Title: String Theory - Review (PHYS 623) - Lecture 5

Date: Jan 29, 2010 11:20 AM

URL: http://pirsa.org/10010093

Abstract:

Conformal Group in D-dimensio

Conformal Group in D-dimensions.

$\rightarrow D = 2$

Conformal Group in D-dimension.

$\rightarrow D = 2$

$ds^2 =$

Conformal Group in D-dimensions.

$\rightarrow D = 2$ $\qquad ds^2 = dx \cdot dx$

$$dx^\mu dx^\nu \eta_{\mu\nu}$$
$$dx^i dx^j \delta_{ij}$$

Conformal Group in D-dimensions.

$\rightarrow D = 2$

$$ds^2 = e^{\omega(x)}\, dx \cdot dx$$

$$dx^\mu dx^\nu \eta_{\mu\nu}$$

$$dx^i dx^j \delta_{ij}$$

Conformal Group in D-dimensions.

$\rightarrow D = 2$

$$ds^2 = e^{\omega(x)} dx \cdot dx$$

Conformally flat.

$$dx^\mu dx^\nu \eta_{\mu\nu}$$

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Conformal Group in D-dimensions.

→ $D = 2$ $\qquad ds^2 = e^{\omega(x)}\, dx \cdot dx$

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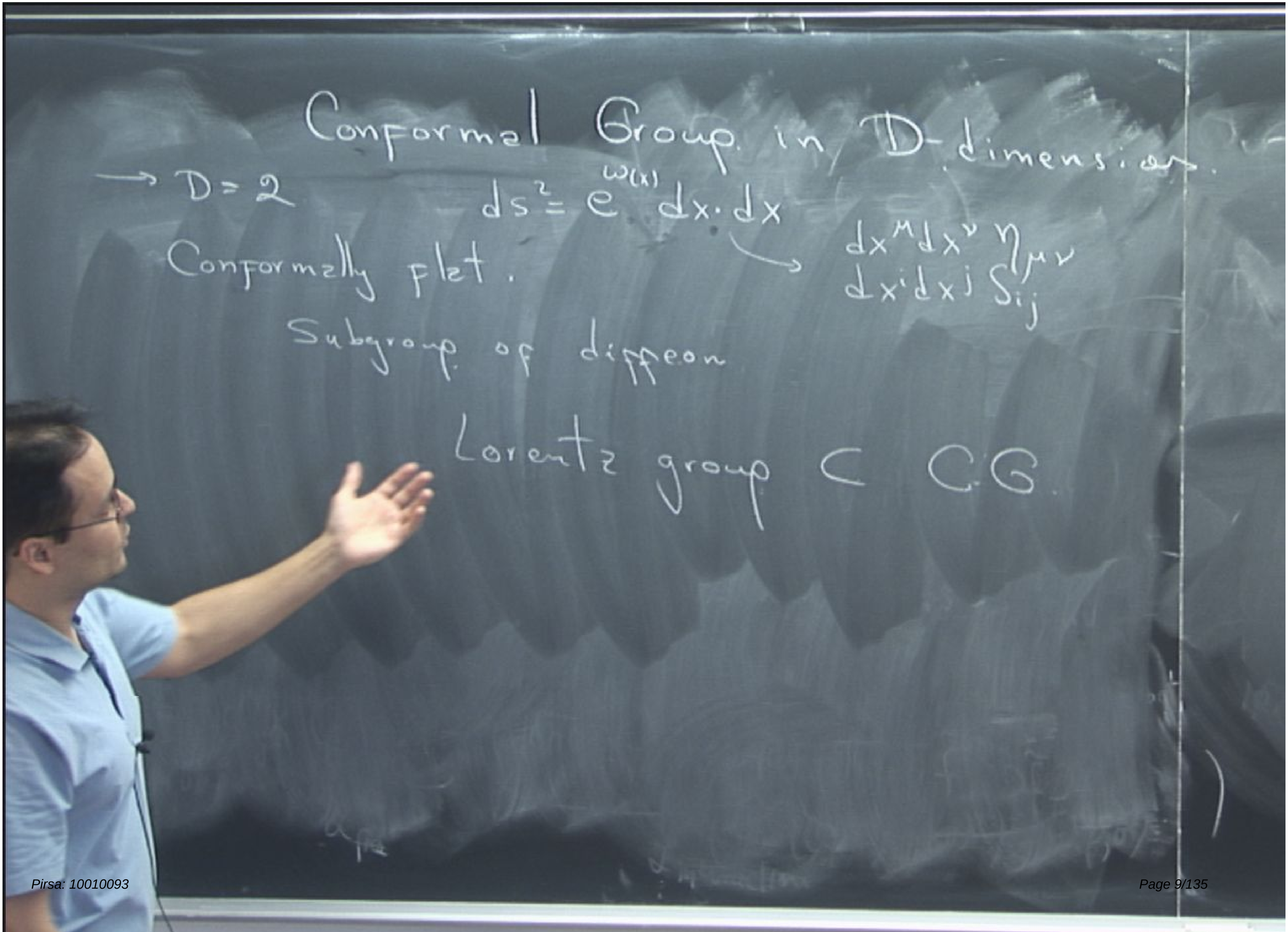
Conformally flat.

Subgroup of diffeon



Conformal Group in D-dimensions.
→ D = 2
$ds^2 = e^{\omega(x)} dx \cdot dx$
Conformally flat.
$dx^\mu dx^\nu \eta_{\mu\nu}$
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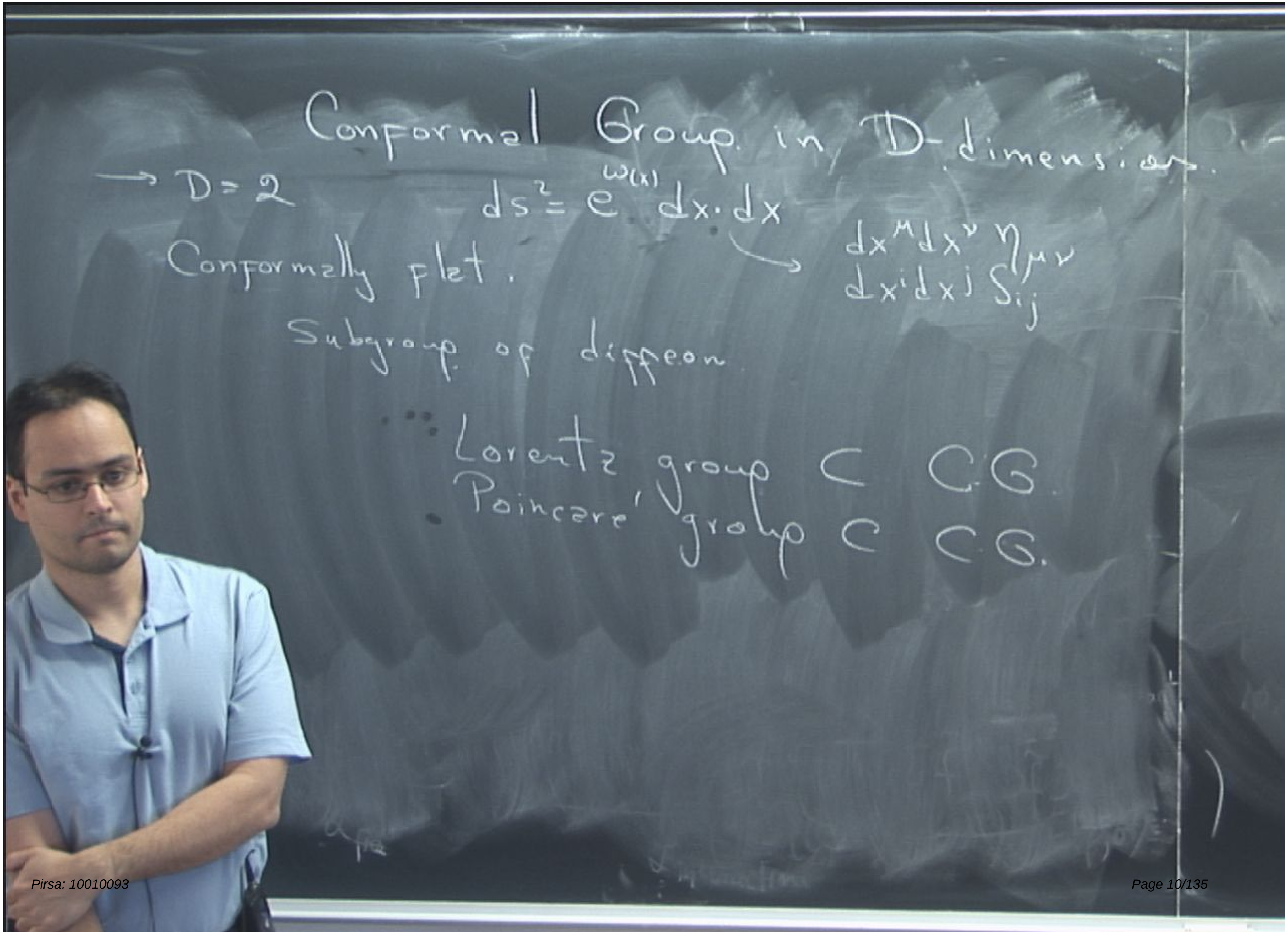
Subgroup of diffeom
Lorentz group ⊂ C.G.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→ D = 2
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Subgroup of diffeom
Lorentz group ⊂ C.G.
Poincaré group ⊂ C.G.

Conformal Group in D-dimensions.

→ D = 2 $\quad ds^2 = e^{\omega(x)}\, dx \cdot dx$

Conformally flat. $\quad \longrightarrow dx^\mu dx^\nu \eta_{\mu\nu}$

$dx^i dx^j \delta_{ij}$

Subgroup of diffeon

Lorentz group ⊂ C.G.

Poincare' group ⊂ C.G.

Conformal Group in D-dimensio

→ $D = 2$

$$ds^2 = e^{\omega(x)} dx \cdot dx \to \begin{array}{l} dx^\mu dx^\nu \eta_{\mu\nu} \\ dx^i dx^j \delta_{ij} \end{array}$$

Conformally flat.

Subgroup of diffeom

- Lorentz group $\subset$ C.G.
- Poincaré group $\subset$ C.G.
- Scale Transformation.

Conformal Group in D-dimension.

$\rightarrow D = 2$ $\quad ds^2 = e^{\omega(x)} dx \cdot dx \rightarrow dx^\mu dx^\nu \eta_{\mu\nu}$

$dx^i dx^j \delta_{ij}$

Conformally flat.

Subgroup of diffeom

Lorentz group $\subset$ C.G.

Poincaré group $\subset$ C.G.

Scale Transformation

$$x^\mu \rightarrow \lambda x^\mu$$

Inversion $x^\mu \rightarrow \frac{x^\mu}{x^2}$

Conformal Group in D-dimensions.

$\rightarrow D=2$ $\quad ds^2 = e^{\omega(x)} dx\cdot dx$

Conformally flat. $\quad dx^\mu dx^\nu \eta_{\mu\nu}$ $\quad dx^i dx^j \delta_{ij}$

Subgroup of diffeom

Lorentz group $\subset$ C.G.

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Scale Transformation.

$$X^\mu \rightarrow \lambda X^\mu$$

Inversion $\quad X^\mu \rightarrow \dfrac{X^\mu}{X^2}$

Special conformal transformation

Inversion — Translation b^μ — Inversion

$$x^\mu \longrightarrow \frac{x^\mu}{x^2} \qquad \underbrace{\frac{x^\mu}{x^2}}_{y^\mu} \longrightarrow \underbrace{\frac{x^\mu}{x^2} + b^\mu}_{y^\mu + b^\mu} \longrightarrow$$

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Inversion — Translation b^μ — Inversion.

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Infinitesimal :

Special conformal transformation

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Infinitesimal:

$$\delta x^\mu = \omega^\mu{}_\nu x^\nu \qquad \omega_{\mu\nu} = -\omega_{\nu\mu} \longrightarrow$$

D-dimens

Special conformal transformation

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Infinitesimal :

D-dimens

$$\delta x^\mu = \omega^\mu{}_\nu x^\nu \qquad \omega_{\mu\nu} = -\omega_{\nu\mu} \longrightarrow \frac{D(D-1)}{2}$$

$$\delta x^\mu =$$

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Infinitesimal: D-dimens.

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$\frac{1}{2}(D+1)(D+2)$

Special conformal transformation

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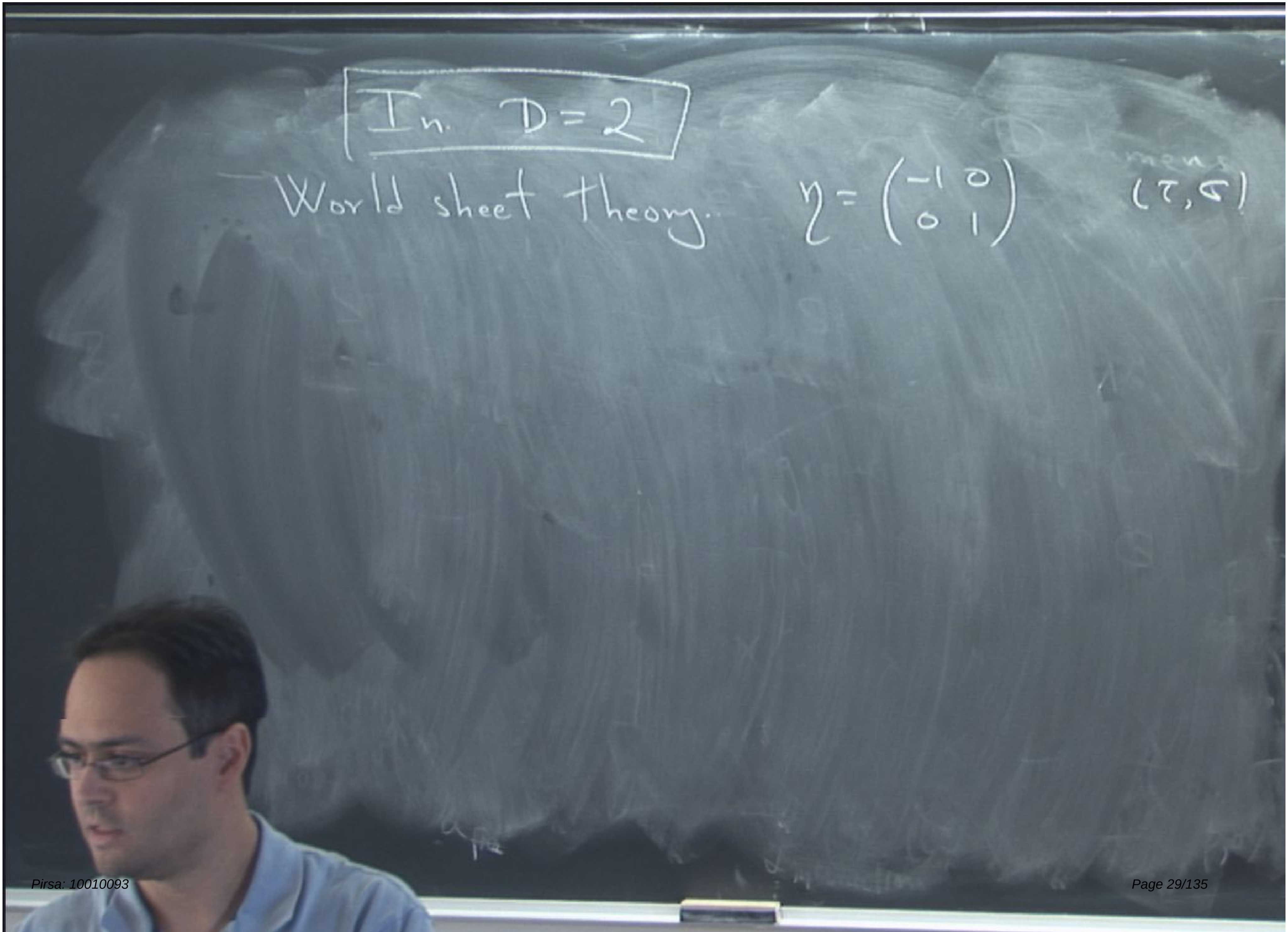
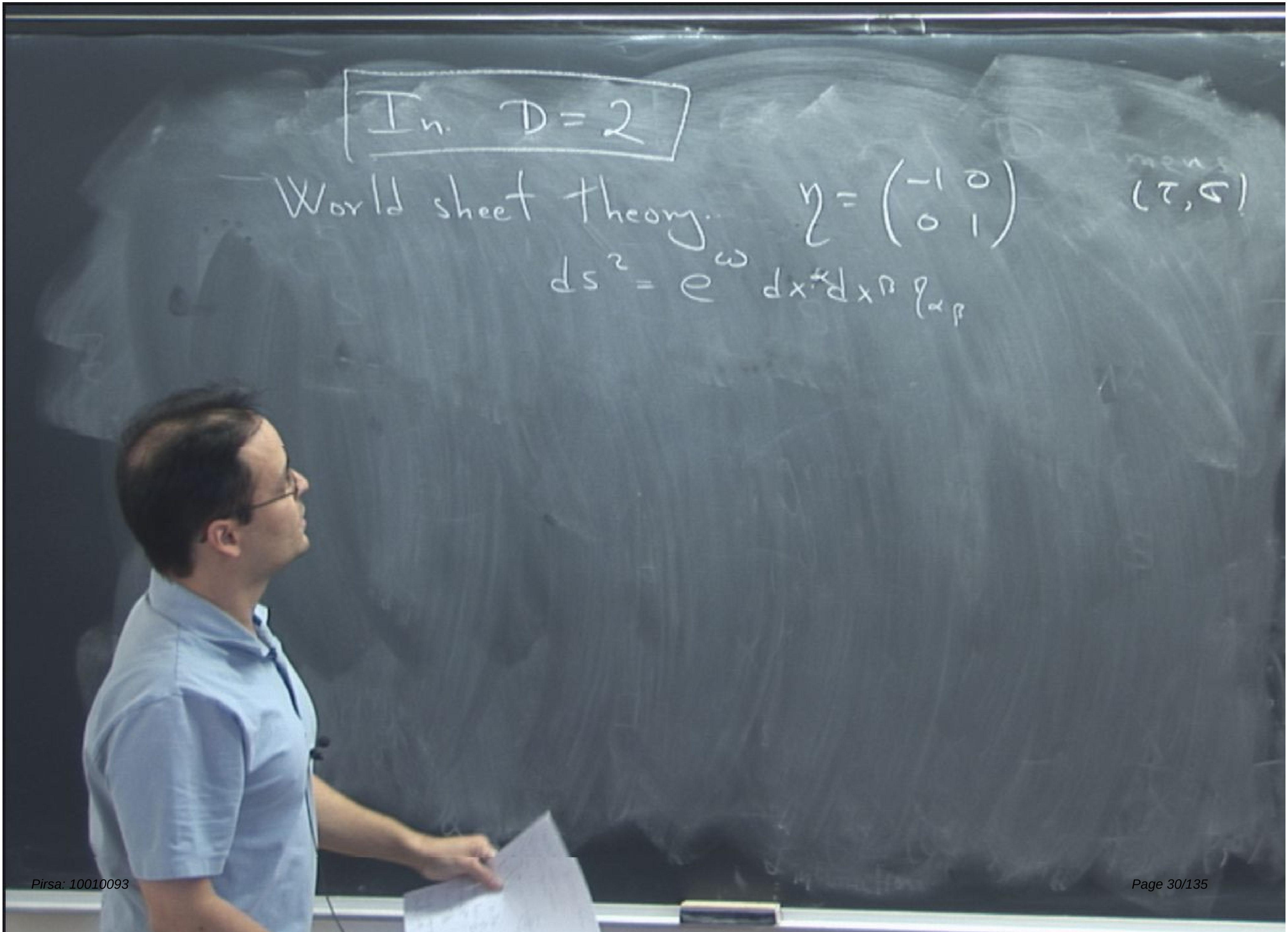

In D = 2
World sheet theory.
$\eta = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
(τ, σ)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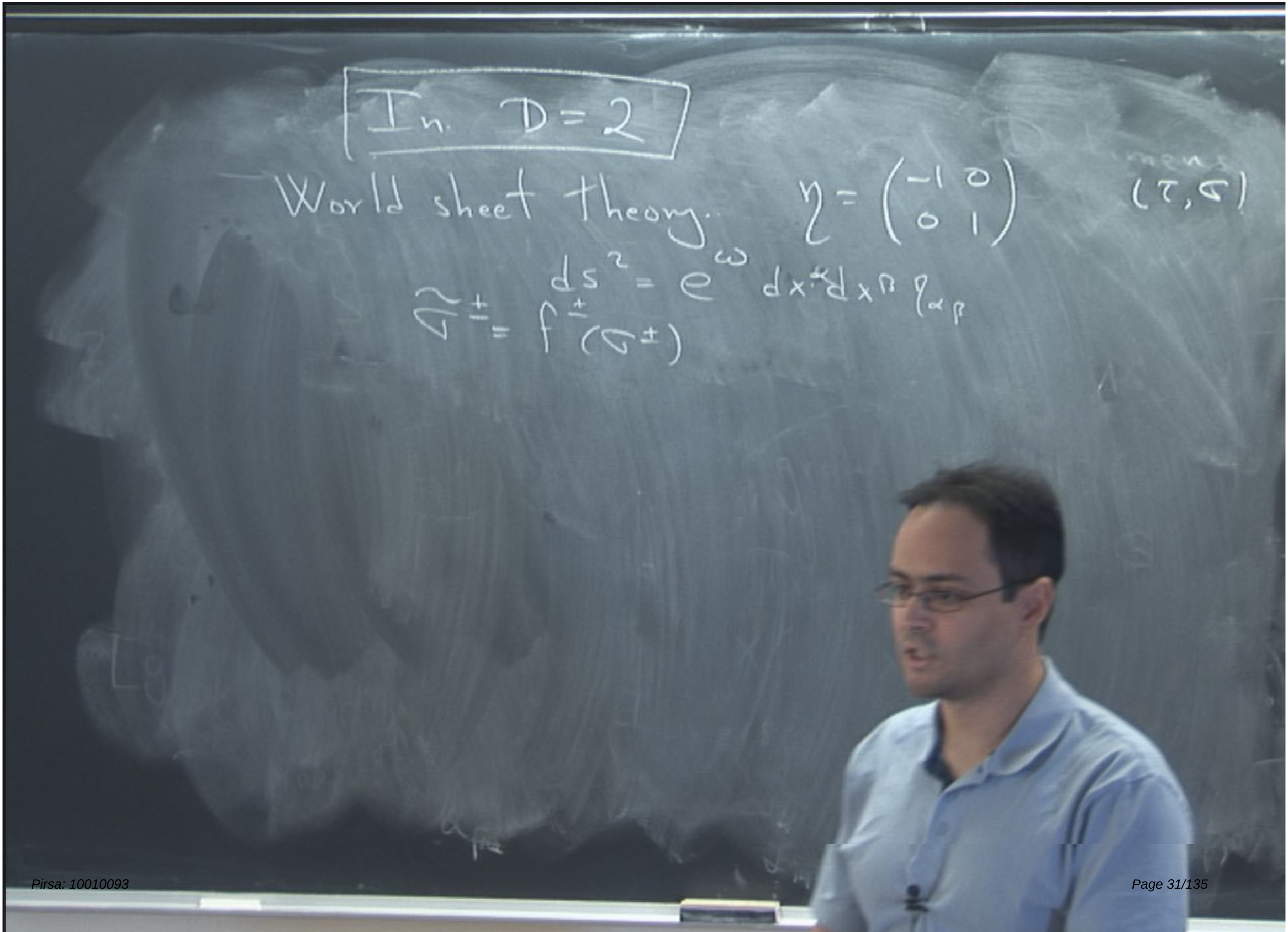
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$\tilde{\sigma}^\pm = f^\pm(\sigma^\pm)$

