Title: The U(N) structure of Loop Quantum Gravity

Date: Feb 16, 2010 12:30 PM

URL: http://pirsa.org/10020027

Abstract: It has recently uncovered that the intertwiner space for LQG carries a natural representation of the U(N) unitary group. I will describe this U(N) action in details and show how it can be used to compute the LQG black hole entropy, to define coherent intertwiner states and to reformulate the LQG dynamics in new terms.

The U(N) Structure of Loop Quantum Gravity

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February 2010 at the Perimeter Institute

An old idea with F. Girelli,
then mostly based on work with L. Freidel,
with more recent work with E. Borja, J. Diaz-Polo and I. Garay

Aim: Looking closer at the Structure of $\mathrm{SU}(2)$ Intertwiners for Loop Quantum Gravity.

1. A $\mathrm{U}(N)$ Action on the Space of $\mathrm{SU}(2)$ Intertwiners:
   - defines area-preserving diffeomorphisms at the discrete level.
2. Counting Intertwiners and compare to Black Hole Entropy
3. The Intertwiner Space as a L^2 space
4. Creation/Annihilation Operators and Coherent Intertwiners
5. A new Approach to LQG Dynamics

A useful tool: Schwinger Representation for $su(2)$

The Object: the space of intertwiners with N legs i.e of $\mathrm{SU}(2)$-invariant states in the tensor product of irreps $V^{j_1} \otimes .. \otimes V^{j_N}$ for arbitrary values of spins $j_i \in \mathbb{N}/2$.

First Step: Write the spaces V^j as Hilbert spaces for a system of two harmonic oscillators at fixed total energy.

$$[a, a^\dagger] = [b, b^\dagger] = 1,$$

$$J_z = \frac{1}{2}(a^\dagger a - b^\dagger b)$$

$$J_+ = a^\dagger b, \quad J_- = a b^\dagger,$$

$$E = \frac{1}{2}(a^\dagger a + b^\dagger b)$$

$$\Rightarrow$$

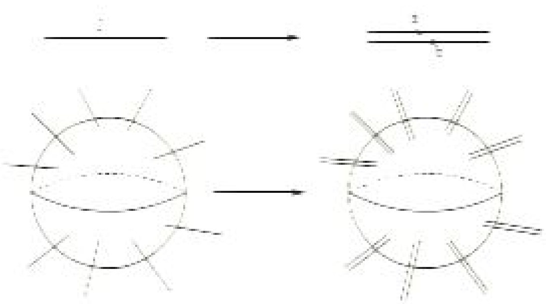
$$[J_z, J_\pm] = \pm J_\pm, \quad [J_+, J_-] = 2J_z,$$

$$\mathcal{C} = \vec{J}^2 = E(E+1),$$

$$|j, m\rangle = |n_a, n_b\rangle_{OH},$$

$$j = \frac{1}{2}(n_a + n_b), \quad m = \frac{1}{2}(n_a - n_b).$$

SU(2) Invariant Operators and the u(N) Algebra



Second Step: We consider intertwiners with N legs, thus we take $2 \times N$ oscillators a_i, b_i.

We look for invariant observables, i.e operators that commute with global $\mathrm{SU}(2)$ transformations generated by $\vec{J} = \sum_{i=1}^{N} \vec{J}^{(i)}$.
The standard operators that characterize intertwiners are the scalar product operators, $\mathcal{O}_{ij} \equiv \vec{J}^{(i)} \cdot \vec{J}^{(j)}$, which are quartic in the a, b's.
A problem: The commutators of the $\vec{J}^{(i)} \cdot \vec{J}^{(j)}$'s are cubic in the J's (volume) and generate an infinite tower of higher order operators. How to build semi-classical coherent states??
A solution: Build invariant operators which form a closed algebra?

