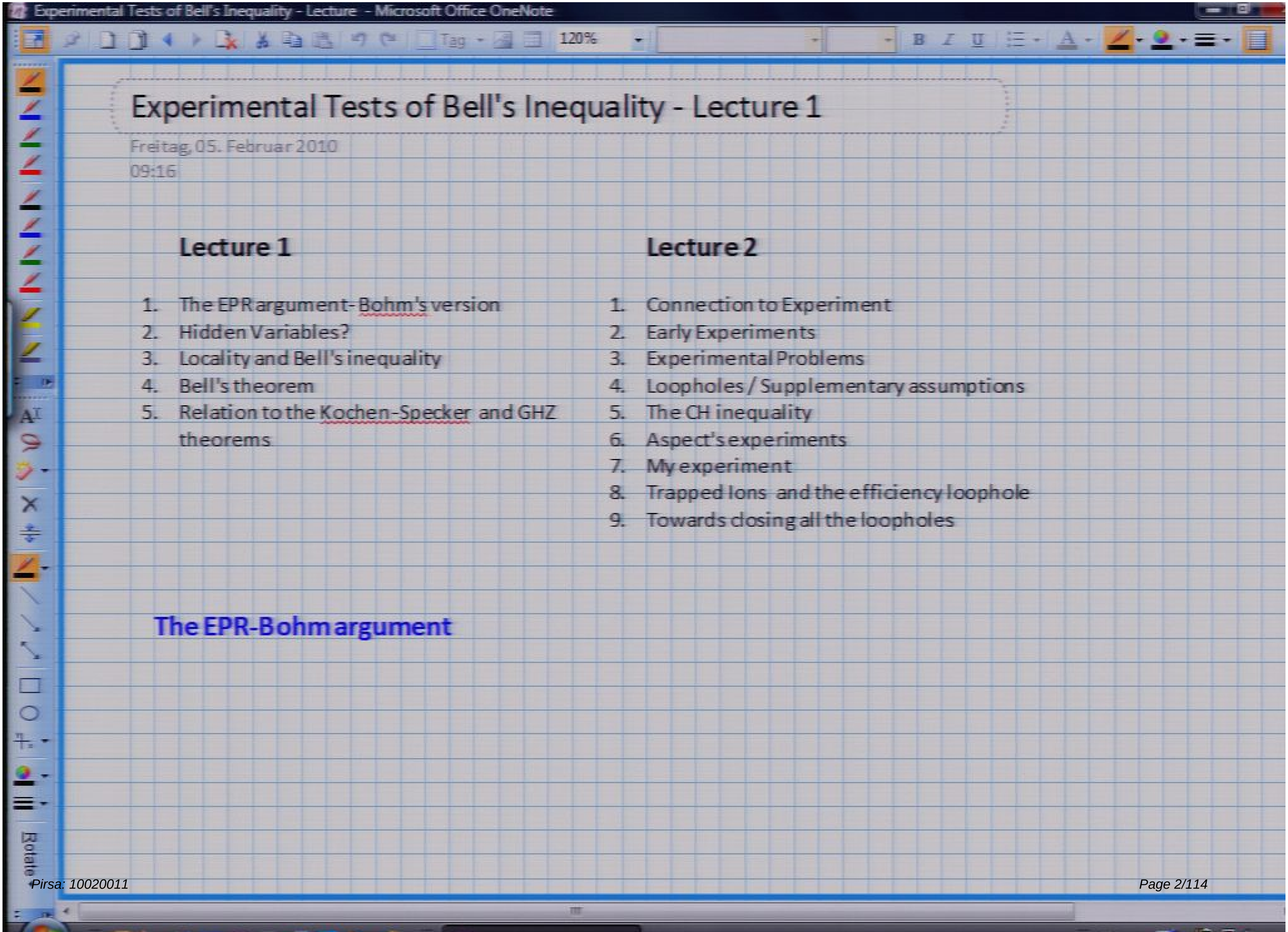
Title: Foundations and Interpretation of Quantum Theory - Lecture 8

Date: Feb 09, 2010 02:30 PM

URL: http://pirsa.org/10020011

Abstract: After a review of the axiomatic formulation of quantum theory, the generalized operational structure of the theory will be introduced (including POVM measurements, sequential measurements, and CP maps). There will be an introduction to the orthodox (sometimes called Copenhagen) interpretation of quantum mechanics and the historical problems/issues/debates regarding that interpretation, in particular, the measurement problem and the EPR paradox, and a discussion of contemporary views on these topics. The majority of the course lectures will consist of guest lectures from international experts covering the various approaches to the interpretation of quantum theory (in particular, many-worlds, de Broglie-Bohm, consistent/decoherent histories, and statistical/epistemic interpretations, as time permits) and fundamental properties and tests of quantum theory (such as entanglement and experimental tests of Bell inequalities, contextuality, macroscopic quantum phenomena, and the problem of quantum gravity, as time permits).



Experimental Tests of Bell's Inequality - Lecture 1

Freitag, 05. Februar 2010
09:16

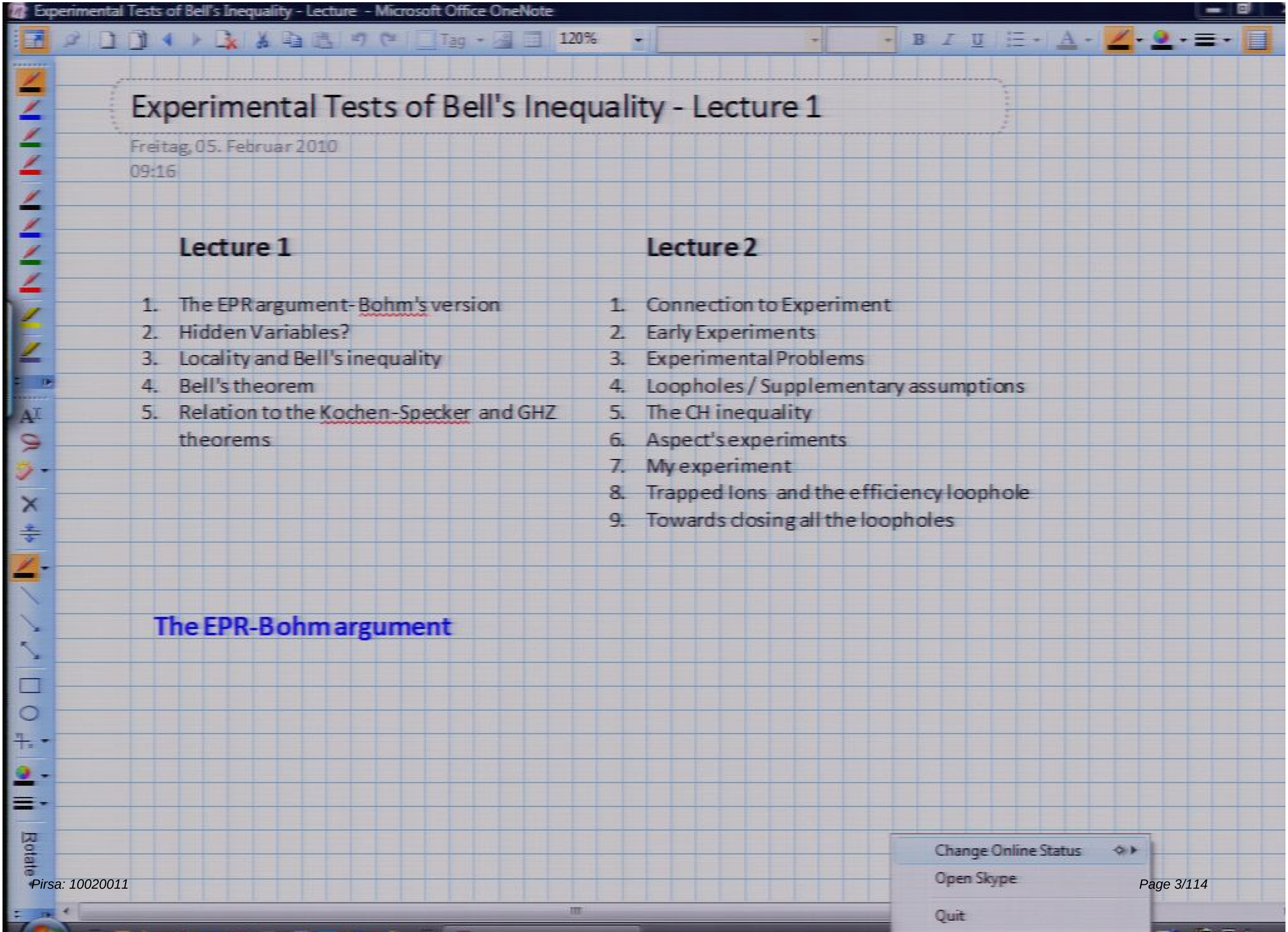
Lecture 1

1. The EPR argument- Bohm's version
2. Hidden Variables?
3. Locality and Bell's inequality
4. Bell's theorem
5. Relation to the Kochen-Specker and GHZ theorems

Lecture 2

1. Connection to Experiment
2. Early Experiments
3. Experimental Problems
4. Loopholes / Supplementary assumptions
5. The CH inequality
6. Aspect's experiments
7. My experiment
8. Trapped Ions and the efficiency loophole
9. Towards closing all the loopholes

The EPR-Bohm argument



Experimental Tests of Bell's Inequality - Lecture 1

Freitag, 05. Februar 2010
09:16

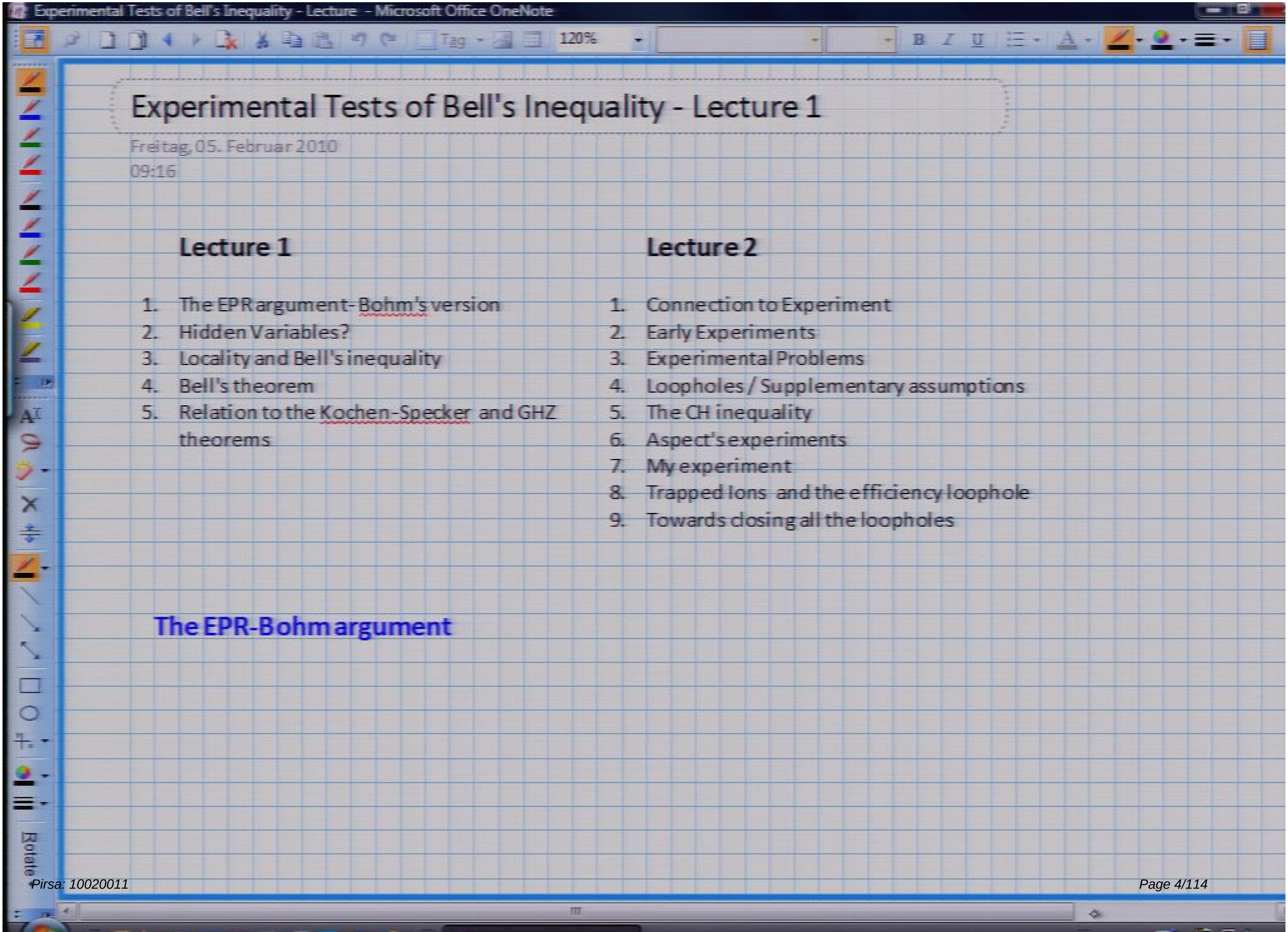
Lecture 1

1. The EPR argument- Bohm's version
2. Hidden Variables?
3. Locality and Bell's inequality
4. Bell's theorem
5. Relation to the Kochen-Specker and GHZ theorems

Lecture 2

1. Connection to Experiment
2. Early Experiments
3. Experimental Problems
4. Loopholes / Supplementary assumptions
5. The CH inequality
6. Aspect's experiments
7. My experiment
8. Trapped Ions and the efficiency loophole
9. Towards closing all the loopholes

The EPR-Bohm argument



Experimental Tests of Bell's Inequality - Lecture 1

Freitag, 05. Februar 2010
09:16

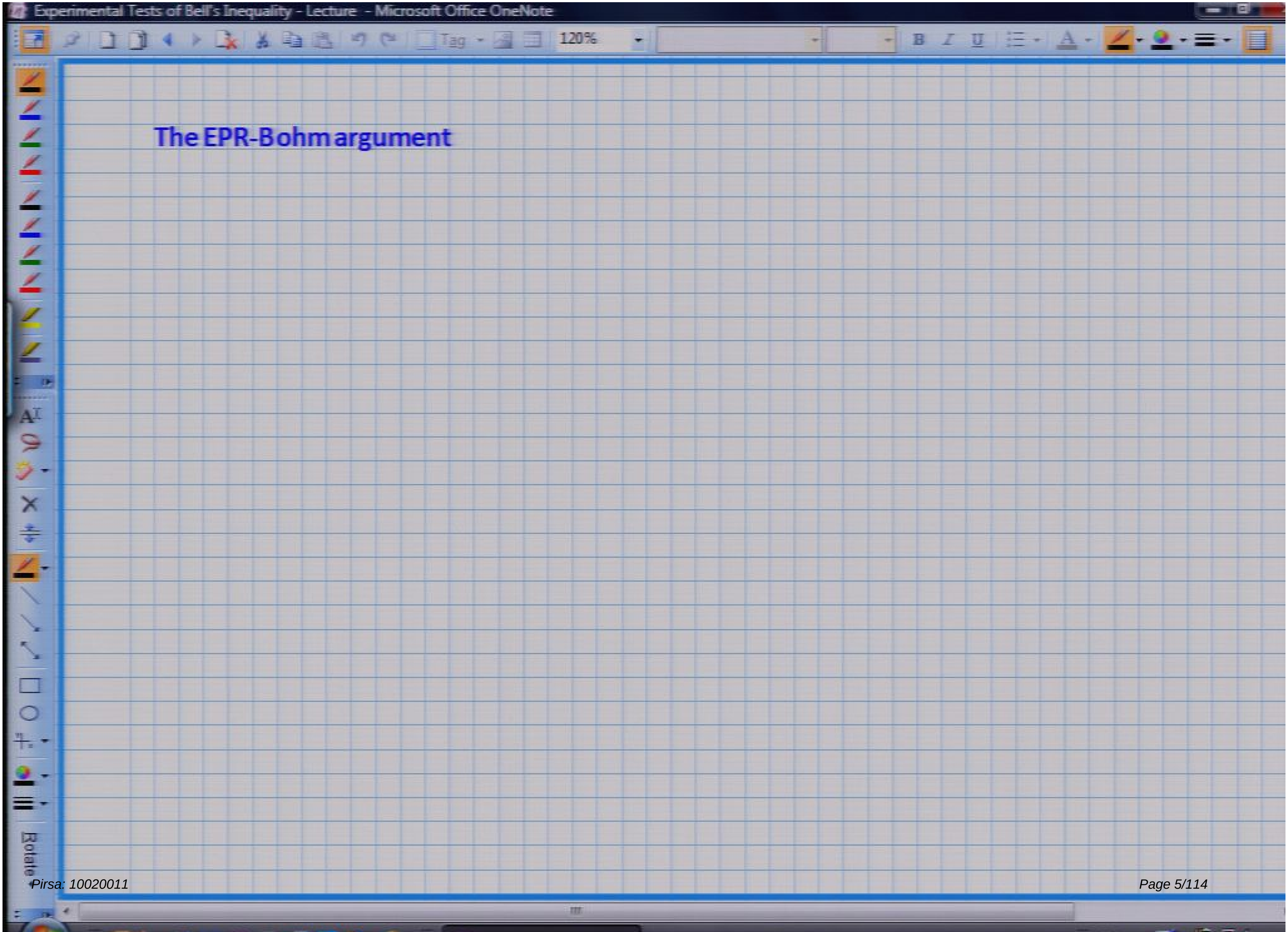
Lecture 1

1. The EPR argument - Bohm's version
2. Hidden Variables?
3. Locality and Bell's inequality
4. Bell's theorem
5. Relation to the Kochen-Specker and GHZ theorems

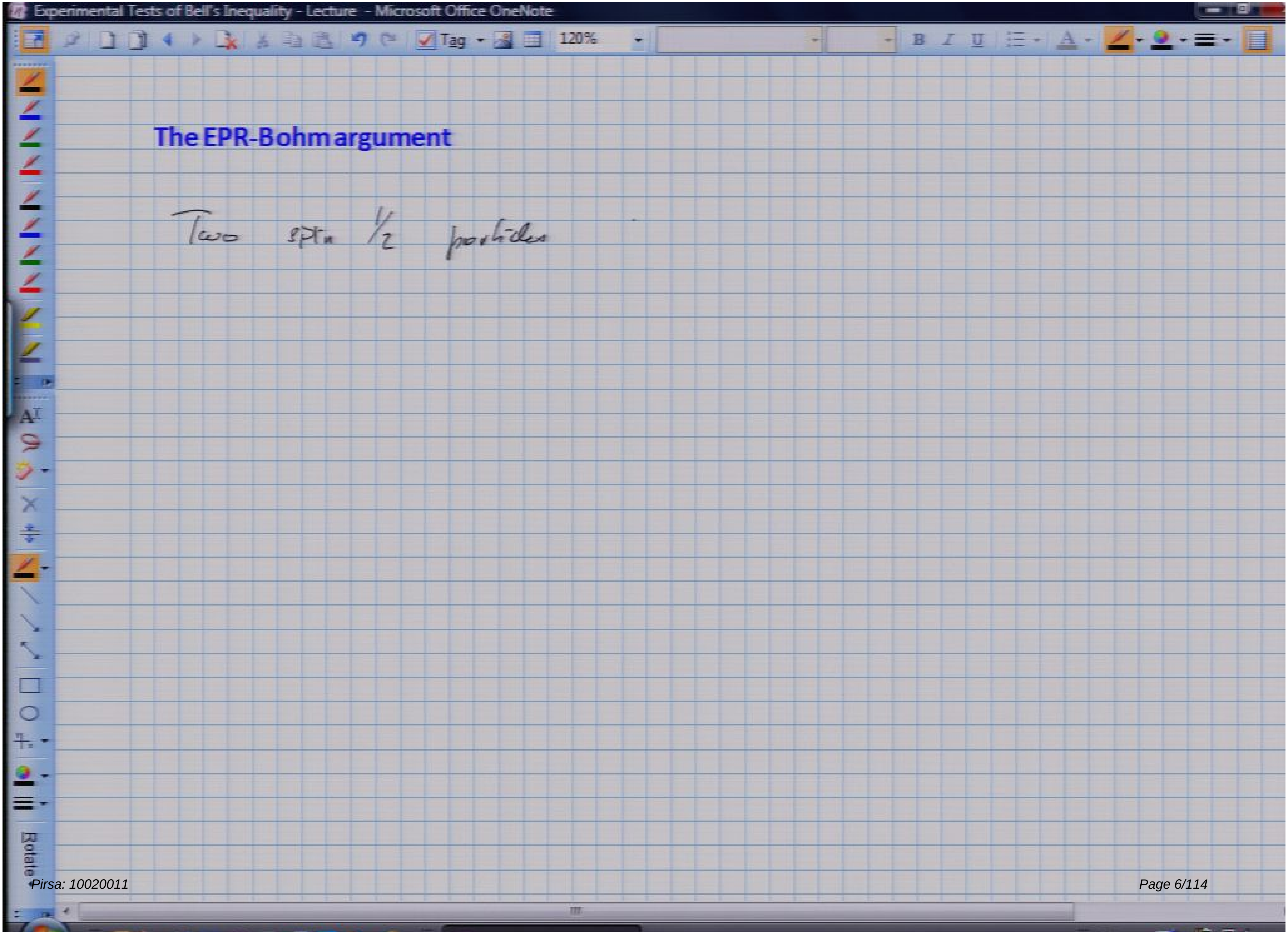
Lecture 2

1. Connection to Experiment
2. Early Experiments
3. Experimental Problems
4. Loopholes / Supplementary assumptions
5. The CH inequality
6. Aspect's experiments
7. My experiment
8. Trapped Ions and the efficiency loophole
9. Towards closing all the loopholes

The EPR-Bohm argument

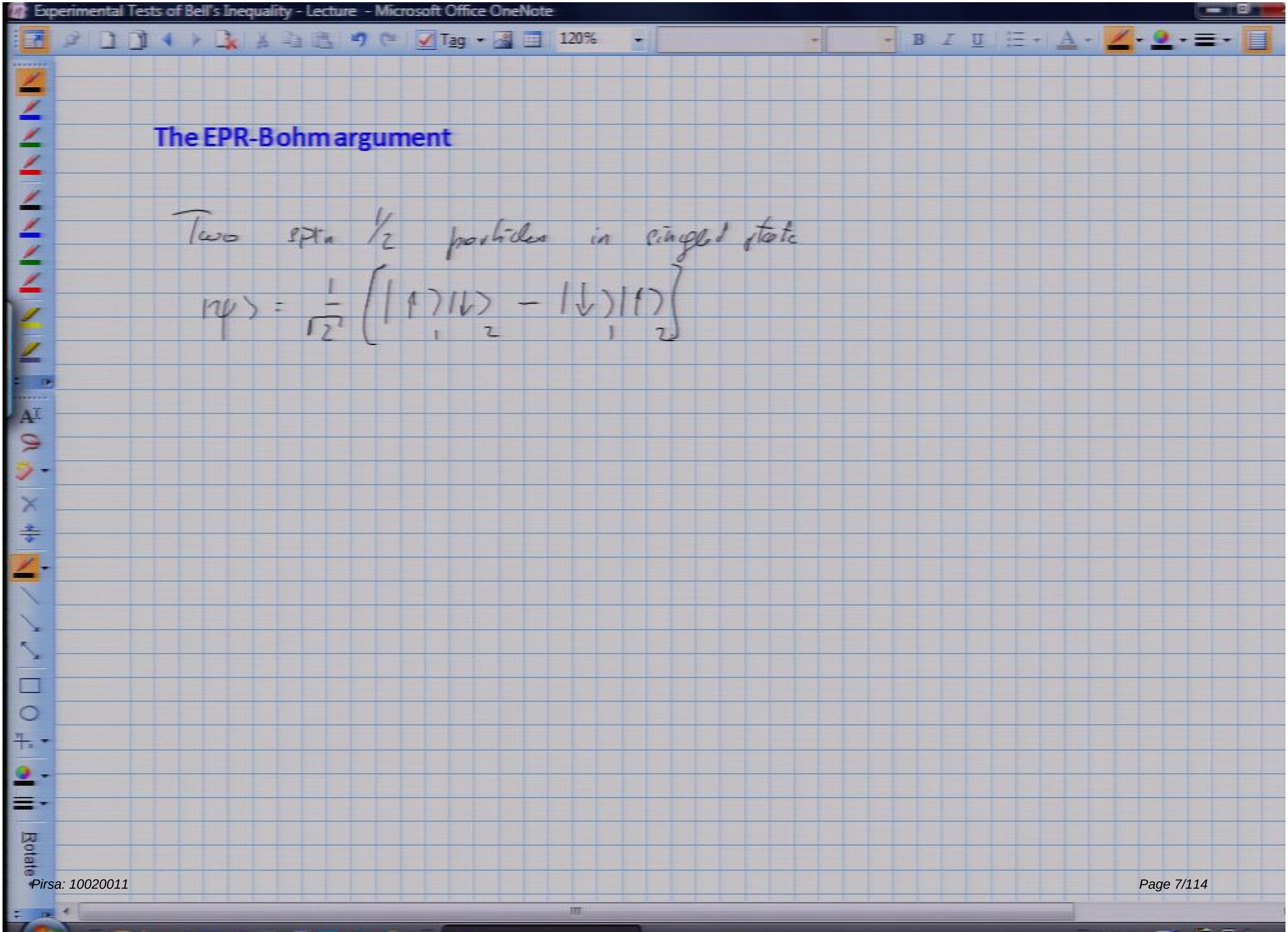


The EPR-Bohm argument



The EPR-Bohm argument

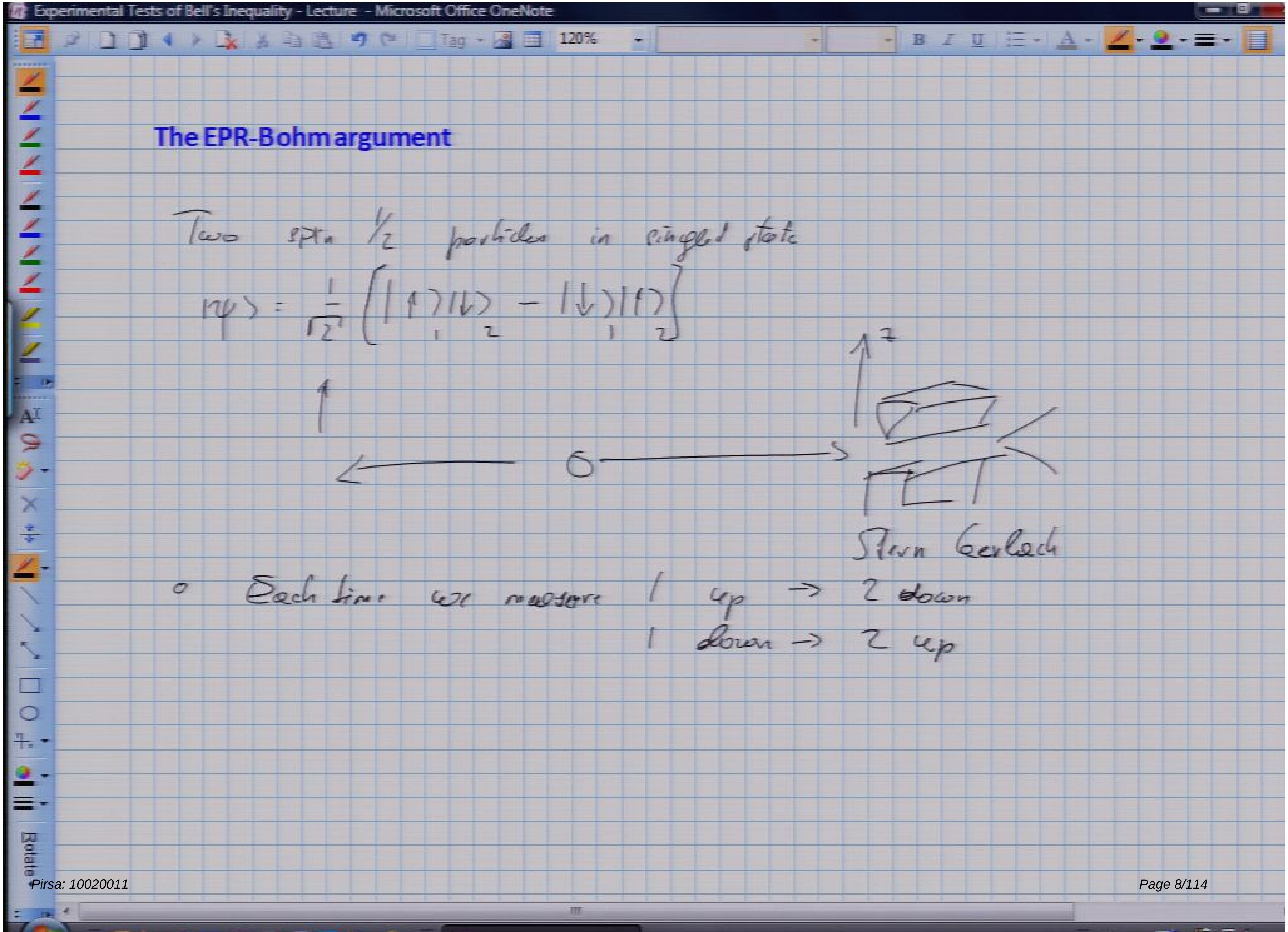
Two spin 1/2 particles



The EPR-Bohm argument

Two spin 1/2 particles in singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right)$$

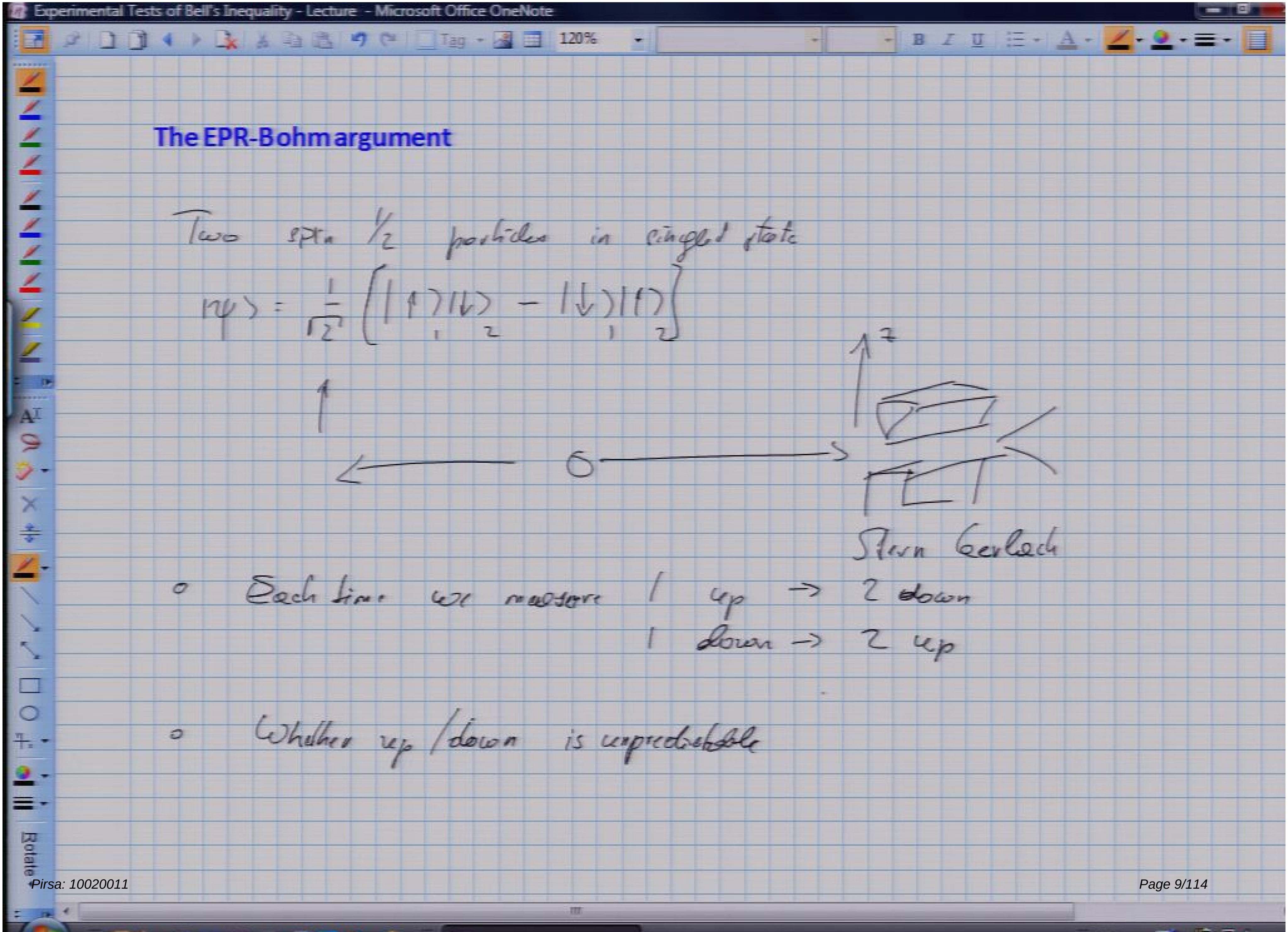


The EPR-Bohm argument

Two spin 1/2 particles in singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right]$$

- Each time we measure 1 up → 2 down
  1 down → 2 up



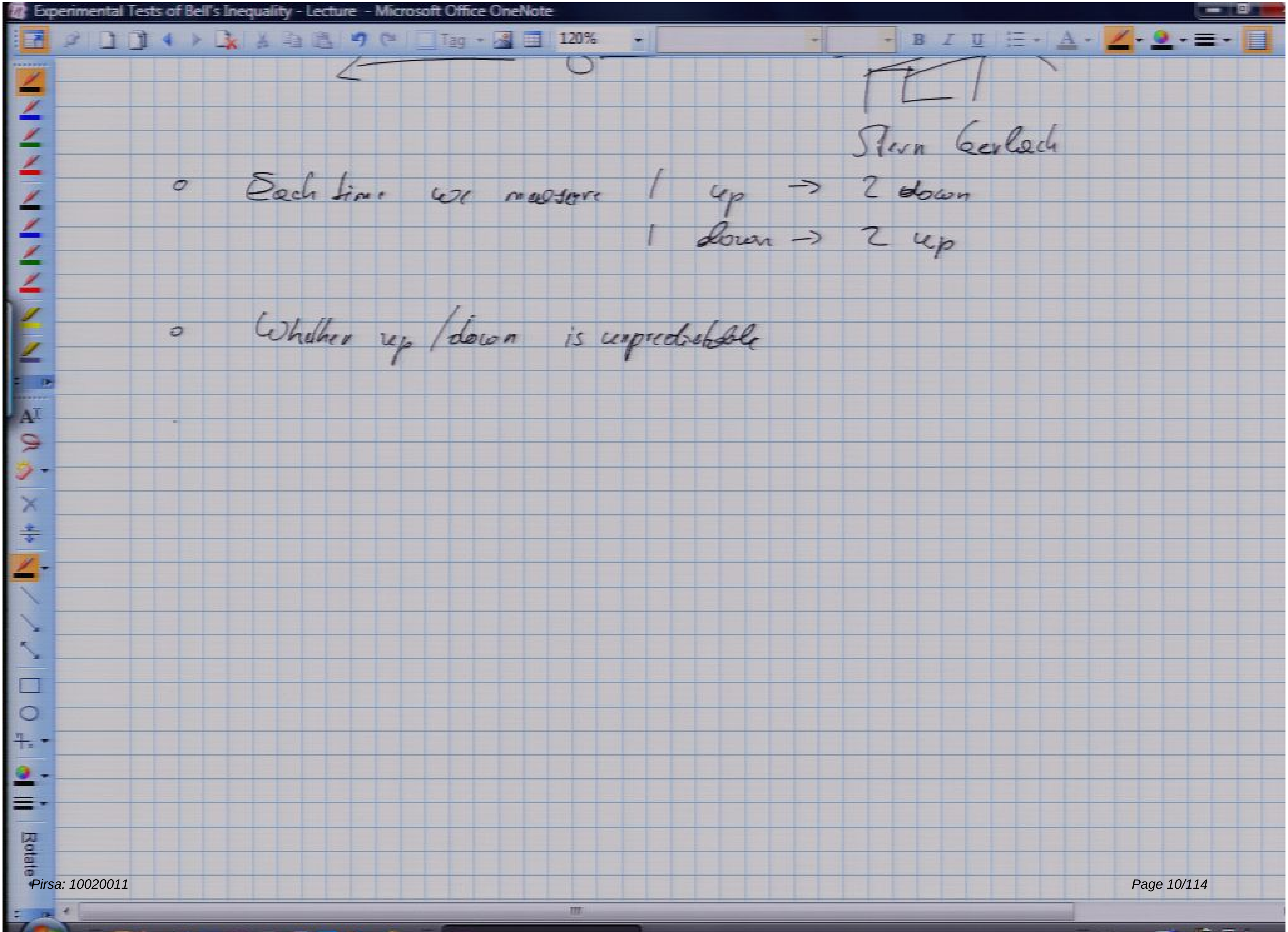
The EPR-Bohm argument

Two spin 1/2 particles in singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \right]$$

- Each time we measure 1 up → 2 down
  1 down → 2 up

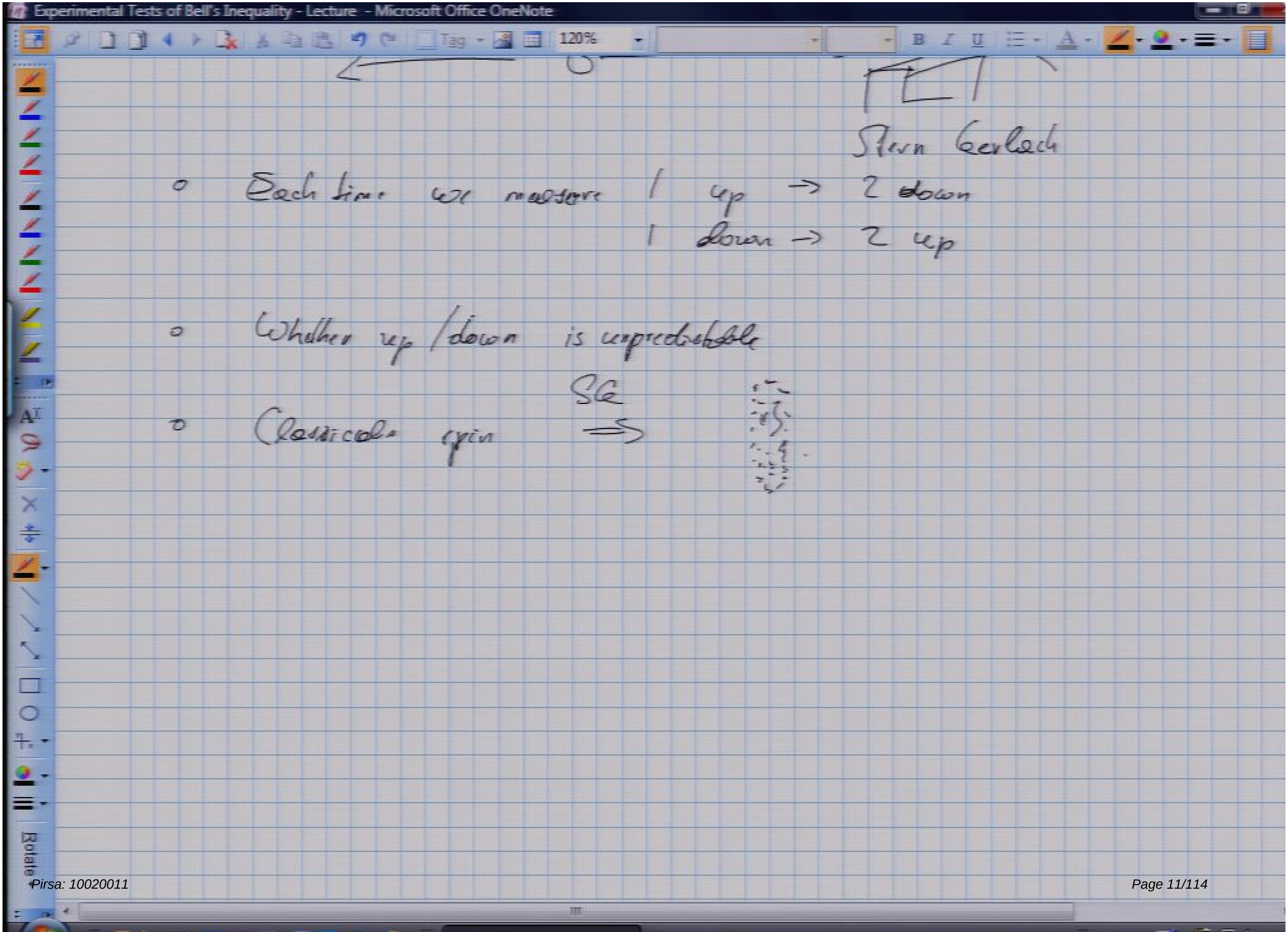
- Whether up/down is unpredictable



Stern Gerlach

○ Each time we measure 1 up → 2 down
1 down → 2 up

○ Whether up/down is unpredictable

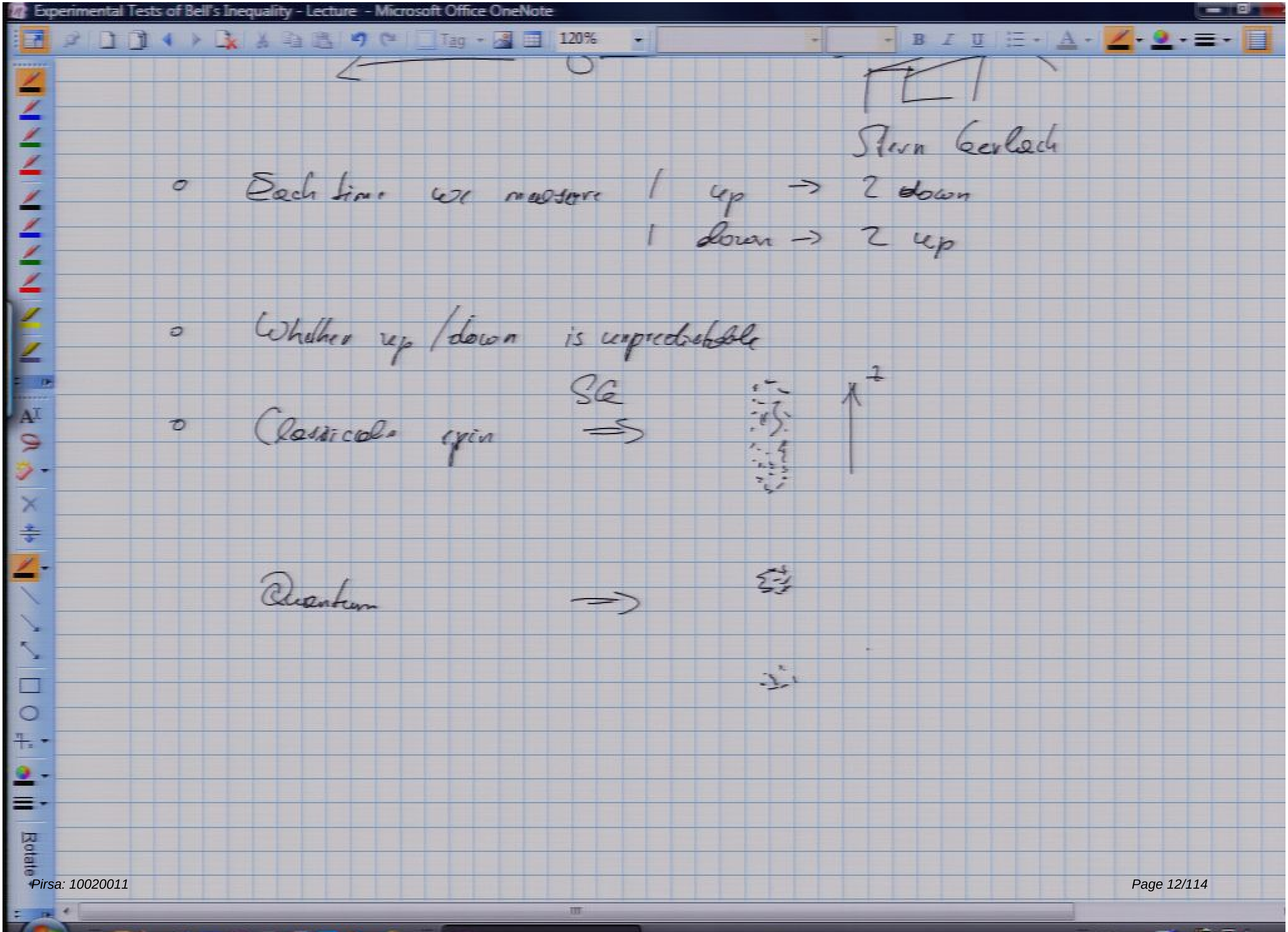


Stern Gerlach

- Each time we measure 1 up → 2 down
  1 down → 2 up

- Whether up/down is unpredictable

- Classical spin $\xRightarrow{SG}$



Stern Gerlach

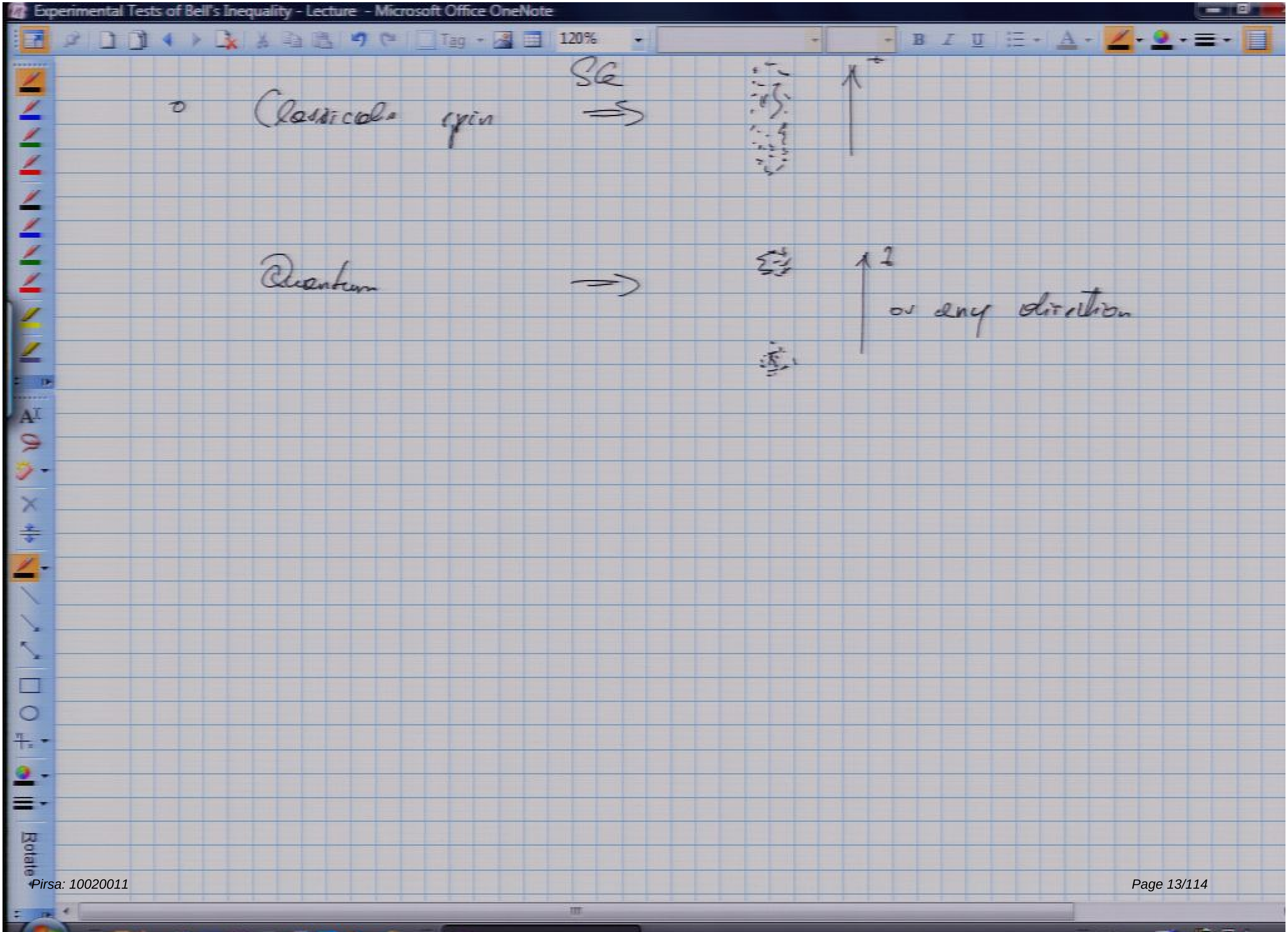
- Each time we measure 1 up → 2 down
  1 down → 2 up

- Whether up/down is unpredictable

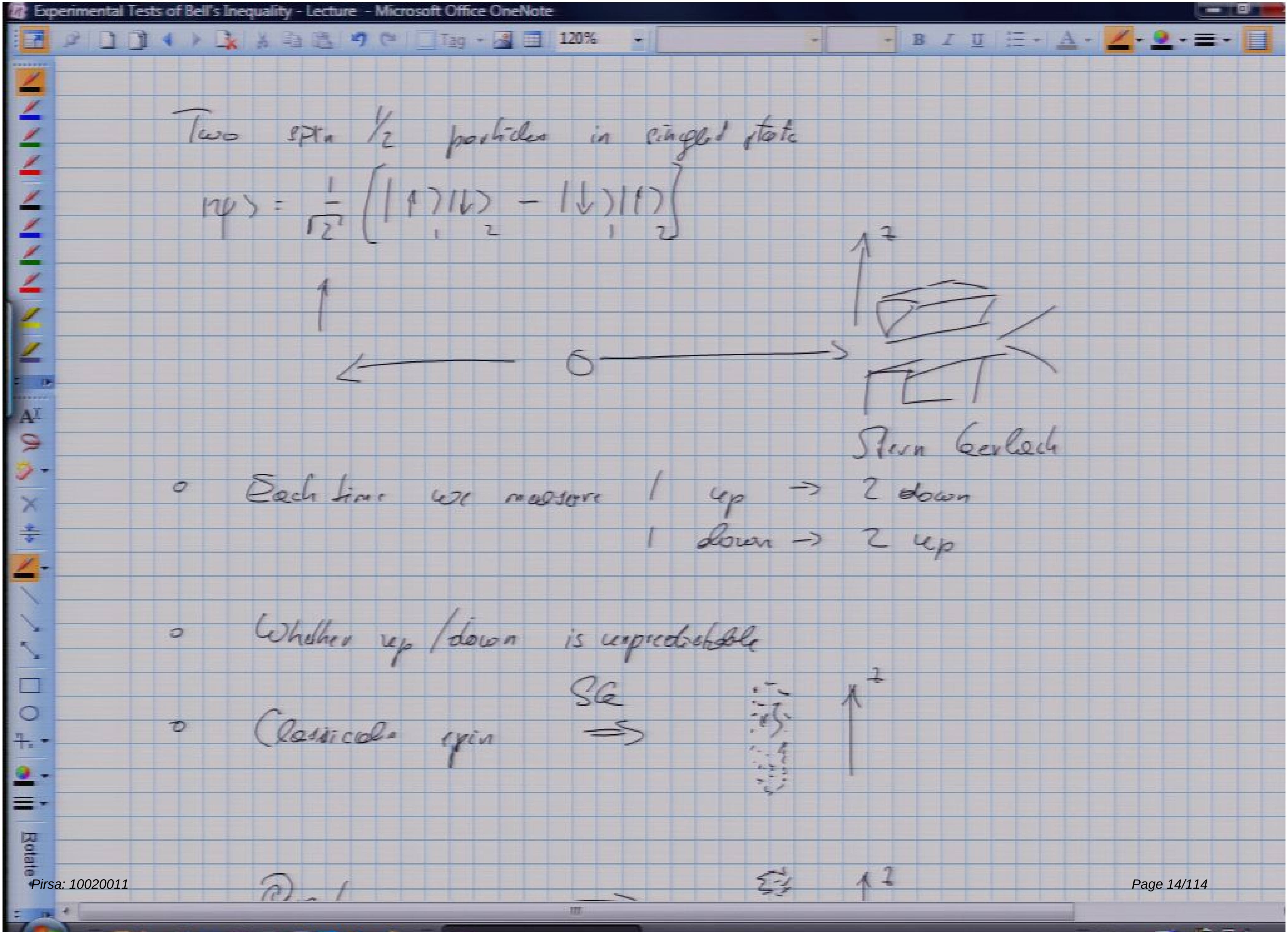
SG

- Classical spin ⟹

Quantum ⟹



Experimental Tests of Bell's Inequality - Lecture - Microsoft Office OneNote



Two spin ½ particles in singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right)$$