$\boxed{\text{In } D=2}$

World sheet theory. $\quad \eta = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (\tau, \sigma)$

$$ds^2 = e^{\omega}\, dx^{\alpha} dx^{\beta}\, \eta_{\alpha\beta}$$

$$\tilde{\sigma}^{\pm} = f^{\pm}(\sigma^{\pm})$$

Wick : $\quad \tau \rightarrow -i\tau$

$$\sigma^{\pm} = \tau \pm \sigma \rightarrow -i\tau \pm \sigma \sim \begin{matrix} z \\ \bar{z} \end{matrix}$$

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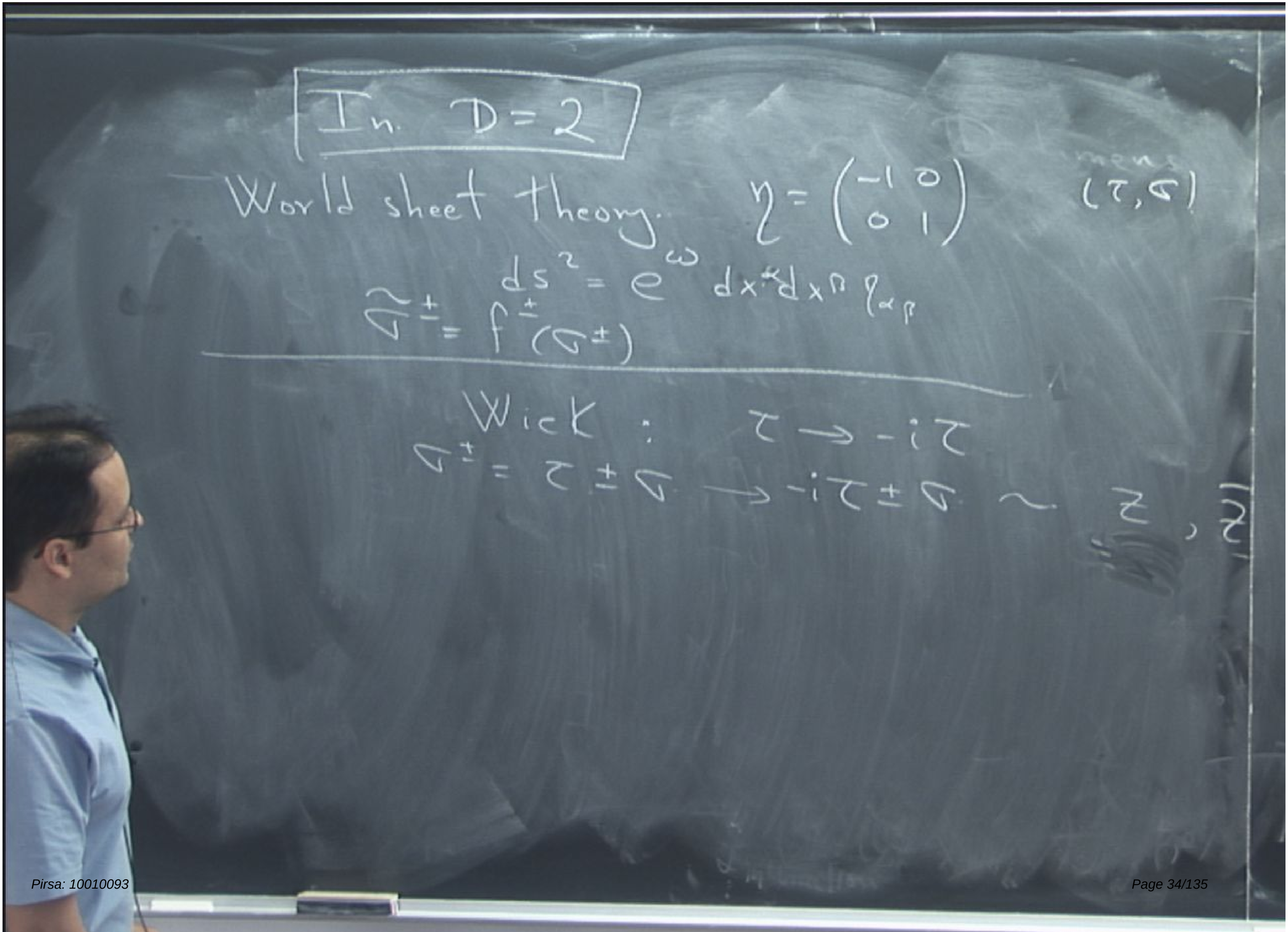
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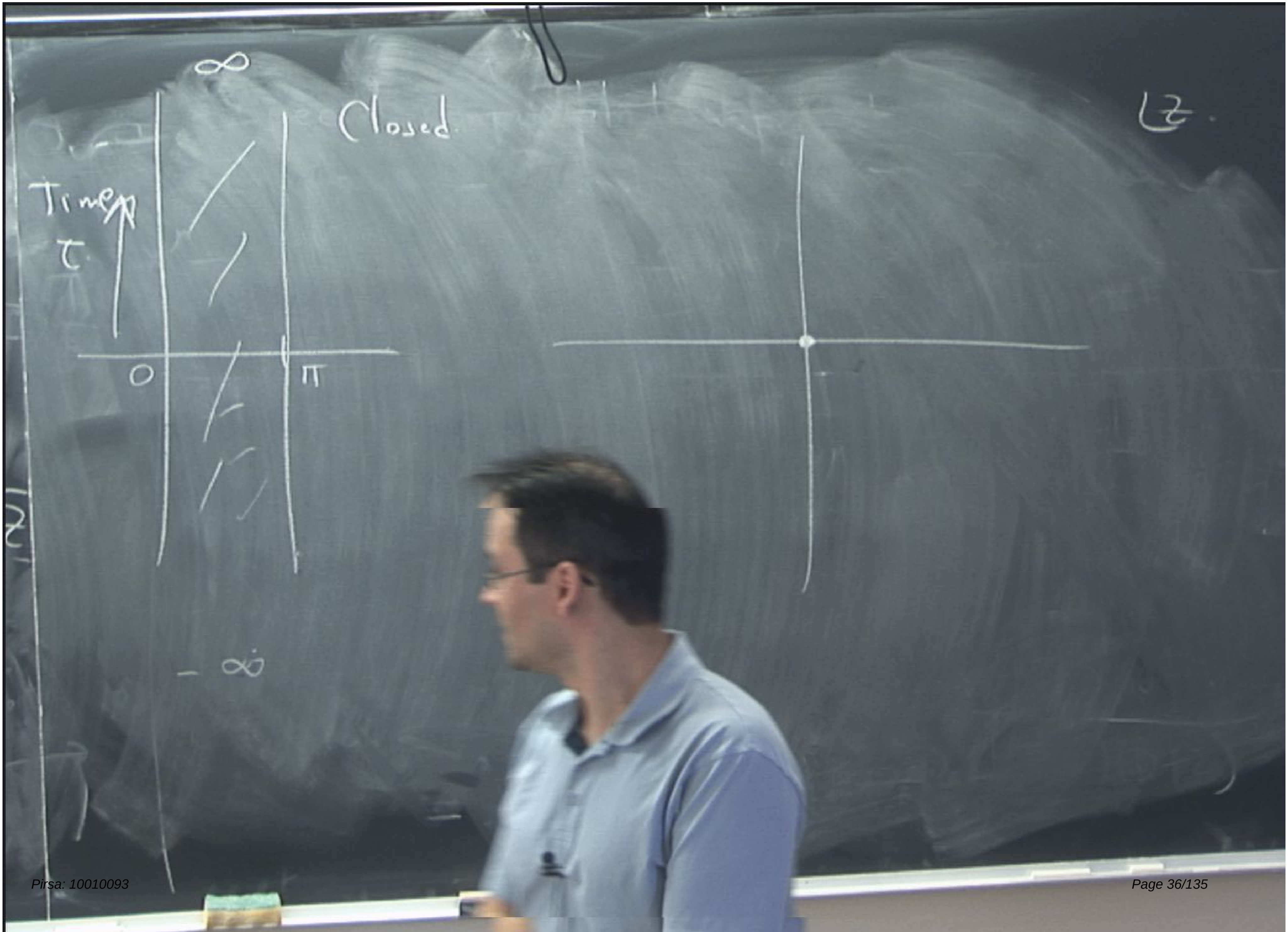
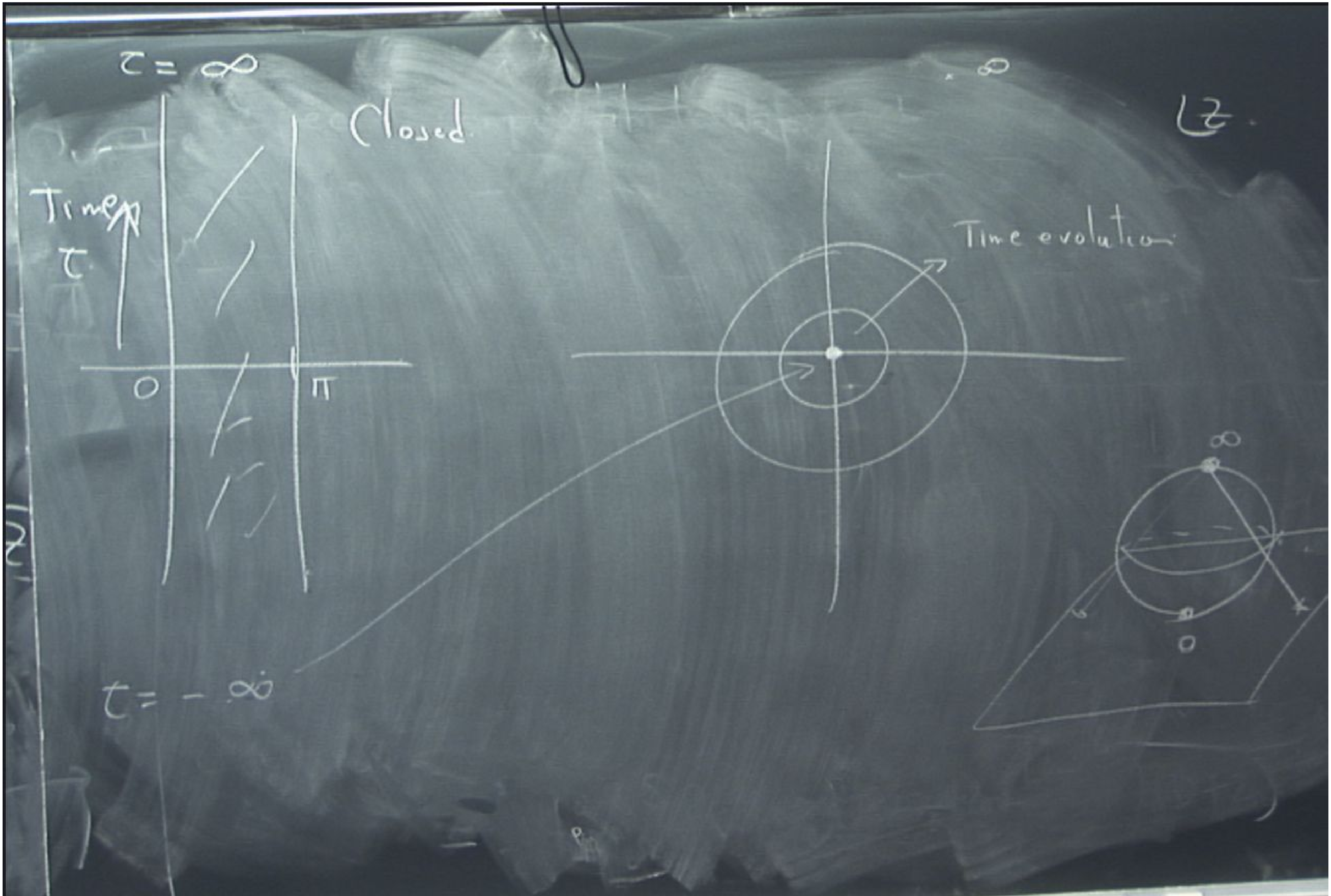
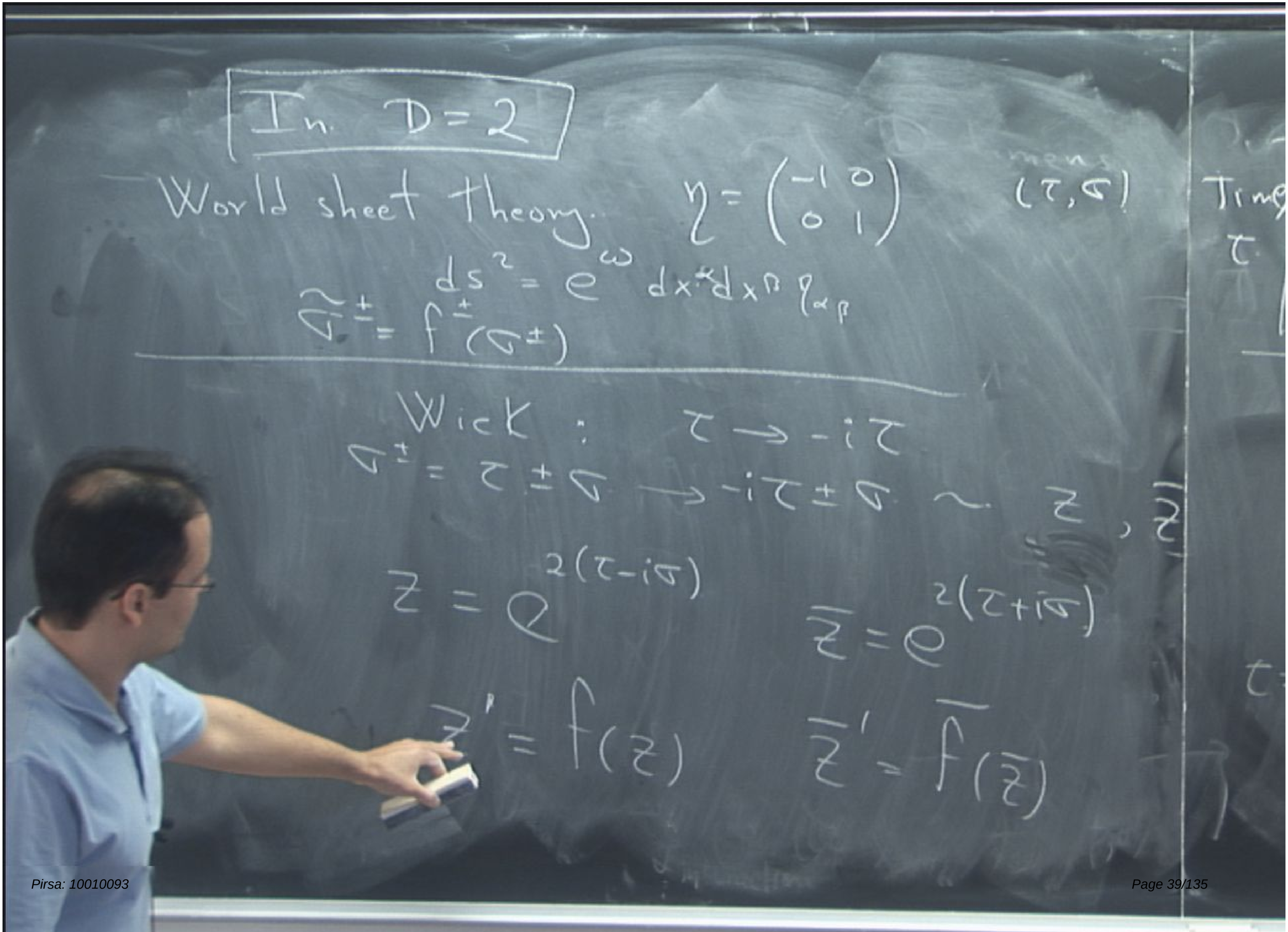
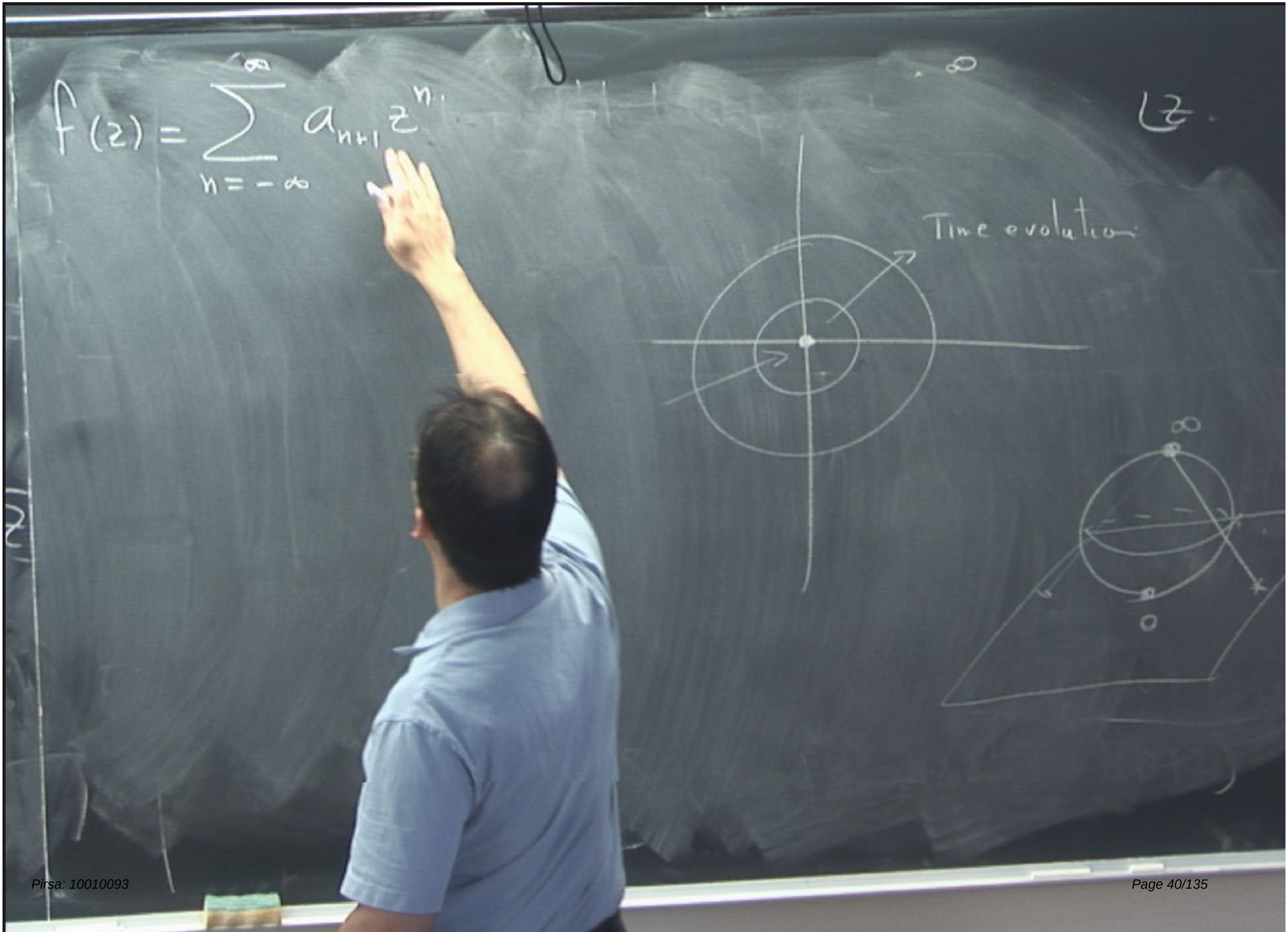
$$z = e^{2(\tau - i\sigma)} \qquad \bar{z} = e^{2(\tau + i\sigma)}$$


∞
Closed.
LZ.
Time
τ
0
π
-∞

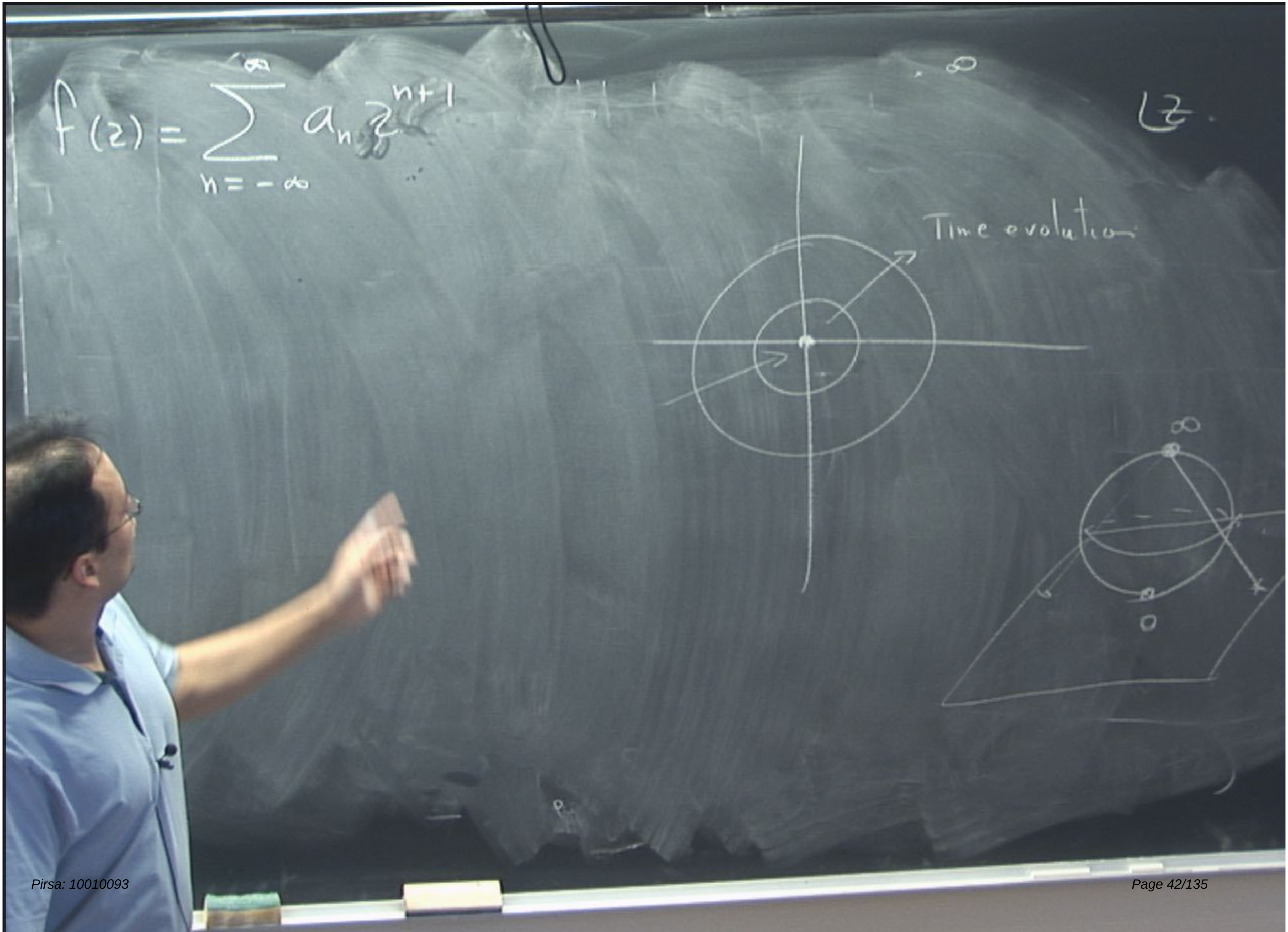
τ = ∞
Closed.
Time
τ
0
π
τ = −∞
LZ
∞
0


τ = ∞
Closed.
Time
τ
0
π
τ = −∞
Time evolution
∞
0


In $D=2$
World sheet theory. $\eta = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ (τ, σ)
Time
τ
$ds^2 = e^{\omega} dx^\alpha dx^\beta \eta_{\alpha\beta}$
$\tilde{\sigma}^\pm = f^\pm(\sigma^\pm)$
Wick : $\tau \to -i\tau$
$\sigma^\pm = \tau \pm \sigma \to -i\tau \pm \sigma \sim z, \bar{z}$
$z = e^{2(\tau - i\sigma)}$
$\bar{z} = e^{2(\tau + i\sigma)}$
$z' = f(z)$
$\bar{z}' = \bar{f}(\bar{z})$


