Title: Foundations and Interpretation of Quantum Theory - Lecture 9

Date: Feb 11, 2010 02:30 PM

URL: http://pirsa.org/10020029

Abstract: After a review of the axiomatic formulation of quantum theory, the generalized operational structure of the theory will be introduced (including POVM measurements, sequential measurements, and CP maps). There will be an introduction to the orthodox (sometimes called Copenhagen) interpretation of quantum mechanics and the historical problems/issues/debates regarding that interpretation, in particular, the measurement problem and the EPR paradox, and a discussion of contemporary views on these topics. The majority of the course lectures will consist of guest lectures from international experts covering the various approaches to the interpretation of quantum theory (in particular, many-worlds, de Broglie-Bohm, consistent/decoherent histories, and statistical/epistemic interpretations, as time permits) and fundamental properties and tests of quantum theory (such as entanglement and experimental tests of Bell inequalities, contextuality, macroscopic quantum phenomena, and the problem of quantum gravity, as time permits).

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Foundations and Interpretations of Quantum Mechanics

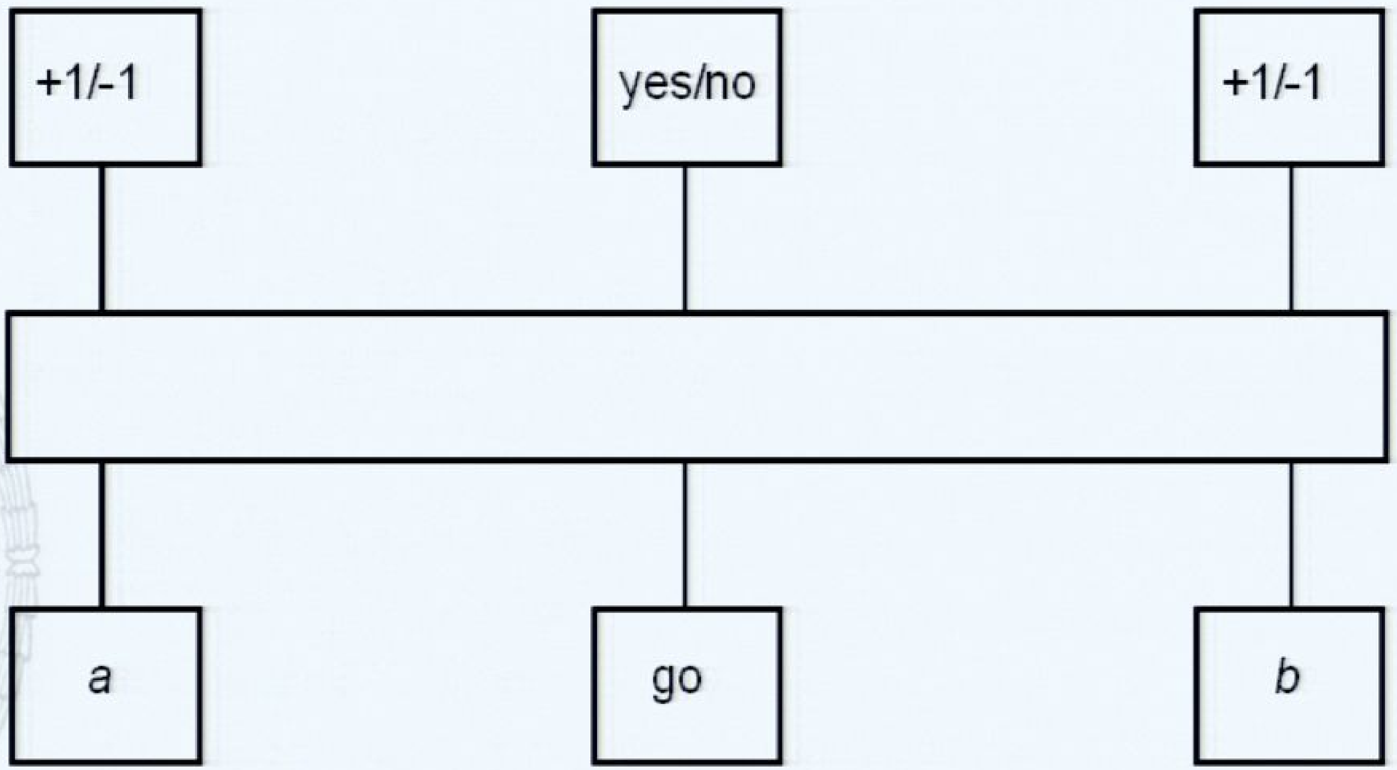
Experimental Tests of Bell's inequalities

Gregor Weihs

Lecture 2

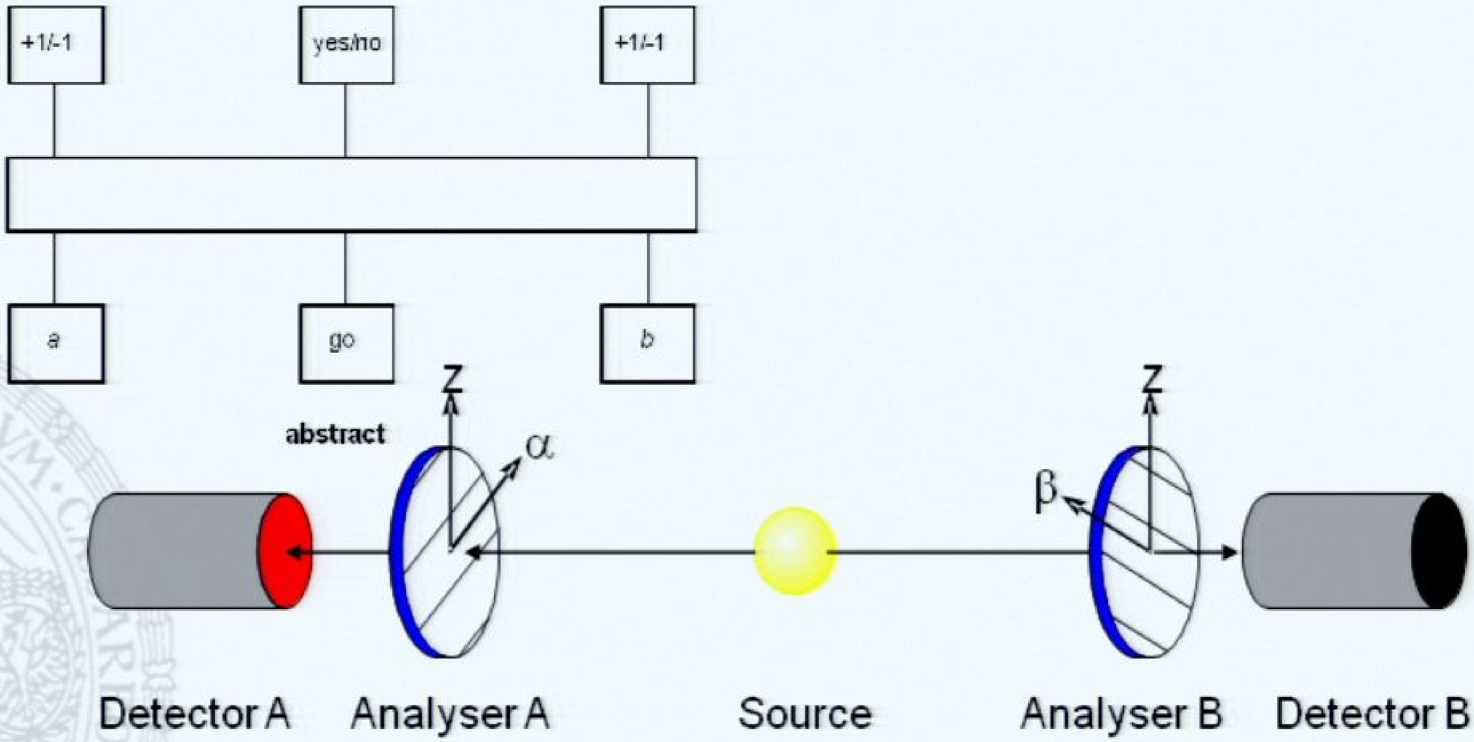
- Connection to Experiment
- Early Experiments
- Experimental Problems
- Loopholes / Supplementary assumptions
- The CH inequality
- Aspect's experiments
- The locality loophole
- Freedom of choice
- Trapped Ions and the efficiency loophole
- Conclusions

From theory to experiment



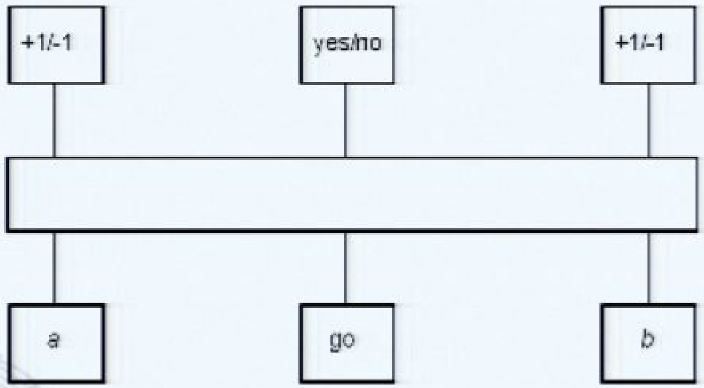
abstract

From theory to experiment

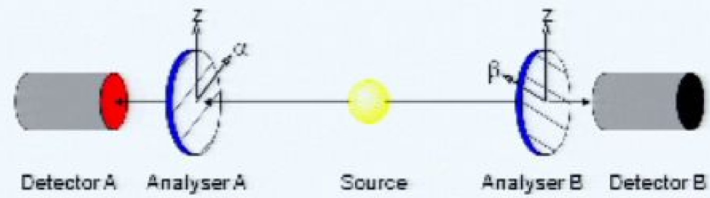


physical

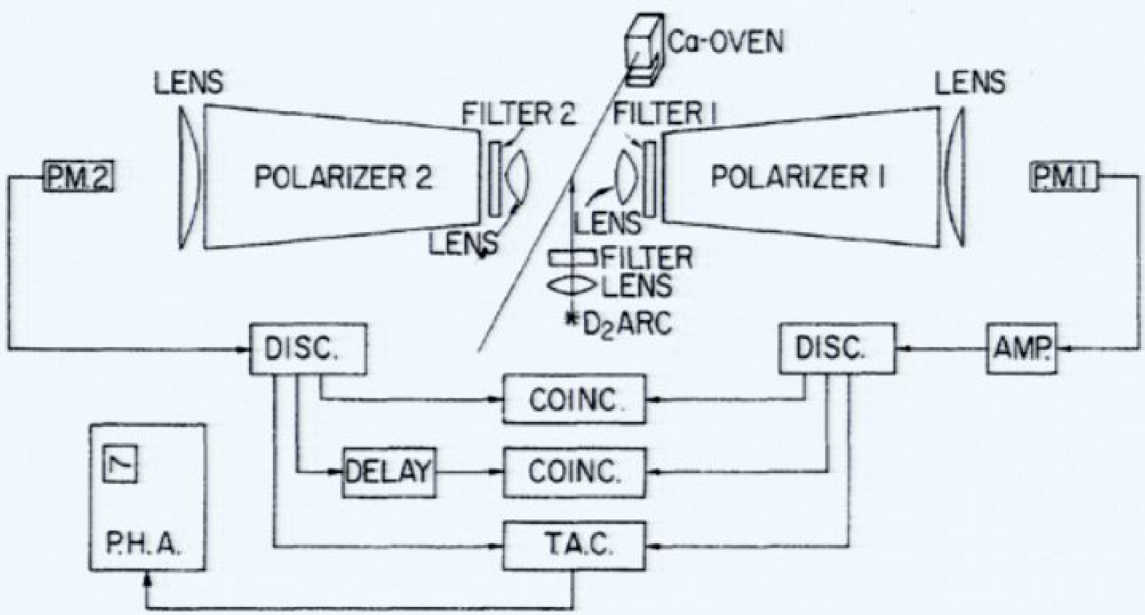
From theory to experiment



abstract

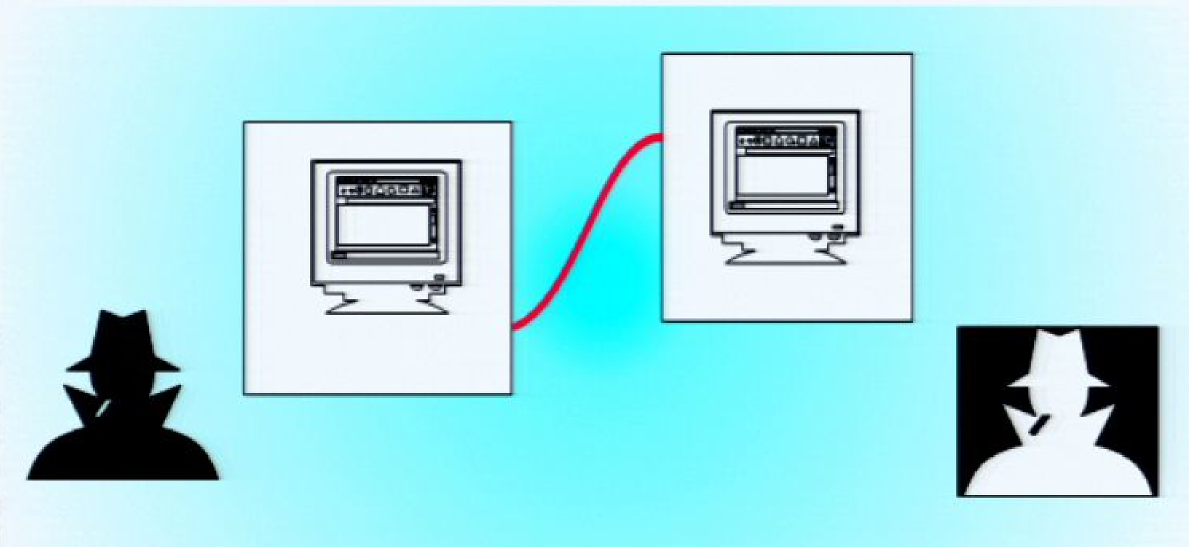


physical

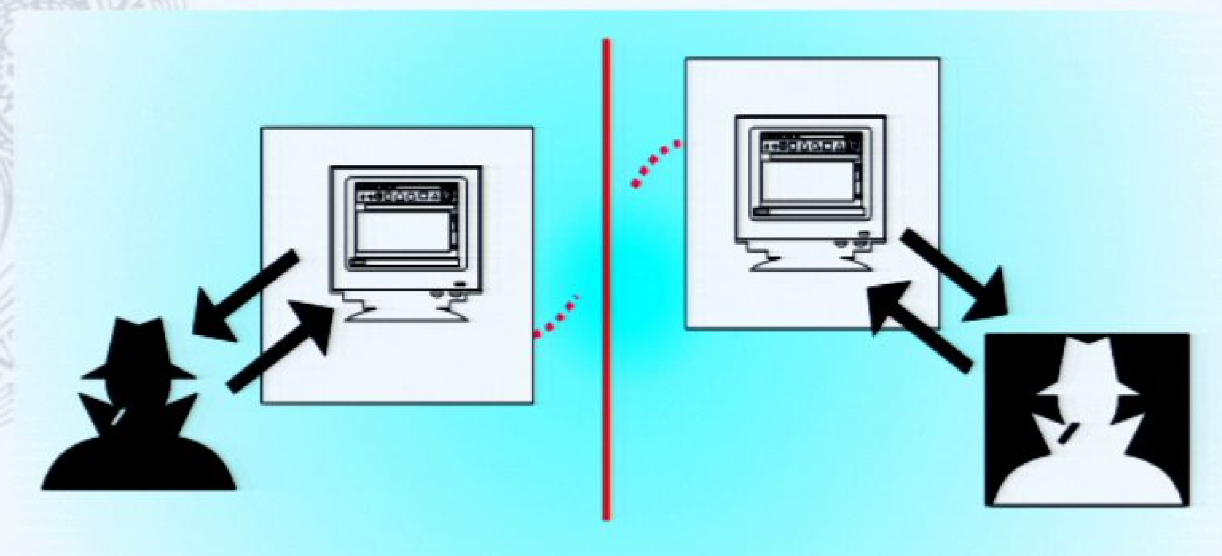


real

Local realism – the Accardi-Gill bet (€1000,-)

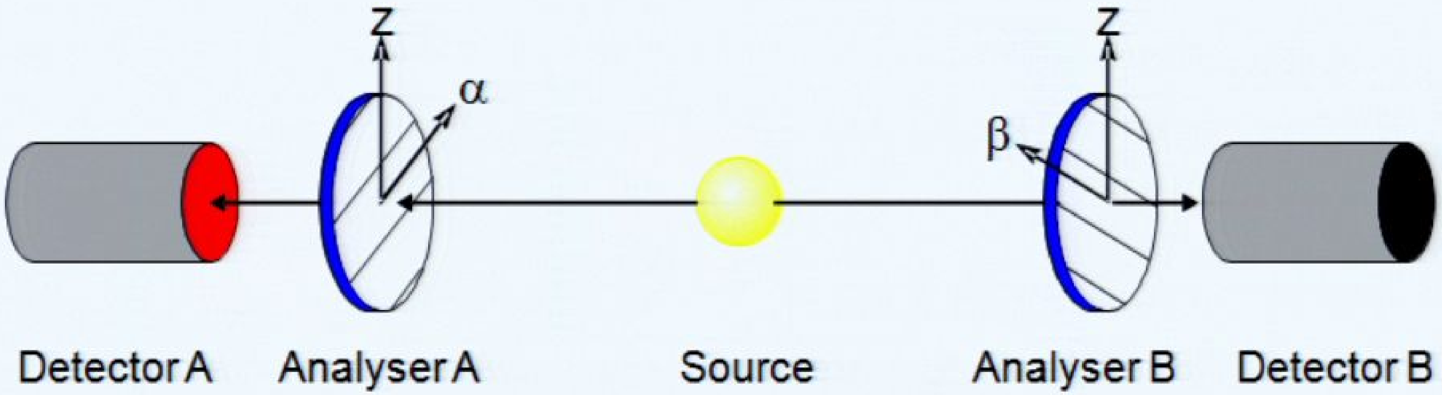


Step 1: computers connected, devise strategy



Step 2: without communication computers respond to 2 possible inputs

CHSH Inequality



$$E(\mathbf{a},\mathbf{b}) = p_{++}(\mathbf{a},\mathbf{b}) + p_{--}(\mathbf{a},\mathbf{b}) - p_{+-}(\mathbf{a},\mathbf{b}) - p_{-+}(\mathbf{a},\mathbf{b})$$

