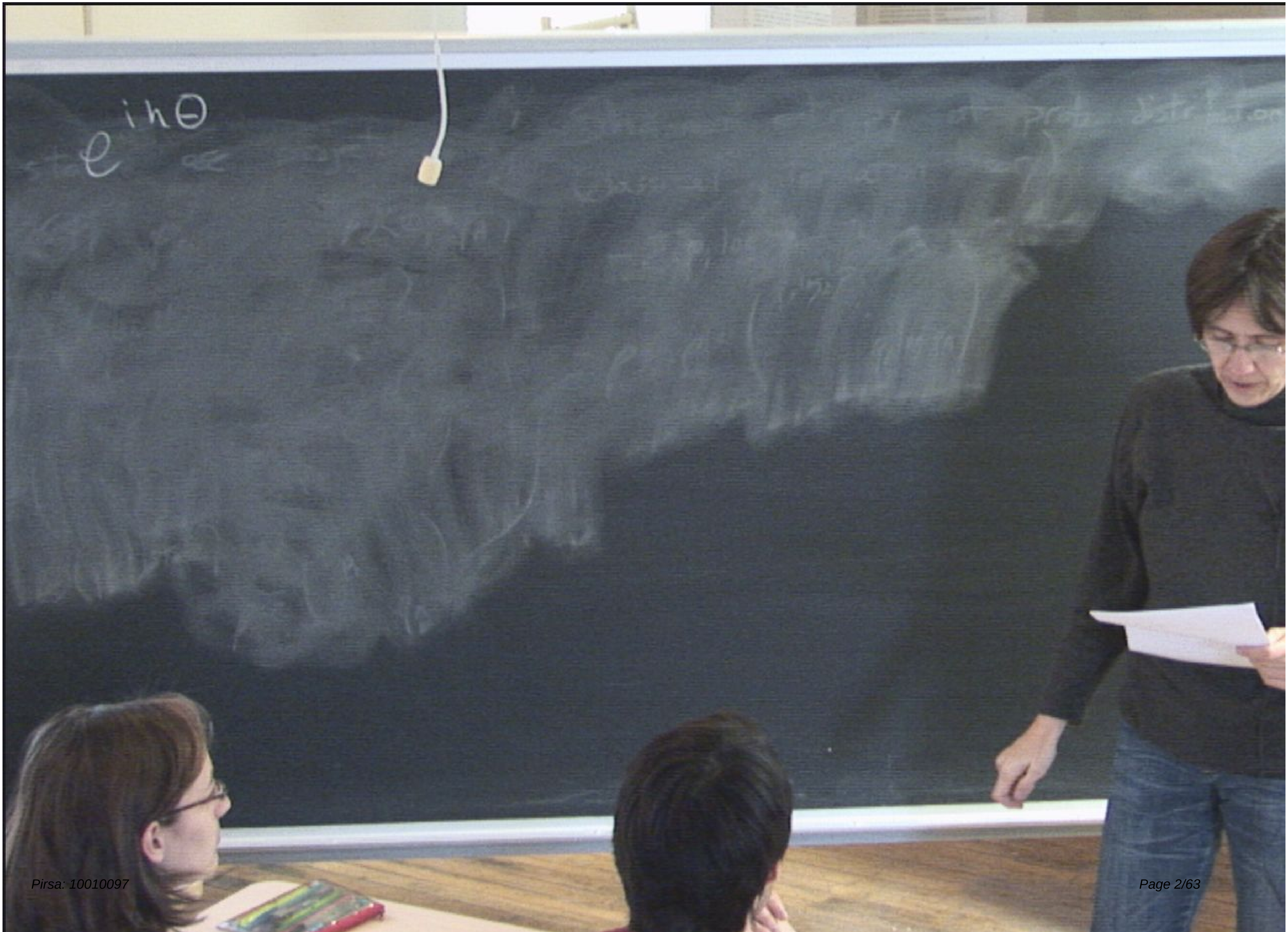
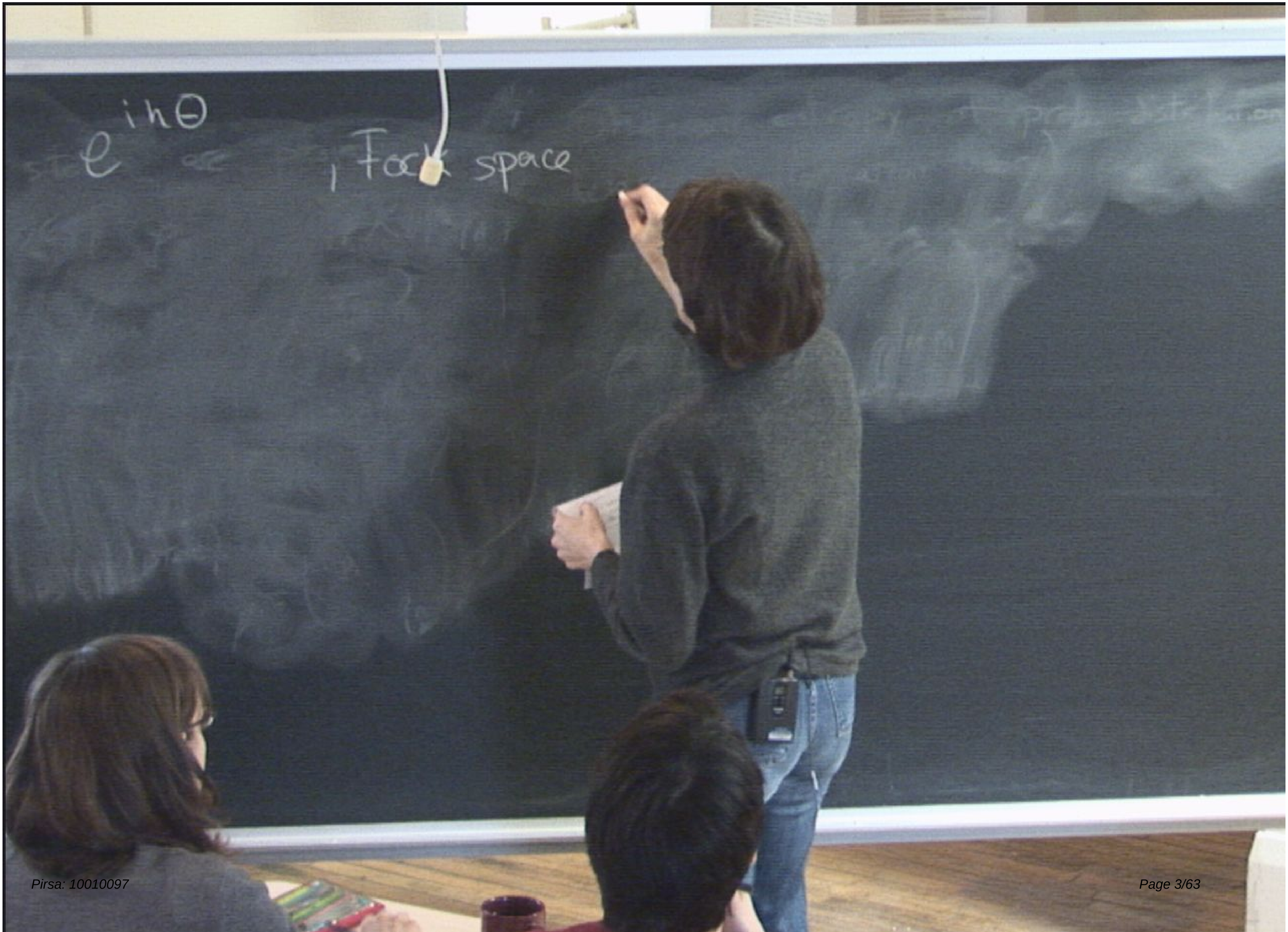
Title: Quantum Gravity - Review (PHYS 638) - Lecture 4

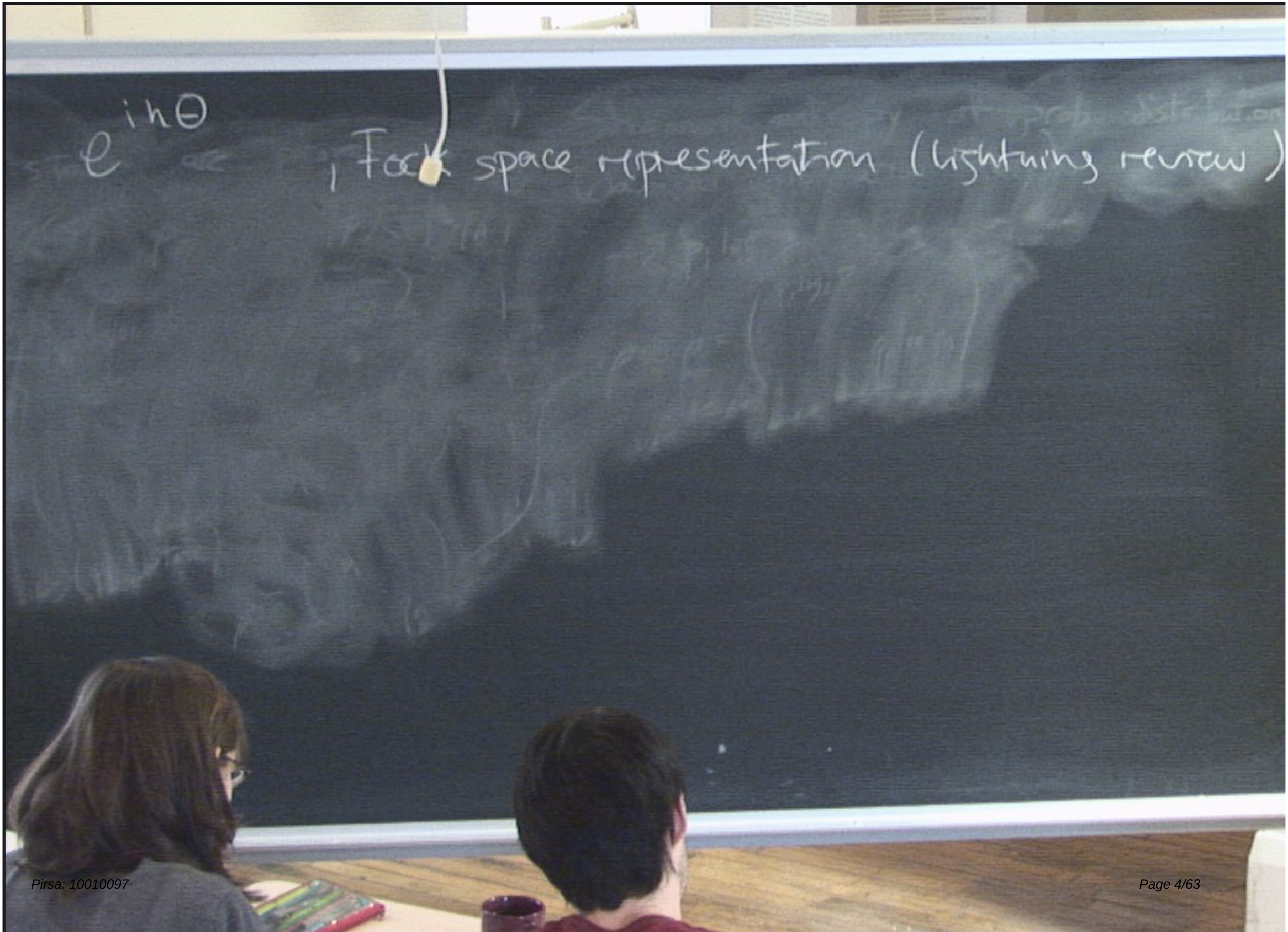
Date: Jan 28, 2010 10:00 AM

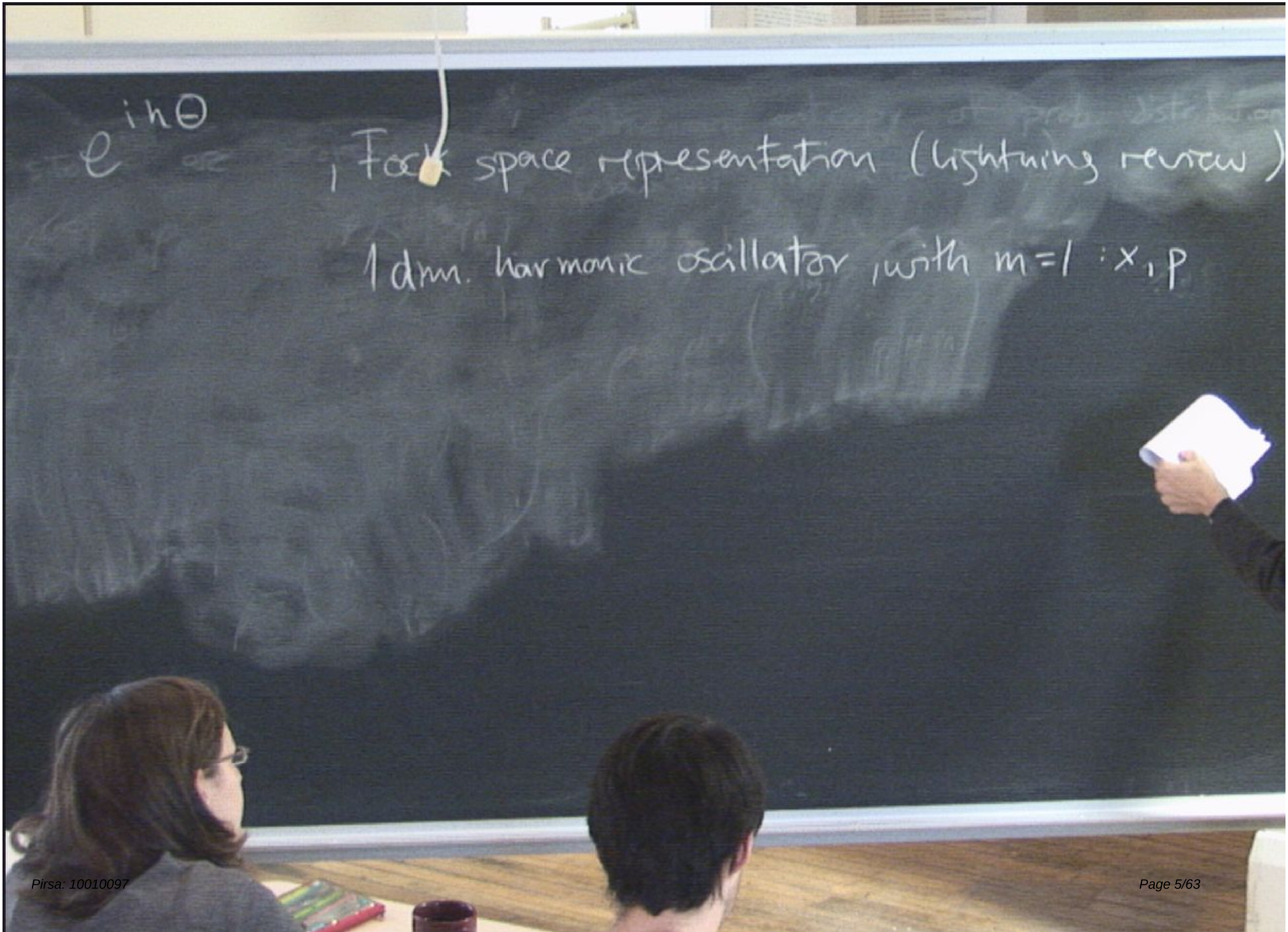
URL: http://pirsa.org/10010097

Abstract:


$e^{in\Theta}$


e inΘ
, Fock space


e^{inΘ}
; Fock space representation (lightning review)

