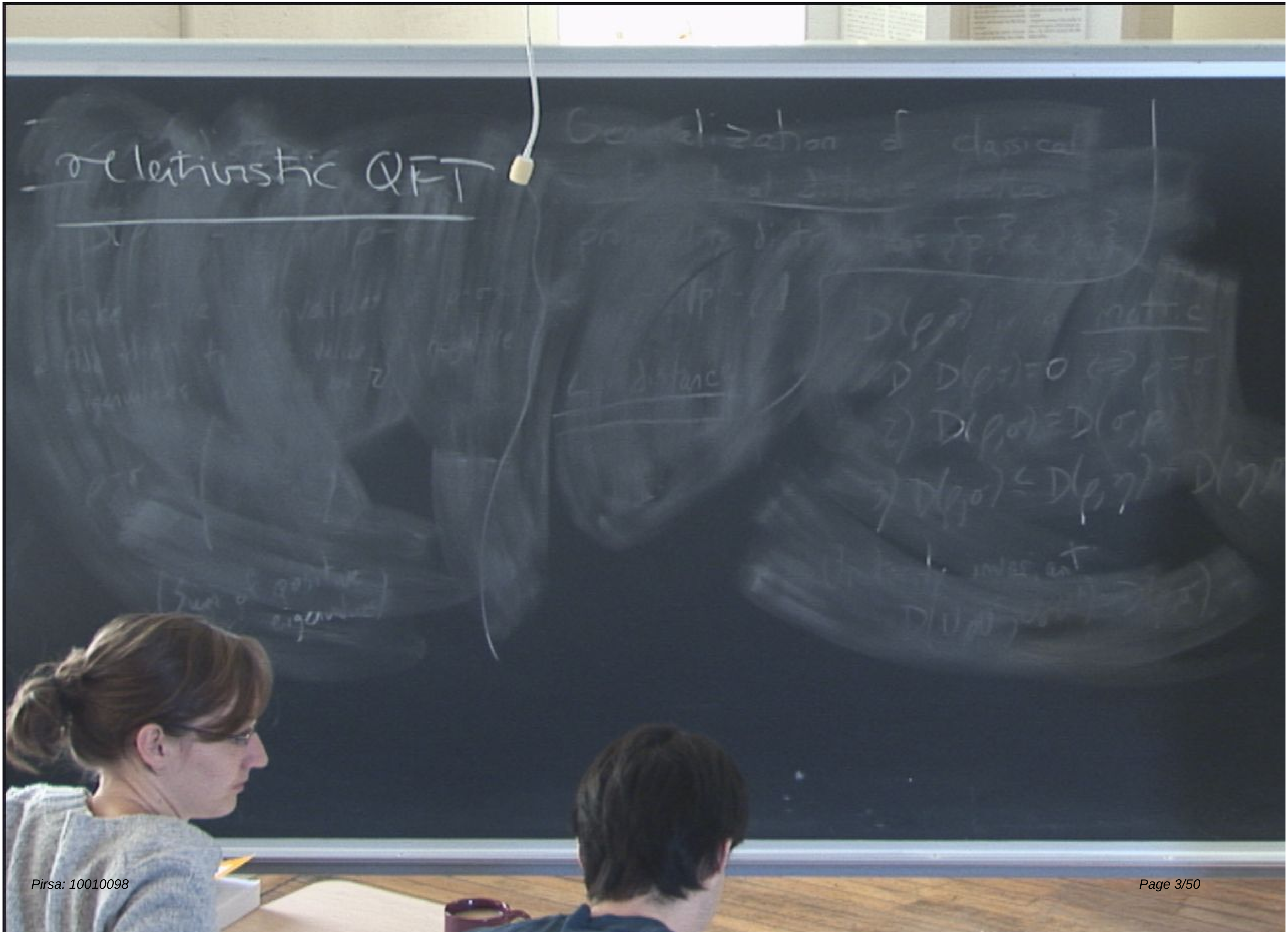
Title: Quantum Gravity - Review (PHYS 638) - Lecture 5

