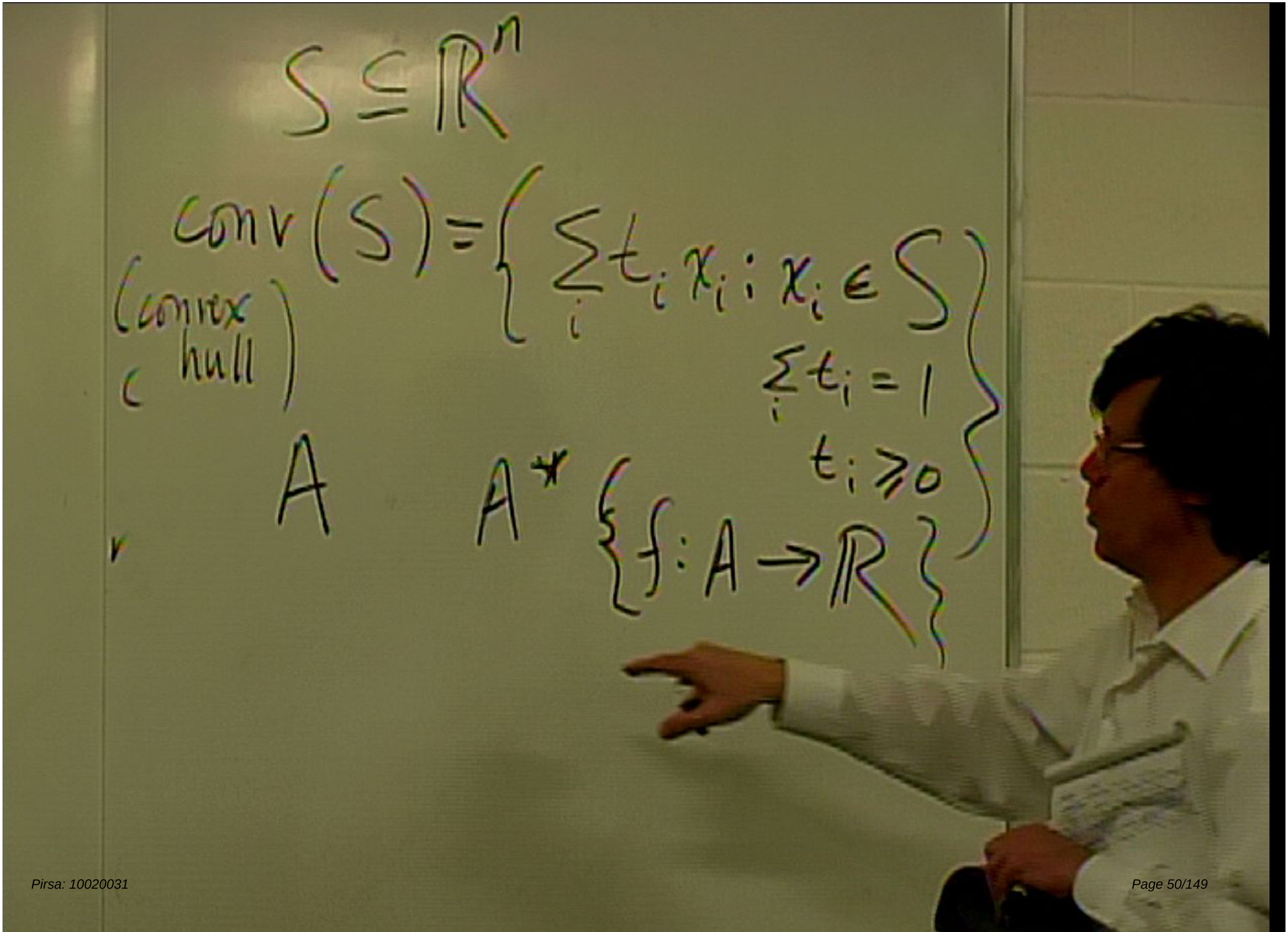
(convex hull)

$$S \subseteq \mathbb{R}^n$$

$$\mathrm{conv}(S) = \left\{ \sum_i t_i x_i : \begin{array}{r} x_i \in S \\ \sum_i t_i = 1 \\ t_i \geqslant 0 \end{array} \right\}$$

(convex hull)



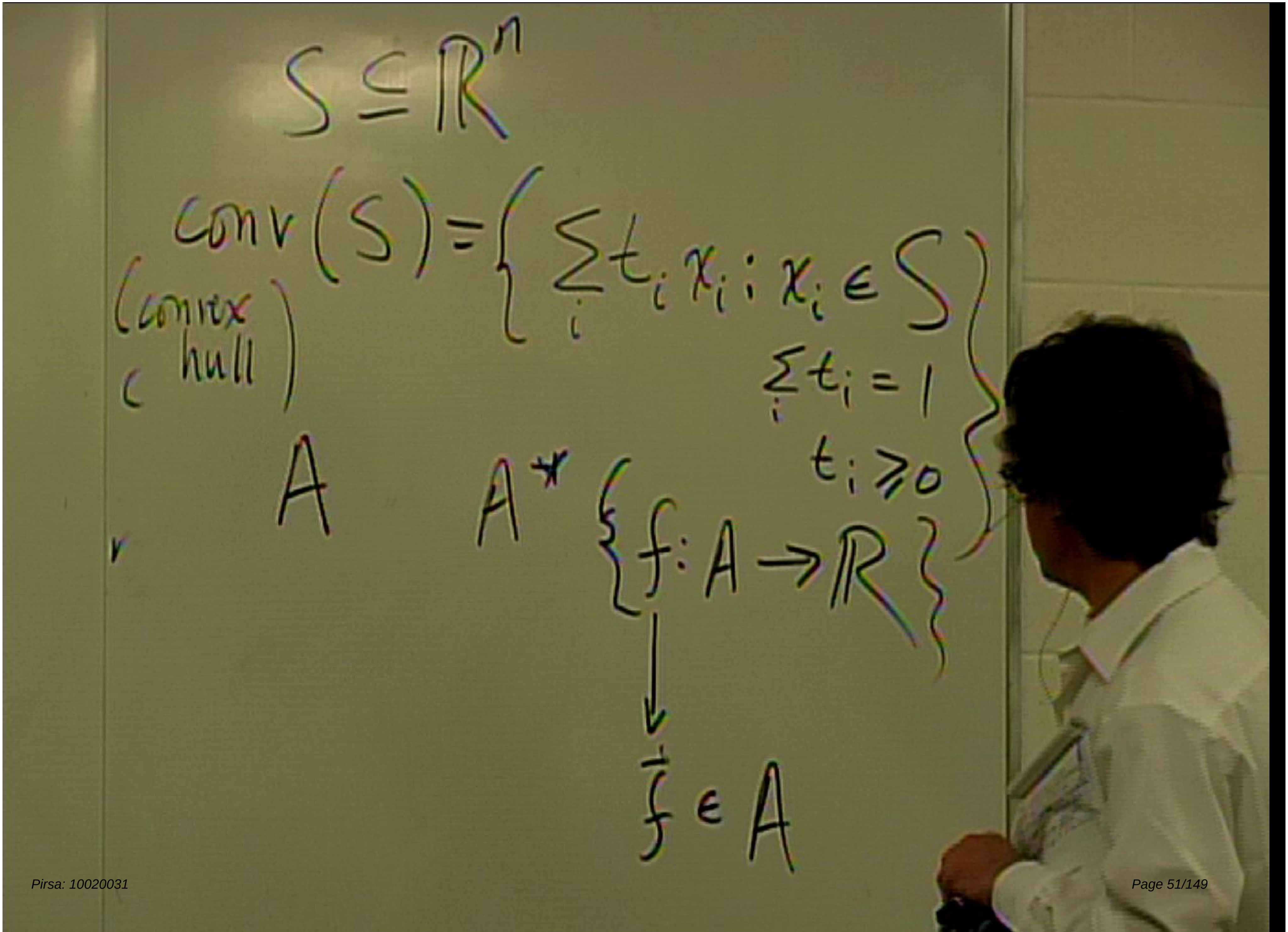


$S \subseteq \mathbb{R}^n$

$$\mathrm{conv}(S) = \left\{ \sum_i t_i x_i : x_i \in S, \ \sum_i t_i = 1, \ t_i \geq 0 \right\}$$

(convex hull)

$A \qquad A^* \ \{ f : A \to \mathbb{R} \}$



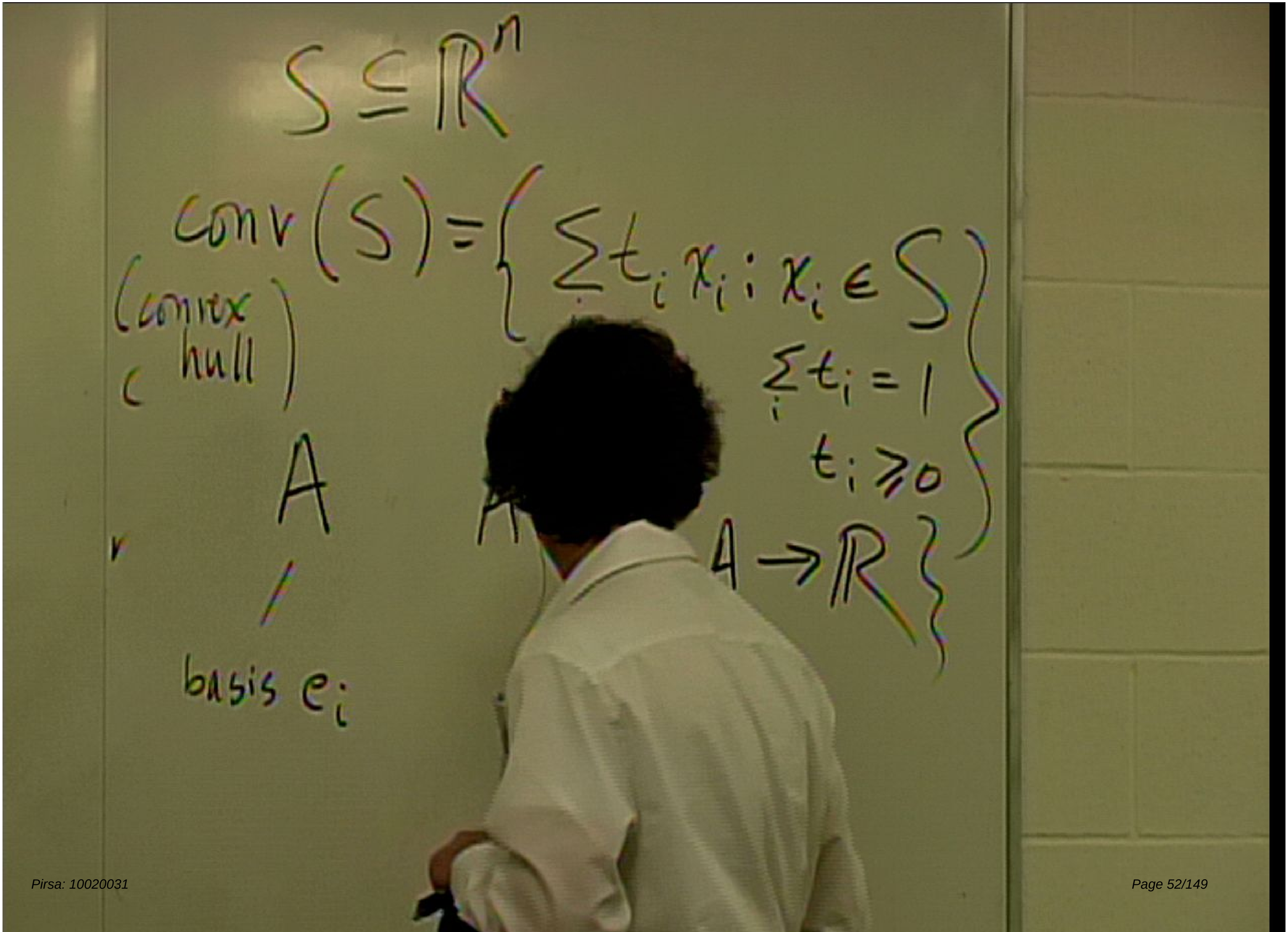
$$S \subseteq \mathbb{R}^n$$

$$\mathrm{conv}(S) = \left\{ \sum_i t_i x_i : \begin{array}{r} x_i \in S \\ \sum_i t_i = 1 \\ t_i \geq 0 \end{array} \right\}$$

(convex hull)

$$A \qquad A^* \left\{ f : A \to \mathbb{R} \right\}$$

$$\begin{tikzcd} f \arrow[d] \\ \vec{f} \in A \end{tikzcd}$$



$S \subseteq \mathbb{R}^n$

$\mathrm{conv}(S) = \left\{ \sum t_i x_i : x_i \in S, \ \sum_i t_i = 1, \ t_i \geq 0 \right\}$

(convex hull)

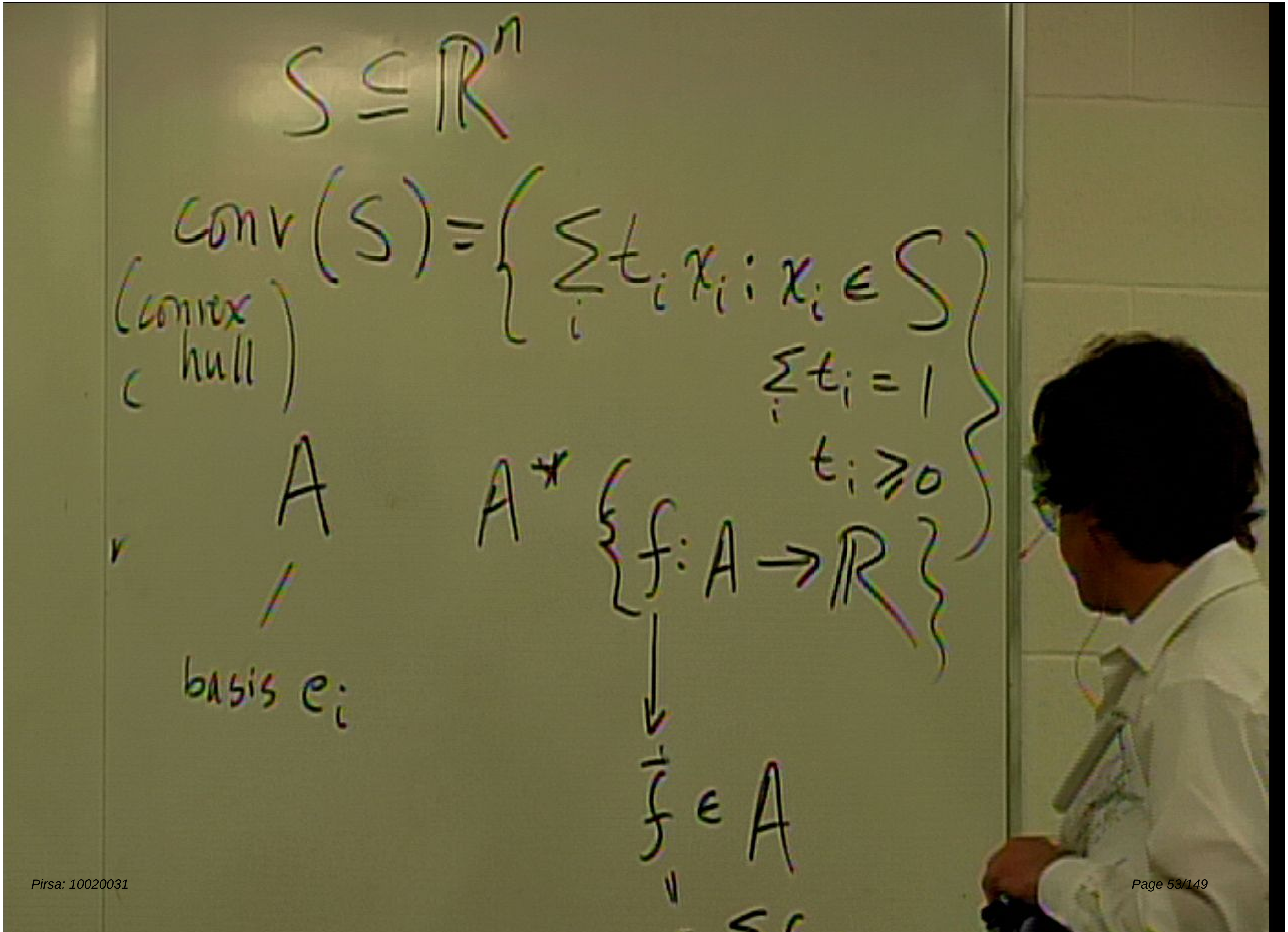
A

A

$A \to \mathbb{R} \}$

/

basis e_i



$$S \subseteq \mathbb{R}^n$$

$$\mathrm{conv}(S) = \left\{ \sum_i t_i x_i : x_i \in S, \ \sum_i t_i = 1, \ t_i \geqslant 0 \right\}$$

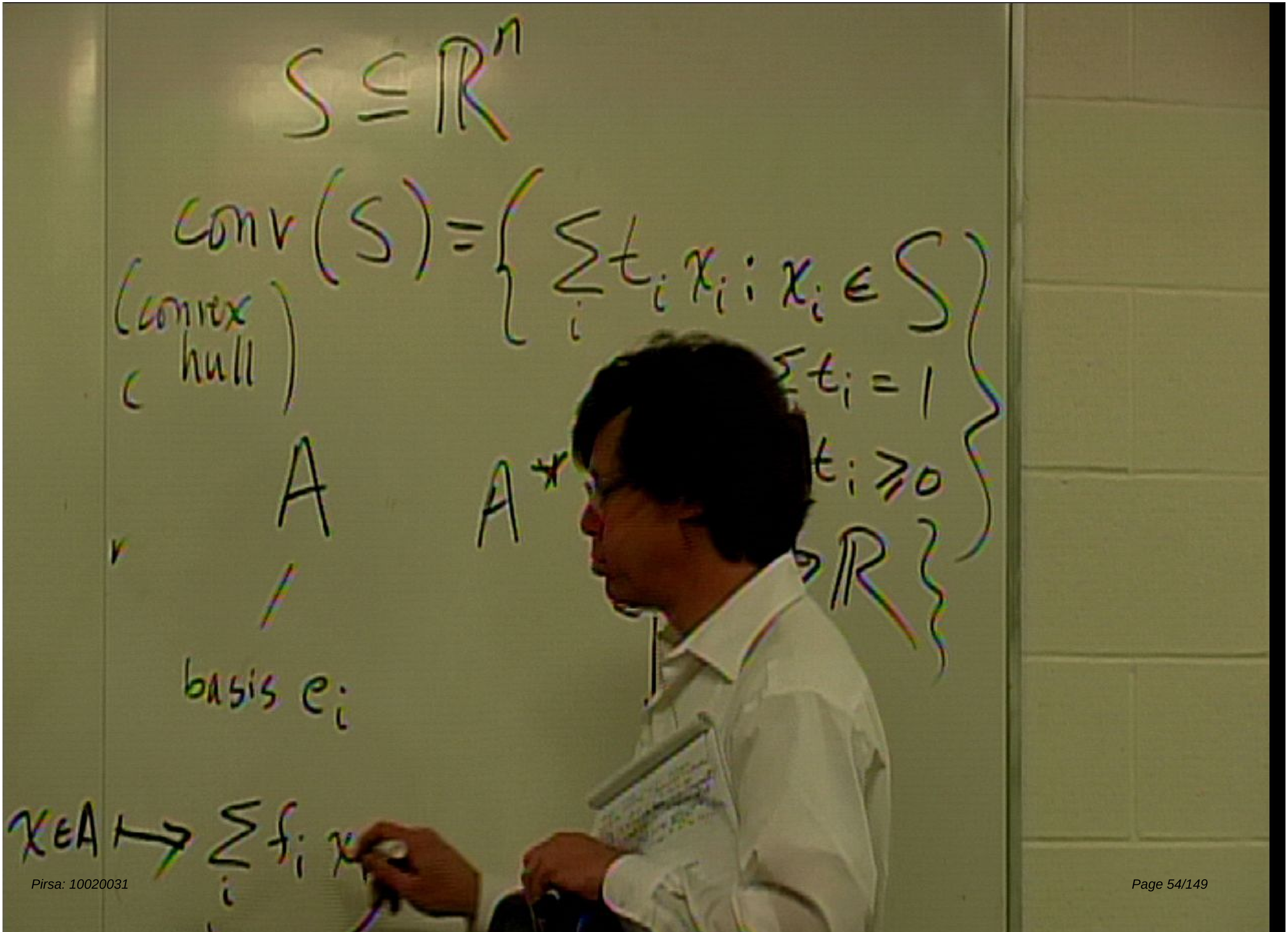
(convex hull)

$$A \qquad A^* \quad \{ f : A \to \mathbb{R} \}$$

basis e_i

$$\downarrow$$

$$\vec{f} \in A$$



$S \subseteq \mathbb{R}^n$

$\mathrm{conv}(S) = \left\{ \sum_i t_i x_i : x_i \in S \right.$

(convex hull)

$\sum t_i = 1$

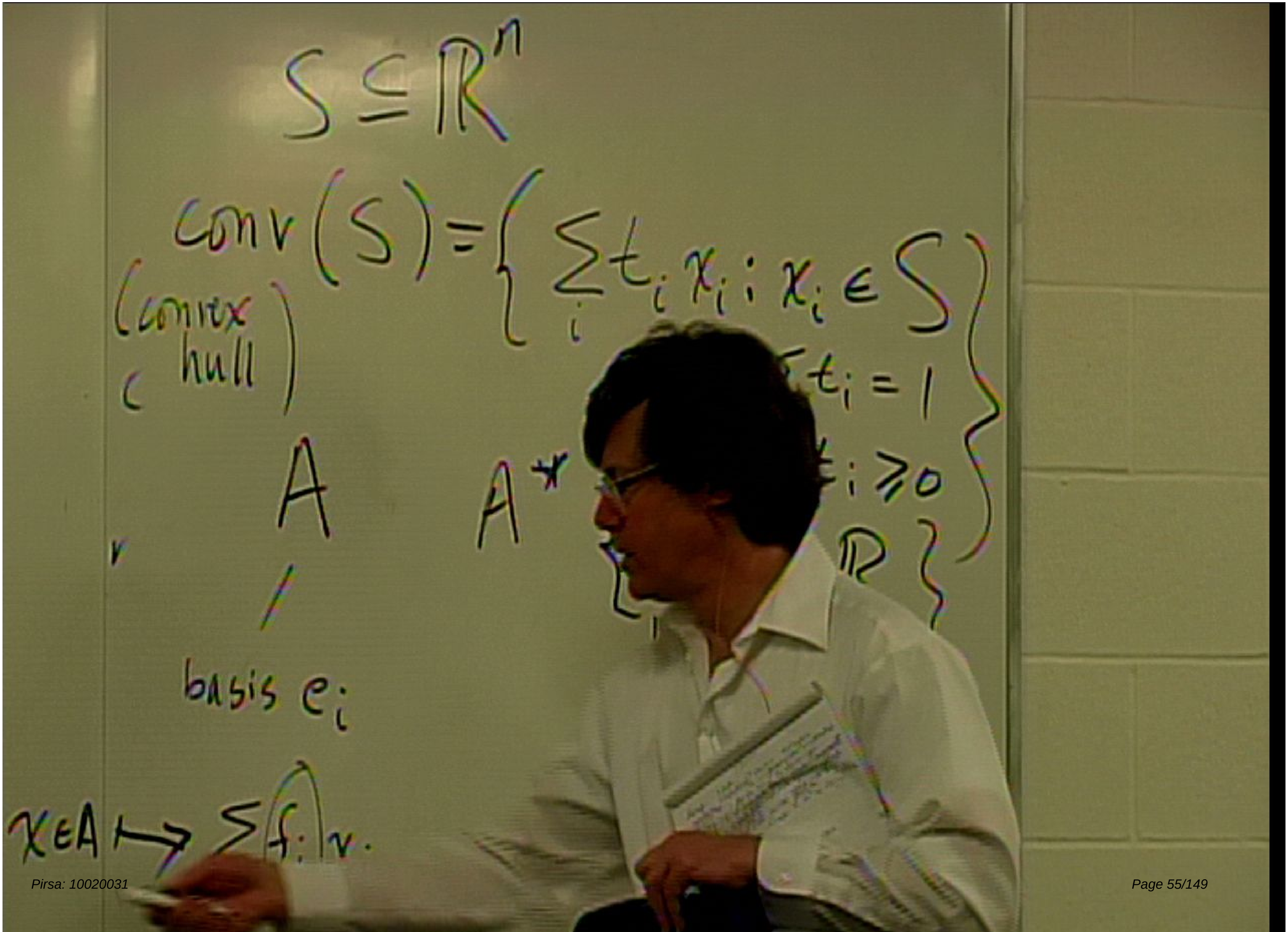
$t_i \geq 0$

$\mathbb{R}\}$

$A \qquad A^*$

basis e_i

$x \in A \mapsto \sum_i f_i x_i$



$S \subseteq \mathbb{R}^n$

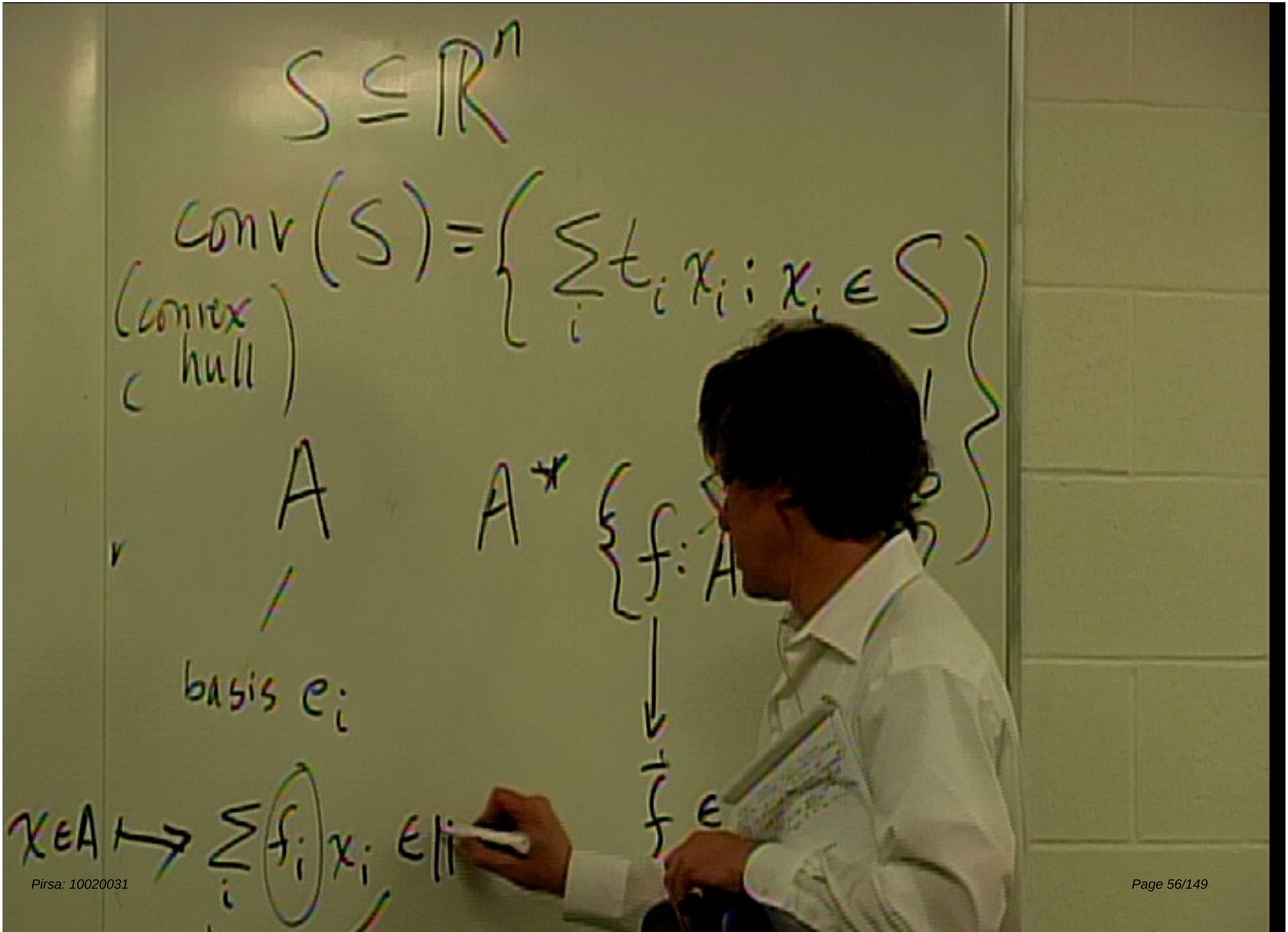
$\mathrm{conv}(S) = \left\{ \sum_i t_i x_i : x_i \in S, \; t_i = 1, \; i \geq 0 \right\}$

(convex hull)

A $\qquad A^*$ $\qquad \{ \quad \mathbb{R} \}$

basis e_i

$x \in A \mapsto \sum f_i v_i$



$S \subseteq \mathbb{R}^n$

$\mathrm{conv}(S) = \left\{ \sum_i t_i x_i : x_i \in S \right.$

(convex hull)

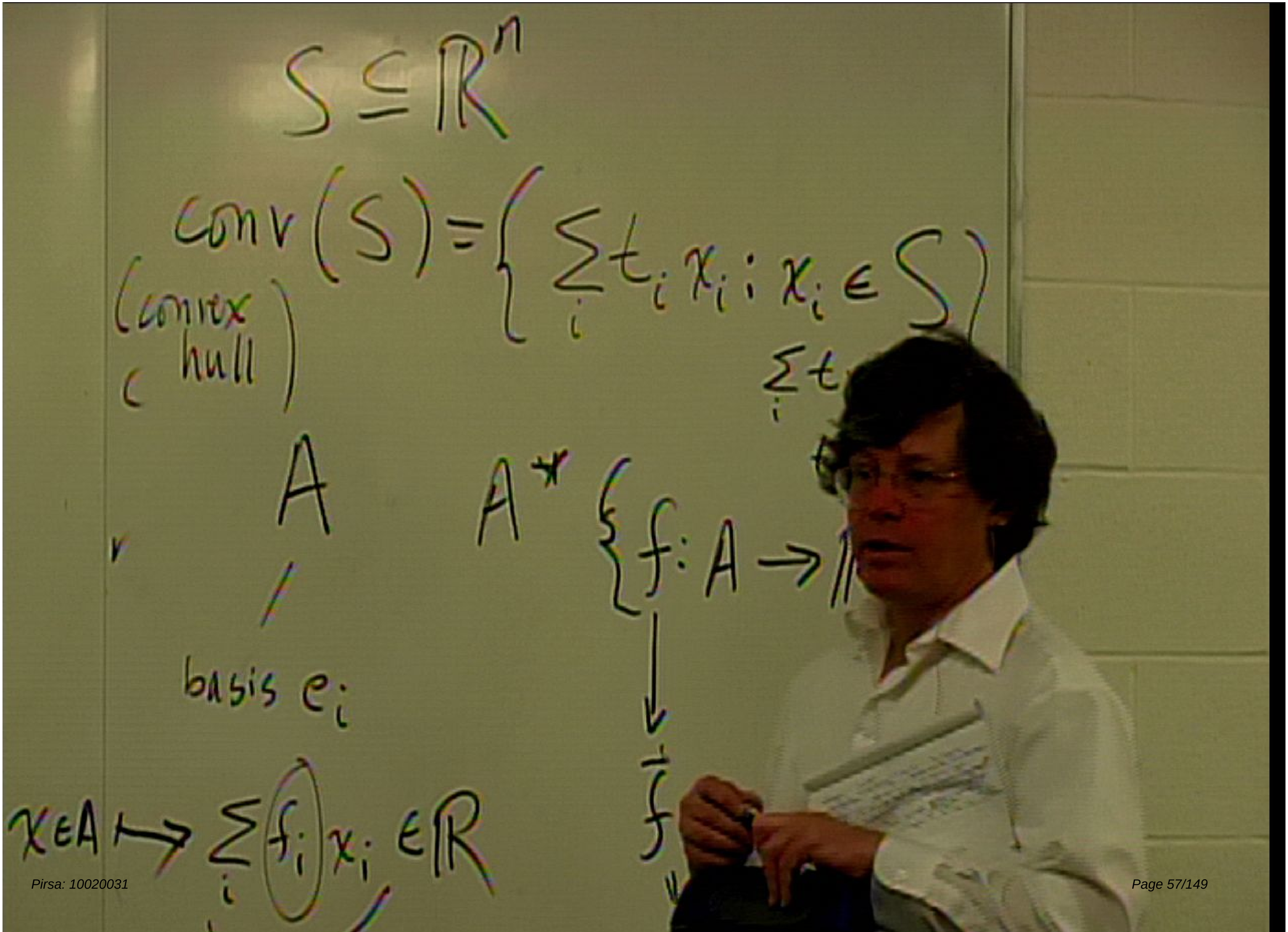
$A \qquad A^* \left\{ f : A \right.$

basis e_i

$\downarrow$

$\bar{f} \in$

$x \in A \mapsto \sum_i \boxed{f_i}\, x_i \in$



$$S \subseteq \mathbb{R}^n$$

$$\mathrm{conv}(S) = \left\{ \sum_i t_i x_i : x_i \in S \right\}$$

(convex hull)

$$\sum_i t$$

$$t$$

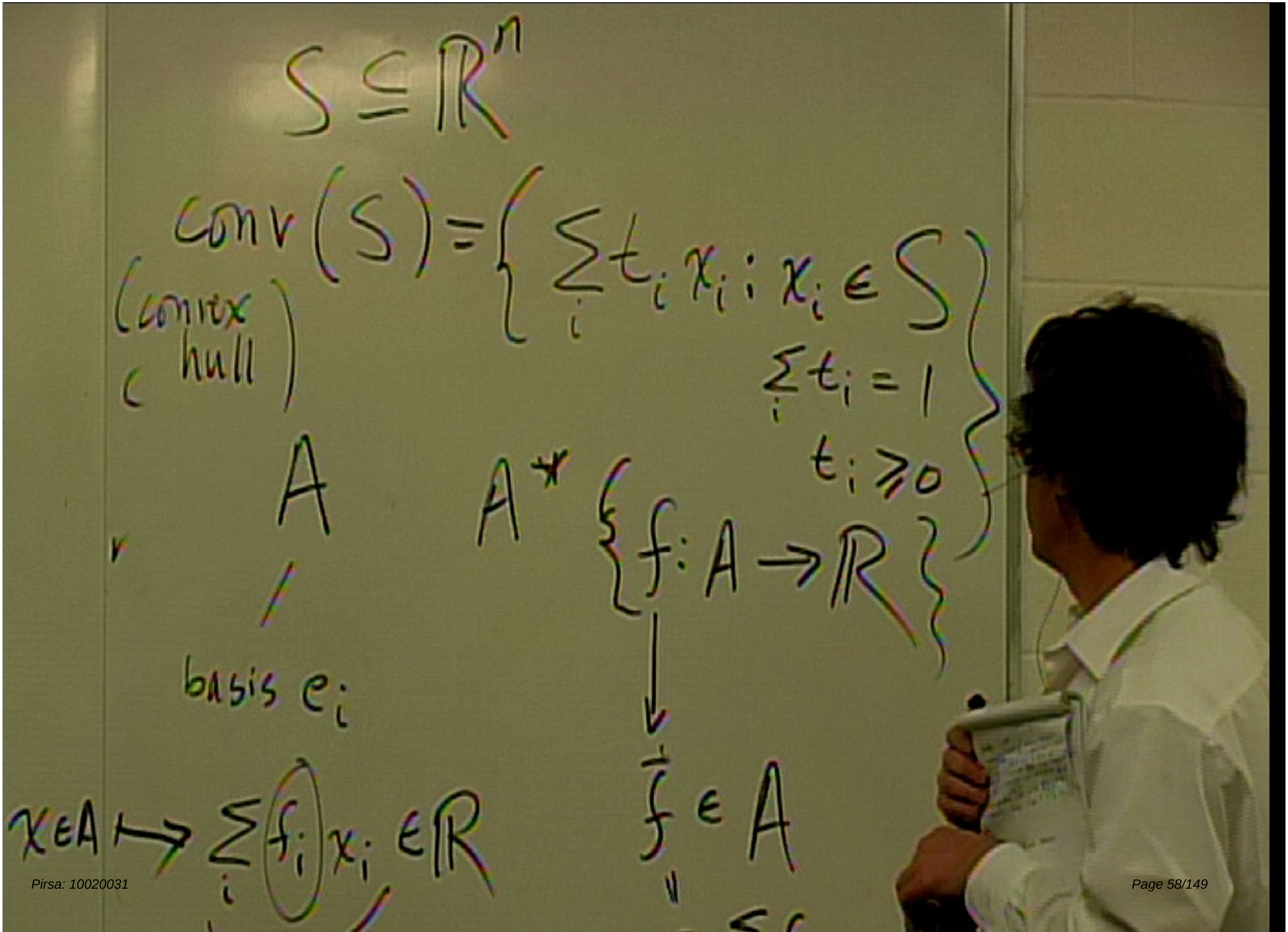
$$A \qquad A^* \left\{ f : A \to \mathbb{R} \right.$$

basis e_i

$$x \in A \mapsto \sum_i \boxed{f_i}\, x_i \in \mathbb{R}$$

$$\downarrow$$

$$\vec{f}$$



$$S \subseteq \mathbb{R}^n$$

$$\mathrm{conv}(S) = \left\{ \sum_i t_i x_i : x_i \in S,\ \sum_i t_i = 1,\ t_i \geq 0 \right\}$$

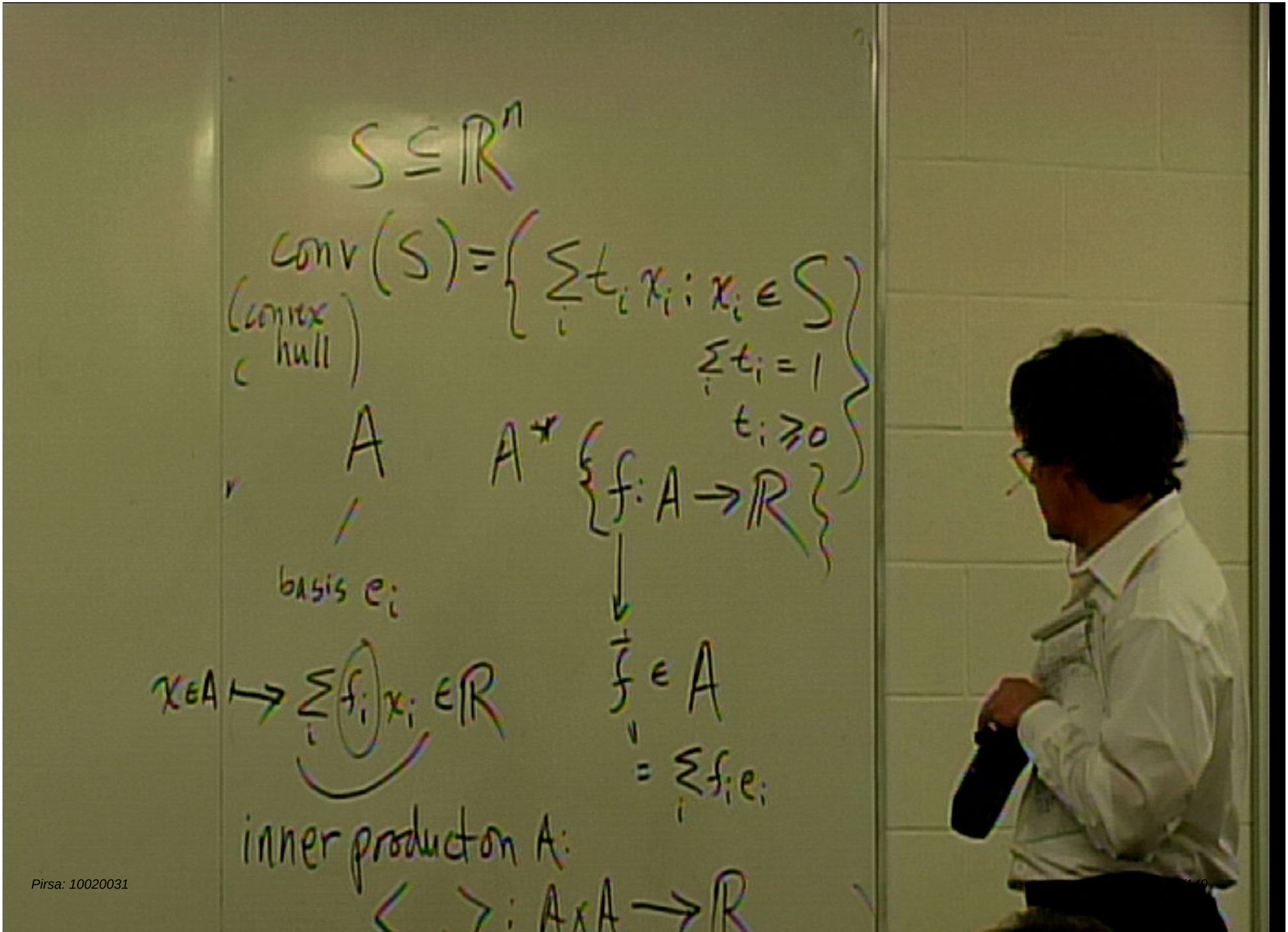
(convex hull)

A $\qquad$ A^*

$$\begin{tikzcd} \{f : A \to \mathbb{R}\} \arrow[d] \\ \vec{f} \in A \end{tikzcd}$$

basis e_i

$$x \in A \mapsto \sum_i f_i x_i \in \mathbb{R}$$



$S \subseteq \mathbb{R}^n$

$\mathrm{conv}(S) = \left\{ \sum_i t_i x_i : x_i \in S,\ \sum_i t_i = 1,\ t_i \geqslant 0 \right\}$

(convex hull)

