Title: Quantum Field Theory for Cosmology - Lecture 12

Date: Feb 25, 2010 05:00 PM

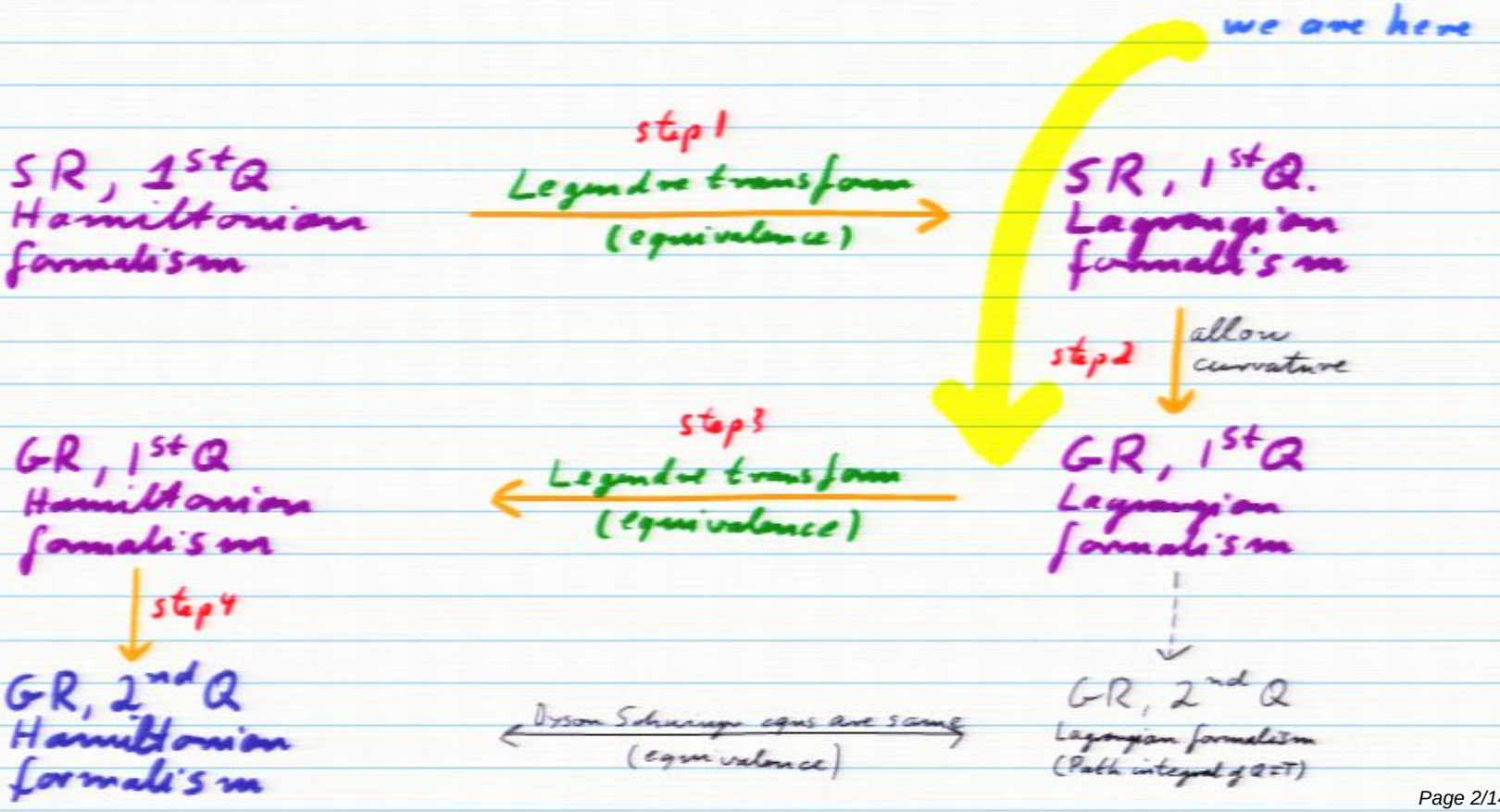
URL: http://pirsa.org/10020021

Abstract: This course begins with a thorough introduction to quantum field theory. Unlike the usual quantum field theory courses which aim at applications to particle physics, this course then focuses on those quantum field theoretic techniques that are important in the presence of gravity. In particular, this course introduces the properties of quantum fluctuations of fields and how they are affected by curvature and by gravitational horizons. We will cover the highly successful inflationary explanation of the fluctuation spectrum of the cosmic microwave background - and therefore the modern understanding of the quantum origin of all inhomogeneities in the universe (see these amazing visualizations from the data of the Sloan Digital Sky Survey. They display the inhomogeneous distribution of galaxies several billion light years into the universe: Sloan Digital Sky Survey).

QFT for Cosmology, Achim Kempf, Winter 2010, Lecture 12

Note Title 2/13/2006

Recall strategy:



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Recall:

General relativistic covariant Klein Gordon theory in the Lagrangian formulation (neglecting the potential):

* **The Klein Gordon eqn:**

$$\frac{1}{\sqrt{|g(x)|}} \frac{\partial}{\partial x^\mu} \left(g^{\mu\nu}(x) \sqrt{|g(x)|}\, \phi_{,\nu}(x) \right) + m^2 \phi(x) = 0 \qquad (KG)$$

arises via $\frac{\delta S_{KG}}{\delta \phi} = 0$ from **the action**:

$$S_{KG} = \frac{1}{2} \int_{\mathbb{R}^4} \left(g^{\mu\nu}(x)\, \phi_{,\mu}(x)\, \phi_{,\nu}(x) - m^2 \phi^2(x) \right) \sqrt{|g|}\, d^4x$$

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$$L_{KG}(t) = \frac{1}{2}\int_{\mathbb{R}^3}\left(g^{\mu\nu}(x,t)\,\phi_{,\mu}(x,t)\,\phi_{,\nu}(x,t) - m^2\phi^2(x,t)\right)\sqrt{|g|}\,d^3x$$

Step 3: Legendre transform back to the Hamiltonian form

* The transform:

$$L(\phi, \phi_{,0}, t) \xrightarrow[\text{Legendre transform}]{\pi(x,t) := \frac{\delta L}{\delta \phi_{,0}(x,t)}} H(\phi, \pi, t)$$

* Thus, the canonically conjugate field $\pi(x,t)$ reads:

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→ Exercise: Calculate $H(\phi,\pi,t)$ and simplify the expression, as far as possible.