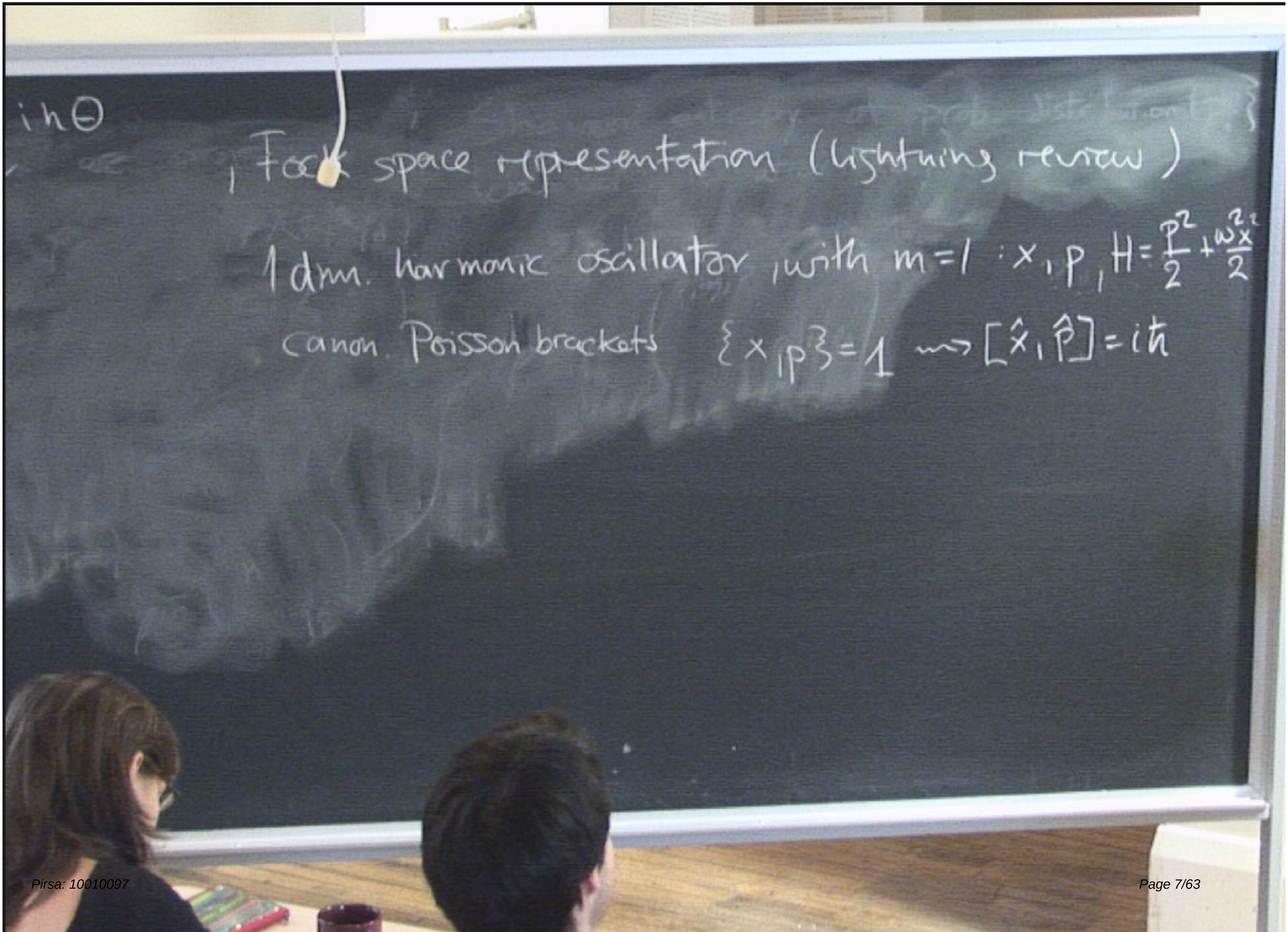

$e^{in\Theta}$
; Fock space representation (lightning review)
1 dim. harmonic oscillator, with m=1 : x, p

$i\hbar\Theta$

Fock space representation (lightning review)

1 dim. harmonic oscillator, with $m=1$: x, p, $H = \frac{p^2}{2} + \frac{\omega^2 x^2}{2}$

canon. Poisson brackets $\{x, p\} = 1$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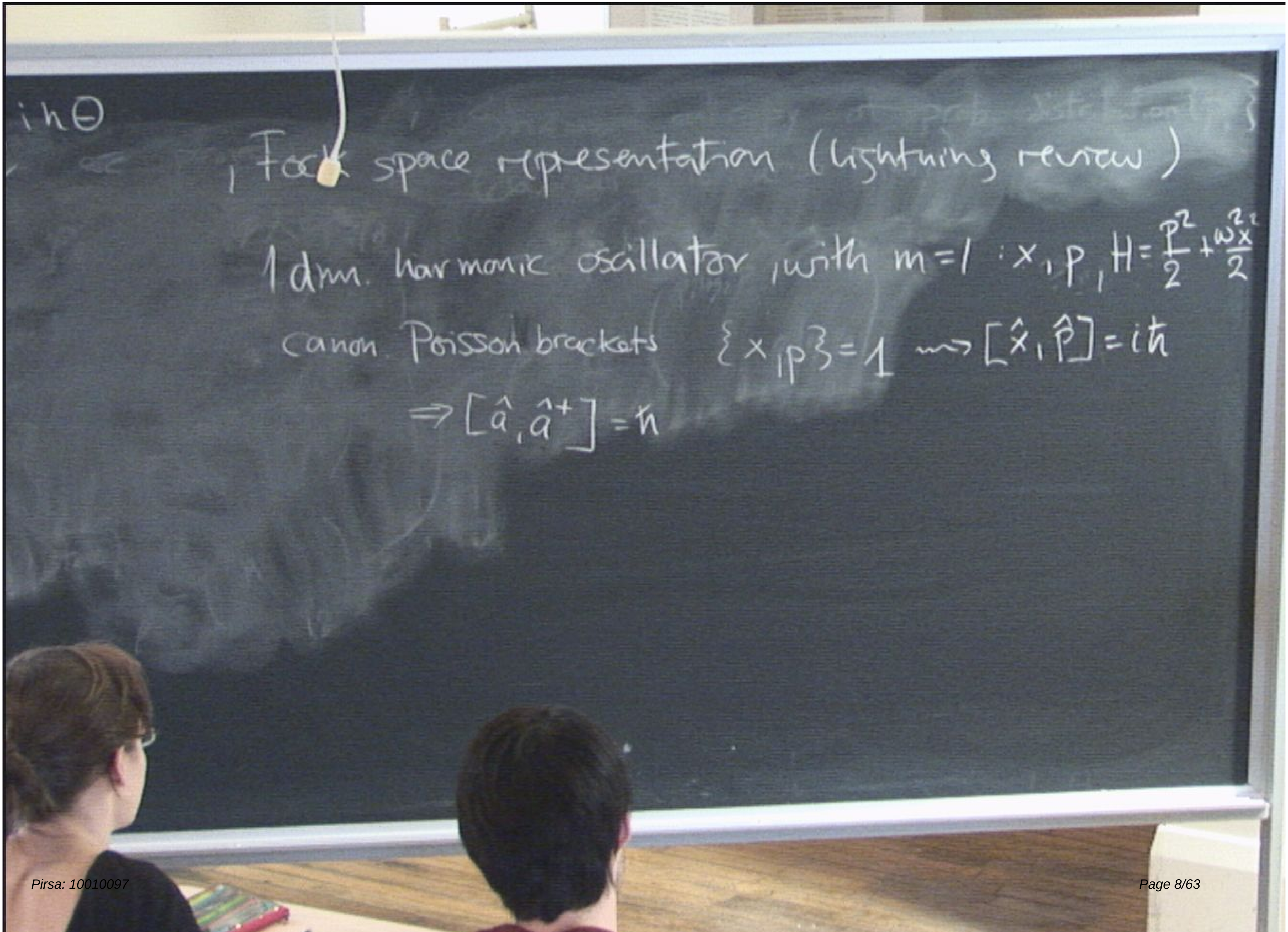
canon. Poisson brackets $\{x,p\}=1 \rightsquigarrow [\hat{x},\hat{p}] = i\hbar$



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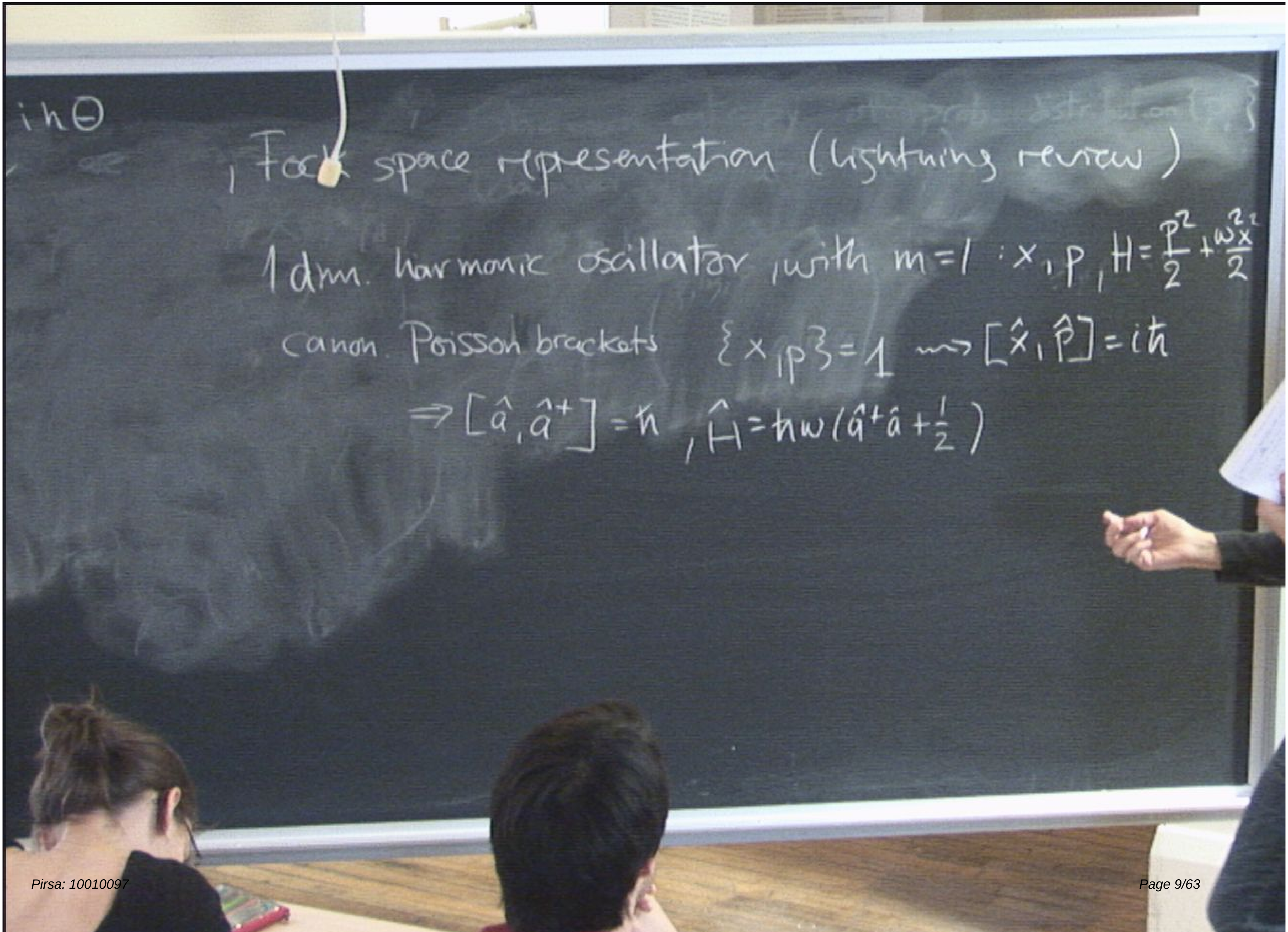
$\Rightarrow [\hat{a},\hat{a}^+] = \hbar$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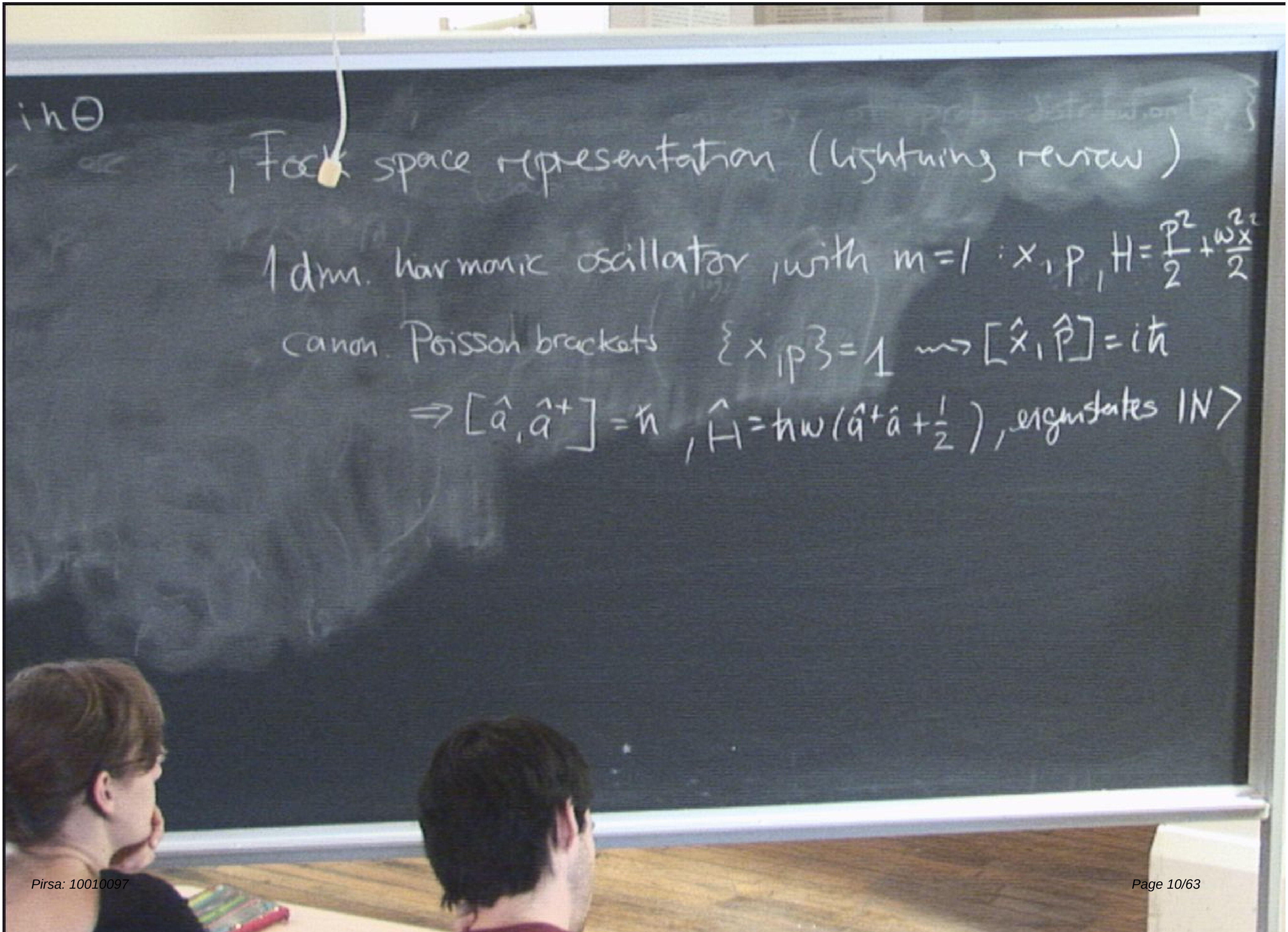