The $\mathfrak{u}(N)$ Algebra as the Underlying Structure of the Intertwiner Space

Harmonic oscillators allow quadratic invariant operators, [Girelli, EL 05]

$$E_{ij} \equiv (a_i^\dagger a_j + b_i^\dagger b_j), \quad [E_{ij}, E_{kl}] = \delta_{jk} E_{il} - \delta_{il} E_{kj}.$$

The new operators E_{ij} form a closed $\mathfrak{u}(N)$ algebra !

$$E_i = E_{ii} = \text{ spin } 2j_i, \; E = \sum E_i = 2 \times \text{ total area.}$$

At fixed number N of legs, the $\mathfrak{u}(N)$ transformations change the individual spins j_i on each leg, but they still commute with the $\mathrm{U}(1)$ Casimir $E = 2\sum_i j_i$ which gives the (total) area.

$\mathrm{U}(N)$ is identified to the group of area-preserving diffeomorphisms on the discrete sphere defined as the space of $\mathrm{SU}(2)$ intertwiners with N legs. [Freidel, EL 09]

Here, the natural definition is Area $= \sum_i j_i$.

Third Step: Which $\mathrm{U}(N)$ representation(s)?

The $\mathfrak{u}(N)$ Algebra from Harmonic Oscillators

In fact, standard construction in mathematics !

- Take P sets of N harmonic oscillators $a_i^{(p)}$, $1 \leq i \leq N$, $1 \leq p \leq P$, then $E_{ij} = \sum_p a_i^{(p)\dagger} a_j^{(p)}$ form a $\mathfrak{u}(N)$ Lie algebra.

Taking $P = 1$ gives a highest weight $[l, 0, 0, ..]$ and a Young tableau with a single line, then we tensor such representations P times.

For $P < N$, this gives irreducible representations of $\mathrm{U}(N)$ with highest weight $[l_1, ..., l_P, 0, 0, ..]$ or equivalently Young tableaux with P horizontal lines.

Or more explicitly. . .

Casimir Equation and Highest Weights

For **P=1**, we get quadratic relations on the $\mathfrak{u}(N)$ generators:

$$\forall i.\ \sum_j E_{ij}E_{ji} = E_i(E+N-1). \quad \sum_{i,j} E_{ij}E_{ji} = E(E+N-1).$$

Taking a highest weight vector v, s.t. $E_i v = l_i v$ with the weights $l_i \in \mathbb{N}$ and $E_{ij}v = 0$ for all $i<j$, with $l_1 \geq l_2 \geq .. \geq l_N \geq 0$, the Casimir equation implies that $l_2 = .. = l_N = 0$.

For **P=2**, we relate the Casimirs of $\mathrm{SU}(N)$, $\mathrm{U}(1)$ and $\mathrm{SU}(2)$:

$$\sum_{i,j} E_{ij}E_{ji} = E(\frac{E}{2}+N-2) + 2\vec{J}\cdot\vec{J}.$$

$\Rightarrow$ highest weight $[l_1, l_2, 0, 0, ..]$ with $\begin{cases} l_1 - l_2 = 2\mathcal{J} \\ \vec{J}\cdot\vec{J} = \mathcal{J}(\mathcal{J}+1) \end{cases}$

SU(2) Intertwiner Spaces as Representations of U(N)

Intertwiners correspond to $\mathcal{J} = 0$, thus $l_1 = l_2$. The representation is defined by the highest weight $[l, l, 0, ..]$ with $\mathrm{U}(1)$ Casimir is $E = 2l$, i.e Area$= l = \sum_i j_i$.

Meaning of the highest weight??

A bivalent intertwiner! ▶ A completely squeezed sphere.

The Young tableau is two lines of equal length l.

$$\text{hook formula} \Rightarrow \dim_N[l] = \frac{1}{l+1}\begin{pmatrix} N+l-1 \\ l \end{pmatrix}\begin{pmatrix} N+l-2 \\ l \end{pmatrix}.$$

- $\mathrm{U}(N)$ acts on the space of intertwiners at fixed total area l and number of punctures N, including trivial irreps $j_i = 0$.
- $\dim_N[l]$ gives the number of such intertwiners.

The Generating Functionals for the U(N) Dimensions...