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Why don't we need a factor of $\frac{1}{N!}$ for covariance here? Because π has it built in!

$= L(\phi, \phi_{,0}(\phi,\pi), t)$

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We know from the general properties of the Legendre transform that the equations of motion now take the form:

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* Exercise: Verify that these eqns are equivalent to (KG).

We are now ready to 2nd quantize:

step 1

SR, 1st Q → Legendre transform → SR, 1st Q.

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SR, 1st Q Hamiltonian formalism
→ step 1: Legendre transform (equivalence) →
SR, 1st Q. Lagrangian formalism
→ step 2: allow curvature ↓
GR, 1st Q Lagrangian formalism
← step 3: Legendre transform (equivalence) ←
GR, 1st Q Hamiltonian formalism

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step 4

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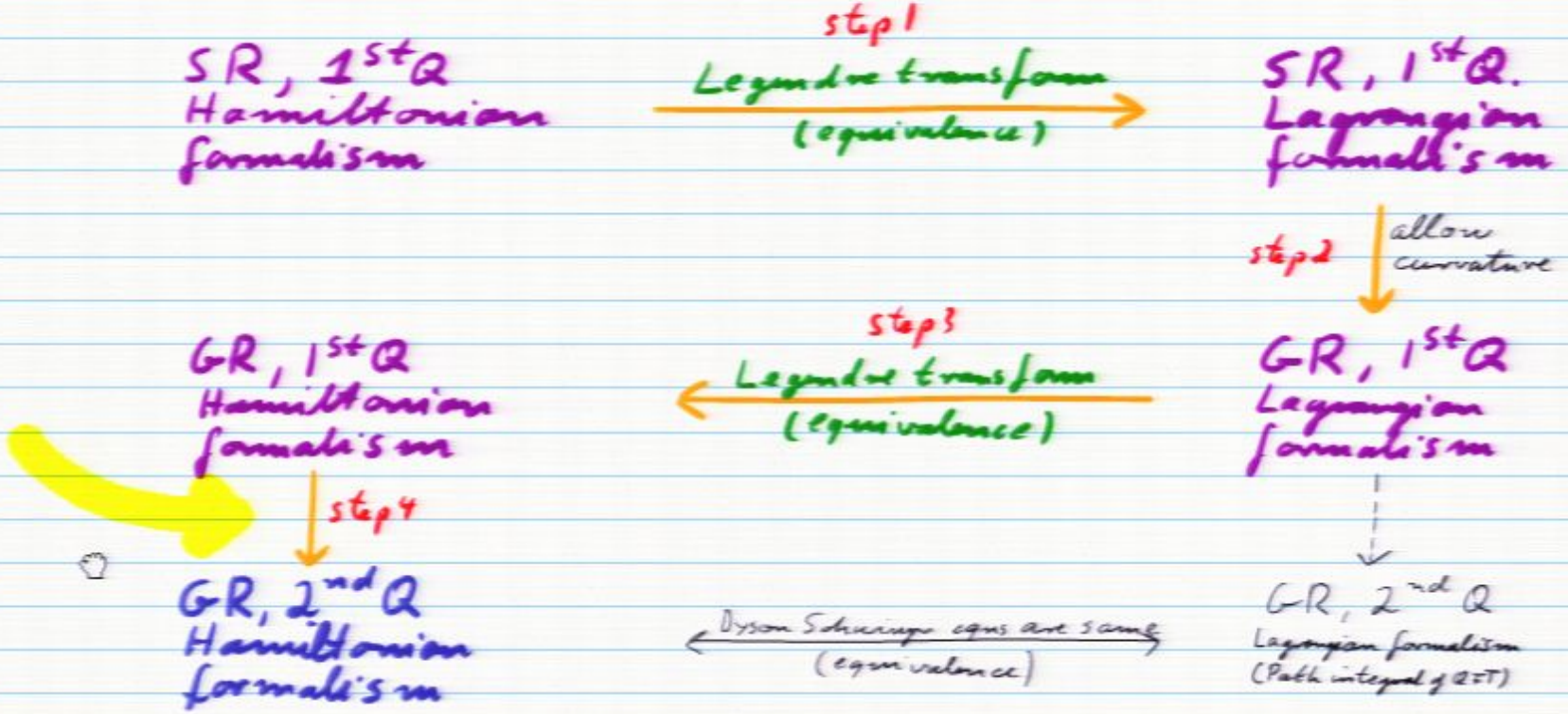

Solving the quantized theory is to solve:

