Title: String Theory - Review (PHYS 623) - Lecture 4

Date: Jan 28, 2010 11:20 AM

URL: http://pirsa.org/10010092

Abstract:

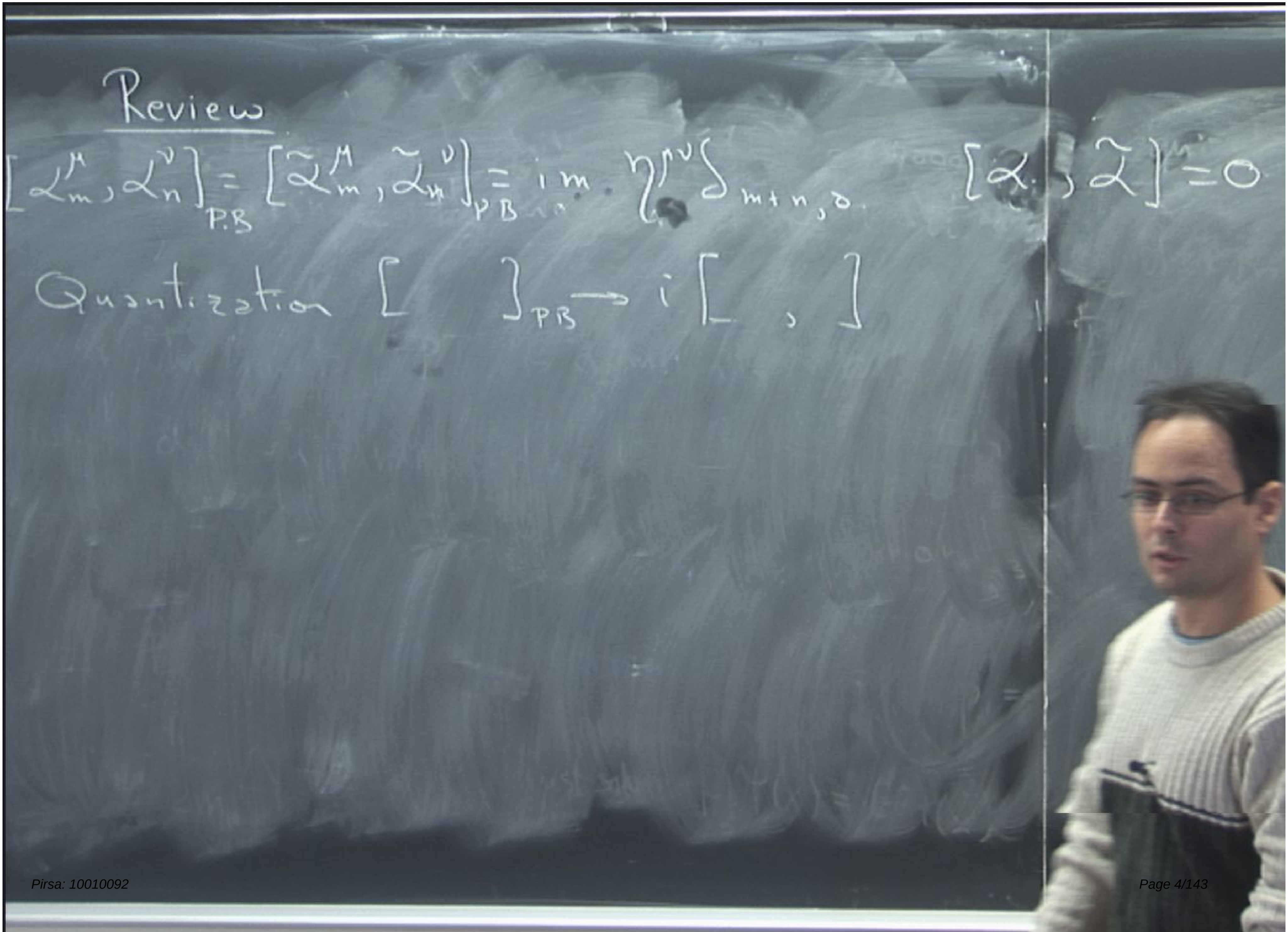
Review

$$[\alpha^\mu_m, \alpha^\nu_n]_{P.B} = [\tilde{\alpha}^\mu_m, \tilde{\alpha}^\nu_n]_{P.B} = i m \, \eta^{\mu\nu} \delta_{m+n,0}.$$

Review

$$[\alpha^\mu_m, \alpha^\nu_n]_{P.B} = [\tilde{\alpha}^\mu_m, \tilde{\alpha}^\nu_n]_{PB} = i m \,\eta^{\mu\nu} \delta_{m+n,0} \qquad [\alpha, \tilde{\alpha}] = 0$$

C


Review
$[\alpha^\mu_m, \alpha^\nu_n]_{P.B} = [\tilde\alpha^\mu_m, \tilde\alpha^\nu_n]_{PB} = i\, m\, \eta^{\mu\nu} \delta_{m+n,0}$
$[\alpha, \tilde\alpha] = 0$
Quantization $[\quad]_{PB} \to i[\ ,\ ]$

$[\alpha, \tilde{\alpha}]^{\mu} = 0$

$m = 0$ $\alpha^{\mu}_{0} = p^{\mu}$

zero mode

Review

$$[\alpha^\mu_m, \alpha^\nu_n]_{P.B} = [\tilde{\alpha}^\mu_m, \tilde{\alpha}^\nu_n]_{PB} = i m\, \eta^{\mu\nu} \delta_{m+n,0} \qquad [\alpha$$

Quantization $[\ \ ]_{PB} \to i[\ ,\ ]$

$$\alpha^{\mu\,\dagger}_m = \alpha^\mu_{-m} \qquad m > 0$$

$\eta^{\mu\nu}\delta_{m+n,0}$ $[\alpha, \tilde{\alpha}]^{\mu} = 0$ $m=0$ $\alpha^{\mu}_{0} = p^{\mu}$ $\longrightarrow$ zero m

$i[\;,\;]$ $\longrightarrow$ $[a^{\mu}_{m}, a^{\nu\dagger}_{n}] = [$

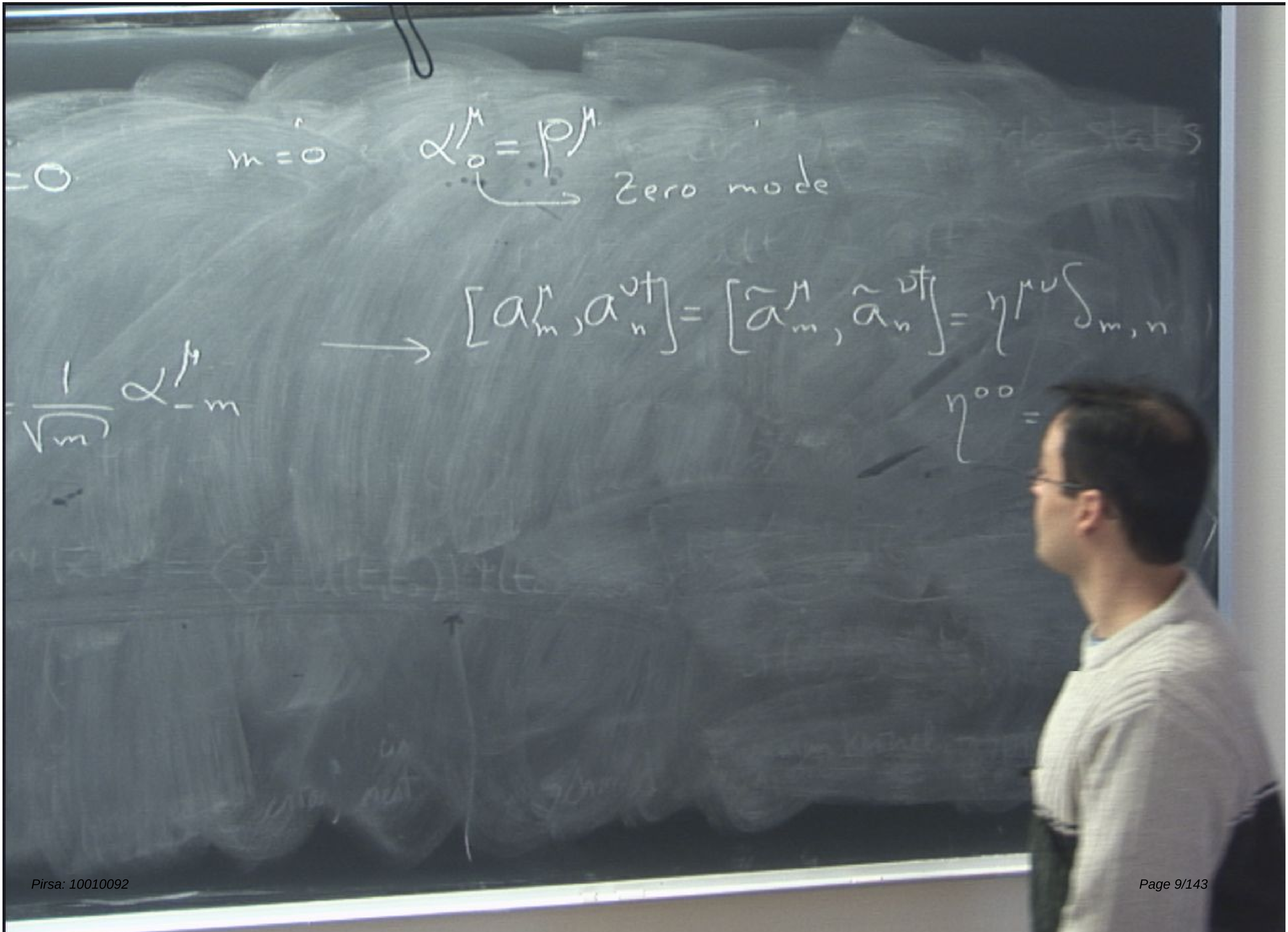
$a^{\mu}_{m} = \frac{1}{\sqrt{m}}\alpha^{\mu}_{m}$ $a^{\mu\dagger}_{m} = \frac{1}{\sqrt{m}}\alpha^{\mu}_{-m}$

$= 0$ $\quad m = 0$ $\quad \alpha_0^\mu = p^\mu$

$\alpha_0^\mu \to$ zero mode

$$\to [a_m^\mu, a_n^{\nu\dagger}] = [\tilde{a}_m^\mu, \tilde{a}_n^{\nu\dagger}] = \eta^{\mu\nu}\delta_{m,n}$$

$$= \frac{1}{\sqrt{m}}\alpha_{-m}^\mu$$


$=0$
$m=0$
$\alpha_0^\mu = p^\mu$
zero mode
$[a_m^\mu, a_n^{\nu\dagger}] = [\tilde{a}_m^\mu, \tilde{a}_n^{\nu\dagger}] = \eta^{\mu\nu}\delta_{m,n}$
$\frac{1}{\sqrt{m}}\alpha^\mu_{-m}$
$\eta^{00} =$

$= 0$ $\quad m = 0 \quad \alpha^\mu_0 = p^\mu$

$\xrightarrow{}$ zero mode

$$\frac{1}{\sqrt{m}}\alpha^\mu_{-m} \longrightarrow [a^\mu_m, a^{\nu\dagger}_n] = [\tilde{a}^\mu_m, \tilde{a}^{\nu\dagger}_n] = \eta^{\mu\nu}\delta_{m,n}$$

$$\eta^{00} = -1$$

$=0$ $\quad m=0 \quad \alpha^\mu_0 = p^\mu$

$\underbrace{\alpha^\mu_0}_{\text{zero mode}}$

$$= \frac{1}{\sqrt{m}} \alpha^\mu_{-m} \longrightarrow [a^\mu_m, a^{\nu\dagger}_n] = [\tilde{a}^\mu_m, \tilde{a}^{\nu\dagger}_n] = \eta^{\mu\nu}\delta_{m,n}$$

$$[a^0_m, a^{0\dagger}_n] = -\delta_{m,n} \qquad \eta^{00} = -1$$

Review

$$[\alpha^\mu_m, \alpha^\nu_n]_{P.B} = [\tilde{\alpha}^\mu_m, \tilde{\alpha}^\nu_n]_{PB} = i\, m\, \eta^{\mu\nu} \delta_{m+n,0} \qquad [\alpha, \tilde{\alpha}] = 0$$

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$$\alpha^{\mu\dagger}_m = \alpha^\mu_{-m} \quad m > 0 \qquad a^\mu_m = \frac{1}{\sqrt{m}} \alpha^\mu_m \qquad a^{\mu\dagger}_m = \frac{1}{\sqrt{m}}$$

$$|0\rangle \qquad \langle 0|0\rangle = 1$$

$$a^\mu_m |0\rangle = 0 \quad m > 0 \qquad |\phi\rangle =$$

$\alpha]^{\mu}=0$ $\quad m=0 \quad \alpha^{\mu}_{0}=p^{\mu}$

$\longrightarrow$ zero mode

$$a^{\mu\dagger}_{m}=\frac{1}{\sqrt{m}}\alpha^{\mu}_{-m} \longrightarrow [a^{\mu}_{m},a^{\nu\dagger}_{n}]=[\tilde{a}^{\mu}_{m},\tilde{a}^{\nu\dagger}_{n}]=\eta^{\mu\nu}\delta_{m,n}$$

$$[a^{0}_{m},a^{0\dagger}_{n}]=-\delta_{m,n} \qquad \eta^{00}=-1$$

$$|\phi\rangle = a^{\mu_1\dagger}_{m_1}\cdots a^{\mu_n\dagger}_{m_n}|0:k\rangle \qquad p^{\mu}|0:k\rangle=k^{\mu}$$

$]^{\mu} = 0$ $\quad m = 0 \quad \alpha_0^\mu = p^\mu$

$\alpha_0^\mu \longrightarrow$ zero mode

$$[a_m^\mu, a_n^{\nu\dagger}] = [\tilde{a}_m^\mu, \tilde{a}_n^{\nu\dagger}] = \eta^{\mu\nu}\delta_{m,n}$$

$${}^{\mu\dagger}_{m} = \frac{1}{\sqrt{m}}\alpha^\mu_{-m} \longrightarrow$$

$$[a_m^0, a_n^{0\dagger}] = -\delta_{m,n} \qquad \eta^{00} = -1$$

$$\phi\rangle \; {}^{\dagger}_{m_1} \dots a^{\mu_n \dagger}_{m_n} |0:k\rangle \qquad p^\mu|0:k\rangle = k^\mu|\;\rangle$$

Review

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Review

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odd

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Virasoro operators.

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$$L_m = \frac{1}{2} \sum_{n=-\infty}^{\infty} \alpha_{m-n} \cdot \alpha_n$$

Virasoro operators.

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} :\alpha_{m-n}\,\alpha_n: \qquad m \neq 0$$

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$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} :\alpha_{m-n}\cdot\alpha_n: \qquad m \neq 0$$

$$L_0 = \frac{1}{2}\alpha_0^2 + \sum_{n=1}^{\infty} :\alpha_{-n}\cdot\alpha_n:$$

Virasoro operators.

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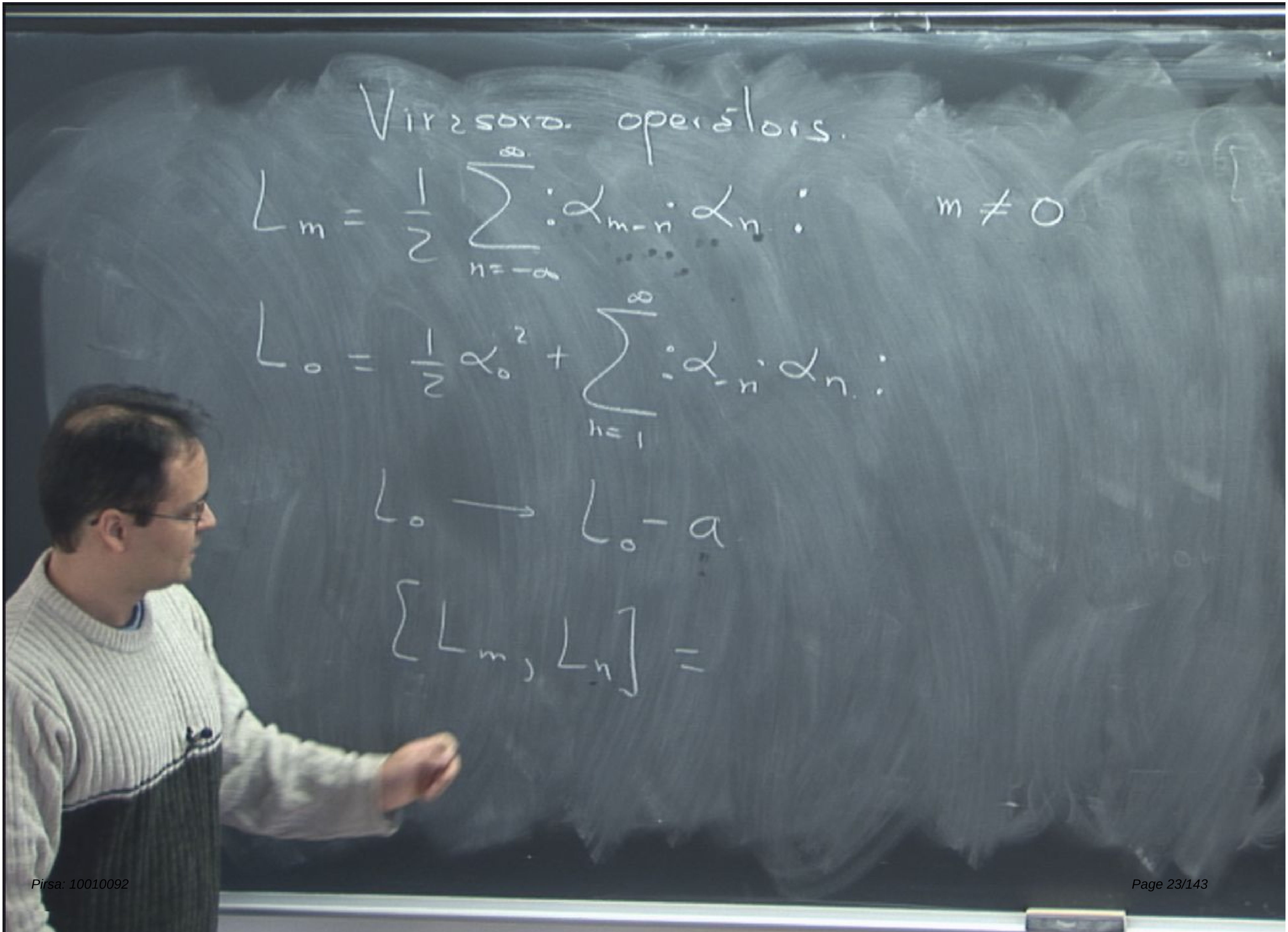
$$L_0 \longrightarrow L_0 - a$$


Virasoro operators.
$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} :\alpha_{m-n}\cdot\alpha_n: \quad m \neq 0$
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$L_0 \longrightarrow L_0 - a$
$[L_m, L_n] =$

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$$[L_m, L_n] = (m-n) L_{m+n}$$

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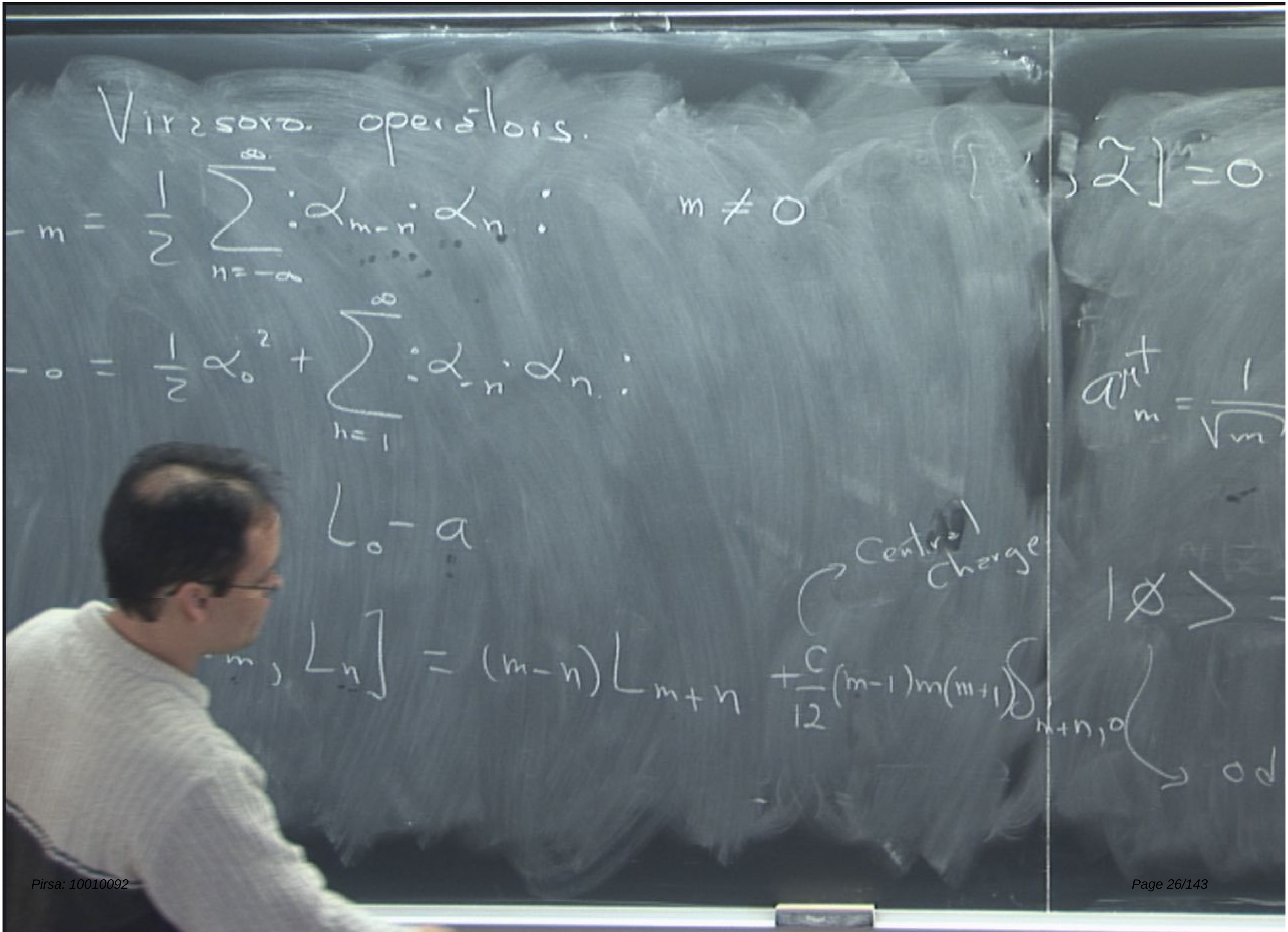
$$[L_m, L_n] = (m-n)L_{m+n} + \delta_{m+n,0}(\ \ $$

$$[\ \ ,\tilde{\alpha}] = 0$$

$$a^{\mu\dagger}_m = \frac{1}{\sqrt{m}}$$

$$|\phi\rangle =$$

od


Virasoro operators.
$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} :\alpha_{m-n}\cdot\alpha_n:$
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$L_0 - a$
Central charge
$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m-1)m(m+1)\delta_{m+n,0}$
$|\emptyset\rangle$
od

Virasoro operators.

