Title: Quantum Field Theory for Cosmology - Lecture 8

Date: Feb 04, 2010 05:00 PM

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Abstract: This course begins with a thorough introduction to quantum field theory. Unlike the usual quantum field theory courses which aim at applications to particle physics, this course then focuses on those quantum field theoretic techniques that are important in the presence of gravity. In particular, this course introduces the properties of quantum fluctuations of fields and how they are affected by curvature and by gravitational horizons. We will cover the highly successful inflationary explanation of the fluctuation spectrum of the cosmic microwave background - and therefore the modern understanding of the quantum origin of all inhomogeneities in the universe (see these amazing visualizations from the data of the Sloan Digital Sky Survey. They display the inhomogeneous distribution of galaxies several billion light years into the universe: Sloan Digital Sky Survey).

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QFT for Cosmology, Achim Kempf, Winter 2010, Lecture 8

The Unruh effect

(W.G. Unruh, 1976)

((Can be interpreted as showing that the very existence or non-existence of particles is sometimes observer dependent.)

The Unruh effect is the observation, by accelerated observers, of particles, even when the field is in the vacuum state in Minkowski space, i.e., even if inertial observers don't see particles.

Intuition 1:

- A monochromatic wave in an inertial frame is not monochromatic for an accelerated observer.
- Thus, the accelerated observer's modes are coupled oscillators: he sees wavelengths change.
- These oscillator's ground state is now different.

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- Consider an accelerated detector of particles.
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$\Rightarrow$ We expect field excitation, i.e. radiation of particles.

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- Consider an observer with a particle detector.

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- Definition: We write the accelerated path as

$$x^\mu(\tau) = (x^0(\tau), \vec{x}(\tau))$$

- Note: Here, x^0 and $\vec{x} = (x^1, x^2, x^3)$ are the observer and detector's coordinates in a cartesian

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* An observer at rest has: $x^\mu(\tau) = (\tau, 0, 0, \ldots)$

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* Case of constant acceleration in the x-direction:

$$x^0(\tau) = \alpha \sinh^2(\tau/\alpha)$$

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$$x^2(\tau) = x^3(\tau) = 0$$

Exercise: □ verify that $\ddot{x}^\mu \ddot{x}_\mu = \text{const}$
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$$\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int e^{i\vec{k}\vec{x}}\, \hat{\phi}_k(x^0)\, d^3k \quad \text{with} \quad \hat{\phi}_k(x^0) = \frac{1}{\sqrt{2}} \left(v_k^*(x^0)\, a_k + v_k(x^0)\, a_{-k}^+ \right)$$

$$\text{and} \quad v_k(x^0) = \frac{1}{\sqrt{\omega_k}}\, e^{i\omega_k x^0} \quad \text{with} \quad \omega_k = \sqrt{\vec{k}^2 + m^2}.$$

- Thus:

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  - An atom
  - An oscillator such as the diatomic molecule H_2.

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Detector efficiency (can also be used as on/off switch)

An observable of the detector's quantum system

The field $\hat{\phi}$ at the current detector location

- Example: $\hat{H}^{int}(\tau) = \varepsilon(\tau)\, \hat{S}_{\cdot}(\tau)\, \hat{B}_{\cdot}(\tau)$

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(can also be used as on/off switch) of the detector's quantum system at the current detector location

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If we (realistically) assume that the detector efficiency $\varepsilon(\tau)$ is small, we can use perturbation theory.

