Title: Quantum Field Theory for Cosmology - Lecture 11

Date: Feb 23, 2010 04:00 PM

URL: http://pirsa.org/10020017

Abstract: This course begins with a thorough introduction to quantum field theory. Unlike the usual quantum field theory courses which aim at applications to particle physics, this course then focuses on those quantum field theoretic techniques that are important in the presence of gravity. In particular, this course introduces the properties of quantum fluctuations of fields and how they are affected by curvature and by gravitational horizons. We will cover the highly successful inflationary explanation of the fluctuation spectrum of the cosmic microwave background - and therefore the modern understanding of the quantum origin of all inhomogeneities in the universe (see these amazing visualizations from the data of the Sloan Digital Sky Survey. They display the inhomogeneous distribution of galaxies several billion light years into the universe: Sloan Digital Sky Survey).

QFT for Cosmology, Achim Kempf, Winter 2010, Lecture 11

Recall strategy:

we are here

SR, 1^{st}Q
Hamiltonian
formalism

step 1
Legendre transform
(equivalence)

SR, 1^{st}Q.
Lagrangian
formalism

step 2
allow
curvature

GR, 1^{st}Q
Hamiltonian
formalism

step 3
Legendre transform
(equivalence)

GR, 1^{st}Q
Lagrangian
formalism

step 4

GR, 2^{nd}Q
Hamiltonian
formalism

Dyson Schwinger eqns are same
(equivalence)

GR, 2^{nd} Q
Lagrangian formalism
(Path integral of QFT)

QFT for Cosmology, Achim Kempf, Winter 2010, Lecture 11

Recall strategy:

we are here

SR, 1^{st}Q Hamiltonian formalism $\xrightarrow[\text{(equivalence)}]{\text{step 1: Legendre transform}}$ SR, 1^{st}Q. Lagrangian formalism

SR, 1^{st}Q. Lagrangian formalism $\xrightarrow{\text{step 2: allow curvature}}$ GR, 1^{st}Q Lagrangian formalism

GR, 1^{st}Q Hamiltonian formalism $\xleftarrow[\text{(equivalence)}]{\text{step 3: Legendre transform}}$ GR, 1^{st}Q Lagrangian formalism

GR, 1^{st}Q Hamiltonian formalism $\xrightarrow{\text{step 4}}$ GR, 2^{nd}Q Hamiltonian formalism

GR, 1^{st}Q Lagrangian formalism $\dashrightarrow$ GR, 2^{nd}Q Lagrangian formalism (Path integral of QFT)

GR, 2^{nd}Q Hamiltonian formalism $\xleftrightarrow[\text{(equivalence)}]{\text{Dyson Schwinger eqns are same}}$ GR, 2^{nd}Q Lagrangian formalism (Path integral of QFT)

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SR, 1^{st}Q
Hamiltonian formalism

step 1: Legendre transform (equivalence) →

SR, 1^{st}Q.
Lagrangian formalism

step 2 ↓ allow curvature

GR, 1^{st}Q
Lagrangian formalism

← step 3: Legendre transform (equivalence)

GR, 1^{st}Q
Hamiltonian formalism

step 4 ↓

GR, 2^{nd}Q
Hamiltonian formalism

⟷ Dyson Schwinger eqns are same (equivalence)

GR, 2^{nd}Q
Lagrangian formalism
(Path integral of QFT)

Step 2 so far:

We started with the Klein Gordon action in special

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- We started with the Klein Gordon action in special relativity:

$$S[\phi] = \frac{1}{2}\int_{\mathbb{R}^4} \eta^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - m^2\phi^2 - V(\phi,\varphi_i)\, d^4x \qquad (a)$$

This formulation is correct for any inertial observer using a rectangular coordinate system, and only for those observers.