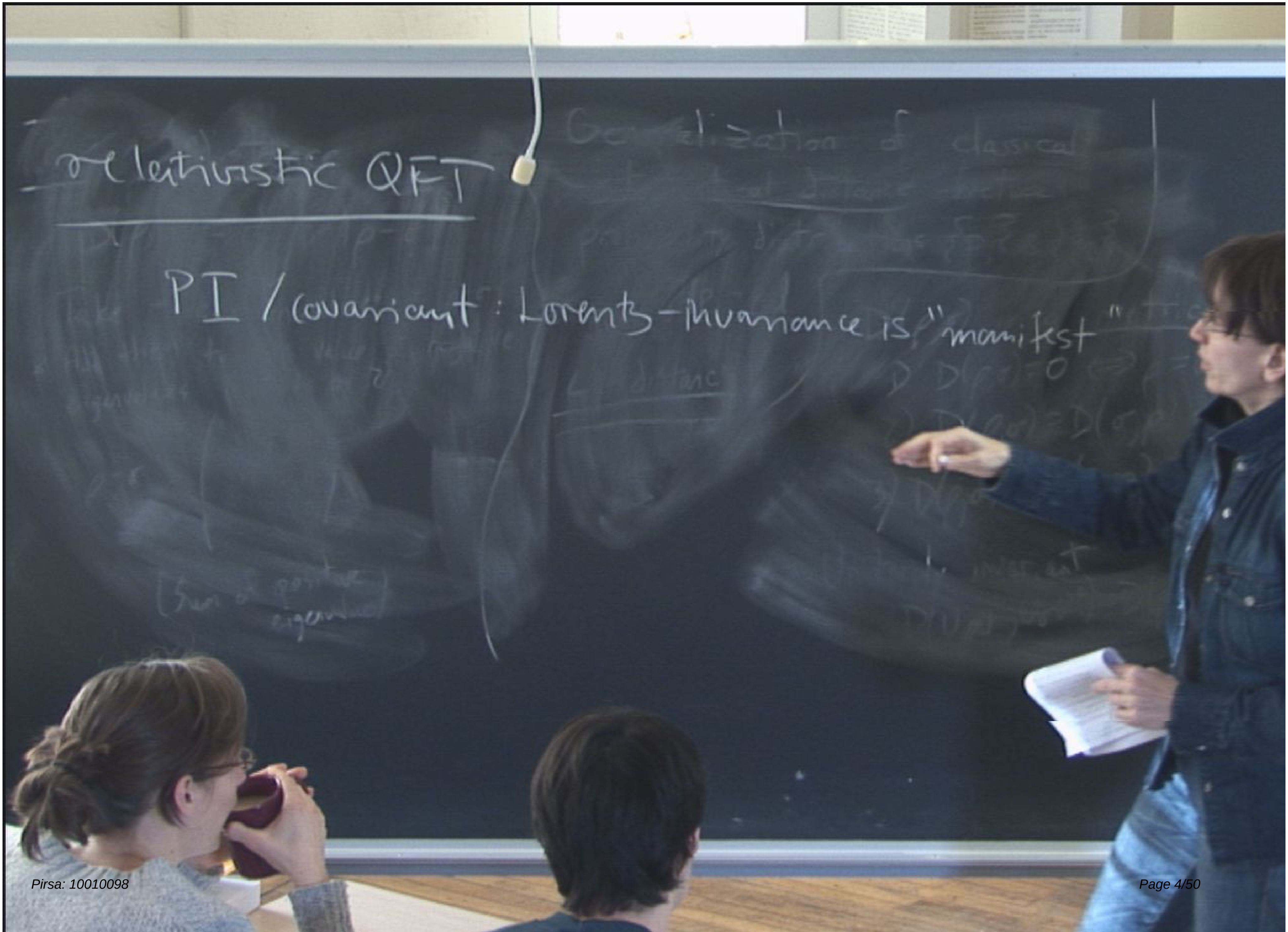
Date: Jan 29, 2010 10:00 AM

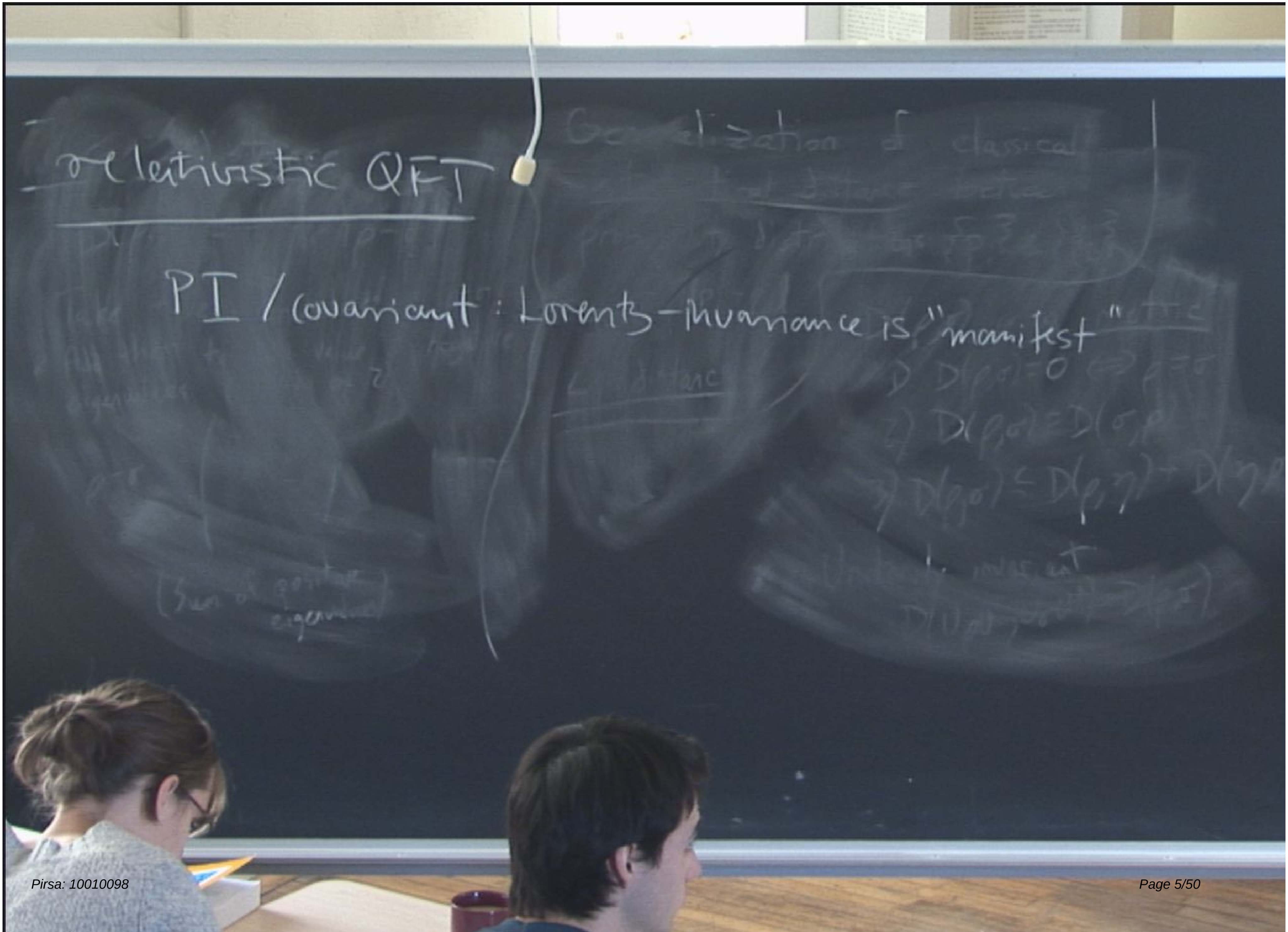
URL: http://pirsa.org/10010098

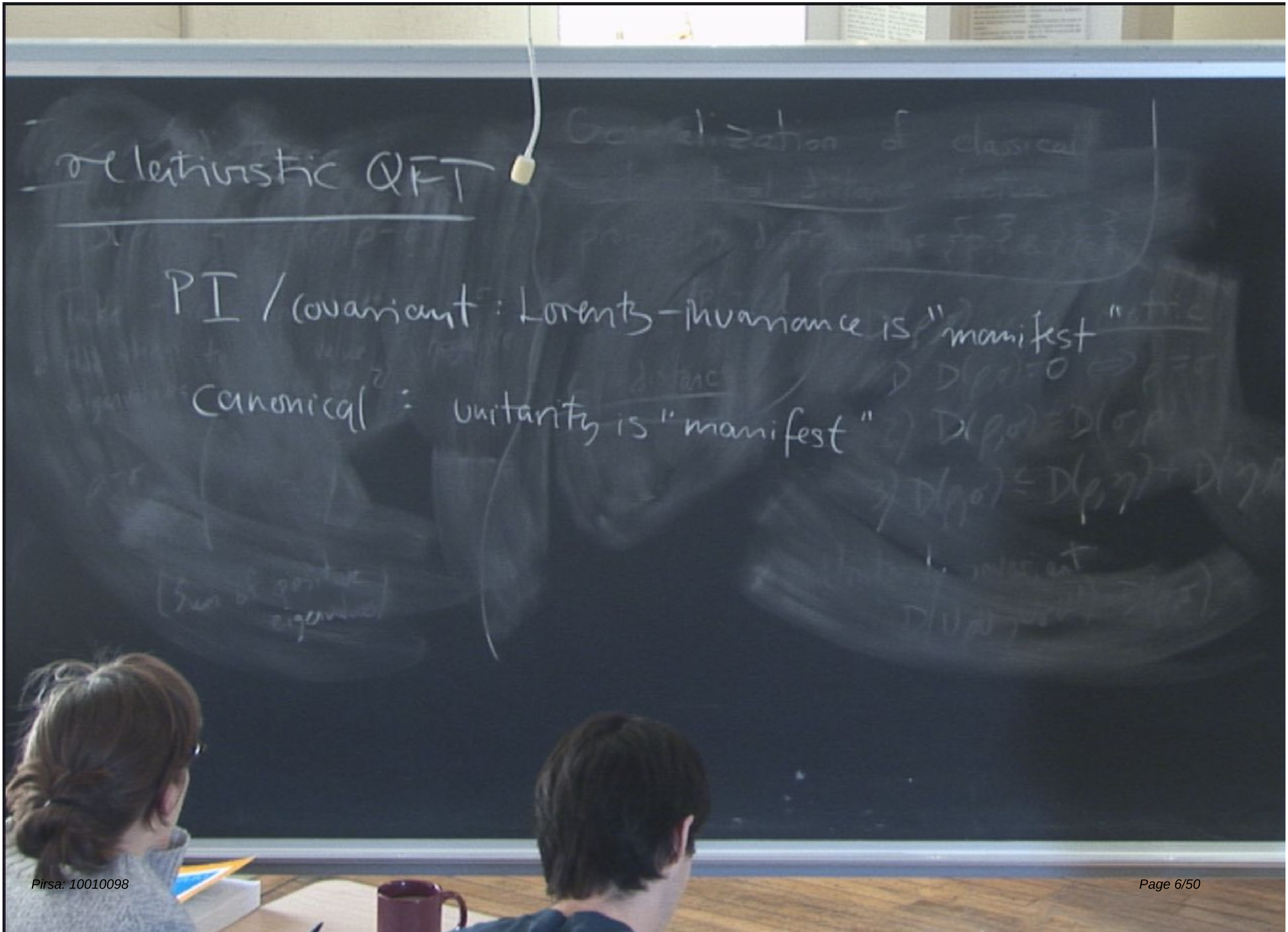
Abstract:

perimeter scholars
international


relativistic QFT
Generalization of classical
metric
1) D(ρ,σ)=0 ⟺ ρ=σ
2) D(ρ,σ)=D(σ,ρ)
3) D(ρ,σ) ≤ D(ρ,η)+D(η,σ)

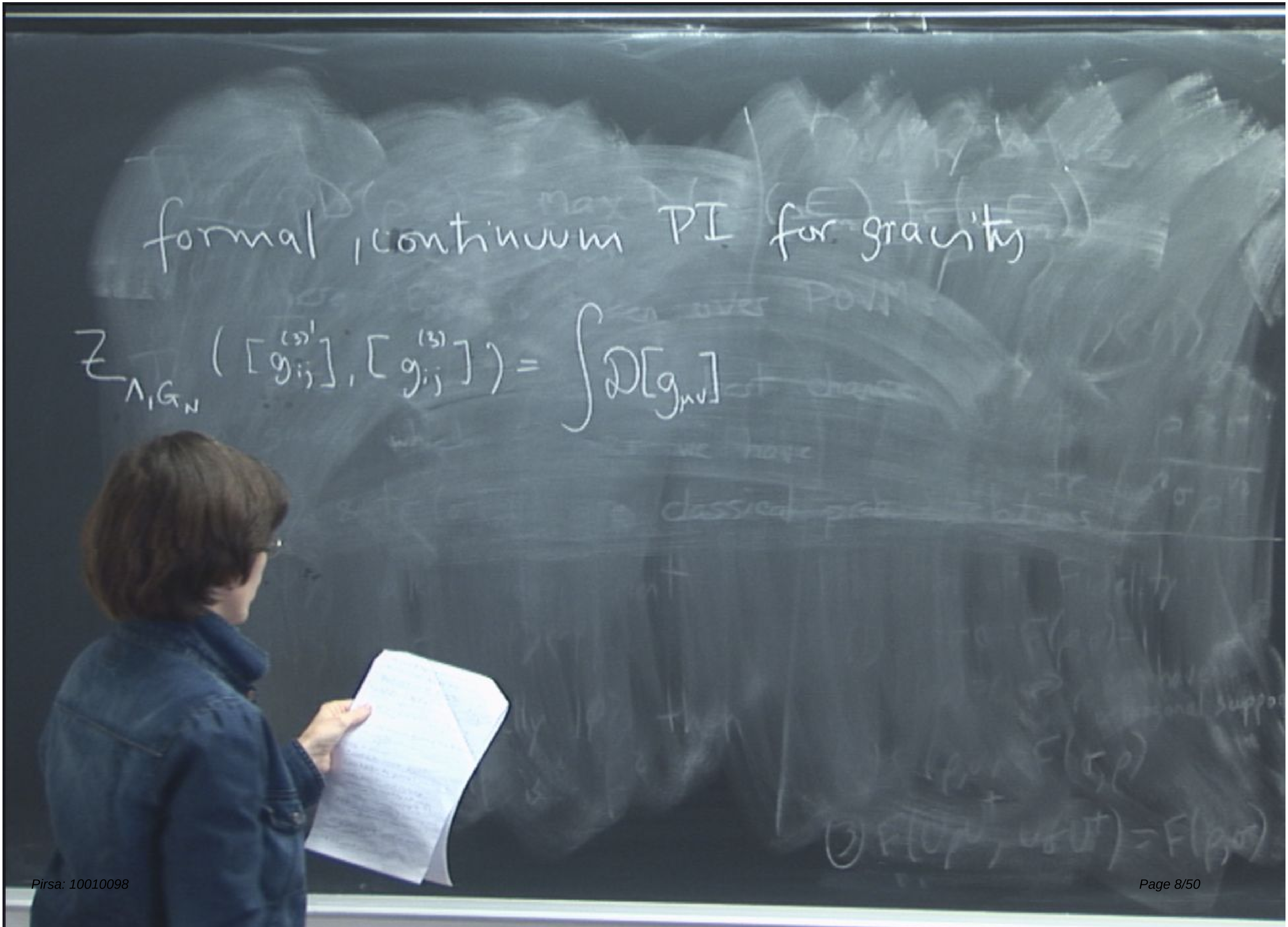

relativistic QFT
PI / covariant: Lorentz-invariance is "manifest"


relativistic QFT
PI / covariant: Lorentz-invariance is "manifest"


relativistic QFT
PI / covariant : Lorentz-invariance is "manifest"
canonical : unitarity is "manifest"

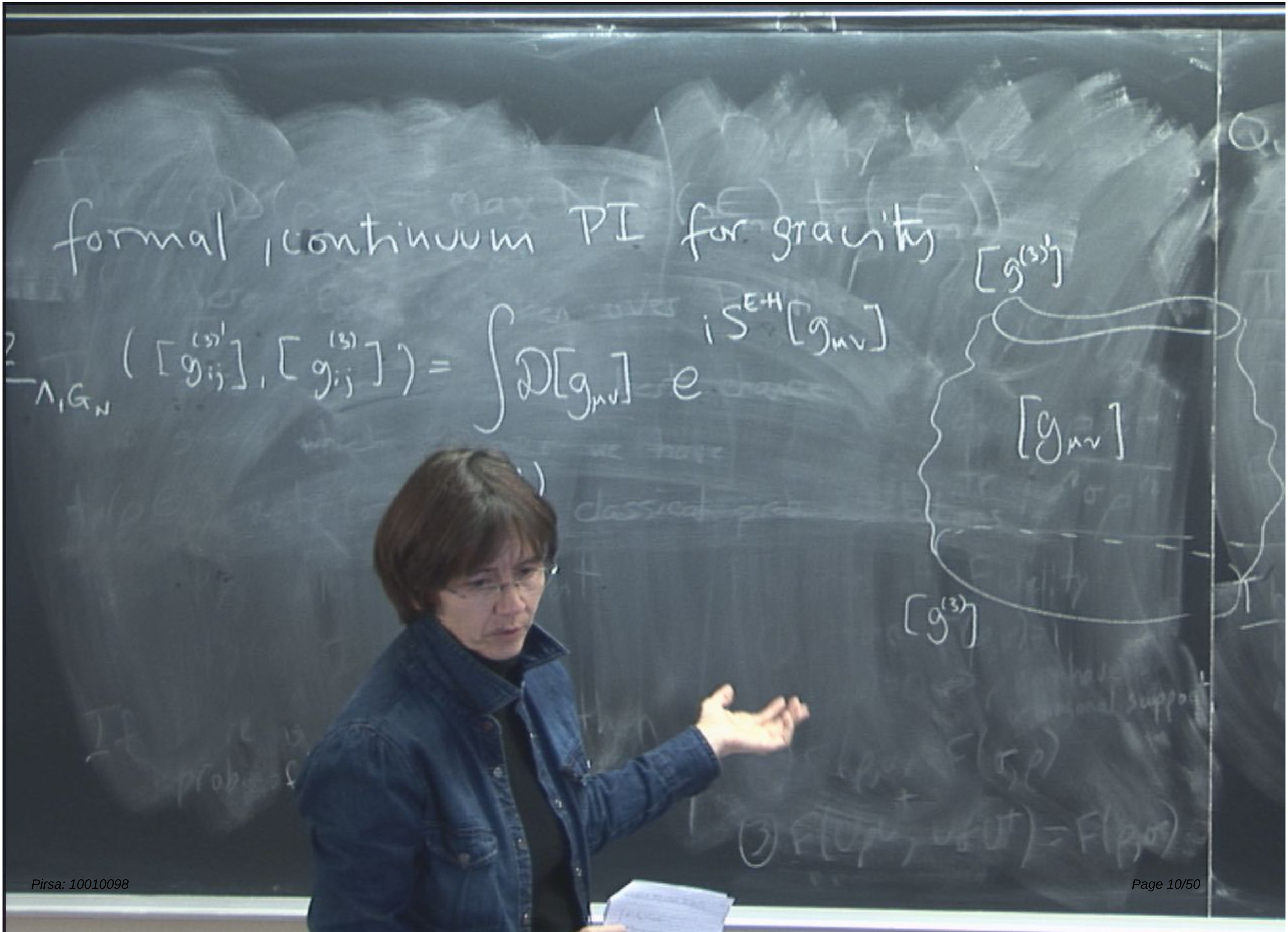
formal, continuum PI for gravity

$$Z_{\Lambda, G_N}\left([g^{(3)'}_{ij}], [g^{(3)}_{ij}]\right) = \int$$


formal, continuum PI for gravity
$Z_{\Lambda, G_N}([g^{(3)'}_{ij}], [g^{(3)}_{ij}]) = \int \mathcal{D}[g_{\mu\nu}]$