f(z) = ∑_{n=-∞}^{∞} a_{n+1} z^n
Time evolution
∞
∞
0

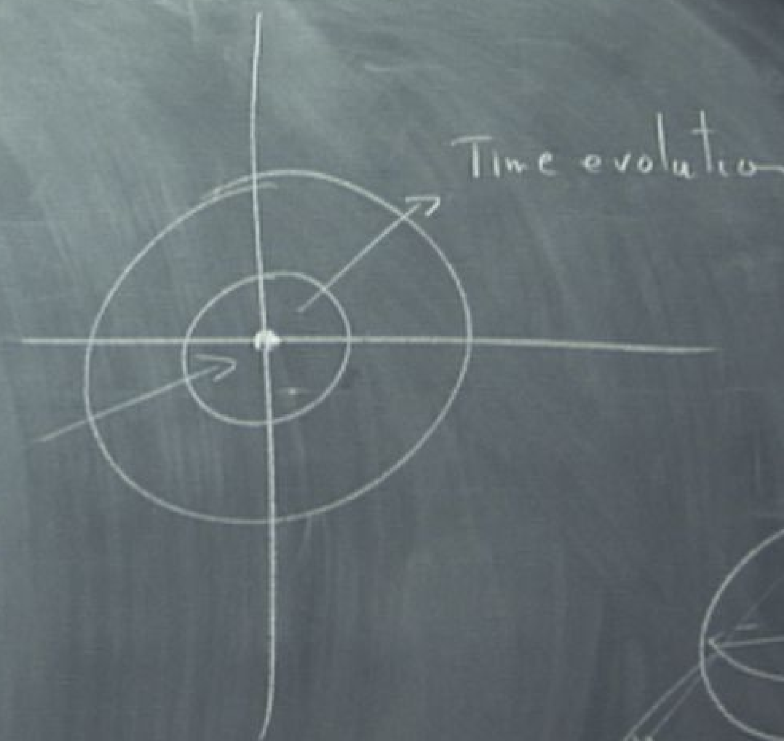

$f(z) = \sum_{n=-\infty}^{\infty} a_n z^{n+1}$
∞
Time evolution
∞
0

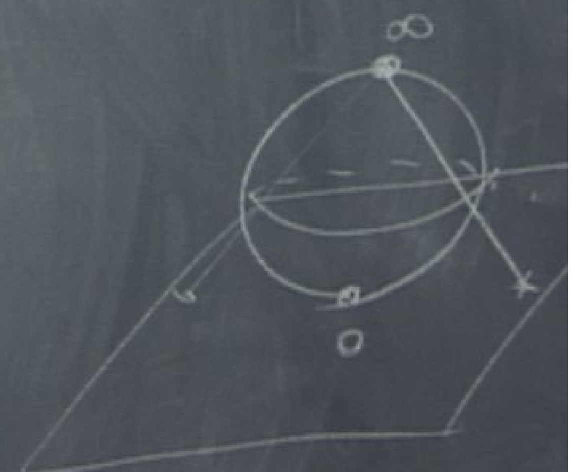

$f(z) = \sum_{n=-\infty}^{\infty} a_n z^{n+1}$
Time evolution
∞
∞
0

$$f(z) = \sum_{n=-\infty}^{\infty} a_n z^{n+1}$$

$\mathbb{C}_z$

Generators:



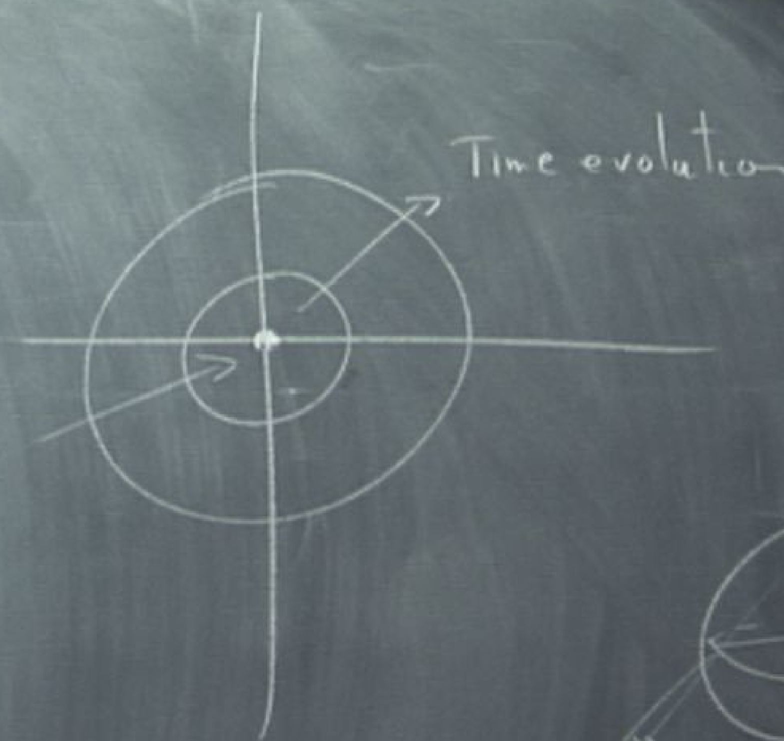


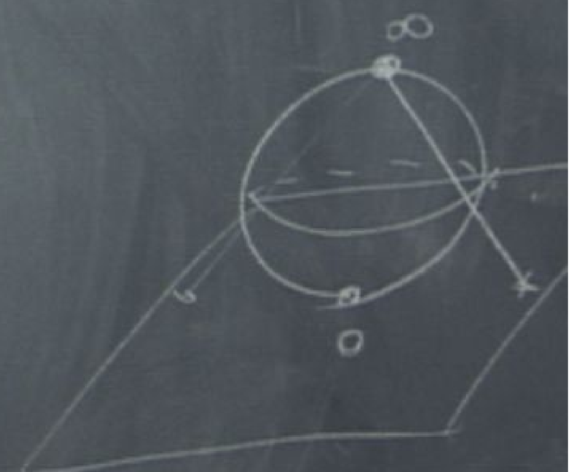
Lz.

$$f(z) = \sum_{n=-\infty}^{\infty} a_n z^{n+1}$$

Generators:

$$z \to z' = z - \epsilon_n z^{n+1}$$





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Generators:

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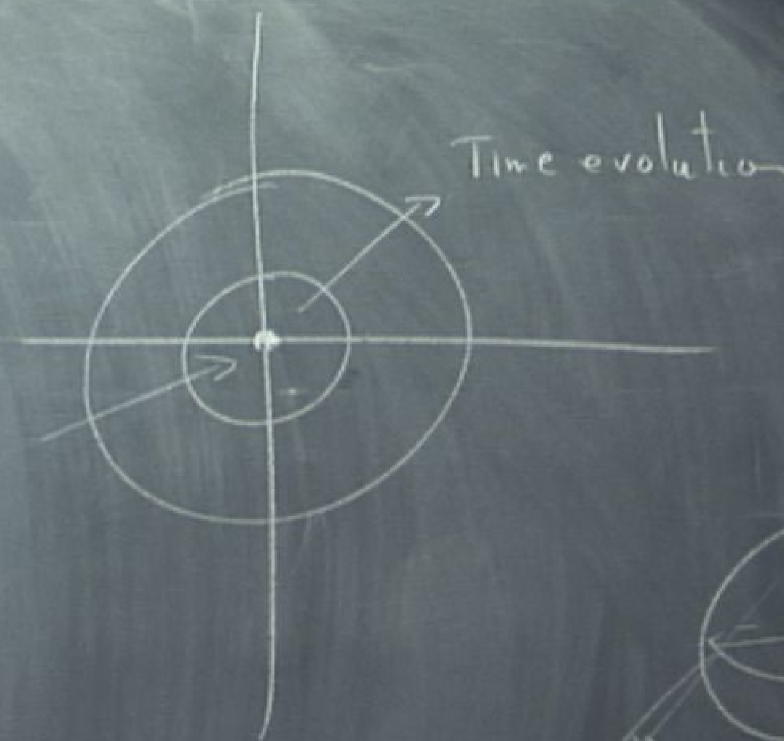
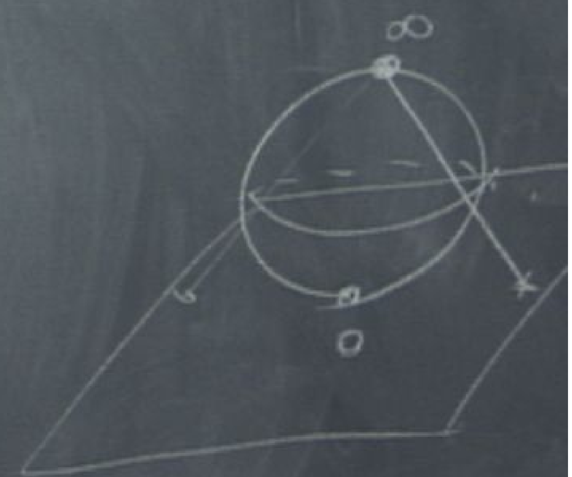
$$\delta z = l_n z$$





$$f(z) = \sum_{n=-\infty}^{\infty} a_n z^{n+1}$$

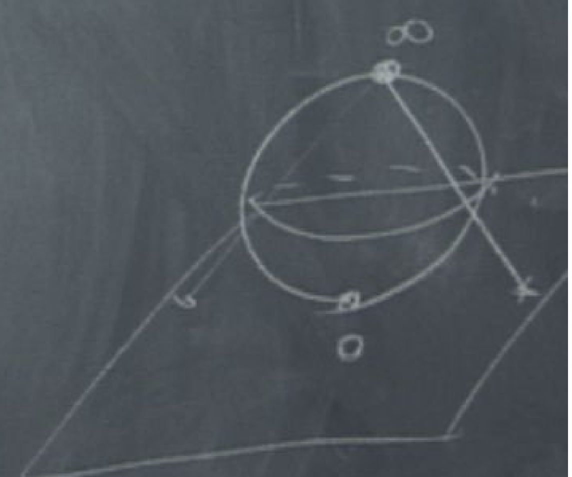
Generators:

$$z \to z' = z - \epsilon_n z^{n+1}$$

$$\delta z = l_n z$$

$$l_n = -z^{n+1}\frac{\partial}{\partial z}$$

$\mathcal{L}z$.



$$f(z) = \sum_{n=-\infty}^{\infty} a_n z^{n+1}$$

$\mathbb{C}$.

Generators:

$$z \to z' = z - \epsilon_n z^{n+1}$$

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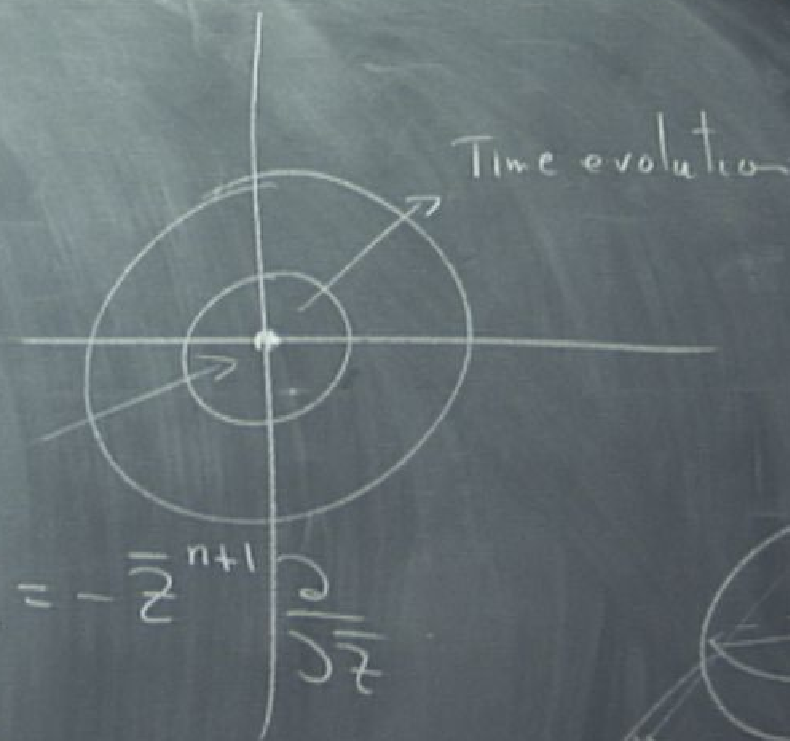

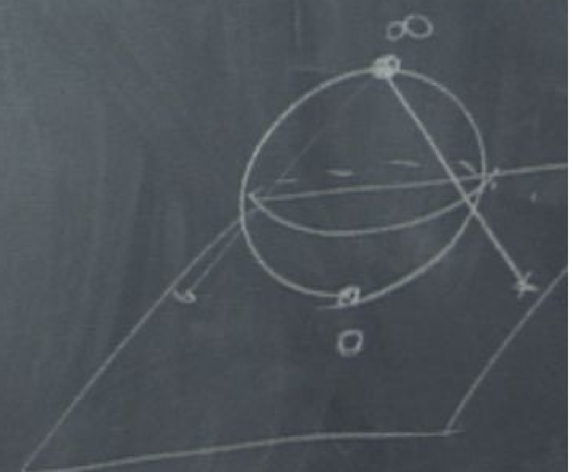

$$f(z) = \sum_{n=-\infty}^{\infty} a_n z^{n+1}$$

Generators:

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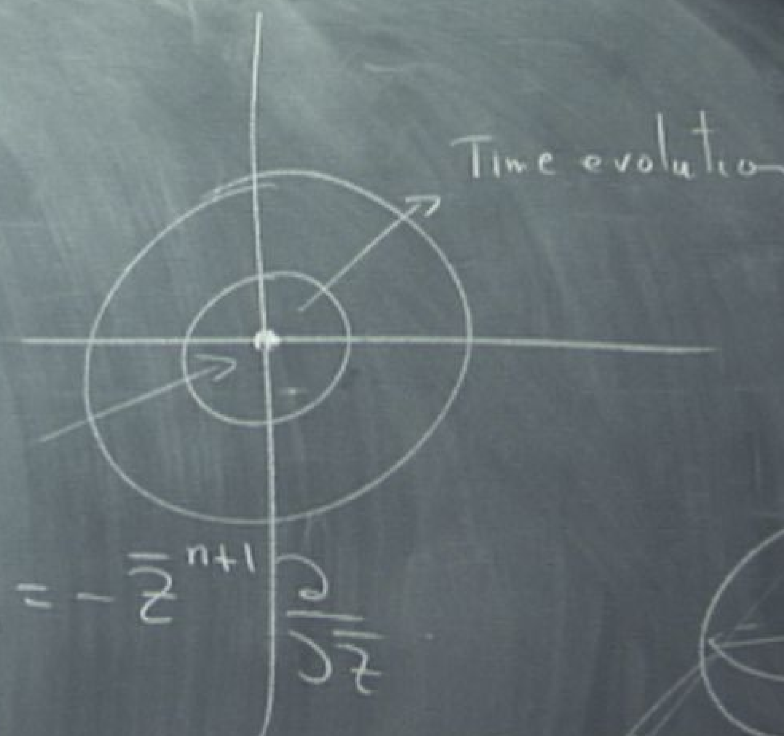
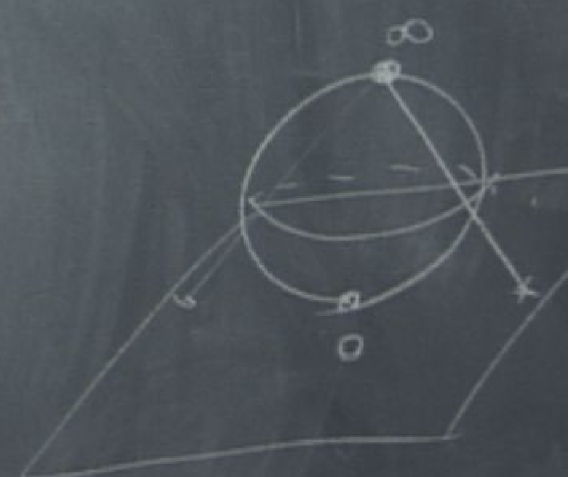
$$\ell_n = -z^{n+1}\frac{\partial}{\partial z} \qquad \hat{\ell}_n = -\bar{z}^{n+1}\frac{\partial}{\partial \bar{z}}$$

$$[\ell_n, \ell_m] = (n-m)\,\ell_{m+n}$$





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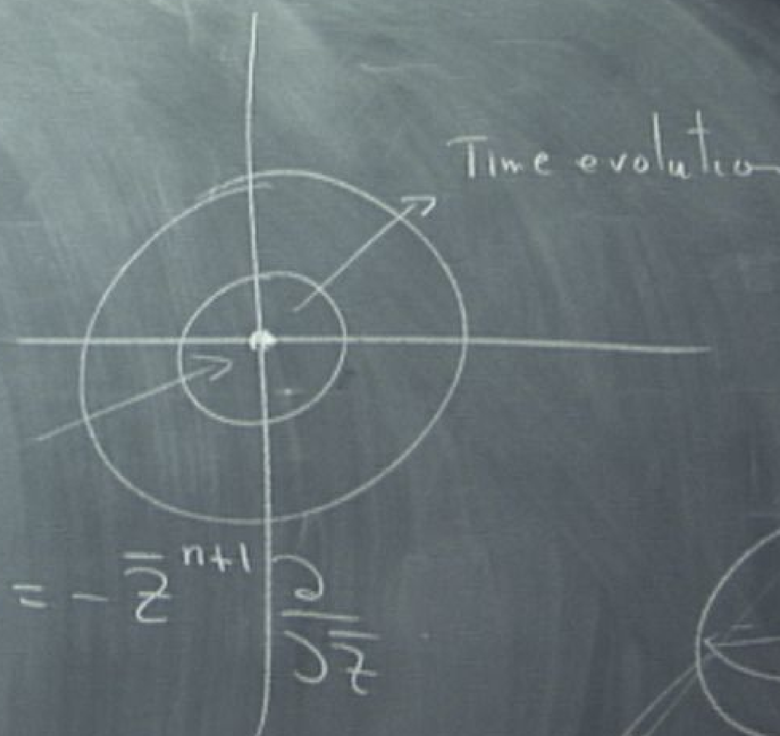
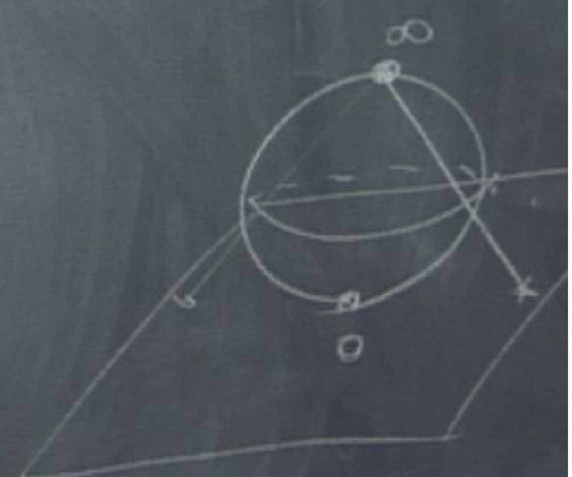
$$[l_n, l_m] = (n-m)\, l_{m+n}$$

$$[l_n, \tilde{l}_m] = 0$$





$\mathbb{C}$

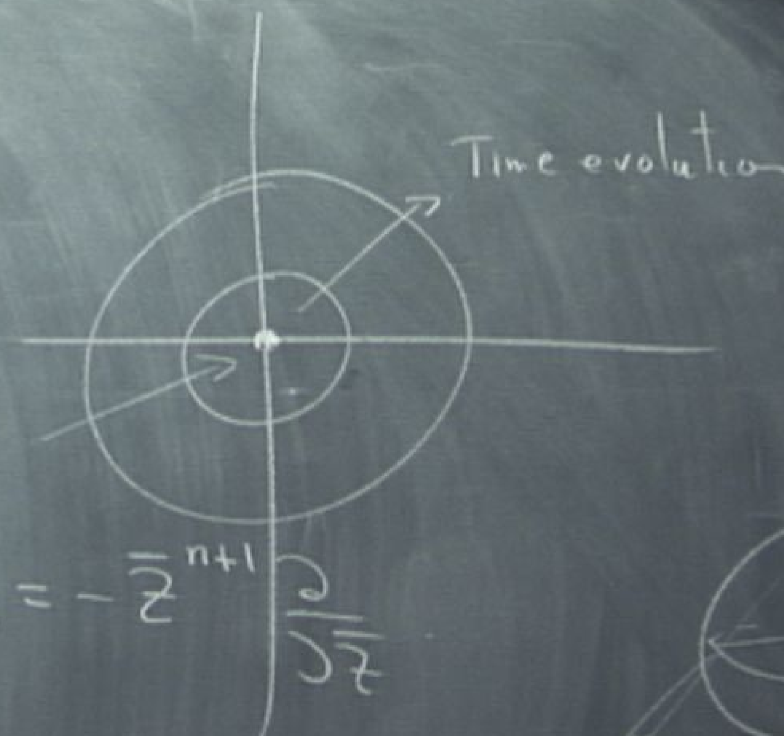
$$f(z) = \sum_{n=-\infty}^{\infty} a_n z^{n+1}$$

Generators:

$$z \to z' = z - \epsilon_n z^{n+1}$$

$$\delta z = \ell_n z$$

$$\ell_n = -z^{n+1}\frac{\partial}{\partial z} \qquad \bar{\ell}_n = -\bar{z}^{n+1}\frac{\partial}{\partial \bar{z}}$$



