

Title: Quantum Gravity - Review (PHYS 638) - Lecture 5

Date: Jan 29, 2010 10:00 AM

URL: <http://pirsa.org/10010098>

Abstract:



perimeter scholars
INTERNATIONAL

Relativistic QFT

Generalization of classical

action at distance between

particles $\{p, \gamma\}$ and $\{p', \gamma'\}$

$L(p, \gamma)$

Light distance

$D(p, \gamma)$ is a metric

$$D(p, \gamma) = 0 \Leftrightarrow p = \gamma$$

$$D(p, \gamma) = D(\gamma, p)$$

$$D(p, \gamma) \leq D(p, \eta) + D(\eta, \gamma)$$

Triangle inequality

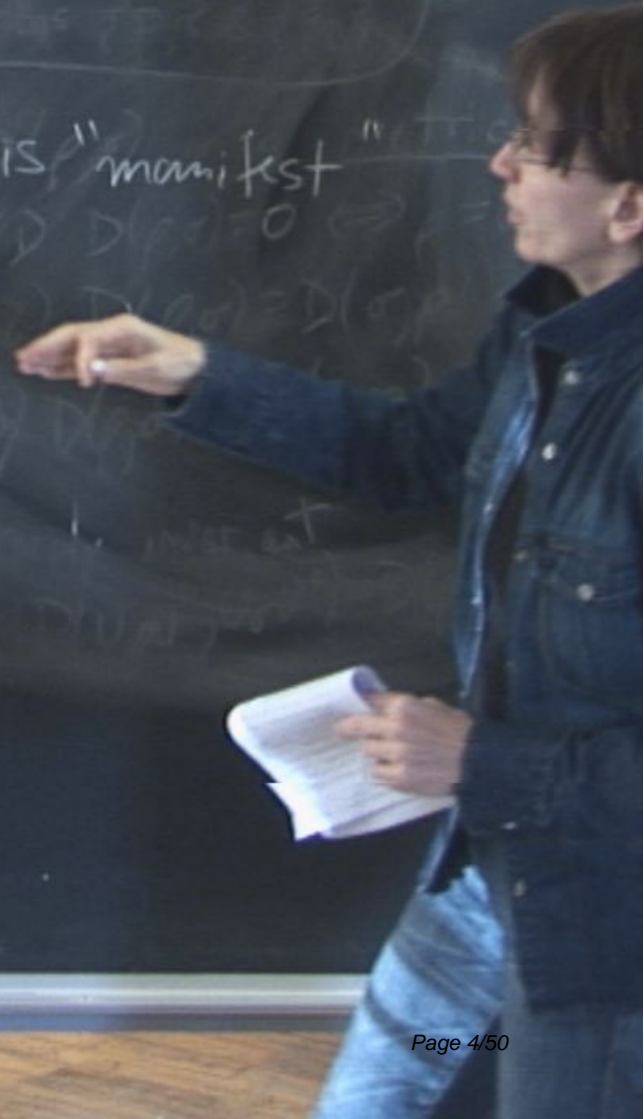
$$D(p, \gamma) \leq D(p, \eta) + D(\eta, \gamma)$$

Relativistic QFT

PI / covariant: Lorentz-invariance is "manifest"

Generalization of classical
action at distance between
particles

Local interaction



Relativistic QFT

PI / covariant: Lorentz-invariance is "manifest"

$$D(p, \sigma) = 0 \Leftrightarrow p = \sigma$$

$$D(p, \sigma) \in D(\sigma, p)$$

$$D(p, \sigma) = D(p, \eta) + D(\eta, \sigma)$$

Under invariant

$$D(p, \sigma) = D(p, \eta) + D(\eta, \sigma)$$

Generalization of classical

action functional between

momentum & position $\{p, q\}$

Legendre

(Sum of positive eigenvalues)

Relativistic QFT

Generalization of classical
action principle
 $\mathcal{L} = \mathcal{L}(\phi, \partial_\mu \phi)$

PI / covariant: Lorentz-invariance is "manifest" metric

Canonical: Unitarity is "manifest"

(Sum of positive eigenvalues)