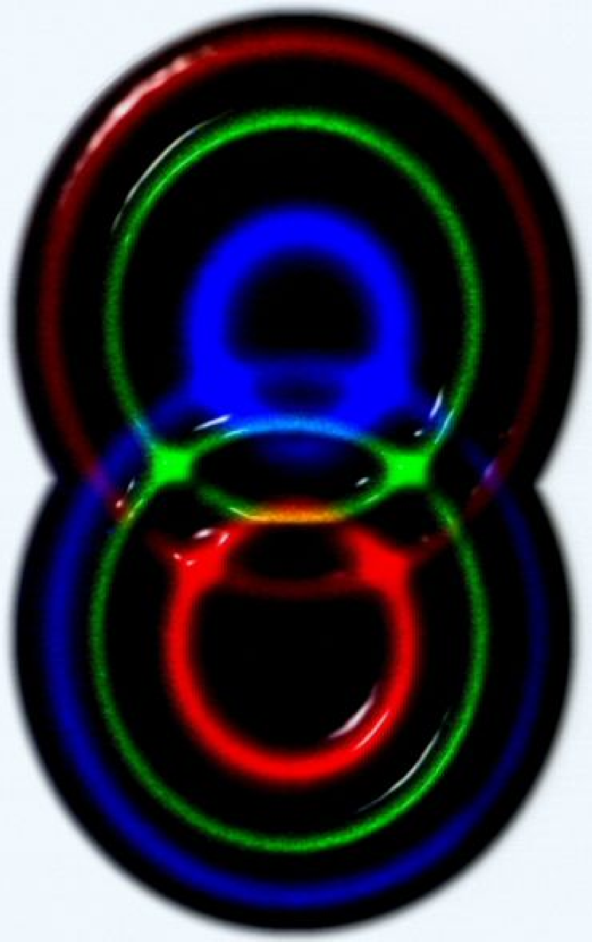
$$S(\mathbf{a},\mathbf{b},\mathbf{a}',\mathbf{b}') := |E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}')| + |E(\mathbf{a}',\mathbf{b}') + E(\mathbf{a}',\mathbf{b})| \leq 2$$

$$E^{\mathrm{qm}}(\mathbf{a},\mathbf{b}) = -\cos(\beta - \alpha)$$

$$\begin{aligned} S(0^\circ, 45^\circ, 90^\circ, 135^\circ) &= |-\cos(45^\circ) + \cos(135^\circ)| + |-\cos(45^\circ) - \cos(-45^\circ)| \\ &= |-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}| + |-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}| = 2\sqrt{2} \geq 2 \end{aligned}$$

Experiments

- **Produce** entangled state
  - Ready signal
- **Collect** particles
  - Collection efficiency
- **Analyze**
  - Accuracy
  - Randomness
- **Detect**
  - Efficiency
  - Simultaneity
  - Background, Extra events
- **Record** values
  - When is the measurement complete?
- **Correlate** values

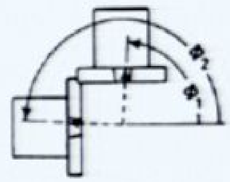


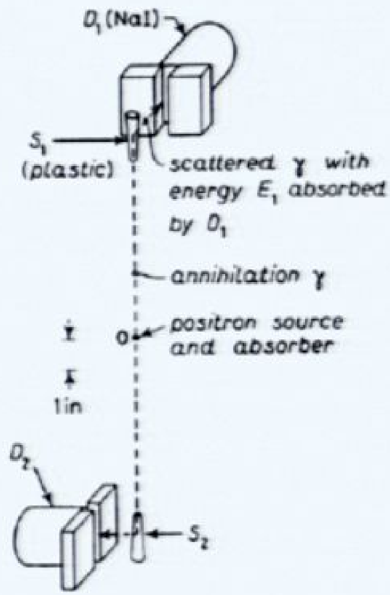
Problems

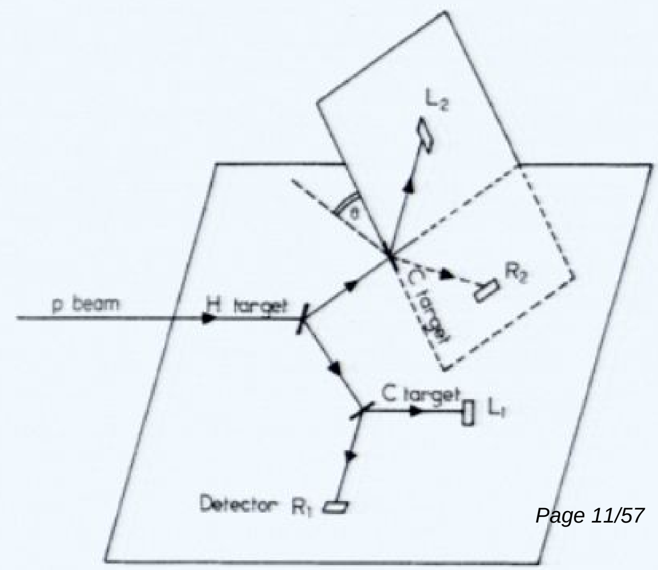
- Statistics: can only measure each particle under a particular angle
- Normalization: can only measure absolute frequencies
- How do we treat unmeasured pairs (at finite efficiency)?
- How do we treat multiple detection on one side?

Systems

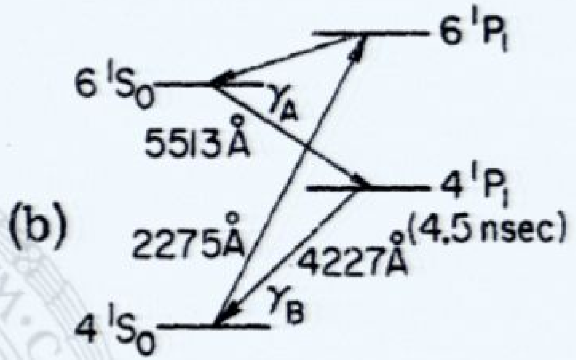
- Low Energy Photons
  - Good analyzers
  - Good sources
  - Poor detection
  - All degrees of freedom useable
- High Energy Photons from positronium annihilation
  - Poor analyzers
  - Good detection
- Proton-Proton Scattering
- Y(4S) ➔ B^0-B^0 Decay
- Trapped Ions

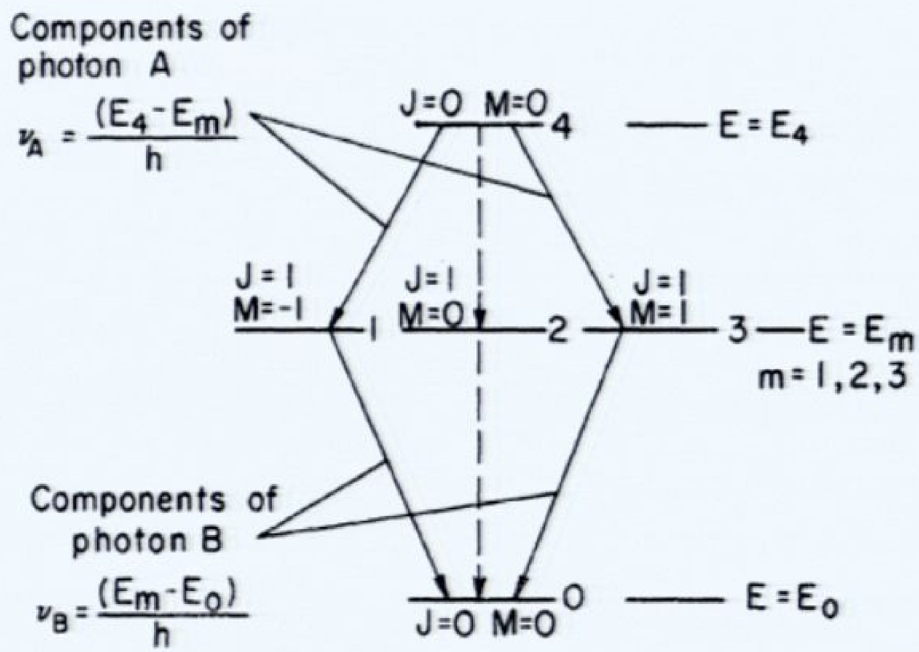






Atomic Sources

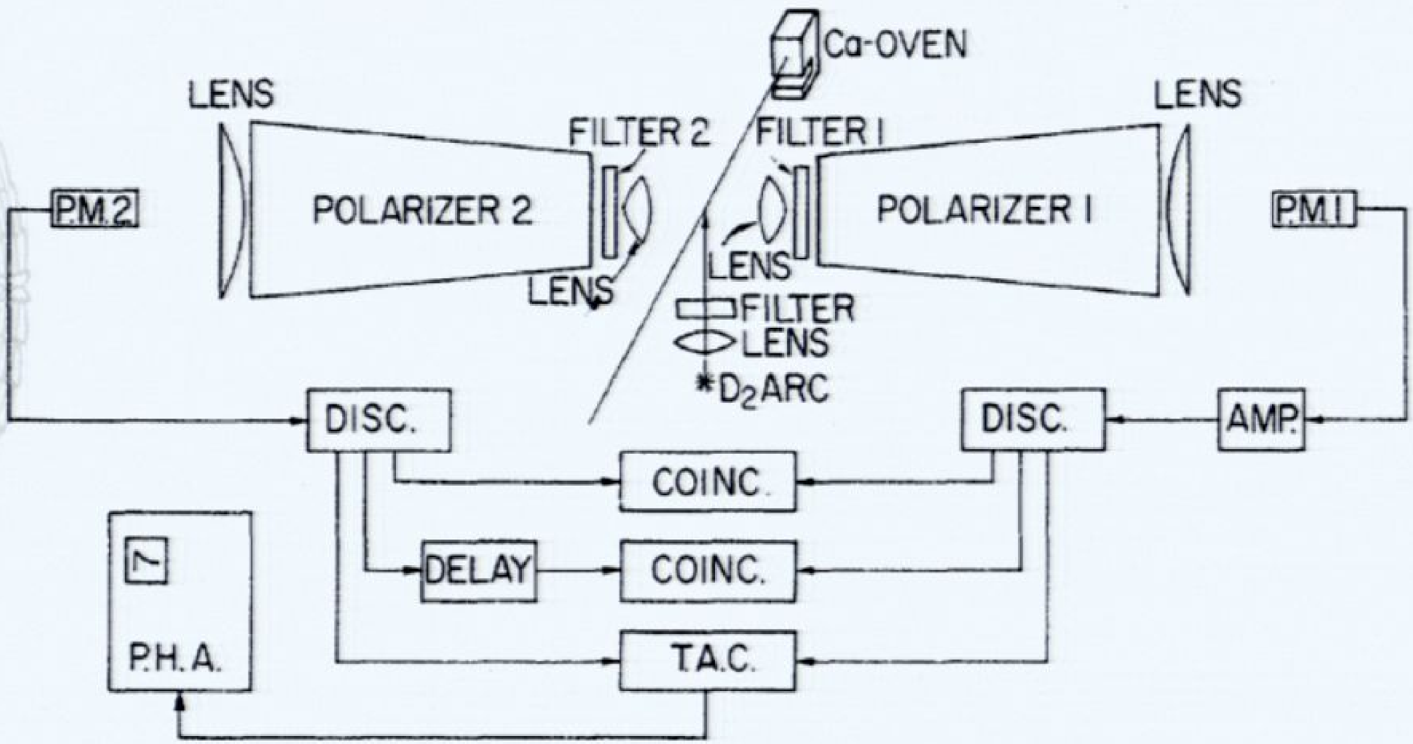




- Two indistinguishable decay paths yield polarization entanglement
- Three-body process results in poor directional correlation ➔ collection efficiency
- Lamp pumped sources (e.g. Clause): 0.3 detected pairs / second
- Laser pumped sources (e.g. Aspect): 100 detected pairs / second

Freedman & Clauser (1972)

- Ca-Cascade pumped by D_2 arc lamp
- No directional correlation of the photon pairs (3-body process)
- 0.2 coincidences / s
- 0.05 accidental coincidences /s



S. J. Freedman & J. F. Clauser, *Experimental test of local hidden-variable theories,*
Phys. Rev. Lett. **28**, 938-941 (1972)

Loopholes: Efficiency / Rotational Invariance

1. Choose settings a, a', b, b' independently and randomly
2. Count events
3. Normalize by total number of **coincident** events
   = assuming **fair sampling**

$$p_{++}(a,b) = \frac{N_{++}(a,b)}{N(a,b)} \rightarrow p_{++}(a,b) = \frac{N_{++}(a,b)}{N_{++}+N_{+-}+N_{-+}+N_{--}}$$

4. Calculate correlations

$$E(a,b) = p_{++}(a,b)+p_{--}(a,b)-p_{+-}(a,b)-p_{-+}(a,b)$$

$$S_{\mathrm{CHSH}} = |E(a,b) - E(a,b')| + |E(a',b) + E(a',b')|$$

5. Subtract accidental coincidences???
6. Rotational invariance: the source produces a singlet (rotationally invariant state, therefore only the relative angle is important.

The CH inequality

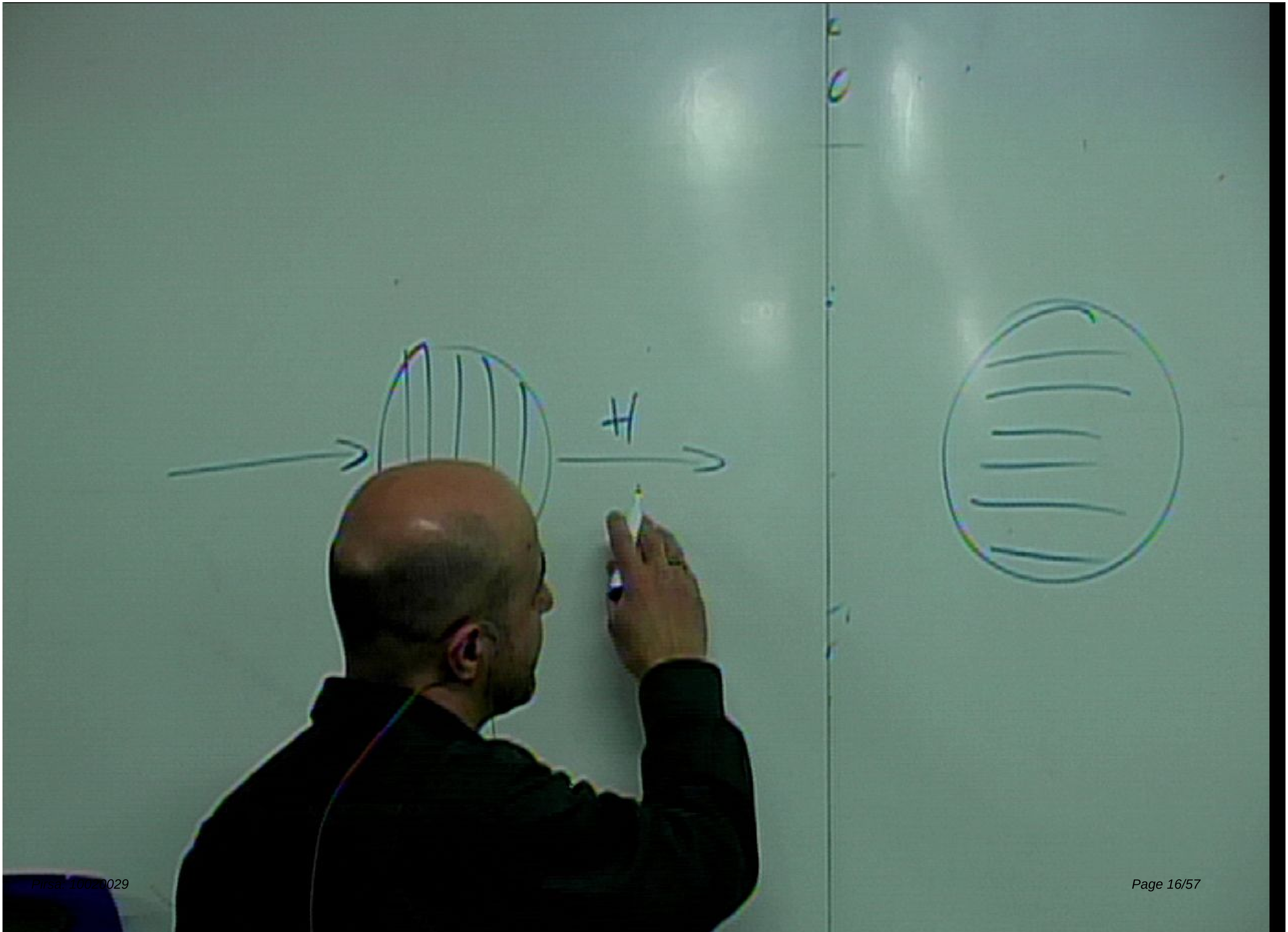
- Fair sampling may be dubious, can we replace it by a weaker assumption?

$$-1 \leq p_{++}(a, b) - p_{++}(a, b') + p_{++}(a', b) + p_{++}(a', b') - p_{+}(a') - p_{+}(b) \leq 0$$

- **No enhancement:** the probability of a particle to be detected behind an analyzer is less or equal than without the analyzer.
- Measure additional coincidence rates with polarizer removed on one side for the following inequality:

$$-p(\infty, \infty) \leq$$
$$p_{++}(a, b) - p_{++}(a, b') + p_{++}(a', b) + p_{++}(a', b') - p_{+}(a', \infty) - p_{+}(\infty, b) \leq 0$$