$A \qquad A^* \{ f : A \to \mathbb{R} \}$

$\downarrow$

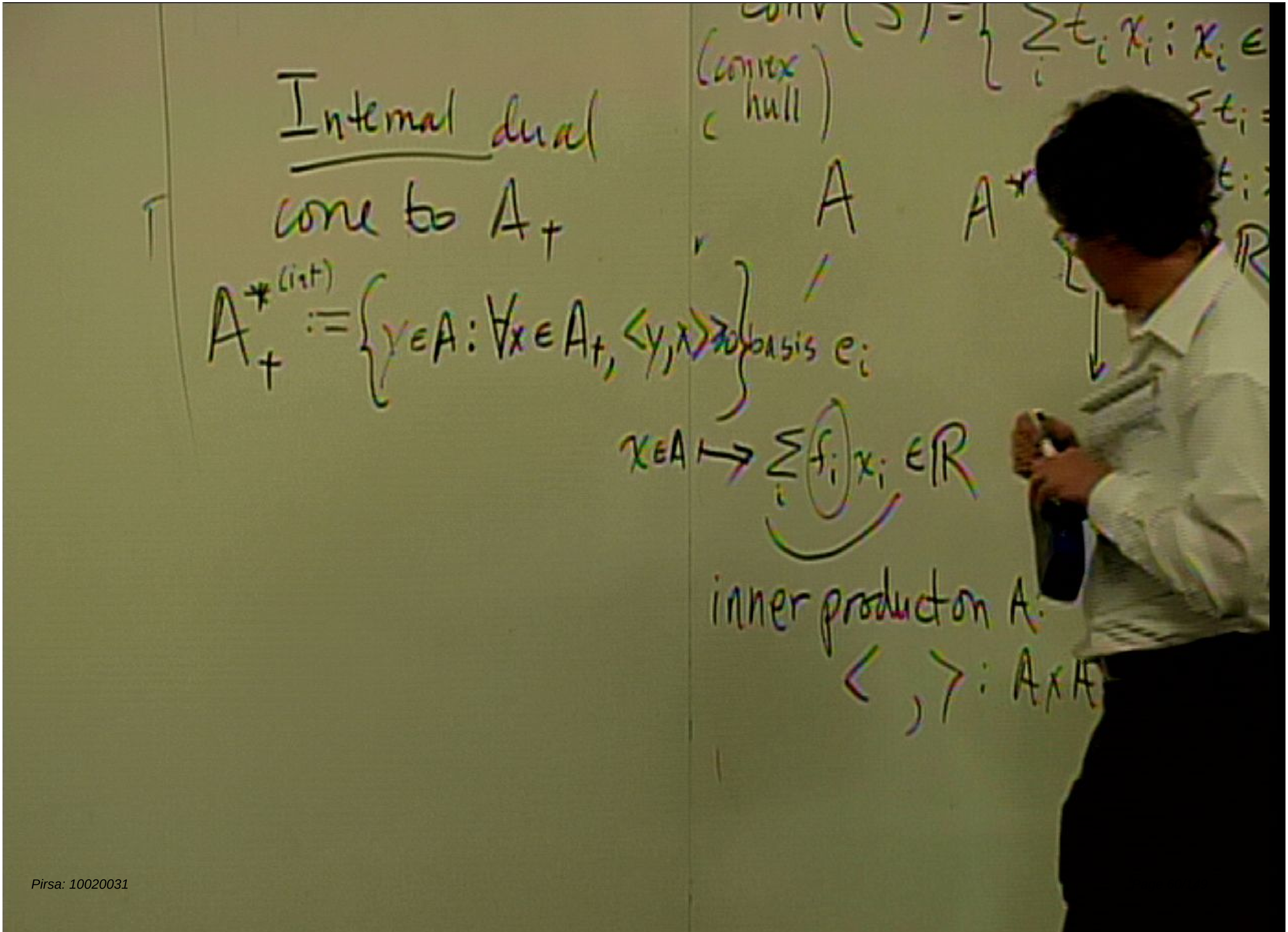
basis e_i

$x \in A \mapsto \sum_i f_i x_i \in \mathbb{R} \qquad \bar{f} \in A$

$= \sum_i f_i e_i$

inner product on A:

$\langle\ ,\ \rangle : A \times A \to \mathbb{R}$



$\mathrm{conv}(S) = \{ \sum_i t_i x_i : x_i \in$

(convex hull)

$\Sigma t_i =$

t_i

$A \qquad A^*$

$\mathbb{R}$

Internal dual

cone to A_+

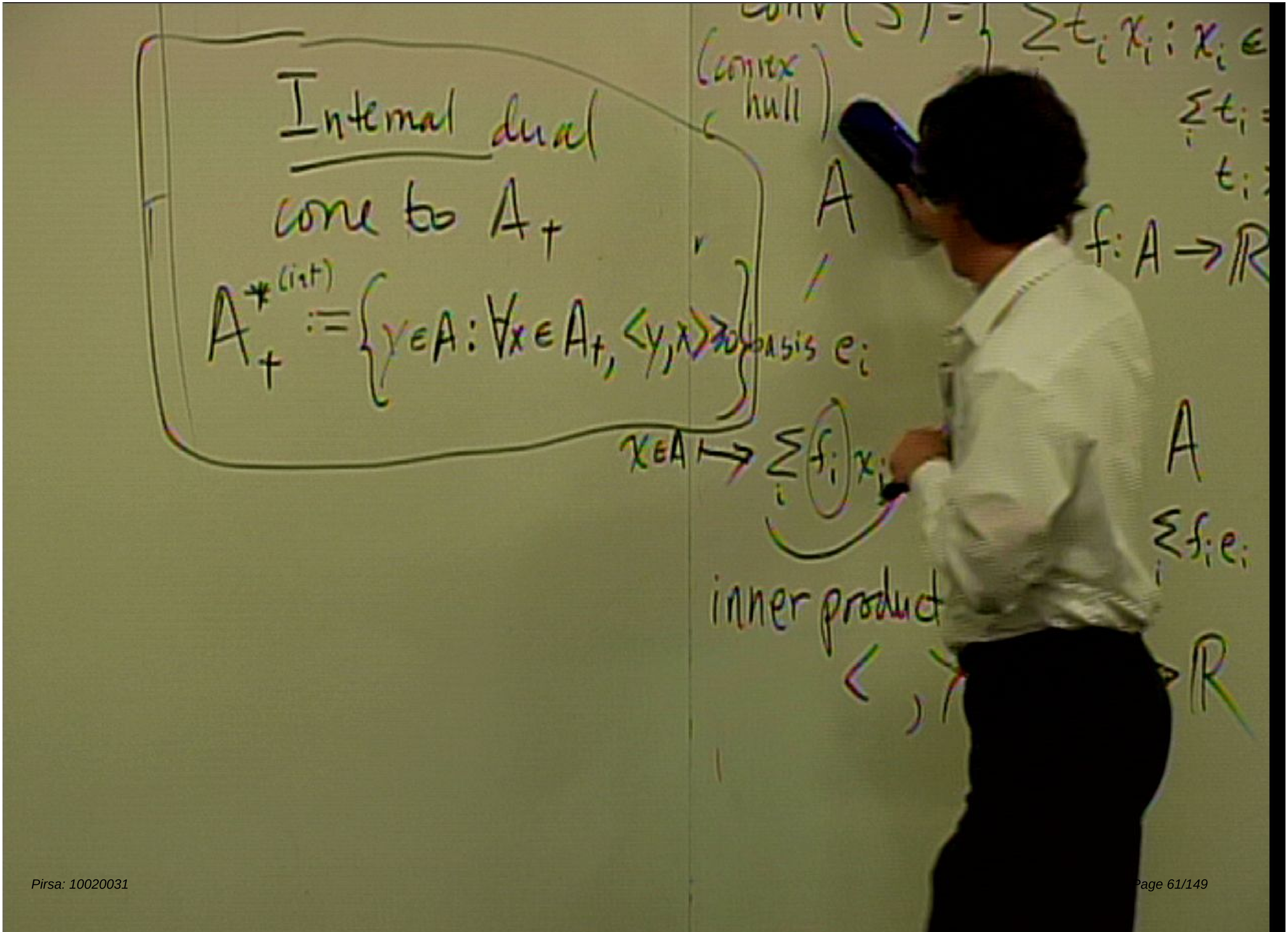
$A_+^{*(\mathrm{int})} := \{ y \in A : \forall x \in A_+, \langle y, x \rangle \geq 0 \}$

basis e_i

$x \in A \mapsto \underbrace{\sum_i \boxed{f_i} x_i} \in \mathbb{R}$

inner product on A:

$\langle \; , \; \rangle : A \times A$



Internal dual cone to A_+

$A_+^{*(int)} := \{ y \in A : \forall x \in A_+, \langle y, x \rangle \geq 0 \}$

$\text{conv}(S) = \{ \sum t_i x_i : x_i \in$

(convex hull)

$\sum_i t_i =$

$t_i \geq$

$f : A \to R$

A

basis e_i

$x \in A \mapsto \sum_i f_i x_i$

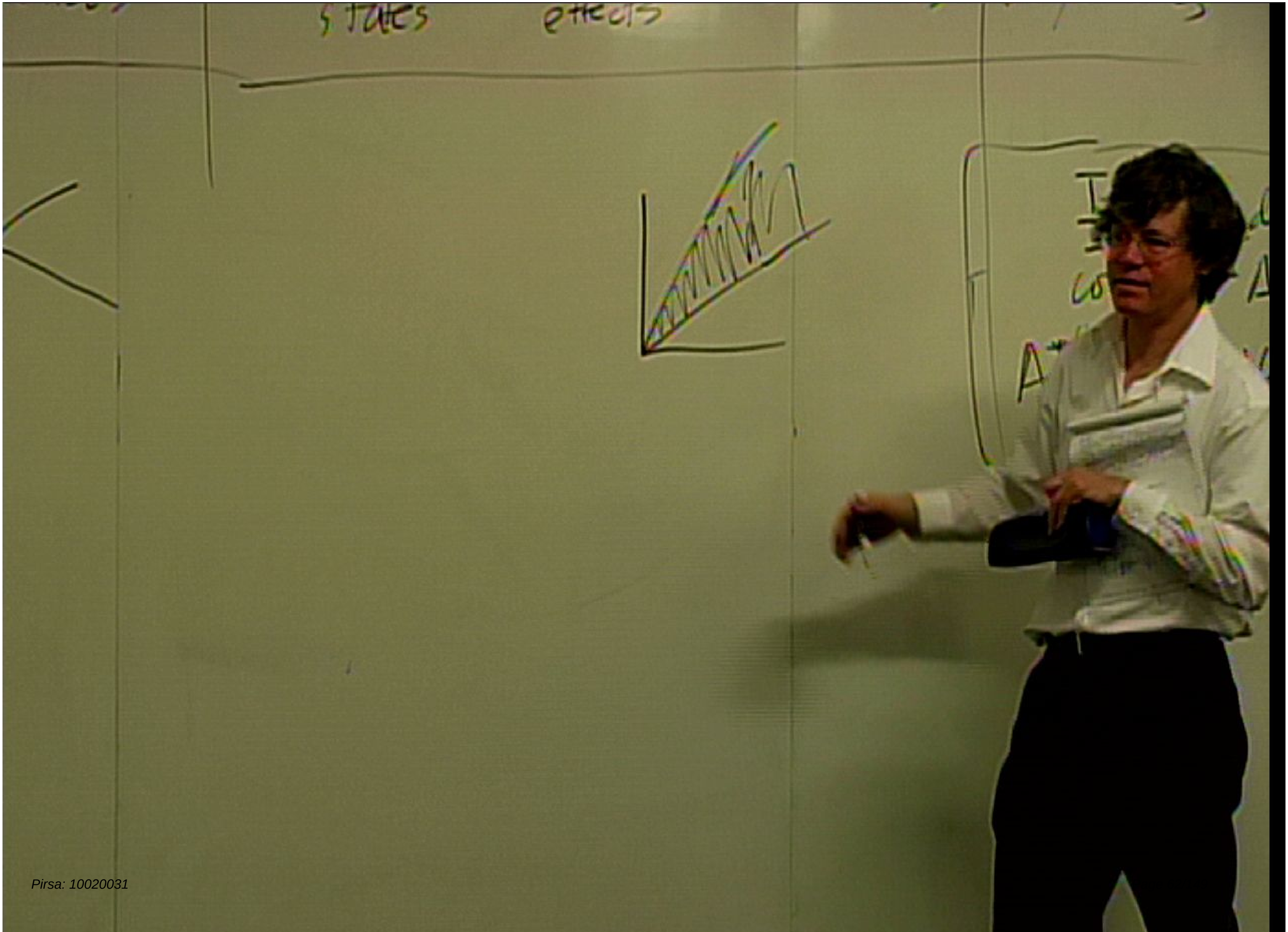
A

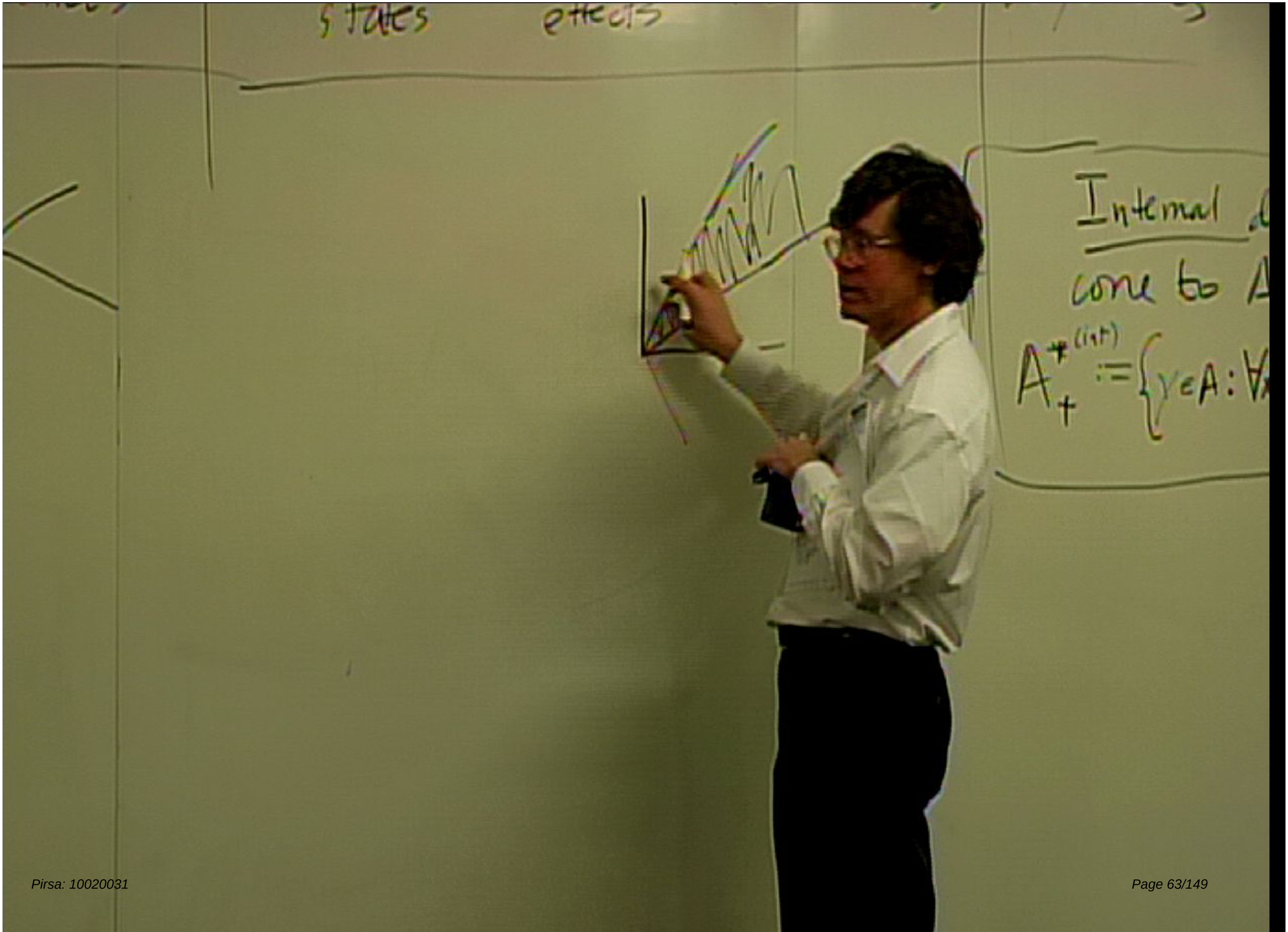
$\sum_i f_i e_i$

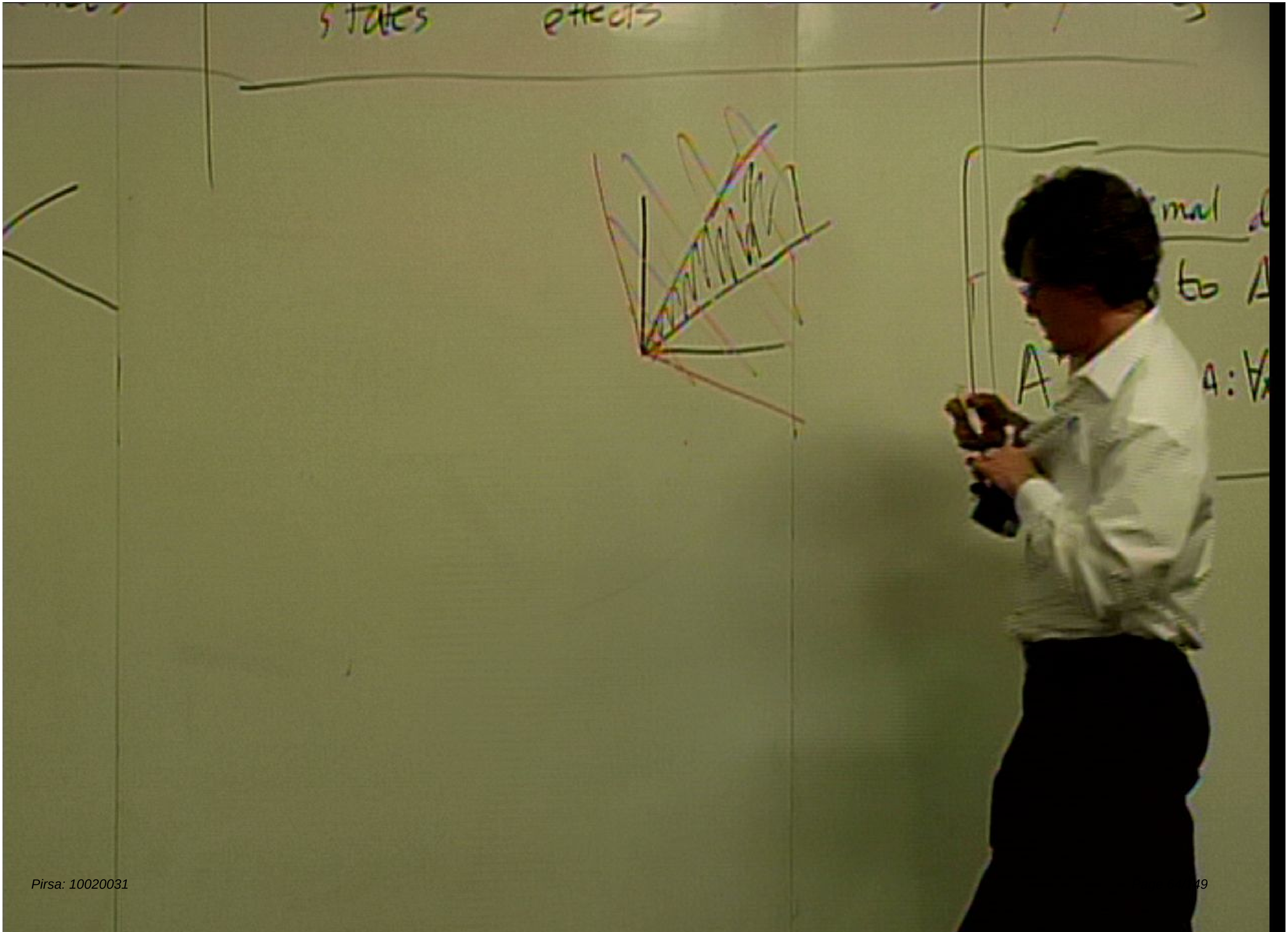
inner product

$\langle \ , \ \rangle$

$\to R$


states
effects


states
effects
Internal
cone to A
A_+^{*(int)} := {γ∈A: ∀


states
effects
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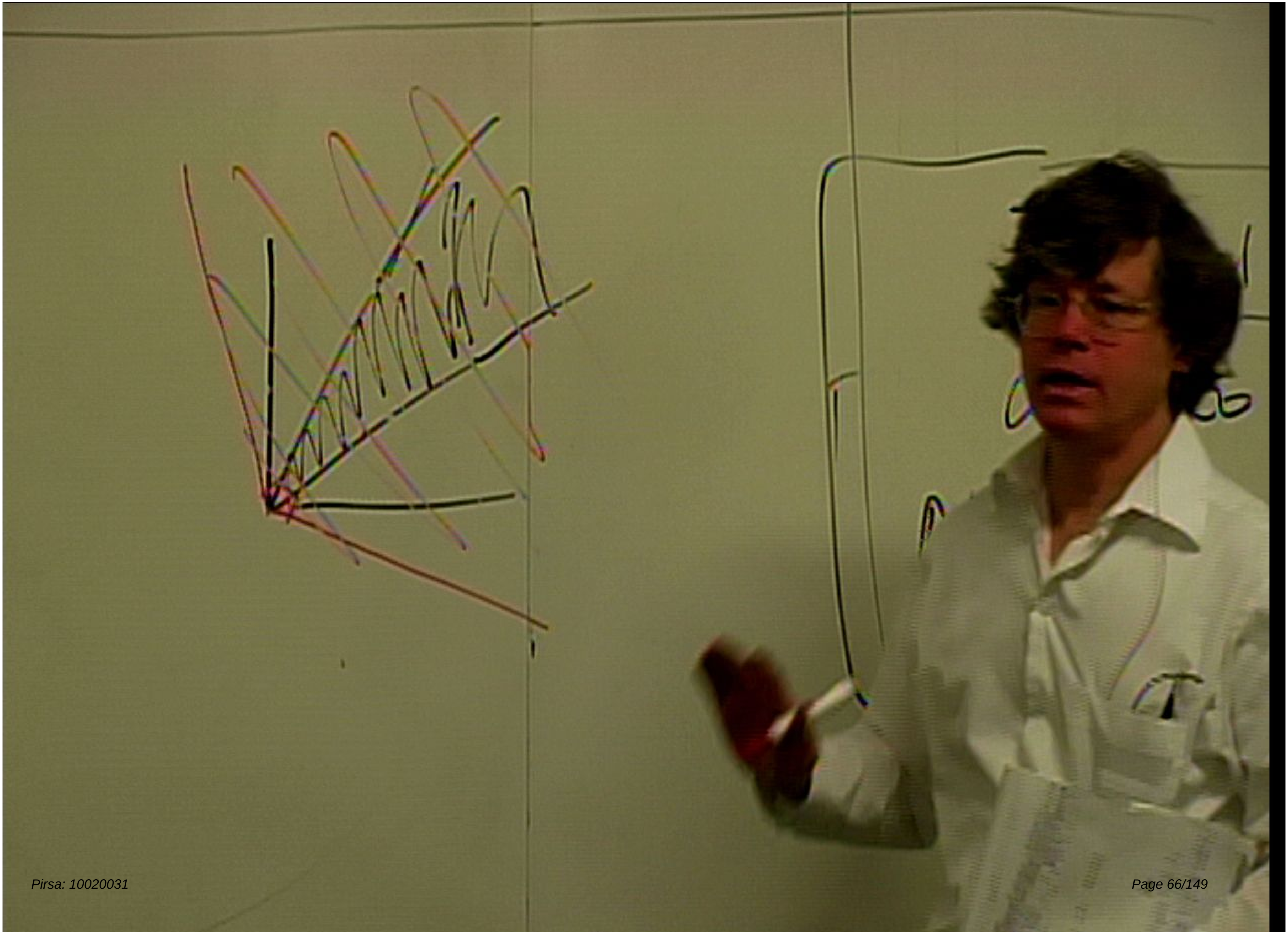
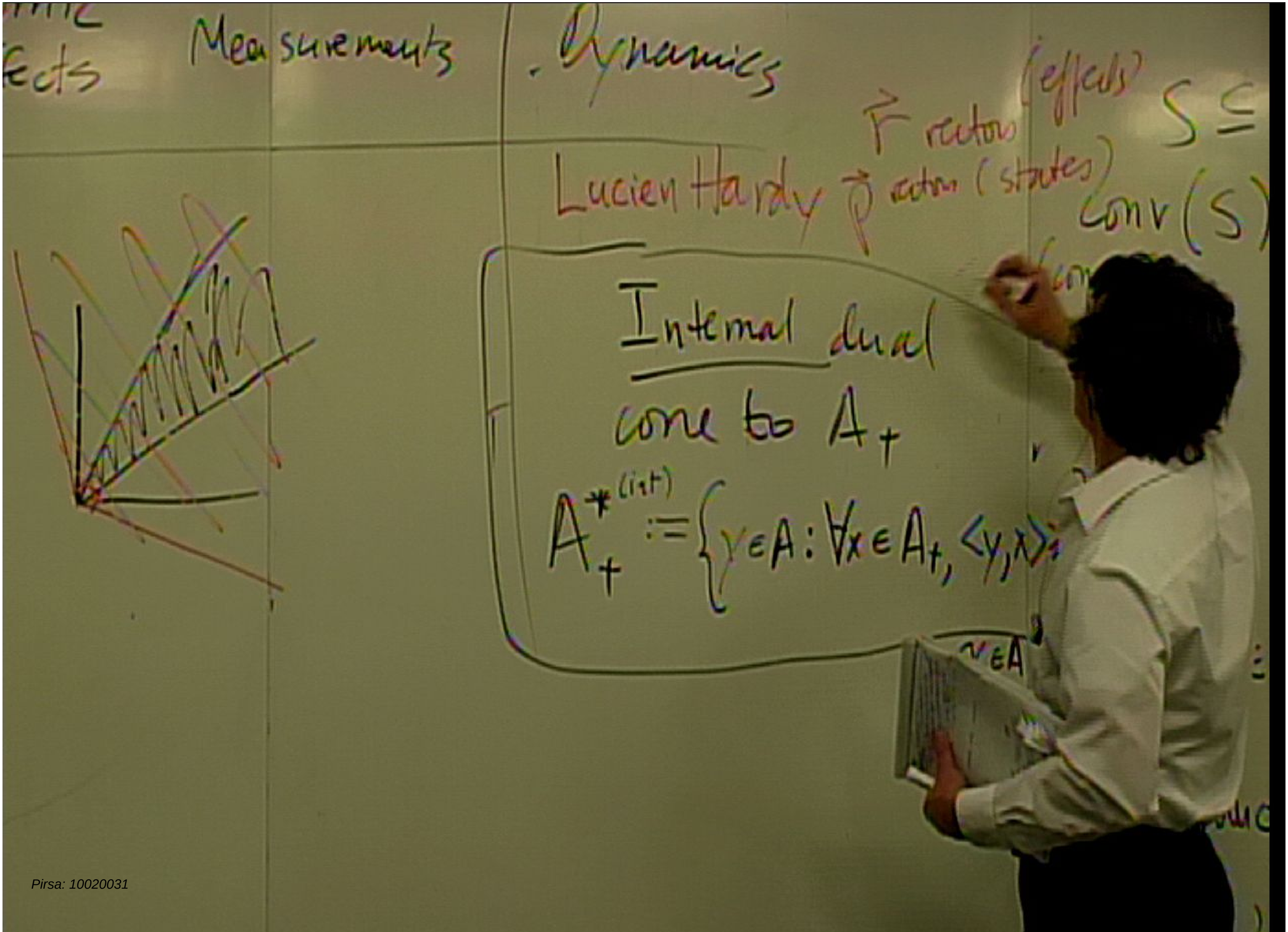
states effects

Internal

cone to

$A_+^{*(int)} := \{ r \in A : \forall$





Measurements

Dynamics

$\vec{F}$ vectors (effects)

Lucien Hardy $\vec{p}$ vectors (states)

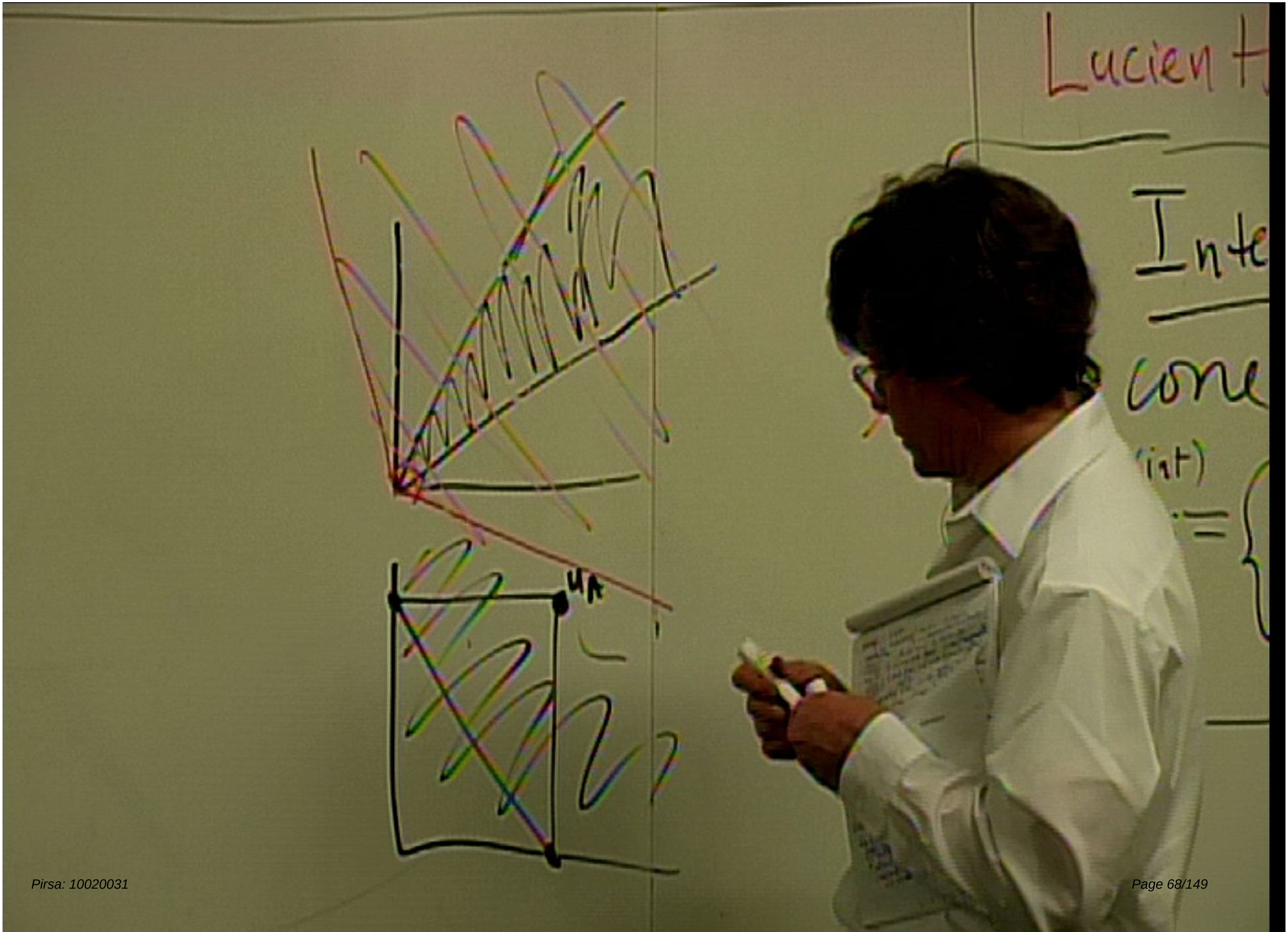
$S \subseteq$

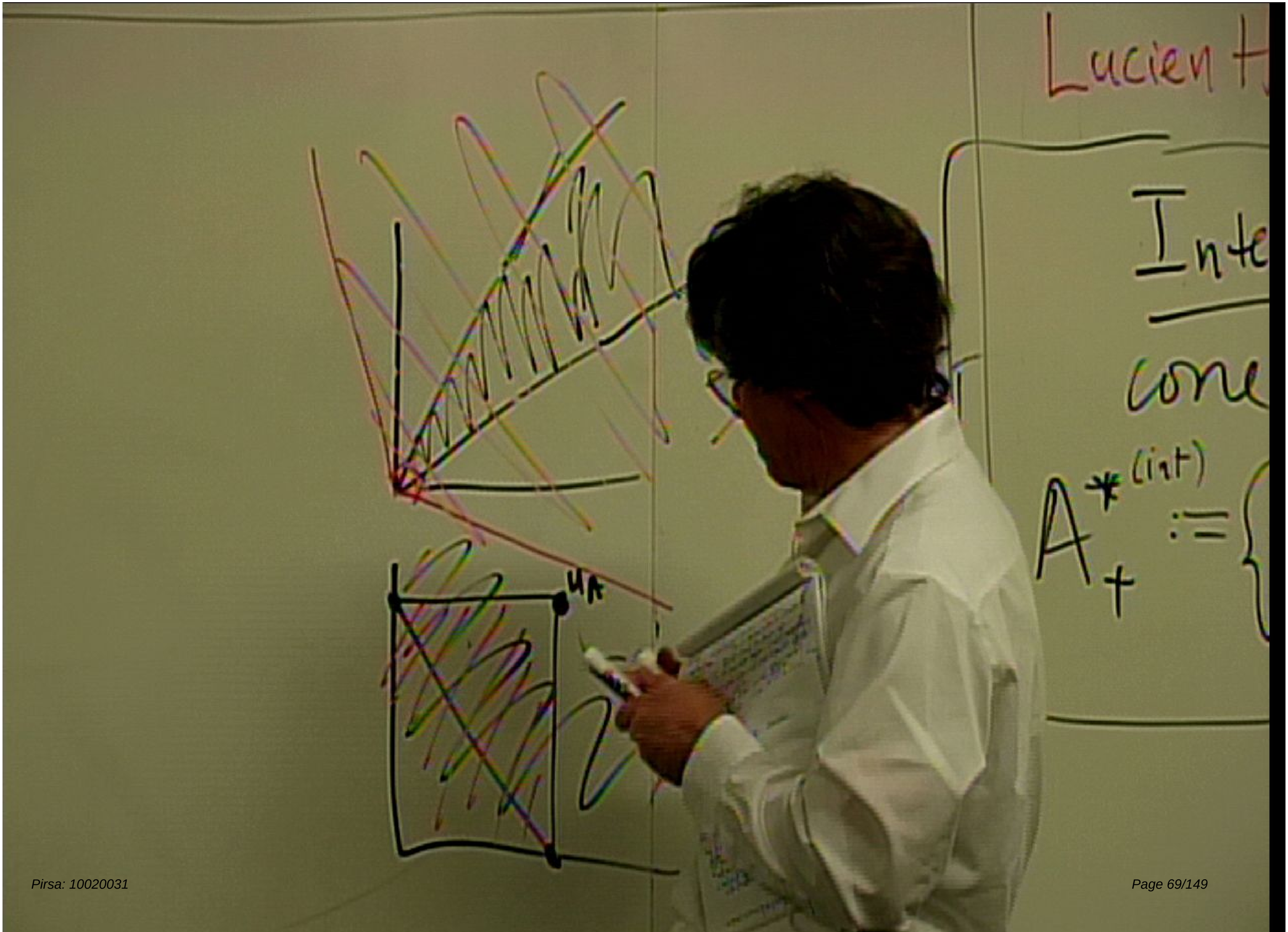
$\mathrm{conv}(S)$

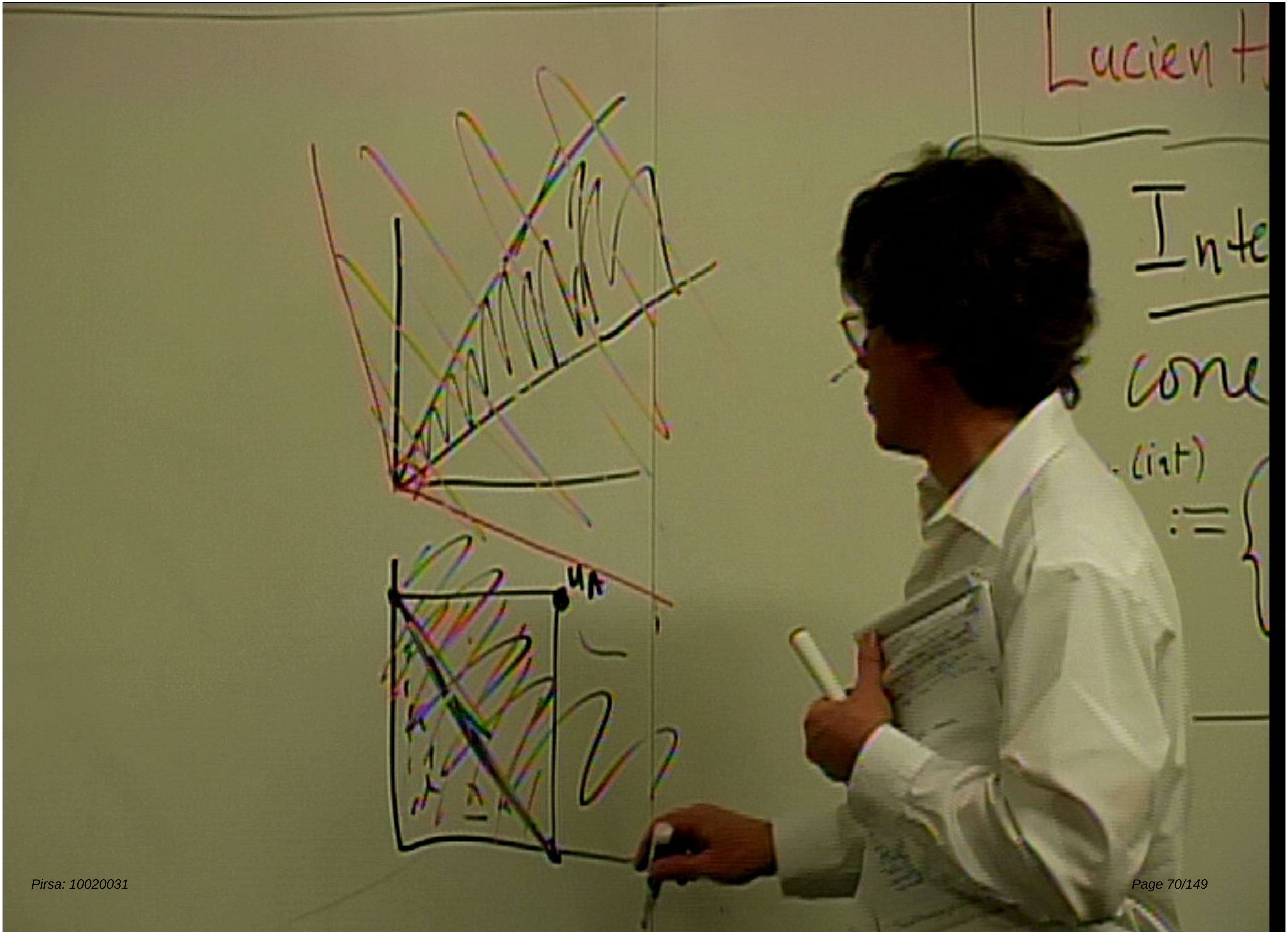
Internal dual cone to A_+

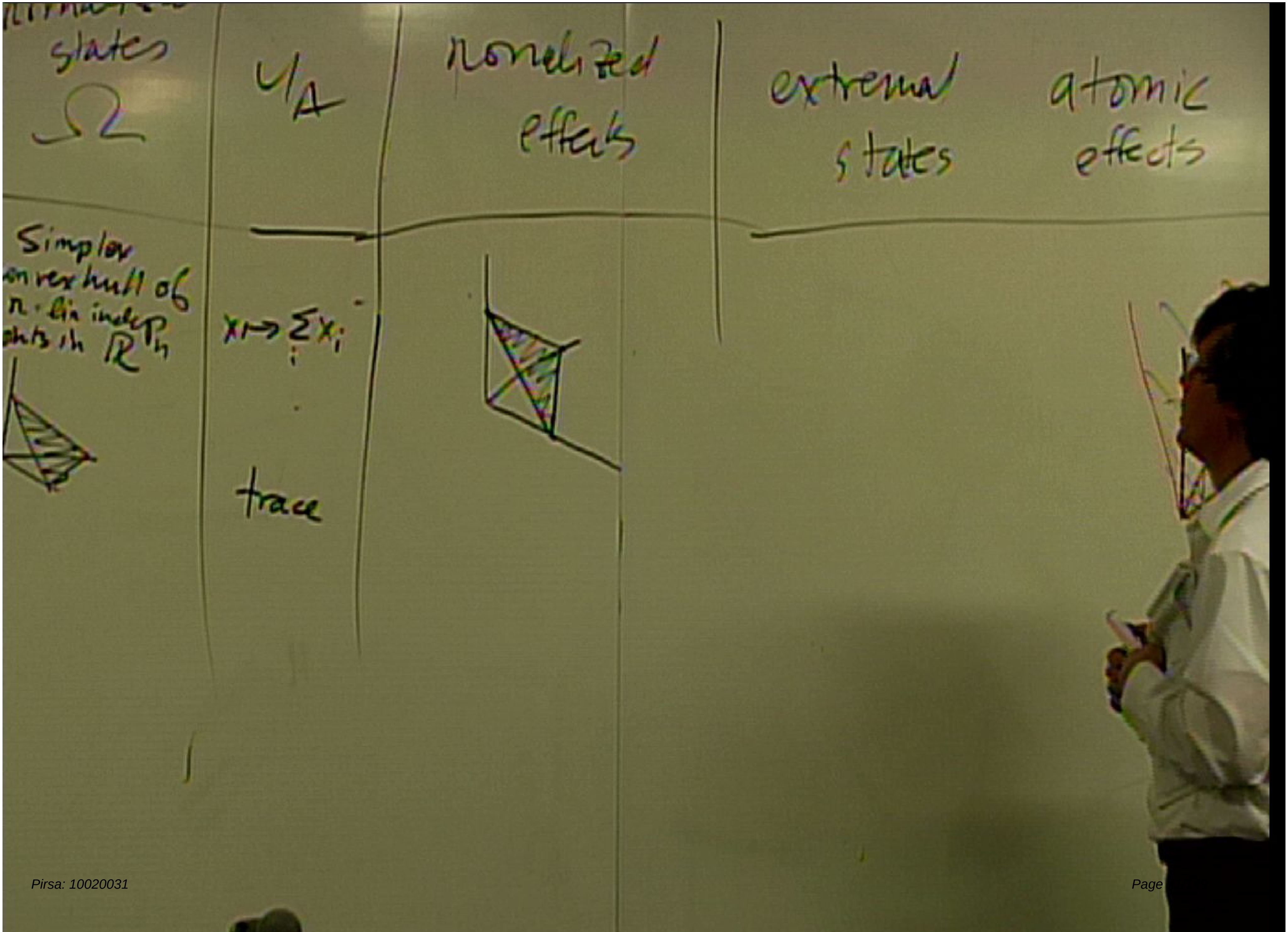
$$A_+^{*(\mathrm{int})} := \{ y \in A : \forall x \in A_+, \langle y, x \rangle$$