Integrating out
 $D(\psi) \int \mathcal{L}(\psi, \phi)$

formal, continuum PI for gravity

$$Z_{\Lambda, G_N}([\overset{(3)}{g}_{ij}], [\overset{(3)}{g}_{ij}]) = \int$$

formal, continuum PI for gravity

$$Z_{\Lambda, G_N}([\overset{(3)}{g}_{ij}], [\overset{(3)}{g}_{ij}]) = \int \mathcal{D}[g_{\mu\nu}]$$

formal, continuum PI for gravity

$$Z_{\Lambda, G_N}([\overset{(3)}{g}_{ij}], [\overset{(3)}{g}_{ij}]) = \int \mathcal{D}[g_{\mu\nu}]$$

$g(M)$

formal, continuum PI for gravity

$$Z_{\Lambda, G_N}([g_{ij}^{(3)'}], [g_{ij}^{(3)}]) = \int \mathcal{D}[g_{\mu\nu}] e^{iS^{EH}[g_{\mu\nu}]}$$

$[g^{(3)'}]$

$[g_{\mu\nu}]$

$[g^{(3)}]$

formal, continuum PI for gravity

$$Z_{\Lambda, G_N}([g_{ij}^{(3)'}], [g_{ij}^{(3)}]) = \int \mathcal{D}[g_{\mu\nu}] e^{iS^{E-H}[g_{\mu\nu}]}$$

$g(M)$

$$S^{E-H} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} R$$

$[g^{(3)'}]$

$[g_{\mu\nu}]$

$[g^{(3)}]$

$$\textcircled{3} F(U, U', \dots) = F(p, q)$$

formal, continuum PI for gravity

$$Z_{\Lambda, G_N}([\bar{g}_{ij}^{(3)}, [g_{ij}^{(3)}]]) = \int \mathcal{D}[g_{\mu\nu}] e^{iS^{E-H}[g_{\mu\nu}]}$$

$g(M)$

S^E

$$\int d^4x \sqrt{|g|} (R - 2\Lambda) +$$

$$+ \frac{1}{8\pi G_N} \int d^3x \sqrt{|\det h|} K$$

$[g^{(3)}]$

$[g_{\mu\nu}]$

$[g^{(3)}]$

formal, continuum PI for gravity

$$Z_{\Lambda, G_N}([g_{ij}^{(3)'}], [g_{ij}^{(3)}]) = \int_{\mathcal{G}(M)} \mathcal{D}[g_{\mu\nu}] e^{iS^{EH}[g_{\mu\nu}]}$$

$$S^{EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} (R - 2\Lambda) +$$

$$+ \frac{1}{8\pi G_N} \int_{\mathcal{M}} d^3x \sqrt{|\det h|} K$$

$[g^{(3)'}$

$[g_{\mu\nu}]$

$[g^{(3)}$

formal, continuum PI for gravity

$$Z_{\Lambda, G_N}([g_{ij}^{(3)'}], [g_{ij}^{(3)}]) = \int \mathcal{D}[g_{\mu\nu}] e^{iS^{EH}[g_{\mu\nu}]}$$

$g(M)$

S

$$\int d^4x \sqrt{|g|} (R - 2\Lambda) +$$

$$+ \frac{1}{8\pi G_N} \int d^3x \sqrt{|\det h|} K$$

$$\textcircled{2} F(U^{\mu\nu}, \partial_\mu U^\nu) = F(p, \sigma)$$

$[g^{(3)'}]$

$[g_{\mu\nu}]$

$[g^{(3)}]$

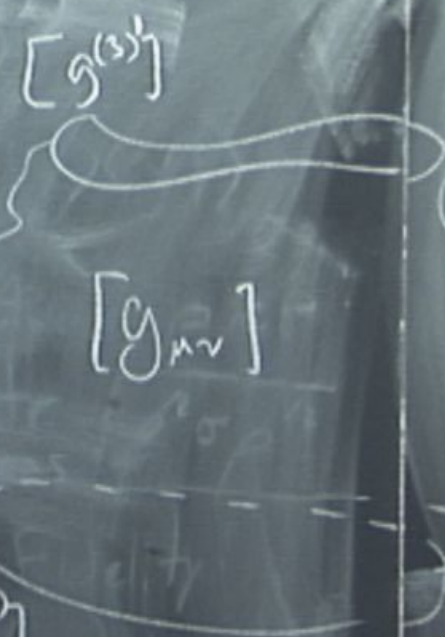
formal continuum PI for gravity

$$Z_{\Lambda, G_N}([g_{ij}^{(3)'}], [g_{ij}^{(3)}]) = \int \mathcal{D}[g_{\mu\nu}] e^{iS^{E-H}[g_{\mu\nu}]}$$