For fixed N, we just established the equality:

$$\dim_N[l] = I^{(N)}[l] \equiv \sum_{j_1+..+j_N=l} \dim_0[j_1.\dots j_N].$$

We can check this by introducing the generating functionals $F_N(t) = \sum_l t^{2l}\dim_N[l]$ and $\widetilde{F}_N(t) = \sum_l t^{2l} I^{(N)}[l]$. $\widetilde{F}_N$ can be computed directly as an integral over $\mathrm{SU}(2)$:

$$\widetilde{F}_N(t) = \int dg\, \left[\sum_j t^{2j}\chi_j(g)\right]^N = \frac{2}{\pi}\int_0^\pi d\theta\, \frac{\sin^2\theta}{(1-2t\cos\theta+t^2)^N}.$$

We check that both functional satisfy the same 2nd order (hypergeometric) diff eqn:

$$\Delta^{(N)}F_N = \Delta^{(N)}\widetilde{F}_N = 0. \quad \text{with}\dots$$

... The Generating Functionals for the U(N) Dimensions

$$\Delta^{(N)} \equiv \frac{1}{4}(1-t^2)(t\partial_t^2 + 3\partial_t) - N(N-1)t - (N-1)t^2\partial_t.$$

We show this using two different methods:

- a recursion relation:
  $(l+1)(l+2)\,\dim_N[l+1] = (N+l)(N+l-1)\,\dim_N[l]$.
- a brute force calculation on the integral:

$$\Delta^{(N)}\widetilde{F}_N = \frac{N(1-t^2)}{\pi}\int_0^\pi d\theta\,\partial_\theta \frac{\sin^3\theta}{(1-2t\cos\theta+t^2)^{N+1}} = 0.$$

At the end of the day, $F_N(t)$ can be expressed in term of the first derivative of the Legendre polynomials.

The Big Generating Functional

Now, we introduce the full generating functional by also performing the sum over the number N of legs: $F(u,t) = \sum_{N,l} u^N t^{2l} \dim_N[l]$. And we can compute explicitly from the integral representation:

$$F(u,t) = \frac{1}{2t^2}\left[t^2(u+2) - u^2 + u - \frac{u[t^2(t^2-2u-2)+(u-1)^2]}{\sqrt{((1+t)^2-u)((1-t)^2-u)}}\right].$$

For fixed $0 < u < 1$, the first pole is $t_c = 1 - \sqrt{u}$, thus the asymptotics:

$$\log \sum_N u^N \dim_N[l] \underset{l\to\infty}{\sim} -2l \log(1-\sqrt{u}).$$

Probing Further with U(N) Charaters

Beyond computing the dimensions of the U(N) representation, we look at the U(N) characters. For group elements conjugated to $e^{i(s_1 E_1 + .. + s_N E_N)}$, we get a Van Der Monde determinant in $t_i = e^{is_i}$:

$$\chi_{[l_i]}(t_1 \dots t_N) = \frac{\det(t_k^{l_i+N-i})_{ik}}{\det(t_k^{N-i})_{ik}}.$$

For highest weights $[l, l, 0, ..]$, this corresponds to

$$\chi_l(t_1 \dots t_N) = \sum_{j_1+..+j_N=J} \prod_i t_i^{2j_i} \dim_0[j_1 \dots j_N].$$

It allows to distinguish the values of different punctures. We can also introduce multi-index generating functionals:

$$F_N(t_1 \dots t_N) = \sum_J \chi_J(t_1 \dots t_N) \quad = \frac{1 - \prod_i t_i}{\prod_{i<j}(1 - t_i t_j)} \text{for } N = 4.$$

Black Hole Entropy?

$\dim_N[I]$ counts the number of $\mathrm{SU}(2)$ intertwiners, so it gives the black hole entropy? But...

