Title: Connes-Kreimer algebraic structures in field theory and quantum gravity

Date: Mar 04, 2010 11:00 AM

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Abstract: The concept of renormalization lies at the heart of fundamental physics. I introduce in this talk the Connes-Kreimer algebraic approach for expressing renormalizability in quantum field theories as well as the extension of these notions to the manifestly non-local framework of noncommutative quantum field theories. Finally, I will present an attempt to further generalize these concepts to quantum gravity models. based on: 0909.5631 [gr-qc], Class. Quant. Grav. (in press)

Plan

- Introduction: renormalizability in quantum field theory (QFT)
- Connes-Kreimer approach for commutative QFT

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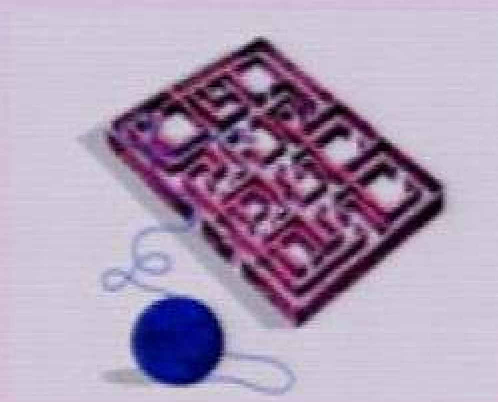
- Connes-Kreimer approach for commutative QFT
- Noncommutative QFT (NCQFT) and renormalizability
- Connes-Kreimer approach for NCQFT

Renormalizability

how to chose between all possible physical models?

what should be Ariadne's thread in this labyrinth?



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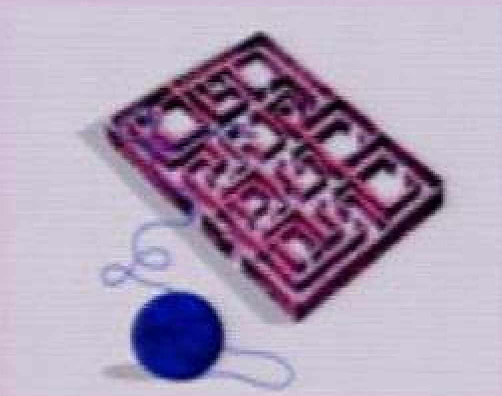


use of *renormalizability*

renormalizable theories - generic building blocks of physics

concept of renormalizability - at the heart of fundamental physics

Main ingredients of renormalizability in QFT

1. power counting theorem: indicates which Feynman graphs are primitively divergent
   superficial degree of divergence ω - should not depend on the internal structure
   exemple: the ϕ^4 model

$$\omega = N - 4.$$

   N - number of external legs of the graph
   primitively divergent graphs: $2-$ and $4-$point graphs
2. locality

$\hookrightarrow$ Bogoliubov subtraction operator R

subtraction of divergences

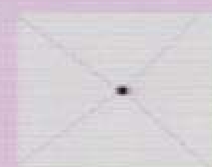
The physical principle of locality (Feynman graph level)

connected graphs can be reduced to points

graph made of internal propagators of high energy - local

example:

→



Connes-Kreimer Hopf algebra

A. Connes and D. Kreimer, *Commun. Math. Phys.*, '00

$\hookrightarrow$ definition of a coproduct Δ

$\mathcal{H}$ - the algebra generated by the 1PI Feynman graphs
multiplication: disjoint union of graphs

$$\Delta : \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}, \;\; \Delta(G) = G \otimes 1 + 1 \otimes G + \sum_{\gamma \in \underline{G}} \gamma \otimes G/\gamma,$$

$\underline{G}$ – primitively divergent subgraphs of G

(renormalization as a factorization issue)

$$\varepsilon : \mathcal{H} \to \mathbb{K}, \;\; \varepsilon(1) = 1, \; \varepsilon(G) = 0, \; \forall G \neq 1,$$

$$S : \mathcal{H} \to \mathcal{H},$$

$$S(1_{\mathcal{H}}) = 1_{\mathcal{H}}, \; G \mapsto -G - \sum_{\gamma \in \underline{G}} S(\gamma) G/\gamma.$$

$\Rightarrow$ $(\mathcal{H}, \Delta, \varepsilon, S)$ *is a Hopf algebra.*

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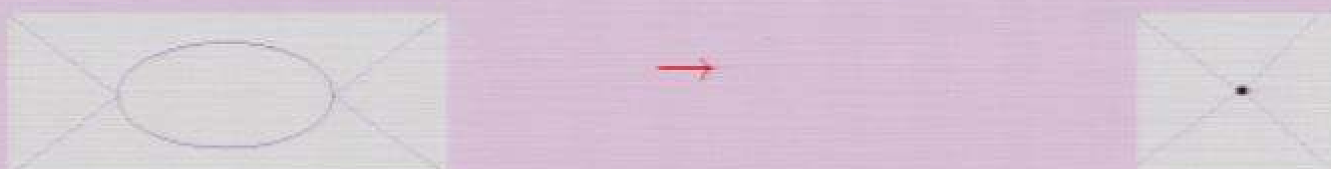

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Algebraic framework for renormalization

R - the map which given a formal integral returns it evaluated at the subtraction point
(implementation of the renormalization scheme)
$R\mathcal{A}(G)$ - the singular part of the Feynman amplitude $\mathcal{A}(G)$

$$S_R^{\mathcal{A}}(1_{\mathcal{H}}) = (1),$$
$$S_R^{\mathcal{A}}(G) = -R(\mathcal{A}(G)) - \sum_{\gamma \in \underline{G}} S_R^{\mathcal{A}}(\gamma) R(\mathcal{A}(G/\gamma)). \qquad (1)$$

the renormalized amplitude of the graph

$$\mathcal{A}_R := S_R^{\mathcal{A}} \star \mathcal{A}.$$

Connes-Kreimer Hopf algebra structure - the combinatorial backbone of renormalization

Core Hopf algebra

(S. Bloch and D. Kreimer, *Commun.Num.Theor.Phys.*, '08, T. Krajewski and P. Martinetti, 0806.4309 [hep-th]

the coproduct sums over *any* subgraph

richer combinatoric structure: contains the renormalization Hopf algebra as a quotient algebra

Hopf primitives (graphs with trivial coproduct): 1−loop graph

pertubative gravity: pertinent structure

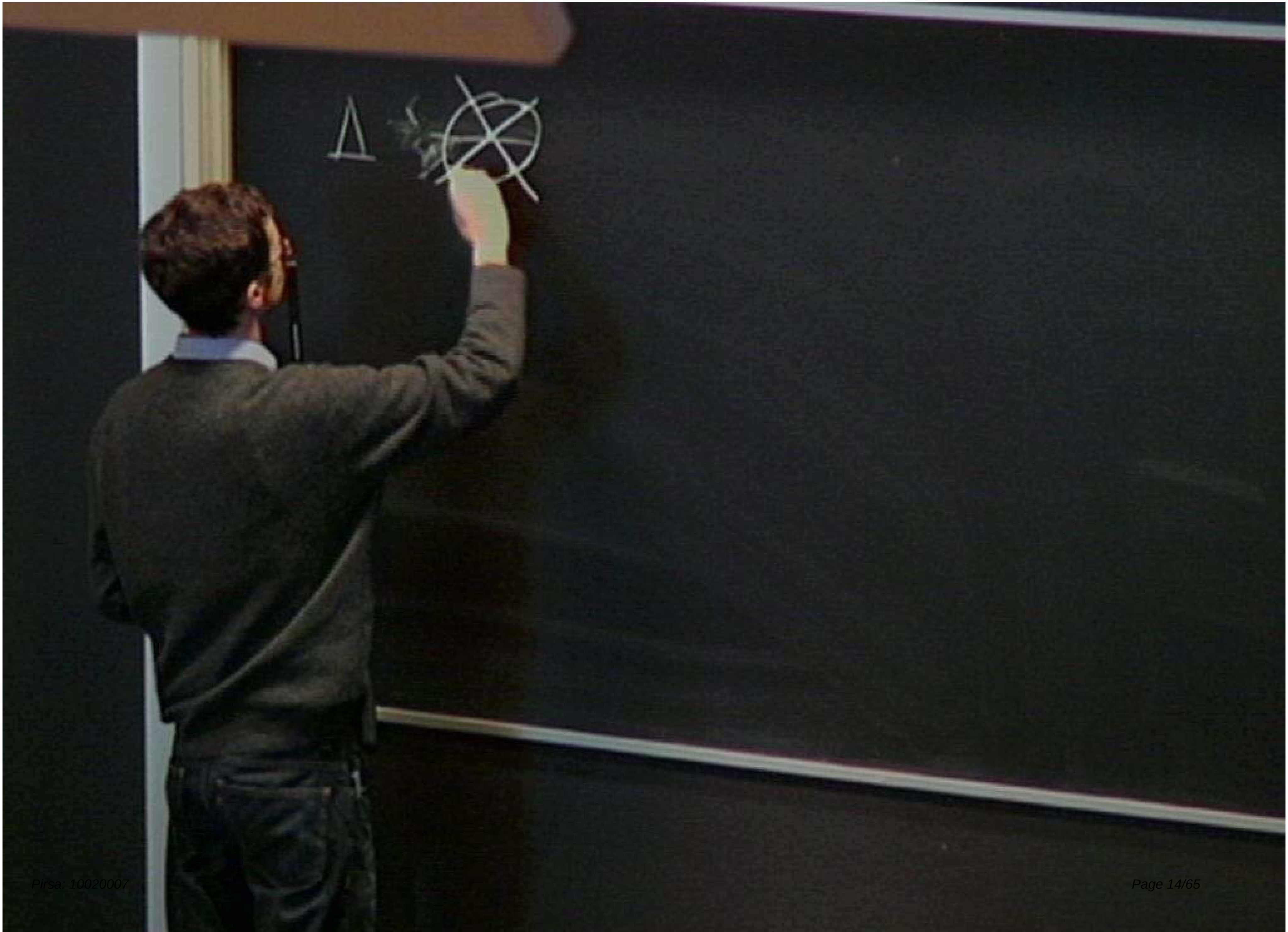
(D. Kreimer and W. Van Sujlekom, *Nucl. Phys.* B, '09)