$g(M)$

$$S^{E-H} = \frac{1}{16\pi G} \int d^4x \sqrt{|g|} (R - 2\Lambda) +$$

$$+ \frac{1}{8\pi G_N} \int d^3x \sqrt{|\det h|} K$$



formal, continuum PI for gravity

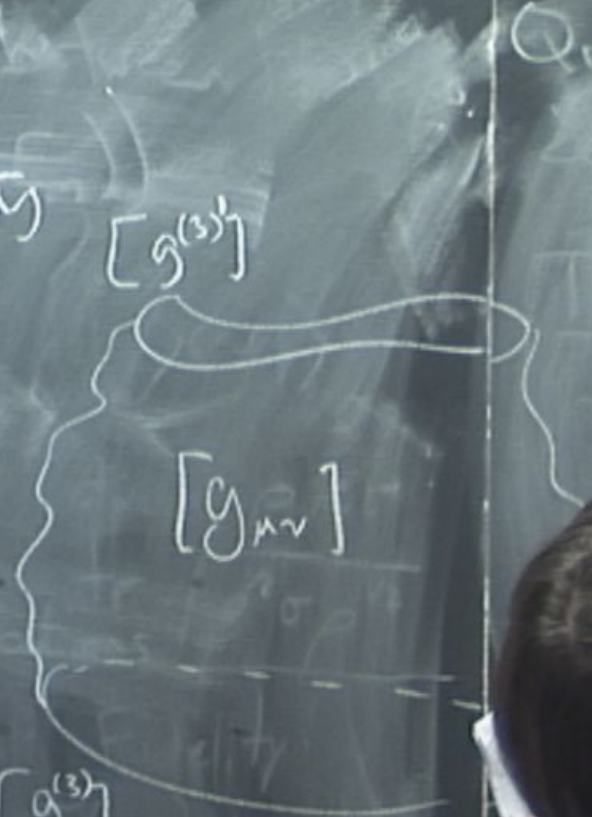
$$Z_{\Lambda, G_N}([g_{ij}^{(3)}], [g_{ij}^{(3)}]) = \int \mathcal{D}[g_{\mu\nu}] e^{iS^{E-H}[g_{\mu\nu}]}$$

$g(M)$

$$S^{E-H} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} (R - 2\Lambda) +$$

$[g_{\mu\nu}] \sim$ diffeomorphism-equivalence class of metrics

$$+ \frac{1}{8\pi G_N} \int_{\partial M} d^3x \sqrt{|\det h|} K$$



Quantity $g(M) = \frac{Lor(M)}{Diff(M)}$

Thm (obtaining th...)

$g^{(3)}$

$[g_{mv}]$

$\det h$ K

$J = F(p, q)$

$F(p, q) = \max_{x \in \mathcal{X}} f(x, y)$

where f is a function in system
 \mathcal{X}, \mathcal{Y} are states in \mathbb{R}^n

T_R (14) $T_R = \{0\} \cup \dots$

between p & q
 a fidelity

IF \dots
 SWAP \dots

Q. $f(M) = \frac{\text{Lor}(M)}{\text{Diff}(M)}$

Thm. (Obtaining the)

$F(p_{30}) = \max_{p \in \mathcal{P}} f(p)$

* what is the measure $D[g_{mv}]$?

system Q
R ⊂ Q

* Thm. $F(p_{30}) = \max_{p \in \mathcal{P}} f(p)$

between
the field

Q. Let $f(M) = \frac{Lor(M)}{Diff(M)}$ Then (obtain the)

* what is the measure $D[g_{mv}]$?