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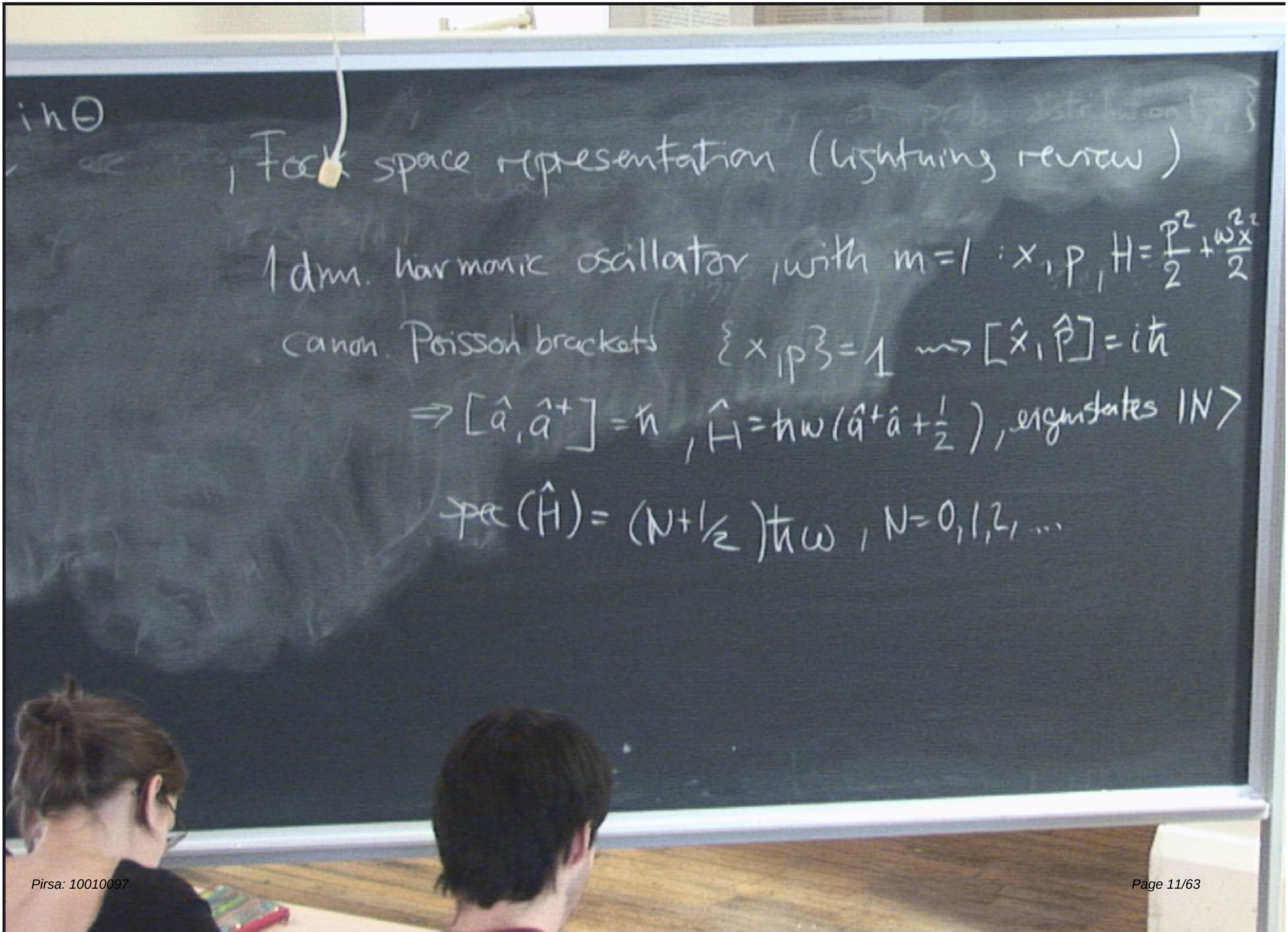
$\Rightarrow [\hat{a},\hat{a}^+]=\hbar$, $\hat{H}=\hbar\omega(\hat{a}^+\hat{a}+\frac{1}{2})$, eigenstates $|N\rangle$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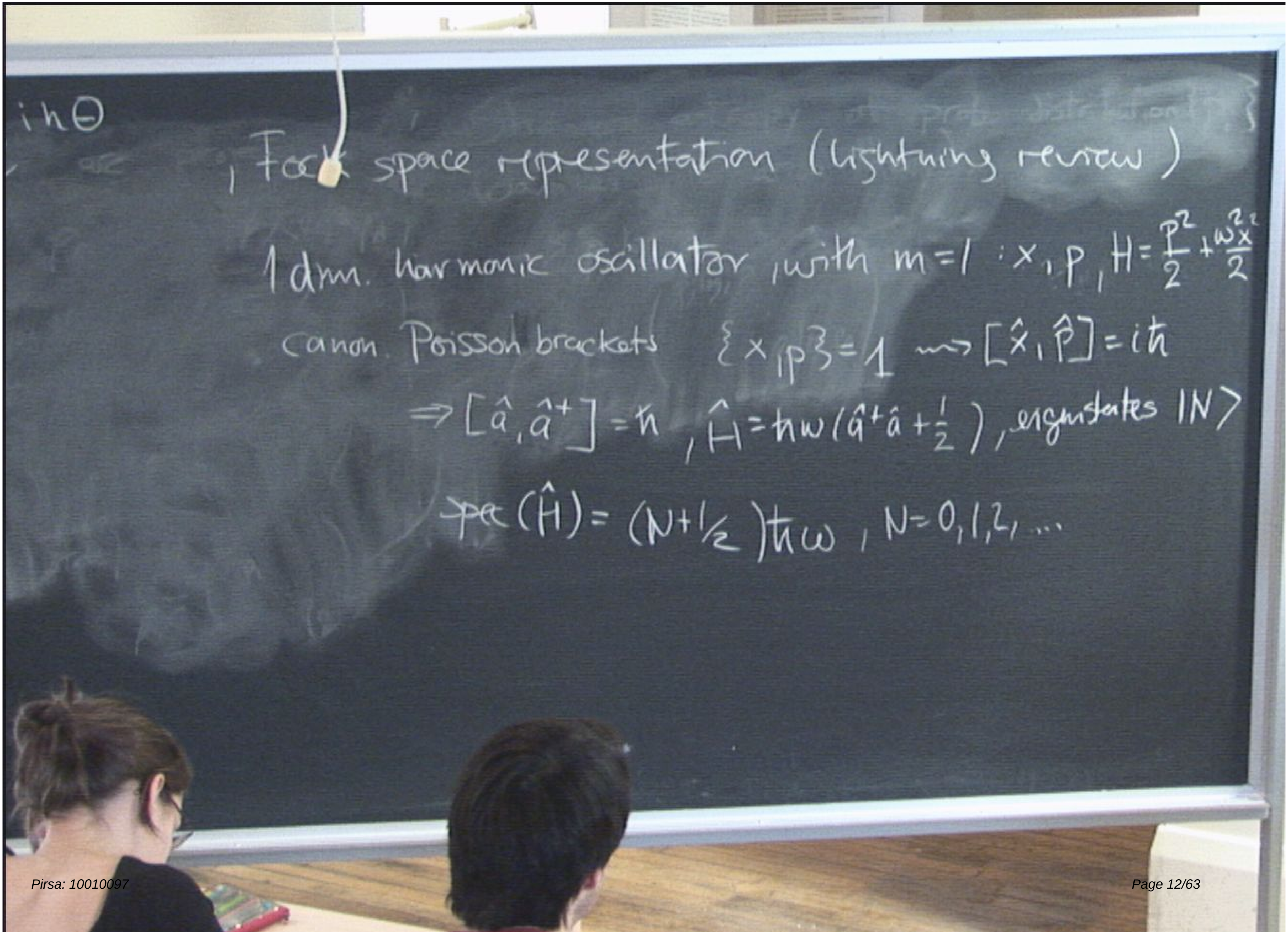
$\Rightarrow [\hat{a}, \hat{a}^+] = \hbar$, $\hat{H} = \hbar\omega(\hat{a}^+\hat{a} + \frac{1}{2})$, eigenstates $|N\rangle$
$\mathrm{spec}(\hat{H}) = (N + 1/2)\hbar\omega$, $N = 0, 1, 2, \ldots$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$\Rightarrow [\hat{a}, \hat{a}^+]=\hbar$, $\hat{H}=\hbar\omega(\hat{a}^+\hat{a}+\frac{1}{2})$, eigenstates $|N\rangle$
$\mathrm{spec}(\hat{H})=(N+\frac{1}{2})\hbar\omega$, $N=0,1,2,\ldots$

$e^{ih\Theta}$

$0\rangle = 0$

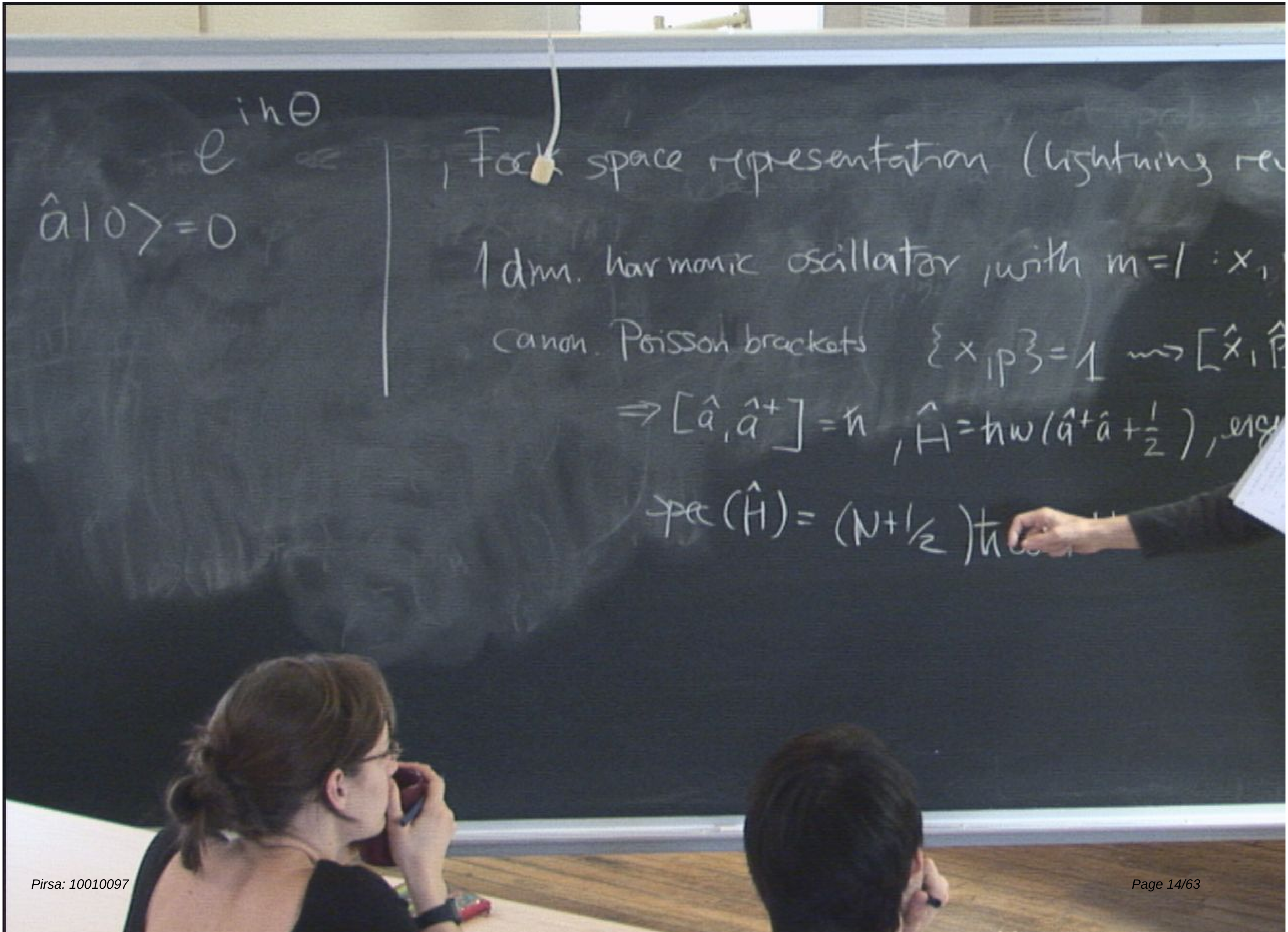
, Fock space representation (lightning review

1 dim. harmonic oscillator, with $m=1$: $x, p, H=$

canon. Poisson brackets $\{x, p\} = 1 \rightsquigarrow [\hat{x}, \hat{p}] = i$

$\Rightarrow [\hat{a}, \hat{a}^+] = \hbar$, $\hat{H} = \hbar\omega(\hat{a}^+\hat{a} + \frac{1}{2})$, eigenstates

spec$(\hat{H}) = (N + 1/2)\hbar\omega$, $N = 0, 1, 2, \dots$, $|0\rangle$



$e^{i\hbar\Theta}$
$\hat{a}|0\rangle = 0$
, Fock space representation (lightning
1 dim. harmonic oscillator, with $m=1$: x,
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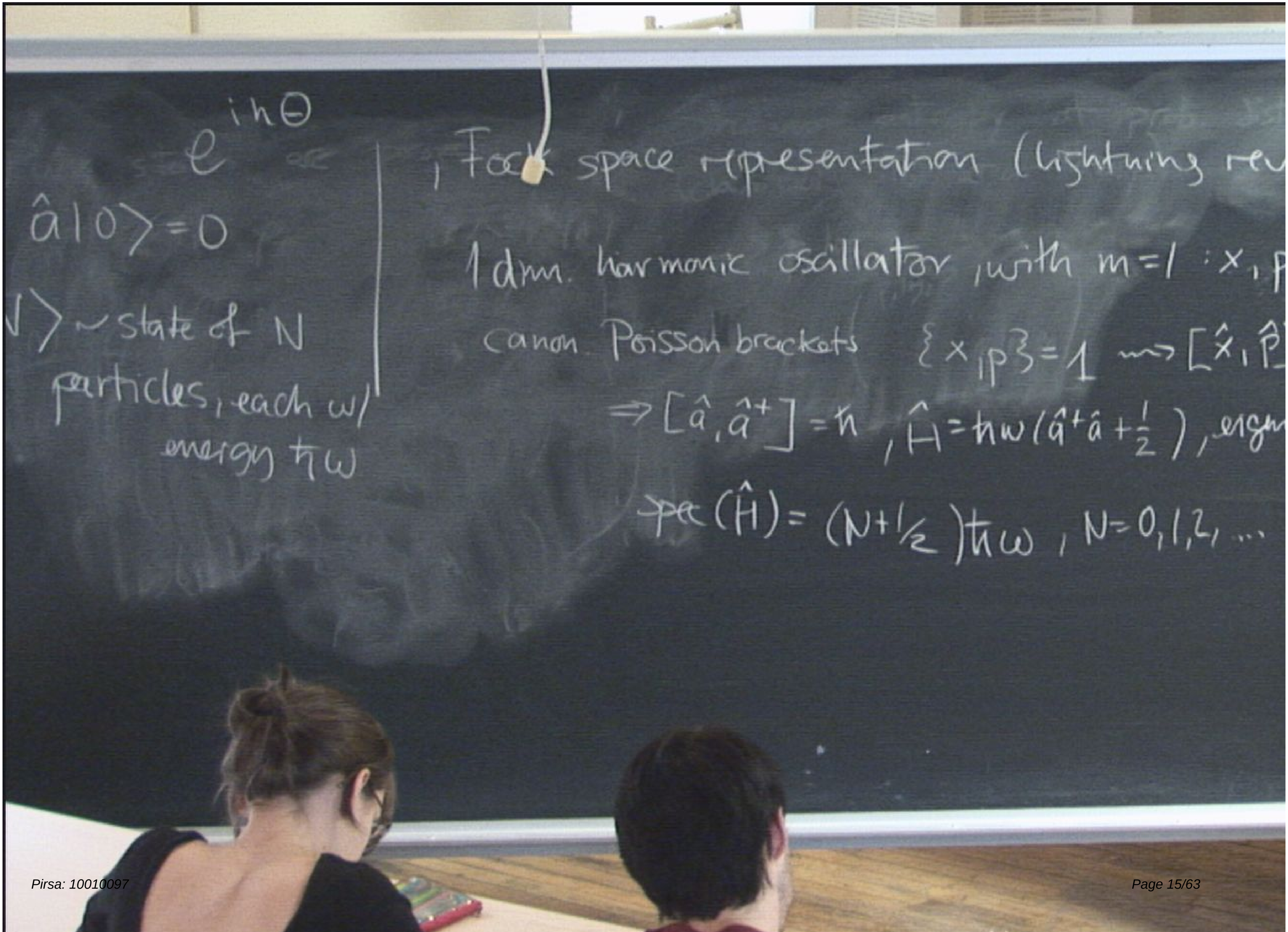
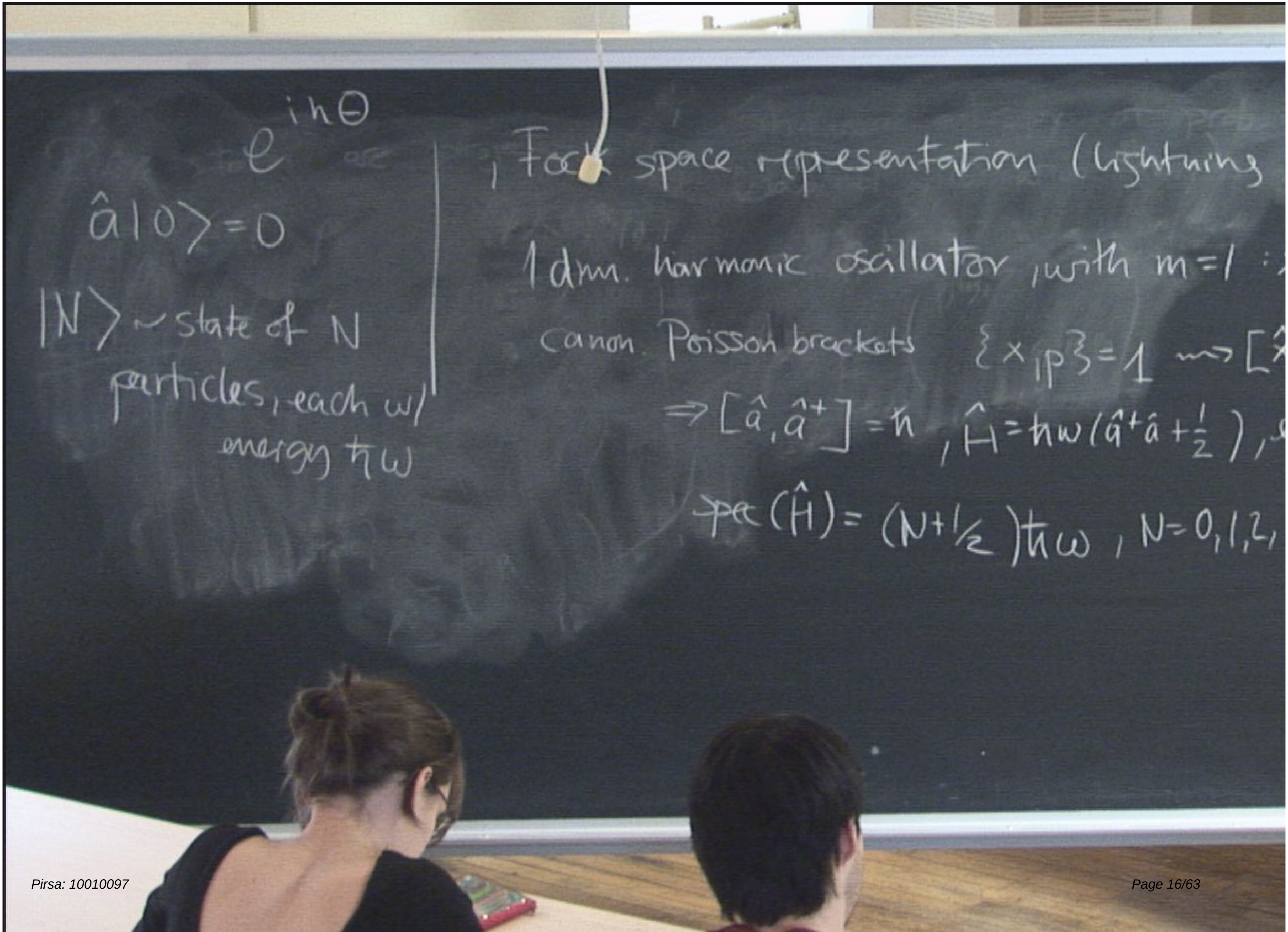