formal, continuum PI for gravity

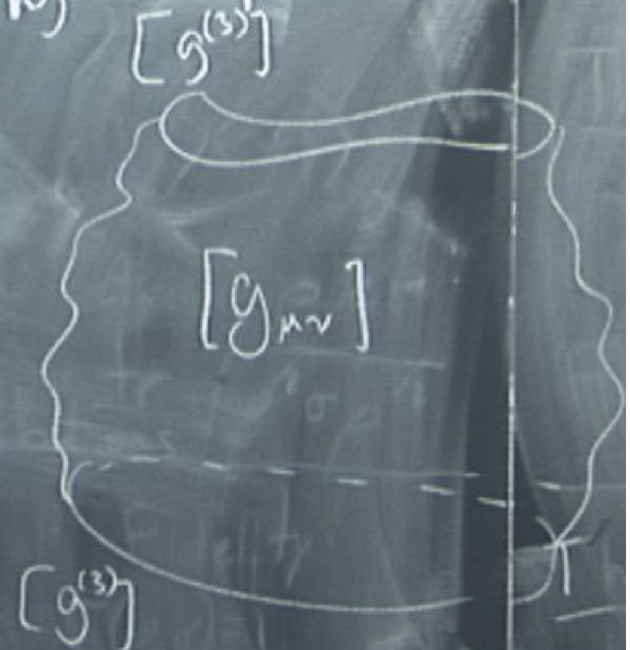
$$Z_{\Lambda, G_N}\left([g^{(3)'}_{ij}], [g^{(3)}_{ij}]\right) = \int_{\mathcal{G}(M)} \mathcal{D}[g_{\mu\nu}]$$

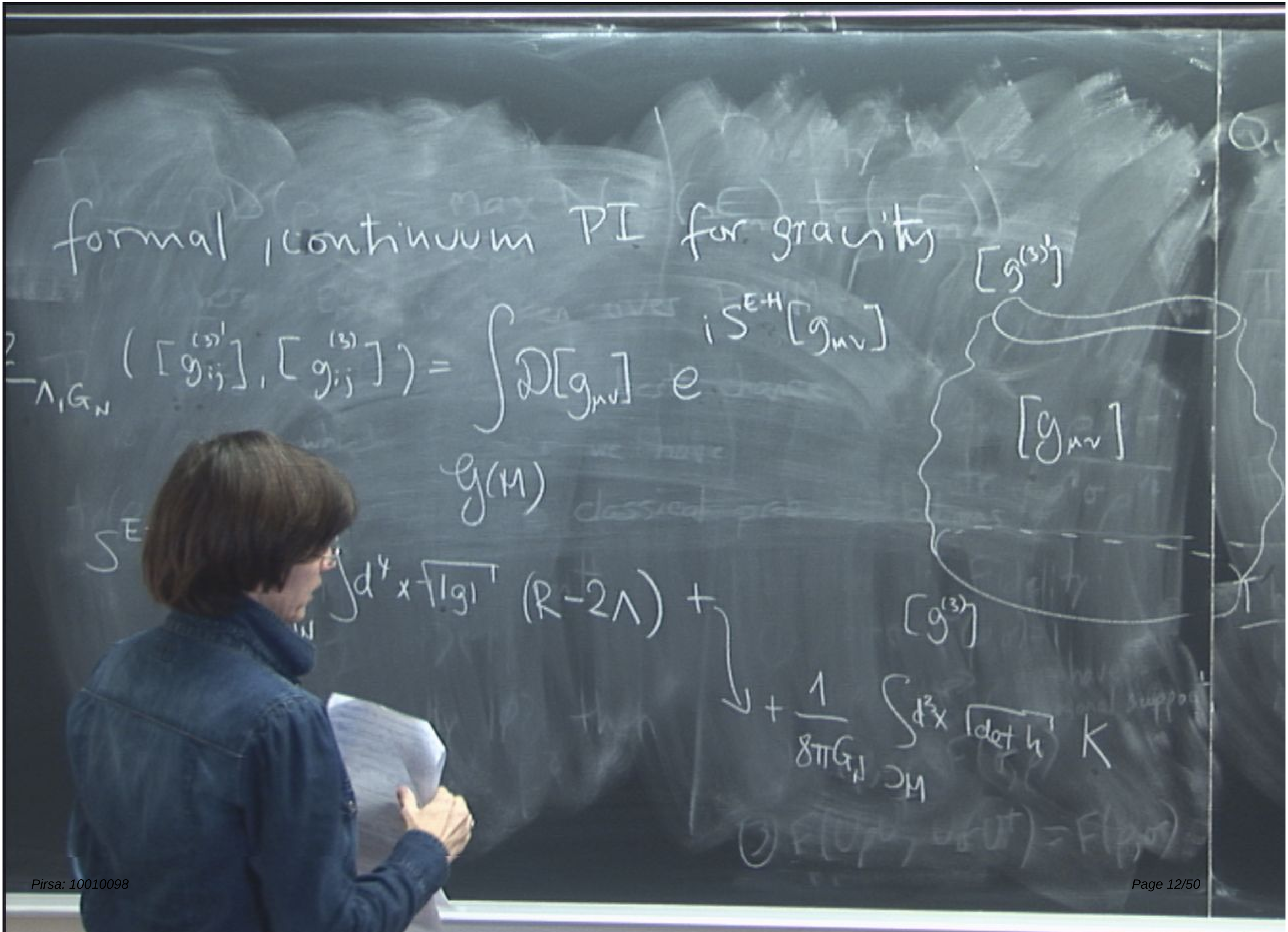

formal, continuum PI for gravity
$Z_{\Lambda, G_N}([g^{(3)'}_{ij}], [g^{(3)}_{ij}]) = \int \mathcal{D}[g_{\mu\nu}]\, e^{iS^{E\text{-}H}[g_{\mu\nu}]}$
$[g^{(3)'}]$
$[g_{\mu\nu}]$
$[g^{(3)}]$

formal, continuum PI for gravity

$$Z_{\Lambda,G_N}\left([g^{(3)'}_{ij}],[g^{(3)}_{ij}]\right) = \int_{\mathcal{G}(M)} \mathcal{D}[g_{\mu\nu}]\; e^{\,iS^{E\text{-}H}[g_{\mu\nu}]}$$

$$S^{E\text{-}H} = \frac{\Lambda}{16\pi G_N}\int d^4x\,\sqrt{|g|}$$

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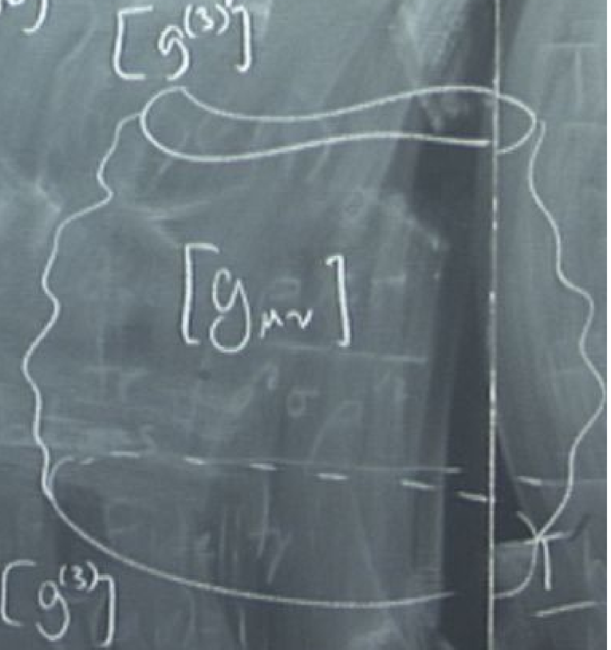
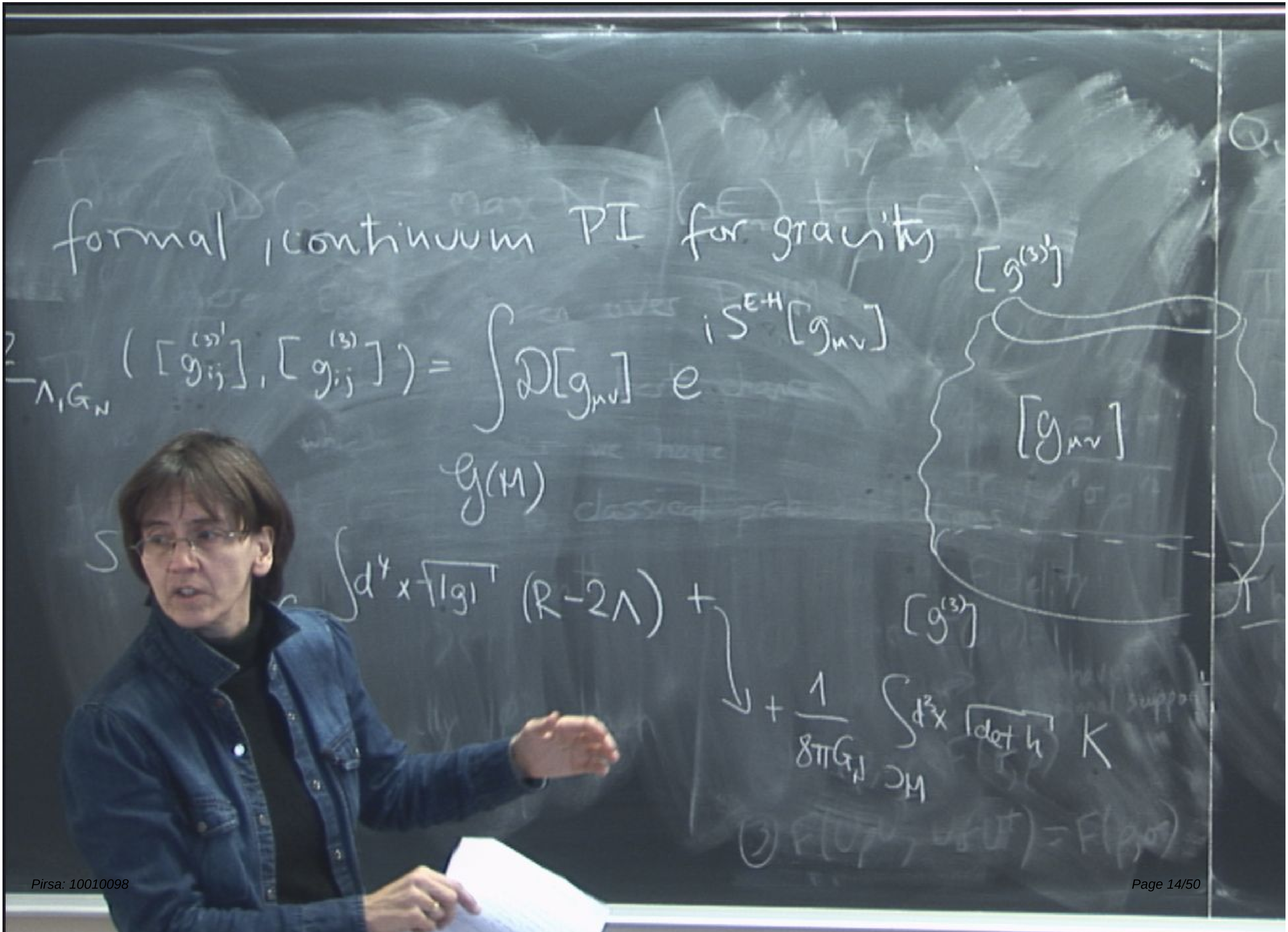
$S^{E}$
$\int d^4x \sqrt{|g|}\,(R-2\Lambda) +$
$+ \frac{1}{8\pi G_N} \int_{\partial M} d^3x \sqrt{\det h}\; K$
$[g^{(3)'}]$
$[g_{\mu\nu}]$
$[g^{(3)}]$