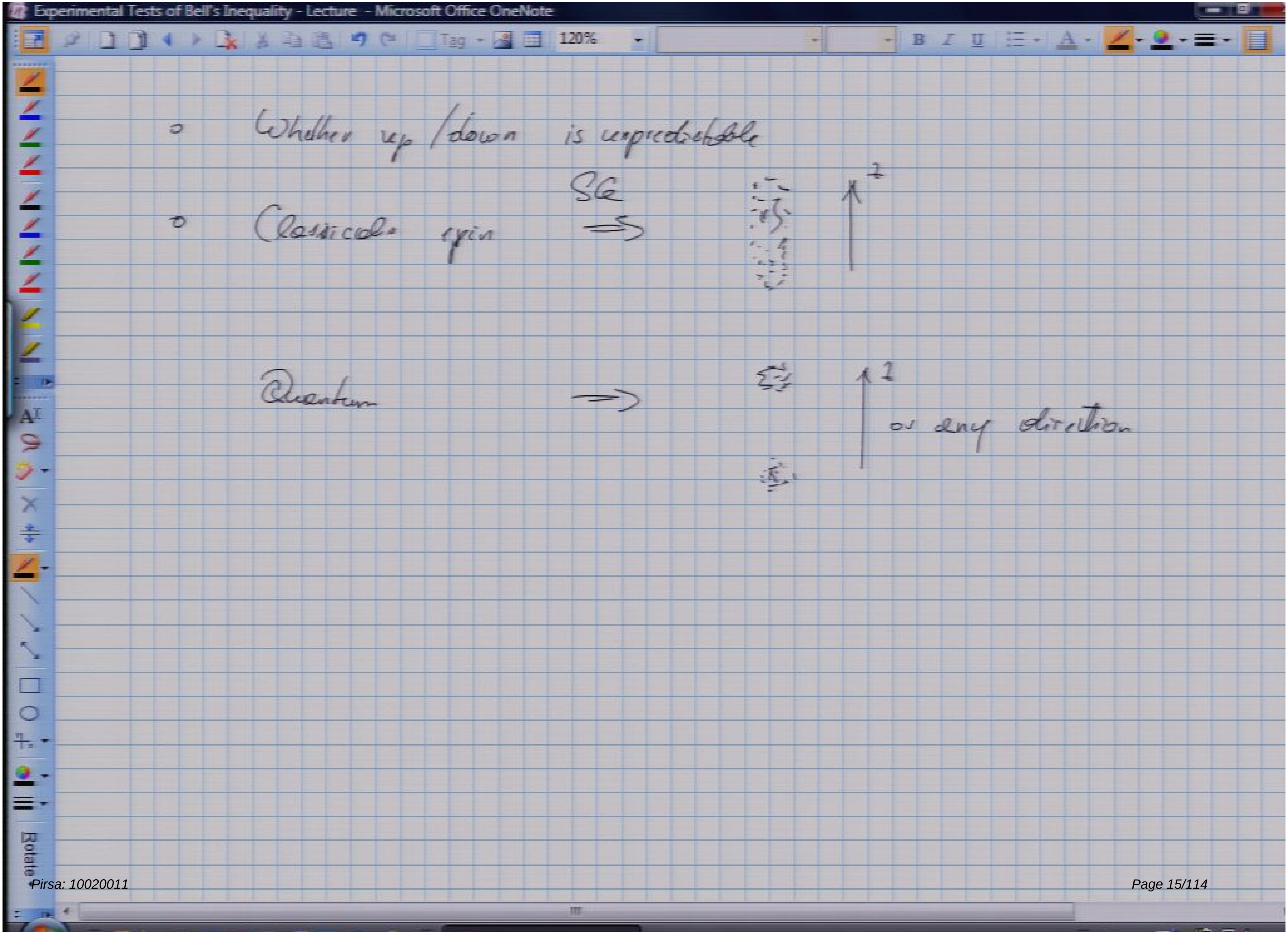
Stern Gerlach

- Each time we measure 1 up → 2 down
  1 down → 2 up

- Whether up/down is unpredictable

SG ⇒

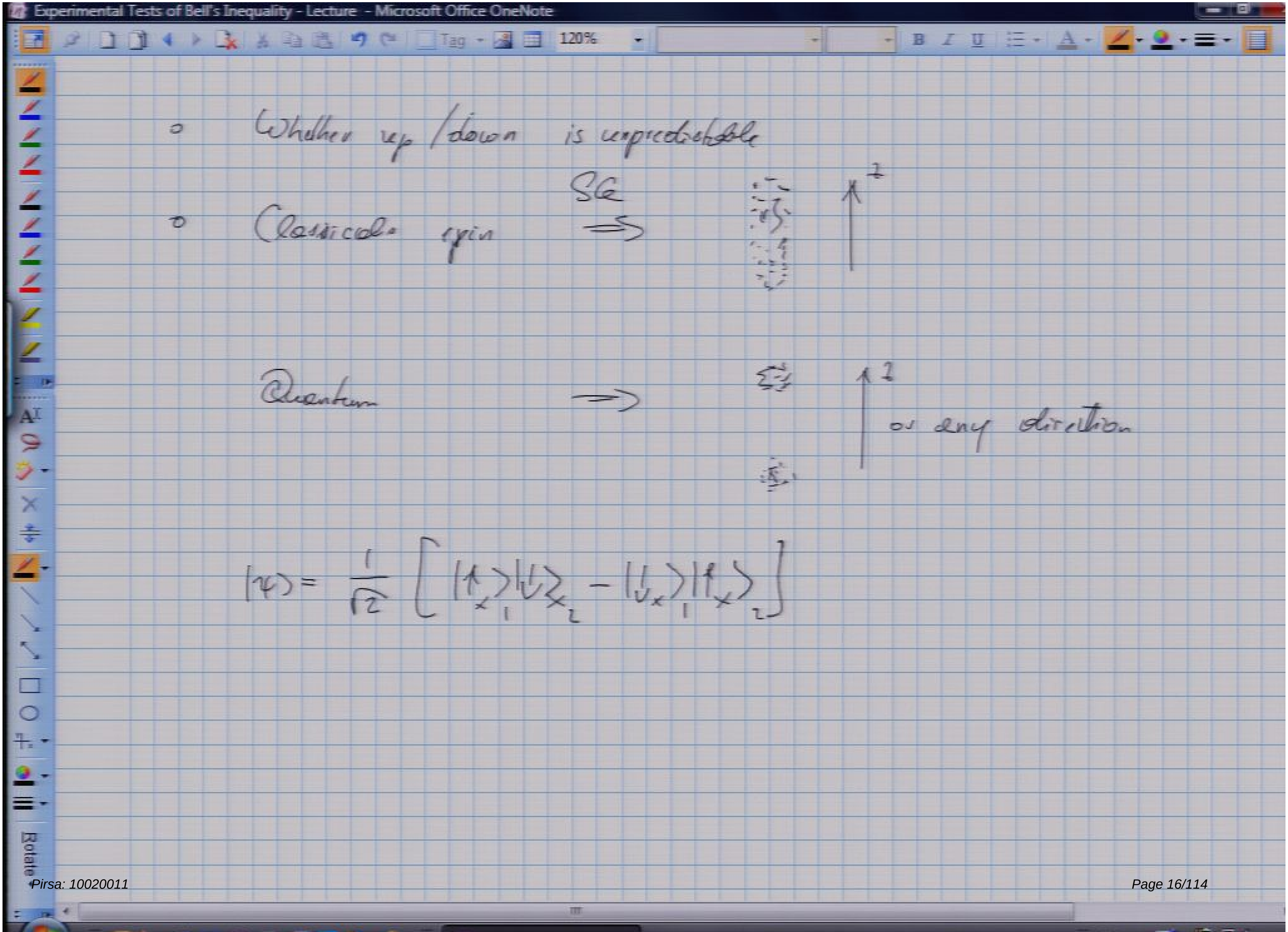
- Classical spin



- Whether up/down is unpredictable
- Classical spin

Quantum

or any direction

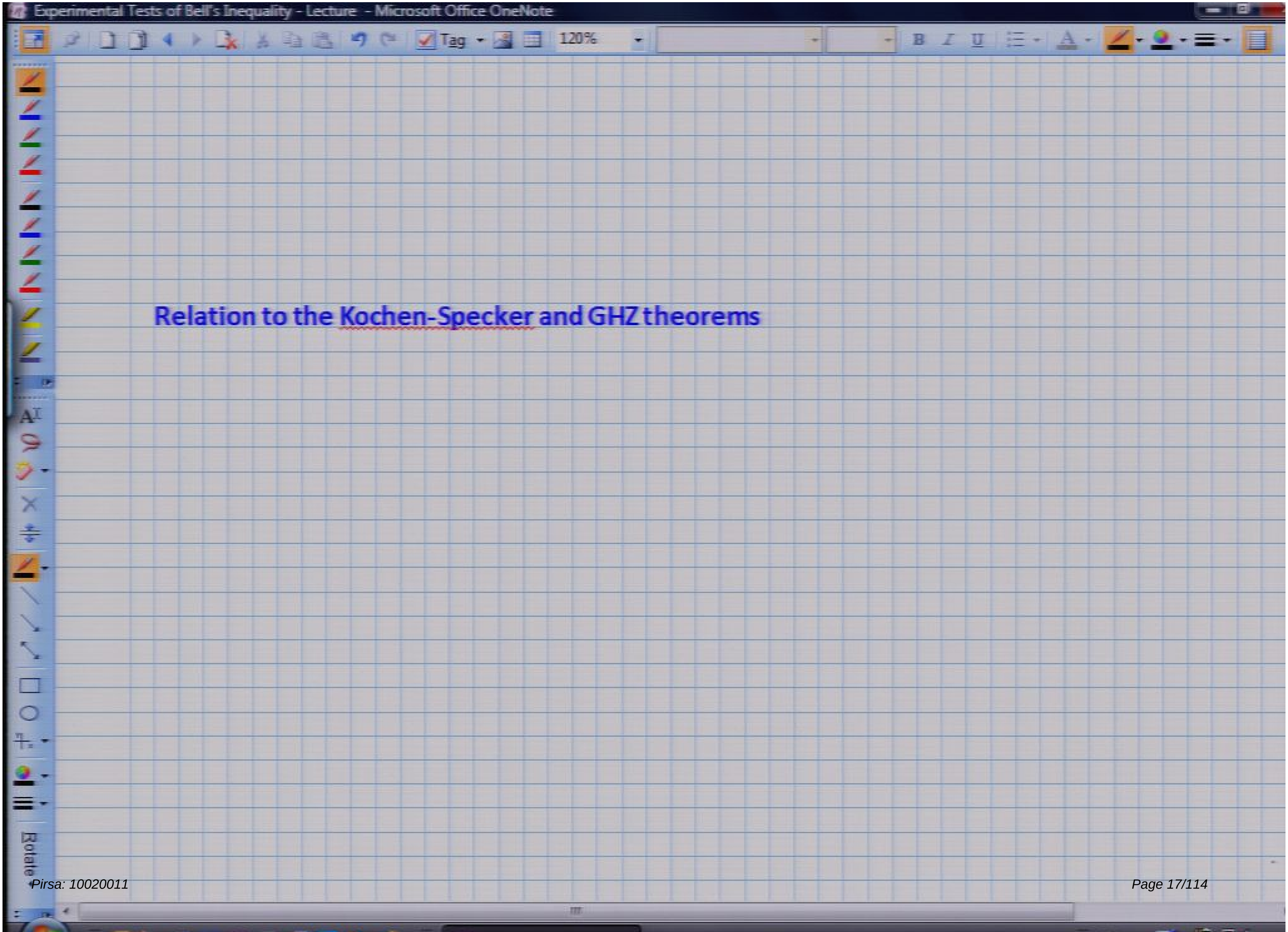


- Whether up/down is unpredictable

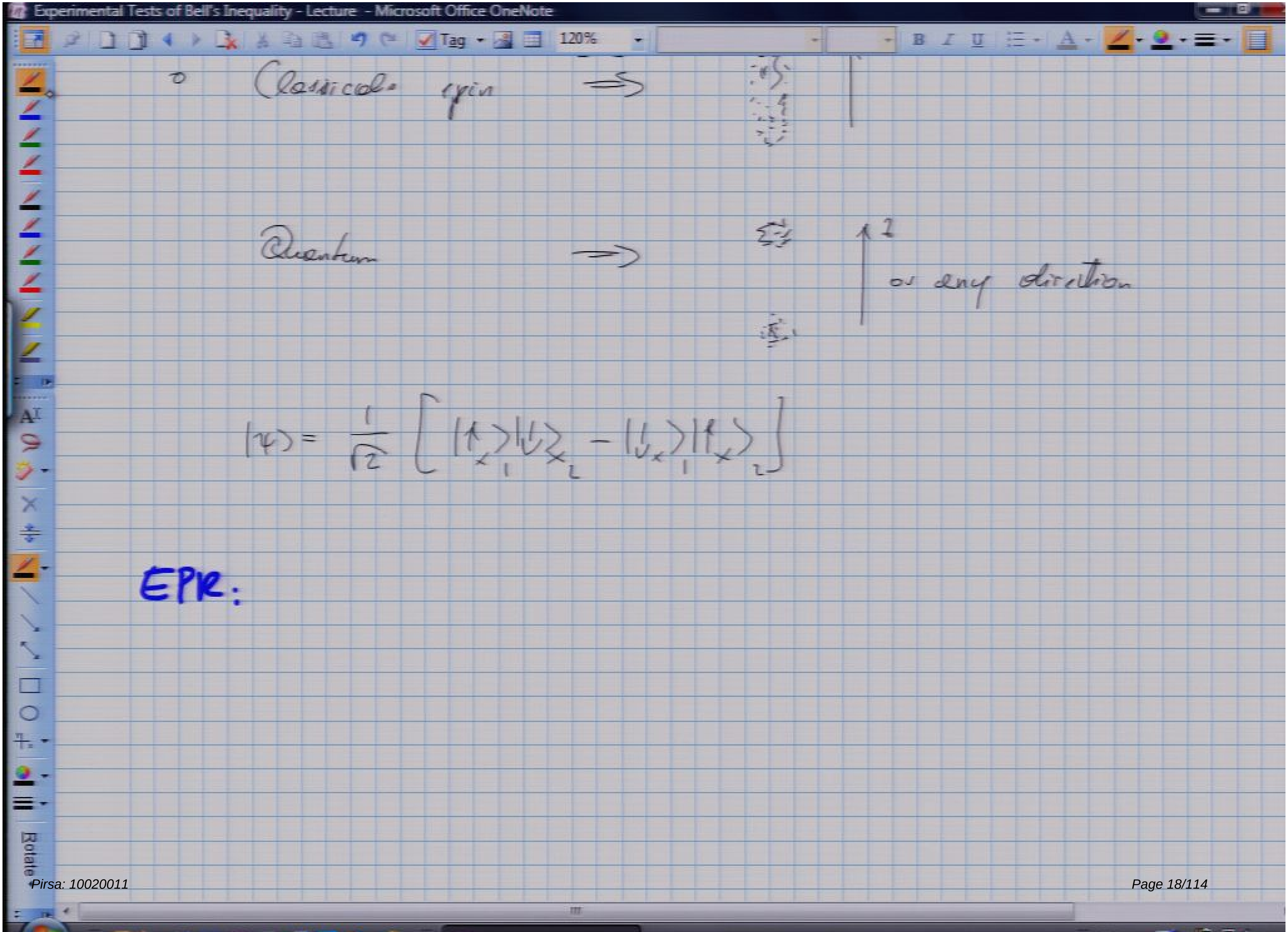
- Classical spin $\xRightarrow{SG}$

Quantum $\Longrightarrow$ or any direction

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_x\rangle_1|\downarrow_x\rangle_2 - |\downarrow_x\rangle_1|\uparrow_x\rangle_2\right]$$



Relation to the Kochen-Specker and GHZ theorems



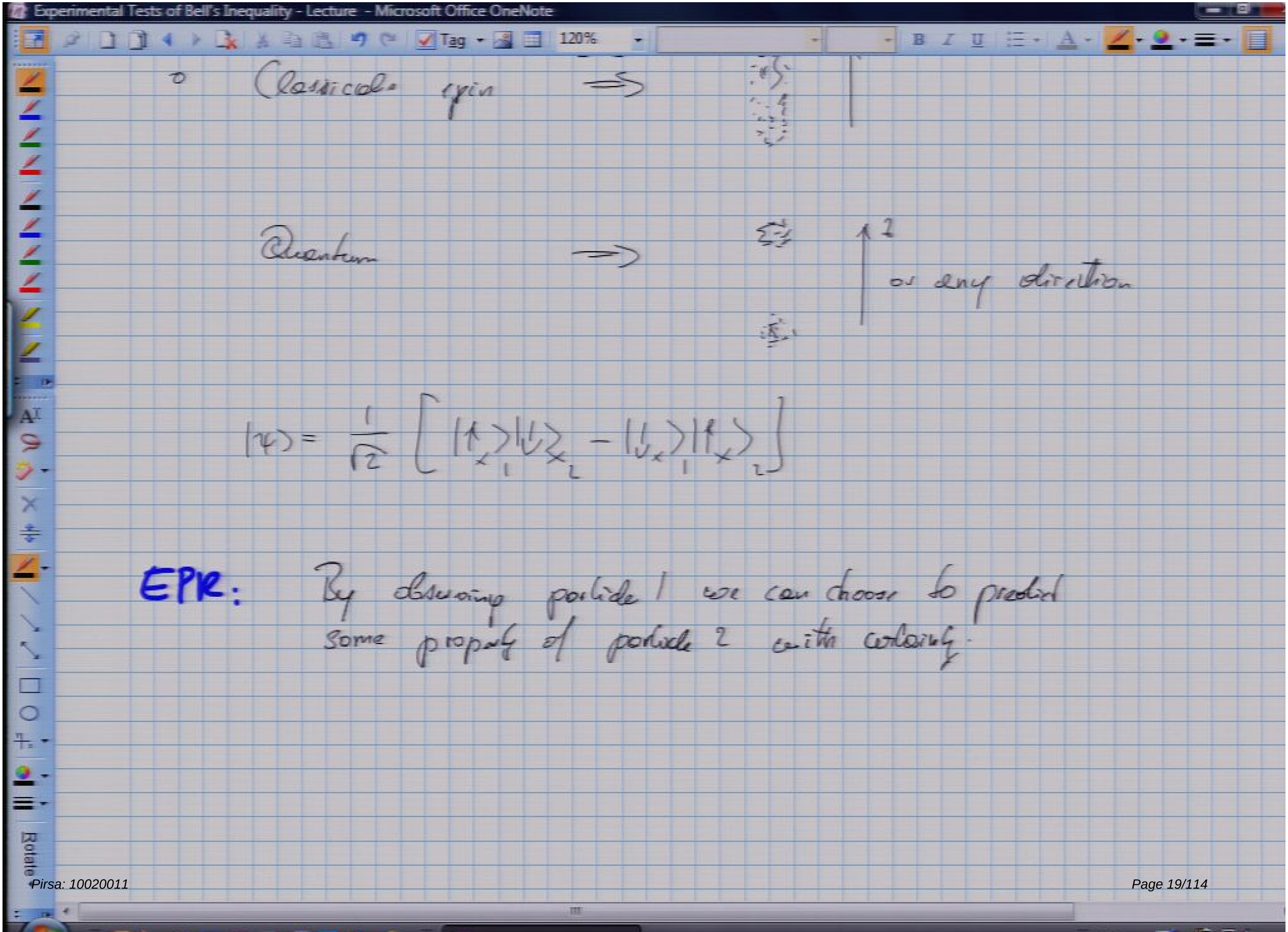
Experimental Tests of Bell's Inequality - Lecture - Microsoft Office OneNote

o Classical spin $\Longrightarrow$

Quantum $\Longrightarrow$ $\uparrow z$ or any direction

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_x\rangle_1|\downarrow_x\rangle_2 - |\downarrow_x\rangle_1|\uparrow_x\rangle_2\right]$$

EPR:



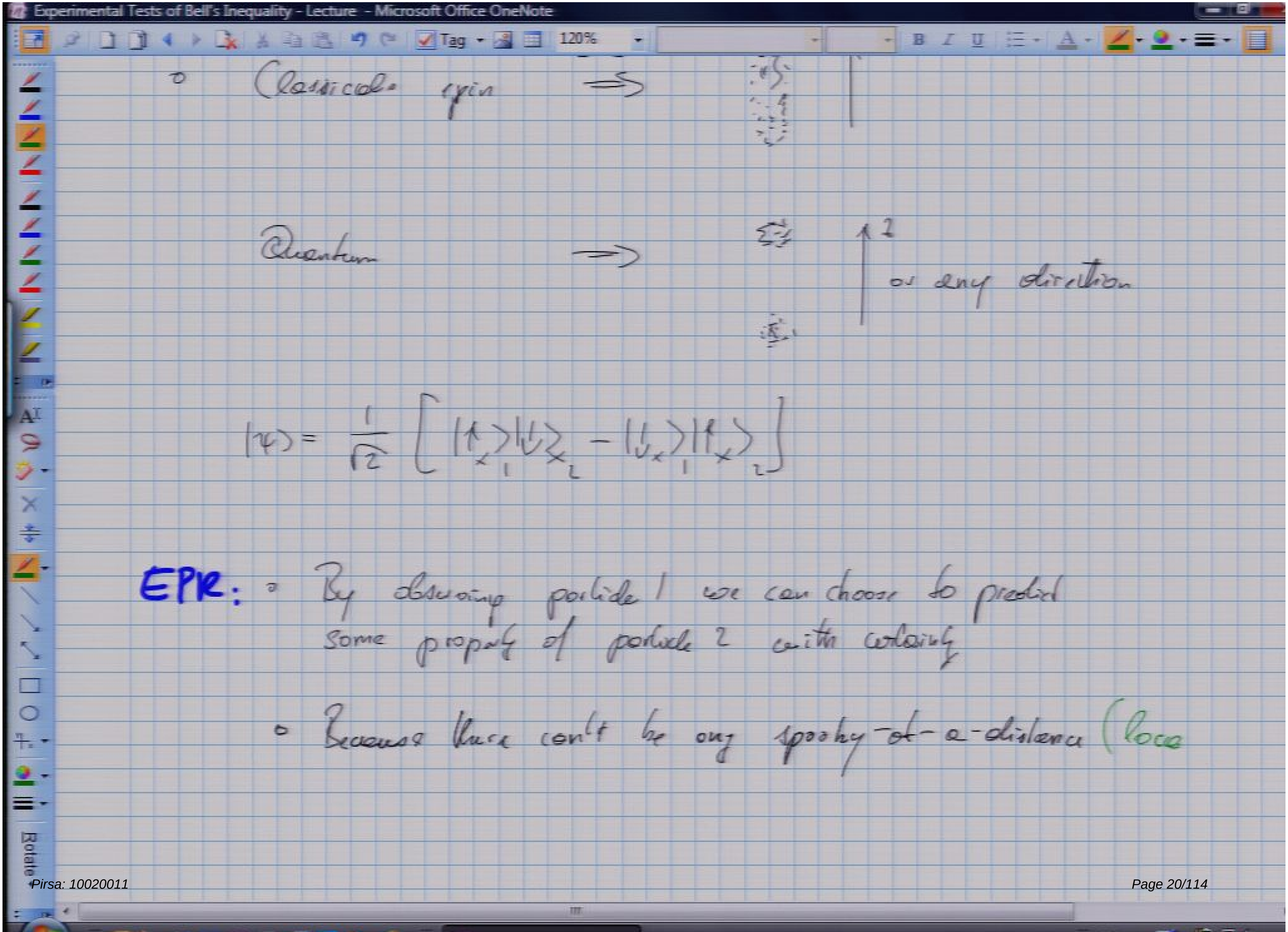
Experimental Tests of Bell's Inequality - Lecture - Microsoft Office OneNote

o Classical spin $\Rightarrow$

Quantum $\Rightarrow$ z or any direction

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_x\rangle_1|\downarrow_x\rangle_2 - |\downarrow_x\rangle_1|\uparrow_x\rangle_2\right]$$

EPR: By observing particle 1 we can choose to predict some property of particle 2 with certainty.



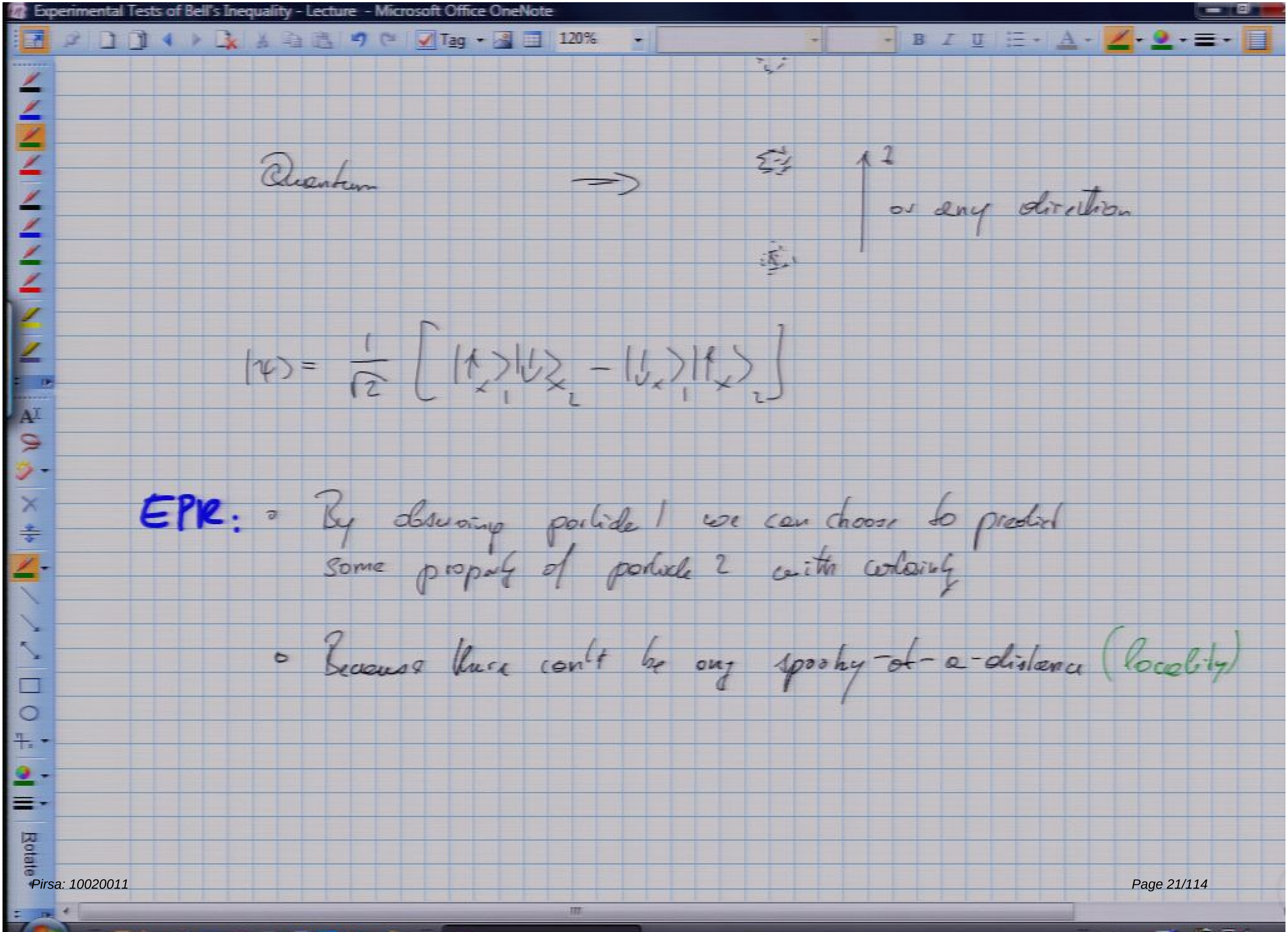
○ Classical spin ⟹

Quantum → ↑ z or any direction

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_x\rangle_1|\downarrow_x\rangle_2 - |\downarrow_x\rangle_1|\uparrow_x\rangle_2\right]$$

EPR: ○ By observing particle 1 we can choose to predict some property of particle 2 with certainty

○ Because there can't be any spooky-at-a-distance (loca



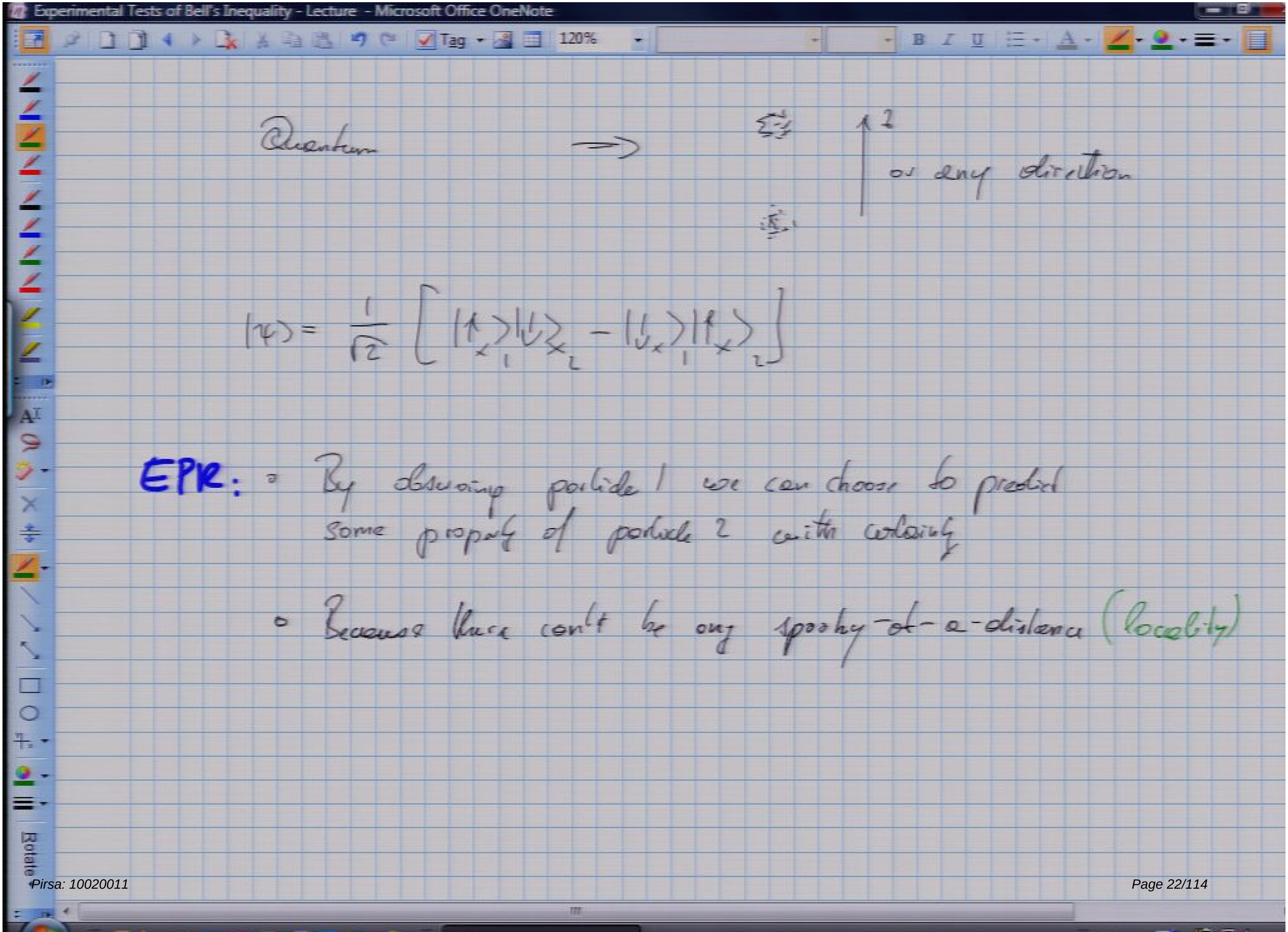
Quantum →

↑ z or any direction

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_x\rangle_1|\downarrow_x\rangle_2 - |\downarrow_x\rangle_1|\uparrow_x\rangle_2\right]$$

EPR:

- By observing particle 1 we can choose to predict some property of particle 2 with certainty
- Because there can't be any spooky-at-a-distance (locality)



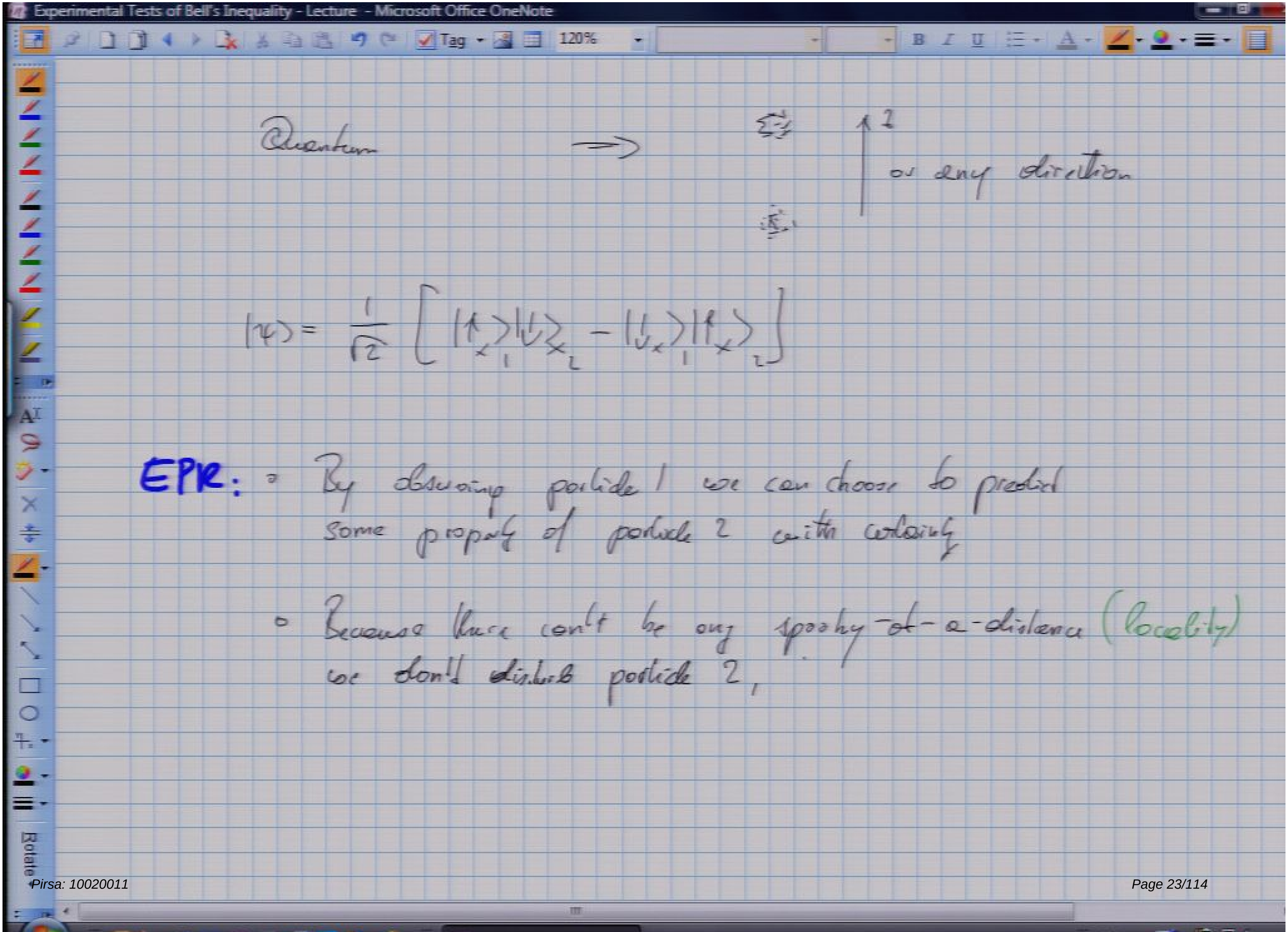
Quantum →

or any direction

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_x\rangle_1|\downarrow_x\rangle_2 - |\downarrow_x\rangle_1|\uparrow_x\rangle_2\right]$$

EPR:

- By observing particle 1 we can choose to predict some property of particle 2 with certainty
- Because there can't be any spooky-at-a-distance (locality)

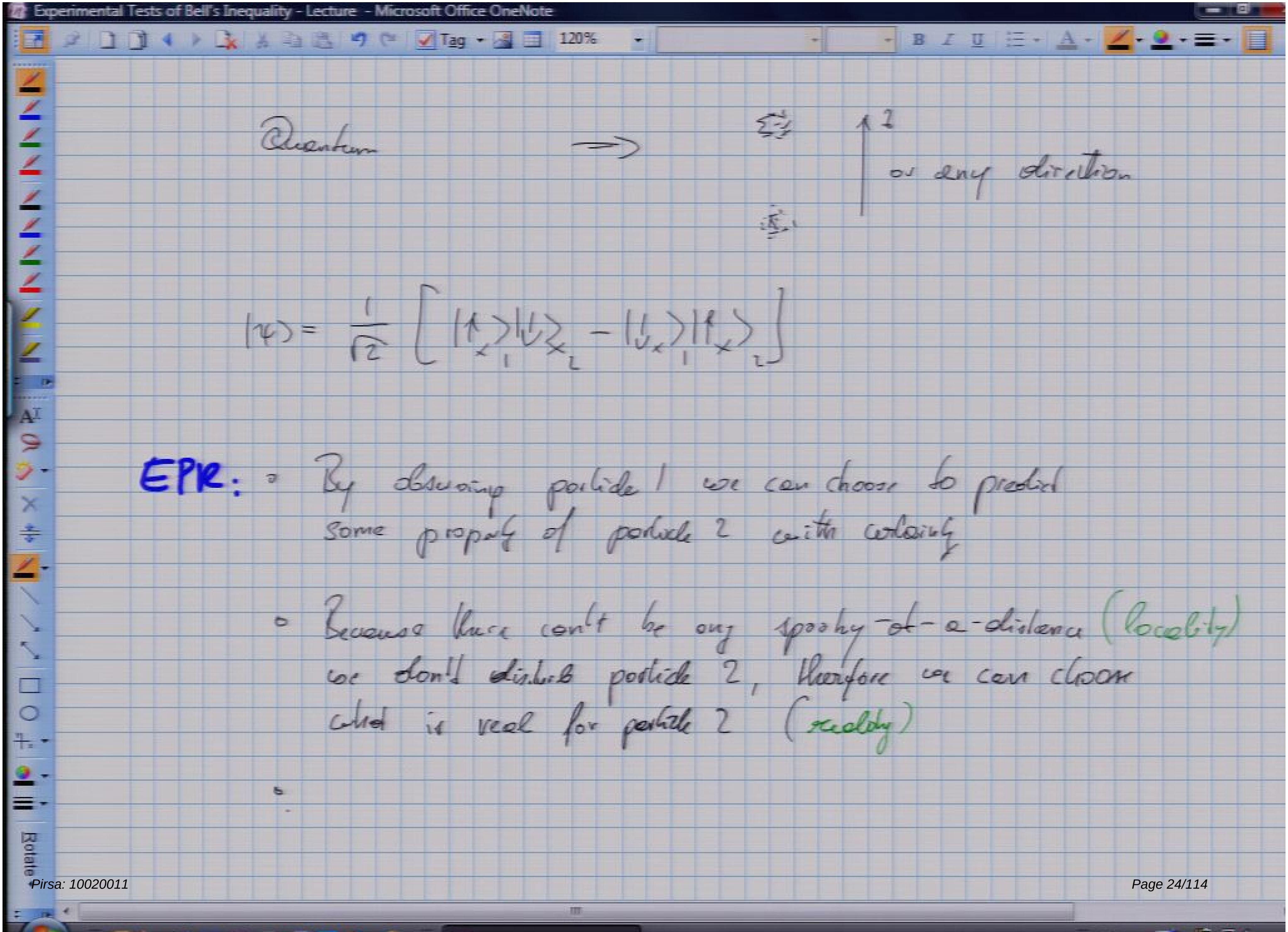


Quantum $\longrightarrow$ or any direction

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_x\rangle_1|\downarrow_x\rangle_2 - |\downarrow_x\rangle_1|\uparrow_x\rangle_2\right]$$

EPR:

- By observing particle 1 we can choose to predict some property of particle 2 with certainty
- Because there can't be any spooky-at-a-distance (locality) we don't disturb particle 2,



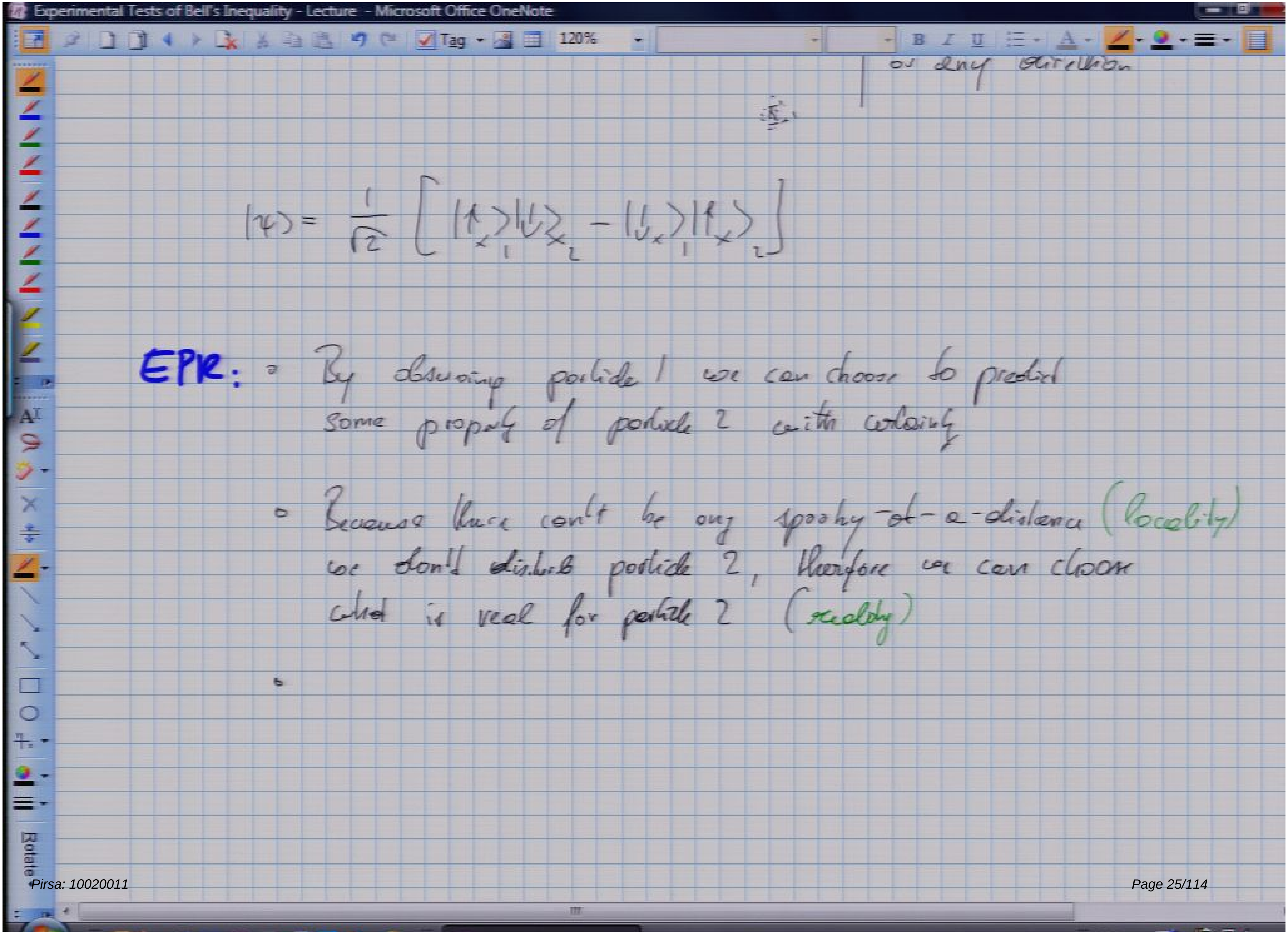
Quantum →

or any direction

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_x\rangle_1|\downarrow_x\rangle_2 - |\downarrow_x\rangle_1|\uparrow_x\rangle_2\right]$$

EPR:

- By observing particle 1 we can choose to predict some property of particle 2 with certainty
- Because there can't be any spooky-at-a-distance (locality) we don't disturb particle 2, therefore we can choose what is real for particle 2 (reality)
-

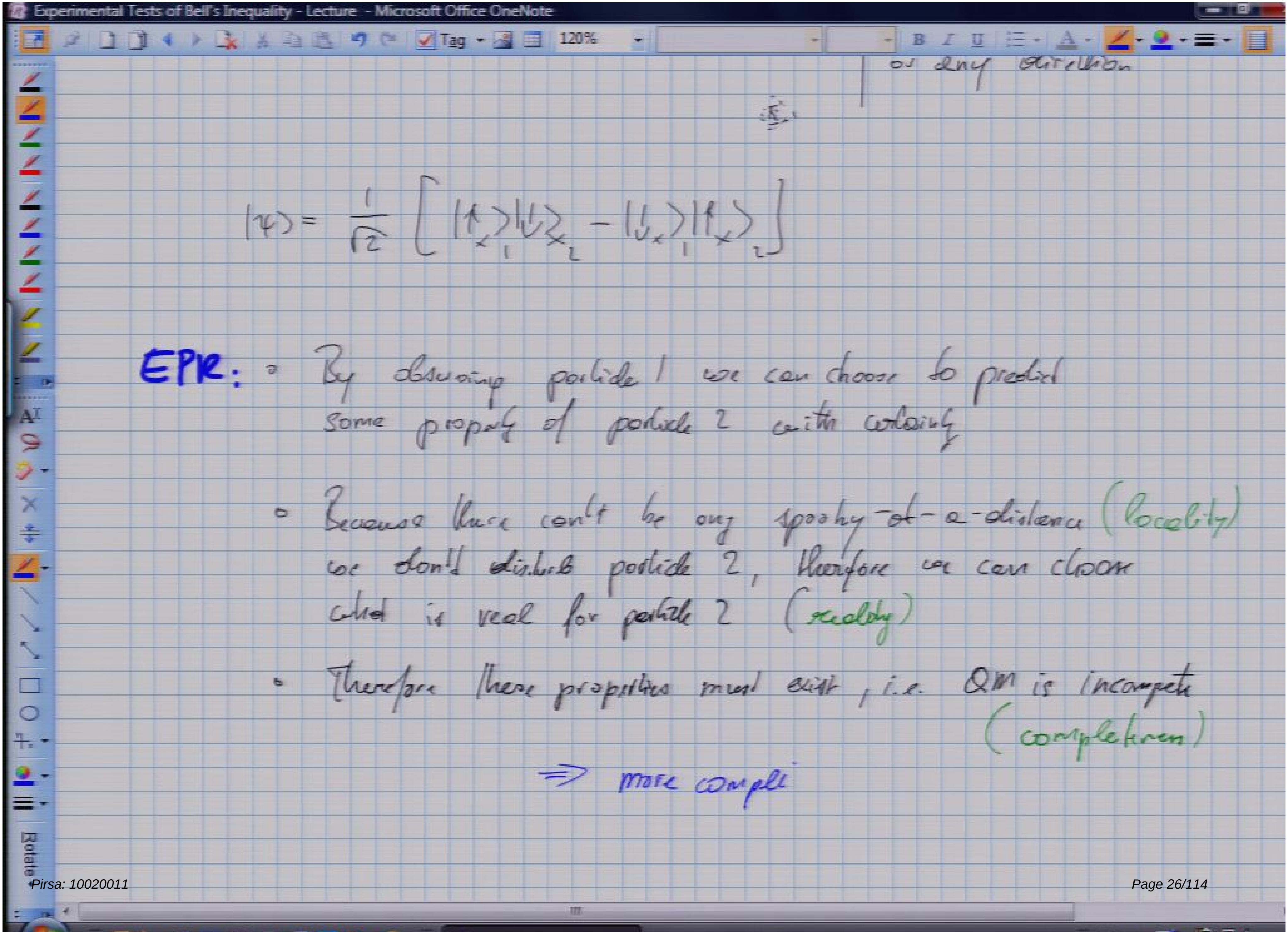


or any direction

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_x\rangle_1|\downarrow_x\rangle_2 - |\downarrow_x\rangle_1|\uparrow_x\rangle_2\right]$$

EPR:

- By observing particle 1 we can choose to predict some property of particle 2 without disturbing
- Because there can't be any spooky-at-a-distance (locality) we don't disturb particle 2, therefore we can choose what is real for particle 2 (reality)
-



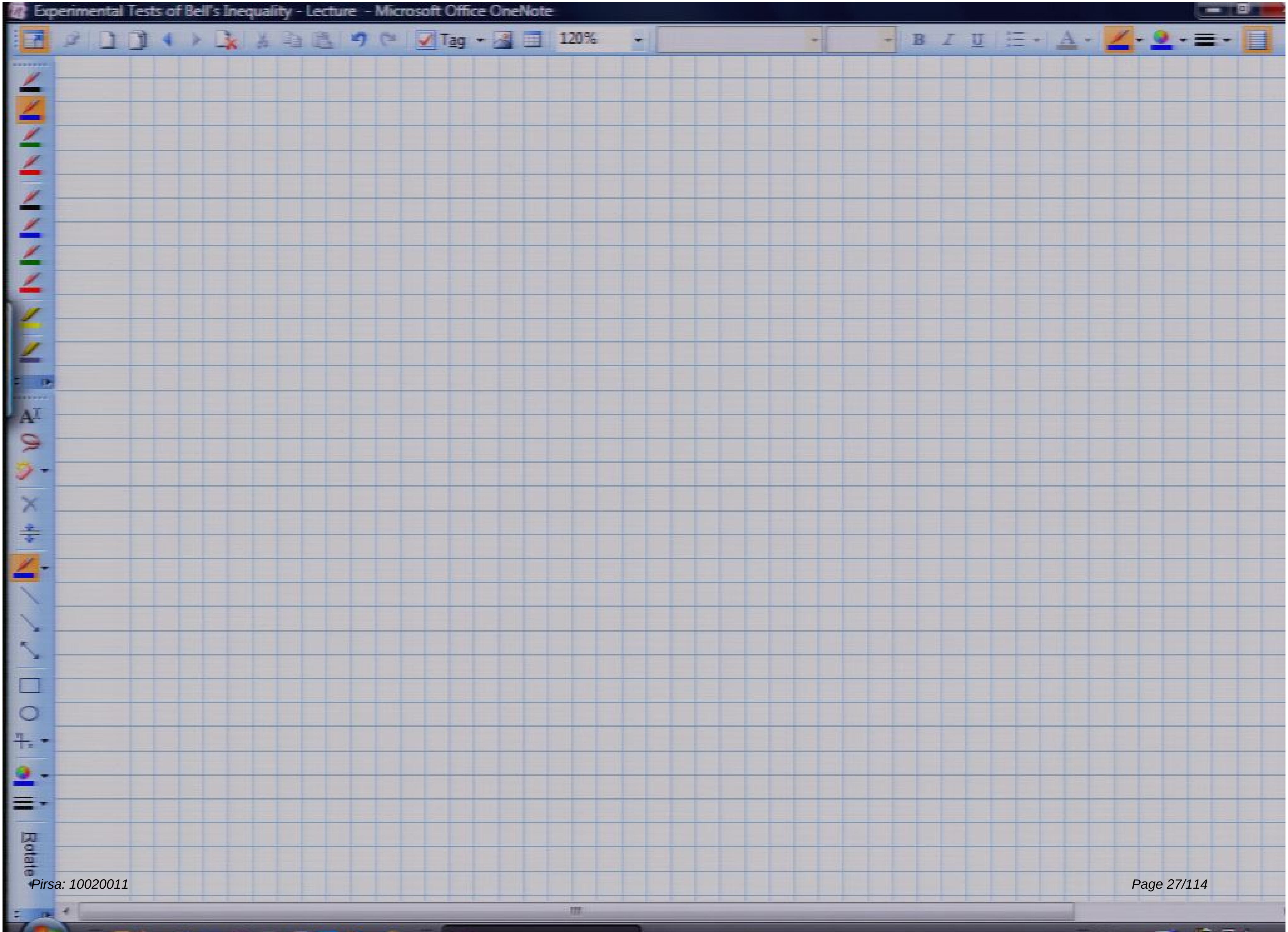
or any direction

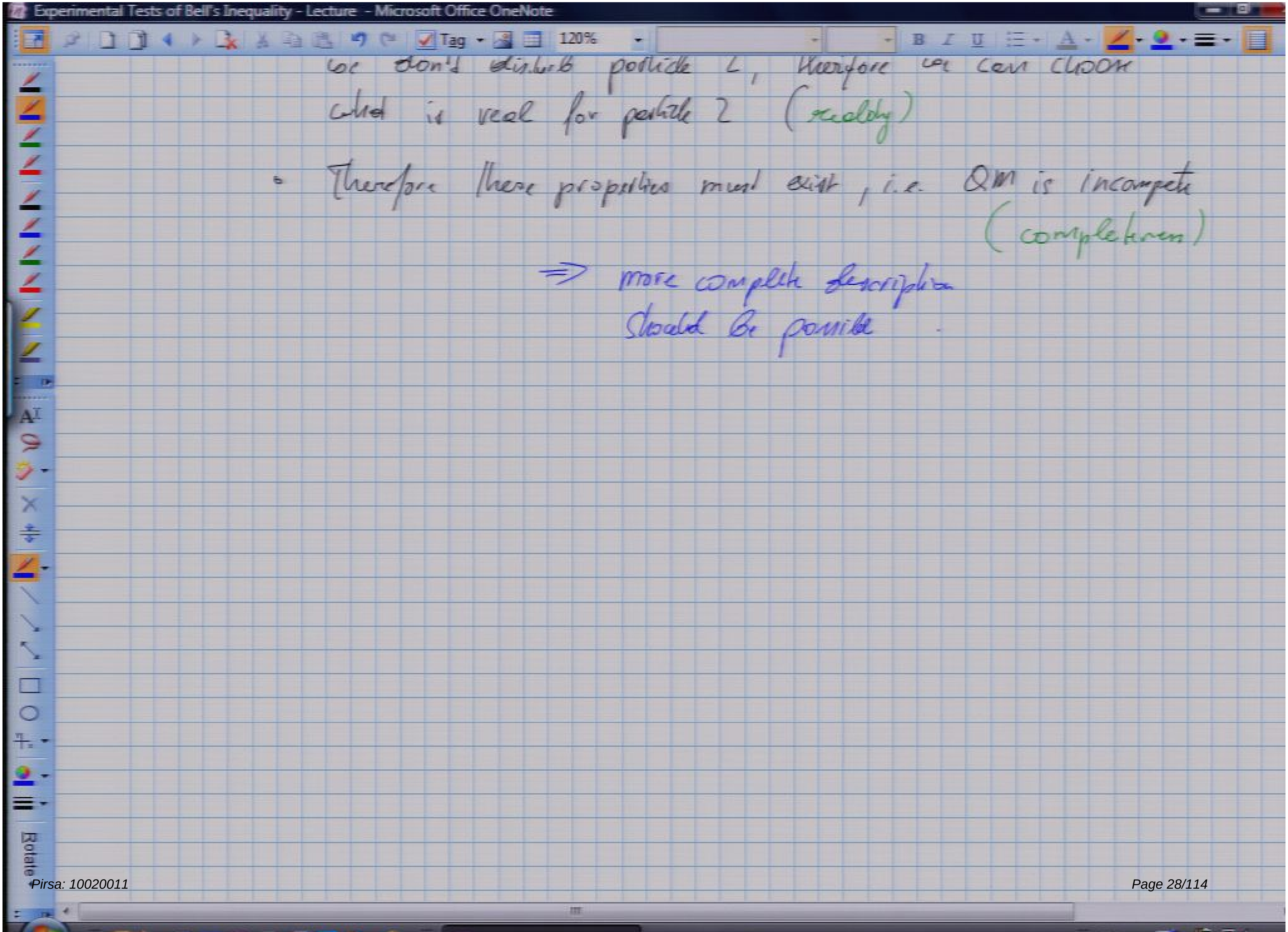
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_x\rangle_1|\downarrow_x\rangle_2 - |\downarrow_x\rangle_1|\uparrow_x\rangle_2\right]$$

EPR:

- By observing particle 1 we can choose to predict some property of particle 2 with certainty
- Because there can't be any spooky-at-a-distance (locality) we don't disturb particle 2, therefore we can choose what is real for particle 2 (reality)
- Therefore these properties must exist, i.e. QM is incomplete (completeness)

⇒ more complete

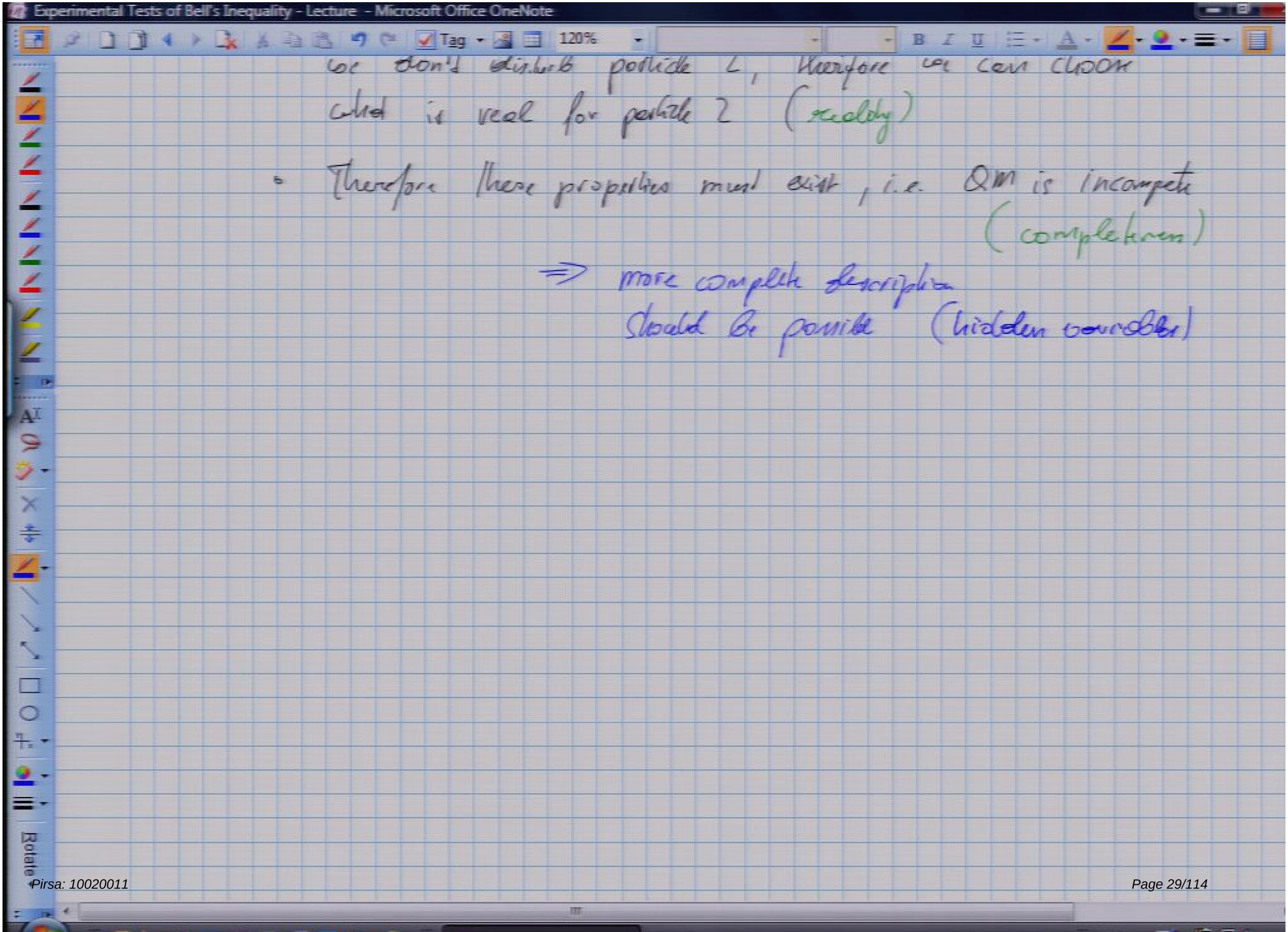

Experimental Tests of Bell's Inequality - Lecture - Microsoft Office OneNote
Tag
120%
Rotate



we don't disturb particle 1, therefore we can know what is real for particle 2 (reality)

- Therefore these properties must exist, i.e. QM is incomplete (completeness)