- Standard LQG isolated horizon counts $\mathrm{U}(1)$ intertwiners
  ▶ but see recent work [Engle,Noui,Perez 09] but also [Kaul,Majumdar Krasnov,Rovelli Dreyer,Smolin EL,Terno Agullo,Barbero,Borja,Diaz-Polo,Villasenor]
- The total area is $\sum_i j_i$ instead of the standard $\sum_i \sqrt{j_i(j_i+1)}$
  ▶ but $\sum_i j_i$ is the area preserved by the $\mathrm{U}(N)$ transformations.
- We are counting many intertwiners which carry trivial legs !
- The dimension depends on the number of punctures N
  ▶ but we can argue that N is given by the graph of the outside spin network state.
  ▶ or we can sum over the number of punctures...

Asymptotics of $\dim_N[l]$

First we give the asymptotics of $S_N[l] \equiv \ln \dim_N[l]$ for large area and large number of punctures N.

- **Large Area Limit:**
  at fixed N, $S_N[l] \underset{l\to\infty}{\sim} (2N-4)\ln l$
- **"Continuum" Limit:**
  at fixed l, $S_N[l] \underset{N\to\infty}{\sim} 2l \ln N$
- **Linear Regime:** Scale number of punctures as $N \sim \lambda\, l$
  $\Rightarrow$ holographic behavior:

$$S_N[l] \underset{l\to\infty}{\sim} 2[(1+\lambda)\ln(1+\lambda) - \lambda \ln \lambda]\, l - 2\ln l$$

Removing Trivial Punctures

We count the irreps: $[j_1 \ldots j_N]$ — $\{(j, k_j)\}$ with $\left| \begin{array}{l} I = \sum_j j k_j \\ N = \sum_j k_j \end{array} \right.$.

$\dim_0[j_1 \ldots j_N]$ depends only the occurrence nbs k_j for $j > 0$.

Thus we separate trivial punctures, $K \equiv N - k_0$:

$$\Rightarrow \dim_N[I] = \sum_{K=0}^{N} \binom{N}{K} D_K[I].$$

$$D_K[I] = \sum_{\sum_{j \geq 1} k_j = K} \frac{K!}{\prod_j k_j!} \dim_0[\{k_j\}].$$

Final Counting

We want the number of intertwiners at fixed area without trivial punctures:

$$D[l] = \sum_K D_K[l]. \quad \blacktriangleright \text{ Sum is finite: } K \leq 2l$$

$D_K[l]$ is the binomial transform of $\dim_N[l]$:

$$D_K[l] = \sum_{N=0}^{K} (-1)^{K-N} \begin{pmatrix} K \\ N \end{pmatrix} \dim_N[l].$$

We introduce the generating functionals, which can be computed as integrals as before:

$$G_K(t) = \sum_l t^{2l} D_K[l]. \quad G(u,t) = \sum_{K,l} u^K t^{2l} D_K[l].$$

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Final Result

The binomial transform has a simple generating functional !

$$G(u.t) = \frac{1}{1+u} F(\frac{u}{1+u}.t).$$

This gives $D[l]$ in term of the original dimensions $\dim_N[l]$:

$$\sum_l t^{2l} D[l] = G(1.t) = \frac{1}{2} F(\frac{1}{2}.t). \quad D[l] = \sum_N \frac{1}{2^{N+1}} \dim_N[l].$$

$$\Rightarrow S_\emptyset[l] = \ln D[l] \underset{l\to\infty}{\sim} l \ln \alpha - \frac{3}{2} \ln l.$$

with $\alpha = \frac{1}{\left(1-\sqrt{\frac{1}{2}}\right)^2} = 6 + 4\sqrt{2} \simeq 11.6568.$

More details?

We give the exact expression for $\mathcal{F}(t) = \sum_l t^{2l} D[l]$:

$$\begin{aligned}\mathcal{F}(t) &= \frac{5}{8} + \frac{1}{16t^2}\left(1 - \sqrt{1 - 12t^2 + 4t^4}\right) \\ &= 1 + t^2 + 6t^4 + 44t^6 + 360t^8 + 3152t^{10} + \dots\end{aligned}$$

It satisfies a 1st order diff eqn in term of $T \equiv t^2$:

$$T(1 - 12T + 4T^2)\partial_T \mathcal{F} + (1 - 6T)\mathcal{F} + (4T - 1) = 0.$$

which translates to a recursion relation on the dimension $D[l]$:

$$D[0] = D[1] = 1. \; D[l] = \frac{1}{l+1}\left(6(2l-1)D[l-1] - 4(l-2)D[l-2]\right).$$

The Intertwiner Space as a L^2 Space...

