Title: Viscosity, quark gluon plasma, and string theory

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Abstract: Viscosity is a very old concept which was introduced to physics by Navier in the 19th century. However, in strongly coupled systems, the viscosity is usually difficult to compute. In this talk I will describe how gauge/gravity duality, a by-product of string theory, allows one to compute the viscosity for a class of strongly interacting fluids not too dissimilar to the quark gluon plasma. I will also describe efforts to measure the viscosity and other physical properties of the quark gluon plasma at the Relativistic Heavy Ion Collider.

Viscosity, quark gluon plasma, and string theory

D. T. Son

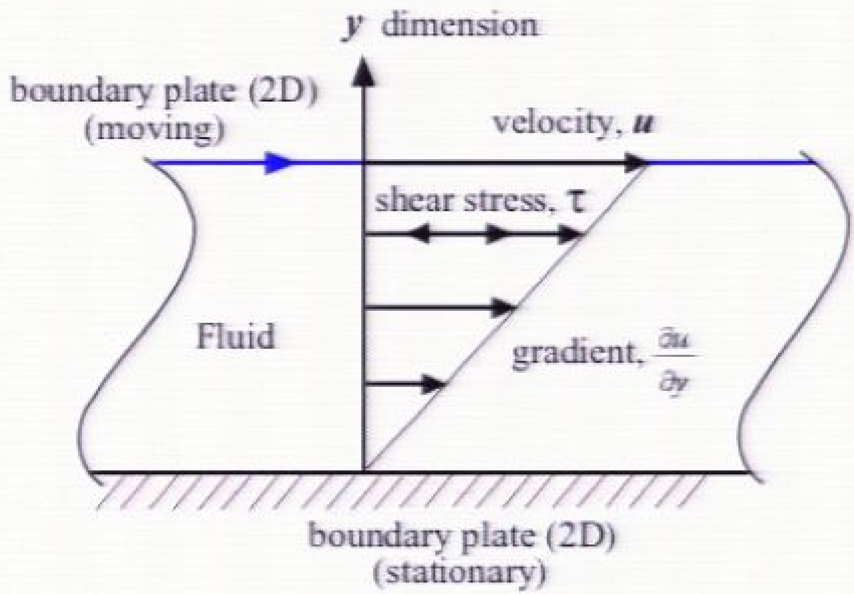
Institute for Nuclear Theory, University of Washington

Plan

- Viscosity
- Heavy ion collisions and the quark gluon plasma
- Gauge/gravity duality: a surpising by-product of string theory
- Applications of gauge/gravity duality

Viscosity

Viscosity: introduced by Claude Navier in 1822 into what would be later called the Navier-Stokes equation.



Friction force between two plates:

$$F = \eta A \frac{\partial u_x}{\partial y}$$

Interlude: viscosity and the strength of interaction

Viscosity *diverges* when interaction is turned off

Reason: large mean free path $\rightarrow$ easy transport of momentum.

Apparent paradox: when interaction is turned off, there is no dissipation. Why the viscosity is infinite?

Answer: the following two limits do not commute:

- The limit of infinitely weak interaction
- The hydrodynamic limit of macroscopic distances

To find the viscosity as interaction $\rightarrow 0$, one needs to do experiments at larger and larger distances, so that the size of the system is always larger than the mean free path

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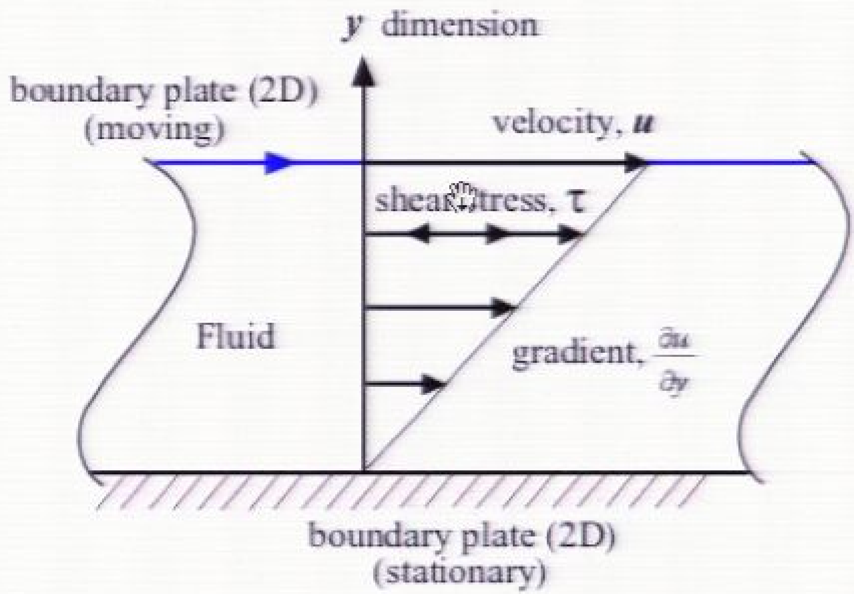

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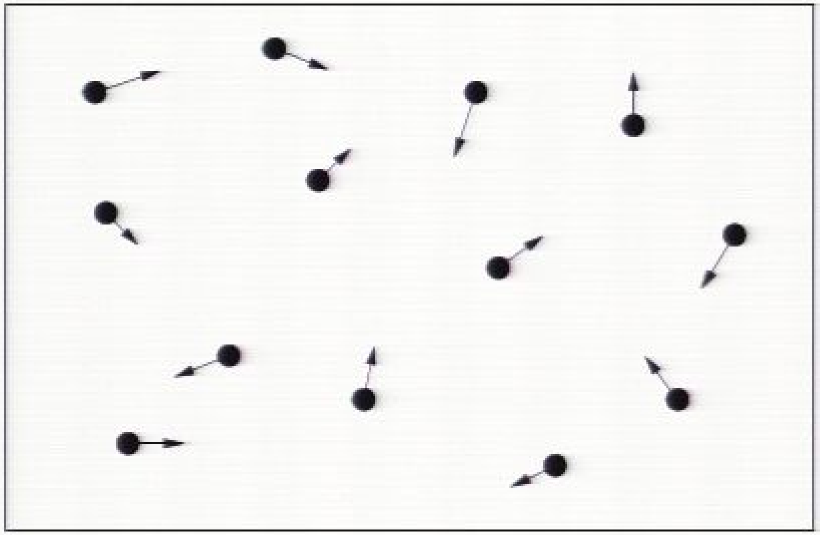
Viscosity and kinetic theory of gas



- Maxwell: in the kinetic theory of gases, viscosity is due to the collisions between gas molecules.
- Hydrodynamics appears as an effective description, valid on length scales $\gg$ mean free path

Maxwell's estimate of the viscosity:

$$\eta \sim \rho v \ell = \text{mass density} \times \text{velocity} \times \text{mean free path}$$

Consequence: at fixed temperature viscosity is independent of pressure (density).
Contradicts expectation at the time: the denser the gas, the larger the viscosity

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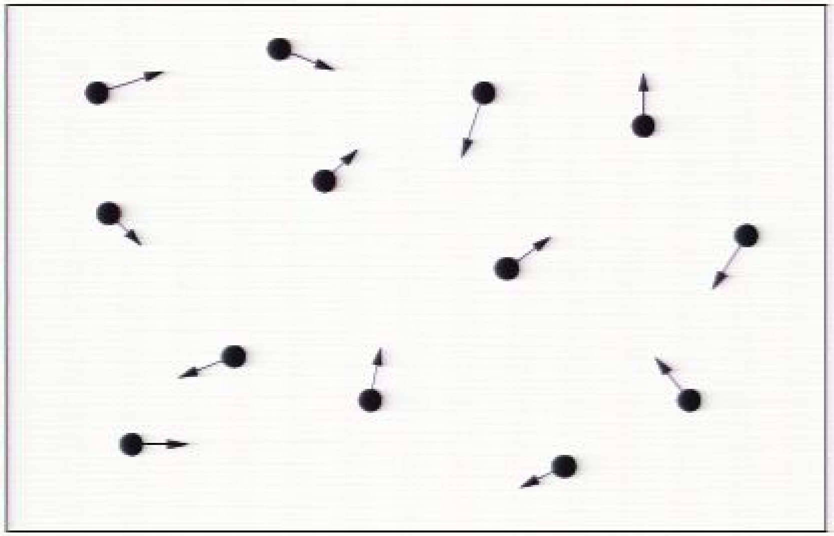

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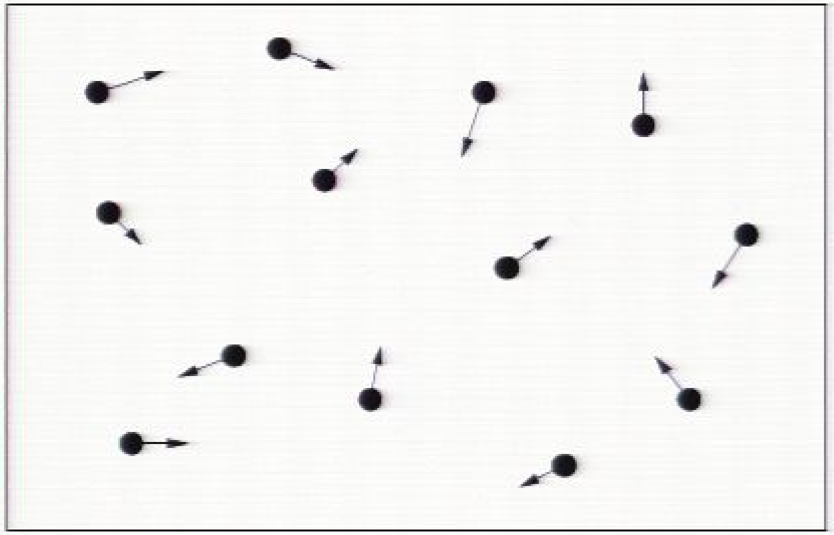

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"Such a consequence of a mathematical theory is very startling, and the only experiment I have met with on the subject does not seem to confirm it" (1860)

Maxwell's own experiment

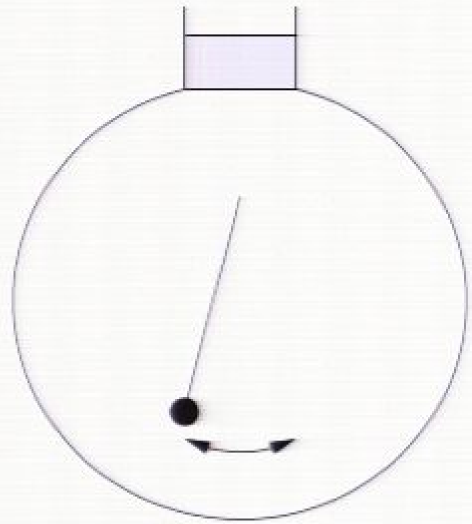
During the next few years Maxwell, with the help of his wife, designed and carried out his own expriment.

Reported result in 1865: viscosity of air is independent of pressure when the latter varies from about 1/60 to one atmosphere.

But perhaps this fact has been stumbled upon long before Maxwell.

Boyle's experiment

Boyle and Hooke's experiment # 26 (1660)



pendulum in a glass container

pump out air: naively one expects oscillations to last longer: less dissipation

Actual result: no appreciable change in damping rate

Consistent with Maxwell's finding.