$y \in A$

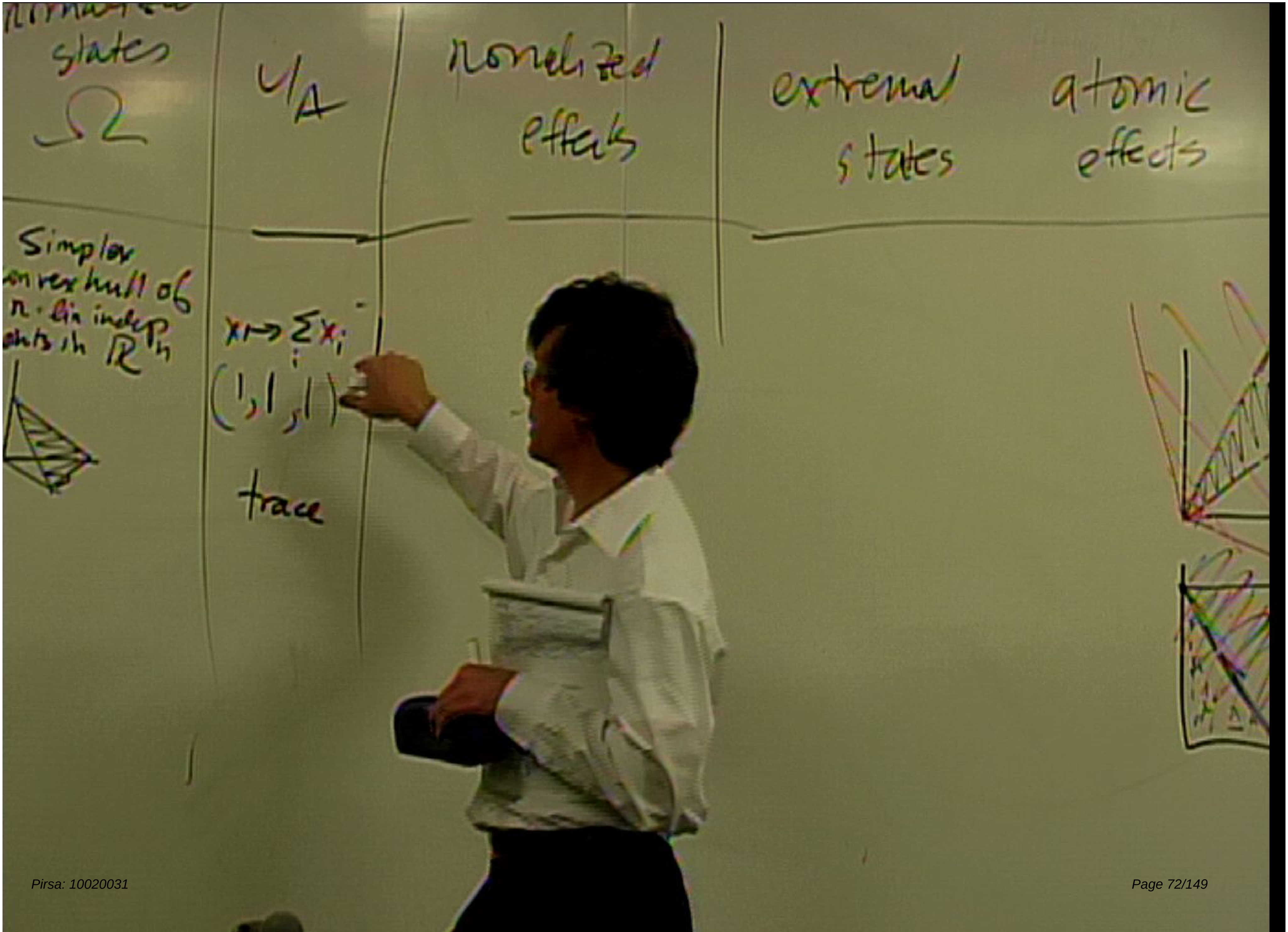

Lucien H
Inte
conc
(int)
=
u_A


Lucien H
Inte
cone
$A_+^{*(int)} := \{$
u_A

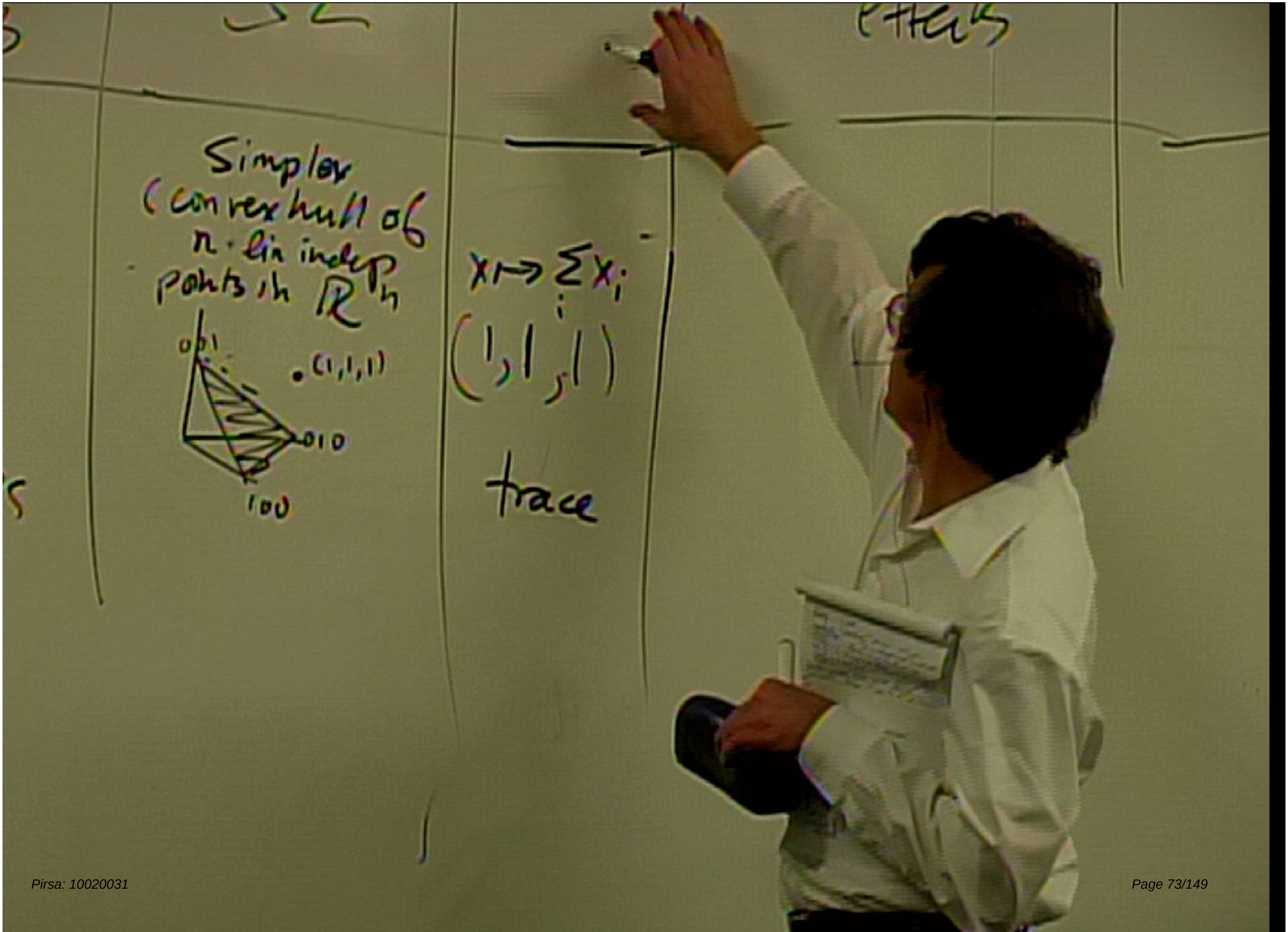

Lucien H
Inte
cone
(int)
:=
uA



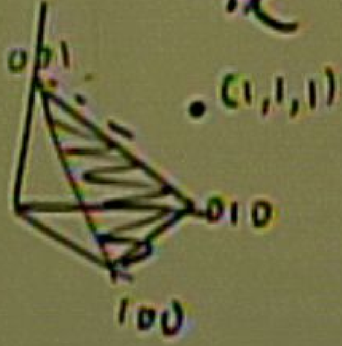
states Ω	u_A	normalized effects	extremal states	atomic effects
Simplex convex hull of n lin indep pts in $\mathbb{R}^n$	$x \mapsto \sum_i x_i$ trace			



normalized states
Ω
u/A
normalized effects
extremal states
atomic effects
Simplex
convex hull of
n lin indep
points in R^n
x ↦ Σ x_i
(1,1,1)
trace


effects
Simplex
(convex hull of
n lin indep
points in R^n
001
(1,1,1)
010
100
x ↦ Σ_i x_i
(1,1,1)
trace

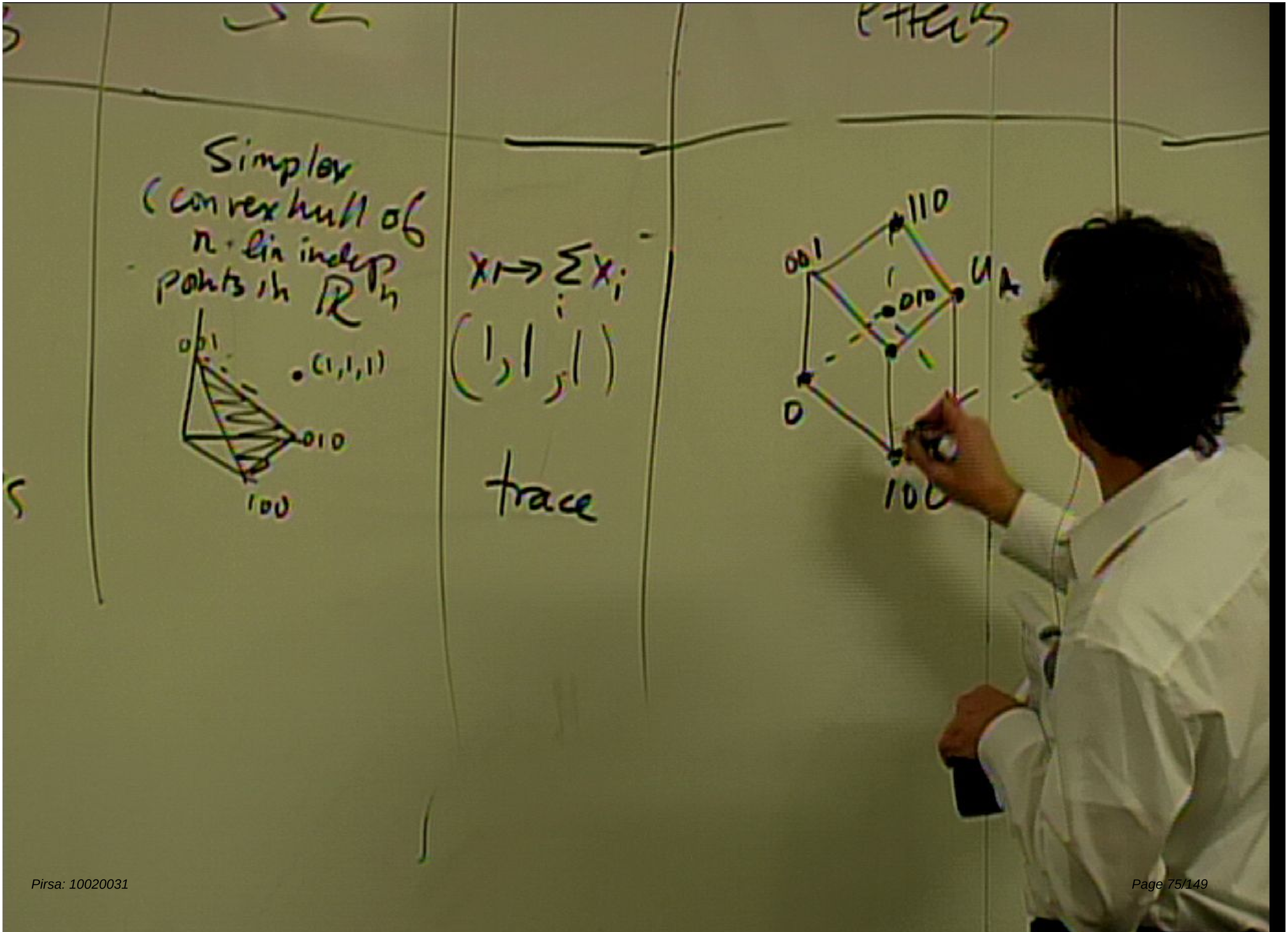
Simplex
(convex hull of
n lin indep
points in $\mathbb{R}^n$



$x \mapsto \sum_i x_i$

$(1,1,1)$

trace



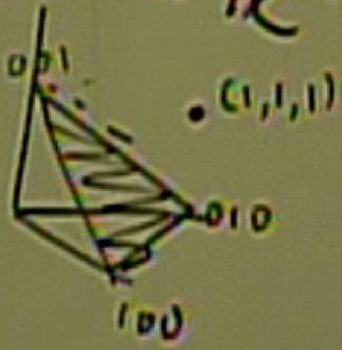
Simplex
(convex hull of
n lin indep
points in $\mathbb{R}^n$

$x \mapsto \sum_i x_i$

$(1, 1, 1)$

trace

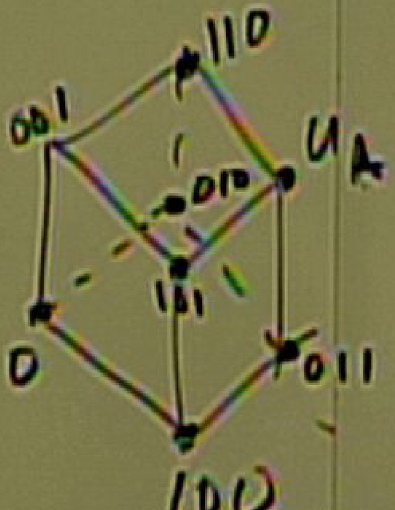
Simplex
(convex hull of
n lin indep
points in $\mathbb{R}^n$

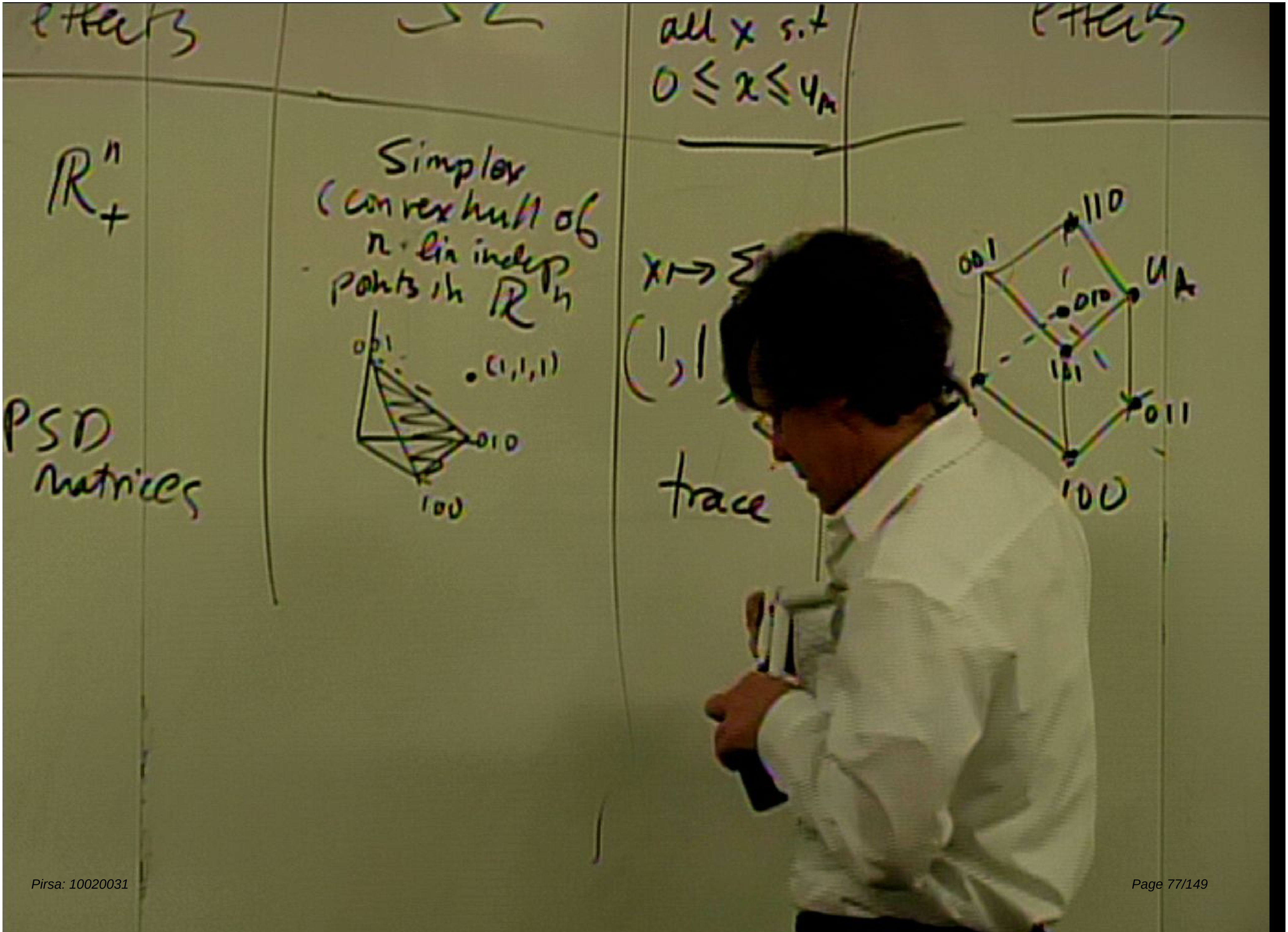


$x \mapsto \sum_i x_i$

$(1,1,1)$

trace





effects

all x s.t
$0 \leq x \leq u_A$

effects

$\mathbb{R}^n_+$

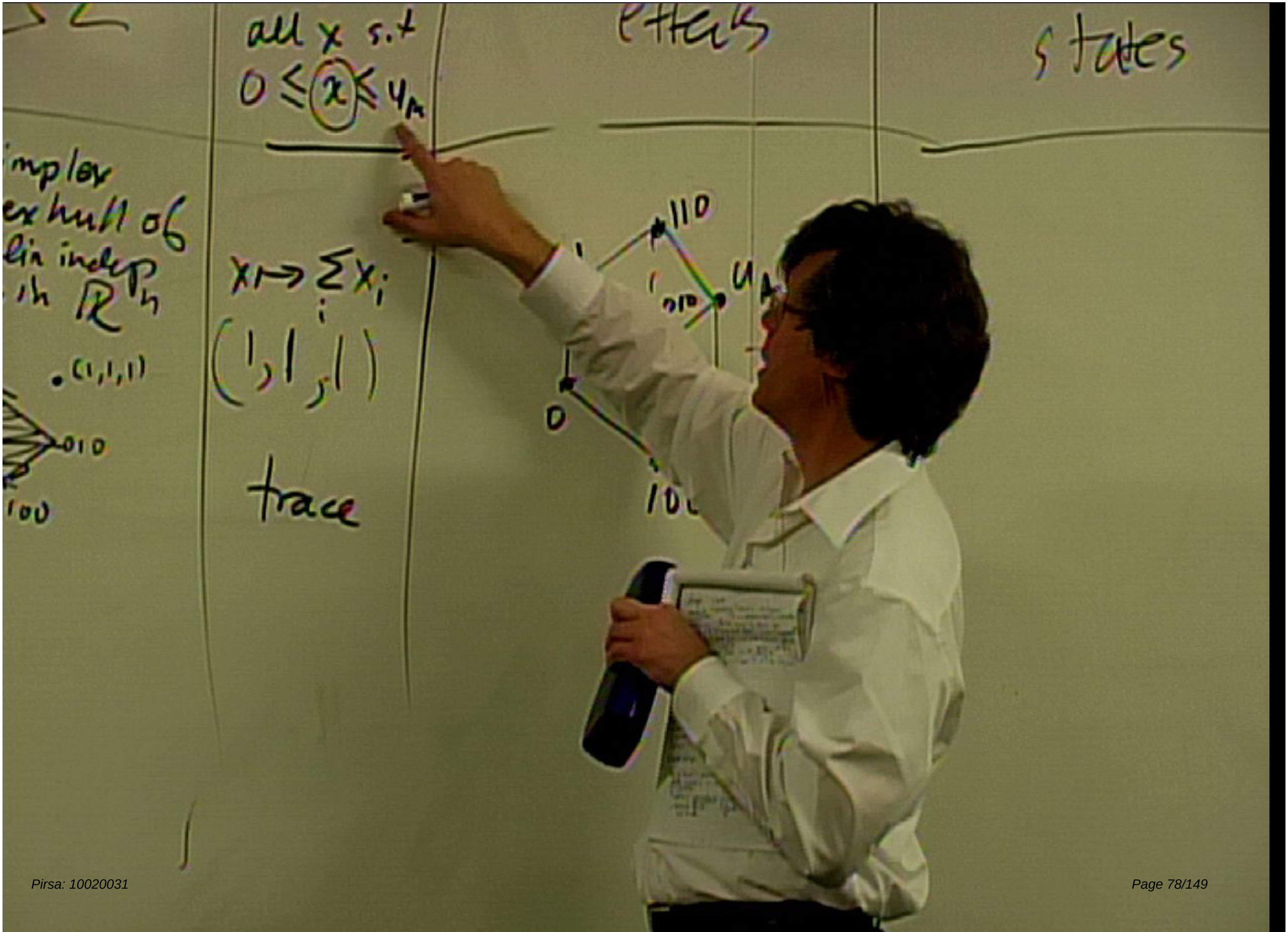
Simplex
(convex hull of
n lin indep
points in $\mathbb{R}^n$

$x \mapsto \Sigma$

(1,1

PSD
matrices

trace


all x s.t
$0 \leq x \leq u_A$
effects
states
implex
ex hull of
lin indep
in $\mathbb{R}^n$
(1,1,1)
010
100
$x \mapsto \sum_i x_i$
(1,1,1)
trace
110
u_A
010
0
100

implex
ex hull of
lin indep
in $\mathbb{R}^n$

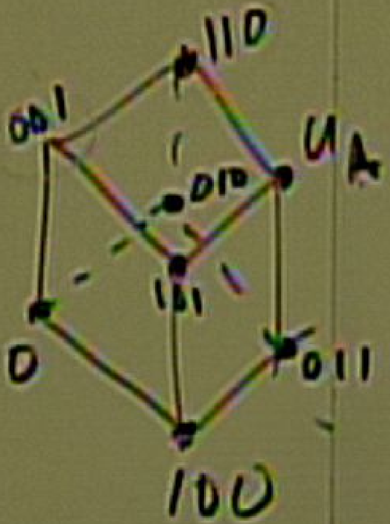
all x s.t
$0 \leq \boxed{x} \leq u_A$

$x \mapsto \sum_i x_i$

$(1, 1, 1)$

trace

effects



$x \geq y \equiv x - y \in \underset{\smile}{K}$

Λ

states

implex
ex hull of
lin indep
in $\mathbb{R}^n$

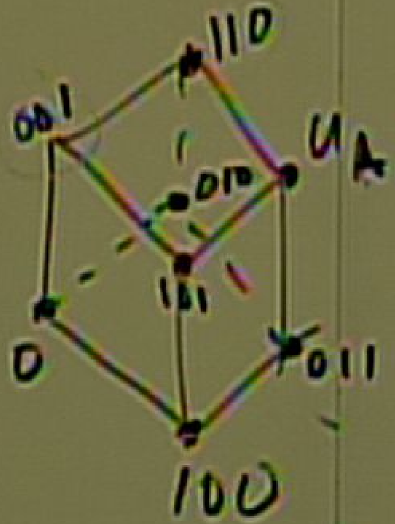
all x s.t
$0 \leq \textcircled{x} \leq u_A$

$x \mapsto \sum_i x_i$

$(1, 1, 1)$

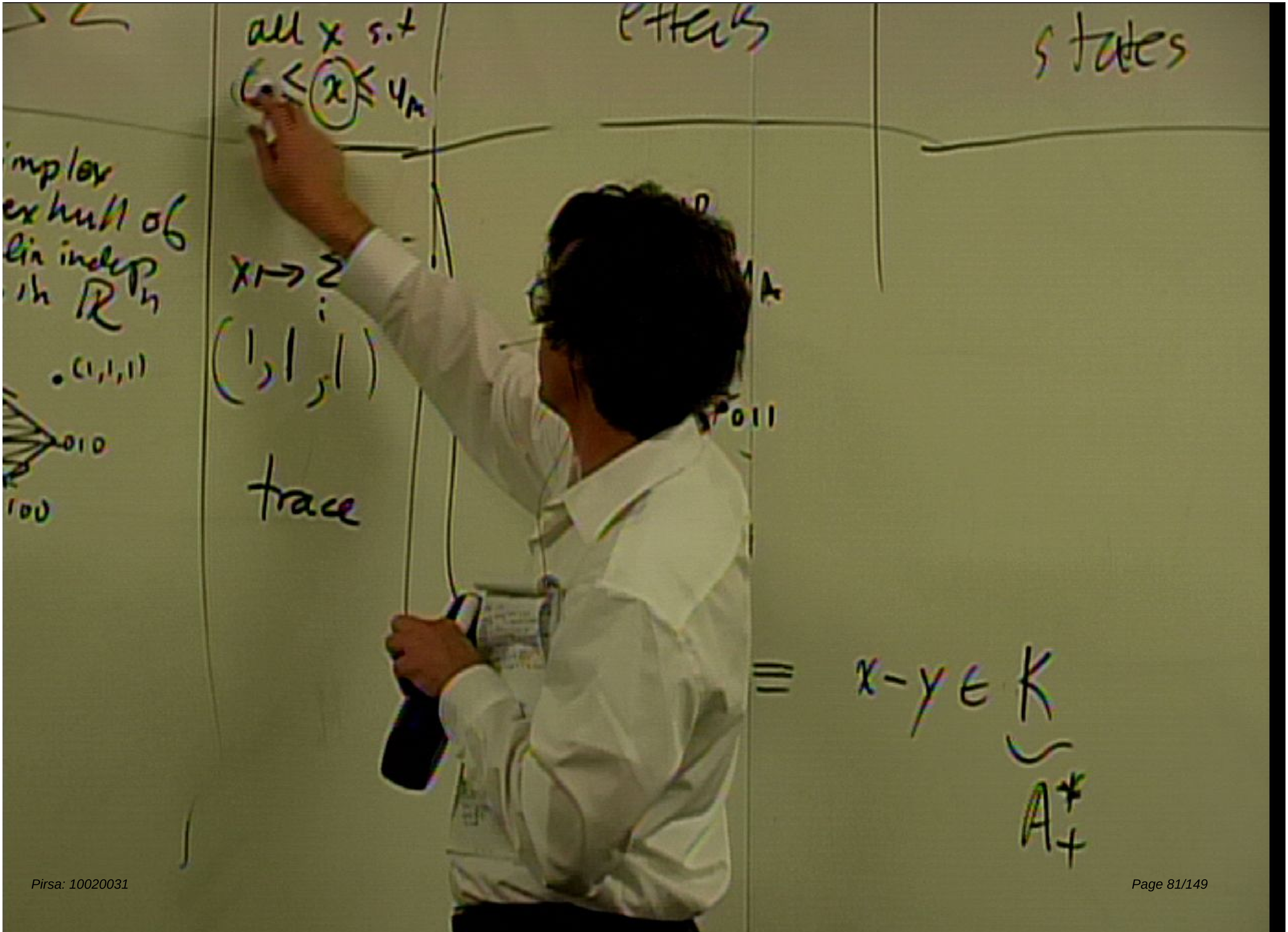
trace

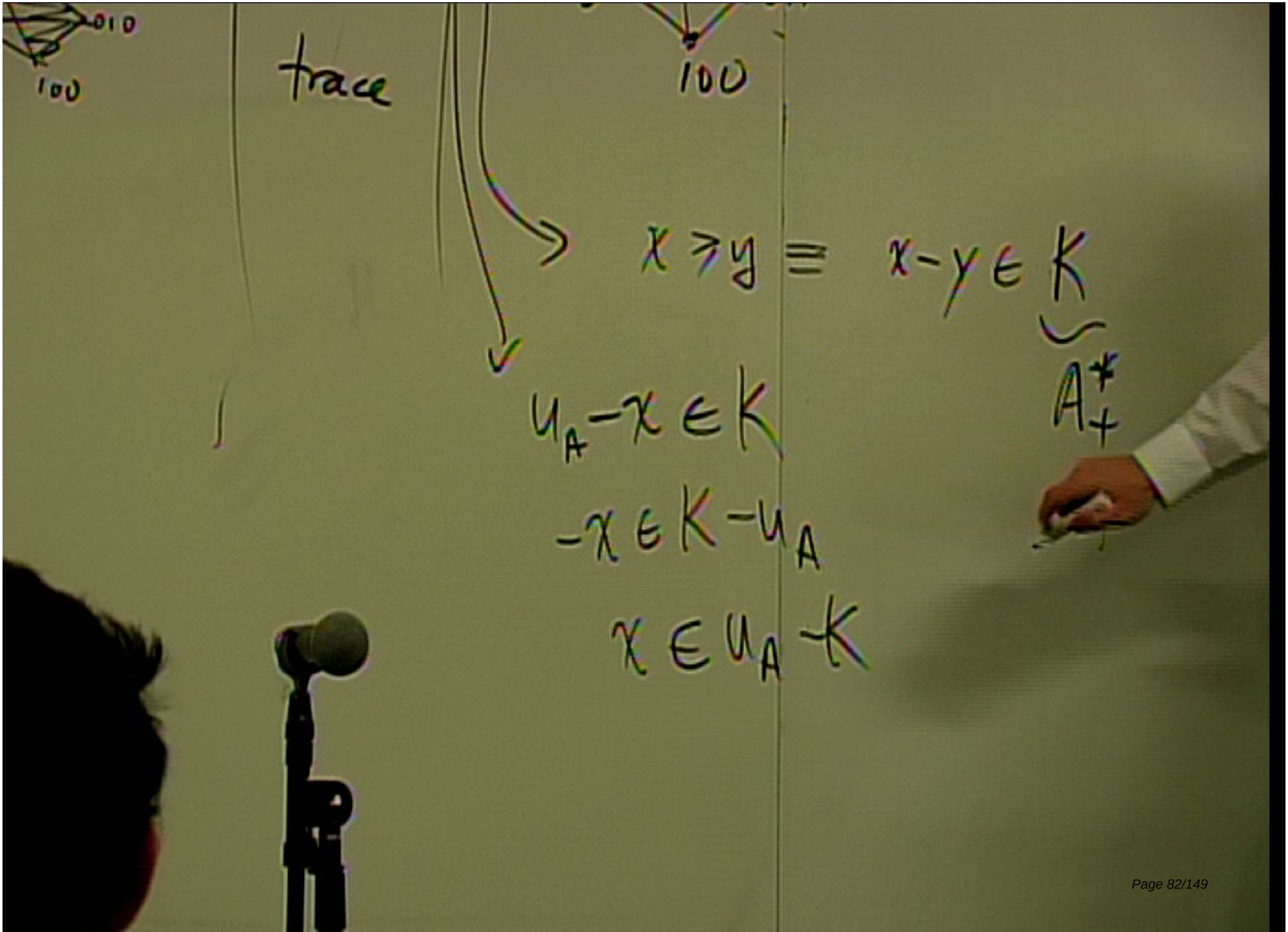
effects



states

$$x \geq y \equiv x - y \in \underbrace{K}_{A^*_+}$$


all x s.t
0 ≤ x ≤ u_A
effects
states
simplex
convex hull of
lin indep
in R^n
(1,1,1)
010
100
x ↦ Σ_i
(1,1,1)
trace
011
≡ x − y ∈ K
A^*_+

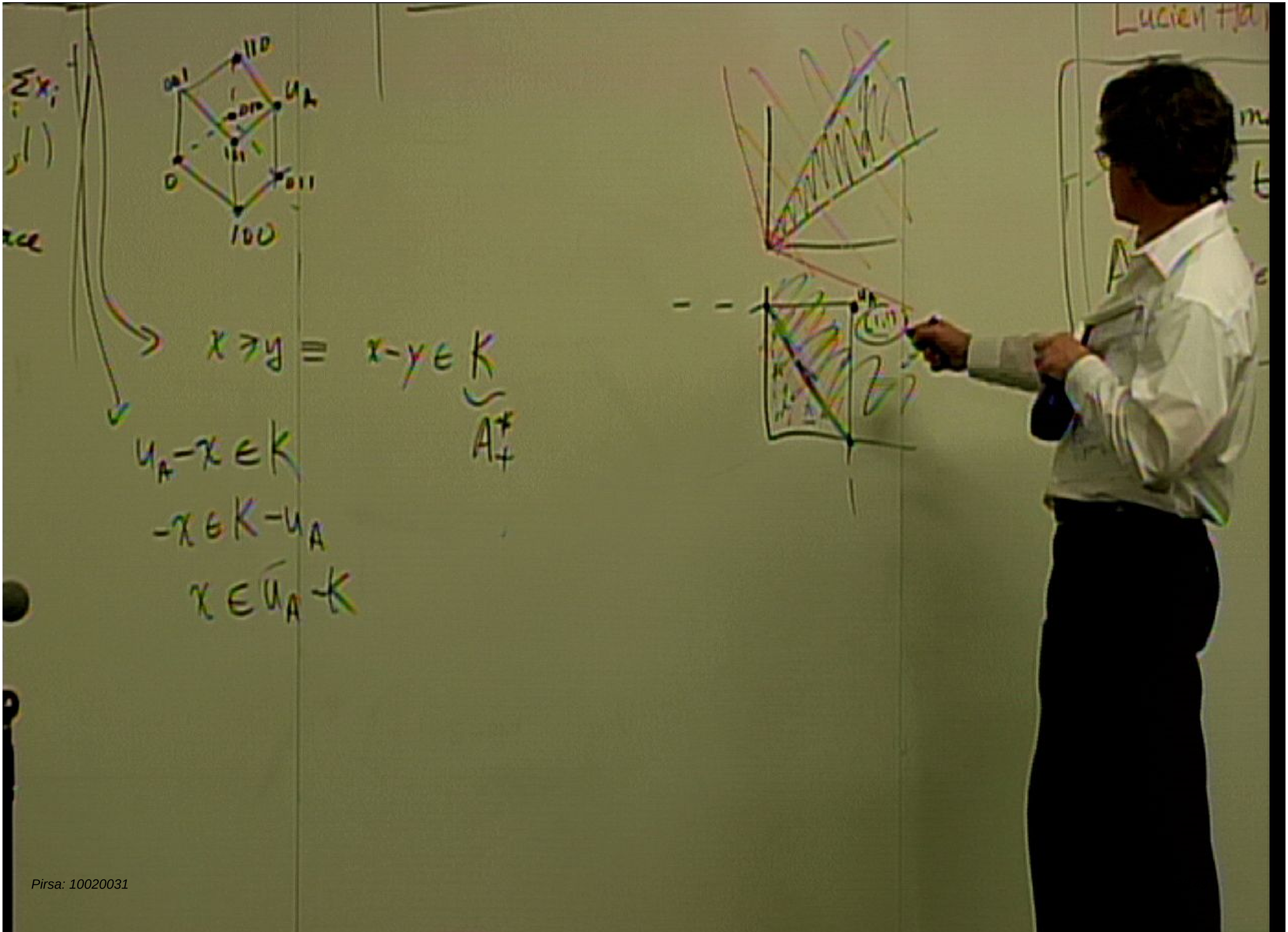


$$x \geq y \equiv x - y \in \underbrace{K}_{A^*_+}$$

$$u_A - x \in K$$

$$-x \in K - u_A$$

$$x \in u_A - K$$

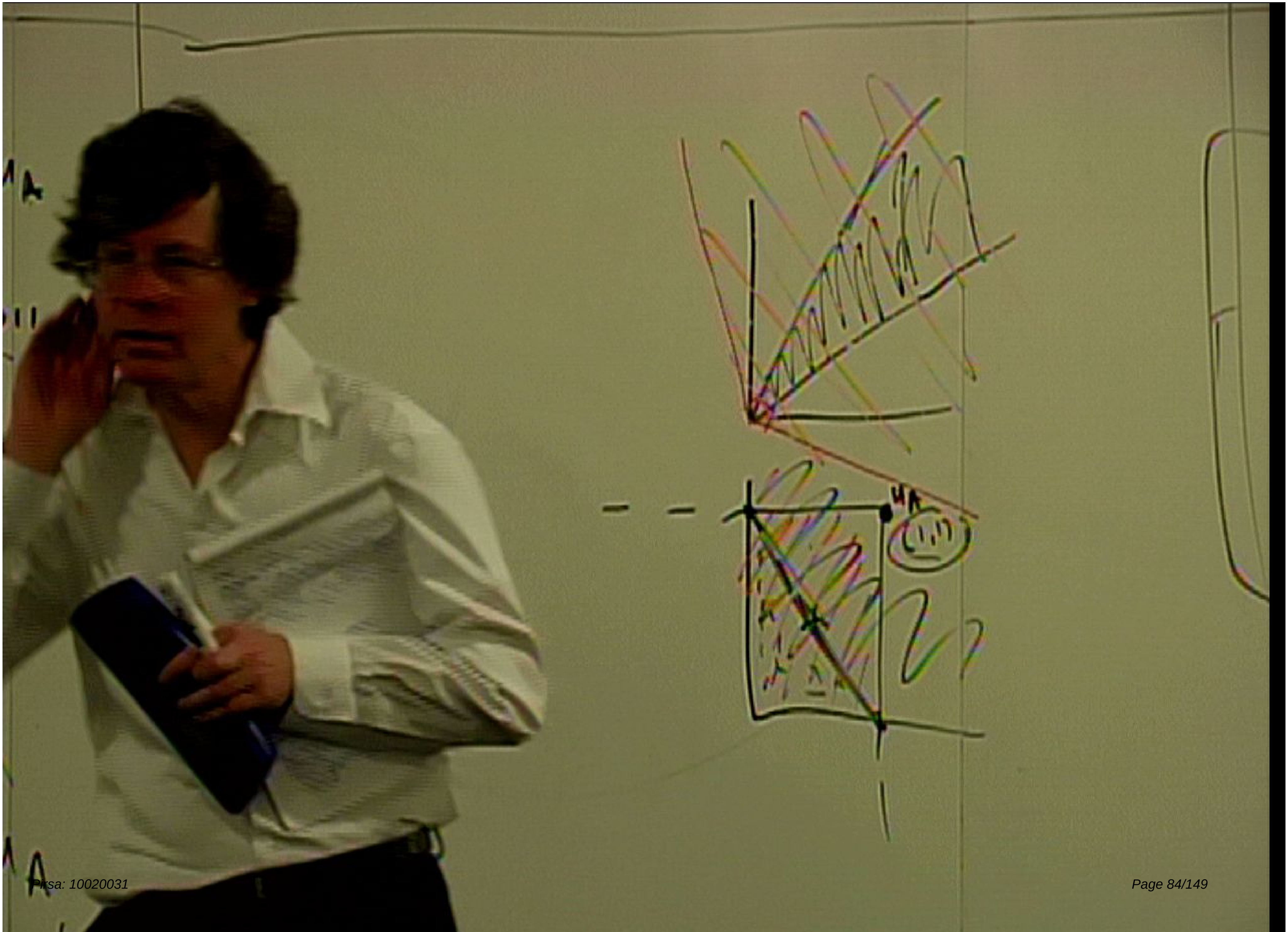


$$x \geq y \equiv x - y \in \underset{\substack{\sim \\ A^*_+}}{K}$$

$$u_A - x \in K$$

$$-x \in K - u_A$$

$$x \in u_A - K$$


uA
(1,1)