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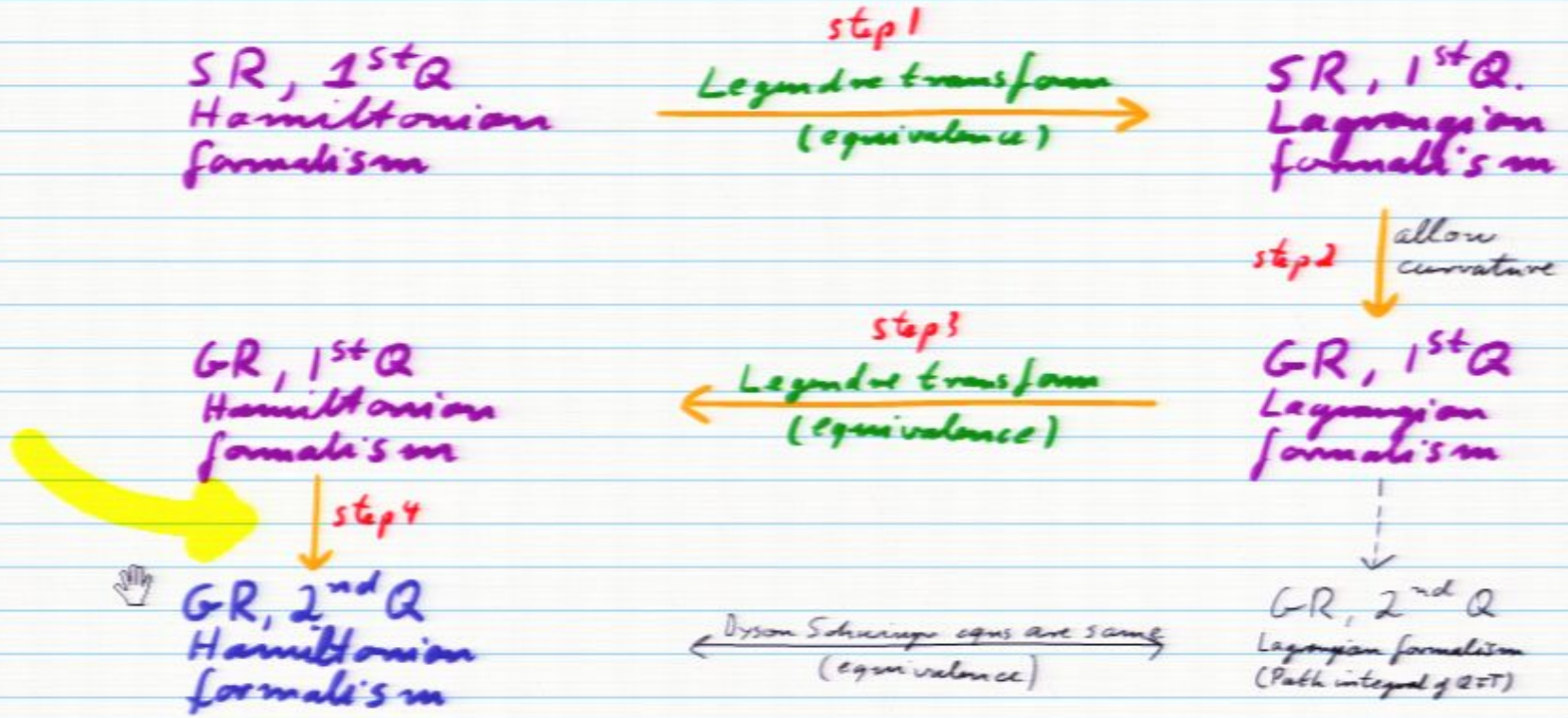

Solving the quantized theory is to solve:

1.) Commutation relations:

We are now ready to 2nd quantize:

SR, 1st Q Hamiltonian formalism

step 1: Legendre transform (equivalence) → SR, 1st Q. Lagrangian formalism

step 2: allow curvature → GR, 1st Q Lagrangian formalism

step 3: Legendre transform (equivalence) → GR, 1st Q Hamiltonian formalism

step 4 → GR, 2nd Q Hamiltonian formalism

GR, 2nd Q Lagrangian formalism (Path integral QFT)

Dyson Schwinger eqns are same (equivalence)

Solving the quantized theory is to solve:

1.) Commutation relations:

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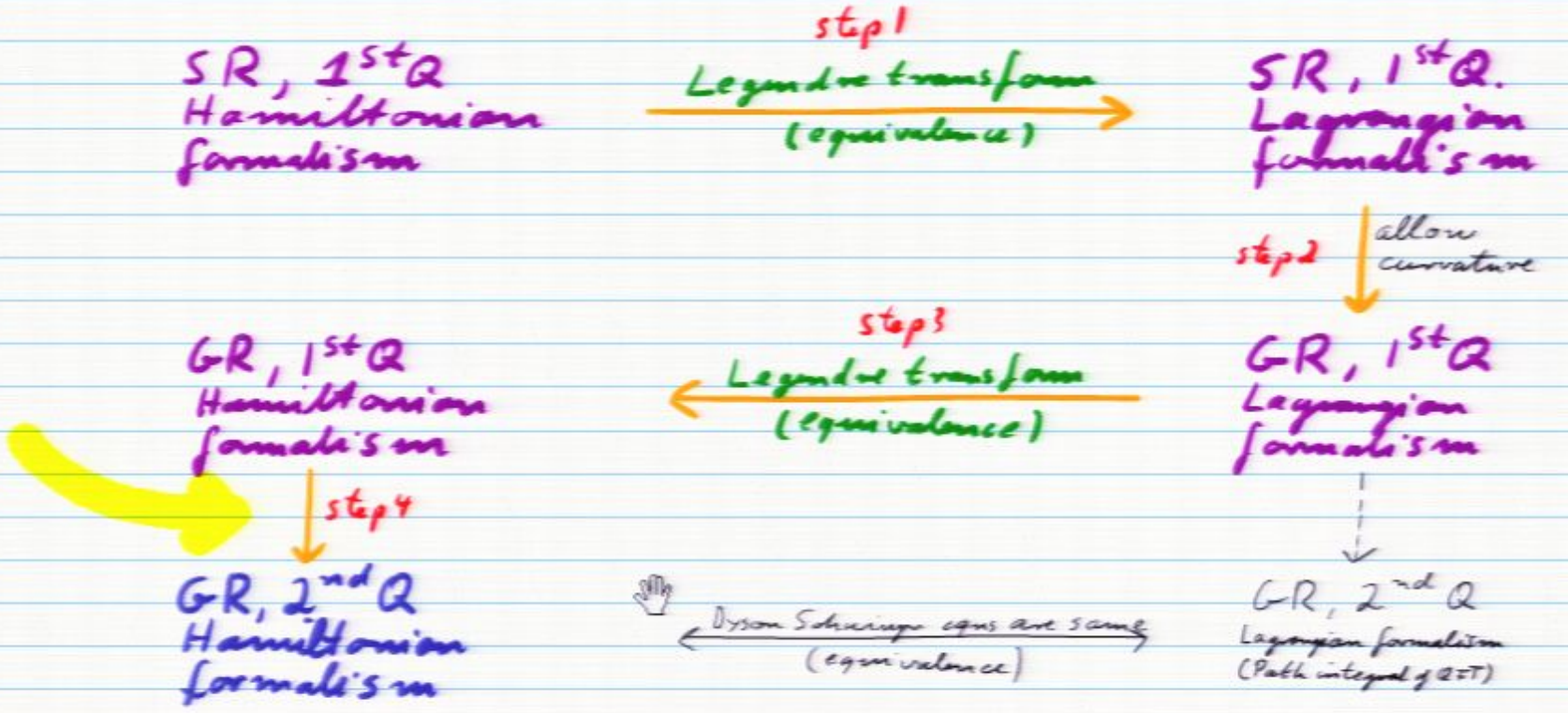

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} (CCRs)

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Exercise: verify

The i in λ is needed for hermiticity,

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all come out formally unchanged (except for the hats: ^).