formal, continuum PI for gravity

$$Z_{\Lambda,G_N}\left([g^{(3)'}_{ij}],[g^{(3)}_{ij}]\right) = \int_{\mathcal{G}(M)} \mathcal{D}[g_{\mu\nu}]\, e^{iS^{E\text{-}H}[g_{\mu\nu}]}$$




$$S^{E\text{-}H} = \frac{1}{16\pi G_N}\int d^4x\sqrt{|g|}\,(R-2\Lambda) + \frac{1}{8\pi G_N}\int_{\partial M} d^3x\sqrt{\det h}\;K$$


formal, continuum PI for gravity
$Z_{\Lambda, G_N}([g^{(3)'}_{ij}], [g^{(3)}_{ij}]) = \int_{\mathcal{G}(M)} \mathcal{D}[g_{\mu\nu}]\, e^{iS^{E\text{-}H}[g_{\mu\nu}]}$
$S = \int d^4x \sqrt{|g|}\,(R - 2\Lambda) +$
$+ \frac{1}{8\pi G_N} \int_{\partial M} d^3x \sqrt{\det h}\, K$
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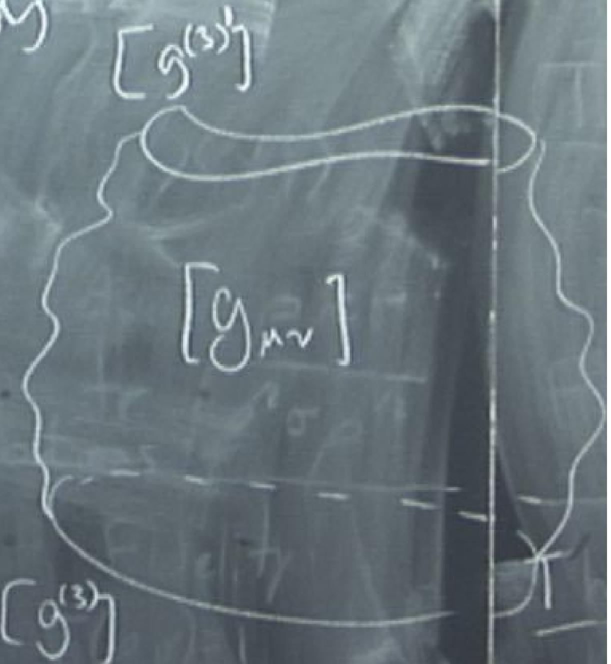
formal, continuum PI for gravity

$$Z_{\Lambda, G_N}\left([g^{(3)'}_{ij}], [g^{(3)}_{ij}]\right) = \int_{\mathcal{G}(M)} \mathcal{D}[g_{\mu\nu}]\; e^{\,i S^{E\text{-}H}[g_{\mu\nu}]}$$

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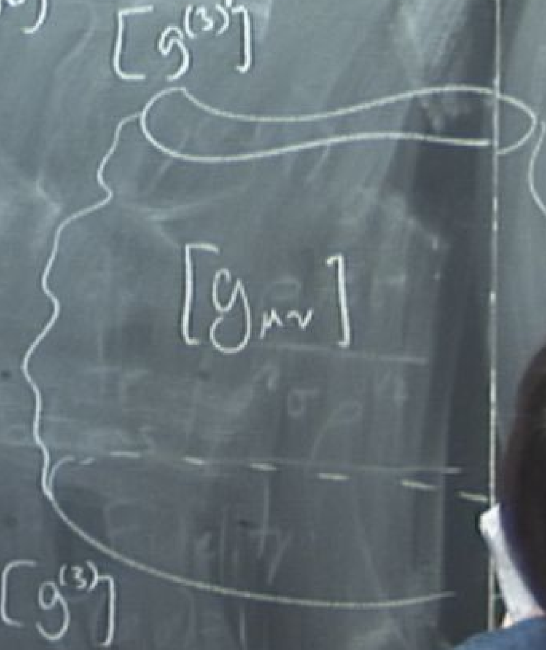



$$S^{E\text{-}H} = \frac{1}{16\pi G_N}\int d^4x \sqrt{|g|}\,(R-2\Lambda) + \frac{1}{8\pi G_N}\int_{\partial M} d^3x \sqrt{\det h}\; K$$

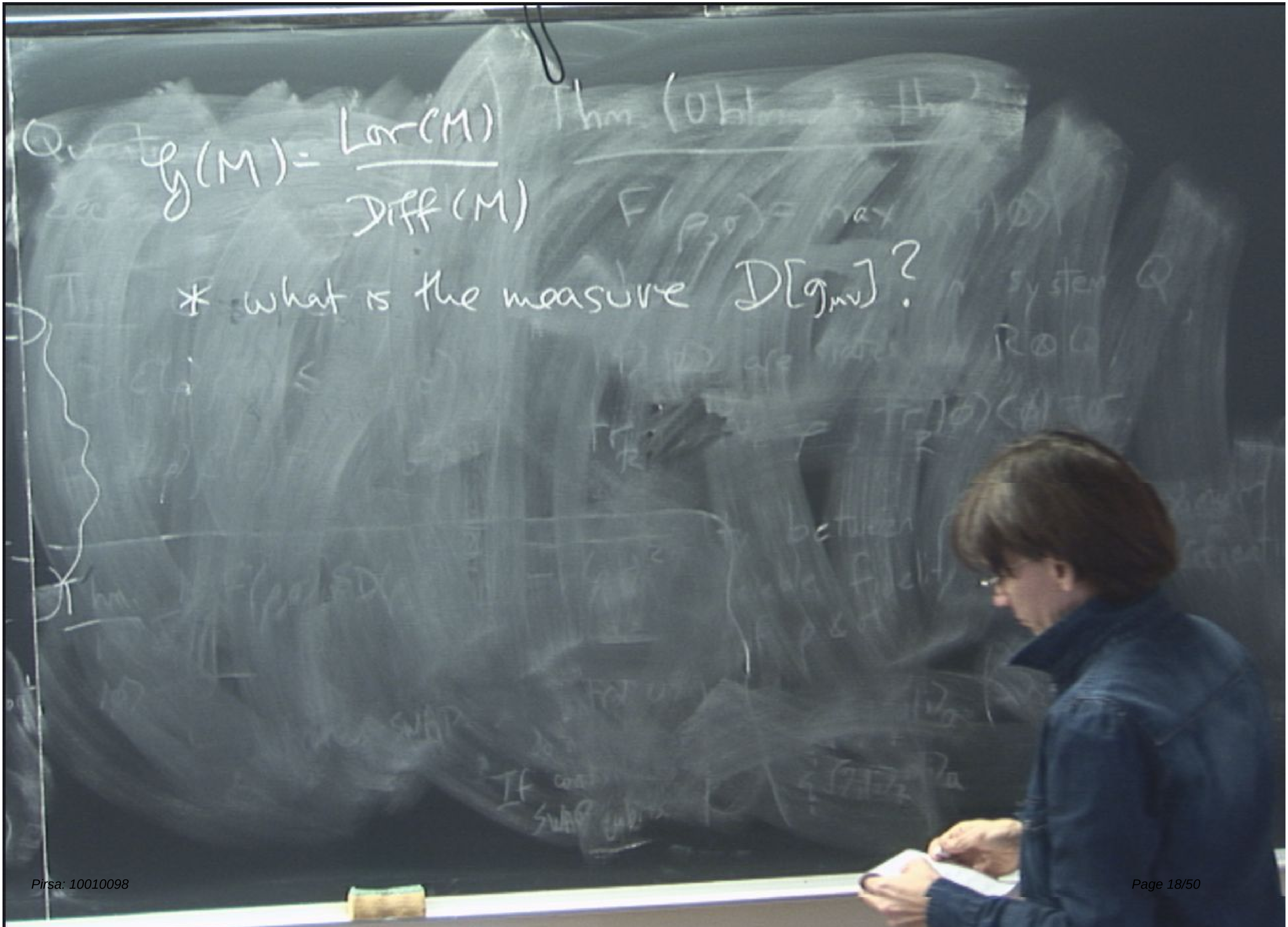
$[g_{\mu\nu}]$ ~ diffeomorphism-equivalence class of metrics

$$\mathcal{G}(M) = \frac{\mathrm{Lor}(M)}{\mathrm{Diff}(M)}$$

$g^{(3)}$]

$[g_{\mu\nu}]$

det h K


$\mathcal{G}(M) = \dfrac{\mathrm{Lor}(M)}{\mathrm{Diff}(M)}$
* what is the measure $D[g_{\mu\nu}]$?

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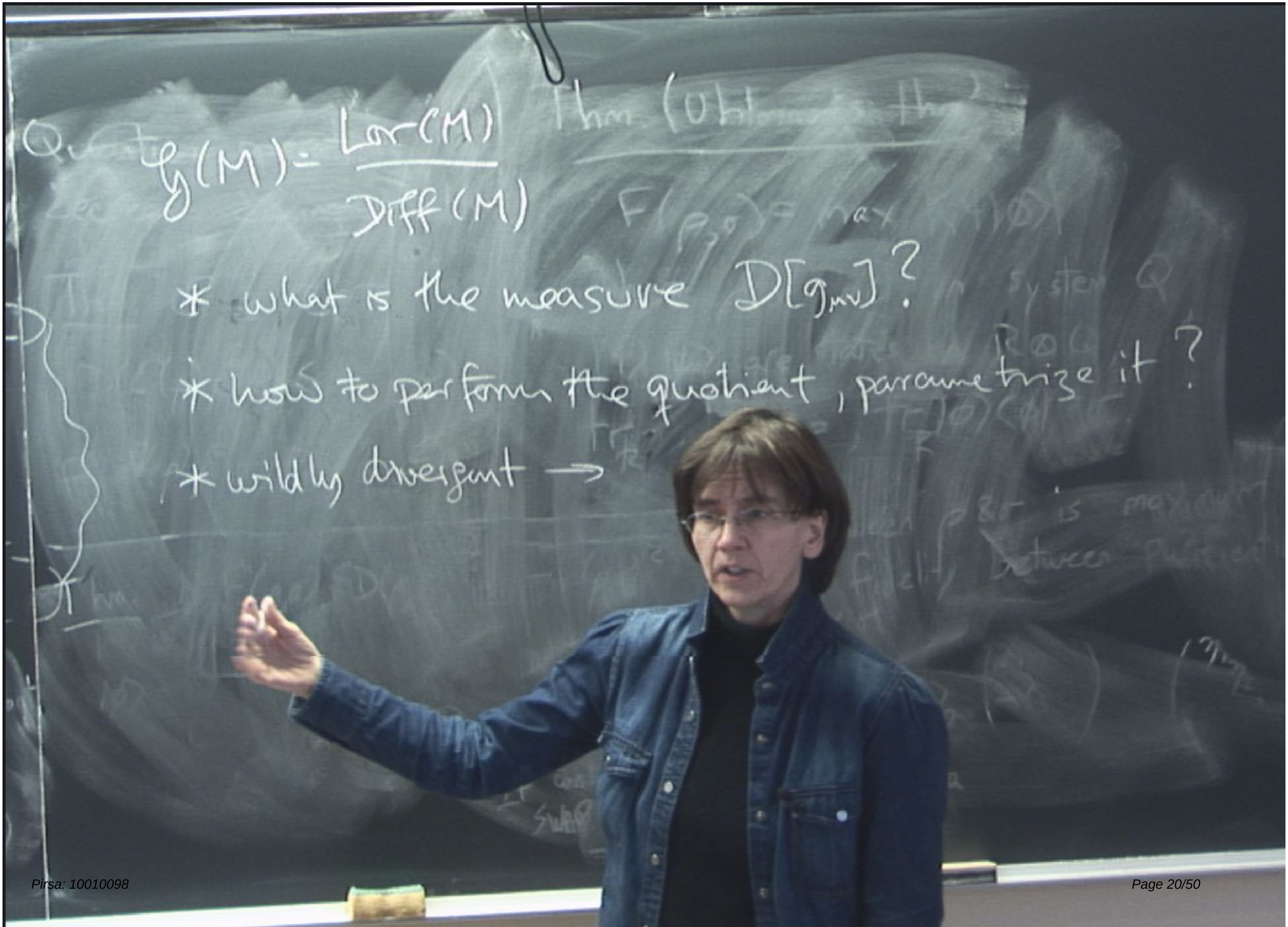
* what is the measure $D[g_{\mu\nu}]$?