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} :\alpha_{m-n}\cdot\alpha_n: \qquad m \neq 0$$

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$$L_0 \longrightarrow L_0 - a$$

Central charge

$$[L_m, L_n] = (m-n)L_{m+n} + \underbrace{\frac{c}{12}(m-1)m(m+1)\delta_{m+n,0}}_{\text{Central extension}}$$

$$[\ ,\ \alpha] = 0$$

$$a^{\mu\dagger}_m = \frac{1}{\sqrt{m}}$$

$$|\emptyset\rangle =$$

od

Virasoro operators.

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} :\alpha_{m-n}\cdot\alpha_n: \qquad m \neq 0$$

$$L_0 = \frac{1}{2}\alpha_0^2 + \sum_{n=1}^{\infty} :\alpha_{-n}\cdot\alpha_n:$$

$$\longrightarrow L_0 - a$$

Central charge

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m-1)m(m+1)\delta_{m+n,0}$$

$$c = \# \text{ of } X_\mu = D \quad \text{- Central exten...}$$

$$[\ ,\ \alpha]^\mu = 0$$

$$a^{\mu\dagger}_m = \frac{1}{\sqrt{m}}$$

$$|\phi\rangle =$$

$\to$ od

Virasoro operators.

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} :\alpha_{m-n}\cdot\alpha_n: \qquad m \neq 0$$

$$[\ ,\ 2] = 0$$

$$L_0 = \frac{1}{2}\alpha_0^2 + \underbrace{\sum_{n=1}^{\infty} :\alpha_{-n}\cdot\alpha_n:} \rightarrow \frac{1}{2}\sum_{n\neq 0}\alpha_{-n}\cdot\alpha_n$$

$$a^{\mu\dagger}_m = \frac{1}{\sqrt{m}}$$

$$L_0 \longrightarrow L_0 - a$$

Central charge

$$|\phi\rangle =$$

$$[L_m, L_n] = (m-n)L_{m+n} + \underbrace{\frac{c}{12}(m-1)m(m+1)\delta_{m+n,0}}$$

c = # of X's $= D$ - Central extension

$\rightarrow$ od

Virasoro operators.

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} :\alpha_{m-n}\,\alpha_n: \qquad m \neq 0$$

$$\sum_{m=-\infty}^{-1}$$

$$L_0 = \frac{1}{2}\alpha_0^2 + \sum_{n=1}^{\infty} :\alpha_{-n}\,\alpha_n: \qquad \frac{1}{2}\sum_{n\neq 0}\alpha_{-n}\alpha_n$$

$$L_0 \to L_0 - a$$

Central charge

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m-1)m(m+1)\delta_{m+n,0}$$

$c = \#$ of X's $= D$ - Central extension

$[\;,\;\partial] = 0$

$a_m^{\dagger} = \frac{1}{\sqrt{m}}$

$|\phi\rangle =$

od

Virasoro operators.

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} :\alpha_{m-n}\alpha_n: \qquad m \neq 0$$

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$$\sum_{m=-\infty}^{-1}$$

$$\frac{1}{2}\sum_{n\neq 0} \alpha_{-n}\alpha_n$$

$$L_0 \longrightarrow L_0 - a$$

Central charge

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$c = \#$ of X's $= D$ - Central extension

$$[\ ,\ 2] = 0$$

$$a^{+}_{m} = \frac{1}{\sqrt{m}}$$

$$|\phi\rangle =$$

od

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$$L_0 \longrightarrow L_0 - a \qquad \sum_{n=1}^{\infty} n$$

Central charge

$$[L_m, L_n] = (m-n)L_{m+n} + \underbrace{\frac{c}{12}(m-1)m(m+1)\delta_{m+n,0}}$$

$c = \#$ of $X_\mu = D$ - Central extension

$$[\ ,\ 2] = 0$$

$$a_m^{\mu\dagger} = \frac{1}{\sqrt{m}}$$

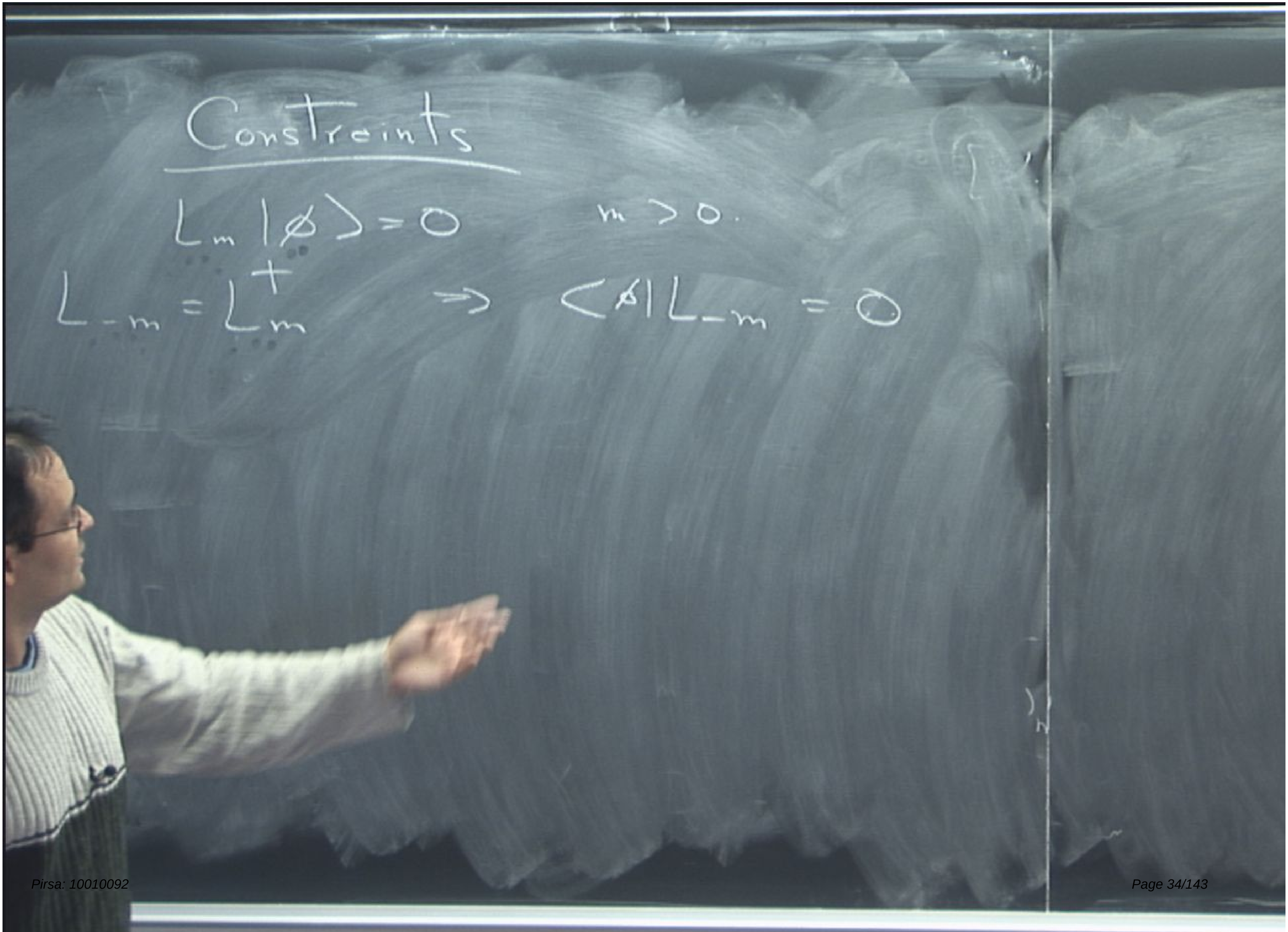
$$|\phi\rangle =$$

od

Constraints

$$L_m |\phi\rangle = 0 \qquad m > 0.$$

$$L_{-m} = L_m^{+}$$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$$\langle 0| L_{-m} L_m |0\rangle = 0$$

$$L_0$$

Constraints

$$L_m |\phi\rangle = 0 \qquad m > 0.$$

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$$(L_0 - a) |\phi\rangle = 0$$

Constraints

$$L_m |\phi\rangle = 0 \qquad m > 0.$$

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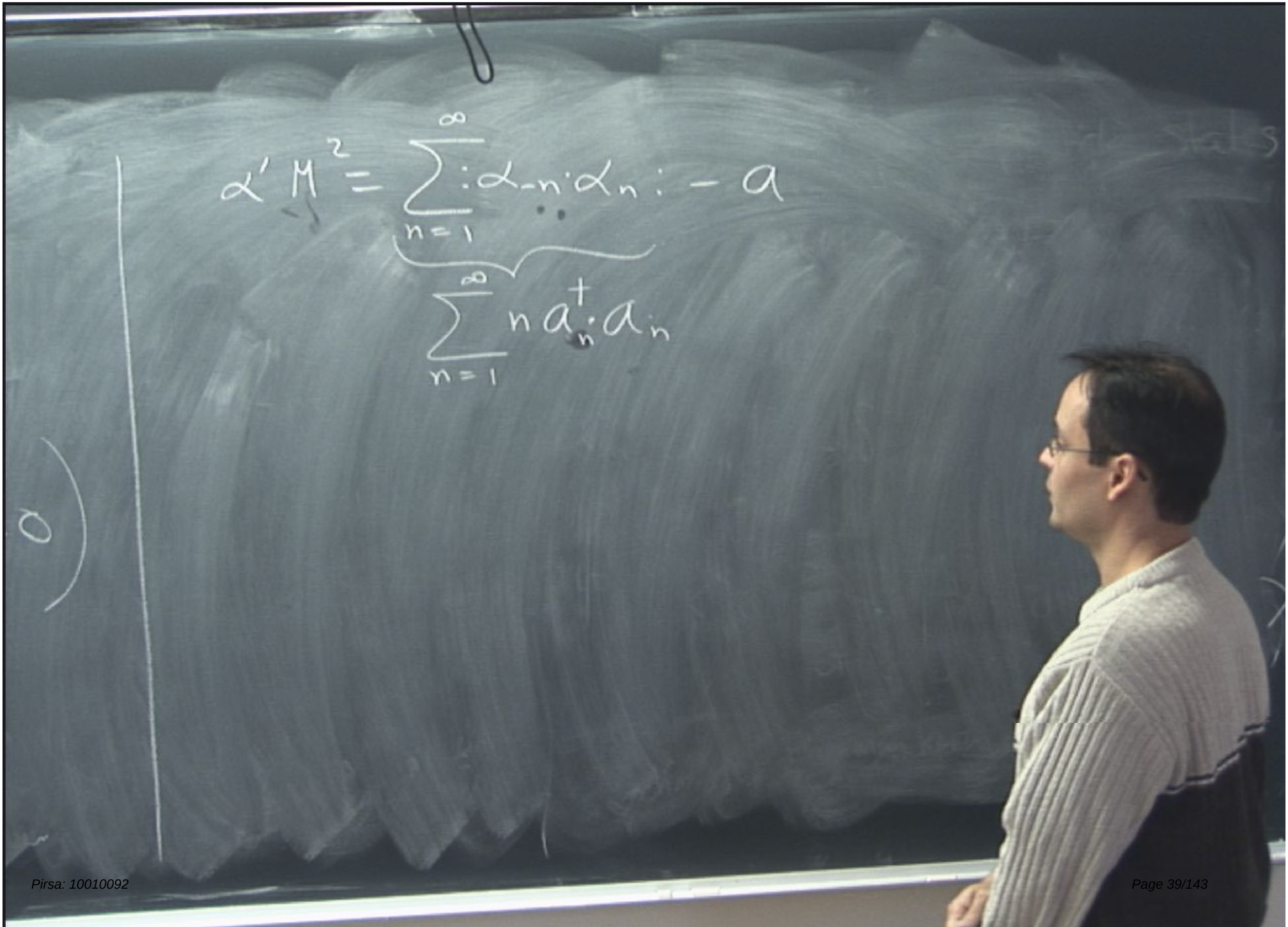
$$(L_0 - a)|\phi\rangle = 0 \qquad \left((\tilde{L}_0 - a)|\phi\rangle = 0 \right)$$



$\alpha' M^2 = \underbrace{\sum_{n=1}^{\infty} :\alpha_{-n} \cdot \alpha_n:}_{\sum_{n=1}^{\infty} n\, a_n^{\dagger} \cdot a_n} - a$

Constraints

$L_m |\phi\rangle = 0 \qquad m > 0.$

$L_{-m} = L_m^{+} \quad \Rightarrow \quad \langle \phi | L_{-m} = 0$

$\langle 0 | L_{-m} L_m | 0 \rangle = 0$

$(L_0 - a)|\phi\rangle = 0 \qquad \left((\bar{L}_0 - a)|\phi\rangle \right.$

Mass shell condition. Open:

$|\phi\rangle \quad |\Phi\rangle$

Constraints

$L_m |\phi\rangle = 0 \qquad m > 0.$

$L_{-m} = L_m^{\dagger} \Rightarrow \langle\phi| L_{-m} = 0$

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$L_m |\phi\rangle = 0 \qquad m > 0$

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$(L_0 - a)|\phi\rangle = 0 \qquad \left((\tilde{L}_0 - a)|\phi\rangle = 0 \right)$

Mass shell condition. Open:

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$$\alpha' M^2 = \underbrace{\sum_{n=1}^{\infty} :\alpha_{-n} \cdot \alpha_n: }_{\underbrace{\sum_{n=1}^{\infty} n\, a_n^{\dagger} \cdot a_n}_{N}} - a$$

N number operator.

Closed.

$$\frac{\alpha'}{4} M^2 = \underbrace{\sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n - a}_{N-a} = \underbrace{\sum_{n=1}^{\infty} \tilde{\alpha}_{-n} \cdot \hat{\alpha}_n - a}_{\tilde{N}-a} =$$

$$(L_0 - \tilde{L}_0)|\phi\rangle = 0$$

Level matching condition. All physical states.

$$\left((\tilde{L}_0 - a)|\phi\rangle = 0 \right)$$

Mass shell condition. Open:

$$|\phi\rangle \quad |\Phi\rangle$$

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$$\alpha' M^2 = \underbrace{\sum_{n=1}^{\infty} :\alpha_{-n}\alpha_n: - a}$$

$$\underbrace{\sum_{n=1}^{\infty} n\, a_n^{\dagger} a_n}$$

N number operator

Closed.

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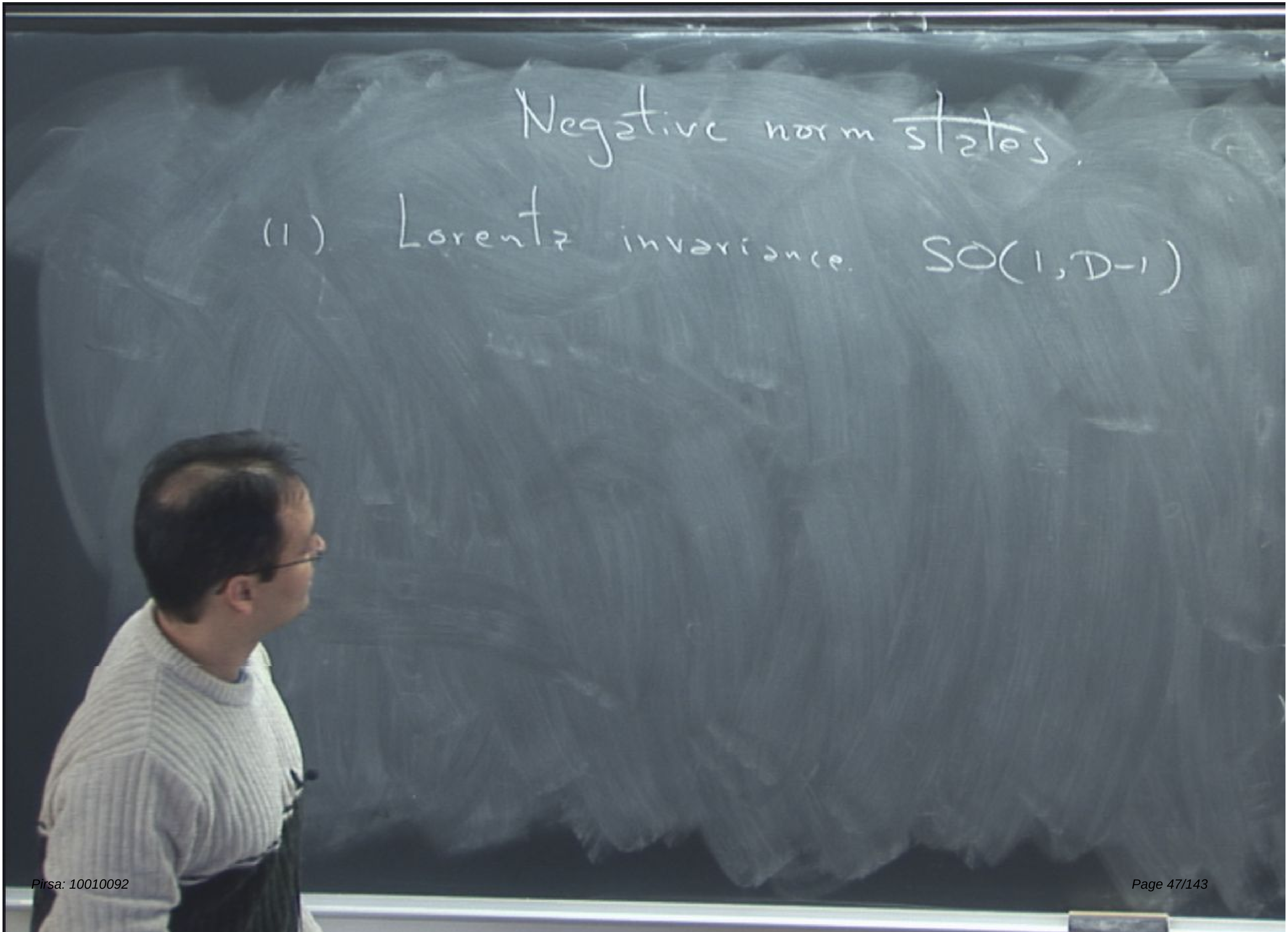
Negative norm states.
(1) Lorentz invariance. SO(1, D-1)

Negative norm states.