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Viscosity of liquids: a huge range

Viscosity of fluids is much poorer undertsood

Table 8.4.1. *Viscosities η for some common materials in units of centipoise (10^{-2} erg s/cm^3).*

Substance	Temperature	Viscosity (cp)
Air	18°C	0.018
Water	0°C	1.8
Water	20°C	1
Water	100°C	0.28
Glycerin	20°C	1500
Mercury	20°C	1.6
n-Pentane	20°C	0.23
Argon	85K	0.28
He^4	4.2K	0.033
Superfluid He^4	< 2.1K	0
Glass		$> 10^{15}$

Note that, by popular convention, the designation "glass" is applied to any disordered material once its viscosity exceeds 10^{15}cp.

(from *Chaikin & Lubensky, Principles of Condensed Matter Physics*)

Viscosity spans many orders of magnitudes, despite the fact that the density does not change much.

Viscosity of liquid glycerin

T (C)	η (mPa s)
0	12070
10	3900
20	1410
30	612
40	284
50	142
60	81.3
70	50.6
80	31.9
100	14.8
120	7.8
140	4.7
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Liquid viscosity can be very large

The pitch-drop experiment, University of Queensland

- 8 drops since 1930
- No one has witnessed a drop fall
- Viscosity $\sim 10^{11}$ times of water
- 2005 Ig Nobel Prize in Physics

Purcell and Weisskopf

- E. Purcell was very puzzled by the fact that liquid viscosity spans such a huge range

Purcell and Weisskopf

- E. Purcell was very puzzled by the fact that liquid viscosity spans such a huge range
- He thought about challenging Weisskopf to explain this fact

Life at low Reynolds number

E. M. Purcell
Lyman Laboratory, Harvard University, Cambridge, Massachusetts 02138
(Received 12 June 1976)

Editor's note: This is a reprint (slightly edited) of a paper of the same title that appeared in the book *Physics and Our World: A Symposium in Honor of Victor F. Weisskopf*, published by the American Institute of Physics (1976). The personal tone of the original talk has been preserved in the paper, which was itself a slightly edited transcript of a tape. The figures

Am. J. Phys. **45**, 3 (1977).

Purcell's question

But it's more mysterious than that, Viki, because if you look at the Chemical Rubber Handbook table you will find that there is almost no liquid with viscosity much lower than that of water. The viscosities have a big range *but they stop at the same place*. I don't understand that. That's what I'm leaving for him.[1]

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Viscosity of liquids is unbounded from above, but seems to be bounded from below

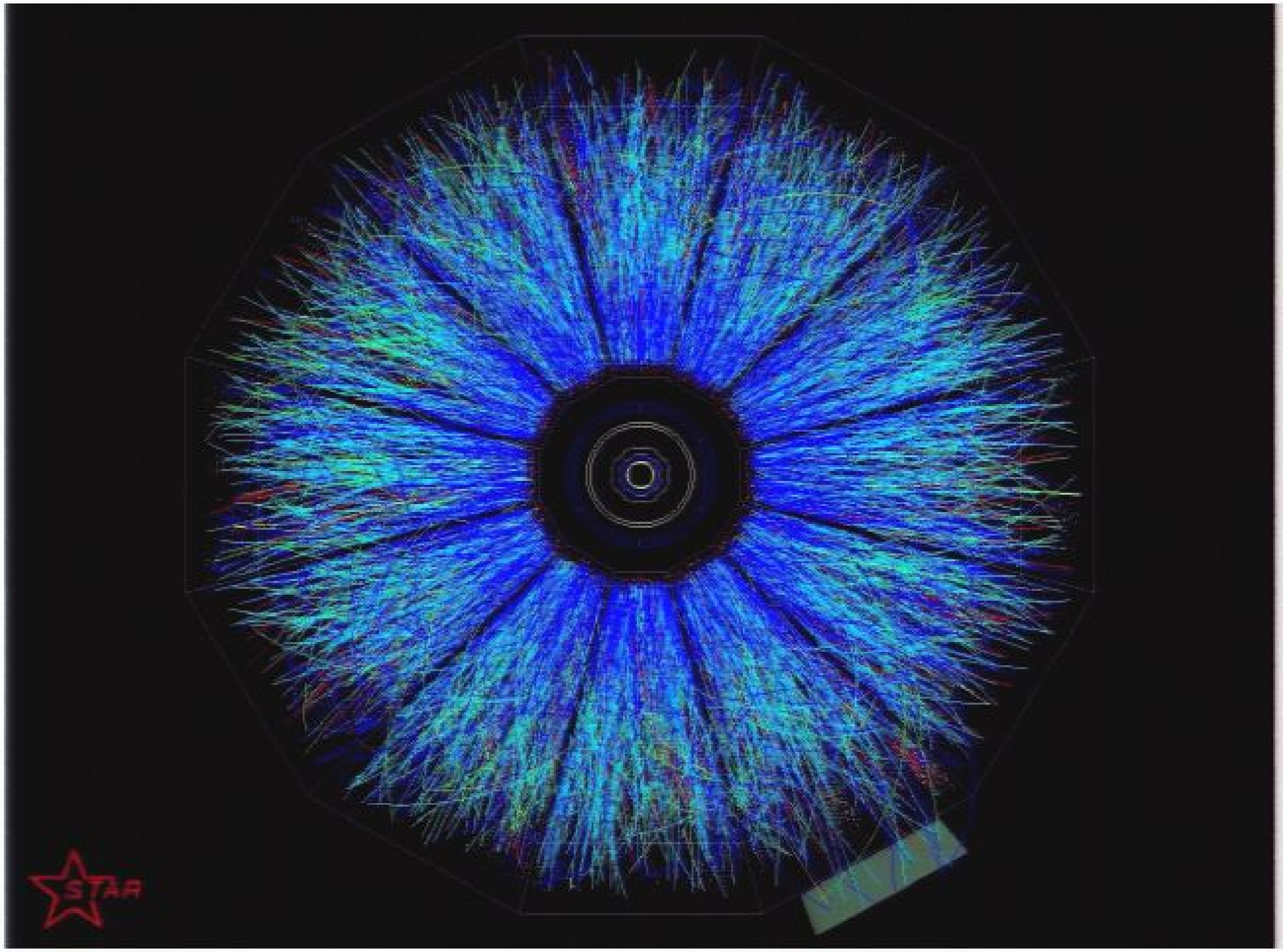
A version of Purcell's question: By playing with the interaction strength, can one make $\eta \to 0$?

Modern context: the quark gluon plasma

Heavy ion collisions

- Experiment at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Lab
- Colliding gold beam with energy 200 GeV per pair of nucleons

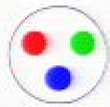
A tybpical heavy ion collision



Goal: to create and study the quark gluon plasma

The quark gluon plasma

Hadrons consists of quarks (3 quarks, or 1 quark and 1 antiquark)



protons, neutrons

mesons (pions etc)

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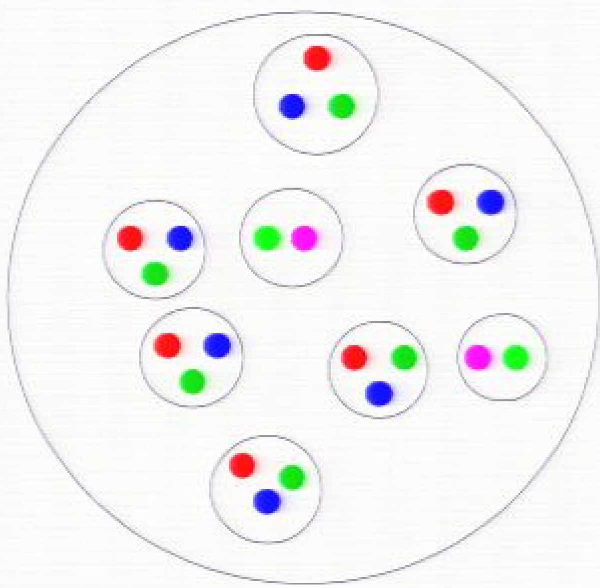
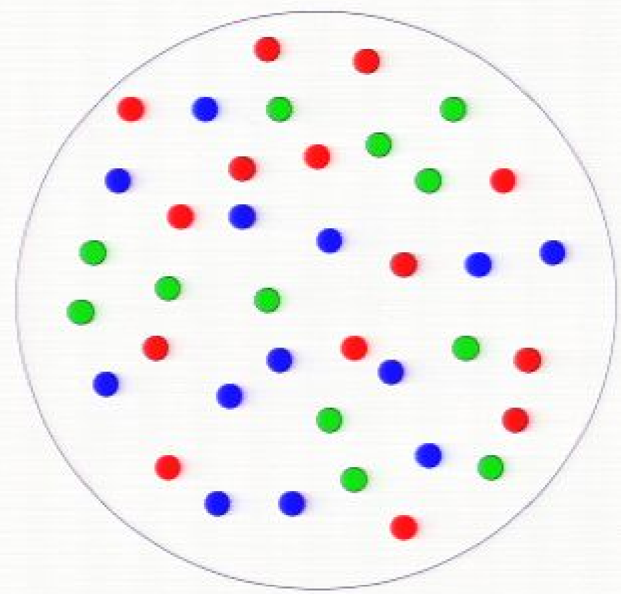