G(M) = Lor(M) / Diff(M)
* what is the measure D[g_μν] ?
* how to perform the quotient, parametrize it ?
* wildly divergent →

$\mathcal{G}(M) = \dfrac{\mathrm{Lor}(M)}{\mathrm{Diff}(M)}$

* what is the measure $D[g_{\mu\nu}]$ ?

* how to perform the quotient, parametrize it ?

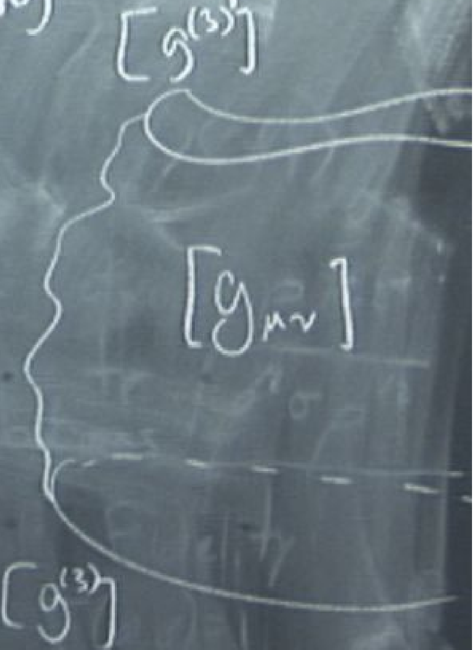
* wildly divergent $\rightarrow$ must regularize

(regularization compatible w/ gauge invariance ?

*

formal, continuum PI for gravity

$$Z_{\Lambda, G_N}\left([g^{(3)'}_{ij}], [g^{(3)}_{ij}]\right) = \int_{\mathcal{G}(M)} \mathcal{D}[g_{\mu\nu}]\, e^{iS^{E-H}[g_{\mu\nu}]}$$




$$S^{E-H} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|}\,(R - 2\Lambda) + \frac{1}{8\pi G_N} \int_{\partial M} d^3x \sqrt{\det h}\, K$$

$[g_{\mu\nu}]$ ~ diffeomorphism-equivalence class of metrics

$$\mathcal{G}(M) = \frac{\mathrm{Lor}(M)}{\mathrm{Diff}(M)}$$

* what is the measure $D[g_{\mu\nu}]$?

* how to perform the quotient, parametrize it?

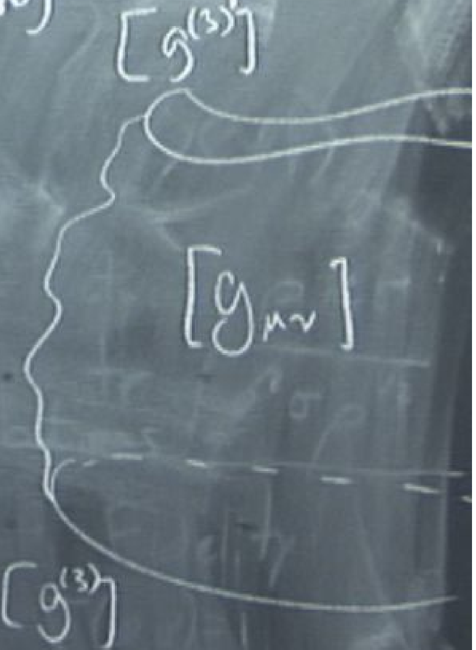
* wildly divergent $\rightarrow$ must regularize

  (regularization compatible w/ gauge invariance?

* factor $i$? $\exists$ Wick rotation?

formal, continuum PI for gravity

$$Z_{\Lambda,G_N}\left([g^{(3)'}_{ij}],[g^{(3)}_{ij}]\right) = \int_{\mathcal{G}(M)} \mathcal{D}[g_{\mu\nu}]\, e^{iS^{E\text{-}H}[g_{\mu\nu}]}$$




$$S^{E\text{-}H} = \frac{1}{16\pi G_N}\int d^4x \sqrt{|g|}\,(R-2\Lambda) + \frac{1}{8\pi G_N}\int_{\partial M} d^3x \sqrt{\det h}\, K$$