(1) Lorentz invariance. $SO(1, D-1)$ is manifest.

$$J^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu - i \sum_{n=1}^{\infty} \frac{1}{n} \left(\alpha_{-n}^\mu \alpha_n^\nu - \alpha_{-n}^\nu \alpha_n^\mu \right)$$

Negative norm states

1) Lorentz invariance $SO(1, D-1)$ is manifest.

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$$[L_m, J^{\mu\nu}] = 0 \implies \text{Representation of } SO(1, D-1)$$

Negative norm states

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$$[L_m, J^{\mu\nu}] = 0 \implies \text{Representation of } SO(1, D-1)$$

• Unitarity is not manifest!

$$L_m |\phi\rangle = 0 \qquad (L_0 - a)|\phi\rangle = 0$$

Negative norm states.

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$$[L_m, J^{\mu\nu}] = 0 \quad \Longrightarrow \quad \text{Representation of } SO(1,D-1)$$

• Unitarity is not manifest!

$$L_m|\phi\rangle = 0 \qquad (L_0 - a)|\phi\rangle = 0$$

(12) Light-cone gauge quantization.

- Unitarity is manifest.
- Lorentz invariance is not manifest.

(2) Light-cone gauge quantization.

- Unitarity is manifest.
- Lorentz invariance is not manifest.

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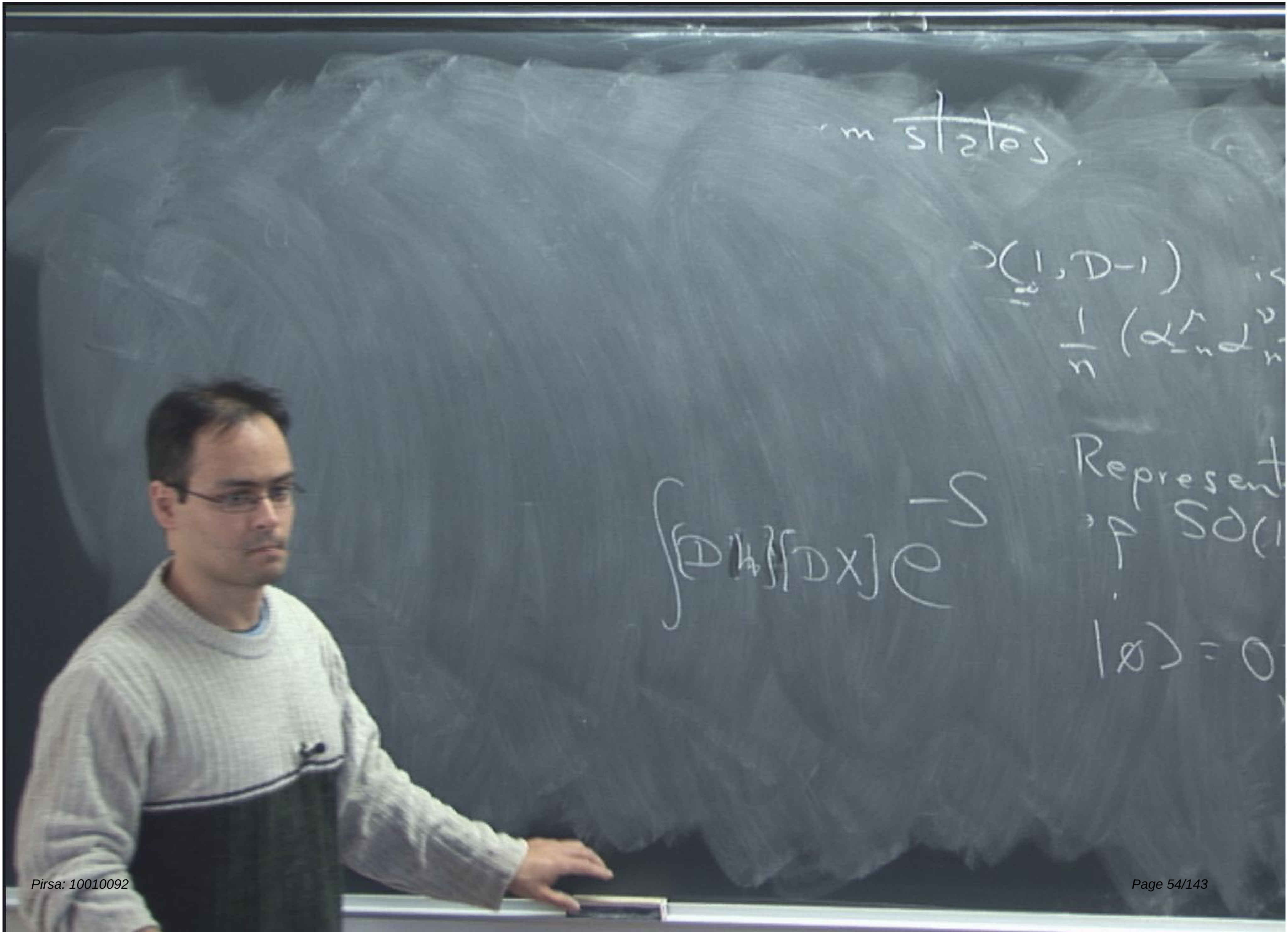
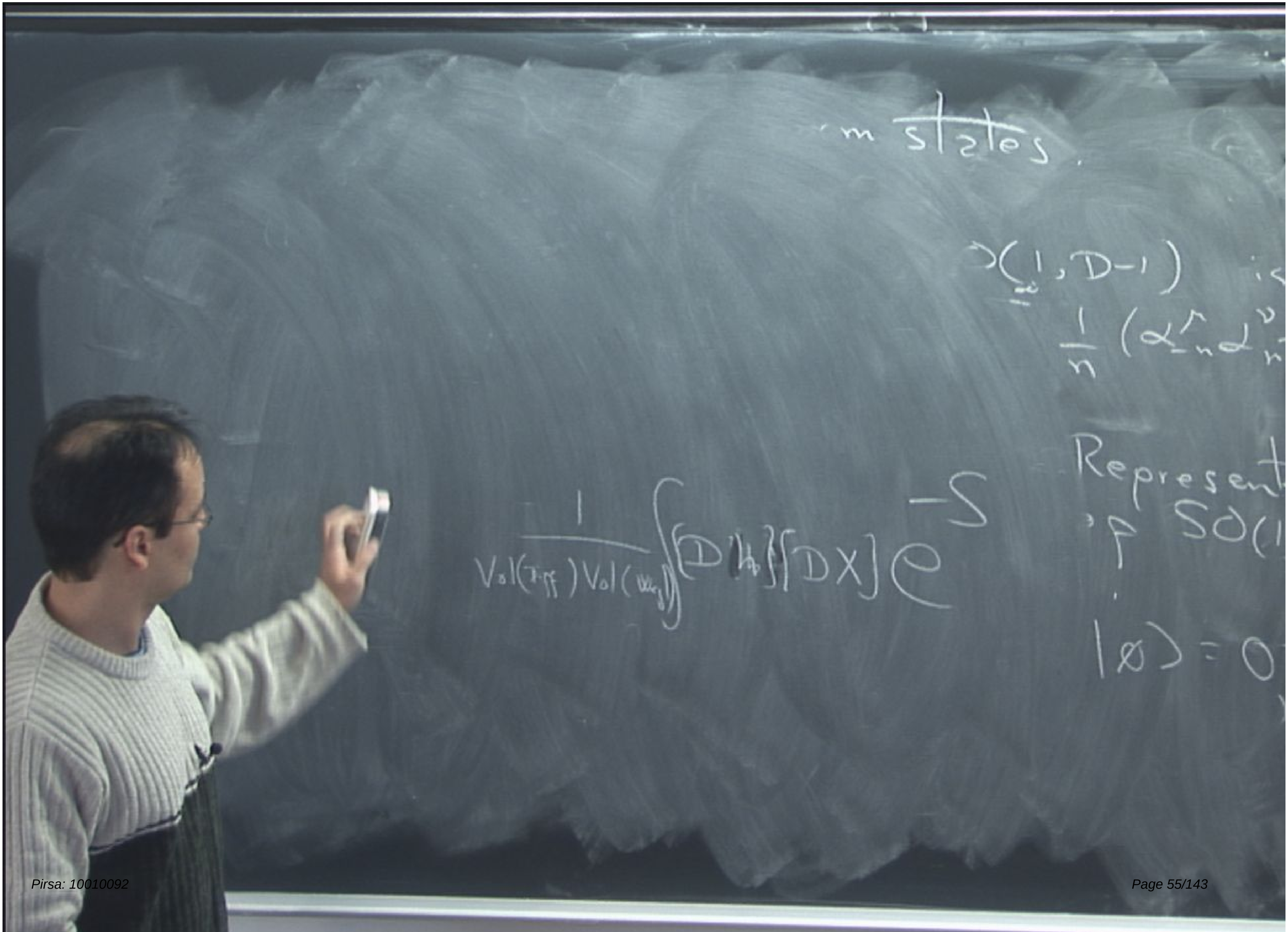
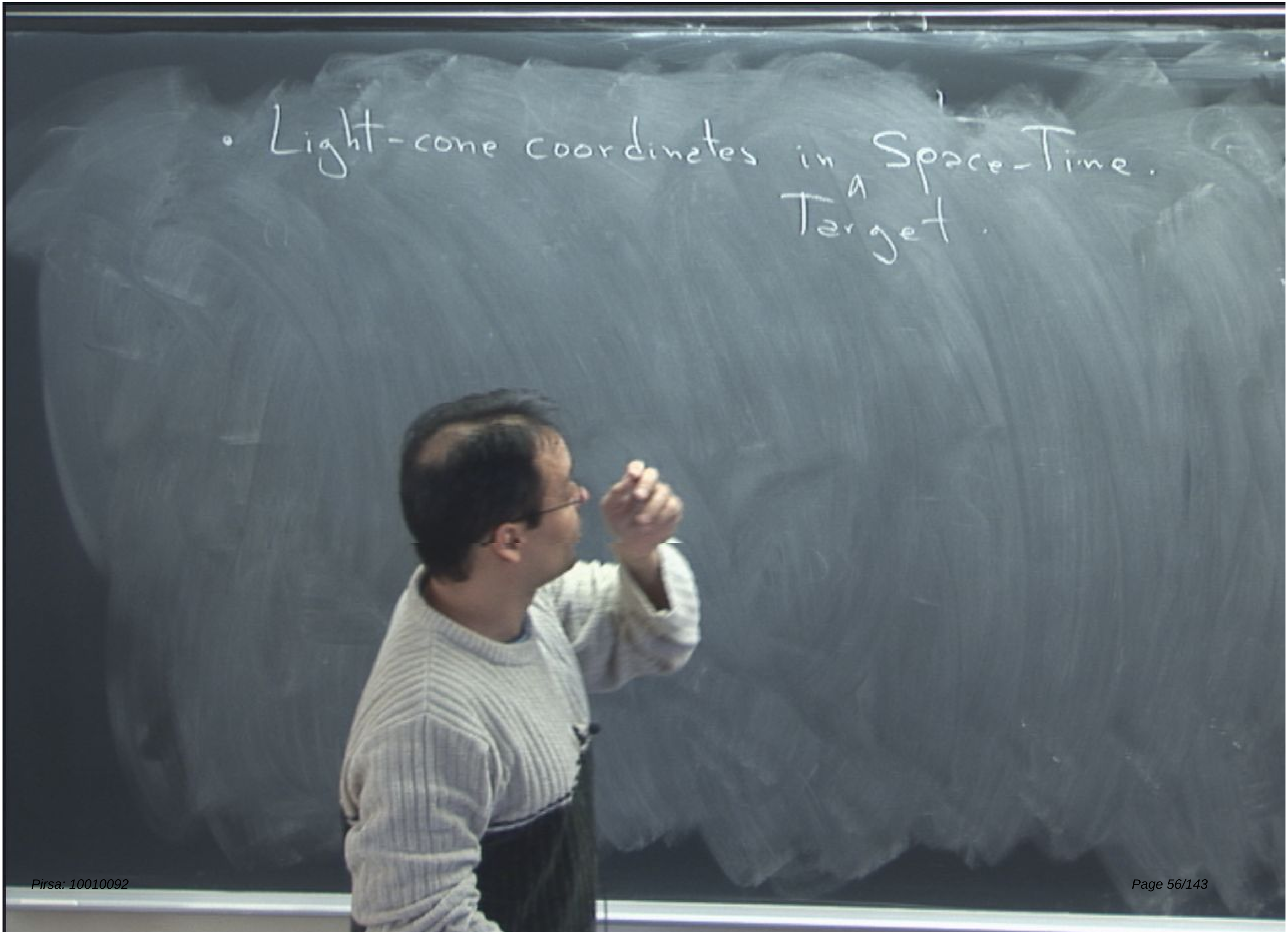
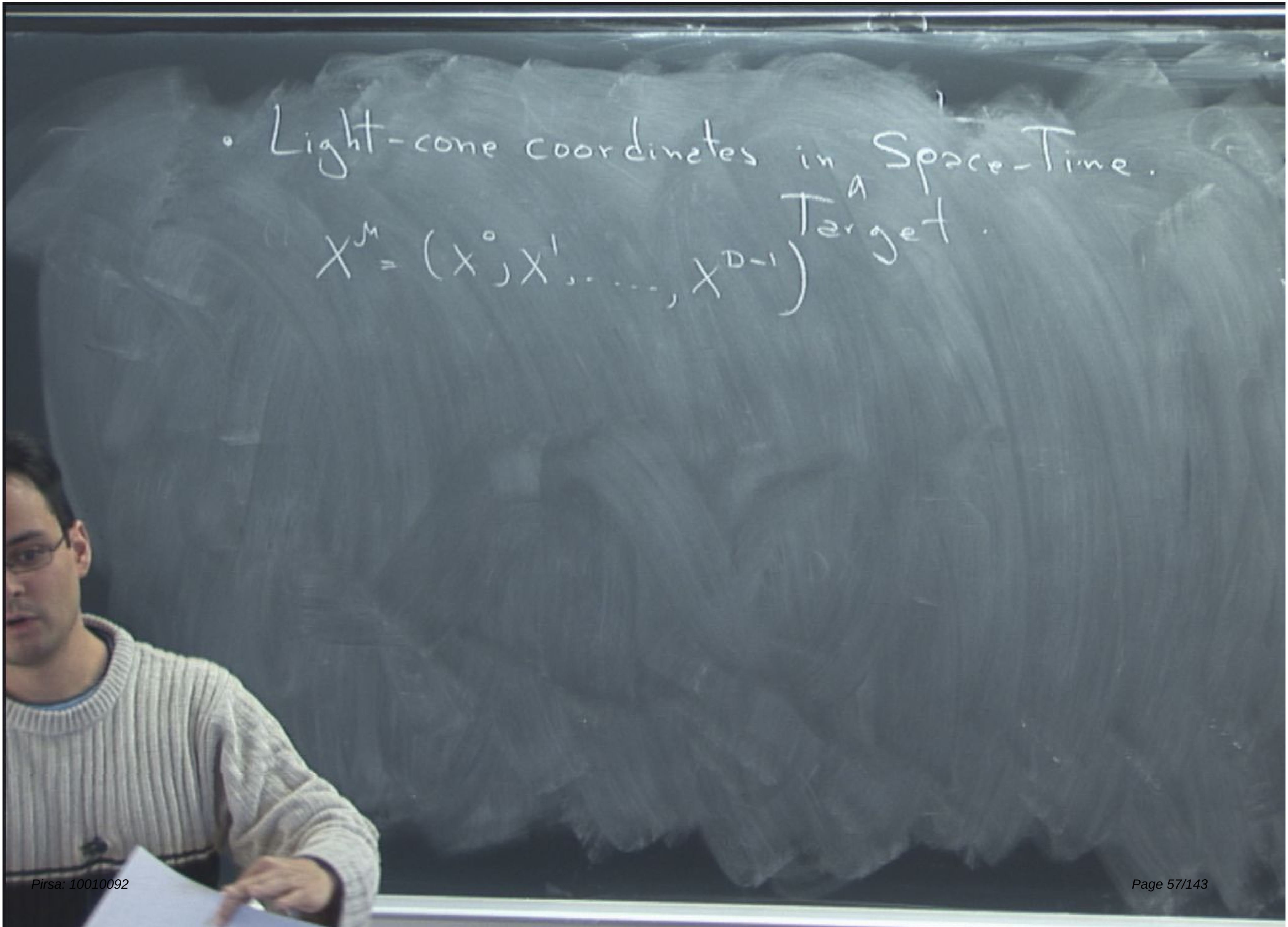

m states
Represent
SO(1


m states
Represent
of SO(1
$\frac{1}{\mathrm{Vol}(\mathrm{Diff})\,\mathrm{Vol}(\mathrm{Weyl})}\int[Dh][DX]\,e^{-S}$


• Light-cone coordinates in Space-Time.
Target .


• Light-cone coordinates in a Target Space-Time.
$X^\mu = (X^0, X^1, \ldots, X^{D-1})$

- Light-cone coordinates in a Target Space-Time.

$$X^\mu = (X^0, X^1, \ldots\ldots, X^{D-1})$$

$$X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^{D-1})$$ Light cone direction

$$X^i \quad i = 1, 2, \ldots, D-2$$ Transverse direct.

- Light-cone coordinates in a Space-Time. Target.

$$X^\mu = (X^0, X^1, \ldots\ldots, X^{D-1})$$

$$X^{\pm} = \frac{1}{\sqrt{2}}\left(X^0 \pm X^{D-1}\right)$$ Light cone direction

$X^i \quad i = 1, 2, \ldots, D-2 \longrightarrow$ Transverse direct.