The space of intertwiners with N legs can be represented as a space of L^2 functions:

$$\mathcal{H}_N = \bigoplus_{\{j_i\}} Inv[j_1 \otimes .. \otimes j_N] = \bigoplus_{J} R^J_N = L^2\left(Gr_{2,N}\right).$$

$Gr_{2,N}$ is a Grassmanian space:

$$Gr_{2,N} \equiv \frac{\mathrm{U}(N)}{\mathrm{U}(N-2)\times \mathrm{SU}(2)}.$$

The subgroup $\mathrm{U}(N-2)\times\mathrm{SU}(2)$ stabilizes the highest weight vector: $\mathrm{U}(N-2)$ is generated by $E_{ij}, i, j \geq 3$ and $\mathrm{SU}(2)$ by $E_{12}, E_{21}, E_1 - E_2$.

...The Intertwiner Space as a L^2 Space

The space $L^2(Gr_{2.N})$ consists in functions on $\mathrm{U}(N)$ satisfying:

$$\forall G \in \mathrm{U}(N),\ \forall H \in \mathrm{U}(N-2)\times\mathrm{SU}(2), \quad f(GH) = f(G).$$

$Gr_{2.N} \sim P_N(A) \times U(1)^N$ can be interpreted as the space of polyhedra with N faces and only trivalent vertices at an arbitrary fixed total area A. Such polyhedra have $3(N-2)$ edges and $2(N-2)$ vertices, so dimensions match:

$$N^2 - (N-2)^2 - 3 = 4N - 7 = 3(N-2) - 1 + N.$$

This provides a geometric interpretation for intertwiners.

Gluing Intertwiners into Spin Networks

Spin networks on a graph Γ are functions of E group elements and satisfying gauge invariance at each vertex:

$$\varphi \in L^2(\mathrm{SU}(2)^E/\mathrm{SU}(2)^V) \; - \; \varphi(\{g_e\}) = \varphi(\{h_{s(e)}^{-1} g_e h_{t(e)}\}).$$

We can shift the degrees of freedom to the vertices:

$$\mathcal{H}_\Gamma = \bigoplus_{\{j_e\}} \bigotimes_{v\in\Gamma} \mathcal{H}_{j_1^v \dots j_{N_v}^v} = L^2\left((\times_e \mathrm{U}_{(e)}(1)) \backslash (\times_v Gr_{2.N_v})\right).$$

where $\mathrm{U}_{(e)}(1)$ generated by $E_e^{s(e)} - E_e^{t(e)}$ ensures the matching on the representations on the same edge e. The gauge invariance now reads on functions $f(K_v)$ with $K_v \in \mathrm{U}(N_v)$:

$$\left| \begin{array}{l} f(\{K_v\}) = f(\{K_v H_v\}) \\ f(\{K_{s(e)}.K_{t(e)}.K_v\}) = f(\{T_e K_{s(e)}.T_e K_{t(e)}.K_v\}). \end{array} \right.$$

But what's the point??

Two interesting consequences:

- We can reformulate the LQG dynamics as directly acting on the intertwiner spaces.
- The intertwiner space is a L^2 space of wave-functions of a classical unitary matrix: the intertwiner dynamics can be described in term of an underlying matrix model. [Borja,Diaz,Garay,EL next month]

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A new Set of SU(2)-invariant Operators

We identify a new set of SU(2)-invariant operators ! [Freidel,EL very soon]

$$F_{ij} \equiv a_i b_j - a_j b_i. \quad F_{ij} = -F_{ji}.$$

They are invariant under SU(2) but do not preserve the total area!!