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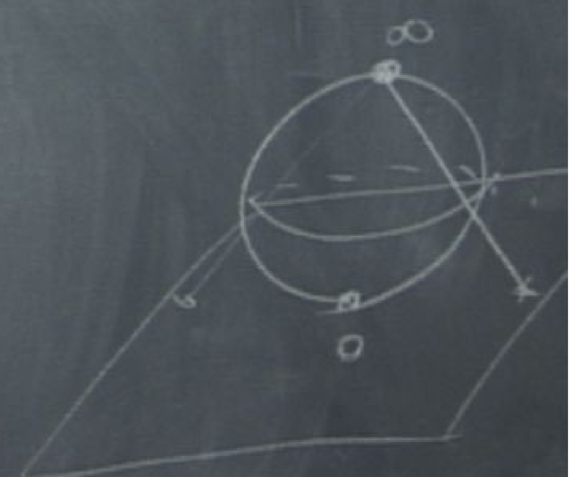
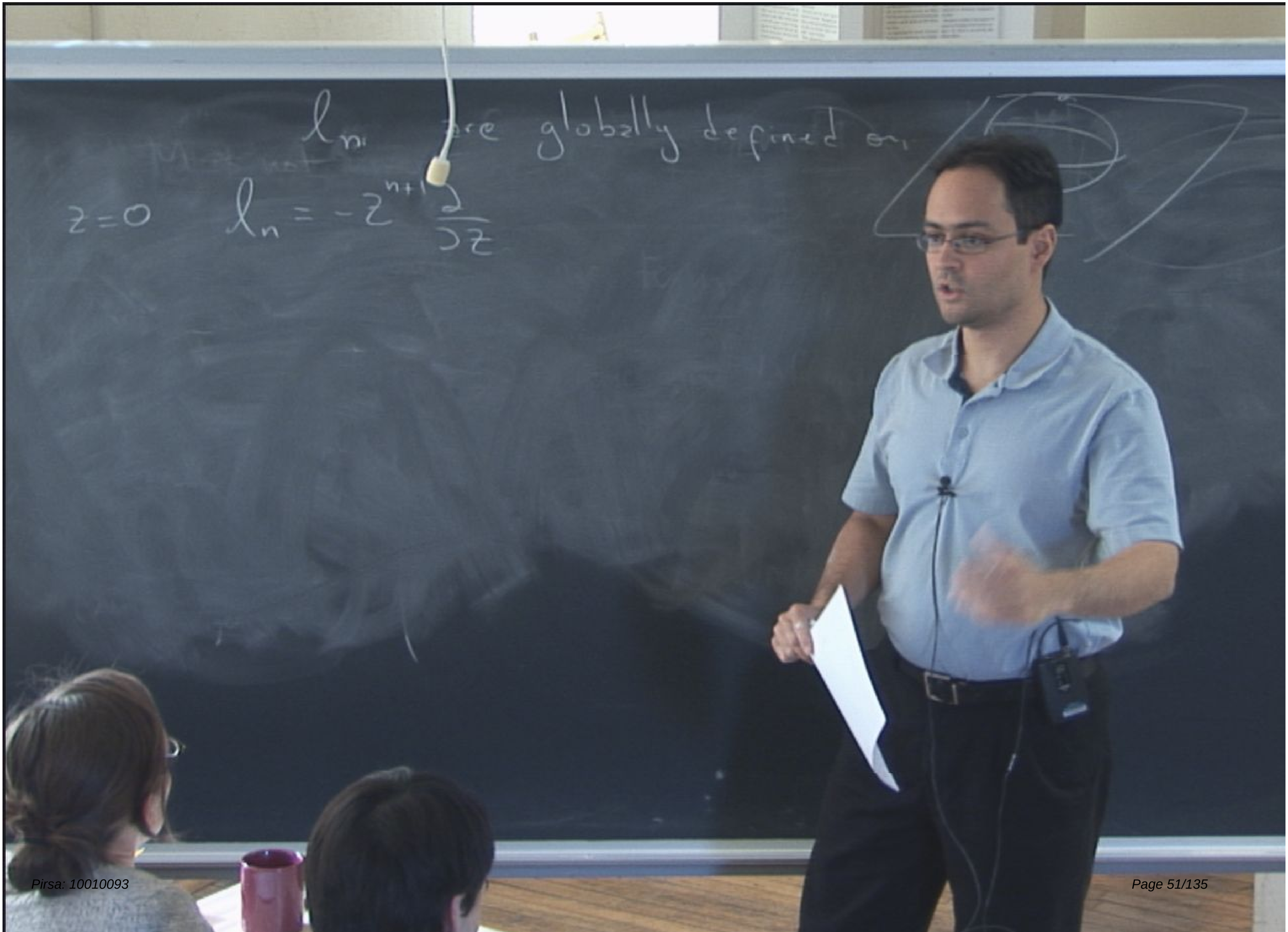
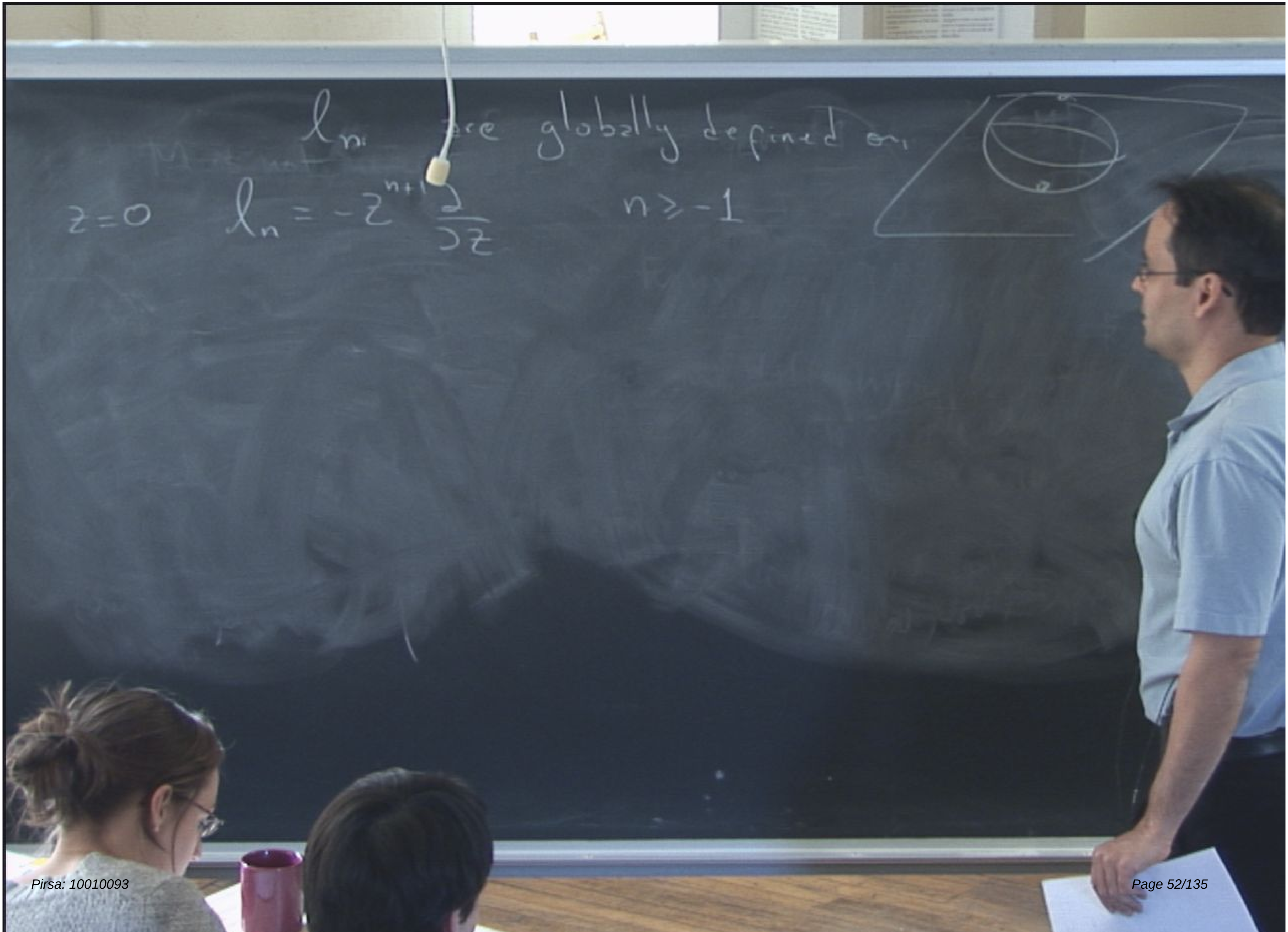



l_n are globally defined on
$z=0$ $l_n = -z^{n+1} \frac{\partial}{\partial z}$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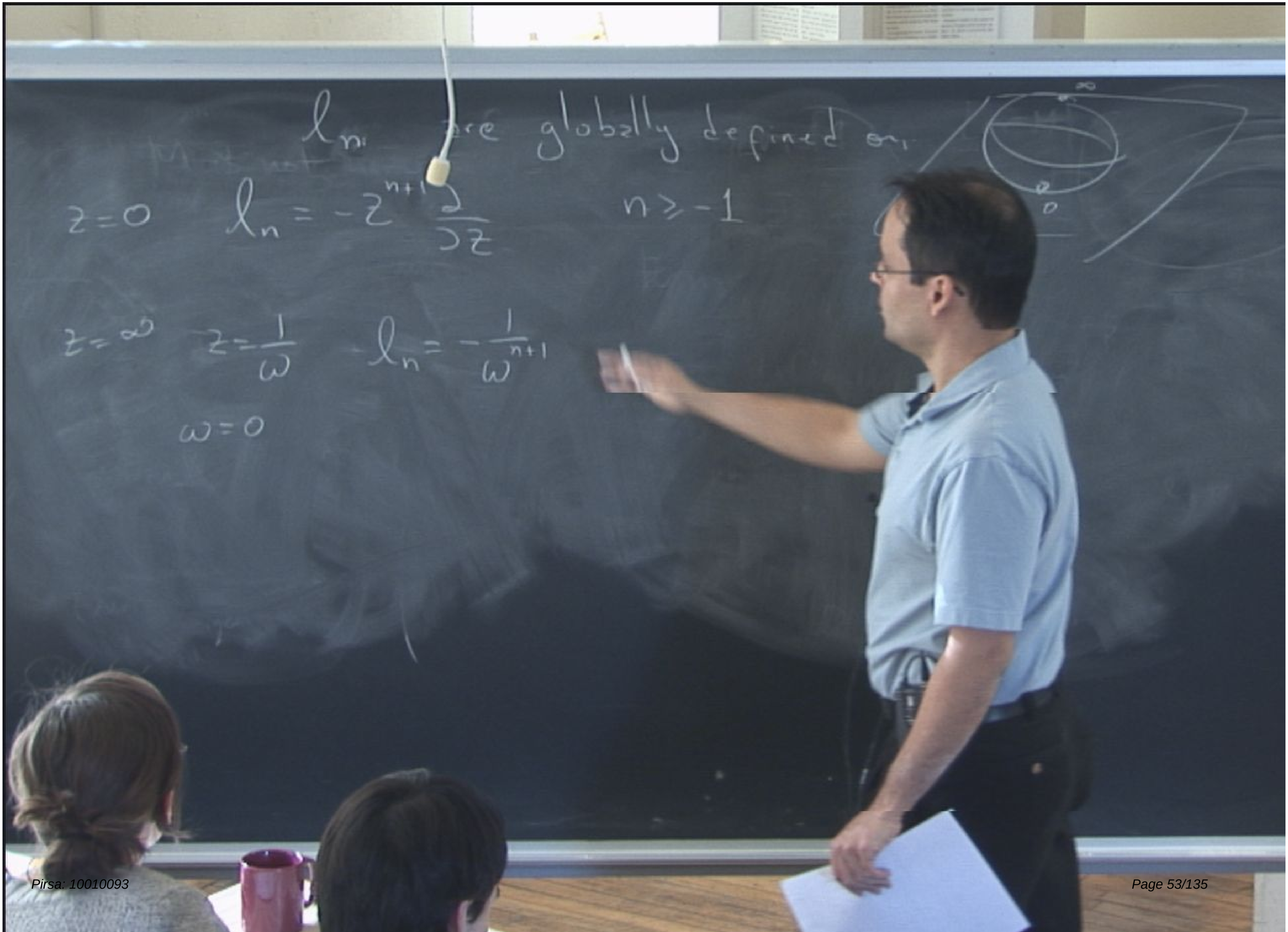
l_n are globally defined on
∞
0
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$\omega=0$

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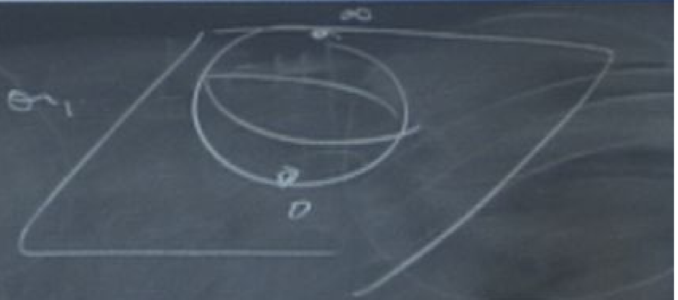

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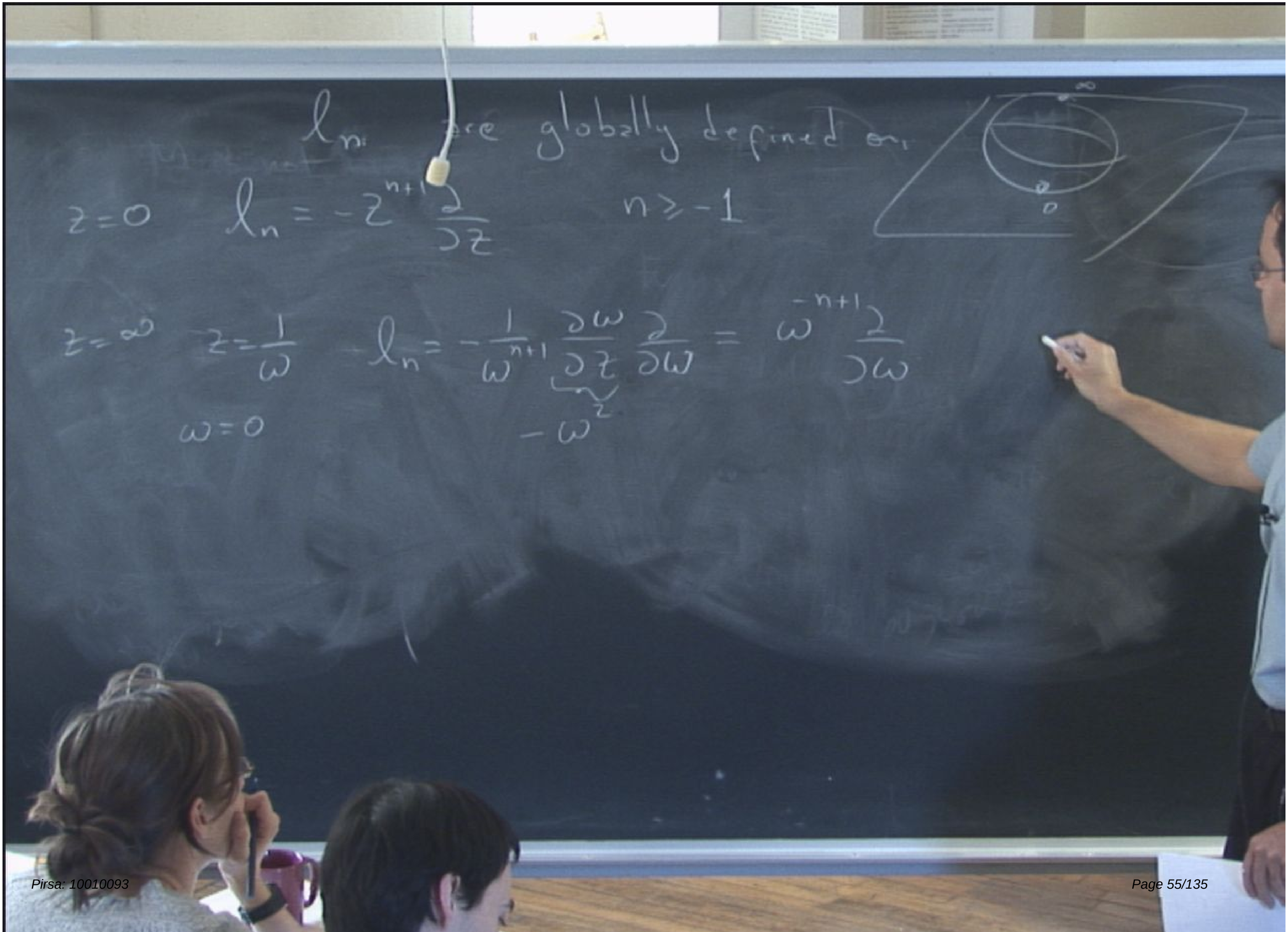
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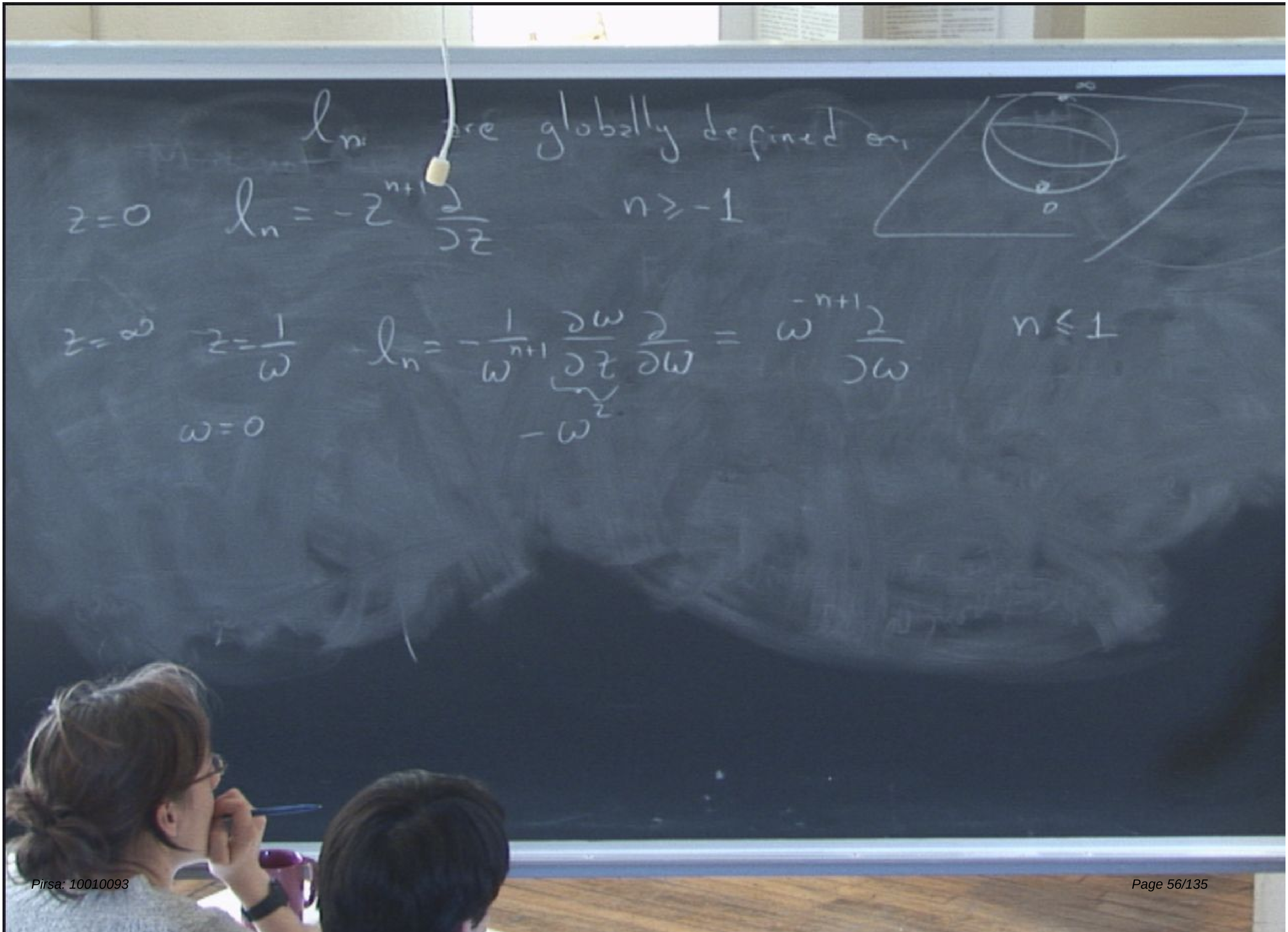
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$Lz.$

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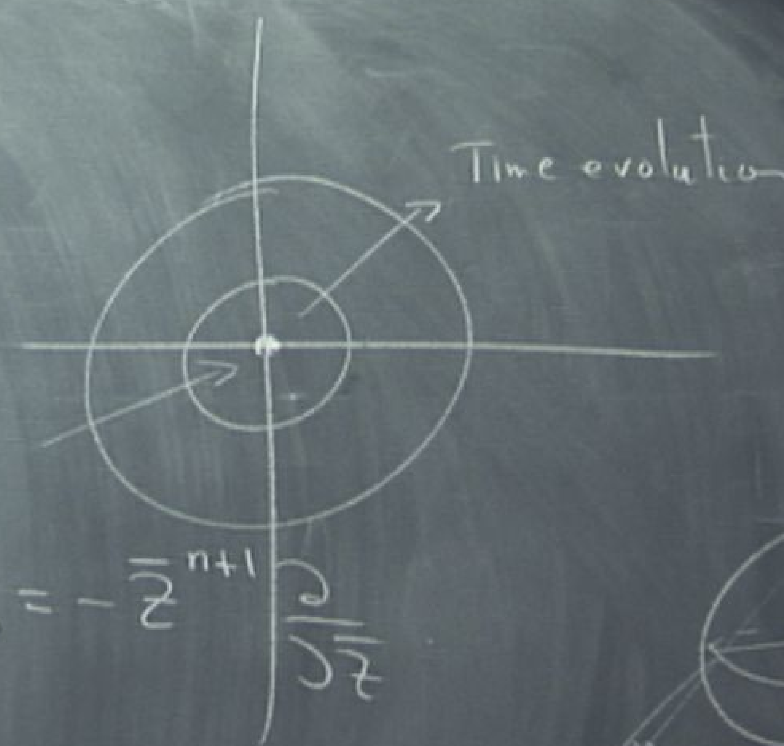

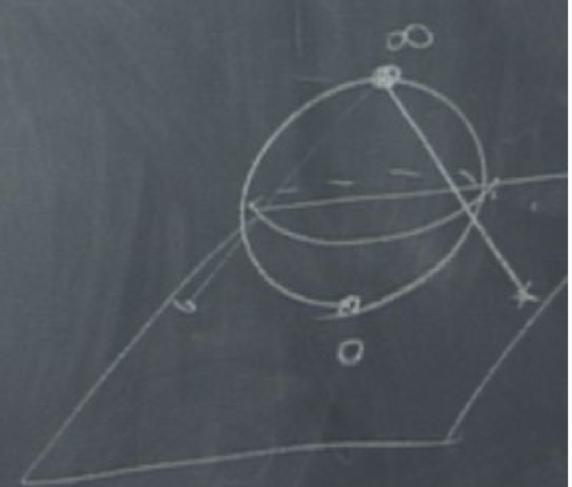

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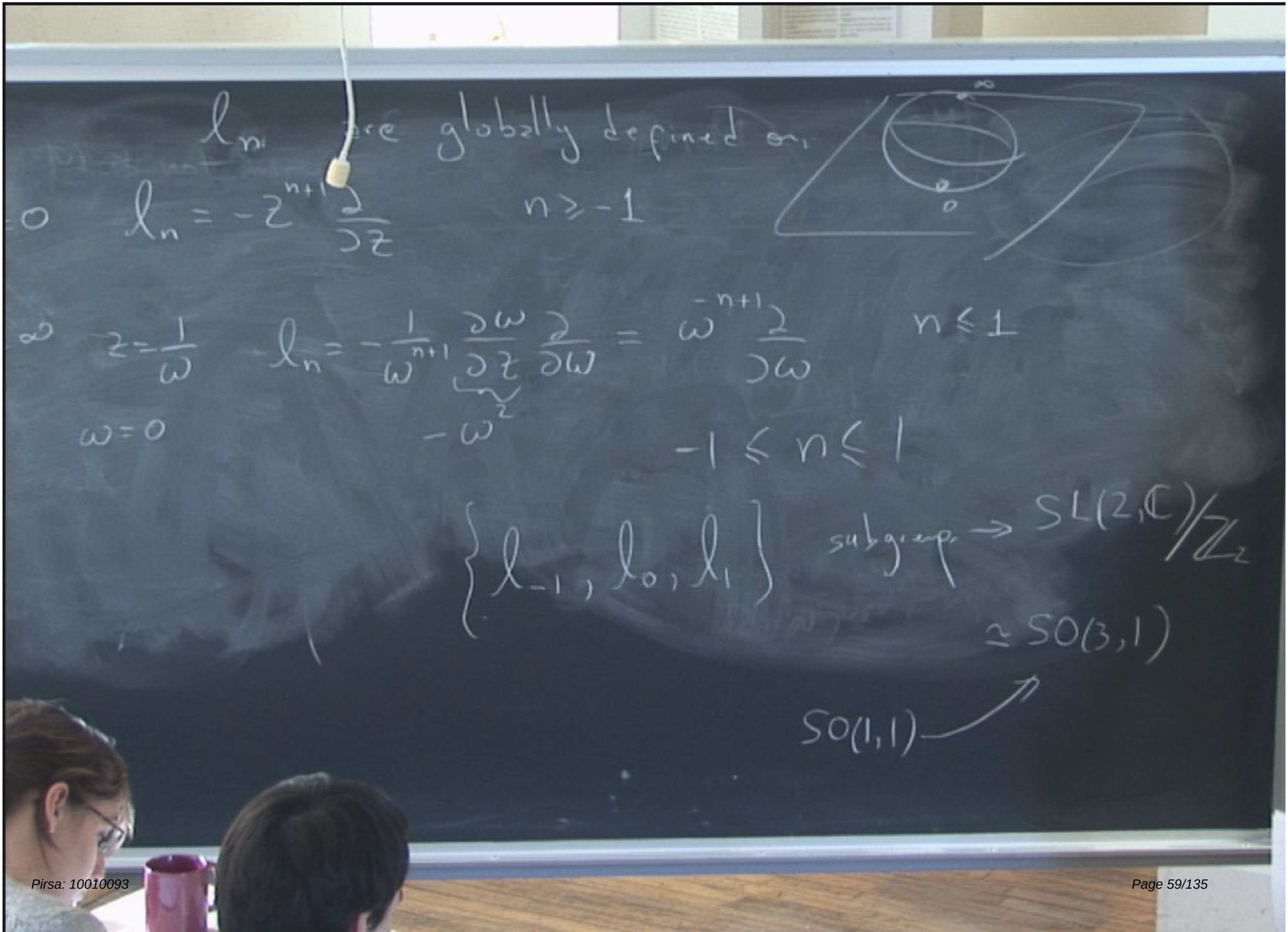
$\omega = 0$

$-1 \leq n \leq 1$

$\{l_{-1}, l_0, l_1\}$ subgroup $\rightarrow SL(2,\mathbb{C})/\mathbb{Z}_2$

$\simeq SO(3,1)$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∞
0

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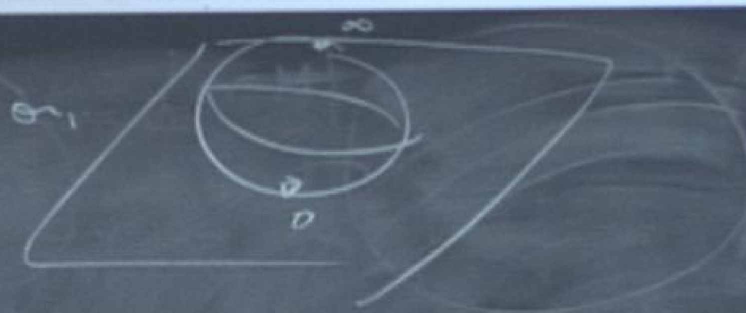

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$SO(2) \nearrow$

Energy-momentum Tensor.