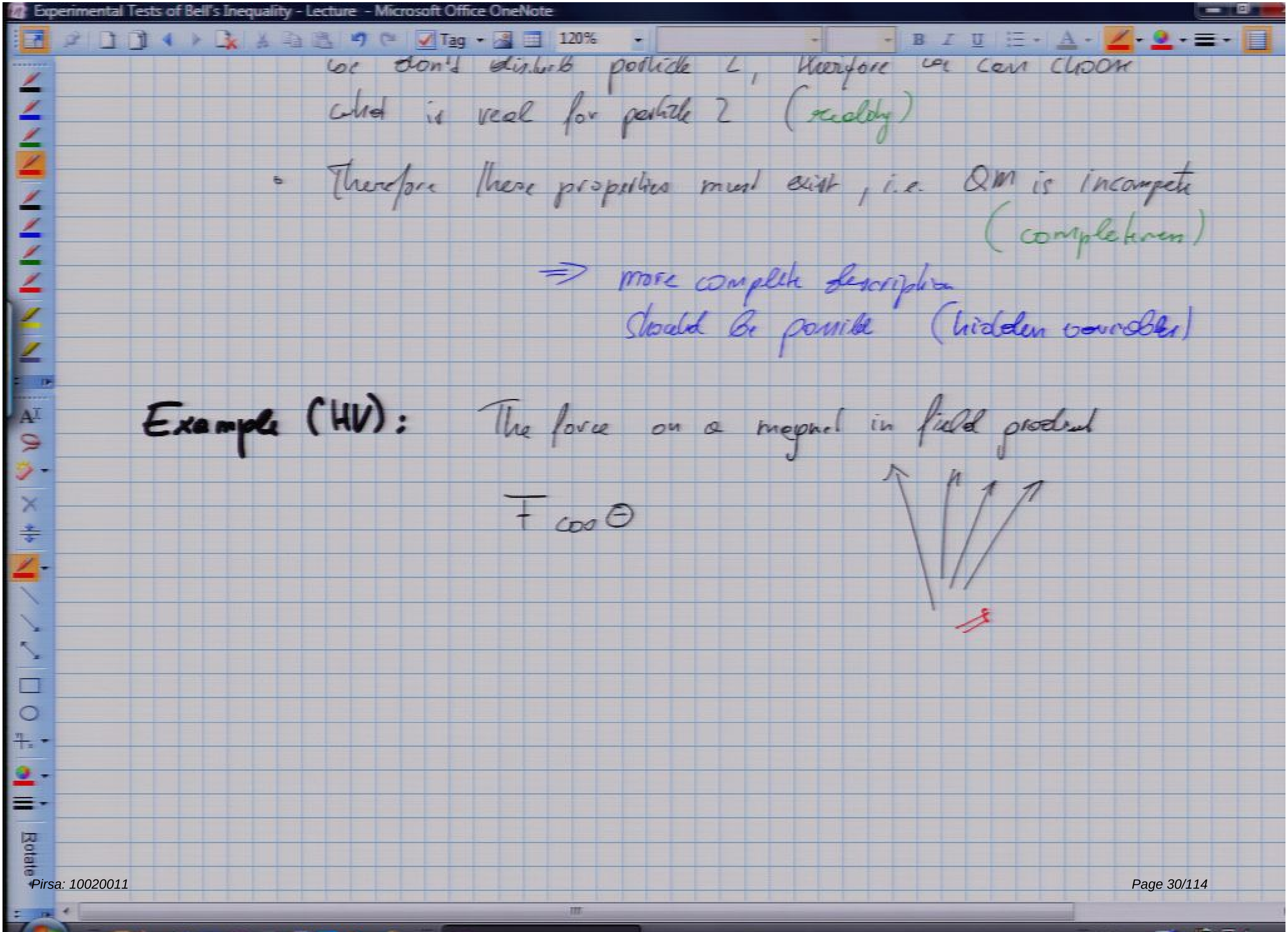
⇒ more complete description should be possible.



we don't disturb particle 1, therefore we can choose what is real for particle 2 (reality)

- Therefore these properties must exist, i.e. QM is incomplete (completeness)

⇒ more complete description should be possible (hidden variables)



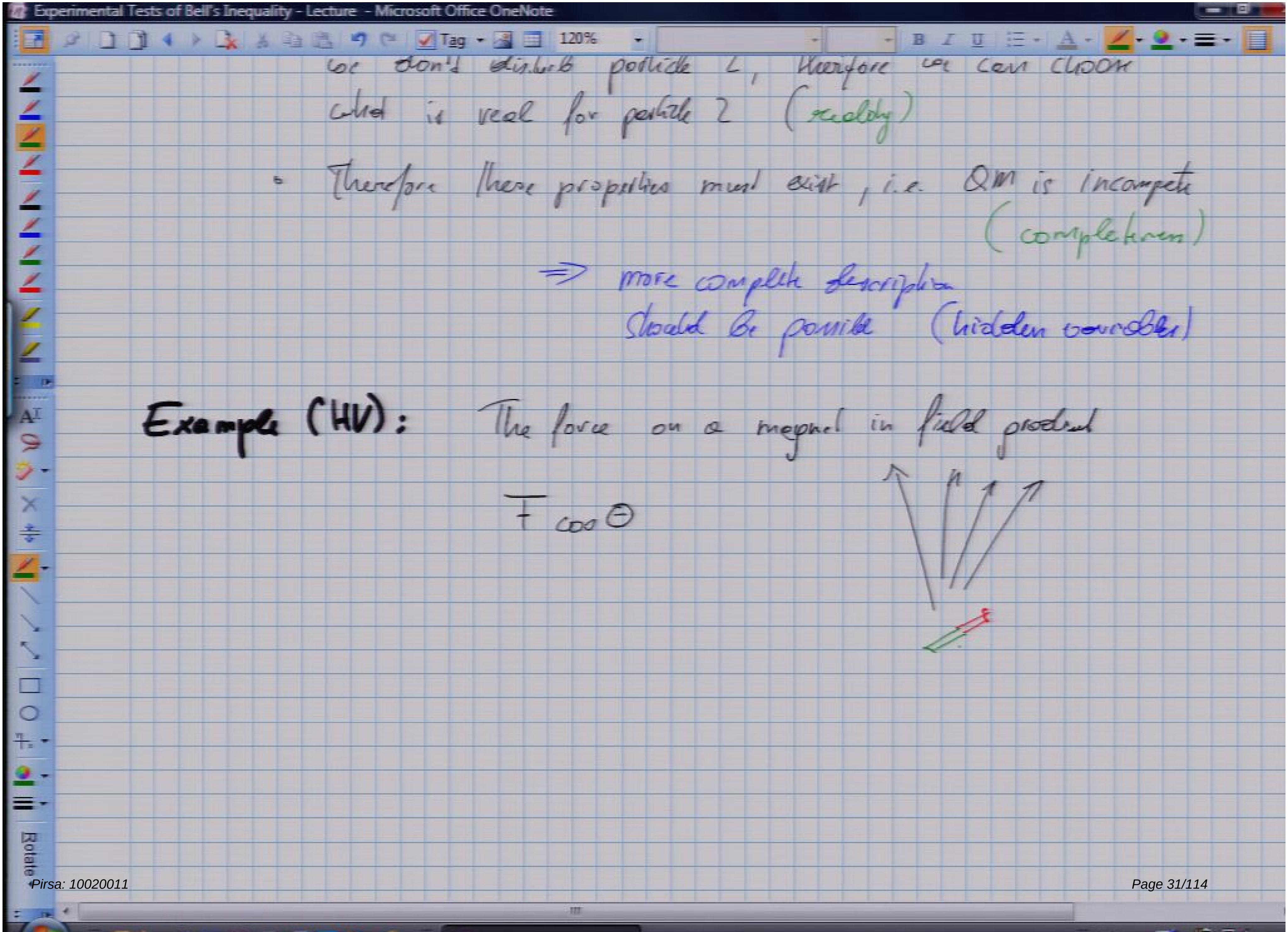
we don't disturb particle 2, therefore we can choose what is real for particle 2 (reality)

- Therefore these properties must exist, i.e. QM is incomplete (completeness)

⇒ more complete description should be possible (hidden variables)

Example (HV): The force on a magnet in field produced

$F \cos\Theta$



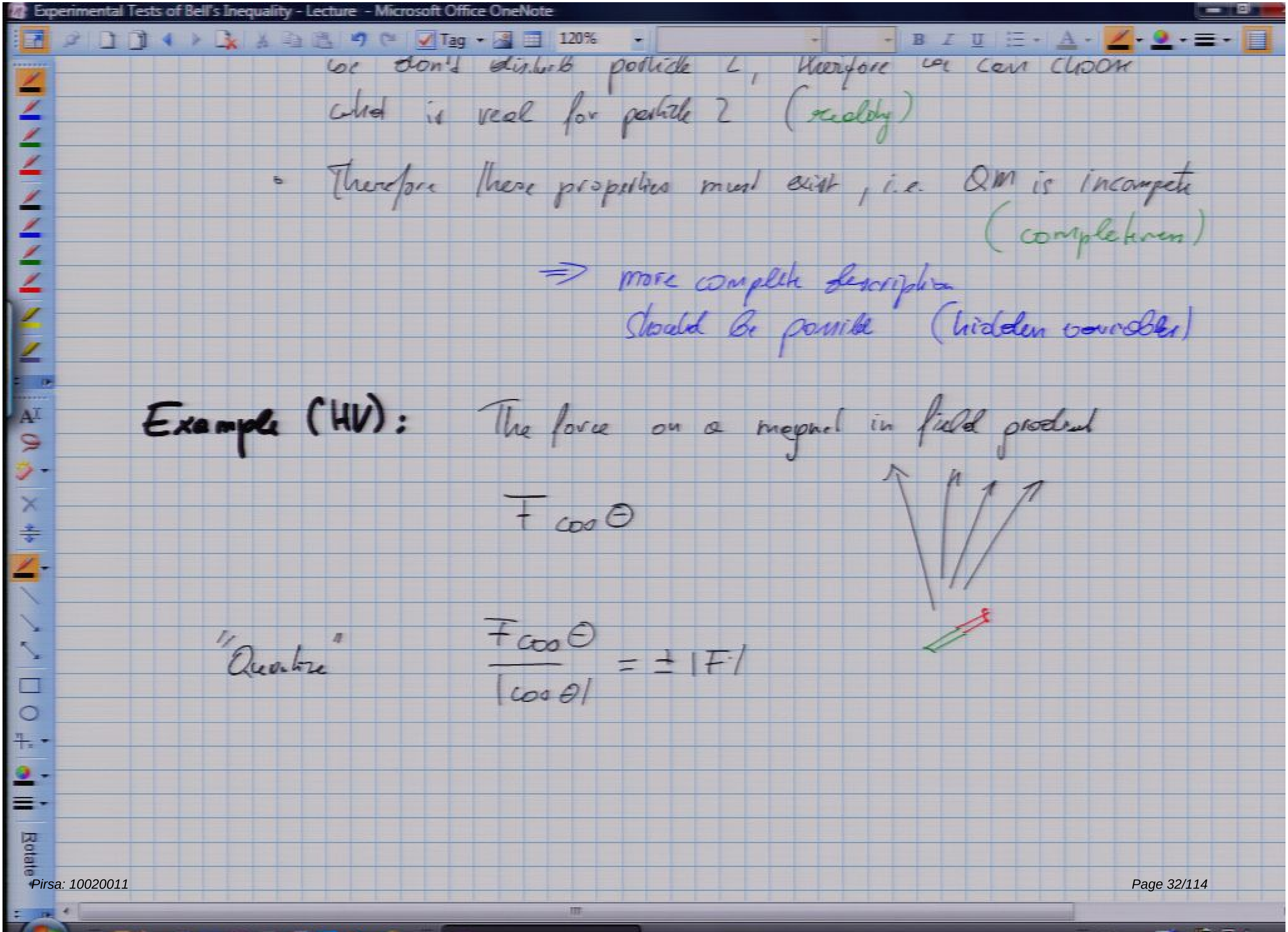
we don't disturb particle 1, therefore we can choose what is real for particle 2 (reality)

- Therefore these properties must exist, i.e. QM is incomplete (completeness)

$\Rightarrow$ more complete description should be possible (hidden variables)

Example (HV): The force on a magnet in field produced

$F \cos\Theta$



we don't disturb particle 2, therefore we can choose what is real for particle 2 (reality)

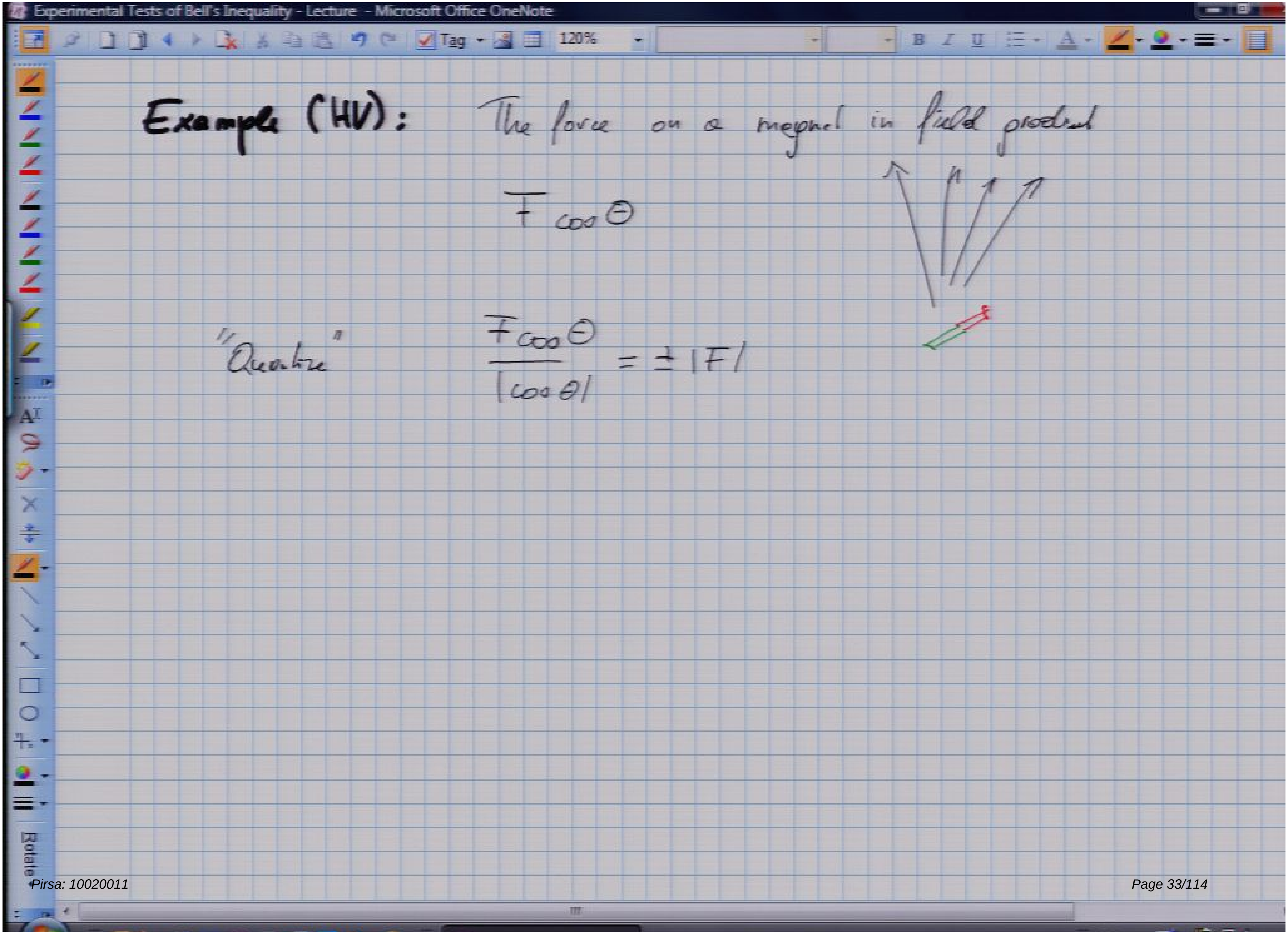
- Therefore these properties must exist, i.e. QM is incomplete (completeness)

$\Rightarrow$ more complete description should be possible (hidden variables)

Example (HV): The force on a magnet in field gradient

$$F \cos\Theta$$

"Quantize" $$\frac{F\cos\Theta}{|\cos\theta|} = \pm|F|$$

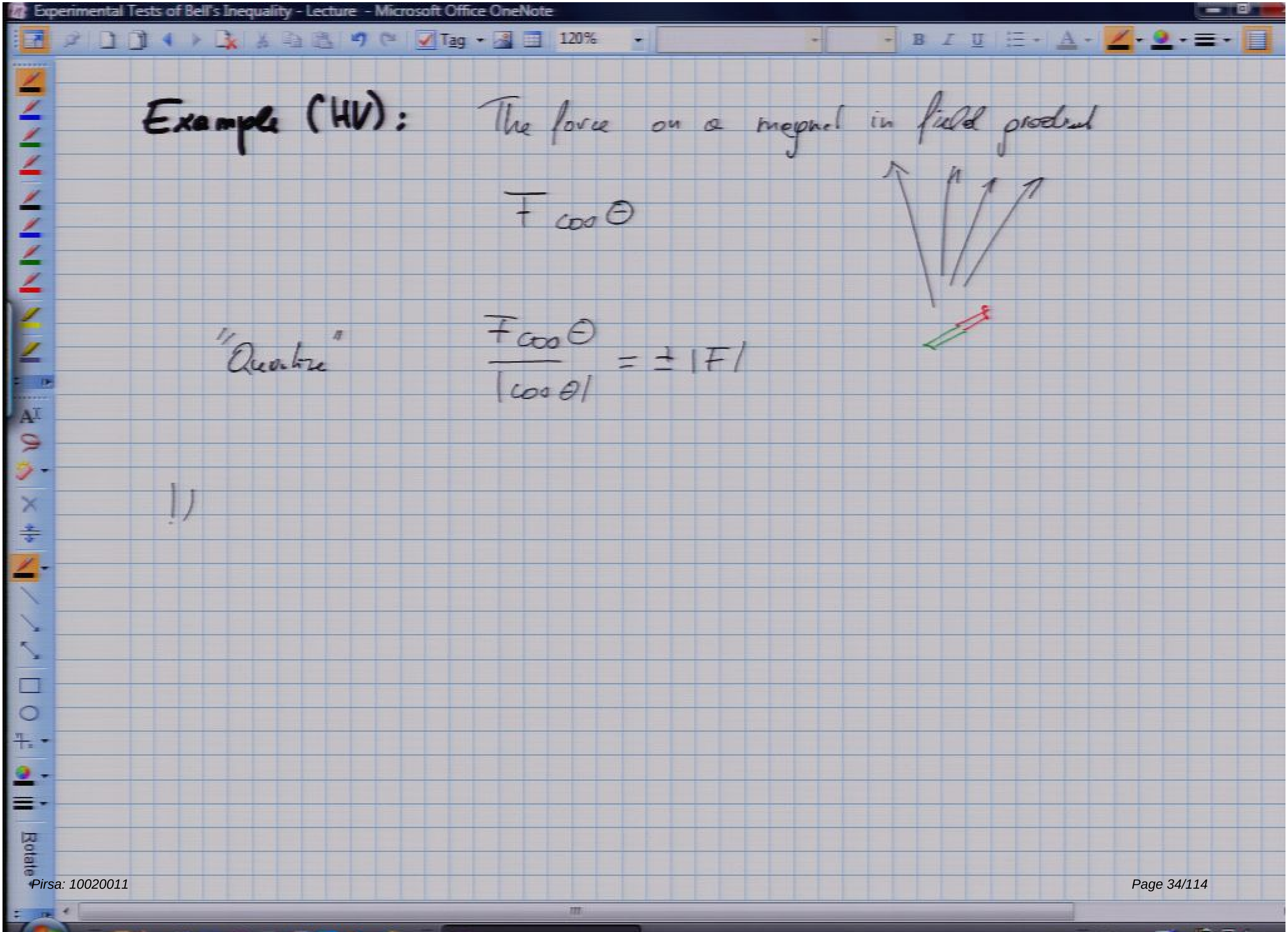


Example (HV): The force on a magnet in field gradient

$F \cos\Theta$

"Quantize"

$$\frac{F\cos\Theta}{|\cos\Theta|} = \pm|F|$$



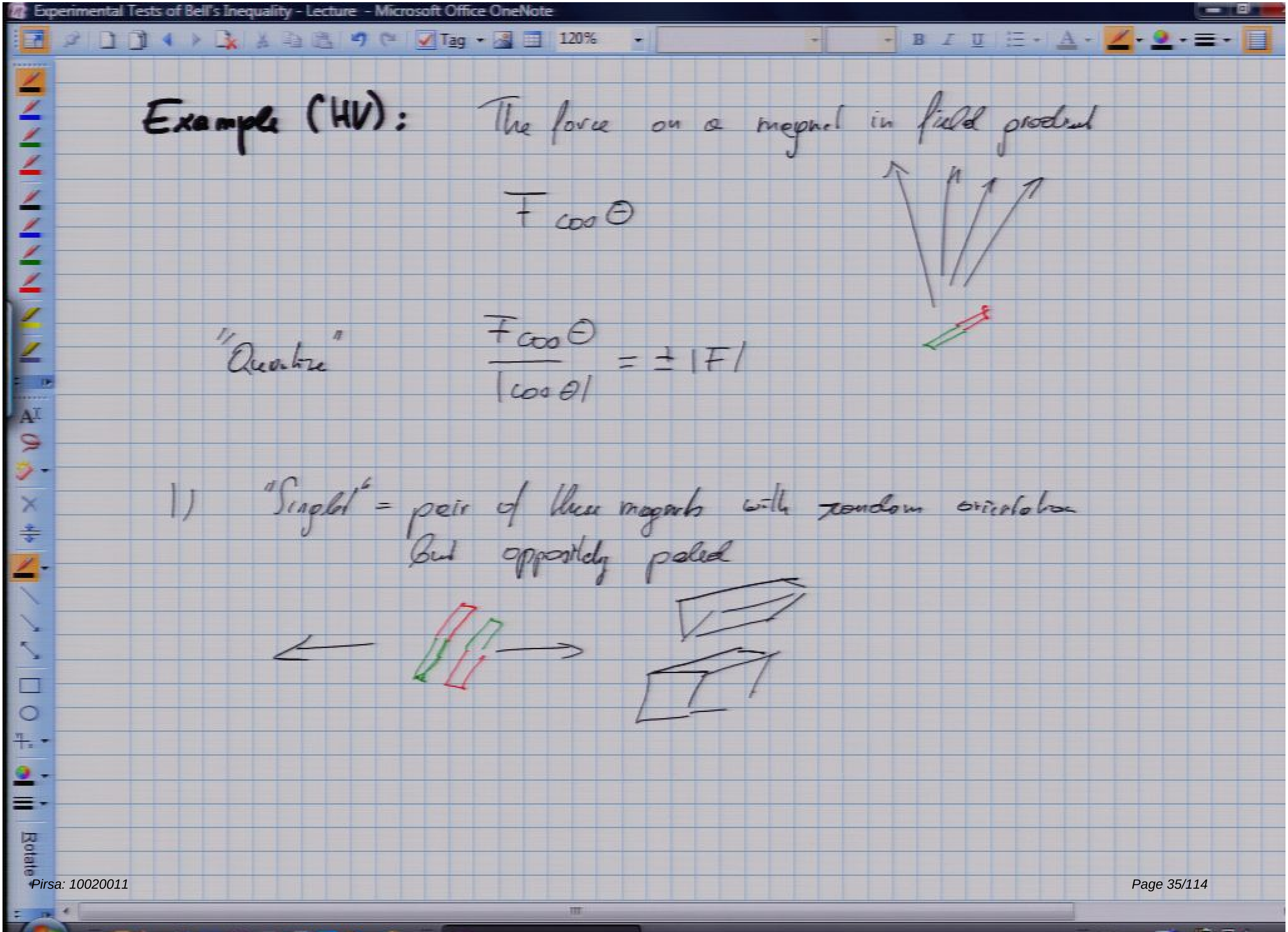
Example (HV): The force on a magnet in field gradient

$$F\cos\theta$$

"Quantize"

$$\frac{F\cos\theta}{|\cos\theta|} = \pm|F|$$

1)



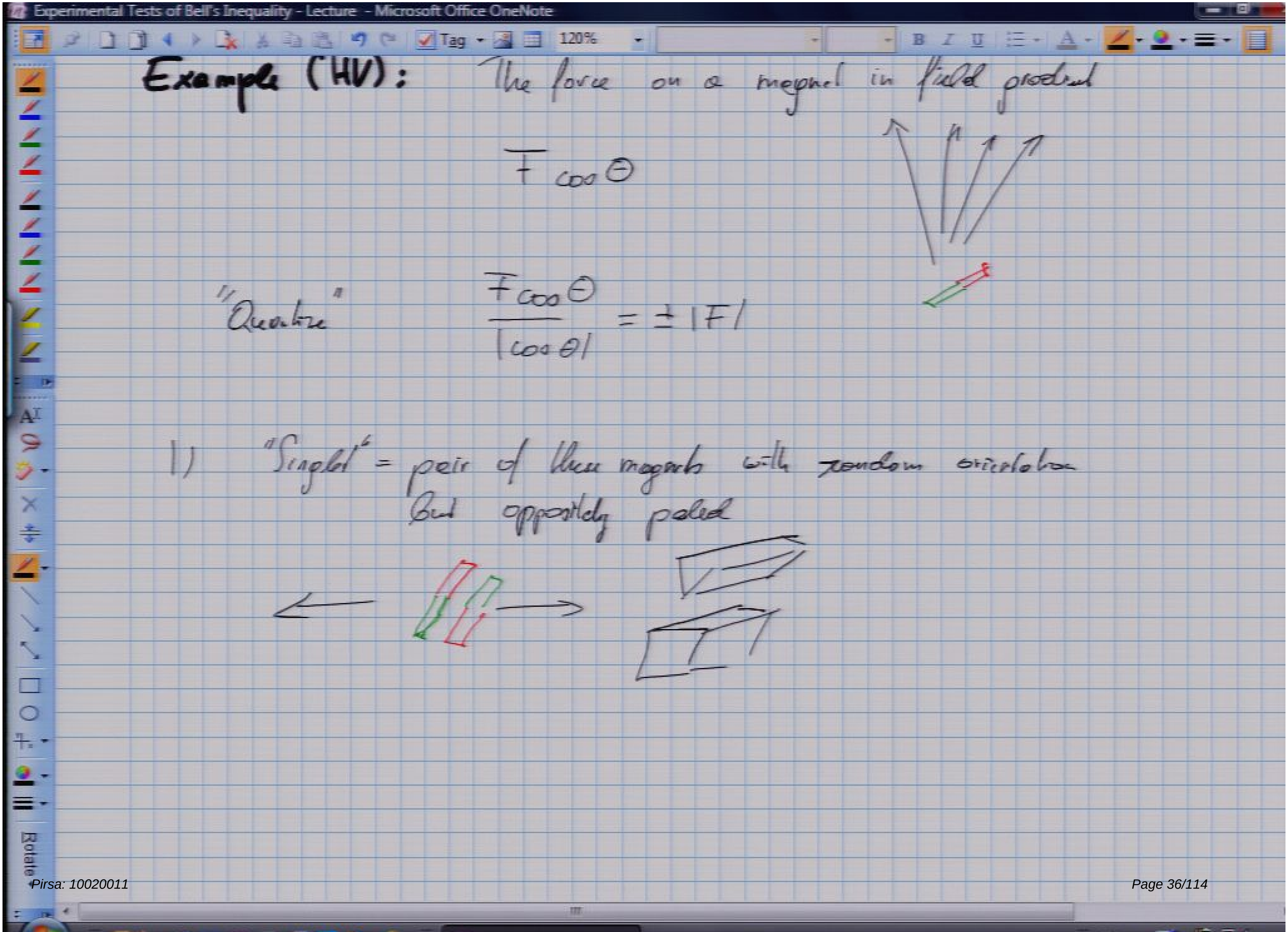
Example (HV): The force on a magnet in field gradient

$$F \cos\theta$$

"Quantize"

$$\frac{F\cos\theta}{|\cos\theta|} = \pm |F|$$

1) "Singlet" = pair of these magnets with random orientation but oppositely poled

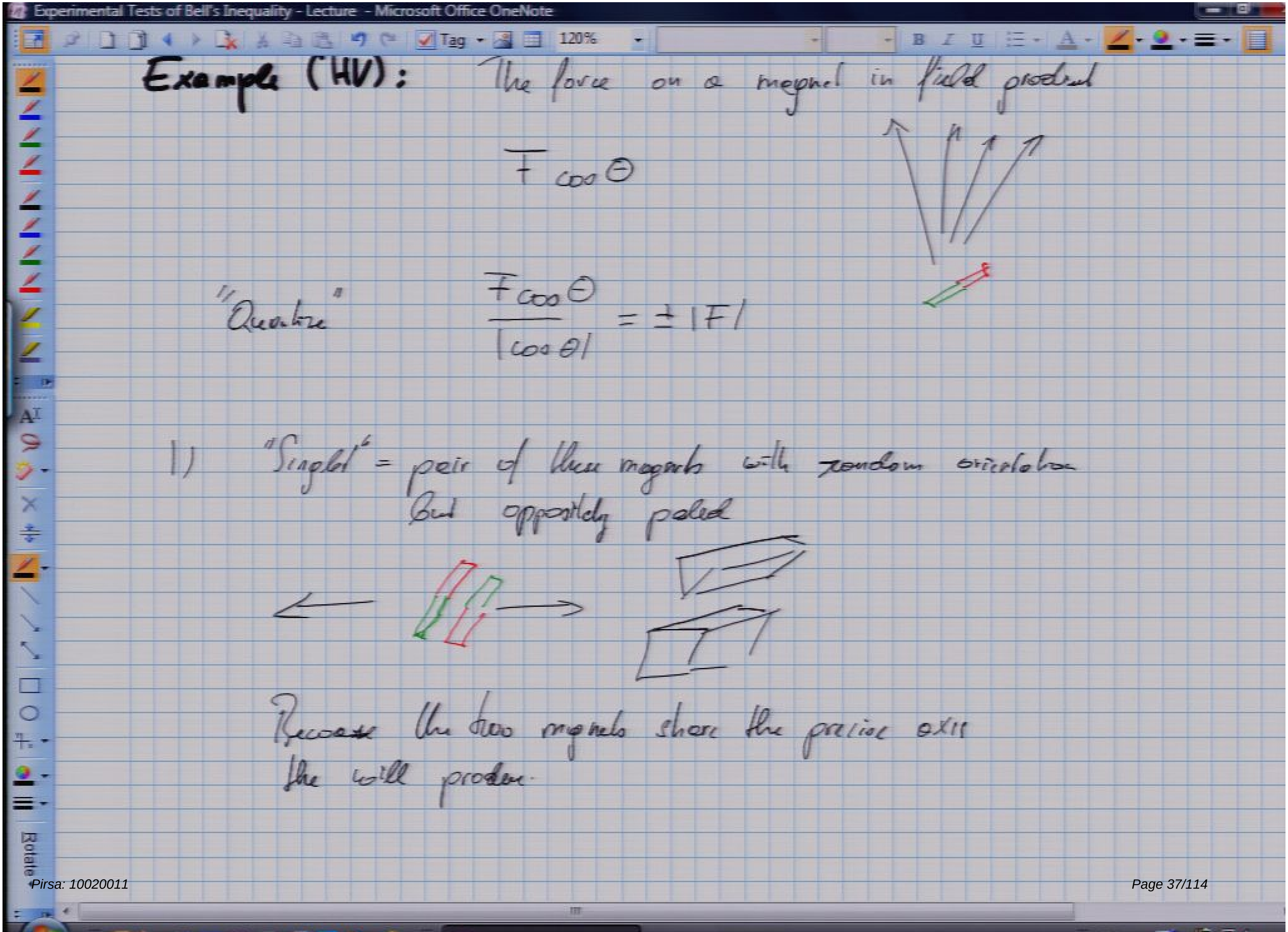


Example (HV): The force on a magnet in field gradient

$$F\cos\Theta$$

"Quantize" $$\frac{F\cos\Theta}{|\cos\theta|} = \pm|F|$$

1) "Singlet" = pair of these magnets with random orientation but oppositely poled



Example (HV): The force on a magnet in field produced

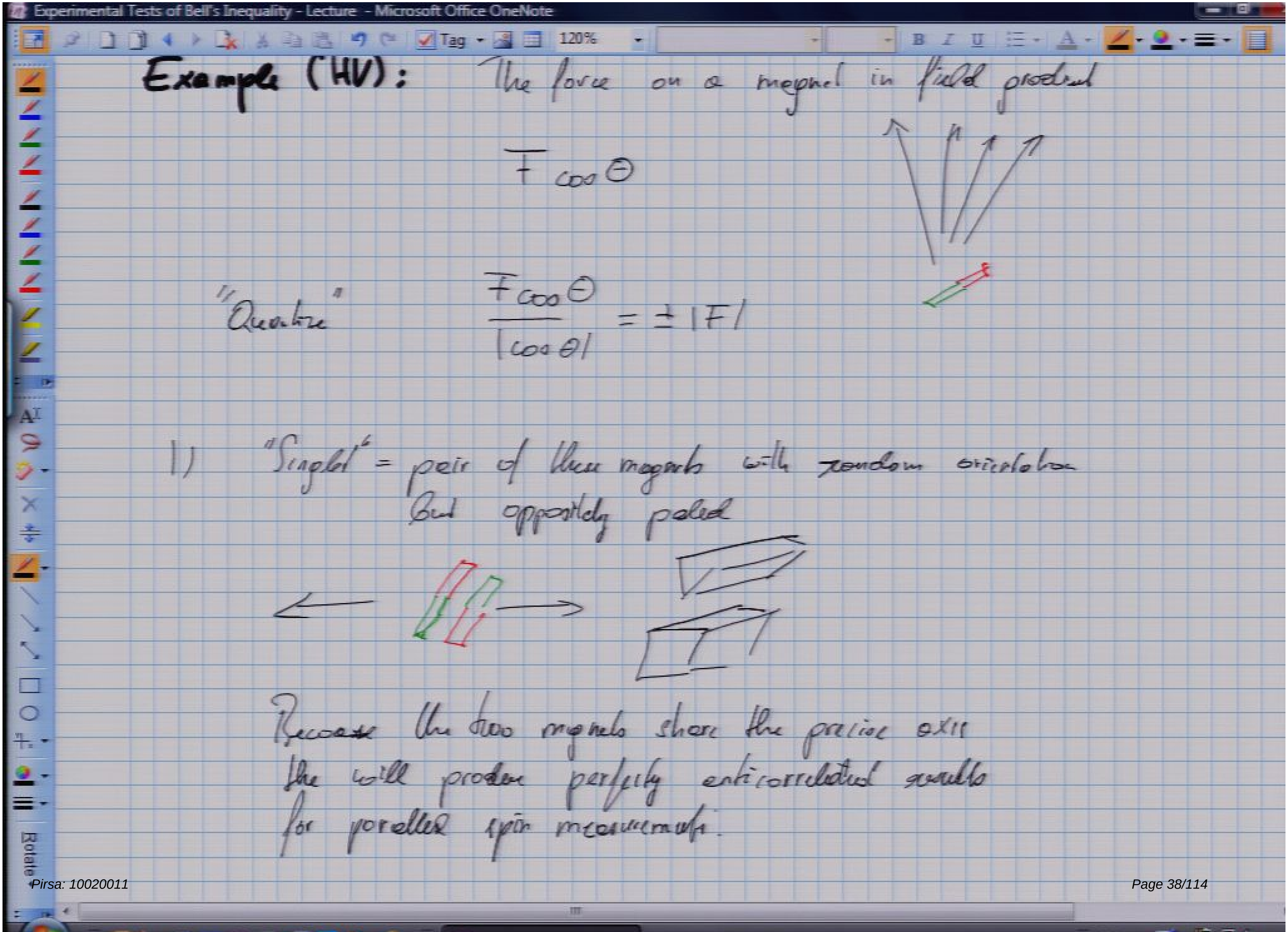
$$F \cos\Theta$$

"Quantize"

$$\frac{F\cos\Theta}{|\cos\theta|} = \pm|F|$$

1) "Singlet" = pair of these magnets with random orientation
but oppositely poled

Because the two magnets share the precise axis
they will produce



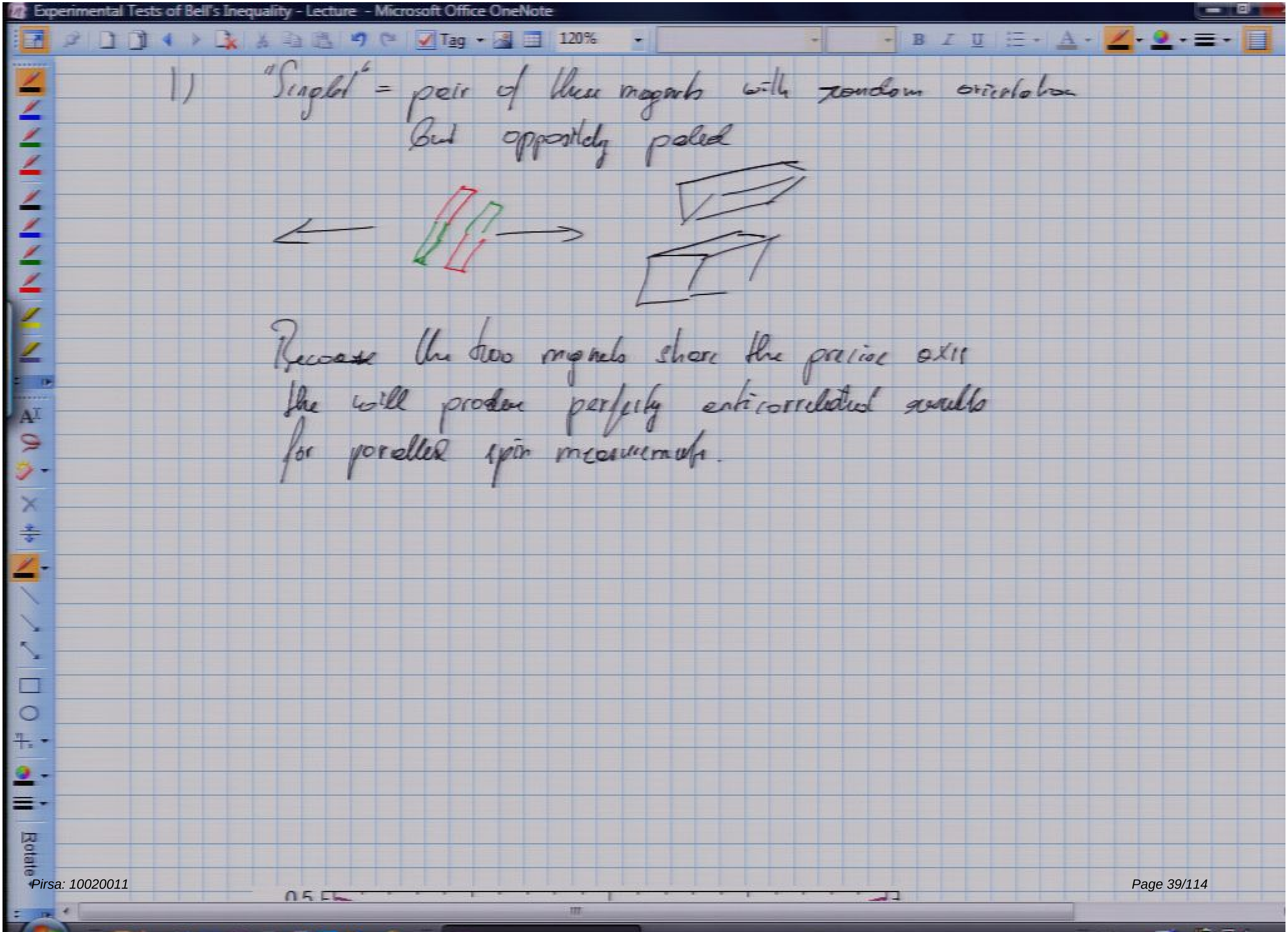
Example (HV): The force on a magnet in field gradient

$F \cos\Theta$

"Quantize" $\frac{F\cos\Theta}{|\cos\theta|} = \pm|F|$

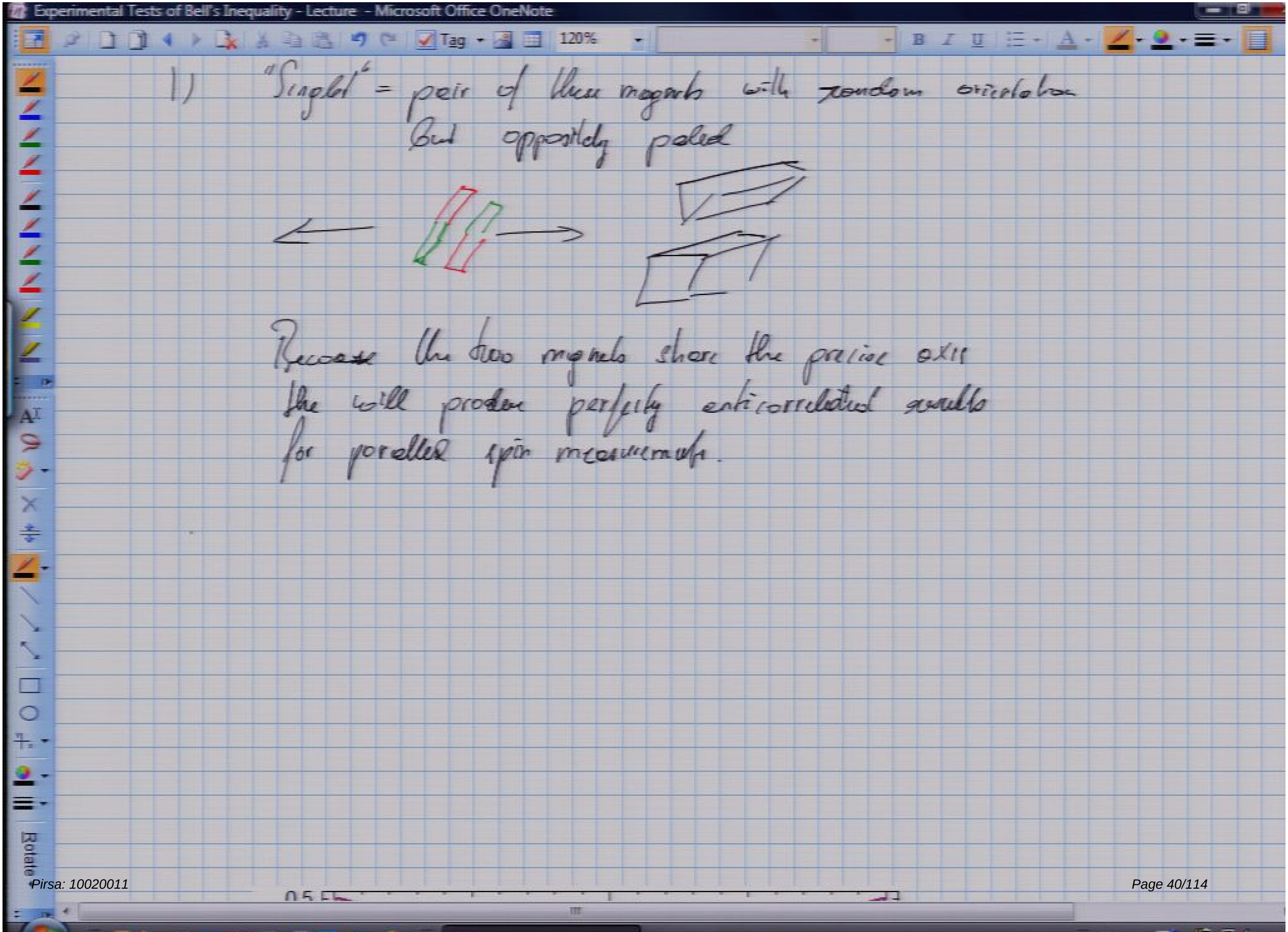
1) "Singlet" = pair of these magnets with random orientation but oppositely poled

Because the two magnets share the precise axis they will produce perfectly anticorrelated results for parallel spin measurements.



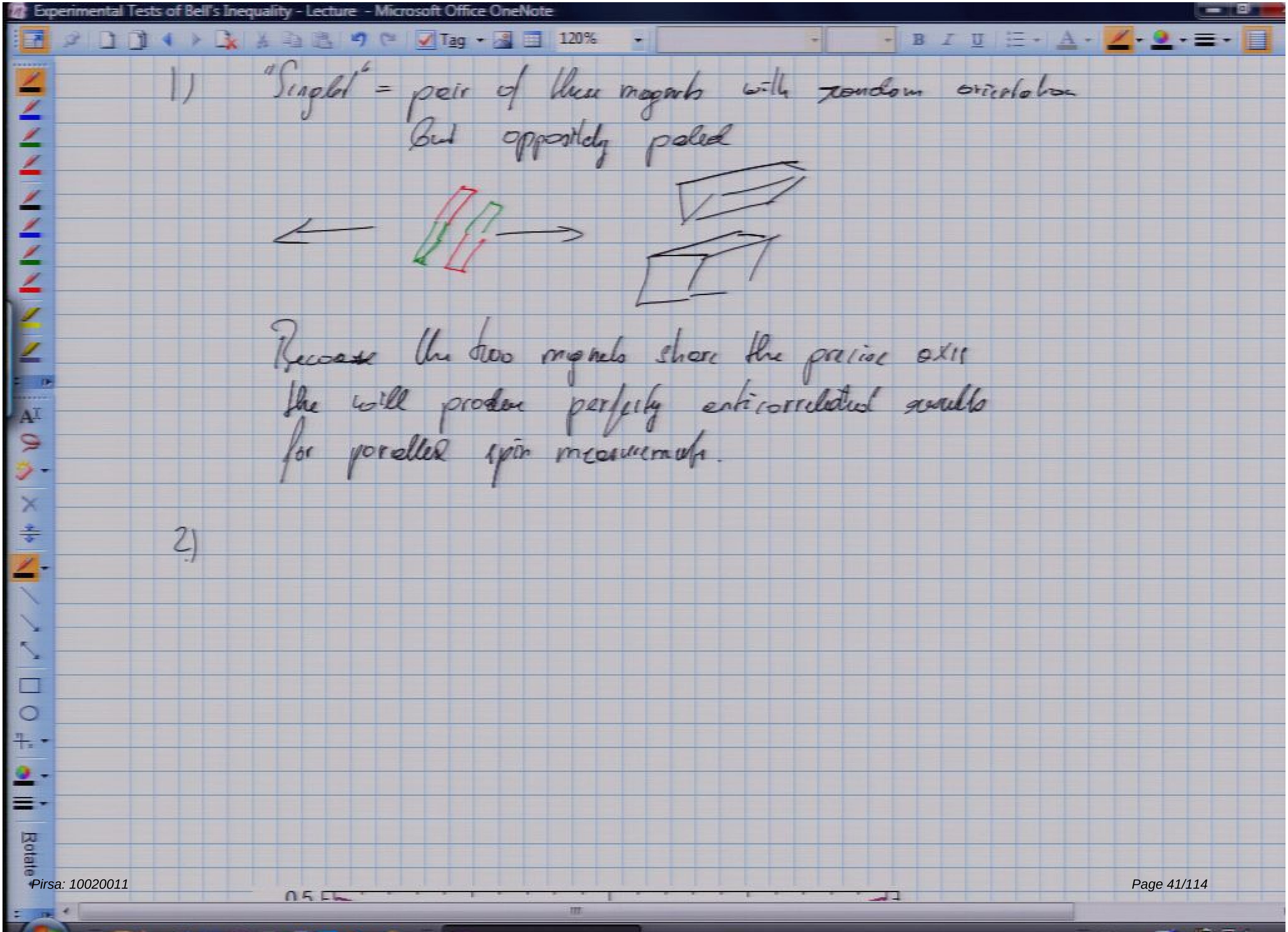
1) "Singlet" = pair of these magnets with random orientation
But oppositely poled

Because the two magnets share the precise axis
the will produce perfectly anticorrelated results
for parallel spin measurements.



1) "Singlet" = pair of these magnets with random orientation
But oppositely poled

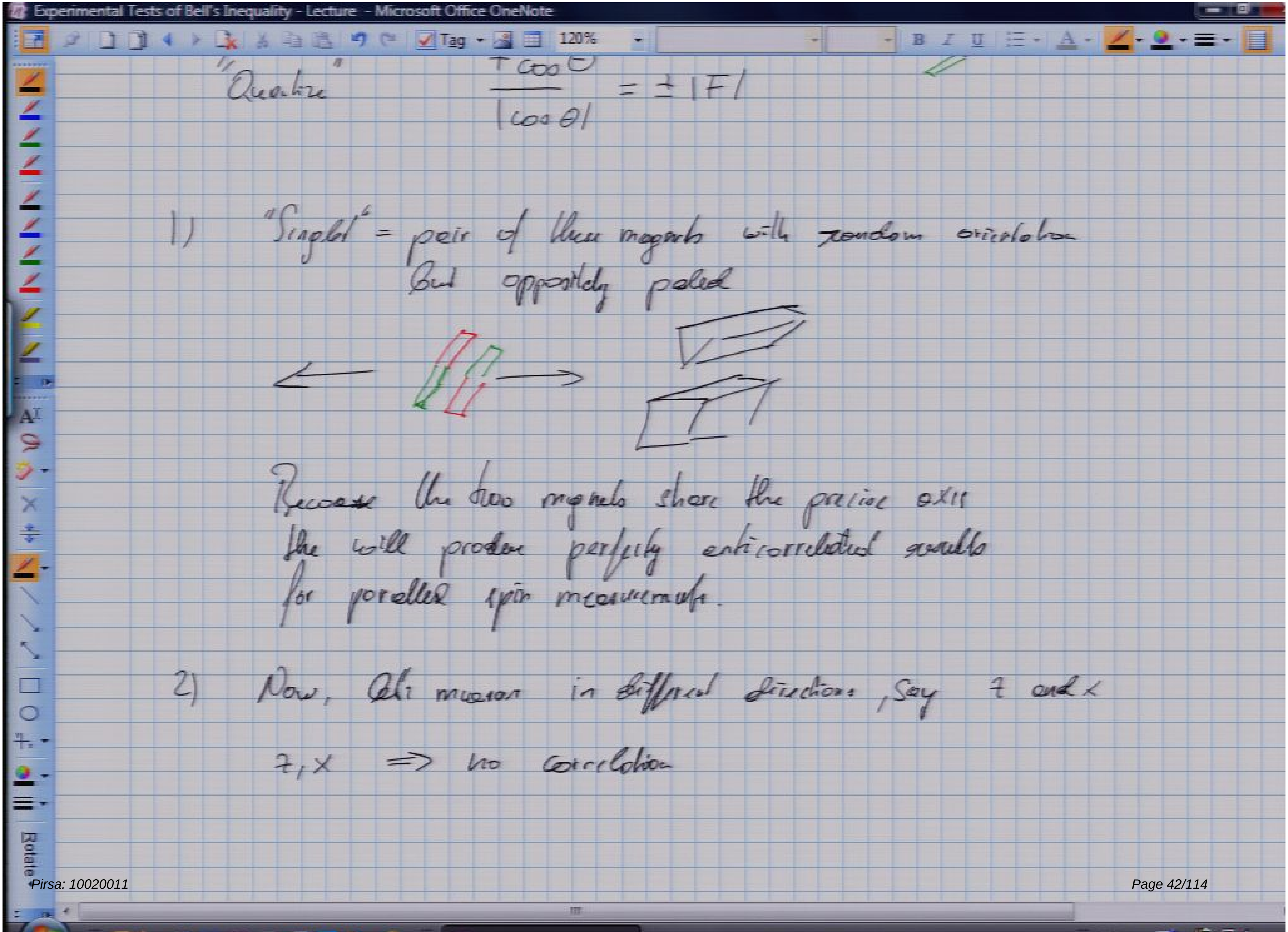
Because the two magnets share the precise axis
they will produce perfectly anticorrelated results
for parallel spin measurements.



1) "Singlet" = pair of these magnets with random orientation
But oppositely poled

Because the two magnets share the precise axis
they will produce perfectly anticorrelated results
for parallel spin measurements.

2)



"Quantize"

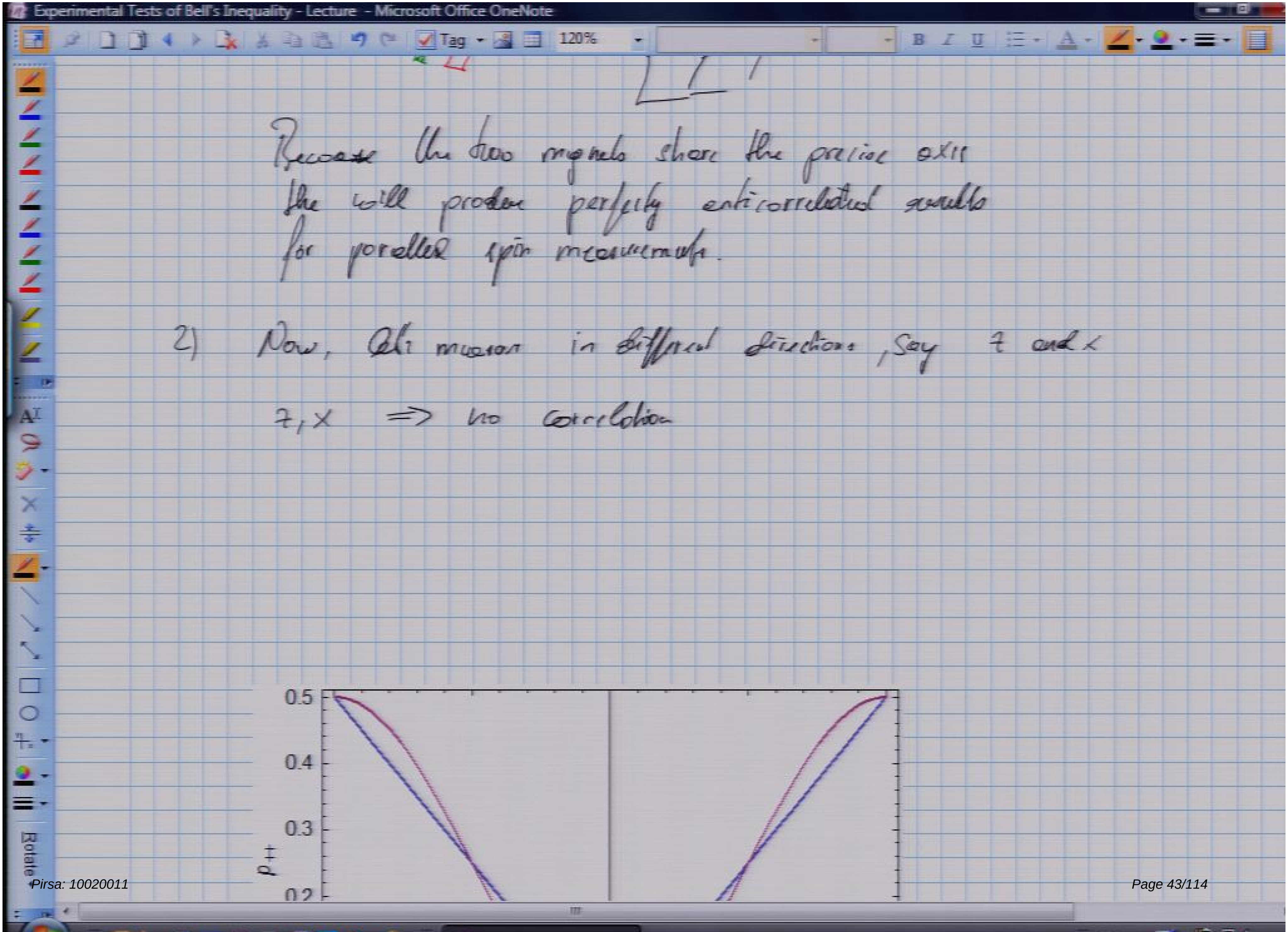
$$\frac{+\cos\theta}{|\cos\theta|} = \pm|F|$$

1) "Singlet" = pair of these magnets with random orientation but oppositely poled

Because the two magnets share the precise axis they will produce perfectly anticorrelated results for parallel spin measurements.