$V = (V^+, V^-, V^i)$ $U = (U^+, U^-, V^i)$

$V \cdot U =$

$$V = (V^+, V^-, V^i) \qquad U = (U^+, U^-, V^i)$$

$$V \cdot U = -V^+U^- - V^-U^+ + \vec{U}\cdot\vec{V}$$

$\rightarrow \mathbb{R}^{D-2}.$

$$V = (V^+, V^-, V^i) \qquad U = (U^+, U^-, V^i)$$

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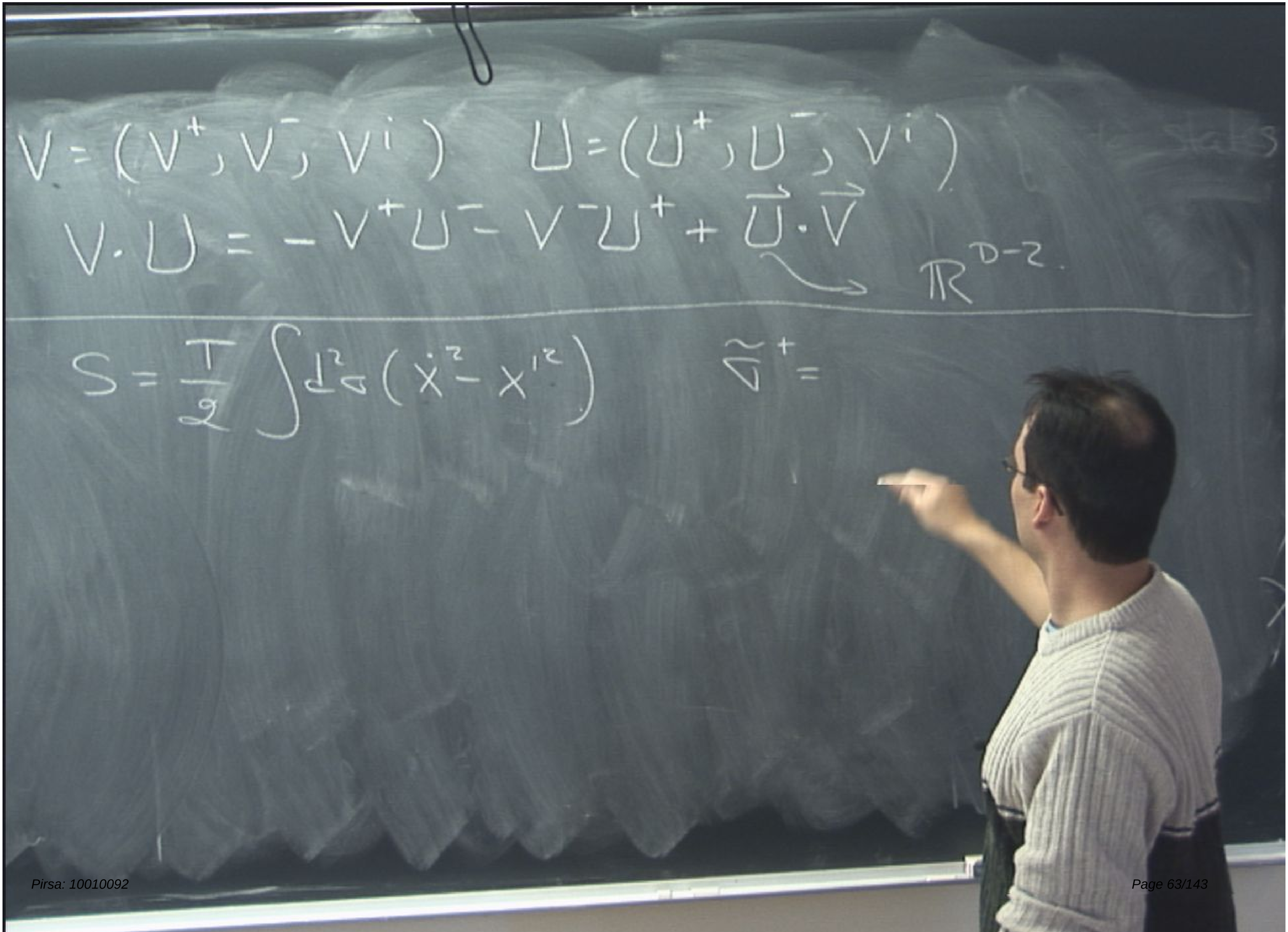
$$S = \frac{T}{2} \int d^2\sigma \left(\dot{X}^2 - X'^2 \right)$$


$V = (V^+, V^-, V^i)$ $U = (U^+, U^-, V^i)$
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$\mathbb{R}^{D-2}$
$S = \frac{T}{2} \int d^2\sigma \left(\dot{X}^2 - X'^2 \right)$
$\tilde{\sigma}^+ =$

$V = (V^+, V^-, V^i) \quad U = (U^+, U^-, V^i)$

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$$S = \frac{T}{2} \int d^2\sigma \left(\dot{X}^2 - X'^2 \right)$$

$$\left.\begin{aligned} \tilde{\sigma}^+ &= f^+(\sigma^+) \\ \tilde{\sigma}^- &= f^-(\sigma^-) \end{aligned}\right\} \longrightarrow \text{Conformal transform}$$

$V = (V^+, V^-, V^i)$ $\quad U = (U^+, U^-, V^i)$

$V \cdot U = -V^+ U^- - V^- U^+ + \vec{U} \cdot \vec{V} \longrightarrow \mathbb{R}^{D-2}$.

$$S = \frac{T}{2} \int d^2\sigma \left(\dot{X}^2 - X'^2 \right)$$

$\tilde{\tau} + \tilde{\sigma}$

$$\left.\begin{aligned} \tilde{\sigma}^+ &= f^+(\sigma^+) \\ \tilde{\sigma}^- &= f^-(\sigma^-) \end{aligned}\right\} \longrightarrow \text{Conformal transform}$$

$\tilde{\tau} - \tilde{\sigma}$

Trick: $\tilde{\tau} =$

$V = (V^+, V^-, V^i) \quad U = (U^+, U^-, V^i)$

$$V \cdot U = -V^+U^- - V^-U^+ + \vec{U}\cdot\vec{V}$$

$\vec{U}\cdot\vec{V} \rightarrow \mathbb{R}^{D-2}$.

$$S = \frac{T}{2}\int d^2\sigma\,(\dot{X}^2 - X'^2)$$

$\tilde{\tau} + \tilde{\sigma}$

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Trick: $\tilde{\tau} = \frac{1}{2}\left(f^+(\sigma^+) + f^-(\sigma^-)\right)$

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$$S = \frac{T}{2}\int d^2\sigma\,(\dot{X}^2 - X'^2)$$

$\tilde{\tau}+\tilde{\sigma}$

$\tilde{\sigma}^+ = f^+(\sigma^+)$

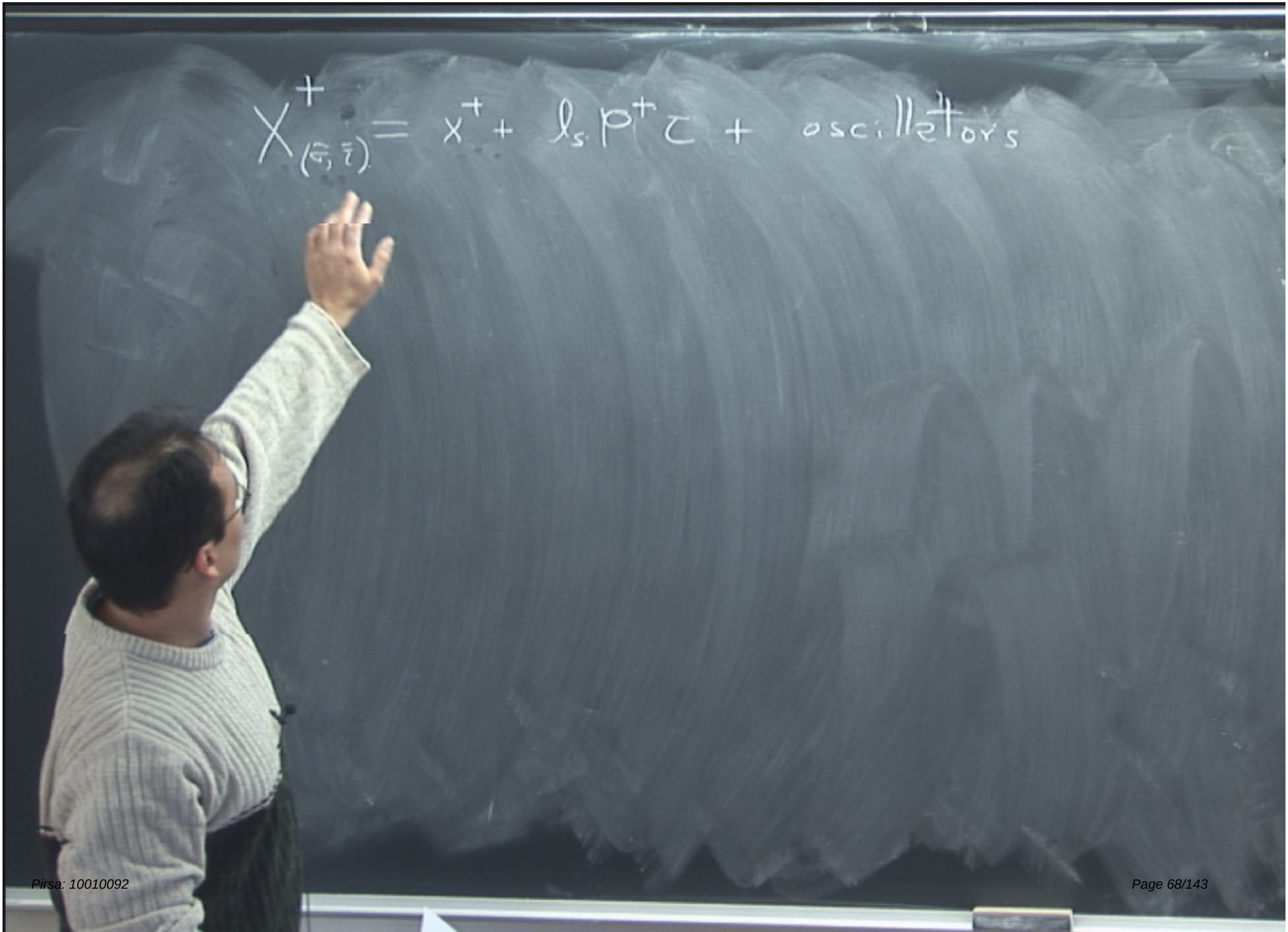
$\tilde{\sigma}^- = f^-(\sigma^-)$ $\Big\} \longrightarrow$ Conformal transform

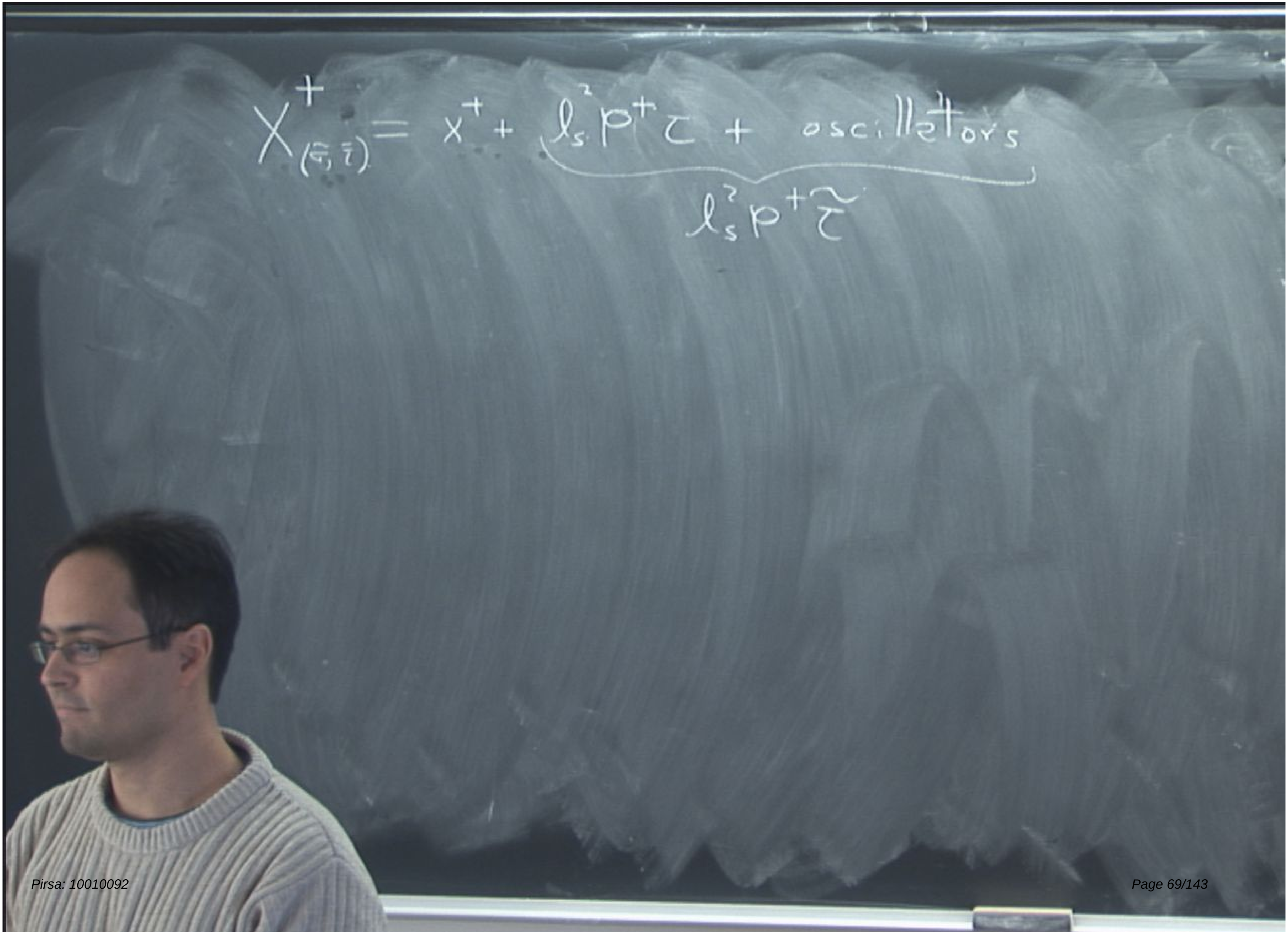
$\tilde{\tau}-\tilde{\sigma}$

Trick: $\tilde{\tau} = \frac{1}{2}\left(f^+(\sigma^+) + f^-(\sigma^-)\right)$

$\partial_+\partial_- X^\mu = 0$

$\partial_+\partial_-\tilde{\tau} = 0$


$X^+_{(\sigma,\tau)} = x^+ + l_s p^+ \tau +$ oscillators


$X^{+}_{(\tilde{\sigma},\tilde{\tau})} = x^{+} + \underbrace{l_s^2 p^{+} \tau + \text{oscillators}}_{l_s^2 p^{+} \tilde{\tau}}$

$$X^+_{(\sigma,\tau)} = x^+ + \underbrace{l_s^2 p^+ \tau + \text{oscillators}}$$

$$= x^+ + l_s^2 p^+ \tilde{\tau} \qquad l_s^2 p^+ \tilde{\tau}$$

Virasoro condition.

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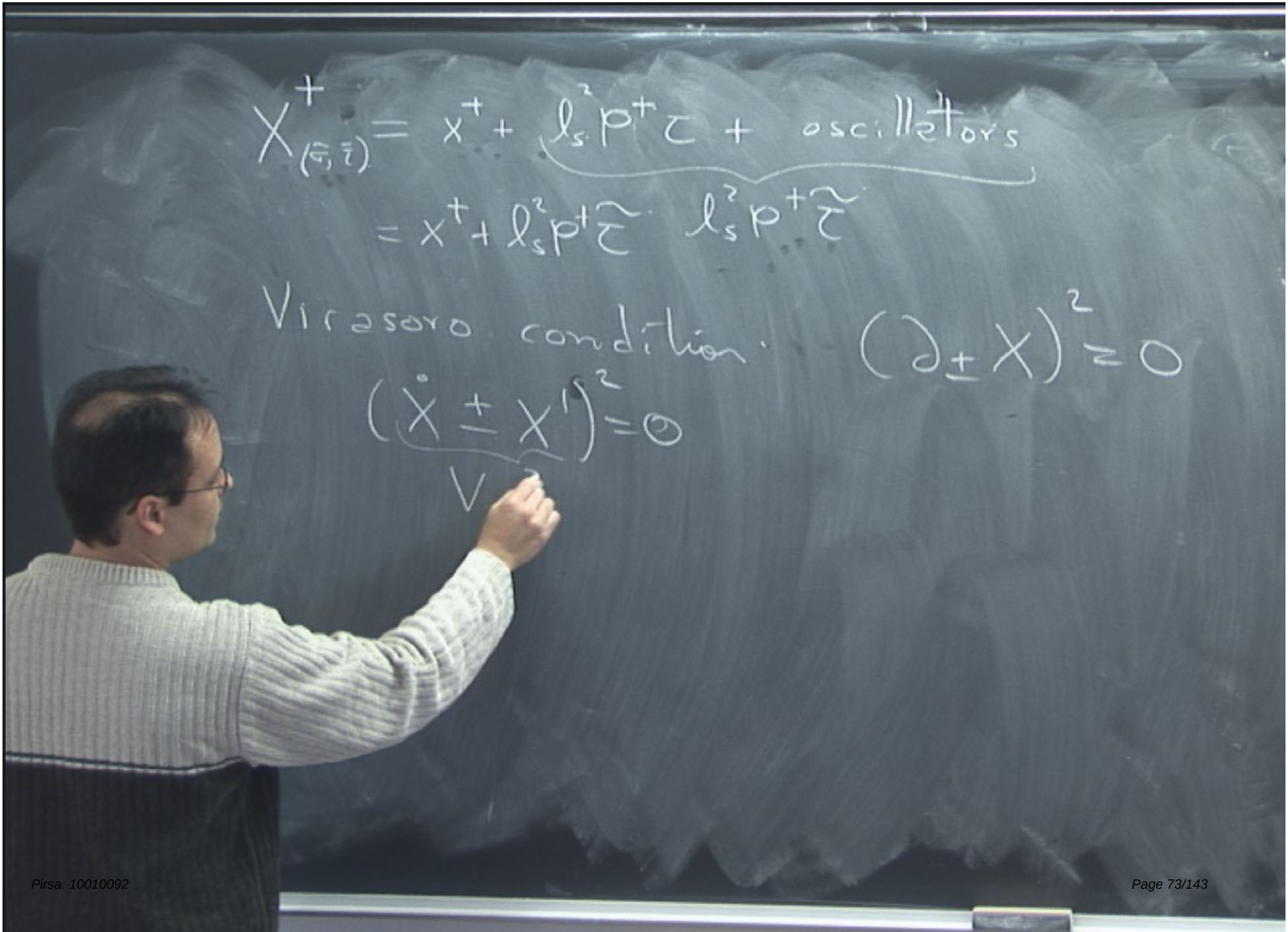
$$(\dot{X} \pm X')^2 = 0$$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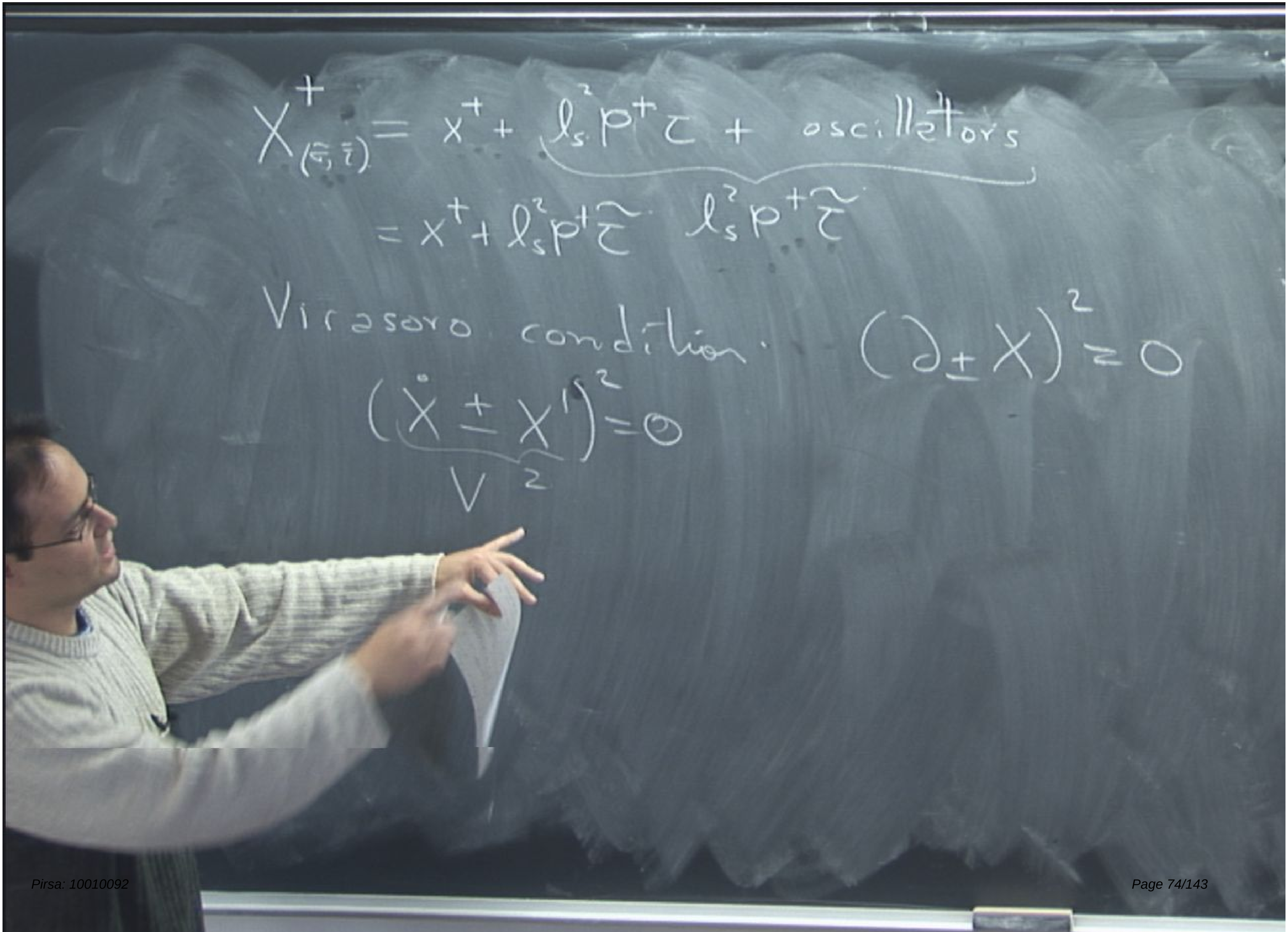
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Virasoro condition: $(\partial_{\pm} X)^2 = 0$

$$(\underbrace{\dot{X} \pm X'}_{V})^2 = 0 \quad 2$$

$$2(\dot{X}^{+} \pm X'^{+})(\dot{X}^{-} \pm X'^{-})$$

$$= (\dot{X}^{i} \pm X'^{i})^2$$

$$X^+_{(\sigma,\tau)} = x^+ + \underbrace{l_s^2 p^+ \tau + \text{oscillators}}$$

$$= x^+ + l_s^2 p^+ \tilde{\tau} \qquad l_s^2 p^+ \tilde{\tau}$$

Virasoro condition:

$$(\underbrace{\dot{X} \pm X'}_{V})^2 = 0 \qquad V^2$$

$$(\partial_\pm X)^2 = 0$$

$$2(\dot{X}^+ \pm \underbrace{\cancel{X'^+}}_{0})(\dot{X}^- \pm X'^-)$$

$$= (\dot{X}^i \pm X'^i)^2$$

$$X^{+}_{(\xi,\tau)} = x^{+} + \underbrace{l_s^2 p^{+} \tau + \text{oscillators}}$$