$T_{++}\ ,\ T_{--} \qquad \partial^{\alpha} T_{\alpha\beta} = 0$

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$\partial_+ T_{+-} + \partial_- T_{--}$ $\qquad \beta \to -$

Energy-momentum Tensor.

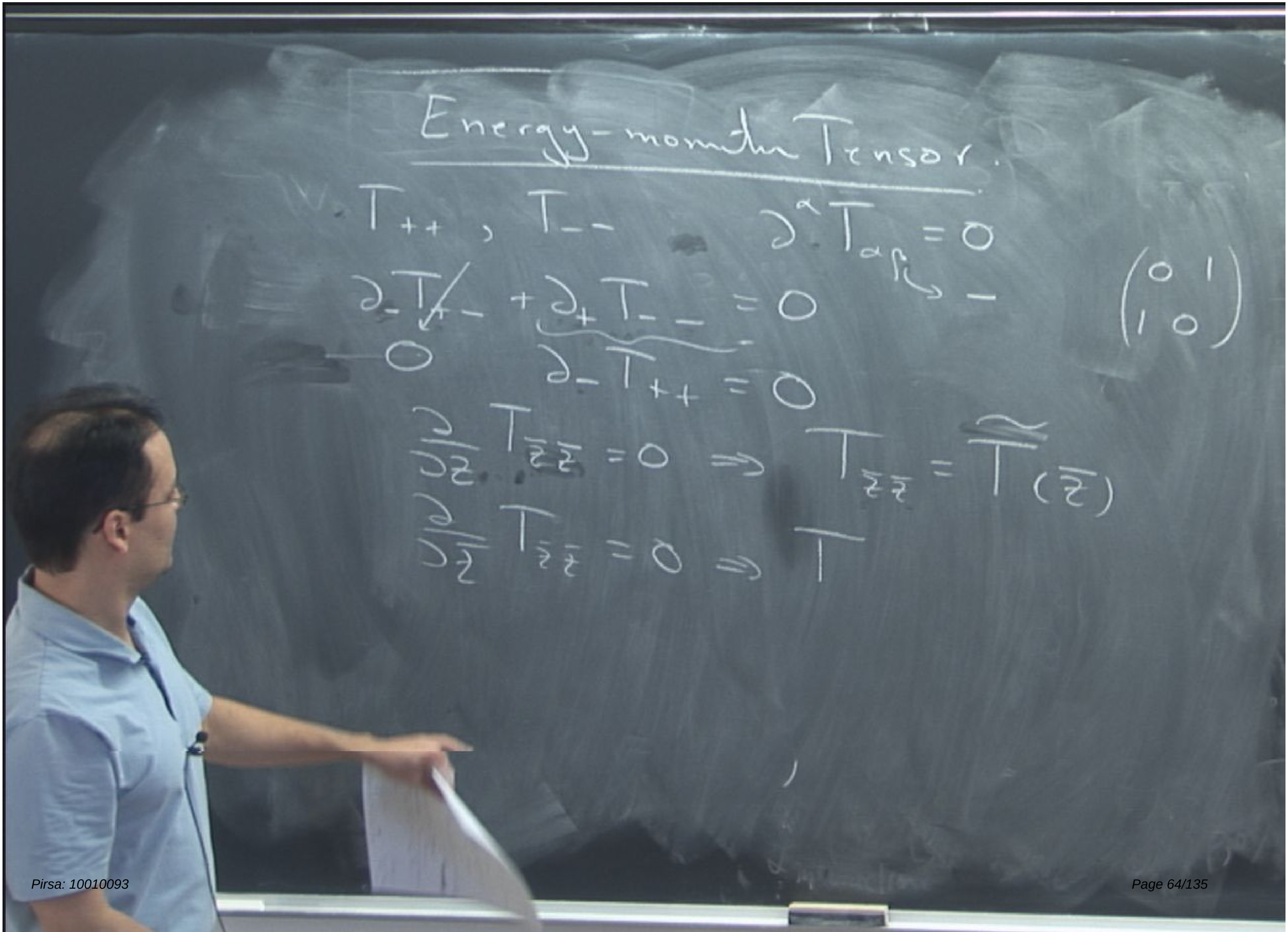
$T_{++}\,,\; T_{--} \qquad \partial^\alpha T_{\alpha\beta} = 0 \qquad \beta \to -$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\underbrace{\partial_- \cancel{T_{+-}}}_{0} + \partial_+ T_{--} = 0$

$\partial_- T_{++} = 0$

$\frac{\partial}{\partial \bar{z}} T_{zz} = 0 \Rightarrow T_{zz} = T(z)$


Energy-momentum Tensor.
T_{++} , T_{--}
$\partial^\alpha T_{\alpha\beta} = 0$
$\partial_- \cancel{T_{+-}} + \partial_+ T_{--} = 0$
0
$\partial_- T_{++} = 0$
$\frac{\partial}{\partial z} T_{\bar z \bar z} = 0 \Rightarrow T_{\bar z \bar z} = \tilde{T}(\bar z)$
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$0 \qquad \partial_- T_{++} = 0$

$$\frac{\partial}{\partial z} T_{\bar z \bar z} = 0 \Rightarrow T_{\bar z \bar z} = \tilde{\bar T}(\bar z)$$

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$$\partial X^\mu = -\frac{i}{2}\sum_{n=-\infty}^{\infty} \alpha^\mu_n z^{-n-1}$$

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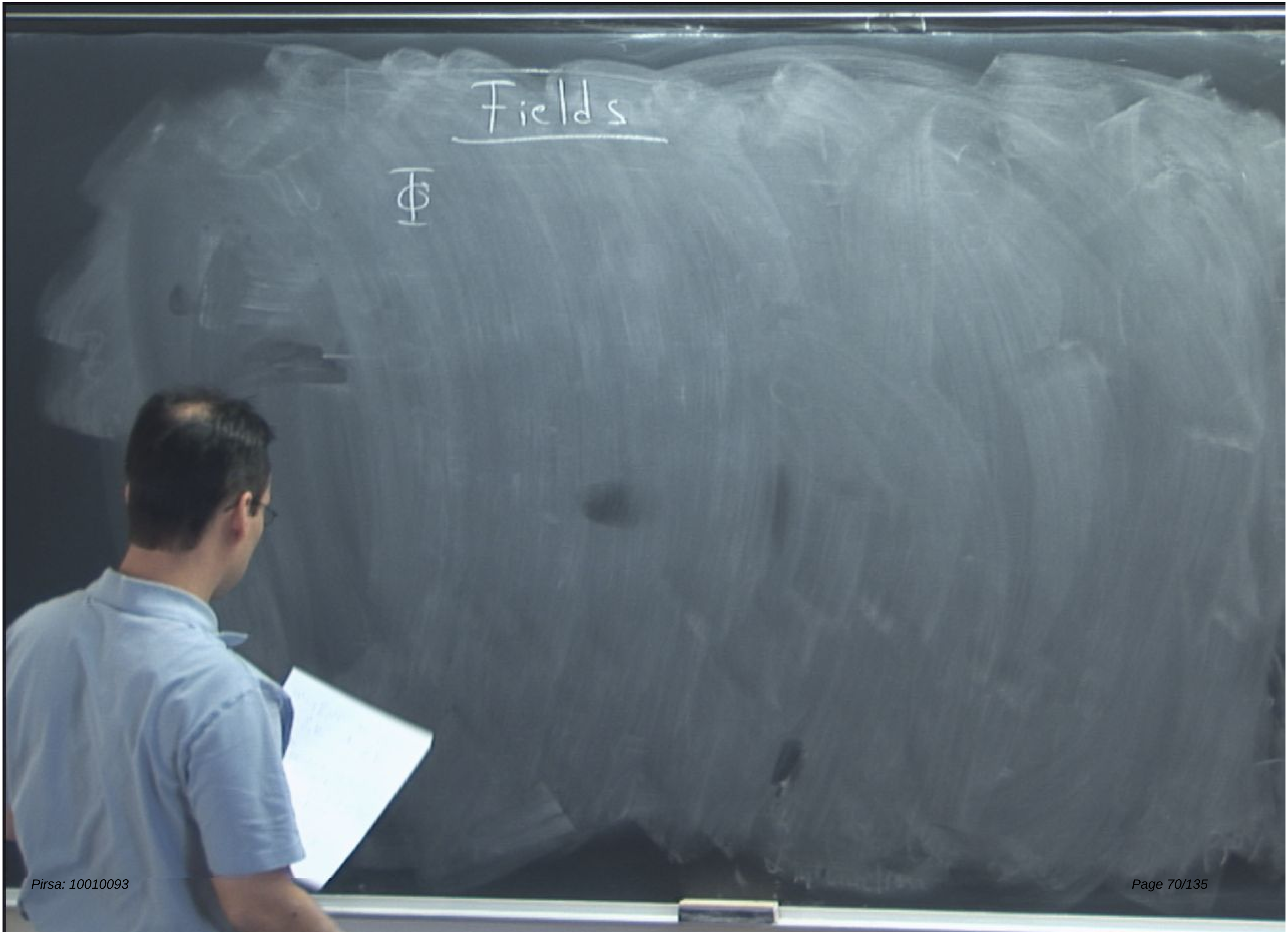
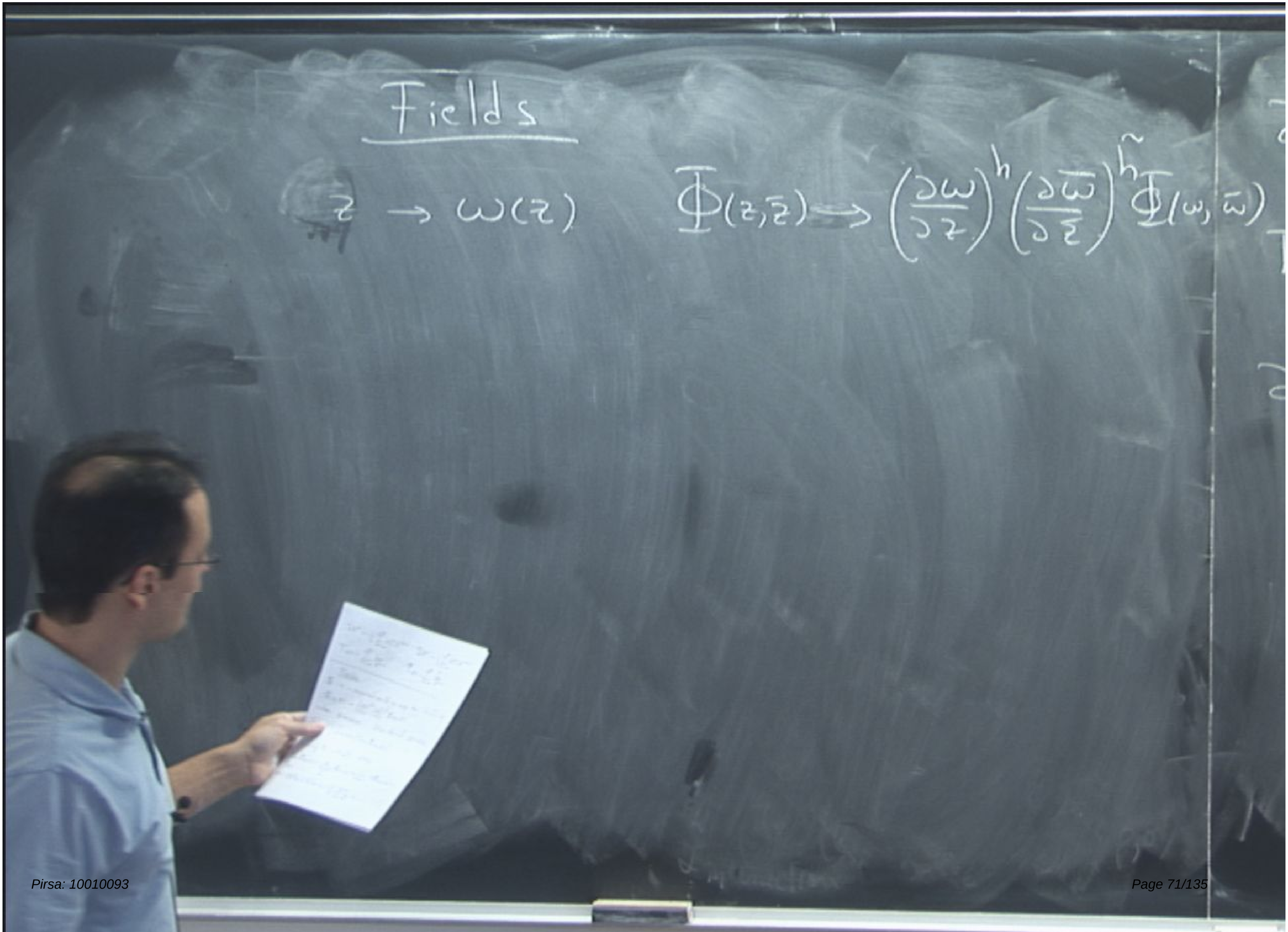

Fields
Φ


Fields
$z \to \omega(z)$
$\Phi(z,\bar z) \to \left(\frac{\partial \omega}{\partial z}\right)^{h}\left(\frac{\partial \bar\omega}{\partial \bar z}\right)^{\tilde h}\Phi(\omega,\bar\omega)$

Fields

$$z \to w(z) \qquad \Phi(z,\bar z) \to \left(\frac{\partial w}{\partial z}\right)^{h}\left(\frac{\partial \bar w}{\partial \bar z}\right)^{\tilde h}\Phi(w,\bar w)$$

conformal fields of conformal dimension $(h, \tilde h)$

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Infinitesimally.

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Infinitesimally: $\delta z = \epsilon(z)$

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$$\delta_\epsilon \Phi = \frac{1}{2\pi i}\oint dz\, \epsilon(z)\left[T(z), \Phi(\omega,\bar\omega)\right]$$

Fields

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, $\bar{\omega})$

OPE

$$T(z)\,\Phi(\omega,\bar{\omega})$$

n

OPE

$$T(z)\,\Phi(w,\bar{w}) = \frac{h}{(z-w)^2}\Phi(w,\bar{w}) + \frac{1}{z-w}\partial\Phi(w,\bar{w}) + \underbrace{\cdots}_{\text{non singular}}$$

OPE

$$T(z)\,\Phi(w,\bar{w}) = \frac{h}{(z-w)^2}\,\Phi(w,\bar{w}) + \frac{1}{z-w}\,\partial\Phi(w,\bar{w}) + \underbrace{\cdots}$$

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non singular

ex: $\partial X^{\mu}(z)\,\partial X^{\nu}(w)$

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ex: $\partial X^\mu(z)\,\partial X^\nu(w) = -\frac{1}{4}\frac{\eta^{\mu\nu}}{(z-w)^2} + \cdots$

OPE

$$T(z)\,\Phi(\omega,\bar\omega) = \frac{h}{(z-\omega)^2}\Phi(\omega,\bar\omega) + \frac{1}{z-\omega}\partial\Phi(\omega,\bar\omega) + \cdots$$

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ex: $\partial X^\mu(z)\,\partial X^\nu(\omega) = -\frac{1}{4}\frac{\eta^{\mu\nu}}{(z-\omega)^2} + \cdots$

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$$\langle 0|\partial X^\mu \partial X^\nu|0\rangle = -\frac{1}{4}\frac{\eta^{\mu\nu}}{(z-\omega)^2}\langle 0|0\rangle$$

$\langle 0|0\rangle = 1$

$$T(z)\,T(\omega) = \frac{c/2}{(z-\omega)^4} + \frac{2}{(z-\omega)^2}T(\omega) + \frac{\partial T(\omega)}{z-\omega} + \cdots +$$

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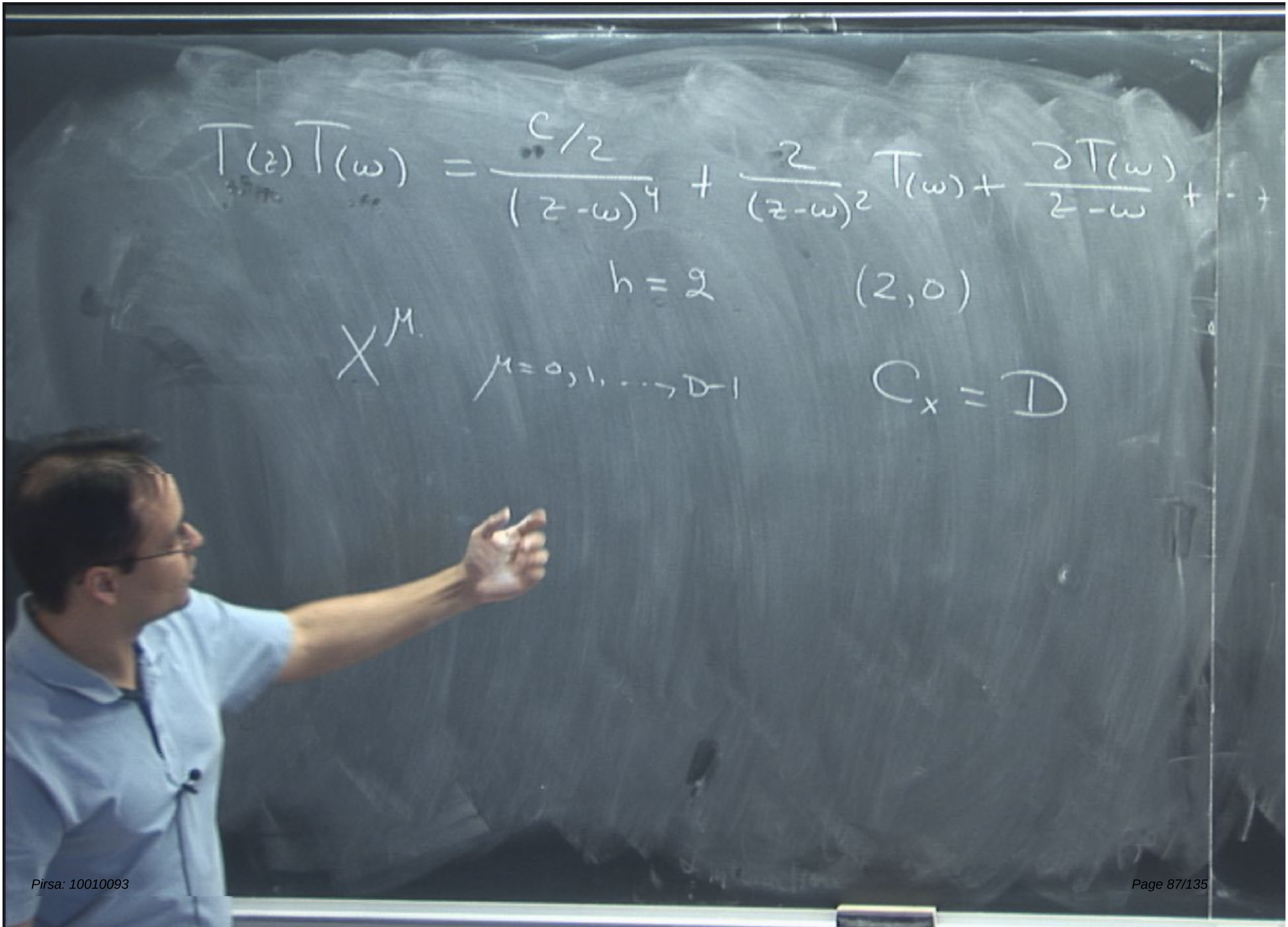
$h = 2 \qquad (2,0)$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$(2,0)$
X^μ
$\mu = 0, 1, \ldots, D-1$
$c_X = D$