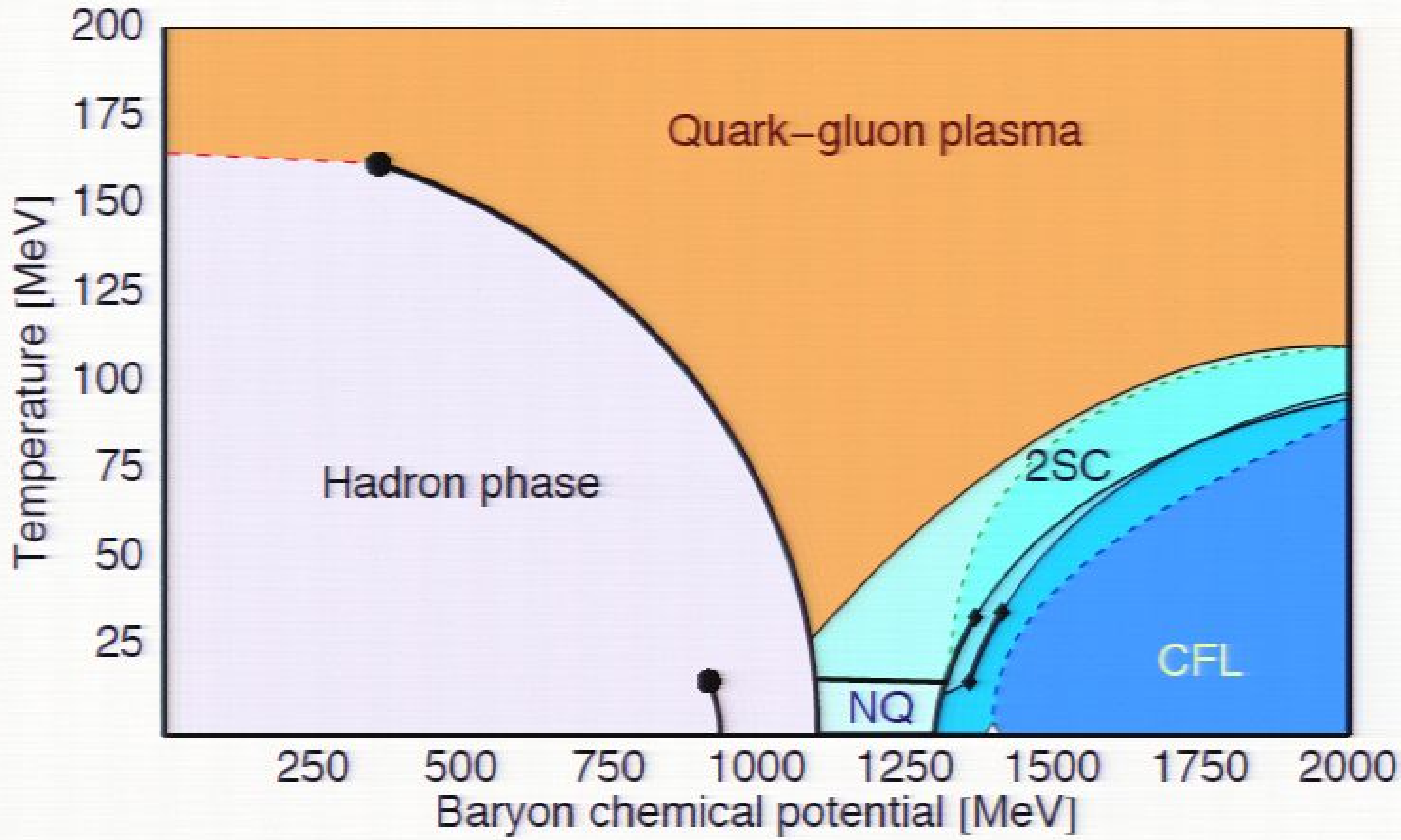
hadron gas



quark–gluon plasma

Quark-gluon plasma: quarks and gluons move relatively freely

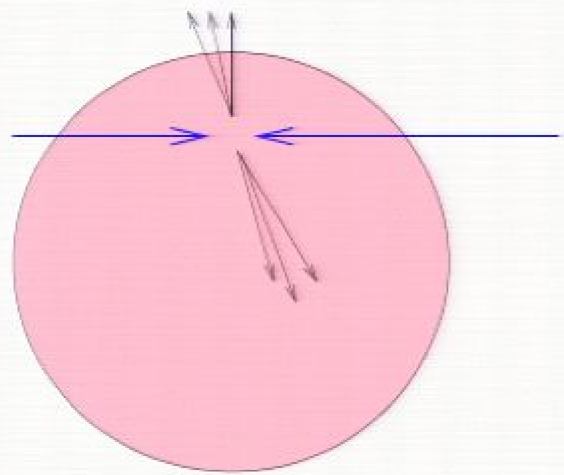
The QCD phase diagram



(from Ruester et al., hep-ph/0503184)

QGP

- Unambiguous identification of QGP difficult
- However, RHIC has created some strongly interacting medium
  - Particle production: well described by thermal distribution
  - Supression of back-to-back jet correlations: signals that some jets are "lost" going through the medium

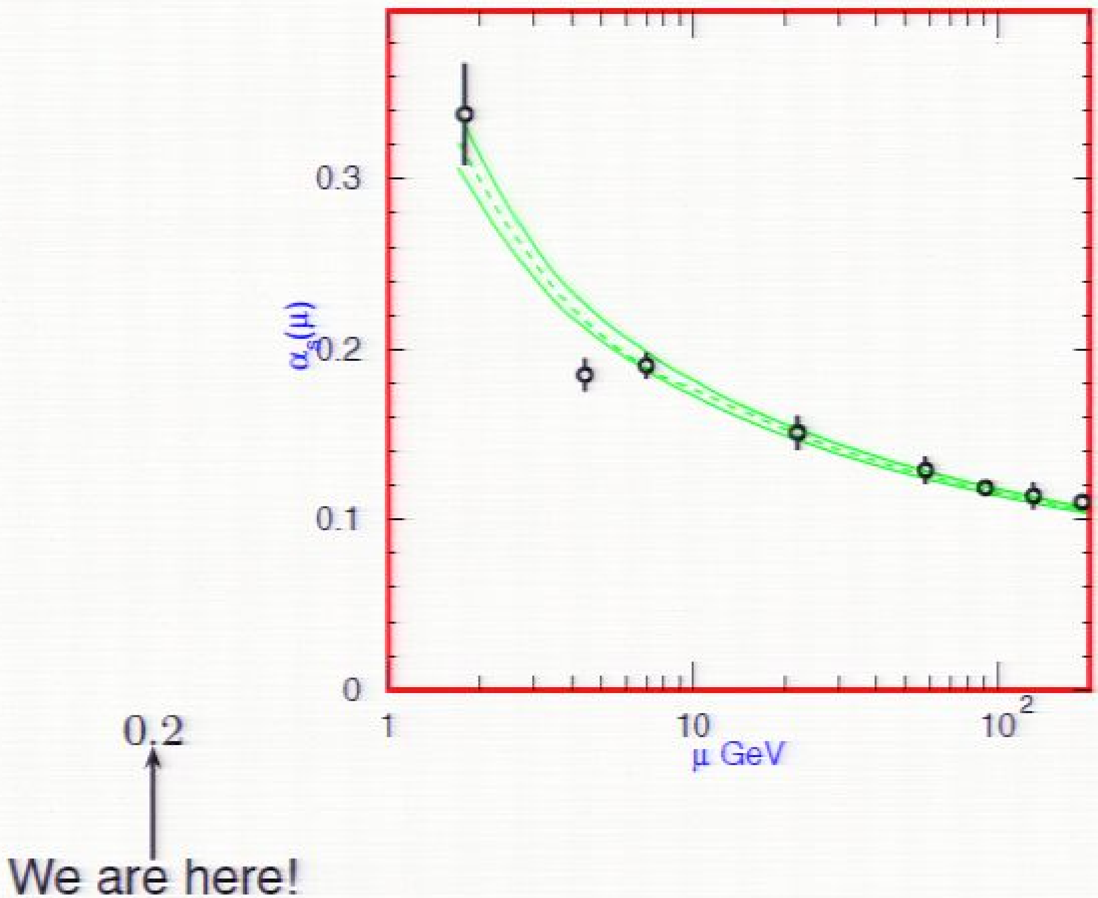


  - "Elliptic flow": signals that particles push each other (more later)

Strong coupling regime

Can one compute theoretically the viscosity of the quark gluon plasma?

- Good: fundamental theory known: Quantum Chromodynamics (QCD)
- Bad: perturbation theory does not work



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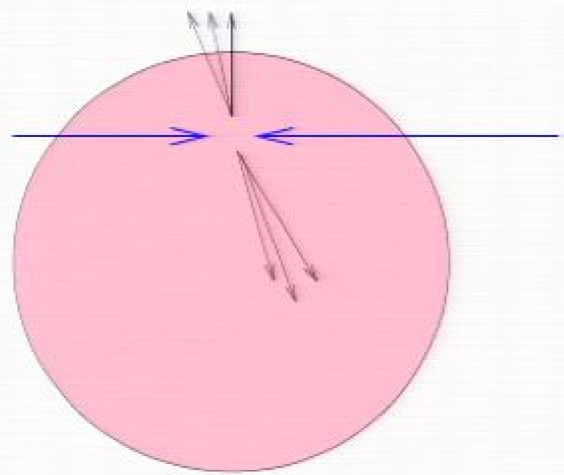

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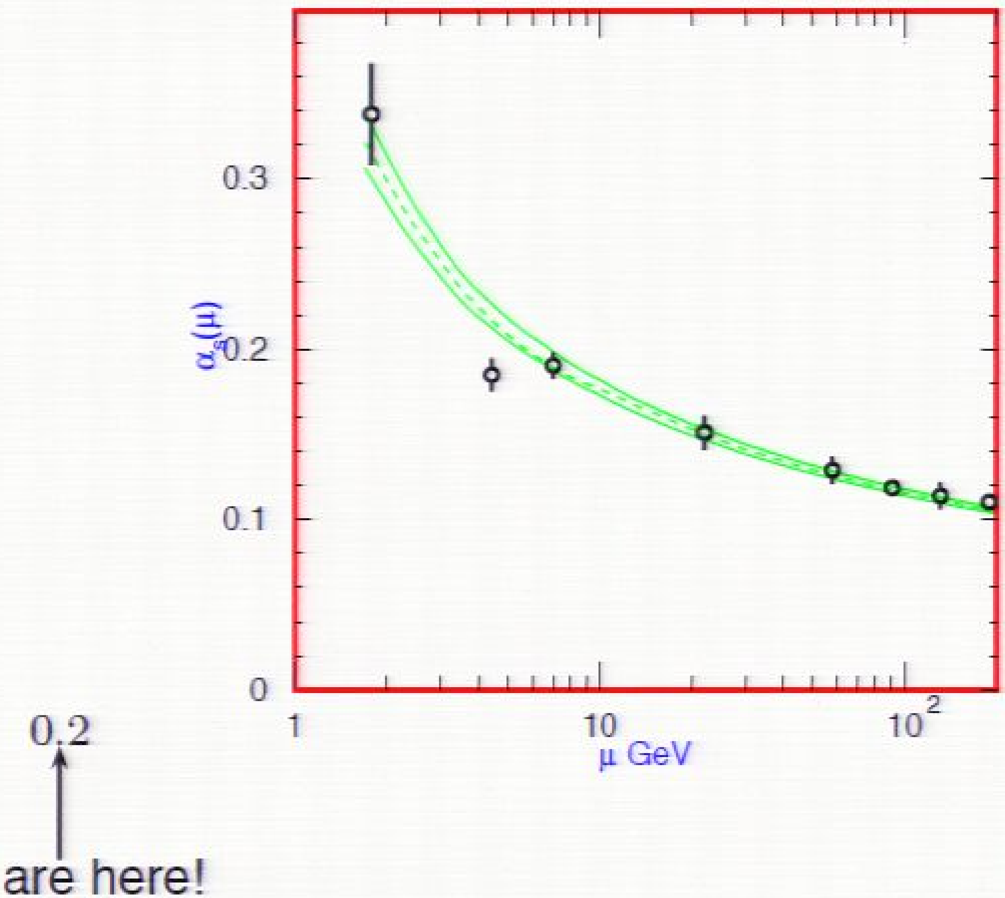
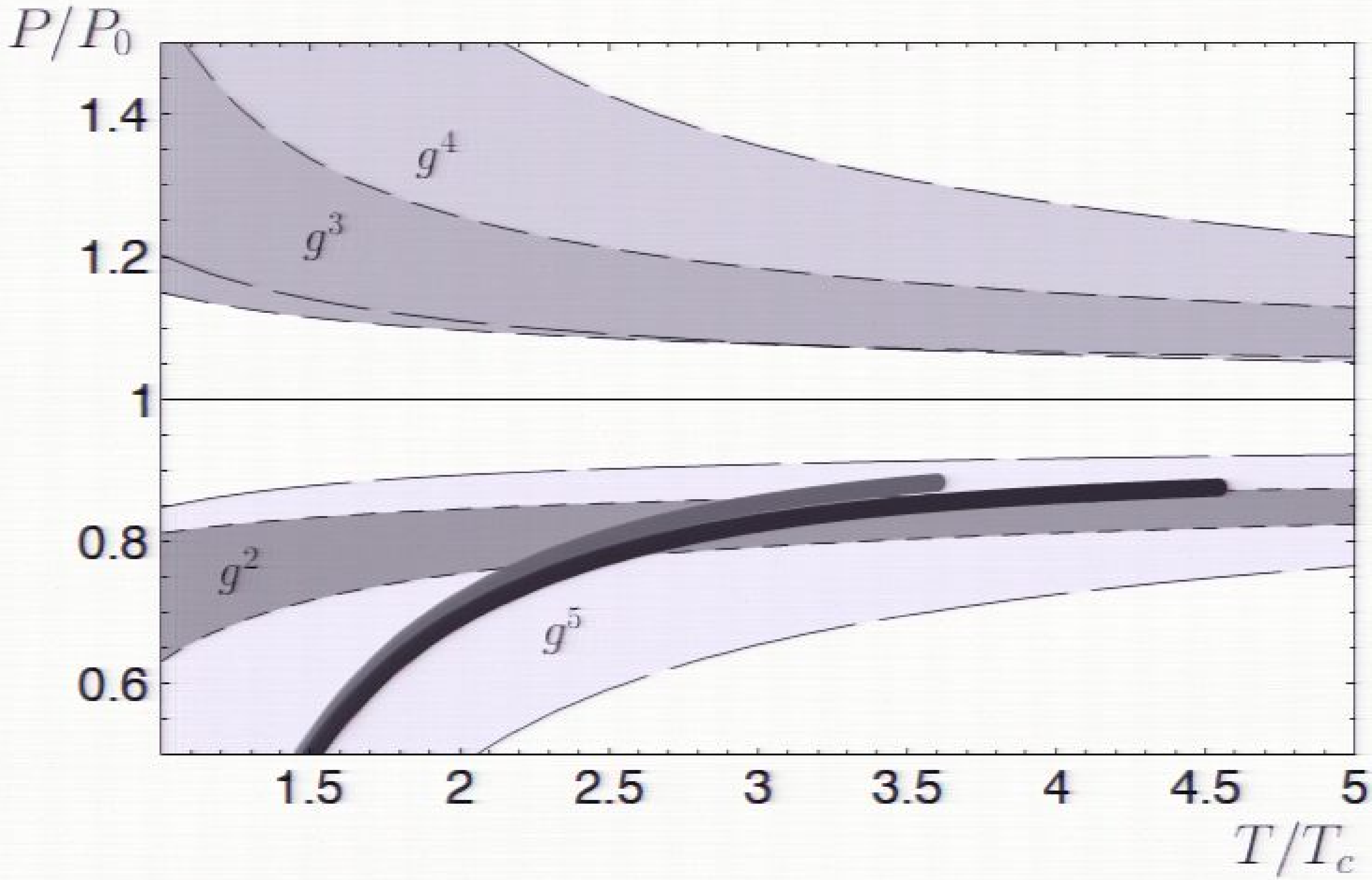