- Problem: Naive counterexample to assumption!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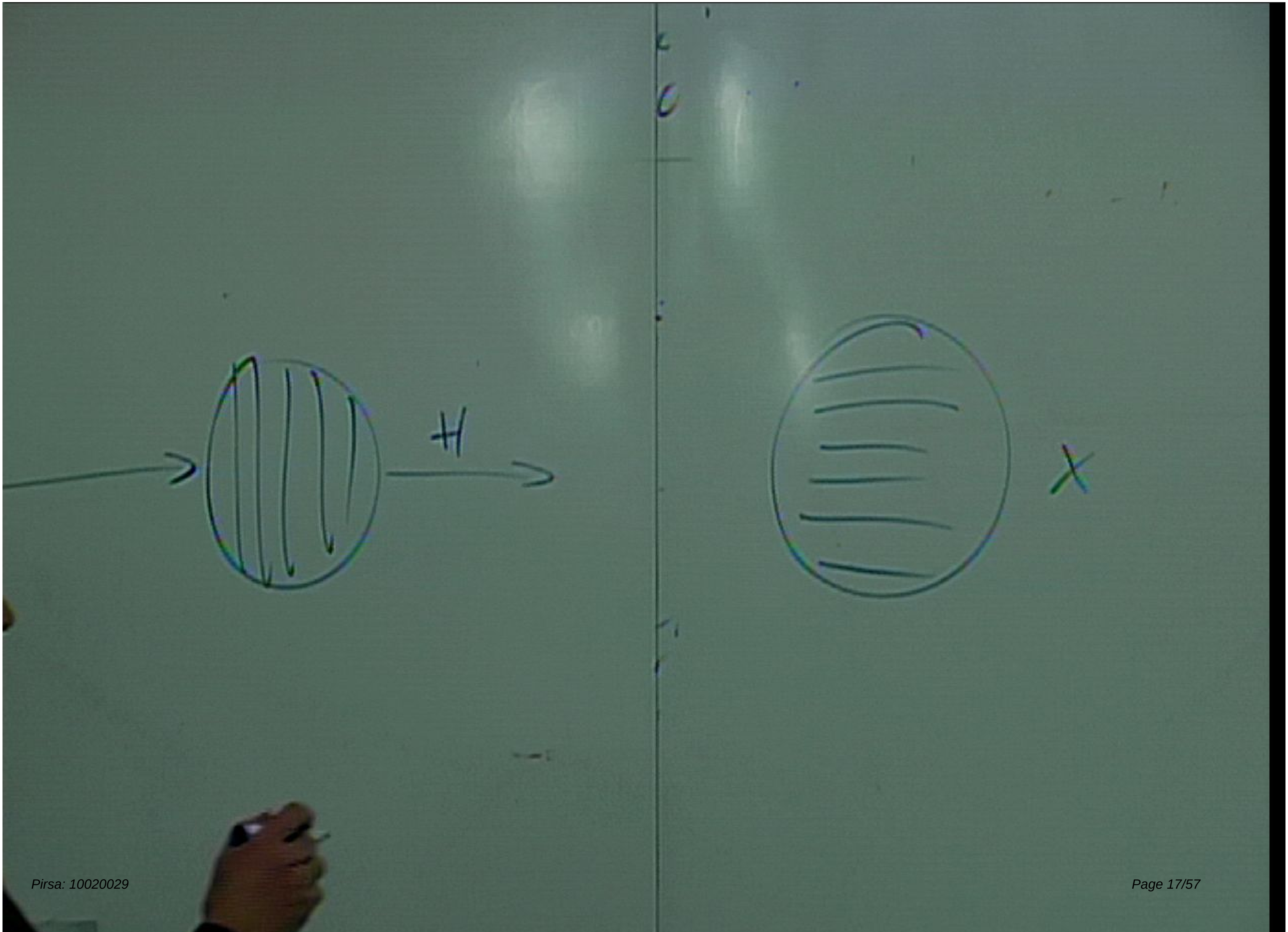
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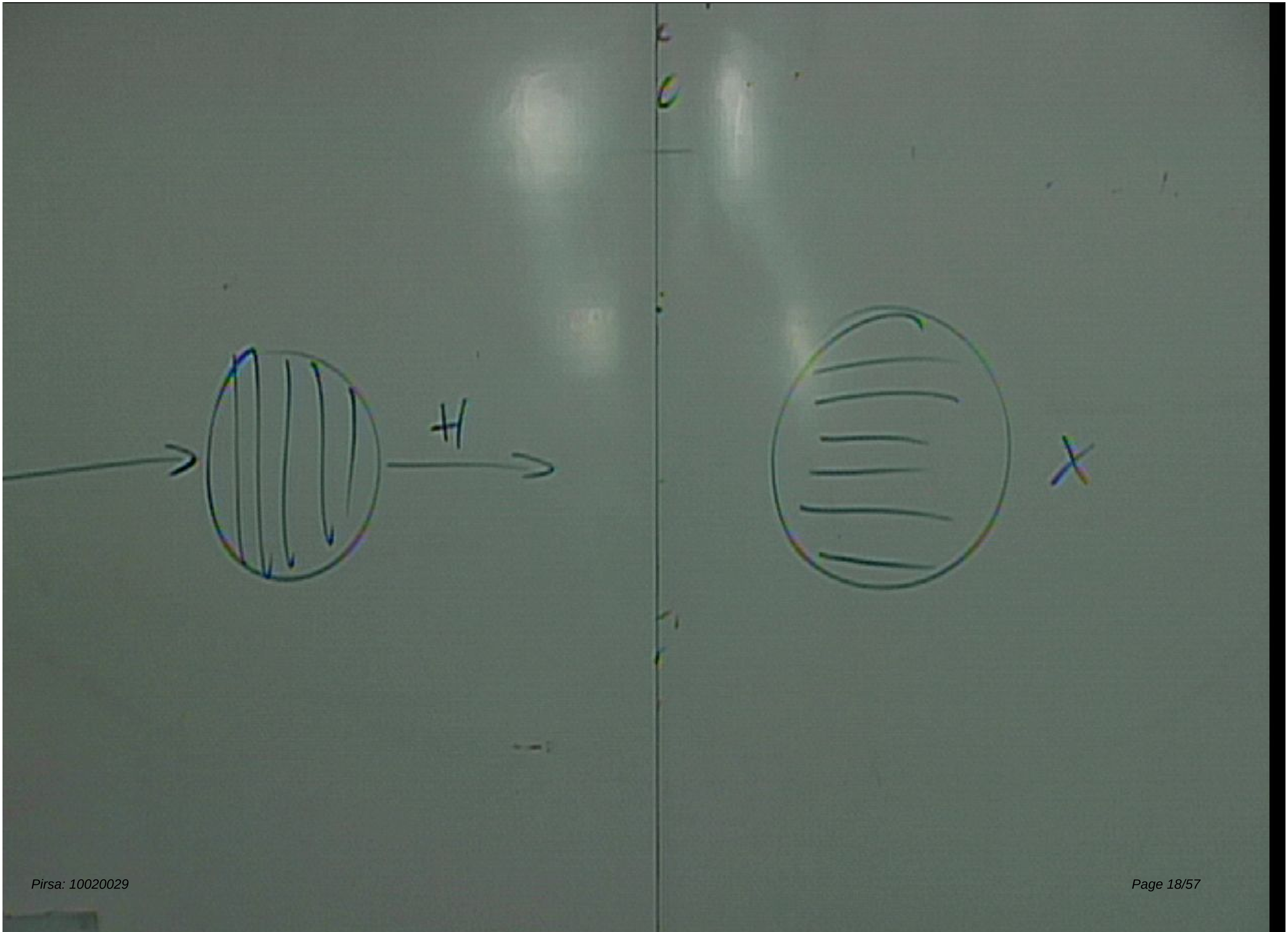
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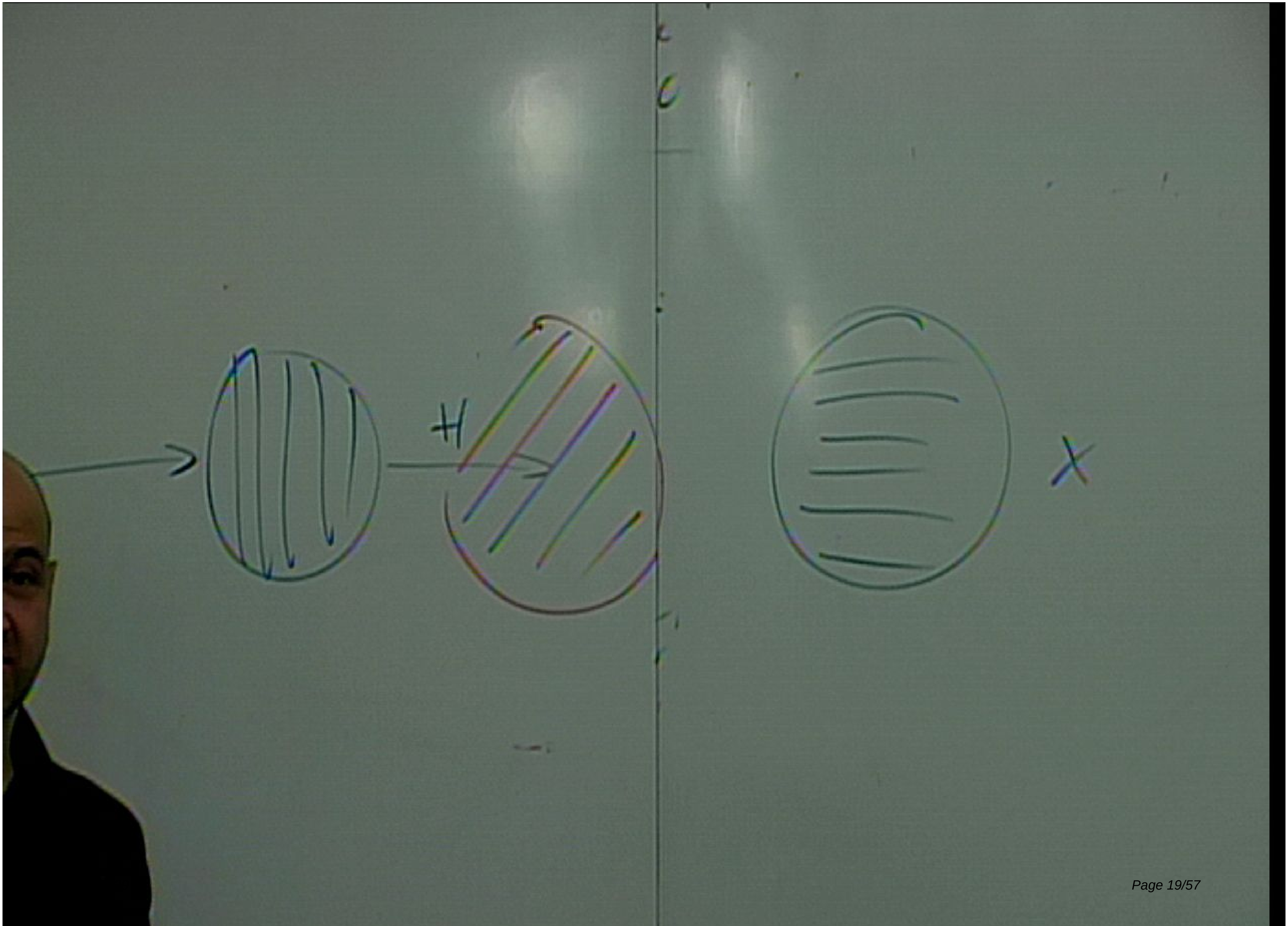
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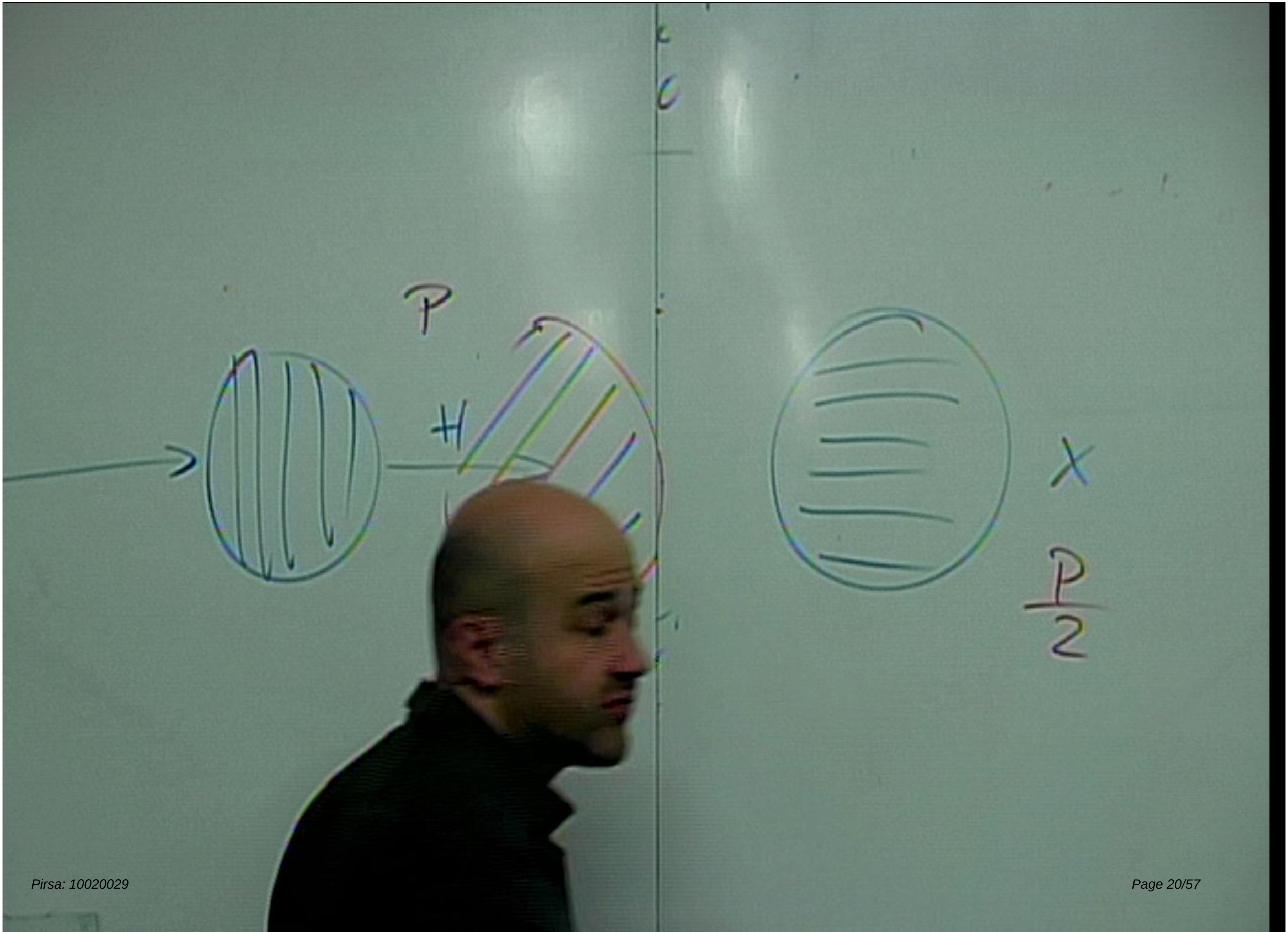
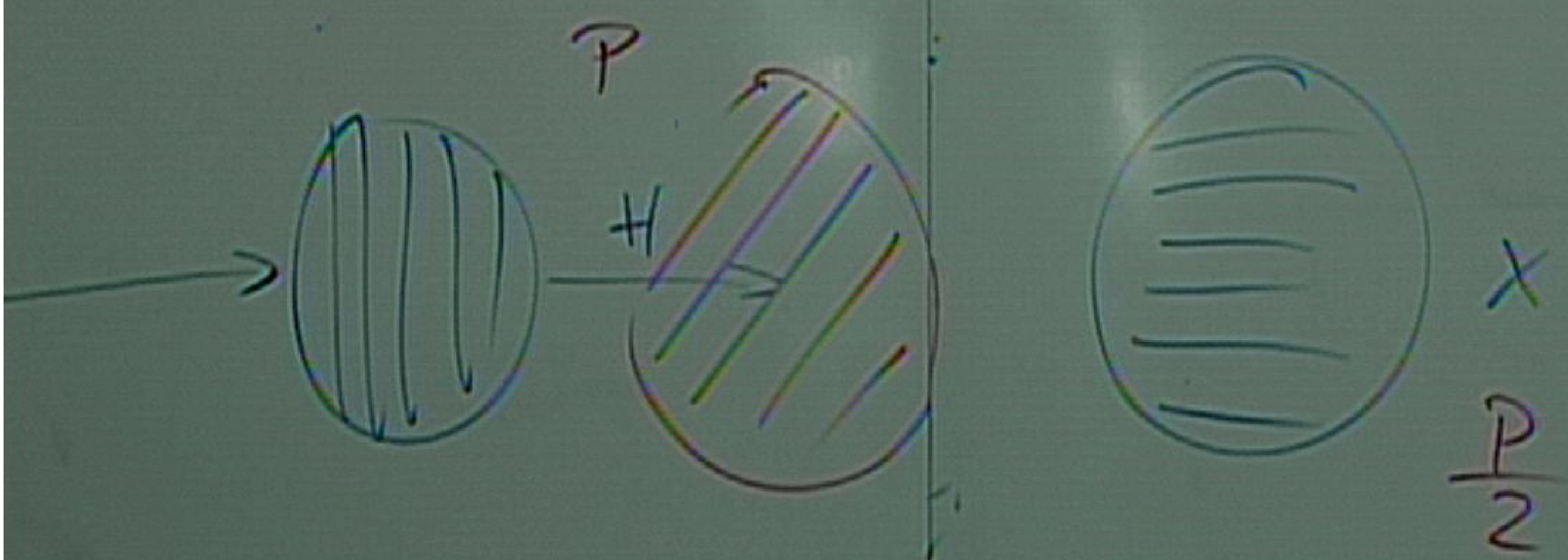

P
H
X
P
2


P
H
X
P
2

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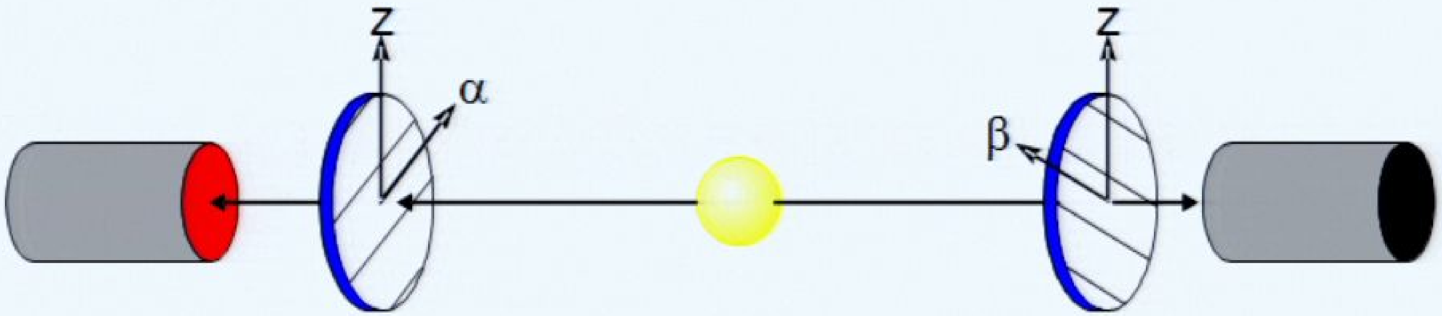

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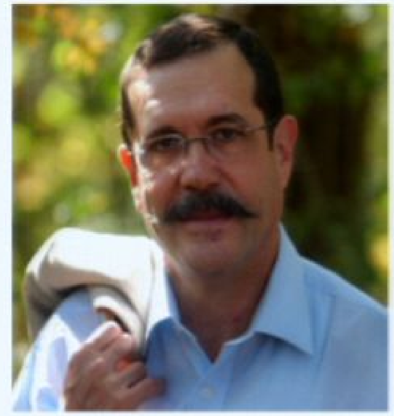
Aspect et al. (1981, 1982)

- Improved source (laser pumped) Ca-cascade
- 100 coinc/s
- 2-channel polarizers
- Fast switching