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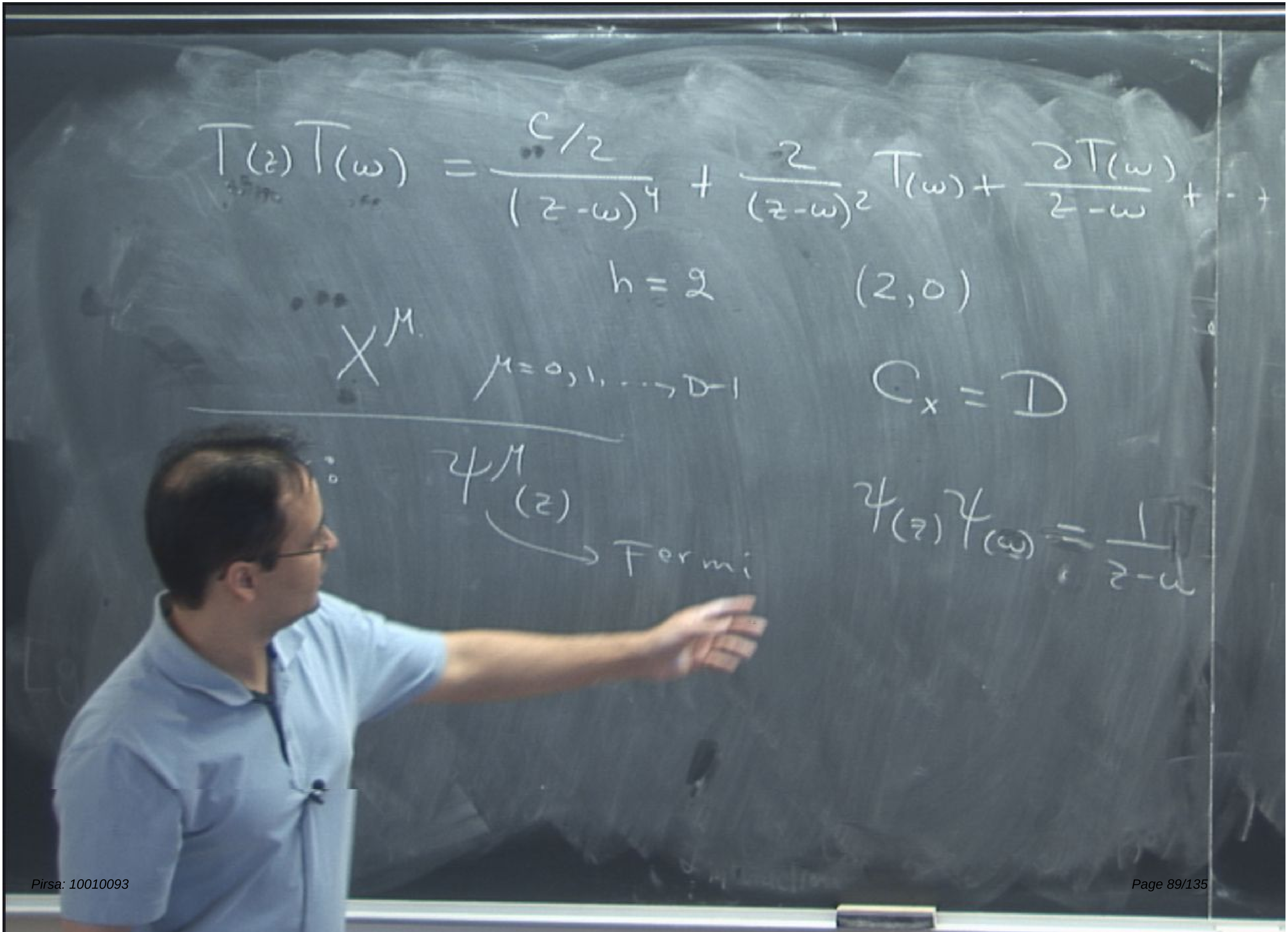
Ex: $\psi^{\mu}(z) \to$ Fermi



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X^μ
$\mu = 0, 1, \ldots, D-1$
$C_X = D$
$\psi^\mu(z)$
Fermi
$\psi(z)\psi(\omega) = \frac{1}{z-\omega}$

$$T(z)T(\omega) = \frac{c/2}{(z-\omega)^4} + \frac{2}{(z-\omega)^2}T(\omega) + \frac{\partial T(\omega)}{z-\omega} + \cdots$$

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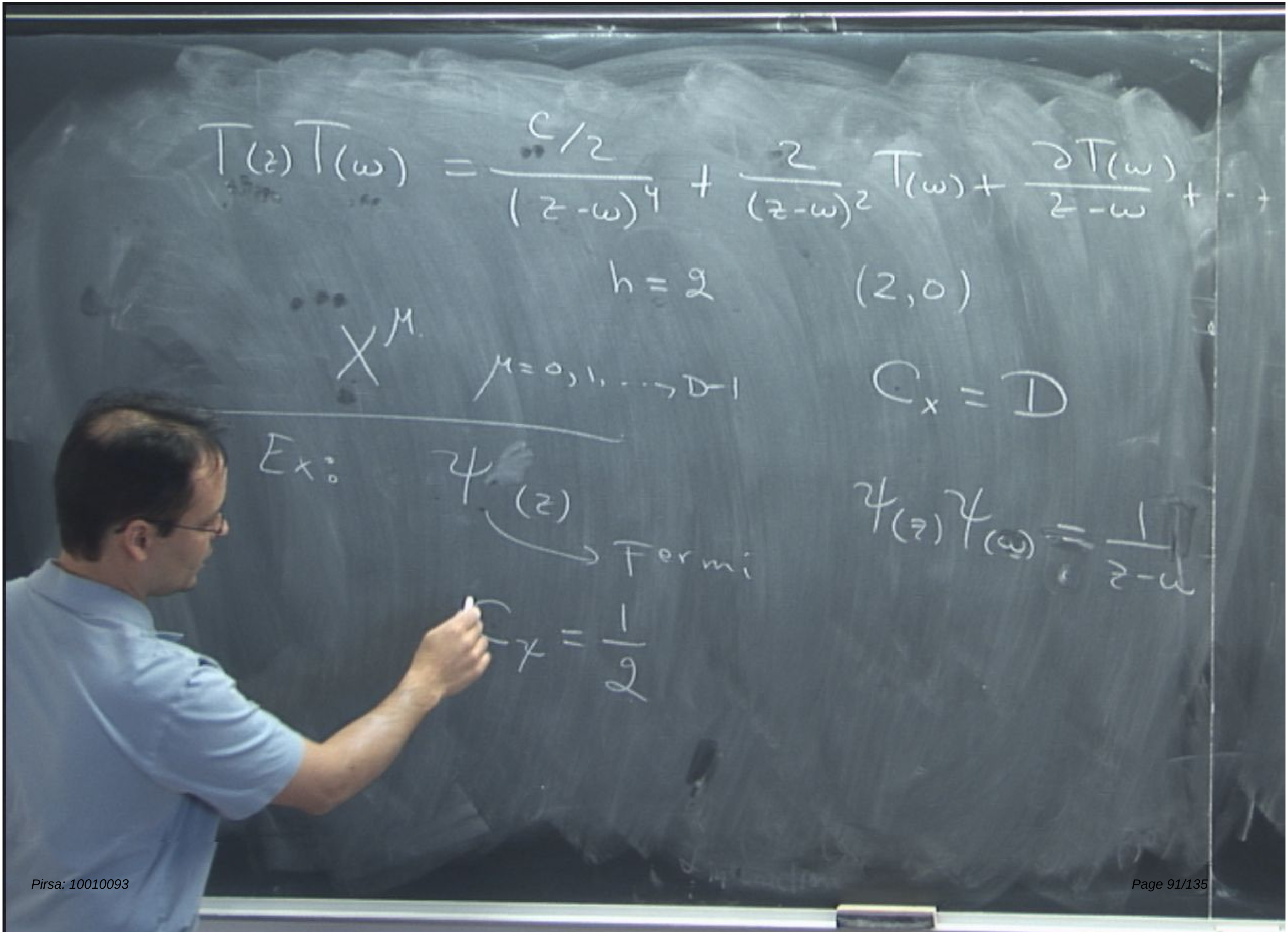
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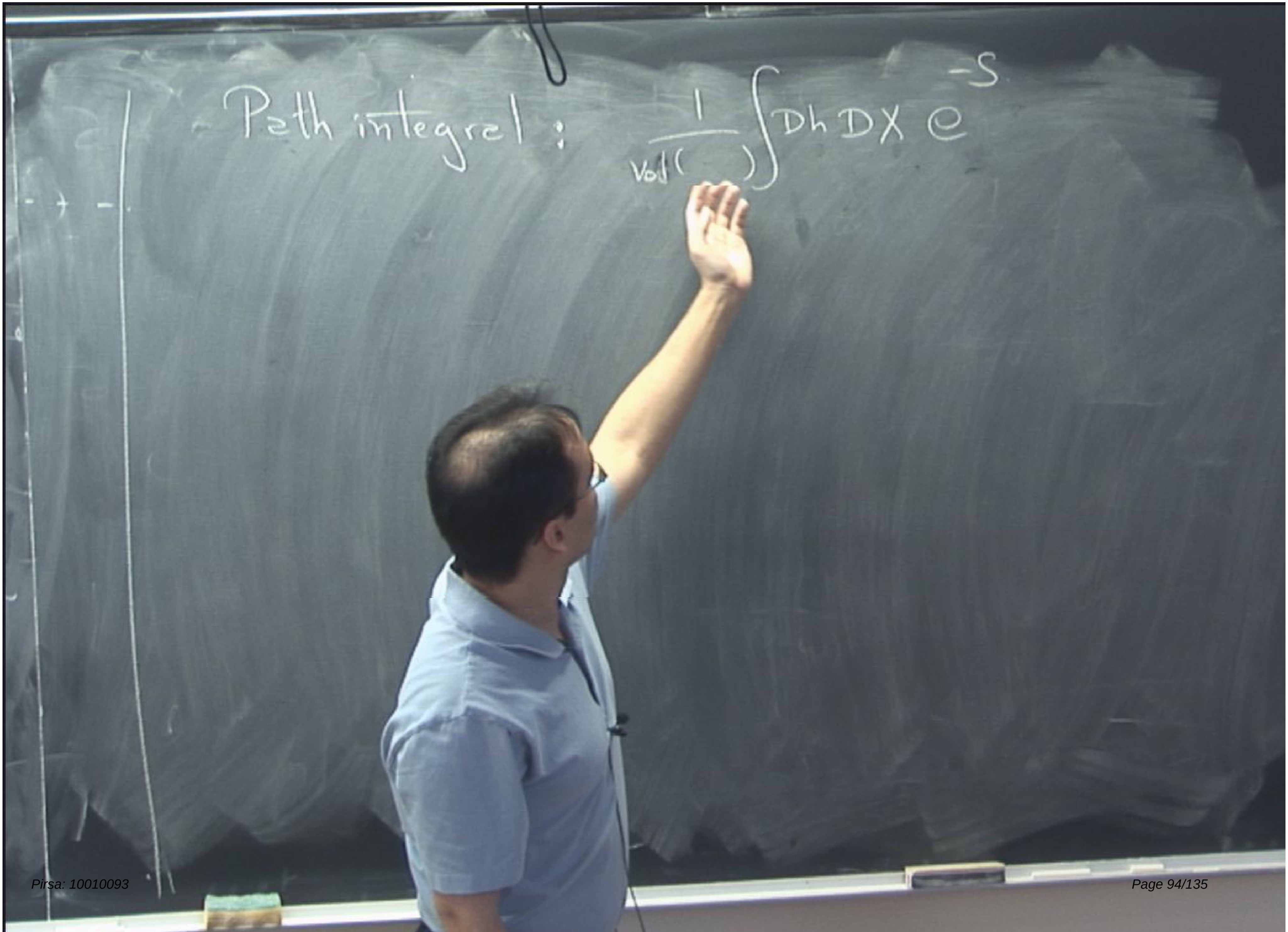
Path integral: $\int Dh\, DX\, e^{-S}$


Path integral : $\frac{1}{\mathrm{Vol}(\ \ )}\int Dh\, DX\, e^{-S}$

- Path integral : $\frac{1}{\mathrm{Vol}(\quad)} \int Dh\, DX\, e^{-S}$

- Faddeev - Popov

Path integral : $\frac{1}{\mathrm{Vol}(\quad)}\int Dh\, DX\, e^{-S}$

Faddeev - Popov

Ghost

Path integral : $\frac{1}{\mathrm{Vol}(\quad)} \int Dh\, DX\, e^{-S}$

Faddeev - Popov

Ghost b, c

Path integral : $\frac{1}{\mathrm{Vol}(\quad)} \int Dh\, DX\, e^{-S_{[X,h]} - S_{F.P.}}$

Faddeev - Popov

Ghost $\quad b_{(z))} \quad c_{(z)}$

$\quad\quad \hookrightarrow 2 \quad\quad \hookrightarrow -1$

Path integral : $\frac{1}{\mathrm{Vol}(\quad)}\int Dh\,DX\,e^{-S_{[X,h]}-S_{F.P.}}$

Faddeev-Popov

Ghost $\quad b_{(z)} \quad c_{(z)} \qquad c_{bc} = -26$

$\quad\hookrightarrow 2 \quad \hookrightarrow -1$

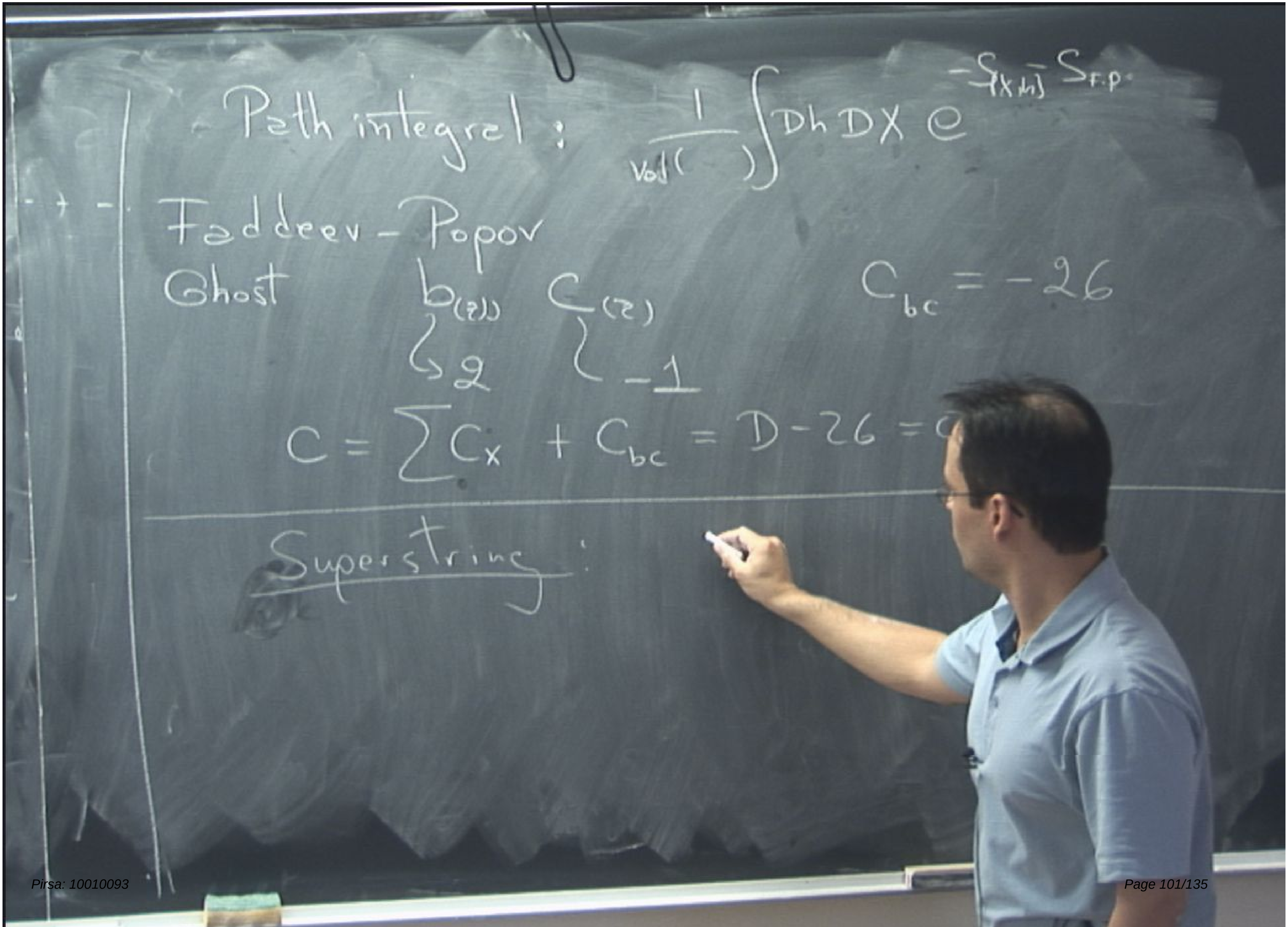
Path integral: $\frac{1}{\mathrm{Vol}(\ \ )}\int Dh\,DX\, e^{-S_{[X,h]}-S_{F.P.}}$

Faddeev–Popov

Ghost $\quad b_{(z)} \quad c_{(z)} \qquad\qquad c_{bc} = -26$

$\quad 2 \qquad -1$

$$c = \sum c_X + c_{bc} = D - 26 = 0$$



Path integral : $\frac{1}{\mathrm{Vol}(\ )}\int Dh\, DX\, e^{-S_{[X,h]}-S_{F.P.}}$
Faddeev-Popov
Ghost
$b_{(z)}$ $c_{(z)}$
2 -1
$c_{bc} = -26$
$c = \sum c_X + c_{bc} = D - 26 =$
Superstring :

Path integral : $\frac{1}{\mathrm{Vol}(\quad)} \int Dh\, DX\, e^{-S_{[X,h]} - S_{F.P.}}$

Faddeev – Popov

Ghost $\quad b_{(z)} \quad c_{(z)} \qquad c_{bc} = -26$

$\quad \hookrightarrow 2 \quad \hookrightarrow -1$

$$c = \sum c_X + c_{bc} = D - 26 = 0$$

Superstring : $\quad c_X + c_\psi = 1 + \frac{1}{2} = \frac{3}{2}$

$$c = \underbrace{\sum (c_X + c_\psi)}_{\frac{3}{2} D}$$

Path integral : $\frac{1}{\mathrm{Vol}(\quad)} \int Dh\, DX\, e^{-S_{\{X,h\}} - S_{F.P.}}$

Faddeev - Popov

Ghost $\quad b_{(z)} \quad c_{(z)} \qquad C_{bc} = -26$

$\quad \hookrightarrow 2 \quad \hookrightarrow -1$

$$C = \sum C_X + C_{bc} = D - 26 = 0$$

Superstring : $\quad C_X + C_\psi = 1 + \frac{1}{2} = \frac{3}{2}$

$$C = \underbrace{\sum (C_X + C_\psi)}_{\frac{3}{2} D} + \underset{-26}{\overset{\|}{C_{bc}}}$$

Path integral : $\frac{1}{\mathrm{Vol}(\quad)} \int Dh\, DX\, e^{-S_{[X,h]} - S_{F.P.}}$

Faddeev – Popov

Ghost $\quad b_{(z)} \quad c_{(z)} \qquad c_{bc} = -26$

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Path integral : $\frac{1}{\mathrm{Vol}(\quad)}\int Dh\, DX\, e^{-S_{[X,h]} - S_{F.P.}}$

Faddeev – Popov

Ghost $\quad b_{(z)} \quad c_{(z)} \qquad\qquad c_{bc} = -26$

$\qquad\quad \hookrightarrow 2 \qquad \hookrightarrow -1$

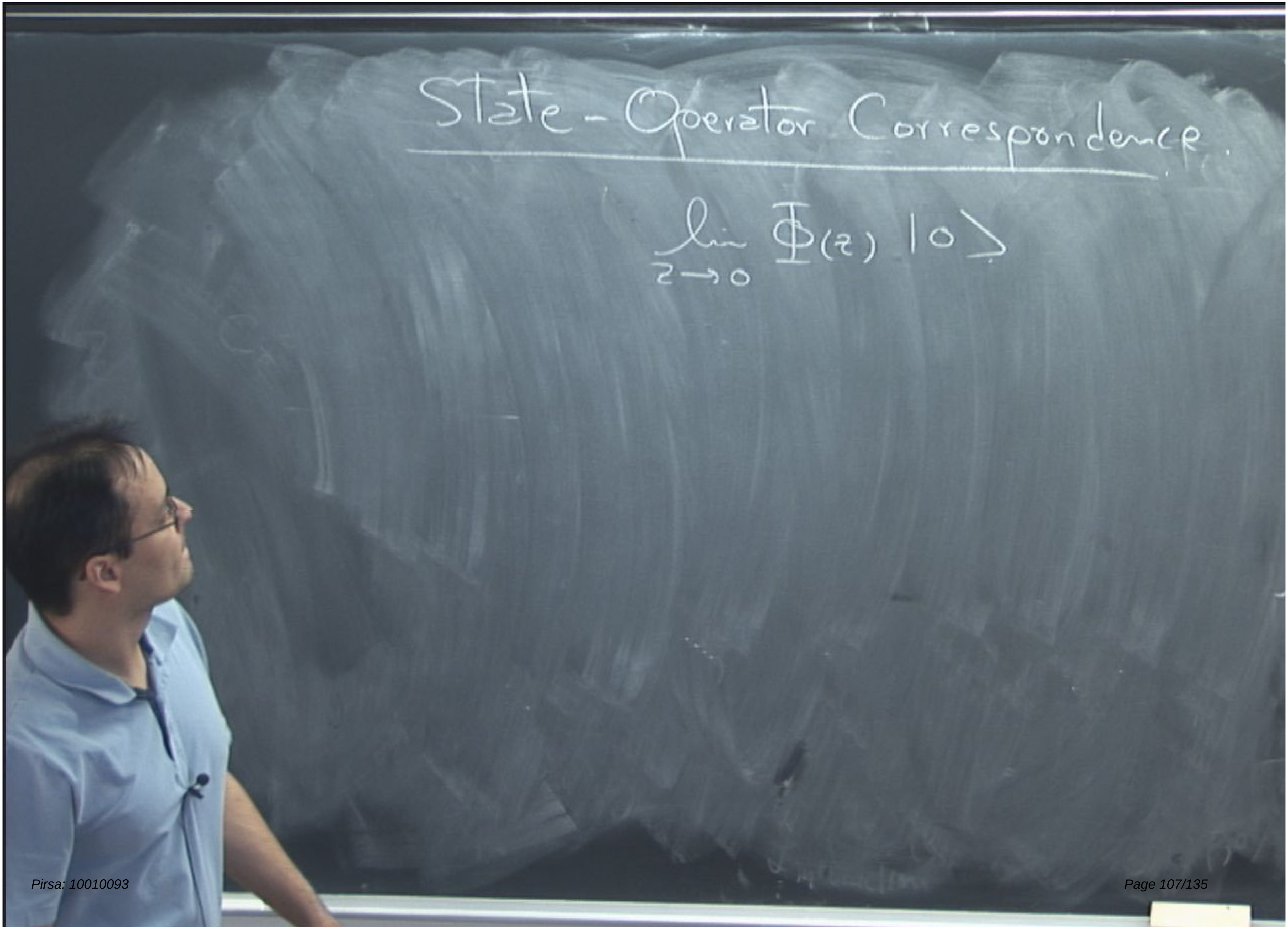
$$c = \sum c_X + c_{bc} = D - 26 = 0$$

Superstring : $\quad c_X + c_\psi = 1 + \frac{1}{2} = \frac{3}{2}$

$$c = \underbrace{\sum (c_X + c_\psi)}_{\frac{3}{2}D} + \underbrace{c_{bc}}_{-26} + \underbrace{c_{\text{New super}}}_{+11} = 0$$