s.t

$\leq u_A$

x_i

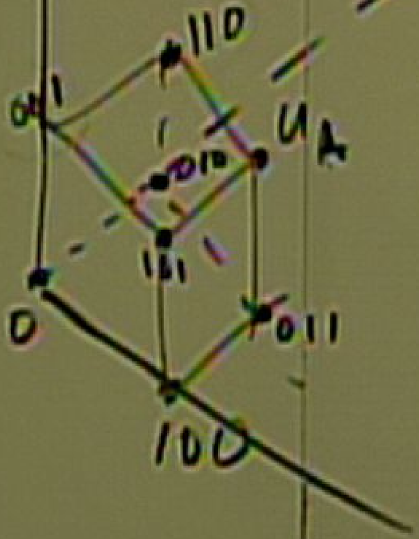
effects

extremal states

atomic effects

on extremal rays

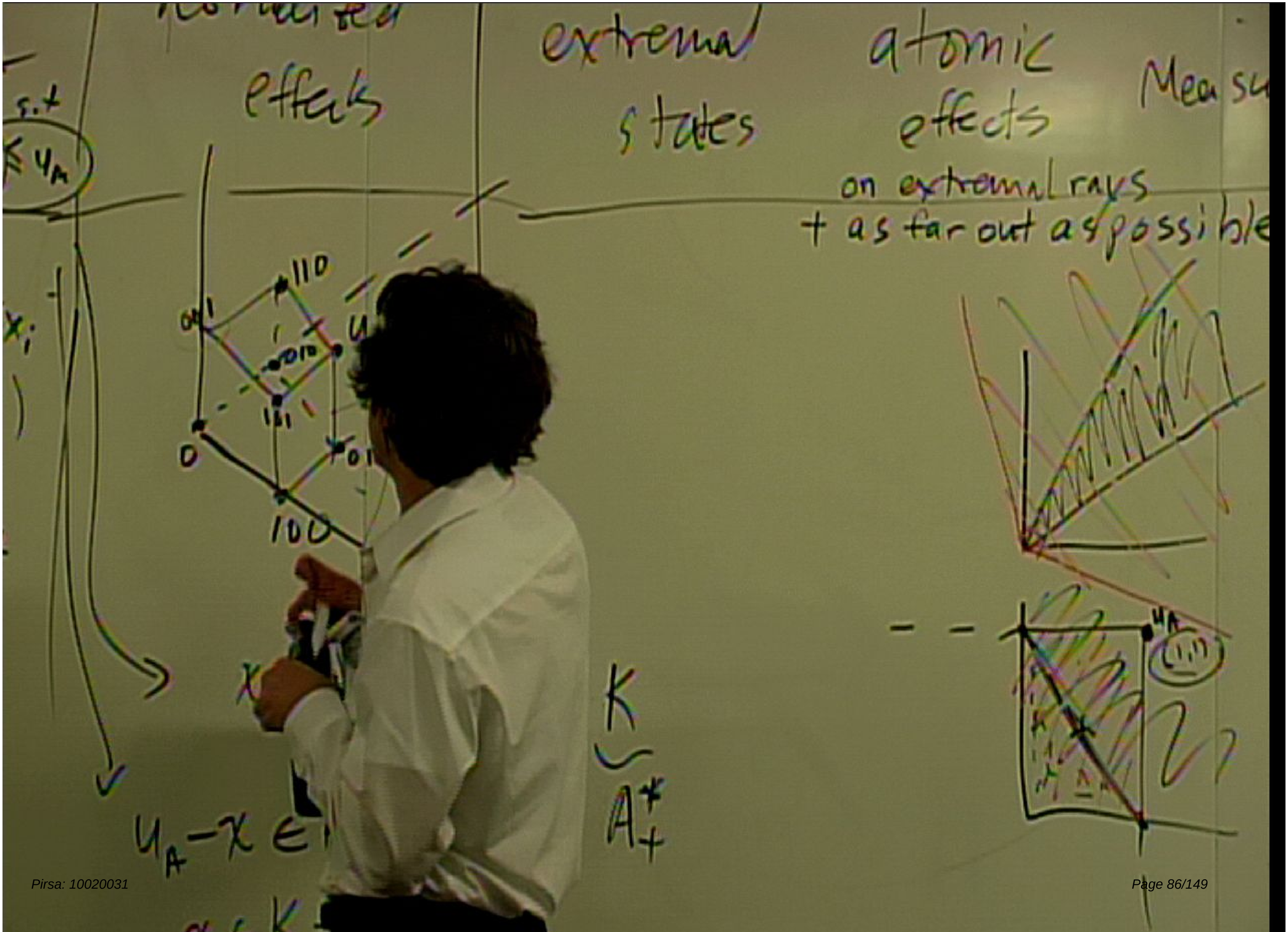
Measu

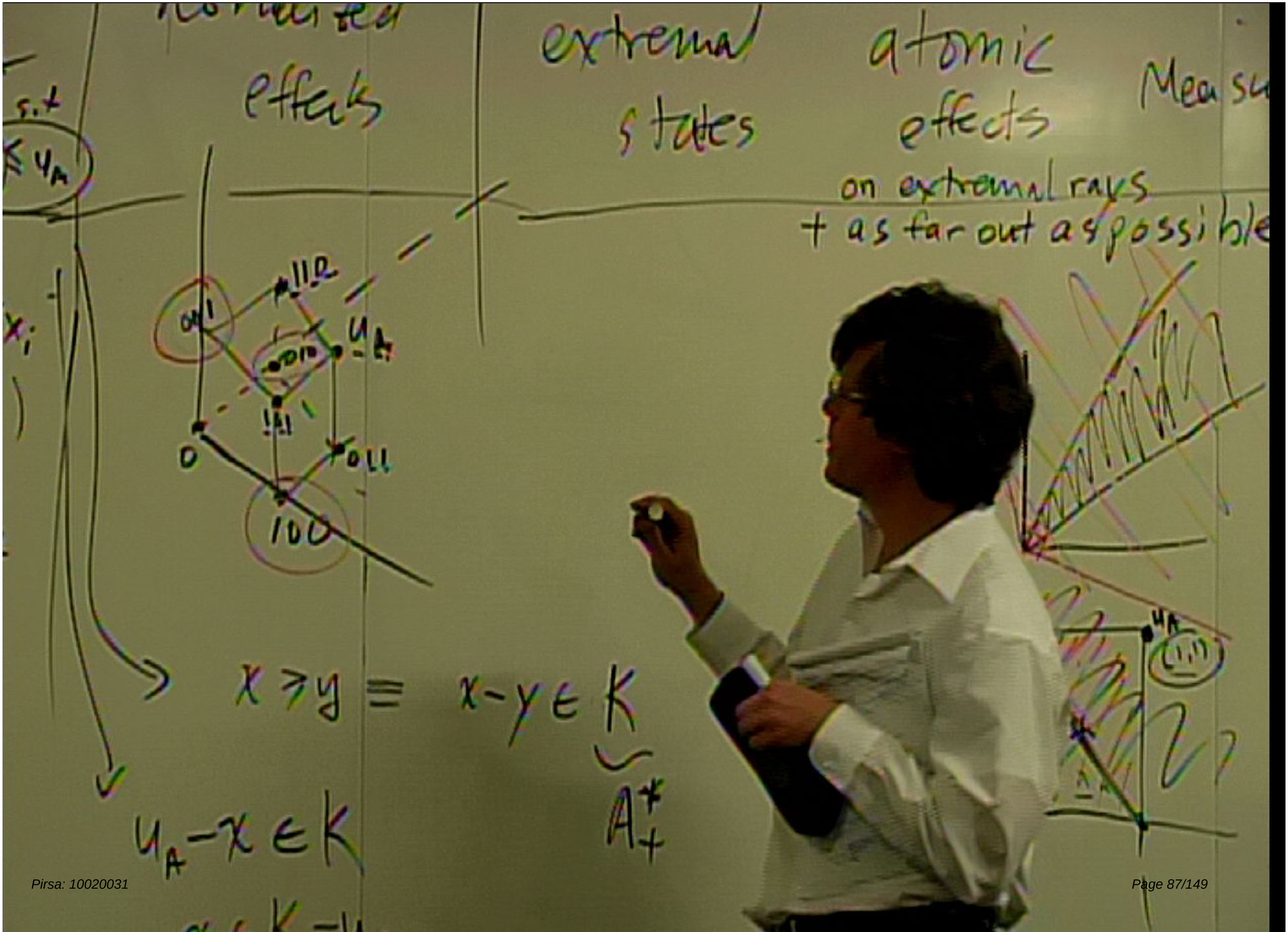


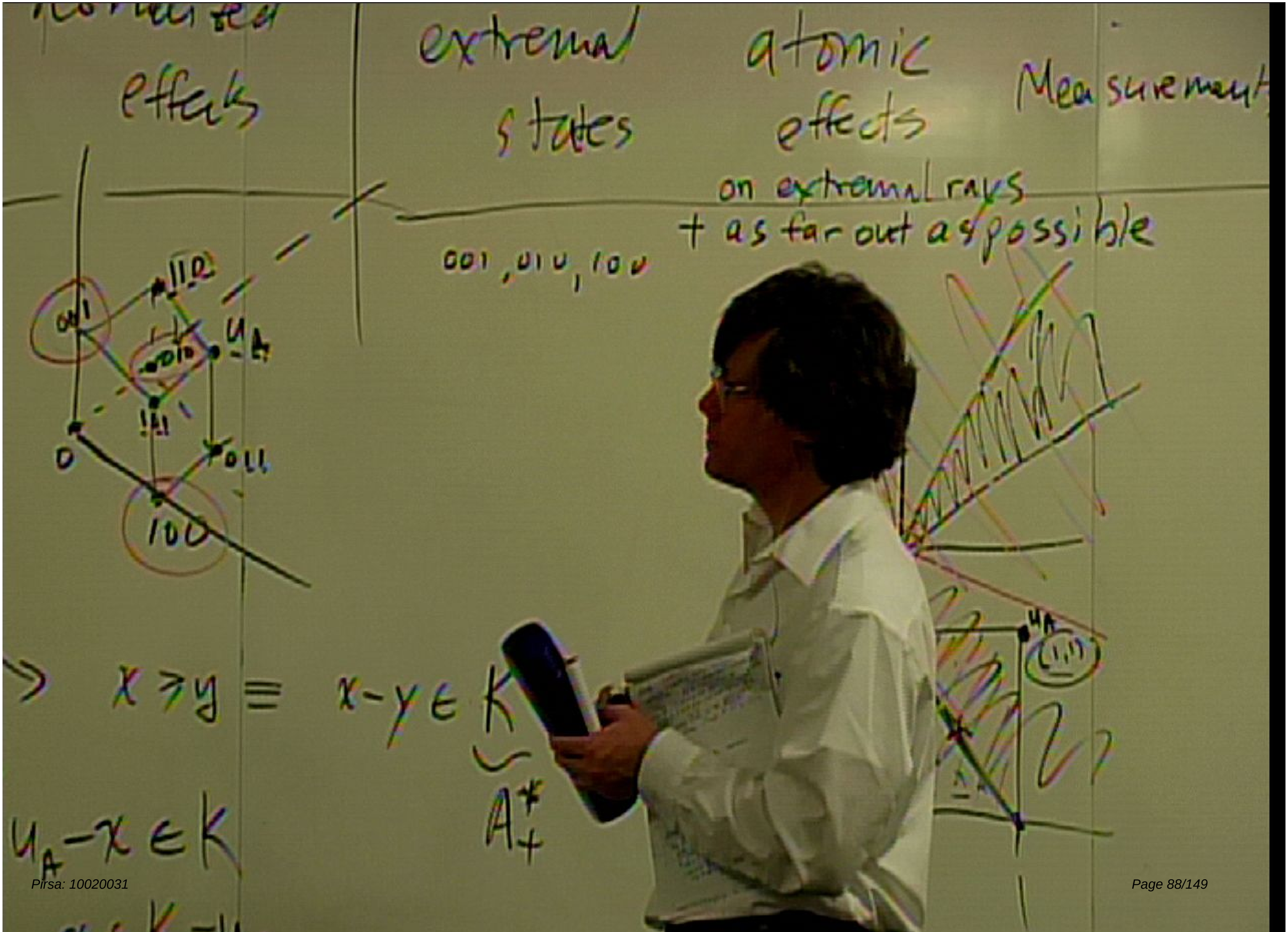
$$x \geq y \equiv x - y \in \underbrace{K}_{A_+^*}$$

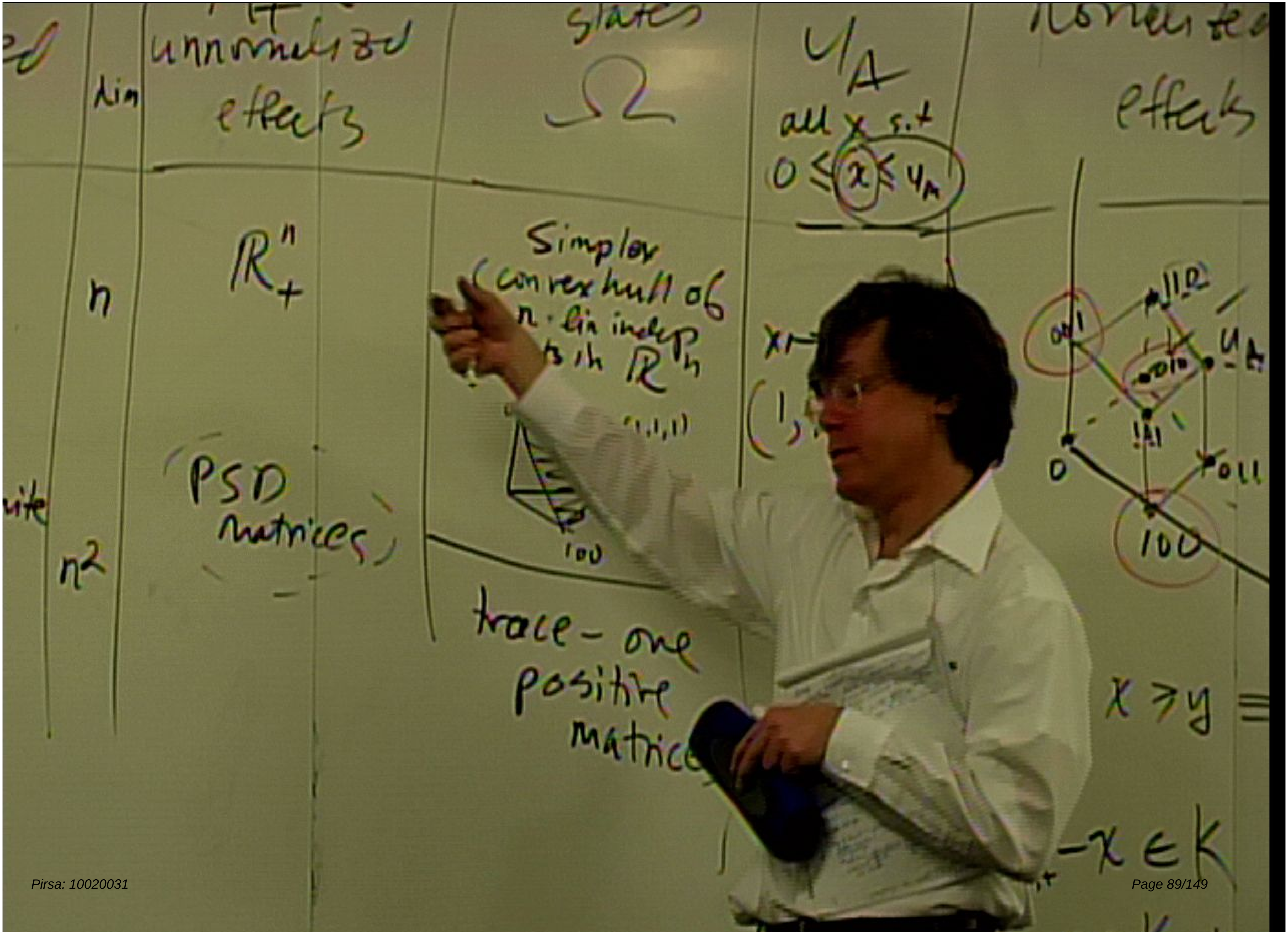
$$u_A - x \in K$$

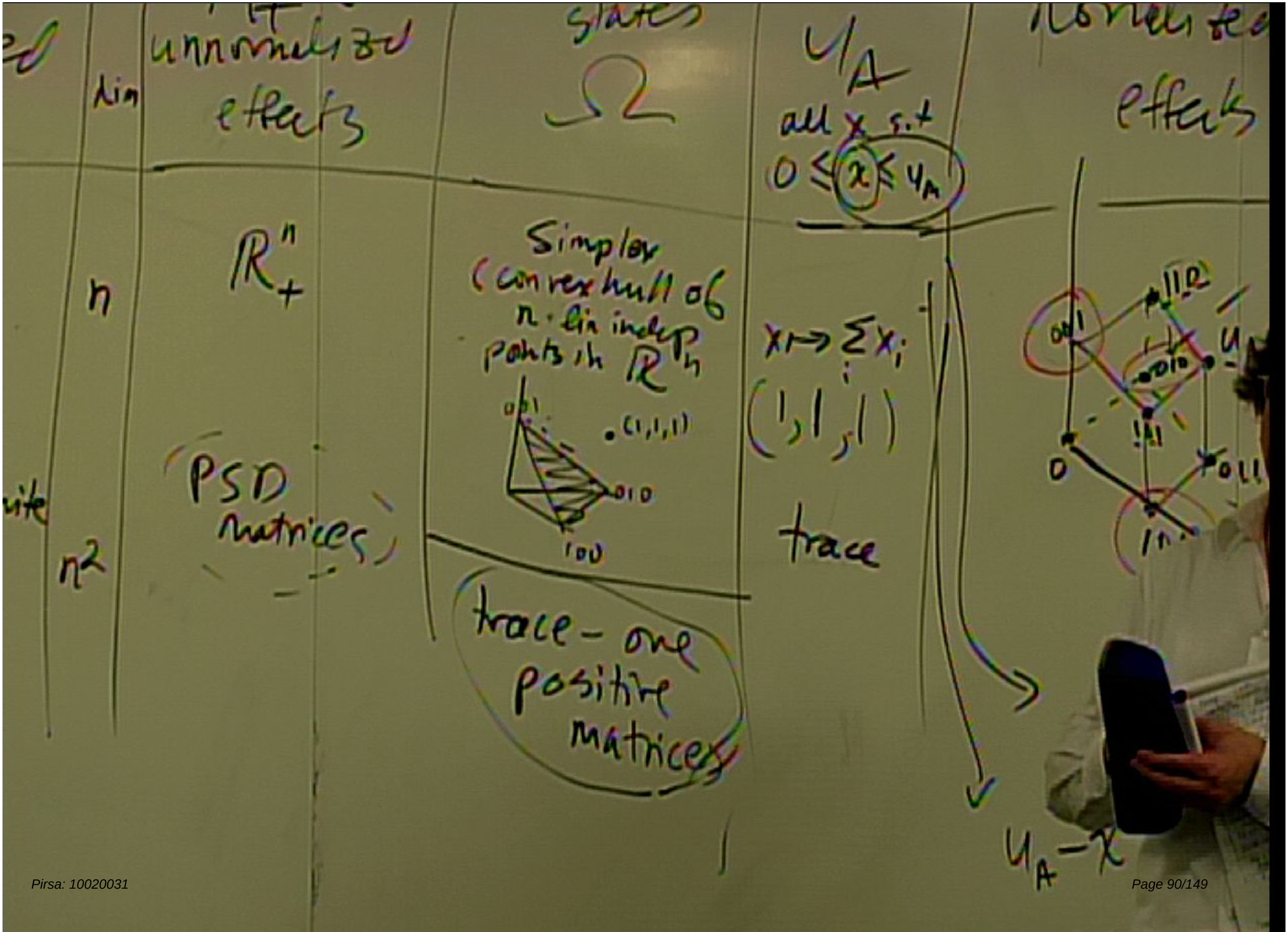
$x \in K - u$


s.t
≤ u_A
x_i
effects
110
001
010
101
0
100
x
u_A − x ∈
K
A*_+
extremal
states
atomic
effects
on extremal rays
+ as far out as possible
Measu
u_A
(1,1)


effects
extremal
states
atomic
effects
Measu
on extremal rays
+ as far out as possible
s.t
$\leq u_A$
x_i
011
110
u_A
010
101
0
011
100
$x \geq y \equiv x - y \in K$
A^*_+
$u_A - x \in K$
u_A
(1,1)


effects
extremal
states
atomic
effects
Measurement
on extremal rays
+ as far out as possible
001, 010, 100
110
001
010
u_A
111
0
011
100
$x \geq y \equiv x - y \in K$
A^*_+
$u_A - x \in K$
u_A
(1,1)


dim
unnormalized effects
states Ω
u/A all x s.t 0 ≤ x ≤ u_A
normalized effects
n
$\mathbb{R}^n_+$
Simplex (convex hull of n lin indep pts in $\mathbb{R}^n$)
(1,1,1)
100
n²
PSD matrices
trace-one positive matrices
001
110
010
u_A
111
0
011
100
x ≥ y
-x ∈ K



ed

dim

unnormalized effects

states

Ω

u_A

all x s.t. $0 \leq x \leq u_A$

normalized

effects

n

$\mathbb{R}^n_+$

Simplex (convex hull of n lin indep points in $\mathbb{R}^n$)

$x \mapsto \sum_i x_i$

$(1,1,1)$

ite

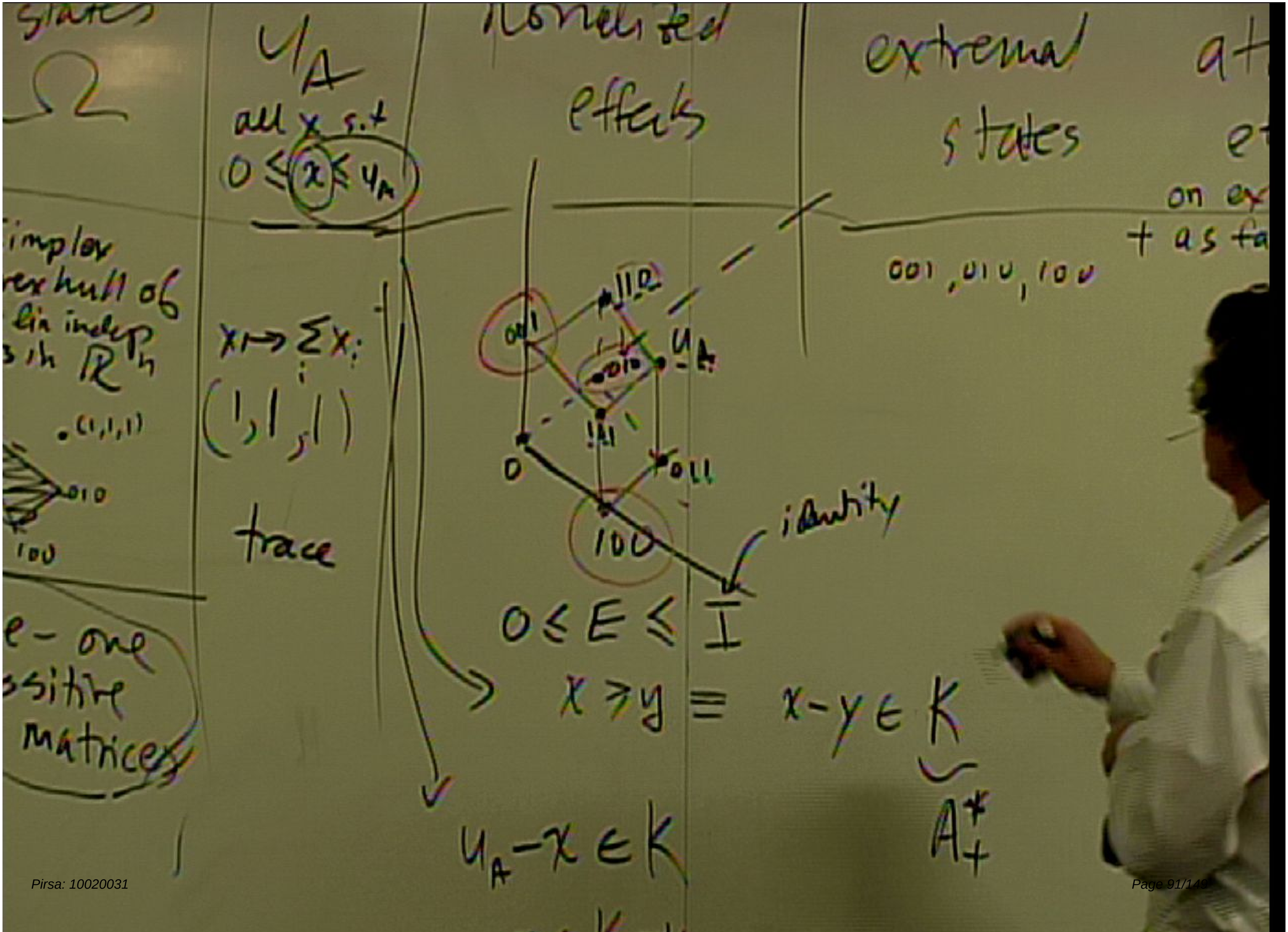
n^2

PSD matrices

trace-one positive matrices

trace

$u_A - x$

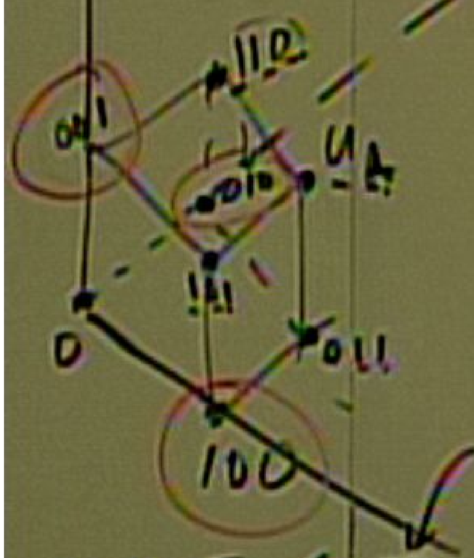


$0 \leq E \leq I$

$x \geq y \equiv x - y \in \underbrace{K}_{A^*_+}$

$u_A - x \in K$

normalized effects



$0 \le E \le I$

$\Rightarrow \quad x \geq y \equiv x - y \in \underbrace{K}_{A_+^*}$

$u_A - x \in K$

extremal states

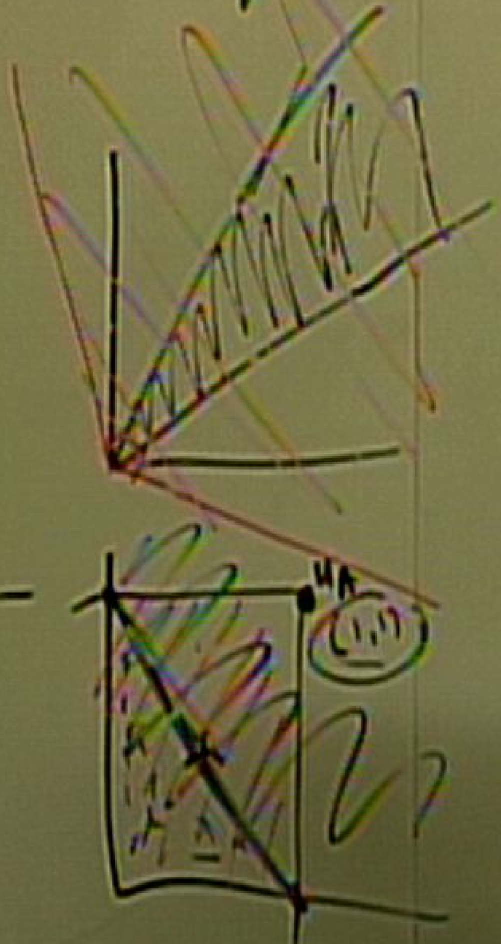
001, 010, 100

rank-one density matrices (projectors of rank-1)

atomic effects

on extremal rays + as far out as possible

Measurements



$\{\sum_i t_i x_i : x_i \in$

(convex hull)

$\sum_i t_i = 1$

$t_i \geq 0$

Internal dual cone to A_+

$A_+^{*(int)} := \{ y \in A : \forall x \in A_+, \langle y, x \rangle \geq 0 \}$

basis e_i

A

$A^* \{ f : A \to \mathbb{R} \}$

$\downarrow$

$\vec{f} \in A$

$= \sum_i f_i e_i$

$x \in A \mapsto \sum_i f_i x_i \in \mathbb{R}$

Quantum: $\langle x, y \rangle = \operatorname{tr} x y$

density matrix

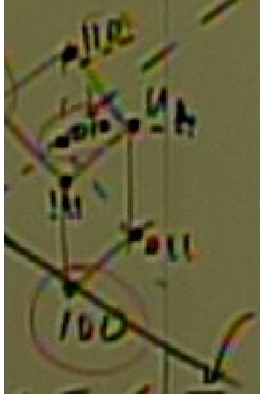
POVM element

$\left(\operatorname{tr} |\psi\rangle\langle\psi| Y = \langle\psi| Y |\psi\rangle \right)$

inner product on A:

$\langle \, , \, \rangle : A \times A \to \mathbb{R}$

...malized effects



$\leq E \leq I$

$x \geq y \equiv x - y \in \underset{\sim}{K}$

A_+^*

$x \in K$

$\in K - u_A$

$x \in u_A - K$

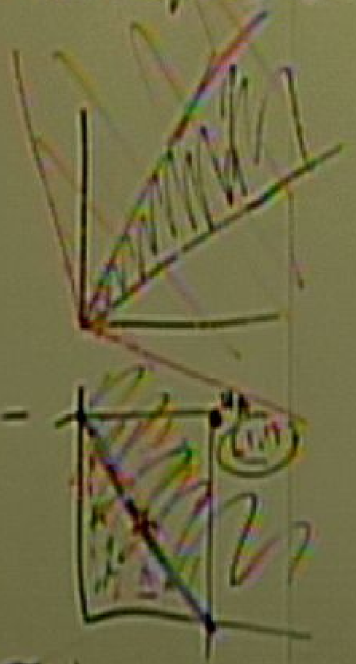
extremal states

001, 010, 100

rank-one density matrices (projectors of rank-1)

atomic effects

on extremal rays + as far out as possible



rank-one projectors

Measurements

Dynamics

$\vec{F}$ vector (eff...

Lucien Hardy $\vec{p}$ atom (state...

Internal dual cone to A_+

$A_+^{*(int)} := \{ y \in A : \forall x \in A_+, \langle y, x \rangle \geq$ ...

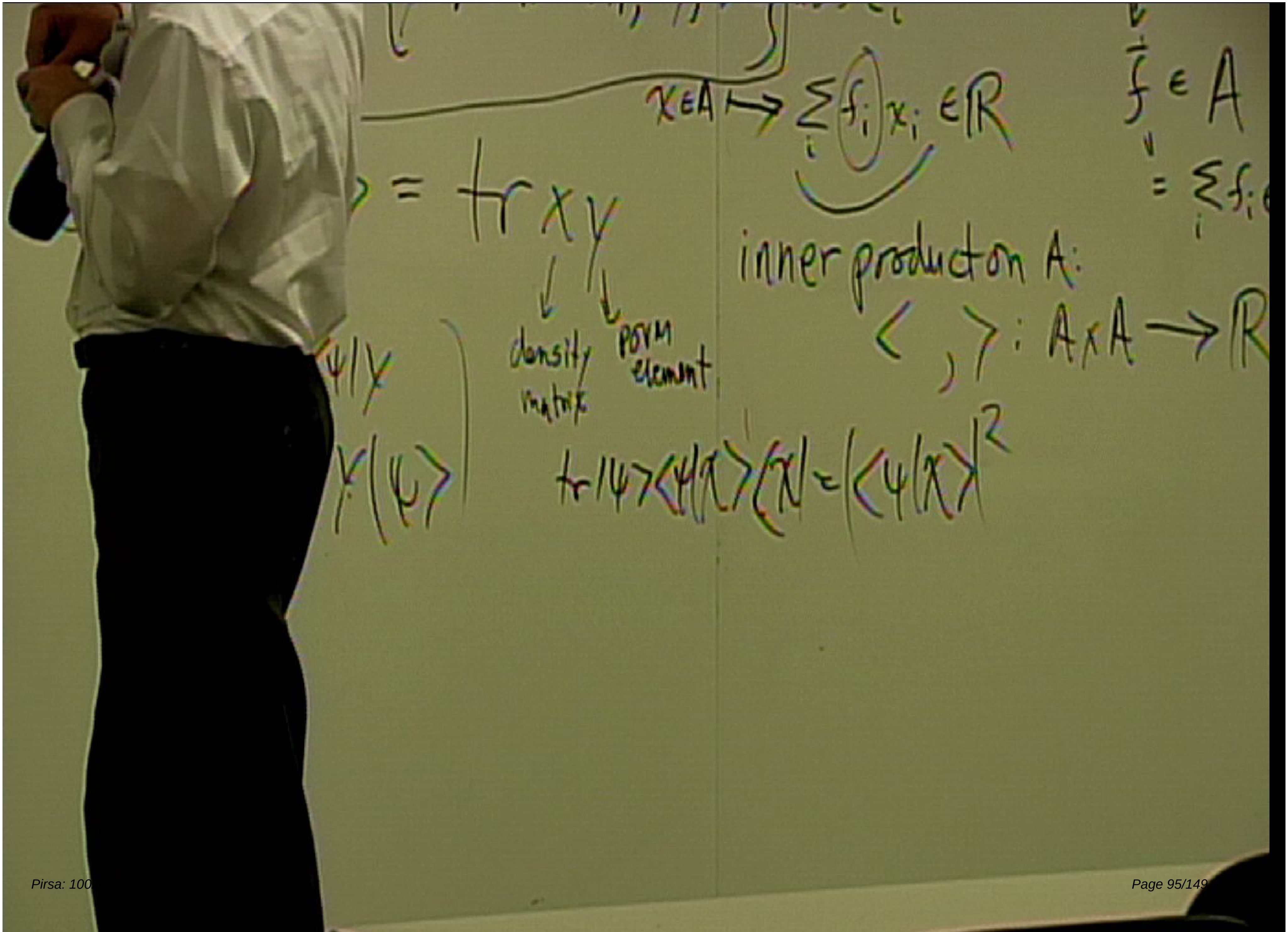
$x \in A \mapsto$

Quantum: $\langle x, y \rangle = \operatorname{tr} xy$

density matrix

POVM element

$\left(\operatorname{tr} |\psi\rangle\langle\psi| Y = \langle\psi|Y|\psi\rangle \right)$



$x \in A \mapsto \sum_i \boxed{f_i}\, x_i \in \mathbb{R}$

$\bar{f} \in A$

$= \sum_i f_i$

$\rangle = \operatorname{tr} x y$

density matrix

POVM element

inner product on A:

$\langle\ ,\ \rangle : A \times A \to \mathbb{R}$

$\psi | y$

$y | \psi \rangle$

$\operatorname{tr} |\psi\rangle\langle\psi|\lambda\rangle\langle\lambda| = |\langle\psi|\lambda\rangle|^2$

$$x \in A \mapsto \underbrace{\sum_i \boxed{f_i}\, x_i}\in \mathbb{R}$$

$$\vec{f} \in A$$

$$\vee$$

$$= \sum_i f_i$$

$$\text{Quantum: } \langle x, y\rangle = \operatorname{tr} x y$$

$x \downarrow$ density matrix

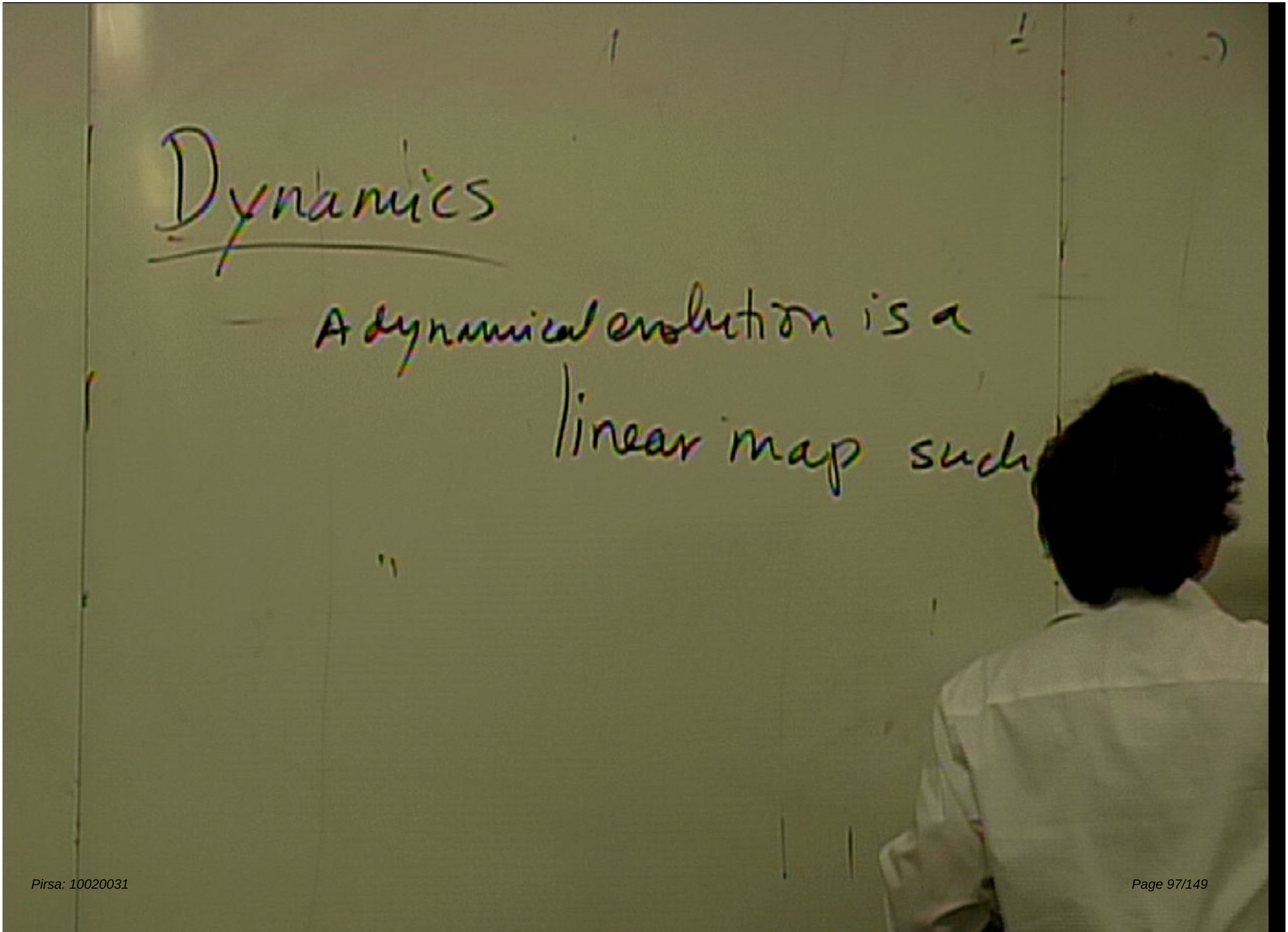
$y \downarrow$ POVM element

$$\left(\operatorname{tr} |\psi\rangle\langle\psi| y = \langle\psi| y |\psi\rangle \right)$$

inner product on A:

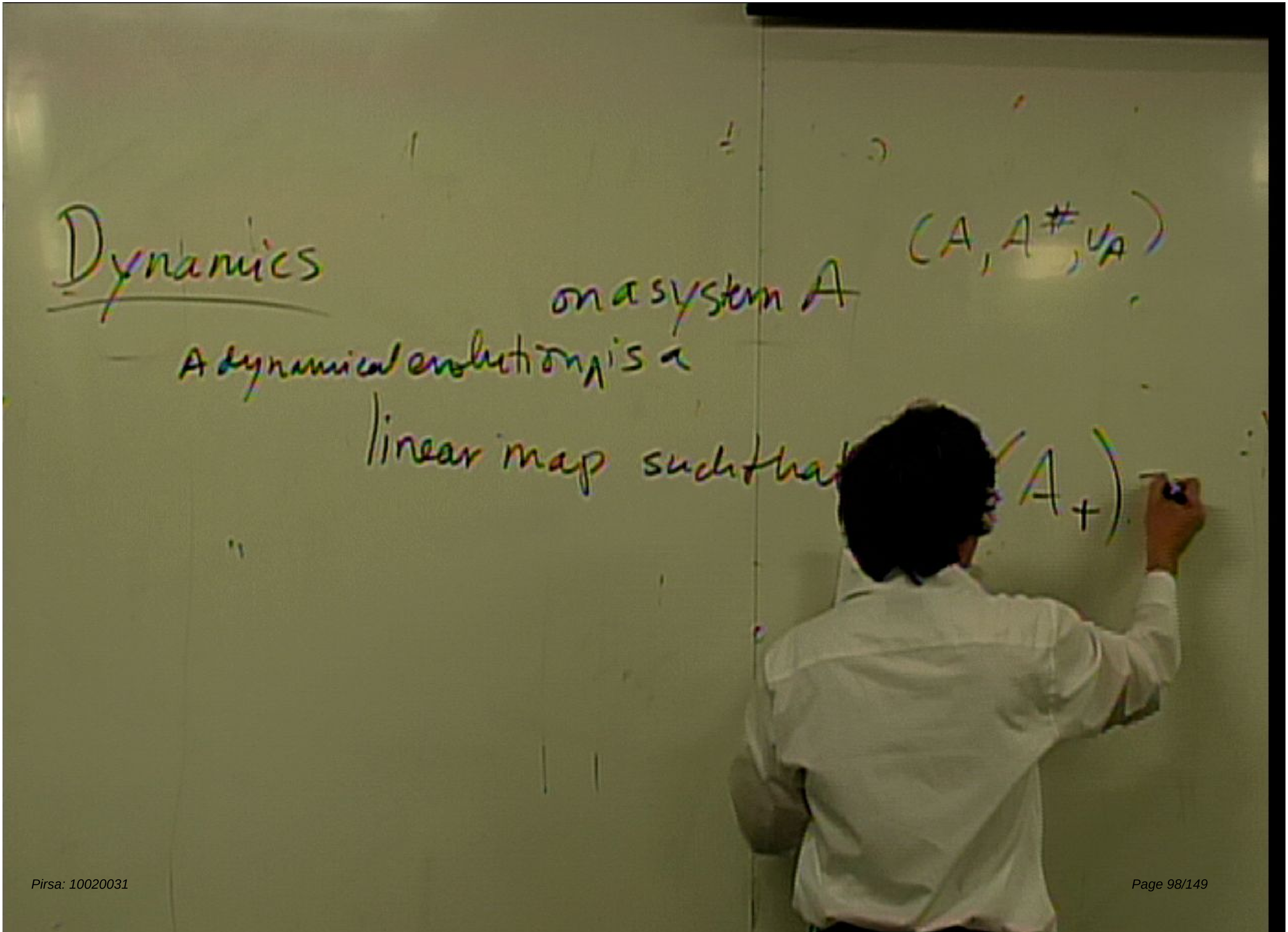
$$\langle \ ,\ \rangle : A \times A \to \mathbb{R}$$

$$\operatorname{tr} |\psi\rangle\langle\psi|\lambda\rangle\langle\lambda| = |\langle\psi|\lambda\rangle|^2$$



Dynamics

A dynamical evolution is a linear map such

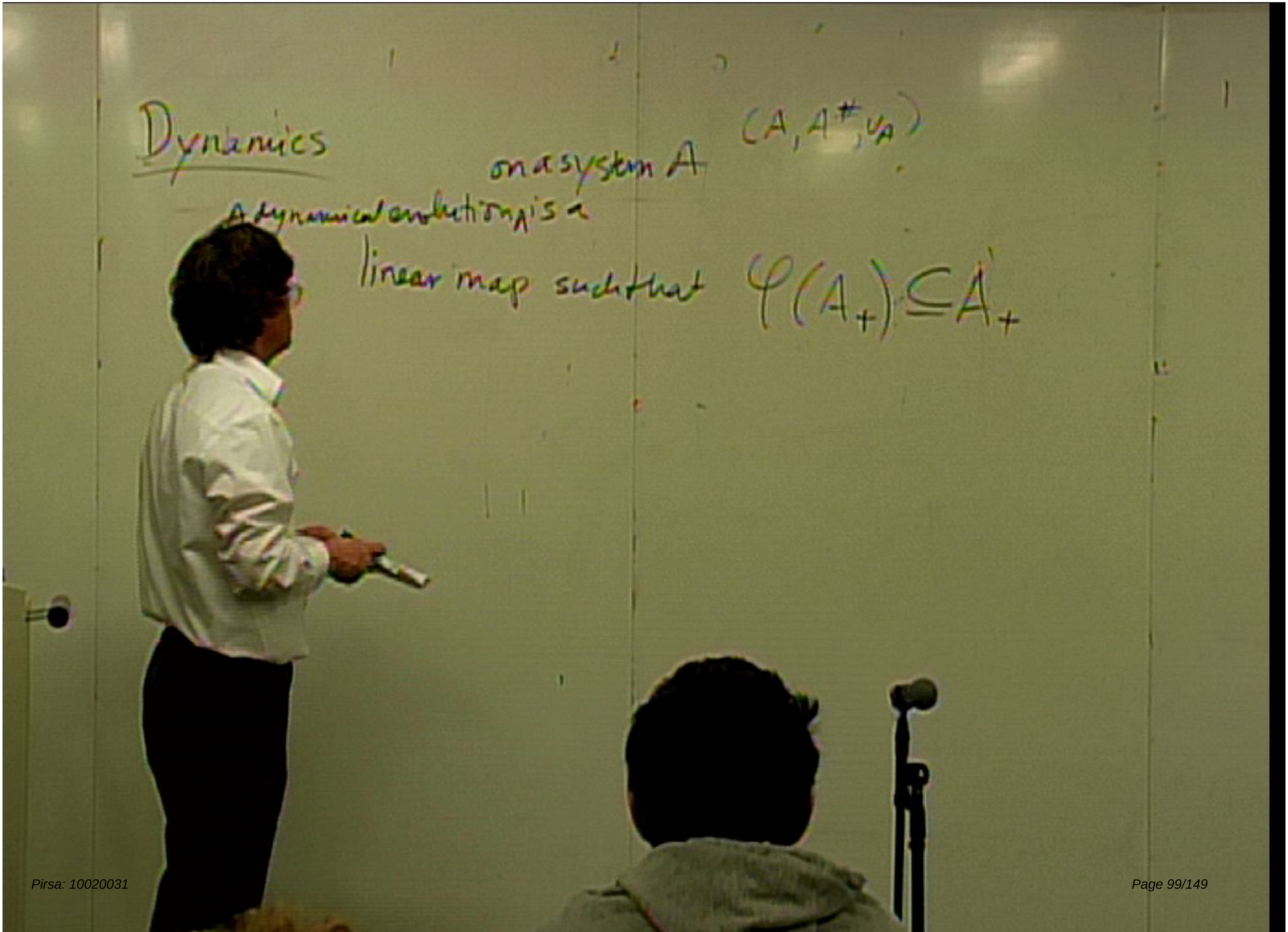


Dynamics

on a system A $(A, A^{\#}, u_A)$

A dynamical evolution is a

linear map such that (A_+)