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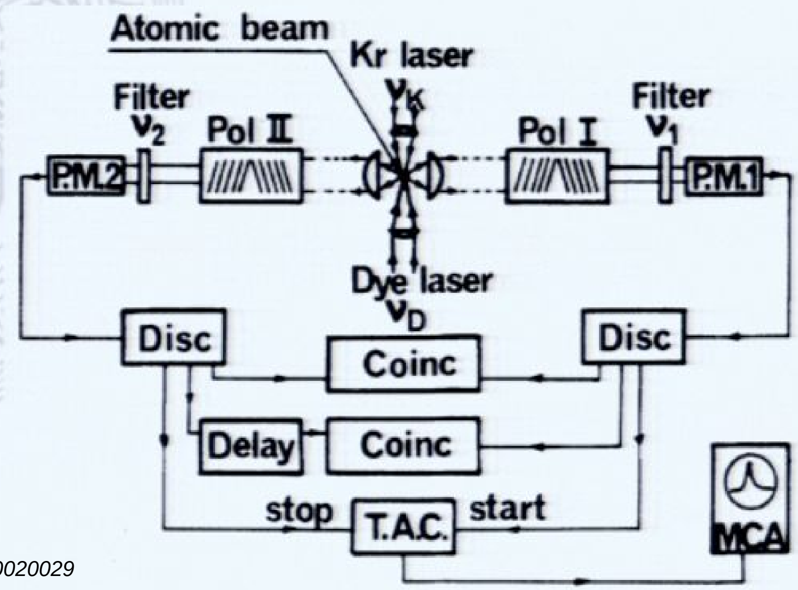
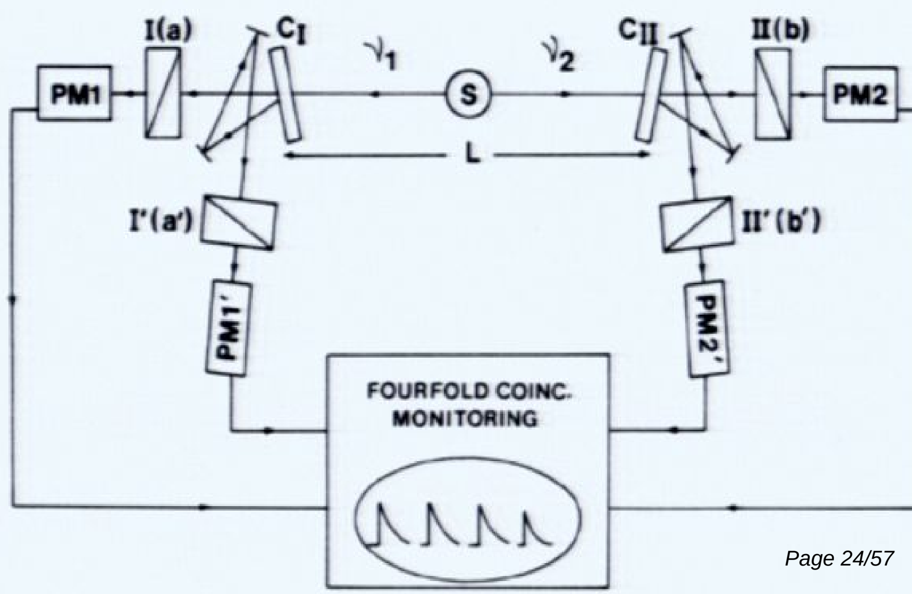
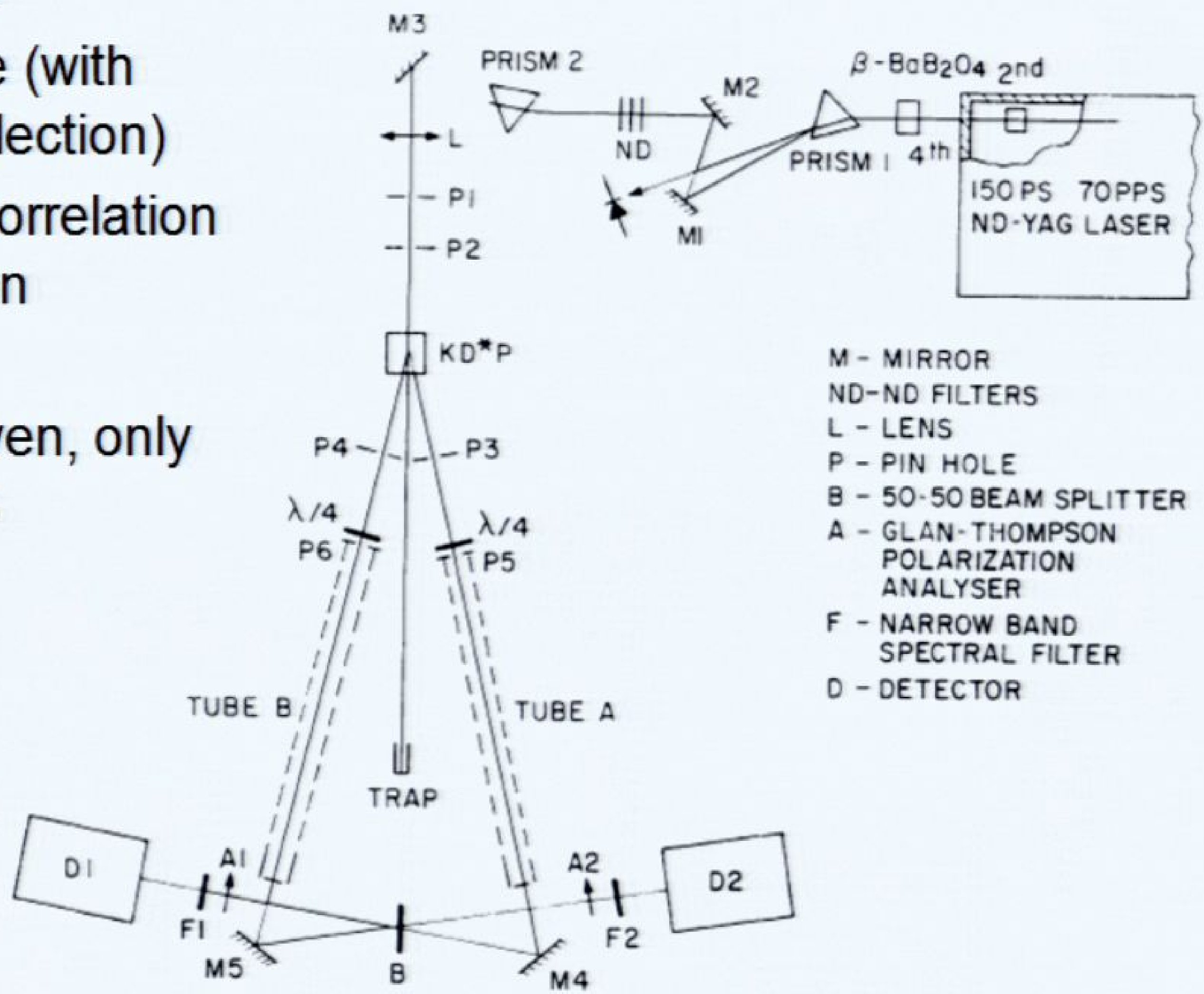




Shih & Alley (1988)

- Parametric down-conversion source (with additional post-selection)
- Strict directional correlation improves collection efficiency
- No count rates given, only 70 pump pulses/s



Loopholes: Locality

First, and those of us who are inspired by Einstein would like this best, quantum mechanics may be *wrong* in sufficiently critical situations. Perhaps Nature is not so queer as quantum mechanics. But the experimental situation is not very encouraging from this point of view[19]. It is true that practical experiments fall far short of the ideal, because of counter inefficiencies, or analyzer inefficiencies, or geometrical imperfections, and so on. It is only with added assumptions, or conventional allowance for inefficiencies and extrapolation from the real to the ideal, that one can say the inequality is violated. Although there is an escape route there, it is hard for me to believe that quantum mechanics works so nicely for inefficient practical set-ups and is yet going to fail badly when sufficient refinements are made. Of more importance, in my opinion, is the complete absence of the vital *time* factor in existing experiments. The analyzers are not rotated during the flight of the particles. Even if one is obliged to admit some long range influence, it need not travel faster than light – and so would be much less indigestible. For me, then, it is of capital importance that Aspect[19,20] is engaged in an experiment in which the time factor is introduced.

from
Bertlmann's socks and the nature of reality,
Journal de Physique, Colloque C2,
suppl. au no. 3, 42 (1981), pp. C2 989-999

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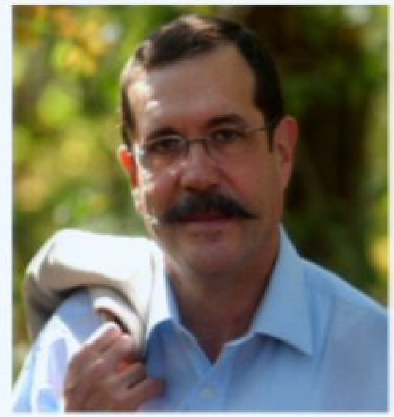

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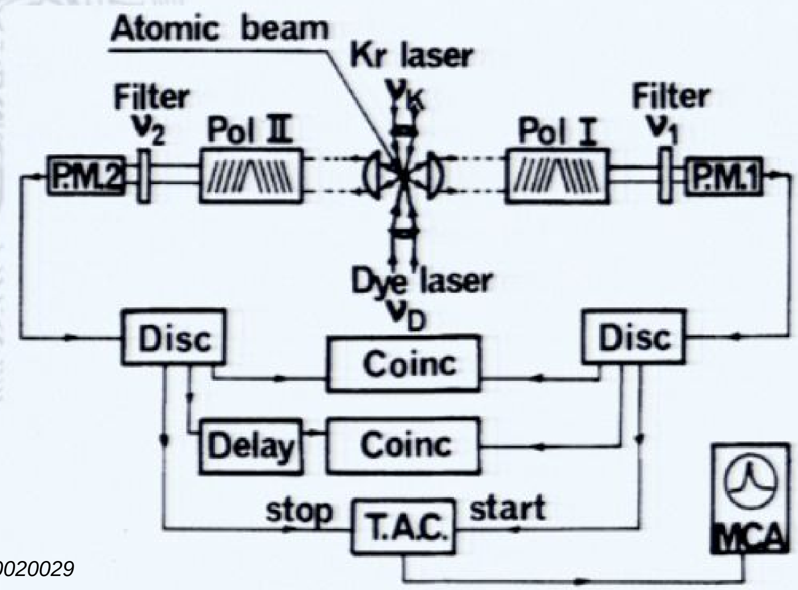

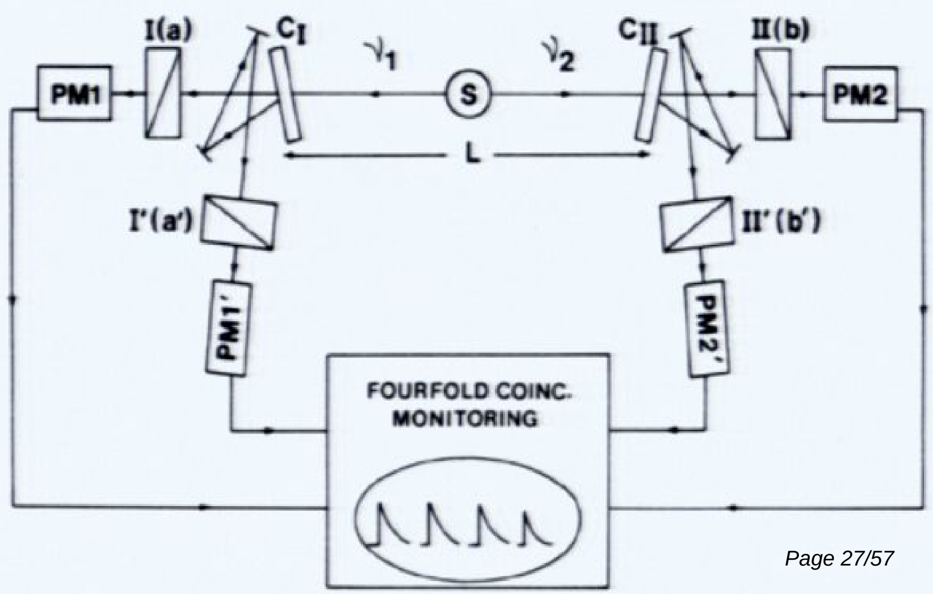

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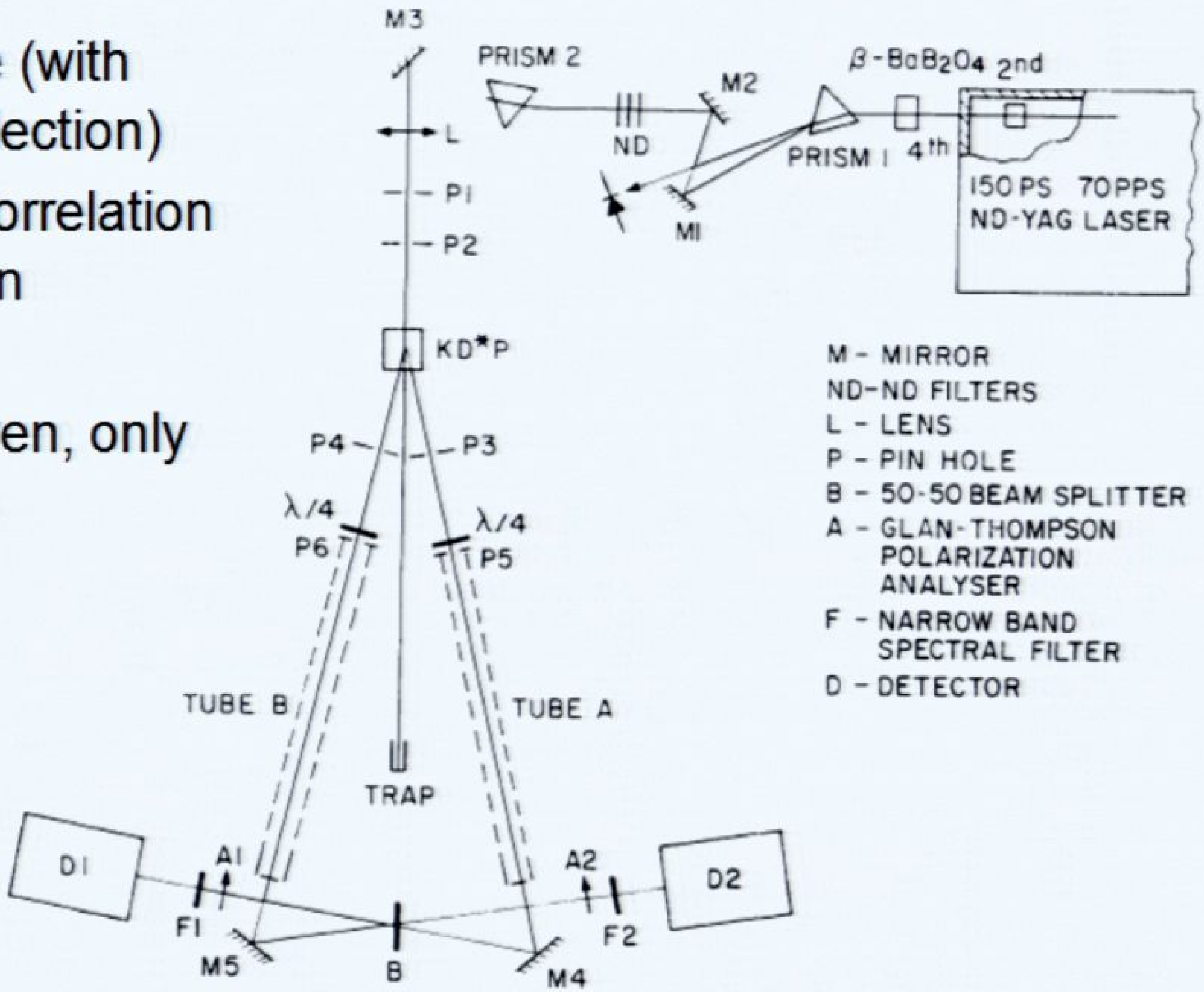
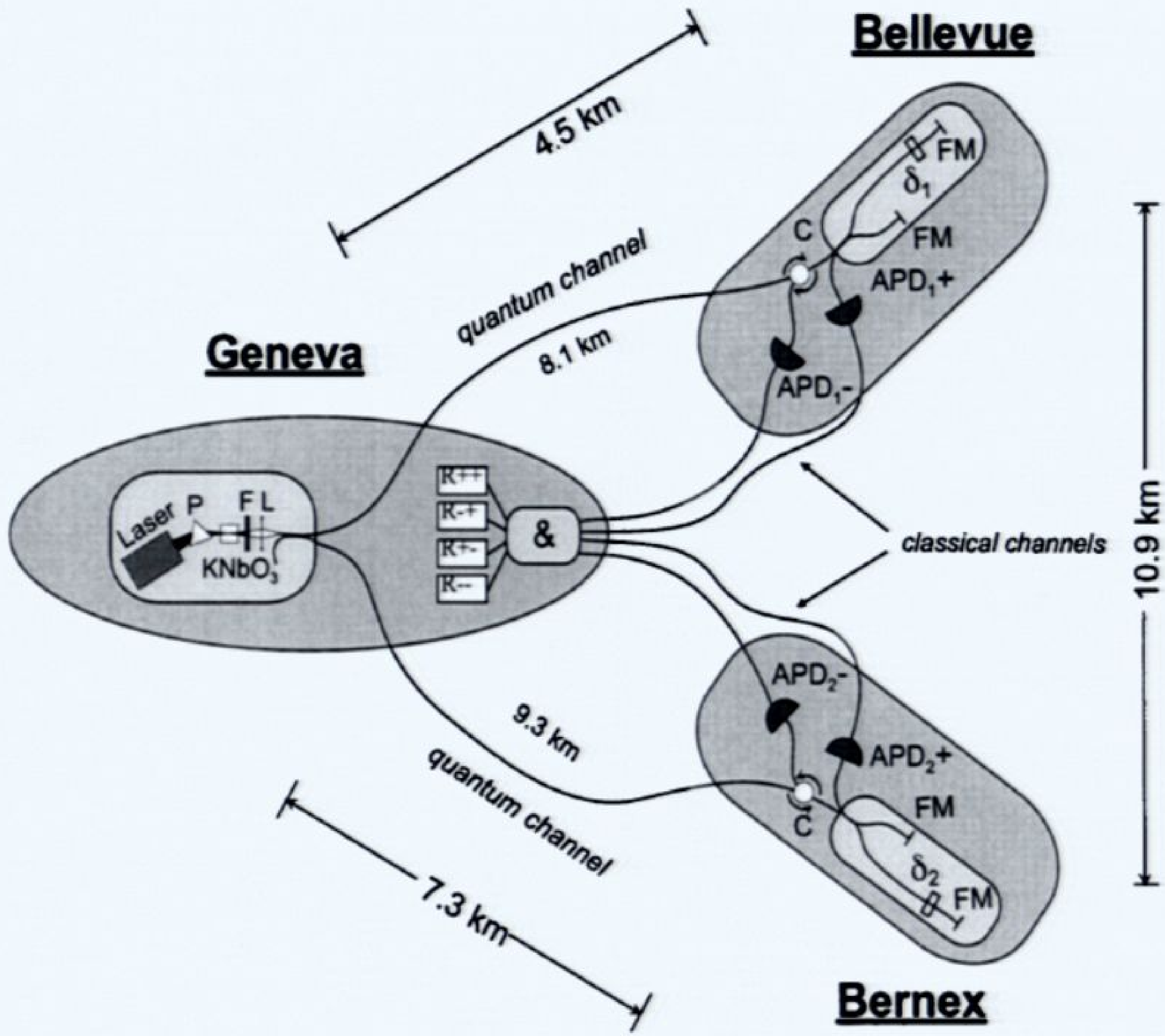


