

Title: Quantum Gravity - Review (PHYS 638) - Lecture 3

Date: Jan 27, 2010 10:00 AM

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Abstract:

Density matrix can
linearized, weak fields

entangled state.

$$\rho = \frac{1}{2} \left(\begin{array}{cc} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{array} \right)$$
$$\rho_A = \text{tr}_B(\rho)$$
$$\rho_B = \text{tr}_A(\rho)$$
$$\rho_{11} = \frac{1}{2} (1 + \langle \sigma_x \rangle)$$
$$\rho_{22} = \frac{1}{2} (1 - \langle \sigma_x \rangle)$$
$$\rho_{12} = \frac{1}{2} \langle \sigma_y \rangle$$
$$\rho_{21} = \frac{1}{2} \langle \sigma_y \rangle$$

Any density matrix can

\Rightarrow density matrix

$\rho = \frac{1}{2} (1 + \langle \sigma_x \rangle \sigma_x + \langle \sigma_y \rangle \sigma_y + \langle \sigma_z \rangle \sigma_z)$

Density matrix can
linearized, weak fields

entails \Rightarrow no backreaction of gravity on matter $\nabla_{\mu} T^{\mu\nu} = 0$

Any density matrix can

Density matrix can
linearized, weak fields

no backreaction of gravity on matter $\nabla_\mu T^{\mu\nu} = 0$

self-consistent inclusion of self-interactions

\Rightarrow full Einstein theory

Any density matrix can
be written as

density matrix = linearized, weak fields

⇒ no backreaction of gravity on matter $\nabla_{\mu} T^{\mu\nu} = 0$

self-consistent inclusion of self-interactions

⇒ full Einstein theory

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Any density matrix can

(Deser, gr-qc/
0411023)

Measurements

under spatial rotations

$$e'_y = \cos \Theta e_y - \sin \Theta e_z$$

$$e'_z = \sin \Theta e_y + \cos \Theta e_z$$

Measurements

Under spatial rotations

$$e'_y = \cos \Theta e_y - \sin \Theta e_z$$

$$e'_z = \sin \Theta e_y + \cos \Theta e_z$$

Circular polarization tensors

$$\hat{e}'_R = e^{-2i\Theta} \hat{e}_R \quad \hat{e}'_L = e^{2i\Theta} \hat{e}_L$$

Measurements
under spatial rotations

$$\hat{e}'_y = \cos \Theta \hat{e}_y - \sin \Theta \hat{e}_z$$

$$\hat{e}'_z = \sin \Theta \hat{e}_y + \cos \Theta \hat{e}_z$$

Circular polarization tensors

$$\hat{e}'_R = e^{-2i\Theta} \hat{e}_R \quad \text{and} \quad \hat{e}'_L = e^{2i\Theta} \hat{e}_L$$

A plane wave

Measurements

Under spatial rotations

$$e'_y = \cos \Theta e_y - \sin \Theta e_z$$
$$e'_z = \sin \Theta e_y + \cos \Theta e_z$$

Circular polarization tensors

$$\hat{e}'_R = e^{-2i\Theta} \hat{e}_R, \quad \hat{e}'_L = e^{2i\Theta} \hat{e}_L$$

Mo A plane wave ψ transforming as $\psi \mapsto e^{ih\Theta} \psi$
Under a rotation by Θ around the axis of propagation
is said to have helicity h .

Measurements

Under spatial rotations

$$e'_y = \cos \Theta e_y - \sin \Theta e_z$$

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Circular polarization tensors

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Mo. A plane wave ψ transforming as $\psi \mapsto e^{ih\Theta} \psi$
Under a rotation by Θ around the axis of propagation
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$$h = \pm 2$$

$\psi = p_1 \psi_1 + p_2 \psi_2 + \dots$
 $\psi_1 = \dots$
 $\psi_2 = \dots$
 $\psi_3 = \dots$
 $\psi_4 = \dots$
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 $\psi_{99} = \dots$
 $\psi_{100} = \dots$

where $h = \pm 2$

!!