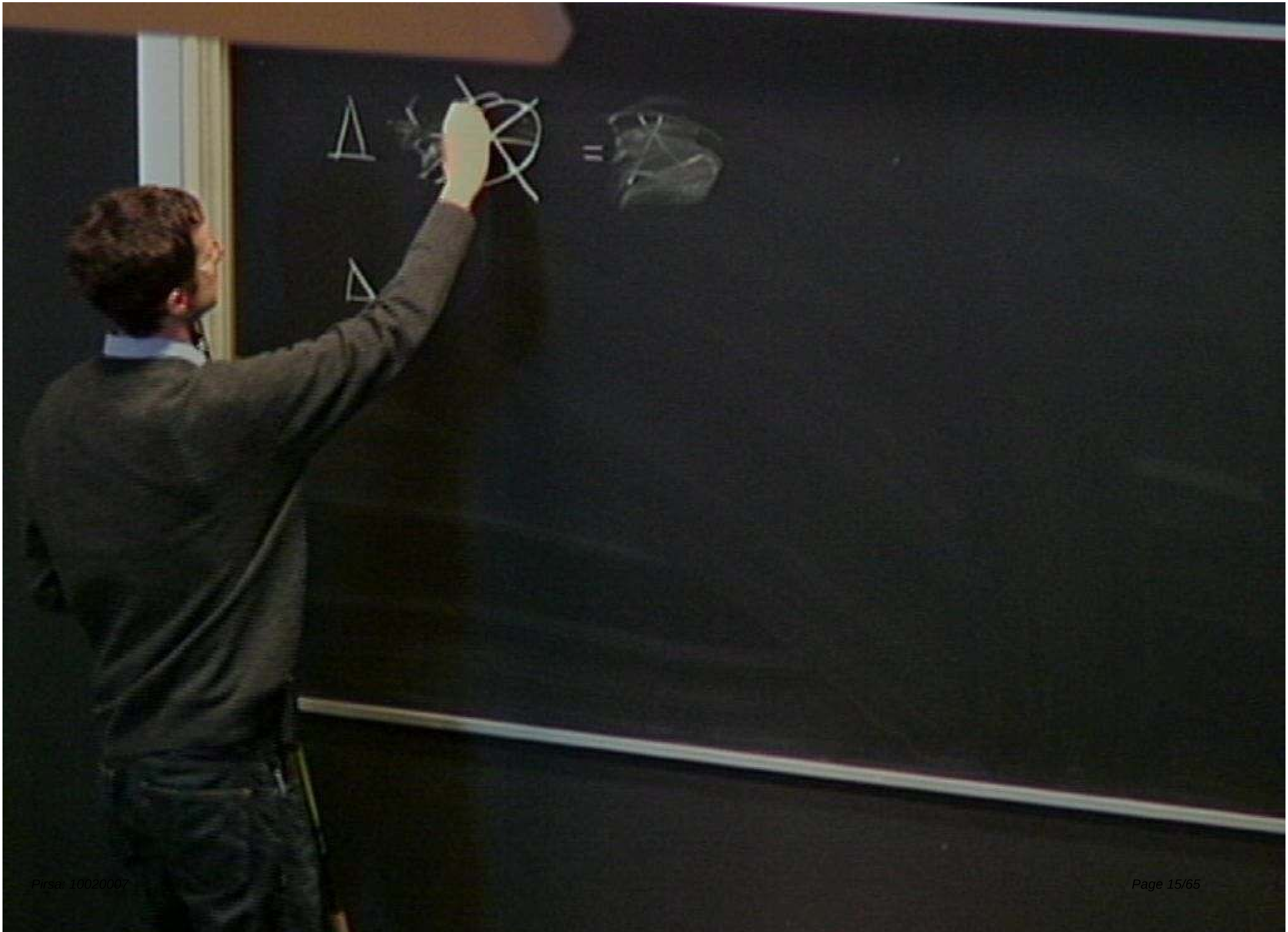
Rôle of Hochschild cohomology

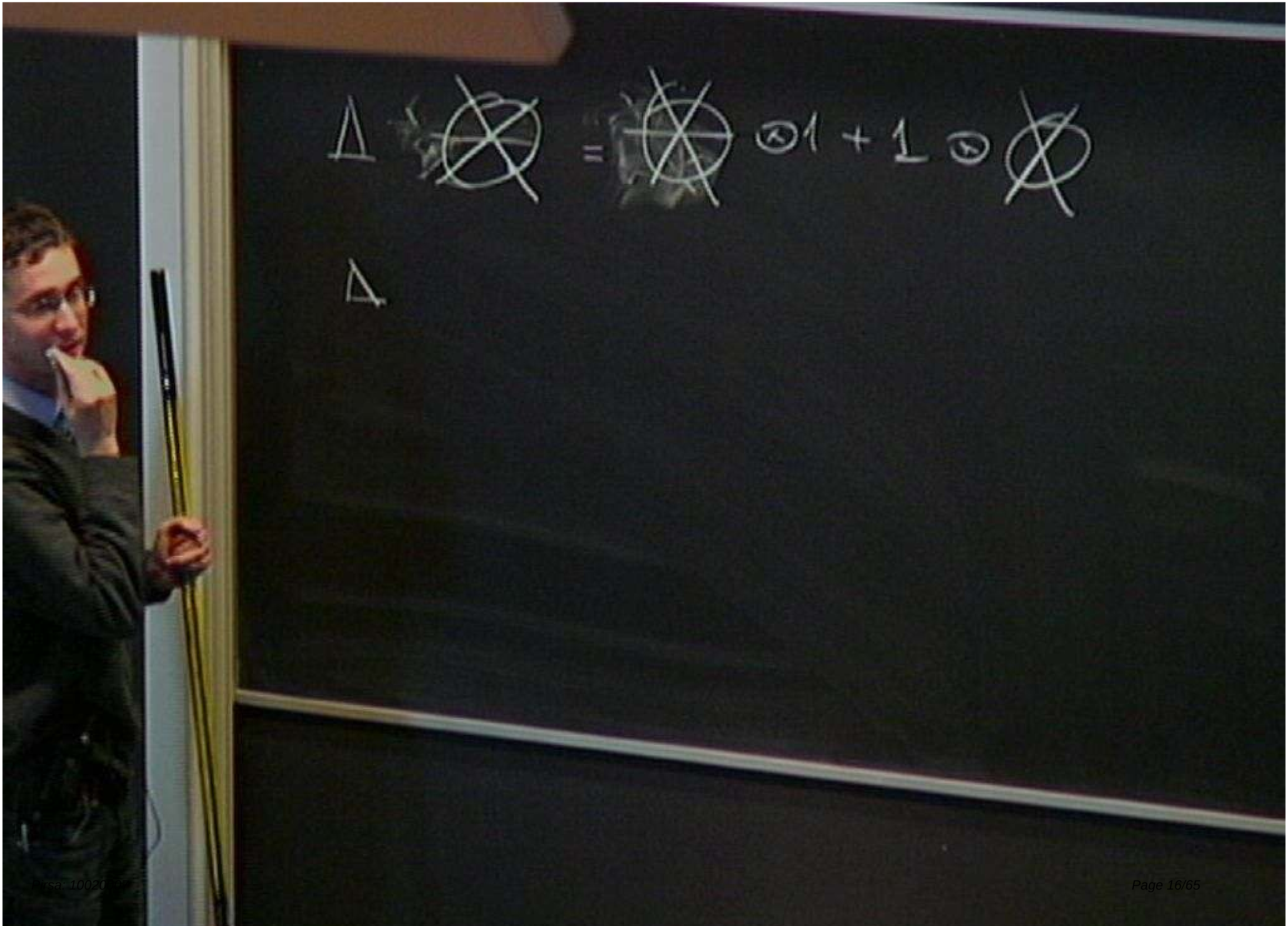
$\hookrightarrow$ perturbative and non-pertubative issues