Converging perturbation series?



$$P/P_0|_{T=2T_c} = 1 - 0.2 + 0.4 + 0.2 - 0.8 + \cdots$$

So what to do?

- Measure the viscosity experimentally, or
- Try a different (simpler theory)

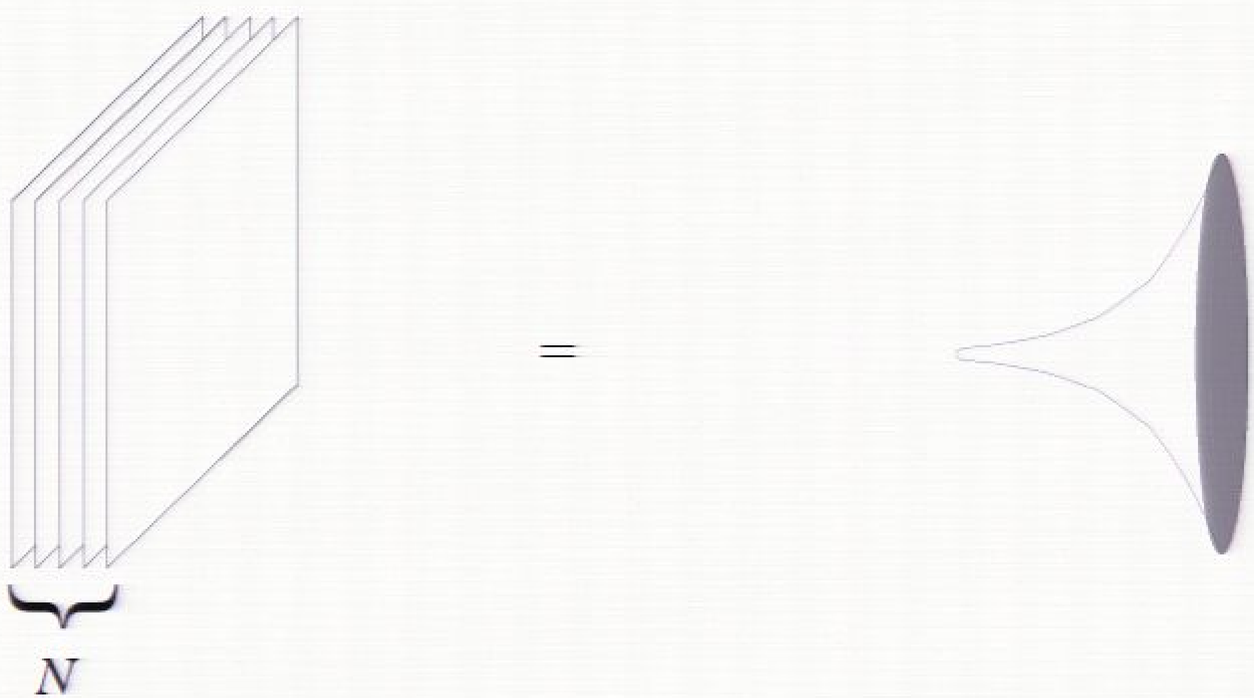
The Gauge/Gravity Duality

Maldacena : stack of N D3-branes in type IIB string theory can be described in two different pictures:

As a quantum field theory describing fluctuations of the branes: $\mathcal{N} = 4$ super-Yang-Mills theory

As string theory on a a curved spacetime called $AdS_5 \times S_5$

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + d\mathbf{x}^2) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$



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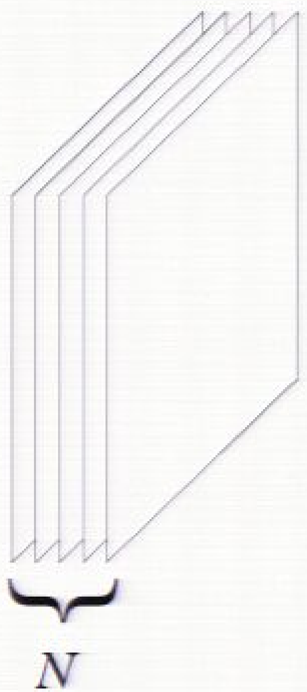

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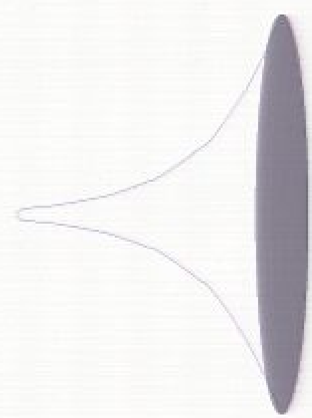
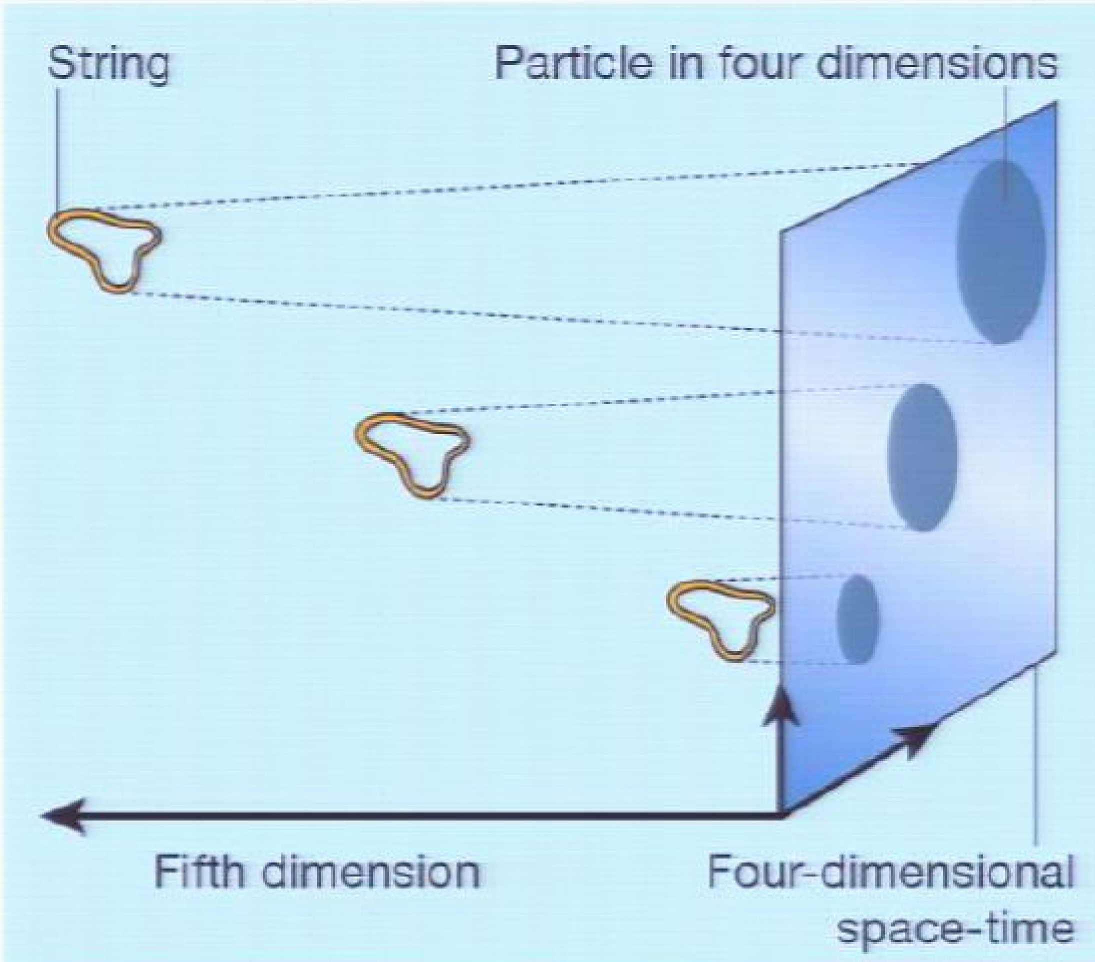


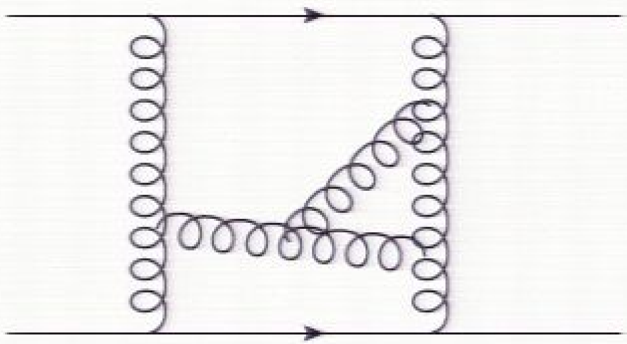
Holography

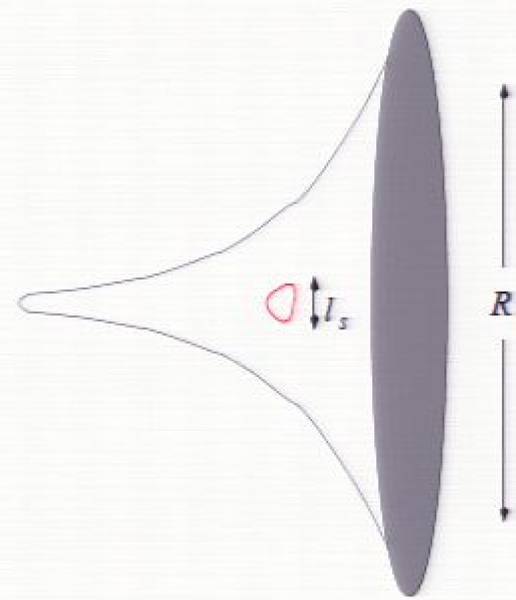
The gauge/gravity duality is a relalization of the idea of holography ('t Hooft, Susskind): a theory without gravity in (3+1)D is equivalent a theory with gravity in higher dimensions



(from Maldacena, Nature 2003)

Mapping of parameters



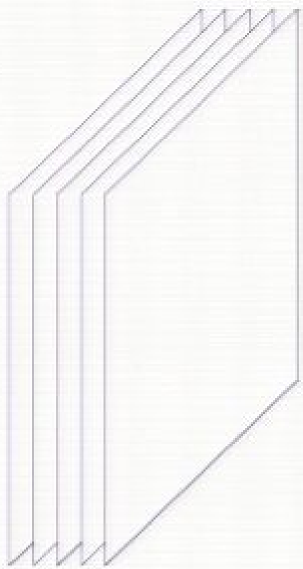


$$g^2 N_c = \frac{R^4}{\ell_s^4}$$

$g^2 N_c \gg 1$:

field theory cannot be solved by perturbation theory
string theory dual $\Longrightarrow$ Einstein's general relativity!