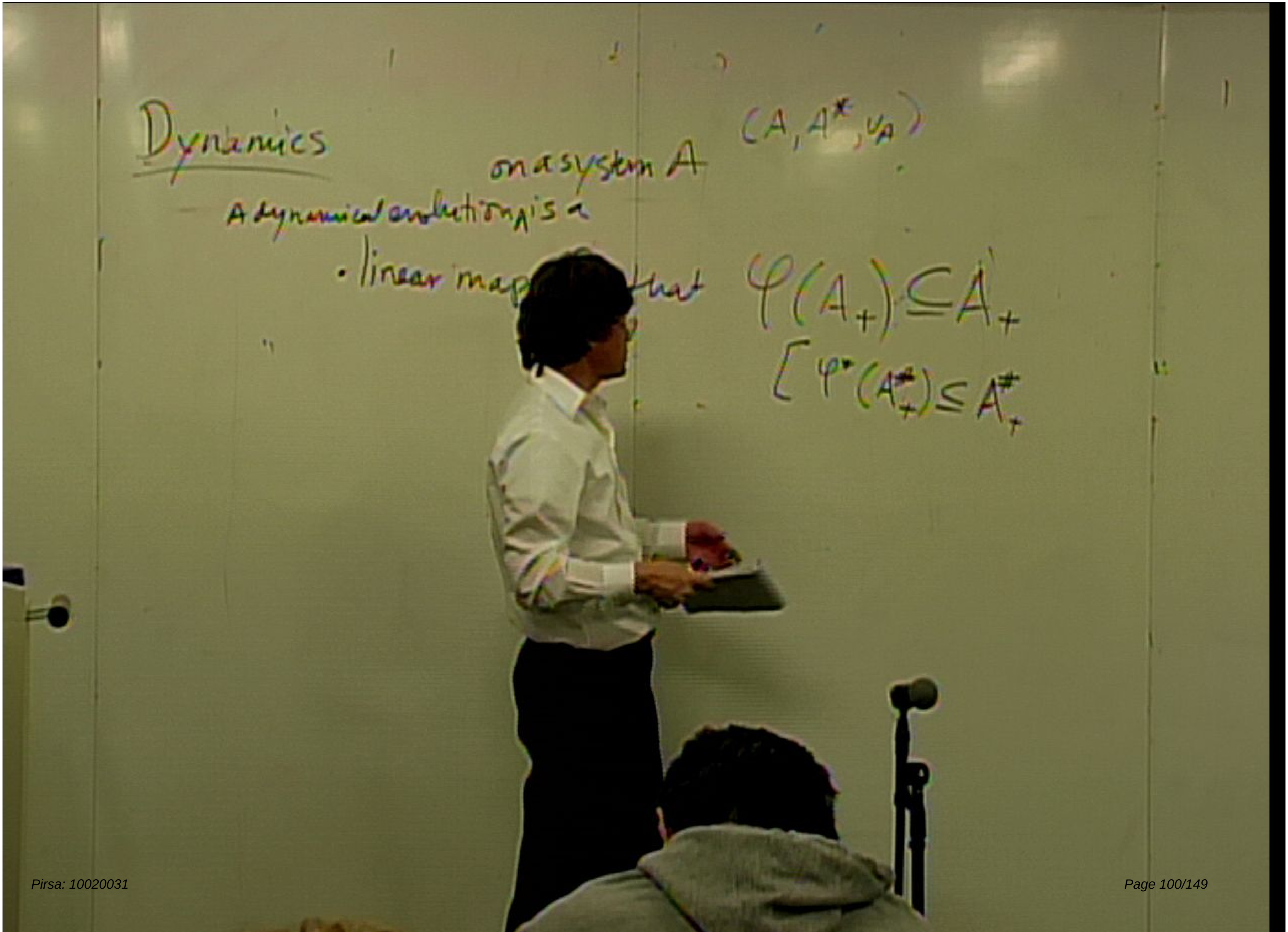


Dynamics

on a system A $(A, A^{\#}, u_A)$

A dynamical evolution$_A$ is a

linear map such that $\varphi(A_+) \subseteq A_+$

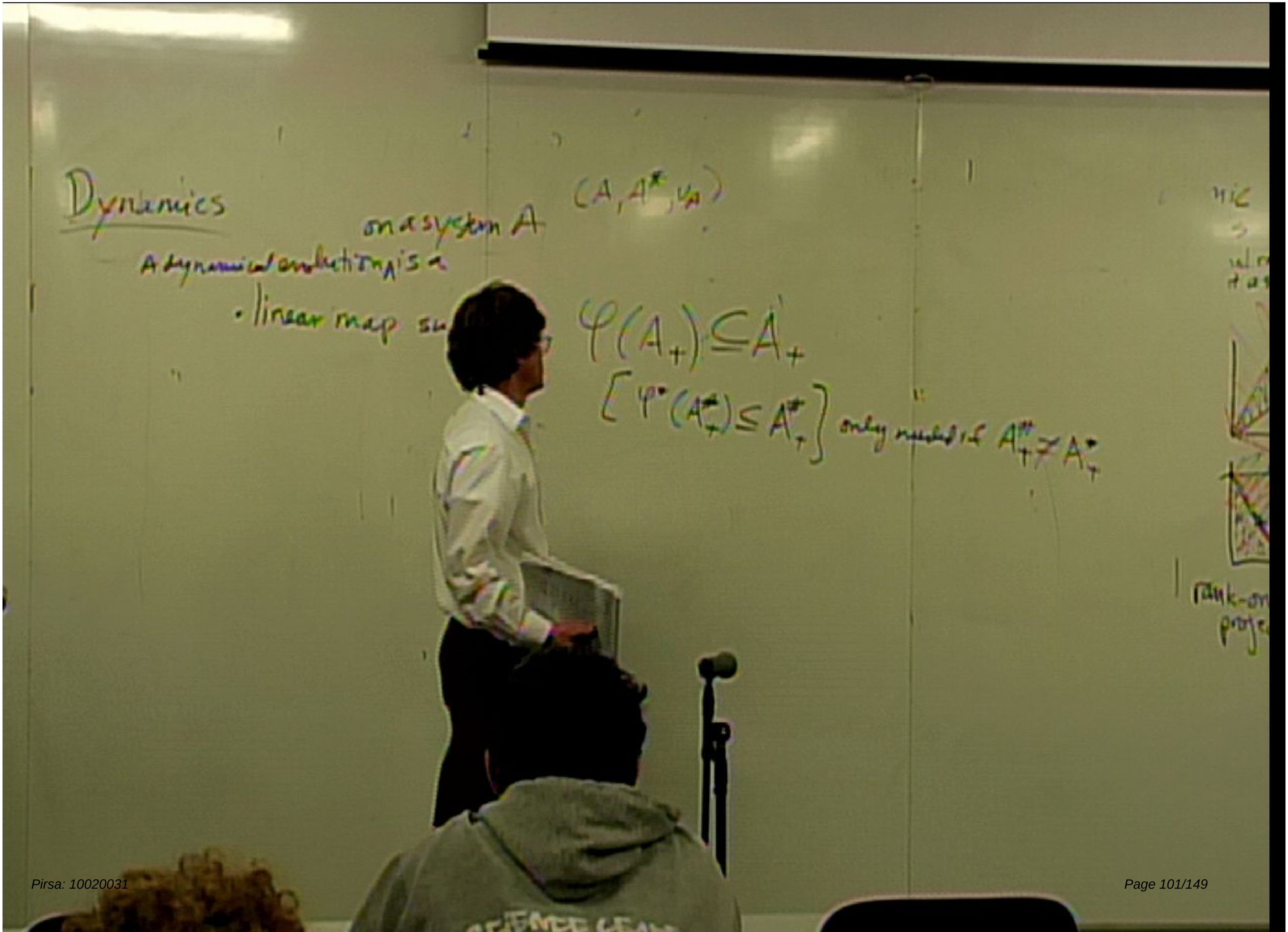


Dynamics on a system A (A, A^*, u_A)

A dynamical evolution$_A$ is a

- linear map [illegible] that $\varphi(A_+) \subseteq A_+$

  $[\ \varphi^*(A^*_+) \subseteq A^*_+$



Dynamics

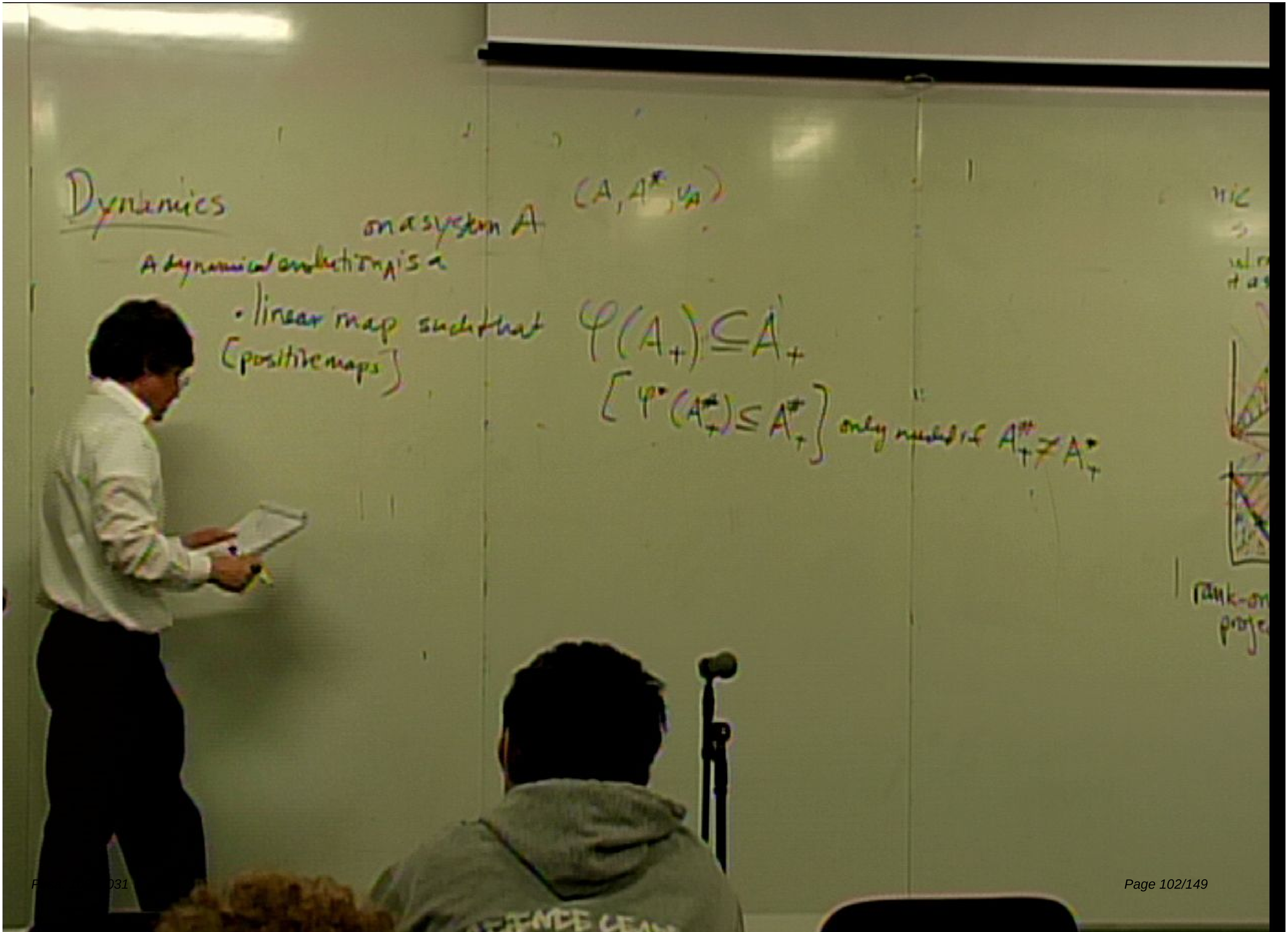
on a system A (A, A^*, u_A)

A dynamical evolution T_A is a

- linear map su

$$\varphi(A_+) \subseteq A_+$$

$$\left[\varphi^*(A^*_+) \le A^*_+\right]$$ only needed if $A^{**}_+ \neq A^*_+$

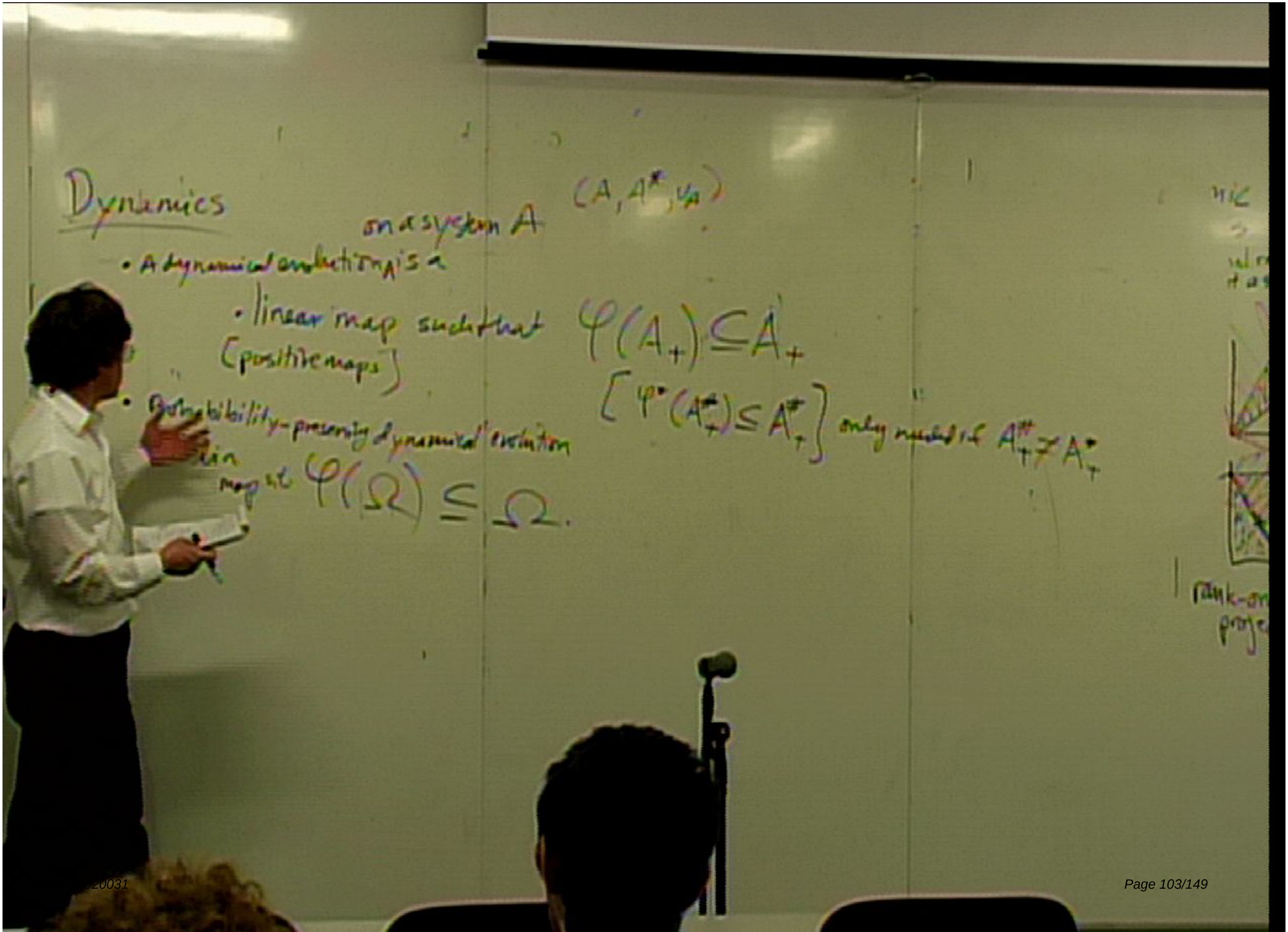


Dynamics on a system A (A, A^{*}, u_A)

A dynamical evolution T_A is a

- linear map such that $\varphi(A_+) \subseteq A_+$ [positive maps]

$[\varphi^{*}(A^{*}_{+}) \leq A^{*}_{+}]$ only needed if $A^{\#}_{+} \neq A^{*}_{+}$

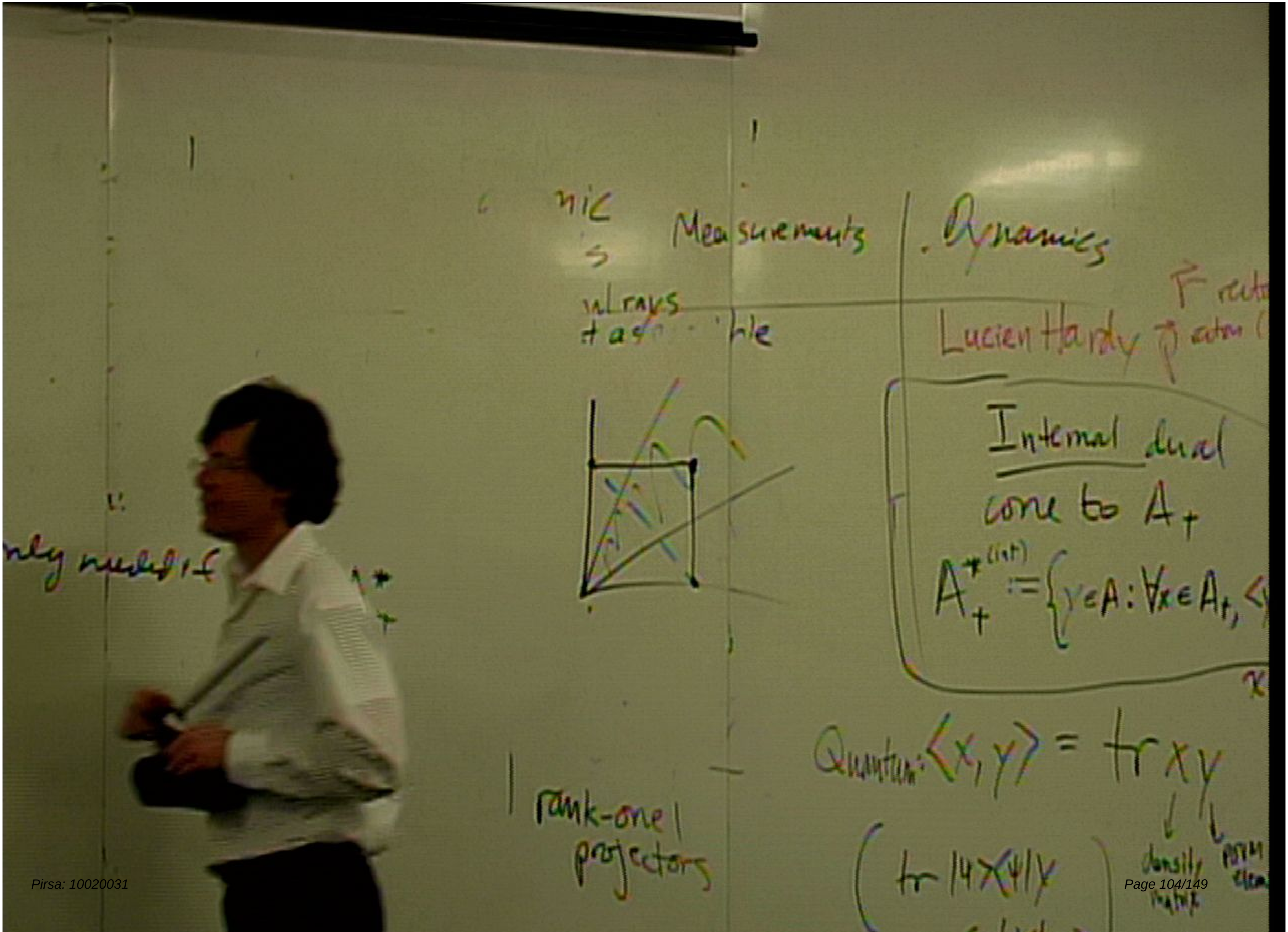


Dynamics

on a system A (A, A^*, u_A)

- A dynamical evolution on A is a
  - linear map such that $\varphi(A_+) \subseteq A_+$

    [positive maps] $[\varphi^*(A^*_+) \subseteq A^*_+]$ only needed if $A^{\#}_+ \neq A^*_+$
- Probability-preserving dynamical evolution is a map s.t. $\varphi(\Omega) \subseteq \Omega$.



Measurements

Dynamics

Lucien Hardy

Internal dual cone to A_+

$A_+^{*(int)} := \{y \in A : \forall x \in A_+, \langle y$

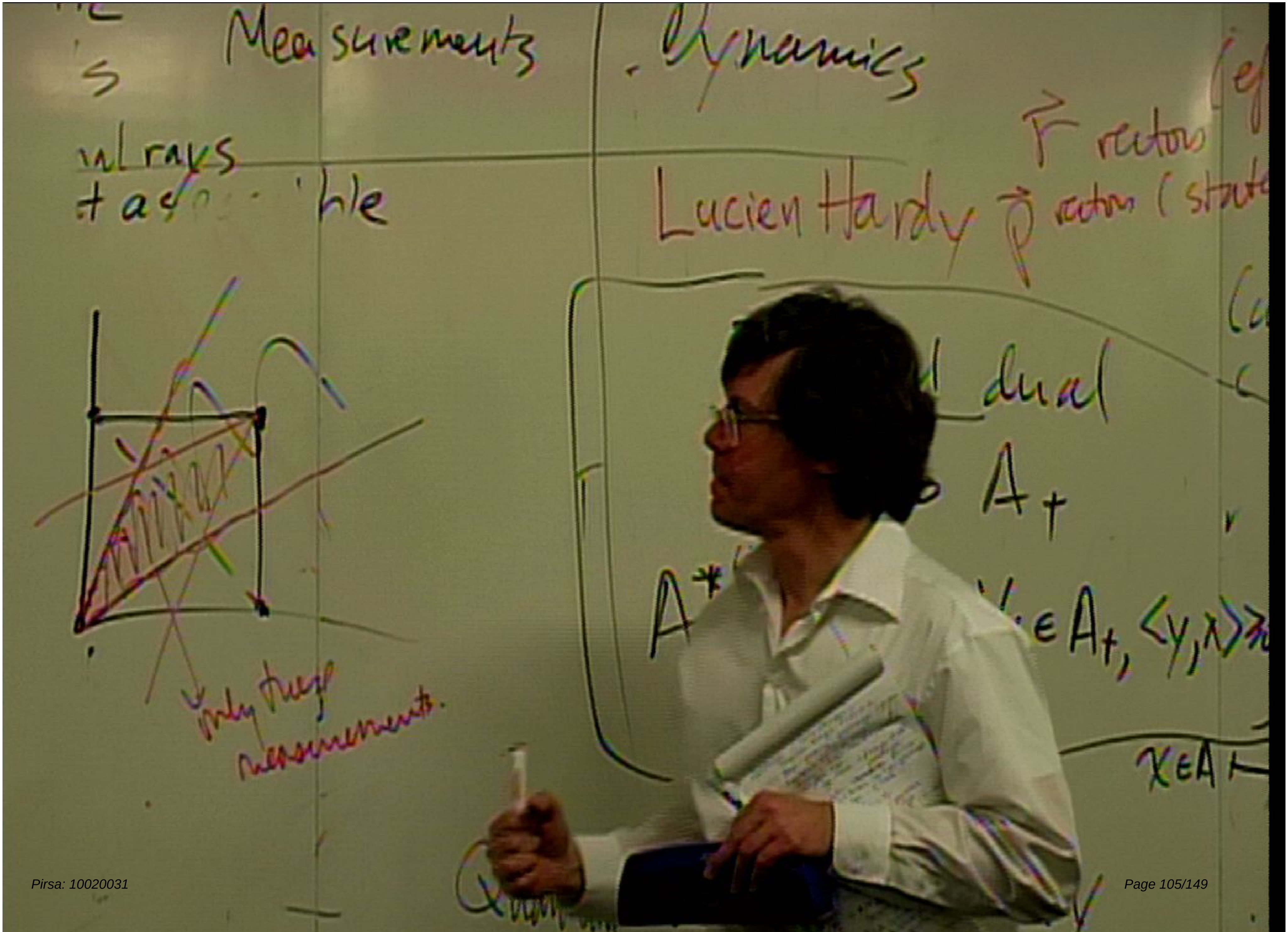
Quantum: $\langle x, y \rangle = \text{tr}\, xy$

density matrix

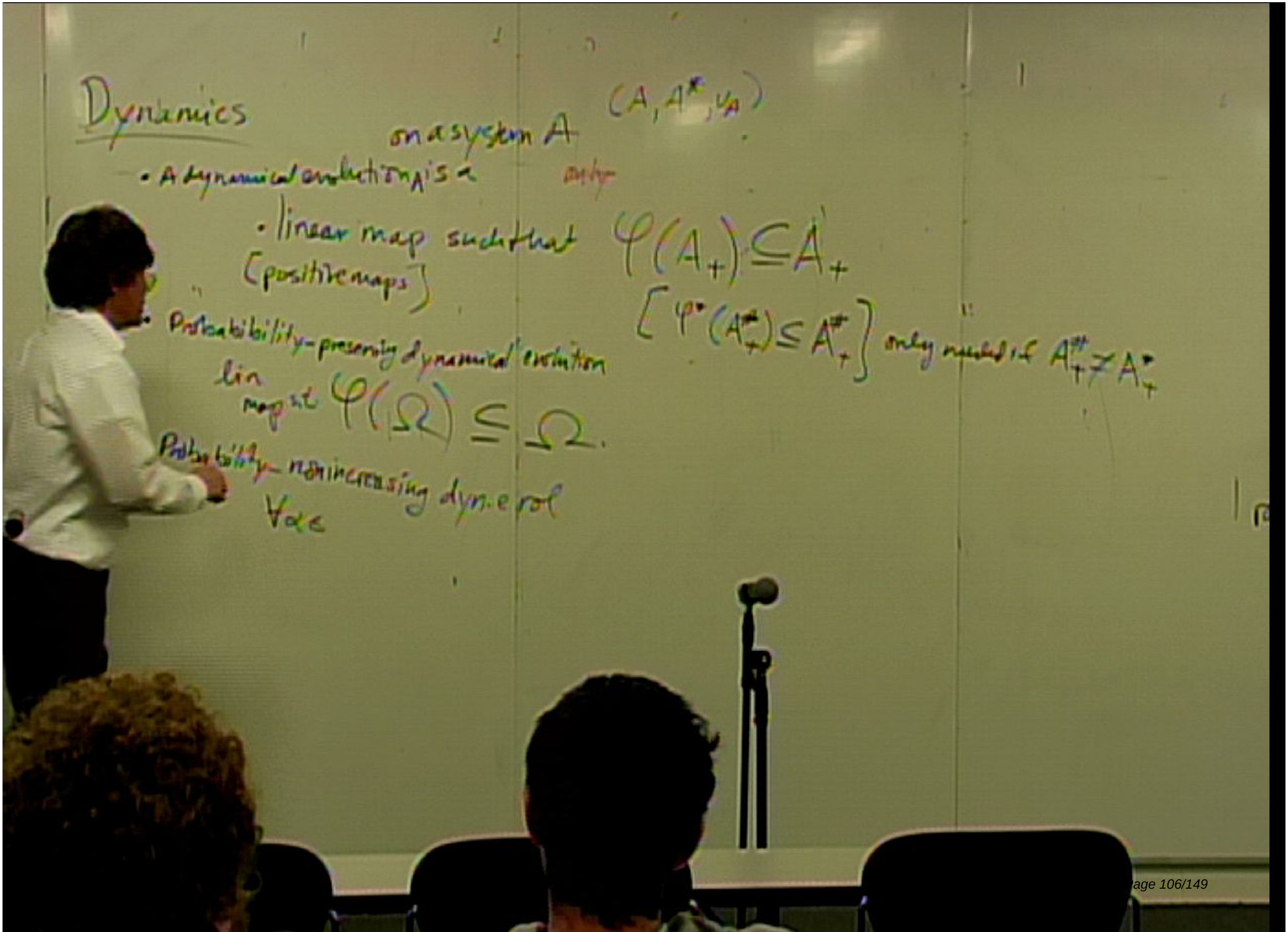
POVM element

rank-one projectors

$\text{tr}\, |\psi\rangle\langle\psi| y$



Measurements
Dynamics
rays
Lucien Hardy
F vectors
p vectors (state
dual
A+
A*
∈ A+, <y,x>
x∈A
only these measurements



Dynamics (A, A^{*}, u_A)

on a system A

- A dynamical evolution$_A$ is a only
  - linear map such that $\varphi(A_+) \subseteq A_+$

[positive maps]

$[\varphi^{*}(A^{\#}_{+}) \leq A^{\#}_{+}]$ only needed if $A^{\#}_{+} \neq A^{*}_{+}$

- Probability-preserving dynamical evolution

lin map s.t. $\varphi(\Omega) \subseteq \Omega$.

Probability-nonincreasing dyn. evol

$\forall \alpha \in$

• Probability-nonincreasing evolution:

$\forall \alpha \in \Omega \; \varphi(\alpha)$

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Measur

$\subseteq A_+$

$) \leq A^{\#}_+ \,]$ only [illegible] $A^{\#}$

• Probability-nonincreasing evolution:

$$\forall \alpha \in \Omega, \quad u_A(\varphi(\alpha)) \leqslant 1$$

nic

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- Probability-nonincreasing evolution:

$$\forall \alpha \in \Omega, \quad u_A(\varphi(\alpha)) \leq 1$$

$$\text{(and } \varphi(K) \subseteq K\text{)}$$

nic

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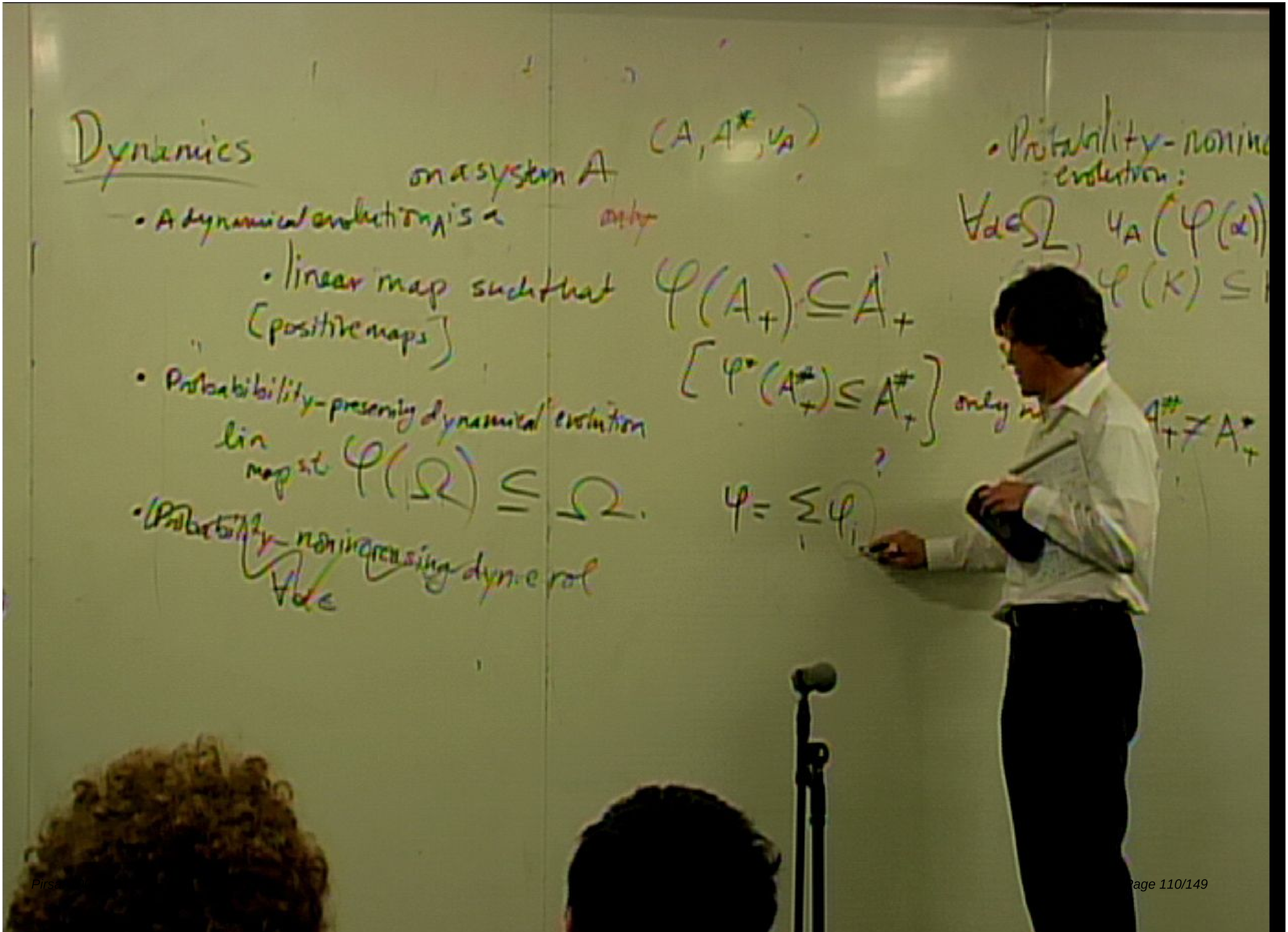


Dynamics

on a system A (A, A^*, u_A)

- A dynamical evolution$_A$ is a
  - linear map such that $\varphi(A_+) \subseteq A_+$ [positive maps]

    $[\varphi^*(A^\#_+) \leq A^\#_+]$ only if ?

- Probability-preserving dynamical evolution lin map s.t. $\varphi(\Omega) \subseteq \Omega$.

$\varphi = \sum_i \varphi_i$

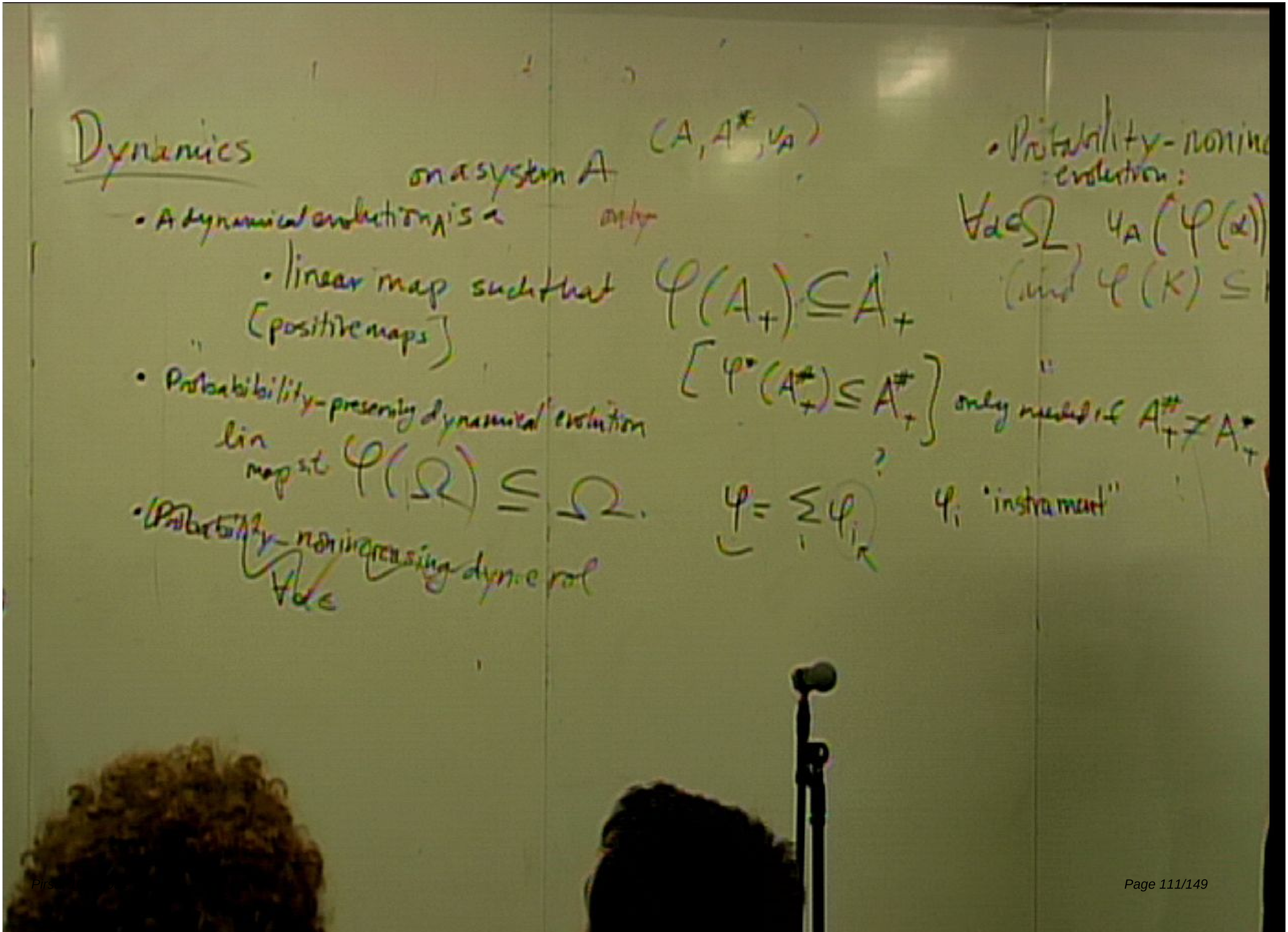
- Probability-nonincreasing dyn. evol $\forall \alpha \in$

- Probability-noninc evolution:

$\forall \alpha \in \Omega, \; u_A(\varphi(\alpha))$

$\varphi(K) \leq$

$A^\#_+ \neq A^*_+$



Dynamics

on a system A (A, A^*, u_A)

- A dynamical evolution$_A$ is a only
  - linear map such that $\varphi(A_+) \subseteq A_+$

    [positive maps]

    $[\varphi^*(A^\#_+) \leq A^\#_+]$ only needed if $A^\#_+ \neq A^*_+$
- Probability-preserving dynamical evolution

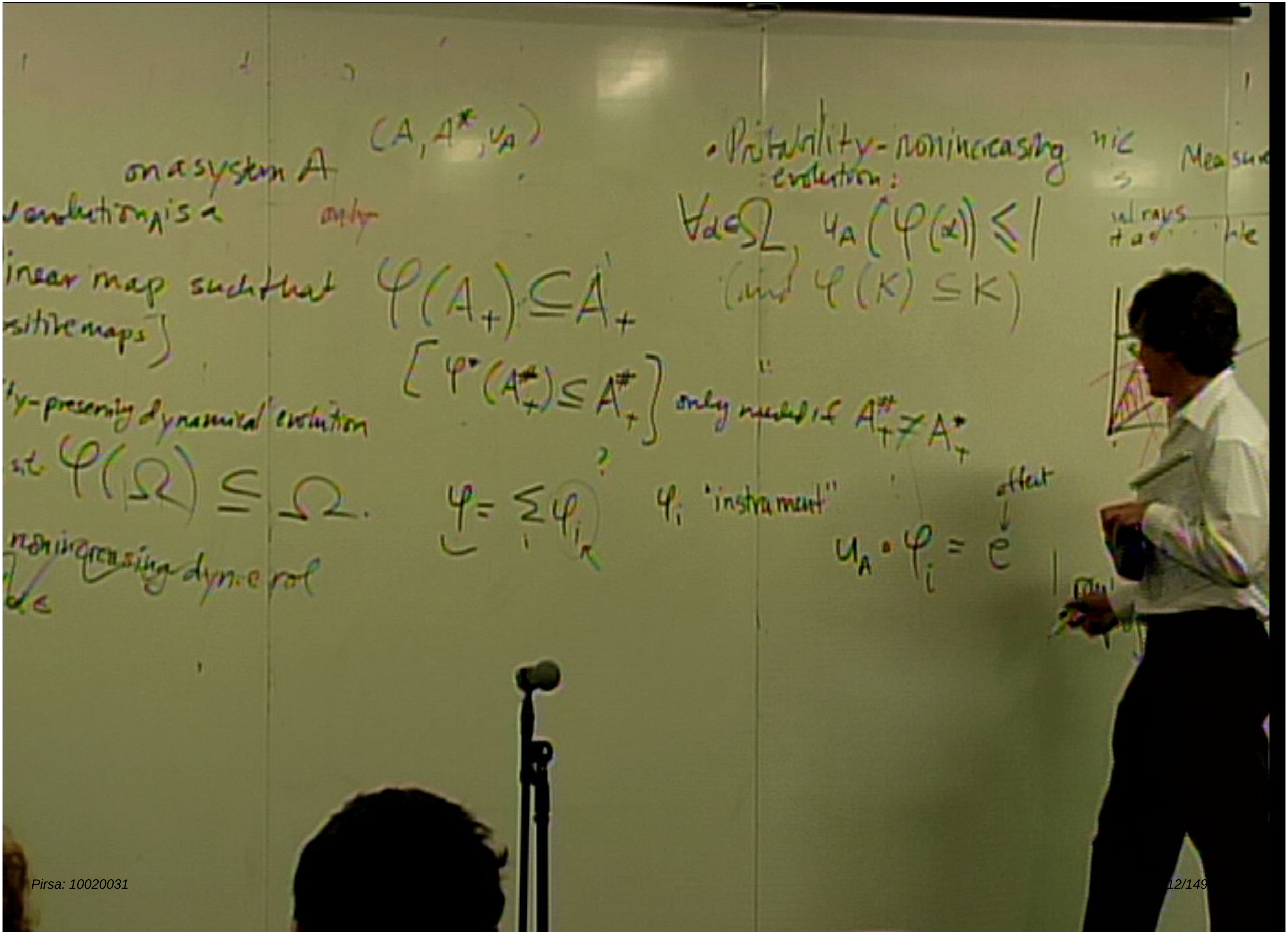
  lin map s.t. $\varphi(\Omega) \subseteq \Omega$.

  $\varphi = \sum_i \varphi_{i_R}$? φ_i "instrument"
- Probability-nonincreasing dyn. e vol

  $\forall \alpha \in$
- Probability-noninc evolution:

  $\forall \alpha \in \Omega, \; u_A(\varphi(\alpha))$

  (and $\varphi(K) \leq$



on a system A (A, A^*, u_A)

evolution$_A$ is a only

linear map such that $\varphi(A_+) \subseteq A_+$

sitive maps]

$[\varphi^*(A^\#_+) \subseteq A^\#_+]$ only needed if $A^\#_+ \neq A^*_+$

ty-preserving dynamical evolution

s.t. $\varphi(\Omega) \subseteq \Omega$.

$\varphi = \sum_i \varphi_i$ φ_i "instrument"

nonincreasing dyn. evol

• Probability-nonincreasing evolution:

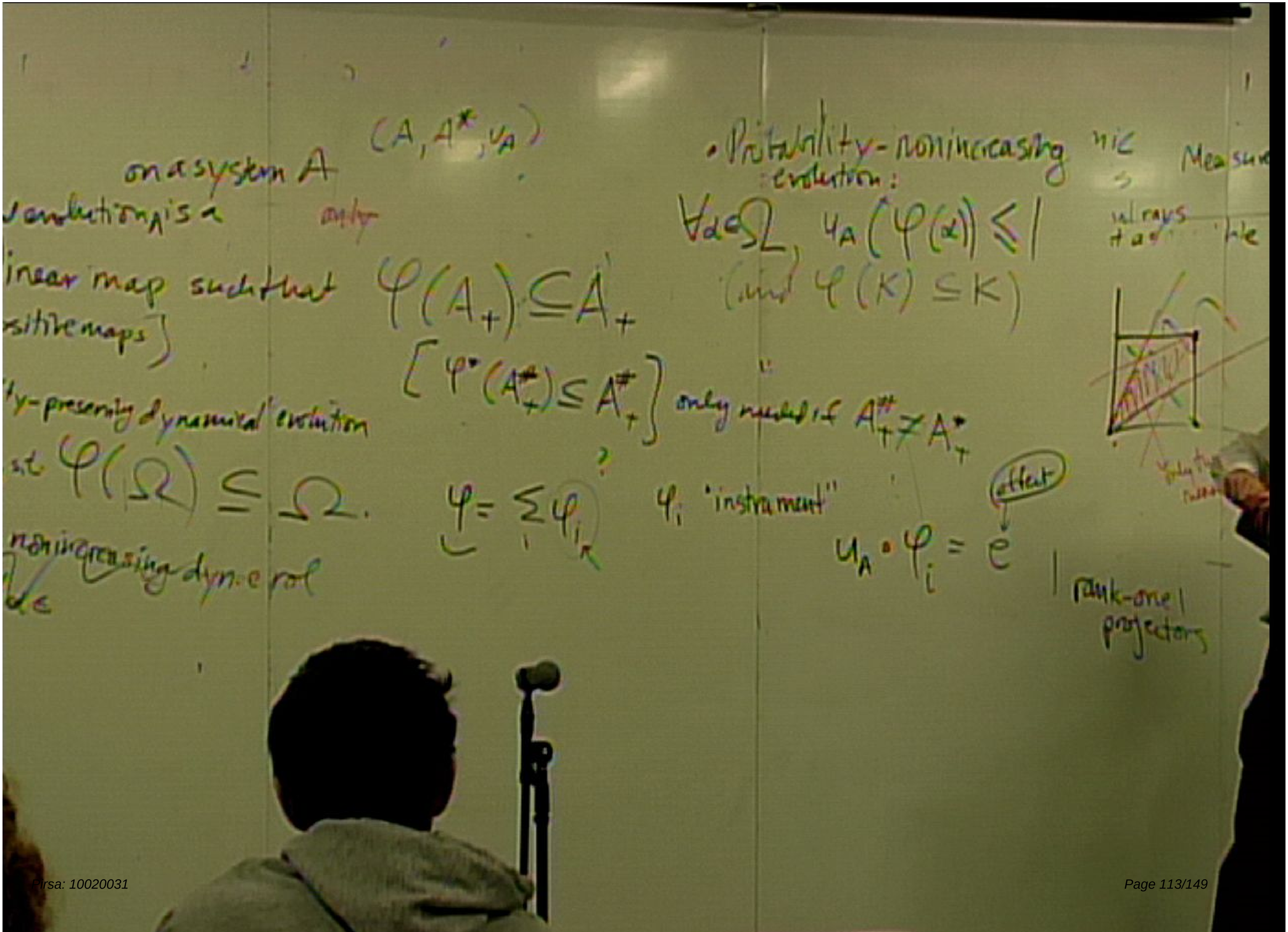
$\forall \alpha \in \Omega, \; u_A(\varphi(\alpha)) \leq 1$

(and $\varphi(K) \subseteq K$)

$u_A \circ \varphi_i = \overset{\text{effect}}{e}$

Measu

always



(A, A^*, u_A)

on a system A

evolution$_A$ is a only

linear map such that $\varphi(A_+) \subseteq A_+$

sitive maps]

$[\varphi^*(A^\#_+) \leq A^\#_+]$ only needed if $A^\#_+ \neq A^*_+$

ty-preserving dynamical evolution

s.t. $\varphi(\Omega) \leq \Omega$.