e^{iħΘ}
â|0> = 0
|N> ~ state of N particles, each w/ energy ħω
, Fock space representation (lightning rev
1 dim. harmonic oscillator, with m = 1 : x, p
canon. Poisson brackets {x, p} = 1 ⇝ [x̂, p̂
⇒ [â, â⁺] = ħ , Ĥ = ħω(â⁺â + 1/2), eigen
spec(Ĥ) = (N + 1/2)ħω , N = 0, 1, 2, ...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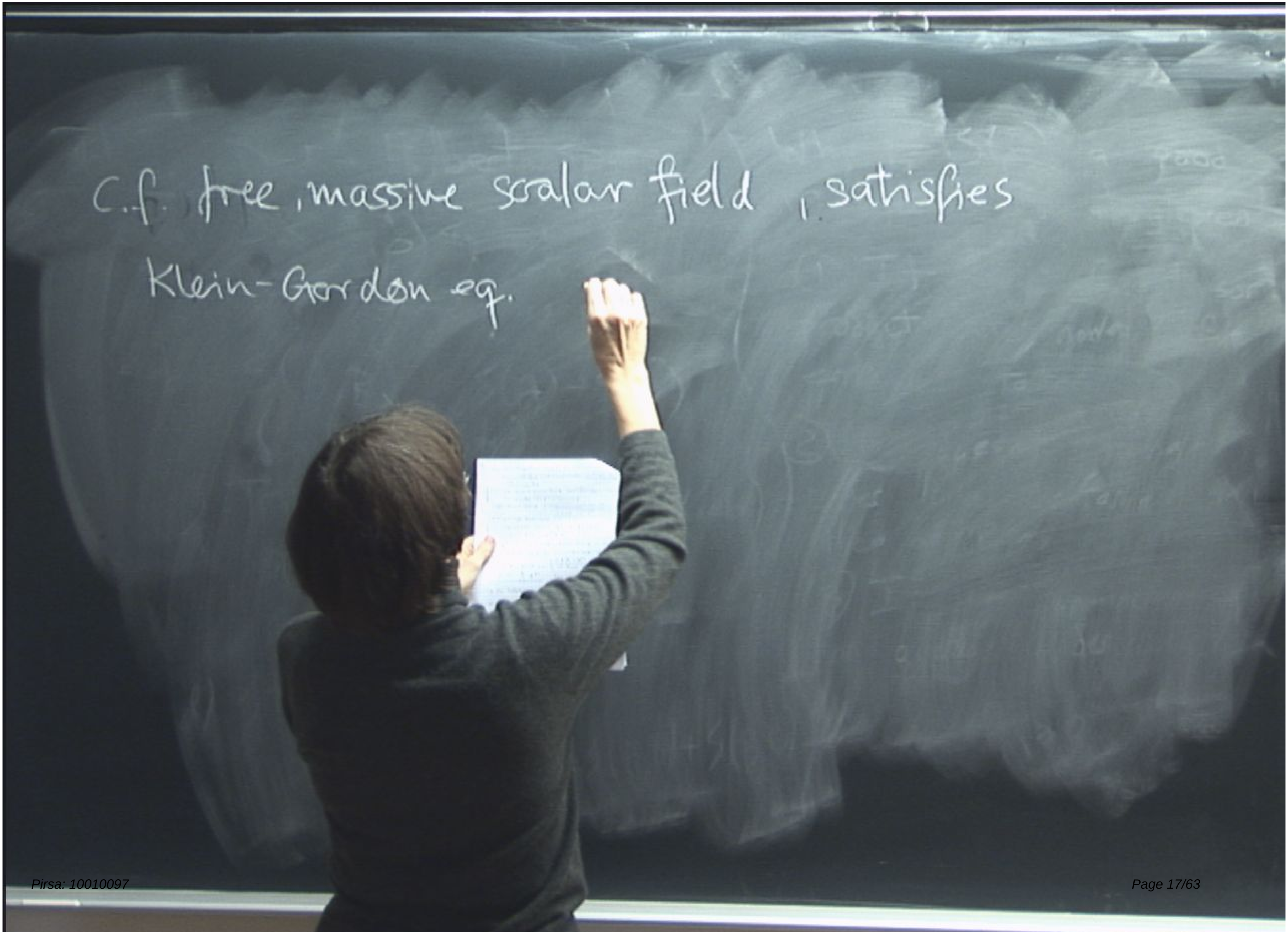
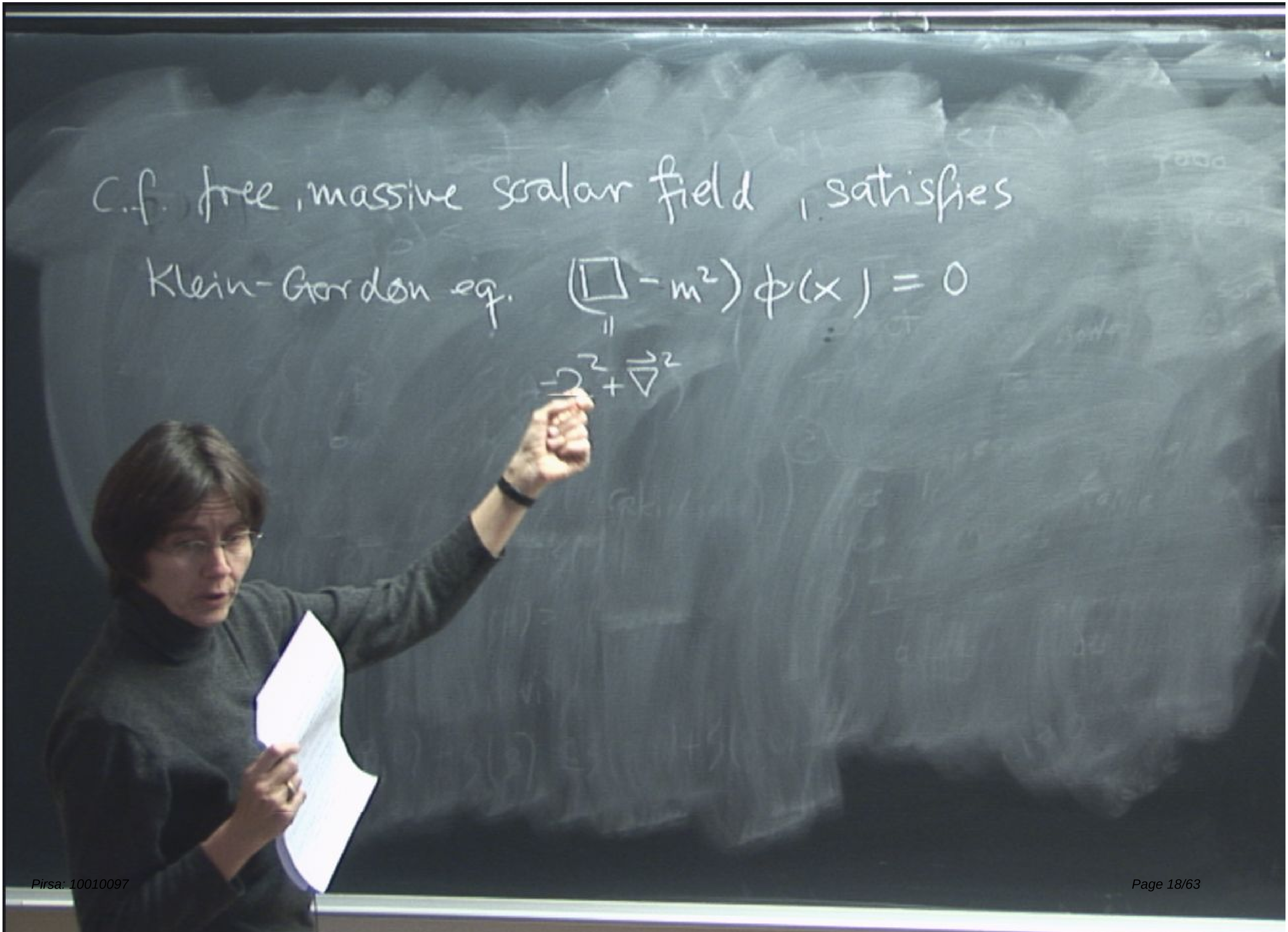
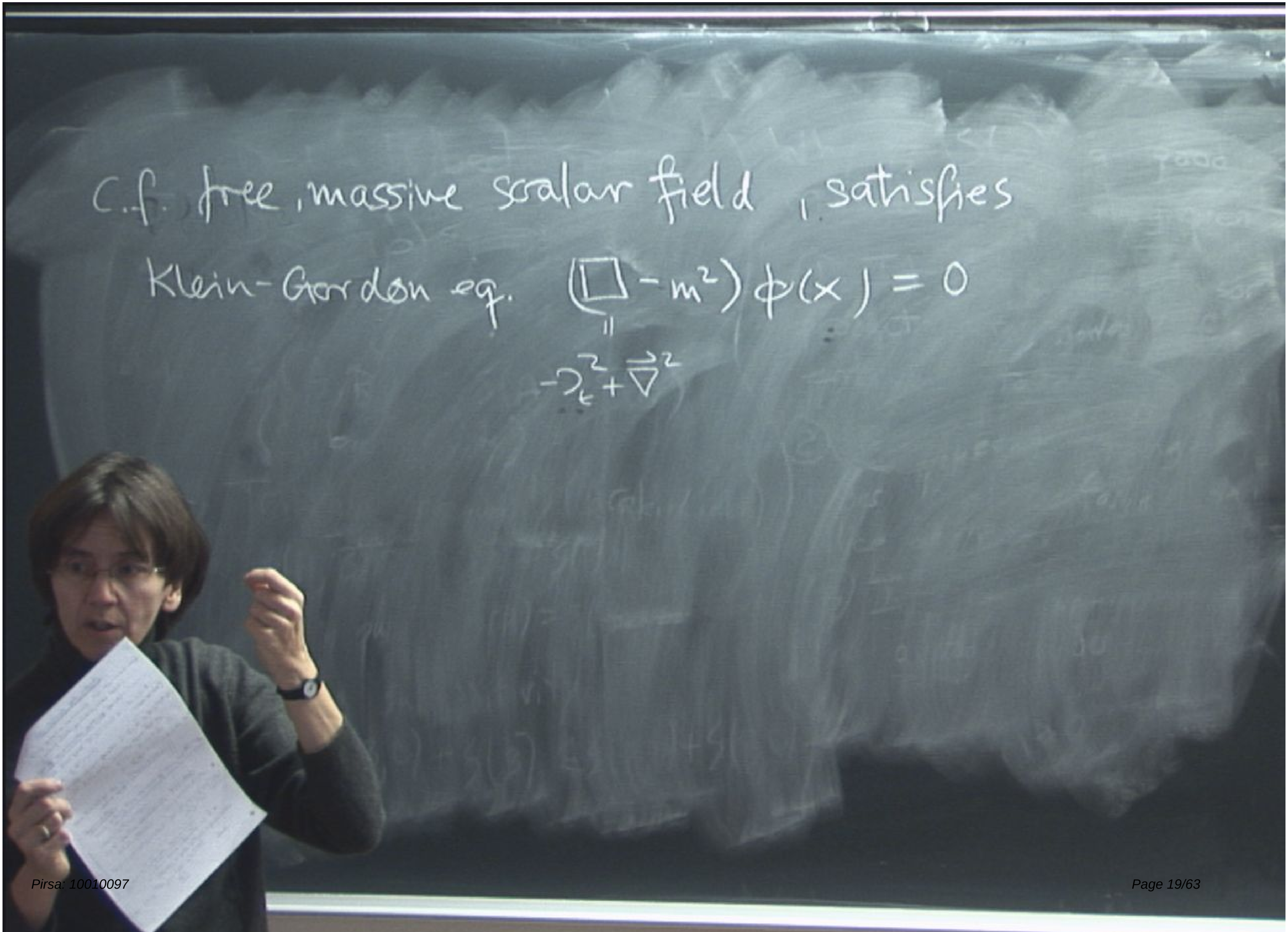

c.f. free, massive scalar field, satisfies
Klein-Gordon eq.