$$[E, F_{ij}] = -2F_{ij}, \qquad [E, F^\dagger_{ij}] = +2F^\dagger_{ij}$$

$\Rightarrow$ annihilation/creation operators

With $E_\rho \equiv \sum_{ij} \rho^{ji} E_{ij}$, $F_\omega \equiv \frac{1}{2}\sum_{ij} \omega^{ji} F_{ij}$, the full algebra is:

$$[E_\alpha, E_\beta] = -E_{[\alpha,\beta]}, \qquad [E_\alpha, F_\omega] = -F_{(\alpha\omega+\omega\alpha^t)},$$

$$[F_{\omega_1}, F^\dagger_{\omega_2}] = E_{\omega_1\omega_2^\dagger}, \qquad [E_\alpha, F^\dagger_\omega] = F^\dagger_{(\alpha\omega+\omega\alpha^t)}.$$

What are Coherent Intertwiners?

We want:

- A over complete basis of intertwiners which transforms consistently under $\mathrm{U}(N)$ transformations.
- Semi-classical intertwiners with peakedness properties.
- A relation with the standard coherent/holomorphic intertwiners. [EL,Speziale 07 Conrady,Freidel 09 Freidel,EL,Krasnov 09]

This will provide an explicit geometric interpretation of the $\mathrm{U}(N)$ transformations.

Remember Spinors

We attach one spinor $|z_i\rangle = \begin{pmatrix} z_i^{(0)} \\ z_i^{(1)} \end{pmatrix}$ to each puncture/leg of the intertwiner. This defines a 3-vector n^i, the "normal vector" to the ith patch, for each leg with normalization $|n_i| = \langle z_i|z_i\rangle$:

$$|z_i\rangle\langle z_i| = \frac{1}{2}\left(\langle z_i|z_i\rangle \mathrm{Id}_2 + \vec{n^i}\cdot\vec{\sigma}\right).$$

With $A(z)$ the "total area", the closure constraints reads:

$$\sum_i n^i = 0 \Leftrightarrow \sum_i |z_i\rangle\langle z_i| = A(z)\mathrm{Id}. \quad A(z) \equiv \frac{1}{2}\sum_i \langle z_i|z_i\rangle.$$

Defining Coherent Intertwiners

We define coherent states as:

$$|J, z_i\rangle \propto \frac{(F_\omega^\dagger)^J}{J!}|0\rangle, \quad \omega_{ij} = \langle z_i|\epsilon z_j\rangle = \overline{z_i^{(1)} z_j^{(0)}} - \overline{z_i^{(0)} z_j^{(1)}}.$$

These are states with total area $E = J$, which behave consistently under $\mathrm{U}(N)$ transformations :

$$U\,|J, z_i\rangle = |J, (u^{-1}z)_i\rangle, \quad \text{where} \quad U = \exp(E_\alpha),\ u = e^\alpha.$$

$\Rightarrow$ They cover the intertwiner space.

Semi-classical states?

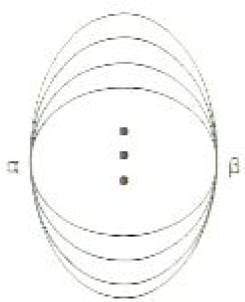
We can compute the expectation value of the E_{kl} :

$$\frac{\langle J.z_i | E_{kl} | J.z_i \rangle}{\langle J.z_i | J.z_i \rangle} = J \frac{\langle z_k | z_l \rangle}{A(z)} = J M_{kl}.$$

We check that M is a rank-two projector with eigenvalues $[1.1.0..]$, which corresponds to our choice of $\mathrm{U}(N)$ irreps. So we have the right mean values.
Then we can relate our new coherent states to the standard coherent intertwiners , which are known to be semi-classical:

$$|J.z_i\rangle \propto \sum_{\sum j_i = J} \frac{1}{\sqrt{(2j_1)!..(2j_N)!}} \int_{\mathrm{SU}(2)} dg\, g \triangleright \bigotimes_{i=1}^{N} |j_i.z_i\rangle$$

Dipolar Cosmology



We consider the simplest class of graphs for LQG with 2 vertices linked with N edges.