$\varphi = \sum_i \varphi_i$?

φ_i "instrument"

nonincreasing dyn. e rol

de

• Probability-nonincreasing evolution:

$\forall \alpha \in \Omega,\ u_A(\varphi(\alpha)) \leq 1$

(and $\varphi(K) \leq K$)

nic

Measure

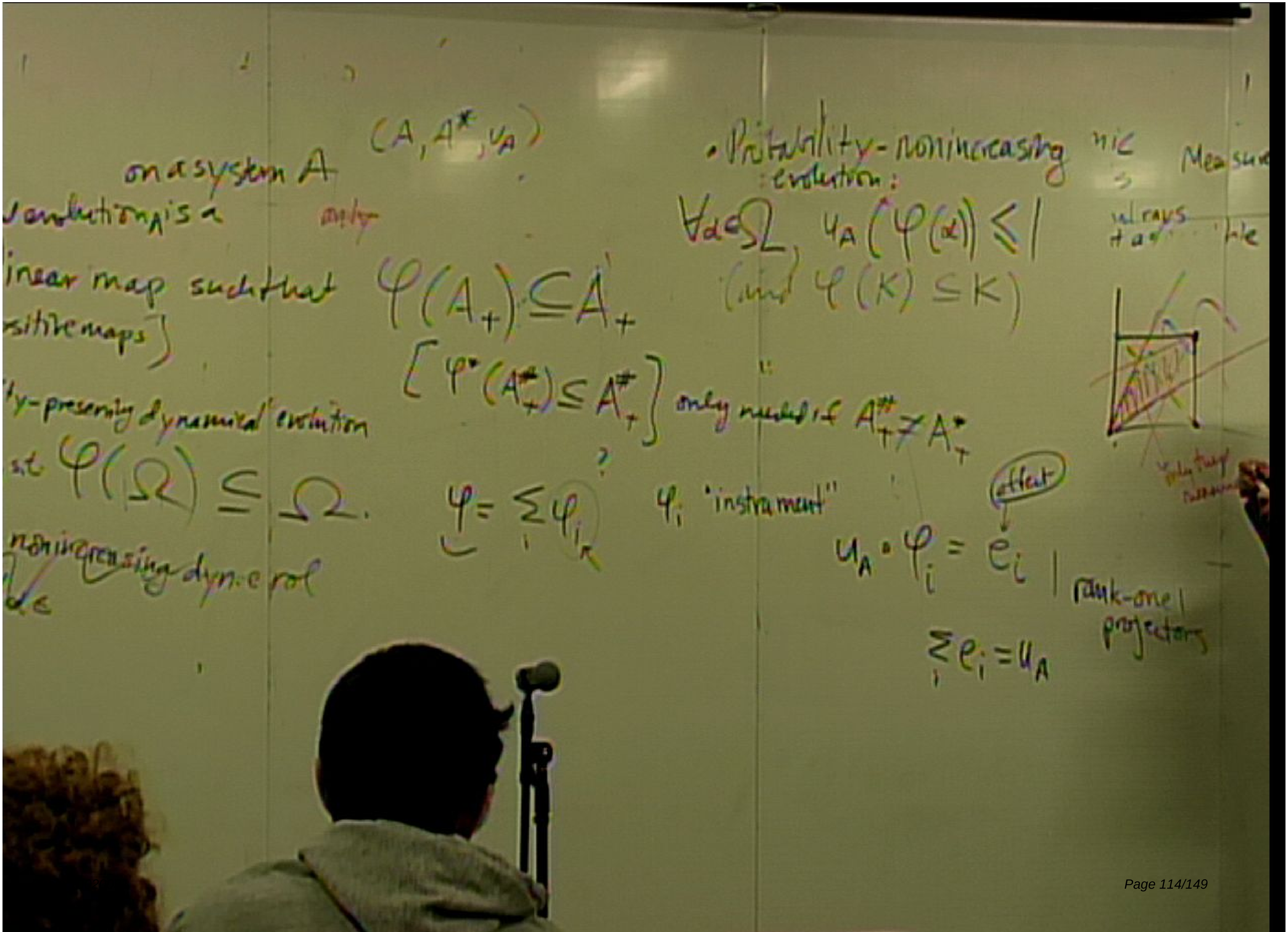
always

like

effect

$u_A \circ \varphi_i = e$

rank-one projectors



(A, A^*, u_A)

on a system A

evolution$_A$ is a … only

linear map such that $\varphi(A_+) \subseteq A_+$

…sitive maps]

$[\varphi^*(A^\#_+) \leq A^\#_+]$ only needed if $A^\#_+ \neq A^*_+$

…ty-preserving dynamical evolution

s.t. $\varphi(\Omega) \leq \Omega.$

$\underset{\smile}{\varphi} = \sum_i \varphi_i$?

φ_i "instrument"

…nonincreasing dyn. e rol

…de

• Probability-nonincreasing evolution:

$\forall \alpha \in \Omega,\ u_A(\varphi(\alpha)) \leq 1$

(and $\varphi(K) \leq K$)

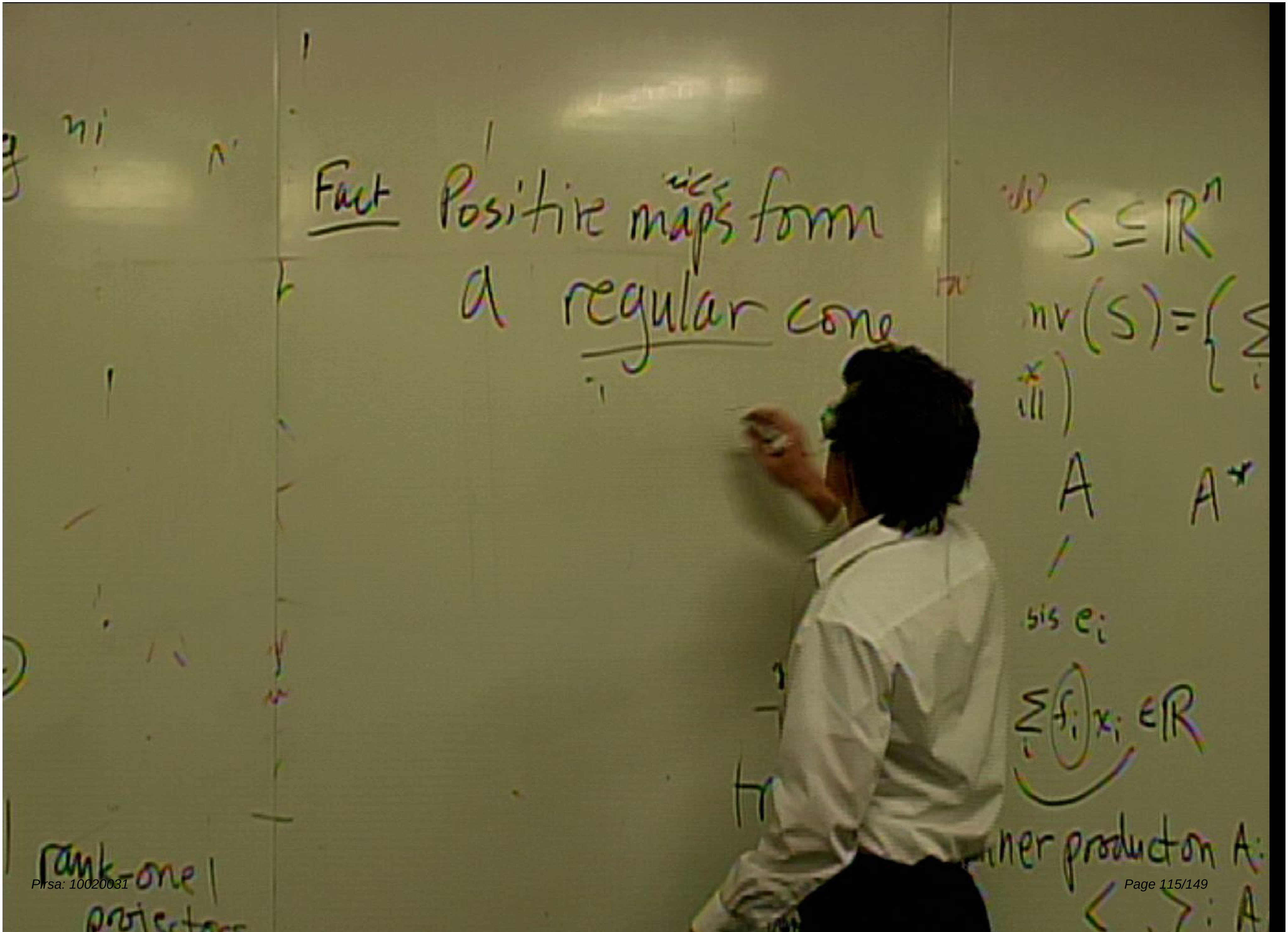
ric

Measure…

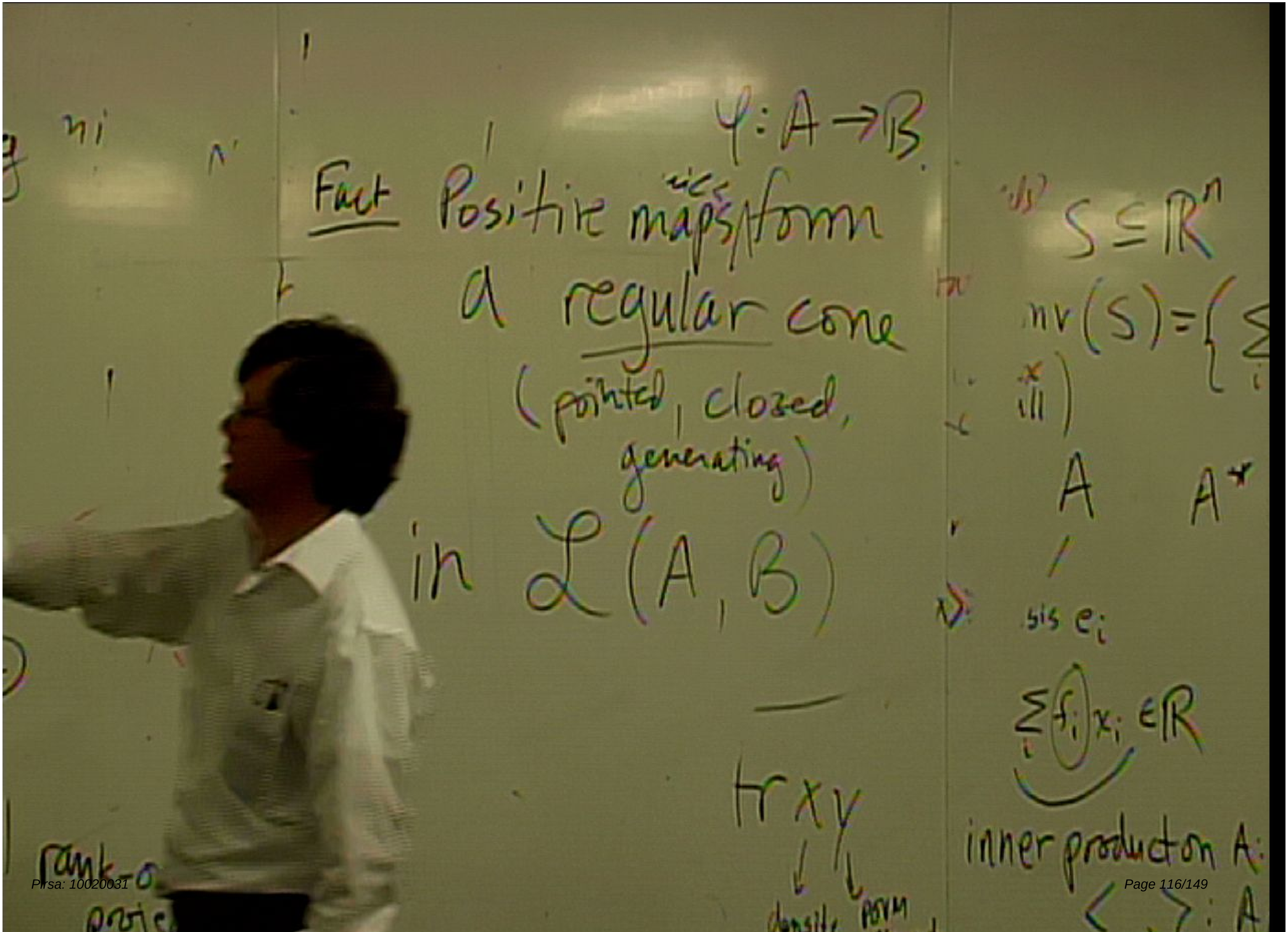
always … like

$u_A \circ \varphi_i = \overset{\text{effect}}{e_i}$ | rank-one projectors

$\sum_i e_i = u_A$



Fact Positive maps form a regular cone



$\varphi : A \to B$

Fact Positive maps form a regular cone (pointed, closed, generating) in $\mathcal{L}(A,B)$

$\mathrm{tr}\, x y$

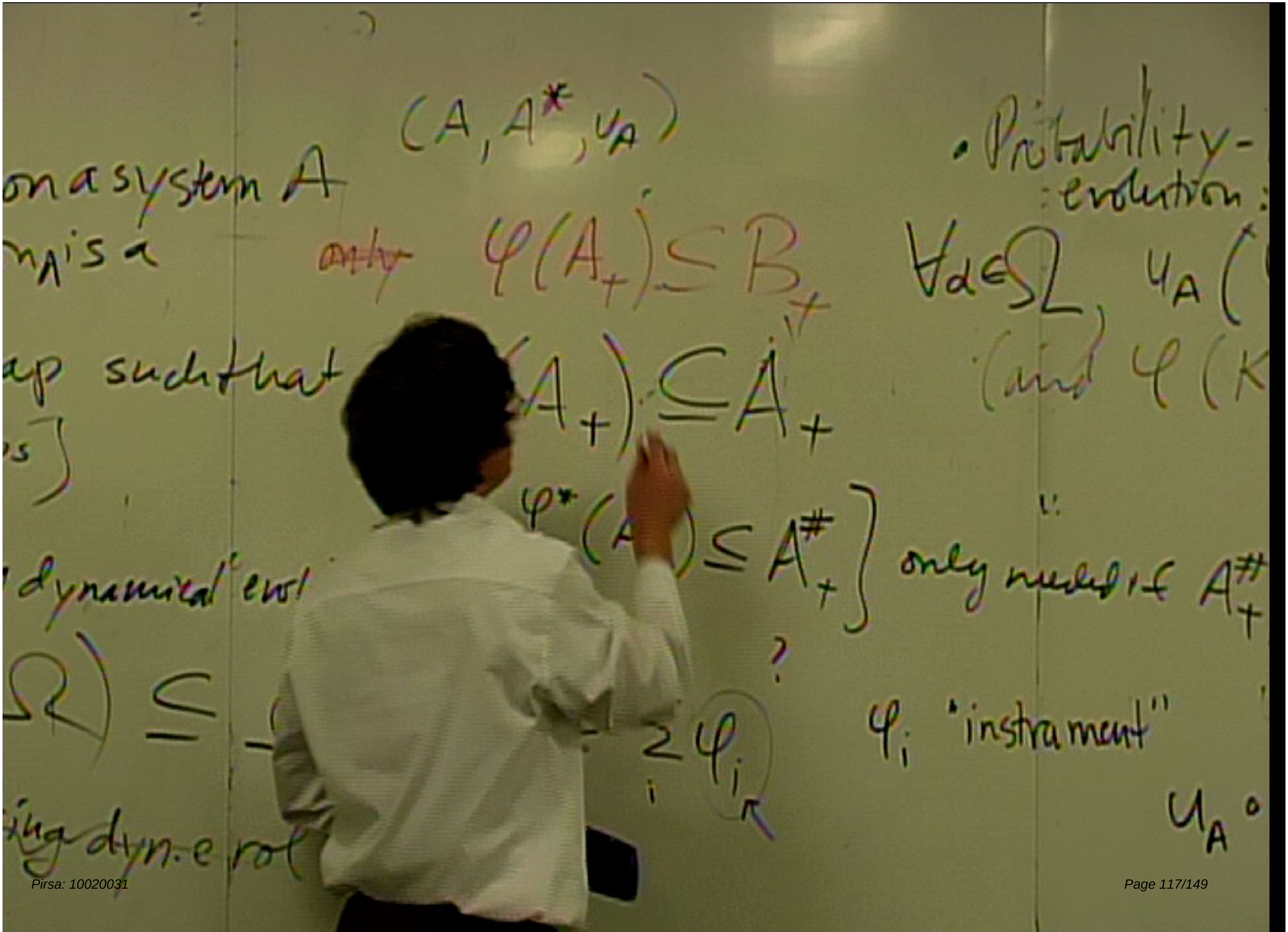
density POVM

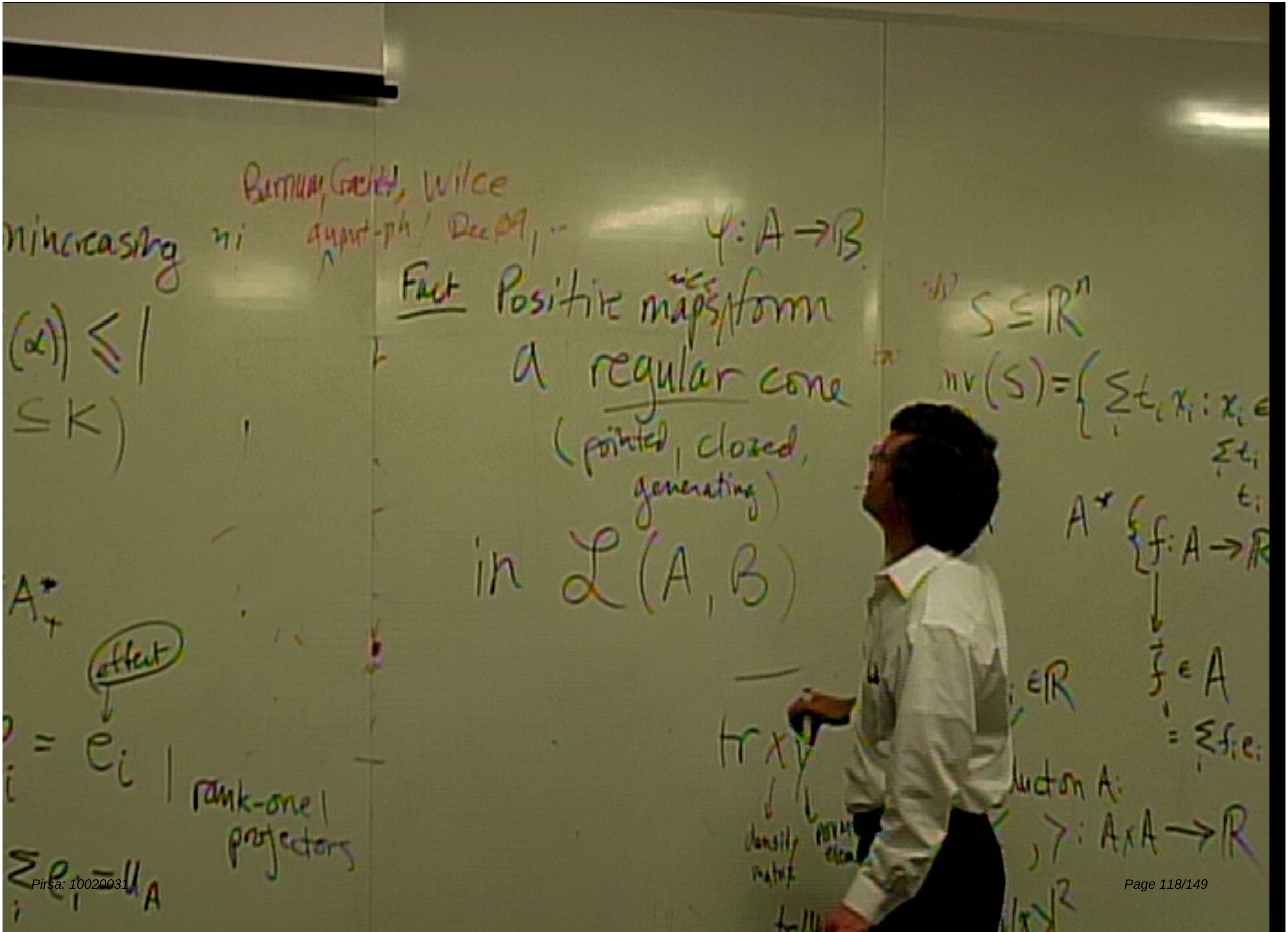
$S \subseteq \mathbb{R}^n$

$\sum_i f_i x_i \in \mathbb{R}$

inner product on A:

$\langle\ ,\ \rangle : A$


(A, A^*, u_A)
on a system A
m_A is a
~~only~~ $\varphi(A_+) \subseteq B_+$
ap such that
$(A_+) \subseteq A_+$
s]
$\varphi^*(A) \leq A^\#_+$ } only needed if $A^\#_+$
dynamical evol
$\Omega) \subseteq$
?
$\sum_i \varphi_i$
φ_i "instrument"
$u_A \circ$
ing dyn. e rol
• Probability-
evolution:
$\forall \alpha \in \Omega, \; u_A($
(and $\varphi(K$



Barnum, Gaebler, Wilce

quant-ph / Dec 09, ...

$\psi : A \to B$

Fact Positive maps form a regular cone (pointed, closed, generating) in $\mathcal{L}(A, B)$

$S \subseteq \mathbb{R}^n$

$\mathrm{cnv}(S) = \{ \sum_i t_i x_i : x_i \in$

$\sum_i t_i$

t_i

$A^* \{ f : A \to \mathbb{R}$

$\vec{f} \in A$

$= \sum_i f_i e_i$

$\in \mathbb{R}$

duction A:

$\langle \, , \rangle : A \times A \to \mathbb{R}$

tr xy

density matrix

normalized effect

nincreasing ni

$(\alpha)) \leq 1$

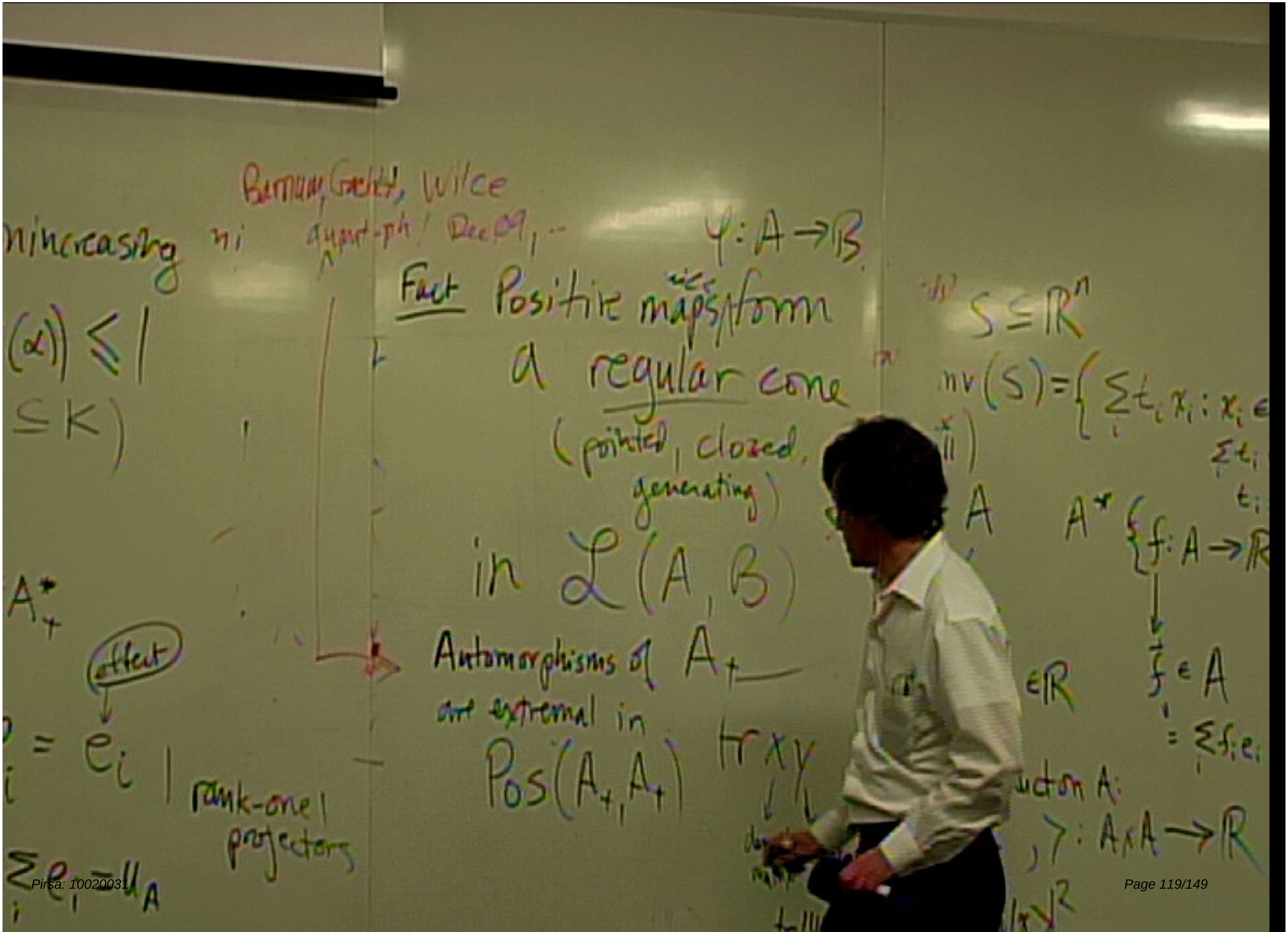
$\leq K)$

A^*_+

effect

$= e_i$ | rank-one projectors

$\sum_i e_i = 1_A$



Barnum, Gaebler, Wilce

quant-ph / Dec 09, ...

Fact Positive maps form a regular cone (pointed, closed, generating) in $\mathcal{L}(A, B)$

Automorphisms of A_+ are extremal in $Pos(A_+, A_+)$

$tr\, xy$

$\varphi : A \to B$

$S \subseteq \mathbb{R}^n$

$nv(S) = \{ \sum_i t_i x_i : x_i \in$

$\sum_i t_i$

t_i

$A \quad A^* \; \{ f : A \to \mathbb{R}$

$\vec{f} \in A$

$= \sum_i f_i e_i$

uction A:

$\langle \, , \, \rangle : A \times A \to \mathbb{R}$

nincreasing n_i

$(\alpha)) \leq 1$

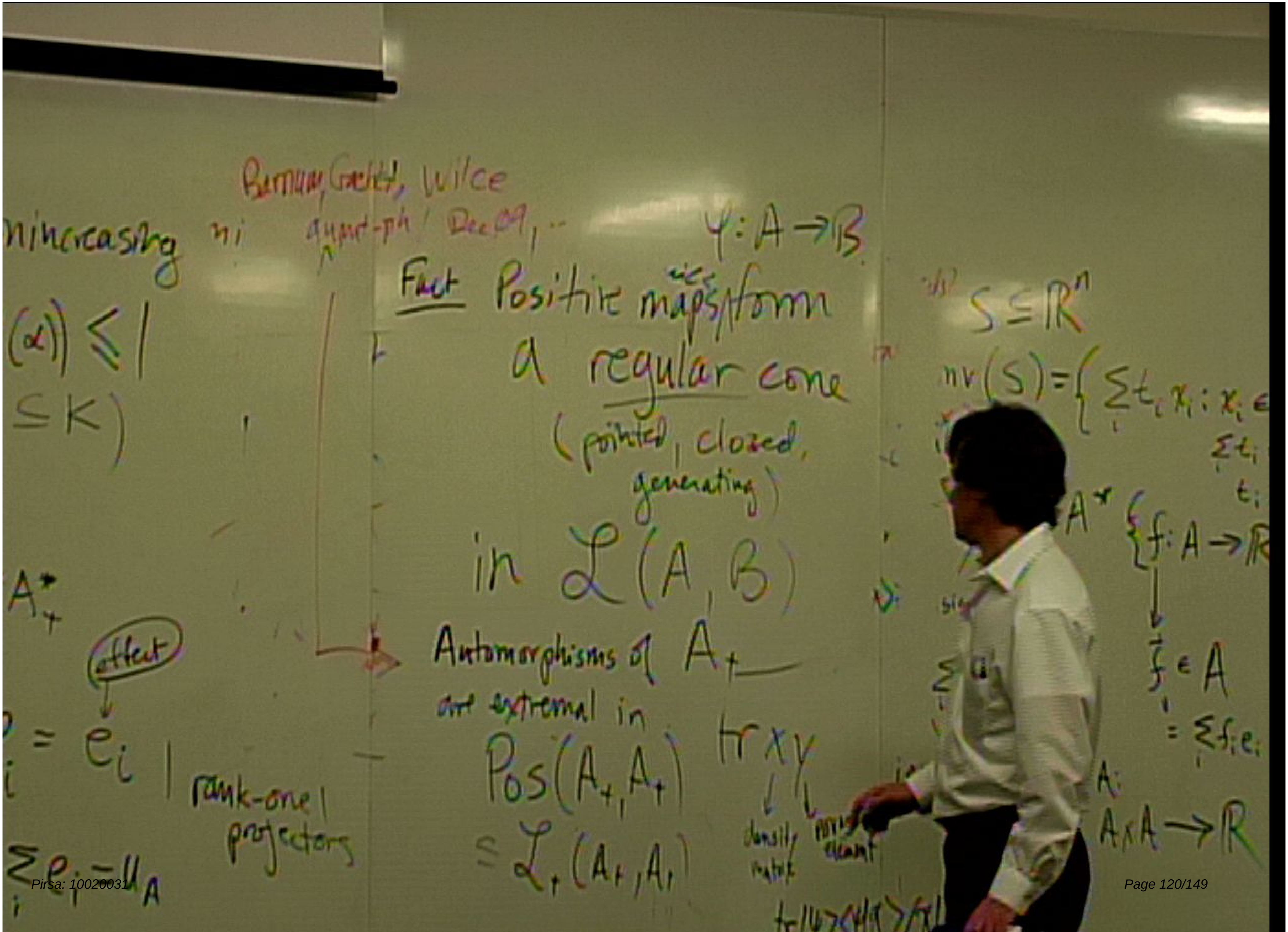
$\subseteq K)$

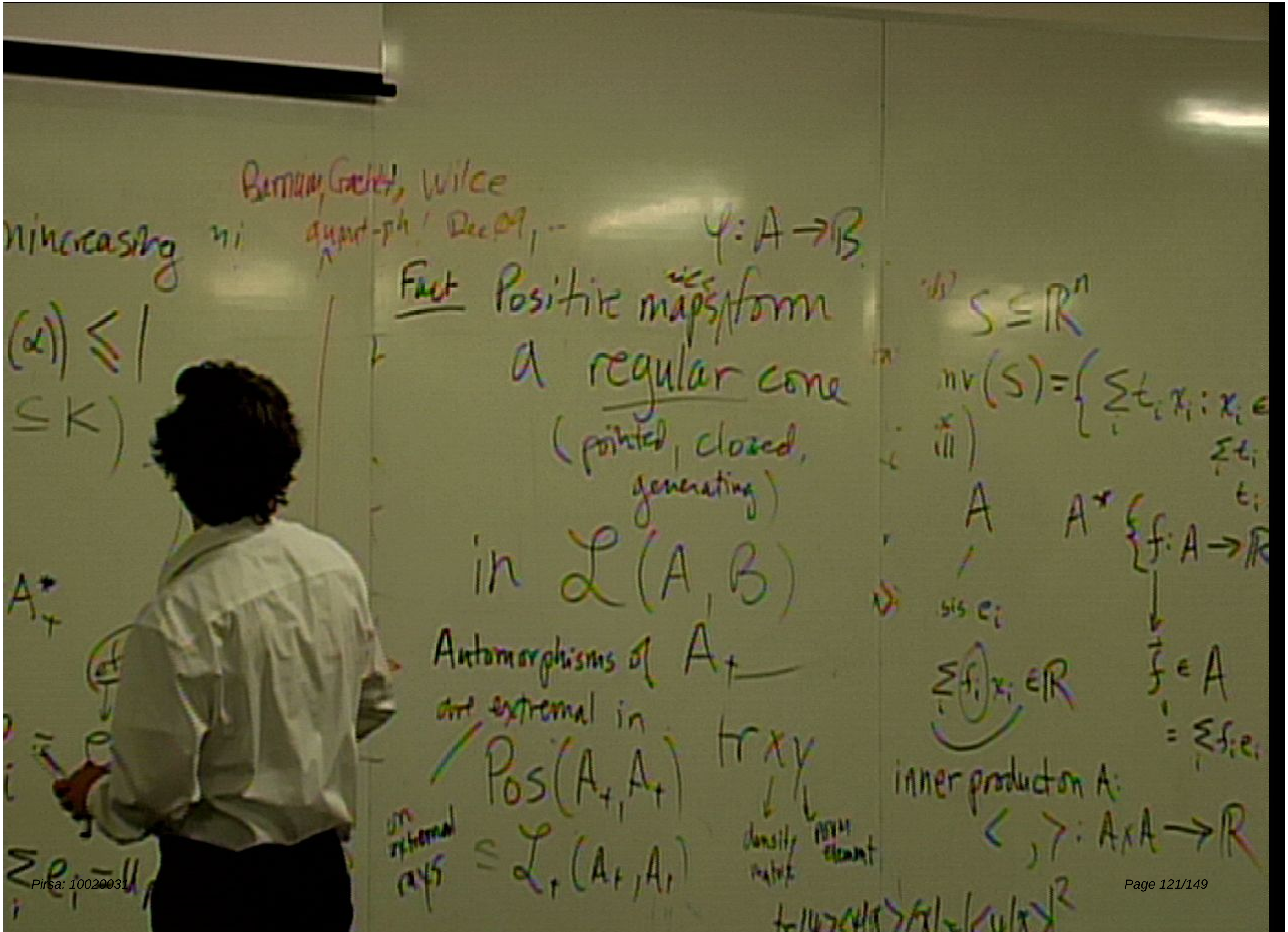
A_+^*

effect

$= e_i$ | rank-one projectors

$\sum_i e_i = 1_A$


Barnum, Gaebler, Wilce
quant-ph / Dec 09, ...
$\varphi: A \to B$
Fact Positive maps form a regular cone
(pointed, closed, generating)
in $\mathcal{L}(A, B)$
Automorphisms of A_+
are extremal in
$Pos(A_+, A_+)$
$\subseteq \mathcal{L}_+(A_+, A_+)$
tr xy
density matrix
nincreasing n_i
$(\alpha)) \leq 1$
$\subseteq K)$
A^*_+
effect
$= e_i$ | rank-one projectors
$\sum_i e_i = 1_A$
$S \subseteq \mathbb{R}^n$
$\text{conv}(S) = \{ \sum_i t_i x_i : x_i \in$
$\sum t_i$
t_i
$A^* \{ f : A \to \mathbb{R}$
$\vec{f} \in A$
$= \sum_i f_i e_i$
A:
$A \times A \to \mathbb{R}$


Barnum, Gaebler, Wilce

quant-ph / Dec 09, ...