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- Strategy: Try an ansatz which
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Checking the CCRs:

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Checking the CCRs:

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- Express $\hat{\phi}$ in terms of the ansatz:

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- Express $\hat{\pi}$ in terms of the ansatz:

$$\hat{\pi}(x,t) := \sqrt{|g|}\,g^{0\nu}\frac{\partial}{\partial x^\nu}\hat{\phi}(x,t) \quad \Rightarrow$$

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Conclusion so far:

Our ansatz

$$\hat{\phi}(x,t) := \sum u_n(x,t)\,a_n + u_n^*(x,t)\,a_n^+$$

solves the QFT, i.e., HC, EoM and CCR

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(8.1)

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- Then, such a set of functions $\{u_n\}$ can be shown to exist.

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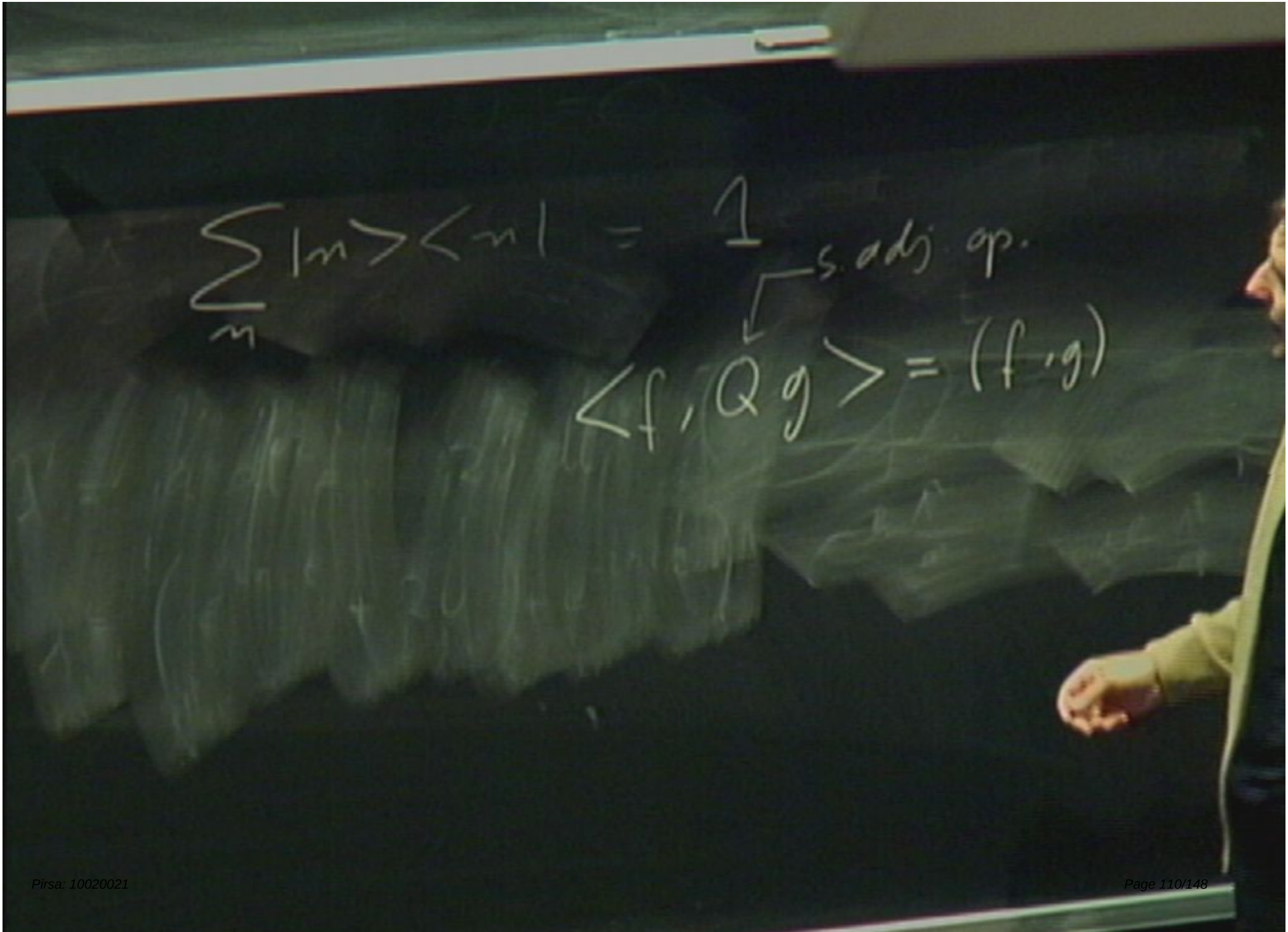
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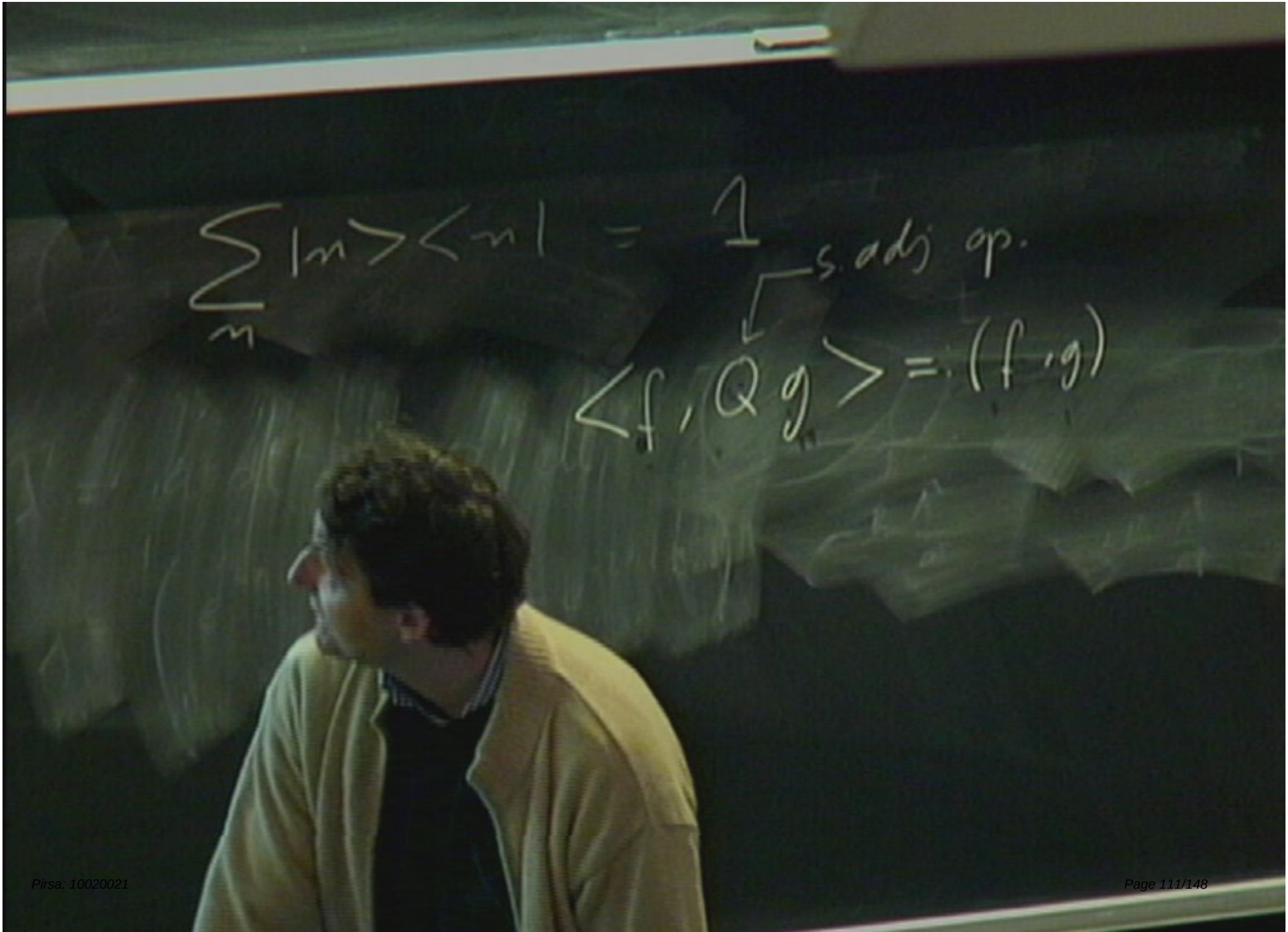
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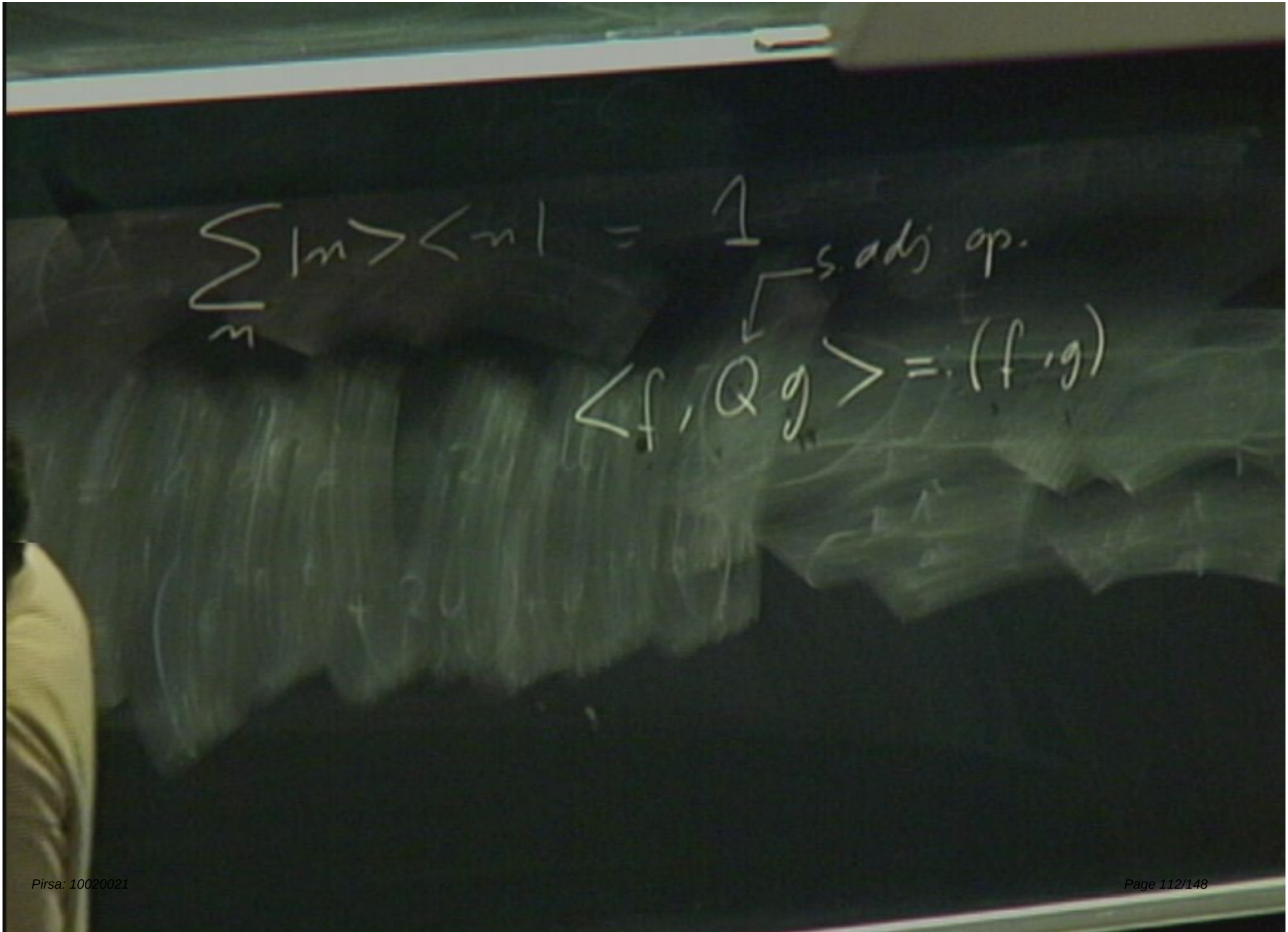
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Σ: any spacelike hypersurface i.e. set of points of equal time.