C.f. free, massive scalar field, satisfies
Klein-Gordon eq. $(\Box - m^2)\phi(x) = 0$
$\Box = -\partial^2 + \vec{\nabla}^2$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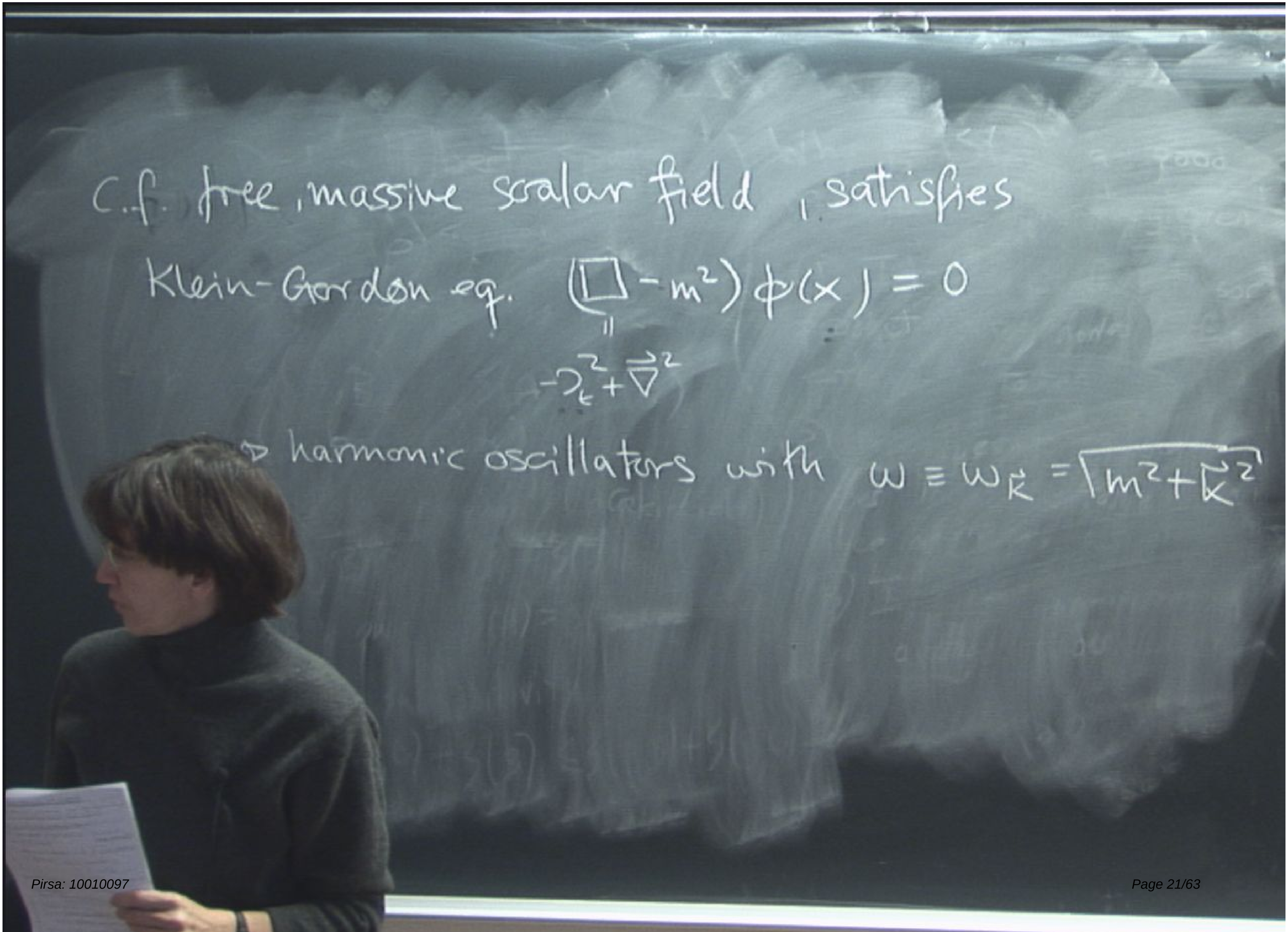
c.f. free, massive scalar field, satisfies

Klein-Gordon eq. $(\underset{\substack{\| \\ -\partial_t^2+\vec{\nabla}^2}}{\Box} - m^2)\,\phi(x) = 0$

$\hookrightarrow$ harmonic oscillators with $\omega \equiv \omega_{\vec{k}} = \sqrt{m^2+\vec{k}^2}$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$\hookrightarrow$ creation

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$\hookrightarrow$ creation / annih. op.s $\hat{a}^+(\vec{k})$, $\hat{a}(\vec{k})$

$$[\hat{a}(\vec{k}), \hat{a}^+(\vec{k}')] = \delta^{(3)}(\vec{k} - \vec{k}') \qquad \text{CCR}$$

$|0\rangle$

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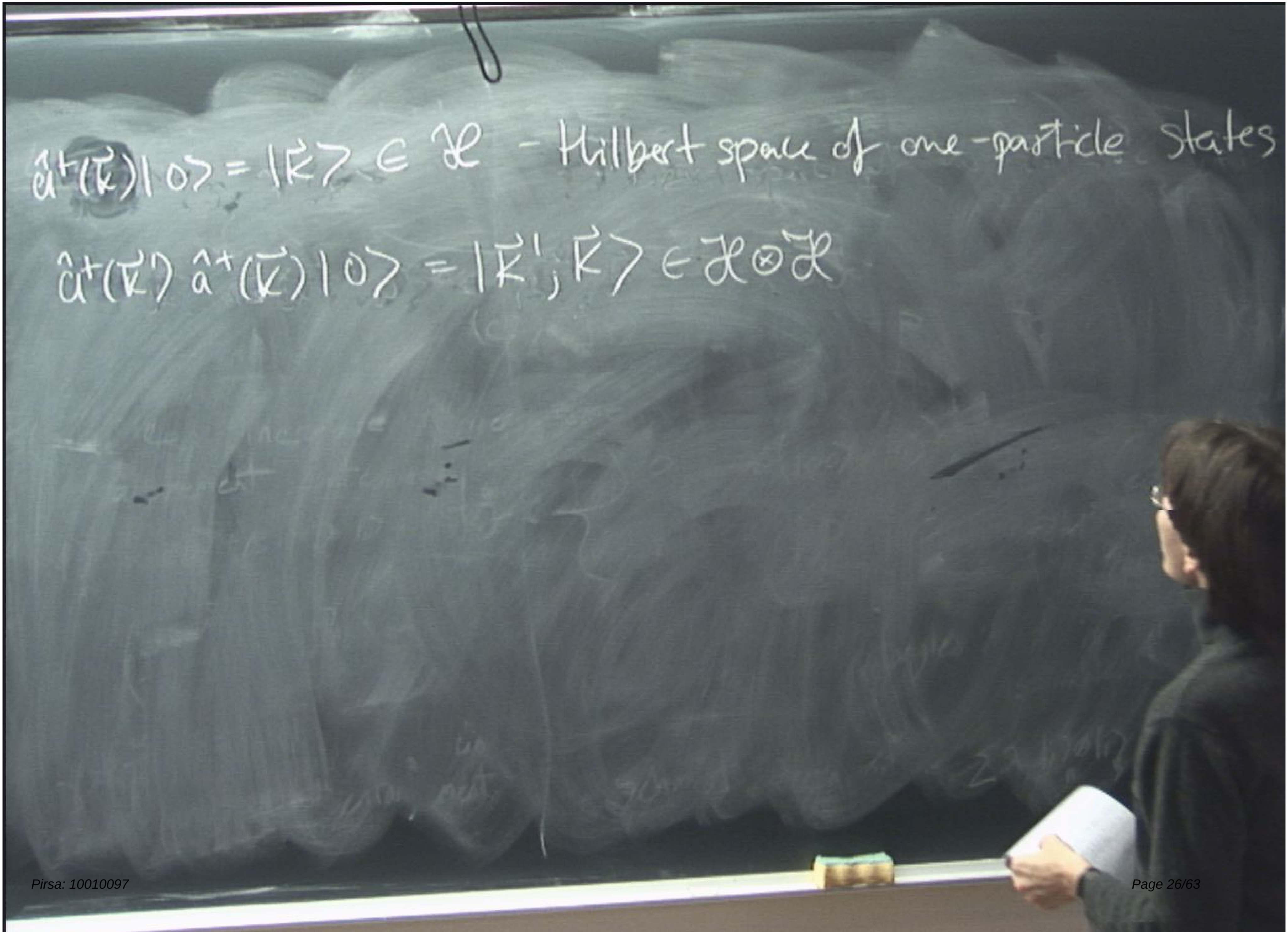
$|0\rangle$ ~ vacuum

$\hat{a}^+(\vec{k})|0\rangle = |\vec{k}\rangle \in \mathcal{H}$ - Hilbert space of one-particle states

$\hat{a}^+(\vec{k}')\,\hat{a}^+(\vec{k})|0\rangle$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$\mathcal{F}(\mathcal{H})$

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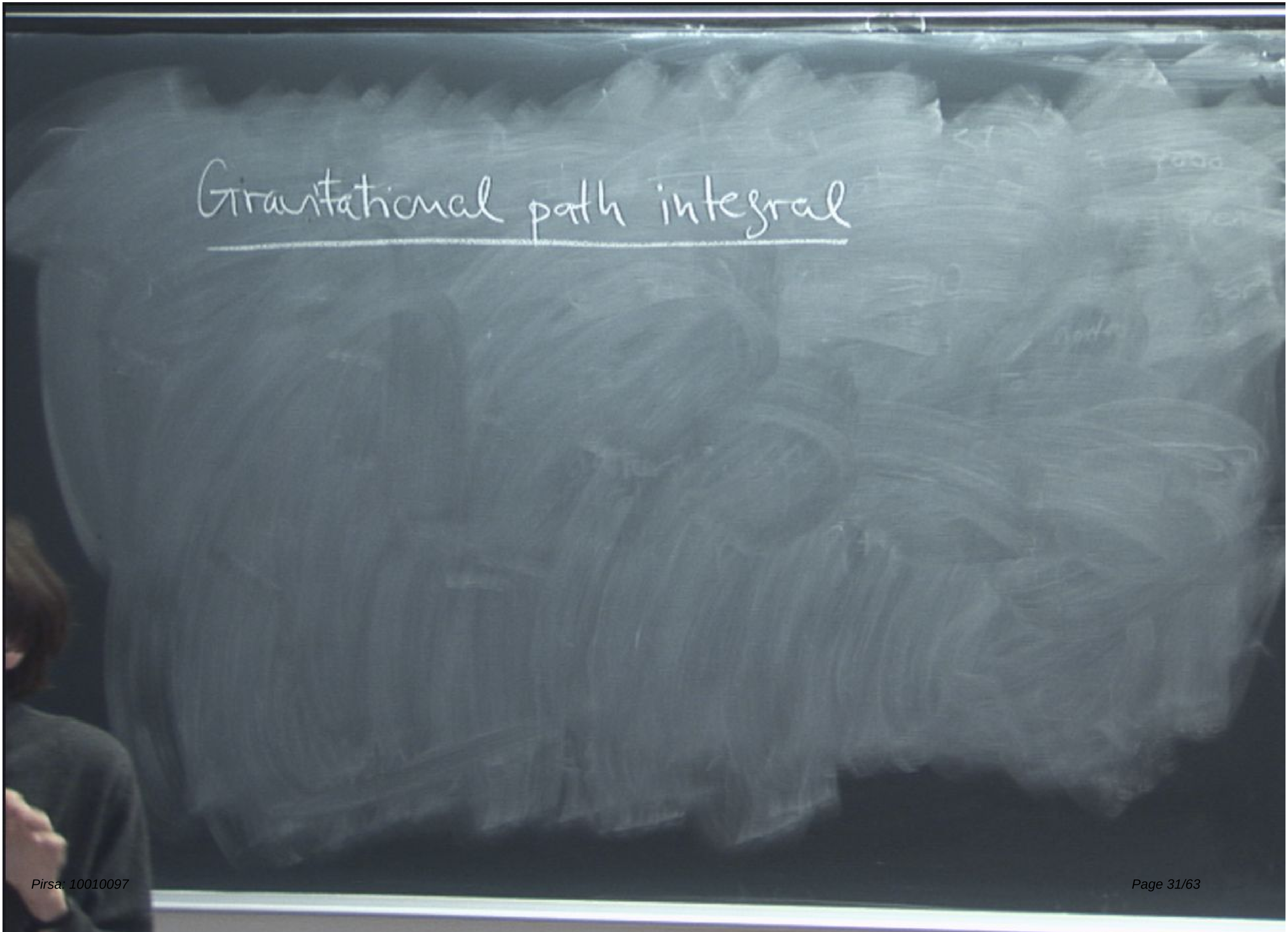
$\mathcal{F}(\mathcal{H}) = \bigoplus_{n=0}^{\infty} \mathcal{H}^{\otimes n}$ Fock space


Gravitational path integral

Gravitational path integral

QM of nonrelativistic particle (3d)

class. phase space (x_i, p_i), $i = 1,2,3$, $\{x_i, p_j\} = \delta_{ij}$

Gravitational path integral

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$$[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$$

Gravitational path integral

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$[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$, unique

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$[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$, unique representation $\hat{x}_i = x_i$, $\hat{p}_i = -i\hbar \frac{\partial}{\partial x_i}$

Gravitational path integral

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$$H=\frac{m}{2}\dot{\vec{x}}^2+V(\vec{x}) \xrightarrow{\wedge} \hat{x}_i, \hat{p}_i, \hat{H} \text{ on } \mathcal{H}=L^2(\mathbb{R}^3, d\vec{x})$$

$[\hat{x}_i, \hat{p}_j]=i\hbar\delta_{ij}$, unique representation $\hat{x}_i=x_i$, $\hat{p}_i=-i\hbar\frac{\partial}{\partial x_i}$

$$\hat{H}=-\frac{\hbar^2}{2m}\Delta$$

Gravitational path integral

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Gravitational path integral

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$[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$, unique representation $\hat{x}_i = x_i$, $\hat{p}_i = -i\hbar\frac{\partial}{\partial x_i}$

$\hat{H} = -\frac{\hbar^2}{2m}\Delta + V(\hat{\vec{x}})$, must solve $\hat{H}\psi(x) = E\psi(x)$

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$$\hat{H} = -\frac{\hbar^2}{2m}\Delta + V(\hat{\vec{x}})$$, must solve $\hat{H}\Psi(x) = E\Psi(x)$

equivalently, define unitary time evolution op.

$$U(t,t_0) = e^{-i\hat{H}(t-t_0)/\hbar} \, , \quad |\psi(t)\rangle = U(t,t_0)\,|\psi(t_0)\rangle$$

$d\vec{x})$

$i\hbar \frac{\partial}{\partial x_i}$

<u>equivalently</u>, define unitary time evolution op.

$$U(t,t_0) = e^{-i\hat{H}(t-t_0)/\hbar}, \quad |\psi(t)\rangle = U(t,t_0)\,|\psi(t_0)\rangle$$

w.r.t. position eigenstates $|\vec{x}\rangle$: $\hat{\vec{x}}\,|\vec{x}\rangle = \vec{x}\,|\vec{x}\rangle$

$d\vec{x})$

$i\hbar\frac{\partial}{\partial x}$

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$$\psi(\vec{x},t) = \langle\vec{x}|\psi(t)\rangle$$

$d\vec{x})$

$i\hbar\frac{\partial}{\partial x_i}$

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$$\psi(\vec{x},t) = \langle\vec{x}|\psi(t)\rangle$$

$$\psi(\vec{x}',t) = \langle\vec{x}'|U(t,t_0)|\psi(t_0)\rangle$$

$d\vec{x})$

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$$\psi(\vec{x}',t) = \langle\vec{x}'|U(t,t_0)|\psi(t_0)\rangle = \int d\vec{x}''$$

$d\vec{x})$

$i\hbar\frac{\partial}{\partial x_i}$

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w.r.t. position eigenstates $|\vec{x}\rangle$: $\hat{\vec{x}}|\vec{x}\rangle = \vec{x}|\vec{x}\rangle$

$$\psi(\vec{x},t) = \langle\vec{x}|\psi(t)\rangle$$

$$\psi(\vec{x}',t) = \langle\vec{x}'|U(t,t_0)|\psi(t_0)\rangle = \int d\vec{x}''\langle\vec{x}'|U(t,t_0)|\vec{x}''\rangle\langle\vec{x}''|\psi(t_0)\rangle$$

∂x_i

equivalently, define unitary time evolution op.