2) Now, Ali measure in different directions, say z and x

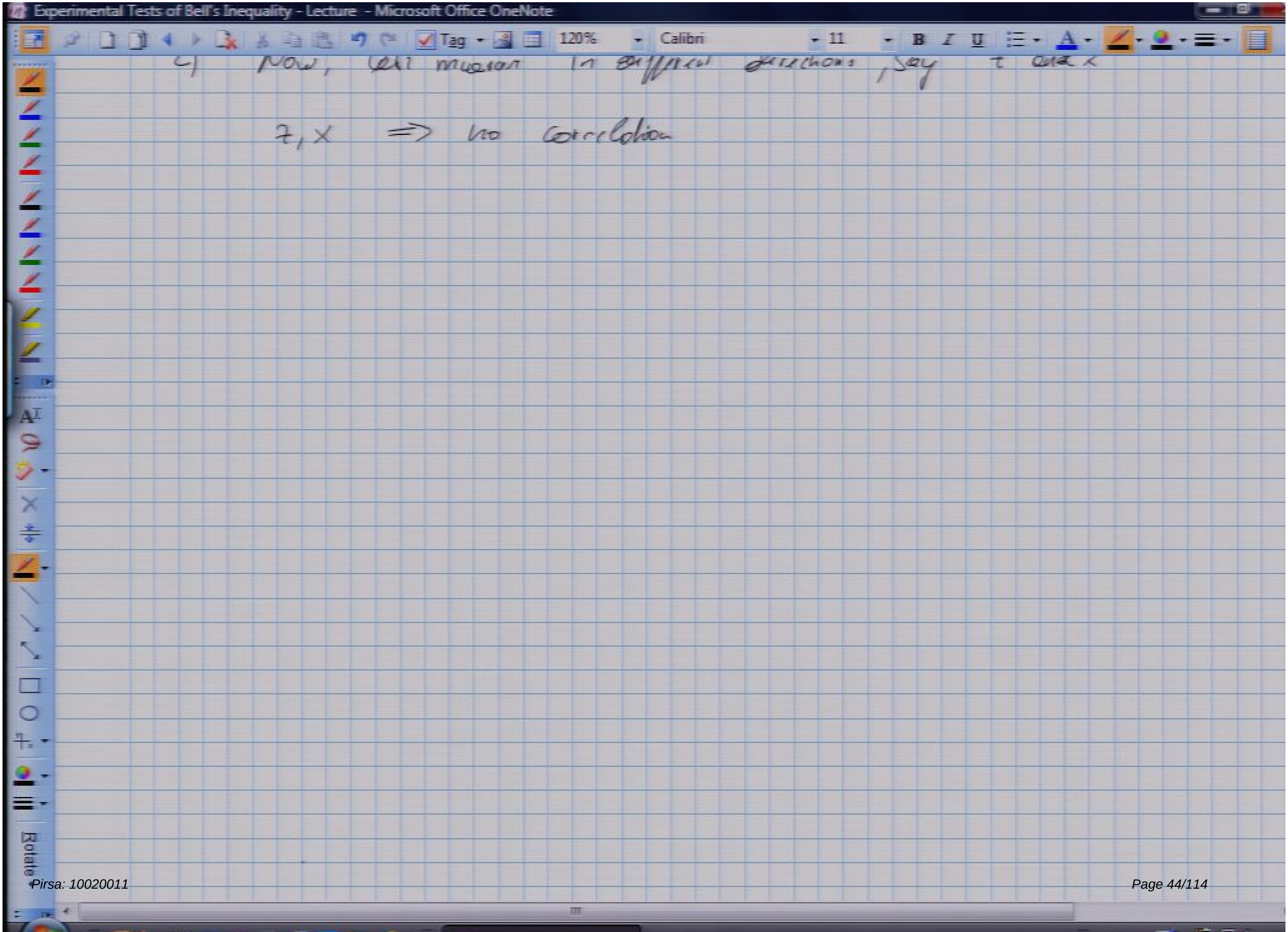
z, x ⇒ no correlation



Because the two magnets share the precise axis they will produce perfectly anticorrelated results for parallel spin measurements.

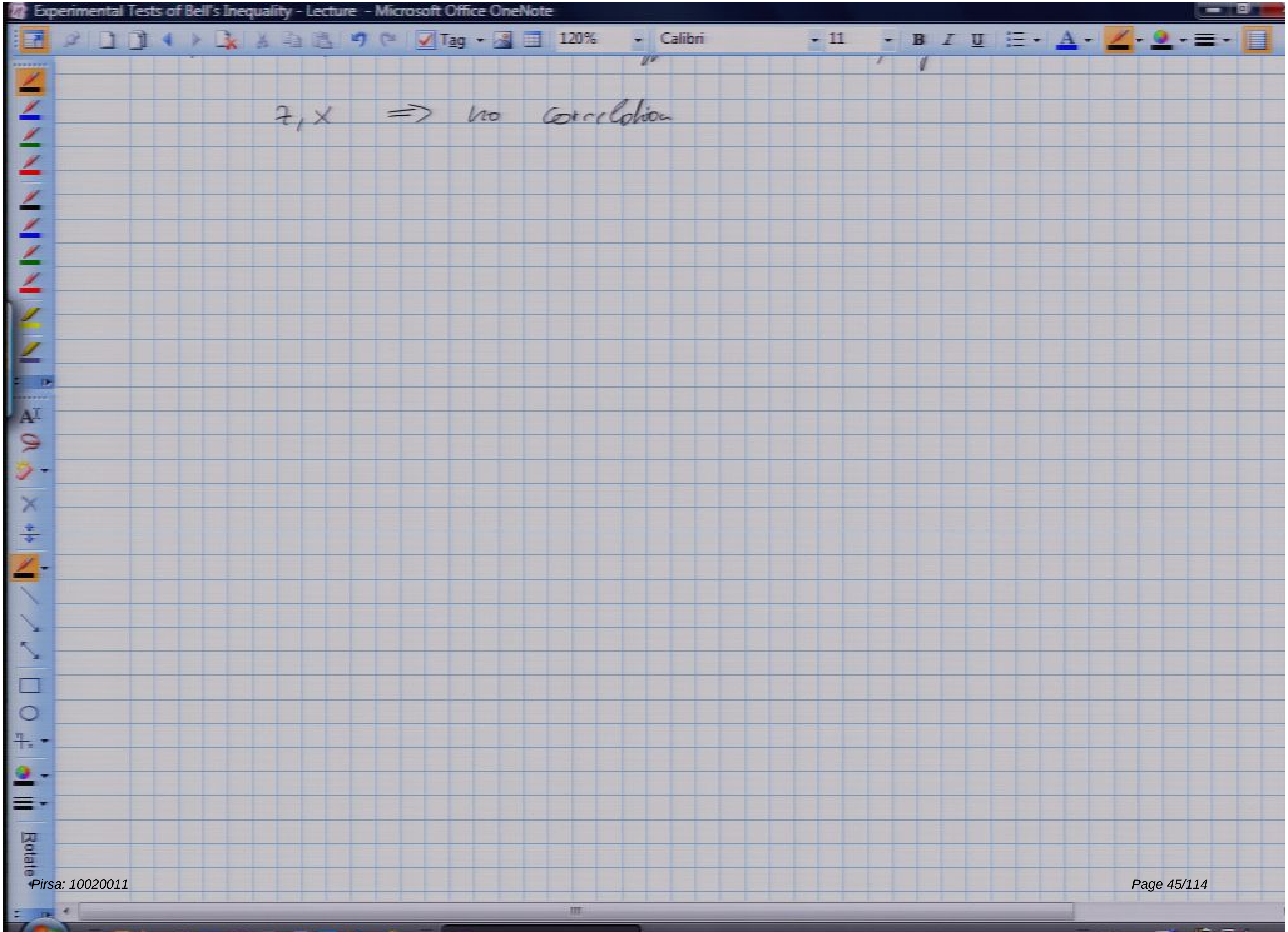
2) Now, Ali measures in different directions, say z and x

z, x ⇒ no correlation

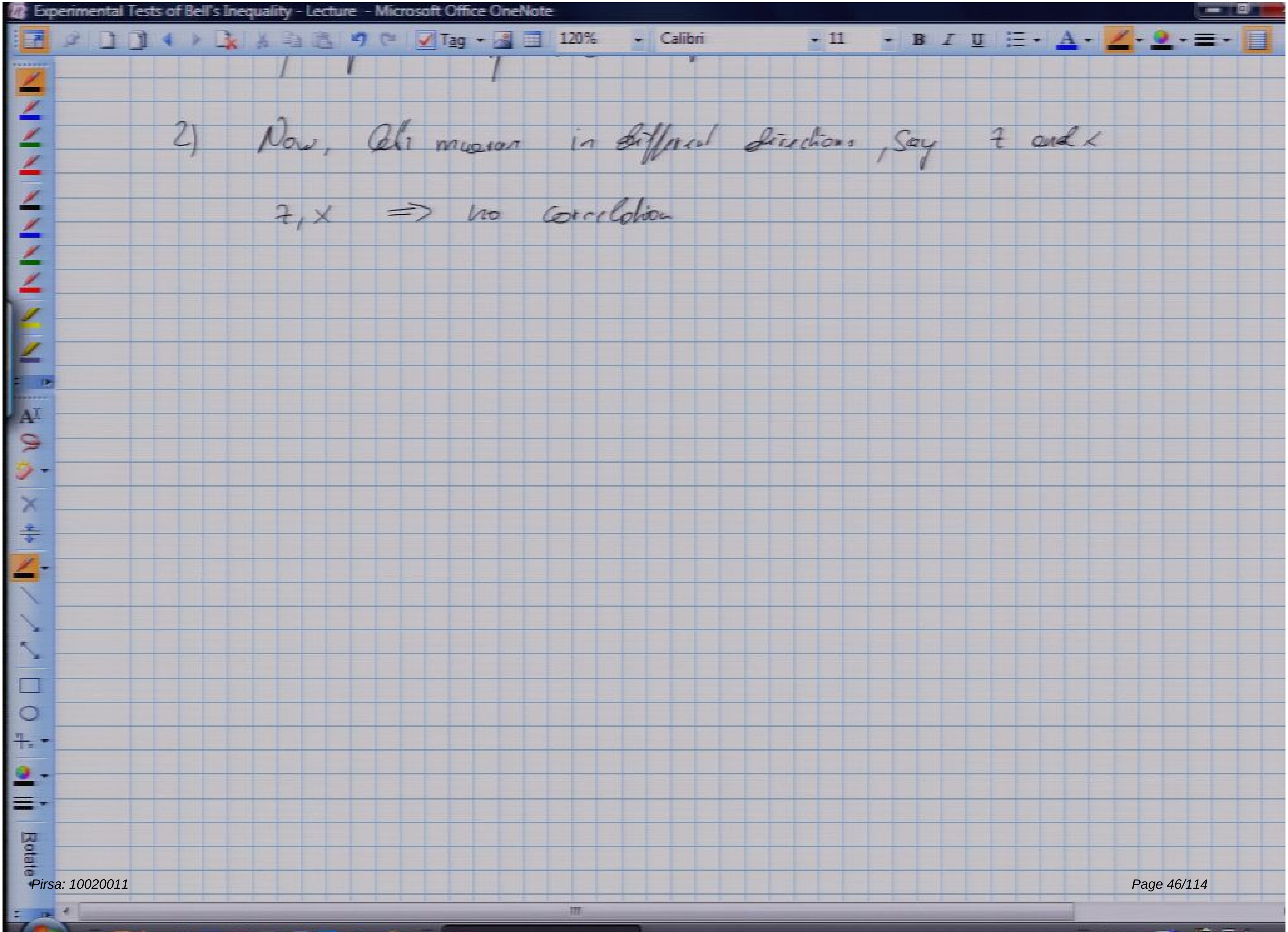


c) Now, cell measure in different directions, say z and x

$z, x \Rightarrow$ no correlation

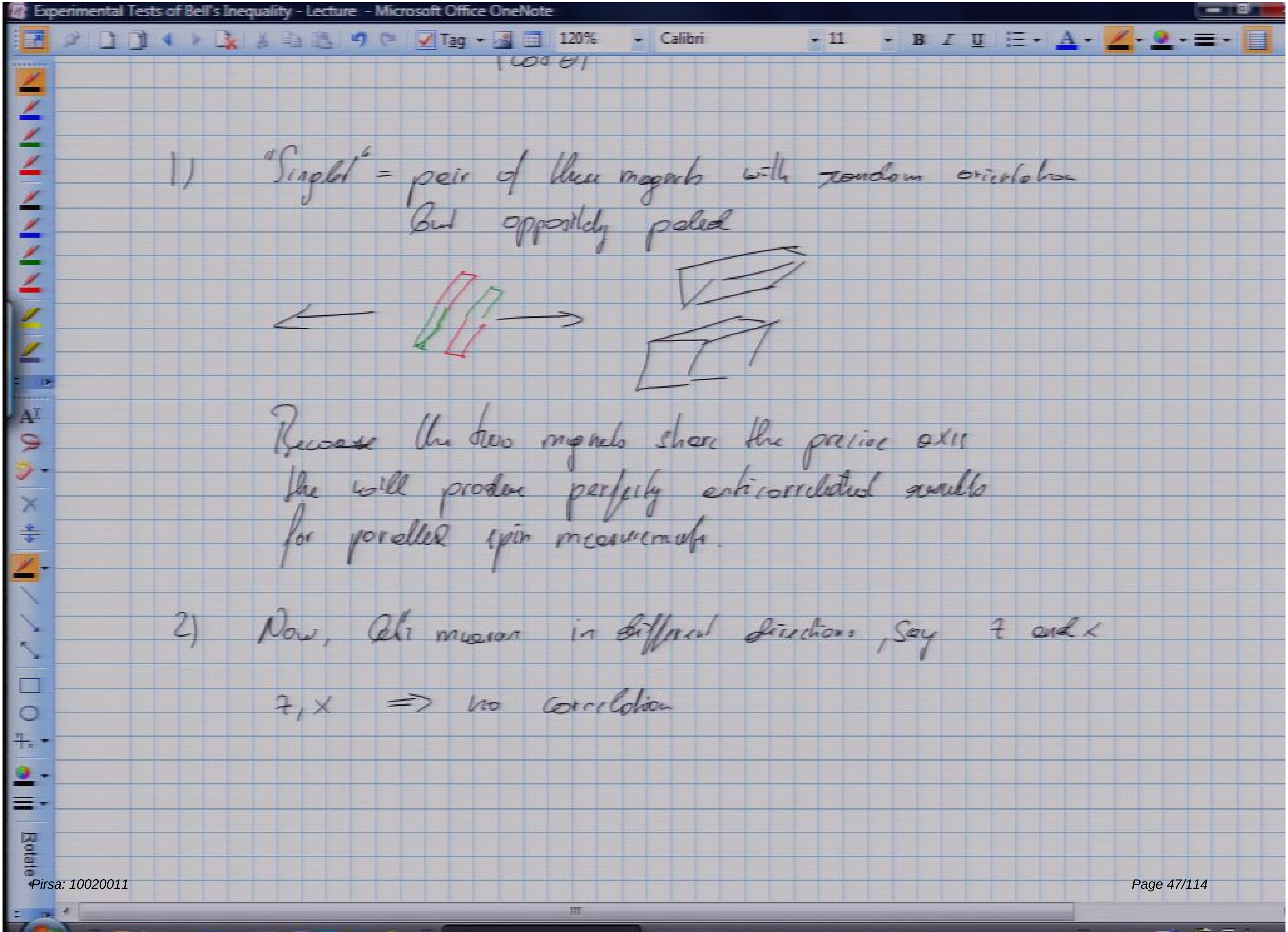


$z, x \Rightarrow$ no correlation



2) Now, Alice measures in different directions, say z and x

$z, x \Rightarrow$ no correlation



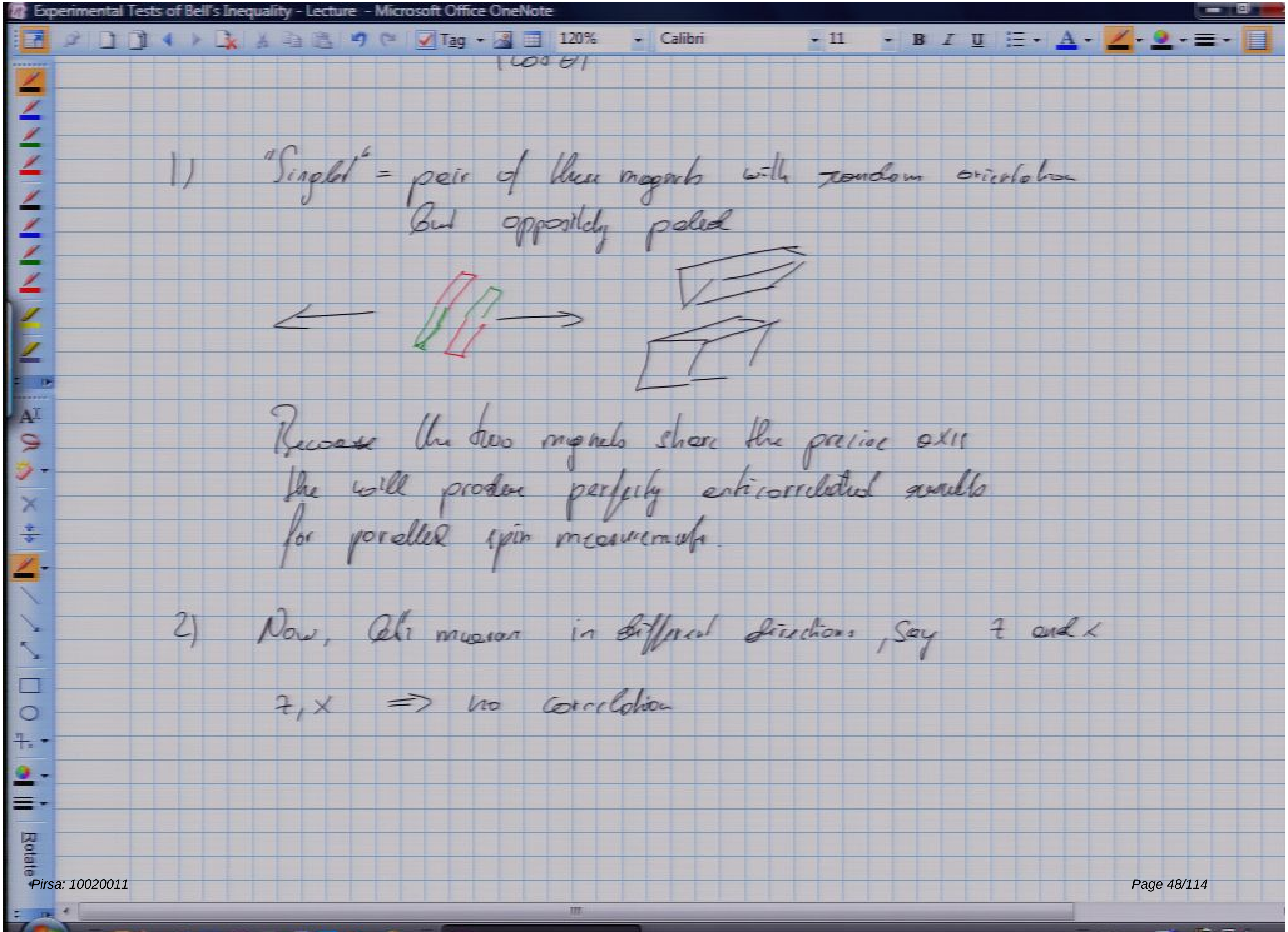
$|\cos\theta|$

1) "Singlet" = pair of these magnets with random orientation
But oppositely poled

Because the two magnets share the precise axis
they will produce perfectly anticorrelated results
for parallel spin measurements.

2) Now, Ali measure in different directions, say z and x

z, x ⇒ no correlation



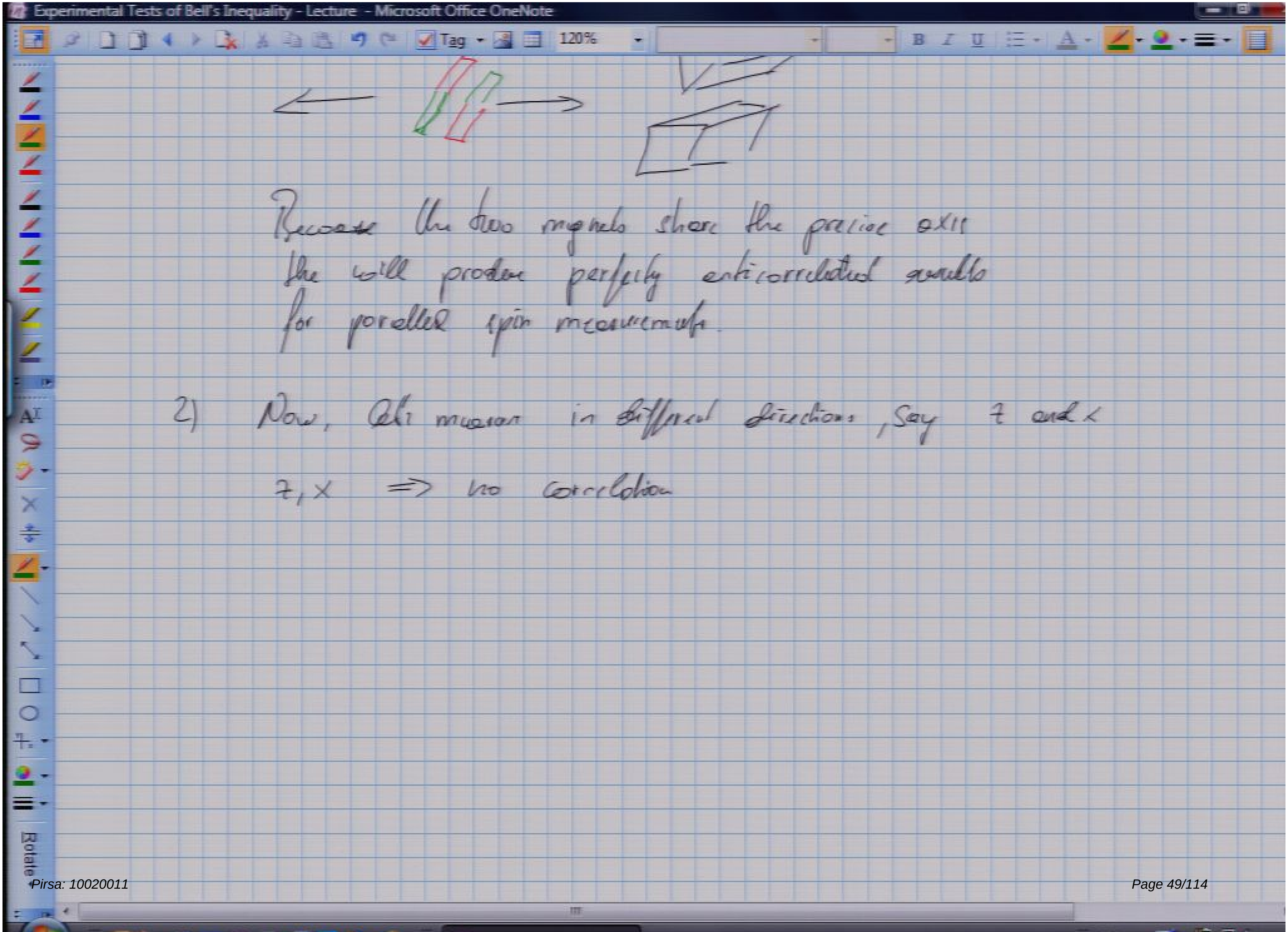
$|\cos\theta|$

1) "Singlet" = pair of these magnets with random orientation
But oppositely poled

Because the two magnets share the precise axis
they will produce perfectly anticorrelated results
for parallel spin measurements.

2) Now, Alice measures in different directions, say z and x

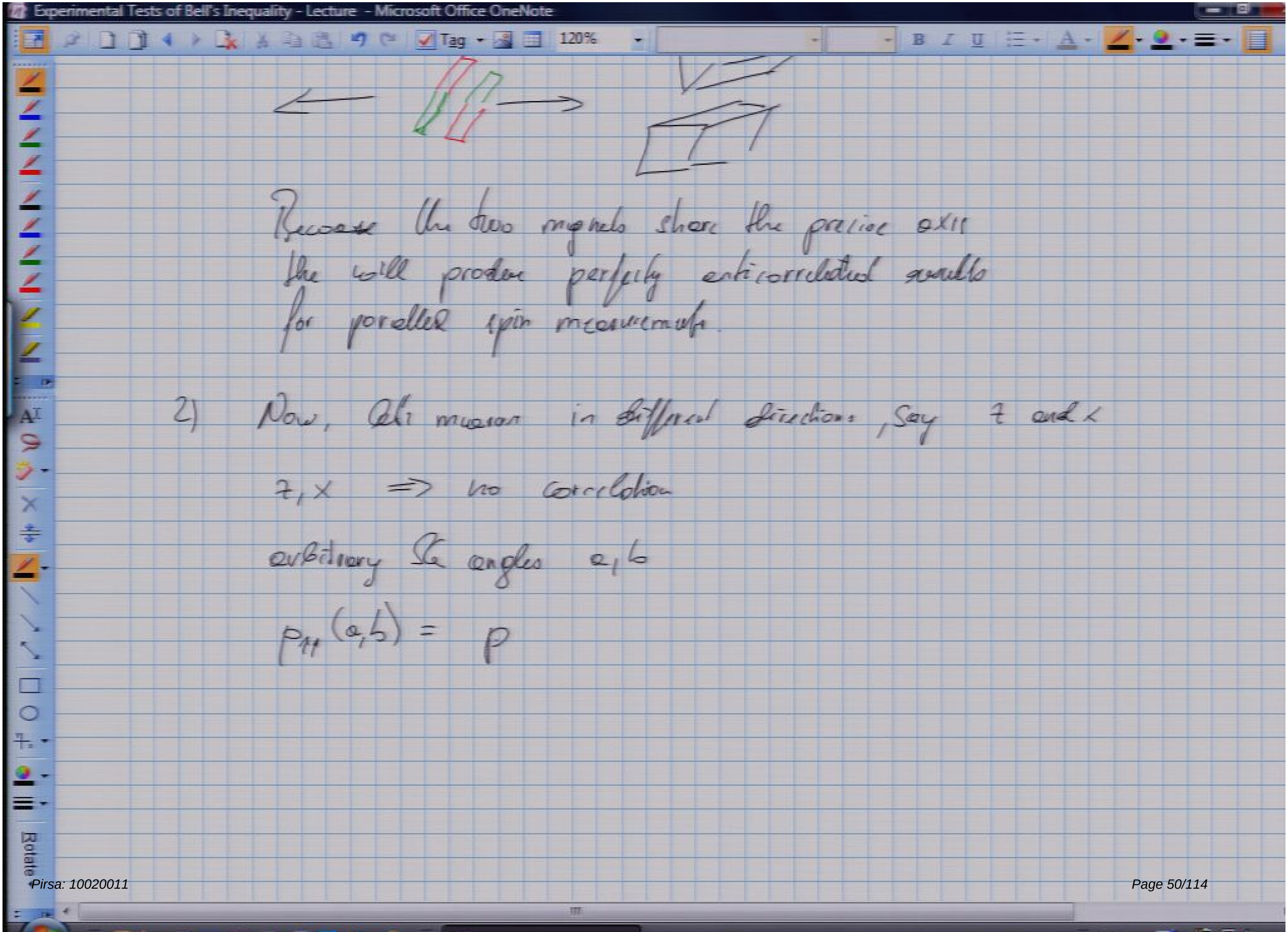
z, x ⇒ no correlation



Because the two magnets share the precise axis they will produce perfectly anticorrelated results for parallel spin measurements.

2) Now, Alice measure in different directions, say z and x

z, x ⇒ no correlation



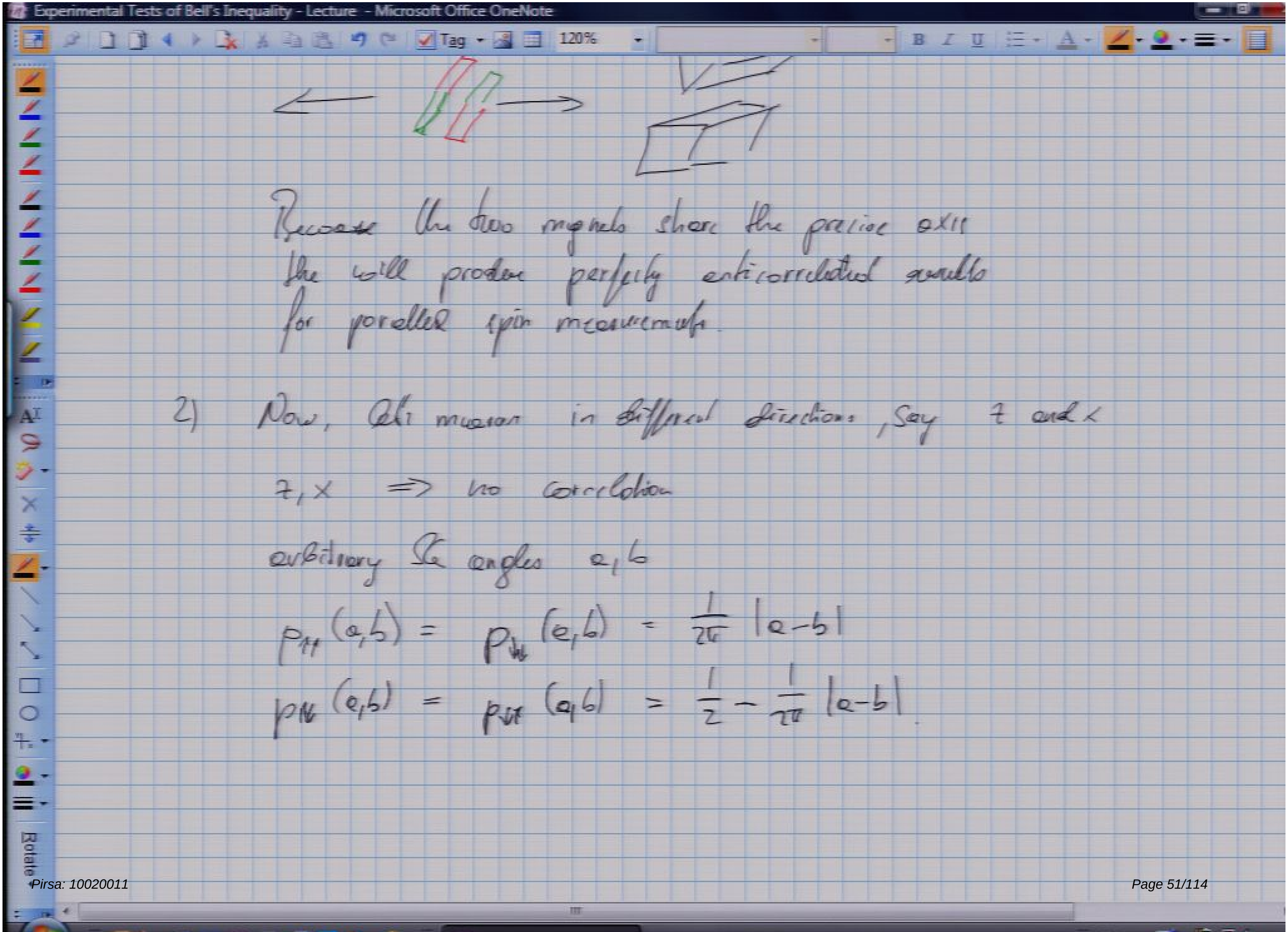
Because the two magnets share the precise axis they will produce perfectly anticorrelated results for parallel spin measurements.

2) Now, Alice measures in different directions, say z and x

z, x ⇒ no correlation

arbitrary SG angles a, b

$p_{++}(a,b) = p$



Because the two magnets share the precise axis
they will produce perfectly anticorrelated results
for parallel spin measurements.

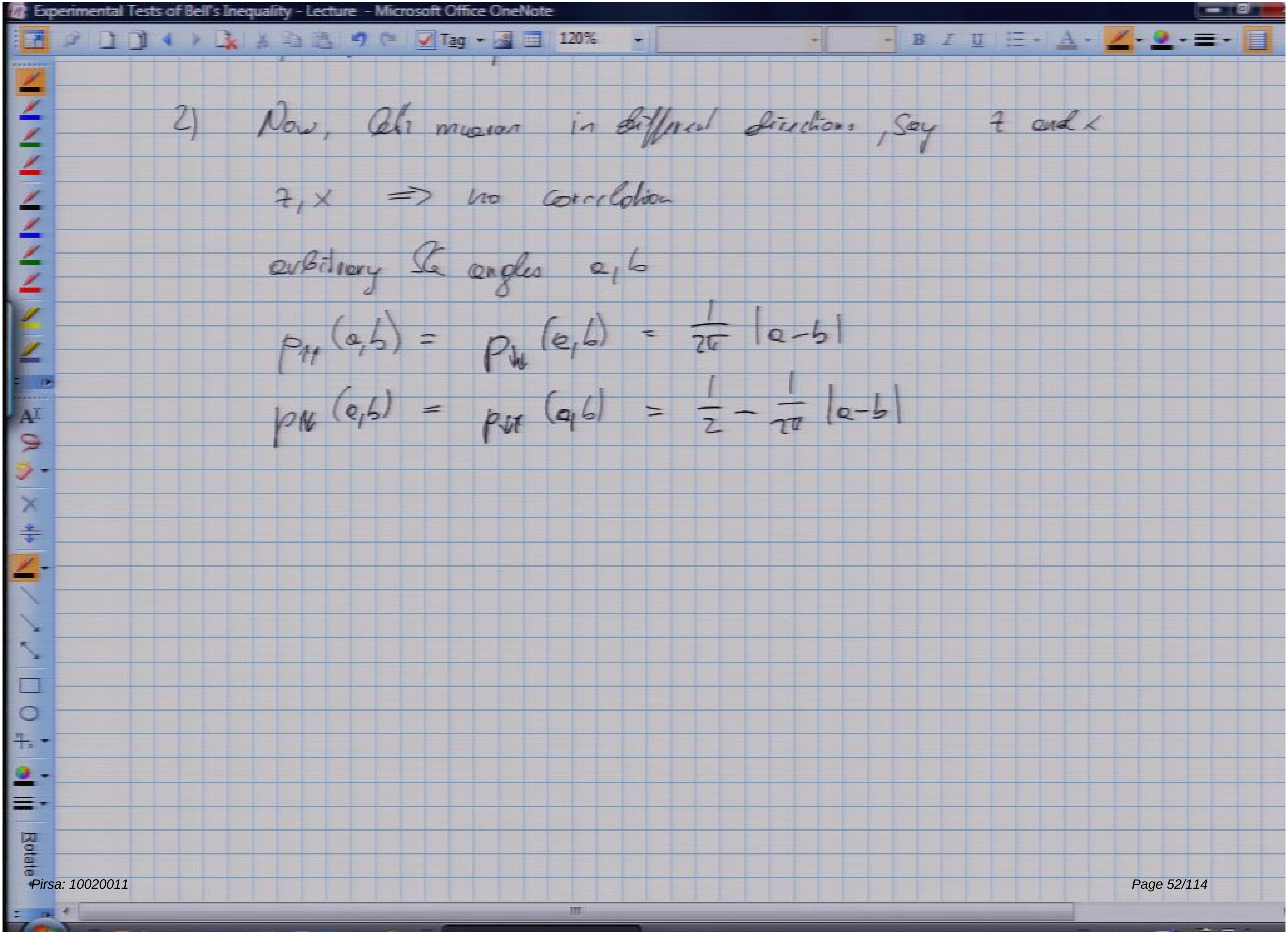
2) Now, let's measure in different directions, say z and x

z, x ⇒ no correlation

arbitrary angles a, b

$$P_{\uparrow\uparrow}(a,b) = P_{\downarrow\downarrow}(a,b) = \frac{1}{2\pi}|a-b|$$

$$P_{\uparrow\downarrow}(a,b) = P_{\downarrow\uparrow}(a,b) = \frac{1}{2} - \frac{1}{2\pi}|a-b|$$



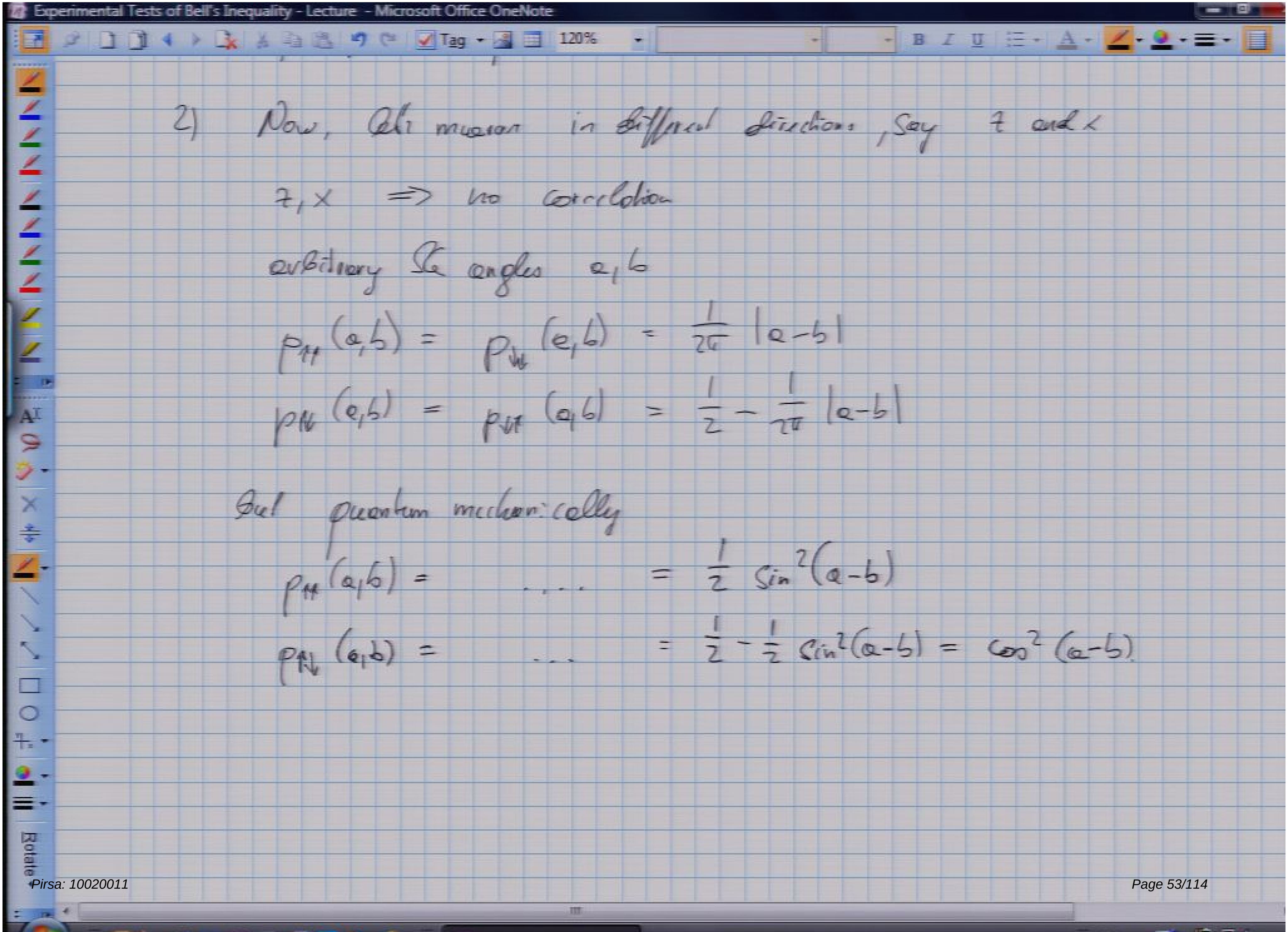
2) Now, Ali measures in different directions, say z and x

z, x ⇒ no correlation

arbitrary S.G. angles a, b

$$p_{\uparrow\uparrow}(a,b) = p_{\downarrow\downarrow}(a,b) = \frac{1}{2\pi}|a-b|$$

$$p_{\uparrow\downarrow}(a,b) = p_{\downarrow\uparrow}(a,b) = \frac{1}{2} - \frac{1}{2\pi}|a-b|$$



2) Now, [illegible] measure in different directions, say z and x

z, x $\Rightarrow$ no correlation

arbitrary SG angles a, b

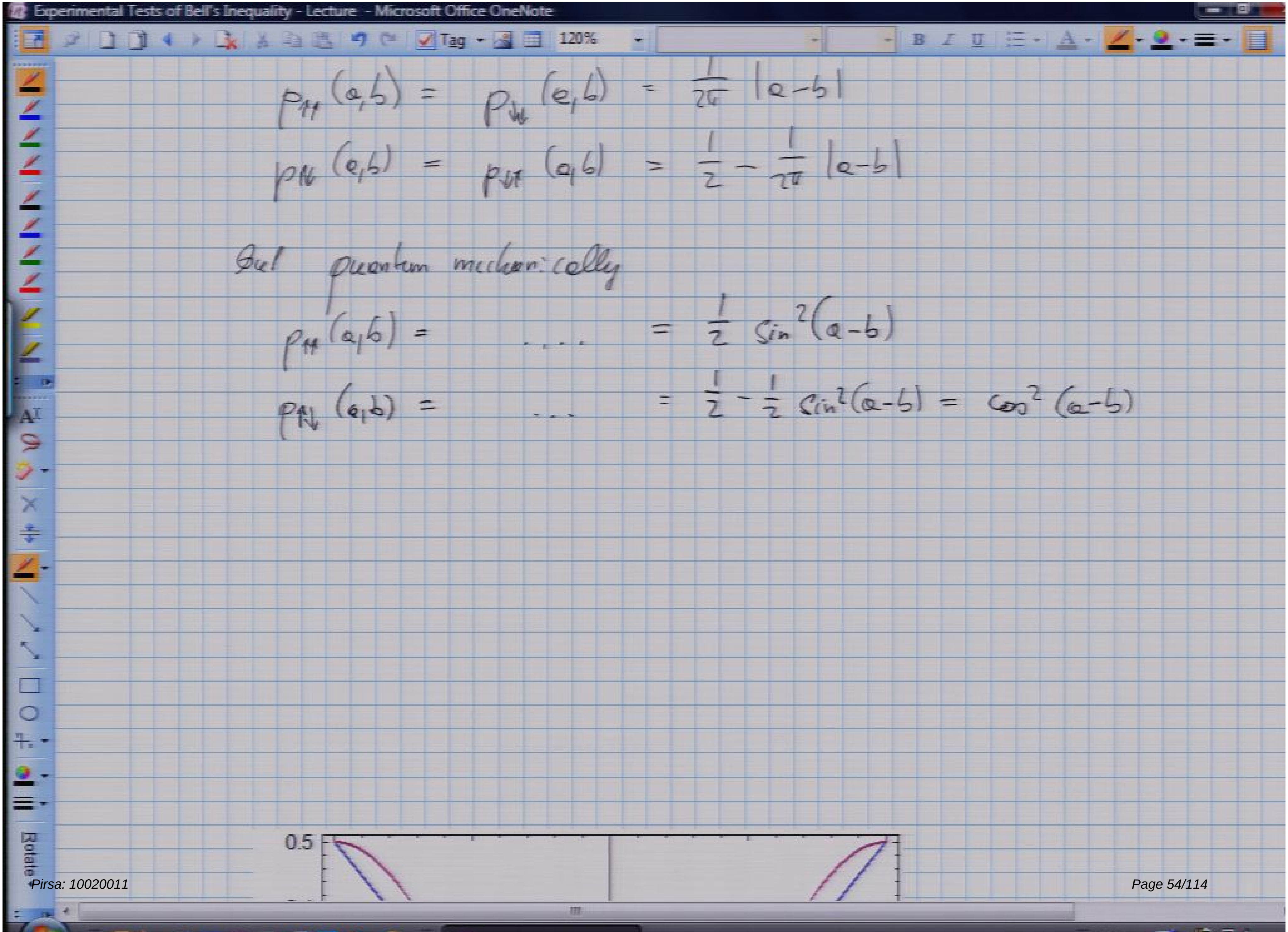
$$p_{\uparrow\uparrow}(a,b) = p_{\downarrow\downarrow}(a,b) = \frac{1}{2\pi}|a-b|$$

$$p_{\uparrow\downarrow}(a,b) = p_{\downarrow\uparrow}(a,b) = \frac{1}{2} - \frac{1}{2\pi}|a-b|$$

But quantum mechanically

$$p_{\uparrow\uparrow}(a,b) = \quad .... \quad = \frac{1}{2}\sin^2(a-b)$$

$$p_{\uparrow\downarrow}(a,b) = \quad ... \quad = \frac{1}{2} - \frac{1}{2}\sin^2(a-b) = \cos^2(a-b)$$



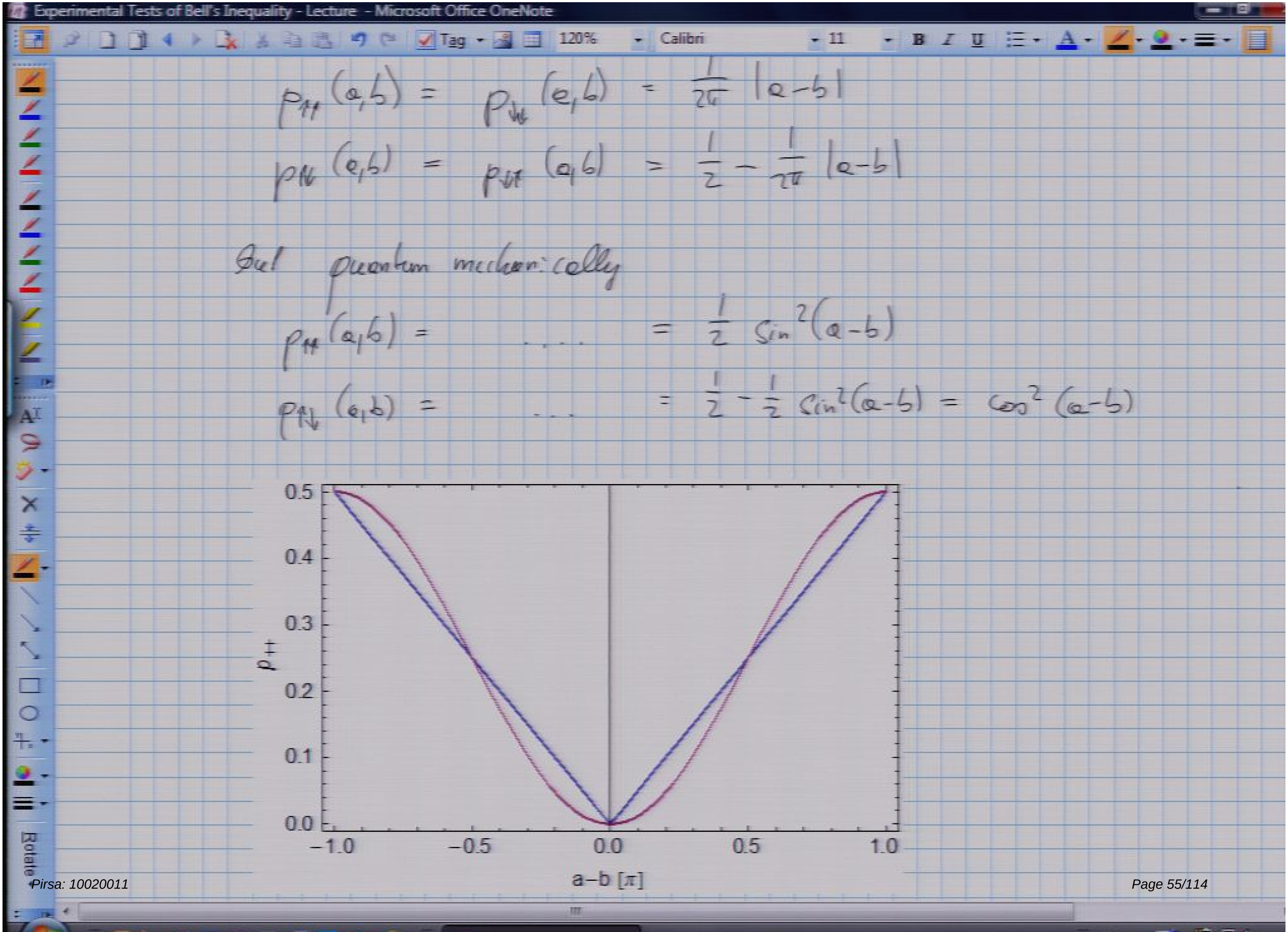
$$p_{\uparrow\uparrow}(a,b) = p_{\downarrow\downarrow}(a,b) = \frac{1}{2\pi}|a-b|$$

$$p_{\uparrow\downarrow}(a,b) = p_{\downarrow\uparrow}(a,b) = \frac{1}{2} - \frac{1}{2\pi}|a-b|$$

But quantum mechanically

$$p_{\uparrow\uparrow}(a,b) = \quad .... \quad = \frac{1}{2}\sin^2(a-b)$$

$$p_{\uparrow\downarrow}(a,b) = \quad ... \quad = \frac{1}{2} - \frac{1}{2}\sin^2(a-b) = \cos^2(a-b)$$



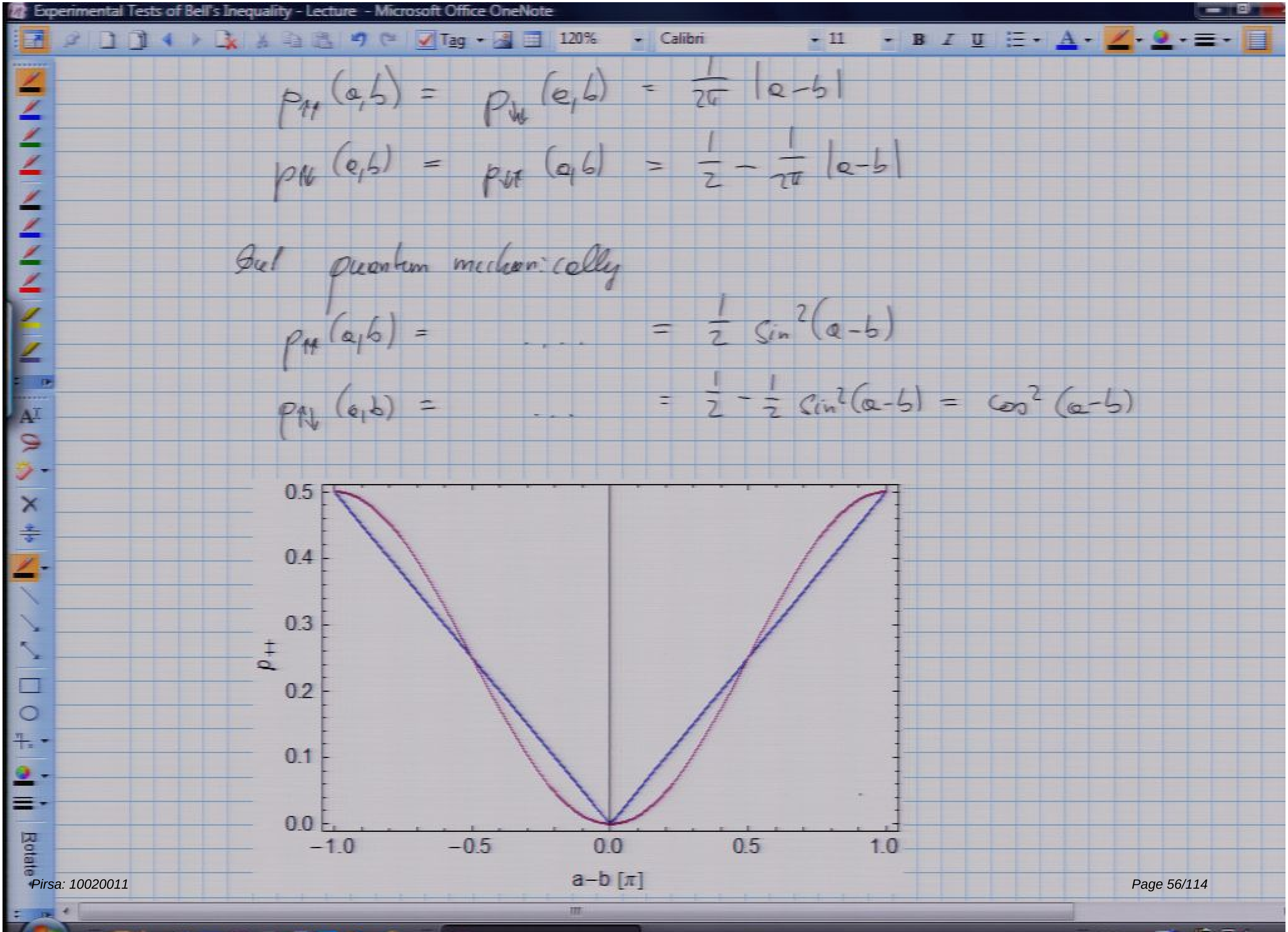
$$p_{\uparrow\uparrow}(a,b) = p_{\downarrow\downarrow}(a,b) = \frac{1}{2\pi}|a-b|$$

$$p_{\uparrow\downarrow}(a,b) = p_{\downarrow\uparrow}(a,b) = \frac{1}{2} - \frac{1}{2\pi}|a-b|$$

But quantum mechanically

$$p_{\uparrow\uparrow}(a,b) = \quad \dots \quad = \frac{1}{2}\sin^2(a-b)$$

$$p_{\uparrow\downarrow}(a,b) = \quad \dots \quad = \frac{1}{2} - \frac{1}{2}\sin^2(a-b) = \cos^2(a-b)$$



$$p_{\uparrow\uparrow}(a,b) = p_{\downarrow\downarrow}(a,b) = \frac{1}{2\pi}|a-b|$$

$$p_{\uparrow\downarrow}(a,b) = p_{\downarrow\uparrow}(a,b) = \frac{1}{2} - \frac{1}{2\pi}|a-b|$$

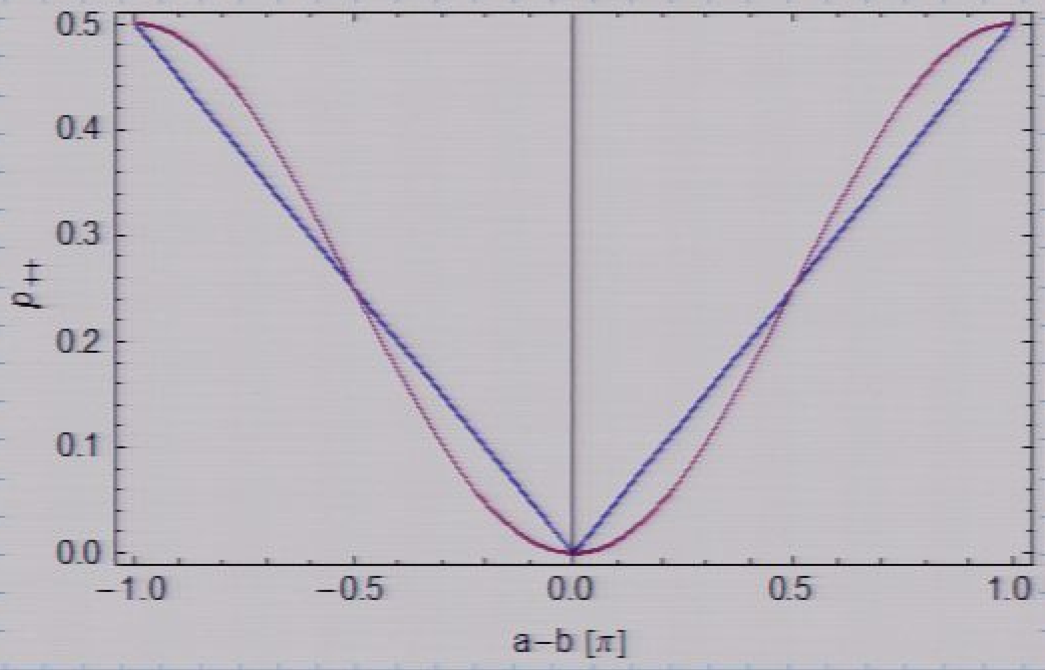
But quantum mechanically

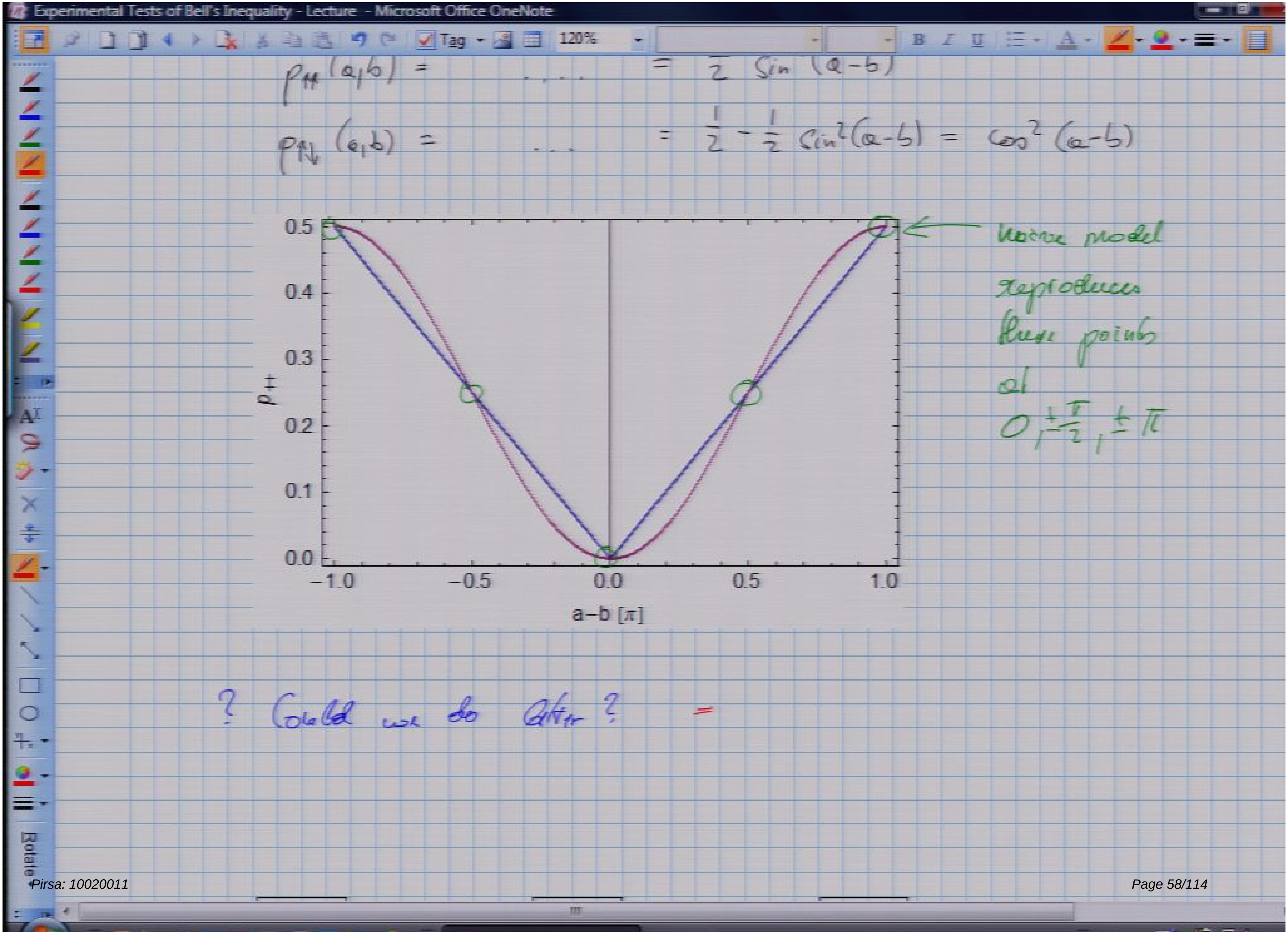
$$p_{\uparrow\uparrow}(a,b) = \quad \dots \quad = \frac{1}{2}\sin^2(a-b)$$

$$p_{\uparrow\downarrow}(a,b) = \quad \dots \quad = \frac{1}{2} - \frac{1}{2}\sin^2(a-b) = \cos^2(a-b)$$

$$\rho_{\uparrow\uparrow}(a,b) = \quad \dots\dots \quad = \frac{\ }{2} \sin (a-b)$$

$$\rho_{\uparrow\downarrow}(a,b) = \quad \dots \quad = \frac{1}{2} - \frac{1}{2}\sin^2(a-b) = \cos^2(a-b)$$