In this case, the Dirac picture of time evolution is convenient:

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* Operators evolve according to

$$\hat{H}^{free} = \hat{H}^{detector} \otimes 1 + 1 \otimes \hat{H}^{field}$$

For example:

$$\hat{Q}(\tau) = e^{i\hat{H}^{free}\tau} (\hat{Q}_0 \otimes 1) e^{-i\hat{H}^{free}\tau}$$

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$$\hat{U}(\tau) = T\exp\left(i\int_{\tau_i}^{\tau_f} \hat{H}^{interaction}(\tau')\,d\tau'\right)$$

the time-ordering symbol

Perturbative ansatz

For small detector efficiency $\varepsilon(\tau)$ we can expand:

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Perturbative ansatz

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$$\hat{U}(\tau) = 1 + i\int_{-\infty}^{\tau} \varepsilon(\tau')\,\hat{Q}(\tau')\,\hat{\phi}(x^0(\tau'),\vec{x}(\tau'))\, d\tau' + O(\varepsilon^2)$$

- Note: We can set $\tau_i = -\infty$ since we can always switch

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$$\hat{U}(\tau) = 1 + i\int_{-\infty}^{\tau} \varepsilon(\tau')\,\hat{Q}(\tau')\,\hat{\phi}(x^0(\tau'),\vec{x}(\tau'))\,d\tau' + O(\varepsilon^2)$$

- Note: We can set $\tau_i = -\infty$ since we can always switch $\varepsilon(\tau)$ on or off.

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- We assume that the quantum field $\hat{\phi}$ starts out in a state $|\alpha\rangle$ with $|\alpha\rangle$ = Minkowski vacuum, $|\alpha\rangle = |0\rangle$, or a 1-particle state $|\alpha\rangle = |1_k\rangle$.

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★ States evolve according to $\hat{H}$ (τ), i.e., according to the time evolution operator:

$$\hat{U}(\tau) = T\exp\left(i\int_{\tau_i}^{\tau_f} \hat{H}^{\text{interaction}}(\tau')\, d\tau'\right)$$

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Recall: $\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int \left(\frac{1}{\sqrt{\omega_k}} e^{i\omega_k x^0 + i\vec{k}\vec{x}} a^+_{\vec{k}} + \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k x^0 + i\vec{k}\vec{x}} a_{\vec{k}} \right) d^3k$

The case $|\alpha\rangle = |0\rangle$:

* In $\hat{\phi}(x)$, only the terms $\sim a^+_k$ can contribute, because $a_k|0\rangle = 0$

* Thus, in $|\Omega\rangle$ only the one-particle components contribute:

$$|\Omega\rangle = \cancel{\Omega_0 |0\rangle} + \int \Omega_k a^+_k |0\rangle d^3k + \cancel{\iint \Omega_{k k'} a^+_k a^+_{k'} |0\rangle d^3k\, d^3k'} + \cancel{\dots}$$

* Thus, let us consider a $|\Omega\rangle = a^+_{\vec{k}}|0\rangle$:

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Recall: $\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int \left(\frac{1}{\sqrt{\omega_k}} e^{i\omega_k x^0 + i\vec{k}\vec{x}} a^+_{\vec{k}} + \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k x^0 + i\vec{k}\vec{x}} a_{\vec{k}} \right) d^3k$

The case $|\alpha\rangle = |0\rangle$:

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* Thus, let us consider a $|\Omega\rangle = a^+_{\tilde{k}} |0\rangle$:

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because $a_k|0\rangle = 0$

* Thus, in $|\Omega\rangle$ only the one-particle components contribute:

$$|\Omega\rangle = \cancel{\Omega_0|0\rangle} + \int \Omega_k a_k^+|0\rangle d^3k + \cancel{\iint \Omega_{k k'} a_k^+ a_{k'}^+|0\rangle d^3k\, d^3k' + \dots}$$

* Thus, let us consider a $|\Omega\rangle = a_{\tilde{k}}^+|0\rangle$:

$$\Rightarrow \quad p(\infty) = i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty}\varepsilon(\tau)e^{i(E_n-E_0)\tau}\langle 0|a_{\tilde{k}}\int\frac{e^{i\omega_k x_0(\tau)-i\vec{k}\vec{x}(\tau)}}{\sqrt{2\omega_k}}a_k^+ d^3k|0\rangle d\tau$$

leads to:

$$\langle 0|a_{\tilde{k}}a_k^+|0\rangle = \langle 0|a_k^+a_{\tilde{k}} + \delta^3(\tilde{k}-k)|0\rangle = \delta^3(\tilde{k}-k)$$