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P1) Well defined, i.e., independent of choice of Σ.

This is where we need global hyperbolicity. We'll prove P1 later (and see that e.g. its $-$ sign is needed!).

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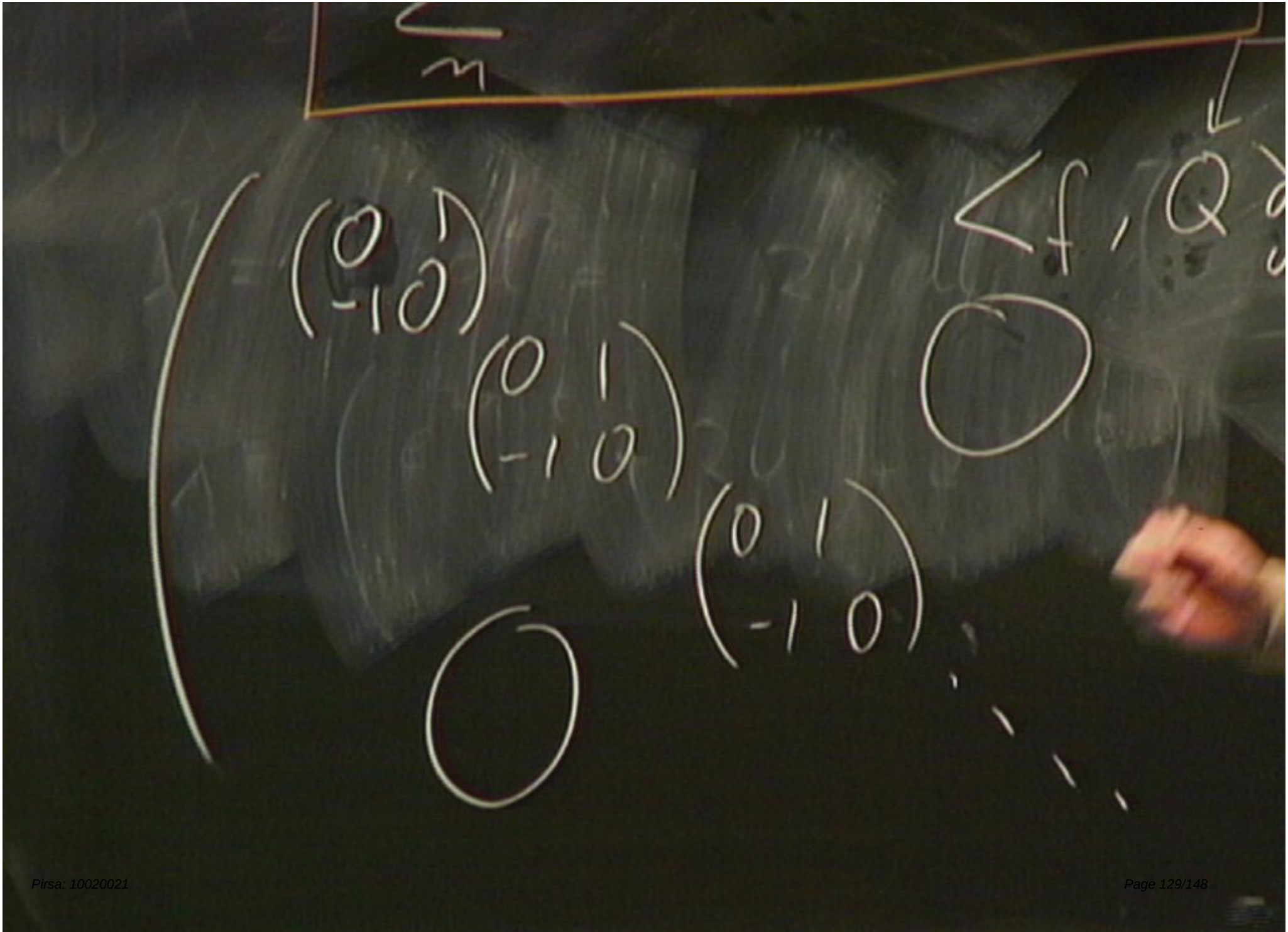
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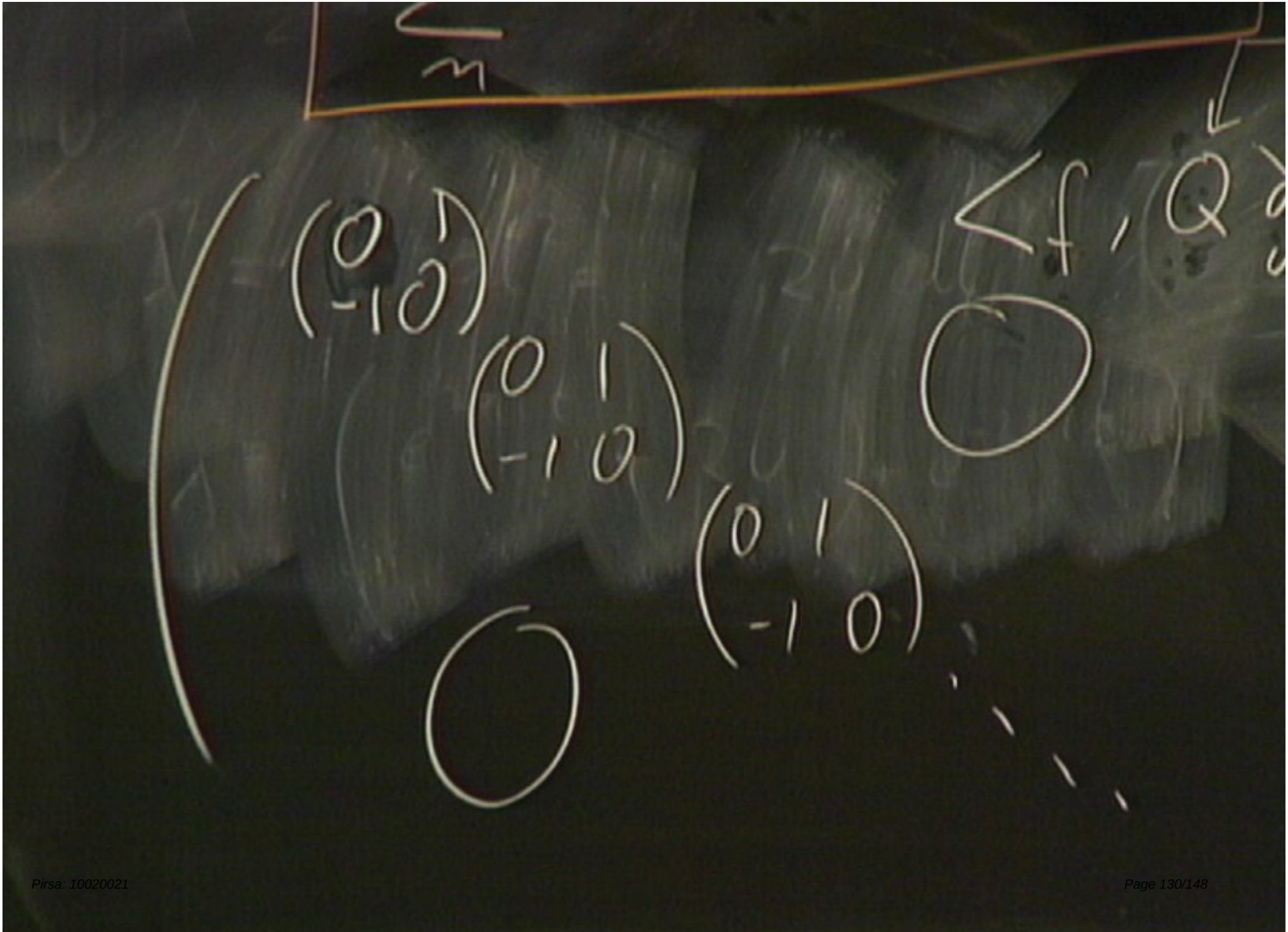
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Now assume we picked such a basis $\{V_n\}$ in V.