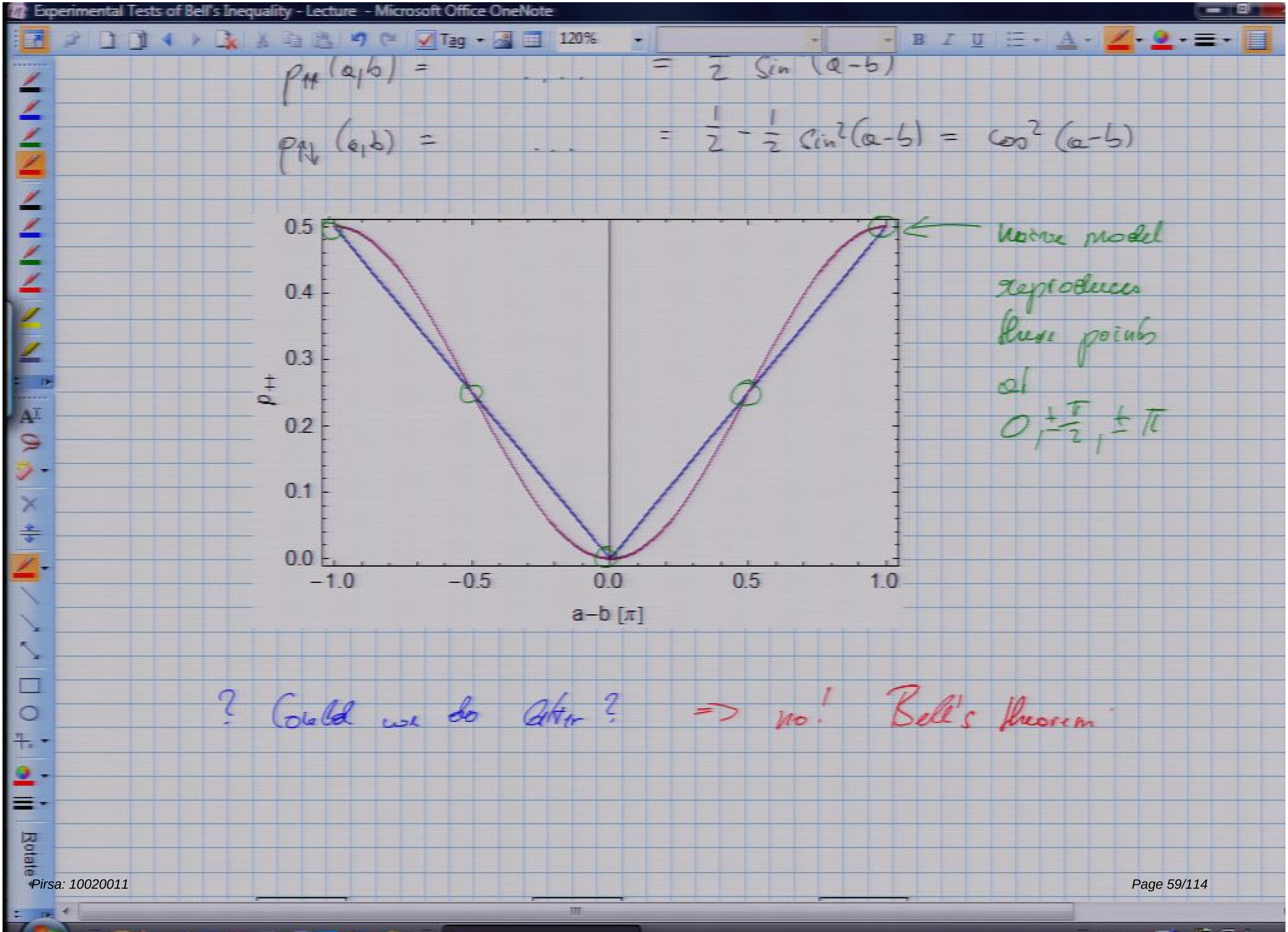


$p_{\uparrow\uparrow}(a,b) = \;\ldots\ldots\; = \frac{1}{2}\sin^2(a-b)$

$p_{\uparrow\downarrow}(a,b) = \;\ldots\; = \frac{1}{2} - \frac{1}{2}\sin^2(a-b) = \cos^2(a-b)$

← naive model reproduces these points at $0, \pm\frac{\pi}{2}, \pm\pi$

? Could we do better ? =

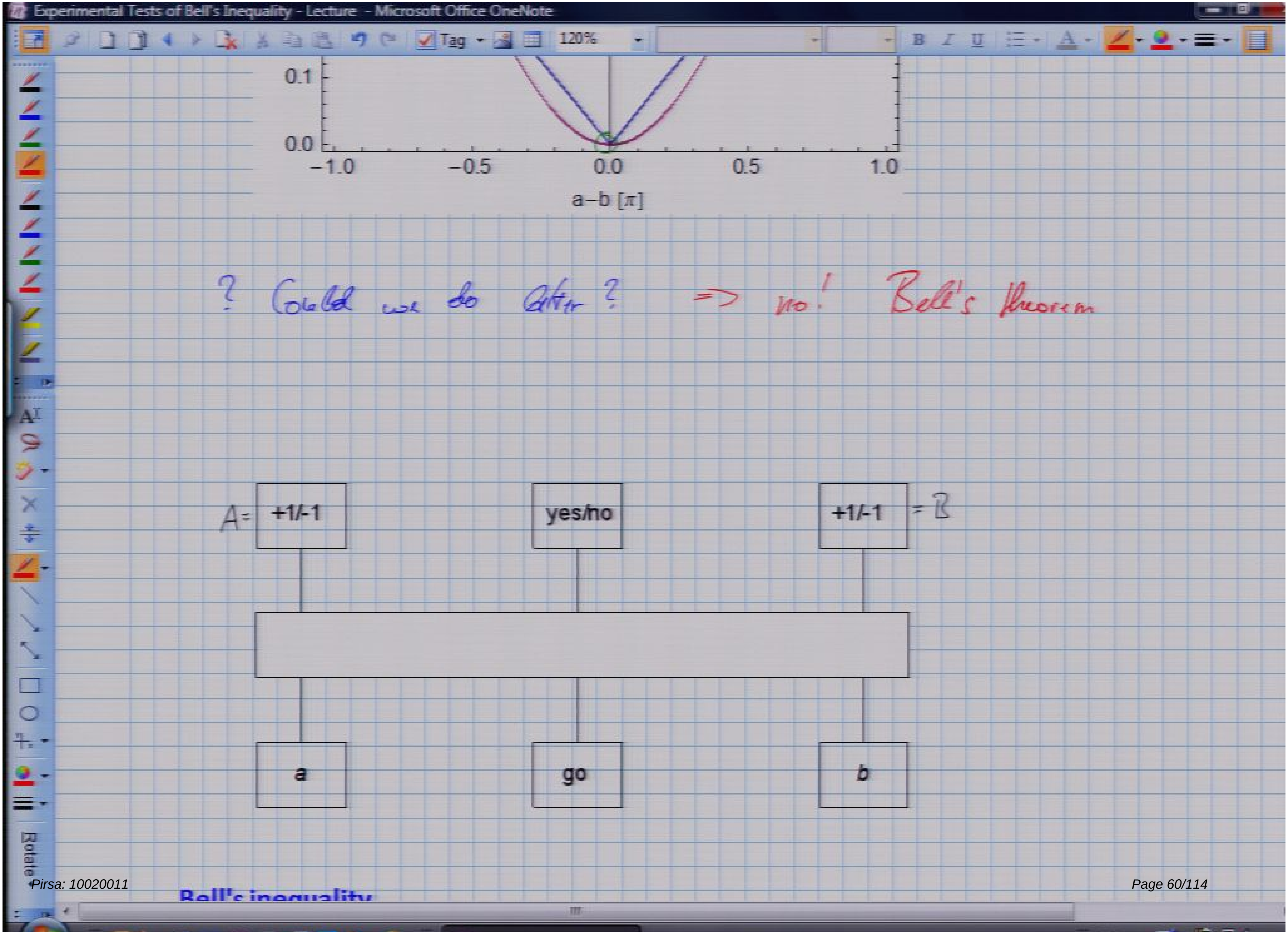


$p_{\uparrow\uparrow}(a,b) = \quad \dots \quad = \frac{}{2} \sin (a-b)$

$p_{\uparrow\downarrow}(a,b) = \quad \dots \quad = \frac{1}{2} - \frac{1}{2} \sin^2(a-b) = \cos^2(a-b)$

naive model reproduces these points at $0, \pm\frac{\pi}{2}, \pm\pi$

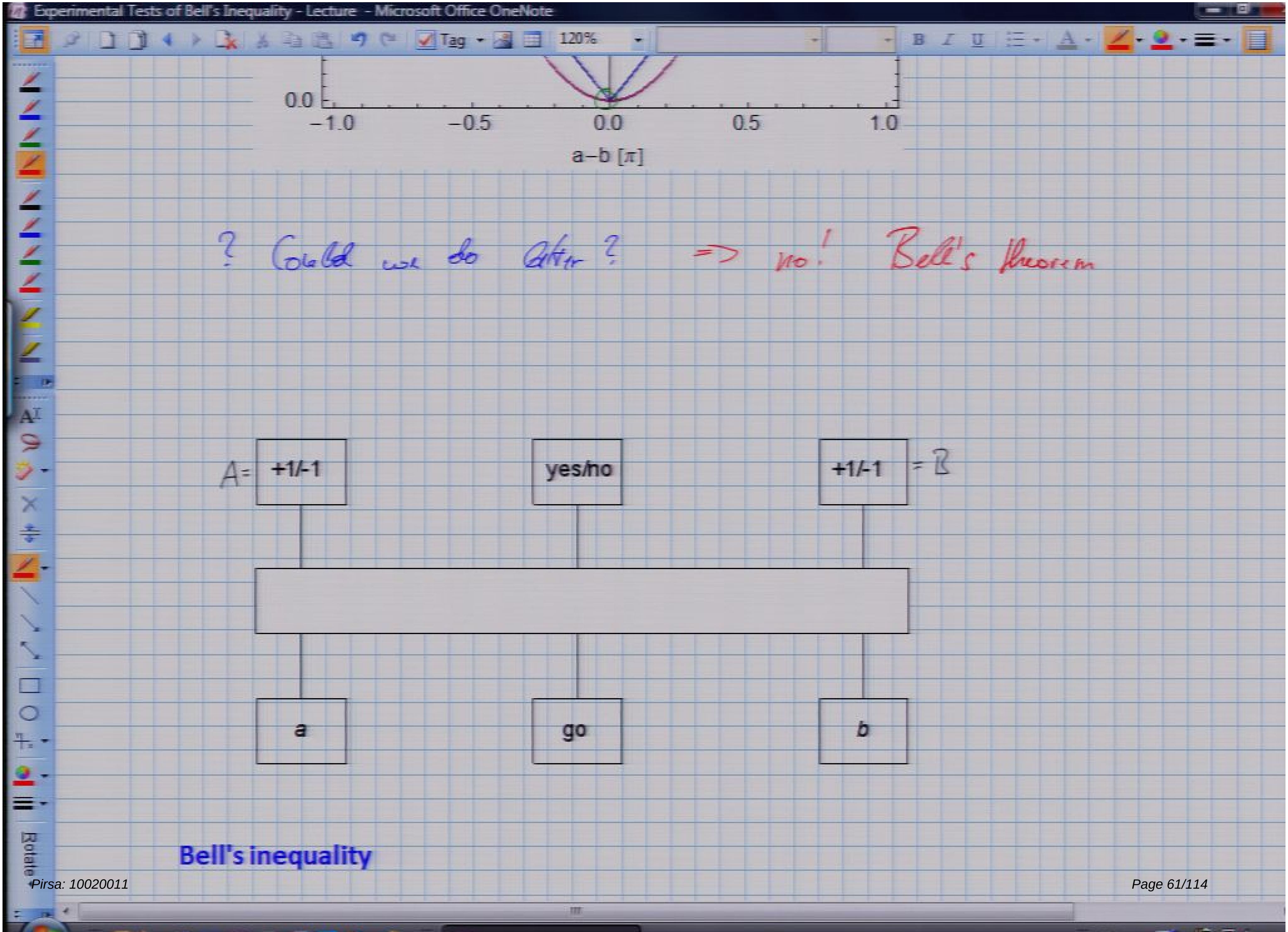
? Could we do better ? => no! Bell's theorem



? Could we do better ? => no! Bell's theorem

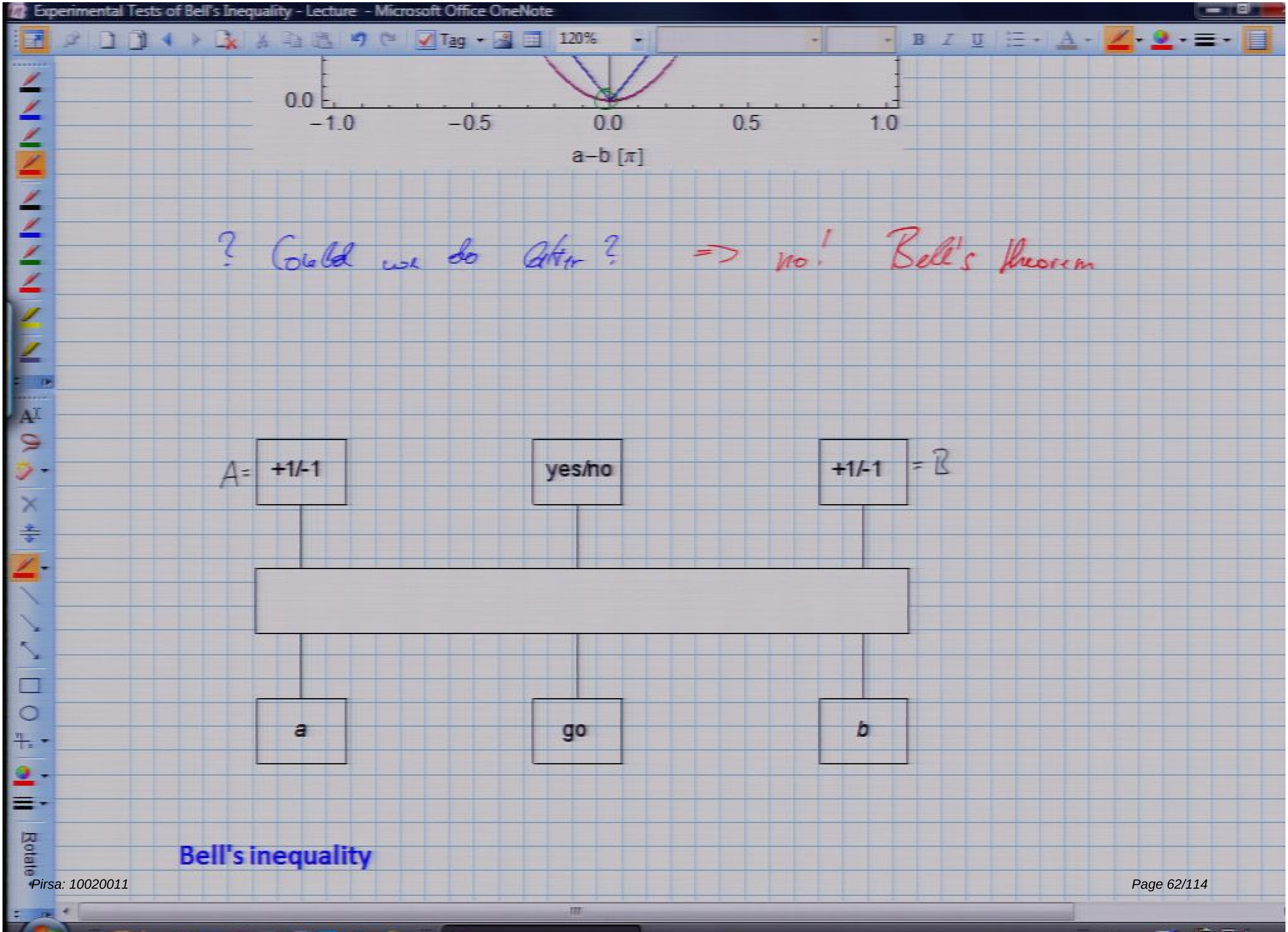
Bell's inequality

Experimental Tests of Bell's Inequality - Lecture - Microsoft Office OneNote



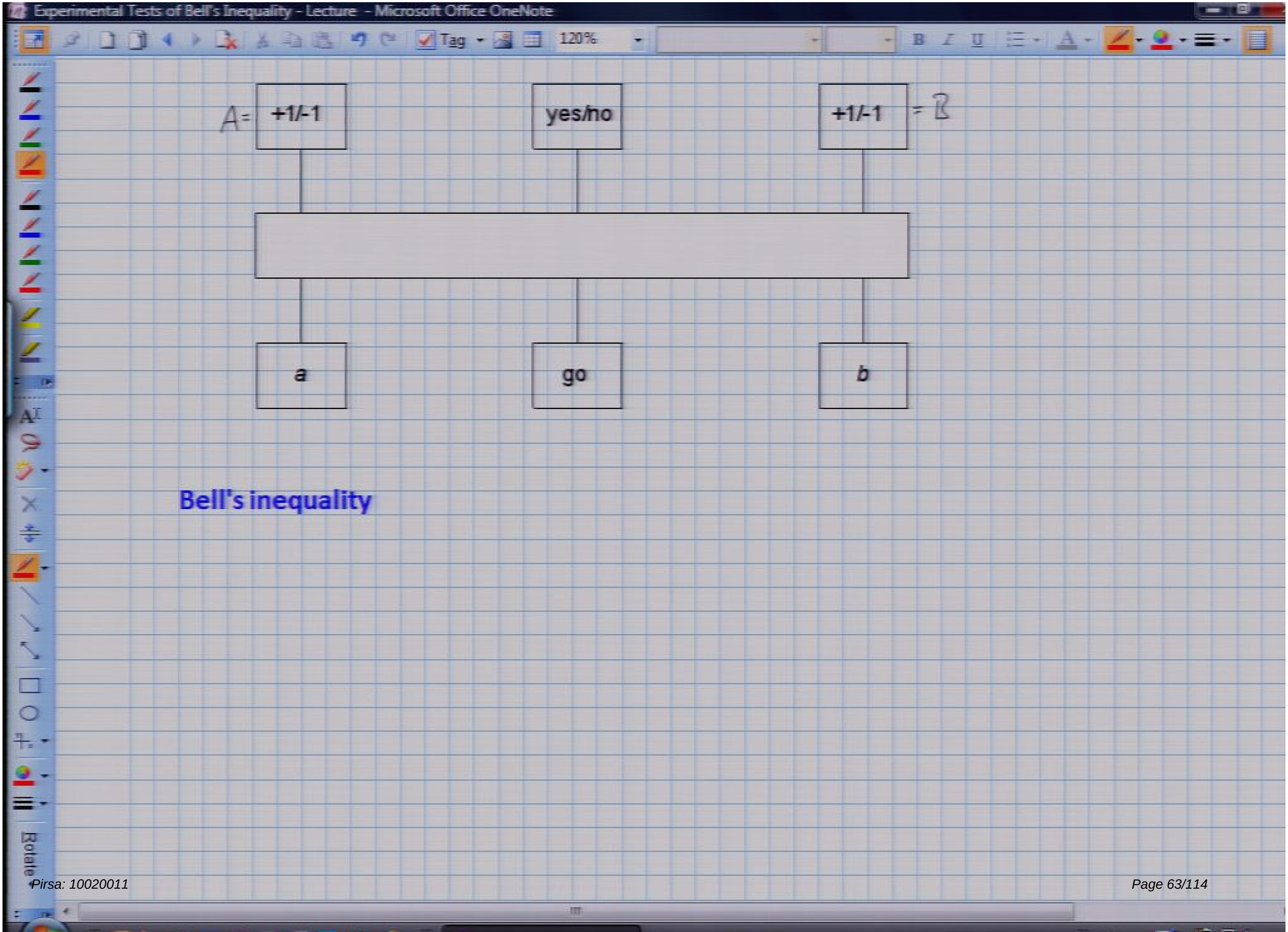
? Could we do better? => no! Bell's theorem

Bell's inequality

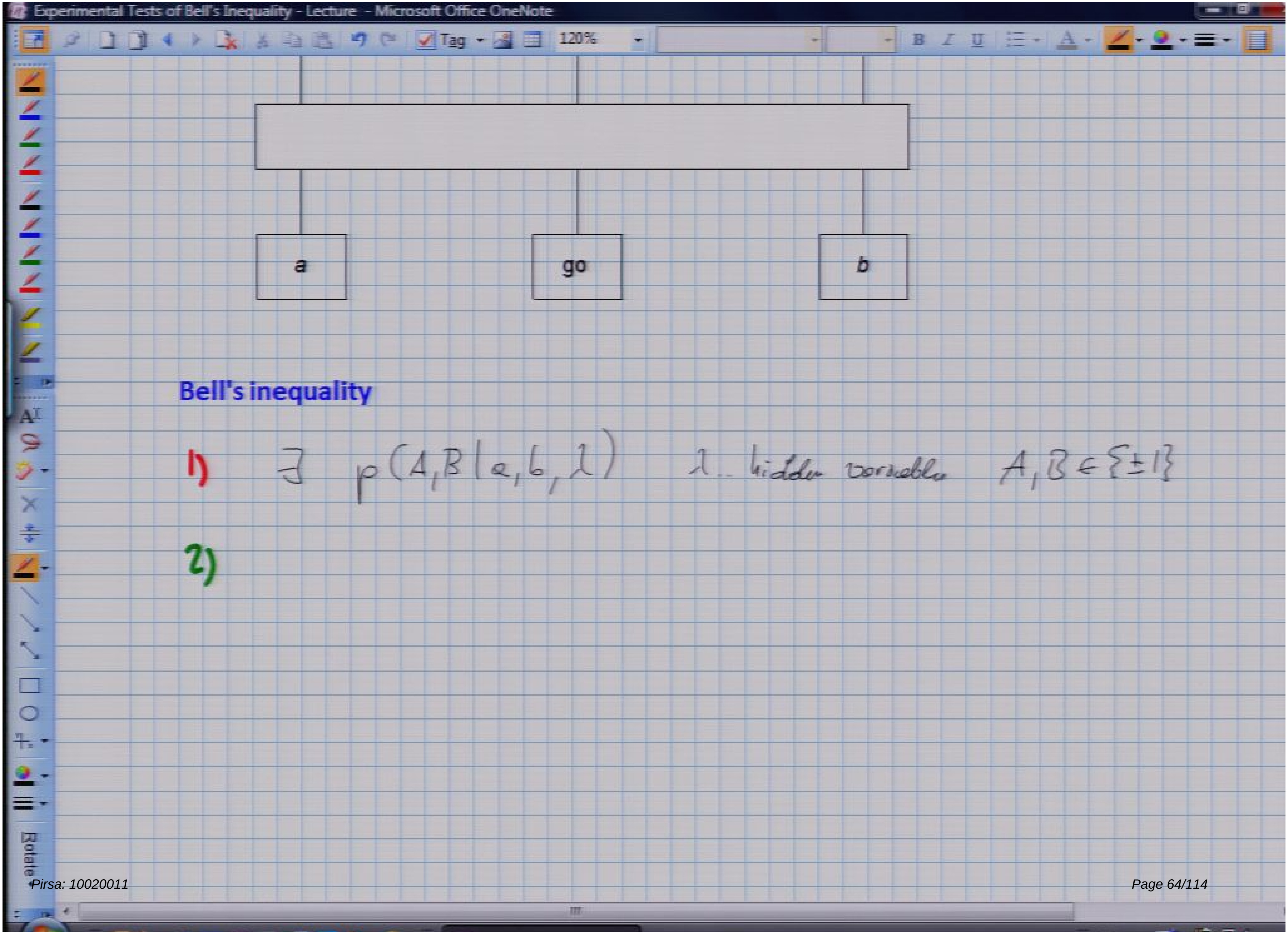


? Could we do better ? ⇒ no! Bell's theorem

Bell's inequality



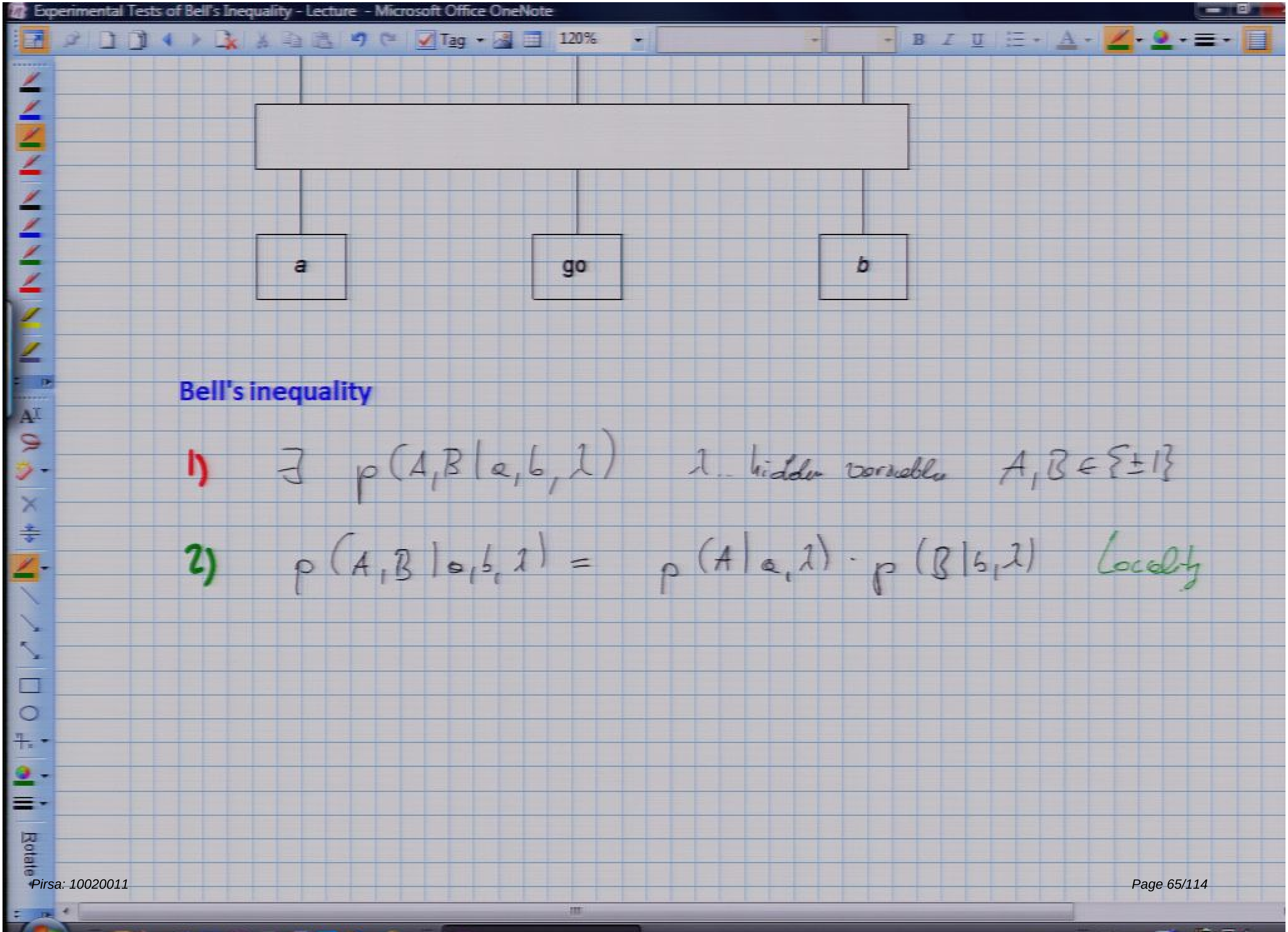
Bell's inequality



Bell's inequality

1) $\exists \; p(A,B|a,b,\lambda)$ λ .. hidden variable $A,B \in \{\pm 1\}$

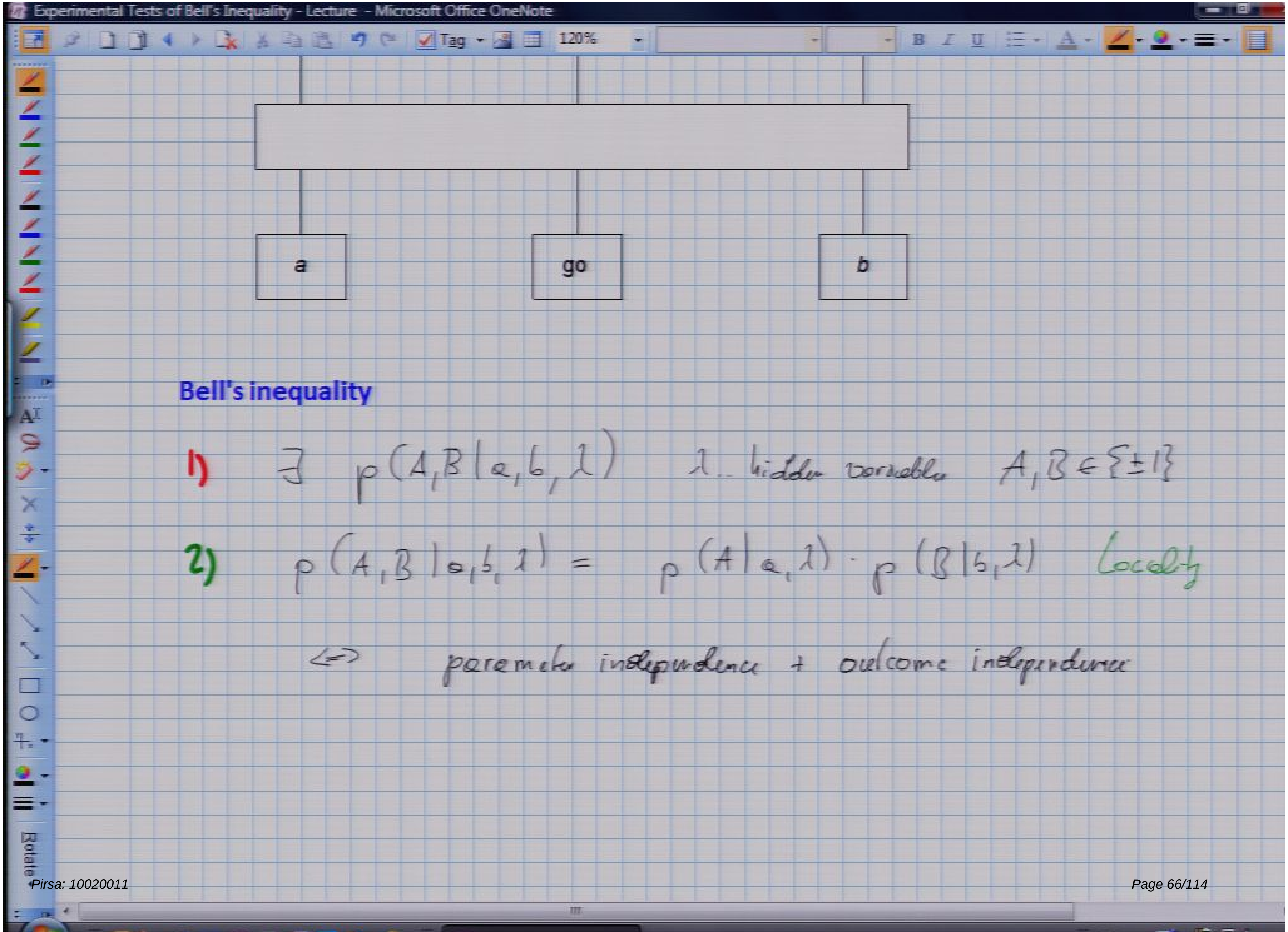
2)



Bell's inequality

1) $\exists \; p(A,B|a,b,\lambda)$ λ.. hidden variables $A,B \in \{\pm 1\}$

2) $p(A,B|a,b,\lambda) = p(A|a,\lambda) \cdot p(B|b,\lambda)$ Locality

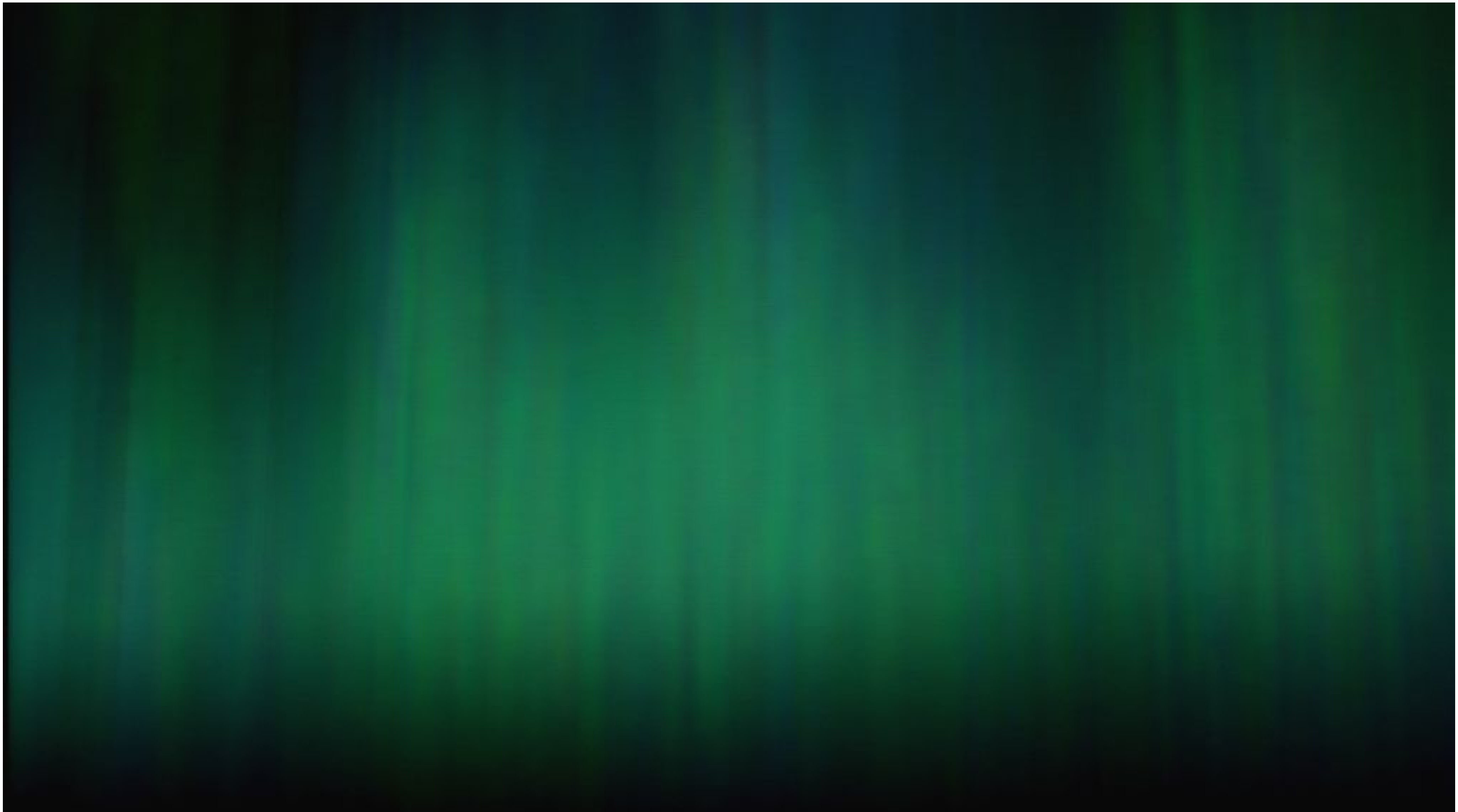


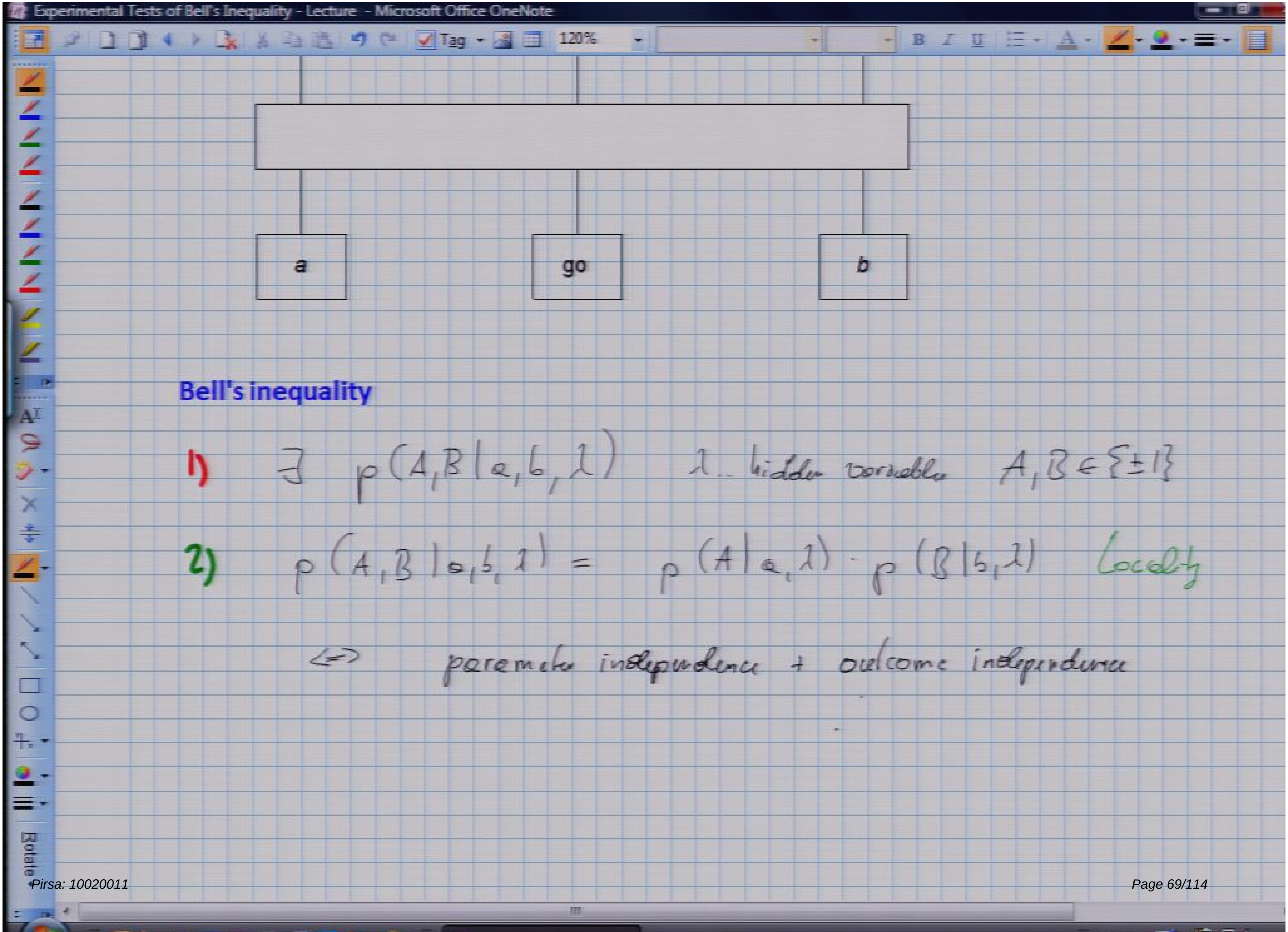
Bell's inequality

1) $\exists \; p(A,B|a,b,\lambda)$ λ.. hidden variable $A,B \in \{\pm 1\}$

2) $p(A,B|a,b,\lambda) = p(A|a,\lambda) \cdot p(B|b,\lambda)$ Locality

$\Leftrightarrow$ parameter independence + outcome independence



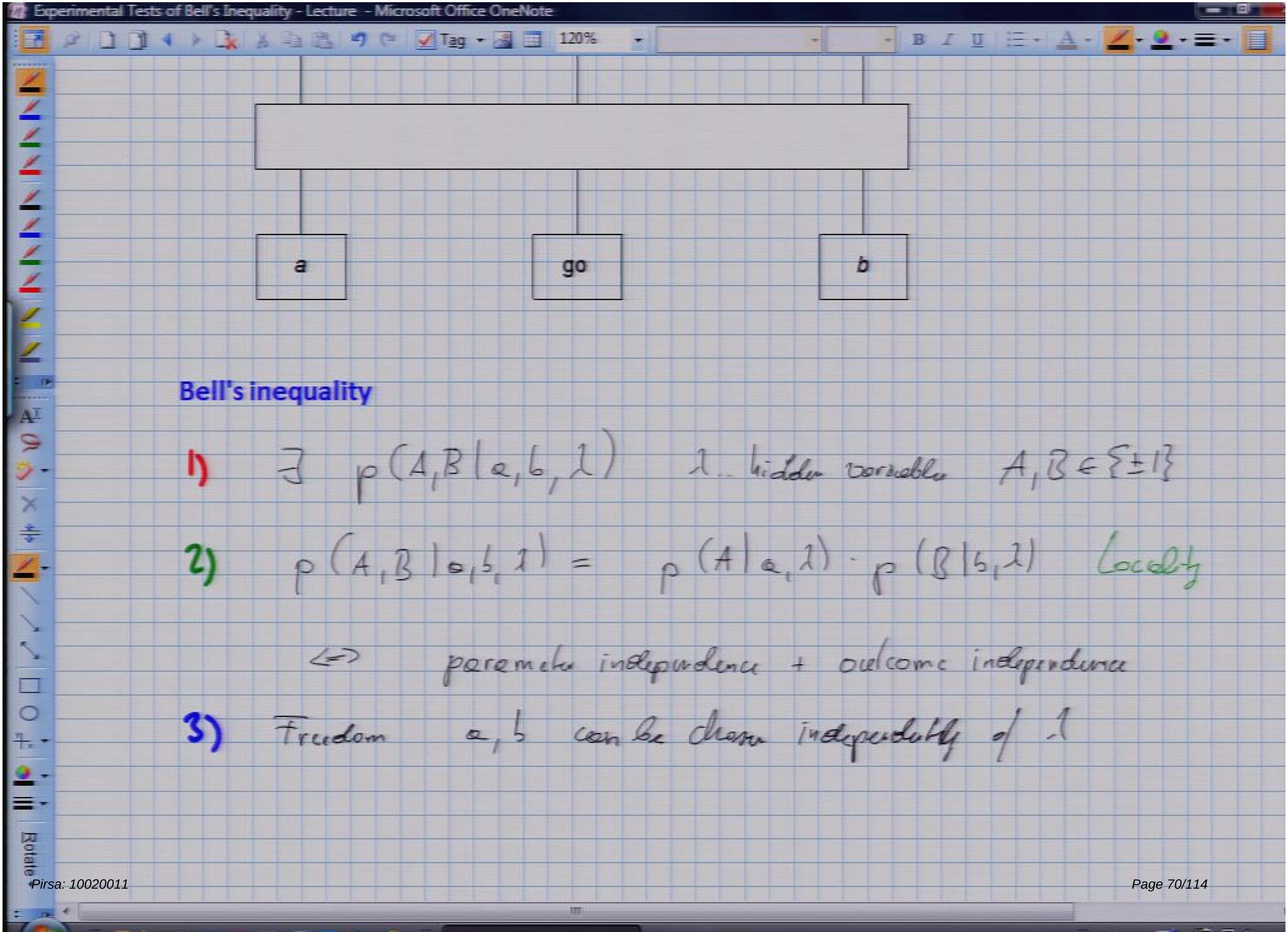


Bell's inequality

1) $\exists \; p(A,B|a,b,\lambda)$ λ.. hidden variables $A,B \in \{\pm 1\}$

2) $p(A,B|a,b,\lambda) = p(A|a,\lambda) \cdot p(B|b,\lambda)$ Locality

$\Longleftrightarrow$ parameter independence + outcome independence



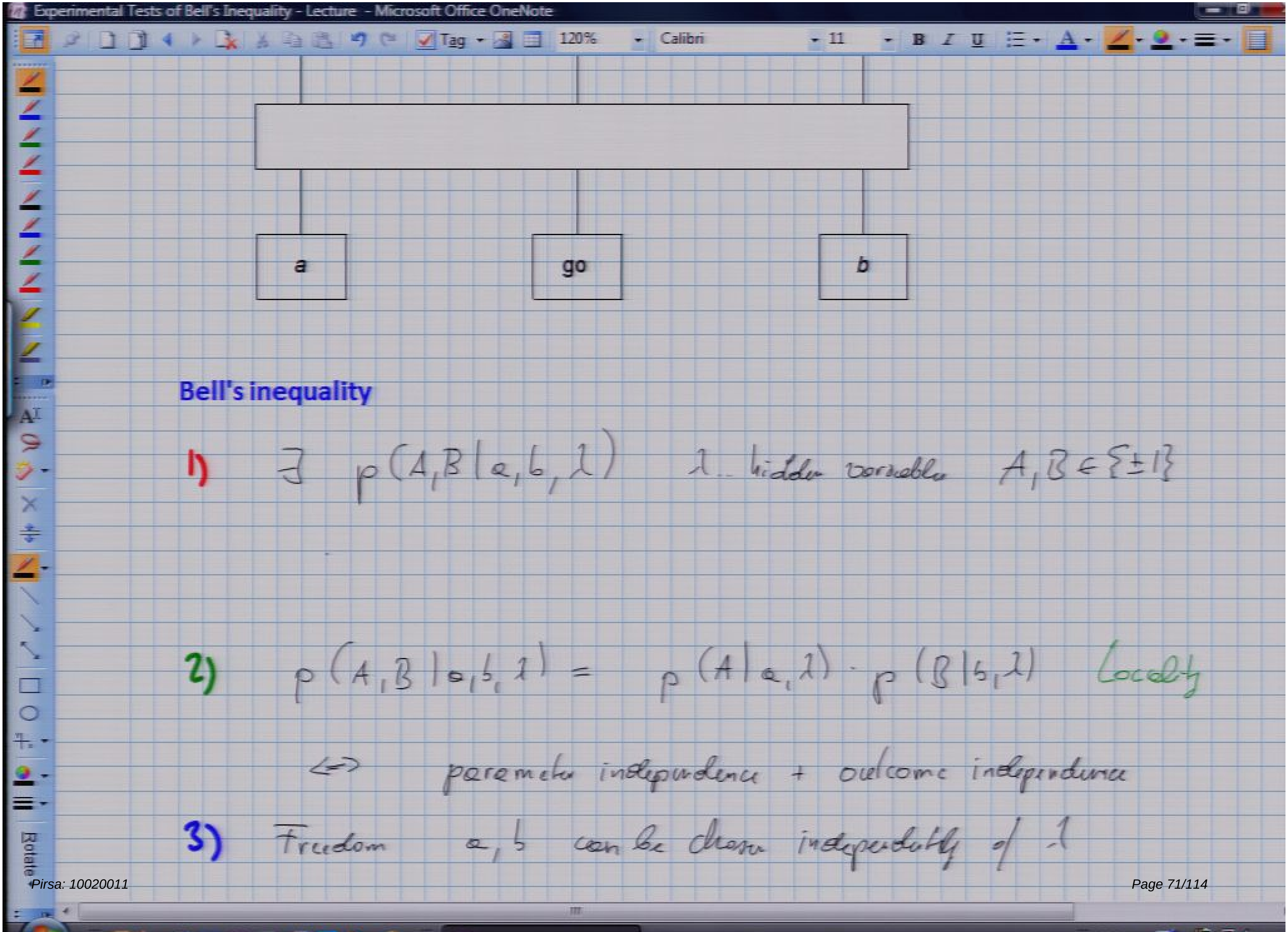
Bell's inequality

1) $\exists \; p(A,B|a,b,\lambda)$ λ.. hidden variables $A,B \in \{\pm 1\}$

2) $p(A,B|a,b,\lambda) = p(A|a,\lambda) \cdot p(B|b,\lambda)$ Locality

$\Leftrightarrow$ parameter independence + outcome independence

3) Freedom a,b can be chosen independently of λ



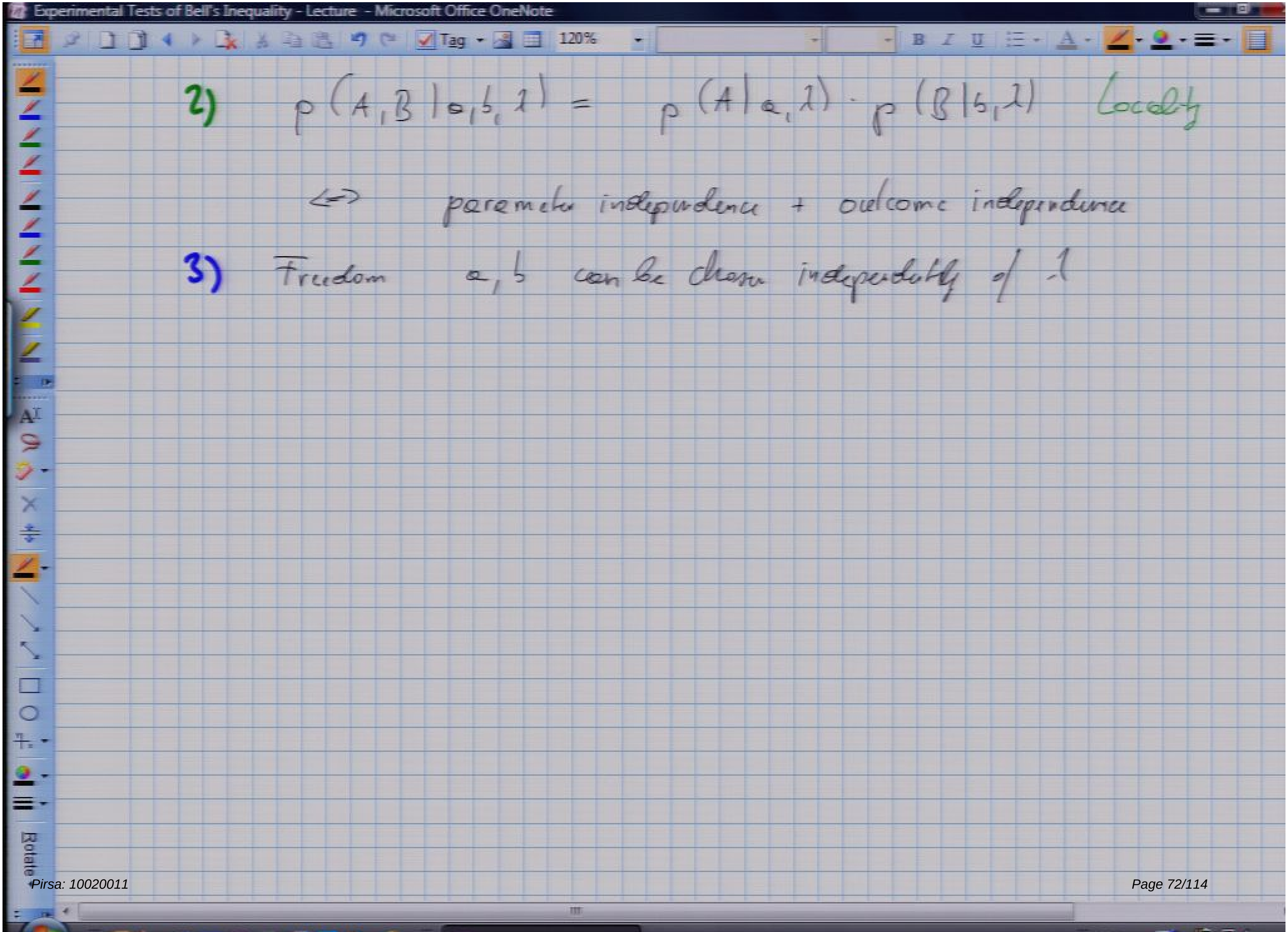
Bell's inequality

1) $\exists \quad p(A,B\,|\,a,b,\lambda)$ $\quad\lambda$ .. hidden variable $\quad A,B \in \{\pm 1\}$

2) $p(A,B\,|\,a,b,\lambda) = p(A\,|\,a,\lambda)\cdot p(B\,|\,b,\lambda)$ Locality

$\Longleftrightarrow$ parameter independence + outcome independence

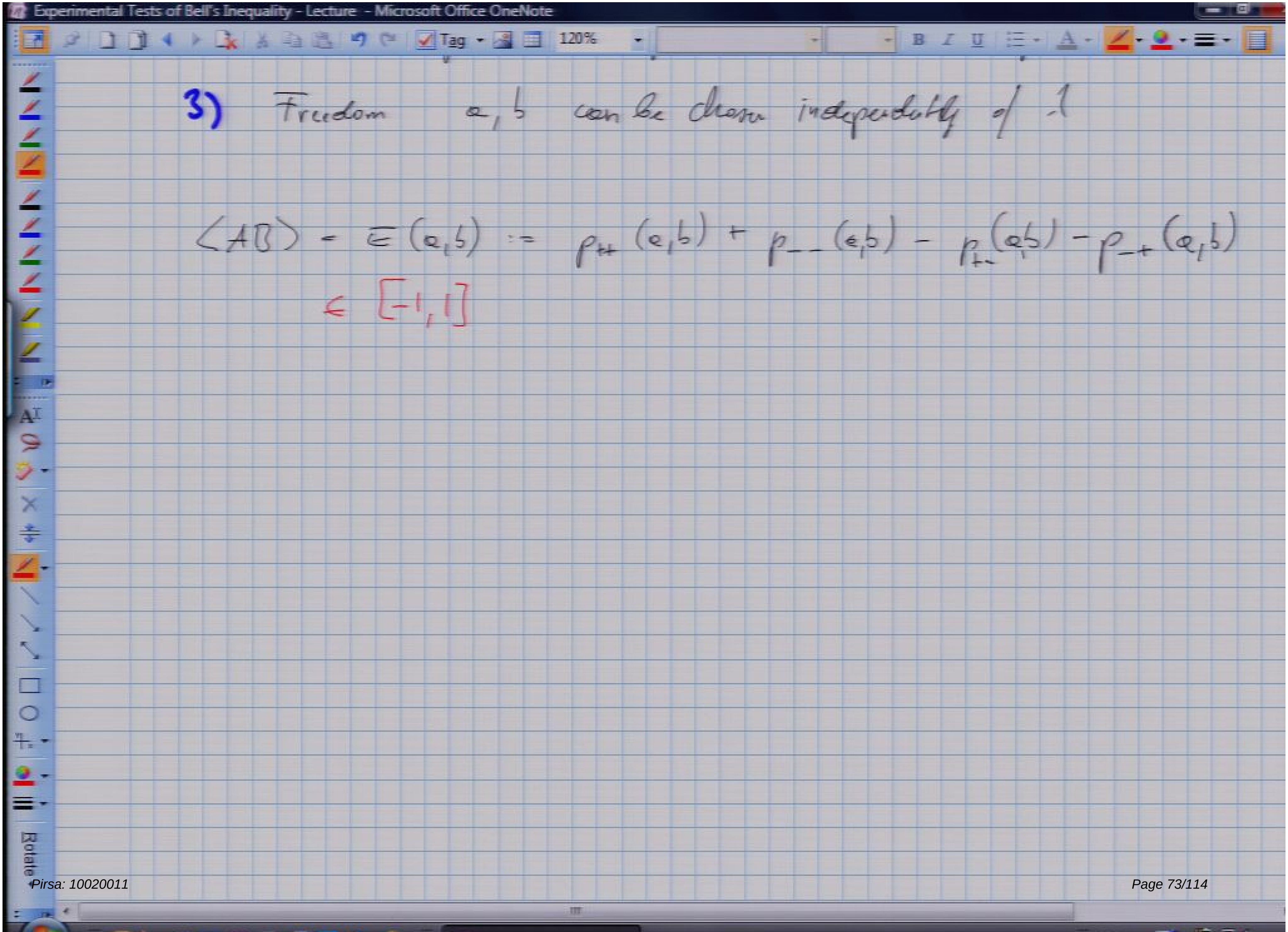
3) Freedom a, b can be chosen independently of λ