$U_{h,1}$

$U_{h,2}$

P

B_3

halls

(2) $U_{h,1}$ is h -regular

(3) $U_{h,1}$ is completely positive

$U_{h,2}$ is h -regular and $U_{h,2}$ is completely positive

$U_{h,1} \oplus U_{h,2}$

where $h = \pm 2$

relativistic QFT : Poincaré symmetry
global symmetry of flat Minkowski spacetime

X

where $h = \pm 2$

relativistic QFT: Poincaré symmetry
global symmetry of flat Minkowski spacetime

$$x^M \mapsto y^M = \Lambda^M_{\nu} x^{\nu} + a^M$$

where $h = \pm 2$

relativistic QFT: Poincaré symmetry
global symmetry of flat Minkowski spacetime

$$x^M \mapsto y^M = \Lambda^M_{\nu} x^{\nu} + a^M$$

6 rot. s

where $h = \pm 2$

relativistic QFT: Poincaré symmetry
global symmetry of flat Minkowski spacetime

$$x^M \mapsto y^M = \Lambda^M_{\nu} x^{\nu} + a^M$$

6 rot. s

4 transl.

where $h = \pm 2$

relativistic QFT: Poincaré symmetry
global symmetry of flat Minkowski space time

$$x^M \mapsto y^M = \Lambda^M_{\nu} x^{\nu} + a^M$$

6 rot. s 4 transl.

10-dim noncompact Lie group

where $h = \pm 2$

relativistic QFT: Poincaré symmetry
global symmetry of flat Minkowski spacetime

$$x^M \mapsto y^M = \Lambda^M_{\nu} x^{\nu} + a^M$$

6 rot. s 4 transl.

→ 10-dim noncompact Lie group

unitary action on states $\psi \mapsto U(\Lambda, a) \psi$

→ classify "particles" according to their transf. behaviour

where $h = \pm 2$

relativistic QFT: Poincaré symmetry
global symmetry of flat Minkowski spacetime

$$x^M \mapsto y^M = \Lambda^M_{\nu} x^{\nu} + a^M$$

6 rot. s 4 transl.

→ 10-dim noncompact Lie group

unitary action on states $\psi \mapsto U(\Lambda, a) \psi$

→ classify "particles" according to their transf behaviour

\Leftrightarrow Unitary

matrix
fields

Any density matrix

Hermitian
 $\rho = \rho^\dagger$
 $\text{Tr}(\rho) = 1$
 $\rho \geq 0$

\Leftrightarrow Unitary irreps of Poincaré group

Any density matrix

transform

$\rho \rightarrow U \rho U^\dagger$

$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

$\rho = \sum_i p_i U |\psi_i\rangle\langle\psi_i| U^\dagger$

$\rho = \sum_i p_i |\psi_i'\rangle\langle\psi_i'|$

\Leftrightarrow Unitary irreps of Poincaré group (mass, spin) by matrix

coupled fields

Field equations

Energy-momentum tensor

Angular momentum

Spin

Helicity

Massless particles

Representation theory

↔ Unitary irreps of Poincaré group (mass, spin) density matrix

$$P_\mu, J_{\mu\nu}$$

\leftrightarrow Unitary irreps of Poincaré group (mass, spin) \rightarrow fields

P_μ, J one-particle states $|\vec{p}, \sigma\rangle$

\Leftrightarrow Unitary irreps of Poincaré group (mass, spin) matrix

P_μ, J fields
one-particle states $|\vec{p}, \sigma\rangle$

\Leftrightarrow Unitary irreps of Poincaré group (mass, spin) by matrix

$P_M, J_{\mu\nu}$; one-particle states $|\vec{p}, \sigma\rangle$

$$P^M |\vec{p}, \sigma\rangle = p^M |\vec{p}, \sigma\rangle$$

\Leftrightarrow Unitary irreps of Poincaré group (mass, spin) by matrix

$P_\mu, J_{\mu\nu}$, one-particle states $|\vec{p}, \sigma\rangle$

$$P^\mu |\vec{p}, \sigma\rangle = p^\mu |\vec{p}, \sigma\rangle, \quad U(1, a) |\vec{p}, \sigma\rangle = e^{i a \cdot p} |\vec{p}, \sigma\rangle$$