$\varphi : A \to B$

Fact Positive maps form a regular cone (pointed, closed, generating) in $\mathcal{L}(A, B)$

Automorphisms of A_+ are extremal in $Pos(A_+, A_+)$ tr xy

density matrix

pure element

on extremal rays $\subseteq \mathcal{L}_1(A_+, A_+)$

$S \subseteq \mathbb{R}^n$

conv$(S) = \{ \sum_i t_i x_i : x_i \in$

$\sum t_i$

t_i

A A^* $\{ f : A \to \mathbb{R}$

$\vec{f} \in A$

$= \sum f_i e_i$

basis e_i

$\sum_i f_i x_i \in \mathbb{R}$

inner product on A:

$\langle \, , \rangle : A \times A \to \mathbb{R}$

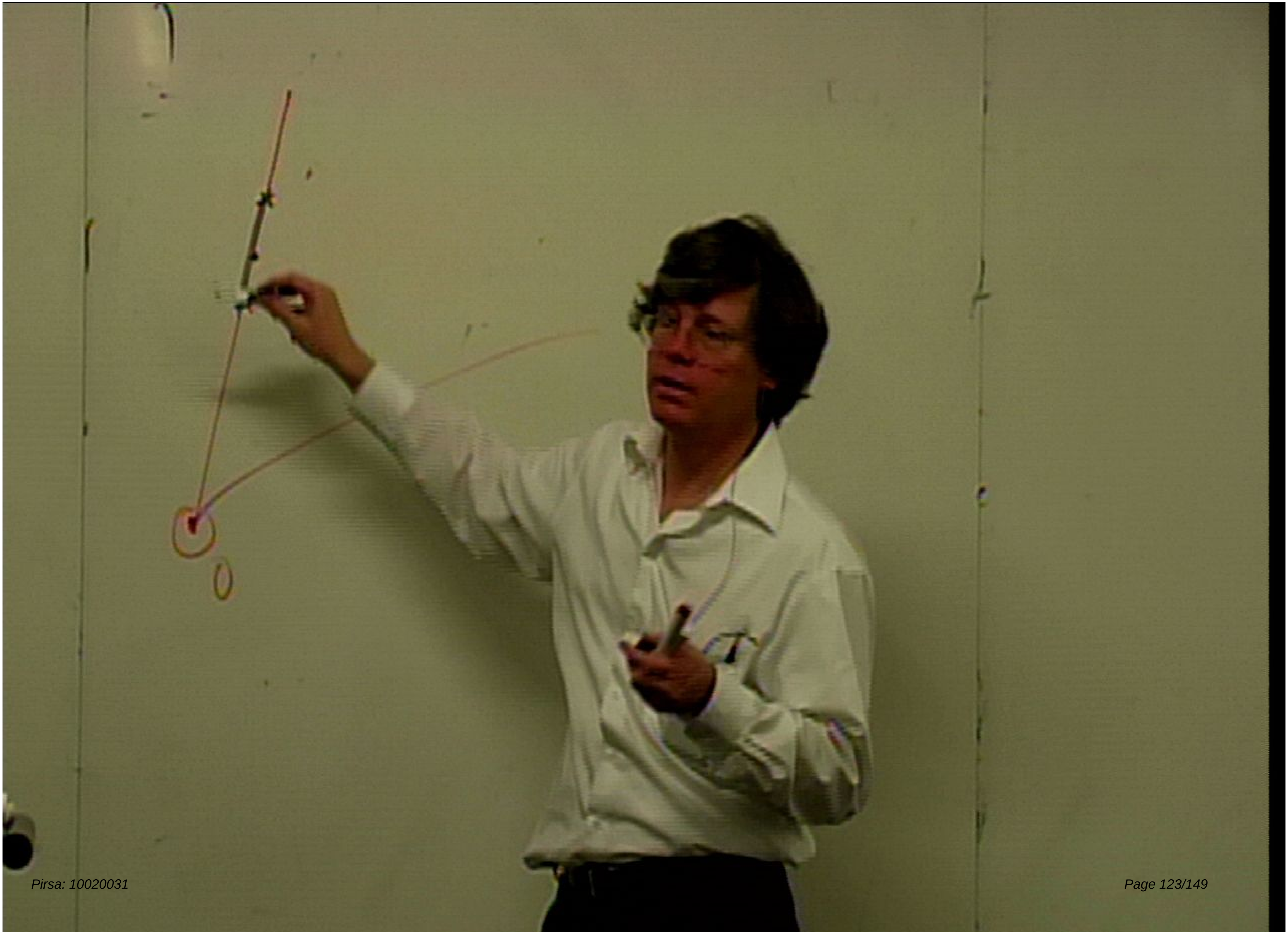
nincreasing

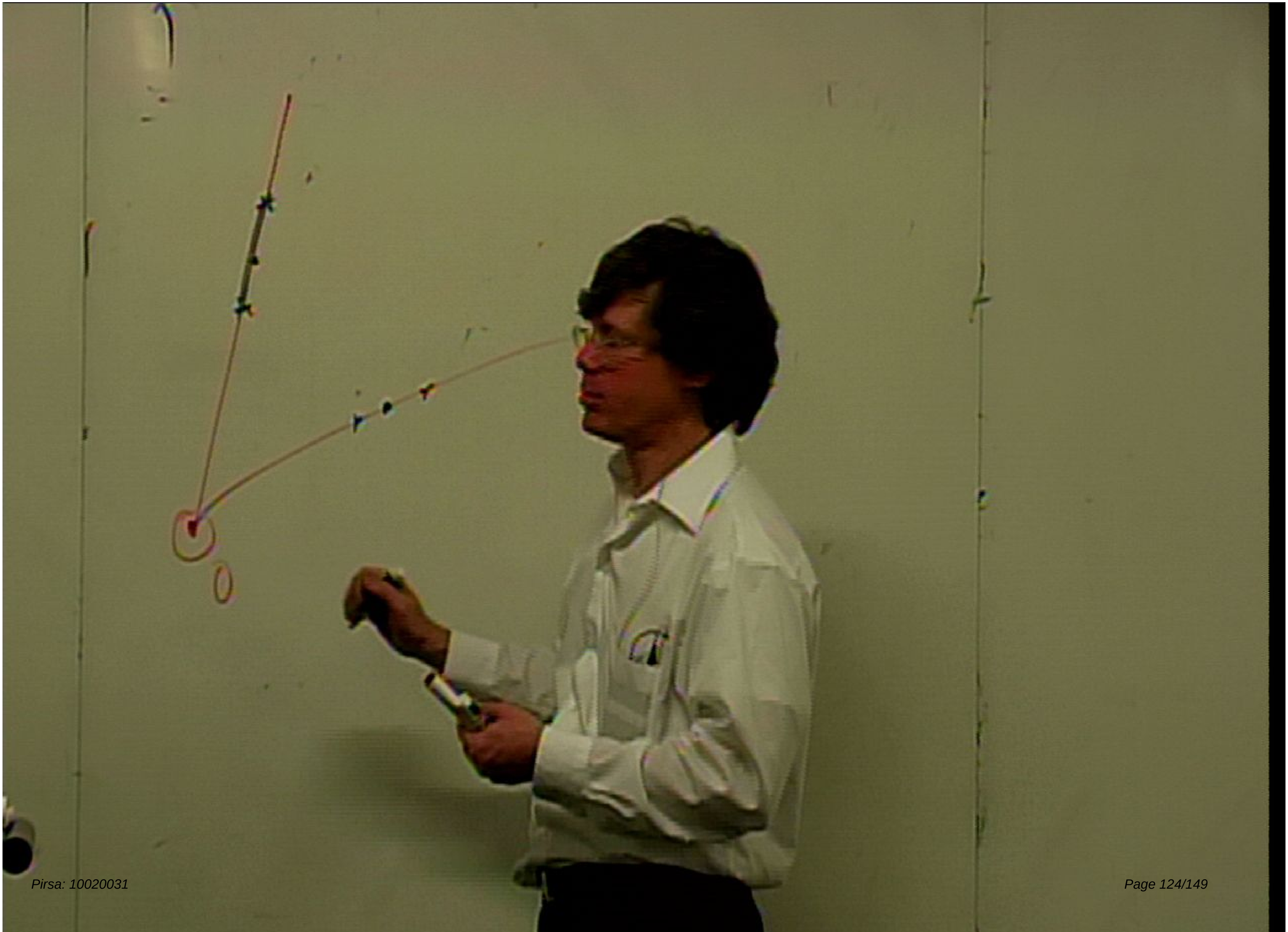
$(\alpha)) \leq 1$

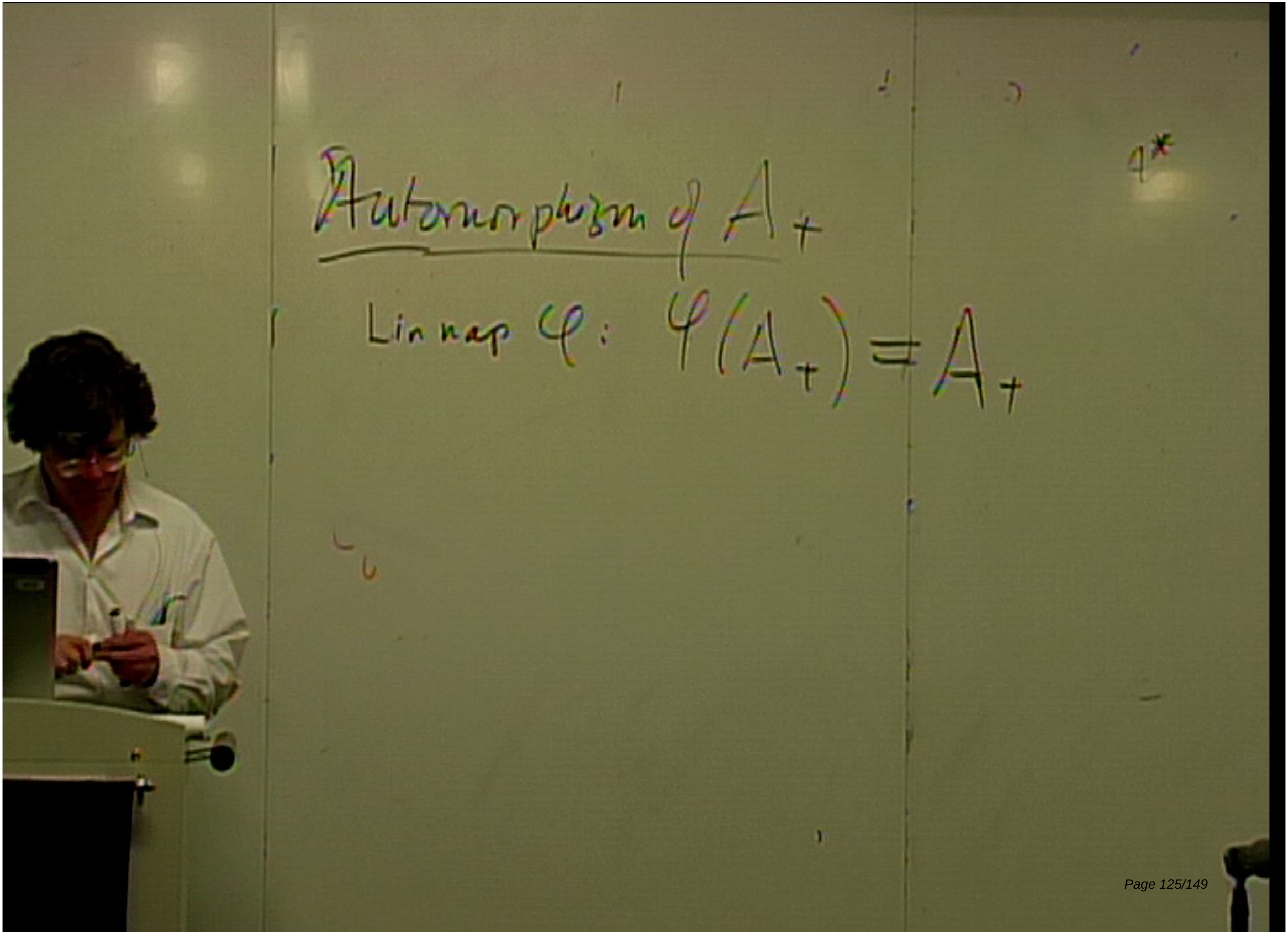
$\subseteq K)$

A^*_+

$\sum_i e_i = u$

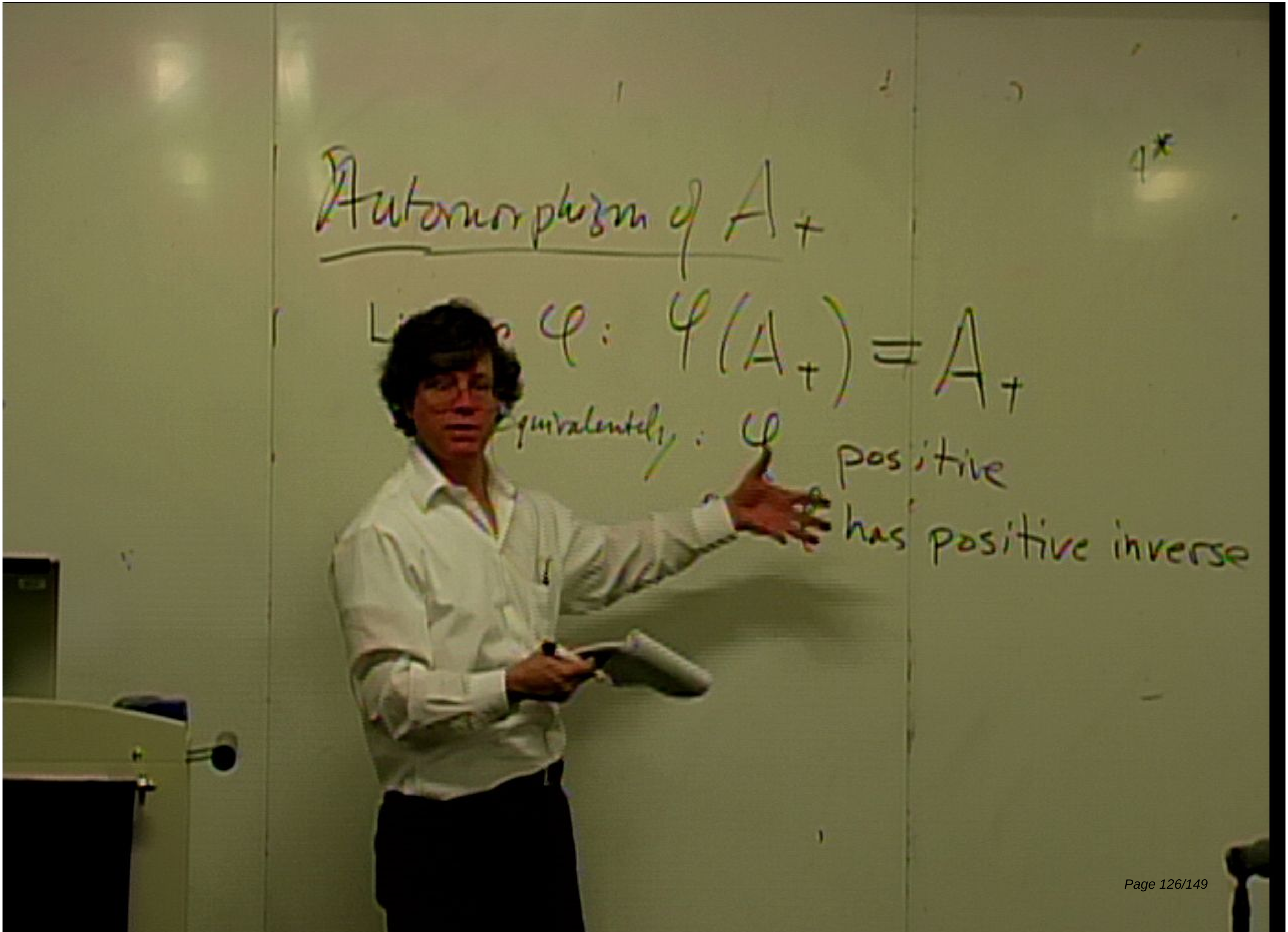






Automorphism of A_+

Lin map φ: $\varphi(A_+) = A_+$



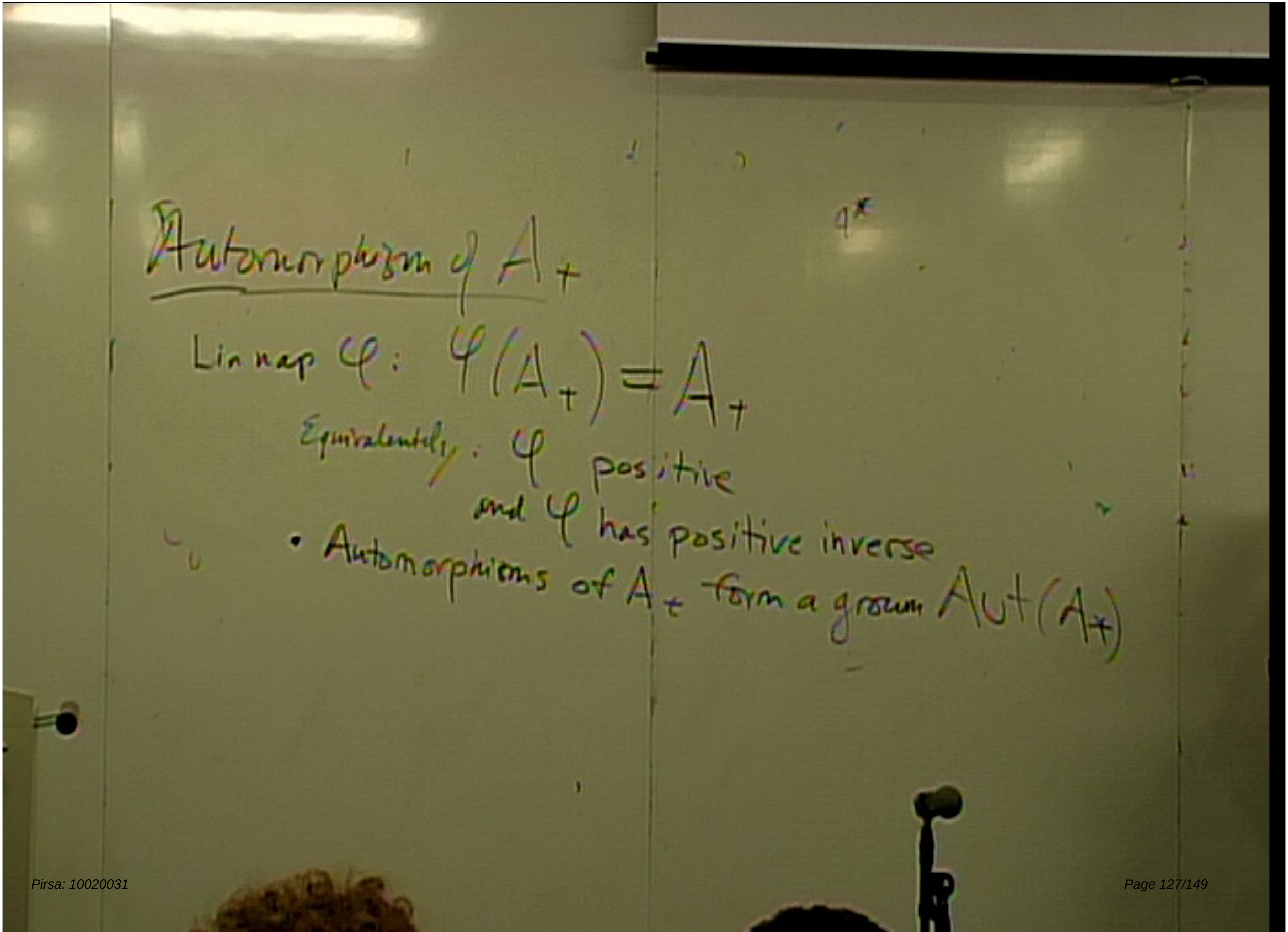
Automorphism of A_+

q^*

Li[illegible] φ: $\varphi(A_+) = A_+$

equivalently: φ positive

φ has positive inverse



q^*

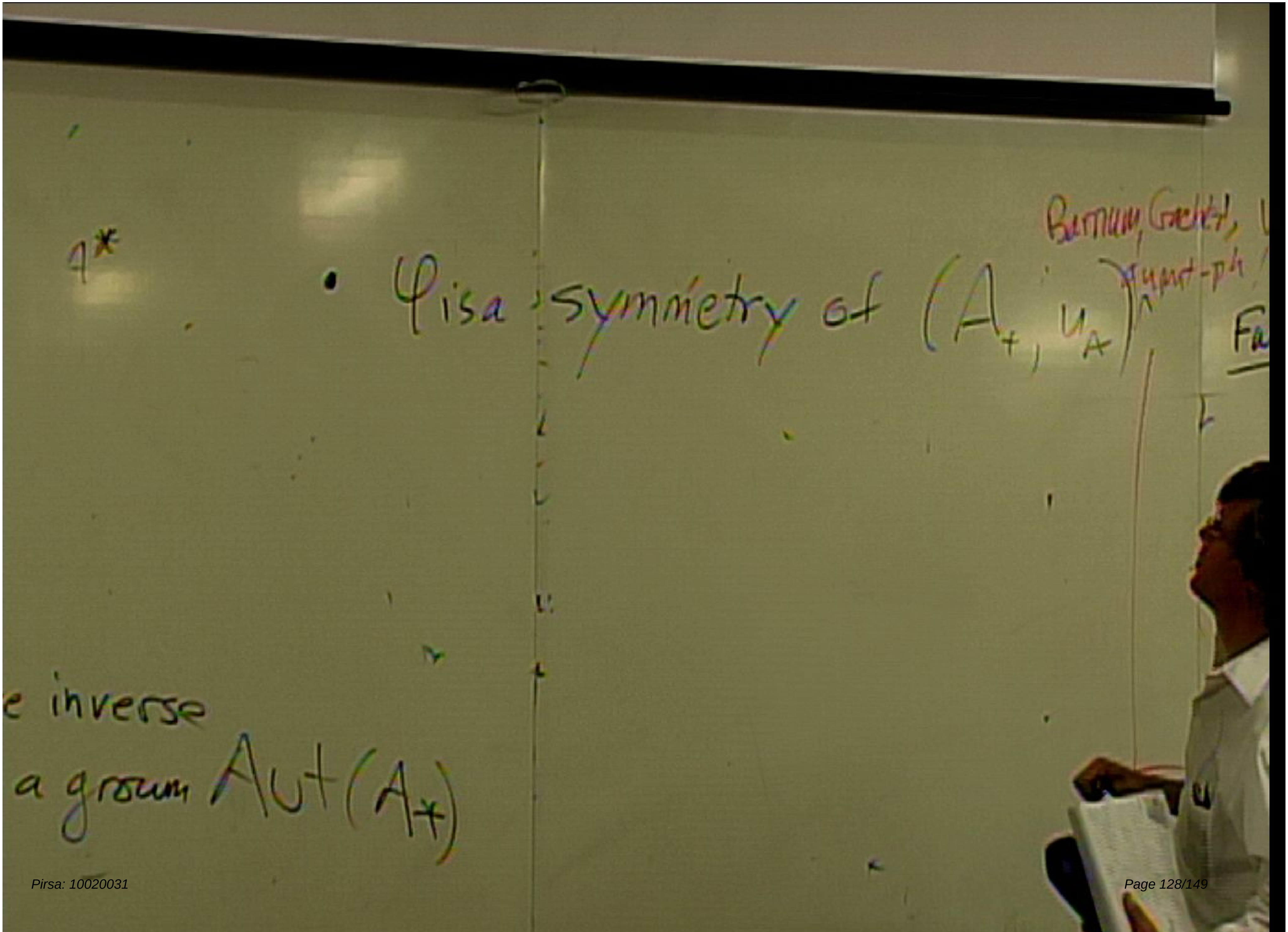
Automorphism of A_+

Lin map φ: $\varphi(A_+) = A_+$

Equivalently: φ positive

and φ has positive inverse

- Automorphisms of A_+ form a group $\mathrm{Aut}(A_+)$



q^*

- φ is a symmetry of $(A_*, u_A)^\wedge$

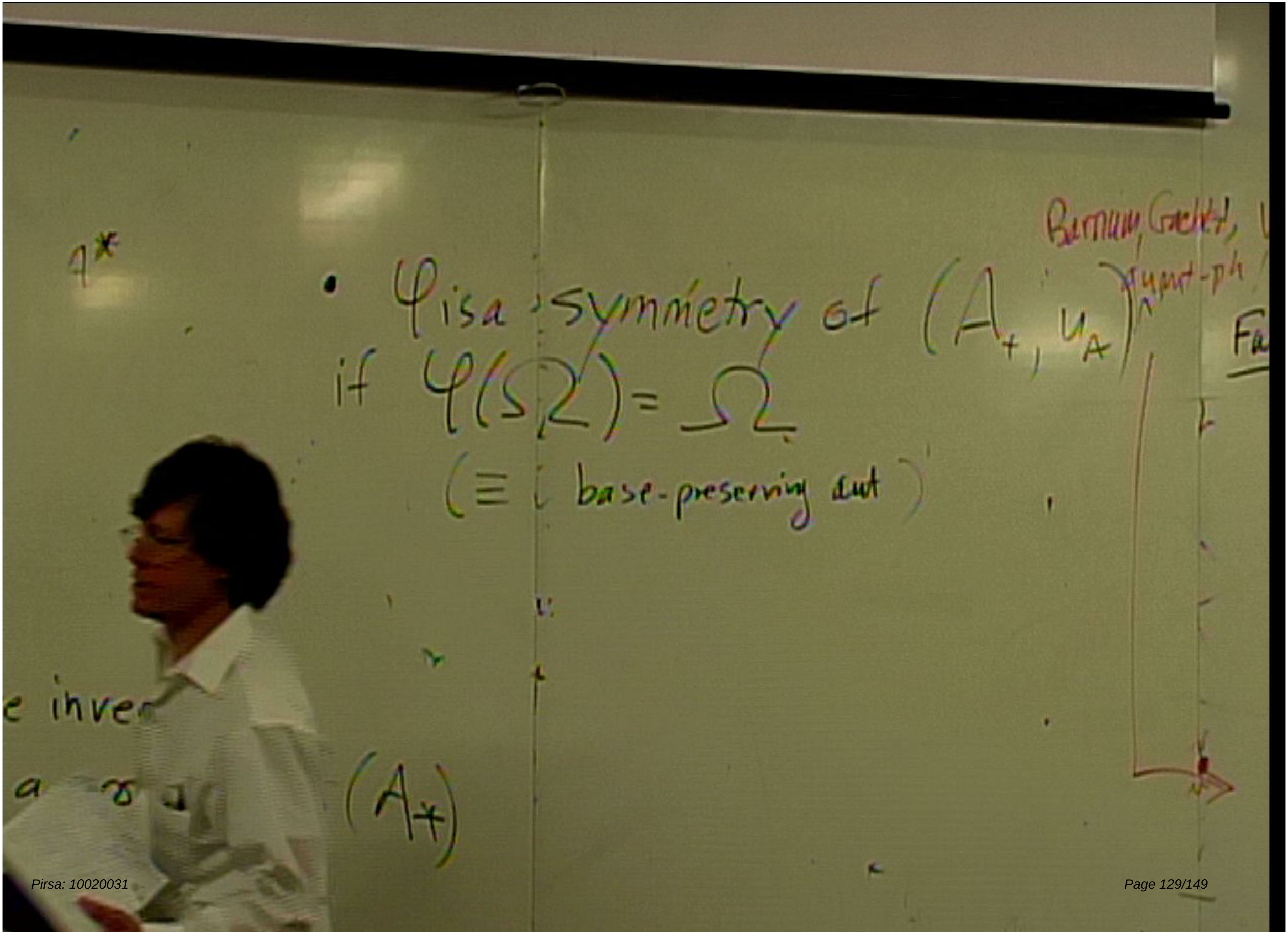
Barnum, Gaebler, W

symmt-ph

Fa

e inverse

a group $\mathrm{Aut}(A_*)$



q^*

- φ is a symmetry of (A_*, u_A)

if $\varphi(\Omega) = \Omega$

$(\equiv$ base-preserving aut $)$

Barnum, Gaebler,

quant-ph

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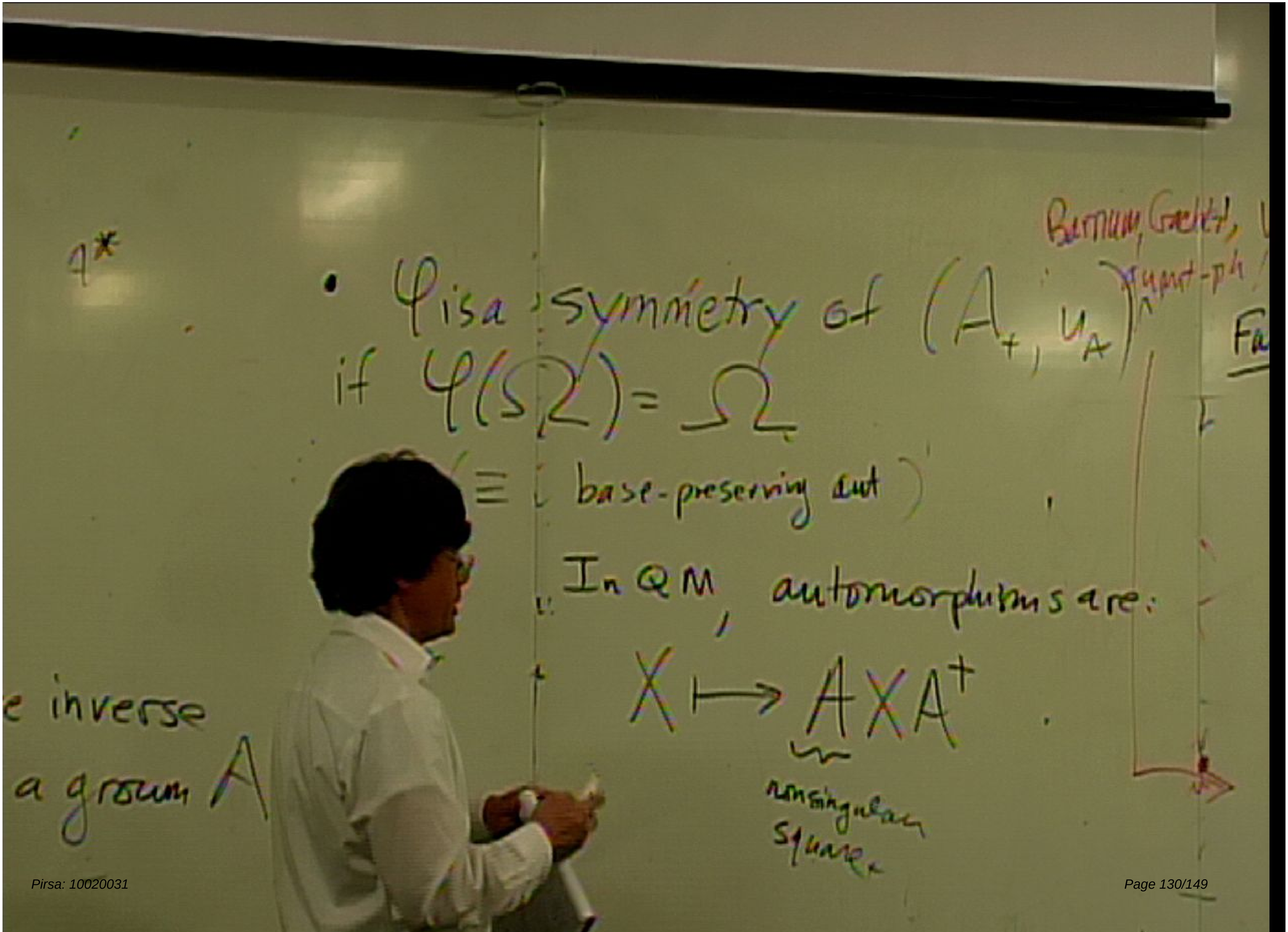
(A_*)



q^*

- φ is a symmetry of (A_+, u_A) if $\varphi(\Omega) = \Omega$

($\equiv$ base-preserving aut)

In QM, automorphisms are:

$$X \longmapsto \underset{\sim}{A} X A^\dagger .$$

nonsingular square

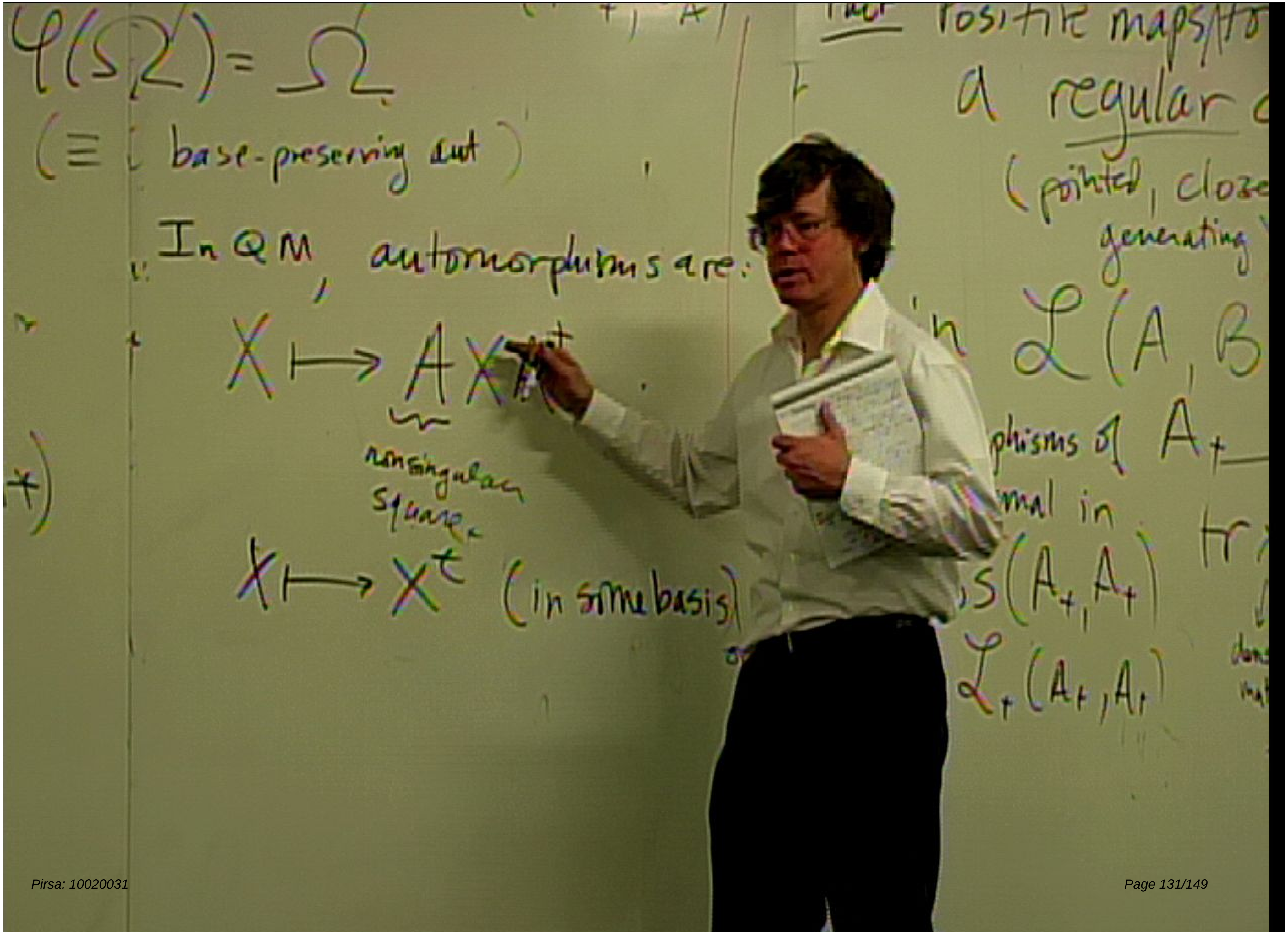
e inverse

a group A

Barnum, Gaebler,

quant-ph

Fa



$\varphi(s\Omega) = \Omega$

($\equiv$ base-preserving aut)

In QM, automorphisms are:

$X \longmapsto \underset{\sim}{A} X A^t$

nonsingular square

$X \longmapsto X^t$ (in some basis)

positive maps

a regular

(pointed, close

generating

in $\mathcal{L}(A, B$

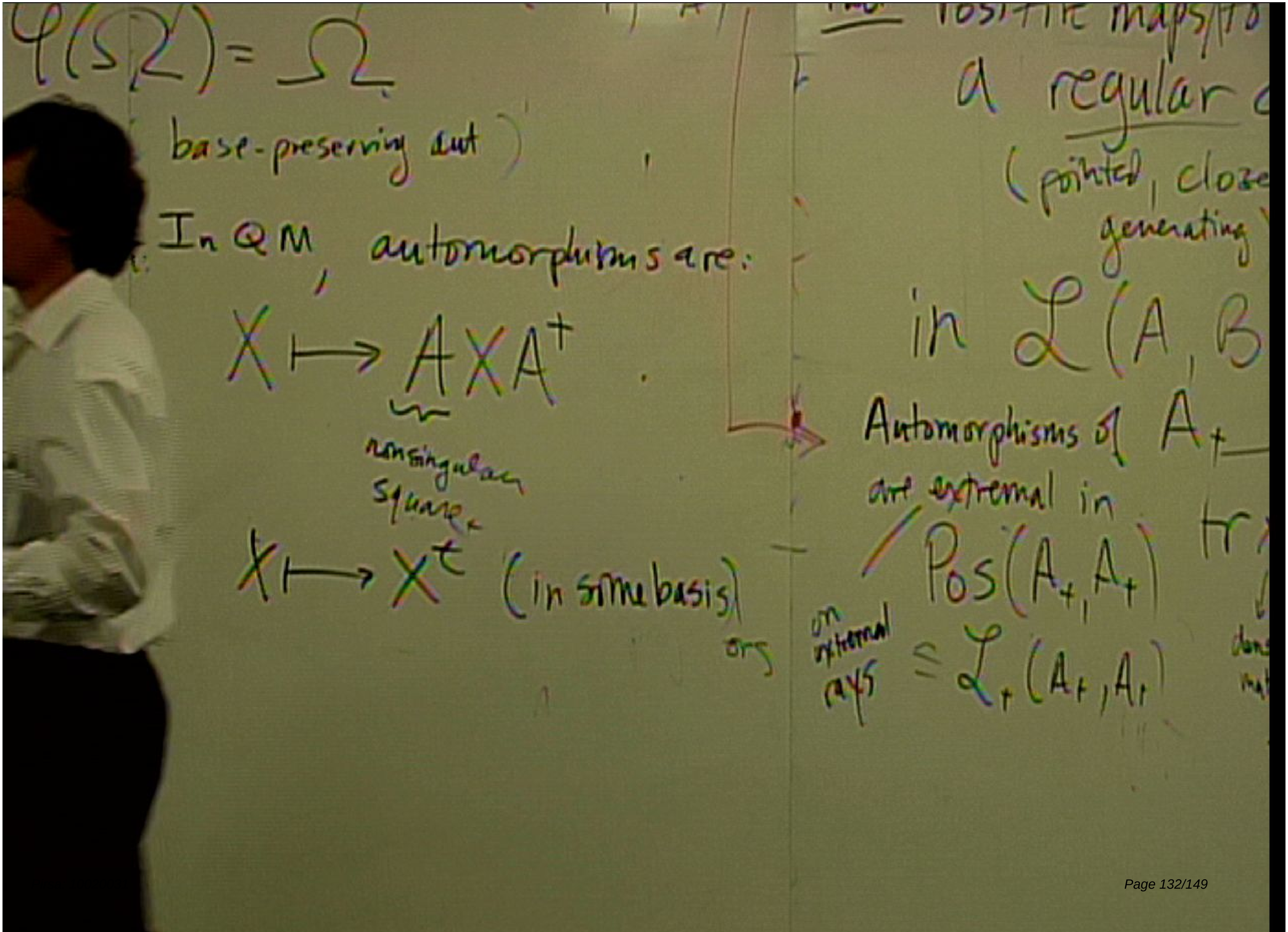
phisms of A_+

mal in

$s(A_+, A_+)$

tr

$\mathcal{L}_+(A_+, A_+)$



$\varphi(S\Omega) = \Omega$

base-preserving aut)

In QM, automorphisms are:

$X \longmapsto \underbrace{A}_{\substack{\text{nonsingular}\\ \text{square}}} X A^{\dagger}$.

$X \longmapsto X^{t}$ (in some basis)

Positive maps

a regular

(pointed, close

generating

in $\mathcal{L}(A, B$

Automorphisms of A_+

are extremal in $\mathrm{Pos}(A_+, A_+)$ tr

ors on extremal rays $\subseteq \mathcal{L}_+(A_+, A_+)$

(is a symmetry of (A_+, u_A))

if $\varphi(\Omega) = \Omega$

($\equiv$ base-preserving aut)

In QM, automorphisms are:

$$X \longmapsto \underset{\sim}{A} X A^{\dagger} .$$

nonsingular square

not completely positive $\left\{ X \longmapsto X^{t} \right.$ (in some basis)

se

$\mathrm{Aut}(A_+)$

Fact Positi

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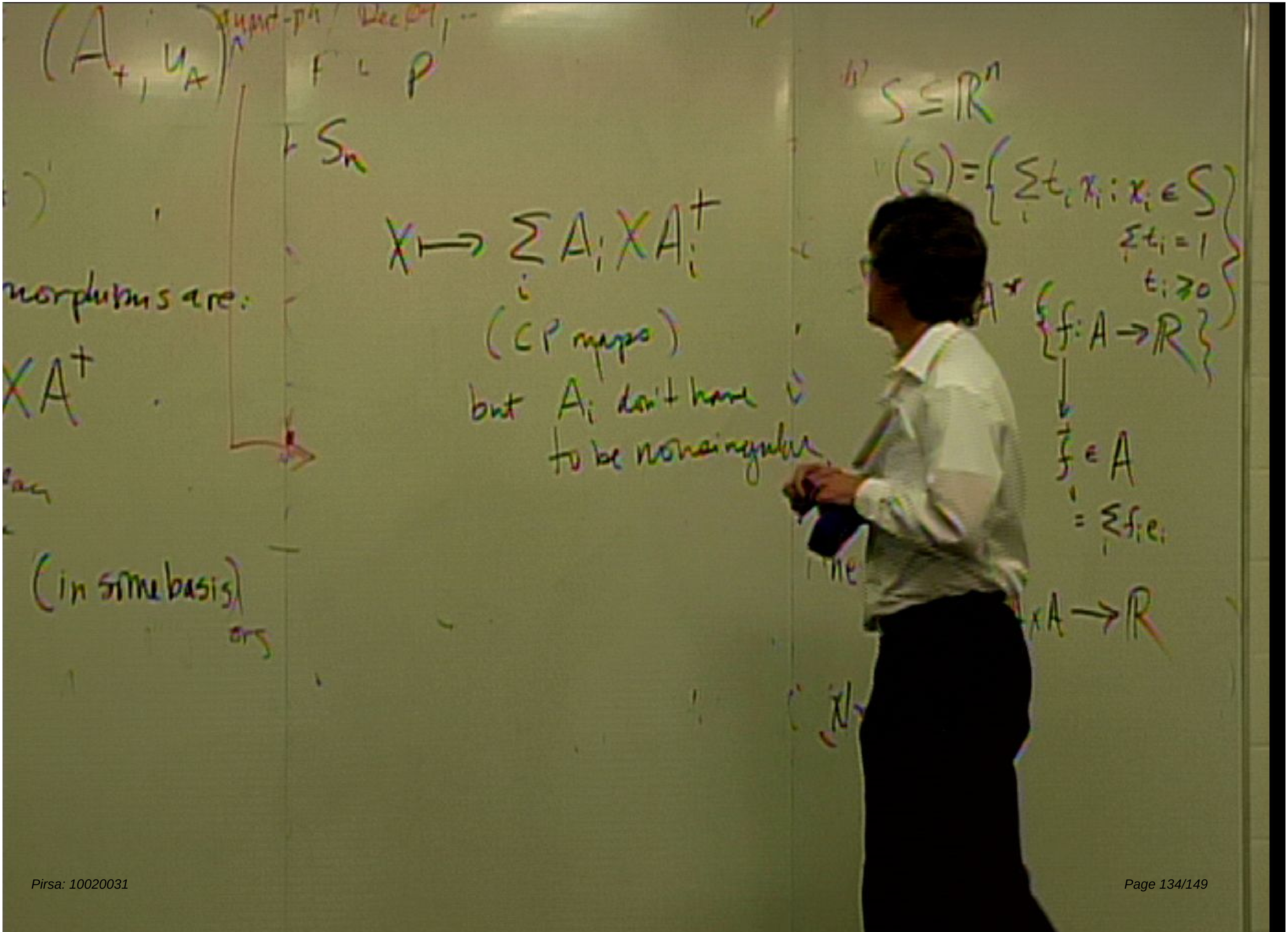
Automorphisms are extremal

$/ \; \mathrm{Pos}(A_+$

or on extremal rays $\cong \mathcal{L}_+(A$



(A_+, u_A) quant-ph / Dec 04, ...

ρ

S_n

norphisms are:

$XA^\dagger$

(in some basis)

ors

$$X \mapsto \sum_i A_i X A_i^\dagger$$

(CP maps)

but A_i don't have to be nonsingular

$S \subseteq \mathbb{R}^n$

$$(S) = \left\{ \sum_i t_i x_i : x_i \in S,\ \sum_i t_i = 1,\ t_i \geq 0 \right\}$$

$A^* \{ f : A \to \mathbb{R} \}$

$\vec{f} \in A$

$= \sum_i f_i e_i$

$\times A \to \mathbb{R}$

, Wilce

ph / Dec 09, ...

$f \quad L \quad \rho$

S_n

$$X \longmapsto \sum_i A_i X A_i^{\dagger}$$

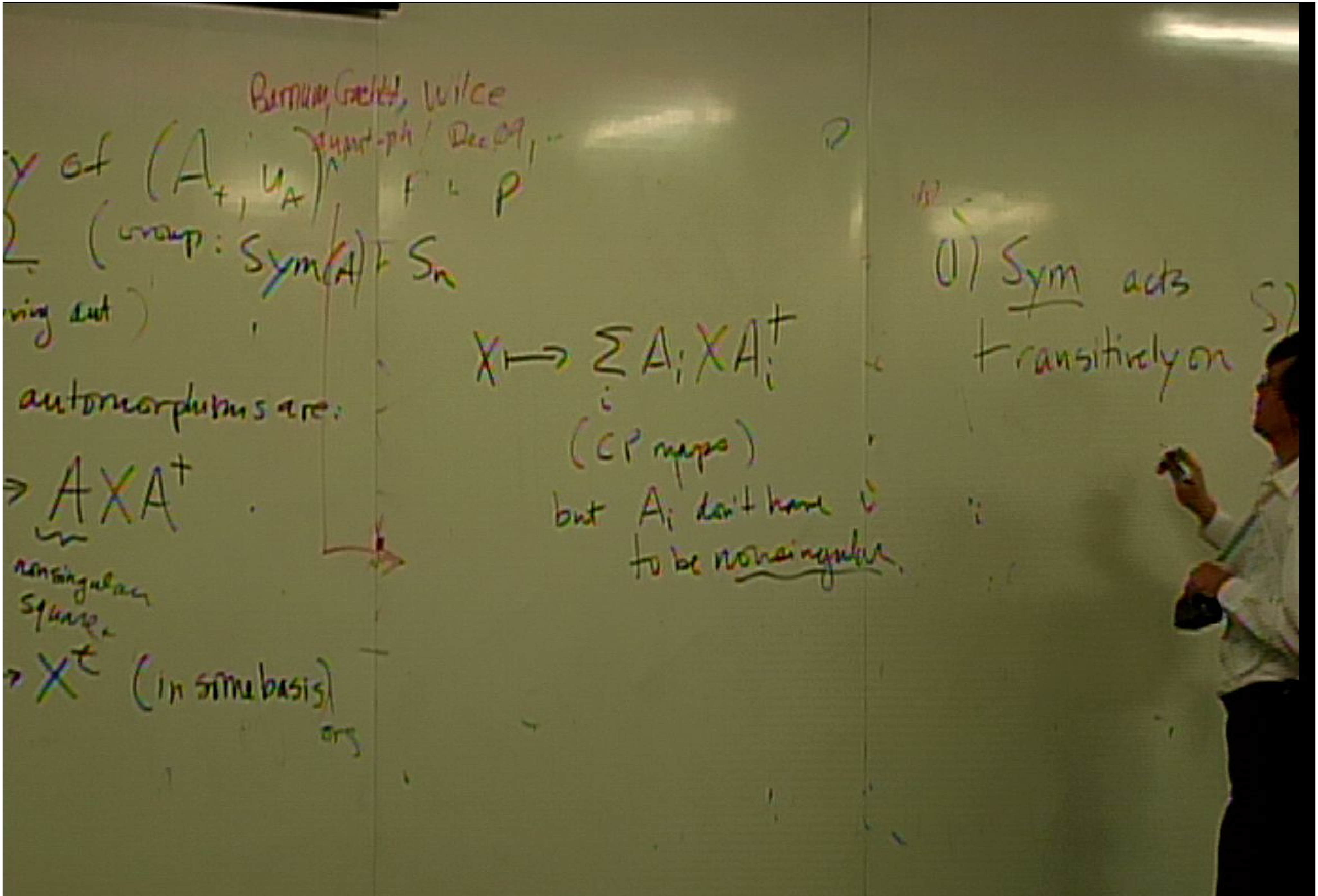
(CP maps)

but A_i don't have

to be nonsingular.