2) $p(A,B|a,b,\lambda) = p(A|a,\lambda)\cdot p(B|b,\lambda)$ Locality

$\Leftrightarrow$ parameter independence + outcome independence

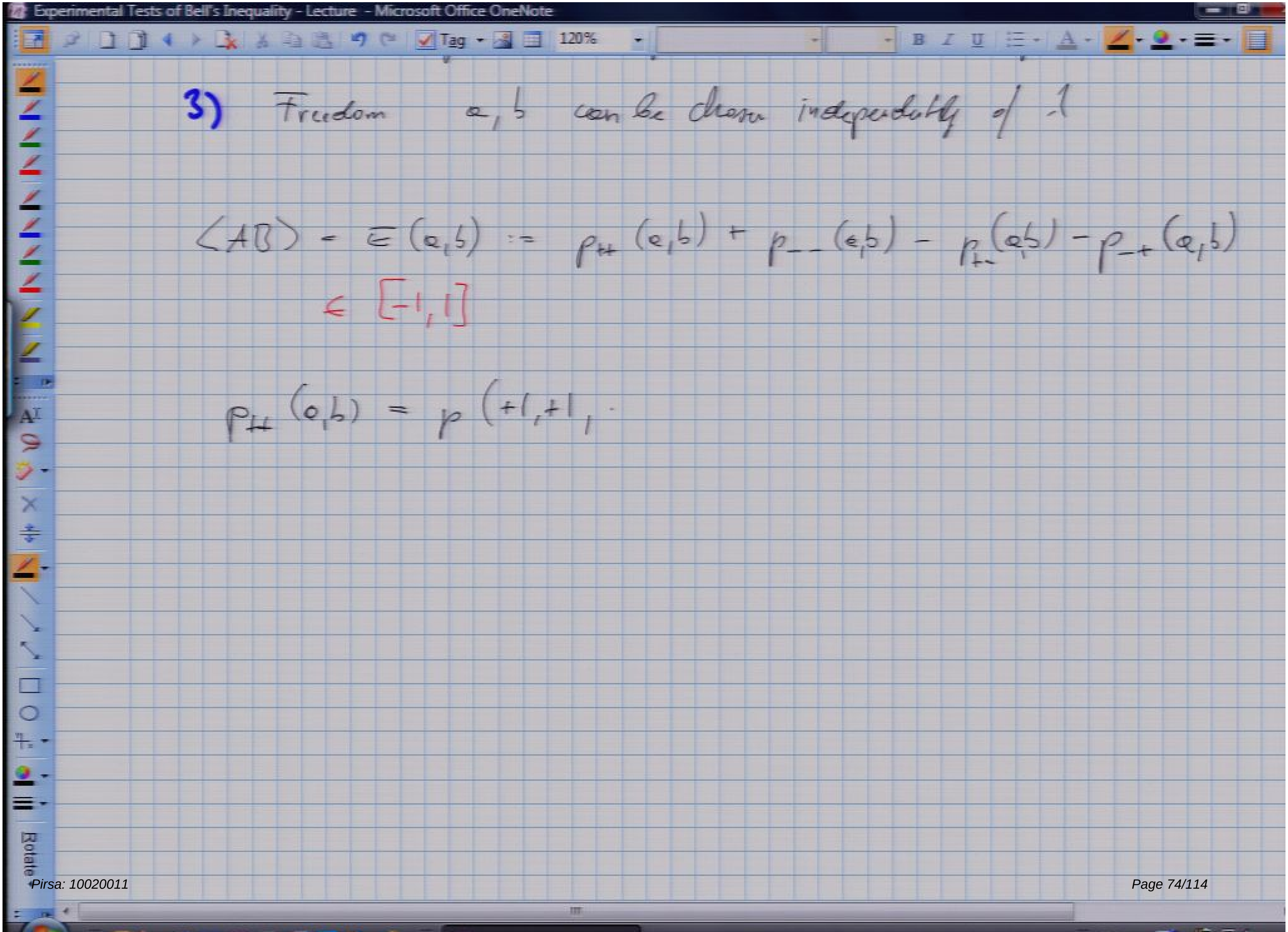
3) Freedom a, b can be chosen independently of λ



3) Freedom a, b can be chosen independently of λ

$$\langle AB\rangle = E(a,b) := p_{++}(a,b) + p_{--}(a,b) - p_{+-}(a,b) - p_{-+}(a,b)$$

$$\in [-1,1]$$

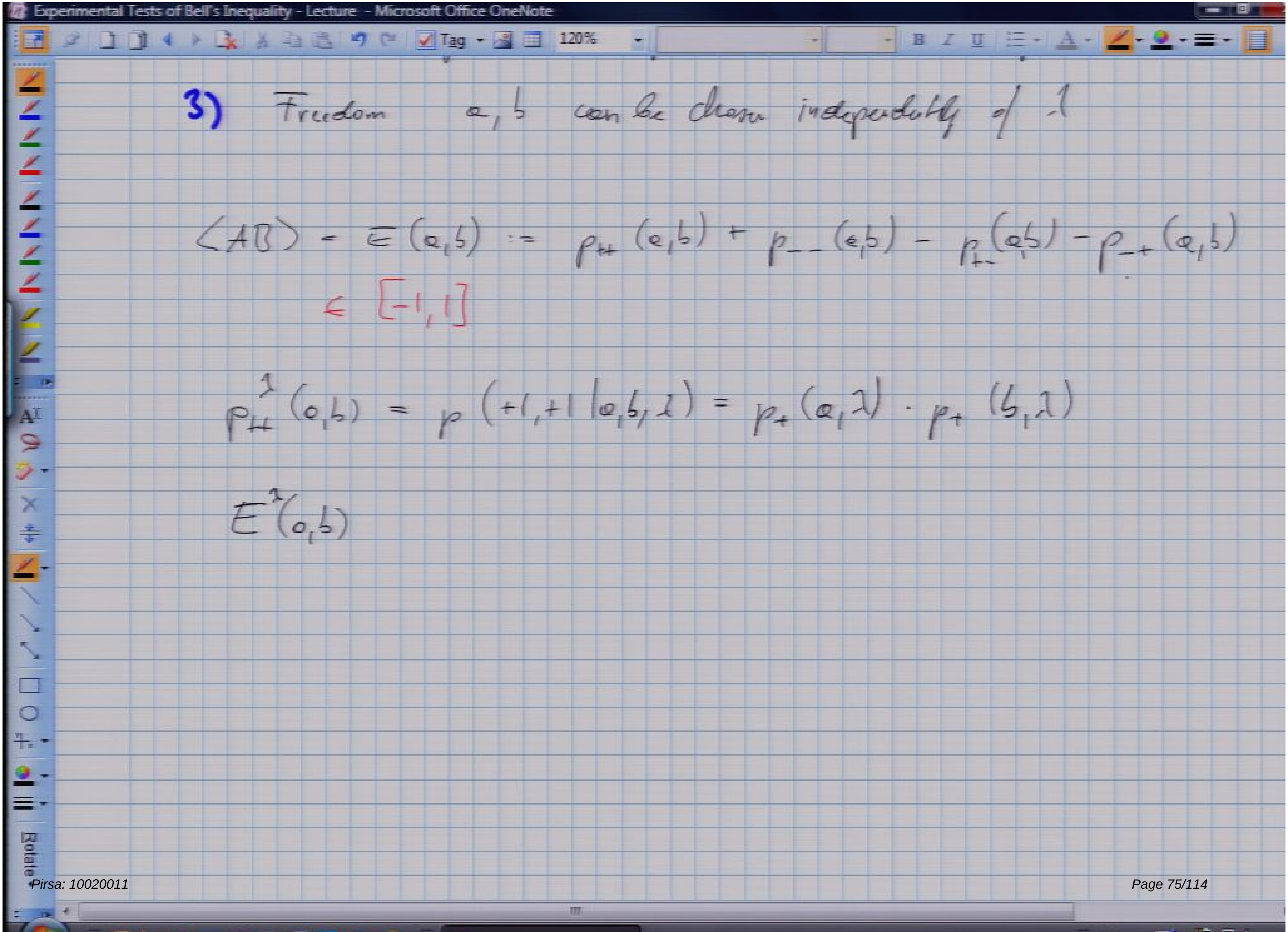


3) Freedom a, b can be chosen independently of λ

$$\langle AB \rangle = E(a,b) := p_{++}(a,b) + p_{--}(a,b) - p_{+-}(a,b) - p_{-+}(a,b)$$

$$\in [-1,1]$$

$$p_{++}(a,b) = p(+1,+1, \cdot$$



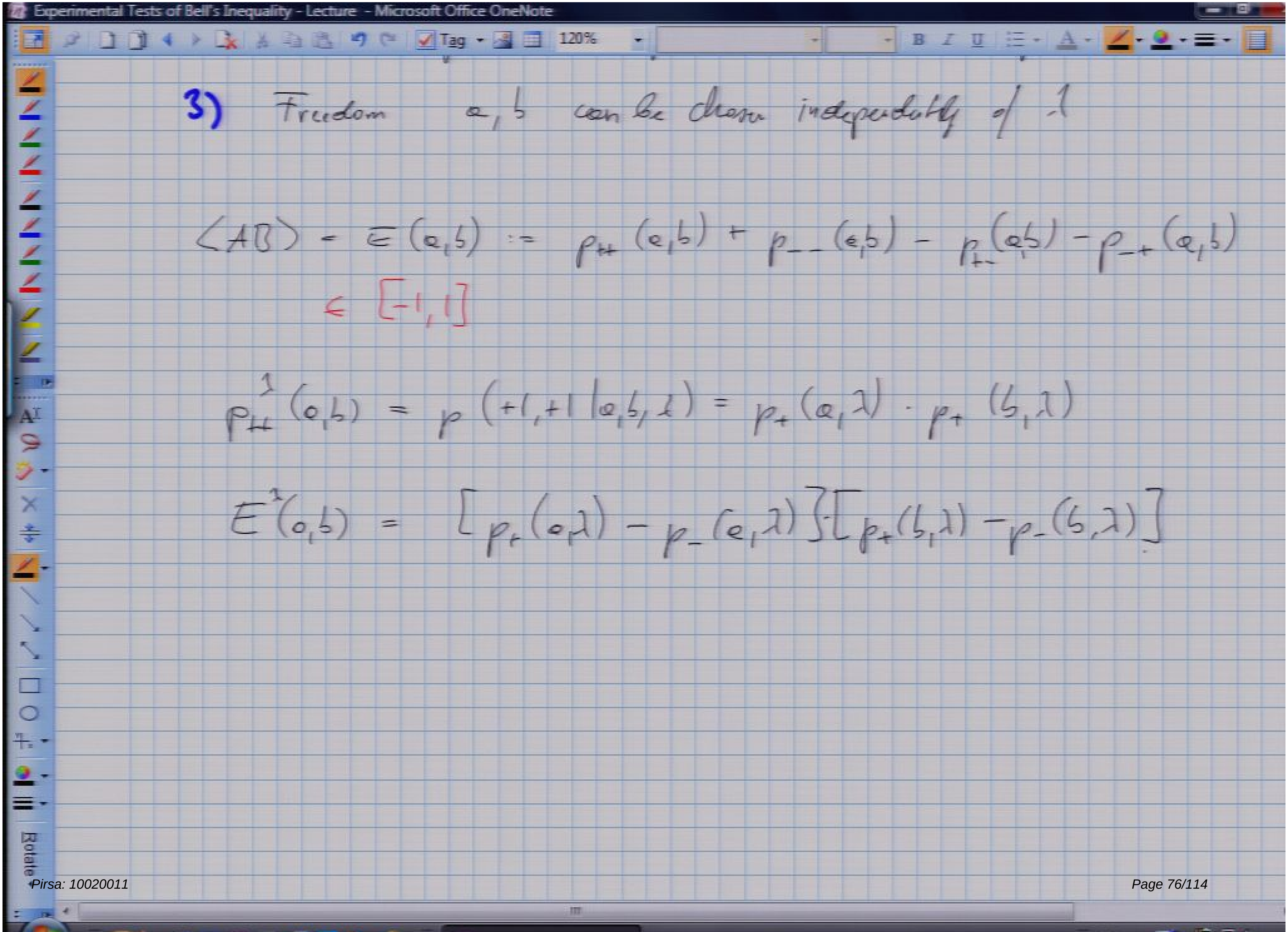
3) Freedom a, b can be chosen independently of λ

$$\langle AB \rangle = E(a,b) := p_{++}(a,b) + p_{--}(a,b) - p_{+-}(a,b) - p_{-+}(a,b)$$

$$\in [-1,1]$$

$$p^{\lambda}_{++}(a,b) = p(+1,+1 \mid a,b,\lambda) = p_{+}(a,\lambda) \cdot p_{+}(b,\lambda)$$

$$E^{\lambda}(a,b)$$



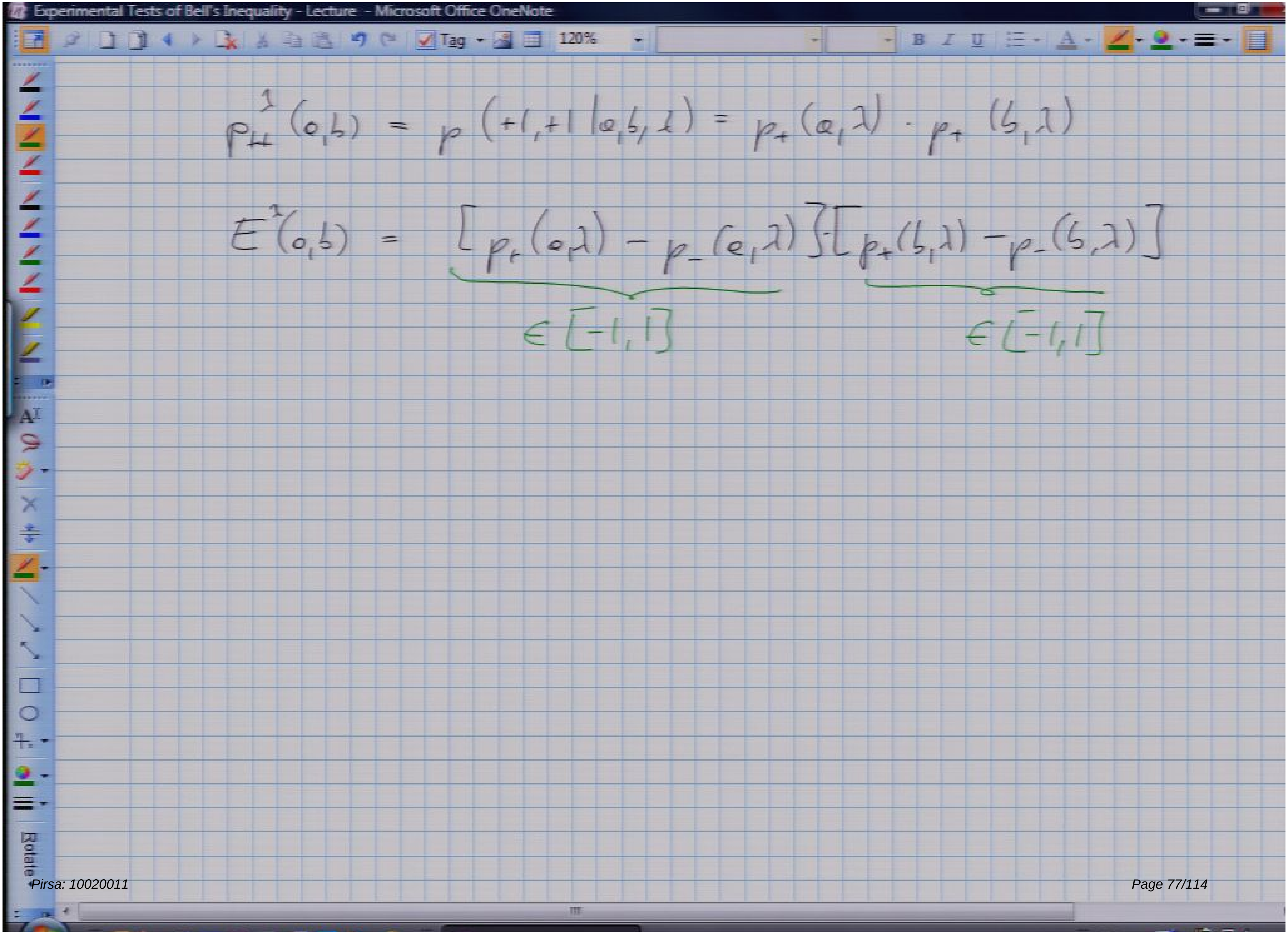
3) Freedom a, b can be chosen independently of λ

$$\langle AB \rangle = E(a,b) := p_{++}(a,b) + p_{--}(a,b) - p_{+-}(a,b) - p_{-+}(a,b)$$

$$\in [-1,1]$$

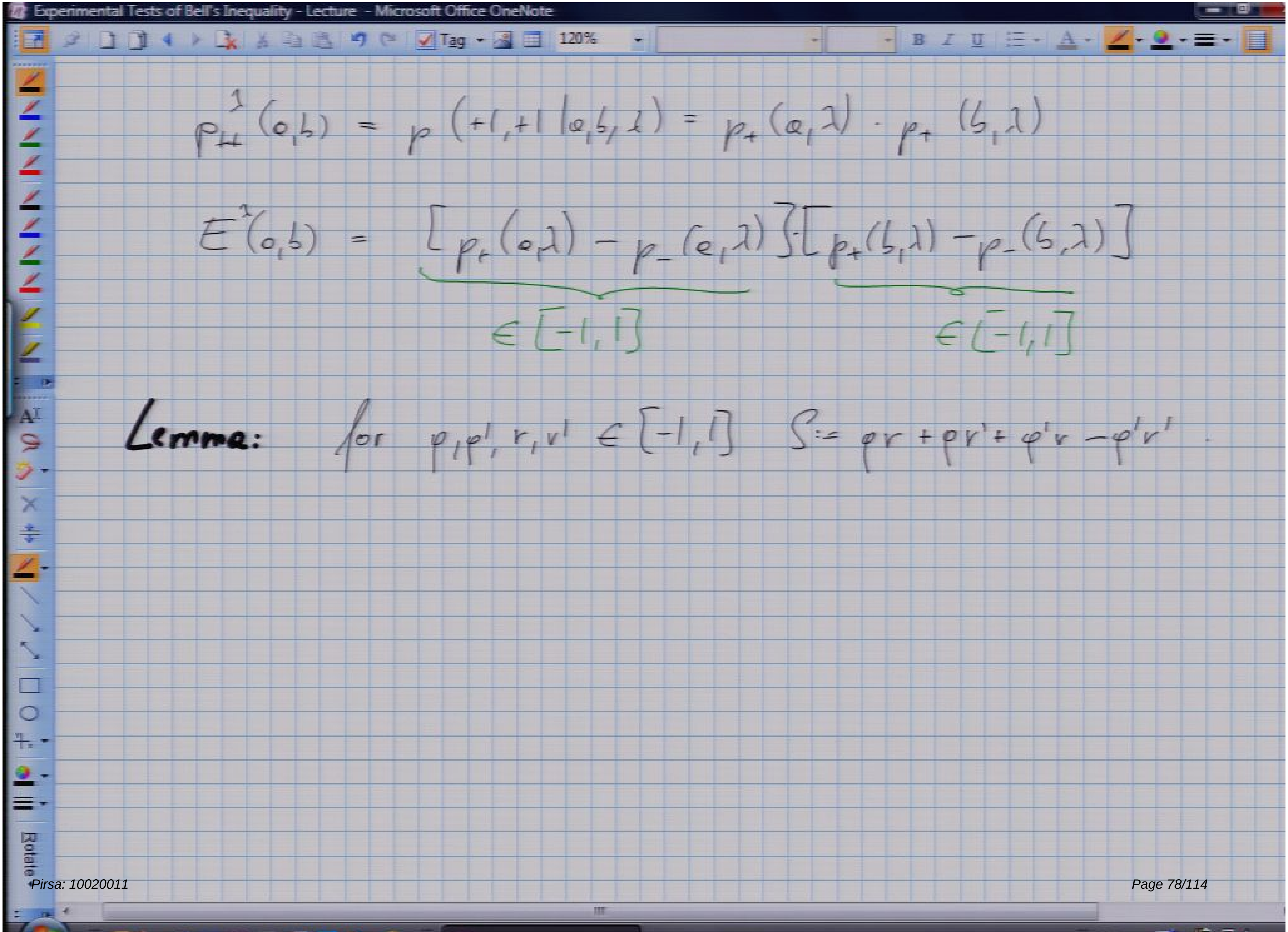
$$p^{\lambda}_{++}(a,b) = p(+1,+1|a,b,\lambda) = p_{+}(a,\lambda) \cdot p_{+}(b,\lambda)$$

$$E^{\lambda}(a,b) = [p_{+}(a,\lambda) - p_{-}(a,\lambda)] \cdot [p_{+}(b,\lambda) - p_{-}(b,\lambda)]$$



$$p^{\lambda}_{++}(a,b) = p(+1,+1\,|\,a,b,\lambda) = p_+(a,\lambda)\cdot p_+(b,\lambda)$$

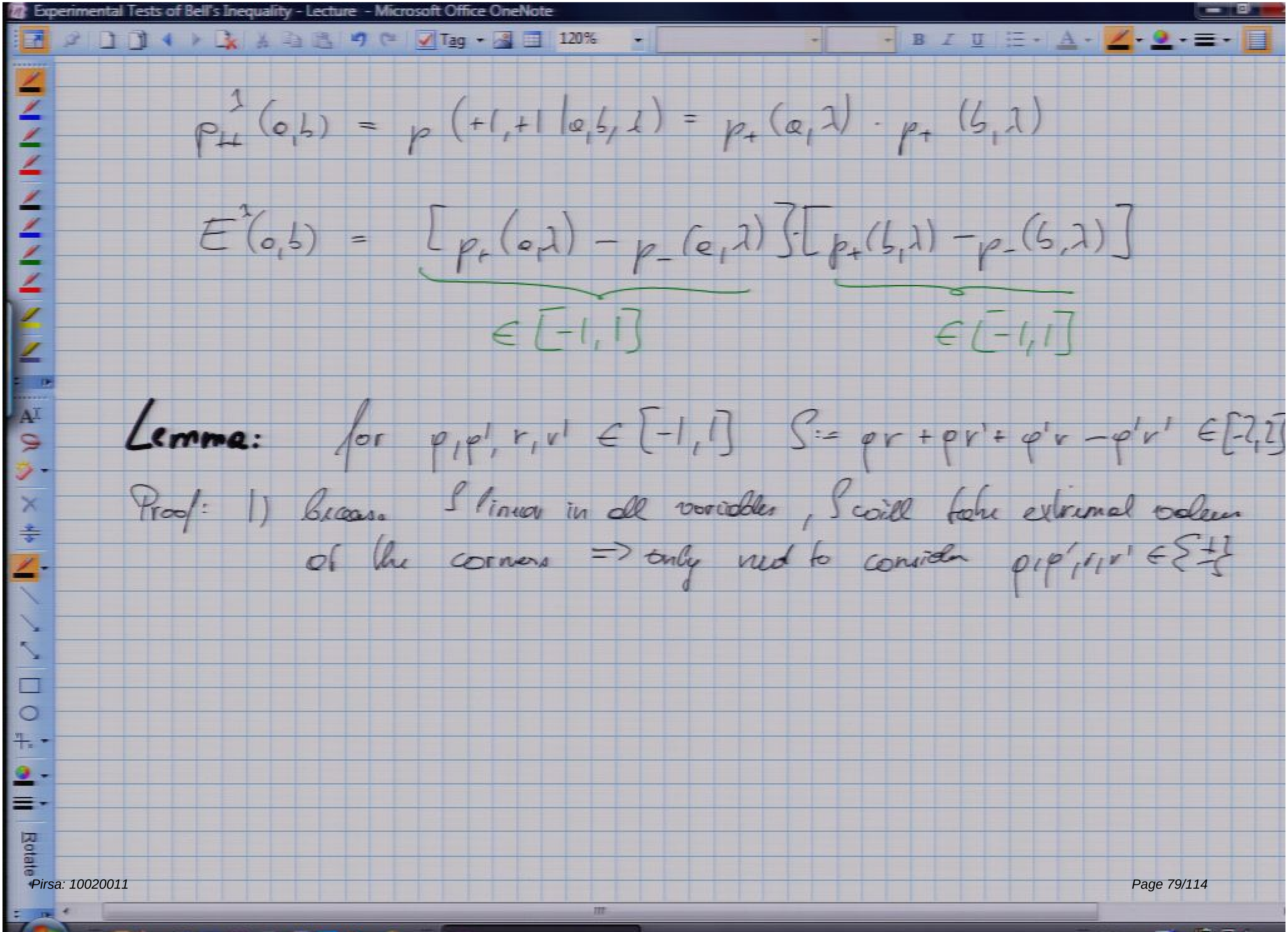
$$E^{\lambda}(a,b) = \underbrace{[p_+(a,\lambda) - p_-(a,\lambda)]}_{\in[-1,1]}\cdot\underbrace{[p_+(b,\lambda) - p_-(b,\lambda)]}_{\in[-1,1]}$$



$$p^{\lambda}_{++}(a,b) = p(+1,+1|a,b,\lambda) = p_+(a,\lambda)\cdot p_+(b,\lambda)$$

$$E^{\lambda}(a,b) = \underbrace{[p_+(a,\lambda) - p_-(a,\lambda)]}_{\in[-1,1]}\cdot\underbrace{[p_+(b,\lambda) - p_-(b,\lambda)]}_{\in[-1,1]}$$

Lemma: for $\varphi,\varphi',r,r' \in [-1,1]$ $\quad S := \varphi r + \varphi r' + \varphi' r - \varphi' r'$.

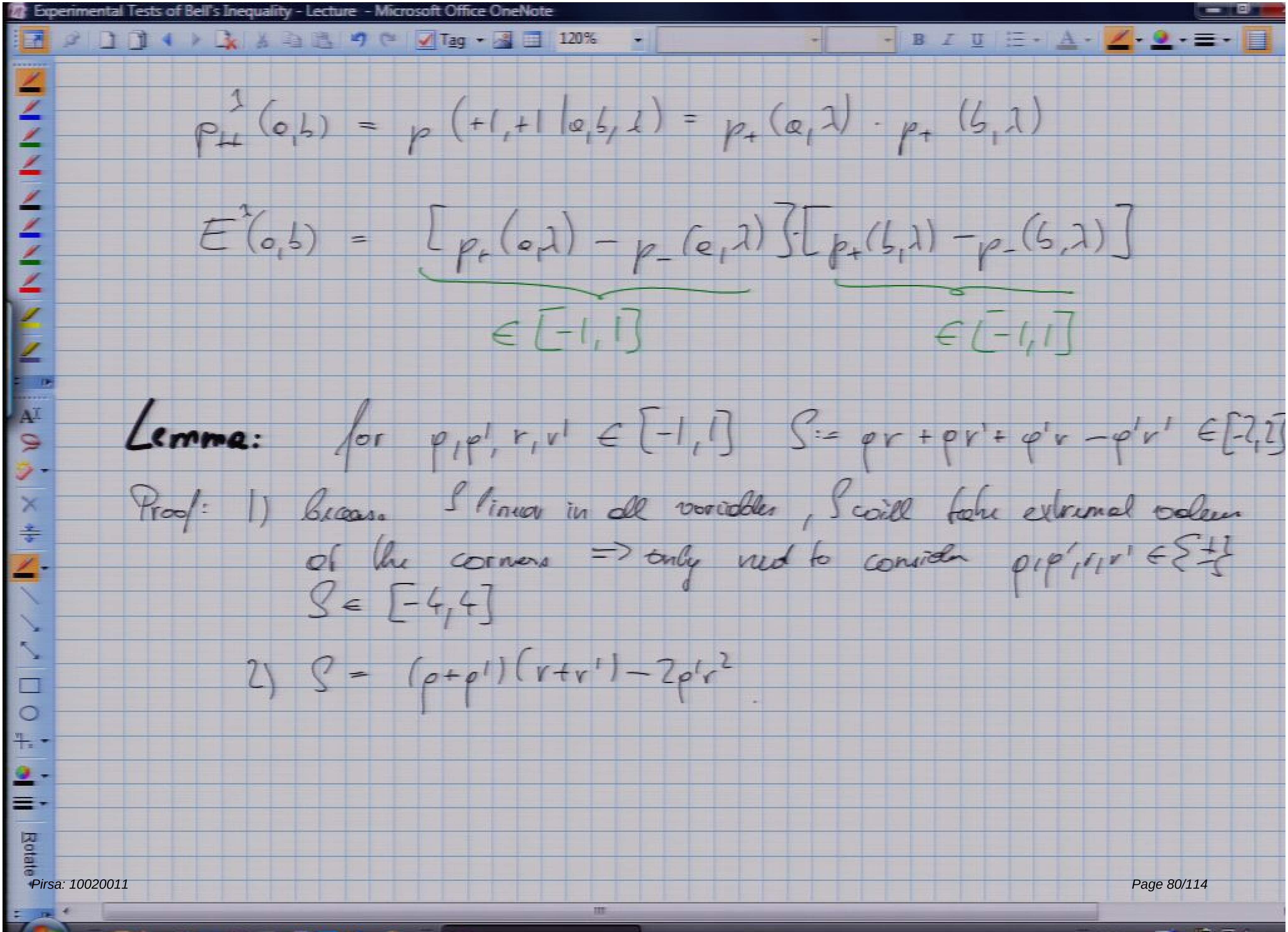


$$p^{\lambda}_{++}(a,b) = p(+1,+1|a,b,\lambda) = p_+(a,\lambda)\cdot p_+(b,\lambda)$$

$$E^{\lambda}(a,b) = \underbrace{[p_+(a,\lambda)-p_-(a,\lambda)]}_{\in[-1,1]}\cdot\underbrace{[p_+(b,\lambda)-p_-(b,\lambda)]}_{\in[-1,1]}$$

Lemma: for $\varphi,\varphi',r,r' \in [-1,1]$ $S := \varphi r + \varphi r' + \varphi' r - \varphi' r' \in [-2,2]$

Proof: 1) Because S linear in all variables, S will take extremal values of the corners $\Rightarrow$ only need to consider $\varphi,\varphi',r,r' \in \{\pm 1\}$



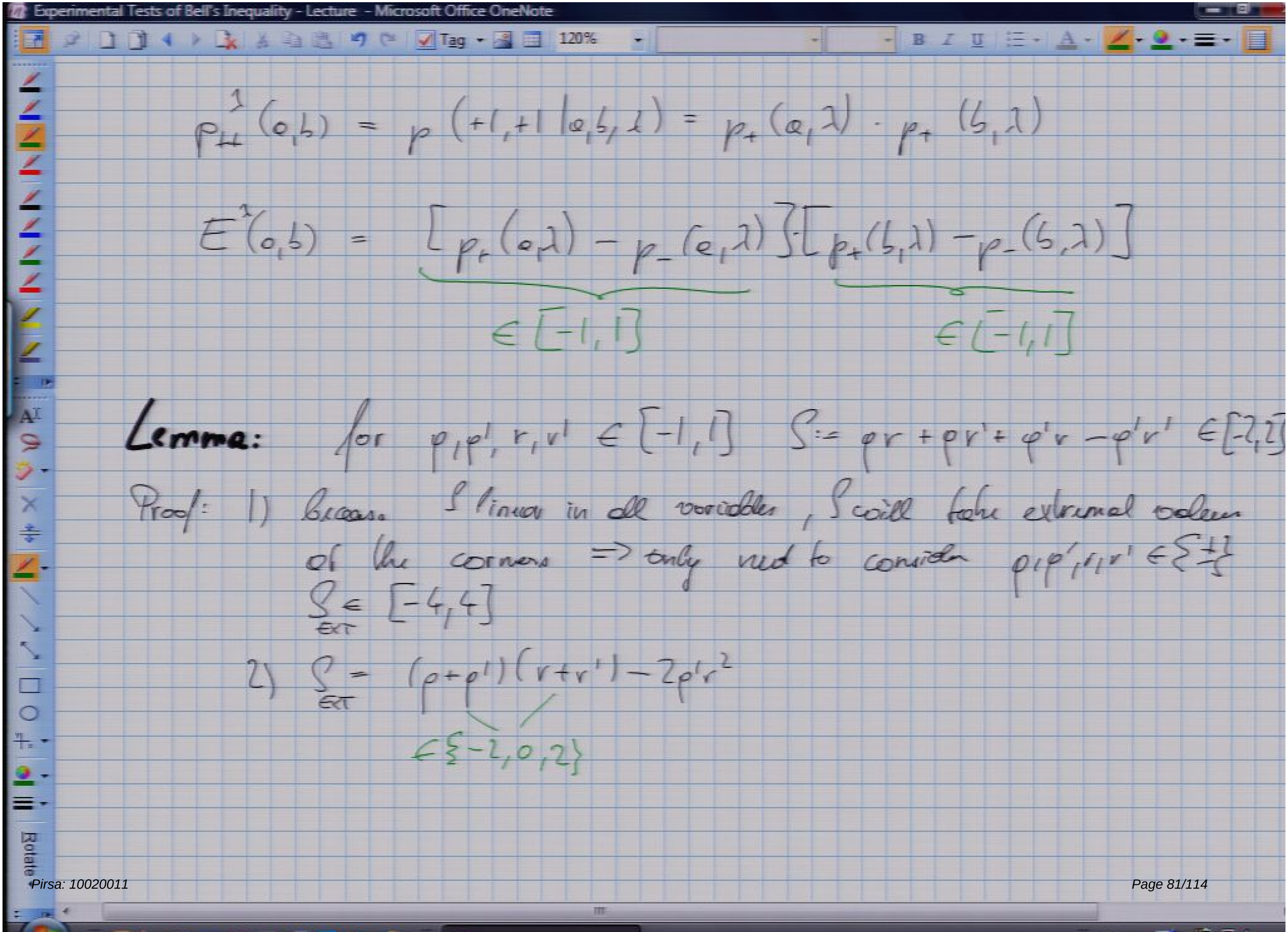
$$p_{++}^{\lambda}(a,b) = p(+1,+1|a,b,\lambda) = p_+(a,\lambda)\cdot p_+(b,\lambda)$$

$$E^{\lambda}(a,b) = \underbrace{[p_+(a,\lambda)-p_-(a,\lambda)]}_{\in[-1,1]}\cdot\underbrace{[p_+(b,\lambda)-p_-(b,\lambda)]}_{\in[-1,1]}$$

Lemma: for $p,p',r,r' \in [-1,1]$ $S := pr + pr' + p'r - p'r' \in [-2,2]$

Proof: 1) because S linear in all variables, S will take extremal values of the corners $\Rightarrow$ only need to consider $p,p',r,r' \in \{\pm 1\}$
$S \in [-4,4]$

2) $S = (p+p')(r+r') - 2p'r^2$.



$$p_{++}^{\lambda}(a,b) = p(+1,+1|a,b,\lambda) = p_+(a,\lambda)\cdot p_+(b,\lambda)$$

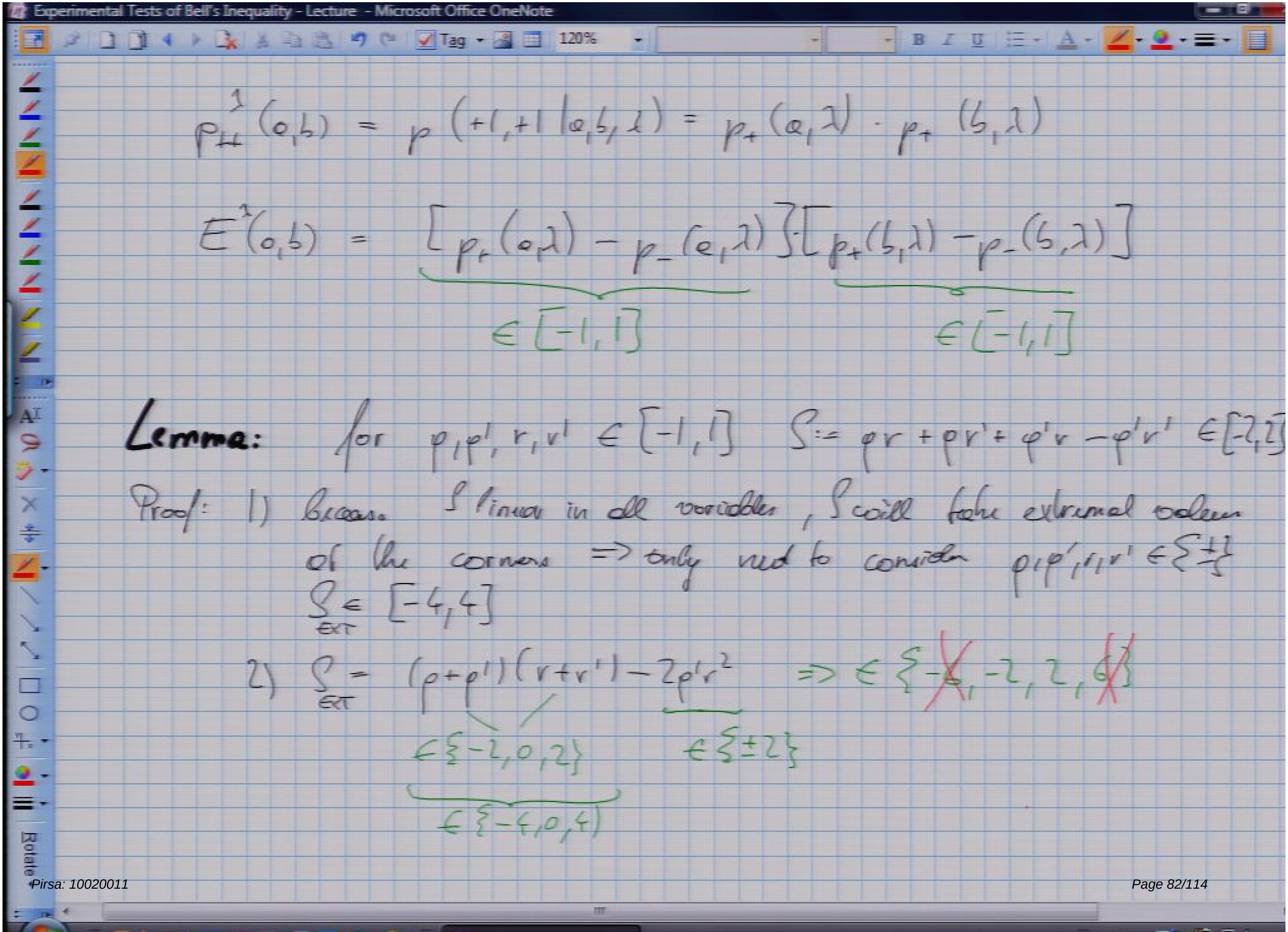
$$E^{\lambda}(a,b) = \underbrace{[p_+(a,\lambda) - p_-(a,\lambda)]}_{\in[-1,1]}\cdot\underbrace{[p_+(b,\lambda) - p_-(b,\lambda)]}_{\in[-1,1]}$$

Lemma: for $\rho,\rho',r,r' \in [-1,1]$ $S := \rho r + \rho r' + \rho' r - \rho' r' \in [-2,2]$

Proof: 1) because S linear in all variables, S will take extremal values of the corners => only need to consider $\rho,\rho',r,r' \in \{\pm 1\}$

$S_{EXT} \in [-4,4]$

2) $S_{EXT} = \underbrace{(\rho+\rho')(r+r')}_{\in\{-2,0,2\}} - 2\rho' r^2$



$$p^{\lambda}_{++}(a,b) = p(+1,+1|a,b,\lambda) = p_+(a,\lambda)\cdot p_+(b,\lambda)$$

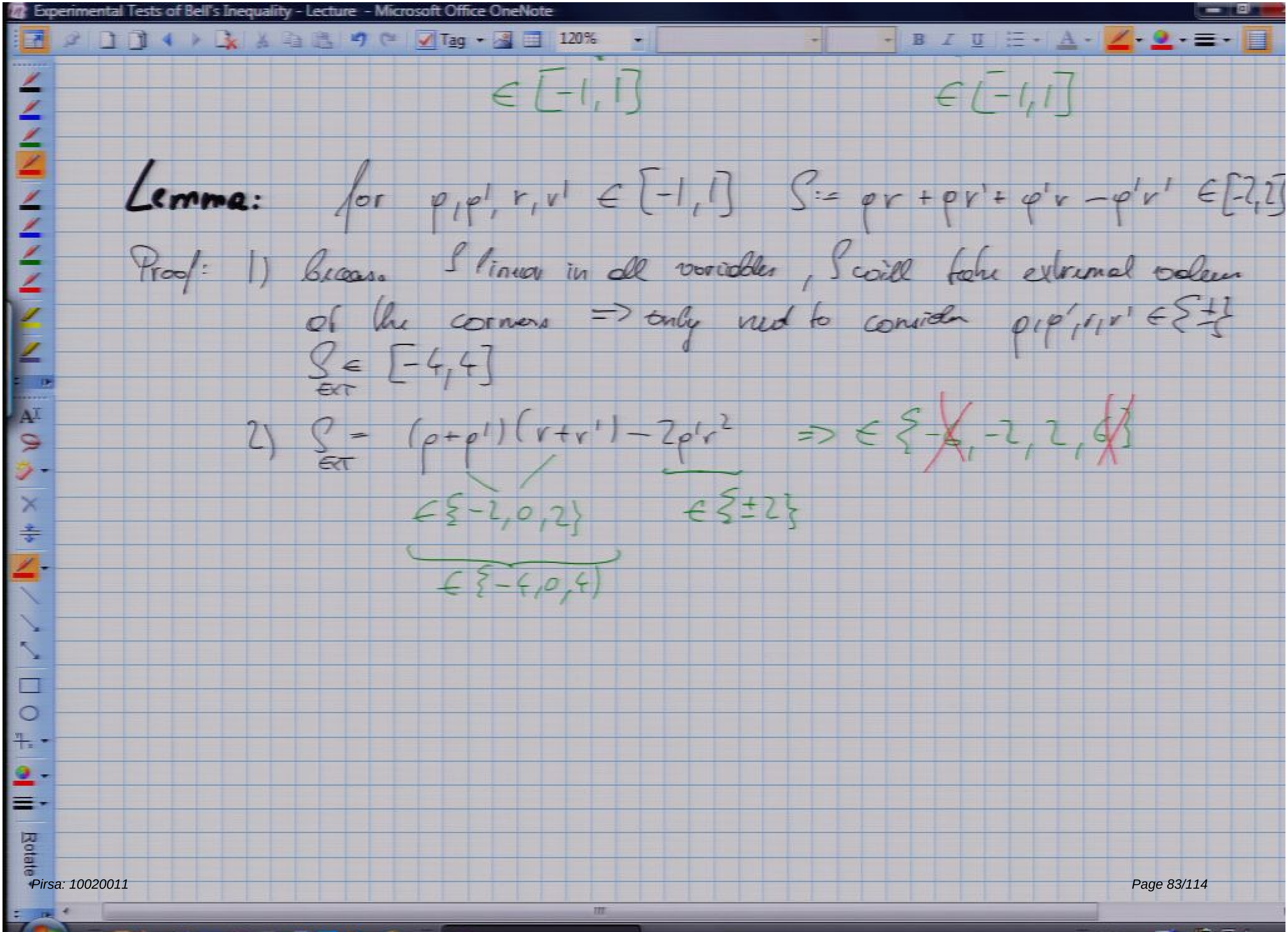
$$E^{\lambda}(a,b) = \underbrace{[p_+(a,\lambda)-p_-(a,\lambda)]}_{\in[-1,1]}\cdot\underbrace{[p_+(b,\lambda)-p_-(b,\lambda)]}_{\in[-1,1]}$$

Lemma: for $\rho,\rho',r,r' \in [-1,1]$ $S := \rho r + \rho r' + \rho' r - \rho' r' \in [-2,2]$

Proof: 1) Because S linear in all variables, S will take extremal values of the corners $\Rightarrow$ only need to consider $\rho,\rho',r,r' \in \{\pm 1\}$

$S_{EXT} \in [-4,4]$

2) $$S_{EXT} = \underbrace{\underbrace{(\rho+\rho')}\underbrace{(r+r')}_{\in\{-2,0,2\}}}_{\in\{-4,0,4\}} - \underbrace{2\rho' r^2}_{\in\{\pm 2\}} \Rightarrow \in\{-\cancel{6},-2,2,\cancel{6}\}$$



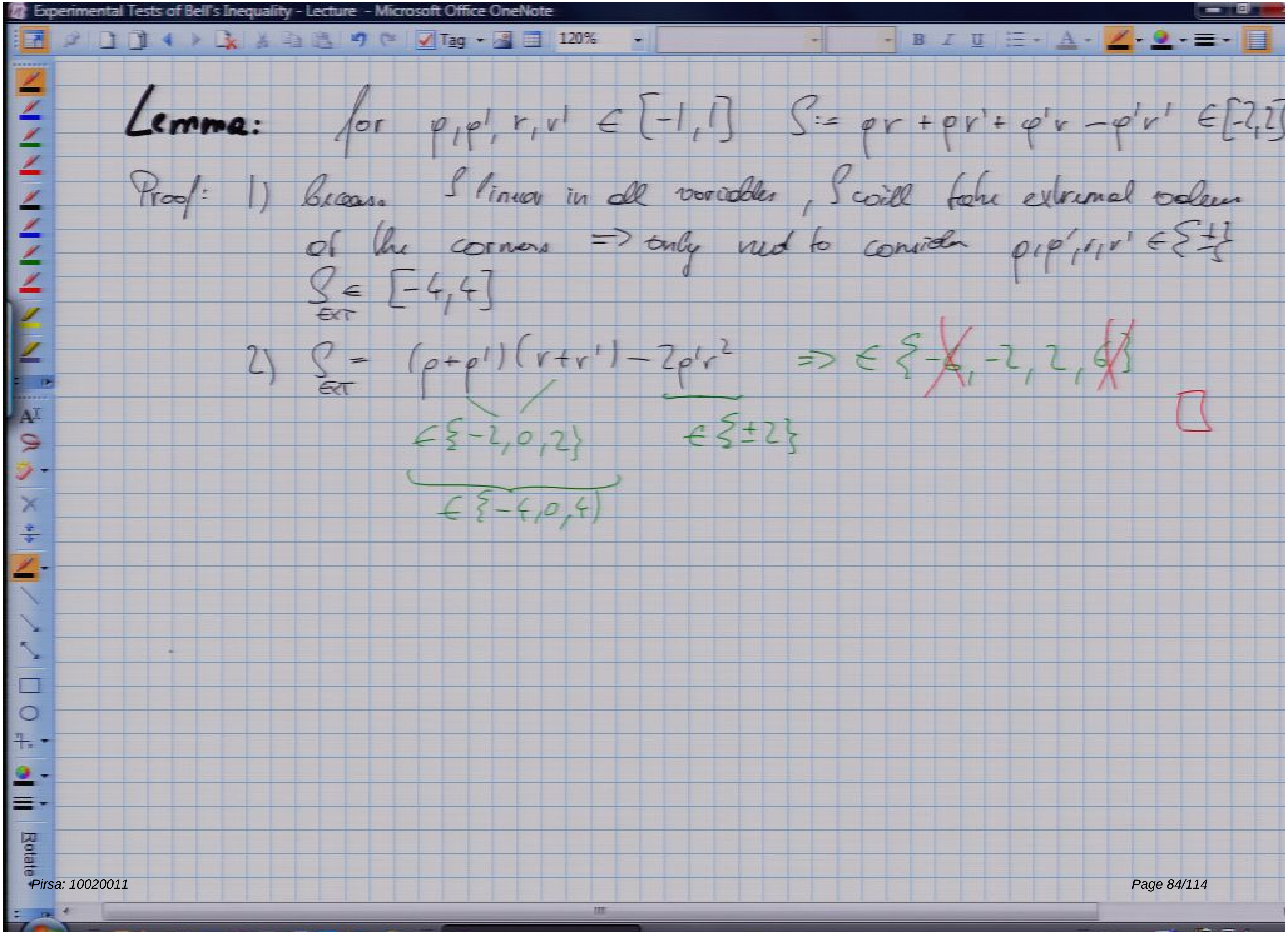
$\in [-1,1]$ $\in [-1,1]$

Lemma: for $\rho,\rho',r,r' \in [-1,1]$ $S := \rho r + \rho r' + \rho' r - \rho' r' \in [-2,2]$

Proof: 1) because S linear in all variables, S will take extremal values of the corners ⇒ only need to consider $\rho,\rho',r,r' \in \{\pm 1\}$

$S_{EXT} \in [-4,4]$

2) $$S_{EXT} = \underbrace{\underbrace{(\rho+\rho')(r+r')}_{\in\{-2,0,2\}}}_{\in\{-4,0,4)} - \underbrace{2\rho' r^2}_{\in\{\pm 2\}} \Rightarrow \in \{\cancel{-6}, -2, 2, \cancel{6}\}$$



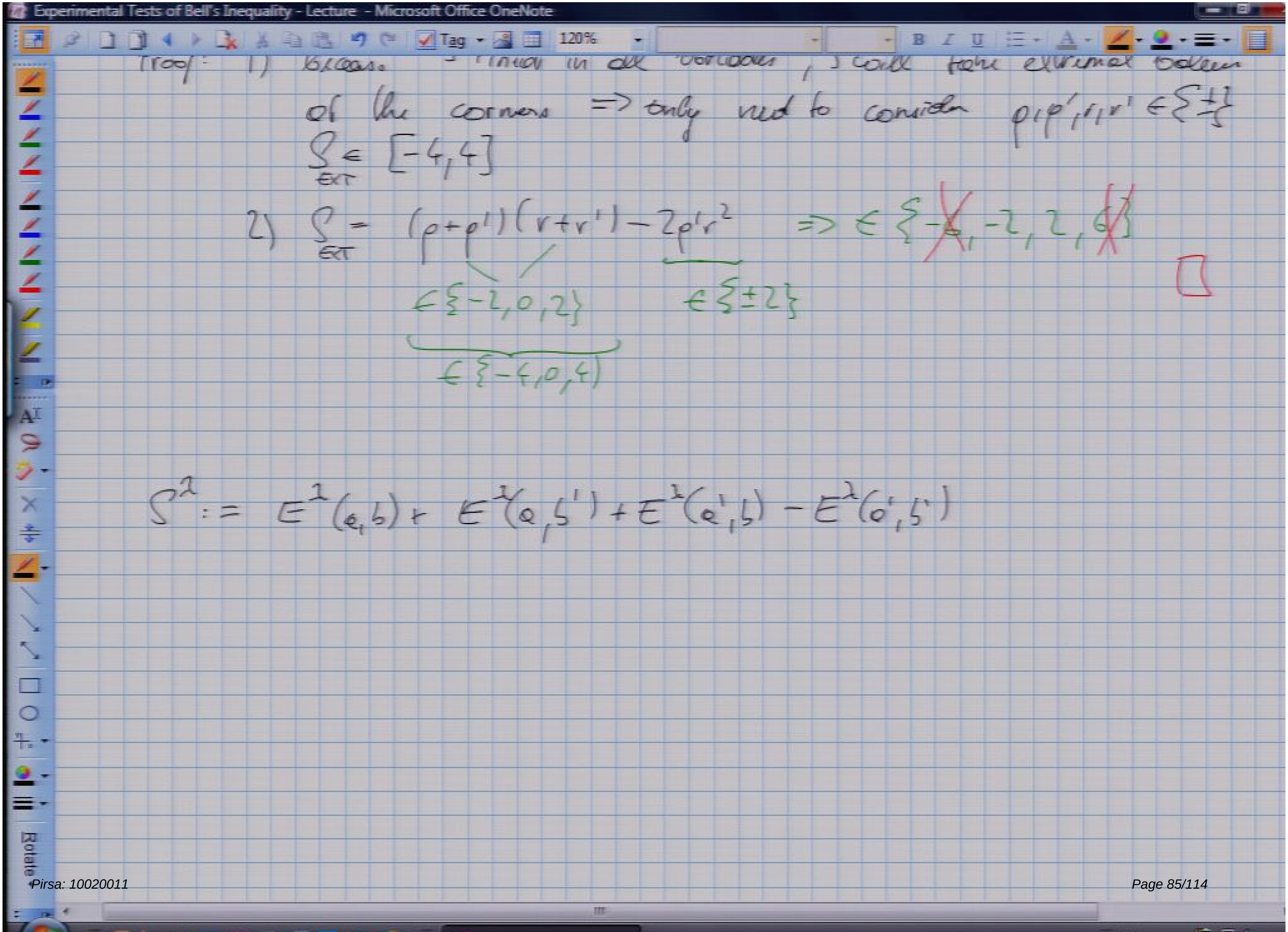
Lemma: for $\rho, \rho', r, r' \in [-1,1]$ $\quad S := \rho r + \rho r' + \rho' r - \rho' r' \in [-2,2]$

Proof: 1) because S linear in all variables, S will take extremal values of the corners $\Rightarrow$ only need to consider $\rho, \rho', r, r' \in \{\pm 1\}$

$S_{EXT} \in [-4,4]$

2) $$S_{EXT} = \underbrace{\underbrace{(\rho+\rho')(r+r')}_{\in\{-2,0,2\}}}_{\in\{-4,0,4)} - \underbrace{2\rho' r^2}_{\in\{\pm 2\}} \Rightarrow \in \{\cancel{-6}, -2, 2, \cancel{6}\}$$

$\square$



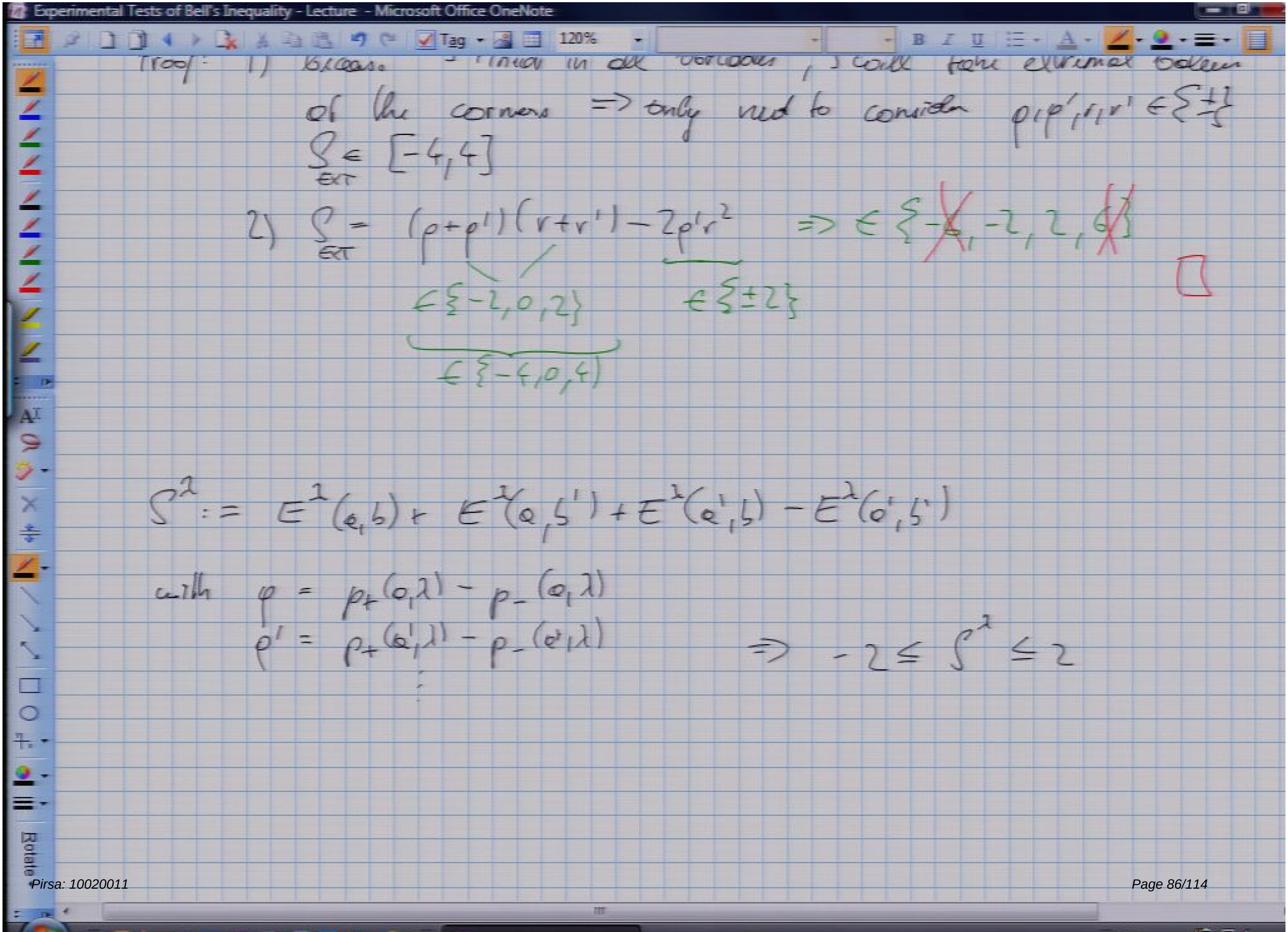
Proof: 1) because S linear in all variables, S will take extremal values of the corners ⇒ only need to consider $\rho, \rho', r, r' \in \{\pm 1\}$