Increasing the distance

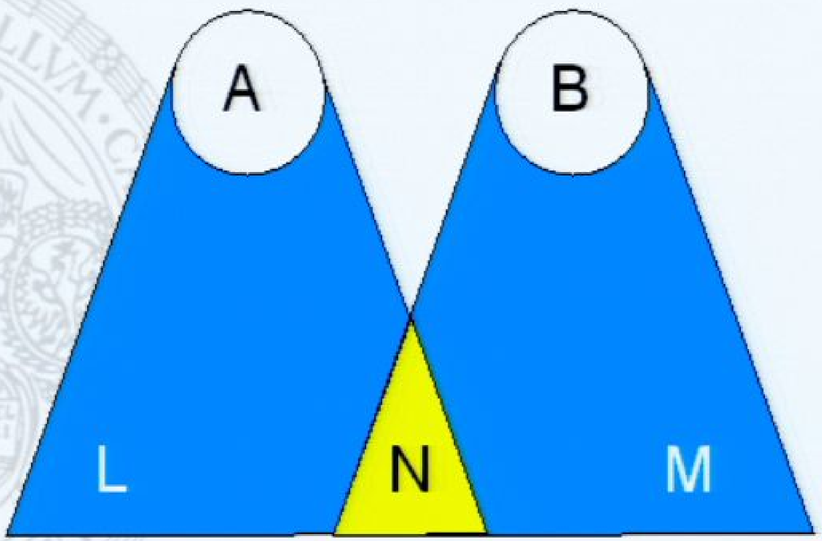


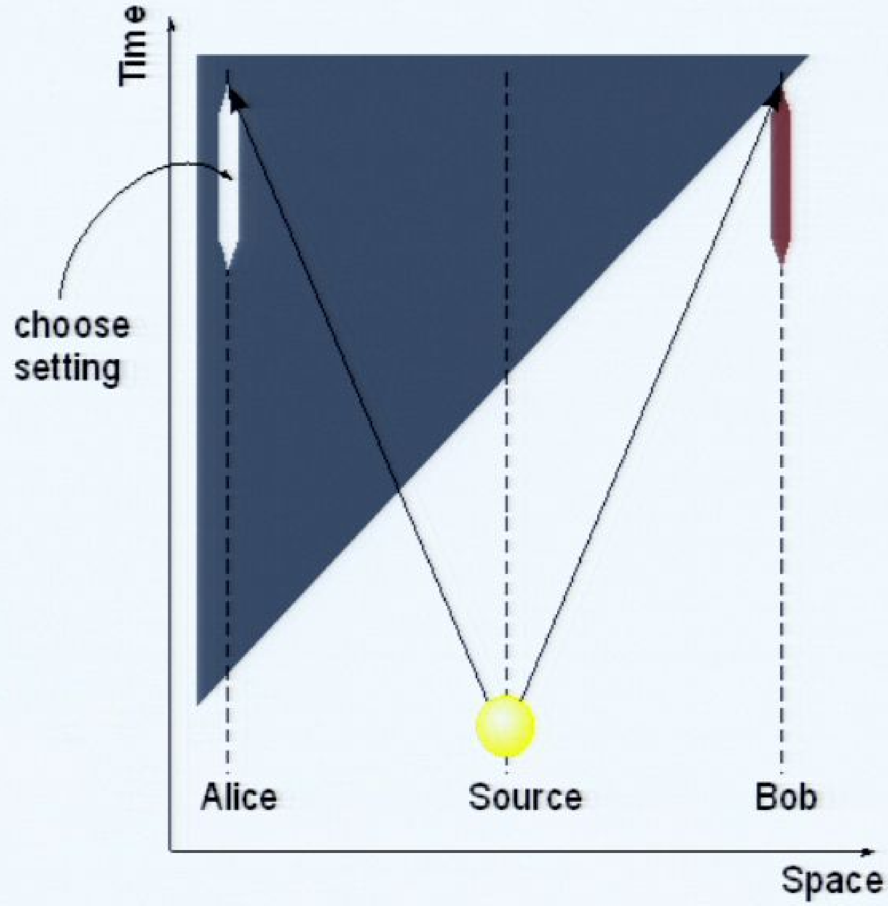
W. Tittel et al., *Violation of Bell Inequalities by Photons More Than 10 km Apart*, PRL **81,** 3564 (1998)

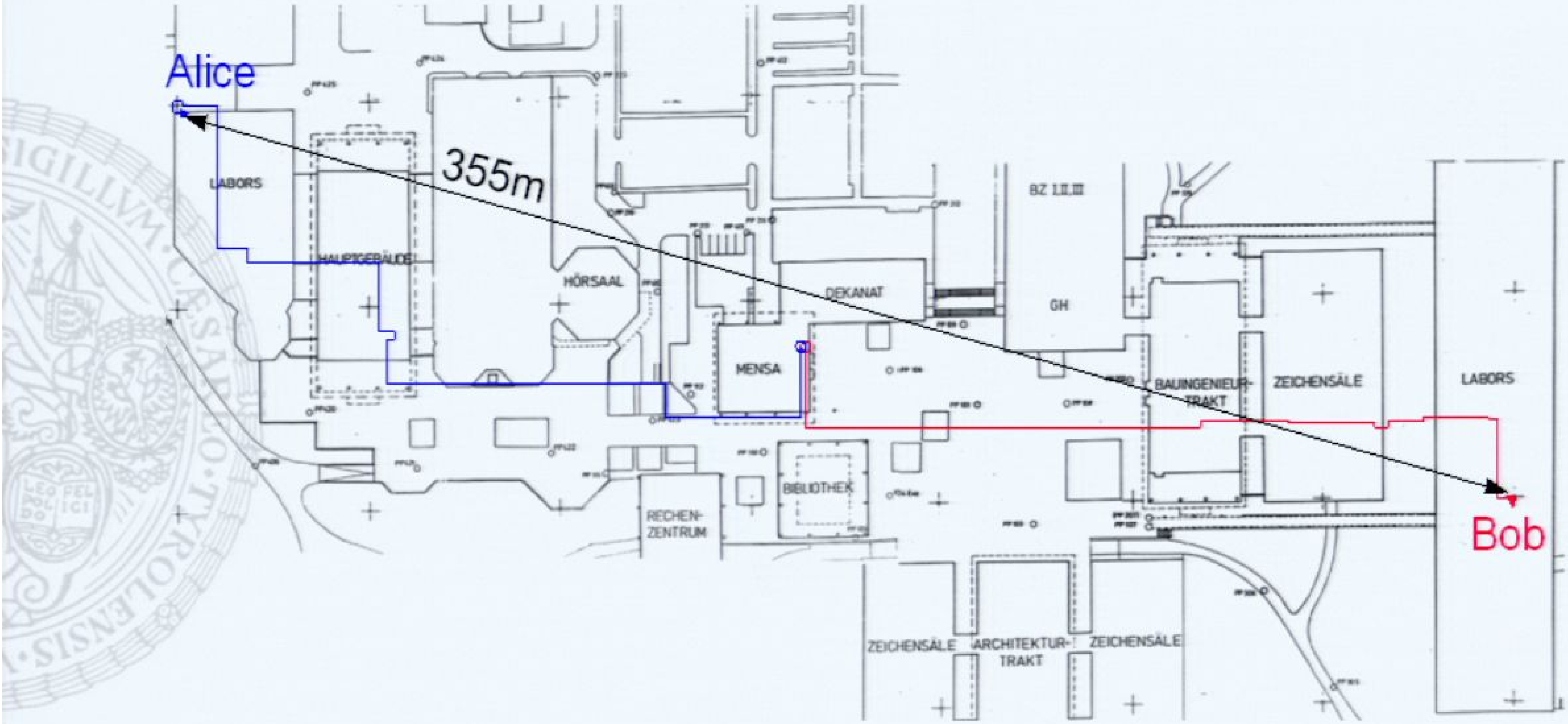
Closing the locality loophole

- New sources enabled long distance experiments with random switching
- Random number generation based on single-photon beam-splitting

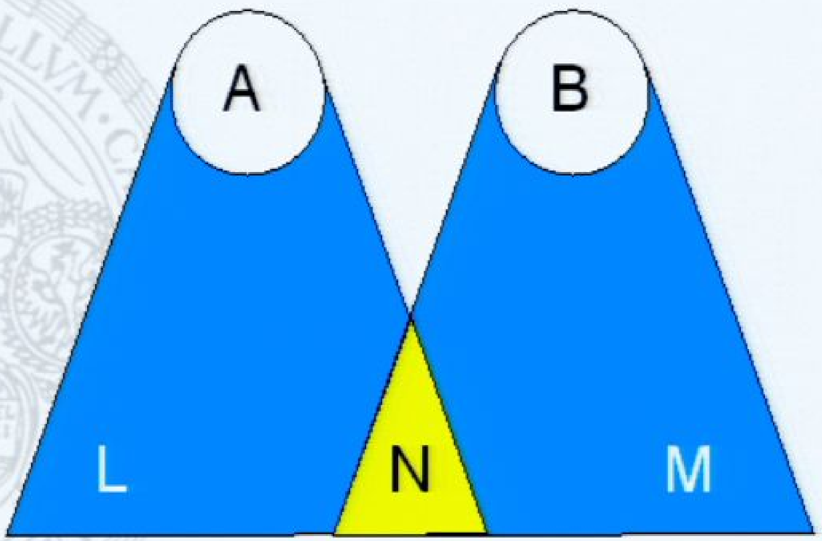
Spacelike separation

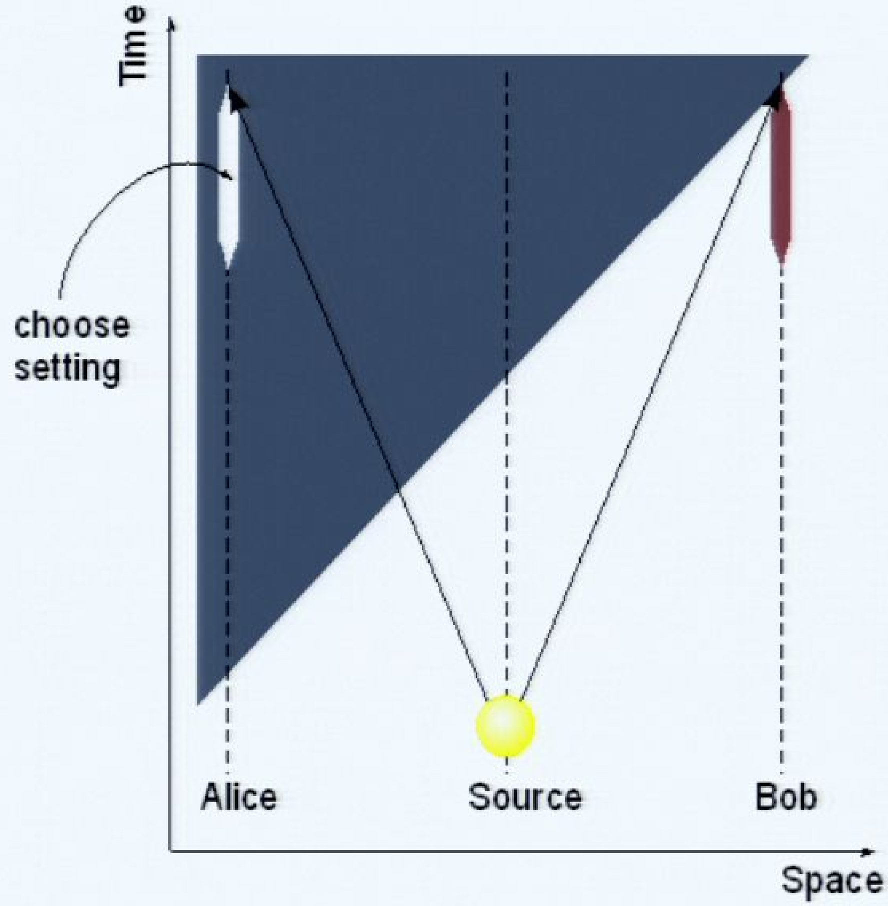


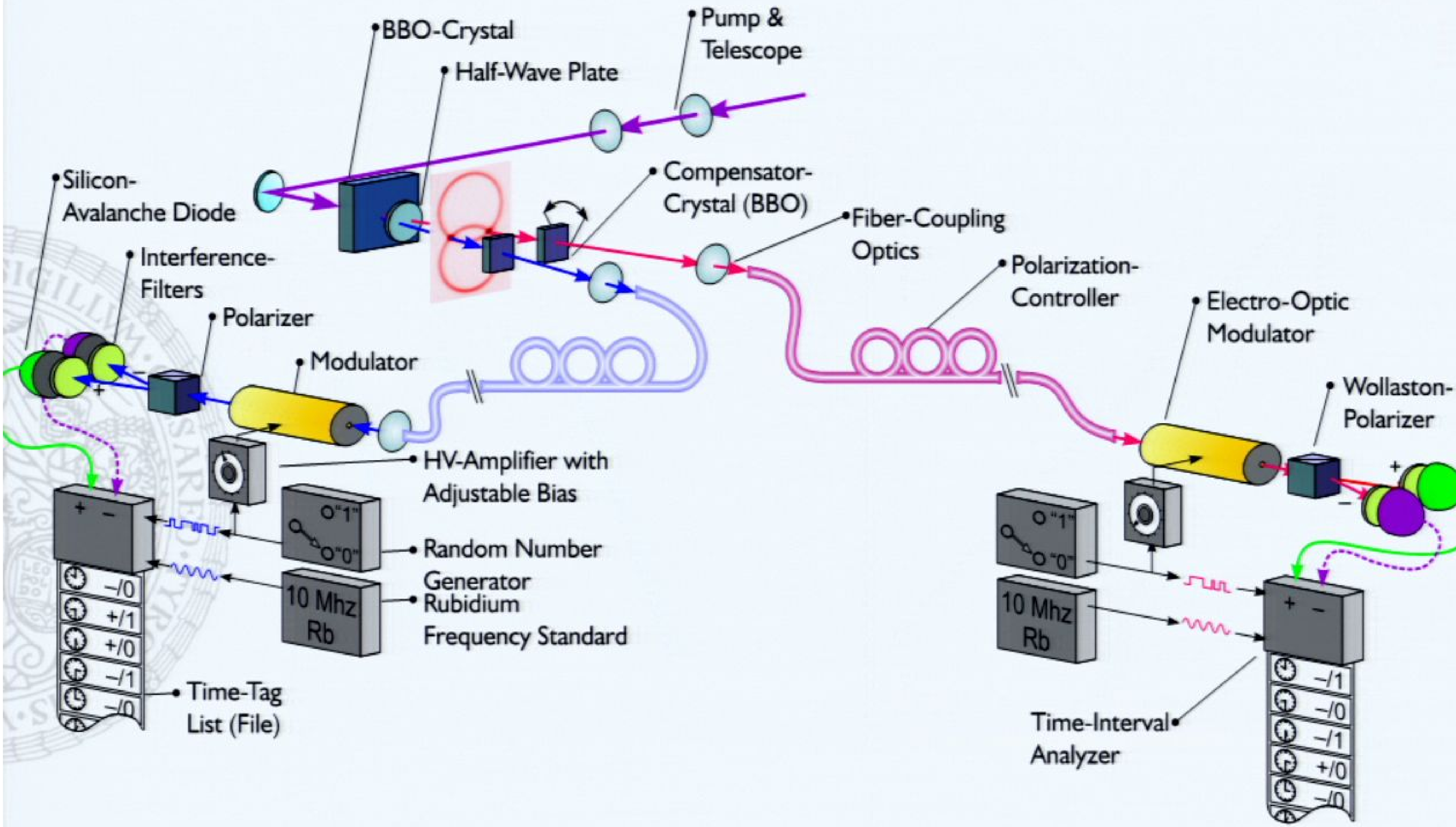



Alice
355m
Bob
LABORS
HAUPTGEBÄUDE
HÖRSAAL
MENSA
DEKANAT
GH
BZ I,II,III
BAUINGENIEUR-TRAKT
ZEICHENSÄLE
LABORS
BIBLIOTHEK
RECHEN-ZENTRUM
ZEICHENSÄLE
ARCHITEKTUR-TRAKT
ZEICHENSÄLE

Spacelike separation






BBO-Crystal
Pump & Telescope
Half-Wave Plate
Silicon-Avalanche Diode
Compensator-Crystal (BBO)
Fiber-Coupling Optics
Interference-Filters
Polarization-Controller
Polarizer
Electro-Optic Modulator
Modulator
Wollaston-Polarizer
HV-Amplifier with Adjustable Bias
Random Number Generator
Rubidium Frequency Standard
10 Mhz Rb
"1"
"0"
Time-Tag List (File)
Time-Interval Analyzer
–/0
+/1
+/0
–/1
–/0
–/1
–/0
–/1
+/0
–/0

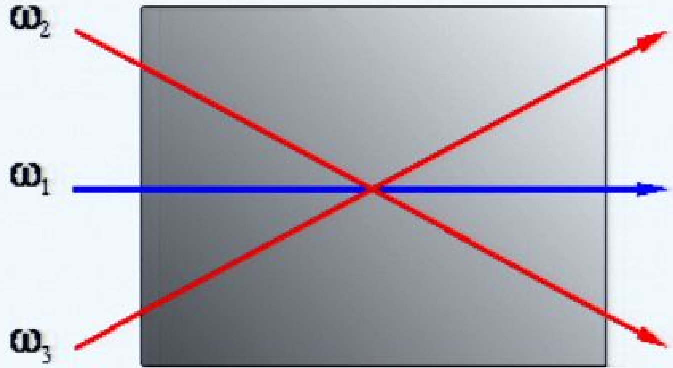
Nonlinear Optics

- Maxwell's equations in vacuum are linear, material response is not
- To lowest order, nonlinearity introduces three-wave mixing

$$e^{i\omega_1 t} \cdot e^{\pm i\omega_2 t} = e^{i\omega_3 t} \qquad \omega_3 = \omega_1 \pm \omega_2$$

- Nonlinear Polarization

$$P_i = \sum \chi_{ij}^{(1)} E_j + \sum \chi_{ijk}^{(2)} E_j E_k + + \dots$$



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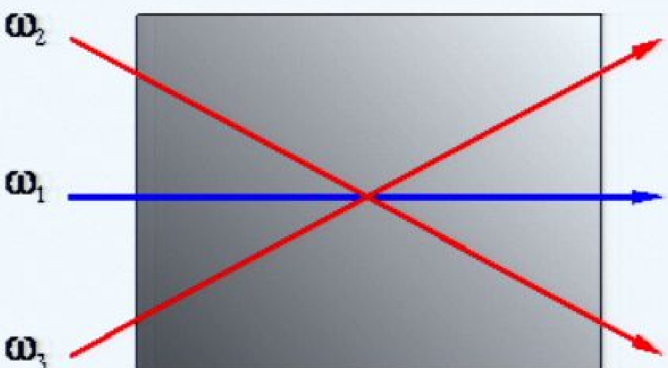
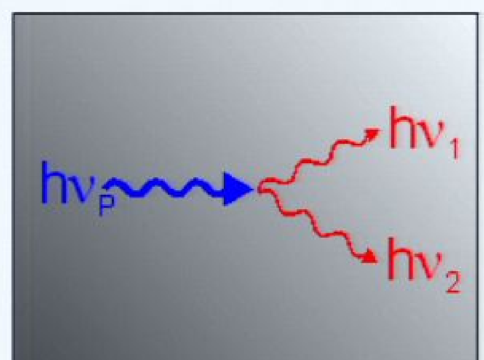
- Nonlinear Polarization $$P_i = \sum \chi_{ij}^{(1)} E_j + \sum \chi_{ijk}^{(2)} E_j E_k + + \ldots$$
- Output quantum state $$|\Psi\rangle = c_0 |0,0\rangle + c_1 |1,1\rangle + c_2 |2,2\rangle + \ldots$$