Gauge/gravity duality at finite temperature



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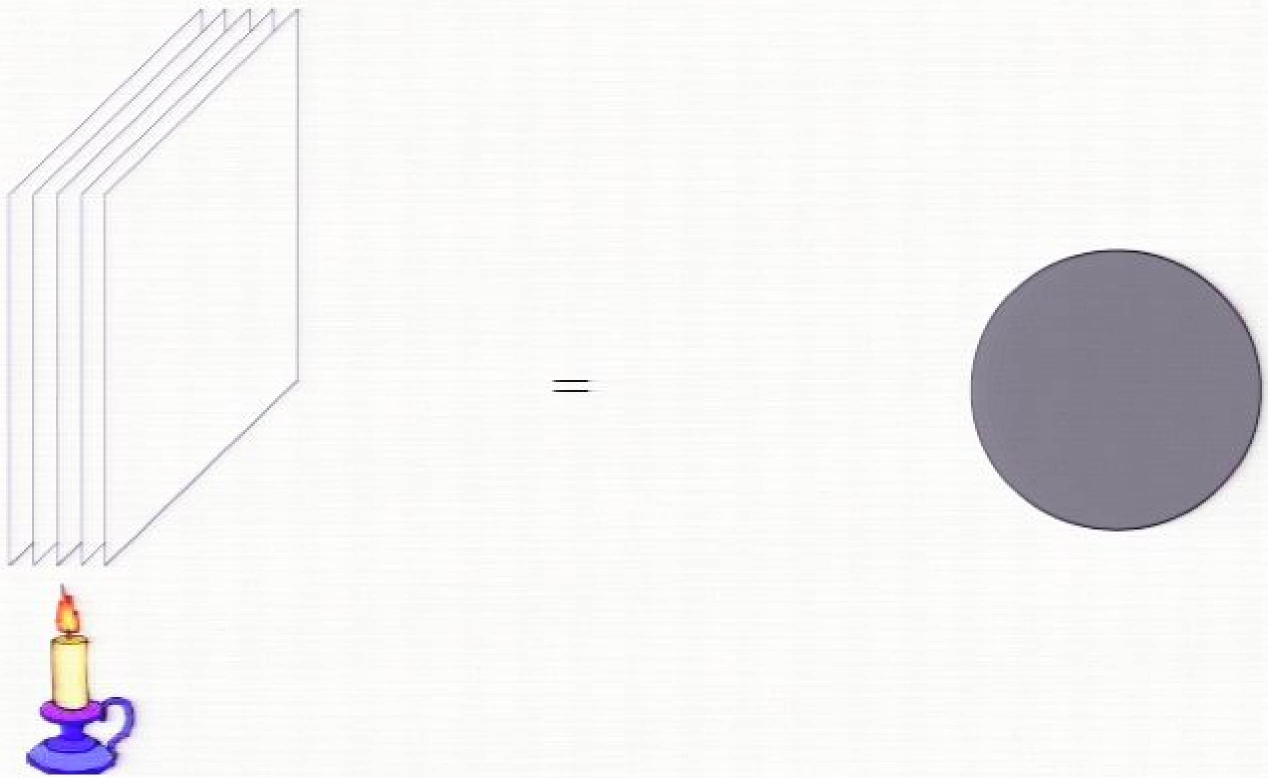

"Quark gluon plasma" = black hole (in anti de-Sitter space)

$$ds^2 = \frac{r^2}{R^2}[-f(r)dt^2 + d\mathbf{x}^2] + \frac{R^2}{r^2 f}dr^2 + R^2 d\Omega_5^2, \quad f(r) = 1 - \frac{r_0^4}{r^4}$$

A theorist's way of measuring viscosity

How do we measure the viscosity of a system?

- Viscosity = response of the fluid under shear
- Theorist: send gravitational wave through the system

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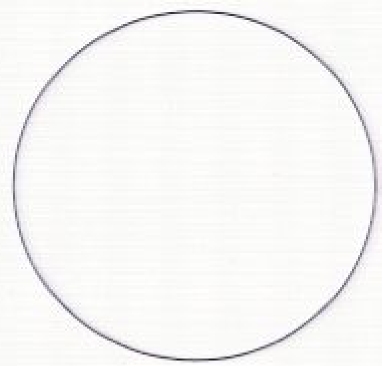

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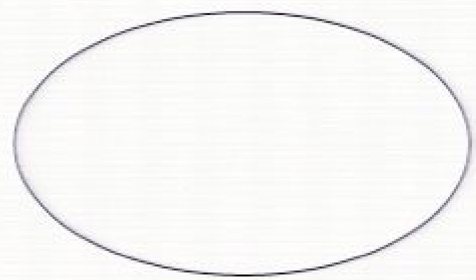

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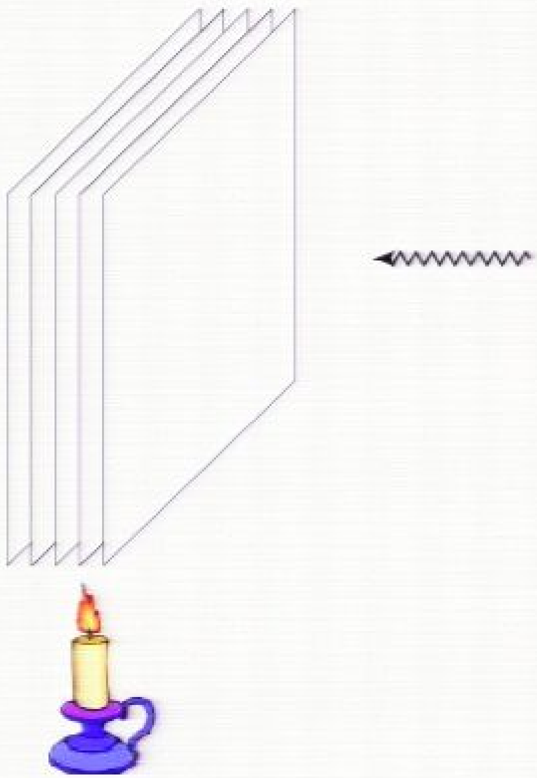
Viscosity on the light of duality

Consider a graviton that falls on this stack of N D3-branes
Will be absorbed by the D3 branes.
The process of absorption can be looked at from two different perspectives:



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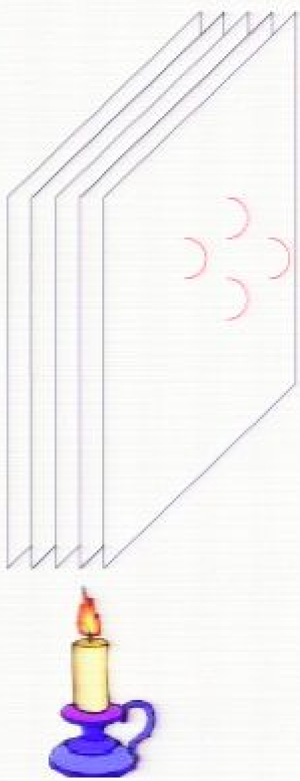

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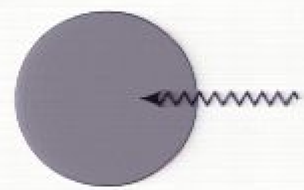

Absorption by D3 branes ($\sim$ viscosity) = absorption by black hole

Viscosity/entropy density ratio

The most surprising result obtained so far has been the universal value of η/s.

- Viscosity $\sim$ absorption cross section for low energy gravitons
- Absorption cross section $=$ area of horizon (a consequence of Einstein equation)
- Entropy: also $\sim$ area of horizon (Bekenstein-Hawking formula)

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

where η is the shear viscosity, s is the entropy per unit volume.

Universality of η/s

Restoring $\hbar$ and c in the formula: a surprise

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}$$

- No velocity of light c
- Finite viscosity at infinitely strong coupling!
- *The same value in all theories with gravity duals* Kovtun, DTS, Starinets; Buchel 2003

Universality of η/s is related to properties of black hole horizons

No exception is found within theories with gravity duals (different number of spacetime dimensions, amount of supersymmetry, etc.)

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Why does η/s have the dimensionality of $\hbar$

In kinetic theory

$$\eta \sim \rho v \ell, \qquad s \sim n = \frac{\rho}{m}$$

$$\frac{\eta}{s} \sim m v \ell \sim \hbar \frac{\text{mean free path}}{\text{de Broglie wavelength}}$$

- Mean free path cannot be much shorter than de Broglie wavelength.
- The stronger the coupling, the smaller is η/s.
- Uncertainty principle: η/s bounded from below by $\#\hbar$, unknown coefficient
- Theories with gravity dual reach $\hbar/4\pi$