$S_{EXT} \in [-4,4]$

2) $S_{EXT} = \underbrace{\underbrace{(\rho+\rho')}_{\in\{-2,0,2\}}(r+r')}_{\in\{-4,0,4)} - \underbrace{2\rho' r^2}_{\in\{\pm 2\}} \Rightarrow \in \{\cancel{-6}, -2, 2, \cancel{6}\}$

$\square$

$$S^\lambda := E^\lambda(a,b) + E^\lambda(a,b') + E^\lambda(a',b) - E^\lambda(a',b')$$



Proof: 1) becaus. linear in all variables, S will take extremal values of the corners => only need to consider $\rho, \rho', r, r' \in \{\pm 1\}$

$S_{EXT} \in [-4,4]$

2) $$S_{EXT} = \underbrace{\underbrace{(\rho+\rho')(r+r')}_{\in\{-2,0,2\}}}_{\in\{-4,0,4\}} - \underbrace{2\rho' r^2}_{\in\{\pm 2\}} \Rightarrow \in \{\cancel{-6}, -2, 2, \cancel{6}\}$$

□

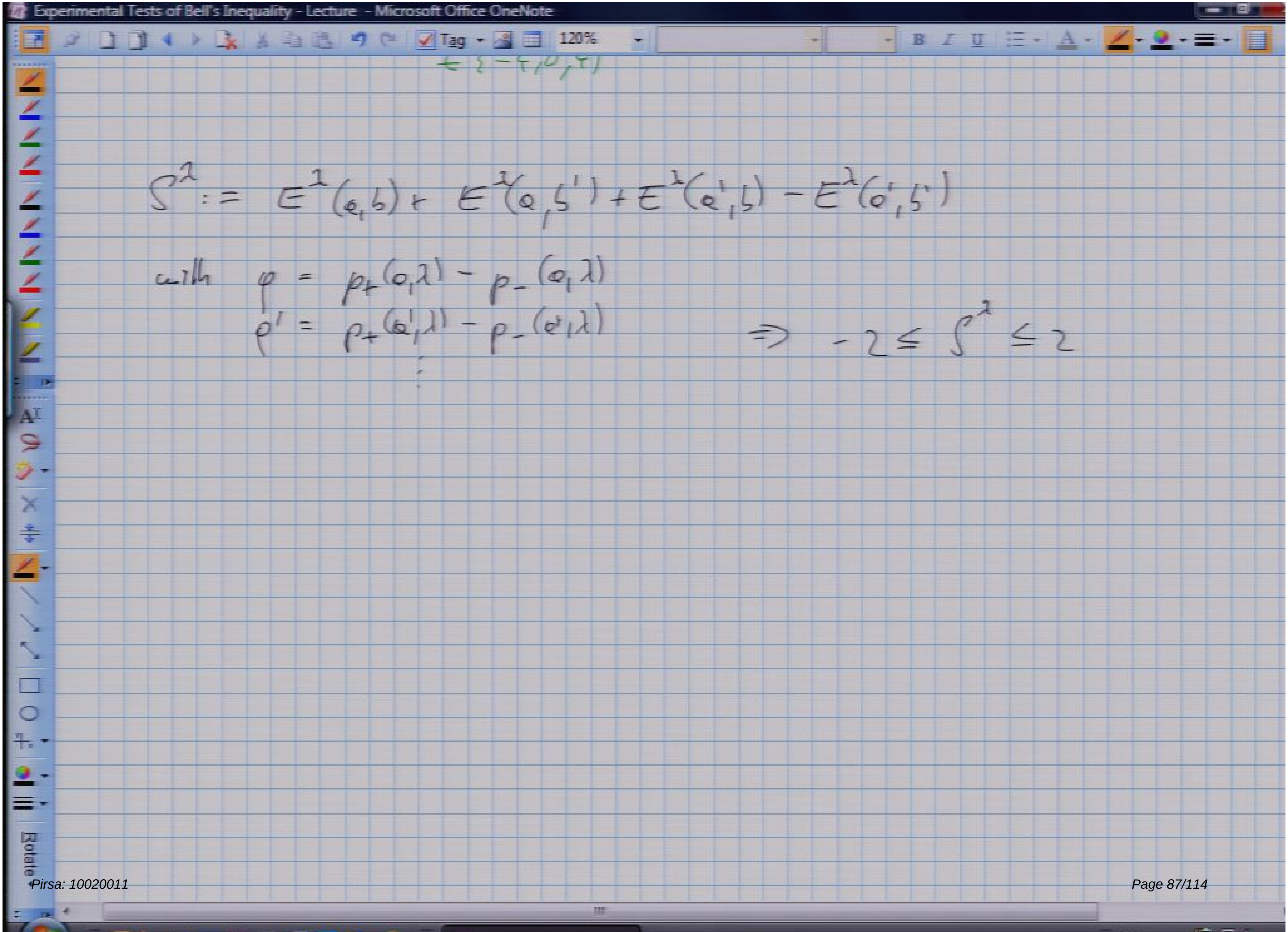
$$S^\lambda := E^\lambda(a,b) + E^\lambda(a,b') + E^\lambda(a',b) - E^\lambda(a',b')$$

with $\rho = p_+(a,\lambda) - p_-(a,\lambda)$

$\rho' = p_+(a',\lambda) - p_-(a',\lambda)$

$\vdots$

$\Rightarrow -2 \le S^\lambda \le 2$



$\in \{-1, 0, 1\}$

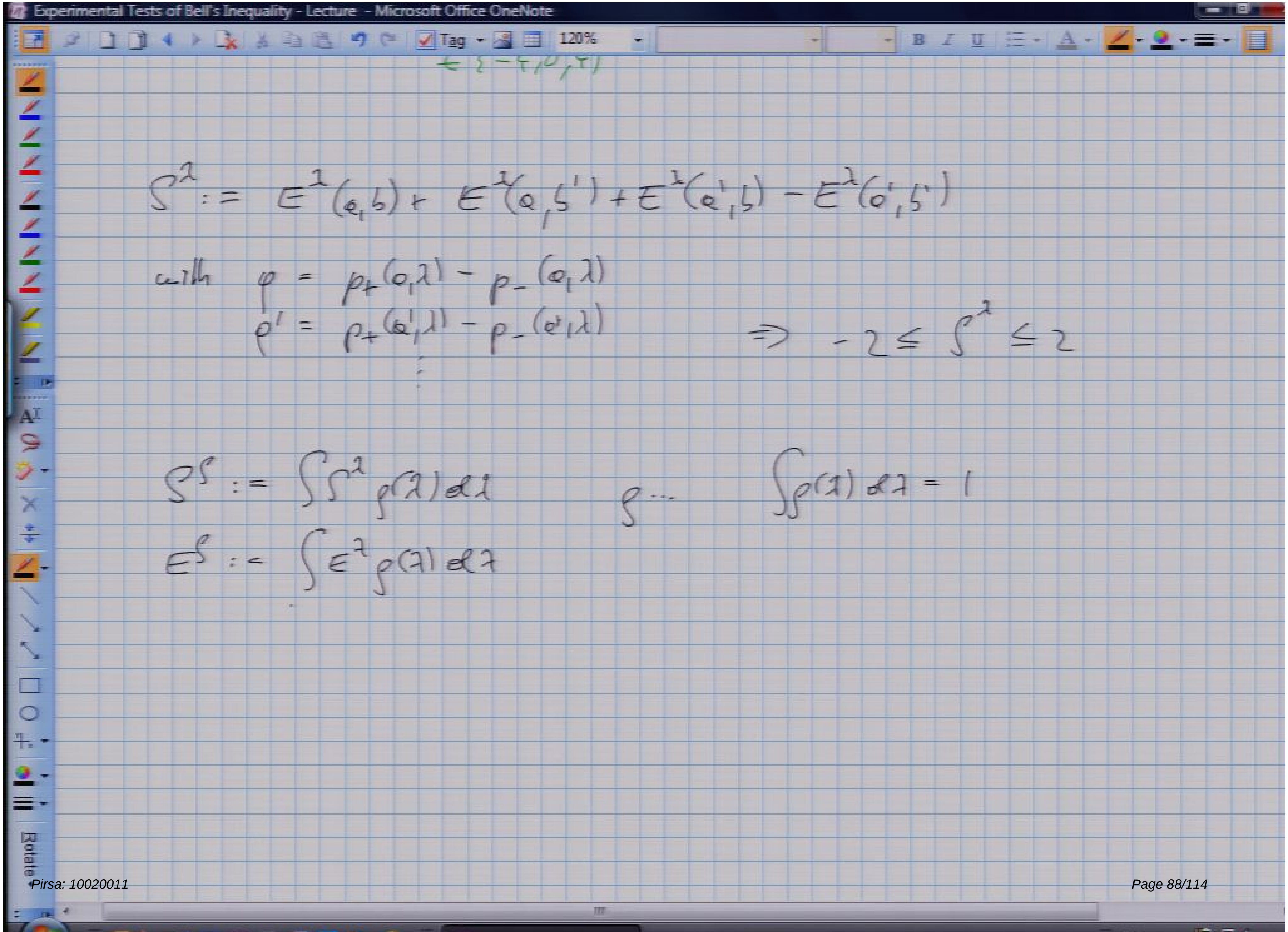
$$S^\lambda := E^\lambda(a,b) + E^\lambda(a,b') + E^\lambda(a',b) - E^\lambda(a',b')$$

with $q = p_+(a,\lambda) - p_-(a,\lambda)$

$q' = p_+(a',\lambda) - p_-(a',\lambda)$

$\vdots$

$$\Rightarrow \quad -2 \le S^\lambda \le 2$$



$\in \{-1, 0, 1\}$

$$S^\lambda := E^\lambda(a,b) + E^\lambda(a,b') + E^\lambda(a',b) - E^\lambda(a',b')$$

with $\varphi = p_+(a,\lambda) - p_-(a,\lambda)$

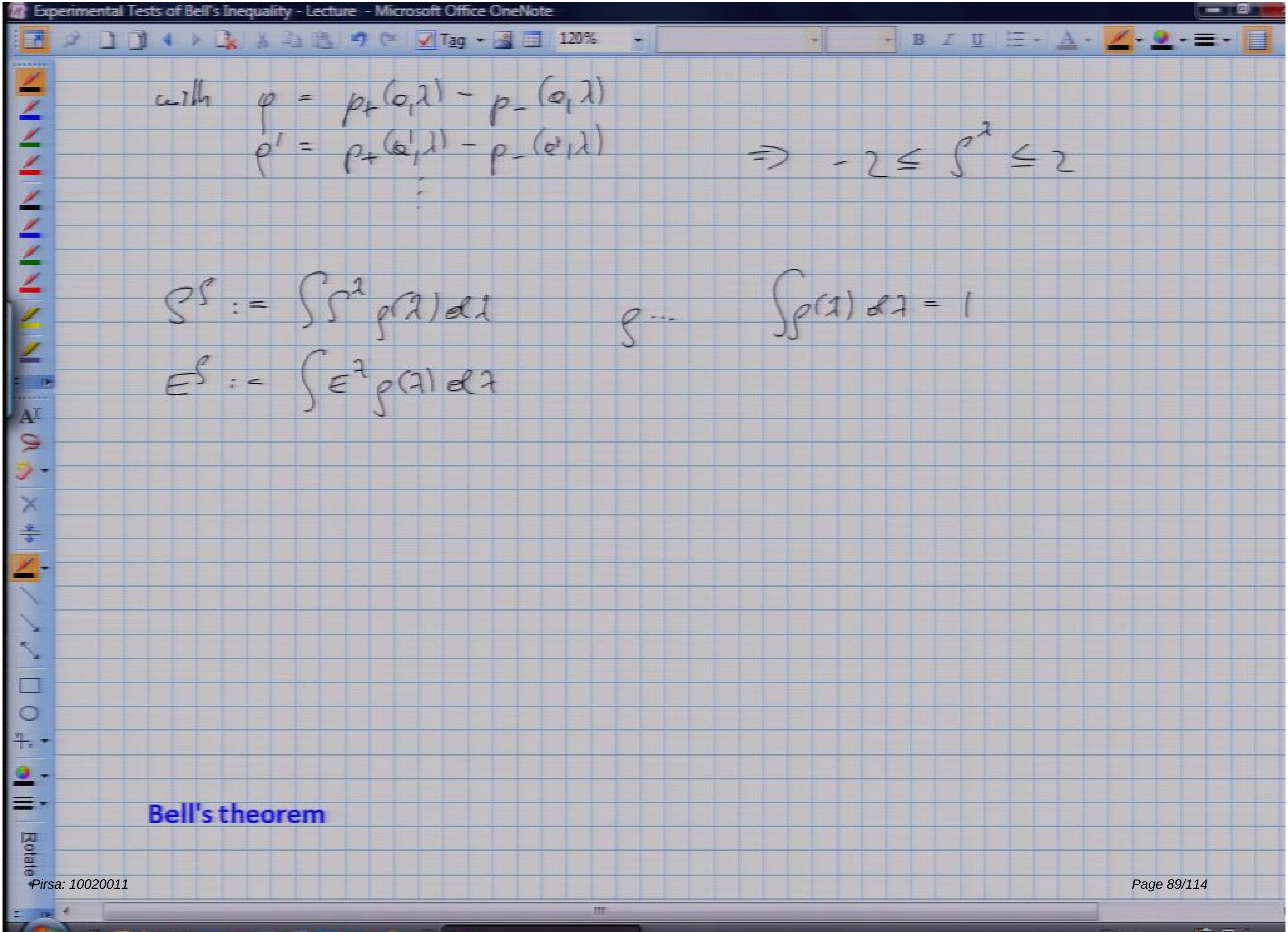
$\rho' = p_+(a',\lambda) - p_-(a',\lambda)$

$\vdots$

$\Rightarrow -2 \leq S^\lambda \leq 2$

$S^\rho := \int S^\lambda \rho(\lambda) d\lambda$ $\rho \cdots$ $\int \rho(\lambda) d\lambda = 1$

$E^\rho := \int E^\lambda \rho(\lambda) d\lambda$



with $\varphi = p_+(a,\lambda) - p_-(a,\lambda)$

$\rho' = p_+(a',\lambda) - p_-(a',\lambda)$

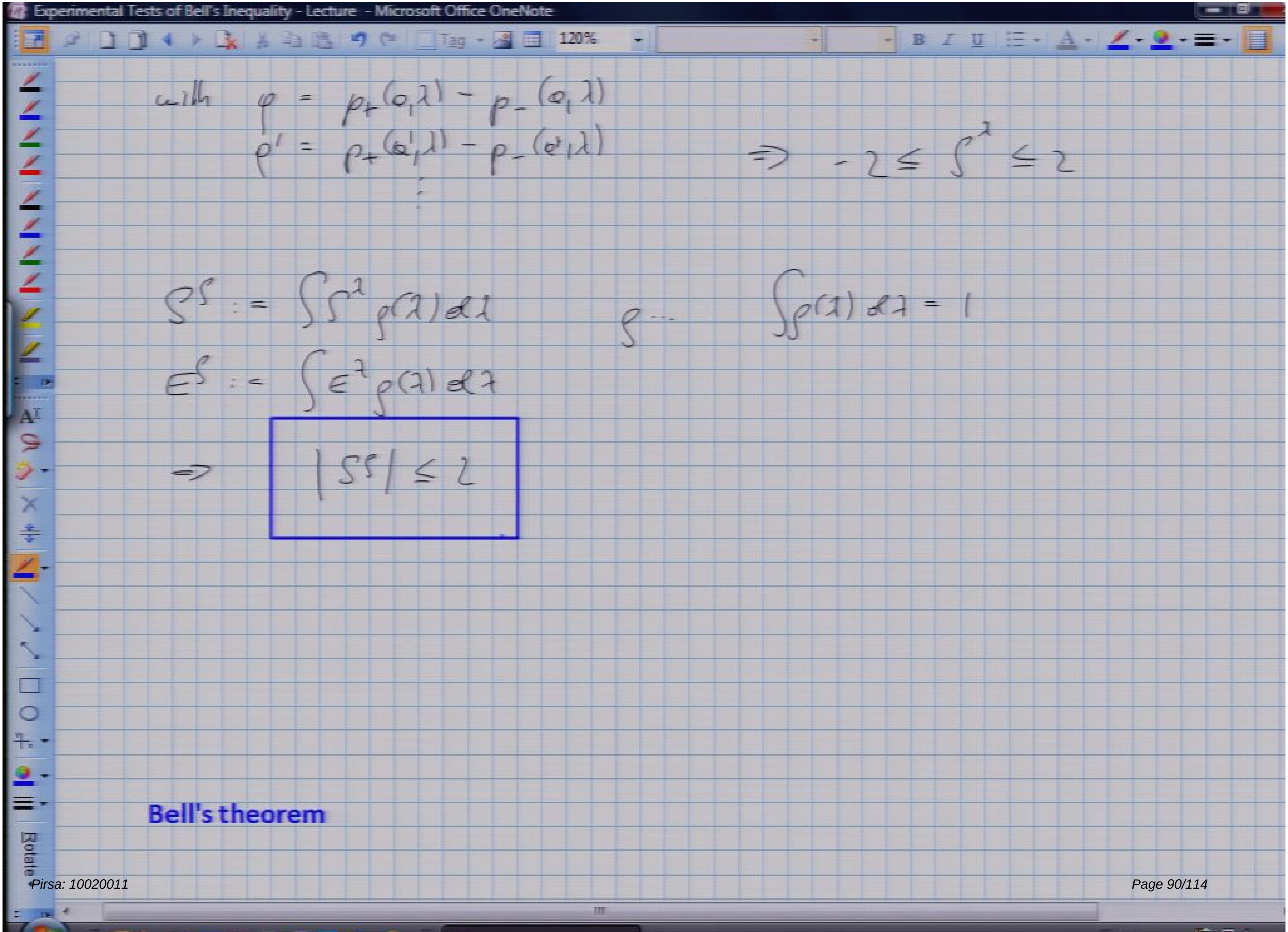
$\vdots$

$\Rightarrow -2 \leq S^\lambda \leq 2$

$S^\rho := \int S^\lambda \rho(\lambda) d\lambda$ $\quad \rho \cdots \quad$ $\int \rho(\lambda) d\lambda = 1$

$E^\rho := \int E^\lambda \rho(\lambda) d\lambda$

Bell's theorem



with $\varphi = p_+(a,\lambda) - p_-(a,\lambda)$

$\rho' = p_+(a',\lambda) - p_-(a',\lambda)$

$\vdots$

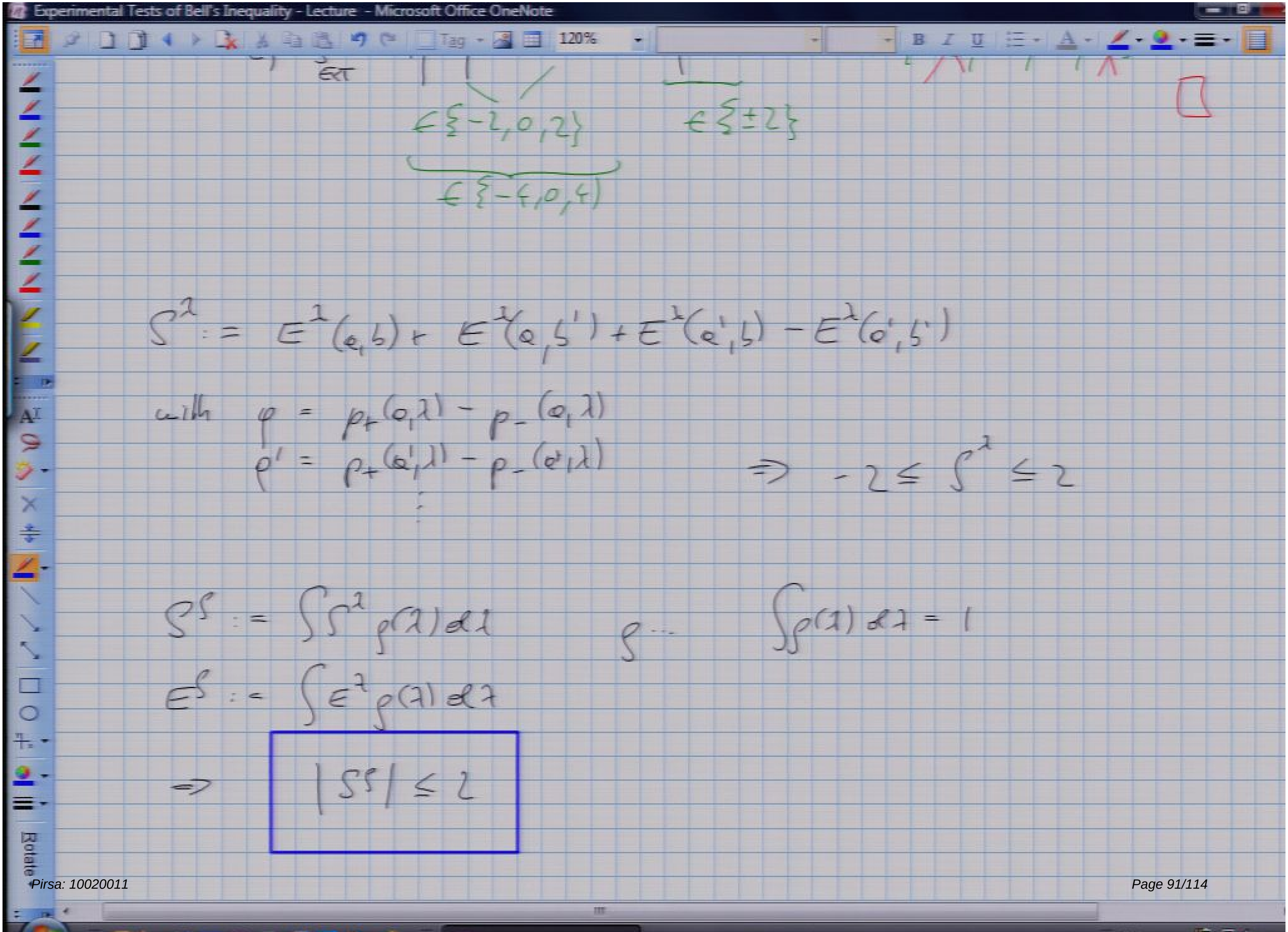
$\Rightarrow \quad -2 \le S^\lambda \le 2$

$$S^\rho := \int S^\lambda \rho(\lambda)\, d\lambda \qquad \rho \cdots \qquad \int \rho(\lambda)\, d\lambda = 1$$

$$E^\rho := \int E^\lambda \rho(\lambda)\, d\lambda$$

$$\Rightarrow \quad \boxed{|S^\rho| \le 2}$$

Bell's theorem



$\in \{-2,0,2\}$ $\qquad \in \{\pm 2\}$

$\in \{-4,0,4)$

$$S^\lambda := E^\lambda(a,b) + E^\lambda(a,b') + E^\lambda(a',b) - E^\lambda(a',b')$$

with $p = p_+(a,\lambda) - p_-(a,\lambda)$

$p' = p_+(a',\lambda) - p_-(a',\lambda)$

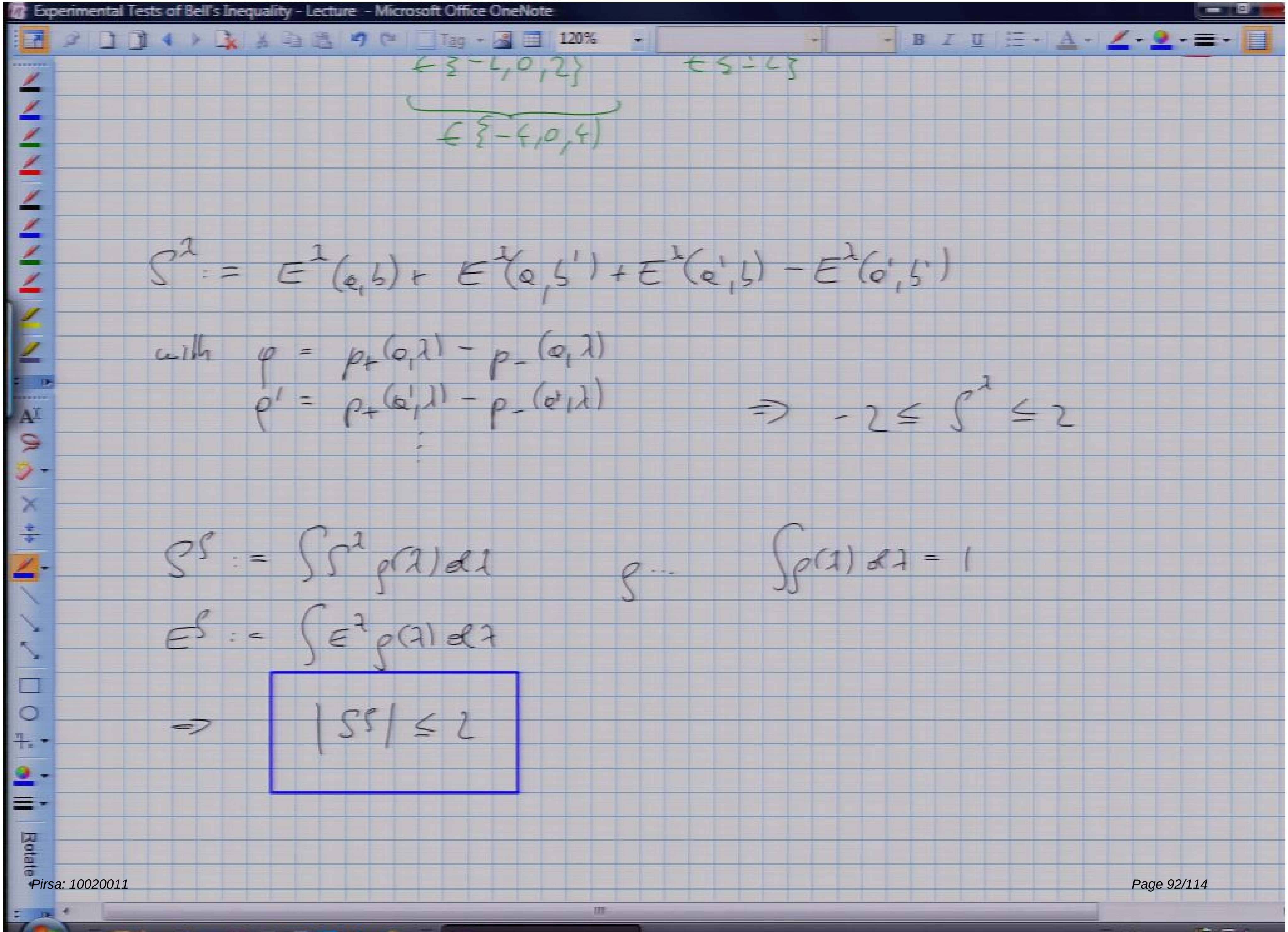
$\vdots$

$\Rightarrow -2 \leq S^\lambda \leq 2$

$S^\rho := \int S^\lambda \rho(\lambda) d\lambda \qquad \rho \cdots \qquad \int \rho(\lambda) d\lambda = 1$

$E^\rho := \int E^\lambda \rho(\lambda) d\lambda$

$\Rightarrow \boxed{|S^\rho| \leq 2}$



$$\underbrace{\in\{-2,0,2\}}_{\in\{-4,0,4\}} \qquad \in\{\pm 2\}$$

$$S^\lambda := E^\lambda(a,b) + E^\lambda(a,b') + E^\lambda(a',b) - E^\lambda(a',b')$$

with $p = p_+(a,\lambda) - p_-(a,\lambda)$

$p' = p_+(a',\lambda) - p_-(a',\lambda)$

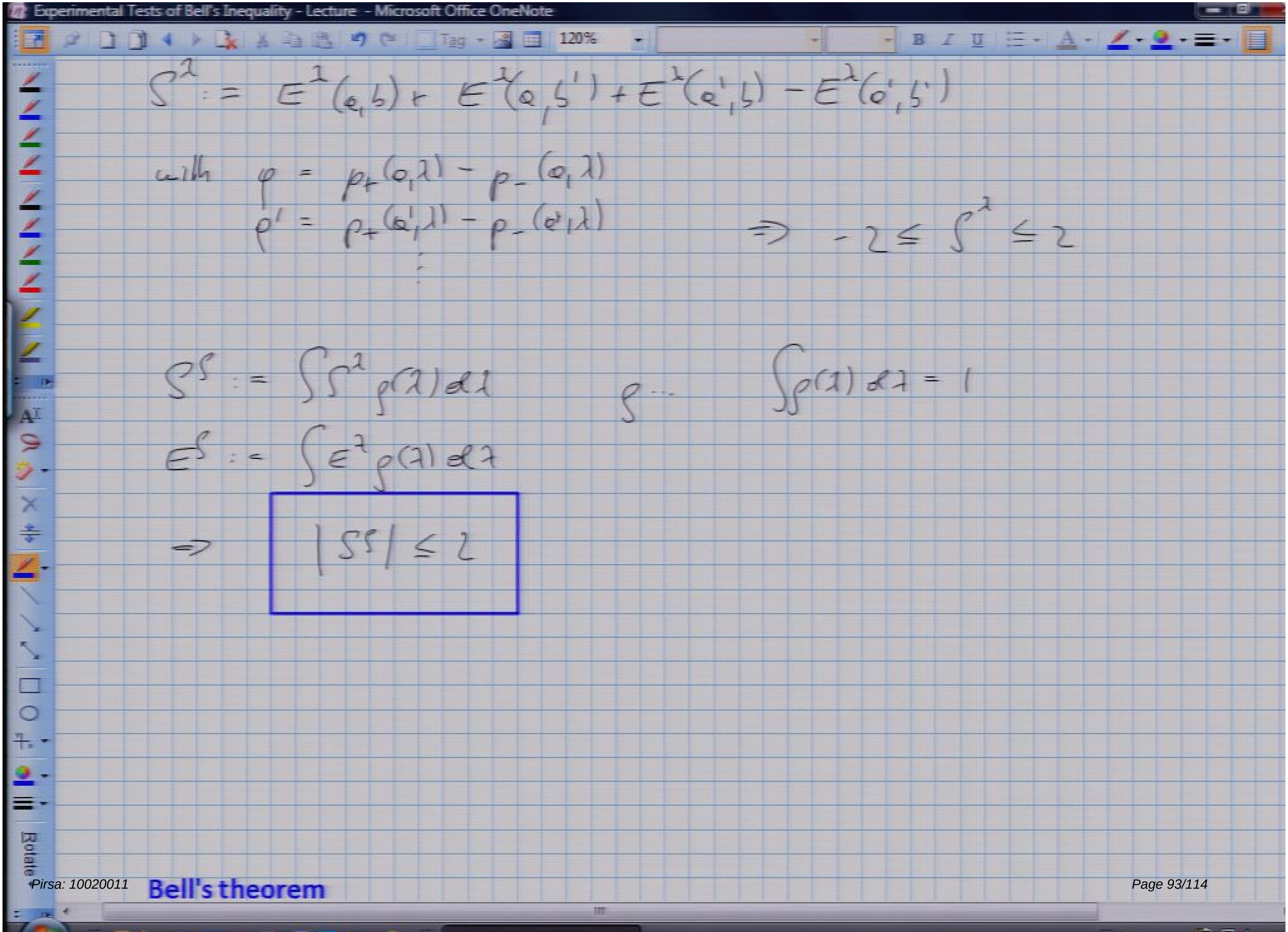
$\vdots$

$\Rightarrow -2 \le S^\lambda \le 2$

$$S^\rho := \int S^\lambda \rho(\lambda)\, d\lambda \qquad \rho \cdots \qquad \int \rho(\lambda)\, d\lambda = 1$$

$$E^\rho := \int E^\lambda \rho(\lambda)\, d\lambda$$

$$\Rightarrow \boxed{|S^\rho| \le 2}$$



$$S^\lambda := E^\lambda(a,b) + E^\lambda(a,b') + E^\lambda(a',b) - E^\lambda(a',b')$$

with $p = p_+(a,\lambda) - p_-(a,\lambda)$

$p' = p_+(a',\lambda) - p_-(a',\lambda)$

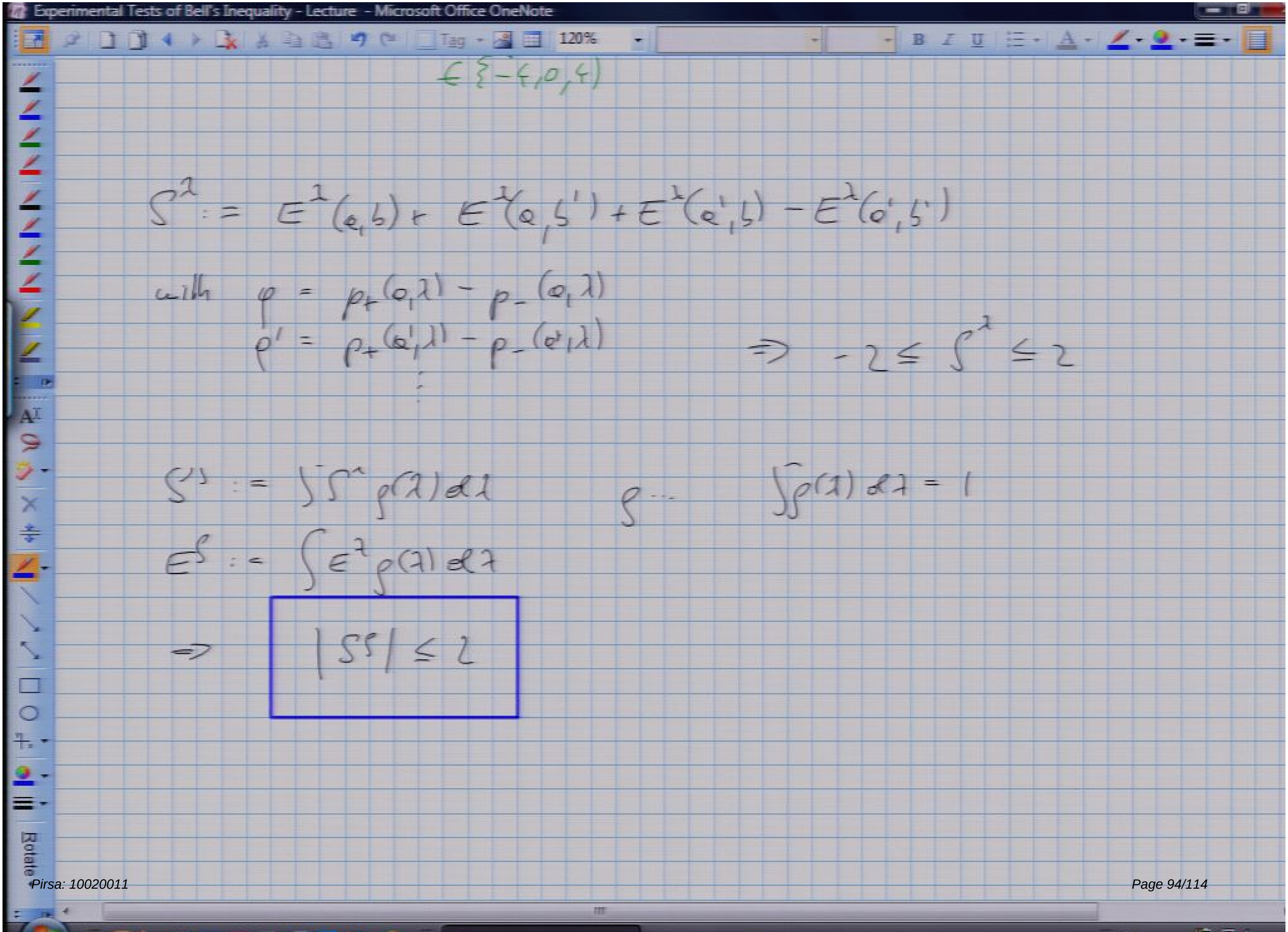
$\vdots$

$$\Rightarrow -2 \le S^\lambda \le 2$$

$$S^\rho := \int S^\lambda \rho(\lambda)\, d\lambda \qquad \rho \cdots \qquad \int \rho(\lambda)\, d\lambda = 1$$

$$E^\rho := \int E^\lambda \rho(\lambda)\, d\lambda$$

$$\Rightarrow \boxed{|S^\rho| \le 2}$$



$\in \{-1, 0, 1\}$

$$S^\lambda := E^\lambda(a,b) + E^\lambda(a,b') + E^\lambda(a',b) - E^\lambda(a',b')$$

with $\varphi = p_+(a,\lambda) - p_-(a,\lambda)$

$\rho' = p_+(a',\lambda) - p_-(a',\lambda)$

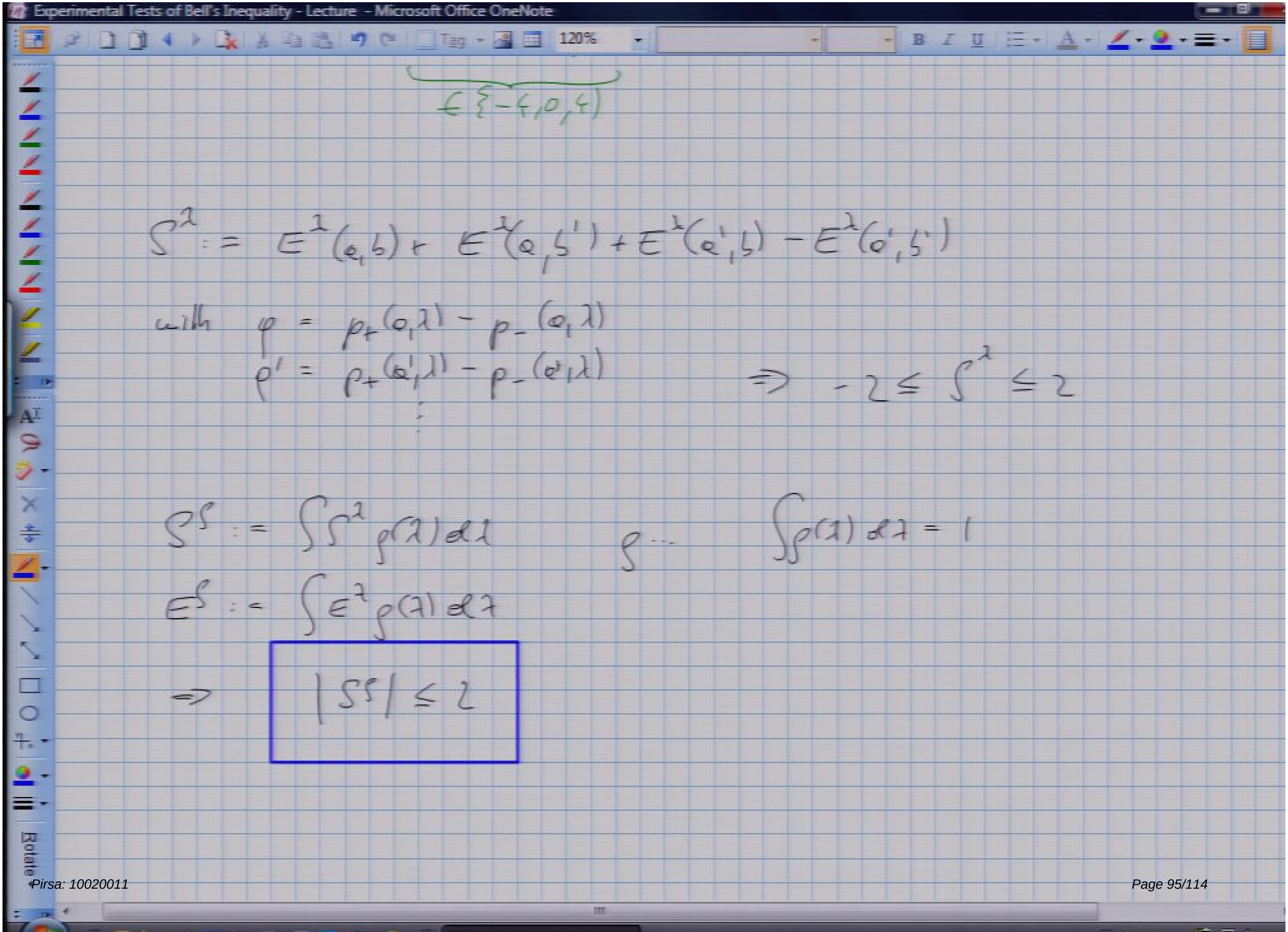
$\vdots$

$\Rightarrow -2 \le S^\lambda \le 2$

$S^\rho := \int S^\lambda \rho(\lambda) d\lambda$ $\qquad \rho \cdots \qquad \int \rho(\lambda) d\lambda = 1$

$E^\rho := \int E^\lambda \rho(\lambda) d\lambda$

$$\Rightarrow \boxed{|S^\rho| \le 2}$$



$$\underbrace{\qquad\qquad}_{\in \{-4,0,4)}$$

$$S^\lambda := E^\lambda(a,b) + E^\lambda(a,b') + E^\lambda(a',b) - E^\lambda(a',b')$$

with $\varphi = p_+(a,\lambda) - p_-(a,\lambda)$

$\rho' = p_+(a',\lambda) - p_-(a',\lambda)$

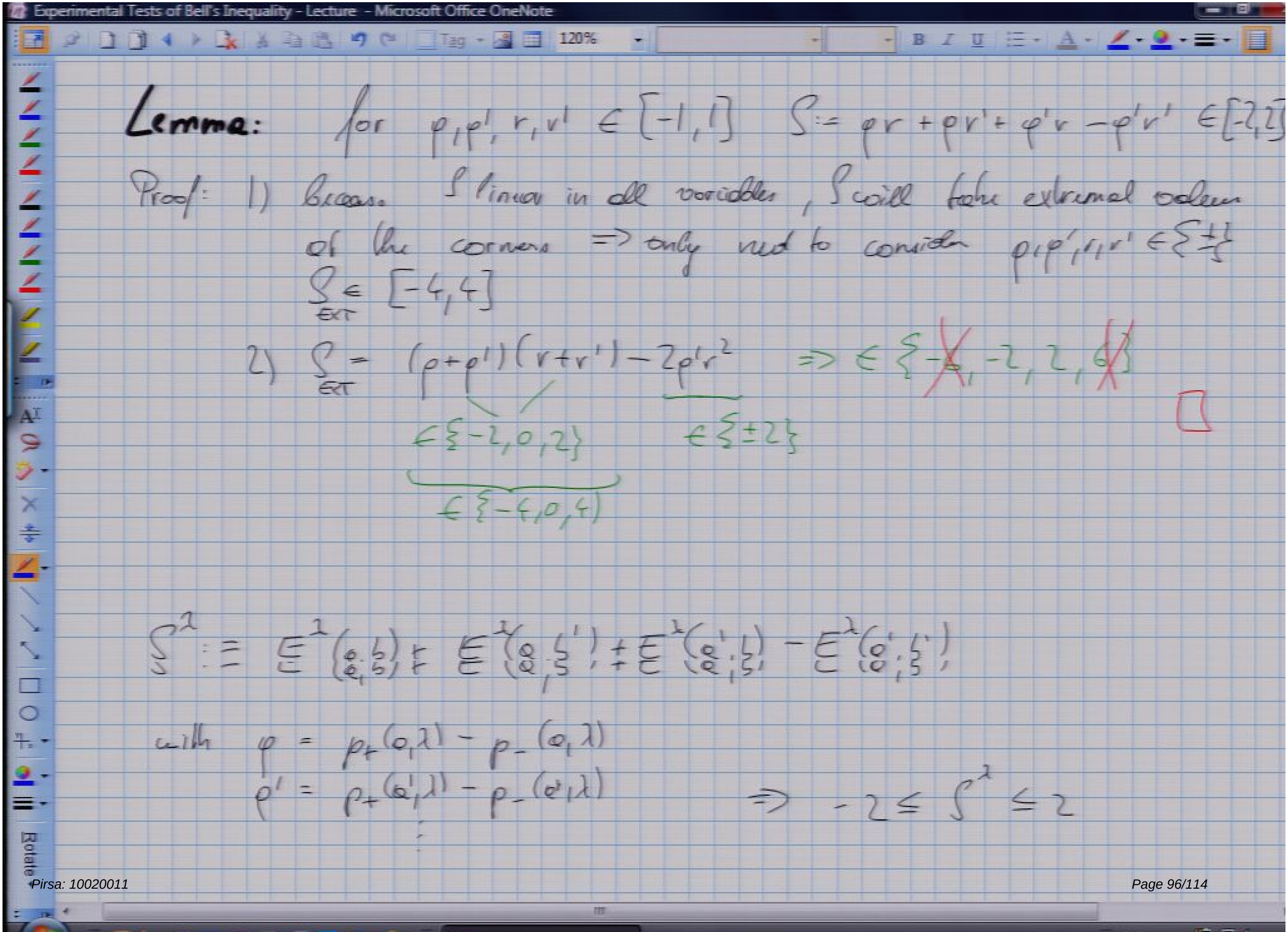
$\vdots$

$\Rightarrow \quad -2 \le S^\lambda \le 2$

$$S^\rho := \int S^\lambda \rho(\lambda)\, d\lambda \qquad \rho \cdots \qquad \int \rho(\lambda)\, d\lambda = 1$$

$$E^\rho := \int E^\lambda \rho(\lambda)\, d\lambda$$

$$\Rightarrow \quad \boxed{|S^\rho| \le 2}$$



Lemma: for $\varphi, \varphi', r, r' \in [-1,1]$ $\quad S := \varphi r + \varphi r' + \varphi' r - \varphi' r' \in [-2,2]$

Proof: 1) Because S linear in all variables, S will take extremal values of the corners $\Rightarrow$ only need to consider $\varphi, \varphi', r, r' \in \{\pm 1\}$

$$S_{EXT} \in [-4,4]$$

2) $$S_{EXT} = \underbrace{(\rho+\rho')(r+r')}_{\in\{-2,0,2\}} - \underbrace{2\rho' r^2}_{\in\{\pm 2\}} \Rightarrow \in \{\cancel{-4}, -2, 2, \cancel{4}\}$$

$$\underbrace{\in\{-2,0,2\}}_{\in\{-4,0,4)}$$

$\square$

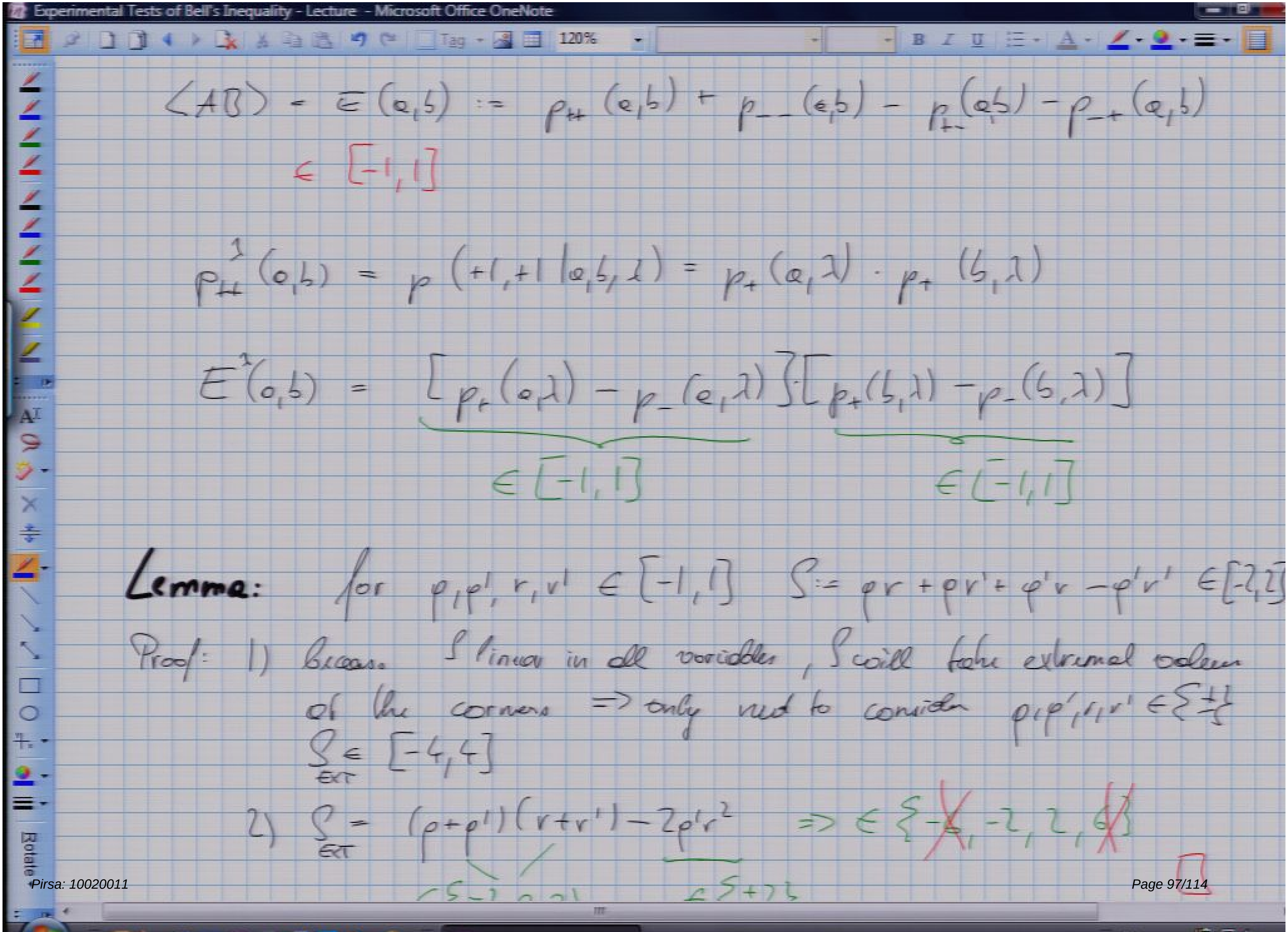
$$S^\lambda := E^\lambda(a,b) + E^\lambda(a,b') + E^\lambda(a',b) - E^\lambda(a',b')$$

with $\varphi = p_+(a,\lambda) - p_-(a,\lambda)$

$\rho' = p_+(a',\lambda) - p_-(a',\lambda)$

$\vdots$

$$\Rightarrow -2 \le S^\lambda \le 2$$



$$\langle AB\rangle = E(a,b) := p_{++}(a,b) + p_{--}(a,b) - p_{+-}(a,b) - p_{-+}(a,b)$$

$$\in [-1,1]$$

$$p^{\lambda}_{++}(a,b) = p(+1,+1|a,b,\lambda) = p_+(a,\lambda)\cdot p_+(b,\lambda)$$

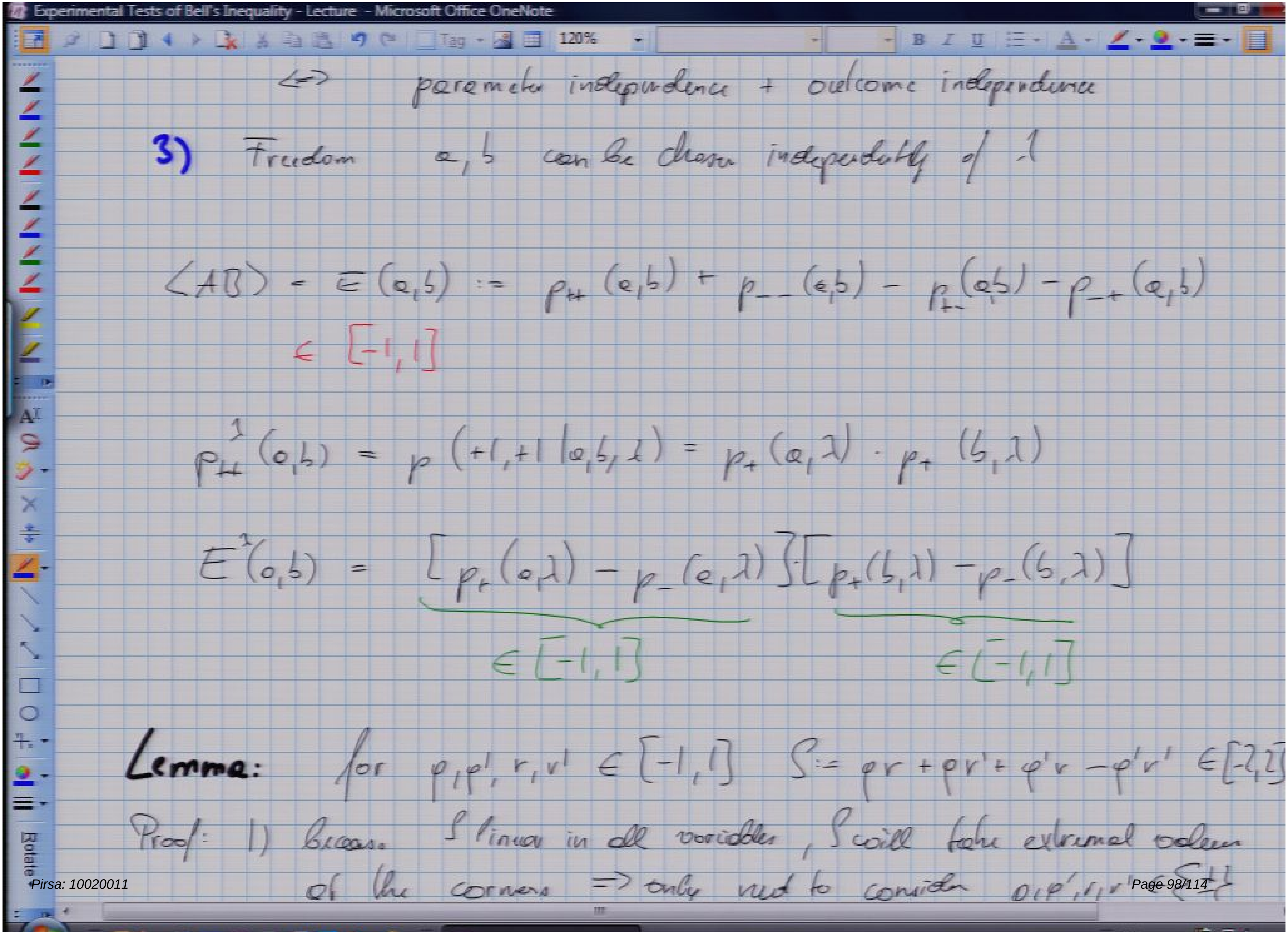
$$E^{\lambda}(a,b) = \underbrace{[p_+(a,\lambda) - p_-(a,\lambda)]}_{\in[-1,1]}\cdot\underbrace{[p_+(b,\lambda) - p_-(b,\lambda)]}_{\in[-1,1]}$$

Lemma: for $\rho, \rho', r, r' \in [-1,1]$ $S := \rho r + \rho r' + \rho' r - \rho' r' \in [-2,2]$

Proof: 1) Because S linear in all variables, S will take extremal values of the corners $\Rightarrow$ only need to consider $\rho, \rho', r, r' \in \{\pm 1\}$
$S_{EXT} \in [-4,4]$

2) $S_{EXT} = (\rho+\rho')(r+r') - \underline{2\rho' r^2}$ $\Rightarrow \in \{\cancel{-6}, -2, 2, \cancel{6}\}$

$\square$



$\Leftrightarrow$ parameter independence + outcome independence

3) Freedom a, b can be chosen independently of λ

$$\langle AB\rangle = E(a,b) := p_{++}(a,b) + p_{--}(a,b) - p_{+-}(a,b) - p_{-+}(a,b)$$

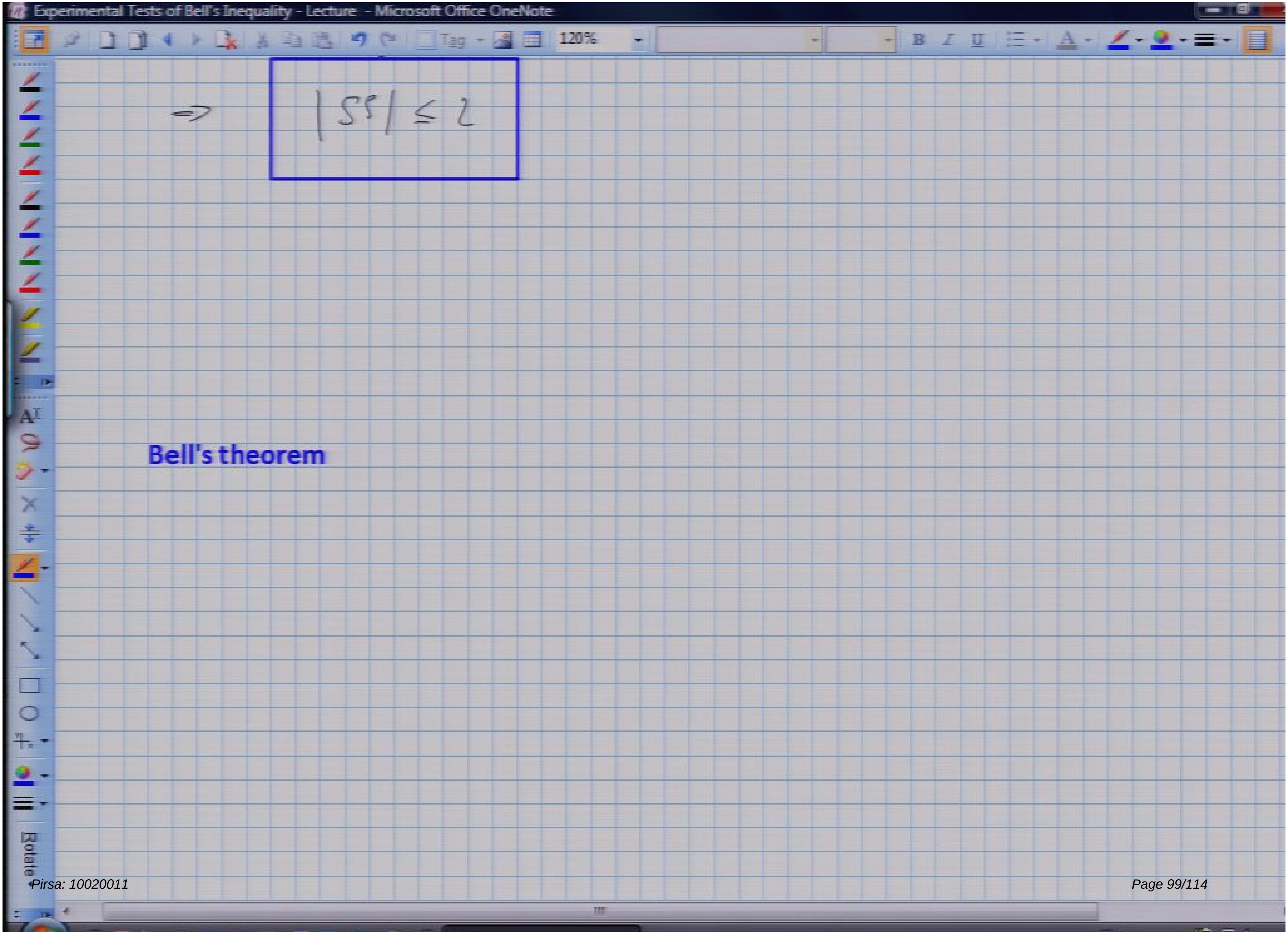
$\in [-1,1]$

$$p^{\lambda}_{++}(a,b) = p(+1,+1|a,b,\lambda) = p_+(a,\lambda)\cdot p_+(b,\lambda)$$

$$E^{\lambda}(a,b) = \underbrace{[p_+(a,\lambda) - p_-(a,\lambda)]}_{\in[-1,1]}\cdot\underbrace{[p_+(b,\lambda) - p_-(b,\lambda)]}_{\in[-1,1]}$$