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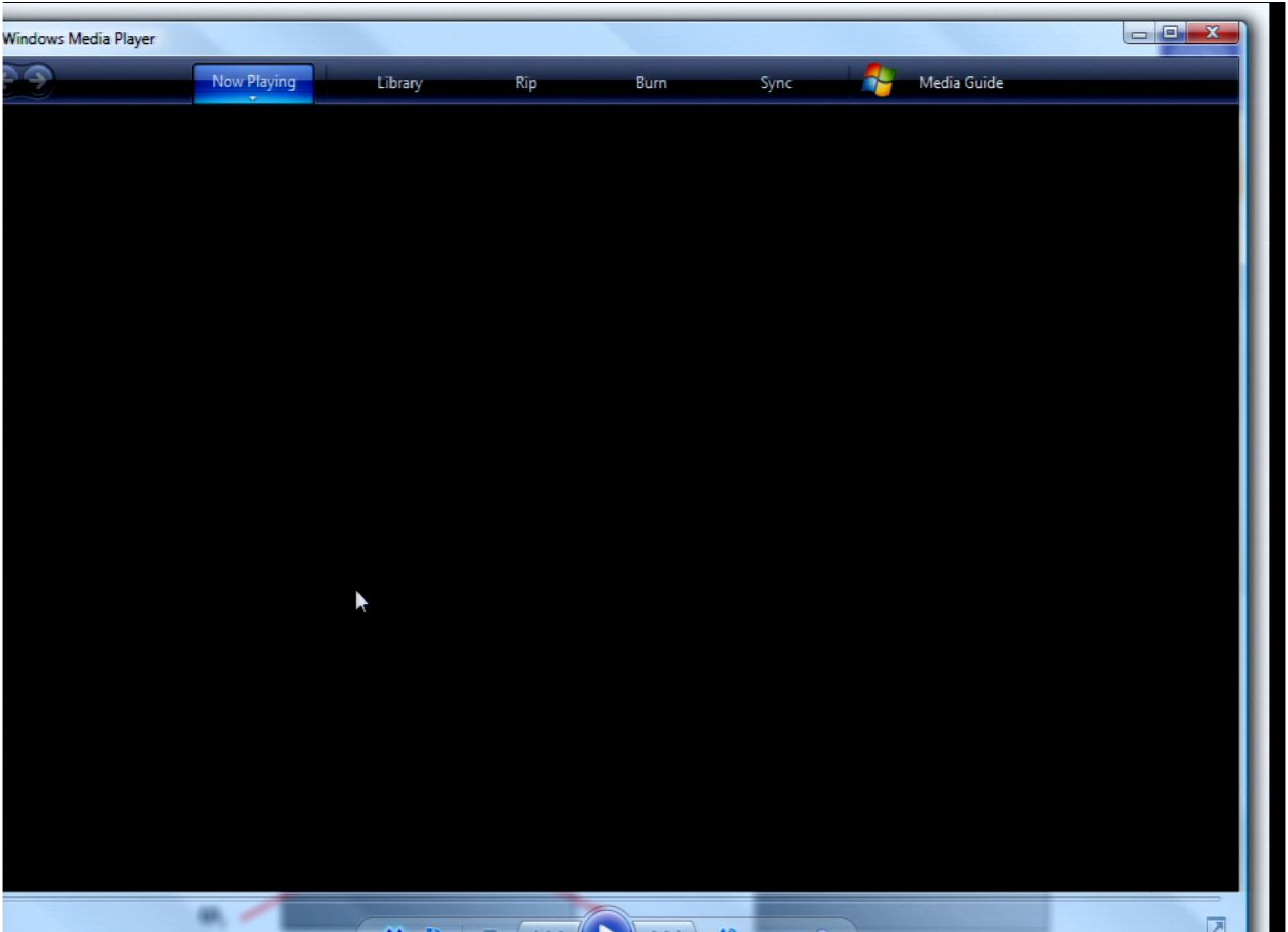
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Now Playing
Library
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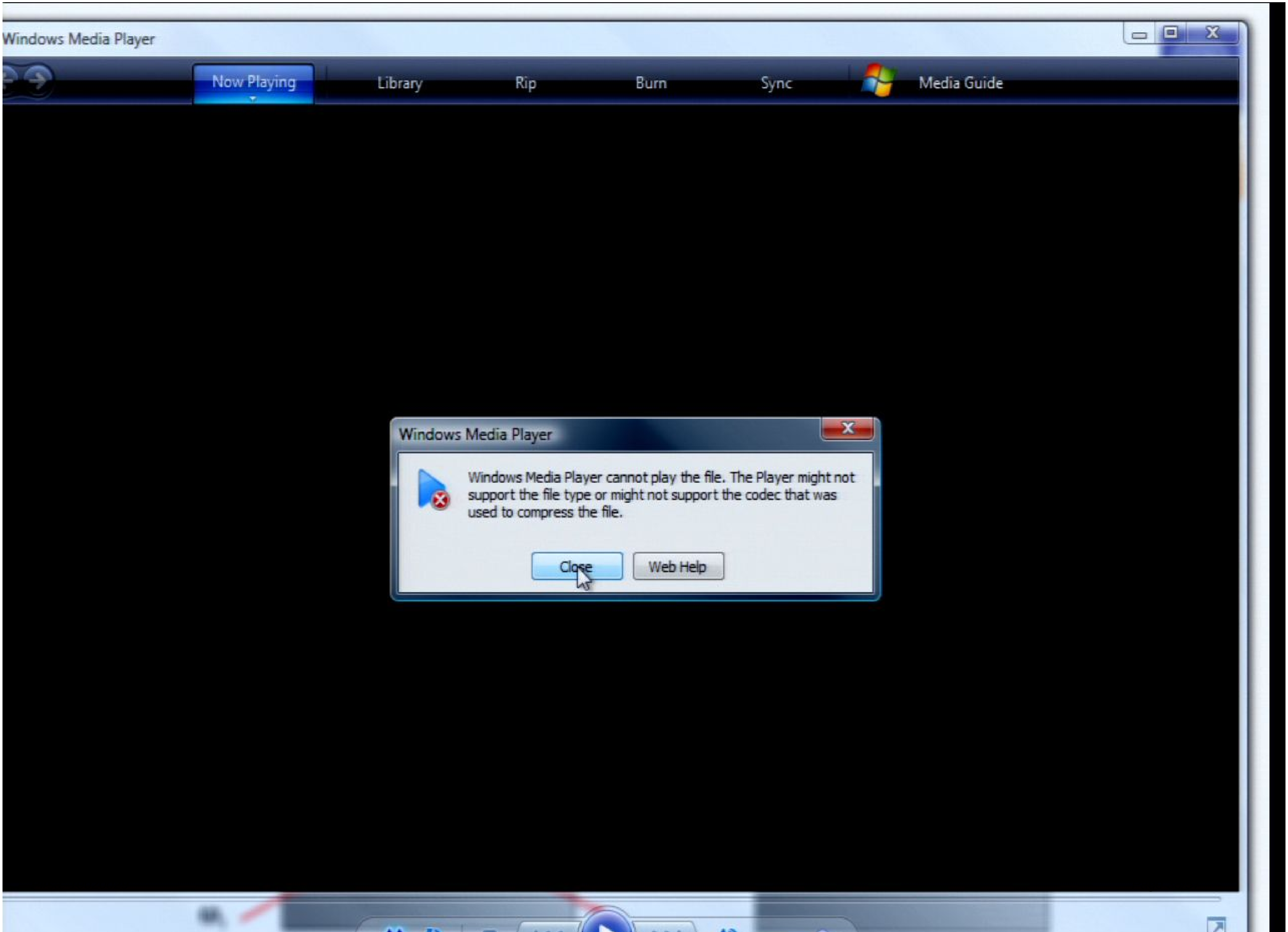
Windows Media Player
Now Playing
Library
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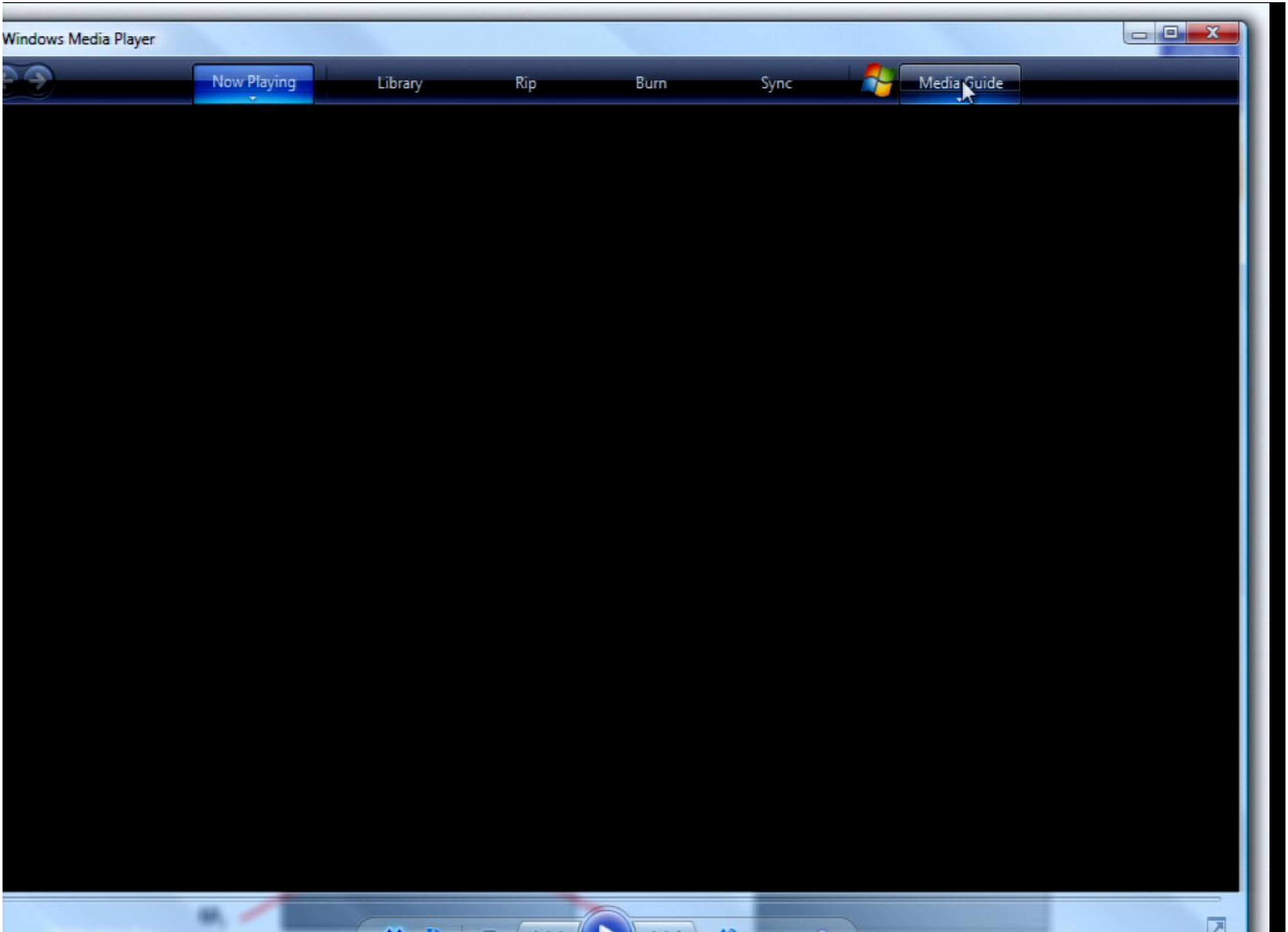
Media Guide
Windows Media Player
Windows Media Player cannot play the file. The Player might not support the file type or might not support the codec that was used to compress the file.
Close
Web Help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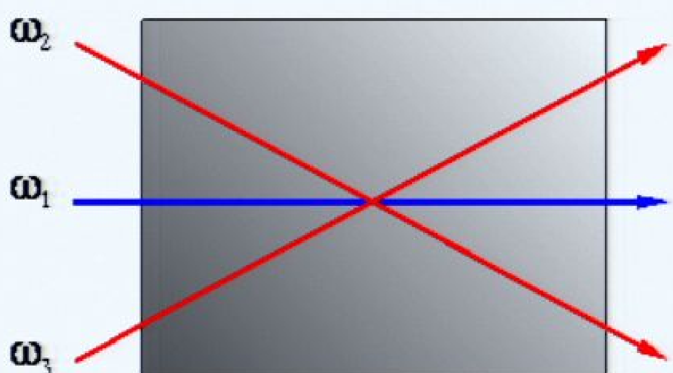
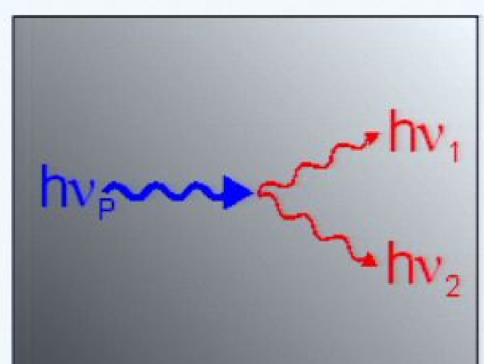
- To lowest order, nonlinearity introduces three-wave mixing

$$e^{i\omega_1 t} \cdot e^{\pm i\omega_2 t} = e^{i\omega_3 t} \qquad \omega_3 = \omega_1 \pm \omega_2$$

- Nonlinear Polarization $\quad P_i = \sum \chi_{ij}^{(1)} E_j + \sum \chi_{ijk}^{(2)} E_j E_k + + \ldots$

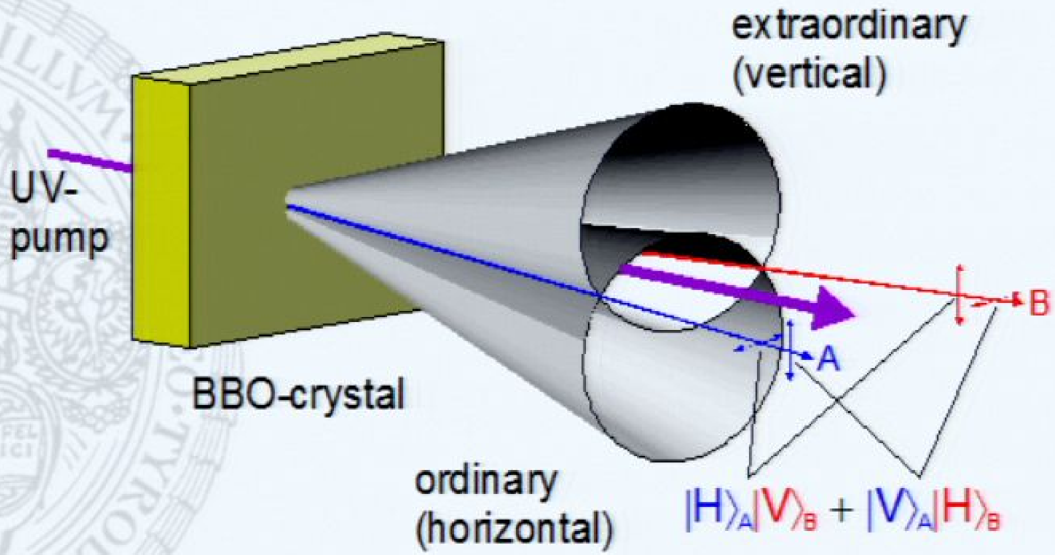
- Output quantum state $\quad |\Psi\rangle = c_0 |0,0\rangle + c_1 |1,1\rangle + c_2 |2,2\rangle + \ldots$



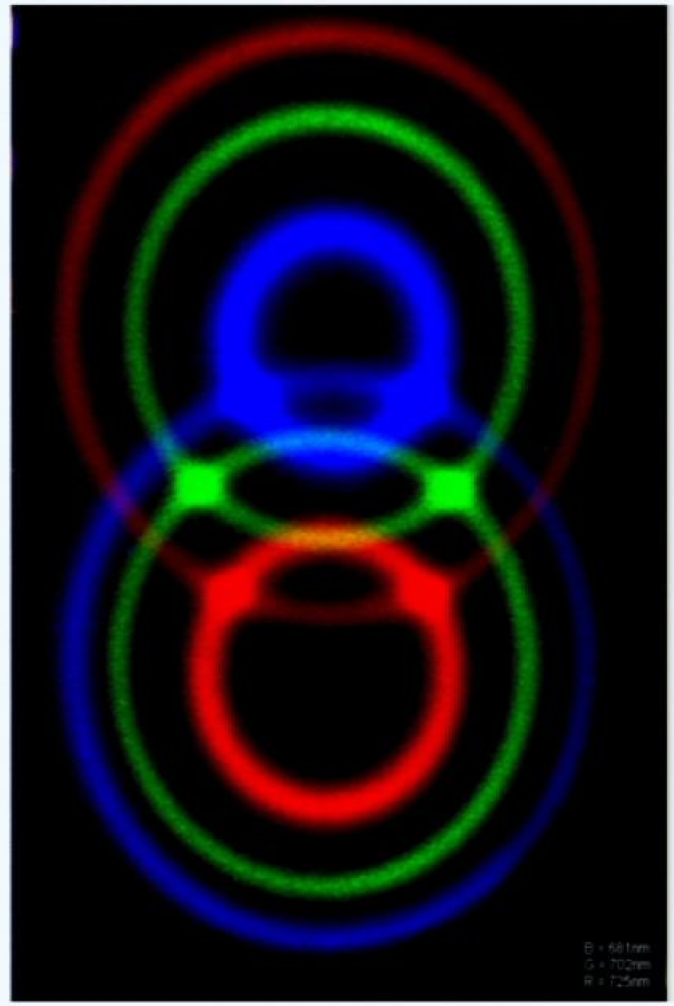


type1s.wmv

Source



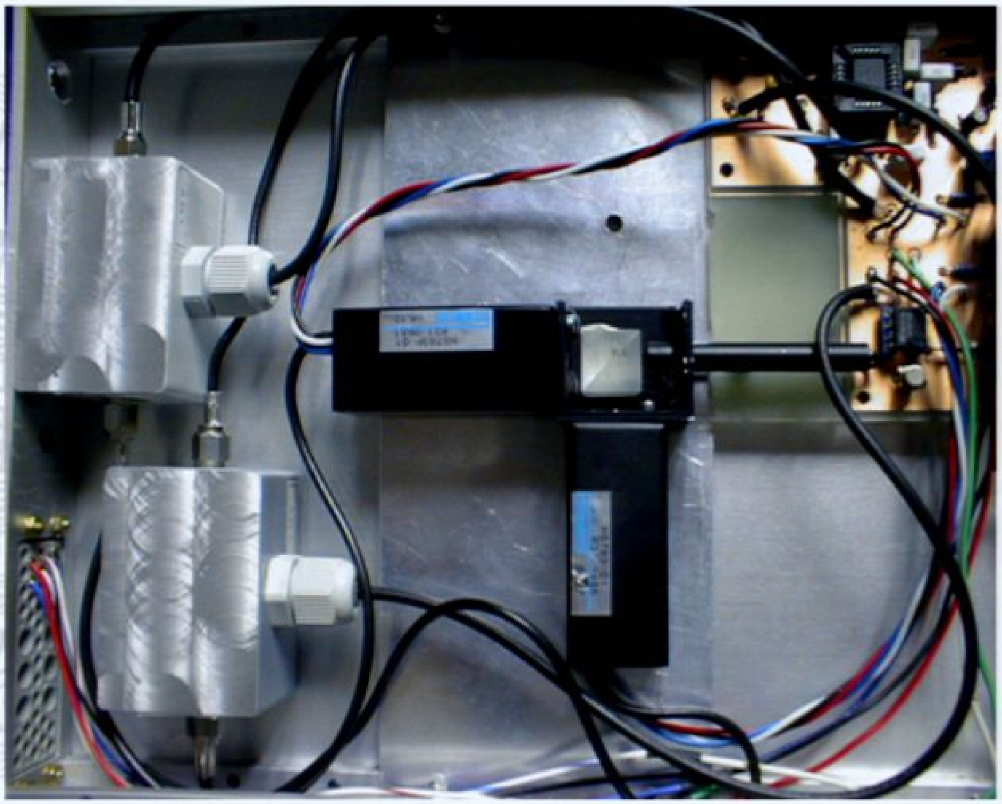
extraordinary
(vertical)
UV-
pump
BBO-crystal
A
B
ordinary
(horizontal)
$|H\rangle_A|V\rangle_B + |V\rangle_A|H\rangle_B$