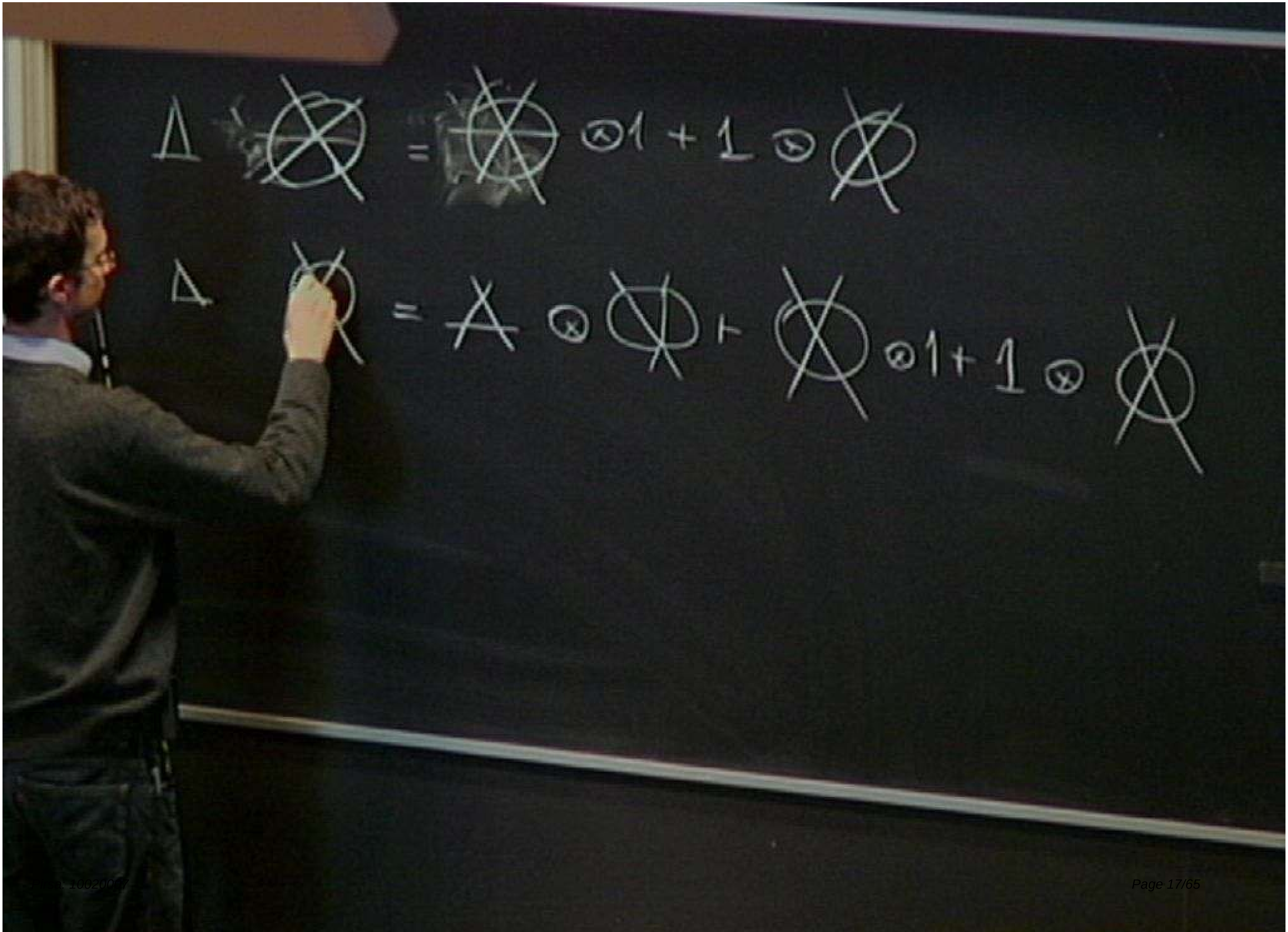
- $\forall$ primitively divergent graph γ - a Hochschild one-cocyle B_+^γ (insertion operator)
- $\forall$ relevant graph (quantum corrections of the propagator or of the vertex) lies in the image of such an insertion operator B_+

expression of the locality principle in this language









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expression of the locality principle in this language

$$i)\ \Gamma = 1 + \sum_{k=1}^{\infty} g^k B_+^k(X_k)$$

combinatorial Dyson-Schwinger equation - recursive equation - power series of the insertion operator B_+

analytic Dyson-Schwinger equation - obtained by applying the Feynman rules

$$ii)\ \Delta(B_+^k(X_k)) = B_+^k(X_k) \otimes \mathbb{I} + (\mathrm{id} \otimes B_+^k)\Delta(X_k)$$

B_+ - Hochschild $1-$cocyle of the Hopf algebra

$$iii)\ \Delta(c_k) = \sum_{j=0}^{k} P_k^n(c) \otimes c_{k-j}^r,$$

$P_k^n(c)$ polynomial in the variables c_m of total degree $n-k$.
c_k - Hopf subalgebras - road toward solutions of the Dyson-Schwinger equation (D. Broadhurst and D. Kreimer, *Nucl. Phys.* B, '01)

Field theory on Moyal space

$\star$ - the noncommutative Moyal product

Φ^4 model:

$$\mathcal{S} = \int d^4x \, [\frac{1}{2}\partial_\mu \Phi \star \partial^\mu \Phi + \frac{1}{2} m^2 \Phi \star \Phi + \frac{\lambda}{4!} \Phi \star \Phi \star \Phi \star \Phi],$$

Implications of the use of the Moyal product in QFT

$$\int d^4x \Phi^{\star 4}(x) \propto \int \prod_{i=1}^{4} d^4x_i \Phi(x_i) \delta(x_1 - x_2 + x_3 - x_4)$$

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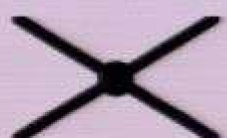
oscillation $\propto$ area of parallelogram



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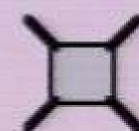

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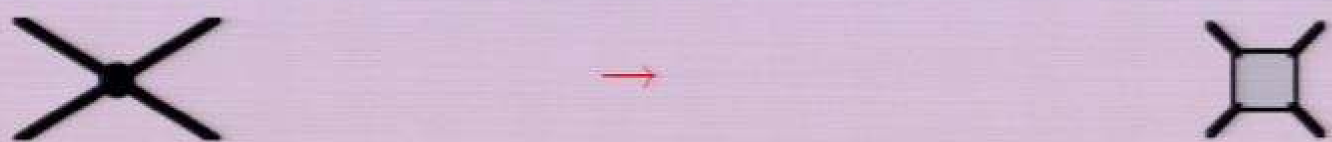
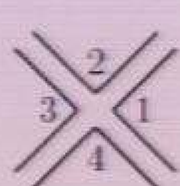


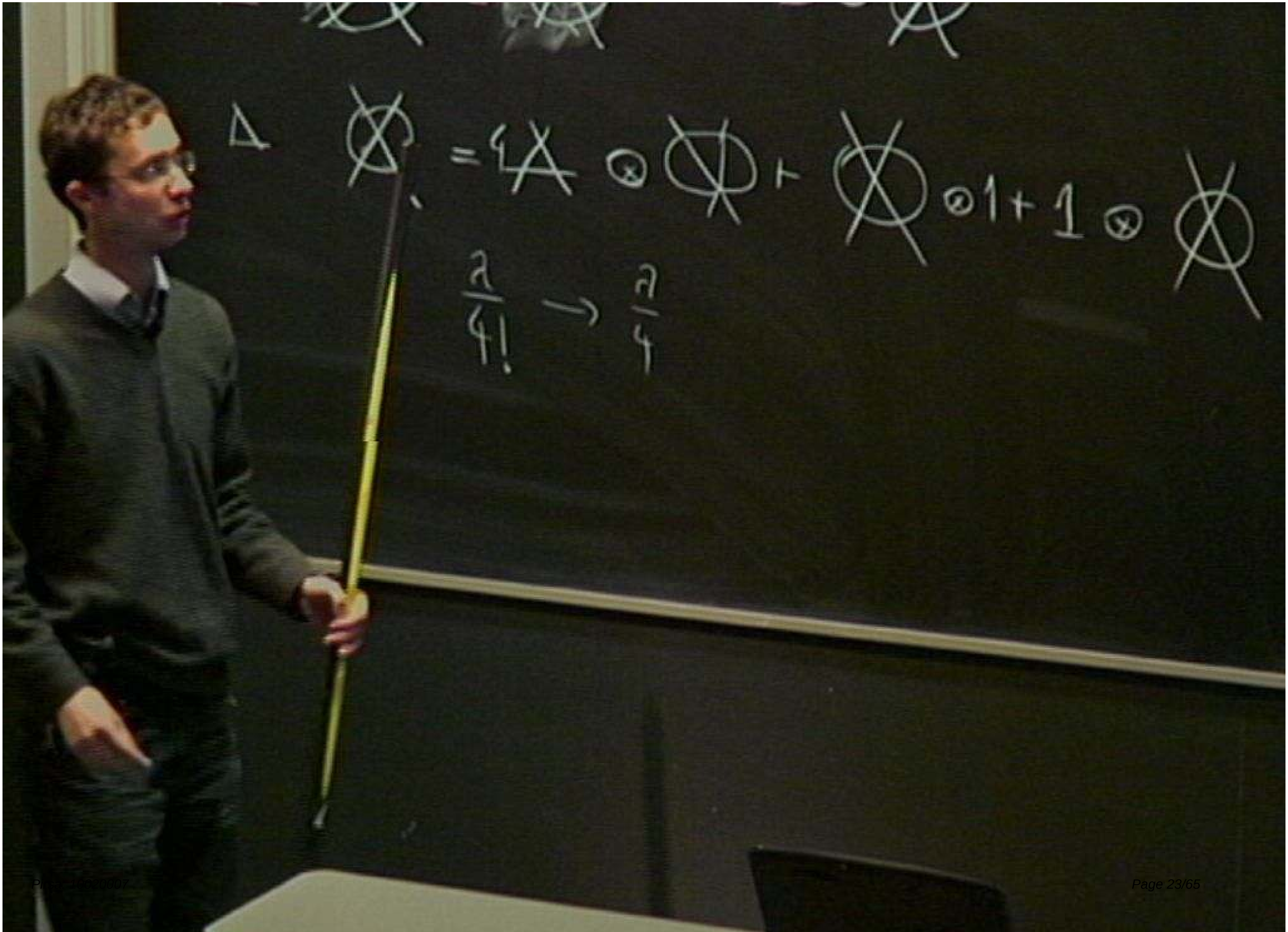
$\hookrightarrow$ non-locality

$\hookrightarrow$ restricted invariance: only under cyclic permutation



$\rightarrow$ ribbon graphs

$\rightarrow$ clear distinction between planar and non-planar graphs



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Feynman graphs in NCQFT

n - number of vertices,
L - number of internal lignes,
F - number of faces,

$$2 - 2g = n - L + F$$

$g \in \mathbb{N}$ - genus

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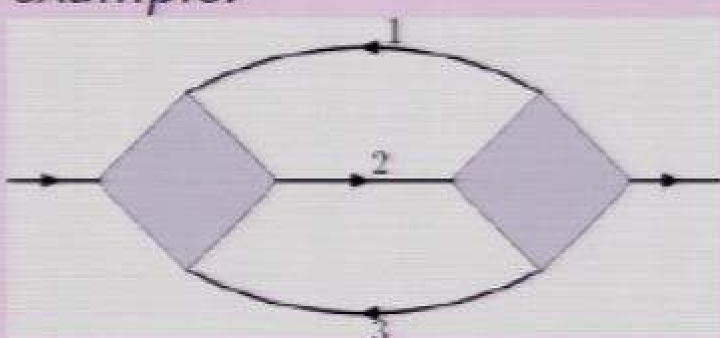
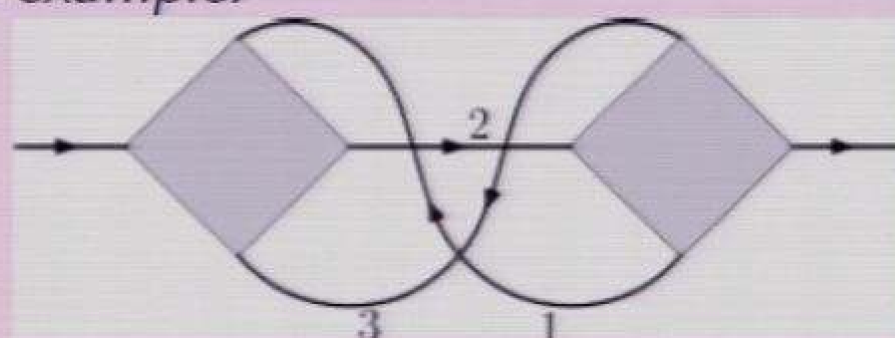
$g = 0$ - planar graph
example:



$n = 2,\ L = 3,\ F = 3,\ g = 0$

$g \geq 1$ - non-planar graph
example:



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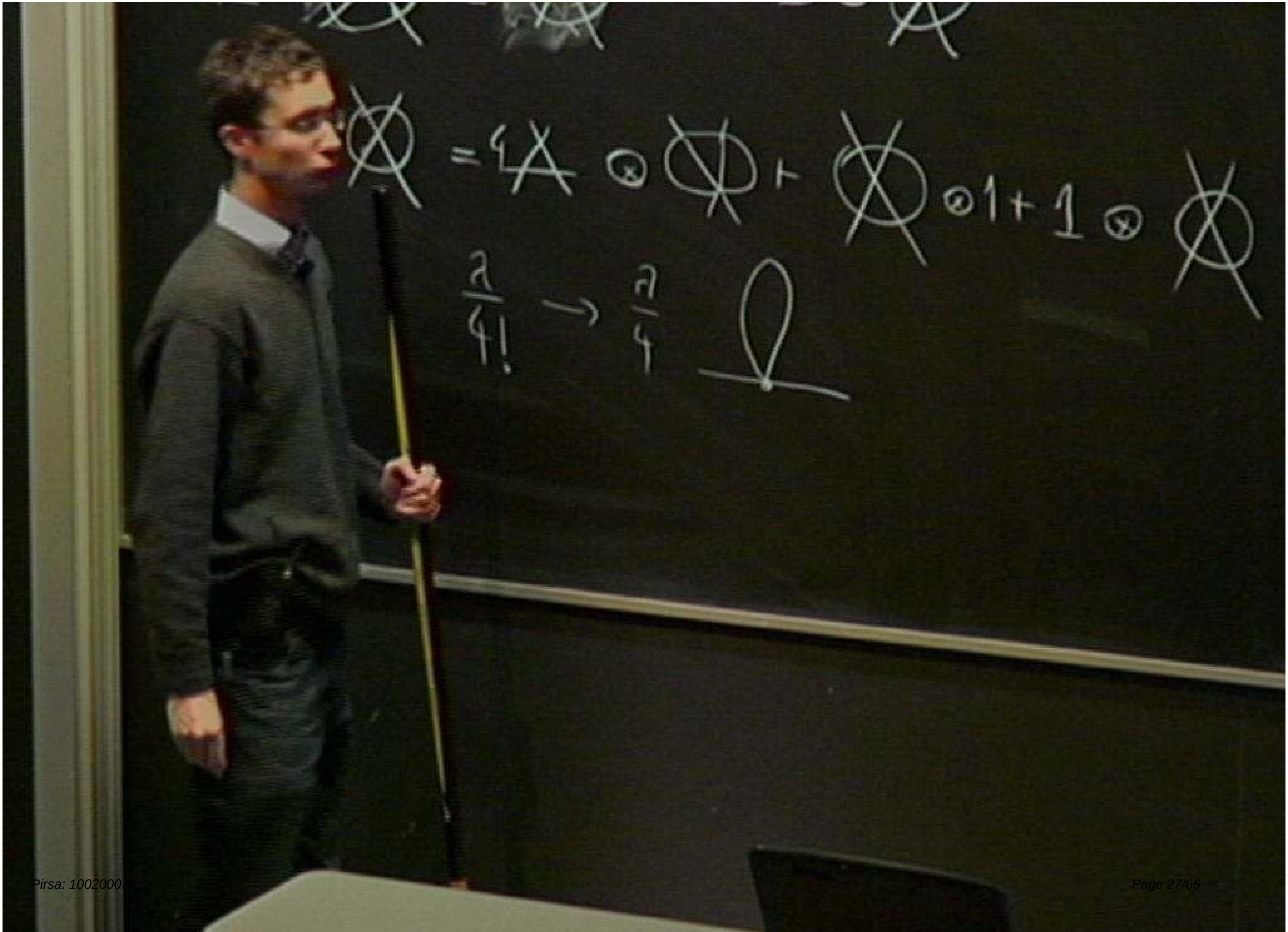