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$$\underbrace{(\dot{X} \pm X')^2}_{V^2} = 0$$

$$2(\dot{X}^{+} \pm \cancel{X'^{+}})(\dot{X}^{-} \pm X'^{-})$$

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$$l_s^2 p^{+}$$

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$$(\underbrace{\dot X \pm X'}_{V})^2 = 0 \qquad V^2$$

$$2(\dot X^+ \pm \cancel{X'^+})(\dot X^- \pm X'^-)$$

$$0$$

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$$2 l_s^2 p^+ (\dot X^- \pm X'^-) =$$

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$$(\underbrace{\dot{X} \pm X'}_{V})^2 = 0 \qquad 2(\dot{X}^+ \pm \cancel{X'^+})(\dot{X}^- \pm X'^-)$$

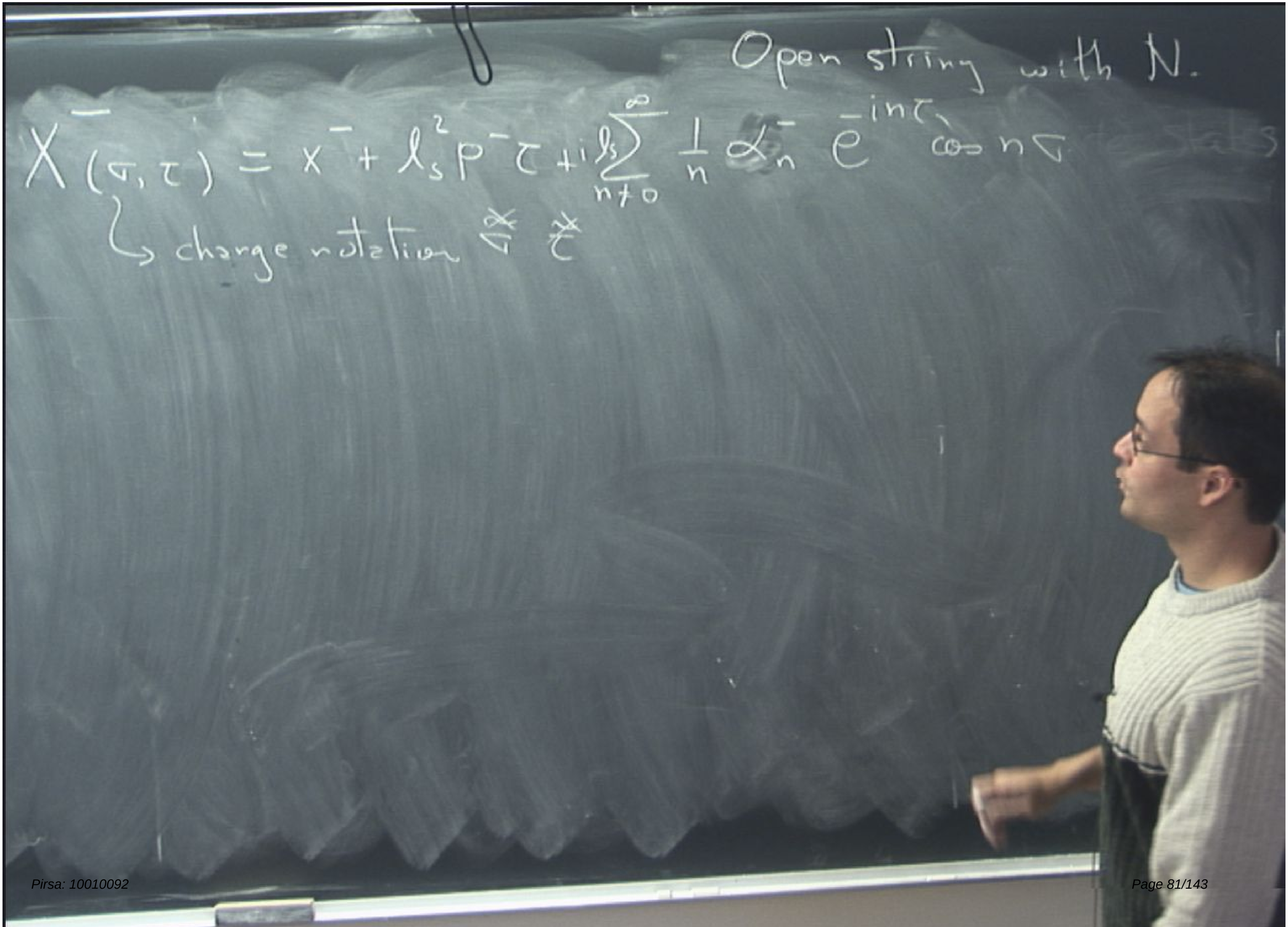
$$= (\dot{X}^i \pm X'^i)^2$$

$$l_s^2 p^+$$

$$2 l_s^2 p^+ (\dot{X}^- \pm X'^-) = (\dot{X}^i \pm X'^i)^2$$

$\bar{X}(\sigma,\tau)$

$\hookrightarrow$ change notation $\overset{\cancel{x}}{\sigma}$ $\overset{\cancel{t}}{\tau}$


Open string with N.
$X^-(\sigma,\tau) = x^- + l_s^2 p^- \tau + i l_s \sum_{n\neq 0}^{\infty} \frac{1}{n} \alpha_n^- e^{-in\tau} \cos n\sigma$
change notation σ τ

Open string with N.

$$X^-(\sigma,\tau) = x^- + l_s^2 p^- \tau + i l_s \sum_{n\neq 0}^{\infty} \frac{1}{n}\alpha_n^- e^{-in\tau} \cos n\sigma$$

↳ change notation $\cancel{\tilde{\sigma}}$ $\cancel{\tilde{\tau}}$

$$\alpha_n^- = \frac{1}{p^+ l_s}\left(\frac{1}{2} \sum_{i=1}^{D-2}\sum_{m=-\infty}^{\infty} :\alpha^i_{n-m}\alpha^i_m: - a\,\delta_{n,0}\right)$$

Open string with N.

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$\hookrightarrow n=0$

M[illegible]hell condition.

$$p^\mu [illegible] \int d\sigma \left.\frac{\partial X^\mu}{\partial \tau}\right|_{\tau=0}$$

Open string with N.

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Mass shell condition.

$$M^2 = -p^2$$

$$p^\mu = T\int_0^\pi d\sigma \frac{\partial X^\mu}{\partial\tau}\Big|_{\tau=0}$$

Open string with N.

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↳ change notation $\overset{\pi}{\sigma}$ $\overset{\pi}{\tau}$

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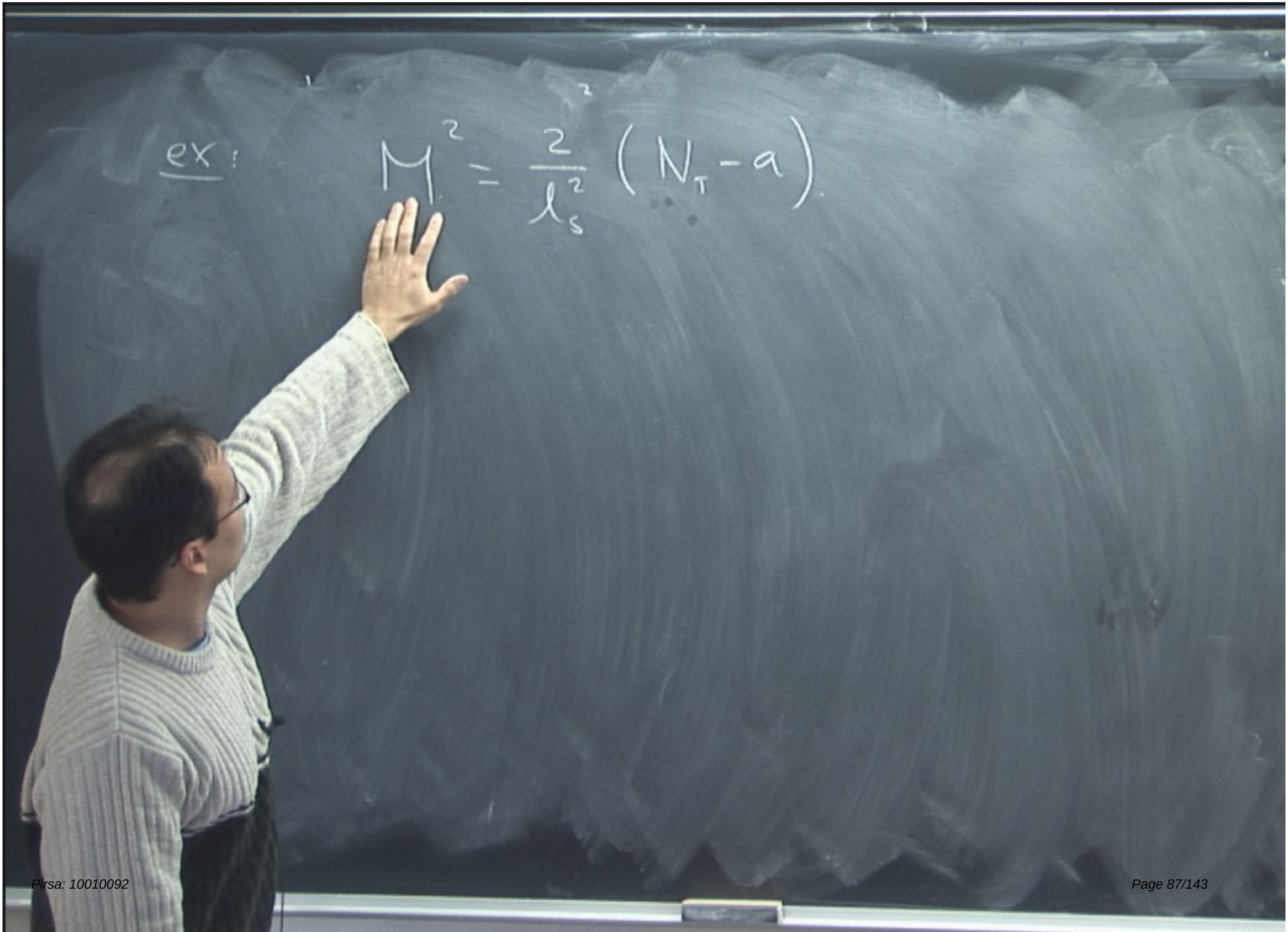
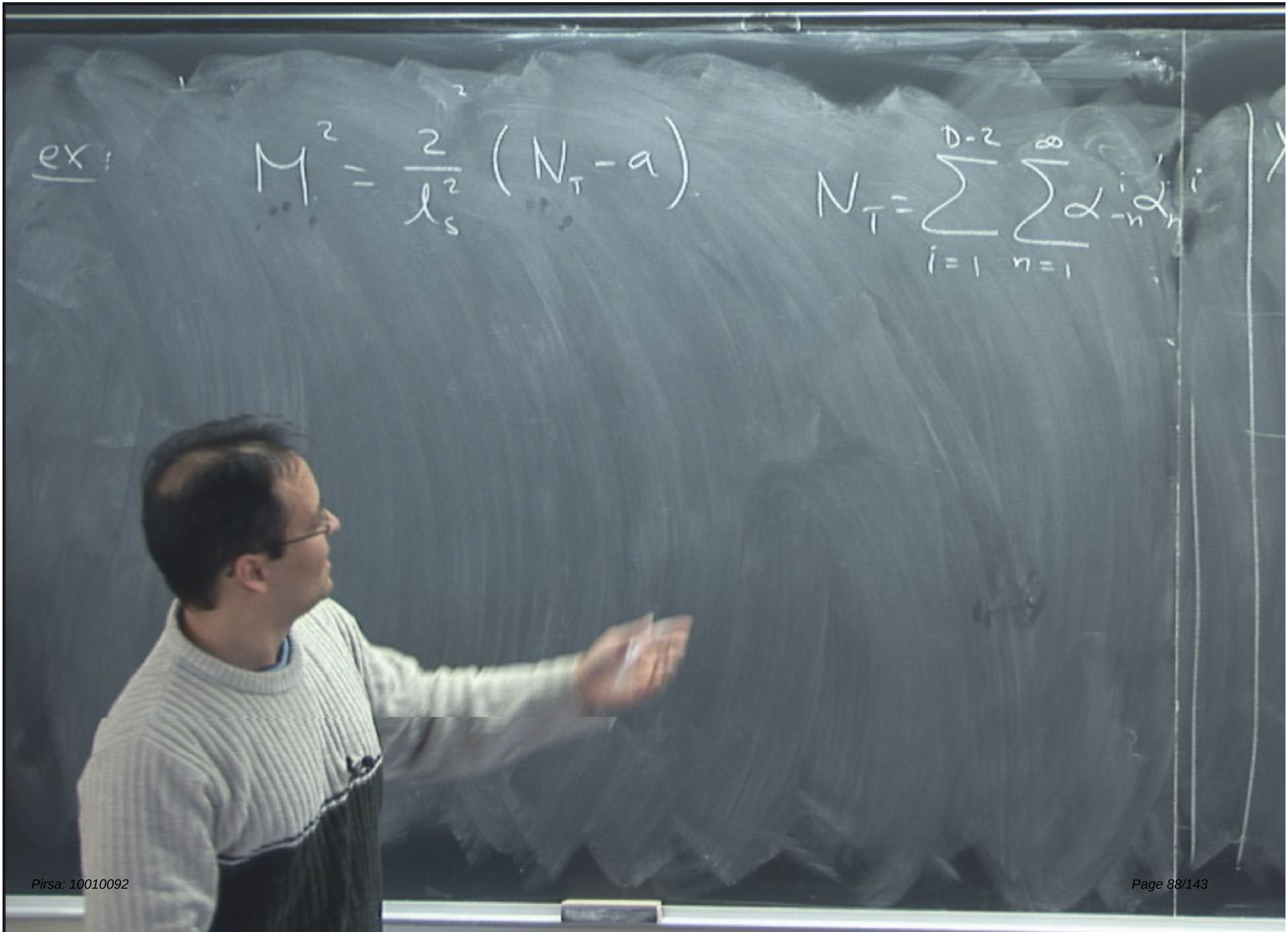

ex:
$M^2 = \frac{2}{l_s^2}(N_T - a)$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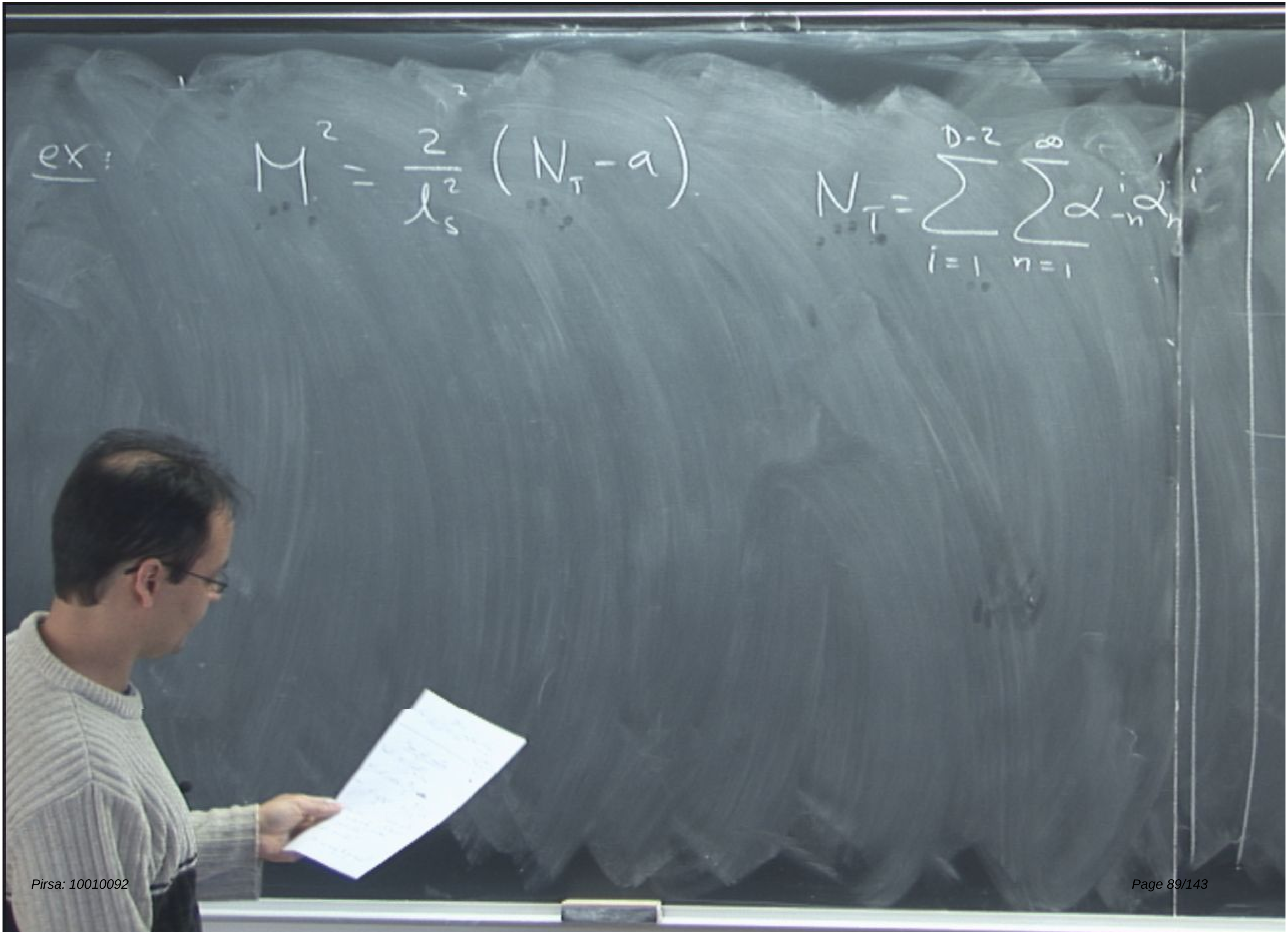
$M^2 = \frac{2}{\ell_s^2}(N_T - a)$
$N_T = \sum_{i=1}^{D-2} \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i$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ex:

$$M^2 = \frac{2}{l_s^2}\left(N_T - a\right).$$

$$N_T = \sum_{i=1}^{D-2}\sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i$$

$$[J^{i-}, J^{j-}]$$

$J^{\mu\nu}$

$$M^2 = \frac{2}{l_s^2}(N_T - a) \qquad N_T = \sum_{i=1}^{D-2}\sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i$$

$J^{\mu\nu}$

$$[J^{i-}, J^{j-}] = 0$$

$$= \sum_{m=1}^{\infty} \Delta_m \left(\alpha_{-m}^i \alpha_m^j - \alpha_{-m}^j \alpha_m^i\right)$$

$$\Delta_m = m\left(\frac{26-D}{12}\right) + \frac{1}{m}\left(\frac{D-26}{12} + 2(1-a)\right)$$

$$M^2 = \frac{2}{l_s^2}(N_T - a) \qquad N_T = \sum_{i=1}^{D-2} \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i$$

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ex: $M^2 = \frac{2}{l_s^2}(N_T - a)$ $N_T = \sum_{i=1}^{D-2} \sum_{n=1}^{\infty} \alpha^i_{-n} \alpha$

$[J^{i-}, J^{j-}] = 0$ J^{μ}

$= \sum_{m=1}^{\infty} \Delta_m (\alpha^i_{-m} \alpha^j_m - \alpha^j_{-m} \alpha^i_m)$