* In $F(g_{mv})$ SD

* SWAP

IF CONTINUED SWAP

between p & r is maximum
between p & r

Quantity $\mathcal{F}(M) = \frac{\text{Lor}(M)}{\text{Diff}(M)}$

Thm (Ohtsuka-Thm)

$F(p_0) = \max_{p \in \mathcal{P}} \mathcal{F}(p)$

* what is the measure $D[g_{\mu\nu}]$?

* how to perform the quotient, parametrize it?

* wildly divergent \rightarrow

Q. $\mathcal{Z}(M) = \frac{\text{Vol}(M)}{\text{Diff}(M)}$

Thm (Obtaining the)

$F(\rho, \sigma) = \max_{\gamma} \sum \rho(\gamma_i) \sigma(\gamma_{i+1})$

* what is the measure $D[g_{\mu\nu}]$?

* how to perform the quotient, parametrize it?

* wildly divergent \rightarrow must regularize

(regularization compatible w/ gauge invariance)

*

formal, continuum PI for gravity

$$Z_{\Lambda, G_N}([g_{ij}^{(3)'}], [g_{ij}^{(3)}]) = \int \mathcal{D}[g_{\mu\nu}] e^{iS^{EH}[g_{\mu\nu}]}$$

$g(M)$

$$S^{EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} (R - 2\Lambda) +$$

$[g_{\mu\nu}] \sim$ diffeomorphism-equivalence class of metrics

$$+ \frac{1}{8\pi G_N} \int_{\partial M} d^3x \sqrt{|\det h|} K$$

$$\mathcal{D}(F(U, \psi, \omega, \sigma)) = F(\mathcal{P}(\sigma))$$

Quantum Gravity (Obtaining the)

$$Z(M) = \frac{\text{Vol}(M)}{\text{Diff}(M)}$$

* what is the measure $D[g_{\mu\nu}]$?

* how to perform the quotient, parametrize it?

* wildly divergent \rightarrow must regularize

(regularization compatible w/ gauge invariance)

* factor i ? \int Wick rotation?

formal, continuum PI for gravity

$$Z_{\Lambda, G_N}([g_{ij}^{(3)'}], [g_{ij}^{(3)}]) = \int_{\mathcal{G}(M)} \mathcal{D}[g_{\mu\nu}] e^{iS^{EH}[g_{\mu\nu}]}$$

$$S^{EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} (R - 2\Lambda) +$$

$[g_{\mu\nu}] \sim$ diffeomorphism-equivalence class of metrics

$$+ \frac{1}{8\pi G_N} \int_{\mathcal{M}} d^3x \sqrt{|\det h|} K$$

$$\textcircled{2} F(\mathcal{U}, \mathcal{V}, \mathcal{W}) = F(\mathcal{W}, \mathcal{U}, \mathcal{V})$$

Generalization of classical
Perturbative path integral for graviton

$$S = S_{\text{free}} + S_{\text{interaction}}$$

Generalization of classical
Perturbative path integral for gravity

$$S = S_{\text{free}} + S_{\text{interaction}}$$

↔ Feynman rules

↔ computing amplitudes
order by order in perturbation

$$S^{E-H} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} R$$

$$S^{E-H} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} R \rightarrow \text{expand in powers of } f_{\mu\nu}$$

schematically $R \sim \partial f \partial f + f \partial f \partial f + f^2 \partial f \partial f + \dots$

$$S^{E-H} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} R \rightarrow \text{expand in powers of } f_{\mu\nu}$$

$$\tilde{f} = \frac{1}{\sqrt{16\pi G_N}} f$$

Schematically $R \sim \partial f \partial f + f \partial f \partial f + f^2 \partial f \partial f + \dots$

$$S^{E-H} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} R \rightarrow \text{expand in powers of } f_{\mu\nu}$$

$$= \int d^4x (\partial \tilde{f} \partial \tilde{f})$$

$$\tilde{f} = \frac{1}{\sqrt{16\pi G_N}} f$$

Schematically $R \sim \partial f \partial f + f \partial f \partial f + f^2 \partial f \partial f + \dots$

$$= \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} R \rightarrow \text{expand in powers of } f_{\mu\nu}$$