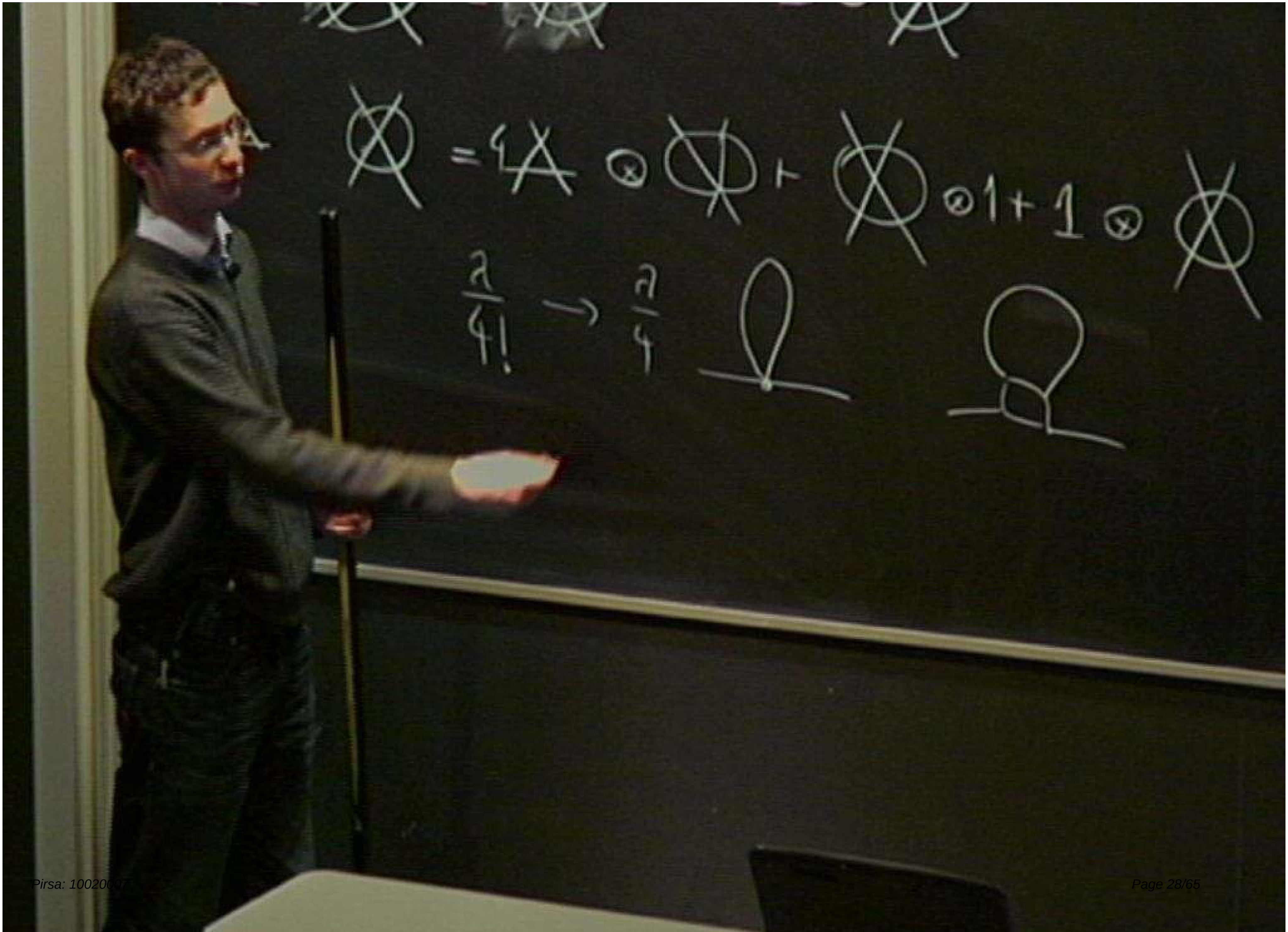

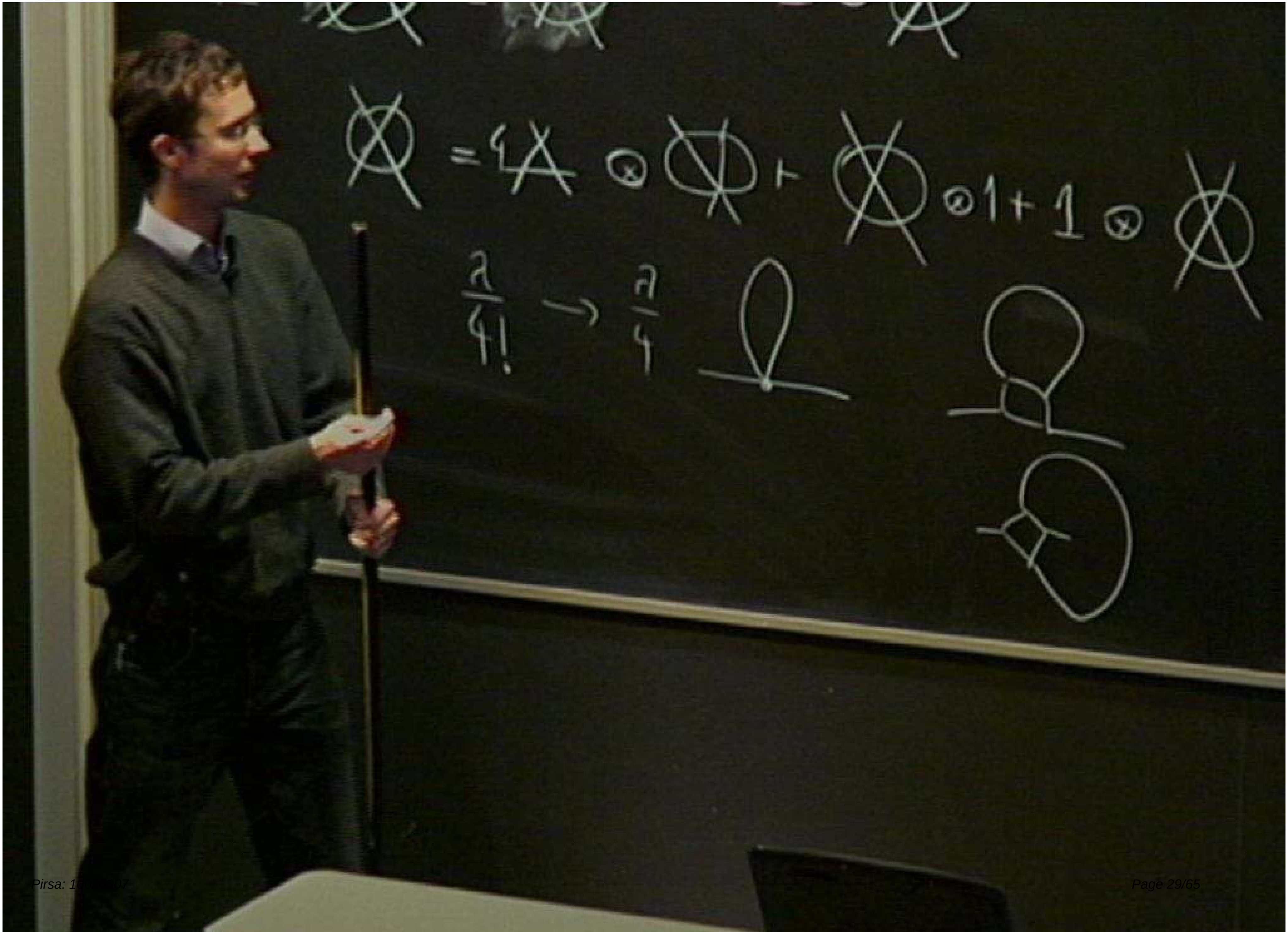
λ/4! → λ/4

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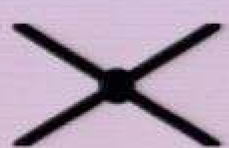

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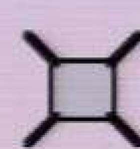

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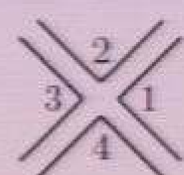

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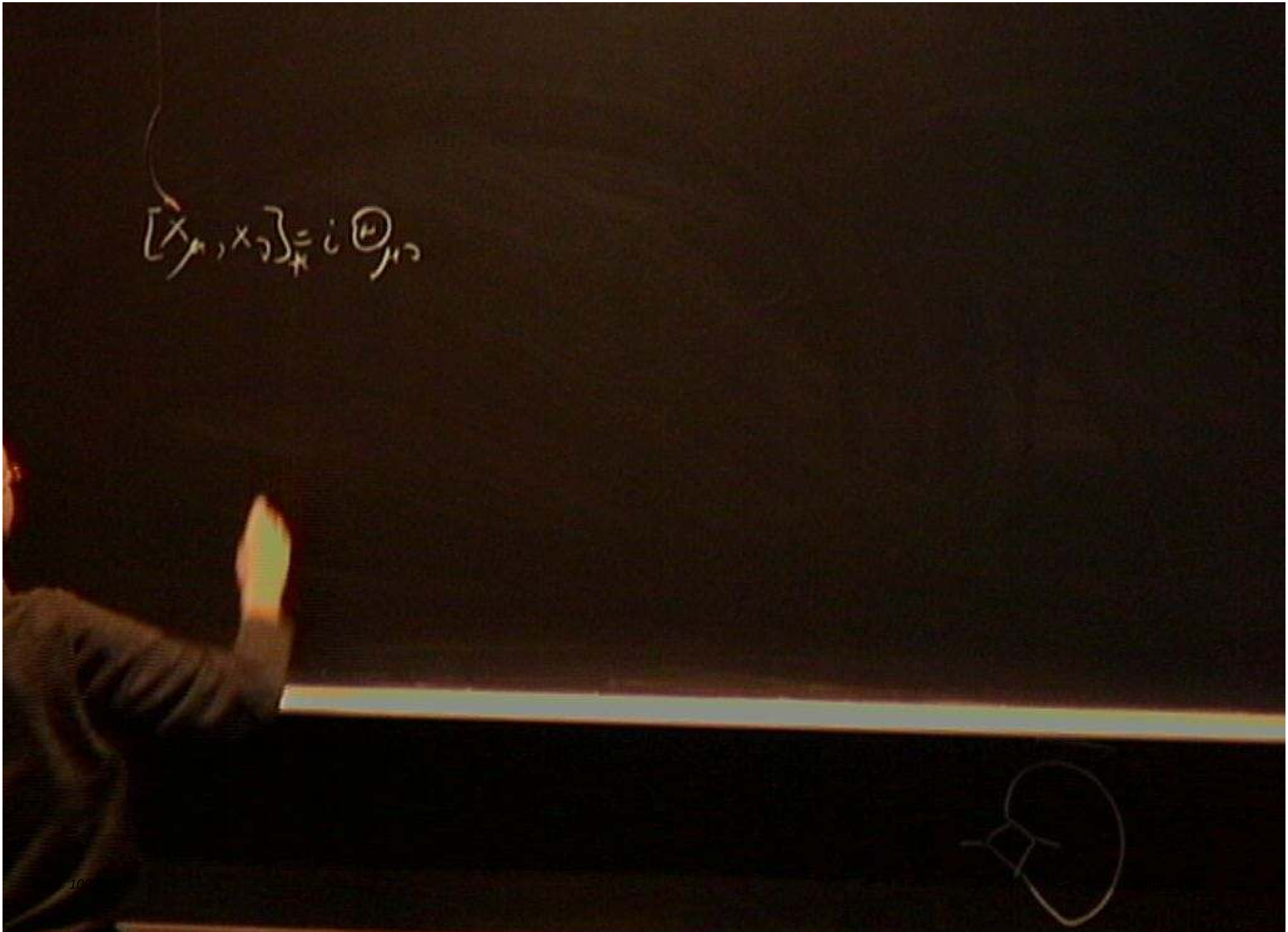
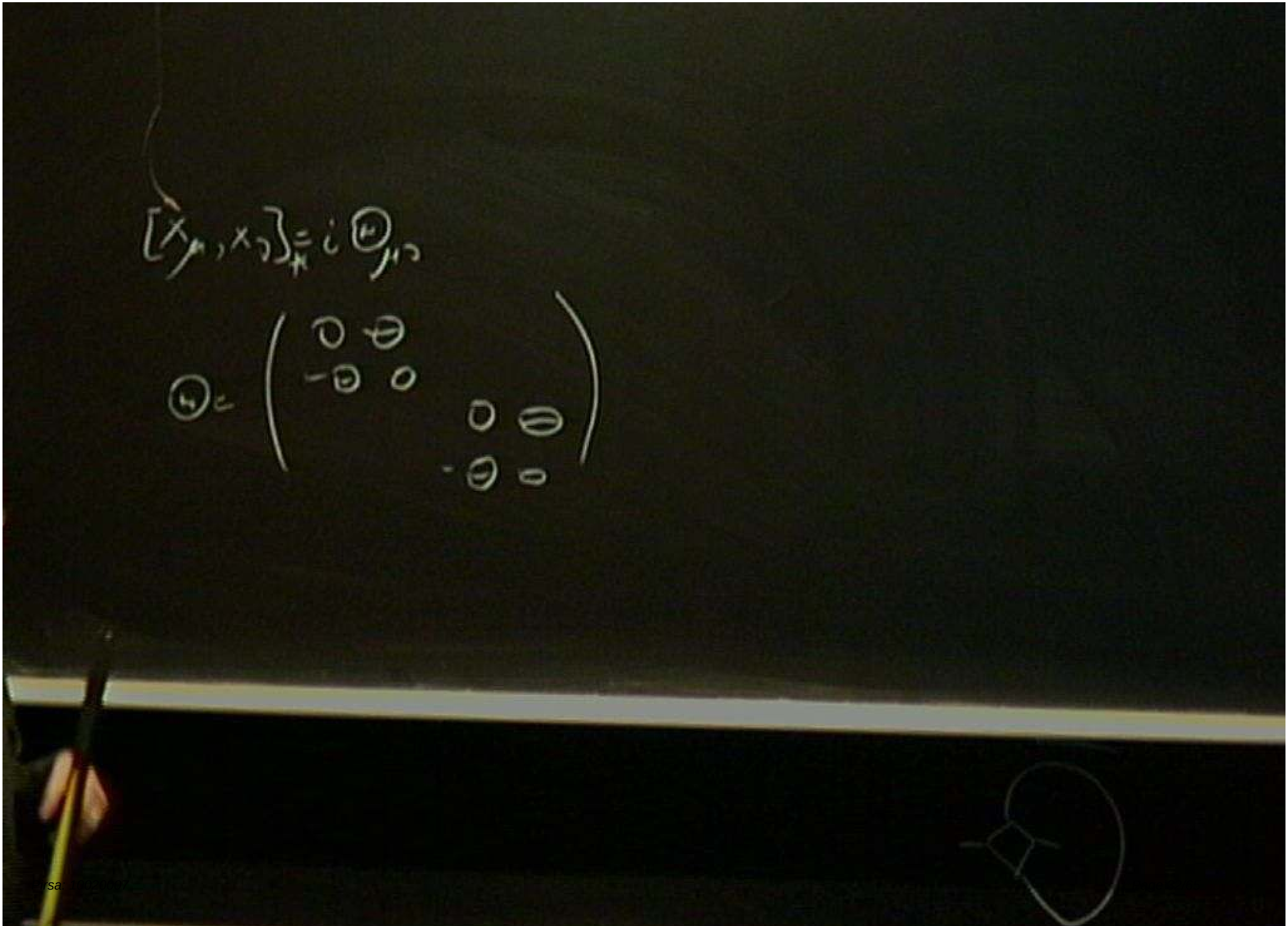