$\Delta_m = m\left(\frac{26-D}{12}\right) + \frac{1}{m}\left(\frac{D-26}{12} + \right.$

$D = 26$ $a = 1$

Excited States

$$|\varphi_1\rangle = \alpha^{i}_{-1}|0;k\rangle$$

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Excited States $\qquad SO(D-1,1)$

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Vector of $SO(D-2)$

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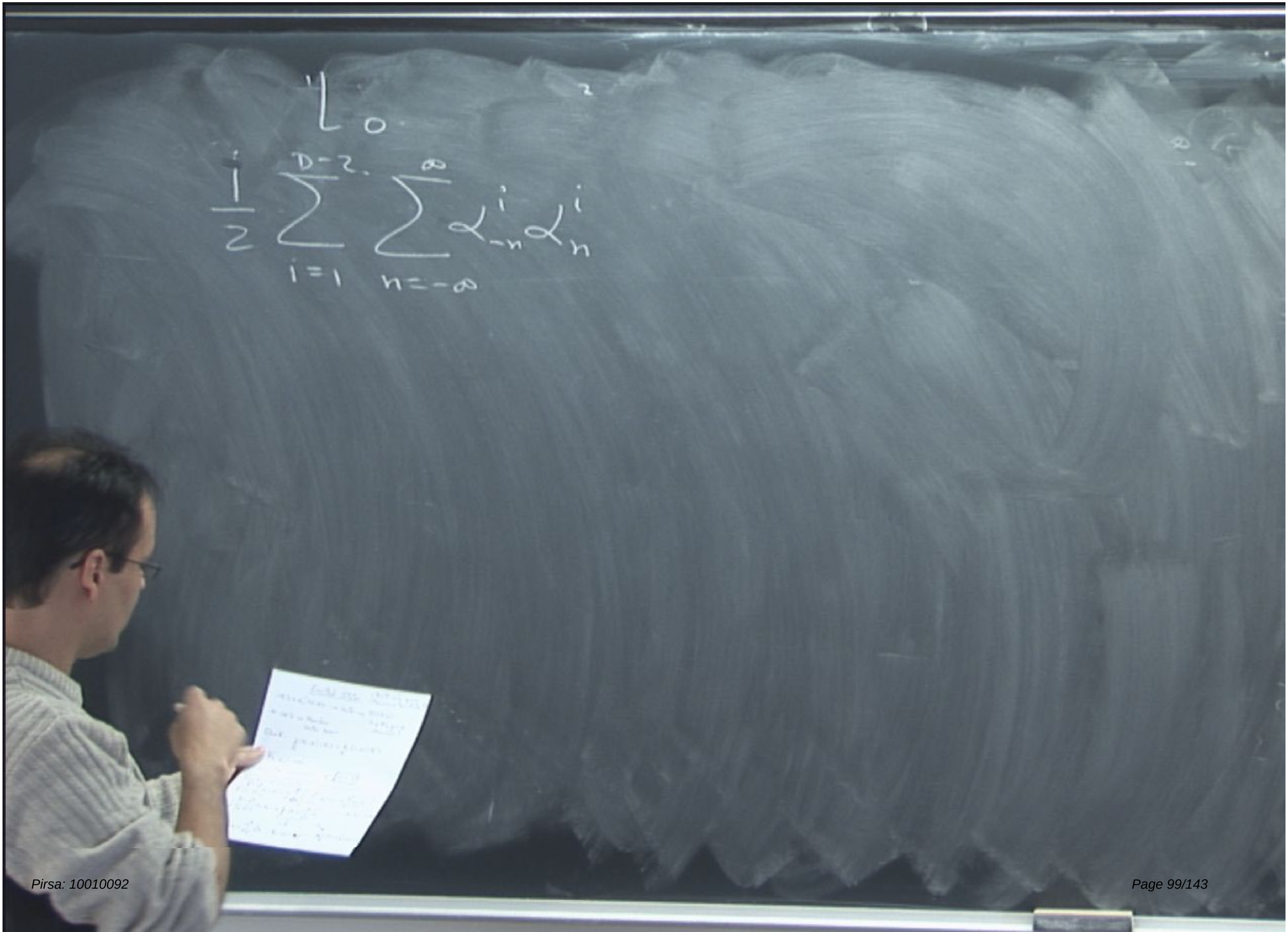
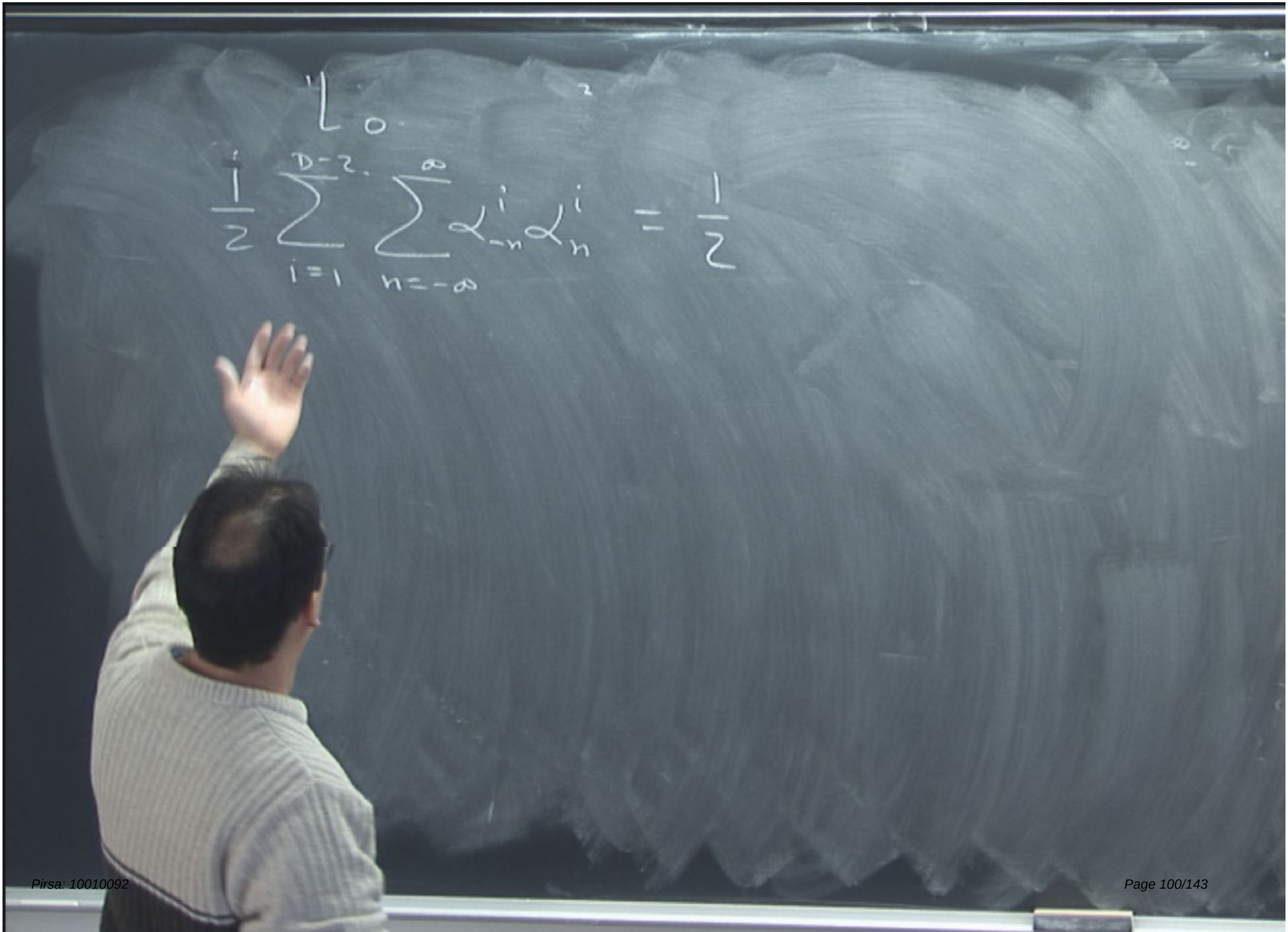
$$\boxed{a = 1}$$

Massless

Poincaré invariance

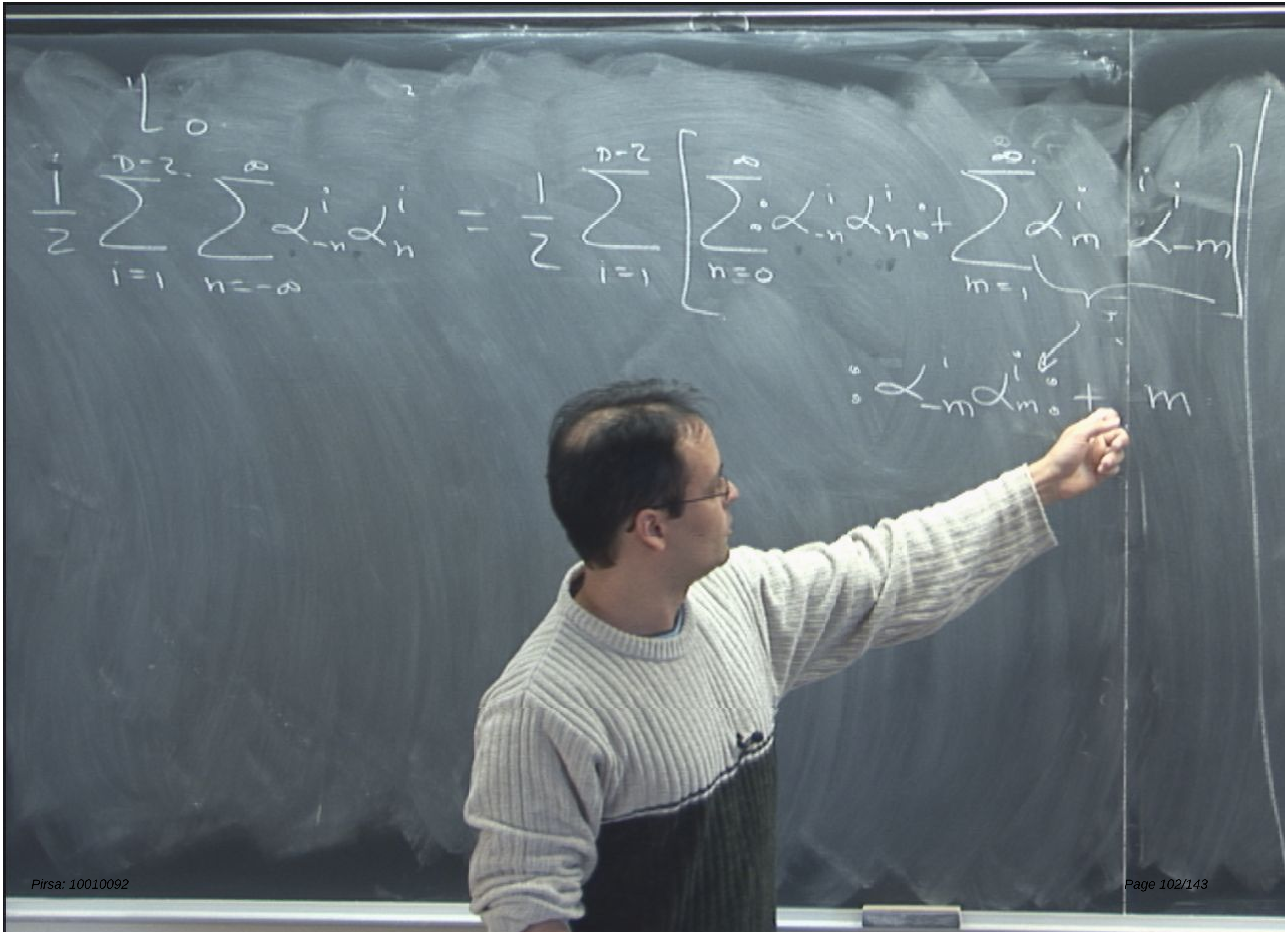
$(1-a)$


L_0
$\frac{1}{2}\sum_{i=1}^{D-2}\sum_{n=-\infty}^{\infty}\alpha^i_{-n}\alpha^i_n$


L_0
$\frac{1}{2}\sum_{i=1}^{D-2}\sum_{n=-\infty}^{\infty}\alpha_{-n}^{i}\alpha_{n}^{i} = \frac{1}{2}$

L_0

$$\frac{1}{2}\sum_{i=1}^{D-2}\sum_{n=-\infty}^{\infty}\alpha^i_{-n}\alpha^i_n = \frac{1}{2}\sum_{i=1}^{D-2}\left[\sum_{n=0}^{\infty}\alpha^i_{-n}\alpha^i_n + \sum_{m=1}^{\infty}\alpha^i_m\,\alpha^i_{-m}\right]$$


L_0
$\frac{1}{2}\sum_{i=1}^{D-2}\sum_{n=-\infty}^{\infty}\alpha^i_{-n}\alpha^i_n = \frac{1}{2}\sum_{i=1}^{D-2}\left[\sum_{n=0}^{\infty}:\alpha^i_{-n}\alpha^i_n: + \sum_{m=1}^{\infty}\underbrace{\alpha^i_m\alpha^i_{-m}}\right]$
$:\alpha^i_{-m}\alpha^i_m: + m$

L_0

$$\frac{1}{2}\sum_{i=1}^{D-2}\sum_{n=-\infty}^{\infty}\alpha^i_{-n}\alpha^i_n = \frac{1}{2}\sum_{i=1}^{D-2}\left[\sum_{n=0}^{\infty}:\alpha^i_{-n}\alpha^i_n: + \sum_{m=1}^{\infty}\underbrace{\alpha^i_m\,\alpha^i_{-m}}_{:\alpha^i_{-m}\alpha^i_m: + m}\right]$$

$$= \sum_{i=1}^{D-2}\sum_{n=1}^{\infty}:\alpha^i_{-n}\alpha^i_n: + \frac{1}{2}\sum_{i=1}^{D-2}\sum_{m=1}^{\infty} m$$

L_0

$$\frac{1}{2}\sum_{i=1}^{D-2}\sum_{n=-\infty}^{\infty}\alpha^i_{-n}\alpha^i_n = \frac{1}{2}\sum_{i=1}^{D-2}\left[\sum_{n=0}^{\infty}:\alpha^i_{-n}\alpha^i_n: + \sum_{m=1}^{\infty}\underbrace{\alpha^i_m\,\alpha^i_{-m}}_{:\alpha^i_{-m}\alpha^i_m: + m}\right]$$

$$= \sum_{i=1}^{D-2}\sum_{n=1}^{\infty}:\alpha^i_{-n}\alpha^i_n: + \underbrace{\frac{1}{2}\sum_{i=1}^{D-2}\sum_{m=1}^{\infty} m}_{\frac{1}{2}(D-2)\sum_{m=1}^{\infty} m}$$

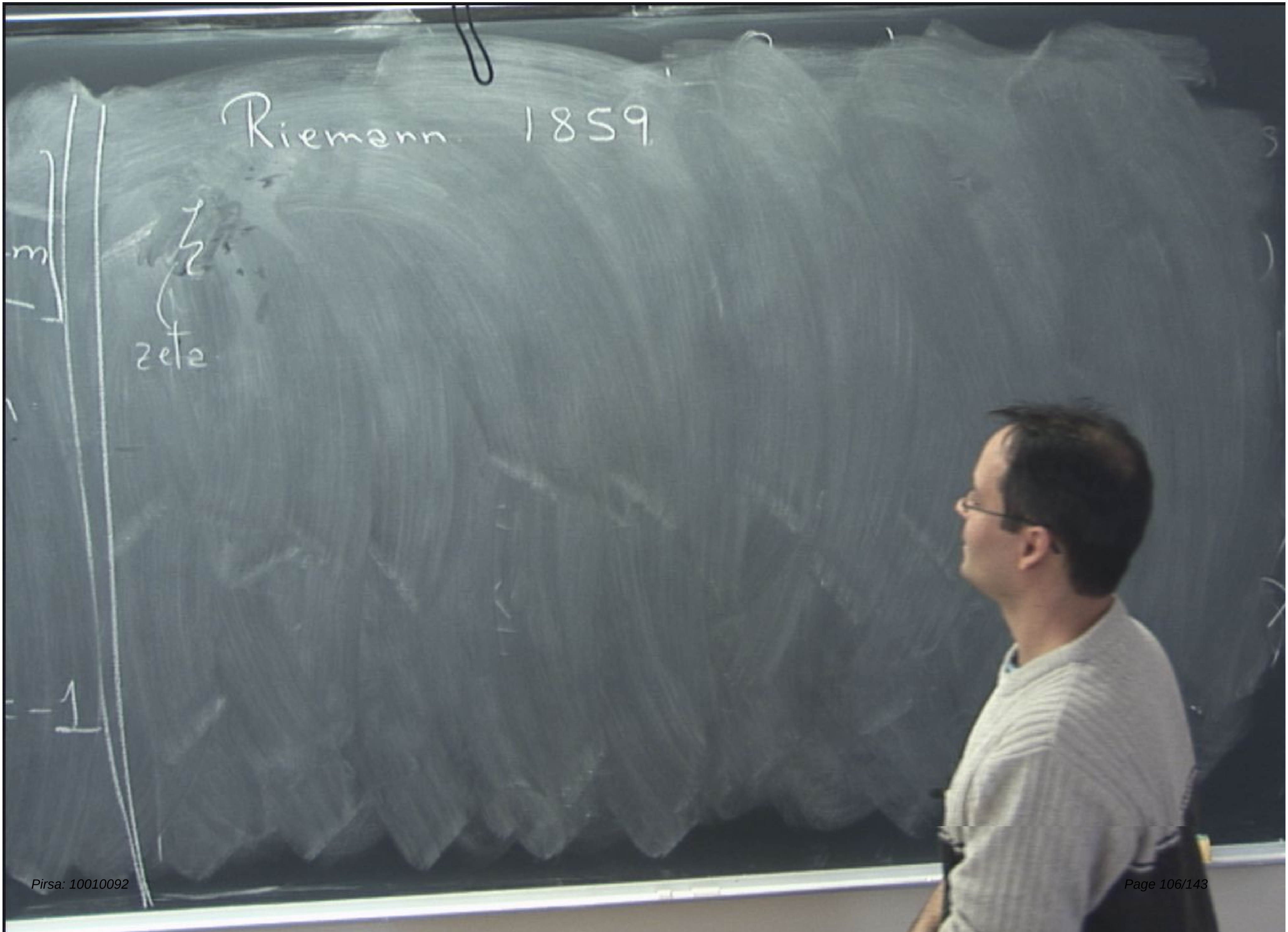
$$L_0$$

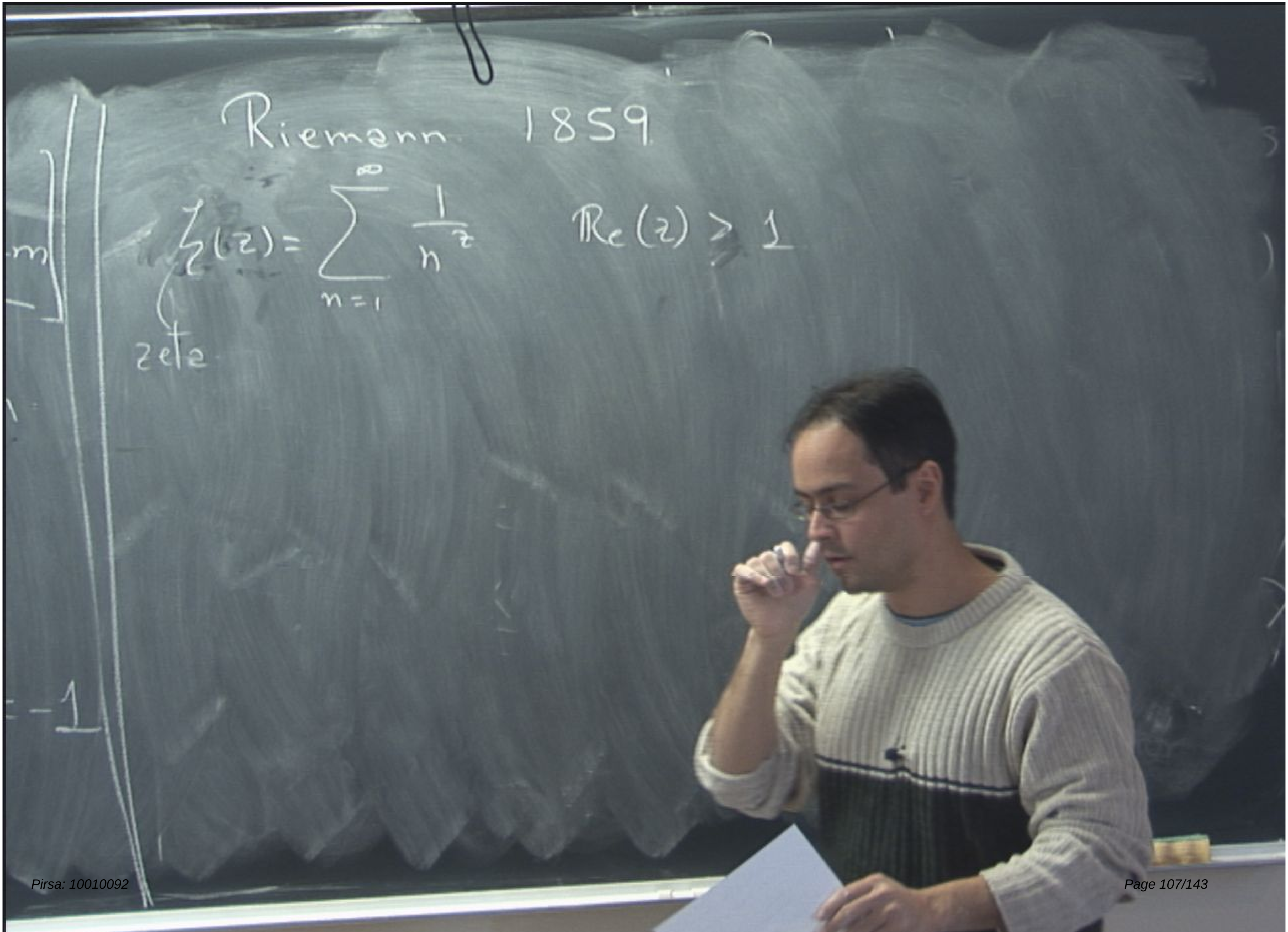
$$\frac{1}{2}\sum_{i=1}^{D-2}\sum_{n=-\infty}^{\infty}\alpha_{-n}^i\alpha_n^i = \frac{1}{2}\sum_{i=1}^{D-2}\left[\sum_{n=0}^{\infty}:\alpha_{-n}^i\alpha_n^i: + \sum_{m=1}^{\infty}\underbrace{\alpha_m^i\alpha_{-m}^i}\right]$$

$$:\alpha_{-m}^i\alpha_m^i: + m$$

$$= \sum_{i=1}^{D-2}\sum_{n=1}^{\infty}:\alpha_{-n}^i\alpha_n^i: + \underbrace{\frac{1}{2}\sum_{i=1}^{D-2}\sum_{m=1}^{\infty} m}$$

$$\frac{1}{2}(D-2)\sum_{m=1}^{\infty} m = -a = -1$$


Riemann 1859
ζ
zeta


Riemann 1859
$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}$
$\mathrm{Re}(z) > 1$
zeta