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$$\langle 0|\,a_{\tilde{k}}\,a^+_k|0\rangle = \langle 0|\,a^+_k a_{\tilde{k}} + \delta^3(\tilde{k}-k)|0\rangle = \delta^3(\tilde{k}-k)$$

$$\Rightarrow \quad p(\infty) = \underbrace{i\,\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}}_{\text{some constant}}\int_{-\infty}^{+\infty} e^{i(E_n-E_0)\tau}\; e^{i(\omega_k x^0(\tau)-\tilde{k}\vec{x}(\tau))}\,\varepsilon(\tau)\,d\tau$$

Special case: $|\alpha\rangle = |0\rangle$ and detector inertial:

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$$\Rightarrow \quad p(\infty) = i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty} \varepsilon(\tau)\, e^{i(E_n-E_0)\tau} \langle 0|a_{\tilde{k}} \int \frac{e^{i\omega_k x_0(\tau) - i\vec{k}\vec{x}(\tau)}}{\sqrt{2\omega_k}} a_k^+ d^3k|0\rangle d\tau$$

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Special case: $|\alpha\rangle = |0\rangle$ and detector inertial:

* Choose the detector's rest frame: $x^\mu(\tau) = (\tau, 0, 0, 0)$

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$\Rightarrow$ $p(\infty) = i \frac{\ldots}{(2\pi)^{3/2}} \int_{-\infty} \ldots \langle 0 | a_{\tilde{k}} \ldots \frac{\ldots}{\sqrt{2\omega_k}} a_k^+ d^3k | 0 \rangle d\tau$

leads to:

$$\langle 0 | a_{\tilde{k}} a_k^+ | 0 \rangle = \langle 0 | a_k^+ a_{\tilde{k}} + \delta^3(\tilde{k}-k) | 0 \rangle = \delta^3(\tilde{k}-k)$$

$\Rightarrow$

$$p(\infty) = i \underbrace{\frac{\langle E_n | \hat{Q}_0 | E_0 \rangle}{(2\pi)^{3/2}}}_{\text{some constant}} \int_{-\infty}^{+\infty} e^{i(E_n - E_0)\tau} \, e^{i(\omega_k x^0(\tau) - \vec{k}\vec{x}(\tau))} \, \varepsilon(\tau) \, d\tau$$

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Special case: $|\alpha\rangle = |0\rangle$ and detector inertial:

* Choose the detector's rest frame: $x^\mu(\tau) = (\tau, 0, 0, 0)$

* Thus:

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* Choose the detector's rest frame: $x^\nu(\tau) = (\tau, 0, 0, 0)$

* Thus:

$$p(\omega) = i \frac{\langle E_n | \hat{Q}_0 | E_0 \rangle}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} e^{i(E_n - E_0)\tau} \, e^{i(\omega_k x^0(\tau) - \vec{k}\vec{x}(\tau))} \, \varepsilon(\tau) \, d\tau$$

assume $\varepsilon(\tau) = 1$, i.e., "always on".

$$= i \frac{\langle E_n | \hat{Q}_0 | E_0 \rangle}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} e^{i(E_n - E_0)\tau} \, e^{i \omega_k \tau} \, d\tau$$

$$= i \frac{\langle E_n | \hat{Q}_0 | E_0 \rangle}{(2\pi)^{3/2}} (2\pi)^{3/2} \, \delta\Big(\underbrace{\overbrace{E_n - E_0}^{>0} + \overbrace{\omega_k}^{\sqrt{k^2+m^2} > 0}}_{\text{this cannot be } 0} \Big)$$

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* Thus:

$$p(\omega) = i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty} e^{i(E_n-E_0)\tau}\, e^{i(\omega_k x^0(\tau)-\vec{k}\vec{x}(\tau))}\,\varepsilon(\tau)\,d\tau$$

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$$= 0$$

$\Rightarrow$ No excitation of the detector, as expected.