$$= \int d^4x (\partial \tilde{f} \partial \tilde{f} + (\sqrt{16\pi G_N} f) \partial \tilde{f} \partial \tilde{f} + (\sqrt{16\pi G_N} f)^2 \partial \tilde{f} \partial \tilde{f} + \dots)$$

$$\tilde{f} = \frac{1}{\sqrt{16\pi G_N}} f$$

schematically $R \sim \partial f \partial f + f \partial f \partial f + f^2 \partial f \partial f + \dots$

$$\int^{E-H} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} R \rightarrow \text{expand in powers of } f_{\mu\nu}$$

$$= \int d^4x \left(\underbrace{\partial \tilde{f} \partial \tilde{f}}_{\mathcal{L}_{\text{free}}} + (\sqrt{16\pi G_N} f) \partial \tilde{f} \partial \tilde{f} + (\sqrt{16\pi G_N} f)^2 \partial \tilde{f} \partial \tilde{f} + \dots \right)$$

$$\tilde{f} = \frac{1}{\sqrt{16\pi G_N}} f$$

schematically $R \sim \partial f \partial f + f \partial f \partial f + f^2 \partial f \partial f + \dots$

$$S^{E-H} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} R \rightarrow \text{expand in powers of } f_{,\mu\nu}$$

$$= \int d^4x \left(\underbrace{\partial \tilde{f} \partial \tilde{f}}_{\mathcal{L}_{\text{free}}} + \underbrace{(\sqrt{16\pi G_N} f) \partial \tilde{f} \partial \tilde{f} + (\sqrt{16\pi G_N} f)^2 \partial \tilde{f} \partial \tilde{f}}_{\mathcal{L}_{\text{interaction}}} + \dots \right)$$

Q. $\sqrt{G_N} \sim \frac{1}{M_{Pl}}$ ~ coupling constant

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Graviton propagator

Q. $\sqrt{G_N} \sim \frac{1}{M_{Pl}}$ \sim coupling constant

Graviton propagator

c.f. QED: $S_{QED} = -\frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} \sim$

Q. $\sqrt{G_N} \sim \frac{1}{M_{Pl}} \sim \text{coupling constant}$

Graviton propagator

cf. QED: $S_{QED} = -\frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} \approx \int d^4x A_\mu$

Q. $\sqrt{G_N} \sim \frac{1}{M_{Pl}} \sim \text{coupling constant}$

Graviton propagator

diff. op.
↓

cf. QED: $S_{QED} = -\frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} \approx \int d^4x A_\mu M^{\mu\nu} A_\nu$

$$\sqrt{G_N} \sim \frac{1}{M_{Pl}} \sim \text{coupling constant}$$

Graviton propagator

diff. op.



c.f. QED: $S_{\text{QED}} = -\frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} \approx \int d^4x A_\mu M^{\mu\nu} A_\nu$

$$\sqrt{G_N} \sim \frac{1}{M_{Pl}} \sim \text{coupling constant}$$

Graviton propagator

c.f. QED: $S_{\text{QED}} = -\frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu}$

gravity: $\mathcal{L}_{\text{FP}} = \frac{1}{2} (\partial_\lambda h_{\mu\nu} - \partial_\nu h_{\lambda\mu}) (\partial^\lambda h^{\mu\nu} - \partial^\nu h^{\lambda\mu}) - \frac{1}{2} \partial_\lambda h^{\lambda\mu} \partial^\lambda h_{\mu\nu} + \frac{1}{2} \partial_\lambda h^{\lambda\mu} \partial^\nu h_{\mu\nu}$

diff. op.



$$\int d^4x A_\mu M^{\mu\nu} A_\nu$$

$$\sqrt{G_N} \sim \frac{1}{M_{Pl}} \sim \text{coupling constant}$$

Graviton propagator

diff. op.