Riemann 1859

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z} \qquad \mathrm{Re}(z) > 1$$

zeta

$$\zeta(z) = 2^z \pi^{z-1} \sin\left(\frac{\pi z}{2}\right) \Gamma(1-z) \zeta(1-z)$$

Riemann 1859

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^{z}} \qquad \mathrm{Re}(z) > 1$$

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$$\zeta(-1) = \frac{1}{2\pi^2} (-1) \Gamma(2) \zeta(2)$$

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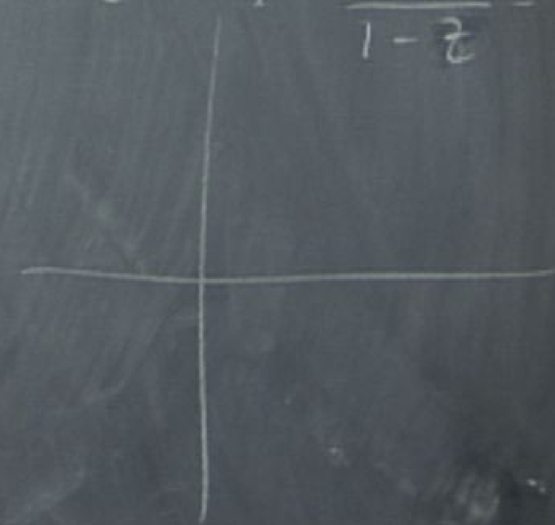
zeta

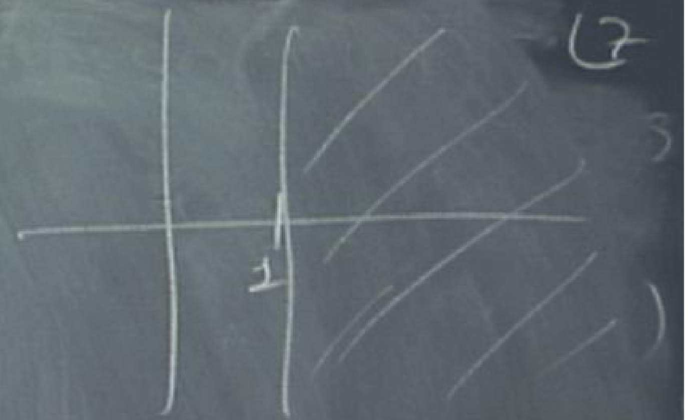
$$\zeta(z) = 2^z \pi^{z-1} \sin\left(\frac{\pi z}{2}\right) \Gamma(1-z)\, \zeta(1-z) \qquad \frac{1}{1-z} =$$

$$\zeta(-1) = \frac{1}{2\pi^2} (-1)\, \Gamma(2)\, \zeta(2)$$



Riemann 1859

$$\zeta(z)=\sum_{n=1}^{\infty}\frac{1}{n^{z}}\qquad \mathrm{Re}(z)>1$$



zeta

$$\zeta(z)=2^{z}\,\pi^{z-1}\sin\left(\frac{\pi z}{2}\right)\Gamma(1-z)\,\zeta(1-z)$$

$$\frac{1}{1-z}=1+z$$

$$\zeta(-1)=\frac{1}{2\pi^{2}}(-1)\,\Gamma(2)\,\zeta(2)$$

Riemann 1859

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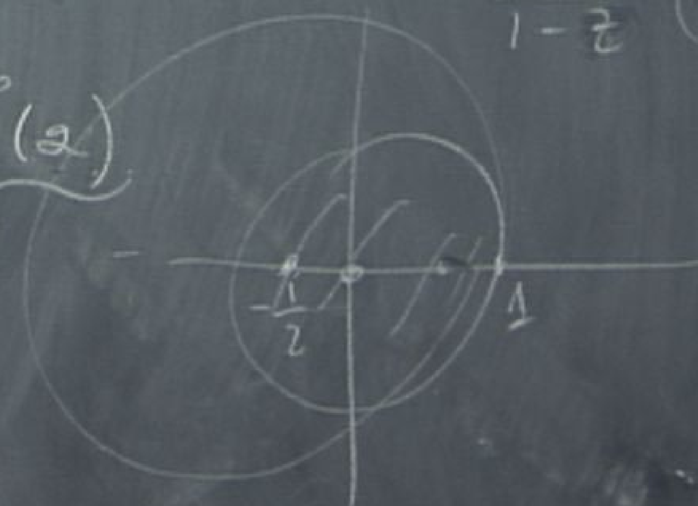


Riemann 1859

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z} \qquad \mathrm{Re}(z) > 1$$

zeta

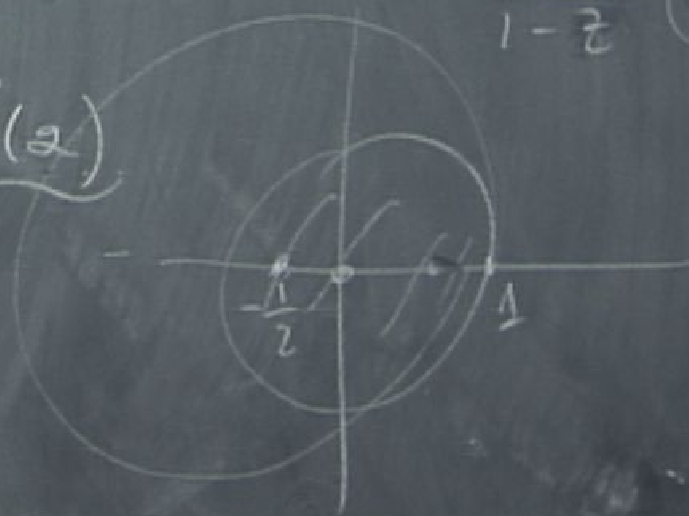
$$\zeta(z) = 2^z \pi^{z-1} \sin\left(\frac{\pi z}{2}\right) \Gamma(1-z)\, \zeta(1-z)$$

$$\frac{1}{1-z} = (1+z+\dots$$

$$\zeta(-1) = \frac{1}{2\pi^2} (-1) \cdot 1 \cdot \underbrace{\zeta(2)}_{= \sum_{n=1}^{\infty} \frac{1}{n^2}}$$

$$= -\frac{1}{2\pi^2} \zeta(2)$$

$$= -1$$



Riemann 1859

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z} \qquad \mathrm{Re}(z) > 1$$

zeta

$$\zeta(z) = 2^z \pi^{z-1} \sin\left(\frac{\pi z}{2}\right) \Gamma(1-z)\, \zeta(1-z) \qquad \frac{1}{1-z} = (1+z \cdots$$

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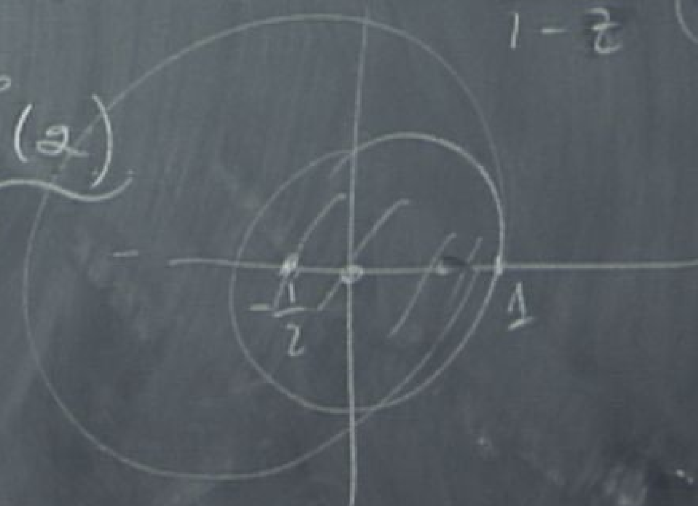
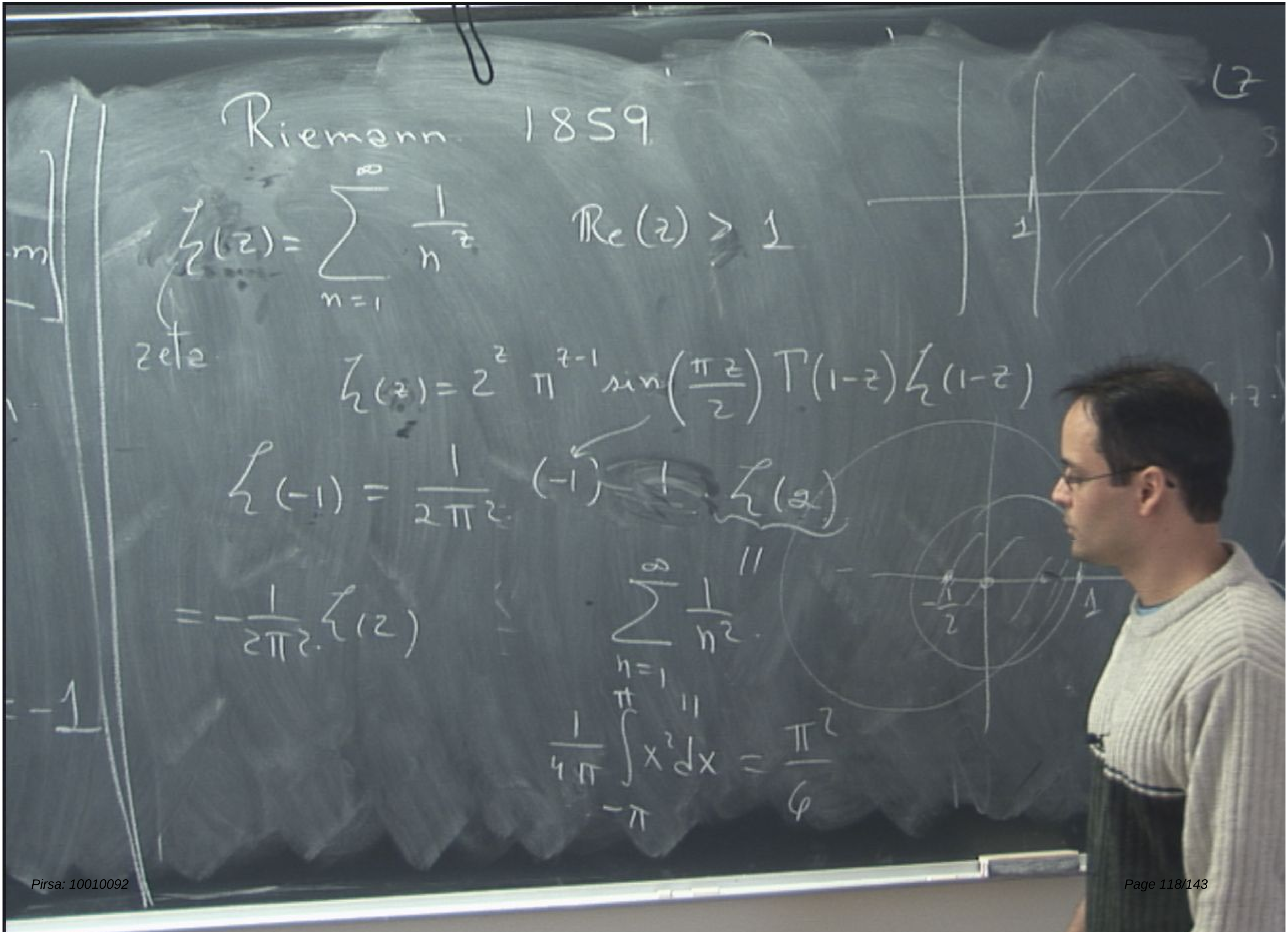
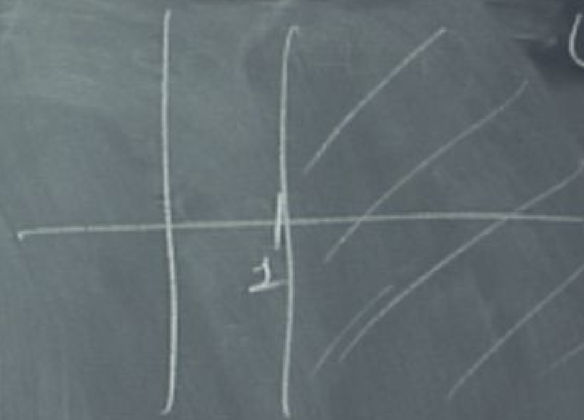



Riemann 1859
$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}$
$\mathrm{Re}(z) > 1$
zeta
$\zeta(z) = 2^z \pi^{z-1} \sin\left(\frac{\pi z}{2}\right) \Gamma(1-z) \zeta(1-z)$
$\zeta(-1) = \frac{1}{2\pi^2} (-1) \cdot 1 \cdot \zeta(2)$
$= -\frac{1}{2\pi^2} \zeta(2)$
$\sum_{n=1}^{\infty} \frac{1}{n^2}$
$\frac{1}{4\pi} \int_{-\pi}^{\pi} x^2 dx = \frac{\pi^2}{6}$
-1
$-\frac{1}{2}$
1

Riemann 1859

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z} \qquad \mathrm{Re}(z) > 1$$

zeta



$$\zeta(z) = 2^z \pi^{z-1} \sin\left(\frac{\pi z}{2}\right) \Gamma(1-z)\, \zeta(1-z)$$

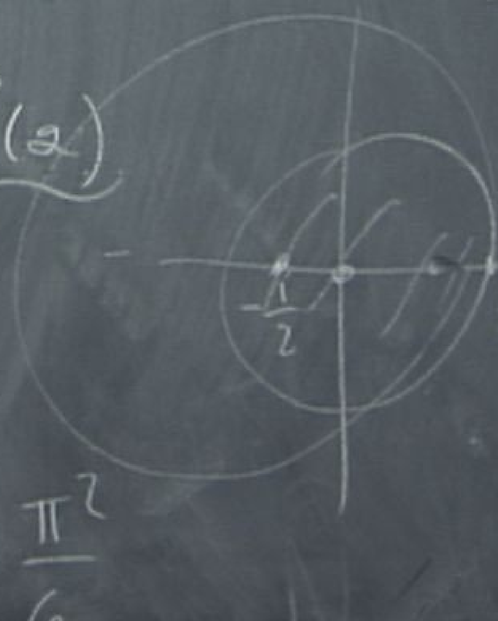
$$\frac{1}{1-z} = (1+z \ldots$$

$$\zeta(-1) = \frac{1}{2\pi^2}(-1)\; 1 \; \zeta(2)$$

$$= -\frac{1}{2\pi^2}\zeta(2) \qquad \zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{4\pi}\int_{-\pi}^{\pi} x^2\,dx = \frac{\pi^2}{6}$$

$$= -\frac{1}{12}$$

$$= -1$$



L_0

$$\frac{1}{2}\sum_{i=1}^{D-2}\sum_{n=-\infty}^{\infty}\alpha^i_{-n}\alpha^i_n = \frac{1}{2}\sum_{i=1}^{D-2}\left[\sum_{n=0}^{\infty} :\alpha^i_{-n}\alpha^i_n: + \underbrace{\sum_{m=1}^{\infty}\alpha^i_m}\right.$$

$$:\alpha^i_{-m}\alpha^i_m: +$$

$$= \sum_{i=1}^{D-2}\sum_{n=1}^{\infty} :\alpha^i_{-n}\alpha^i_n: + \underbrace{\frac{1}{2}\sum_{i=1}^{D-2}\sum_{m=1}^{\infty} m}$$

$$\frac{1}{2}(D-2)\left(-\frac{1}{12}\right) = -1 \qquad \frac{1}{2}(D-2)\underbrace{\sum_{m=1}^{\infty} m}_{\zeta(-1)} = -a$$

$$D-2 = 24 \qquad \boxed{D=26}$$

$$L_0$$

$$\frac{1}{2}\sum_{i=1}^{D-2}\sum_{n=-\infty}^{\infty}\alpha_{-n}^{i}\alpha_{n}^{i} = \frac{1}{2}\sum_{i=1}^{D-2}\left[\sum_{n=0}^{\infty}:\alpha_{-n}^{i}\alpha_{n}^{i}: + \sum_{m=1}^{\infty}\underbrace{\alpha_{m}^{i}\alpha_{-m}^{i}}\right]$$

$$:\alpha_{-m}^{i}\alpha_{m}^{i}: + m$$

$$= \sum_{i=1}^{D-2}\sum_{n=1}^{\infty}:\alpha_{-n}^{i}\alpha_{n}^{i}: + \underbrace{\frac{1}{2}\sum_{i=1}^{D-2}\sum_{m=1}^{\infty} m}$$