Source of Entangled Photon Pairs

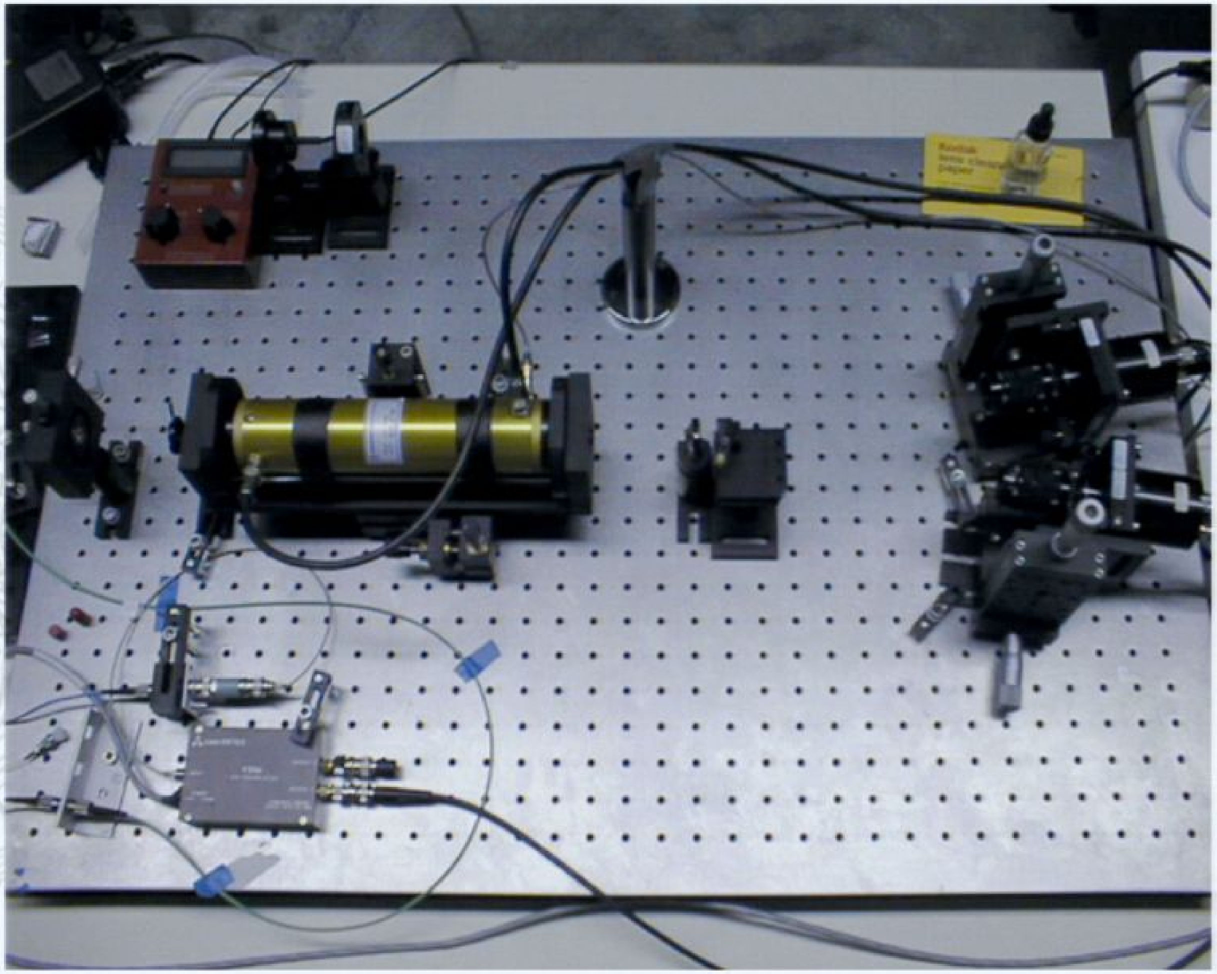
- Type-II Barium Borate
- Pumped by Ar-Ion laser
- Manual polarization compensation

Generation of Random Bits



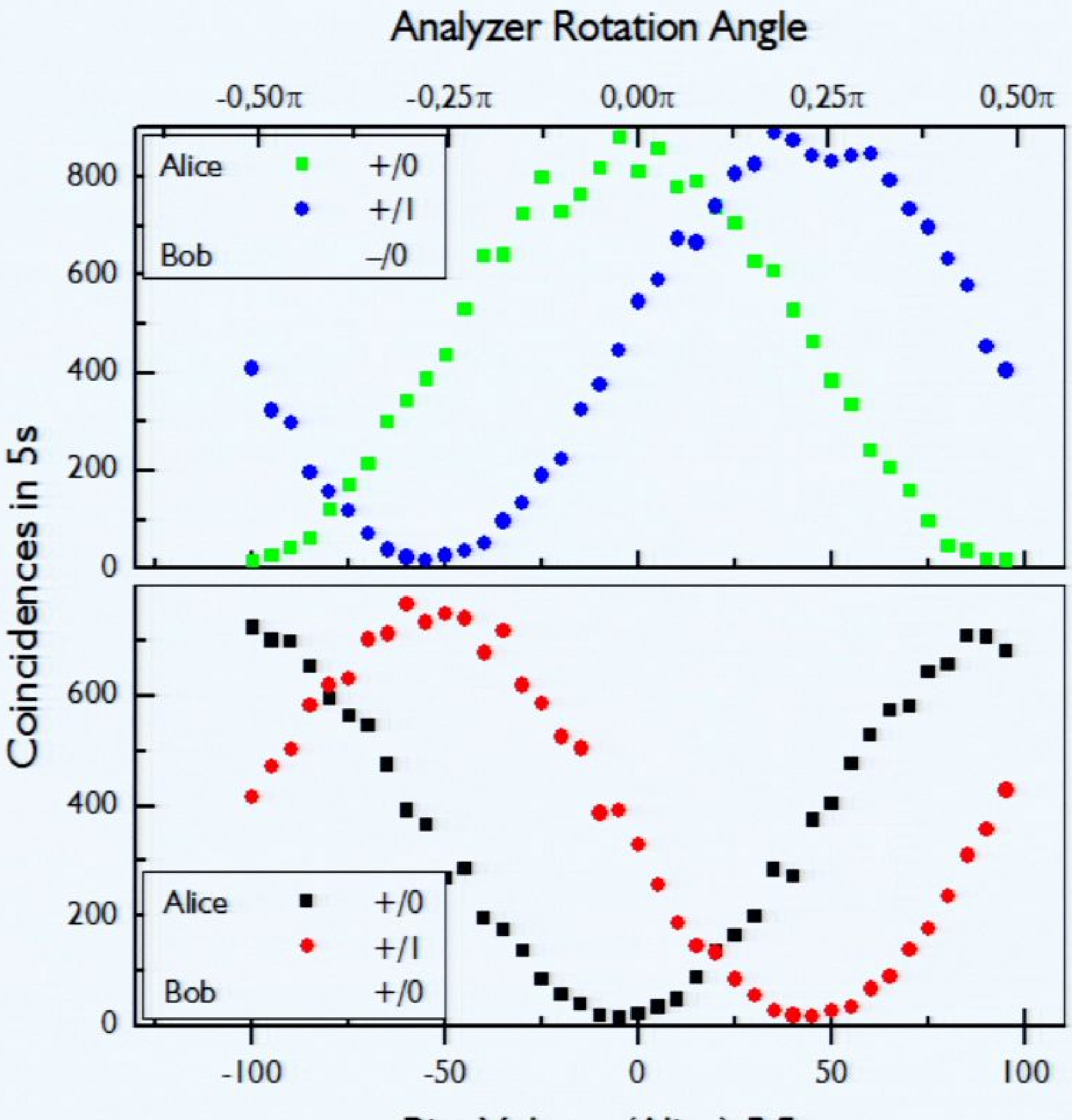
- Mean toggle rate up to 50 MHz
- Balancing difficult
- Autocorrelation time <15 ns
- Very good performance in entropy test (downsampled output)

Analyzer Settings

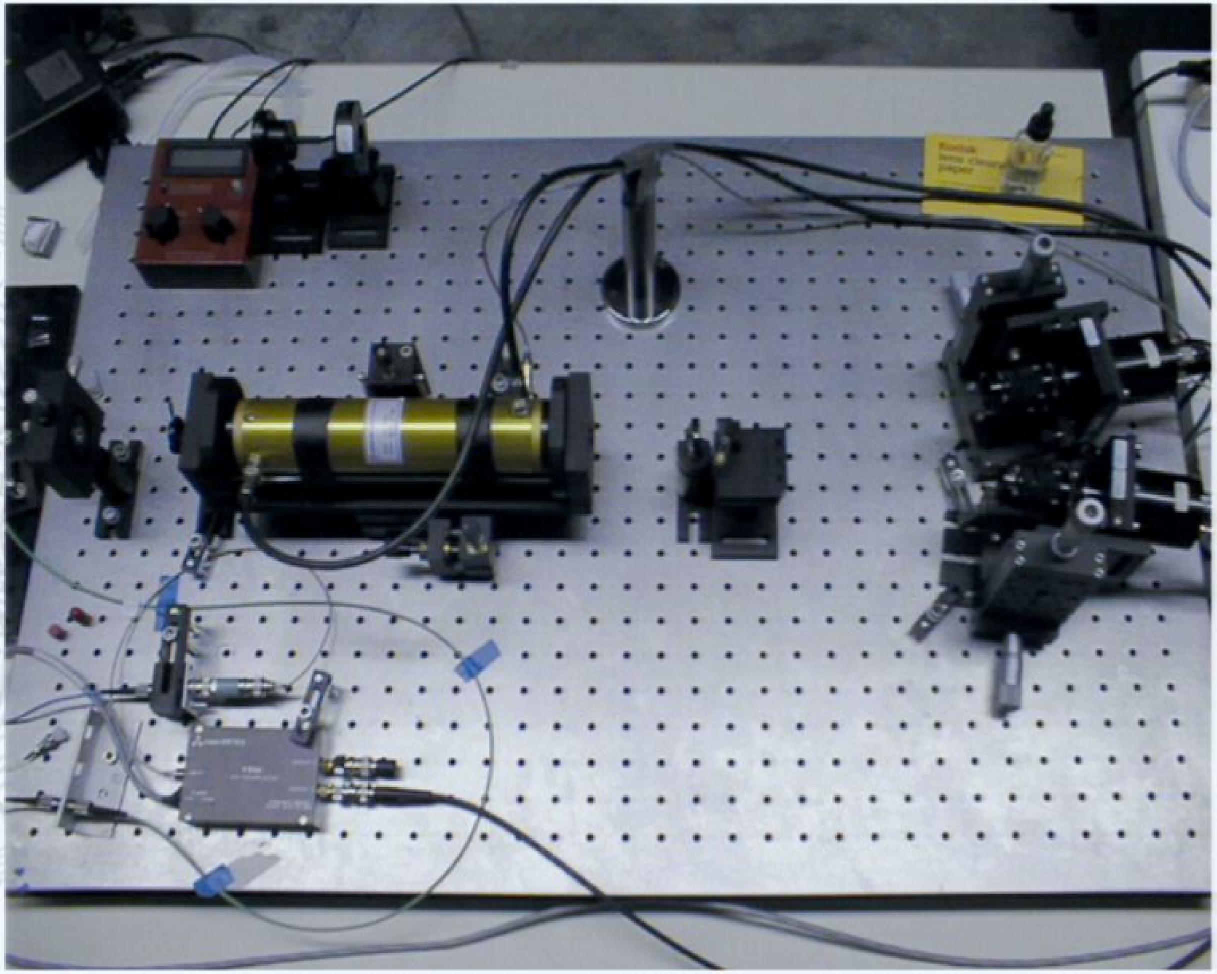


- Electro-Optic Modulators
- Max frequency 30 MHz
- Difficult alignment
- Drifting bias point

Experimental Correlations



Analyzer Settings



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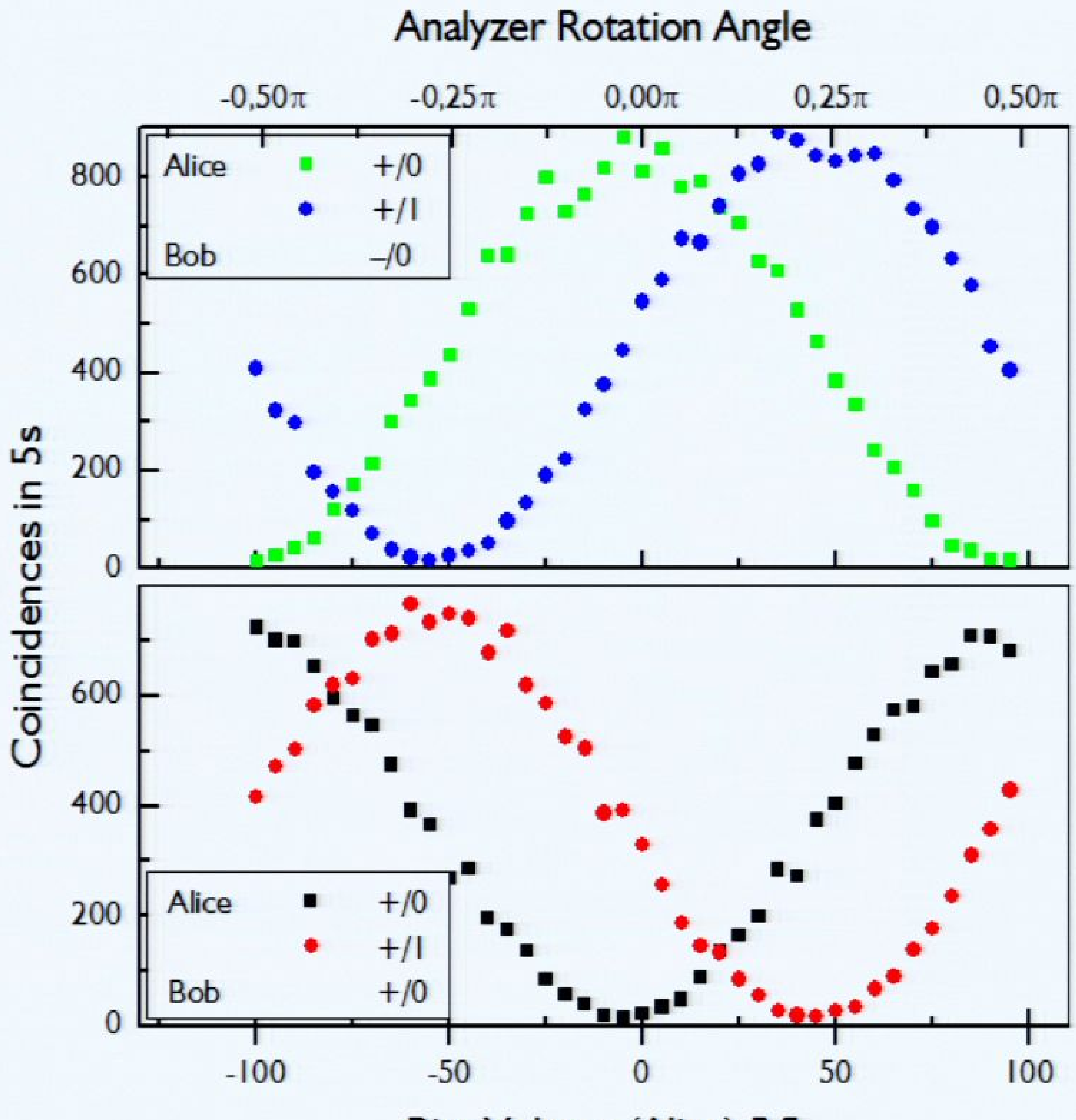
Experimental Correlations

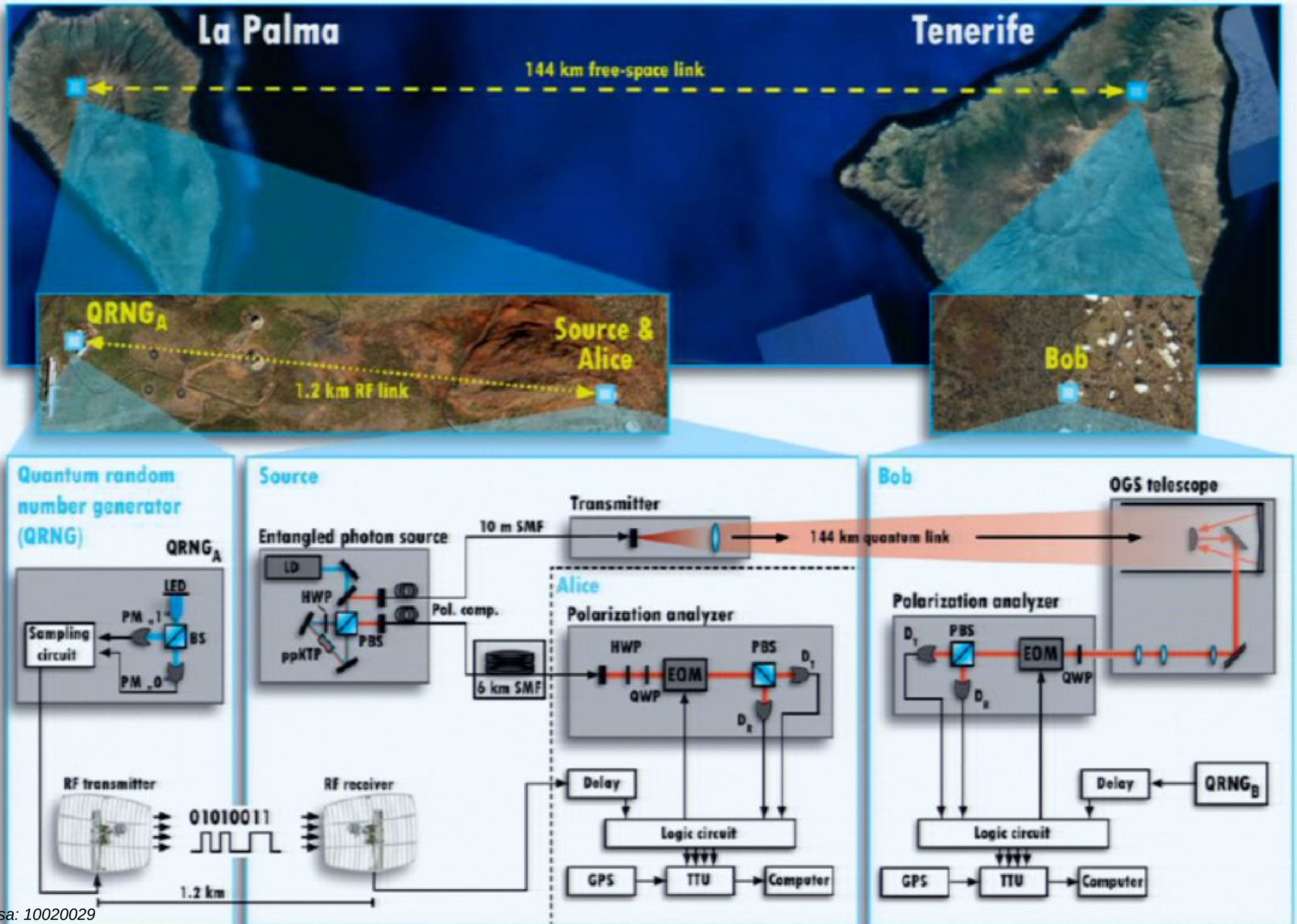


Bell Inequality Violation

			Alice			
			0°	45°	90°	135°
		Counts	104122	93348	100144	90841
Bob	22.5°	77988	**313**	**1636**	**1728**	**179**
	67.5°	75892	**418**	**269**	**1683**	**1100**
	112.5°	74935	**1978**	**294**	**351**	**1143**
	157.5°	73456	**1578**	**1386**	**361**	**156**
			Coincidences			