$[x_\mu, x_\nu]_* = i\,\Theta_{\mu\nu}$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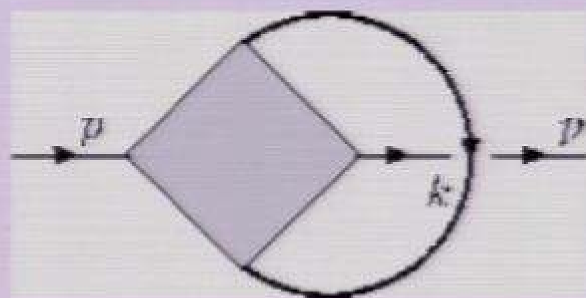
Renormalization on the Moyal space

UV/IR mixing (S. Minwalla *et. al.*, *JHEP*, '00)



$$\lambda \int d^4k \frac{e^{ik_\mu \Theta^{\mu\nu} p_\nu}}{k^2+m^2} \to_{|p|\to 0} \frac{1}{\theta^2 p^2}$$

same type of behavior at any order in perturbation theory

J. Magnen, V. Rivasseau and A. T., *Europhys. Lett.* '09

→ non-renormalizability!

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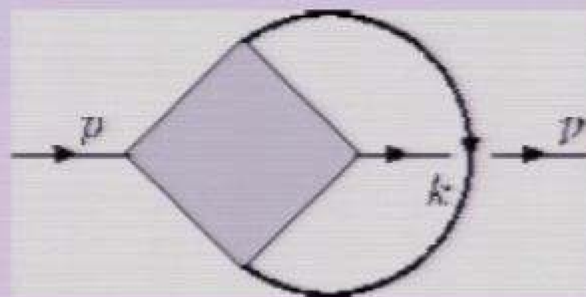

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A first solution to this problem - the Grosse-Wulkenhaar model

additional harmonic term

(H. Grosse and R. Wulkenhaar, *Comm. Math. Phys.*, '05)

$$s[\phi(x)] = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \star \partial^\mu \phi + \frac{\Omega^2}{2} (\tilde{x}_\mu \phi) \star (\tilde{x}^\mu \phi) + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi \right),$$

$$\tilde{x}_\mu = 2(\Theta^{-1})_{\mu\nu} x^\nu.$$

modification of the propagator - the model becomes renormalizable

- most of the techniques of QFT extend to Grosse-Wulkenhaar-like models:
  - the parametric representation
    (R. Gurău and V. Rivasseau, *Commun. Math. Phys.*, '07, A. T. and V. Rivasseau, *Commun. Math. Phys.*, '08, A. T., *J. Phys. Conf. Series*, '08, A. T., solicited by de *Modern Encyclopedia Math. Phys.*)
  - the Mellin representation
    (R. Gurău, A. Malbouisson, V. Rivasseau and A. T., *Lett. Math. Phys.*, '07)
  - dimensional regularization
    (R. Gurău and A. T., *Annales H. Poincaré*, '08)
- study of vacuum configurations (A. de Goursac, A. T. and J-C. Wallet, *EPJ* C, '08)
- gauge model propositions
  $\hookrightarrow$ non-trivial vacuum state

  (A. de Goursac, J-C. Wallet and R. Wulkenhaar *EPJ* C, '07,'08, H. Grosse and M. Wohelegant *EPJ* C, '07)

Translation-invariant renormalizable scalar model

(R. Gurău, J. Magen, V. Rivasseau and A. T., *Commun. Math. Phys.* '09)

the Grosse-Wulkenhaar model breaks translation-invariance !

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the complete propagator:

$$C(p,m,\theta) = \frac{1}{p^2 + a\frac{1}{\theta^2 p^2} + m^2}$$

arbitrary planar irregular 2-point function: same $\frac{1}{p^2}$ behavior !

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$\hookrightarrow$ other modification of the action:

$$S = \int d^4p \, [\frac{1}{2} p_\mu \phi \star p^\mu \phi + \frac{1}{2} a \frac{1}{\theta^2 p^2} \phi \star \phi + \frac{1}{2} m^2 \phi \star \phi + \frac{\lambda}{4!} V^\star[\phi]]. \quad (2)$$

renormalizability at any order in perturbation theory !

NCQFT vs. CMB G. Palma and S. Patil arXiv:0906.4727 [hep-th]

Gribov-Zwanziger result D. Dudal *et. al.*, *Phys. Rev.* D '08

Other translation-invariant field theoretical techniques

- parametric representation (A. T., *J. Phys.* **A** '09)
- relation with Bollobás-Riordan topologic ribbon graph polynomial

  (T. Krajewski, V. Rivasseau, A. T. and Z. Wang, *J. Noncomm. Geom.* (in press))
- renormalization group flow

  (J. Ben Geloun and A. T., *Lett. Math. Phys.* '08)
- commutative limit

  (J. Magnen, V. Rivasseau and A. T., *Lett. Math. Phys.* '08)
- field theories with other noncommutative products

  $\hookrightarrow$ A. T. and P. Vitale, *Phys. Rev.* **D** (in press)

Renormalizability of NCQFT: locality → "Moyality"

QFT	→	NCQFT
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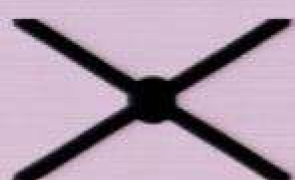

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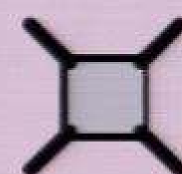

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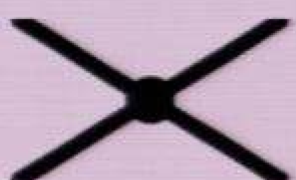

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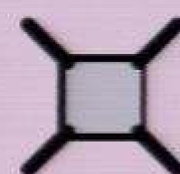
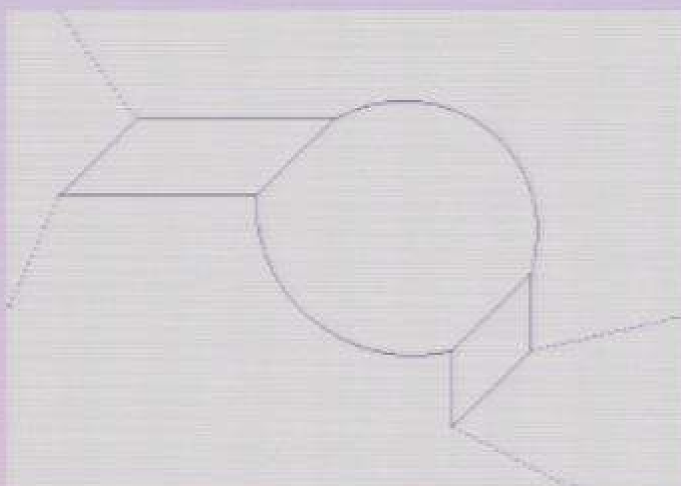
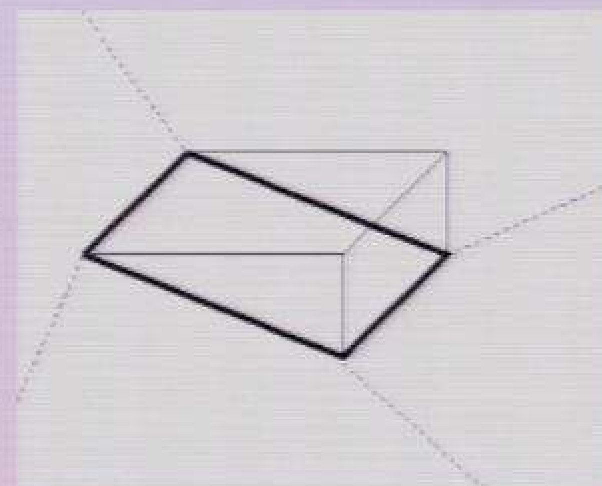


The principle of "Moyality" - ribbon Feynman graph level





valid iff the graph is planar

renormalization necessary only for the planar sector !