Remark: Here, $V(\phi,\varphi_i)$ is a potential. It describes how the ϕ field interacts with other fields $\{\varphi_i\}$ and with itself. The very early universe is assumed to have been dominated by a scalar field ϕ which interacted mainly

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SR, 1^{st}Q Hamiltonian formalism	step 1 Legendre transform → (equivalence)	SR, 1^{st}Q. Lagrangian formalism
		↓ step 2 allow curvature
GR, 1^{st}Q Hamiltonian formalism	step 3 ← Legendre transform (equivalence)	GR, 1^{st}Q Lagrangian formalism
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$$S_{KG}[\phi] := \frac{1}{2} \int_{\mathbb{R}^4} \left(g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - m^2 \phi^2 - V(\phi, \lambda_i) \right) \overbrace{\sqrt{|g|}}^{\sqrt{-\det(g_{\mu\nu})}} d^4x \qquad (b)$$

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If an inertial observer sets up a rectangular coordinate system, then the metric tensor $g_{\mu\nu}(x)$ is of course given by this constant matrix

$$g_{\mu\nu}(x) = \eta_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

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$\Rightarrow$ In special relativity, in arbitrary coordinates, the metric $g_{\mu\nu}$ is a position-dependent matrix of the form (c).

- Question:

* We notice that $g_{\mu\nu}(x)$ is always symmetric $g_{\mu\nu}(x) = g_{\nu\mu}(x)$

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Answer: No! $\rightsquigarrow$ Completion of Step 2:

Introduction of curvature:

- We continue to postulate the coordinate system-independent Klein Gordon action of above:

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- We will allow arbitrary metric tensors subject only to this condition:

We allow that there may not exist coordinates $\tilde{x}$ in which:

$$\tilde{g}_{\mu\nu}(\tilde{x}) = \eta_{\mu\nu} \quad \text{for all } x$$

But for each x_0, there must exist a change of coordinates

$$x \to \tilde{x}$$

so that: $\tilde{g}_{\mu\nu}(\tilde{x}_0) = \eta_{\mu\nu}$

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This ensures that at least locally special relativity holds.

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- This requirement is **The Equivalence Principle**:

  * We postulate that gravity can always locally be eliminated:

  * We assume that if a freely falling observer in a small region sets up a rectangular coordinate system the observer will see arbitrarily small gravity effects if the region is made arbitrarily small.

  * For this to be true, any body's gravitational mass must be equal to its inertial mass, i.e. all bodies must fall equally. (Else the notion of freely falling observer) is not even well defined)

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Identifying Curvature:

* Assume we are given a metric tensor

$$g_{\mu\nu}(x)$$

as an explicit matrix-valued function, in some coordinates.

* How can we determine whether or not this is the metric tensor of flat space-time, i.e., whether or not

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* How can we determine whether or not this is the metric tensor of flat space-time, i.e., whether or not there exist coordinates $\tilde{x}$ in which $\tilde{g}_{\mu\nu}(\tilde{x}) = \eta_{\mu\nu}$?

* This problem is solved in differential geometry:

Define: The "Christoffel symbol functions":

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Identifying Curvature:

* Assume we are given a metric tensor

$$g_{\mu\nu}(x)$$

as an explicit matrix-valued function, in some coordinates.

* How can we determine whether or not this is the metric tensor of flat space-time, i.e., whether or not there exist coordinates $\tilde{x}$ in which $\tilde{g}_{\mu\nu}(\tilde{x}) = \eta_{\mu\nu}$?

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$$\Gamma^{\mu}_{\alpha\beta}(x) := \frac{1}{2} g^{\mu\nu}(x)\left(g_{\alpha\nu,\beta}(x) + g_{\beta\nu,\alpha}(x) - g_{\alpha\beta,\nu}(x)\right)$$

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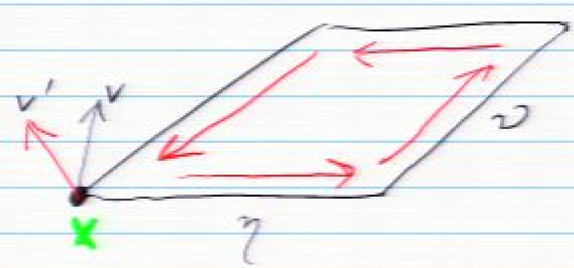

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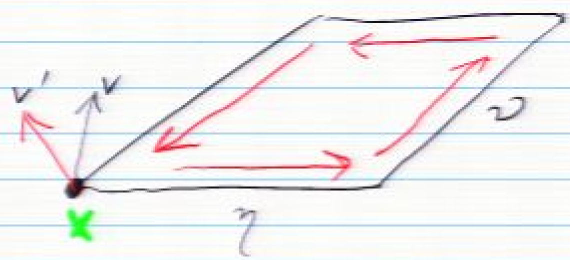