[Borja,Diaz,Garay,EL next month]

This generalizes the dipolar setting by Rovelli & Vidotto for cosmology with some inhomogeneities. [Rovelli,Vidotto 08]

We have two independent intertwiner spaces related by the constraints of having a single $\mathrm{SU}(2)$ spin per link. We define the $\mathfrak{u}(N)$ boundary deformation algebra: $e_{ij} = E^{\alpha}_{ij} - E^{\beta}_{ji}$.

The matching conditions are the diagonal operators $e_i = E^{\alpha}_i - E^{\beta}_i$

An ansatz for dynamics...

Two simple operators that respect the matching conditions: $E^{\alpha}_{ij}E^{\beta}_{ij}$ increases the spin j_i by $+\frac{1}{2}$ and decreases j_j by $\frac{1}{2}$ while $F^{\alpha}_{ij}F^{\beta}_{ij}$ decreases both spins j_i, j_j.

We introduce $f = \sum_{ij} F^{\alpha}_{ij}F^{\beta}_{ij}$ and $g = E^{\alpha}_{ij}E^{\beta}_{ij}$.

Both operators are invariant under boundary deformations: $[e_{ij}, f] = [e_{ij}, g] = 0$.

A simple ansatz for a Hamiltonian: $H \equiv \lambda g + (\sigma f + \bar{\sigma} f^{\dagger})$.

This generalizes the action of $\frac{1}{2}$-holonomy operators of the dynamics of BF theory.

... An ansatz for dynamics

Some simple remarks:

- Meaning of $f^\dagger$? If we apply $(f^\dagger)^J$ to the void state $|0\rangle$, we get a pure state for the coupled system for a fixed boundary area J but the totally mixed state for the individual systems α and β.
  $\Rightarrow$ $(f^\dagger)^J$ is "black hole creation operator".
- Hamiltonian constraint of LQG usually "holonomy with double graspings": new ansatz $\mathcal{H} \sim \sum_{ij} E^\alpha_{ij} E^\alpha_{ji} F^\alpha_{ij} F^\beta_{ij}$ not $\mathfrak{u}(N)$-invariant.
  $\Rightarrow$ invariant under which boundary deformations?
- A simple (cosmological) model to test LQG's dynamics !

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Both operators are invariant under boundary deformations: $[e_{ij}, f] = [e_{ij}, g] = 0$.

A simple ansatz for a Hamiltonian: $H \equiv \lambda g + (\sigma f + \bar{\sigma} f^{\dagger})$.

This generalizes the action of $\frac{1}{2}$-holonomy operators of the dynamics of BF theory.

... An ansatz for dynamics

Some simple remarks:

- Meaning of $f^\dagger$? If we apply $(f^\dagger)^J$ to the void state $|0\rangle$, we get a pure state for the coupled system for a fixed boundary area J but the totally mixed state for the individual systems α and β.
  $\Rightarrow$ $(f^\dagger)^J$ is "black hole creation operator".
- Hamiltonian constraint of LQG usually "holonomy with double graspings": new ansatz $\mathcal{H} \sim \sum_{ij} E^\alpha_{ij} E^\alpha_{ji} F^\alpha_{ij} F^\beta_{ij}$ not $\mathfrak{u}(N)$-invariant.
  $\Rightarrow$ invariant under which boundary deformations?
- A simple (cosmological) model to test LQG's dynamics !

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Conclusion & Outlook

1. $U(N)$ action on intertwiners at fixed nb of legs and fixed area
   Area-preserving Diffeomorphisms
2. We computed $\dim_N[I]$ with trivial punctures and $D[I]$ without trivial punctures and recovered the standard entropy formula .
3. Defined $U(N)$ coherent states and related them to coherent intertwiners
4. Showed how to use this framework to discuss LQG dynamics

- Procedure works for groups $U(M)$, $U_q(M)$ and susy groups
- $N \to \infty$ limit of intertwiner space and $U(N)$ as diffeomorphisms in the continuum?
- Dynamics of Horizons/Surfaces from matrix models?
- Solve and generalize dipolar cosmological model?
- Use $U(N)$ coherent states in spinfoam models?