(Recall: They are solutions $V_n(x,t)$ of the K.G. eqn.)

Notice:

- We'll need a hermitean form so get a diagonalization and corresponding ON basis of V.

- We will likely need complex sols of the KG.

⇝ See what happens when we complexify V, to obtain $\overline{V}$, set of complex lin. comb. of sol. to the K.G. equation.

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Easily find a basis of $\overline{V}$, namely $\{u_n\} \cup \{u_n^*\}$ where:

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$$\langle f, g \rangle = i \int_\Sigma d\Sigma_\mu \sqrt{g}\, g^{\mu\nu} \left(f^* \partial_\nu g - (\partial_\nu f^*) g \right)$$

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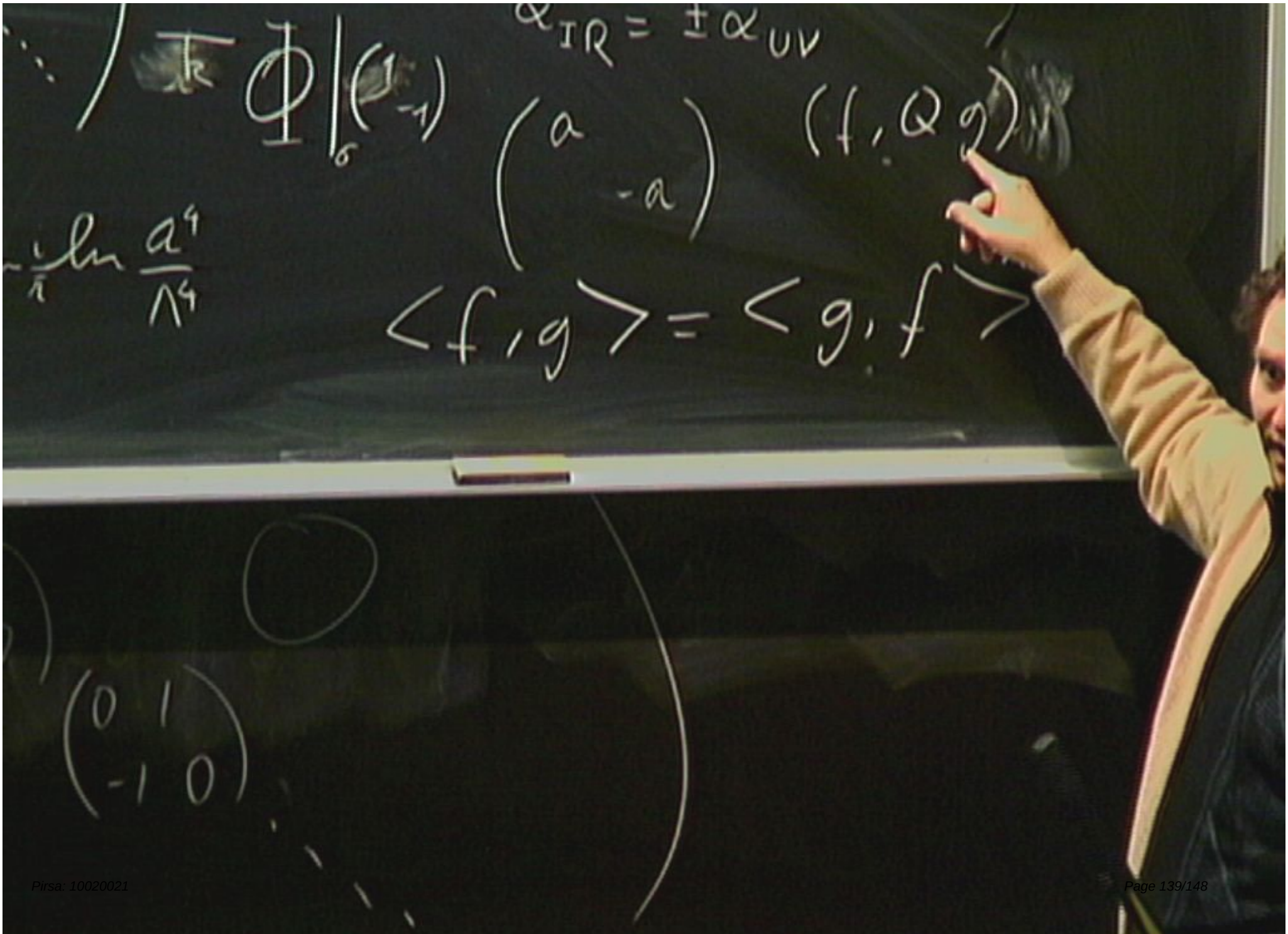
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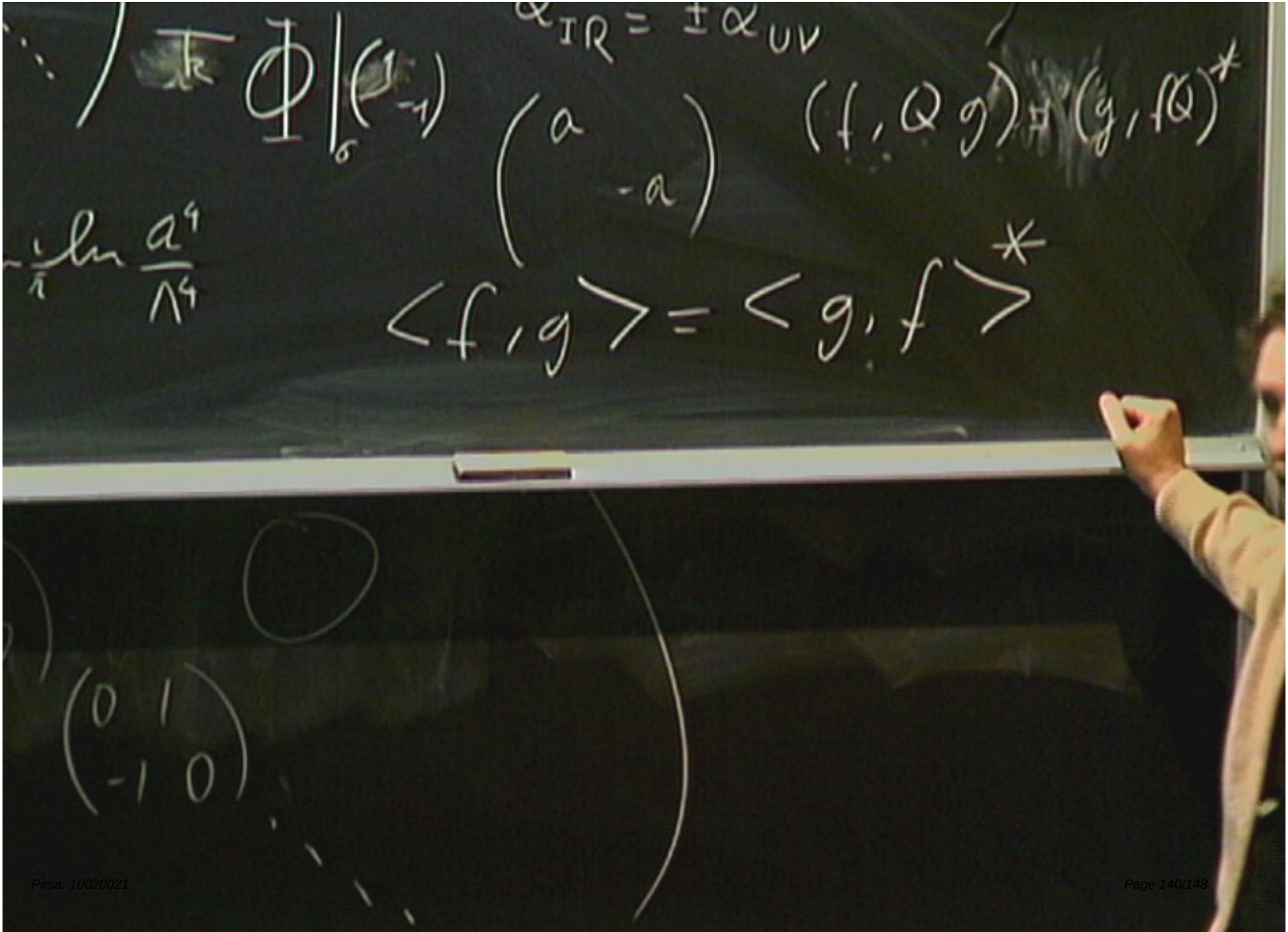
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$$\mathbb{1} = \sum_n |u_n\rangle\langle u_n| + |u_n^*\rangle\langle u_n^*|$$

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Exercise: Writing this out, show that it yields (RI), i.e.:

$$\sqrt{|g|}\, g^{0\nu} \sum \left(u_n(x,t) \frac{\partial}{\partial x^\nu} u_n^*(x',t) - u_n^*(x,t) \frac{\partial}{\partial x^\nu} u_n(x',t) \right) = i\,\delta^3(x-x')$$

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Example:

For Minkowski space, we had found this solution for the noninteracting Klein Gordon field:

$$\hat{\phi}(x,t) = \int_{\mathbb{R}^3} \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_k}} \left(a_k\, e^{-i\omega_k t + ikx} + a_k^+ e^{i\omega_k t - ikx} \right) d^3k$$

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