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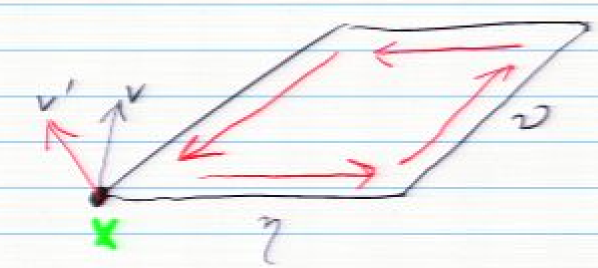

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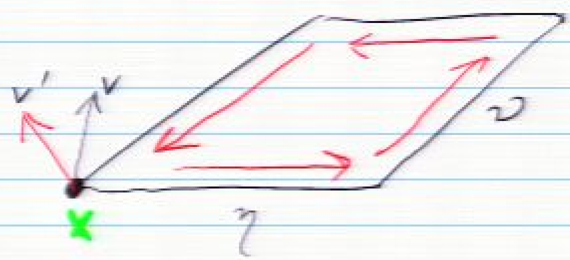

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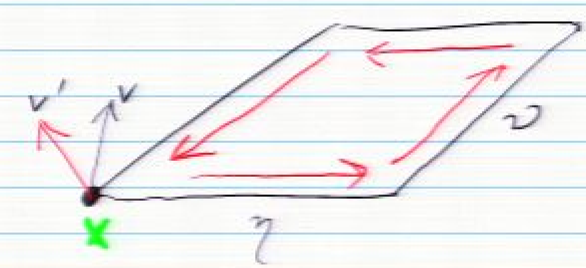

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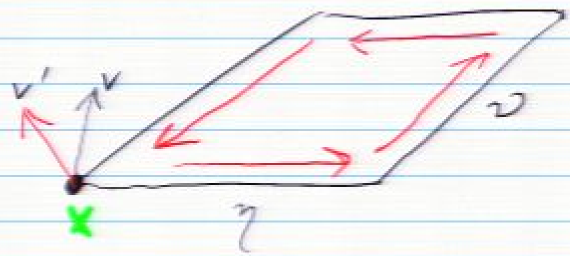

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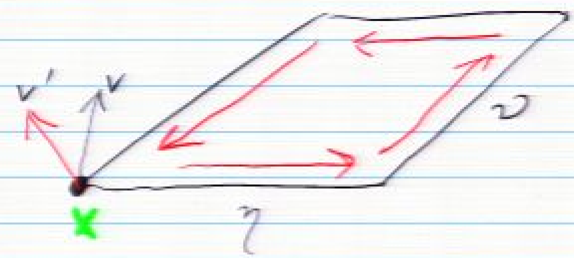

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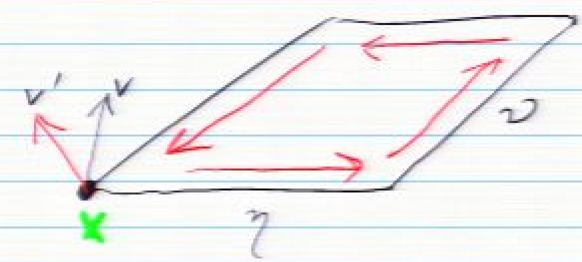

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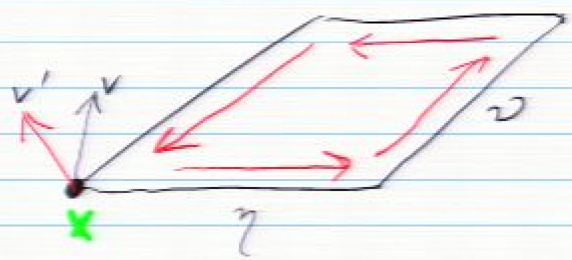

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The dynamics of space-time

Problem:

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Which are the degrees of freedom of curvature for which we have to find an equation of motion?