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$\omega_k = \sqrt{k^2+m^2} > 0$

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some constant $-\infty$

Special case: $|\alpha\rangle = |0\rangle$ and detector inertial:

* Choose the detector's rest frame: $x^\mu(\tau) = (\tau, 0, 0, 0)$

* Thus:

$$p(\infty) = i\frac{\langle E_n|\hat{Q}_d|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty} e^{i(E_n-E_0)\tau}\, e^{i(\omega_k x^0(\tau) - \tilde{k}\vec{x}(\tau))}\, \varepsilon(\tau)\, d\tau$$

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$$\sqrt{k^2+m^2} > 0$$

$$= i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}(2\pi)^{3/2}\,\delta\big(\overbrace{E_n-E_0}^{>0}+\underbrace{\omega_k}_{=\sqrt{k^2+m^2}>0}\big)$$

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* Choose the detector's rest frame: $x^\mu(\tau) = (\tau, 0,0,0)$

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$$p(\infty) = i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty} e^{i(E_n-E_0)\tau}\, e^{i(\omega_k x^0(\tau) - \vec{k}\vec{x}(\tau))}\, \varepsilon(\tau)\, d\tau$$

assume $\varepsilon(\tau)=1$, i.e., "always on":

$$= i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty} e^{i(E_n-E_0)\tau}\, e^{i\,\omega_k\,\tau}\, d\tau$$

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$$= i\,\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}} (2\pi)^{3/2}\, \delta\big(\overbrace{E_n - E_0}^{>0} + \underset{\sqrt{k^2+m^2}\,>\,0}{\underset{\parallel}{\omega_k}}\big)$$

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$$= 0$$

$\Rightarrow$ No excitation of the detector, as expected.

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$\omega_k = \sqrt{k^2+m^2} > 0$

$$= 0$$

$\Rightarrow$ No excitation of the detector, as expected.

Special case: $|\alpha\rangle = |0\rangle$ and detector non-inertial:

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assume $\varepsilon(\tau) = 1$, i.e., "always on".

$$= i \frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} e^{i(E_n - E_0)\tau} \; e^{i\,\omega_k \tau} \, d\tau$$

$$\omega_k = \sqrt{k^2+m^2} > 0$$

$$= i \frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}} (2\pi)^{1/2} \; \delta\big(\underbrace{\overbrace{E_n - E_0}^{>0} + \omega_k}_{\text{this cannot be } 0}\big)$$

$$= 0$$

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Special case: $|\alpha\rangle = |0\rangle$ and detector non-inertial:

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* Thus:

$$\rho(\infty) = i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty} e^{i(E_n-E_0)\tau}\, e^{i(\omega_k x^0(\tau) - \tilde{k}\vec{x}(\tau))}\,\varepsilon(\tau)\, d\tau$$

assume $\varepsilon(\tau)=1$, i.e., "always on".

$$= i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty} e^{i(E_n-E_0)\tau}\, e^{i\,\omega_k \tau}\, d\tau$$

$$= i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}(2\pi)^{1/2}\,\delta\Big(\underbrace{\overbrace{E_n-E_0}^{>0}+\overbrace{\omega_k}^{\sqrt{k^2+m^2}>0}}_{\text{this cannot be } 0}\Big)$$

$$= 0$$

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$$\langle 0|\, a_{\tilde k}\, a^+_k |0\rangle = \langle 0|\, a^+_k a_{\tilde k} + \delta^3(\tilde k - k)|0\rangle = \delta^3(\tilde k - k)$$