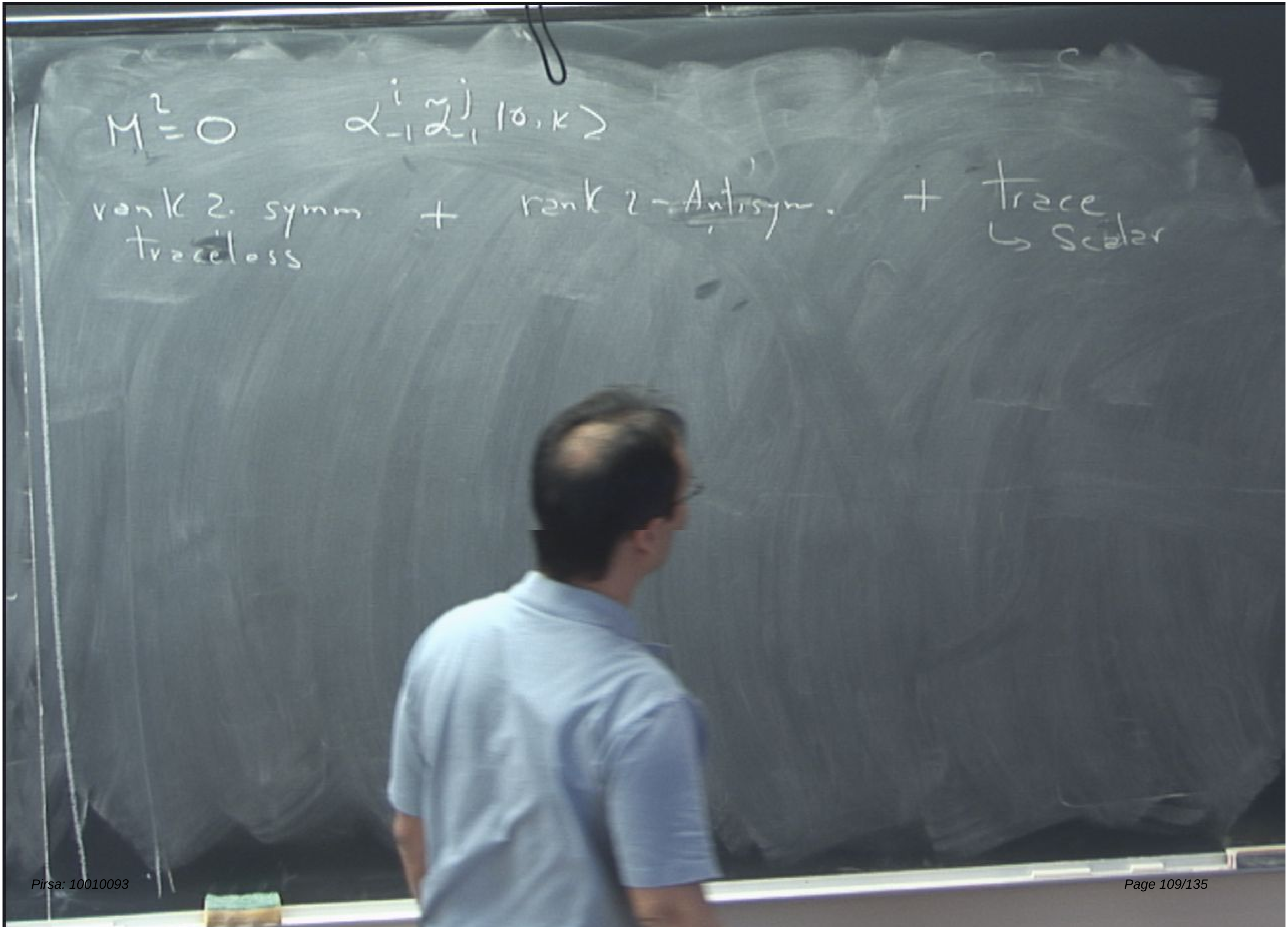
$\frac{3}{2}D = 15$

$\boxed{D = 10}$


State-Operator Correspondence
$\lim_{z \to 0} \Phi(z) |0\rangle$

State - Operator Correspondence

$$|\Phi\rangle = \lim_{z\to 0} \Phi(z)\,|0\rangle$$


$M^2=0$
$\alpha^i_{-1}\tilde{\alpha}^j_{-1}|0,k\rangle$
rank 2 symm traceless
+
rank 2 - Antisym.
+
trace
↳ Scalar

$M^2 = 0 \quad \alpha^i_{-1} \tilde{\alpha}^j_{-1} |0, k\rangle$

rank 2 symm. + rank 2 - Antisym. + trace

(traceless

↳ Scalar

$M^2 = 0 \qquad \alpha^i_{-1}\tilde{\alpha}^j_{-1}|0,k\rangle$

rank 2. symm + rank 2 - Antisym. + trace

traceless

$g_{\mu\nu}(X)$

graviton.

$B_{\mu\nu}(X)$

Ant

2-form

B

Scalar

$\Phi(X)$

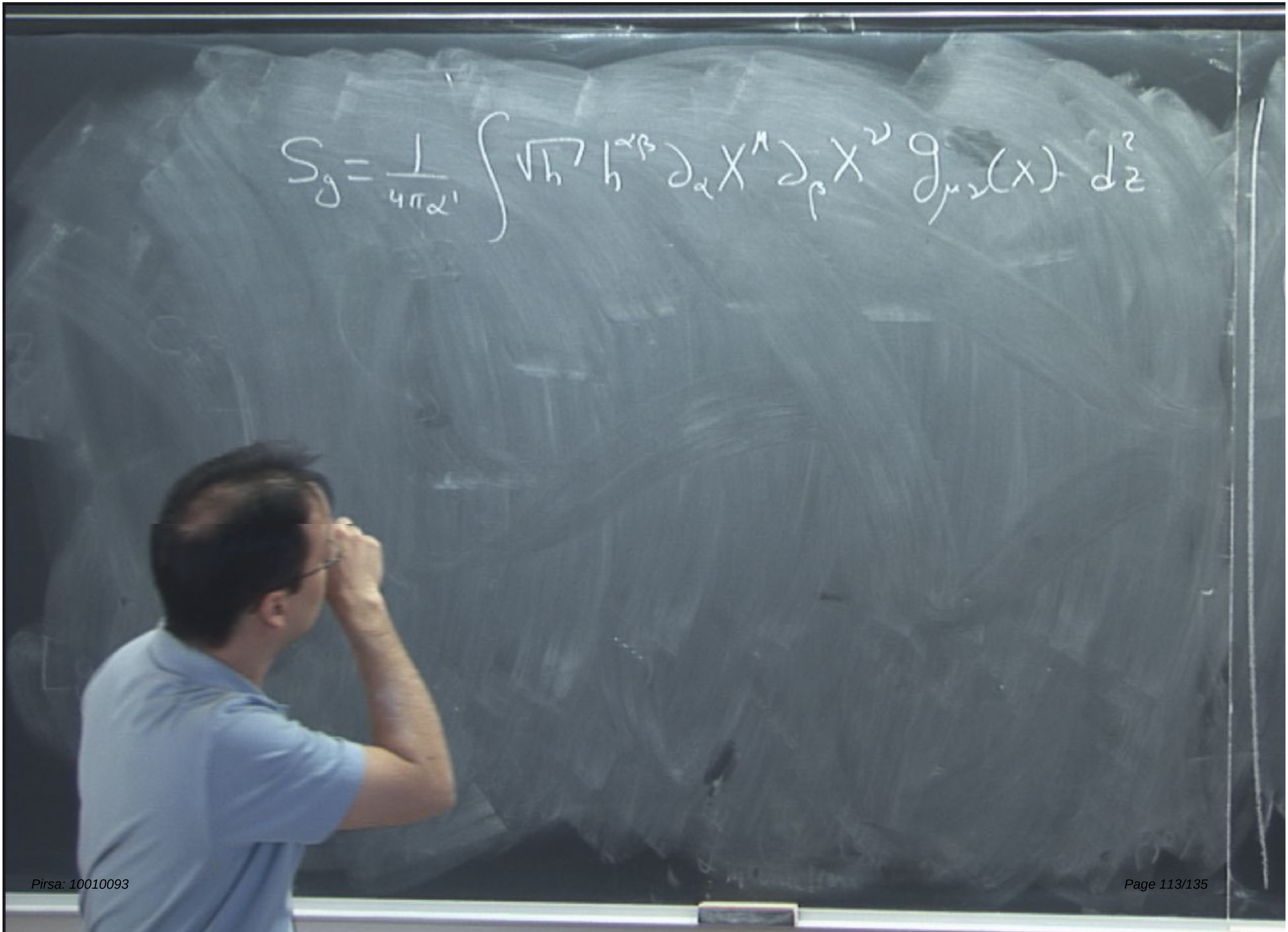
$M^2 = 0 \qquad \alpha_{-1}^i \tilde{\alpha}_{-1}^j |0, k\rangle$

rank 2 symm. + rank 2 - Antisym. + trace

traceless $\to g_{\mu\nu}(X)$ $\to$ graviton.

$\to B_{\mu\nu}(X)$ Ant $\to$ 2-form B

$\to$ Scalar $\Phi(X)$

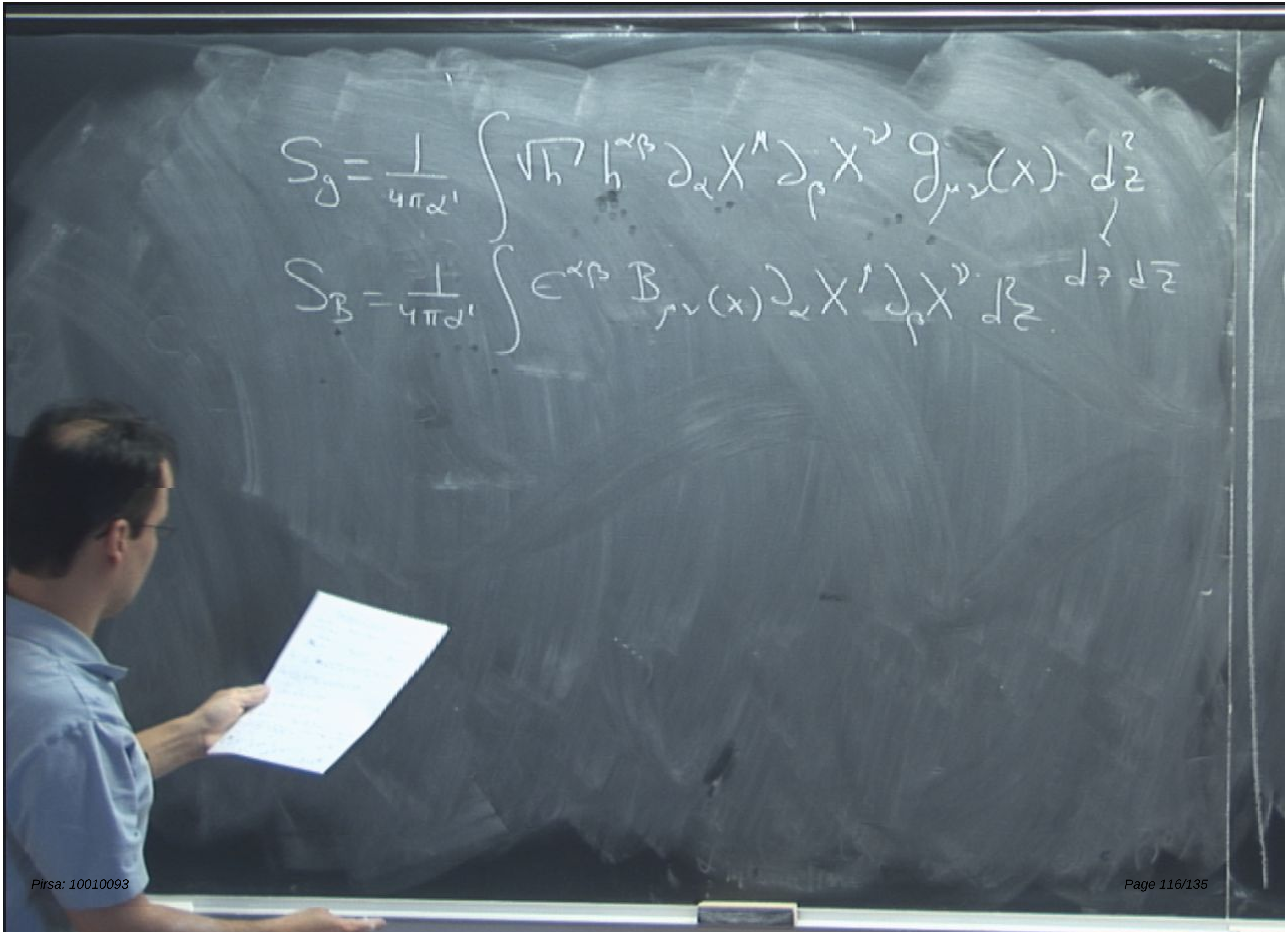

$S_g = \frac{1}{4\pi\alpha'} \int \sqrt{h}\, h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X)\, d^2z$

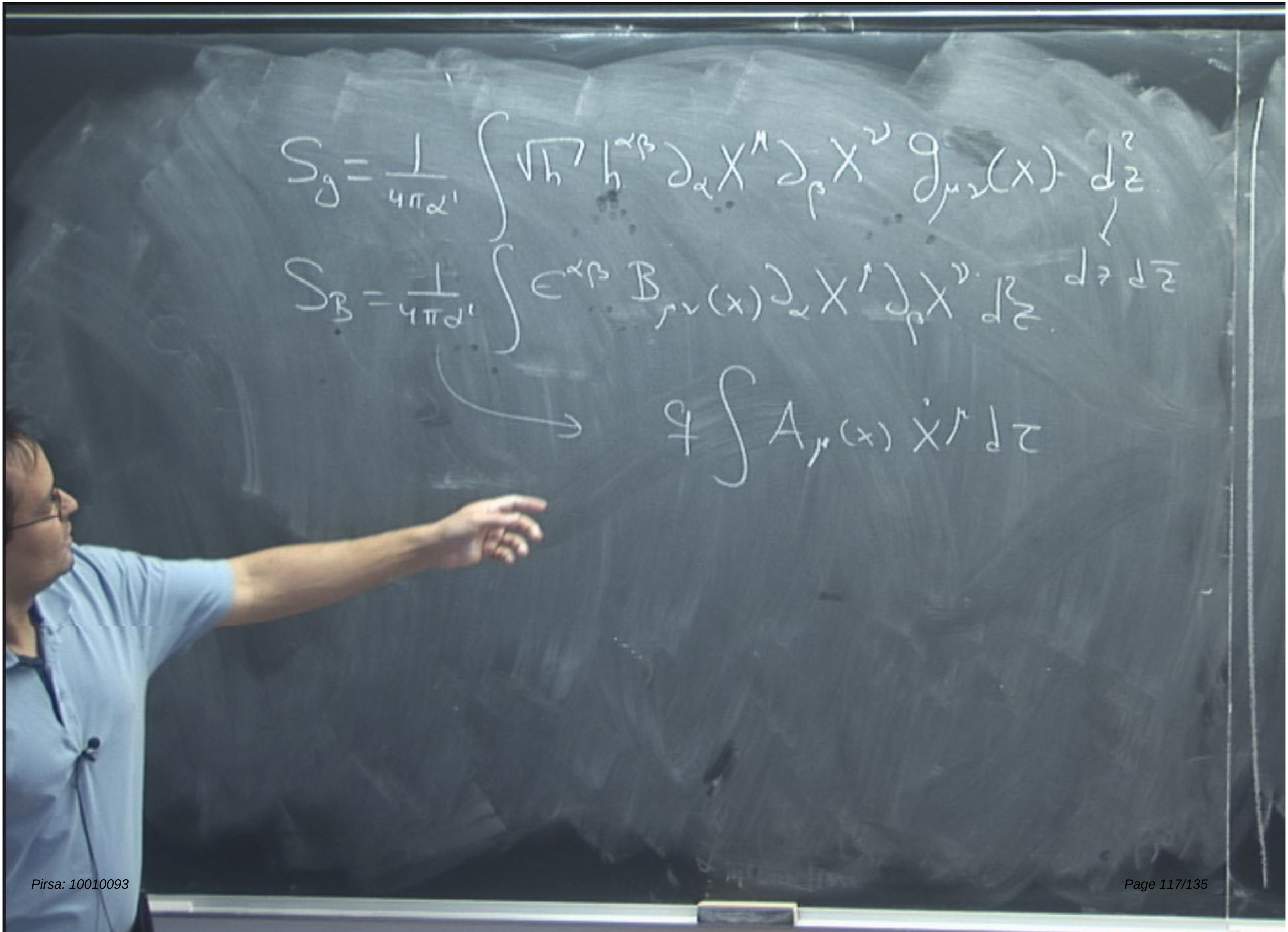
$$S_g = \frac{1}{4\pi\alpha'}\int \sqrt{h}\, h^{\alpha\beta}\,\partial_\alpha X^\mu\, \partial_\beta X^\nu\, g_{\mu\nu}(X)\, d^2 z$$

$$d z\, d\bar{z}$$

$$S_g = \frac{1}{4\pi\alpha'} \int \sqrt{h}\, h^{\alpha\beta}\, \partial_\alpha X^\mu \partial_\beta X^\nu\, g_{\mu\nu}(X)\, d^2 z$$

$$d z\, d\bar{z}$$


$S_g = \frac{1}{4\pi\alpha'} \int \sqrt{h}\, h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X)\, d^2z$
$S_B = \frac{1}{4\pi\alpha'} \int \epsilon^{\alpha\beta} B_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu\, d^2z$
$dz\, d\bar{z}$


$S_g = \frac{1}{4\pi\alpha'} \int \sqrt{h}\, h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X)\, d^2z$
$S_B = \frac{1}{4\pi\alpha'} \int \epsilon^{\alpha\beta} B_{\mu\nu}(x) \partial_\alpha X^\mu \partial_\beta X^\nu\, d^2z$
$dz\, d\bar{z}$
$q \int A_\mu(x)\, \dot{x}^\mu\, d\tau$

$$S_g = \frac{1}{4\pi\alpha'} \int \sqrt{h}\, h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X)\, d^2\sigma$$

$$S_B = \frac{1}{4\pi\alpha'} \int \epsilon^{\alpha\beta} B_{\mu\nu}(x) \partial_\alpha X^\mu \partial_\beta X^\nu\, d^2\sigma \qquad dz\, d\bar{z}$$

$$\rightarrow \quad q \int A_\mu(x)\, \dot{x}^\mu\, d\tau$$

$$S_g = \frac{1}{4\pi\alpha'} \int \sqrt{h}\, h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X)\, d^2z$$

$$S_B = \frac{1}{4\pi\alpha'} \int \epsilon^{\alpha\beta} B_{\mu\nu}(x) \partial_\alpha X^\mu \partial_\beta X^\nu d^2z \qquad dz\, d\bar{z}$$

$$\longrightarrow \quad q \int A_\mu(x)\, \dot{x}^\mu\, d\tau$$

$$S_\Phi = \frac{1}{4\pi} \int \sqrt{h}\, \Phi$$

$$S_g = \frac{1}{4\pi\alpha'} \int \sqrt{h}\, h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X)\, d^2 z$$

$$S_B = \frac{1}{4\pi\alpha'} \int \epsilon^{\alpha\beta} B_{\mu\nu}(x) \partial_\alpha X^\mu \partial_\beta X^\nu\, d^2 z \qquad dz\, d\bar{z}$$

$$\longrightarrow \quad q \int A_\mu(x)\, \dot{x}^\mu\, d\tau$$

$$\frac{1}{4\pi} \int \sqrt{h}\, \Phi(x)\, R^{(2)}(h)\, d^2 z$$

Assume $\Phi(x) = \Phi$ const

$$= \Phi \frac{1}{4\pi} \int \sqrt{h} R\, d^2 z$$

$$S_g = \frac{1}{4\pi\alpha'} \int \sqrt{h}\, h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X)\, d^2 z$$

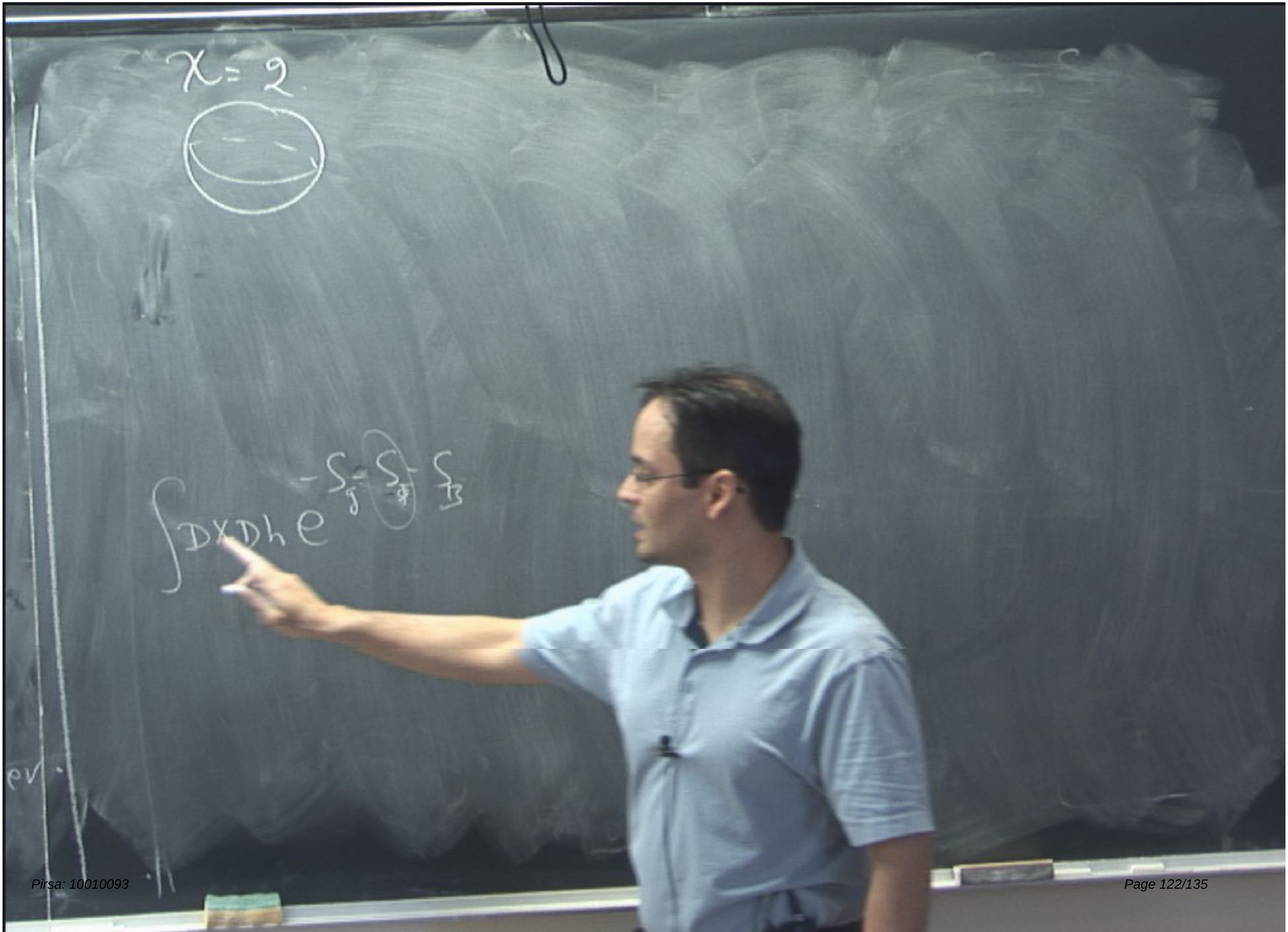
$$S_B = \frac{1}{4\pi\alpha'} \int \epsilon^{\alpha\beta} B_{\mu\nu}(x) \partial_\alpha X^\mu \partial_\beta X^\nu d^2 z \quad dz\, d\bar{z}$$