- We saw that the curvature of space-time is encoded in the matrix-valued metric function:

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Other curvature scalars:

* The simplest scalar that can be formed from the metric alone is $g^{\mu\nu}(x)\, g_{\nu\mu}(x) = 4$.

* The next simplest scalar that can be formed is the Ricci scalar $R(x)$.

* All other scalars made out of g only are composed of higher powers of the Riemann tensor $R^{\mu}{}_{\nu\rho\sigma}(x)$:

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The gravitational action

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The gravitational action

- A priori, the full action now reads:

other "matter" fields for e^-, quarks, photons etc

$$S'_{tot}[g,\phi,e_i] = \underbrace{S_{KG}}_{\text{Klein Gordon action}} + S_{other} + S'_{grav}$$

with: $$S'_{grav}[g] := \int \left(c_0 + c_1 R(x) + c_2 R_{\mu\nu}(x) R^{\mu\nu}(x) + c_3 R^{\cdot\cdot}_{\cdot\cdot} R^{\cdot\cdot}_{\cdot\cdot} R^{\cdot\cdot}_{\cdot\cdot} + \ldots\right) \sqrt{|g|}\, d^4x$$

- Comparison with experiment shows evidence only for the first two terms:

$$S_{grav}[g] = -\frac{1}{16\pi G} \int \left(2\Lambda + R(x)\right) \sqrt{|g(x)|}\, d^4x$$

"Einstein action" — Newton's constant — "Cosmological constant"

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The action principle is to require that the action be extremal with respect to all degrees of freedom:

A) $\frac{\delta S_{tot}}{\delta e_i(x)} = 0$ B) $\frac{\delta S_{tot}}{\delta g_{\mu\nu}(x)} = 0$ C) $\frac{\delta S_{tot}}{\delta \phi(x)} = 0$

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- Comparison with experiment shows evidence only for the first two terms:

$$S_{grav}[g] = -\frac{1}{16\pi G} \int \left(2\Lambda + R(x) \right) \sqrt{|g(x)|}\, d^4x$$

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The gravitational action

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- Recall S_{KG}:

Example of a potential

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Next: Step 3 in

step 1

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\text{SR, } 1^{st}\text{Q Hamiltonian formalism} & \xrightarrow[\text{(equivalence)}]{\text{step 1: Legendre transform}} & \text{SR, } 1^{st}\text{Q. Lagrangian formalism} \\
 & & \text{step 2} \Big\downarrow \text{allow curvature} \\
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$$\sim \frac{\delta S'_{grav}}{\delta g_{\mu\nu}(x)} \qquad \sim - \frac{\delta(S_{other} + S'_{KG})}{\delta g_{\mu\nu}(x)}$$

* Here, $T_{\mu\nu}(x)$ is the "Energy Momentum Tensor".

Neglecting the contribution by the $\ell_i(x)$, one obtains:

$$T^{(K.G.)}_{\mu\nu}(x) = \phi_{,\mu}(x)\,\phi_{,\nu}(x) - g_{\mu\nu}(x)\Big(\frac{1}{2} g^{\alpha\beta}(x)\,\phi_{,\alpha}(x)\,\phi_{,\beta}(x) - \overbrace{V(\phi(x))}^{\text{mass term } m^2\phi^2 \text{ included}}\Big)$$

* **Quantization:** To quantize the Einstein equation is difficult for many reasons:

- For example, it is difficult to separate the curvature degrees of freedom from mere artifacts of the choice of the coordinate system.

This yields the equation of motion for the dynamics of curvature, i.e., the Einstein equation:

(See exercise in Mukhanov's text) →

$$\boxed{R_{\mu\nu}(x) - \frac{1}{2} g_{\mu\nu}(x) R(x) + \Lambda g_{\mu\nu}(x) = +8\pi G\, T_{\mu\nu}(x)}$$