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w.r.t. position eigenstates $|\vec{x}\rangle$: $\hat{\vec{x}}\,|\vec{x}\rangle = \vec{x}\,|\vec{x}\rangle$

$$\psi(\vec{x},t) = \langle \vec{x} | \psi(t)\rangle$$

$$\psi(\vec{x}',t) = \langle \vec{x}' | U(t,t_0) | \psi(t_0)\rangle = \int d\vec{x}'' \langle \vec{x}' | U(t,t_0) | \vec{x}''\rangle \langle \vec{x}'' | \psi(t_0)\rangle$$

)

∂x_i

equivalently, define unitary time evolution op:

$$U(t,t_0) = e^{-i\hat{H}(t-t_0)/\hbar}, \quad |\psi(t)\rangle = U(t,t_0)\,|\psi(t_0)\rangle$$

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Feynman kernel, propagator

∂x_i

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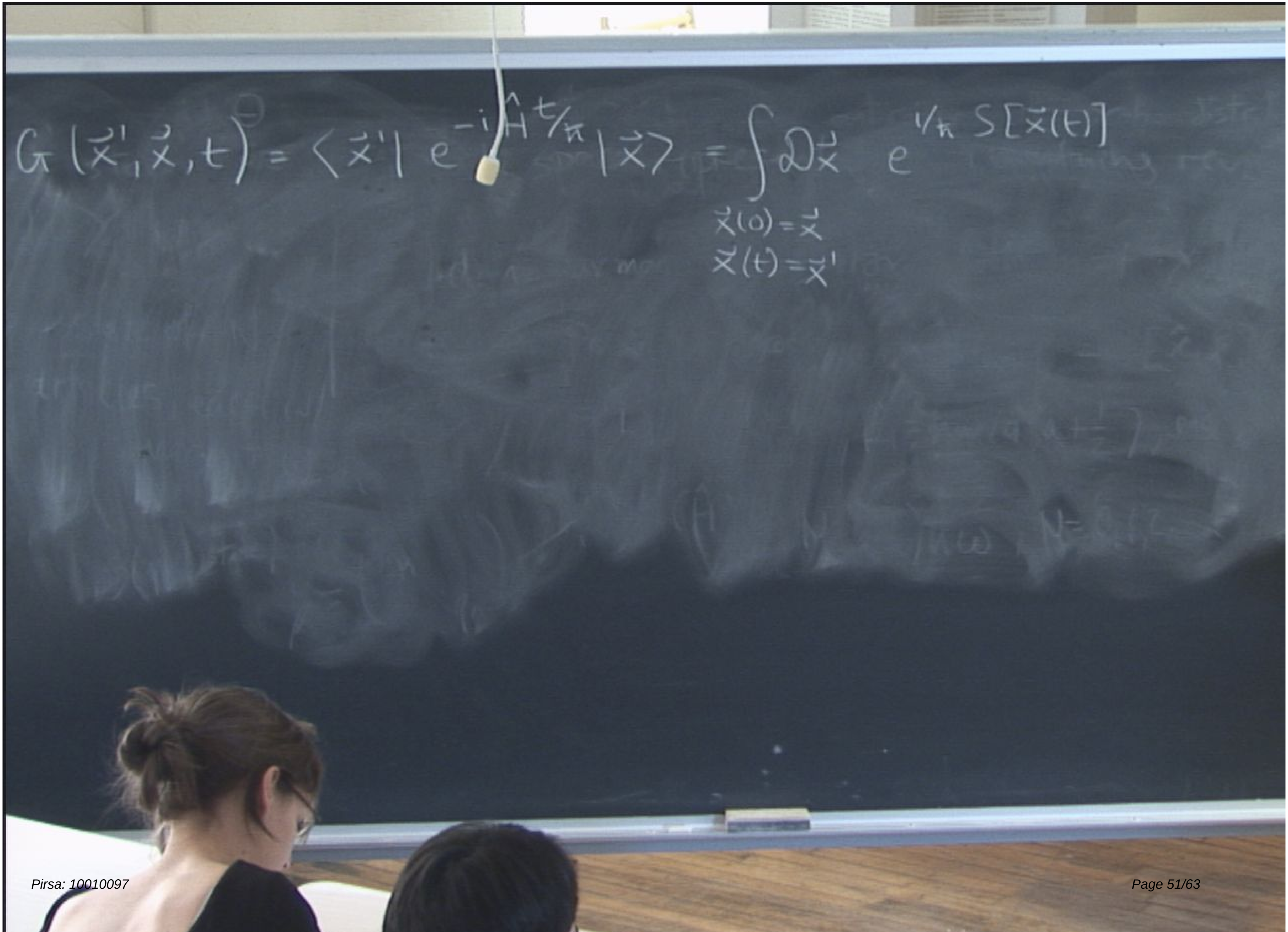
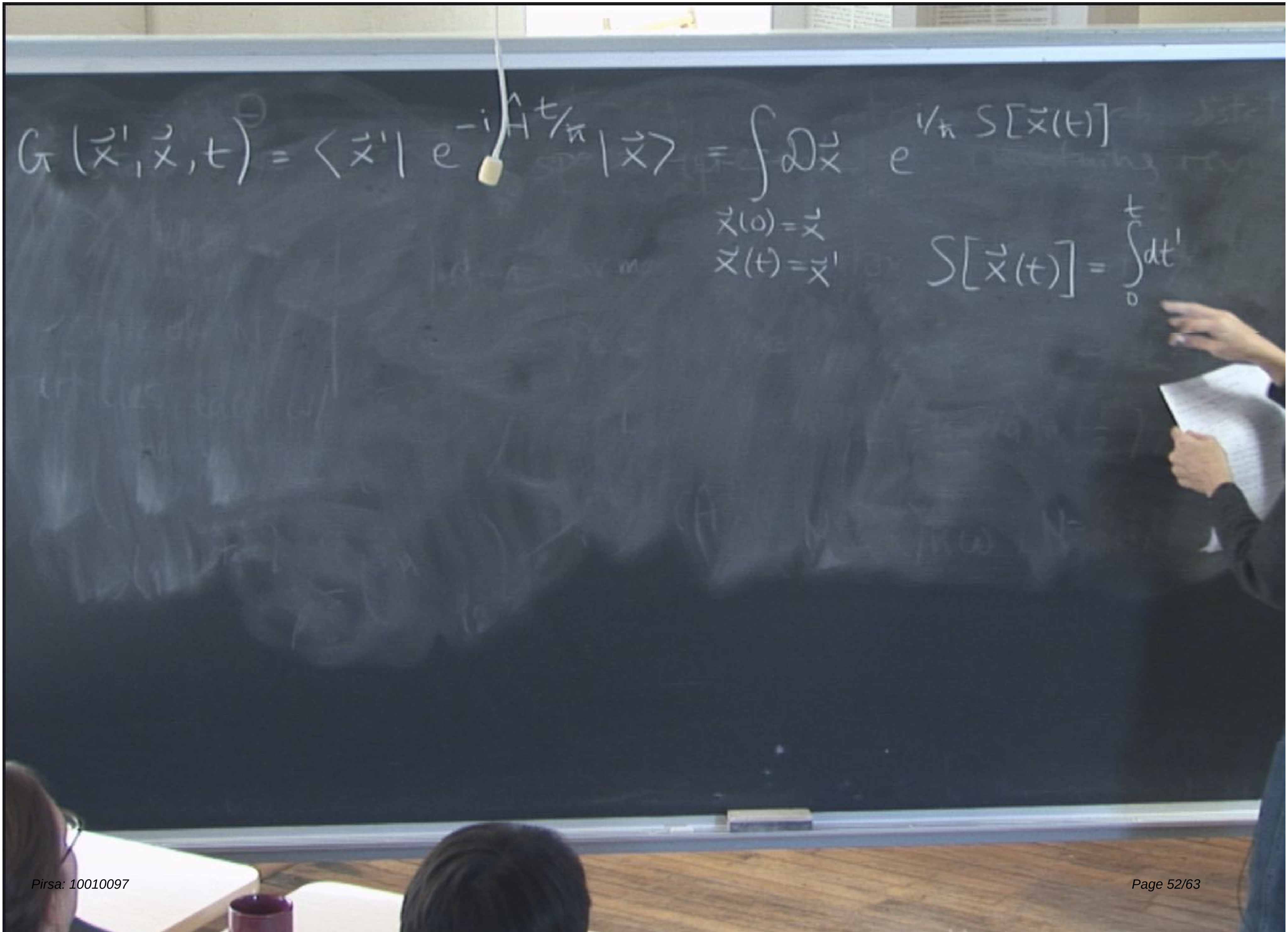
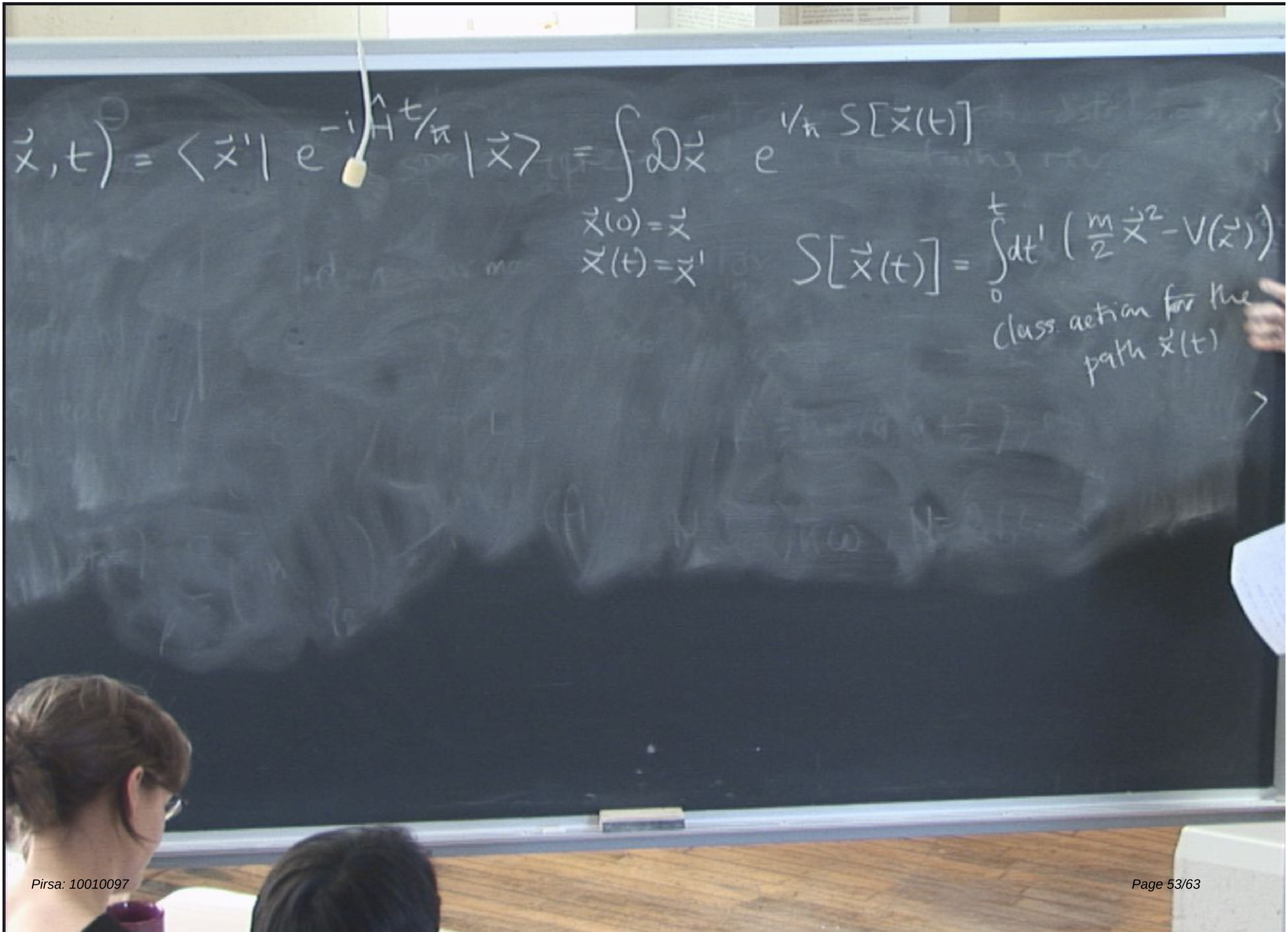
Feynman Kernel, propagator

∂x_i



$G(\vec{x}', \vec{x}, t) = \langle \vec{x}' | e^{-i\hat{H}t/\hbar} | \vec{x} \rangle = \int \mathcal{D}\vec{x} \; e^{i/\hbar \, S[\vec{x}(t)]}$
$\vec{x}(0) = \vec{x}$
$\vec{x}(t) = \vec{x}'$


$G(\vec{x}', \vec{x}, t) = \langle \vec{x}' | e^{-i\hat{H}t/\hbar} | \vec{x} \rangle = \int \mathcal{D}\vec{x} \; e^{i/\hbar \, S[\vec{x}(t)]}$
$\vec{x}(0) = \vec{x}$
$\vec{x}(t) = \vec{x}'$
$S[\vec{x}(t)] = \int_0^t dt'$



$(\vec{x}', t) = \langle \vec{x}' | e^{-i\hat{H}t/\hbar} | \vec{x} \rangle = \int_{\substack{\vec{x}(0)=\vec{x} \\ \vec{x}(t)=\vec{x}'}} \mathcal{D}\vec{x}\; e^{i/\hbar\, S[\vec{x}(t)]}$
$S[\vec{x}(t)] = \int_0^t dt' \left(\frac{m}{2}\dot{\vec{x}}^2 - V(\vec{x}) \right)$
class. action for the path $\vec{x}(t)$

$$\vec{x}, t) = \langle \vec{x}' | e^{-i\hat{H}t/\hbar} | \vec{x} \rangle = \int_{\substack{\vec{x}(0)=\vec{x} \\ \vec{x}(t)=\vec{x}'}} \mathcal{D}\vec{x} \; e^{i/\hbar\, S[\vec{x}(t)]}$$