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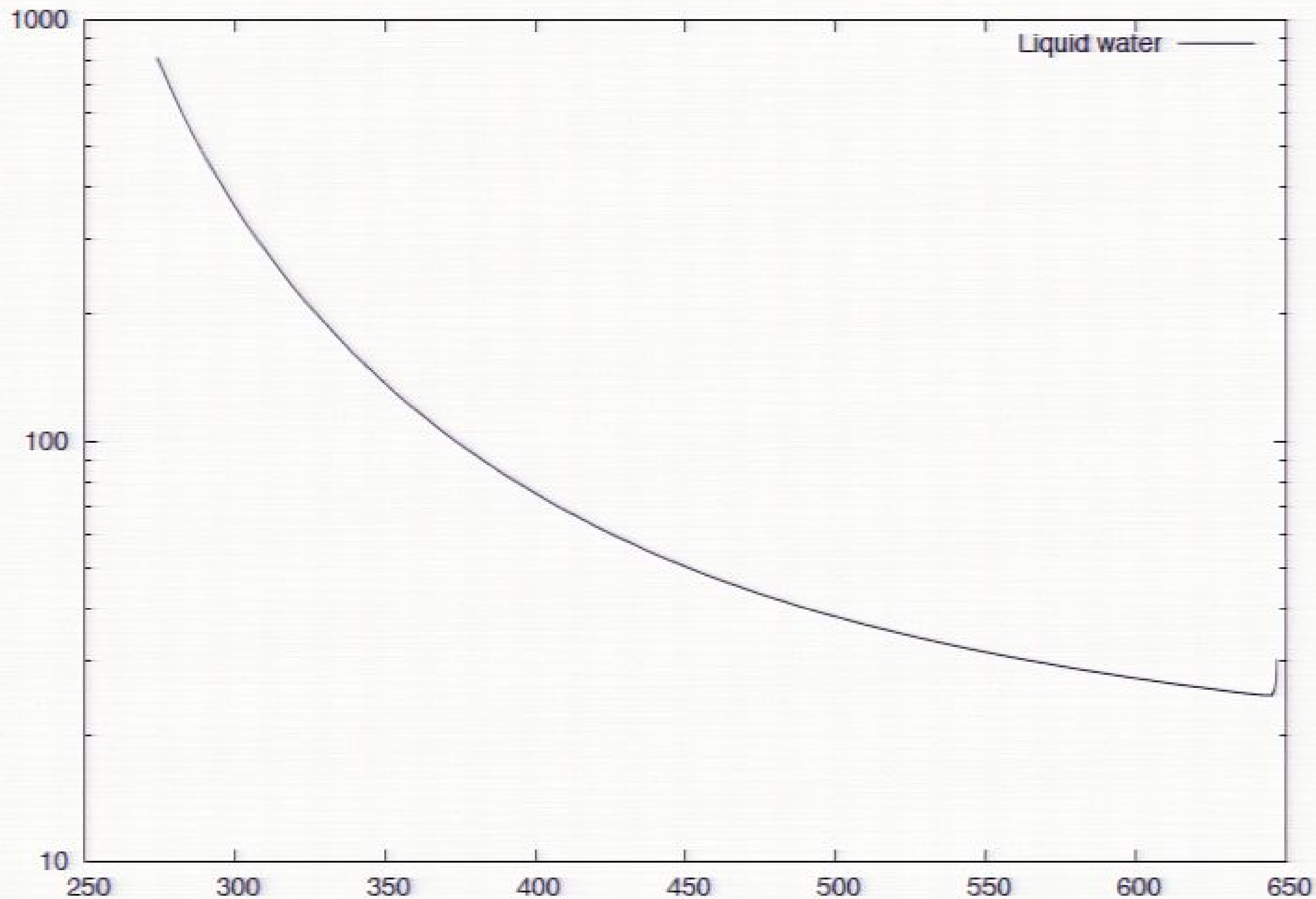
The viscosity bound (conjectured) Kovtun, DTS, Starinets 2003

$$\frac{\eta}{s} \gtrsim \frac{\hbar}{4\pi}$$

Reminds Purcell's question to Weisskopf
No c: $\hbar/(4\pi)$ can be compared with ordinary, nonrelativistic liquids

Ordinary liquids

η/s of liquid water in unit of $\hbar/(4\pi)$, as function of temperature (K), along the saturation curve



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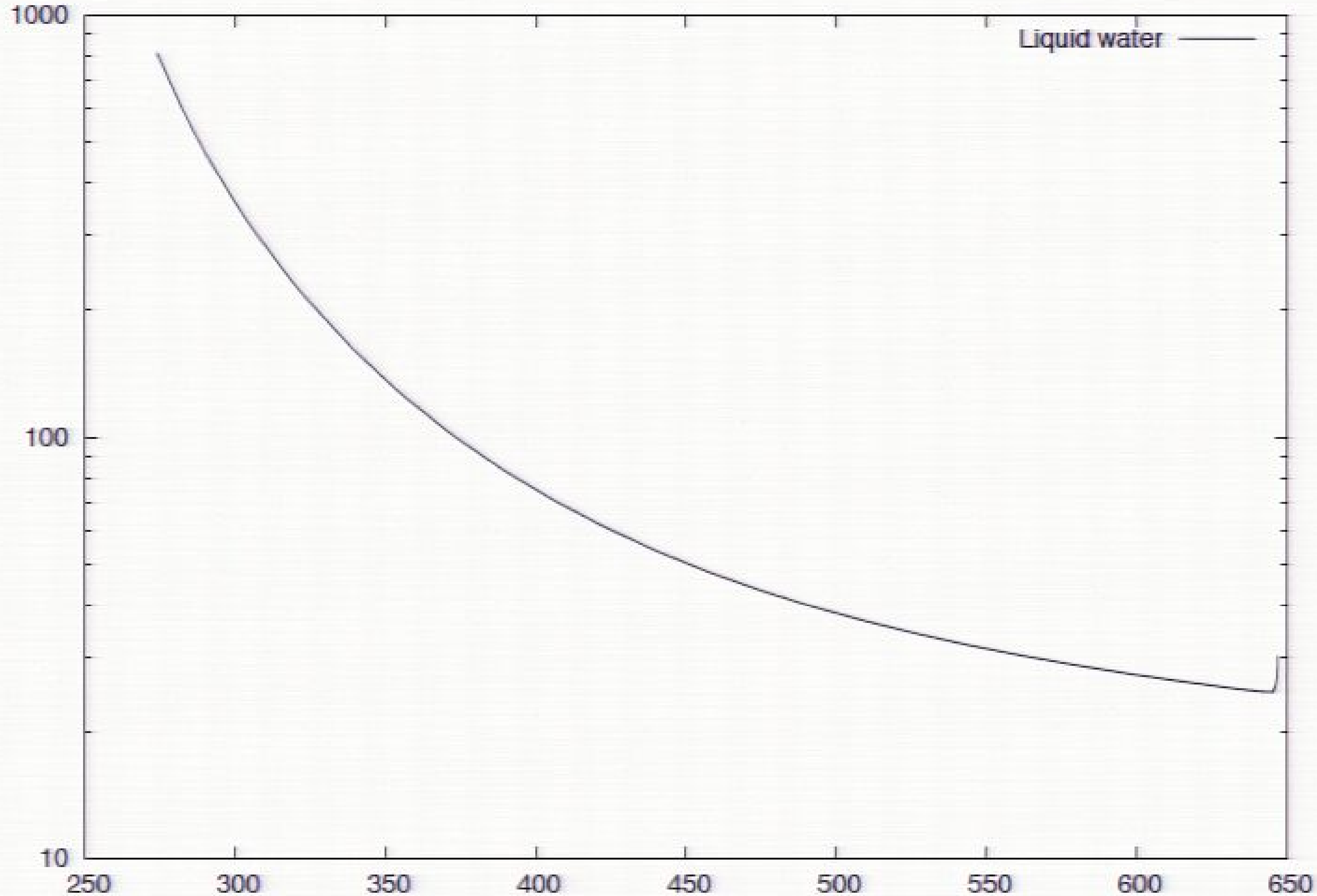
Ordinary liquids

η/s of liquid water in unit of $\hbar/(4\pi)$, as function of temperature (K), along the saturation curve



General observation on liquids

- η/s reaches minimum near the critical point (liquid=gas)
- but not exactly at the critical point: η diverges there according to theory of dynamic critical phenomena
- The minimal value of η/s varies from substance to substance

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Ar	37	H_2S	35	CO_2	32
Kr	57	N_2	23	SO_2	39
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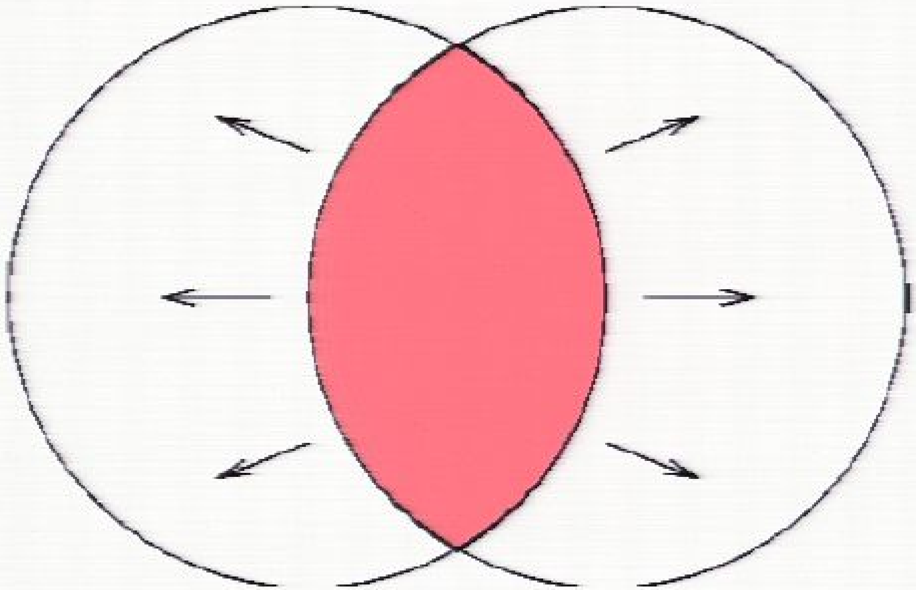
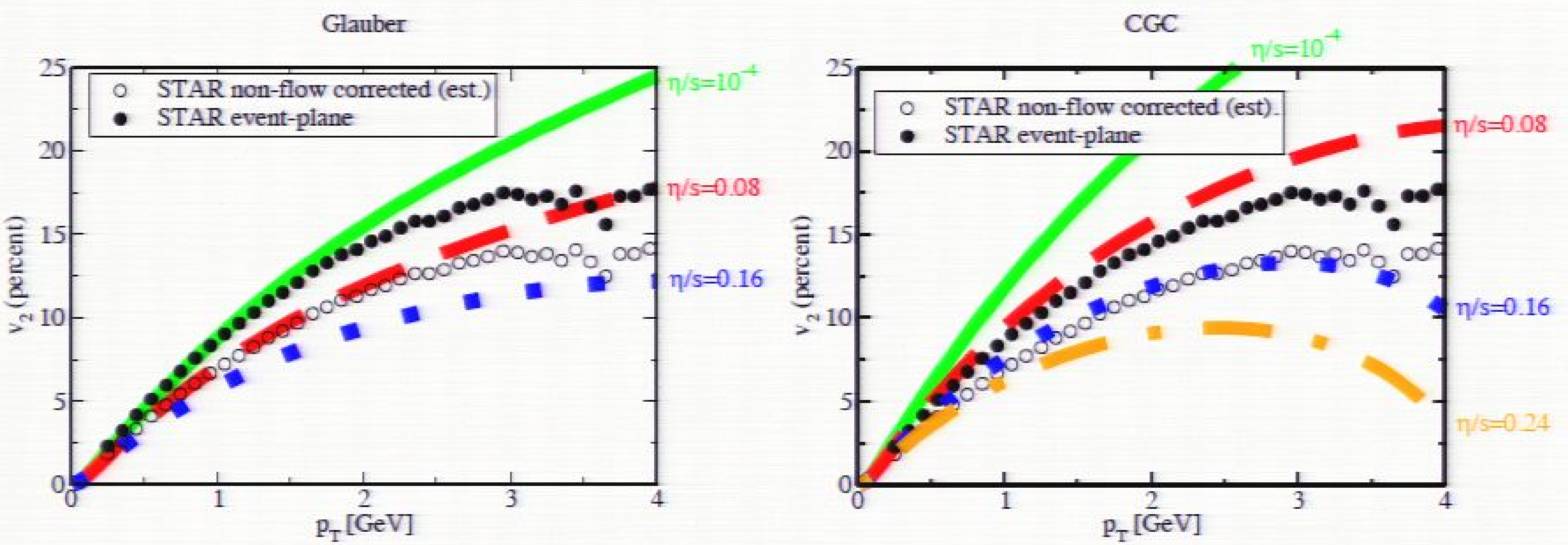
How one can try to determine η of QGP



- Look at scattering events with nonzero impact parameter (select by number of final particles)
- Distribution of particles over momentum is not axially symmetric: characterized by "elliptic flow" parameter v_2
- Naturally explained if the matter acts like a liquid: more pressure along the smaller axis of the hot region.