c.f. QED: $S_{\text{QED}} = -\frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} \simeq \int d^4x A_\mu M^{\mu\nu} A_\nu$

gravity: $\mathcal{L}_{\text{FP}} = \frac{1}{2} (\partial^\lambda h_{\mu\nu}) (\partial_\lambda h^{\mu\nu}) - \frac{1}{2} (\partial_\lambda h^\mu{}_\mu) (\partial^\lambda h^\nu{}_\nu) -$
 $-(\partial^\lambda h^{\mu\nu}) (\partial_\mu h_{\lambda\nu}) + (\partial_\lambda h^{\lambda\mu}) (\partial_\mu h^\nu{}_\nu) \simeq$

If we...
 ...

$$\sqrt{G_N} \sim \frac{1}{M_{Pl}} \sim \text{coupling constant}$$

Graviton propagator

diff. op.



c.f. QED: $S_{\text{QED}} = -\frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} \simeq \int d^4x A_\mu M^{\mu\nu} A_\nu$

gravity: $\mathcal{L}_{\text{FP}} = \frac{1}{2} (\partial^\lambda f_{\mu\nu}) (\partial_\lambda f^{\mu\nu}) - \frac{1}{2} (\partial_\lambda f^\mu{}_\mu) (\partial^\lambda f^\nu{}_\nu) -$

$$- (\partial^\lambda f^{\mu\nu}) (\partial_\mu f_{\lambda\nu}) + (\partial_\lambda f^{\lambda\mu}) (\partial_\mu f^\nu{}_\nu) \simeq$$

$$\simeq \frac{1}{2} f_{\mu\nu} M^{\mu\nu\rho\sigma} f_{\rho\sigma}$$

$$\sqrt{G_N} \sim \frac{1}{M_{Pl}} \sim \text{coupling constant}$$

Graviton propagator

diff. op.



cf. QED: $S_{\text{QED}} = -\frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} \simeq \int d^4x A_\mu M^{\mu\nu} A_\nu$

gravity: $\mathcal{L}_{\text{FP}} = \frac{1}{2} (\partial^\lambda f_{\mu\nu}) (\partial_\lambda f^{\mu\nu}) - \frac{1}{2} (\partial_\lambda f^\mu{}_\mu) (\partial^\lambda f^\nu{}_\nu) -$

$$- (\partial^\lambda f^{\mu\nu}) (\partial_\mu f_{\lambda\nu}) + (\partial_\lambda f^{\lambda\mu}) (\partial_\mu f^\nu{}_\nu) \simeq$$

$$\simeq \frac{1}{2} f_{\mu\nu} M^{\mu\nu} f_{\mu\nu}$$

M is not invertible!

Claim: adding a gauge-fixing term $F_\mu F^\mu$ to L_{FP}

M is not invertible!

Claim: adding a gauge-fixing term $F_\mu F^\mu$ to L_{FP} ,
where $F_\mu[f] = \partial_\nu f_{\mu\nu} - \frac{1}{2} \partial_\mu f^\nu_\nu$, the differential operator
becomes $\tilde{M}^{\sigma\mu\nu} = (-\eta^{\sigma\mu} \eta^{\nu\alpha} + \frac{1}{2} \eta^{\sigma\alpha} \eta^{\mu\nu}) \partial_\alpha$, which is invertible.

"Faddeev - Popov method"

"Faddeev-Popov method"

$$1 = \Delta_{\mathcal{F}}[f] \int \mathcal{D}g \prod_M \delta(\mathcal{F}^M[f])$$

↑
integral over
gauge group

FP determinant

"Faddeev-Popov method"

$$1 = \overbrace{\Delta_{\mathcal{F}}[f]}^{\text{FP determinant}} \int \mathcal{D}g \prod_M \delta(\mathcal{F}^M[f])$$

↑
integral over
gauge group

FP determinant

DeWitt (1964)

"Faddeev - Popov method"

$$1 = \overbrace{\Delta_{\mathcal{F}}[f]}^{\text{FP determinant}} \int \mathcal{D}g \prod_M \delta(\mathcal{F}^M[f])$$

↑
integral over
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FP determinant

DeWitt (1964)

"Faddeev-Popov method"

$$1 = \overbrace{\Delta_{\mathcal{F}}[f]}^{\text{FP determinant}} \int \mathcal{D}g \prod_M \delta(\mathcal{F}^M[f])$$

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DeWitt (1964)