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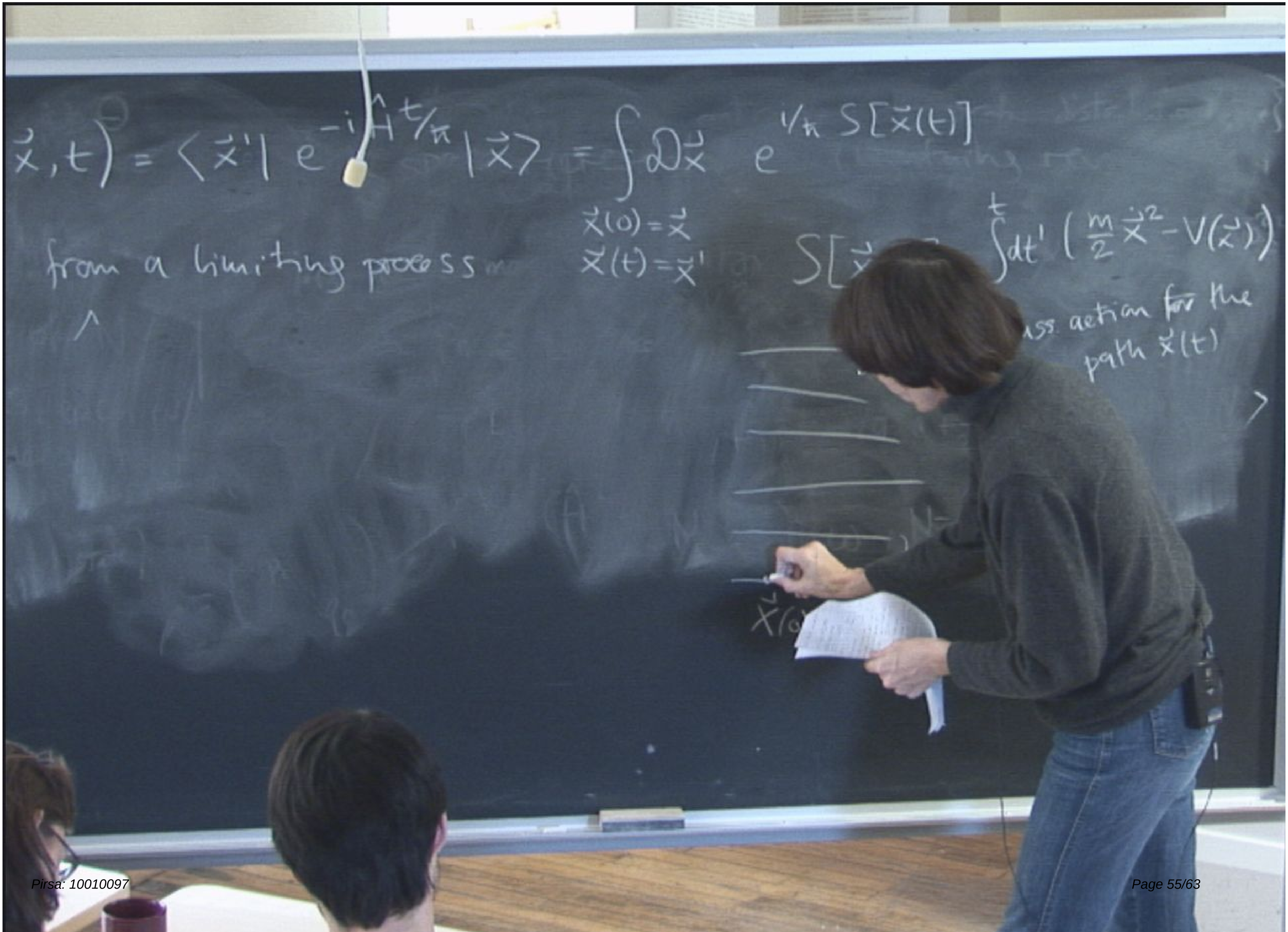
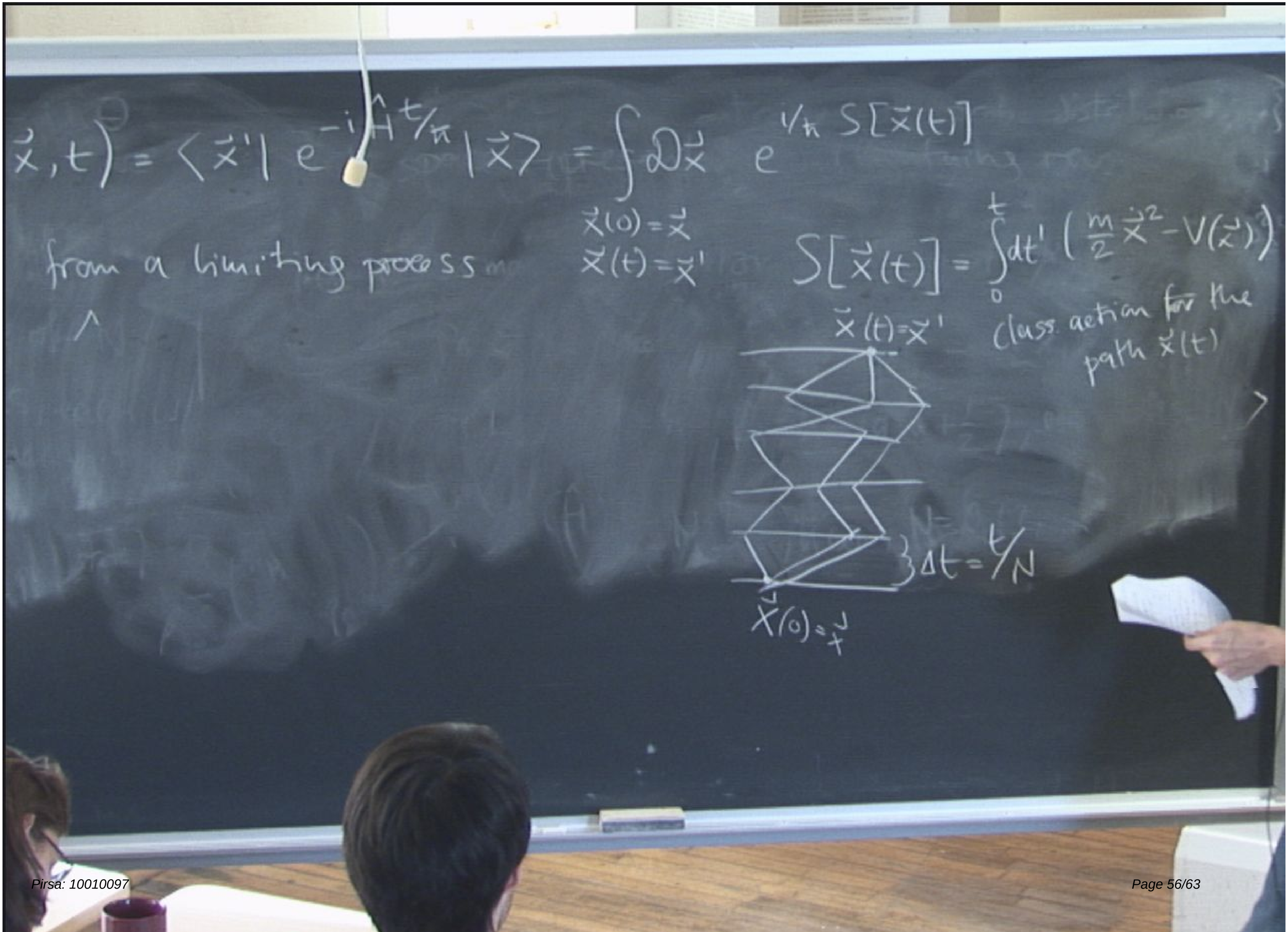
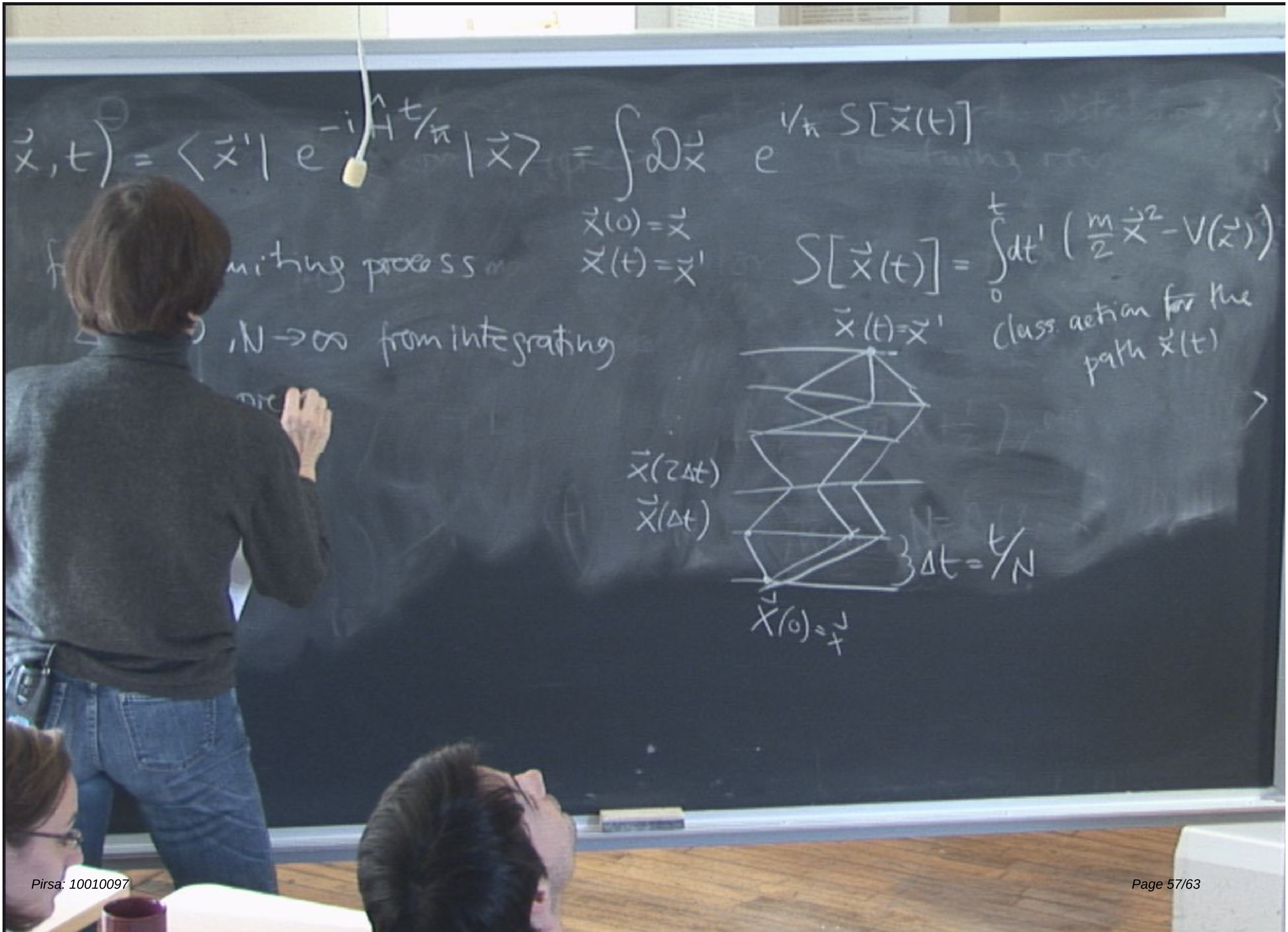
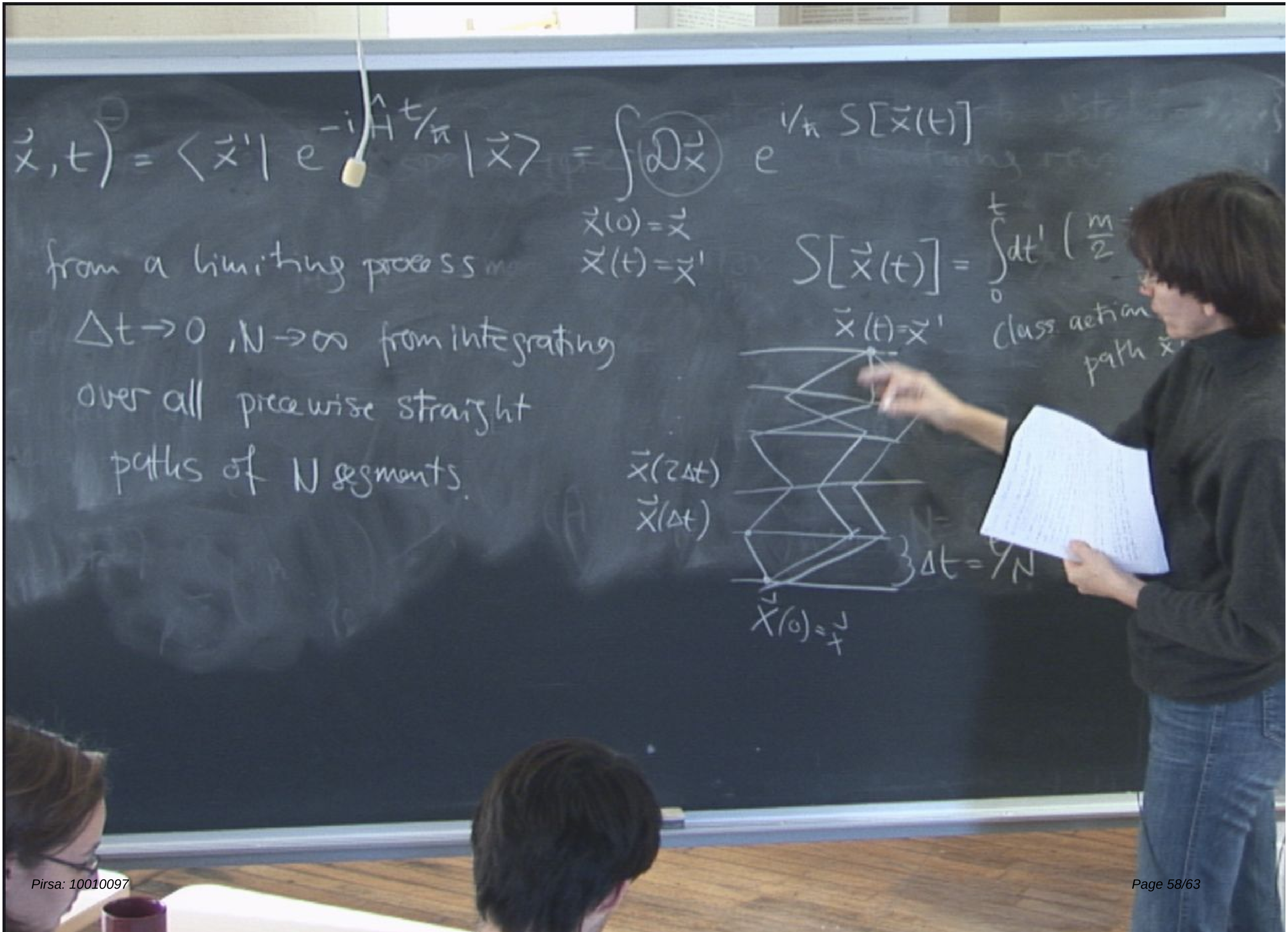
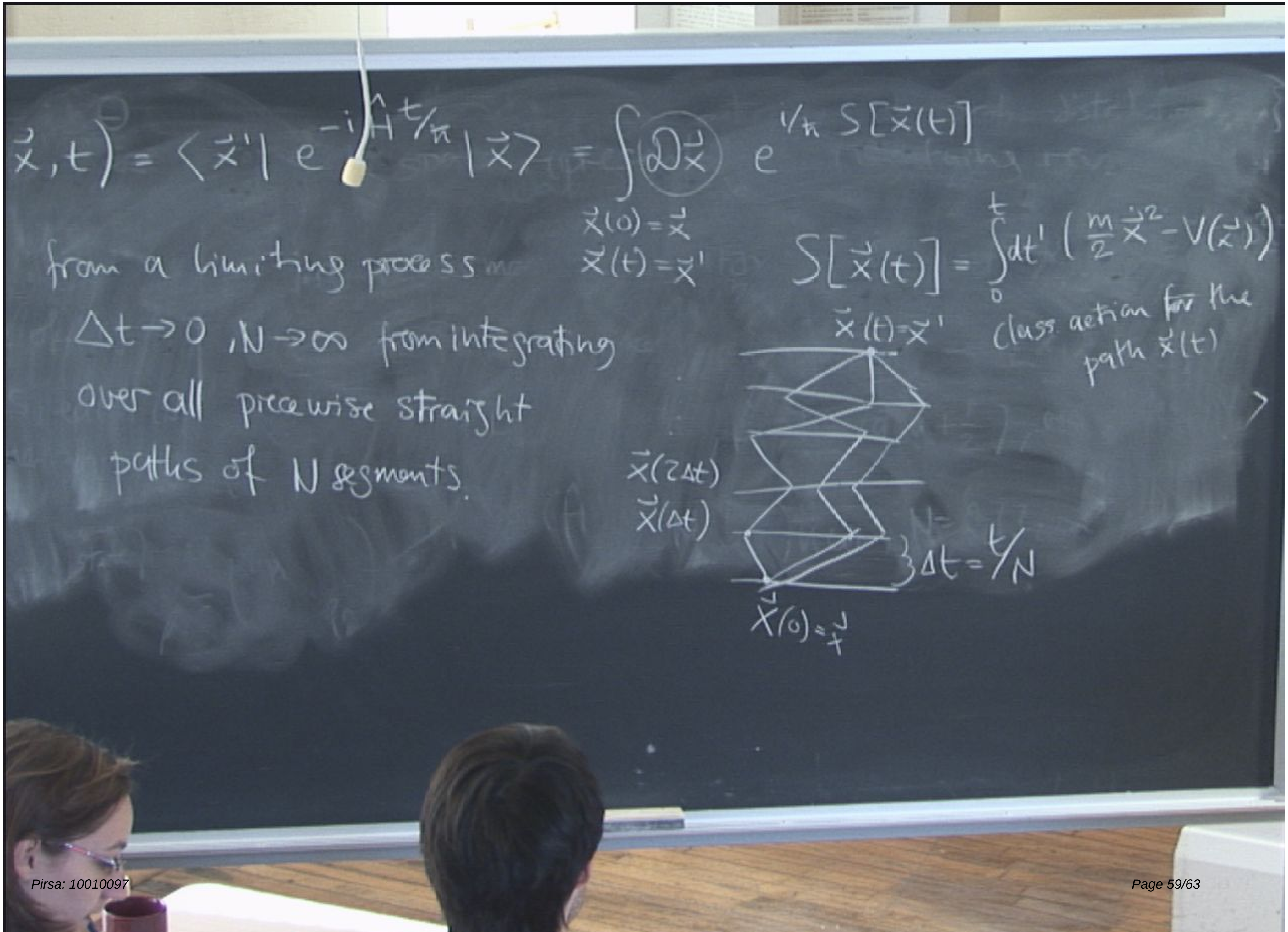
class. action ~~for~~ the path $\vec{x}(t)$


$\vec{x},t) = \langle \vec{x}' | e^{-i\hat{H}t/\hbar} | \vec{x} \rangle = \int \mathcal{D}\vec{x} \; e^{i/\hbar \, S[\vec{x}(t)]}$
$\vec{x}(0) = \vec{x}$
$\vec{x}(t) = \vec{x}'$
from a limiting process
$\int^{t} dt' \left(\frac{m}{2} \dot{\vec{x}}^2 - V(\vec{x}) \right)$
ass. action for the path $\vec{x}(t)$