- Too large viscosity kills v_2
- Hydrodynamic simulations can give estimates for η

Recent numerical simulations



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Note that, by popular convention, the designation "glass" is applied to any disordered material once its viscosity exceeds 10^{15}cp.

Condensed matter analogy

Comparision: Mott's minimal metallic conductivity

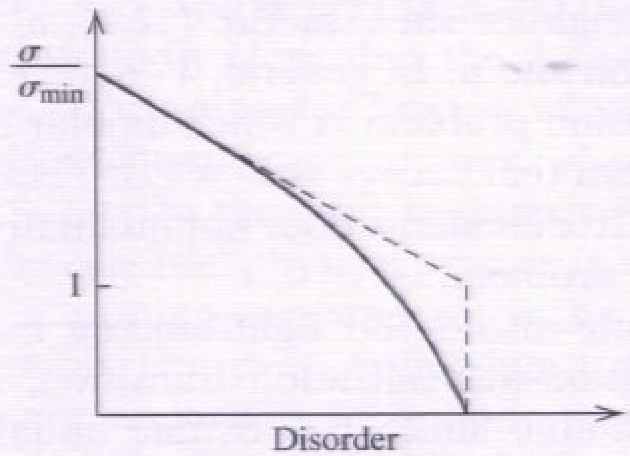


Figure 12.1 Two possibilities for the behavior of the conductivity in the vicinity of the mobility edge of the Anderson transition. The dashed line represents the minimum-metallic conductivity hypothesis of Mott. The solid line, in contrast, depicts the continuous decrease of the conductivity at the Anderson transition. The latter is observed experimentally and predicted by the scaling theory of localization.

(*Phillips, Advanced Solid State Physics*)

Mott's minimal metallic conductivity is a useful concept, but not a real, absolute minimum.

Is there a viscosity bound?

- There exist theories where

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}\left(1 - \frac{c}{N}\right), \qquad N \gg 1$$

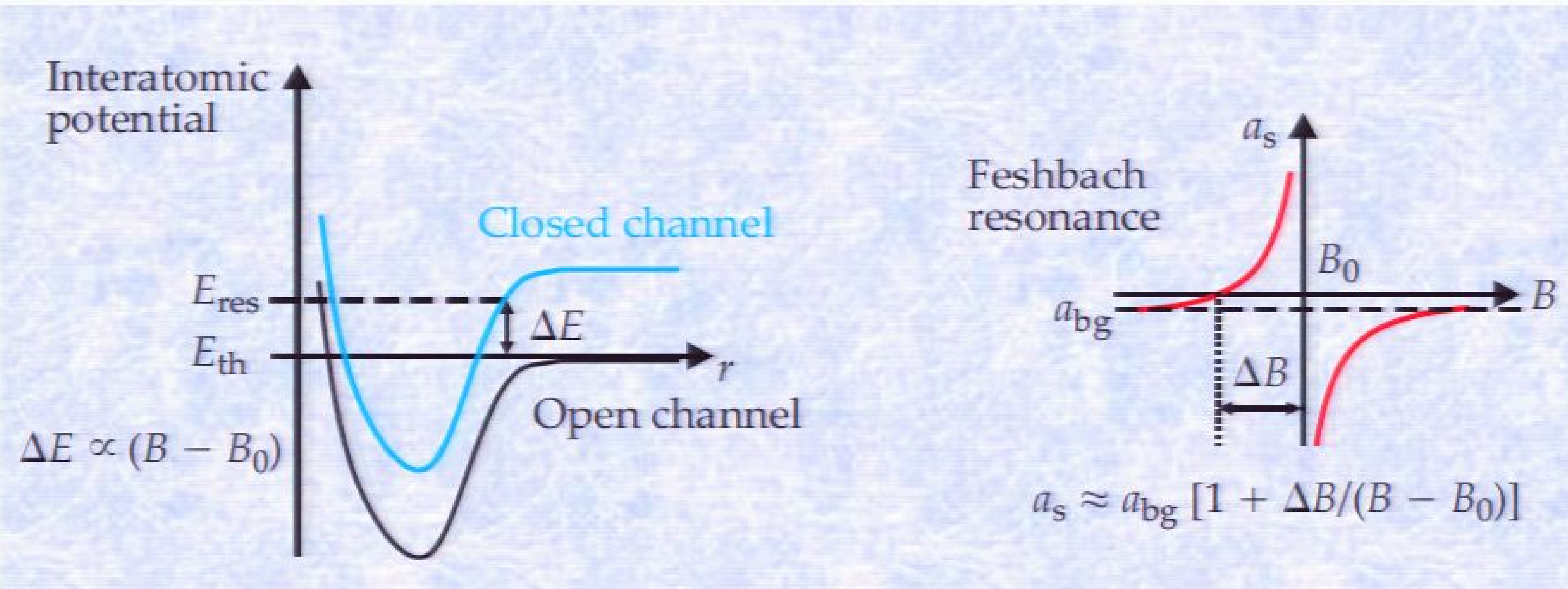
Kats, Petrov; Buchel, Myers, Sinha

- Bad things happen when one tries to dial η/s to 0 (superluminal propagation etc) Brigante etc. PRL 2008
  - Within a model: $\eta/s > \frac{16}{25}\frac{\hbar}{4\pi}$
- What is the real minimum of η/s ?

A condensed matter application?

Fermi gas at unitarity

- Neutrons: $a = -20$ fm, $|a| \gg 1$ fm
- Trapped atom gases, with scattering length a controlled by magnetic field



Further applications

Chiral separation in quark matter:

- Rotating volume of quark matter with nonzero chemical potential
- Chiral current along the axis of rotation:

$$j^{5\mu} = frac\xi 2\epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta$$

- First seen using holography in $\mathcal{N} = 4$, then understood to be general feature of chiral relativistic fluids
- Originates from chiral anomalies
- (more on Friday)

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Conclusion

Surprising applications of string theory to real-world problems

- New perspective on old problems
  - QGP = black hole
  - viscosity = gravity absorption
  - etc
- Suggested η/s as a relevant ratio
- Suggested the value $\hbar/(4\pi)$ as particularly interesting.

Most serious challenge: connect to QCD

The unity of physics

We have also learned a lesson that physics is one single subject:

- Hydrodynamics
- Quantum field theory
- Statistical mechanics
- Gravity
- String theory

Toward a holographic dual of unitarity Fermi gas

A candidate spacetime:

$$ds^2 = \frac{-2dx^+dx^- + dx^i dx^i + dz^2}{z^2} - \frac{2(dx^+)^2}{z^4}$$

Balasubramanian and McGreevy, PRL 2008
DTS, PRD 2008

Realizes geometrically the "Schrödinger symmetry" of unitarity fermions

2D version of this space realizable in string theory

Adams, Balasubramanian, McGreevy
Maldacena, Martelli, Tachikawa
Herzog, Rangamani, Ross (2008)

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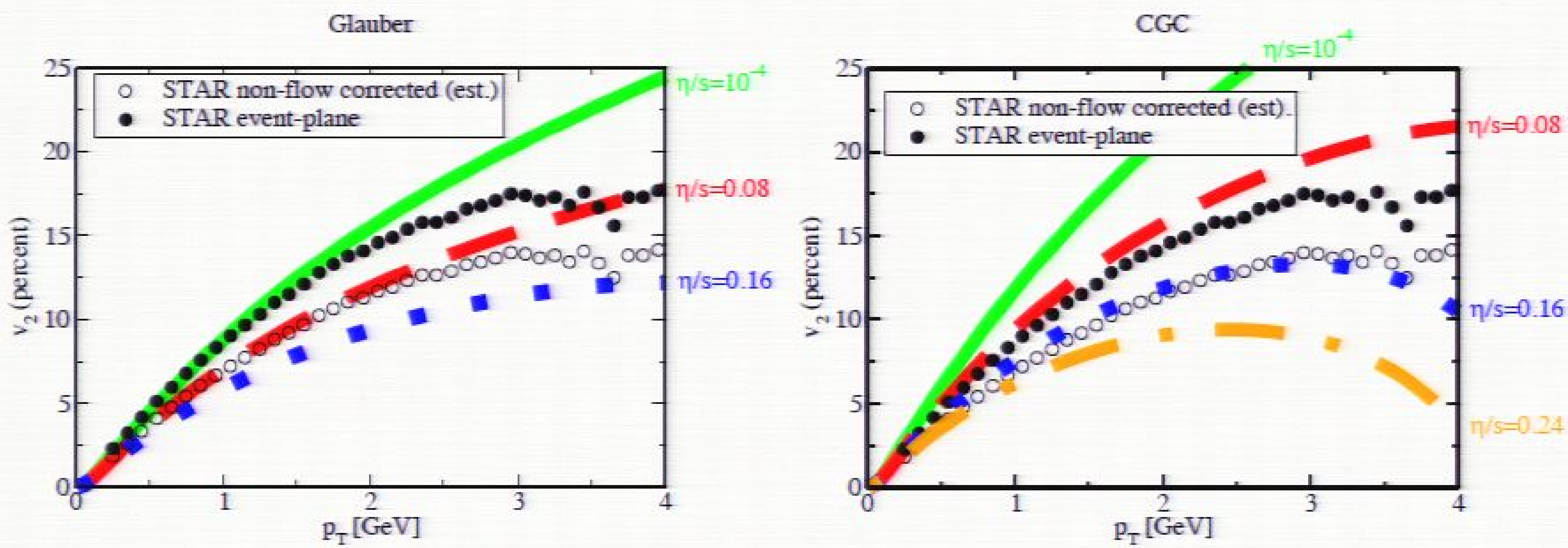

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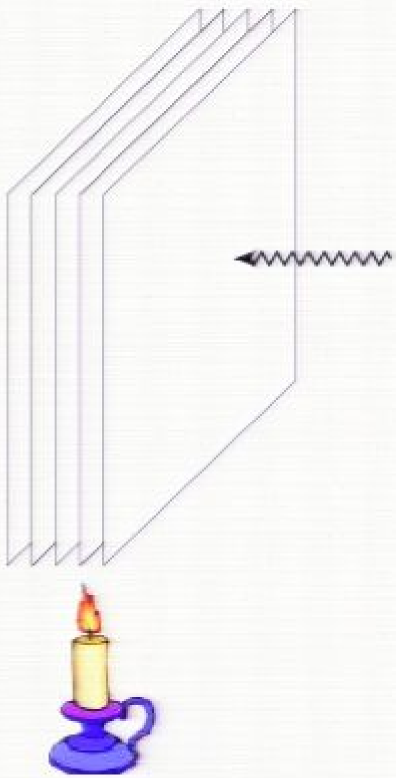

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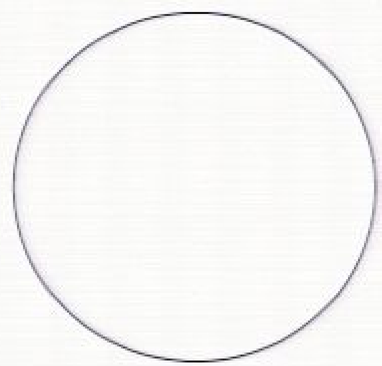

This is the essence of Kubo's formula

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d\mathbf{x} e^{i\omega t} \langle [T^{xy}(t, \mathbf{x}), T^{xy}(0, \mathbf{0})] \rangle$$

Why does η/s have the dimensionality of $\hbar$

In kinetic theory

$$\eta \sim \rho v \ell, \qquad s \sim n = \frac{\rho}{m}$$

$$\frac{\eta}{s} \sim m v \ell \sim \hbar \frac{\text{mean free path}}{\text{de Broglie wavelength}}$$

- Mean free path cannot be much shorter than de Broglie wavelength.
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Universality of η/s

Restoring $\hbar$ and c in the formula: a surprise

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Very high absolute value in normal unit. Almost glass!