Hopf algebra for renormalizable NCQFTs

A. T. and F. Vignes-Tourneret, *J. Noncomm. Geom.*, '08

A. T. and D. Kreimer, arXiv: 0907.2182, submitted

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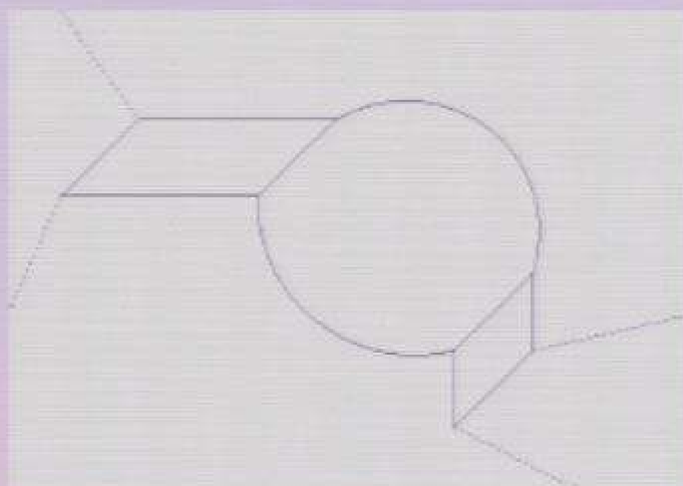

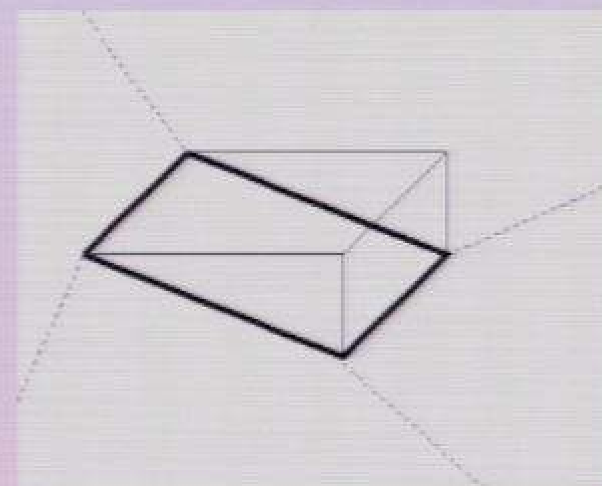

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renormalization necessary only for the planar sector !

Hopf algebra for renormalizable NCQFTs

A. T. and F. Vignes-Tourneret, *J. Noncomm. Geom.*, '08

A. T. and D. Kreimer, arXiv: 0907.2182, submitted

$\hookrightarrow$ definition of a coproduct Δ

$\mathcal{H}$ - the algebra generated by the 1PI Feynman ribbon graphs

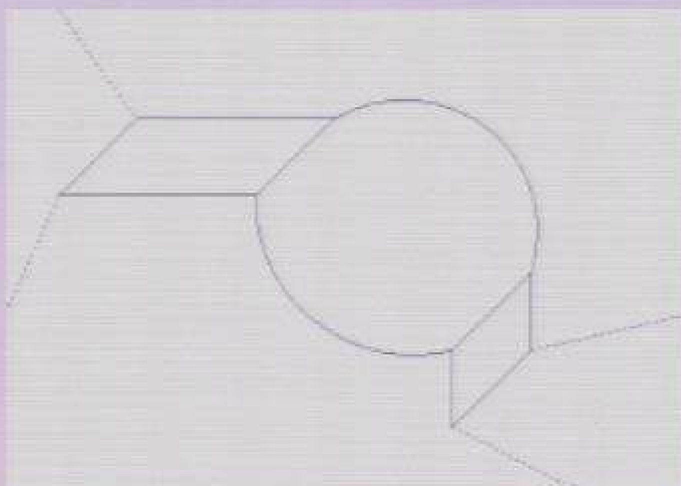
$$\Delta : \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}, \;\; \Delta(G) = G \otimes 1 + 1 \otimes G + \sum_{\gamma \in \underline{G}} \gamma \otimes G/\gamma,$$

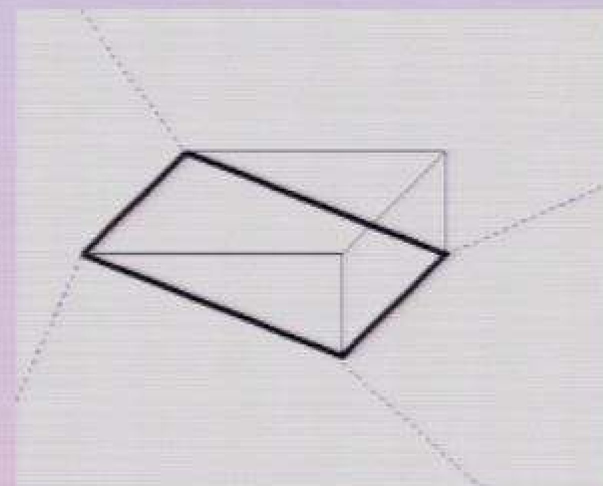
$$\varepsilon : \mathcal{H} \to \mathbb{K}, \;\; \varepsilon(1) = 1, \; \varepsilon(G) = 0, \; \forall G \neq 1,$$

$$S : \mathcal{H} \to \mathcal{H}, \;\; G \mapsto -G - \sum_{\gamma \in \underline{G}} S(\gamma) G/\gamma.$$

$\Rightarrow$ $(\mathcal{H}, \Delta, \varepsilon, S)$ *is a Hopf algebra.*

The principle of "Moyality" - ribbon Feynman graph level





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$\hookrightarrow$ 2– and 4–point graphs (in commutative ϕ^4)

$\rightarrow$ 2– and 4–point planar regular graph

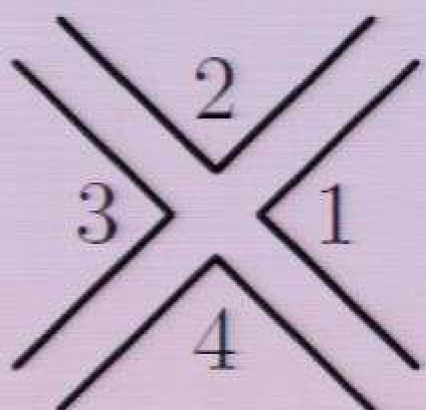
↪ 2− and 4−point graphs (in commutative ϕ^4)

→ 2− and 4−point planar regular graph

this Hopf algebra structure - the combinatorial backbone of noncommutative renormalization

Hochschild cohomology - combinatorial Dyson-Schwinger equation

core Hopf algebra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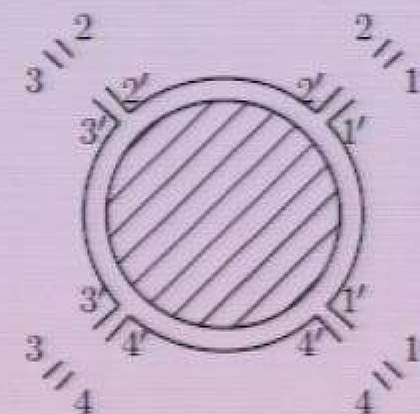
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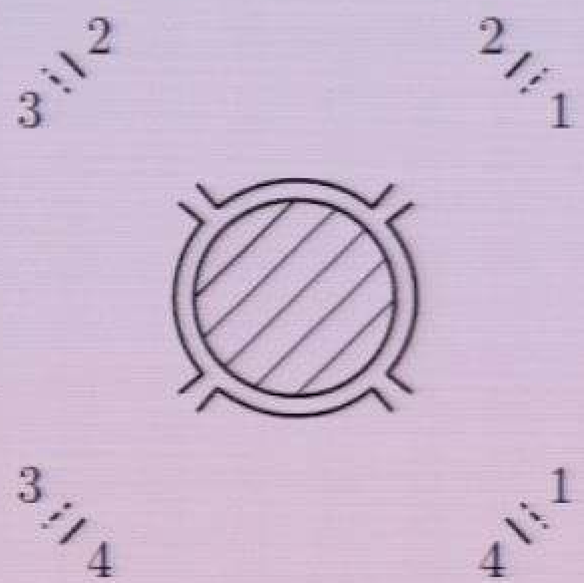

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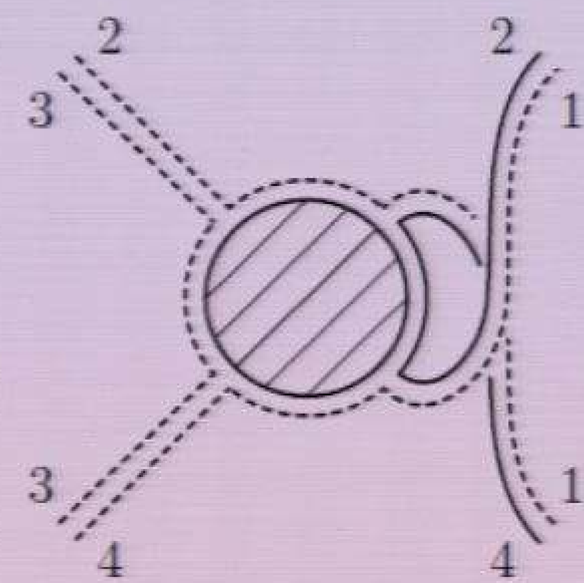

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What about QG? - Markopoulou's construction

(F. Markopoulou, *Class. Quant. Grav.*, '03)