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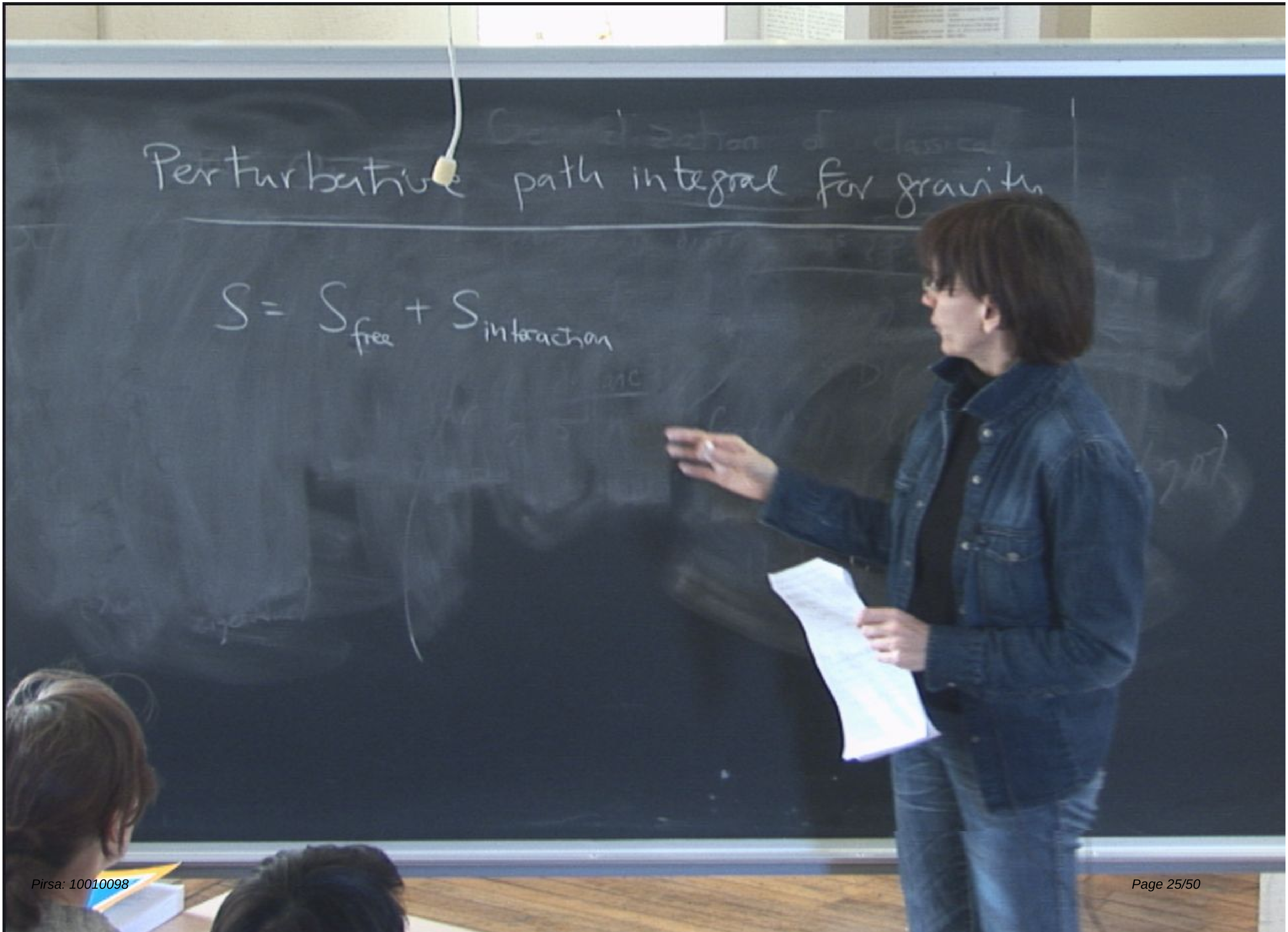
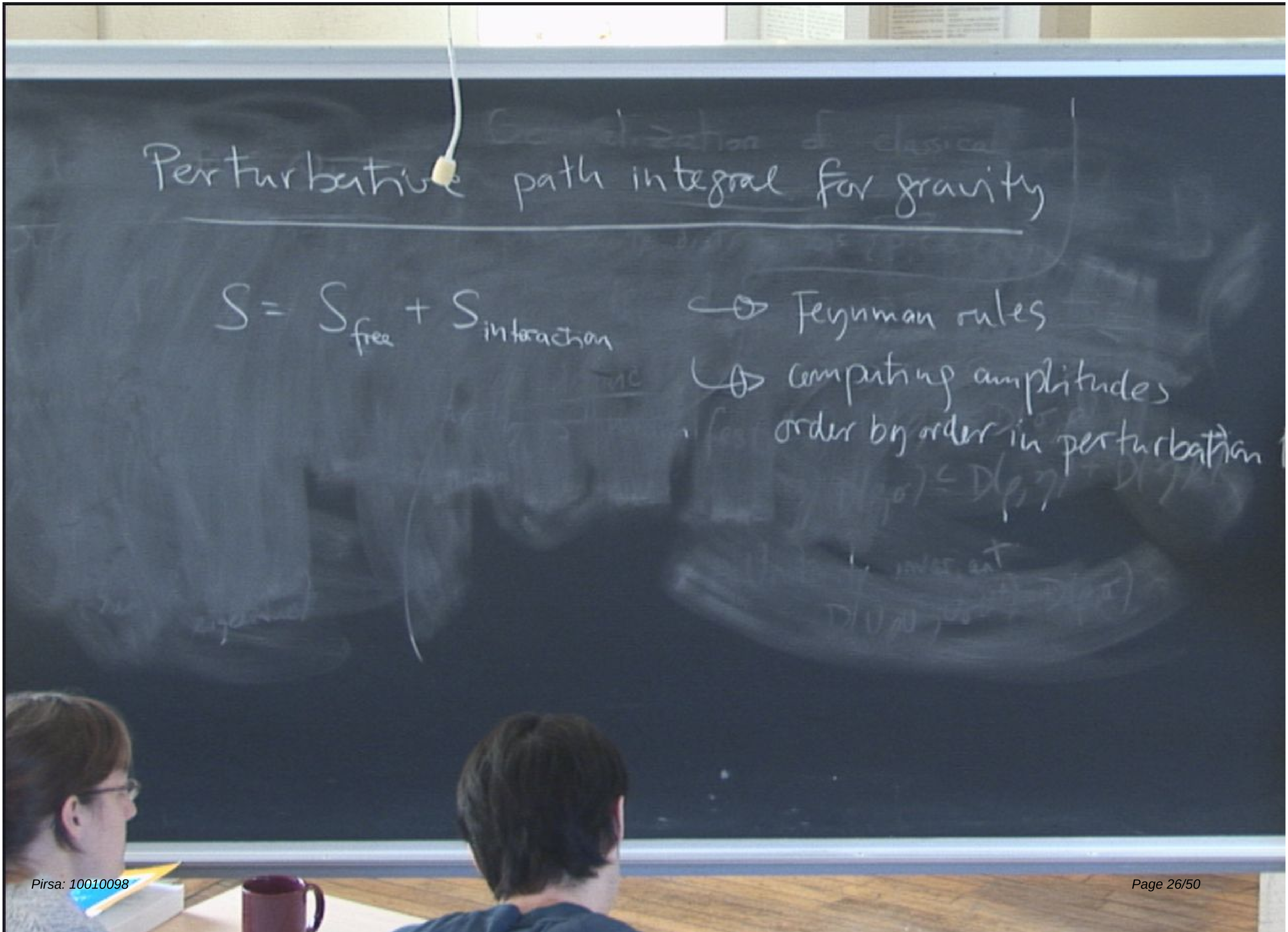
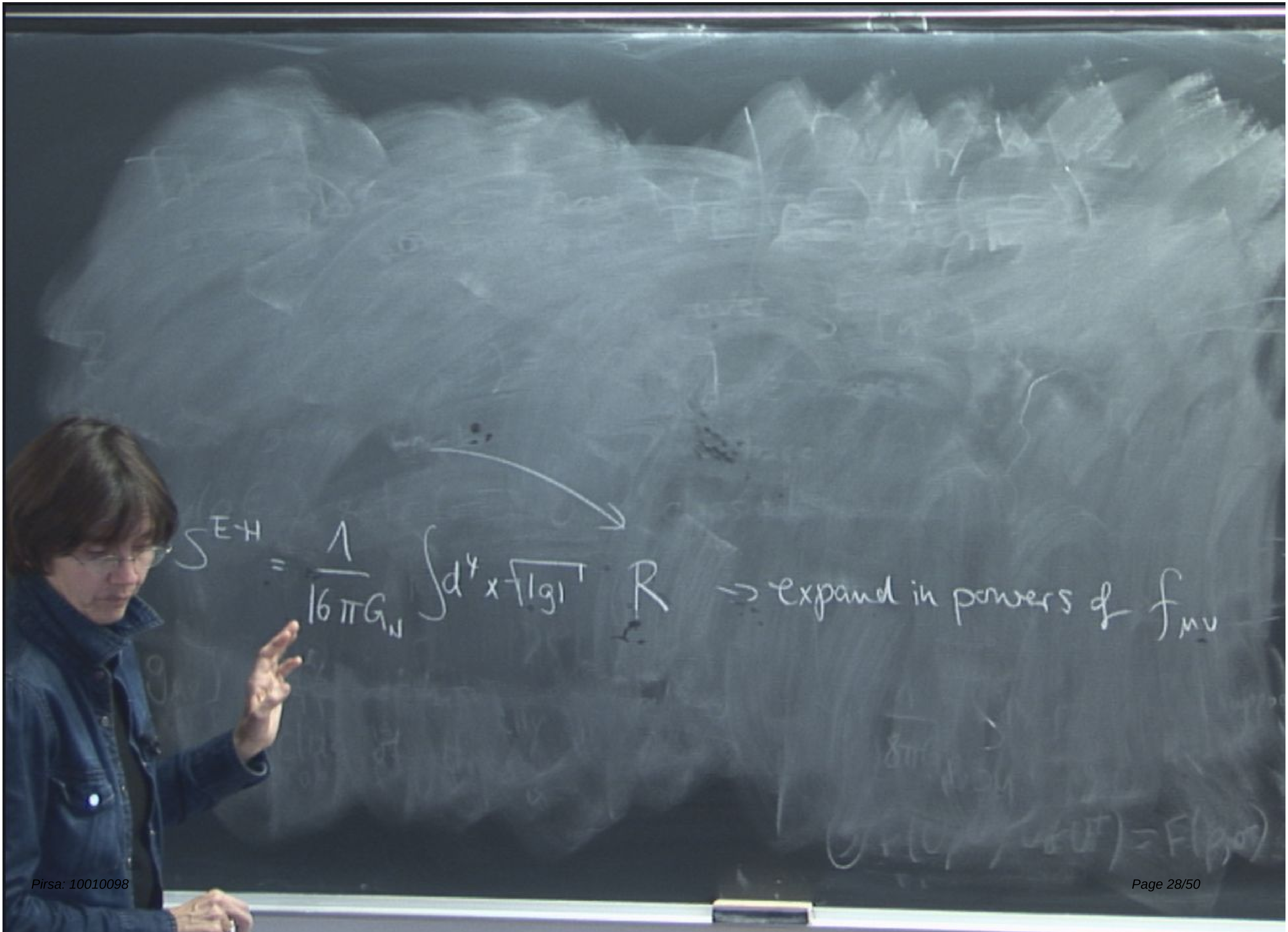

Perturbative path integral for gravity
S = S_free + S_interaction


Perturbative path integral for gravity
S = S_free + S_interaction
Feynman rules
computing amplitudes
order by order in perturbation

$$S^{E\text{-}H} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|}\; R$$


$S^{E\text{-}H} = \frac{1}{16\pi G_N}\int d^4x\sqrt{|g|}\;R \Rightarrow$ expand in powers of $f_{\mu\nu}$

schematically $R \sim \partial f \partial f + f \partial f \partial f + f^2 \partial f \partial f + \dots$

$$S^{E\text{-}H} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|}\; R \implies \text{expand in powers of } f_{\mu\nu}$$

$$\tilde{f} = \frac{1}{\sqrt{16\pi G_N}} f$$

schematically $R \sim \partial f \partial f + f \partial f \partial f + f^2 \partial f \partial f + \ldots$

$$S^{E\text{-}H} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|}\, R \Rightarrow \text{expand in powers of } f_{\mu\nu}$$

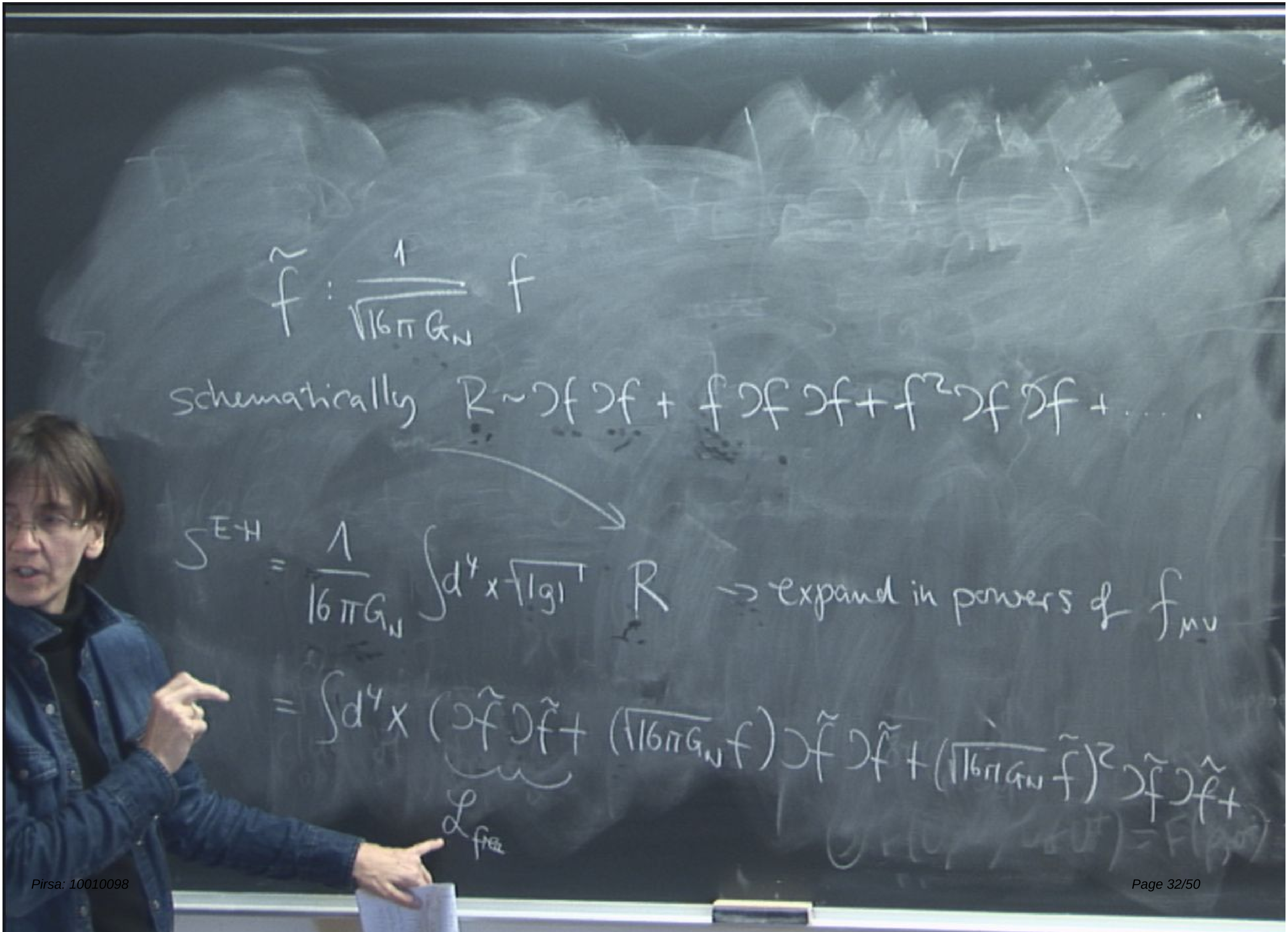
$$= \int d^4x \, (\partial \tilde{f} \partial \tilde{f}$$

$$\tilde{f} : \frac{1}{\sqrt{16\pi G_N}} f$$

schematically $R \sim \partial f \partial f + f \partial f \partial f + f^2 \partial f \partial f + \ldots$

$$= \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|}\ R \Rightarrow \text{expand in powers of } f_{\mu\nu}$$

$$= \int d^4x \left( \partial\tilde{f}\,\partial\tilde{f} + (\sqrt{16\pi G_N}\,\tilde{f})\,\partial\tilde{f}\,\partial\tilde{f} + (\sqrt{16\pi G_N}\,\tilde{f})^2\,\partial\tilde{f}\,\partial\tilde{f} + \right.$$