$\Rightarrow$

$$p(\infty) = i\,\underbrace{\frac{\langle E_n|\hat Q_0|E_0\rangle}{(2\pi)^{3/2}}}_{\text{some constant}} \int_{-\infty}^{+\infty} e^{i(E_n - E_0)\tau}\, e^{i(\omega_k x^0(\tau) - \tilde k \vec x(\tau))}\, \varepsilon(\tau)\, d\tau$$

Special case: $|\alpha\rangle = |0\rangle$ and detector inertial:

* Choose the detector's rest frame: $x^\mu(\tau) = (\tau, 0, 0, 0)$

* Thus:

$$\ldots \frac{\langle E_n|\hat Q_0|E_0\rangle}{\ldots} \int^{+\infty} e^{i(E_n - E_0)\tau}\, e^{i(\omega_k x^0(\tau) - \tilde k \vec x(\tau))} \ldots$$

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* Thus, let us consider a $|\Omega\rangle = a^+_{\tilde{k}}|0\rangle$:

$$\Rightarrow \quad p(\infty) = i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty}\varepsilon(\tau)\, e^{i(E_n-E_0)\tau}\langle 0|a_{\tilde{k}}\int \frac{e^{i\omega_k x_0(\tau)-i\vec{k}\vec{x}(\tau)}}{\sqrt{2\omega_k}}\,a^+_k\, d^3k|0\rangle\, d\tau$$

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$$\langle 0|a_{\tilde{k}}\, a^+_k|0\rangle = \langle 0|a^+_k a_{\tilde{k}} + \delta^3(\tilde{k}-k)|0\rangle = \delta^3(\tilde{k}-k)$$

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* Choose the detector's rest frame: $x^\mu(\tau) = (\tau, 0, 0, 0)$

* Thus:

$$p(\omega) = i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty} e^{i(E_n-E_0)\tau}\, e^{i(\omega_k x^0(\tau) - \vec{k}\vec{x}(\tau))}\,\varepsilon(\tau)\,d\tau$$

assume $\varepsilon(\tau) = 1$, i.e., "always on".

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$\omega_k = \sqrt{k^2+m^2} > 0$

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this cannot be 0

$$= 0$$

$\Rightarrow$ No excitation of the detector, as expected.

Special case: $|\alpha\rangle = |0\rangle$ and detector non-inertial:

- The probability amplitude for the detector to become excited will depend on the excitation energy:

$$E_{ex} := E_n - E_0$$

- Namely:

$$p(\infty) = i \frac{\langle E_n|\hat{Q}_0|E_0\rangle}{\ldots} \int^{+\infty} e^{i(E_n - E_0)\tau} \, e^{i(\omega_k x^0(\tau) - \tilde{k}\vec{x}(\tau))} \, \varepsilon(\tau) \, d\tau$$

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Clearly:

For generic, accelerated detectors the function

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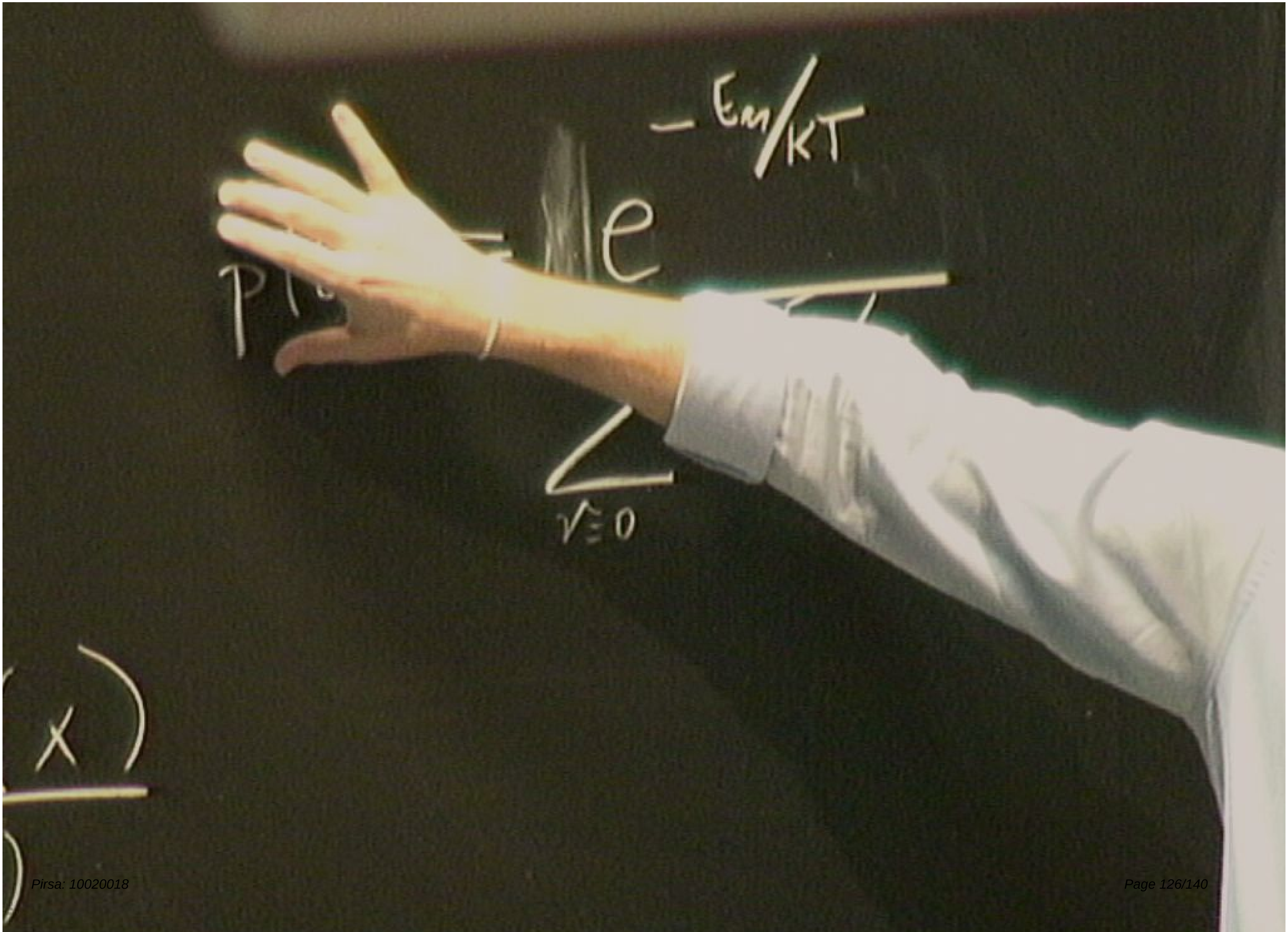
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Prob. amplitude for detector to get excited

Recall: $\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}}\int\left(\frac{1}{\sqrt{\omega_k}}\, e^{i\omega_k x^0 + i\vec{k}\vec{x}}\, a^+_{\vec{k}} + \frac{1}{\sqrt{\omega_k}}\, e^{-i\omega_k x^0 - i\vec{k}\vec{x}}\, a_{\vec{k}}\right) d^3k$

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i.e., not only "detects" a particle

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a.) No contribution if the detector is always on: $\varepsilon \equiv const.$

Hint: Show that

$$p(\infty) \sim \int_{-\infty}^{+\infty} \varepsilon(\tau)\, e^{i(E_m - E_0)\tau + i\omega_k \tau} \frac{1}{\sqrt{\omega_k}} = 0$$

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Why is p generally nonzero if the detector is not always on?

b.) There is generally a finite contribution to p. If the detector is always on, show that the detector gap has to exactly match the particle's energy. Why only if the detector is always on? (Hint: Which uncertainty principle could play a ...

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