rep.s of Poincaré group (mass, spin) by matrix can

one-particle states $|\vec{p}, \sigma\rangle$

$$P^\mu |\vec{p}, \sigma\rangle = p^\mu |\vec{p}, \sigma\rangle, \quad U(1, a) |\vec{p}, \sigma\rangle = e^{-ia_\mu P^\mu} |\vec{p}, \sigma\rangle$$

rep.s of Poincaré group (mass, spin) unitary matrix can

one-particle states $|\vec{p}, \sigma\rangle$

$$P^\mu |\vec{p}, \sigma\rangle = p^\mu |\vec{p}, \sigma\rangle, \quad U(1, a) |\vec{p}, \sigma\rangle = e^{-ia_\mu P^\mu} |\vec{p}, \sigma\rangle$$

reps. of Poincaré group (mass, spin) by matrix can

fields
 $\Gamma_{\mu\nu}$; one-particle states $|\vec{p}, \sigma\rangle$

$$P^\mu |\vec{p}, \sigma\rangle = p^\mu |\vec{p}, \sigma\rangle, \quad U(1, a) |\vec{p}, \sigma\rangle = e^{-ia_\mu P^\mu} |\vec{p}, \sigma\rangle$$

$m^2 = 0, p^0 > 0$; helicity = (component of angular momentum
along 3-momentum $\vec{k} = (0, 0, \omega)$) $J_3 |\vec{k}, \sigma\rangle = \sigma |\vec{k}, \sigma\rangle$

$$U(\Lambda, 0) |\vec{p}, \sigma\rangle = N e^{i\sigma\Theta(\Lambda, p)} |\vec{\Lambda p}, \sigma\rangle$$

transforming the p
 around the axis of propagation
 have helicity σ .

$$U(\Lambda, 0) |\vec{p}, \sigma\rangle = N e^{i\sigma \Theta(\Lambda, \vec{p})} |\vec{\Lambda p}, \sigma\rangle$$

↑
angle about \vec{p}
(contained in Λ)

transforming \vec{p}
around the axis of propagation
helicity σ .

$$U(\Lambda, 0) |\vec{p}, \sigma\rangle = N e^{i\sigma \Theta(\Lambda, \vec{p})} |\vec{\Lambda p}, \sigma\rangle$$

$|\sigma| \sim$ "spin"

↑
angle about \vec{p}
(contained in Λ)

$f_{\mu\nu}$

$$U(\Lambda, 0) |\vec{p}, \sigma\rangle = N e^{i\sigma \Theta(\Lambda, \vec{p})} |\vec{\Lambda}\vec{p}, \sigma\rangle$$

$|\sigma| \sim$ "spin"

↑
angle about \vec{p}
(obtained in Λ)

$$f_{\mu\nu}(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{\vec{k}}}} \sum_{\sigma=\pm 2} [d(\vec{k}, \sigma) a_{\mu\nu}(\vec{k}, \sigma) e^{ikx}]$$

$$\Theta(\Lambda, \vec{p}) \left| \vec{\Lambda} \vec{p}, \sigma \right\rangle$$

↑
angle about \vec{p}
(contained in Λ)

$$\sum_{\sigma=\pm 2} \left[\hat{d}(\vec{k}, \sigma) a_{\mu\nu}(\vec{k}, \sigma) e^{ik \cdot x} + \hat{d}^\dagger(\vec{k}, \sigma) a_{\mu\nu}^*(k, \sigma) e^{-ik \cdot x} \right]$$

$$\Theta(\Lambda, \vec{p}) \left| \vec{\Lambda} \vec{p}, \sigma \right\rangle$$

↑
angle about \vec{p}
(contained in Λ)

$$\sum_{\sigma=\pm 2} \left[\hat{d}(\vec{k}, \sigma) a_{\mu\nu}(\vec{k}, \sigma) e^{ik \cdot x} + \hat{d}^\dagger(\vec{k}, \sigma) a_{\mu\nu}^*(\vec{k}, \sigma) e^{-ik \cdot x} \right]$$