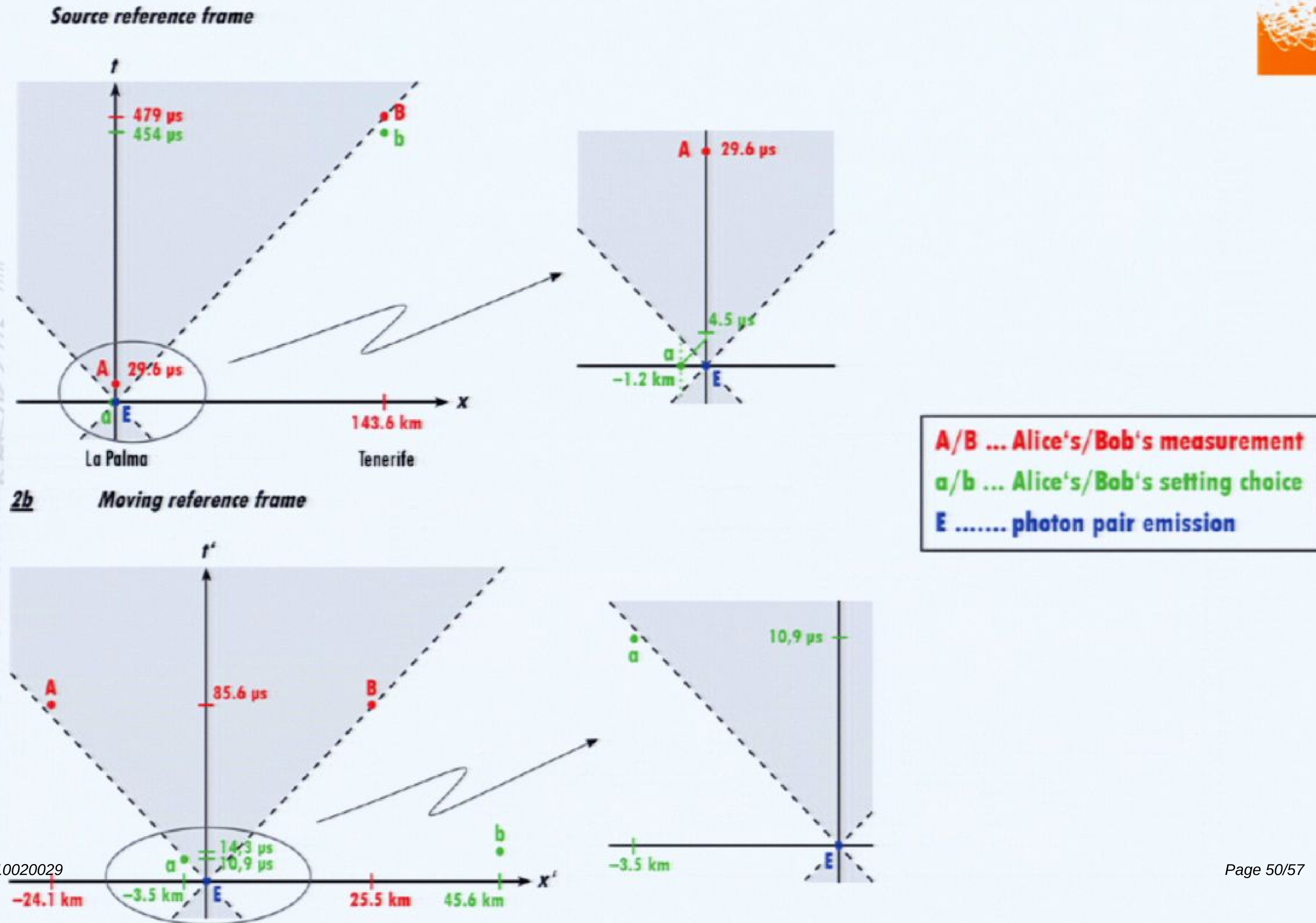
			Quantum Prediction		
Time Shift	3.8	ns			
Coincidence Window	4.0	ns			
Coincidences Total	14573		ideal	at 94%-97% Contrast	
E(0°,22.5°)	-0.70	± 0.01	-0.71	0.68	± 0.02
E(0°,67.5°)	-0.61	± 0.01	-0.71	0.68	± 0.02
E(45°,22.5°)	0.71	± 0.01	0.71	0.68	± 0.02
E(45°,67.5°)	-0.71	± 0.01	-0.71	0.68	± 0.02
S	**2.73**	**± 0.02**	2.82	2.72	± 0.04
Violation	29.8	stddev			

Freedom of choice



T. Scheidl, et al. *Violation of local realism with freedom of choice*, arXiv:0811.3129v1

Spacetime pictures



Closing the detection loophole

- What efficiency do we even need?
- Use CH inequality (others identical) for probabilities normalized by emitted pairs:

$$p_{++}^{\text{ex}}(a,b) = \frac{1}{2}\eta^2 \cos^2(\beta - \alpha)$$

$$p_{+}^{\text{ex}}(a) = p_{+}^{\text{ex}}(b) = \frac{1}{2}\eta.$$

$$\begin{aligned} &\tfrac{1}{4}\eta^2 \left[\cos^2 22.5^\circ - \cos^2 67.5^\circ + \cos^2 22.5^\circ + \cos^2 22.5^\circ\right] - \eta\left[\tfrac{1}{2} + \tfrac{1}{2}\right] = \\ &= \quad \tfrac{1}{2}\eta^2\left[1 + \sqrt{2}\right] - \eta \leq 0 \\ &= \quad \eta \leq \tfrac{2}{1+\sqrt{2}} = 0.828 = \eta_{\text{min}} \end{aligned}$$

Making it „easier“

- P. Eberhard: use non-maximally entangled states and appropriate angles ➔ η = 67%

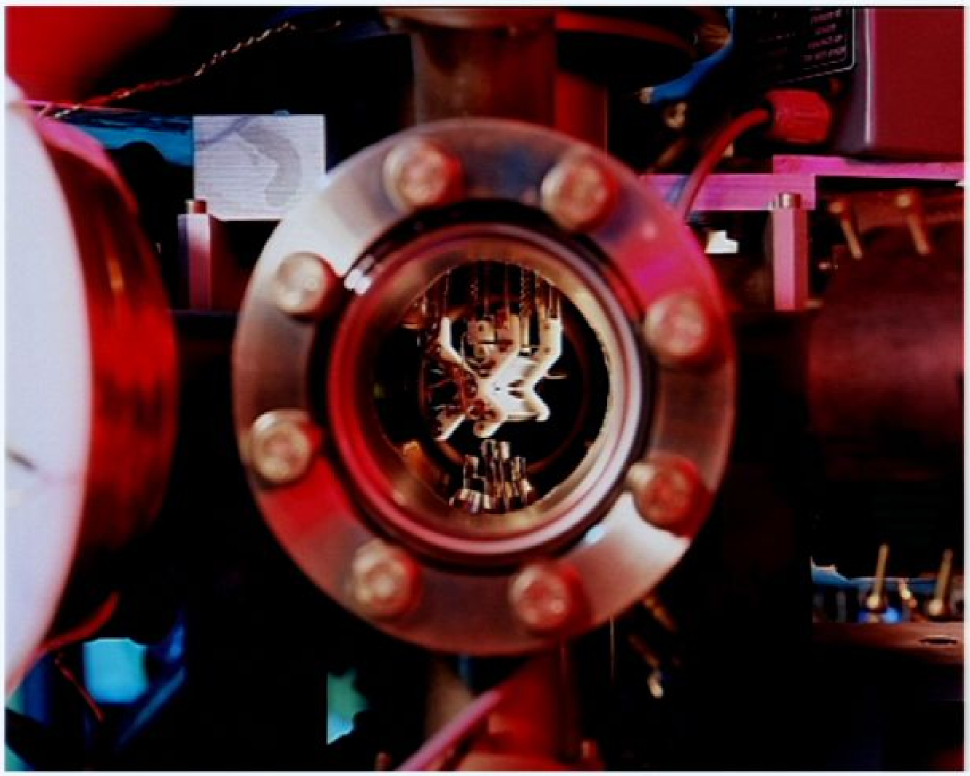
- Brunner *et al.* + Larsson & Cabello:
  Assymetric efficiencies as in combined atom-photon entanglement allows one side to have only 50% efficiency

Some Facts About Photodetectors

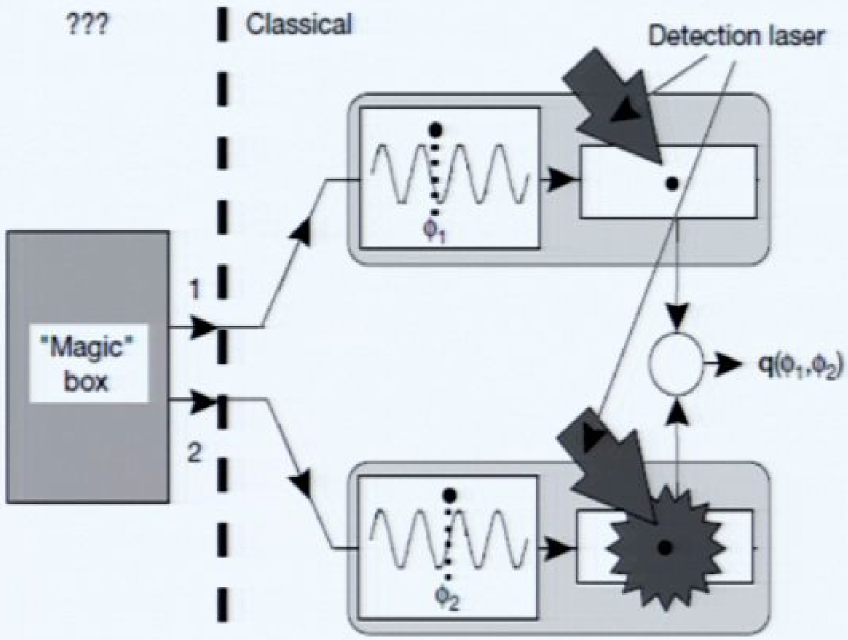
- Detection efficiency = Quantum efficiency * Amplification efficiency
- For red light about 70% available
- Highest combined detection*collection efficiency reported: 30% by Kurtsiefer et al. PRA **64,** 023802 (2001).
- Most common:
  Single Photon Avalanche Diode (SPAD)
- Alternative detectors
  - Visible Light Photon Counter
  - Superconducting Transition Edge Detector

Trapped Ions

- Rowe *et al.*, *Experimental Violation of a Bell's Inequality with Efficient Detection*, Nature **409,** 791 - 794 (2001).



Trapped Ions Concrete



- Use two hyperfine levels of a Be^+ Ion
- Using Lasers entangle the ions (fidelity 88%)
- Apply rotations before analysis
- Analyze and detection = measure state by probe laser on different transition for which one of the levels is bright (scatters photons) and one is dark.
- Can't distinguish the two ions! (only 3 μm separation)

Trapped Ions Results

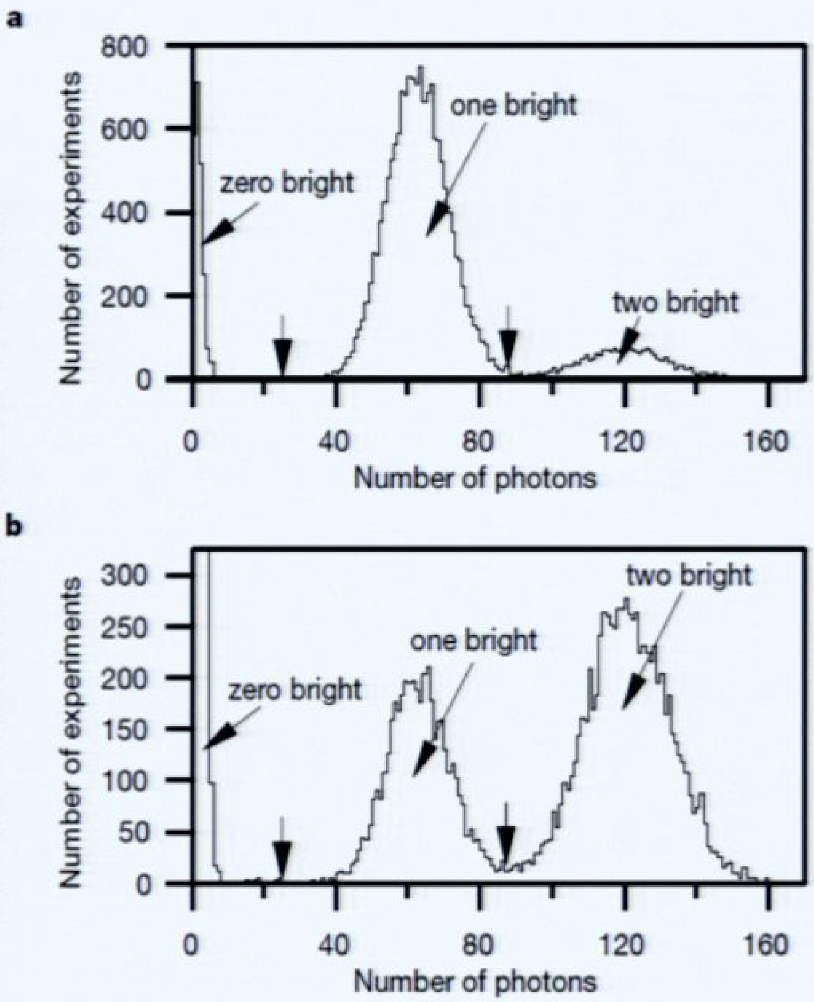


Table 1 The four sets of phase angles used for the Bell's experiment

Experiment input	ϕ_1	ϕ_2	$\Delta\phi$	ϕ_{tot}
$\alpha_1\beta_2$	$-\pi/8$	$-\pi/8$	0	$-\pi/4$
$\alpha_1\gamma_2$	$-\pi/8$	$3\pi/8$	$-\pi/2$	$+\pi/4$
$\delta_1\beta_2$	$3\pi/8$	$-\pi/8$	$+\pi/2$	$+\pi/4$
$\delta_1\gamma_2$	$3\pi/8$	$3\pi/8$	0	$+3\pi/4$

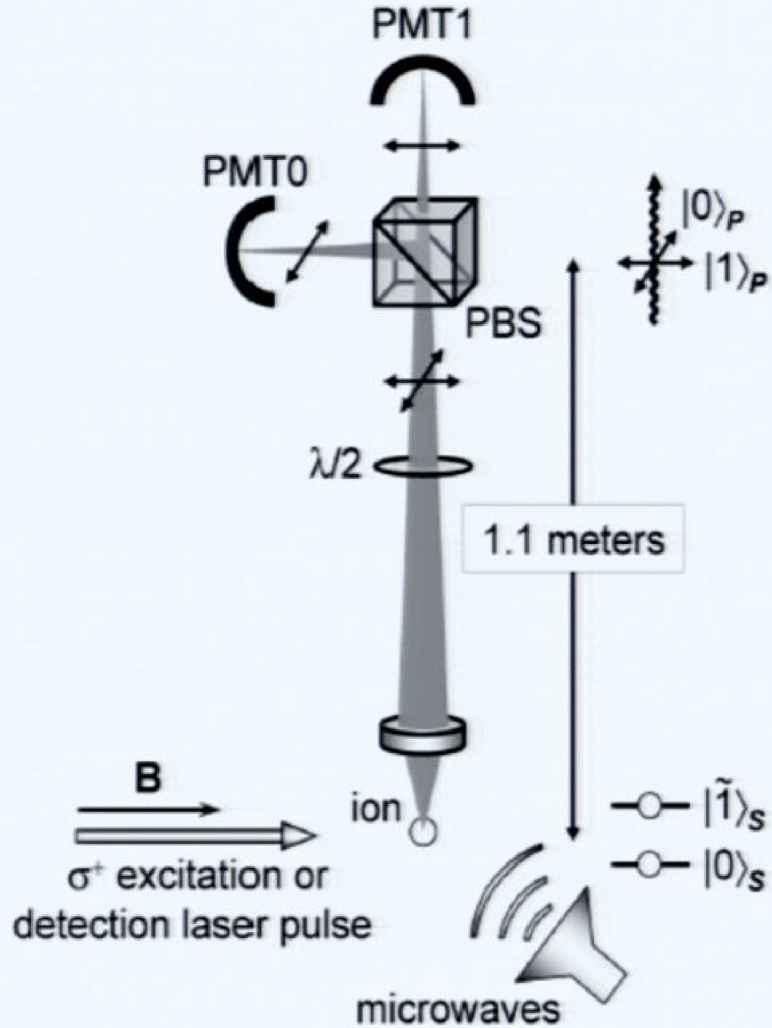
Table 2 Correlation values and resulting Bell's signals for five experimental runs

Run number	$q(\alpha_1, \beta_2)$	$q(\alpha_1, \gamma_2)$	$q(\delta_1, \beta_2)$	$q(\delta_1, \gamma_2)$	$B(\alpha_1, \delta_1, \beta_2, \gamma_2)$
1	0.541	0.539	0.569	−0.573	2.222
2	0.575	0.570	0.530	−0.600	2.275
3	0.551	0.634	0.590	−0.487	2.262
4	0.575	0.561	0.559	−0.551	2.246
5	0.541	0.596	0.537	−0.571	2.245

The experimental angle values were $\alpha_1 = -(\pi/8)$, $\delta_1 = 3\pi/8$, $\beta_2 = -(\pi/8)$, and $\gamma_2 = 3\pi/8$. The statistical errors are 0.006 and 0.012 for the q and B values respectively. The systematic errors (see text) are 0.03 and 0.06 for the q and B values respectively.

Conclusions

- Convincing evidence but
- No completely decisive experiment to date
- New photodetectors
- Remote trapped ions



Moehring et al., PRL **93,** 090410 (2004).