(1) Sym acts

+



Barnum, Gaebler, Wilce
quant-ph / Dec 09, ...

y of (A_+, u_A) F L ρ

Σ (group: Sym(A) F S_n

ving out)

automorphisms are:

$\to \underset{\sim}{A} X A^\dagger$

nonsingular square

$\to X^t$ (in some basis)

org

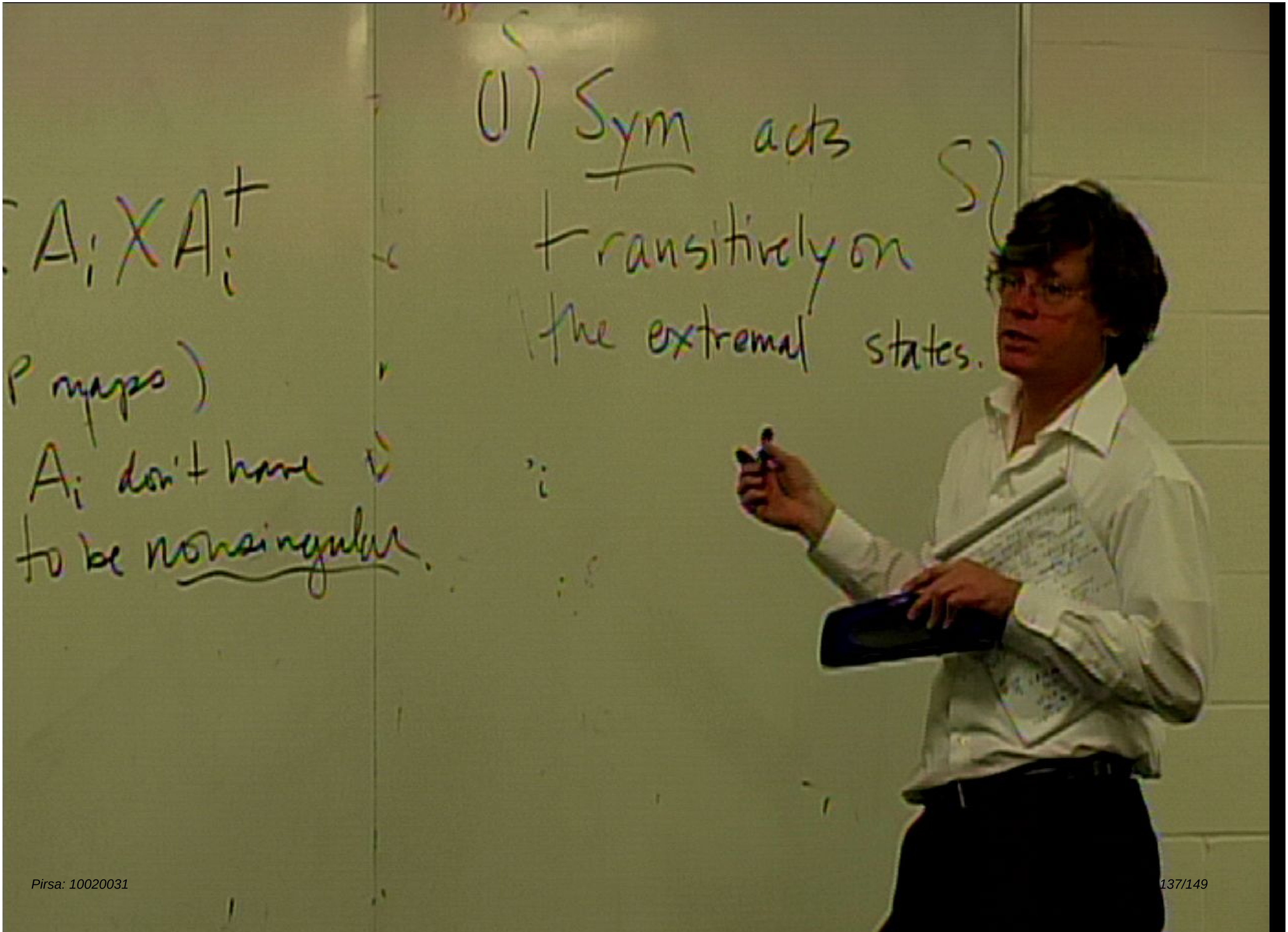
$X \mapsto \sum_i A_i X A_i^\dagger$

(CP maps)

but A_i don't have to be nonsingular.

(1) Sym acts transitively on

S)

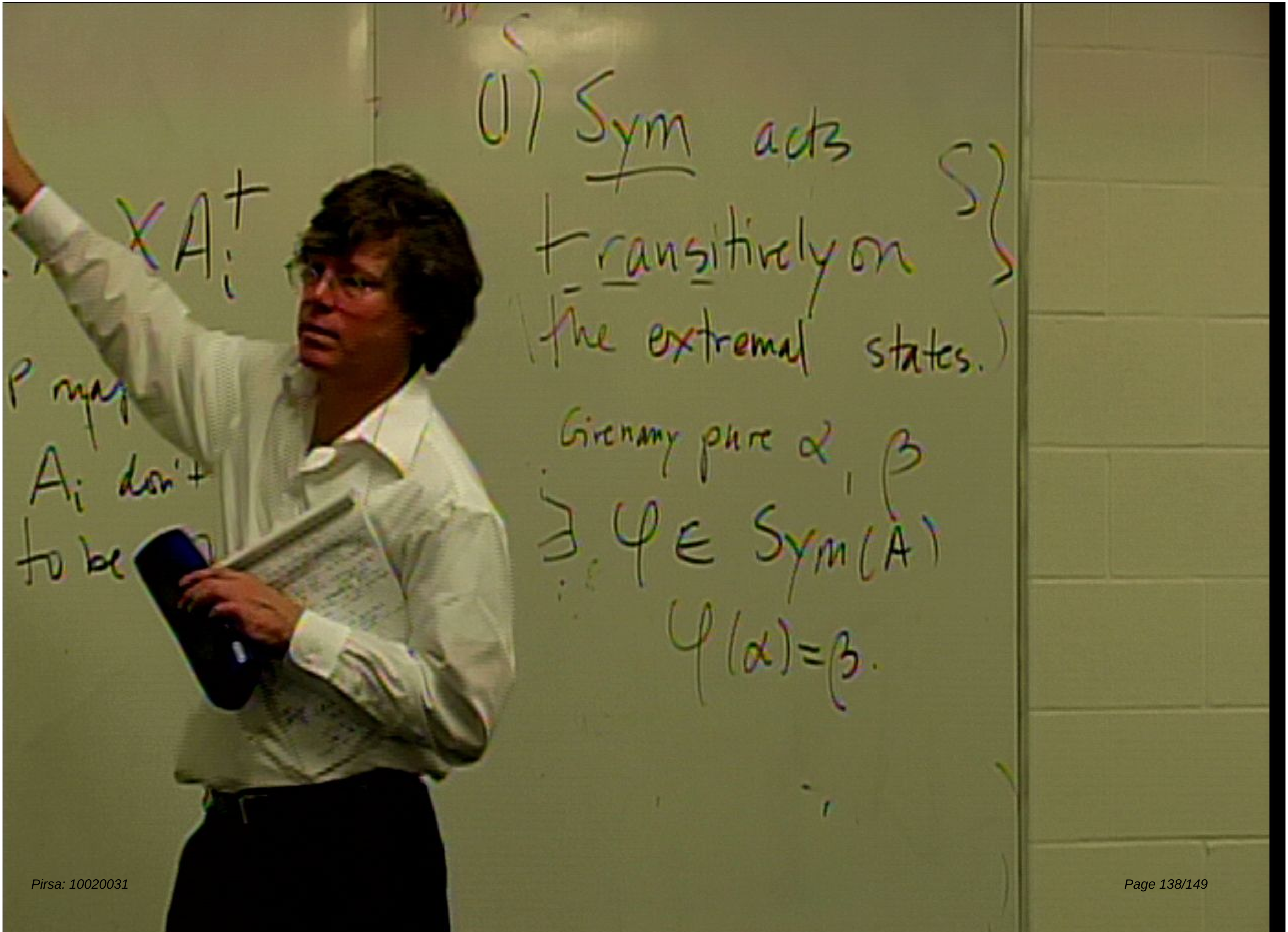


$A_i X A_i^\dagger$

P maps)

A_i don't have to be nonsingular.

(1) Sym acts transitively on the extremal states.



(1) Sym acts transitively on the extremal states.

Given any pure α, β

$\exists\ \varphi \in Sym(A)$

$\varphi(\alpha) = \beta.$

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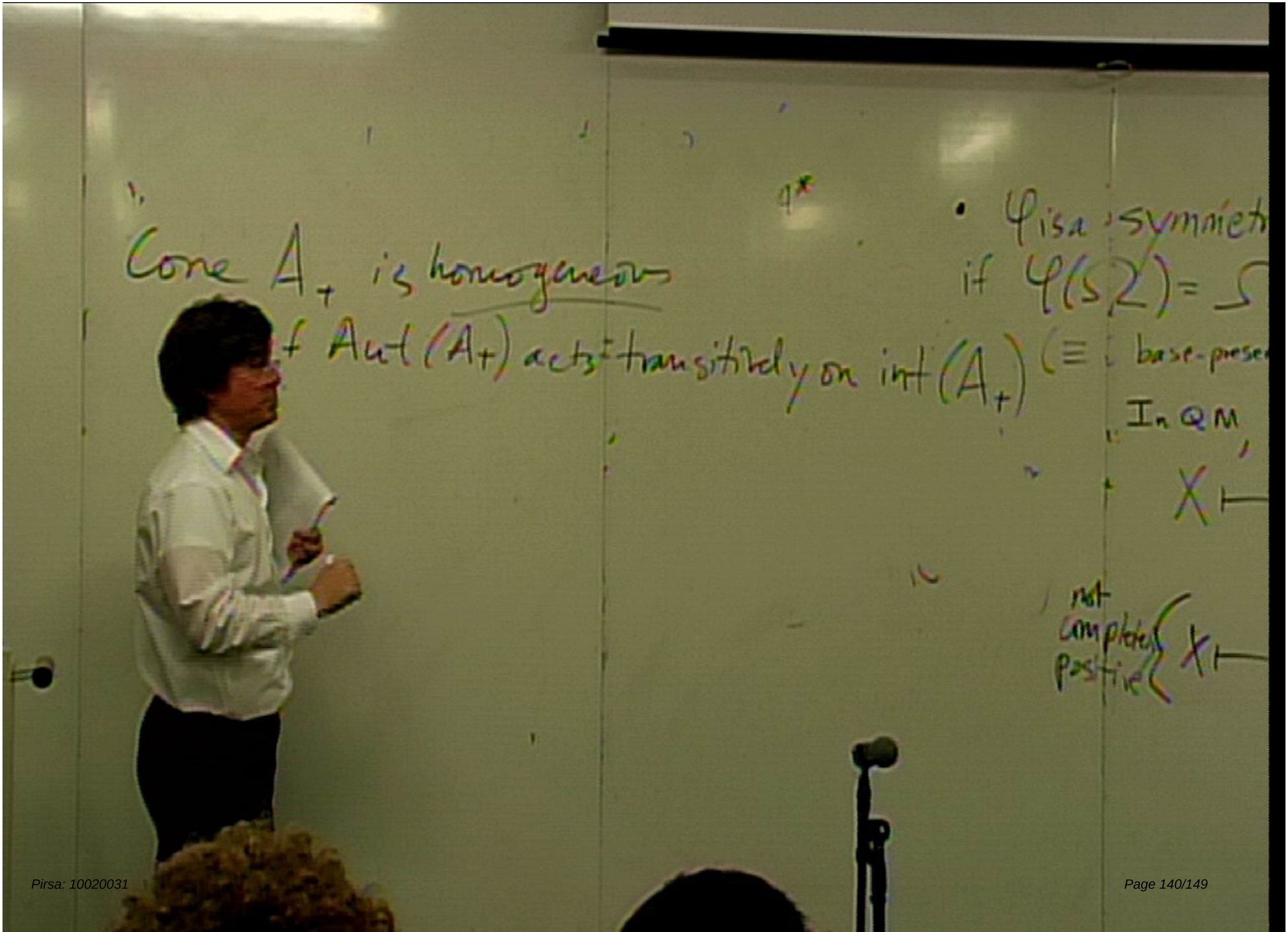


Cone A_+ is homogeneous

if Aut(A_+) acts transitively on int(A_+) $(\equiv$ base-prese

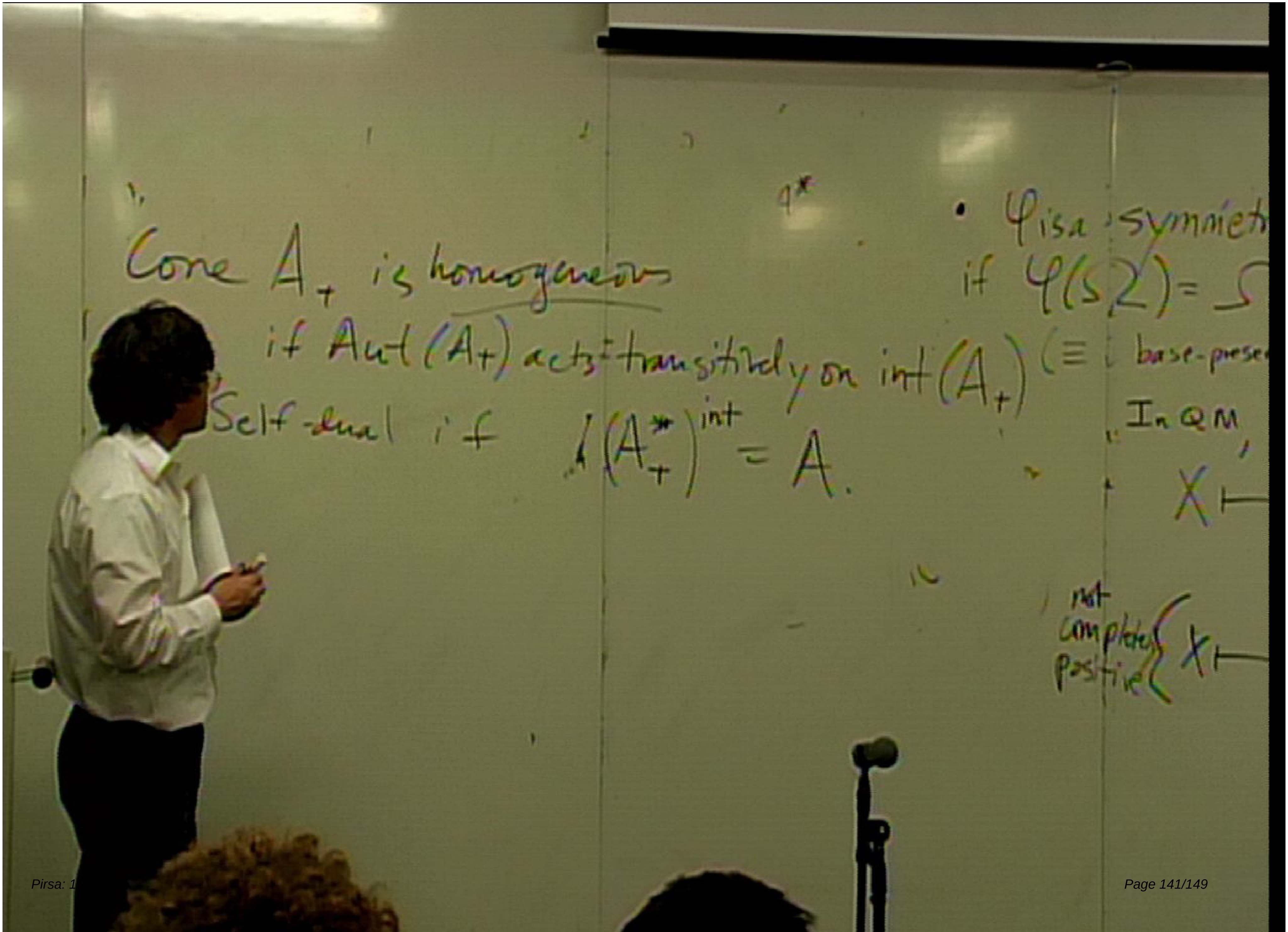
- φ is a symmet

if $\varphi(s, \Sigma) = \Sigma$

In QM,

$X \mapsto$

not completely positive $\{ X \mapsto$



Cone A_+ is homogeneous

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Self-dual if $(A^*_+)^{int} = A$.

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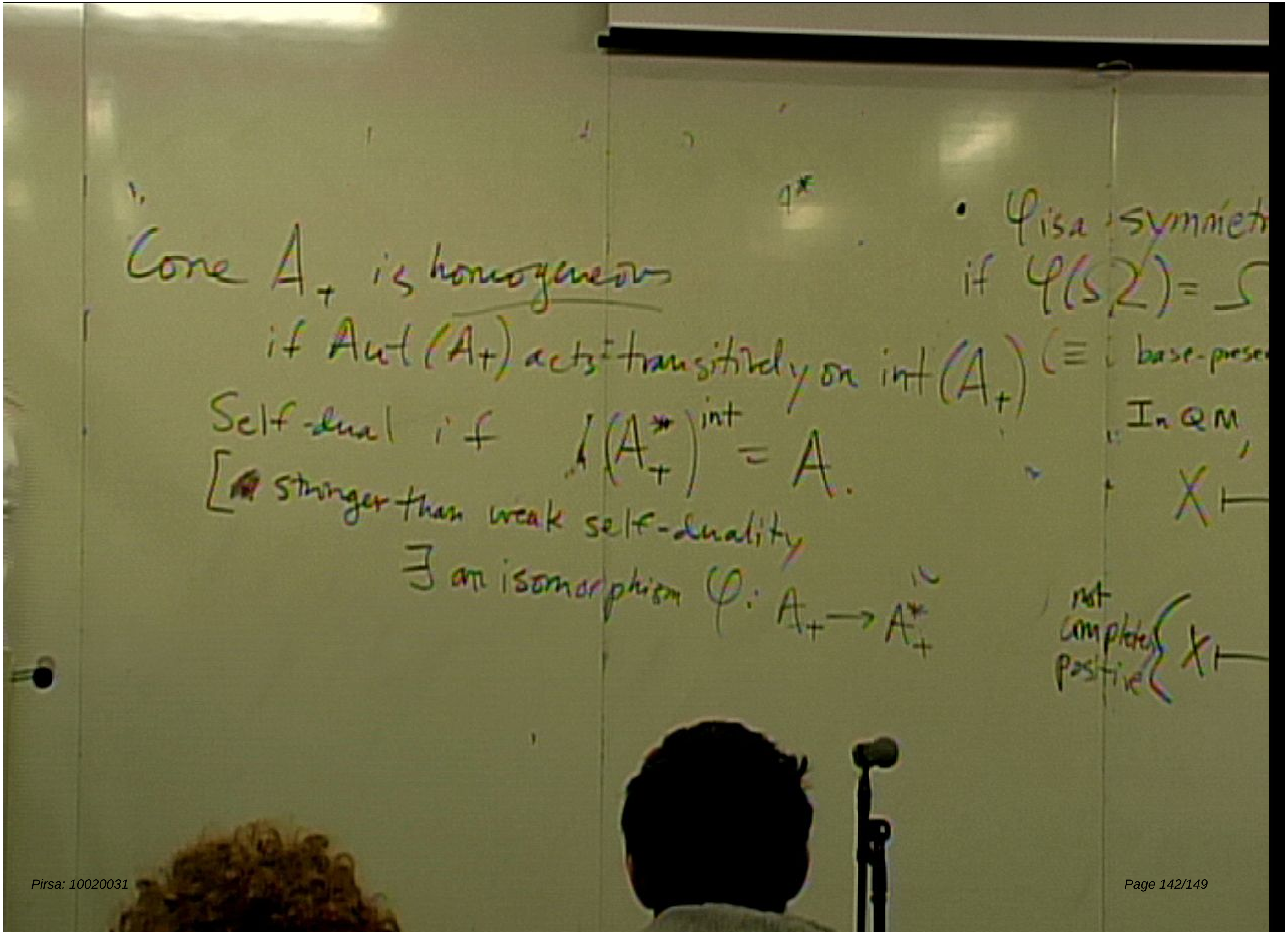

q^*

Cone A_+ is homogeneous

if $\mathrm{Aut}(A_+)$ acts transitively on $\mathrm{int}(A_+)$ $($≡ base-prese

Self-dual if $(A_+^*)^{int} = A$.

[stronger than weak self-duality

$\exists$ an isomorphism $\varphi : A_+ \to A_+^*$

- φ is a symmet

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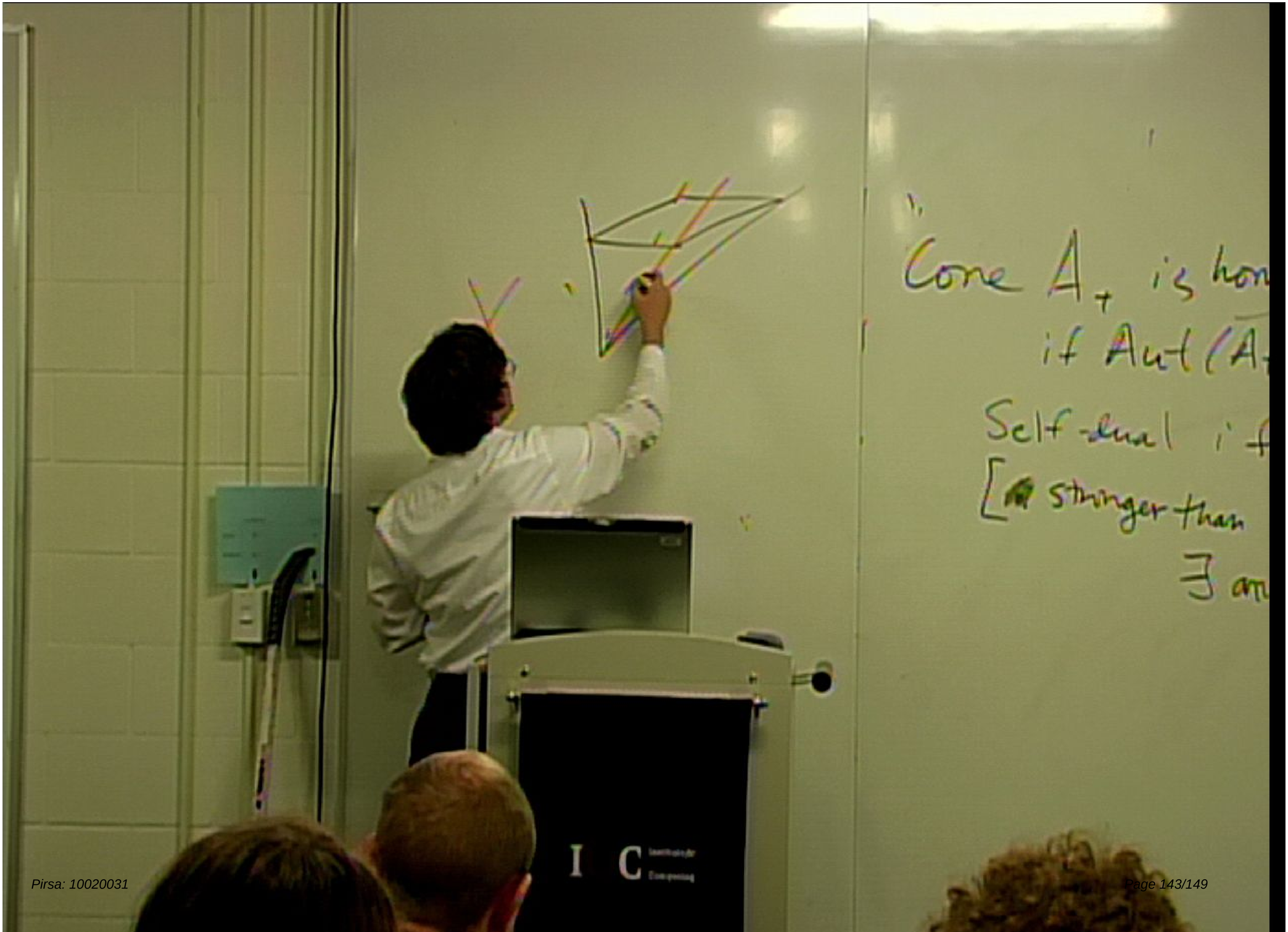

Cone A_+ is hon

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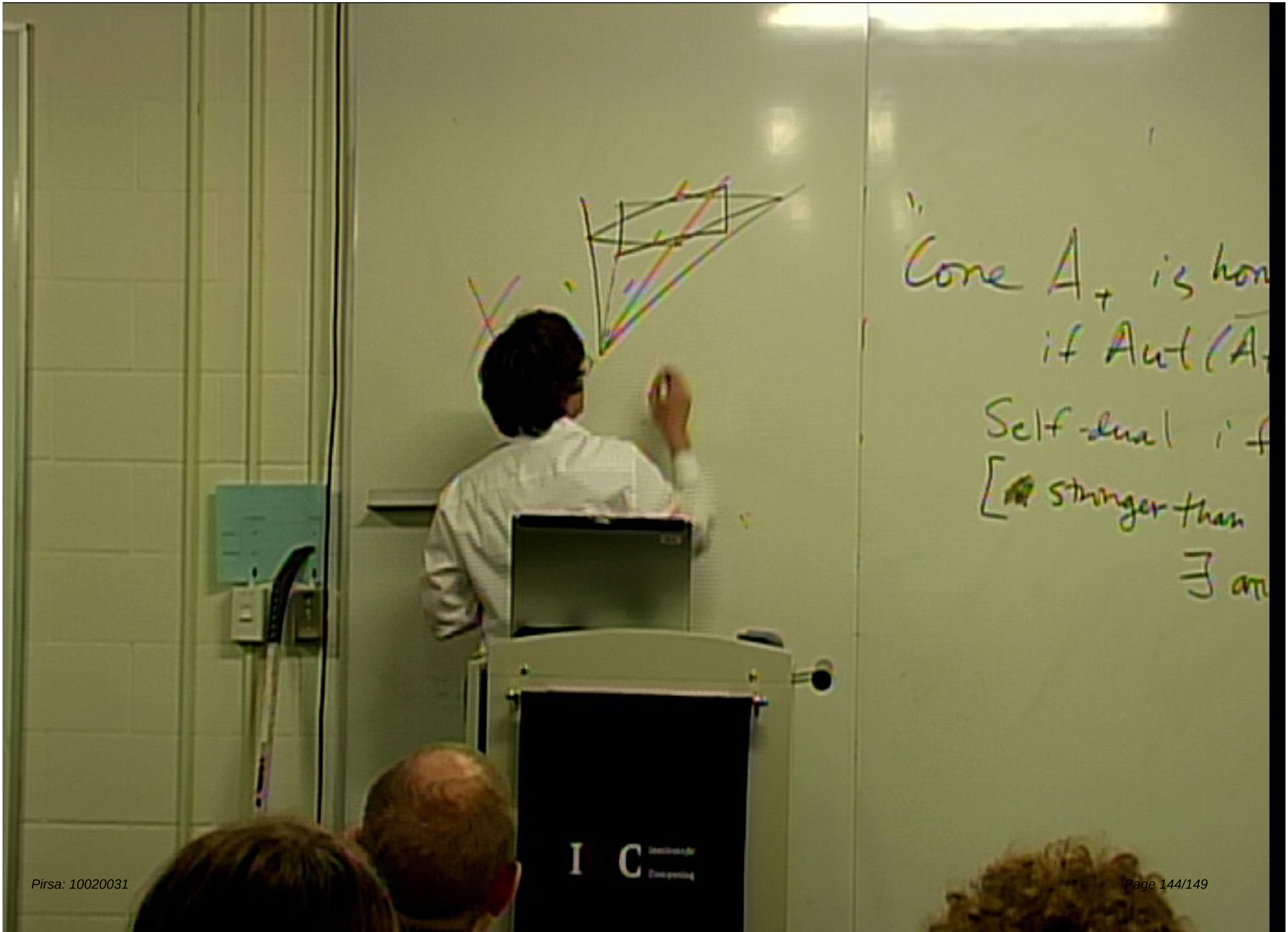
Self-dual if

[stronger than

$\exists$ an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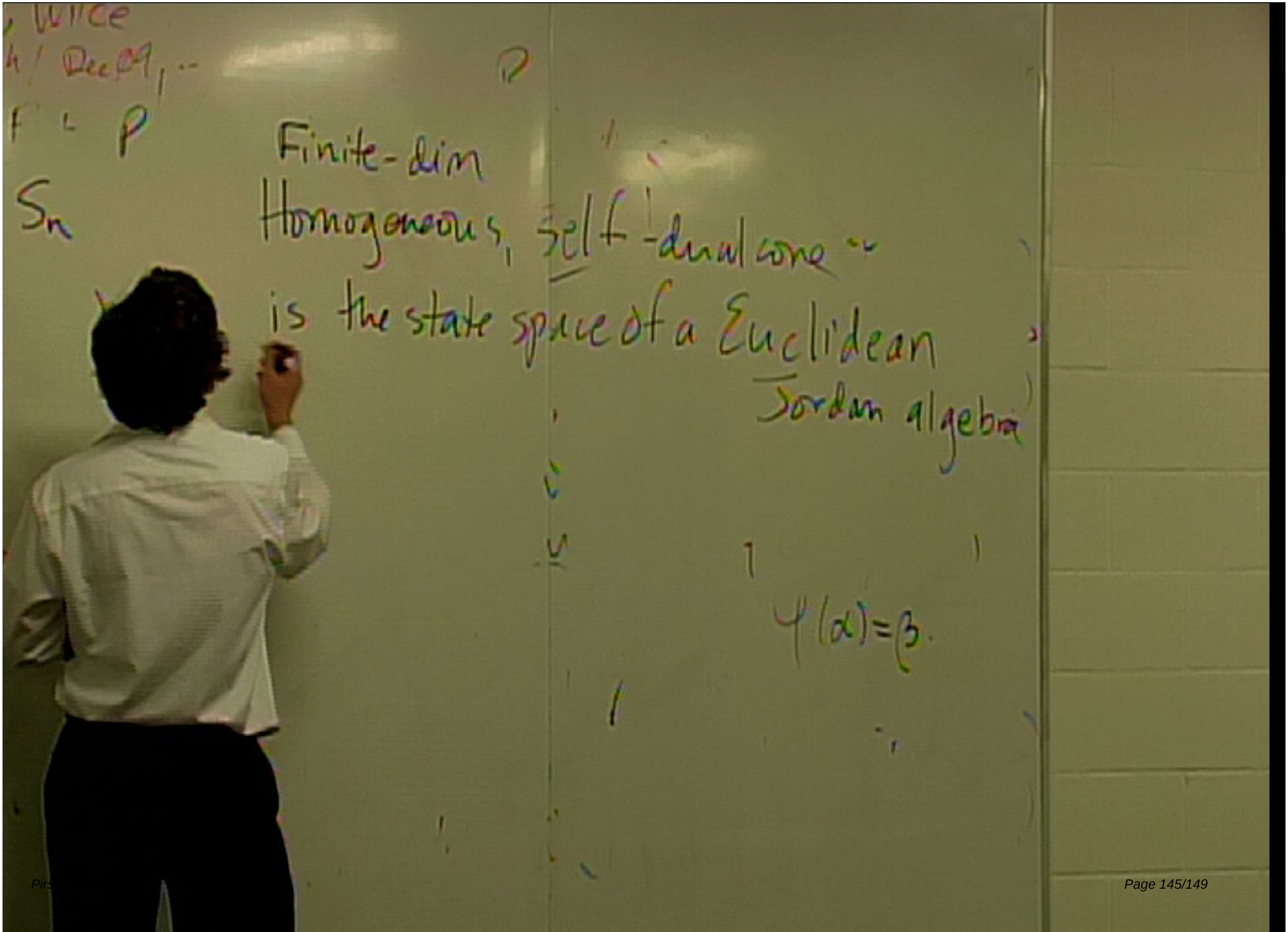

, Wilce

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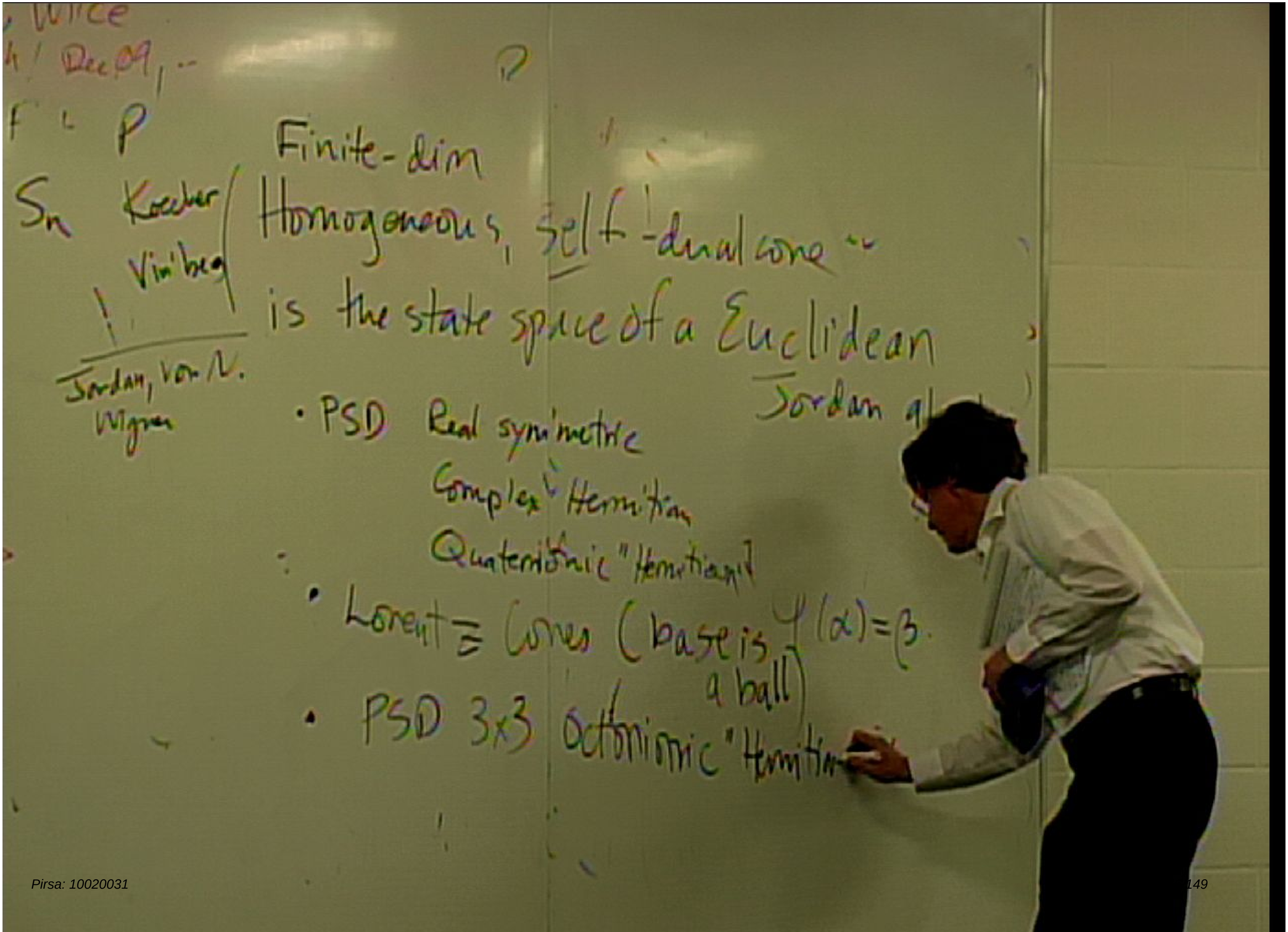
S_n

Finite-dim

Homogeneous, self-dual cone

is the state space of a Euclidean Jordan algebra

$\varphi(\alpha)=\beta$.



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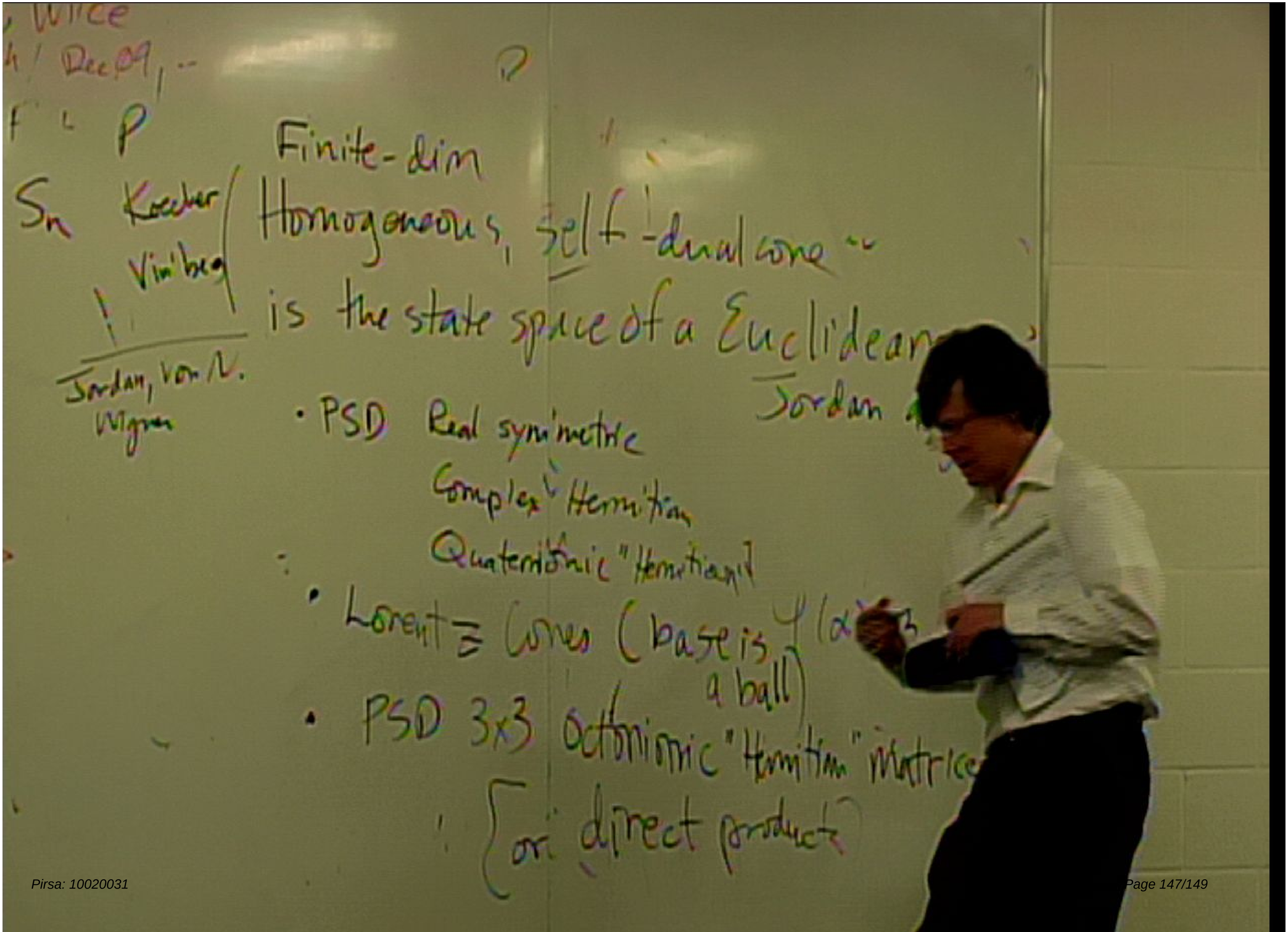
F L P

Sn Koecher / Vin'berg

Jordan, von N.
Wigner

Finite-dim
Homogeneous, self-dual cone ..
is the state space of a Euclidean
Jordan al

- PSD Real symmetric
  Complex Hermitian
  Quaternionic "Hermitian"
- Lorentz cones (base is a ball) $\varphi(\alpha)=\beta$
- PSD 3x3 Octonionic "Hermitian"



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{or direct products

Wilce
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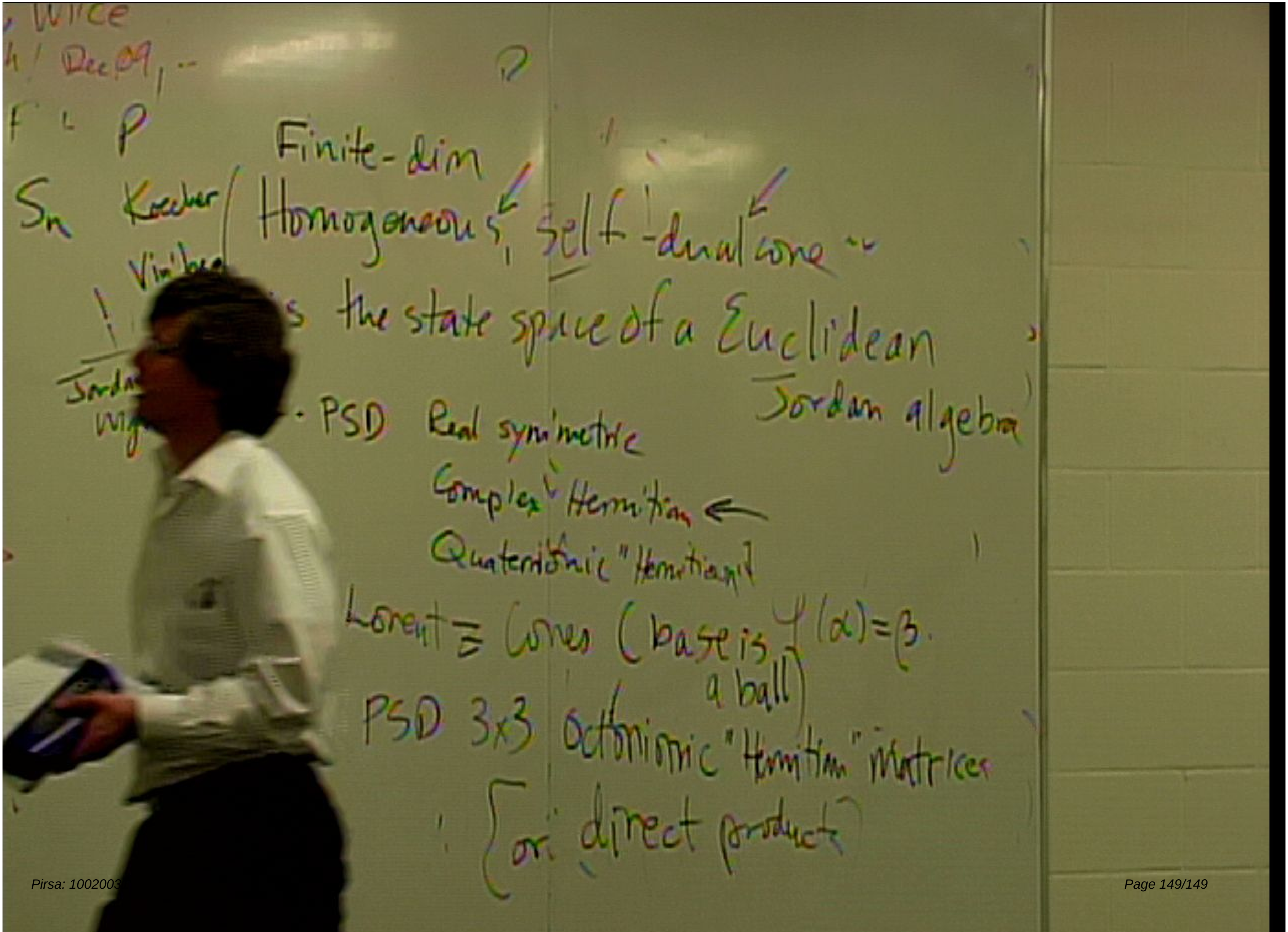

Wilce

Dec 09, ...

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