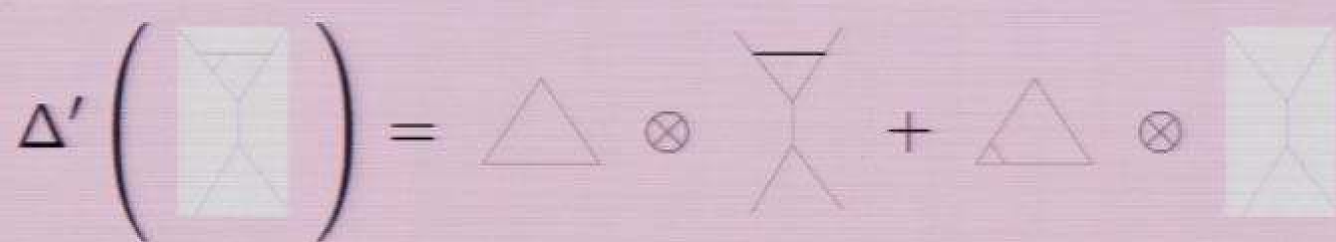
M - the space of spin-foams

$$\Delta\Gamma = \Gamma \otimes 1 + 1 \otimes \Gamma + \sum_{\gamma \subset \Gamma} \gamma \otimes \Gamma/\gamma,$$

the non-trivial part:

$$\Delta'\Gamma = \sum_{\gamma \subset \Gamma} \gamma \otimes \Gamma/\gamma,$$

1 - the empty spin-foam



(A. Tanasa, arXiv:0909.5631, *Class. Quant. Grav.*, (in press))

ex.:

quotiented out (they form a Hopf coideal)

- the quotiented space - graduaded *core Hopf algebra*

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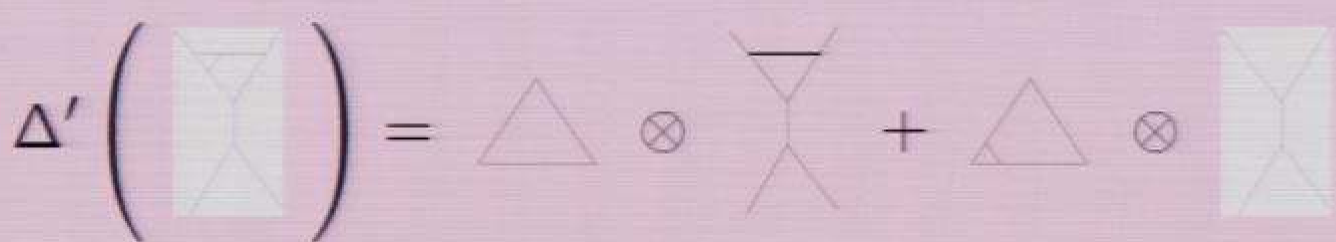

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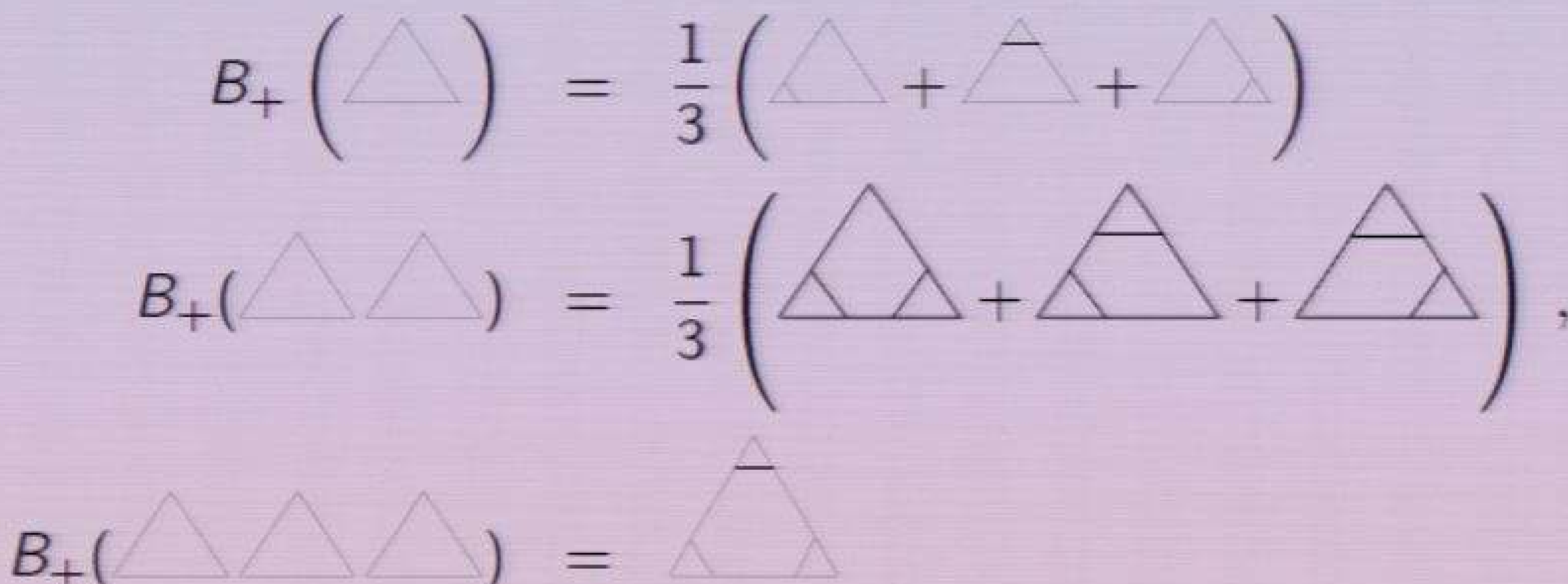
The grafting operator B_+



the grafting operator B_+ - increases the graduation by 1

the internal structure does not play any rôle

Hochschild 1−cocycle on the core Hopf algebra

operator of insertion of Feynman graphs in the QFT frame

Equivalent of the combinatorial Dyson-Schwinger equation

$$X = 1 + tB_+(X^3)$$

cubic combinatorial Dyson-Schwinger equation

ansatz:

$$X = \sum_{n=0}^{\infty} t^n c_n$$

recursive result:

$$c_{n+1} = \sum_{k_1+k_2+k_3=n} B_+(c_{k_1} c_{k_2} c_{k_3}).$$

different of the general combinatorial Dyson-Schwinger equations considered in QFTs

Further results

$$\Delta(B_+) = B_+ \otimes 1 + (\mathrm{id} \otimes B_+)\Delta,$$

$$\Delta(c_n) = \sum_{k=0}^{n} P_k^n \otimes c_k,$$

P_k^n - polynomial in c_ℓ, ($\ell \leq n$), total degree $n - k$.

ex.: $D = 2$

P_n^k	$n=0$	$n=1$	$n=2$	$n=3$
$k=0$	1	c_1	c_2	c_3
$k=1$		1	$3c_1$	$3c_1^2 + 3c_2$
$k=2$			1	$5c_1$
$k=3$				1

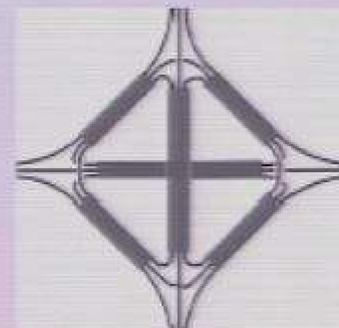
$D = 3$

P_n^k	$n=0$	$n=1$	$n=2$	$n=3$
$k=0$	1	c_1	c_2	c_3
$k=1$		1	$4c_1$	$6c_1^2 + 4c_2$
$k=2$			1	$7c_1$
$k=3$				1

$D = 4$

P_n^k	$n=0$	$n=1$	$n=2$	$n=3$
$k=0$	1	c_1	c_2	c_3
$k=1$		1	$5c_1$	$10c_1^2 + 5c_2$
$k=2$			1	$9c_1$
$k=3$				1

Relation with quantum Group Field Theory (GFT)



bubble
(see for example L. Freidel *et. al.*, *Phys. Rev.* **D** ('09)
- algorithm for identifying bubbles)

$\hookrightarrow$ Feynman amplitude divergent

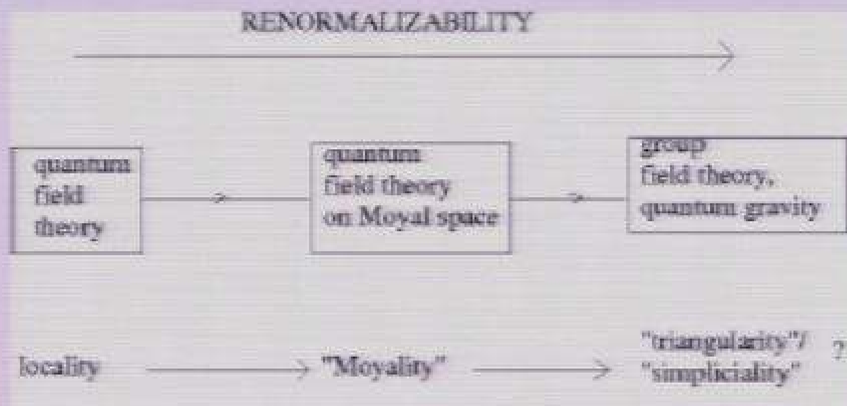
quantum GFT - better framework for renormalizability studies

$\hookrightarrow$ insights on the renormalizability of $3D$ models

(L. Freidel *et. al.*, *Phys. Rev.* **D** '09, R. Gurău, arXiv:0907.2582, J. Magnen *et. al.*, arXiv: 0906.5477, *Class. Quant. Grav.* (in press), J. Ben Geloun *et. al.*, arXiv:0911.1719, arXiv:1002.3592)

Conclusions and perspectives

applications of QFT techniques for the renormalizability study of quantum GFT models



- relation between topological polynomials of tensor graphs (R. Gurău, arXiv:0911.1945) and Symanzik polynomials in linearized colored quantum GFTs (J. Ben Geloun *et. al.*, arXiv:1002.3592, R. Gurău, arXiv:0907.2582)
- Connes-Kreimer analysis in GFT framework: type 1 graphs (L. Freidel *et. al.*, *Phys. Rev.* D ('09)) - Hopf primitives
- renormalization group flow of the Freidel-Krasnov model (quantum GFT formulation) (work in progress with the Paris region group)

"The amount of theoretical work one has to cover before being able to solve problems of real practical value is rather large, but this circumstance is [...] likely to become more pronounced in the theoretical physics of the future."
P.A.M. Dirac, *"The principles of Quantum Mechanics"*, 1930

Thank you for your attention!