Low viscosity in the sense of η/s.

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The viscosity bound (conjectured) Kovtun, DTS, Starinets 2003

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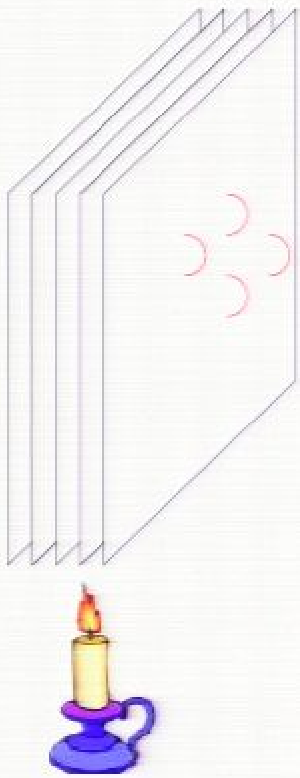

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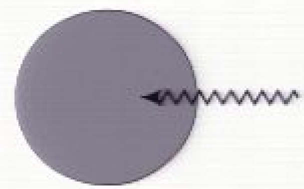

Absorption by D3 branes ($\sim$ viscosity) = absorption by black hole

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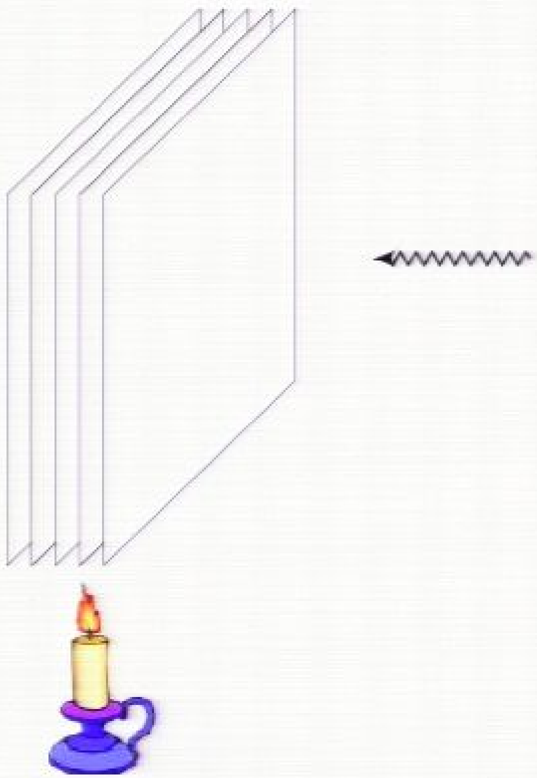

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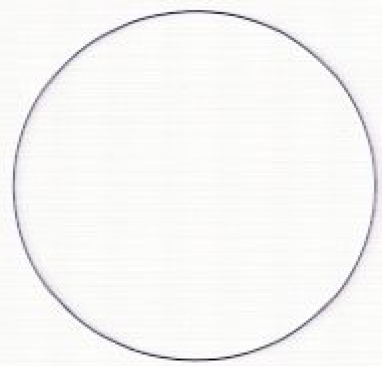
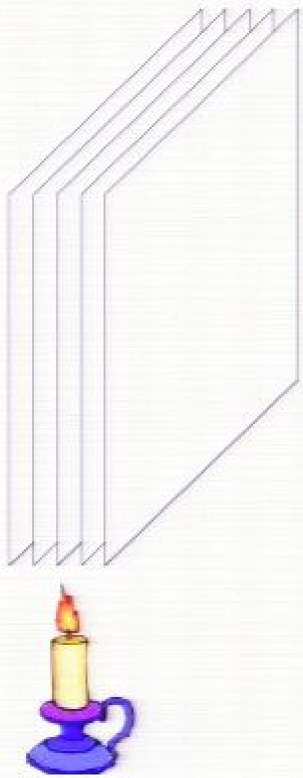


Gauge/gravity duality at finite temperature



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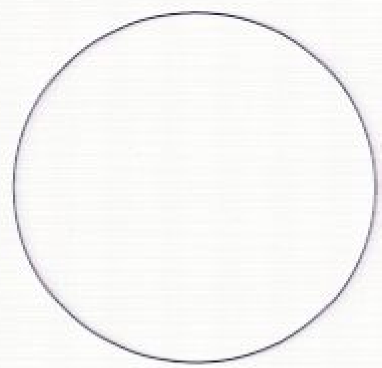

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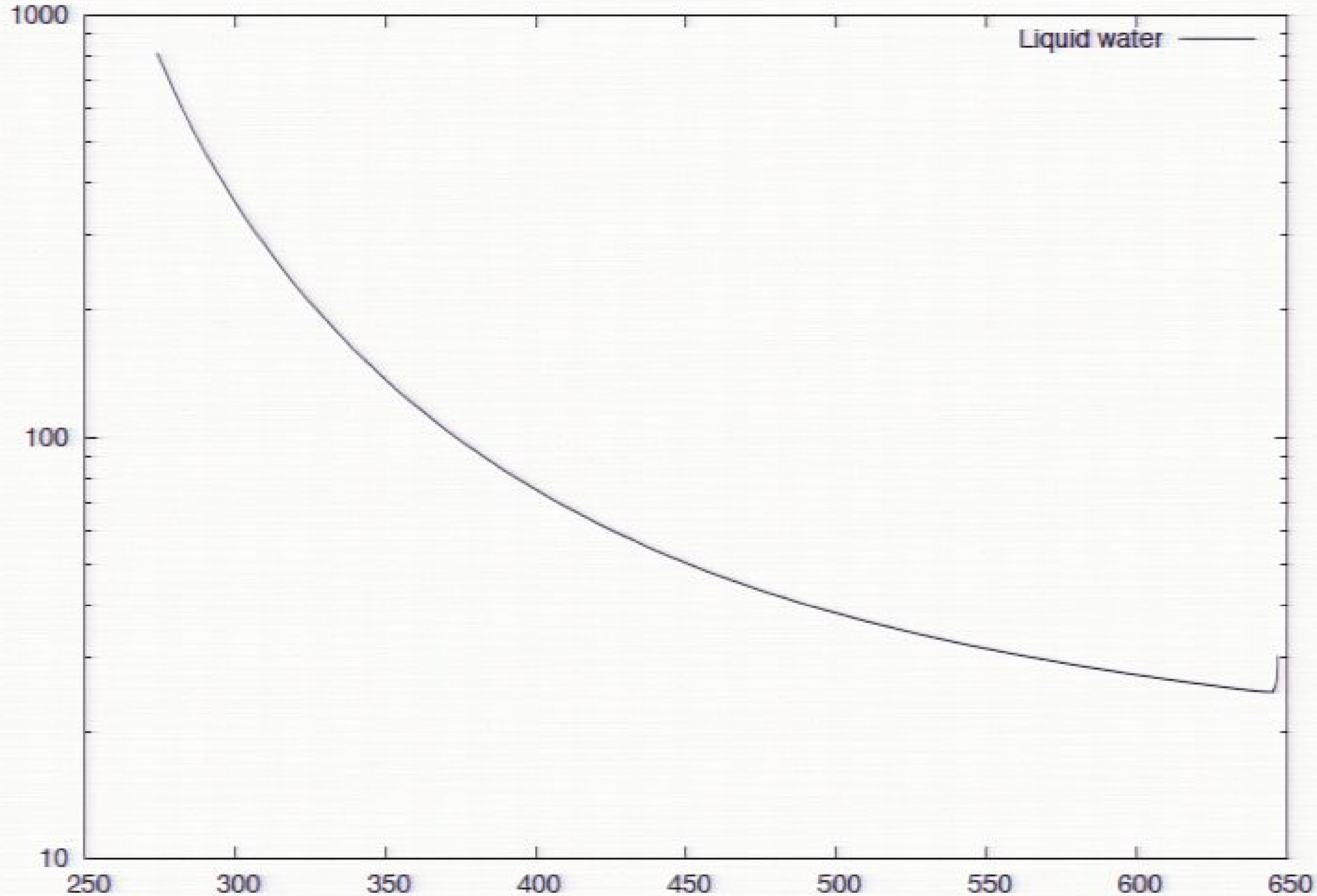

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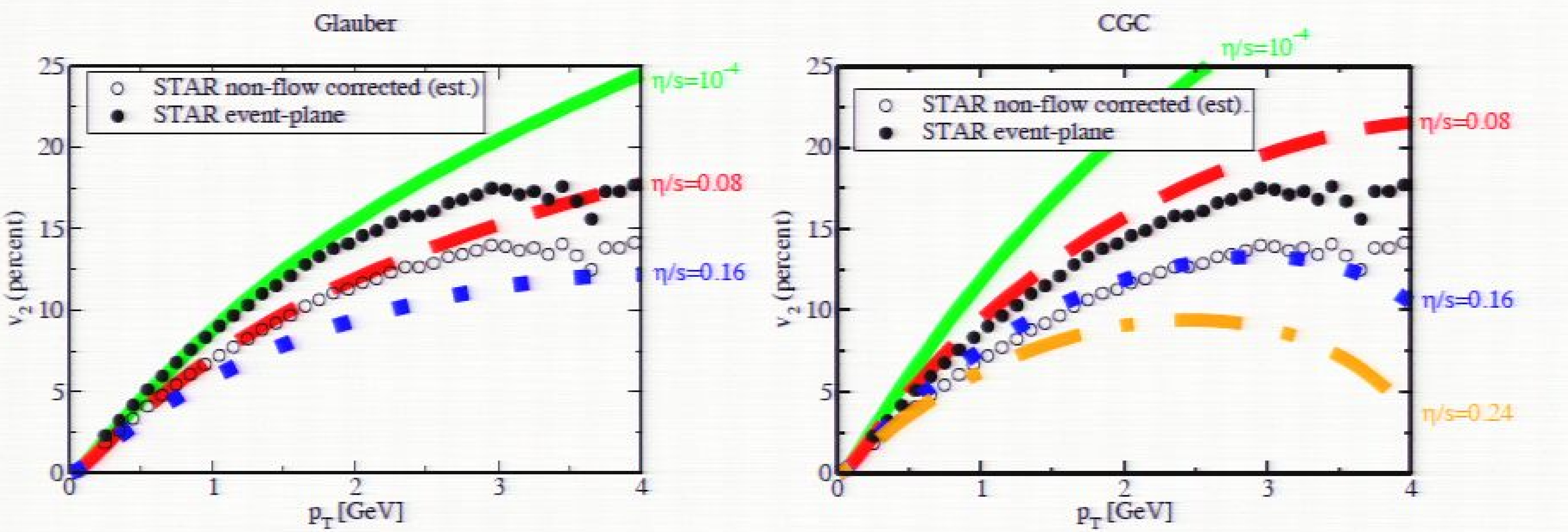

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