$$q \int A_\mu(x)\, \dot{x}^\mu\, d\tau$$

$$S_\Phi = \frac{1}{4\pi} \int \sqrt{h}\, \Phi(x)\, R^{(2)}_{(h)}\, d^2 z$$

Assume $\Phi(x) = \Phi$ const

$$= \Phi \frac{1}{4\pi} \int \sqrt{h}\, R^{(2)}_{(h)} d^2 z$$

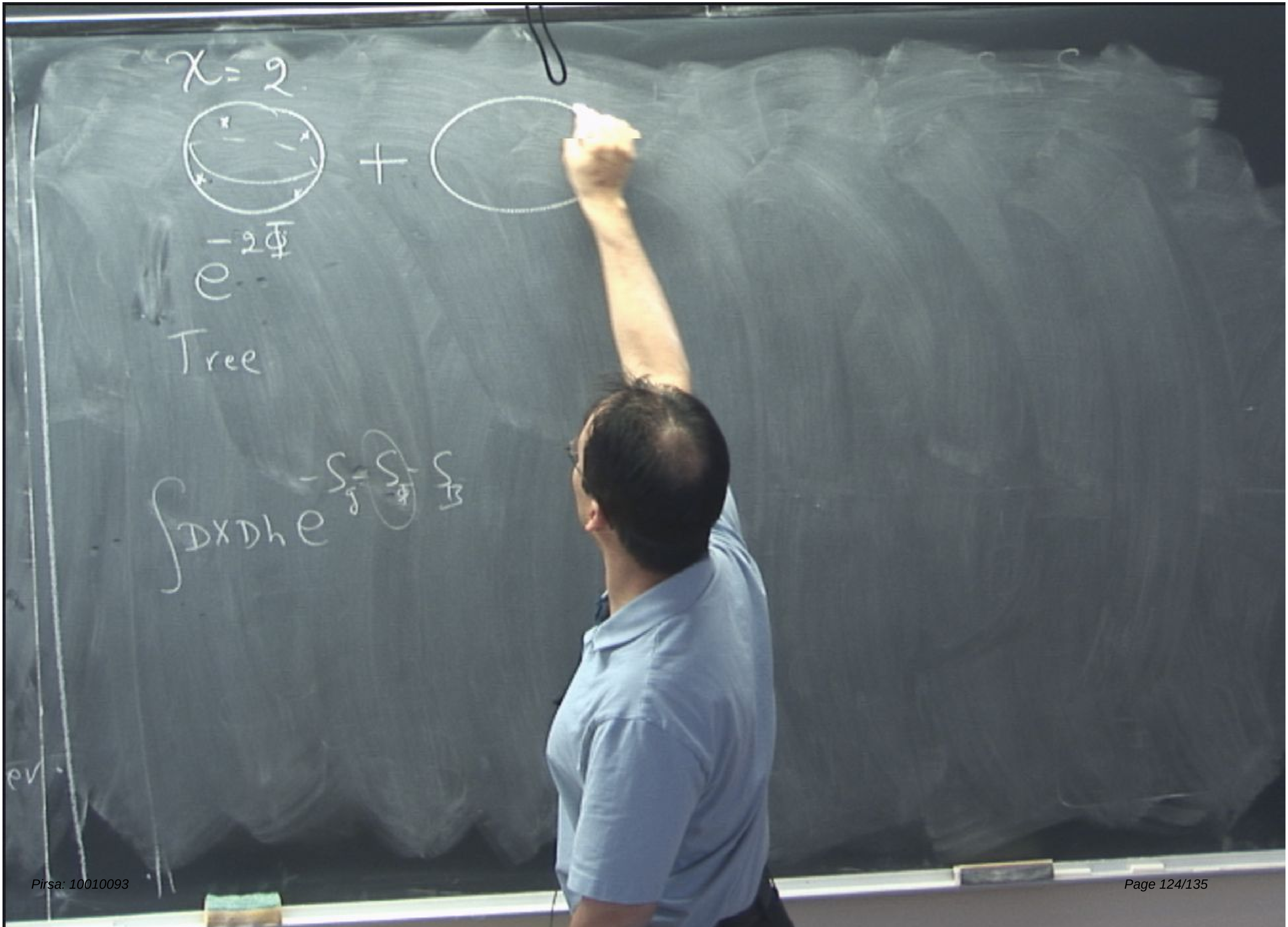

χ = 2
∫DXDh e

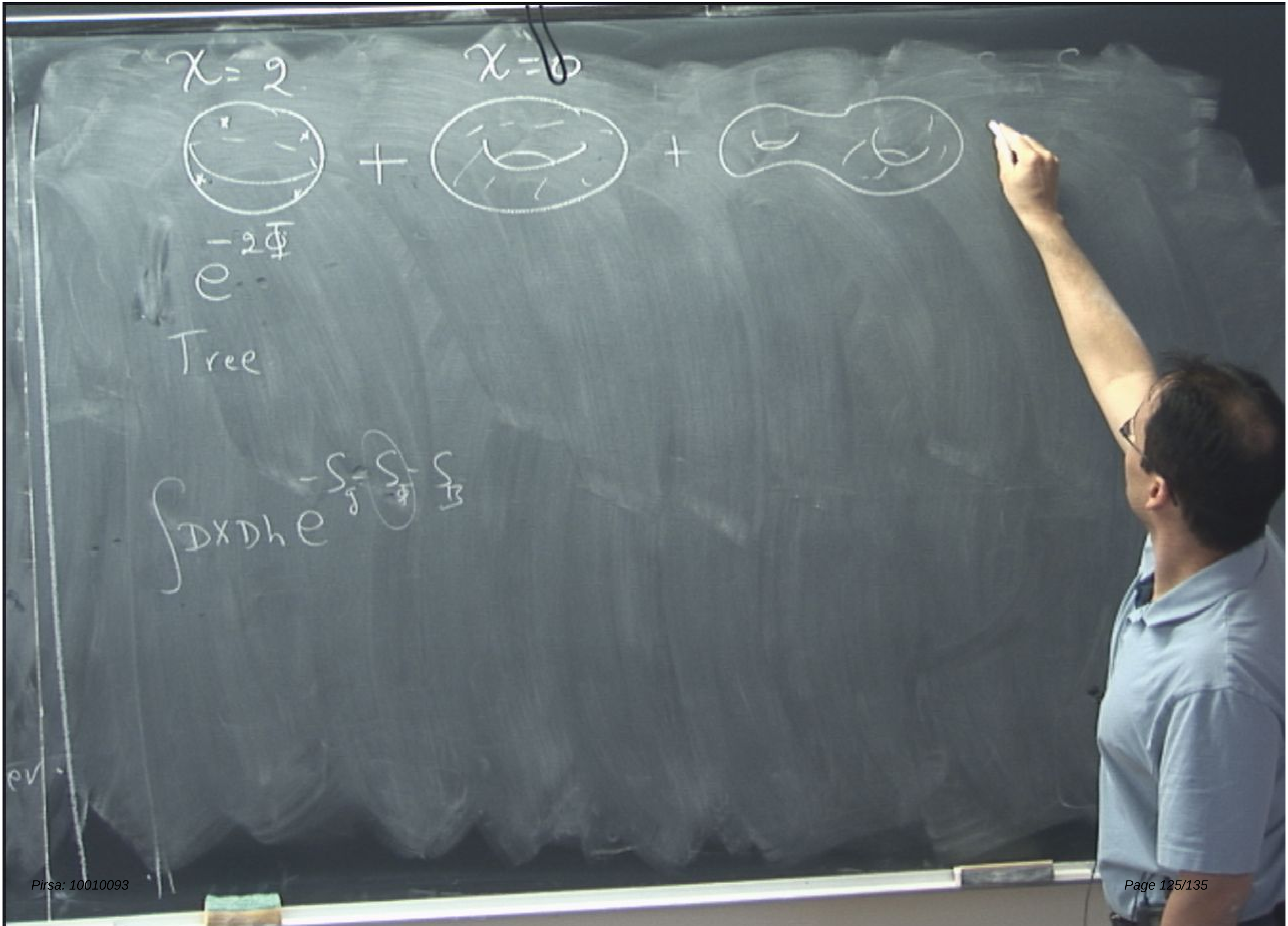
$\chi = 2$

 $+$

$e^{-2\Phi}$

$$\int DX\,Dh\, e^{-S_g - S_\Phi - S_B}$$


χ = 2
e^{-2Φ}
Tree
∫DXDh e^{-S_g - S_Φ - S_B}


χ = 2
χ = 0
e^{-2Φ}
Tree
∫DXDh e^{-S_g - S_Φ - S_B}

$\chi = 2$ $\chi = 0$

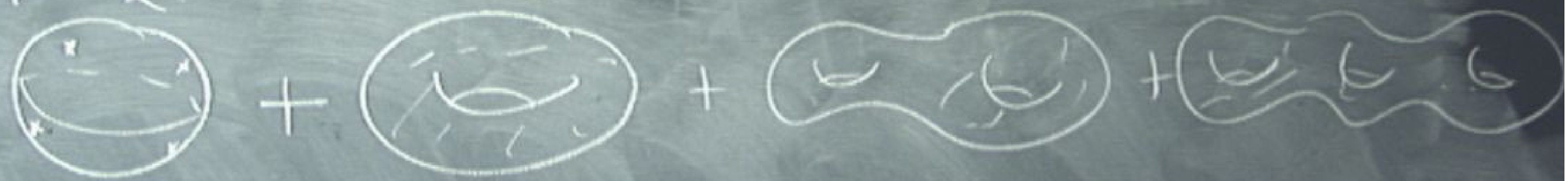
$e^{-2\Phi}$

Tree

$$\int DX\,Dh\, e^{-S_g - S_\Phi - S_B}$$

eV

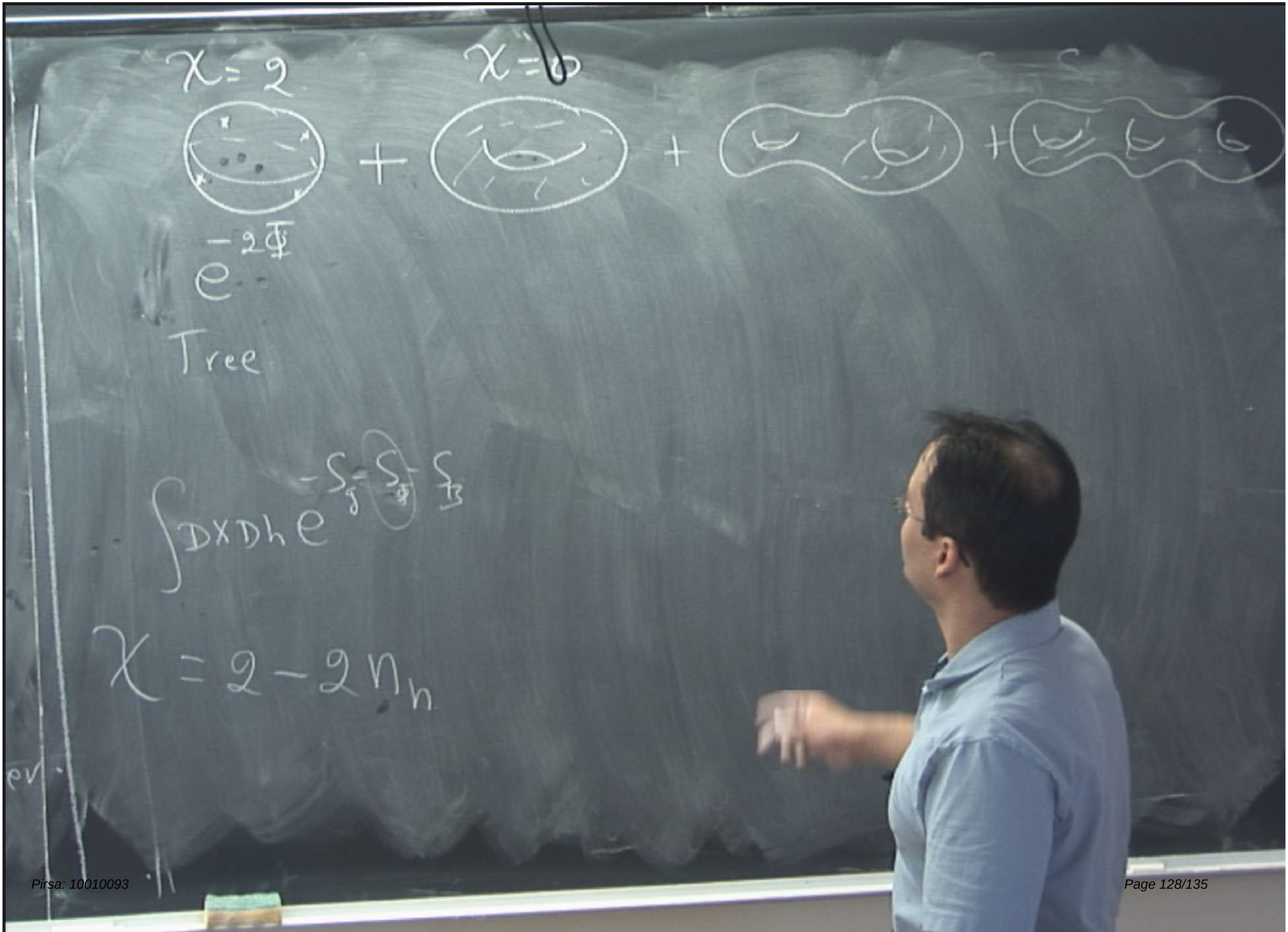
$\chi = 2$ $\qquad\qquad$ $\chi = 0$

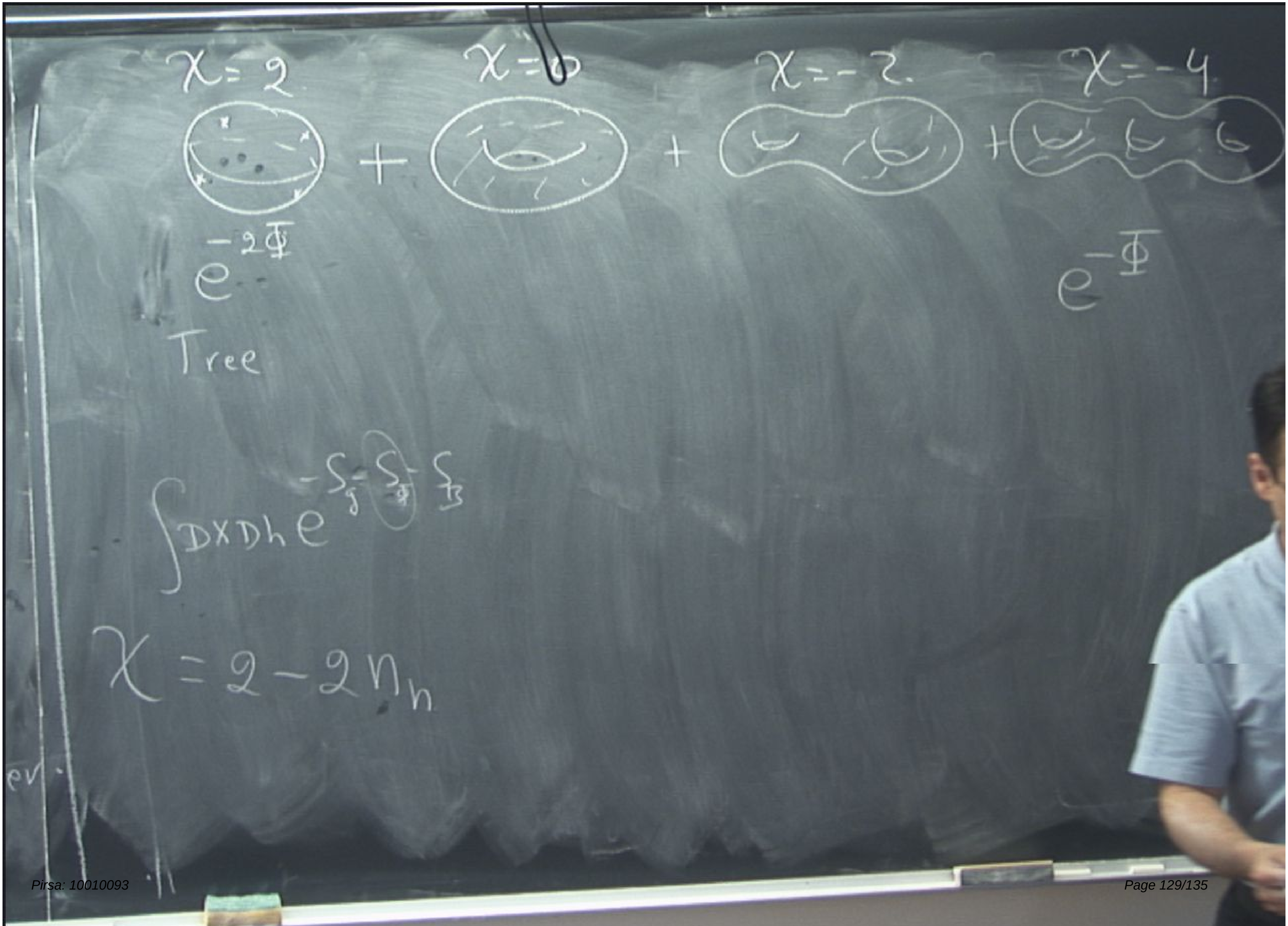


$e^{-2\Phi}$

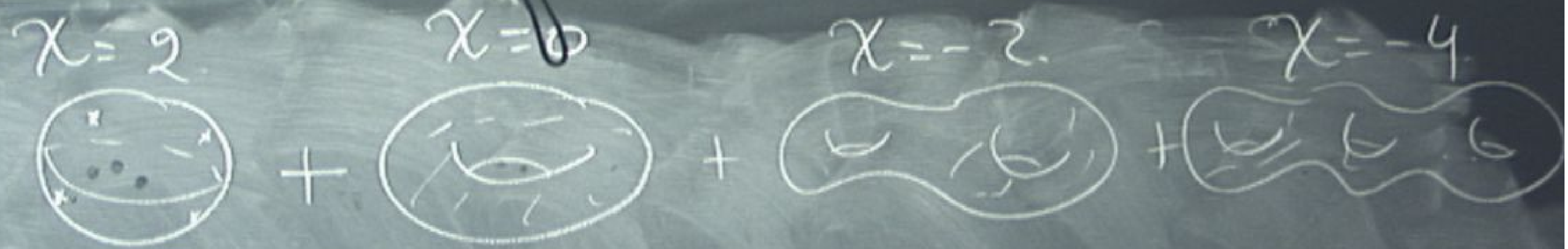
Tree

$$\int DX\, Dh\, e^{-S_g - S_\Phi - S_B}$$


χ = 2
χ = 0
e^{-2Φ}
Tree
∫DXDh e^{-S_g - S_Φ - S_B}
χ = 2 - 2n_h



$\chi = 2$
$\chi = 0$
$\chi = -2$
$\chi = -4$
$e^{-2\Phi}$
$e^{-\Phi}$
Tree
$\int DX\, Dh\, e^{-S_g - S_\Phi - S_B}$
$\chi = 2 - 2n_h$



$e^{-2\Phi}$ $e^{-\Phi}$

Tree

In QFT

$$\int DX\,Dh\, e^{-S_g - S_\Phi - S_B}$$

$$\chi = 2 - 2n_h$$

$\chi = 2$ $\chi = 0$ $\chi = -2$ $\chi = -4$

$e^{-2\Phi}$ $e^{-\Phi}$

Tree

$$\int DX\, Dh\, e^{-S_g - S_\Phi - S_B}$$

$$\chi = 2 - 2n_h$$

In QFT. k^μ. $e^{ik\cdot X}$

$$V = e^{ik\cdot X}$$

$k^2 = 2$ Tachyon

$\chi = 2 \qquad \chi = 0 \qquad \chi = -2 \qquad \chi = -4$

$e^{-2\Phi}$ $\qquad\qquad$ $e^{-\Phi}$

Tree

$h e^{-S_g - S_\Phi - S_B}$

$2 - 2n_h$

In QFT. $\quad k^\mu \quad e^{ik\cdot X}$

$V = e^{ik\cdot X} \qquad k^2 = 2 \quad$ Tachyon

$A(k_1, \ldots, k_n) =$

Scattering Amplitude in Space-Time.

$\chi = 2$ $\quad \chi = 0$ $\quad \chi = -2$ $\quad \chi = -4$

$e^{-2\Phi}$

Vertex operators $\quad e^{-\Phi}$

Tree

In QFT $\quad k^\mu \quad e^{ik\cdot X}$

$V = e^{ik\cdot X}$ $\qquad k^2 = 2$ $\quad$ Tachyon

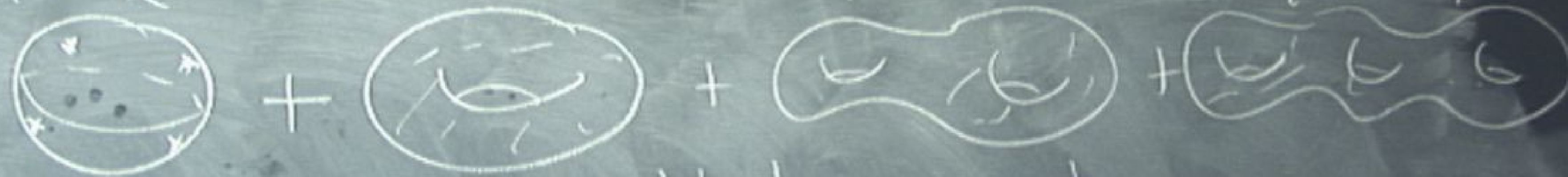
$$\int DX\,Dh\; e^{-S_g - S_\Phi - S_B}$$

$$A(k_1, \dots, k_n) = \langle V_1 \dots V_n \rangle$$

Scattering
Amplitude in
Space-Time

$$\chi = 2 - 2n_h$$

$\chi = 2$ $\chi = 0$ $\chi = -2$ $\chi = -4$



$e^{-2\Phi}$

Tree

Vertex operators $e^{-\Phi}$

In QFT k^μ $e^{ik\cdot X}$

$V = e^{ik\cdot X}$ $k^2 = 2$ Tachyon

$$\int DX\,Dh\, e^{-S_g - S_\Phi - S_B}$$

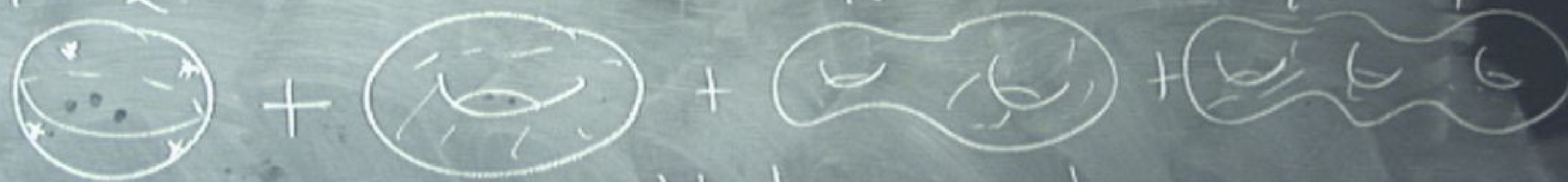
$$A(k_1, \dots, k_n) = \langle V_1 \dots V_n \rangle$$

Scattering Amplitude in Space-Time

Correlation function in World-sheet Theory

$$\chi = 2 - 2n_h$$

$\chi = 2$ $\chi = 0$ $\chi = -2$ $\chi = -4$



$e^{-2\Phi}$

Vertex operators $e^{-\Phi}$

Tree

In QFT k^{μ} $e^{ik\cdot X}$

$V = e^{ik\cdot X}$ $k^2 = 2$ Tachyon

$$\int DX\,Dh\, e^{-S_g - S_\Phi - S_B}$$

$$A(k_1, \dots, k_n) = \langle V_1 \dots V_n \rangle$$

Scattering Amplitude in Space-Time

Correlation Function in World-sheet Theory

$$\chi = 2 - 2n_h$$