$\vec{x}, t) = \langle \vec{x}' | e^{-i\hat{H}t/\hbar} | \vec{x} \rangle = \int \mathcal{D}\vec{x} \; e^{i/\hbar \, S[\vec{x}(t)]}$
$\vec{x}(0) = \vec{x}$
$\vec{x}(t) = \vec{x}'$
from a limiting process
$S[\vec{x}(t)] = \int_0^t dt' \left(\frac{m}{2} \dot{\vec{x}}^2 - V(\vec{x}) \right)$
class. action for the path $\vec{x}(t)$
$\vec{x}(t) = \vec{x}'$
$\Delta t = t/N$
$\vec{x}(0) = \vec{x}$


$\vec{x},t) = \langle \vec{x}'| e^{-i\hat{H}t/\hbar} |\vec{x}\rangle = \int \mathcal{D}\vec{x}\; e^{i/\hbar\, S[\vec{x}(t)]}$
$\vec{x}(0)=\vec{x}$
$\vec{x}(t)=\vec{x}'$
$S[\vec{x}(t)] = \int_0^t dt' \left(\frac{m}{2}\dot{\vec{x}}^2 - V(\vec{x})\right)$
class. action for the path $\vec{x}(t)$
miting process
, $N \to \infty$ from integrating
$\vec{x}(t)=\vec{x}'$
$\vec{x}(2\Delta t)$
$\vec{x}(\Delta t)$
$\Delta t = t/N$
$\vec{x}(0)=\vec{x}$


from a limiting process
Δt → 0 , N → ∞ from integrating
over all piecewise straight
paths of N segments
x(0) = x
x(t) = x'
S[x(t)] = ∫ dt' (m/2
class. action
path x
x(t) = x'
x(2Δt)
x(Δt)
Δt = t/N
x(0) = x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$\vec{x}(0)=\vec{x}$
$\vec{x}(t)=\vec{x}'$
from a limiting process
$S[\vec{x}(t)] = \int_0^t dt' \left(\frac{m}{2}\dot{\vec{x}}^2 - V(\vec{x})\right)$
$\Delta t \to 0$, $N \to \infty$ from integrating
over all piecewise straight
paths of N segments.
class. action for the path $\vec{x}(t)$
$\vec{x}(t)=\vec{x}'$
$\vec{x}(2\Delta t)$
$\vec{x}(\Delta t)$
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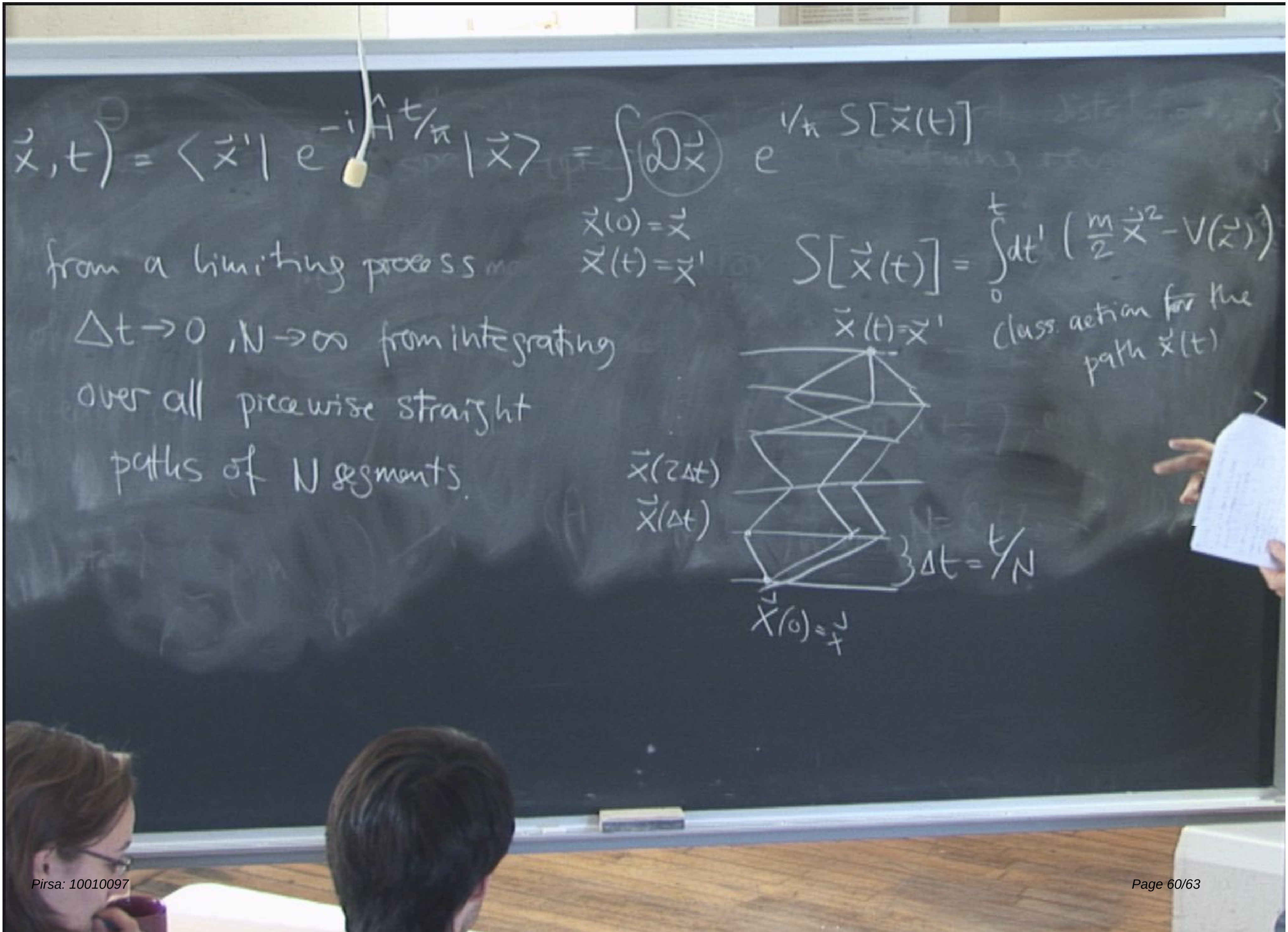
$\vec{x}, t) = \langle \vec{x}' | e^{-i\hat{H}t/\hbar} | \vec{x} \rangle = \int \mathcal{D}\vec{x} \; e^{i/\hbar \, S[\vec{x}(t)]}$
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$\vec{x}(2\Delta t)$
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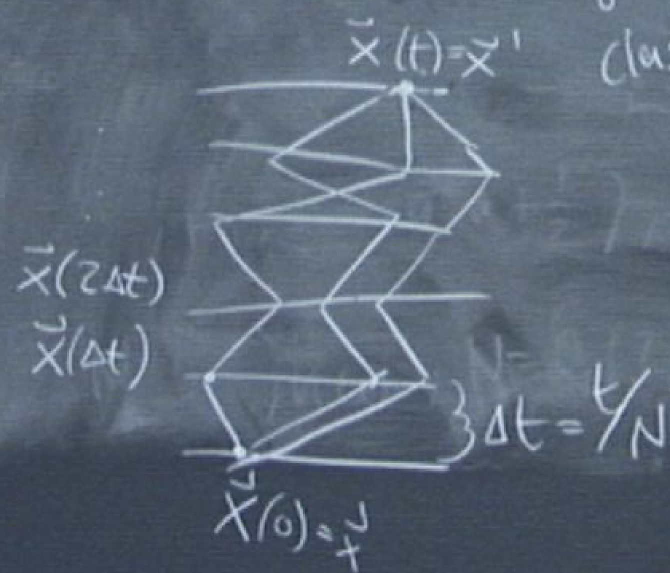
$$\ldots,\vec{x},t) = \langle \vec{x}'|\, e^{-i\hat{H}t/\hbar}\,|\vec{x}\rangle = \int_{\substack{\vec{x}(0)=\vec{x}\\ \vec{x}(t)=\vec{x}'}} \mathcal{D}\vec{x}\; e^{\frac{i}{\hbar} S[\vec{x}(t)]}$$

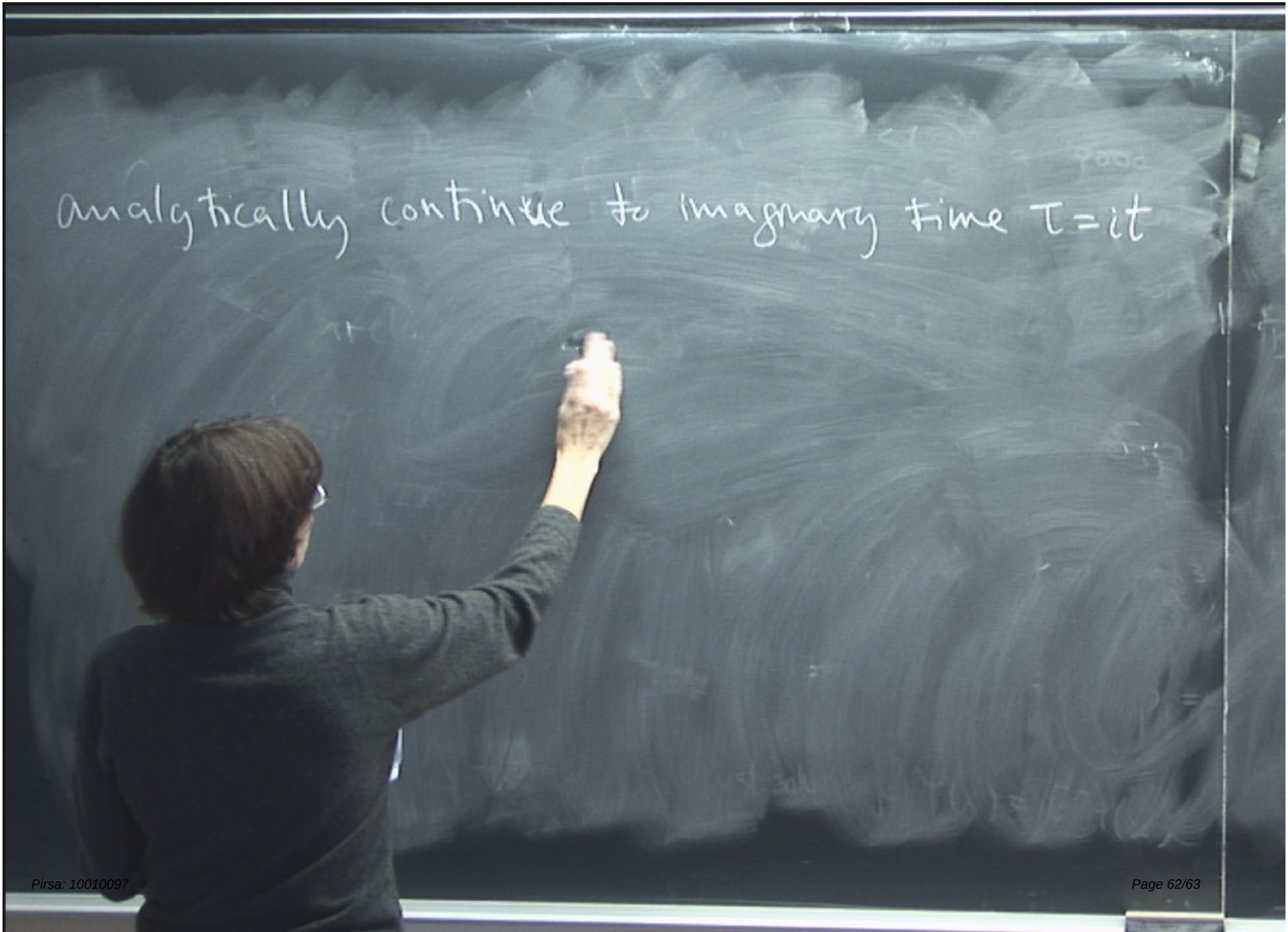
from a limiting process

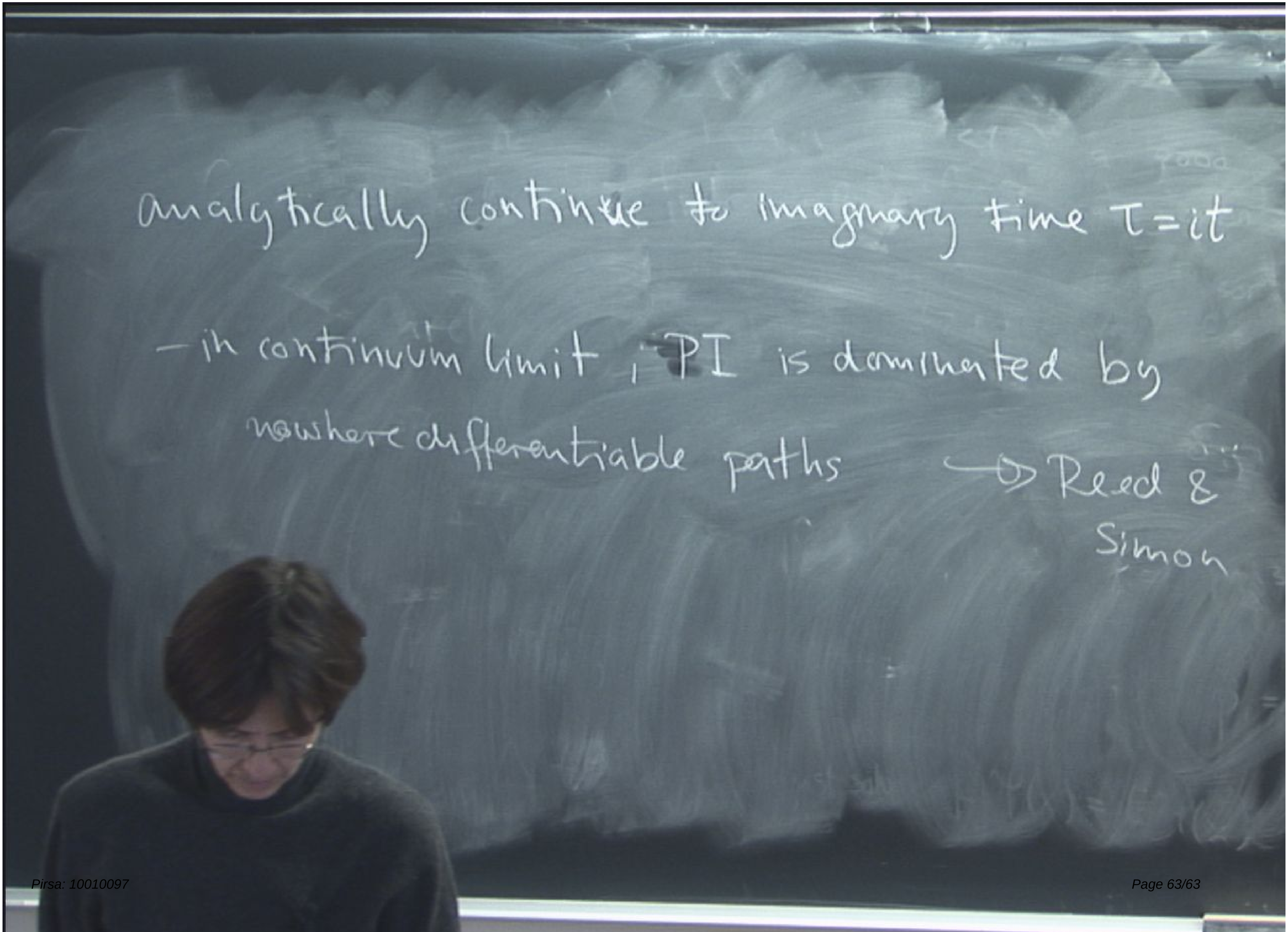
$\Delta t \to 0$, $N \to \infty$ from integrating over all piecewise straight paths of N segments

$$S[\vec{x}(t)] = \int_0^t dt' \left(\frac{m}{2}\dot{\vec{x}}^2 - V(\vec{x}) \right)$$

class. action for the path $\vec{x}(t)$




analytically continue to imaginary time $\tau = it$


analytically continue to imaginary time $\tau = it$
- in continuum limit, PI is dominated by
nowhere differentiable paths
→ Reed & Simon