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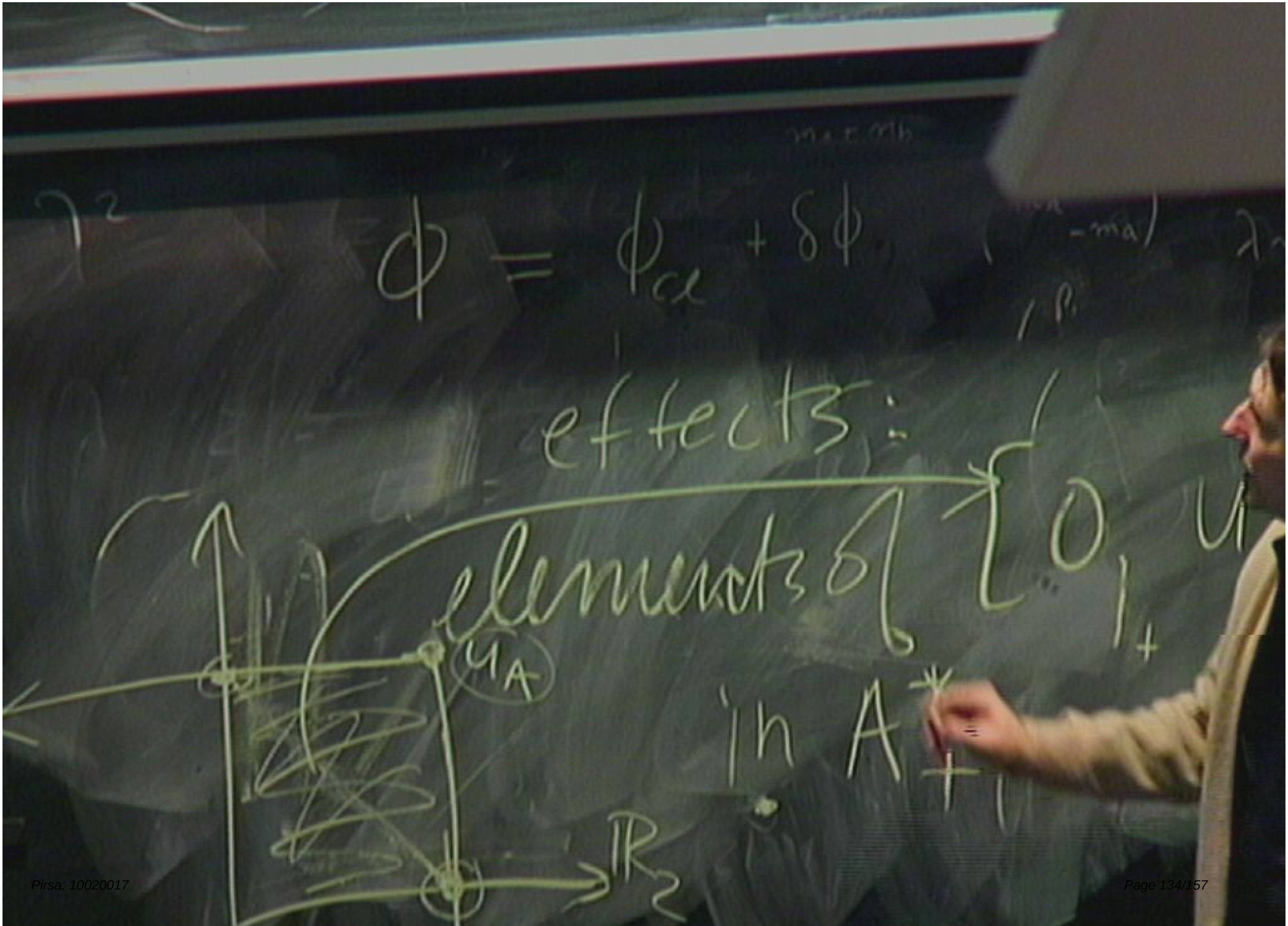
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Next: Step 3 in

SR, 1^{st}Q Hamiltonian formalism $\xrightarrow[\text{(equivalence)}]{\text{step 1: Legendre transform}}$ SR, 1^{st}Q. Lagrangian formalism

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SR, $1^{st}Q$ Hamiltonian formalism $\xrightarrow[\text{(equivalence)}]{\substack{\text{step 1} \\ \text{Legendre transform}}}$ SR, $1^{st}Q$. Lagrangian formalism

step 2 | allow curvature

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Next: Step 3 in

SR, 1^{st}Q Hamiltonian formalism

step 1
Legendre transform
(equivalence)