$\tilde{f} : \frac{1}{\sqrt{16\pi G_N}} f$
schematically $R \sim \partial f \partial f + f \partial f \partial f + f^2 \partial f \partial f + \dots$
$S^{E\text{-}H} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|}\; R$ → expand in powers of $f_{\mu\nu}$
$= \int d^4x \, (\underbrace{\partial \tilde{f} \partial \tilde{f}}_{\mathcal{L}_{free}} + (\sqrt{16\pi G_N} \tilde{f}) \partial \tilde{f} \partial \tilde{f} + (\sqrt{16\pi G_N} \tilde{f})^2 \partial \tilde{f} \partial \tilde{f} +$

$$\tilde f = \frac{1}{\sqrt{16\pi G_N}}\, f$$

schematically $R \sim \partial f \partial f + f \partial f \partial f + f^2 \partial f \partial f + \dots$

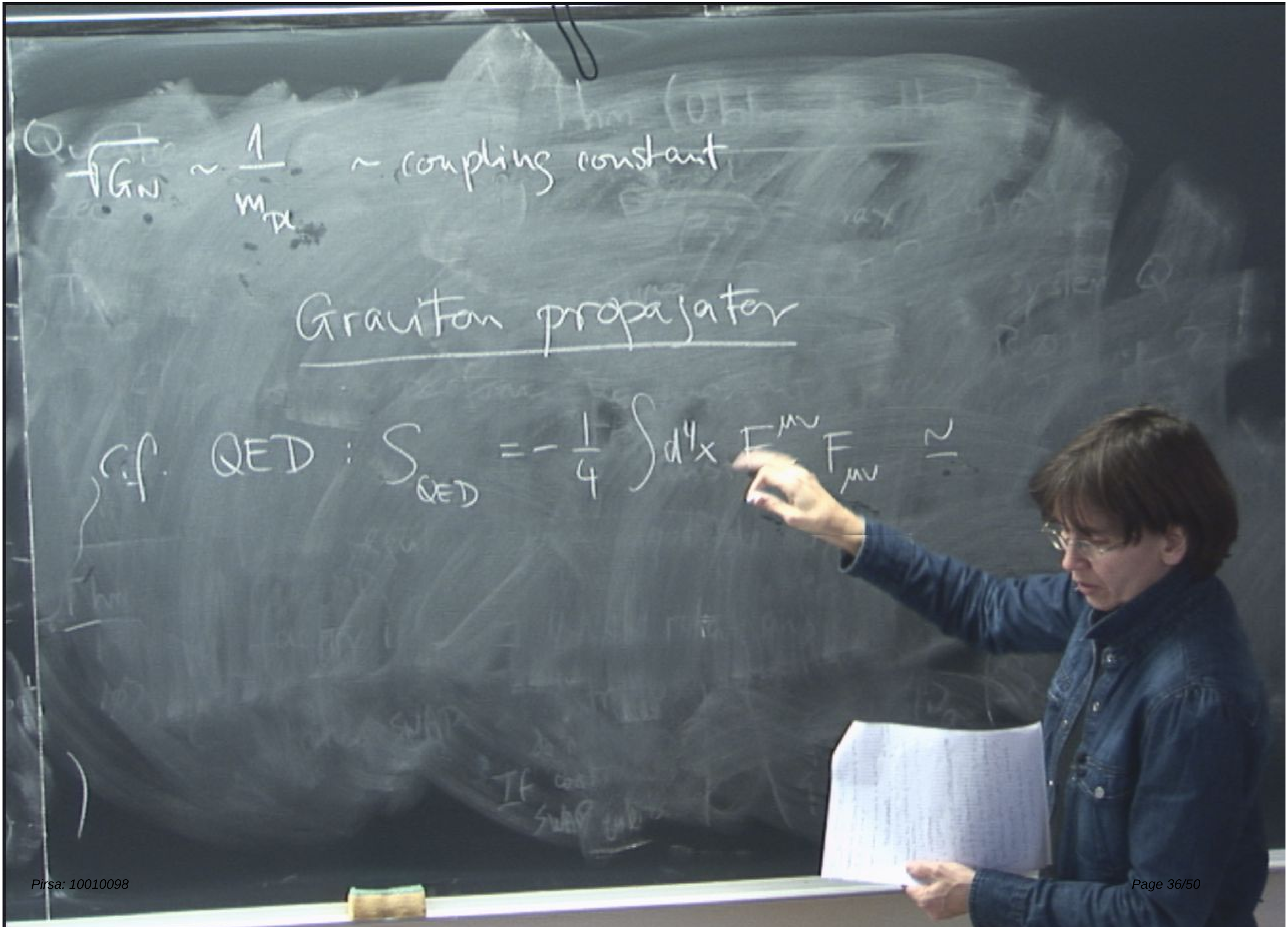
$$S^{E\text{-}H} = \frac{1}{16\pi G_N}\int d^4x \sqrt{|g|}\; R$$ $\Rightarrow$ expand in powers of $f_{\mu\nu}$

$$= \int d^4x \Big(\underbrace{\partial\tilde f\, \partial\tilde f}_{\mathcal{L}_{free}} + \underbrace{(\sqrt{16\pi G_N}\, \tilde f)\, \partial\tilde f\, \partial\tilde f + (\sqrt{16\pi G_N}\, \tilde f)^2\, \partial\tilde f\, \partial\tilde f + \dots}_{\mathcal{L}_{interaction}}$$

$\sqrt{G_N} \sim \frac{1}{m_{Pl}} \sim$ coupling constant

$\sqrt{G_N} \sim \frac{1}{m_{pl}}$ ~ coupling constant

Graviton propagator


$\sqrt{G_N} \sim \frac{1}{m_{Pl}}$ ~ coupling constant
Graviton propagator
c.f. QED : $S_{QED} = -\frac{1}{4}\int d^4x\, F^{\mu\nu}F_{\mu\nu} \simeq$

$\sqrt{G_N} \sim \frac{1}{m_{pl}}$ ~ coupling constant

Graviton propagator

c.f. QED : $S_{QED} = -\frac{1}{4}\int d^4x\, F^{\mu\nu}F_{\mu\nu} \simeq \int d^4x\, A_\mu$

Q: $\sqrt{G_N} \sim \frac{1}{m_{Pl}}$ ~ coupling constant

diff. op.

Graviton propagator

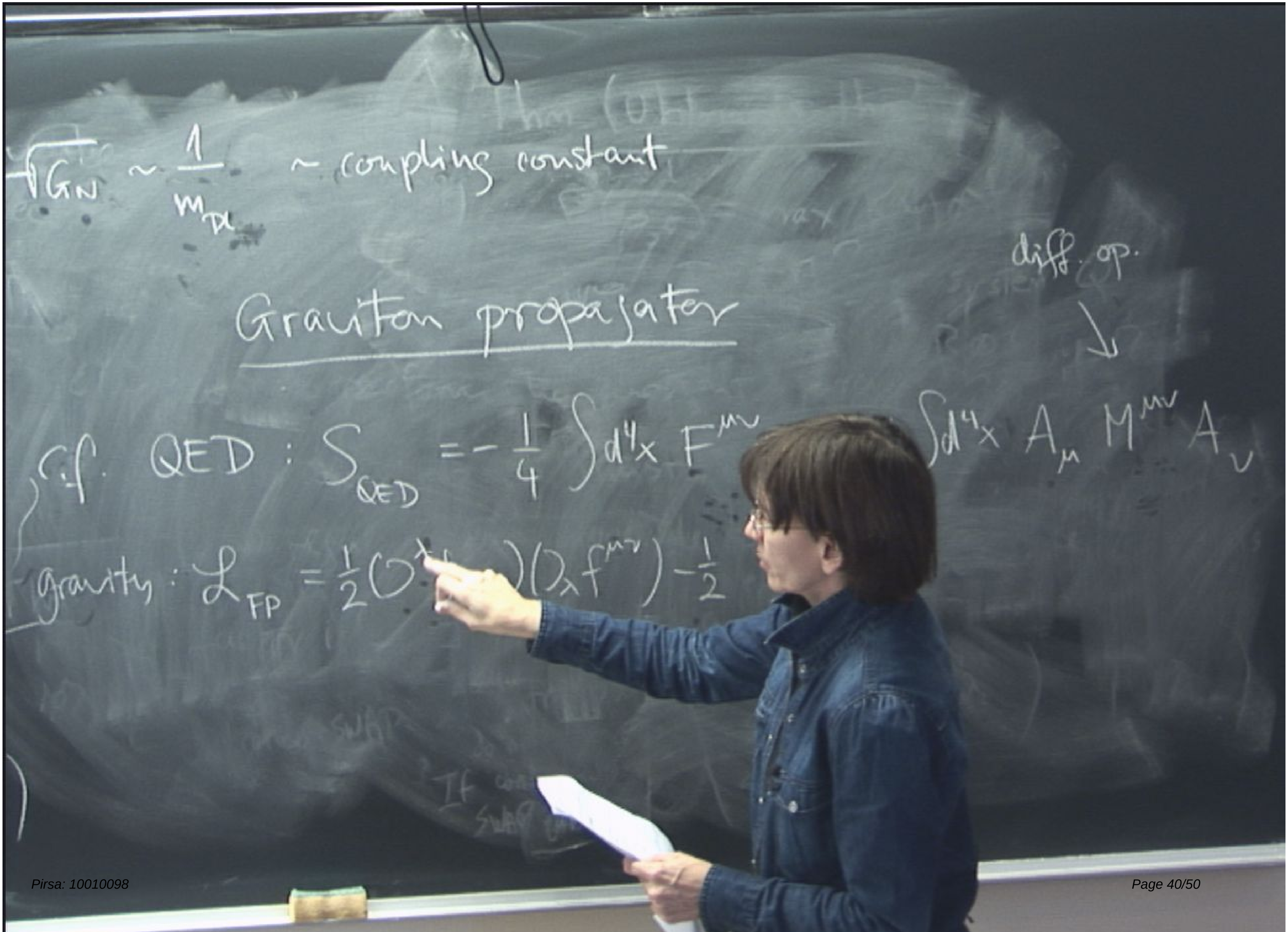
c.f. QED : $S_{QED} = -\frac{1}{4}\int d^4x\, F^{\mu\nu}F_{\mu\nu} \simeq \int d^4x\, A_\mu M^{\mu\nu} A$

$\sqrt{G_N} \sim \frac{1}{m_{Pl}} \sim$ coupling constant

diff. op.

## Graviton propagator

c.f. QED : $S_{QED} = -\frac{1}{4}\int d^4x\, F^{\mu\nu}F_{\mu\nu} \simeq \int d^4x\, A_\mu M^{\mu\nu} A_\nu$



$\sqrt{G_N} \sim \frac{1}{m_{pl}}$ ~ coupling constant
diff. op.
Graviton propagator
c.f. QED : $S_{QED} = -\frac{1}{4} \int d^4x \, F^{\mu\nu}$
$\int d^4x \, A_\mu M^{\mu\nu} A_\nu$
gravity : $\mathcal{L}_{FP} = \frac{1}{2}(\partial^\lambda$
$)(\partial_\lambda f^{\mu\nu}) - \frac{1}{2}$

$\sqrt{G_N} \sim \frac{1}{m_{Pl}} \sim$ coupling constant

diff. op.

## Graviton propagator

c.f. QED : $S_{QED} = -\frac{1}{4}\int d^4x\, F^{\mu\nu}F_{\mu\nu} \simeq \int d^4x\, A_\mu M^{\mu\nu} A_\nu$

gravity : $\mathcal{L}_{FP} = \frac{1}{2}(\partial^\lambda f_{\mu\nu})(\partial_\lambda f^{\mu\nu}) - \frac{1}{2}(\partial_\lambda f^{\mu}{}_{\mu})(\partial^\lambda f^{\nu}{}_{\nu}) -$

$-(\partial^\lambda f^{\mu\nu})(\partial_\mu f_{\lambda\nu}) + (\partial_\lambda f^{\lambda\mu})(\partial_\mu f^{\nu}{}_{\nu}) \simeq$

$\sqrt{G_N} \sim \frac{1}{m_{Pl}} \sim$ coupling constant

diff. op.