↑
annihilation
op.s

↑
(creation op.s)

$$|0\rangle, |\vec{p}, \sigma\rangle = N e^{i\sigma \Theta(\Lambda, \vec{p})} |\Lambda \vec{p}, \sigma\rangle$$

↑
angle about \vec{p}
(contained in Λ)

$|\sigma| \sim$ "spin"

$$\langle x | = \int \frac{d^3 k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_{\vec{k}}}} \sum_{\sigma=\pm 2} \left[\hat{d}(\vec{k}, \sigma) a_{\mu\nu}(\vec{k}, \sigma) e^{i\vec{k}\cdot\vec{x}} + \hat{d}^\dagger(\vec{k}, \sigma) a_{\mu\nu}(\vec{k}, \sigma) e^{-i\vec{k}\cdot\vec{x}} \right]$$

↑
annihilation
op.s

↑
creation

$$[\hat{d}(\vec{k}, \sigma), \hat{d}^\dagger(\vec{k}', \sigma')] = \delta_{\sigma\sigma'} \delta^{(3)}(\vec{k} - \vec{k}')$$

L. Rosenfeld (1930), M. Bornstein (1936)

J. Stachel

$$e^{ik \cdot x} + \left[\hat{d}^\dagger(\vec{k}, \sigma) a_{\mu\nu}^*(k, \sigma) e^{-ik \cdot x} \right]$$

↑
creation op.s

L. Rosenfeld (1930), M. Bornstein (1936)

J. Stachel

How real are gravitons?

$$\hat{d}^\dagger(\vec{k}, \sigma) a_{\mu\nu}^*(k, \sigma) e^{-ik \cdot x}$$

↑
(creation op.)

L. Rosenfeld (1930), M. Bornstein (1936)

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$$\hat{d}^\dagger(\vec{k}, \sigma) a_{\mu\nu}^*(k, \sigma) e^{-ik \cdot x}$$

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$$\hat{d}^\dagger(\vec{k}, \sigma) a_{\mu\nu}^*(k, \sigma) e^{-ik \cdot x}$$

↑
creation op.s

typical gravit. wave

L. Rosenfeld (1930), M. Bornstein (1936)

J. Stachel

$$\hat{d}^\dagger(\vec{k}, \sigma) a_{\mu\nu}^*(k, \sigma) e^{-ik \cdot x}$$

↑
(creation op.)

How real are gravitons?

typical gravit. wave (1000 Hz)

L. Rosenfeld (1930), M. Bornstein (1936)

J. Stachel

How real are gravitons?

$$\hat{d}^\dagger(\vec{k}, \sigma) a_{\mu\nu}^*(k, \sigma) e^{-ik \cdot x}$$

creation op.s

typical gravit. wave (1000 Hz)

graviton number density $\sim 3 \times 10^{14} \text{ cm}^{-3}$

Broughn & Rothman: Γ -Bourbaki group (st. group) by matrix con

Boughin & Rothman:

transition rate hydrogen $3d \xrightarrow{\text{graw}} 1$

Boughn & Rothman

transition rate hydrogen $3d \xrightarrow{\text{gaw}} 1s$; $\Gamma_{\text{gaw}} \approx 5.7 \times 10^7$

ghy & Rothman: H^+ Porin group (acid, solid) by matrix case

transition rate hydrogen $3d \xrightarrow{\text{graw}} 1s$; $T_{\text{graw}} \approx 5.7 \times 10^{-10} \text{ s}$