SR, 1^{st}Q. Lagrangian formalism

step 2
allow curvature

GR, 1^{st}Q Lagrangian formalism

step 3
Legendre transform
(equivalence)

GR, 1^{st}Q Hamiltonian formalism

step 4

GR, 2^{nd}Q Hamiltonian formalism

(PI)

GR, 2^{nd}Q Lagrangian formalism (Path integral of QFT)

Dyson Schwinger eqns are same
(equivalence)

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Next: Step 3 in

SR, 1^{st}Q Hamiltonian formalism $\xrightarrow[\text{(equivalence)}]{\text{step 1: Legendre transform}}$ SR, 1^{st}Q. Lagrangian formalism

step 2 $\downarrow$ allow curvature

GR, 1^{st}Q Hamiltonian formalism $\xleftarrow[\text{(equivalence)}]{\text{step 3: Legendre transform}}$ GR, 1^{st}Q Lagrangian formalism

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Comment on step (PI): 2nd quantization with path integral

- Assume a fixed spacetime is chosen and we are given its metric $g_{\mu\nu}(x)$ in some arbitrary coordinate system.

- Then, for each field $\phi(\vec{x},t)$ we can calculate its action $S_{KG}[\phi,g]$:

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SR, 1^{st}Q Hamiltonian formalism $\xrightarrow[\text{(equivalence)}]{\text{Legendre transform}}$ SR, 1^{st}Q. Lagrangian formalism

step 2: allow curvature

SR, 1^{st}Q. Lagrangian formalism $\downarrow$ GR, 1^{st}Q Lagrangian formalism

step 3

GR, 1^{st}Q Hamiltonian formalism $\xleftarrow[\text{(equivalence)}]{\text{Legendre transform}}$ GR, 1^{st}Q Lagrangian formalism

step 4: GR, 1^{st}Q Hamiltonian formalism $\downarrow$ GR, 2^{nd}Q Hamiltonian formalism

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- With Feynman we also get it from the path integral:

Advantages:

1) Quick derivation of Feynman rules

2) Manifestly covariant.

$$G(\vec{x},t,\vec{x}',t) = N \int_{\text{all } \phi} \phi(\vec{x},t)\,\phi(\vec{x}',t)\, e^{\frac{i}{\hbar} S_{KG}[\phi,g]}\, D[\phi]$$

Problems:

1) Ill-defined due to uncountable number of integrations and related divergences.

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$$S_{\mathrm{kG}}[\phi, g] = \frac{1}{2}\int_{\mathbb{R}^4} \left(g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - m^2\phi^2 - \lambda\phi^4\right)\sqrt{|g|}\, d^4x$$

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- Consider, e.g., the vacuum expectation value of $\phi(\vec{x},t)\,\phi(\vec{x}',t)$, i.e., the correlation function of field amplitudes:

$$G(\vec{x},t,\vec{x}',t) := \langle 0|\,\hat{\phi}(\vec{x},t)\,\hat{\phi}(\vec{x}',t)\,|0\rangle$$

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Advantages:

1) Quick derivation of Feynman rules

2) Manifestly covariant

Problems:

1) Ill defined due to uncountable number of integrations and related divergences.

2) [illegible] these are temporarily regularized, the identification of the vacuum state is ambiguous.

$$G(\vec{x},t,\vec{x}',t) = N \int_{\text{all } \phi} \phi(\vec{x},t)\,\phi(\vec{x}',t)\, e^{\frac{i}{\hbar} S_{\text{kg}}[\phi, g]}\, D[\phi]$$

↑ "Path Integral" over a function space

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SR, 1st Q
Hamiltonian
formalism

Legendre transform
(equivalence)

SR, 1st Q.
Lagrangian
formalism

step 2
allow
curvature

step 3
Legendre transform
(equivalence)

GR, 1st Q
Hamiltonian
formalism

GR, 1st Q
Lagrangian
formalism

step 4

(PI)

GR, 2nd Q
Hamiltonian
formalism

Dyson Schwinger eqns are same
(equivalence)

GR, 2nd Q
Lagrangian formalism
(Path integral of QFT)

Comment on step (PI): 2nd quantization with path integral

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all ϕ

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