Graviton propagator

$$\text{c.f. QED}: \; S_{QED} = -\frac{1}{4}\int d^4x\, F^{\mu\nu}F_{\mu\nu} \simeq \int d^4x\, A_\mu M^{\mu\nu} A_\nu$$

$$\text{gravity}: \; \mathcal{L}_{FP} = \frac{1}{2}(\partial^\lambda f_{\mu\nu})(\partial_\lambda f^{\mu\nu}) - \frac{1}{2}(\partial_\lambda f^\mu{}_\mu)(\partial^\lambda f^\nu{}_\nu) -$$

$$-(\partial^\lambda f^{\mu\nu})(\partial_\mu f_{\lambda\nu}) + (\partial_\lambda f^{\lambda\mu})(\partial_\mu f^\nu{}_\nu) \simeq$$

$$\simeq \frac{1}{2} f_{\rho\sigma} M^{\rho\sigma\mu\nu} f_{\mu\nu}$$

$\sqrt{G_N} \sim \frac{1}{m_{Pl}}$ ~ coupling constant

diff. op.

## Graviton propagator

c.f. QED : $S_{QED} = -\frac{1}{4}\int d^4x \, F^{\mu\nu}F_{\mu\nu} \simeq \int d^4x \, A_\mu M^{\mu\nu} A_\nu$

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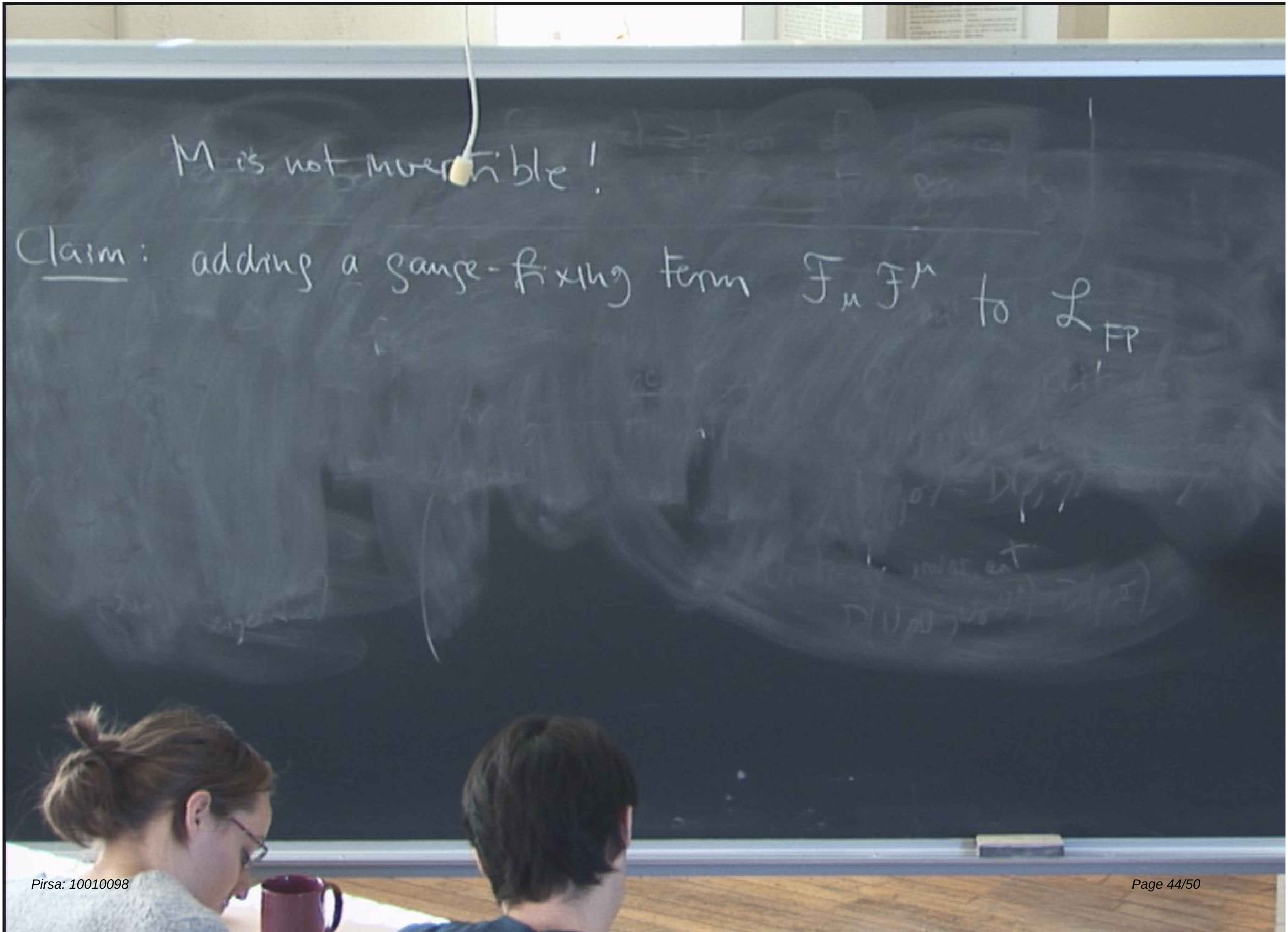
$-(\partial^\lambda f^{\mu\nu})(\partial_\mu f_{\lambda\nu}) + (\partial_\lambda f^{\lambda\mu})(\partial_\mu f^\nu{}_\nu) \simeq$

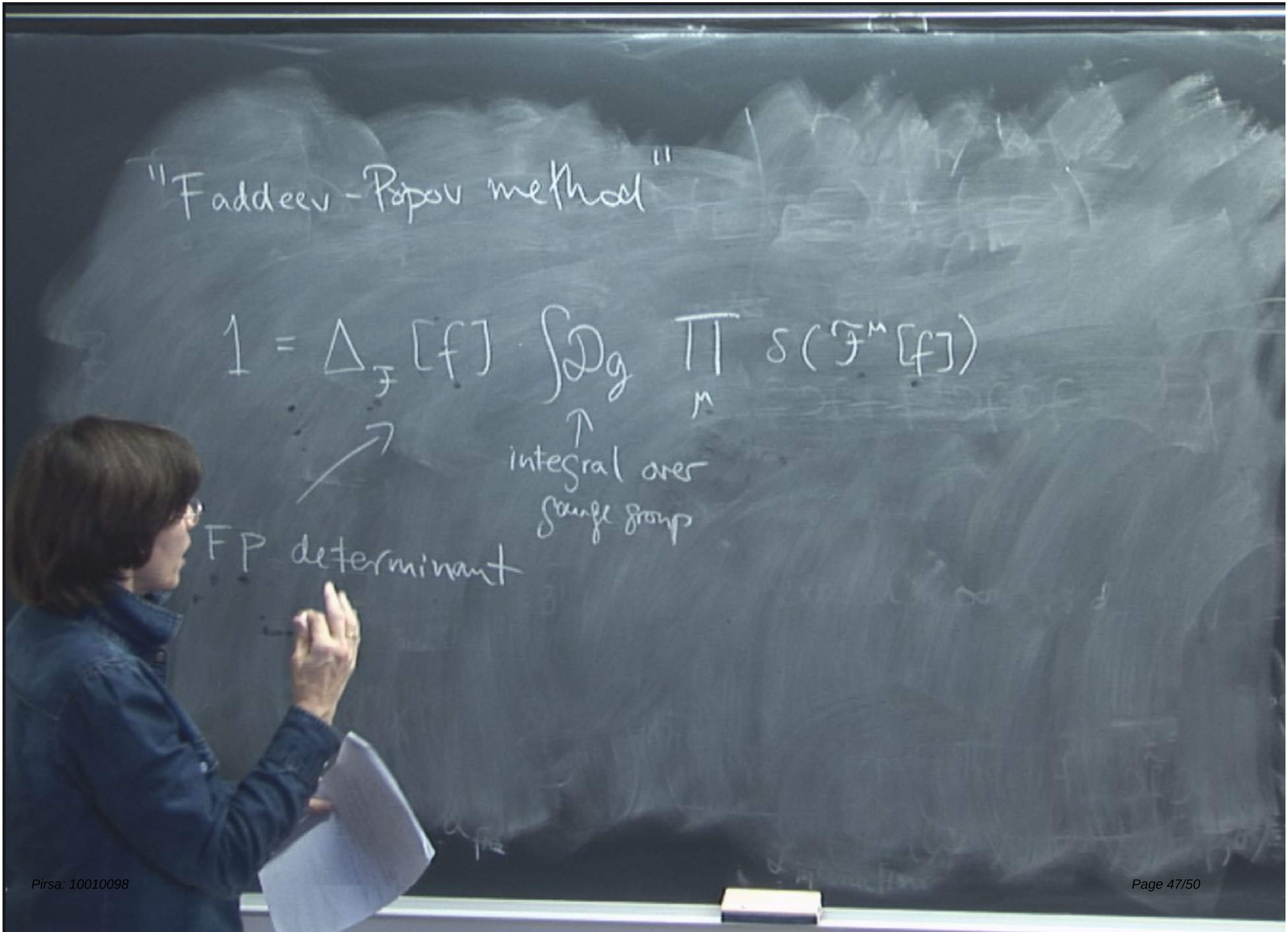
$\simeq \frac{1}{2} f_{\rho\sigma} M^{\rho\sigma\mu\nu} f_{\mu\nu}$


M is not invertible!
Claim: adding a gauge-fixing term $F_\mu F^\mu$ to $\mathcal{L}_{FP}$

$M$ is not invertible!

<u>Claim</u>: adding a gauge-fixing term $\mathcal{F}_\mu \mathcal{F}^\mu$ to $\mathcal{L}_{FP}$, where $\mathcal{F}_\mu[f] = \partial^\nu f_{\mu\nu} - \frac{1}{2}\partial_\mu f^\nu{}_\nu$, the differential operator becomes $\tilde{M}^{\rho\sigma\mu\nu} = \left(-\eta^{\rho\mu}\eta^{\sigma\nu} + \frac{1}{2}\eta^{\rho\sigma}\eta^{\mu\nu}\right)\partial^2$, which is invertible.

"Faddeev - Popov method"


"Faddeev - Popov method"
$1 = \Delta_{\mathcal{F}}[f] \cdot \int \mathcal{D}g \prod_{\mu} \delta(\mathcal{F}^{\mu}[f])$
integral over gauge group
FP determinant

"Faddeev - Popov method"

$$1 = \overbrace{\Delta_{\mathcal{F}}[f]}\cdot \int \mathcal{D}g \prod_{\mu} \delta(\mathcal{F}^{\mu}[f])$$

FP determinant

integral over gauge group

DeWitt (1964)

"Faddeev - Popov method"

$$1 = \overbrace{\Delta_{\mathcal{F}}[f]} \cdot \int \mathcal{D}g \prod_{\mu} \delta(\mathcal{F}^{\mu}[f])$$

FP determinant

integral over gauge group

DeWitt (1964)

"Faddeev-Popov method"

$$1 = \overbrace{\Delta_{\mathcal{F}}[f]} \cdot \int \mathcal{D}g \prod_{\mu} \delta(\mathcal{F}^{\mu}[f])$$

integral over gauge group

FP determinant

DeWitt (1964)