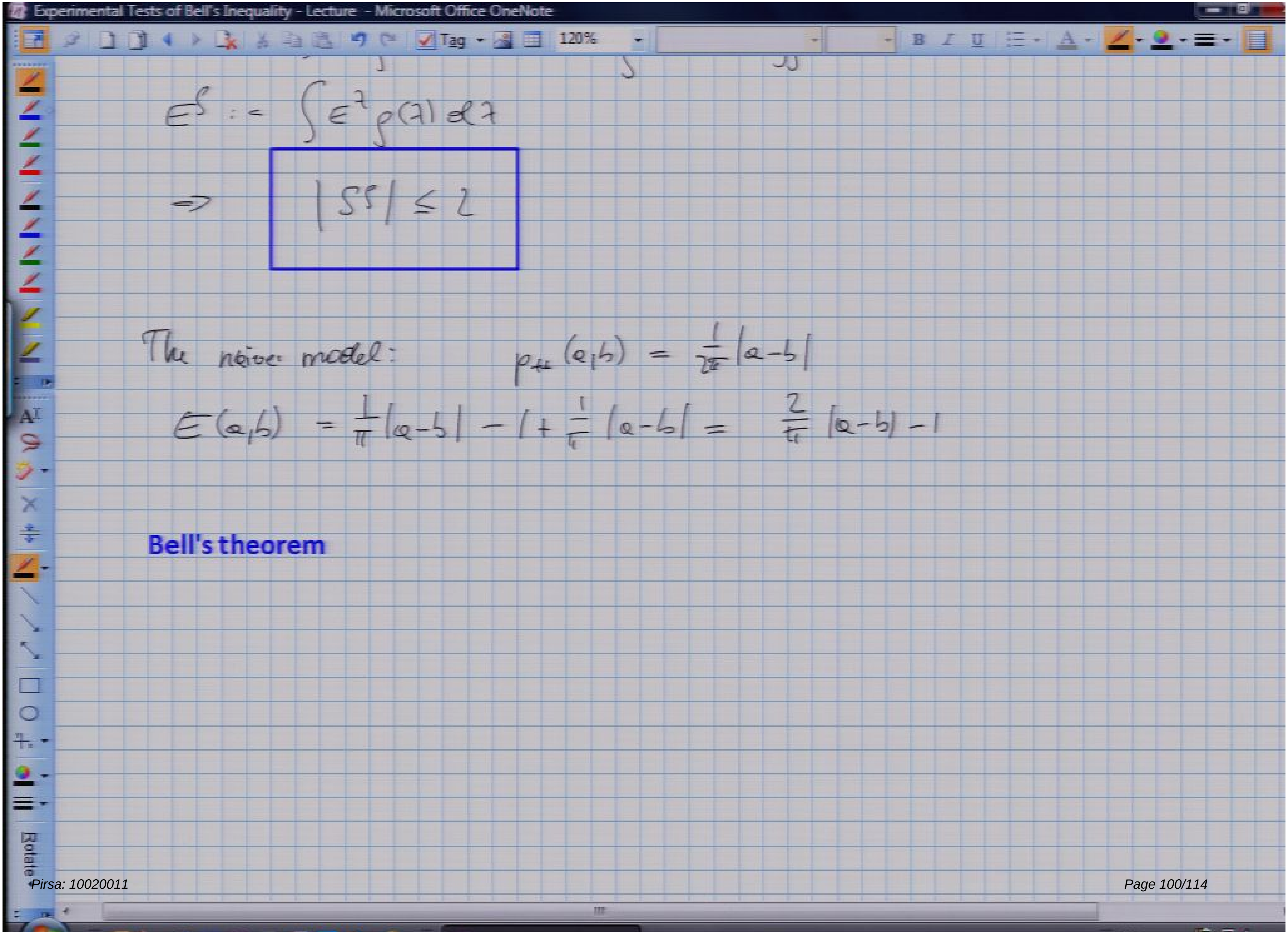
Lemma: for $\varphi, \varphi', r, r' \in [-1,1]$ $S := \varphi r + \varphi r' + \varphi' r - \varphi' r' \in [-2,2]$

Proof: 1) Because S linear in all variables, S will take extremal values of the corners $\Rightarrow$ only need to consider $\varphi, \varphi', r, r' \in \{\pm 1$



$$\Rightarrow \quad \boxed{|S^{\ell}| \leq 2}$$

Bell's theorem



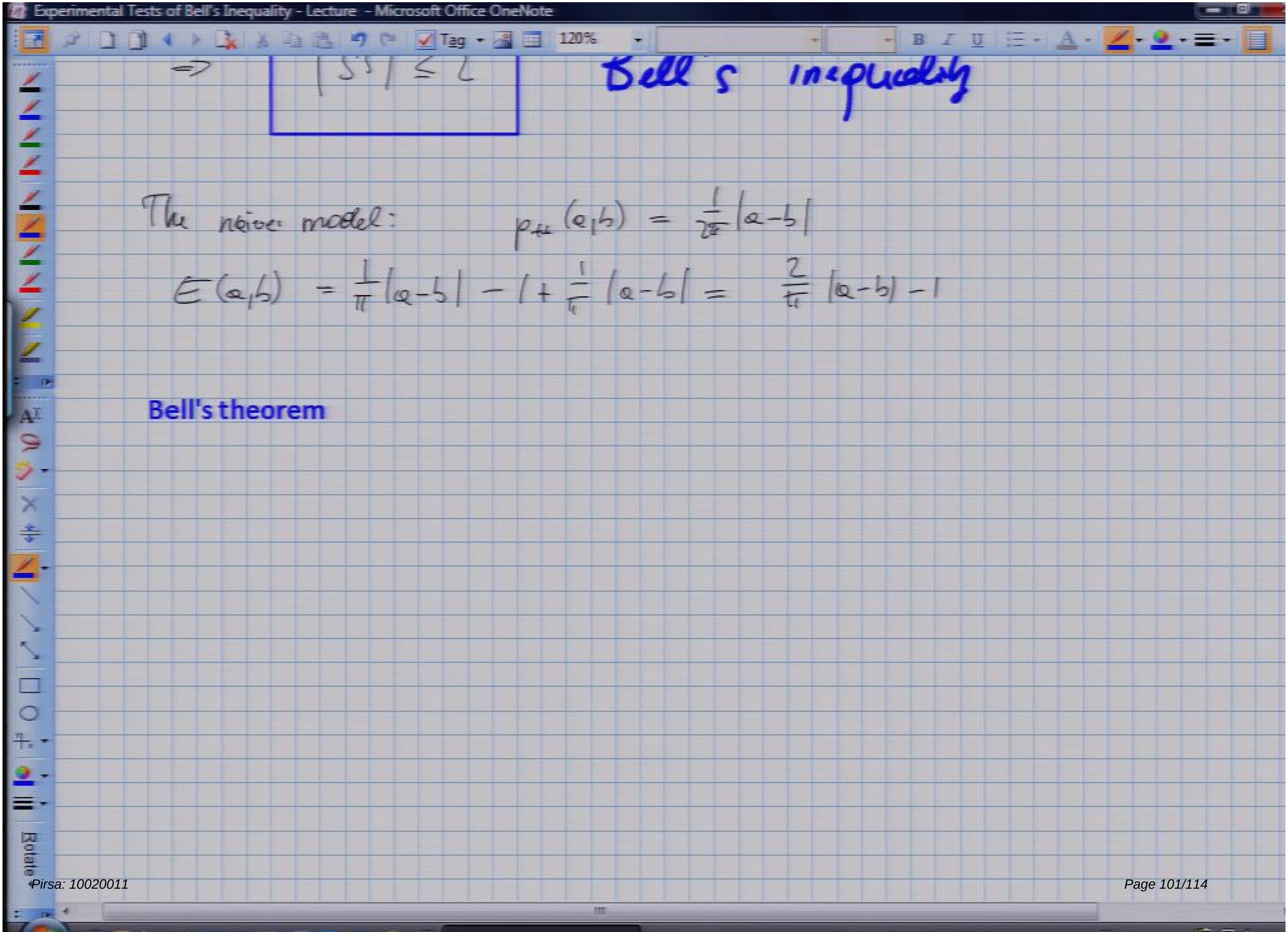
$$E^S := \int E^\lambda \rho(\lambda)\, d\lambda$$

$$\Rightarrow \quad \boxed{|S^S| \le 2}$$

The naive model: $p_{++}(a,b) = \frac{1}{2\pi}|a-b|$

$$E(a,b) = \frac{1}{\pi}|a-b| - 1 + \frac{1}{\pi}|a-b| = \frac{2}{\pi}|a-b| - 1$$

Bell's theorem

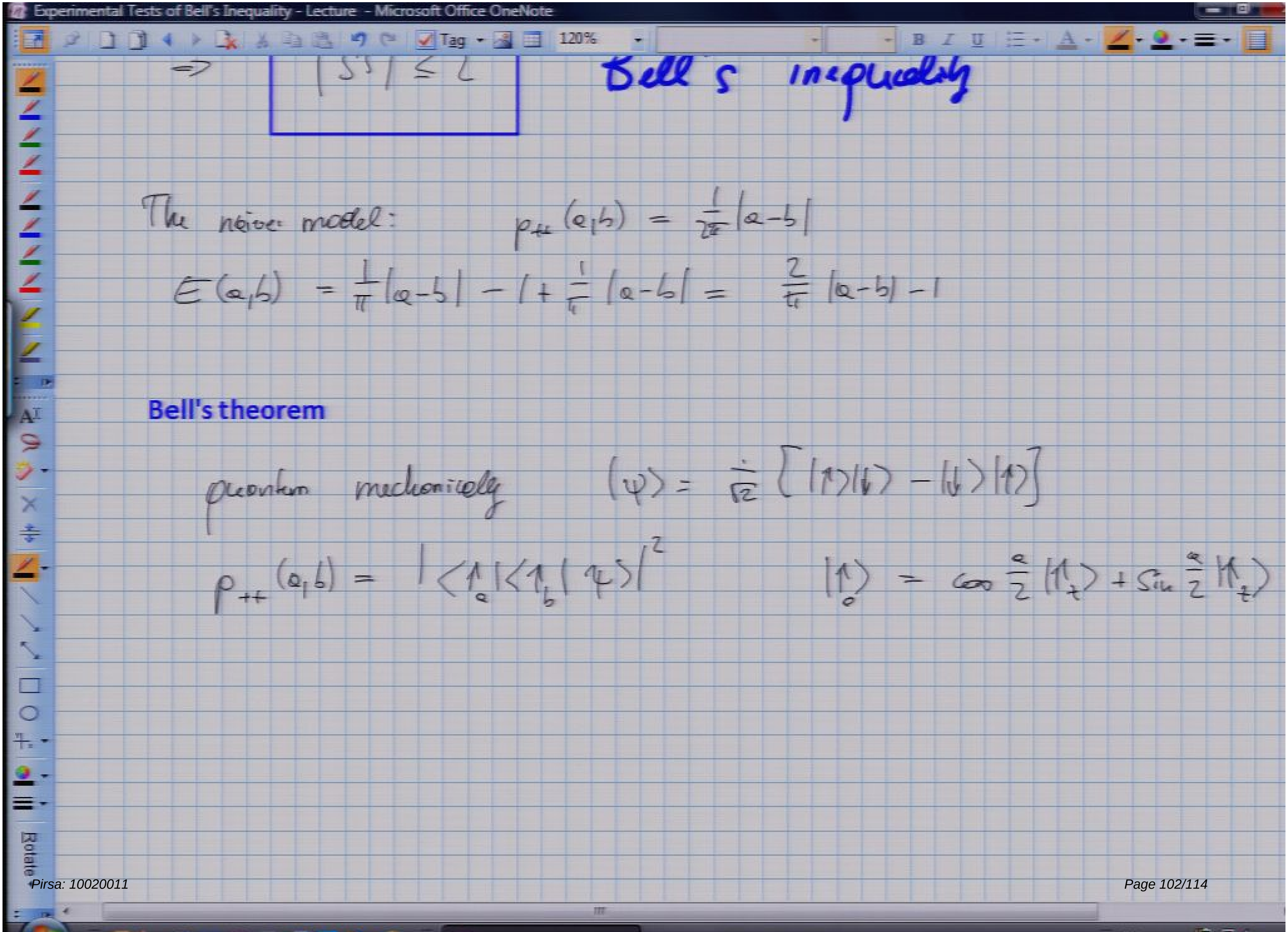


$$\Rightarrow \quad \boxed{|S| \leq 2} \qquad \text{Bell's inequality}$$

The naive model: $p_{++}(a,b) = \frac{1}{2\pi}|a-b|$

$$E(a,b) = \frac{1}{\pi}|a-b| - 1 + \frac{1}{\pi}|a-b| = \frac{2}{\pi}|a-b| - 1$$

Bell's theorem



$$\Rightarrow \quad \boxed{|S| \leq 2} \qquad \text{Bell's inequality}$$

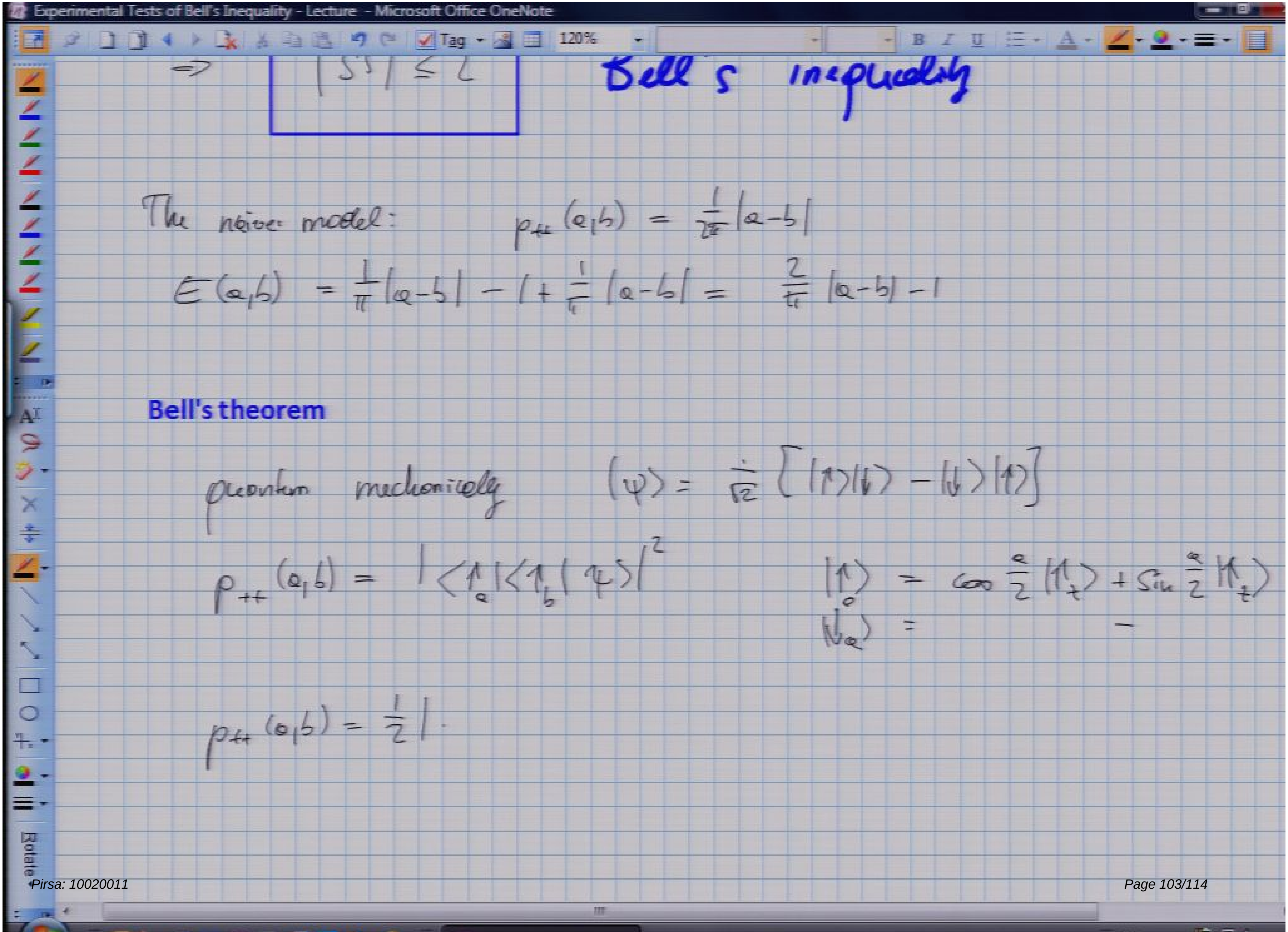
The naive model: $p_{++}(a,b) = \frac{1}{2\pi}|a-b|$

$$E(a,b) = \frac{1}{\pi}|a-b| - 1 + \frac{1}{\pi}|a-b| = \frac{2}{\pi}|a-b| - 1$$

Bell's theorem

quantum mechanically $|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right]$

$$p_{++}(a,b) = |\langle\uparrow_a|\langle\uparrow_b|\psi\rangle|^2 \qquad |\uparrow_a\rangle = \cos\frac{a}{2}|\uparrow_z\rangle + \sin\frac{a}{2}|\uparrow_z\rangle$$



$\Rightarrow$ $\boxed{|S| \leq 2}$ Bell's inequality

The naive model: $p_{++}(a,b) = \frac{1}{2\pi}|a-b|$

$$E(a,b) = \frac{1}{\pi}|a-b| - 1 + \frac{1}{\pi}|a-b| = \frac{2}{\pi}|a-b| - 1$$

Bell's theorem

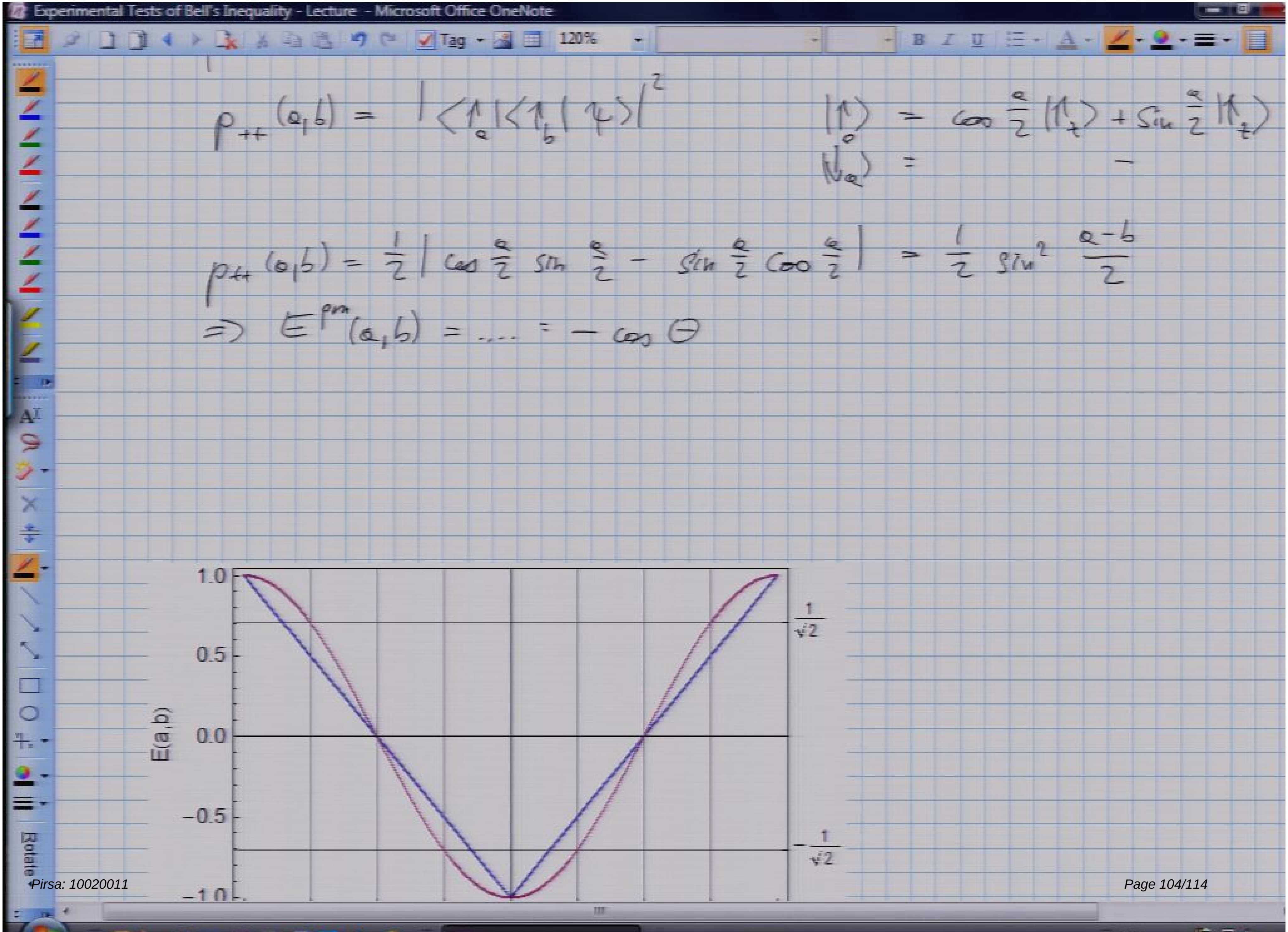
quantum mechanically $|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right]$

$$p_{++}(a,b) = |\langle\uparrow_a|\langle\uparrow_b|\psi\rangle|^2$$

$|\uparrow_a\rangle = \cos\frac{a}{2}|\uparrow_z\rangle + \sin\frac{a}{2}|\uparrow_z\rangle$

$|\downarrow_a\rangle = \qquad\qquad -$

$$p_{++}(a,b) = \frac{1}{2}|$$



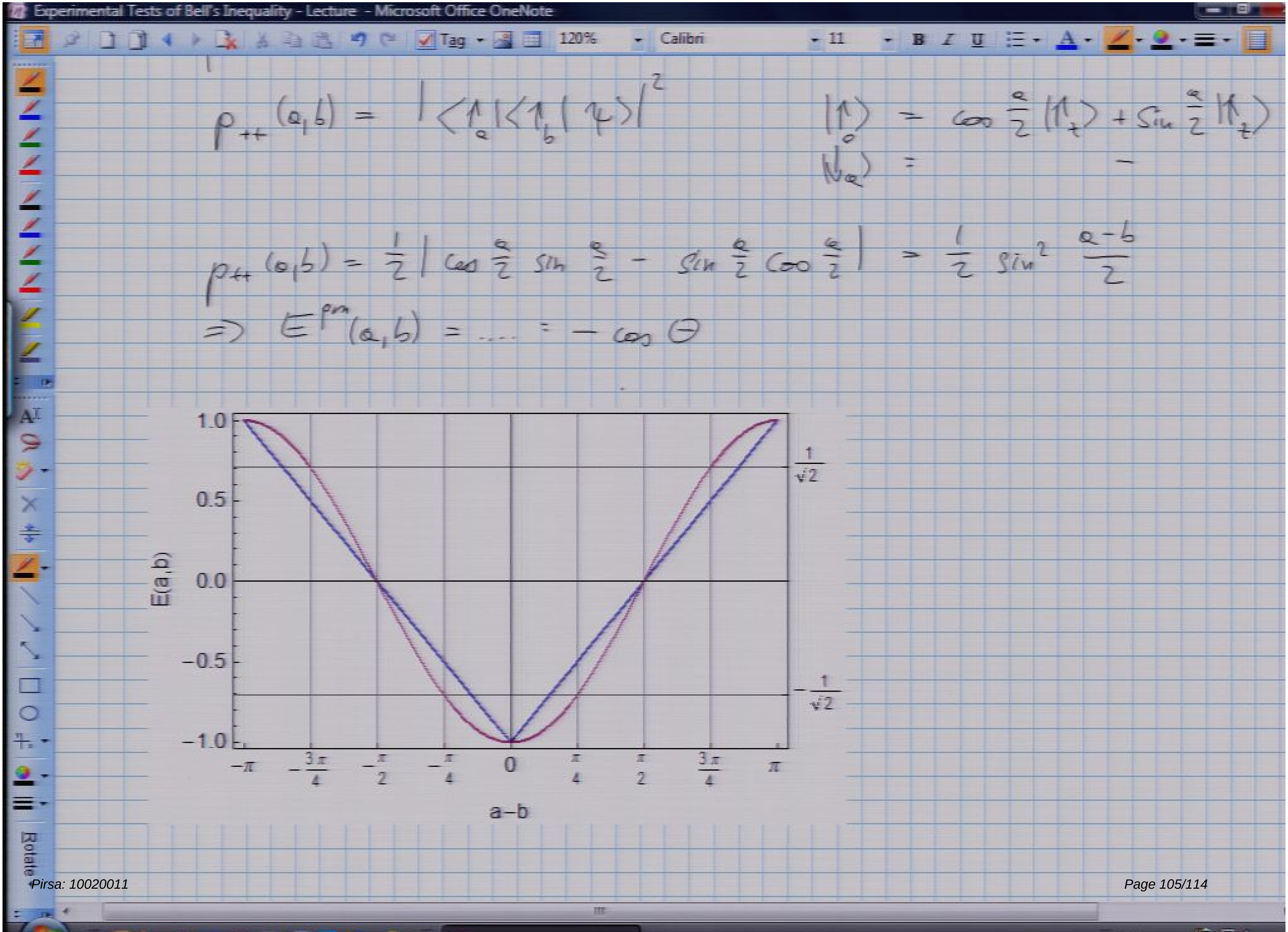
$$p_{++}(a,b) = |\langle\uparrow_a|\langle\uparrow_b|\psi\rangle|^2$$

$$|\uparrow_a\rangle = \cos\frac{a}{2}|\uparrow_z\rangle + \sin\frac{a}{2}|\uparrow_z\rangle$$

$$|\downarrow_a\rangle = \qquad\qquad -$$

$$p_{++}(a,b) = \frac{1}{2}\left|\cos\frac{a}{2}\sin\frac{a}{2} - \sin\frac{a}{2}\cos\frac{a}{2}\right| = \frac{1}{2}\sin^2\frac{a-b}{2}$$

$$\Rightarrow E^{pm}(a,b) = \dots = -\cos\Theta$$



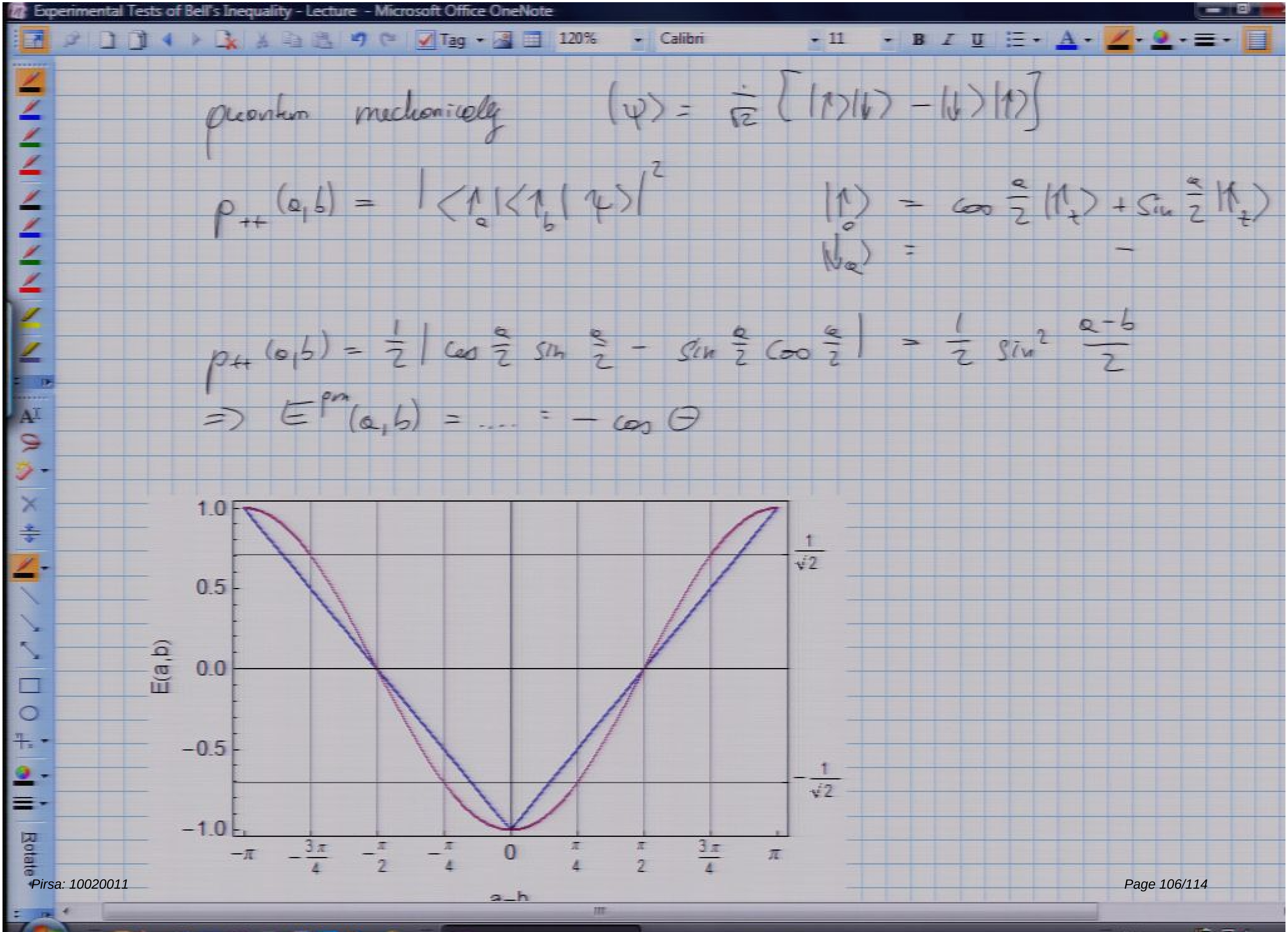
$$p_{++}(a,b) = |\langle\uparrow_a|\langle\uparrow_b|\psi\rangle|^2$$

$$|\uparrow_a\rangle = \cos\frac{a}{2}|\uparrow_z\rangle + \sin\frac{a}{2}|\uparrow_z\rangle$$

$$|\downarrow_a\rangle = \qquad -$$

$$p_{++}(a,b) = \frac{1}{2}\left|\cos\frac{a}{2}\sin\frac{a}{2} - \sin\frac{a}{2}\cos\frac{a}{2}\right| \Rightarrow \frac{1}{2}\sin^2\frac{a-b}{2}$$

$$\Rightarrow E^{QM}(a,b) = \dots = -\cos\Theta$$



quantum mechanically $\quad |\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right]$

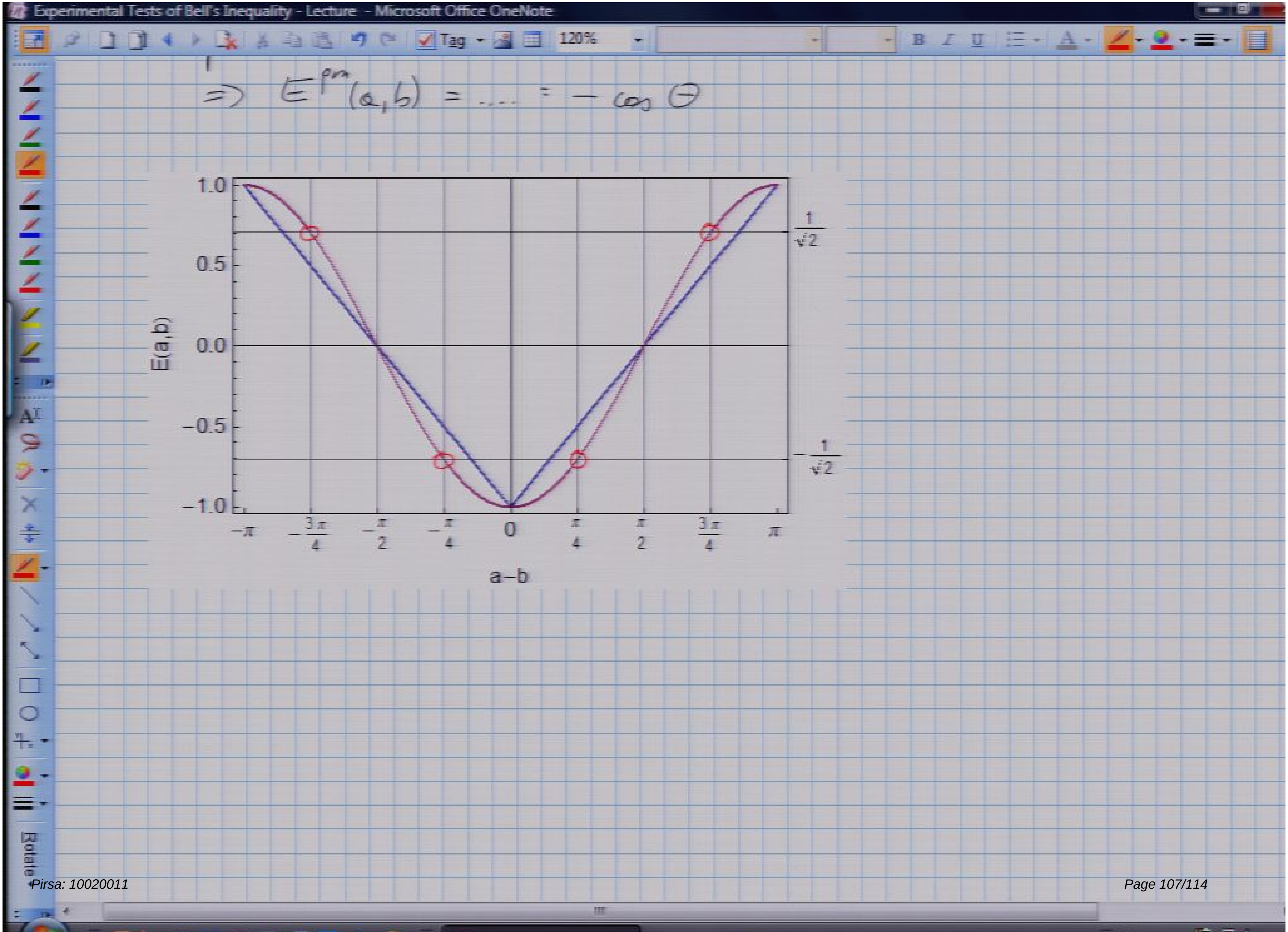
$$P_{++}(a,b) = \left|\langle\uparrow_a|\langle\uparrow_b|\psi\rangle\right|^2$$

$$|\uparrow_a\rangle = \cos\frac{a}{2}|\uparrow_z\rangle + \sin\frac{a}{2}|\uparrow_z\rangle$$

$$|\downarrow_a\rangle = \qquad -$$

$$p_{++}(a,b) = \frac{1}{2}\left|\cos\frac{a}{2}\sin\frac{a}{2} - \sin\frac{a}{2}\cos\frac{a}{2}\right| = \frac{1}{2}\sin^2\frac{a-b}{2}$$

$$\Rightarrow E^{qm}(a,b) = \dots = -\cos\Theta$$

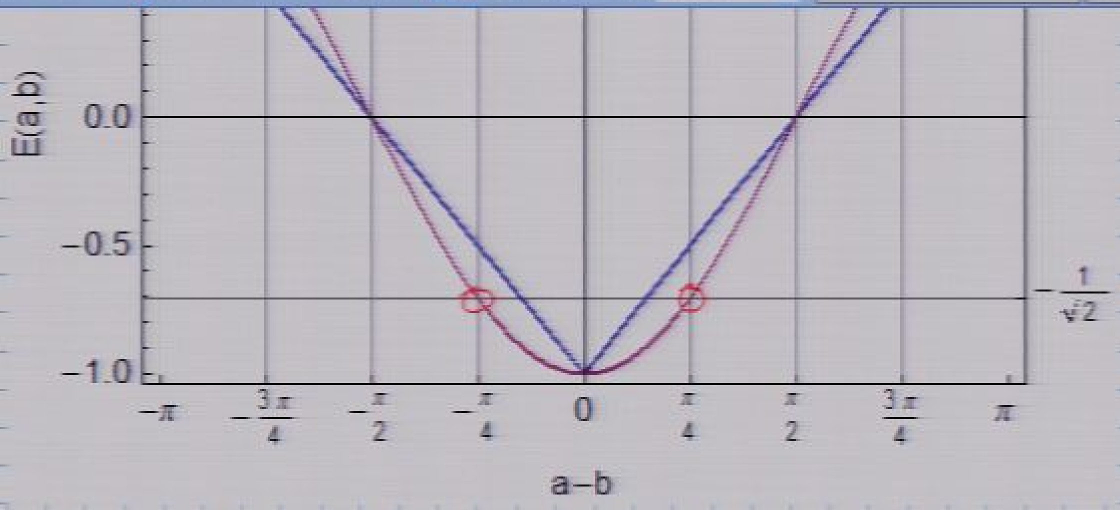


$$\Rightarrow E^{pm}(a,b) = \ldots = -\cos\Theta$$

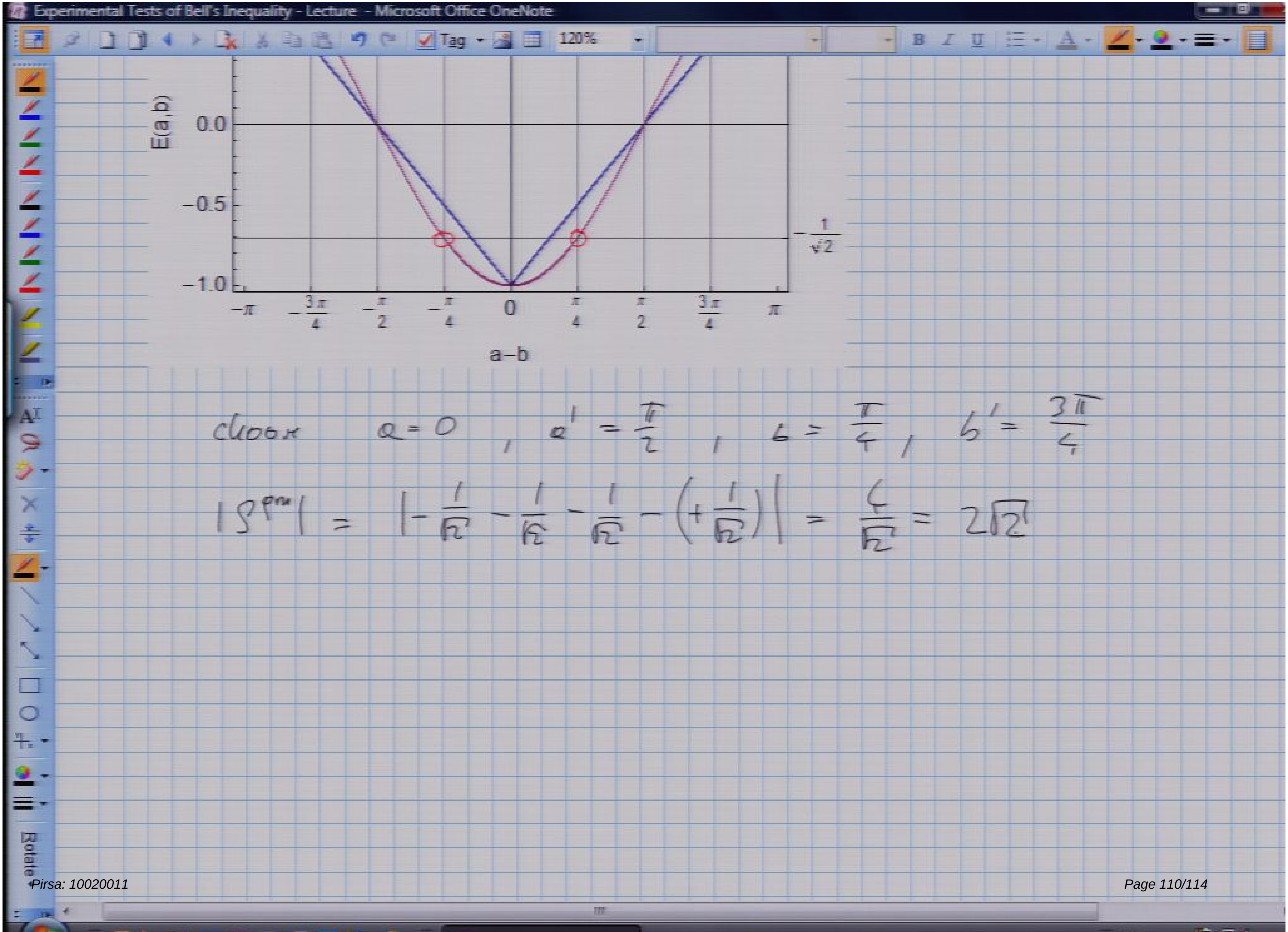
Experimental Tests of Bell's Inequality - Lecture - Microsoft Office OneNote

$$\Rightarrow \quad E^{pm}(a,b) = \dots = -\cos\Theta$$

Experimental Tests of Bell's Inequality - Lecture - Microsoft Office OneNote



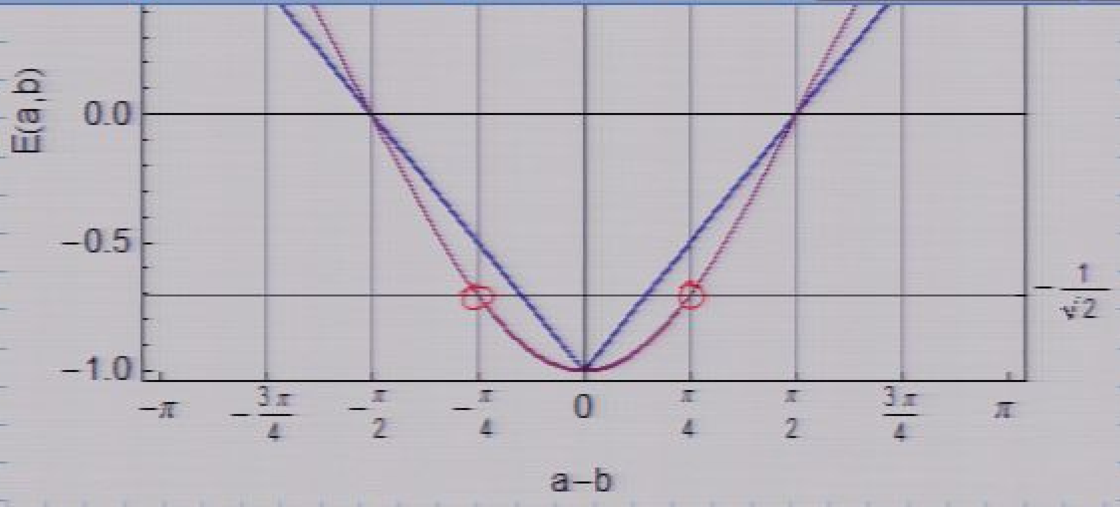
choose $a = 0$, $a' = \frac{\pi}{2}$, $b = \frac{\pi}{4}$, $b' = \frac{3\pi}{4}$



choose $a = 0$, $a' = \frac{\pi}{2}$, $b = \frac{\pi}{4}$, $b' = \frac{3\pi}{4}$

$$|S^{qm}| = \left|-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \left(+\frac{1}{\sqrt{2}}\right)\right| = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Experimental Tests of Bell's Inequality - Lecture - Microsoft Office OneNote

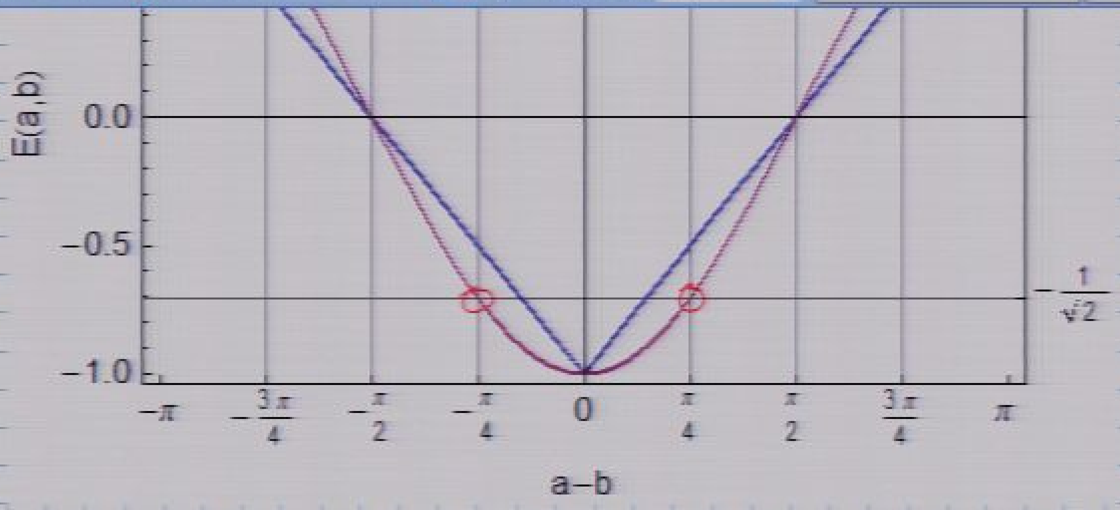


choose $a = 0$, $a' = \frac{\pi}{2}$, $b = \frac{\pi}{4}$, $b' = \frac{3\pi}{4}$

$$|S^{qm}| = \left| -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \left(+ \frac{1}{\sqrt{2}} \right) \right| = \frac{4}{\sqrt{2}} = 2\sqrt{2} > 2$$

- This is the maximal violation of Bell's inequality
- All maximally entangled states give this value.

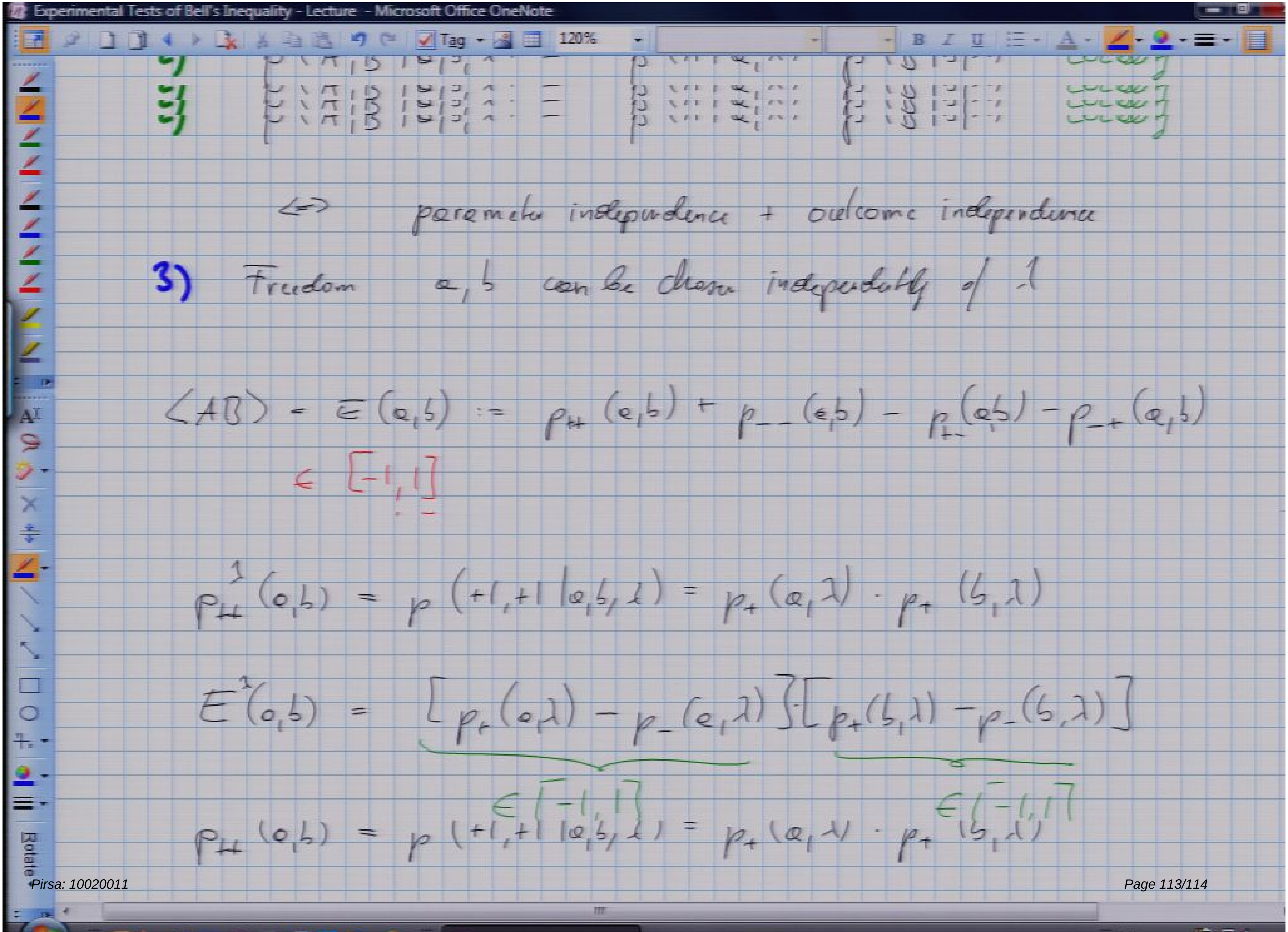
Experimental Tests of Bell's Inequality - Lecture - Microsoft Office OneNote



choose $a = 0$, $a' = \frac{\pi}{2}$, $b = \frac{\pi}{4}$, $b' = \frac{3\pi}{4}$

$$|S^{qm}| = \left|-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \left(+\frac{1}{\sqrt{2}}\right)\right| = \frac{4}{\sqrt{2}} = 2\sqrt{2} > 2$$

- This is the maximal violation of Bell's inequality
- All maximally entangled states give this value
- Separable states don't violate BI



$\Longleftrightarrow$ parameter independence + outcome independence

3) Freedom a, b can be chosen independently of λ

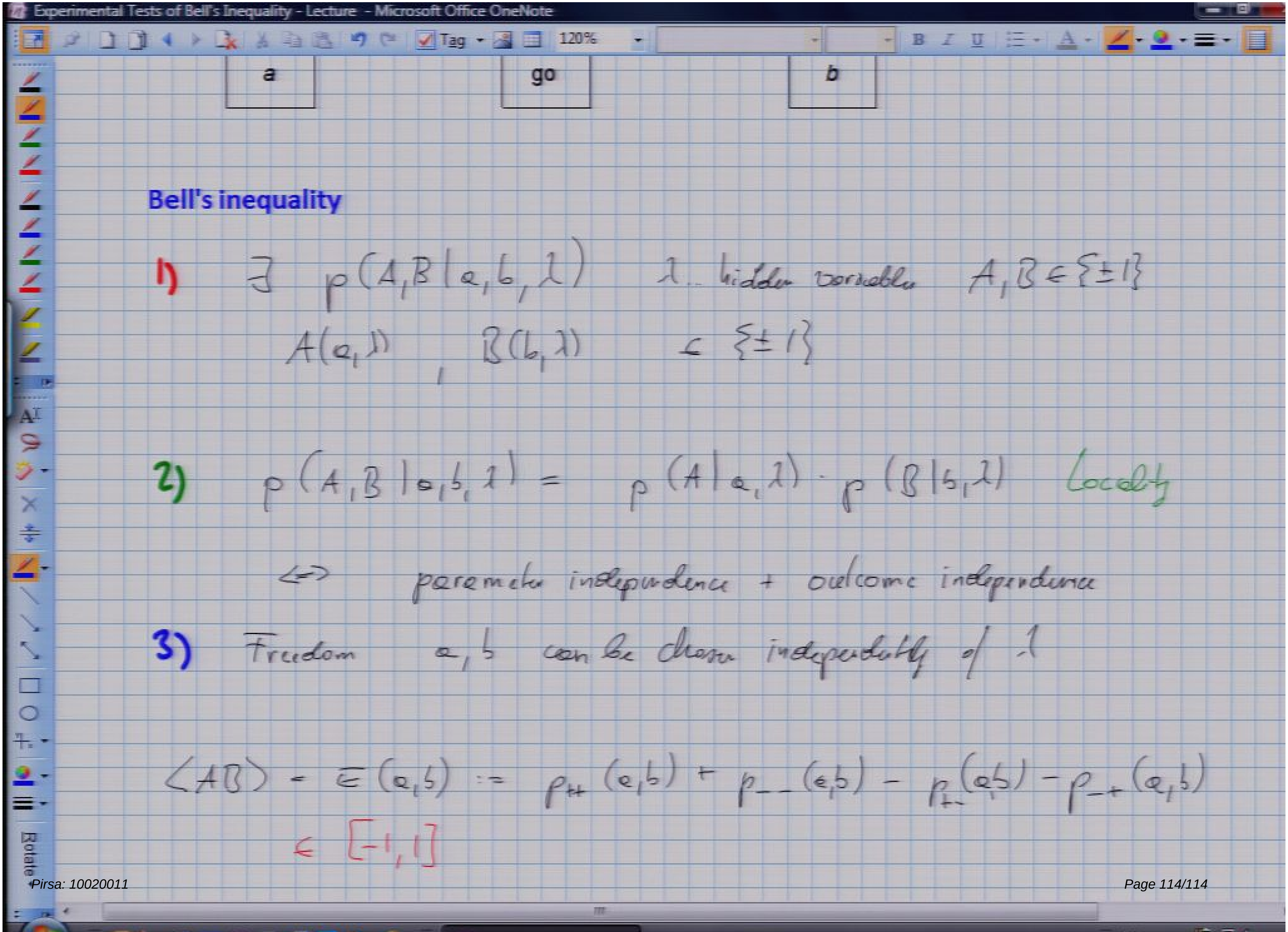
$$\langle AB\rangle = E(a,b) := p_{++}(a,b) + p_{--}(a,b) - p_{+-}(a,b) - p_{-+}(a,b)$$

$$\in [-1,1]$$

$$p^{\lambda}_{++}(a,b) = p(+1,+1\,|\,a,b,\lambda) = p_+(a,\lambda)\cdot p_+(b,\lambda)$$

$$E^{\lambda}(a,b) = \underbrace{[p_+(a,\lambda) - p_-(a,\lambda)]}_{\in[-1,1]}\cdot\underbrace{[p_+(b,\lambda) - p_-(b,\lambda)]}_{\in[-1,1]}$$

$$p_{++}(a,b) = p(+1,+1\,|\,a,b,\lambda) = p_+(a,\lambda)\cdot p_+(b,\lambda)$$



Bell's inequality

1) $\exists \quad p(A,B|a,b,\lambda)$ $\quad \lambda$.. hidden variables $\quad A,B \in \{\pm 1\}$

$A(a,\lambda) \quad , \quad B(b,\lambda) \quad \in \{\pm 1\}$

2) $p(A,B|a,b,\lambda) = p(A|a,\lambda) \cdot p(B|b,\lambda)$ Locality

$\Leftrightarrow$ parameter independence + outcome independence

3) Freedom a,b can be chosen independently of λ

$\langle AB \rangle = E(a,b) := p_{++}(a,b) + p_{--}(a,b) - p_{+-}(a,b) - p_{-+}(a,b)$

$\in [-1,1]$