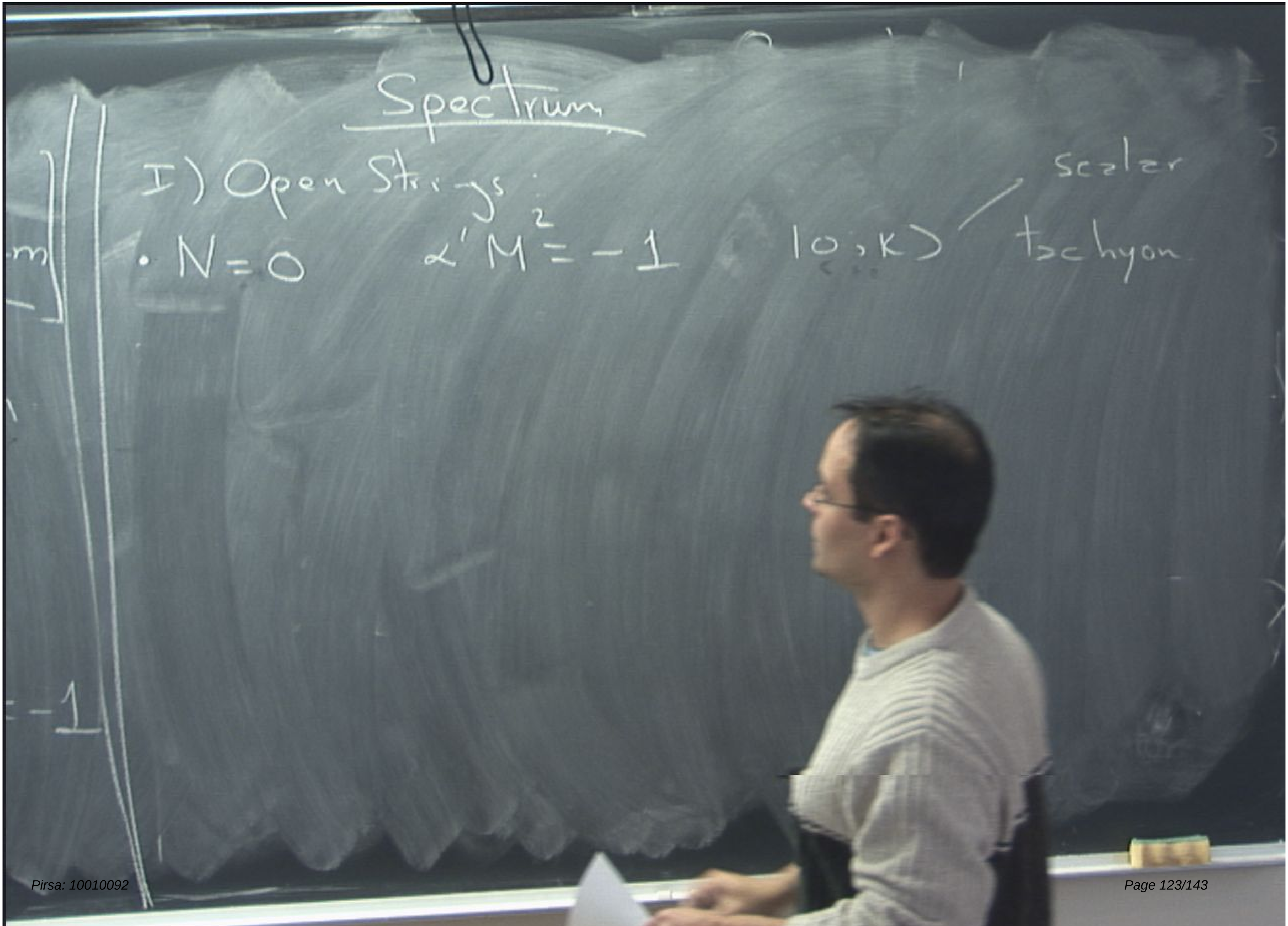
$$\frac{1}{2}(D-2)\left(-\frac{1}{12}\right) = -1 \qquad \frac{1}{2}(D-2)\underbrace{\sum_{m=1}^{\infty} m}_{\zeta(-1)} = -a = -1$$

$$D-2 = 24 \qquad \boxed{D=26}$$

Spectrum

I) Open Strings:

• $N = 0$


Spectrum
I) Open Strings:
• N = 0
α'M² = −1
|0;k⟩
scalar
tachyon

Spectrum

I) Open Strings:

- $N=0 \qquad \alpha' M^2 = -1 \qquad |0;k\rangle$ scalar tachyon
- $N=1 \qquad \alpha' M^2 = 0 \qquad \alpha^i_{-1}|0;k\rangle$ Vector boson.

Spectrum

I) Open Strings:

- $N = 0$ $\quad \alpha' M^2 = -1$ $\quad |0;k\rangle$ tachyon — scalar
- $N = 1$ $\quad \alpha' M^2 = 0$ $\quad \alpha^i_{-1}|0;k\rangle$ Vector boson. $U(1)$
- $N = 2$. $\quad \alpha' M^2 = 1$.

Spectrum

I) Open Strings :

- $N=0 \qquad \alpha' M^2 = -1 \qquad |0;k\rangle$ tachyon / scalar
- $N=1 \qquad \alpha' M^2 = 0 \qquad \alpha^i_{-1}|0;k\rangle$ Vector boson $U(1)$
- $N=2 \qquad \alpha' M^2 = 1$

$$\alpha^i_{-2}|0;k\rangle \,, \quad \alpha^i_{-1}\alpha^j_{-1}|0;k\rangle$$

Spectrum

I) Open Strings:

- $N=0 \quad \alpha' M^2 = -1 \quad |0;k\rangle$ scalar / tachyon
- $N=1 \quad \alpha' M^2 = 0 \quad \alpha^i_{-1}|0;k\rangle$ Vector boson $U(1)$
- $N=2. \quad \alpha' M^2 = 1.$

$$\alpha^i_{-2}|0;k\rangle, \quad \alpha^i_{-1}\alpha^j_{-1}|0;k\rangle$$

$$24 \quad + \quad \frac{24\times 25}{2}$$

Spectrum

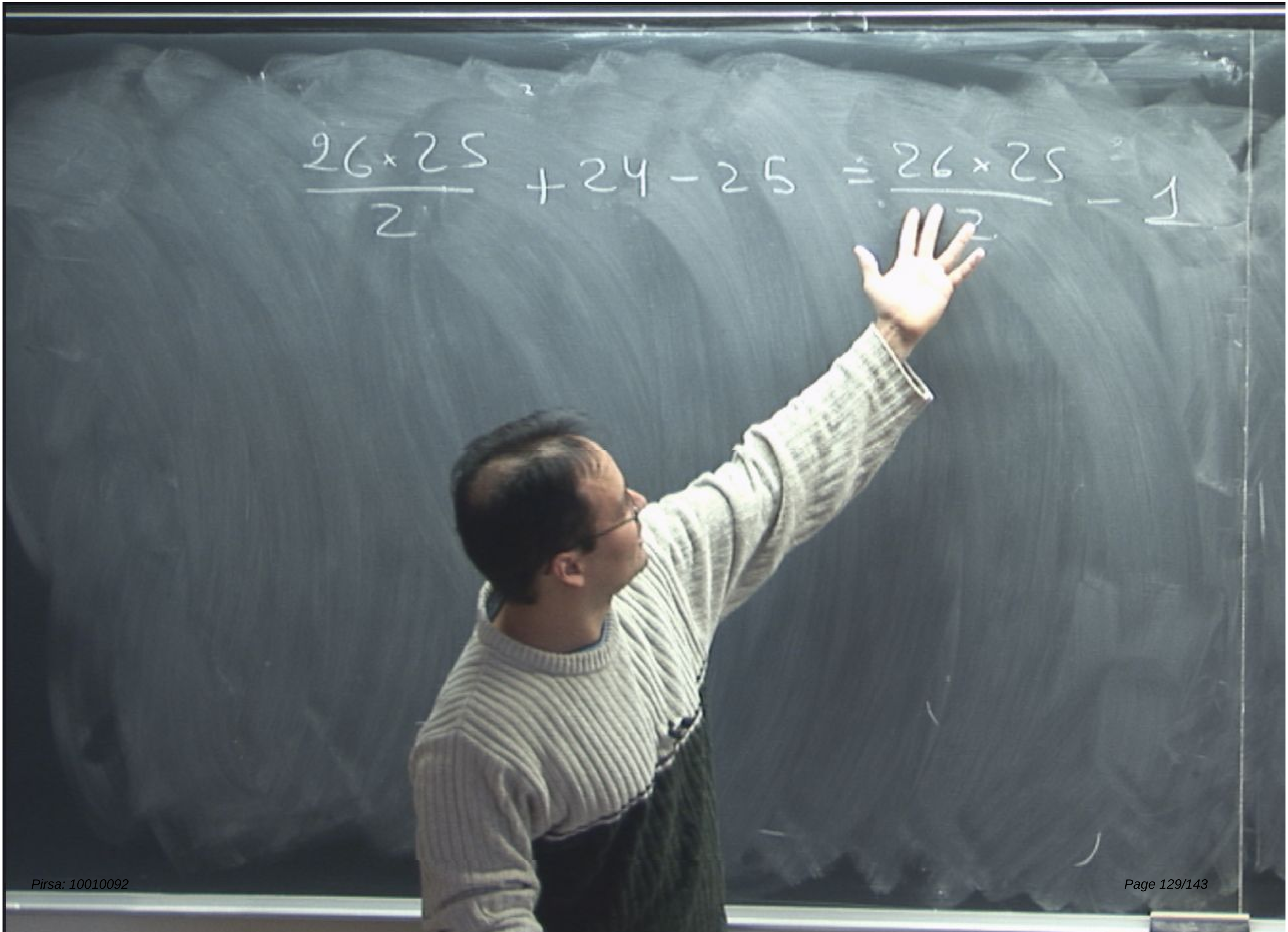
I) Open Strings:

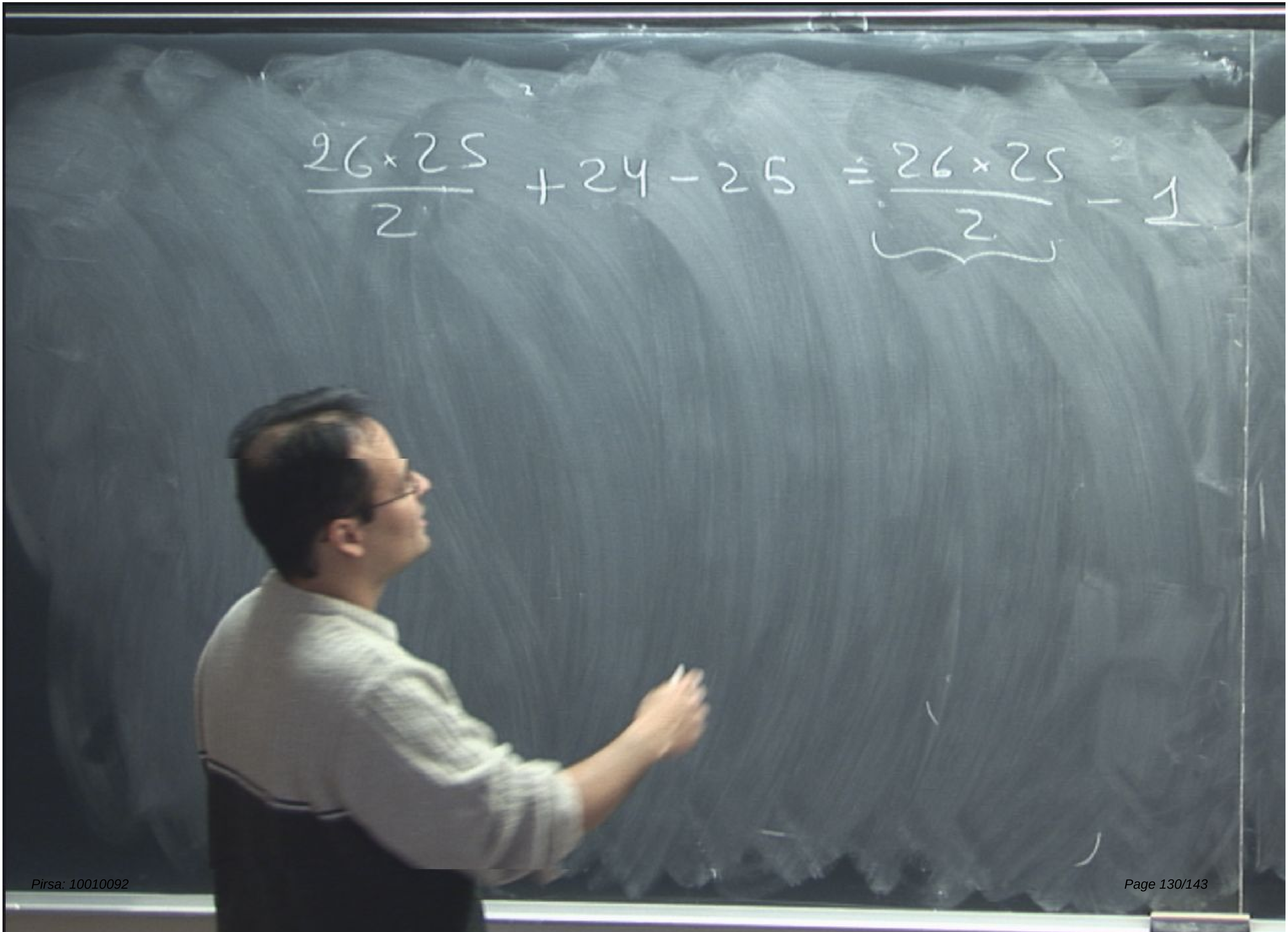
- $N=0$ $\quad \alpha' M^2 = -1$ $\quad |0;k\rangle$ tachyon scalar
- $N=1$ $\quad \alpha' M^2 = 0$ $\quad \alpha^i_{-1}|0;k\rangle$ Vector boson $U(1)$
- $N=2$. $\quad \alpha' M^2 = 1$.

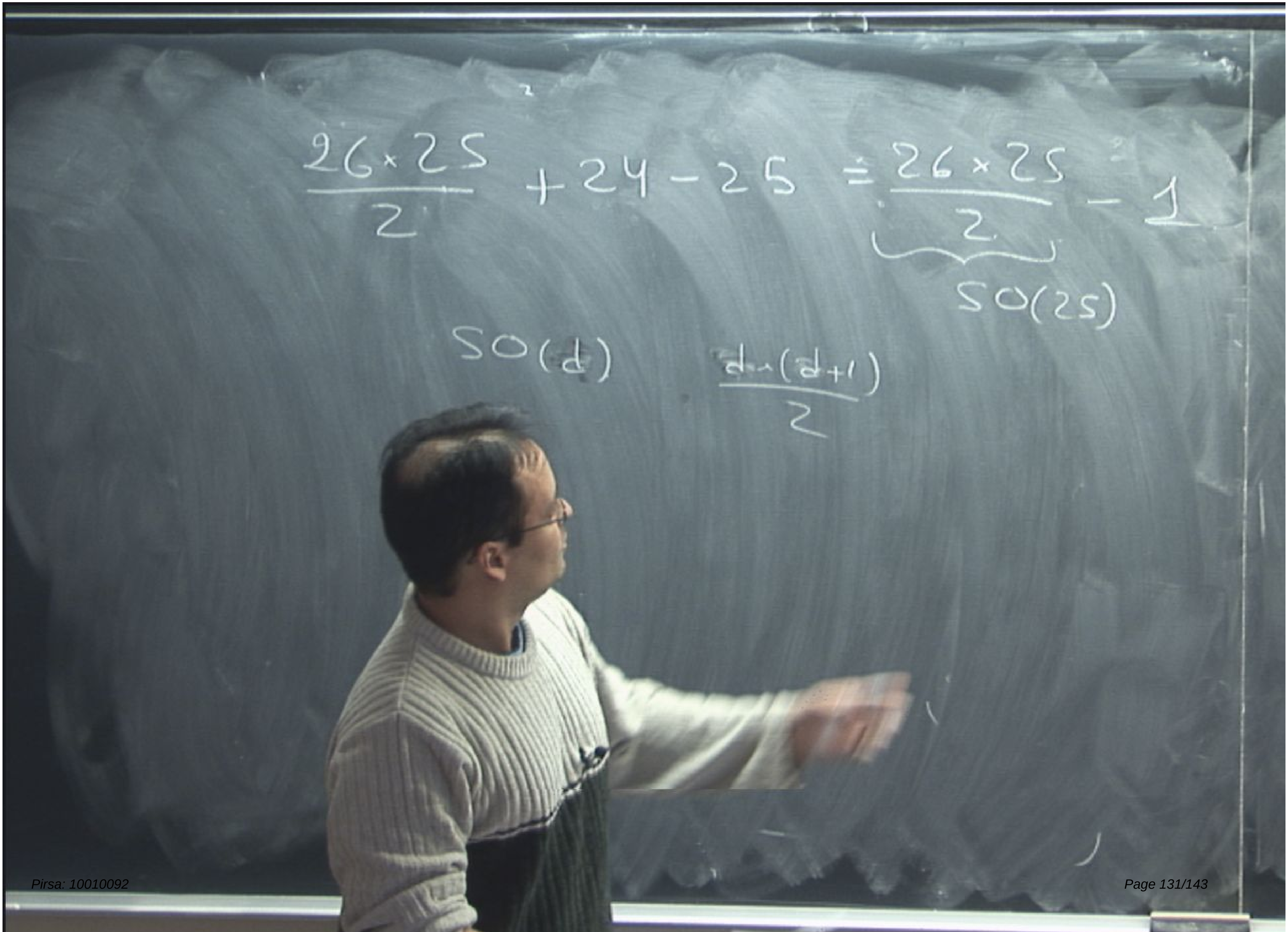
$$\alpha^i_{-2}|0;k\rangle \, , \quad \alpha^i_{-1}\alpha^j_{-1}|0;k\rangle$$

$$24 + \frac{24 \times 25}{2} =$$

$(26-2)$


26×25/2 + 24 − 25 = 26×25/2 − 1


$\frac{26\times 25}{2} + 24 - 25 = \underbrace{\frac{26\times 25}{2}} - 1$


26×25/2 + 24 − 25 = 26×25/2 − 1
SO(25)
SO(d)
d×(d+1)/2

$$\frac{26\times 25}{2} + 24 - 25 = \underbrace{\frac{26\times 25}{2}}_{SO(25)} - 1$$

Traceless

$SO(d)$ $\quad \frac{d\times(d+1)}{2}$

Spectrum

I) Closed Strings:

- $N=0$ $\quad \alpha' M^2 = -4 \quad |0;k\rangle$ tachyon, scalar
- $N=1$ $\quad \alpha' M^2 = 0 \quad \alpha^i_{-1}|0;k\rangle$
- $N=2$ $\quad \alpha' M^2 = 1$

$$\alpha^i_{-2}|0;k\rangle \,,\quad \alpha^i_{-1}\alpha^j_{-1}|0;k\rangle$$

$$24 \quad + \quad \frac{24\times 25}{2} =$$

$$(26-2)$$

Spectrum

I) Closed Strings:

- $N = 0$ $\quad \alpha' M^2 = -4 \quad |0, k\rangle$ tachyon, scalar
- $N = 1$ $\quad \alpha' M^2 = 0 \quad \alpha^i_{-1} \tilde{\alpha}^j_{-1} |0, k\rangle$

24^2 states

9

Spectrum

I) Closed Strings:

- $N=0 \qquad \alpha' M^2 = -4 \qquad |0;k\rangle$ tachyon scalar
- $N=1 \qquad \alpha' M^2 = 0 \qquad \alpha^i_{-1} \tilde{\alpha}^j_{-1} |0;k\rangle$

24^2 states

$$24^2 = \frac{24 \times 23}{2} + \frac{24 \times 25}{2}$$

Spectrum

I) Closed Strings:

- $N=0 \qquad \alpha' M^2 = -4 \qquad |0;k\rangle$ scalar tachyon
- $N=1 \qquad \alpha' M^2 = 0 \qquad \alpha^i_{-1} \tilde{\alpha}^j_{-1} |0;k\rangle$

24^2 states

$$24^2 = \frac{24\times 23}{2} + \left(\frac{24\times 25}{2} - 1\right) + 1$$

Spectrum

I) Closed Strings:

scalar

- $N=0 \qquad \alpha' M^2 = -4 \qquad |0,k\rangle$ tachyon
- $N=1 \qquad \alpha' M^2 = 0 \qquad \alpha^i_{-1} \tilde{\alpha}^j_{-1} |0,k\rangle$

24^2 states

$$24^2 = \frac{24 \times 23}{2} + \underbrace{\left(\frac{24 \times 25}{2} - 1\right)}_{SO(24)} + 1$$

symmetry rank 2 traceless

$g_{\mu\nu} \rightarrow$ Grav

Spectrum

I) Closed Strings:

- $N = 0 \qquad \alpha' M^2 = -4 \qquad |0, k\rangle$ tachyon / scalar
- $N = 1 \qquad \alpha' M^2 = 0 \qquad \alpha^i_{-1} \tilde{\alpha}^j_{-1} |0, k\rangle$

24^2 states

$$24^2 = \underbrace{\frac{24 \times 23}{2}}_{\text{Antisymm. rank 2 tensor}} + \underbrace{\left(\frac{24 \times 25}{2} - 1\right)}_{SO(24)} + 1$$

Scalar Dilaton ϕ

symmetry rank 2 traceless

$g_{\mu\nu} \rightarrow$ Graviton

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

Spectrum

I) Closed Strings:

- $N=0$ $\quad \alpha' M^2 = -4$ $\quad |0;k\rangle$ tachyon — scalar
- $N=1$ $\quad \alpha' M^2 = 0$ $\quad \alpha^i_{-1} \tilde{\alpha}^j_{-1} |0;k\rangle$

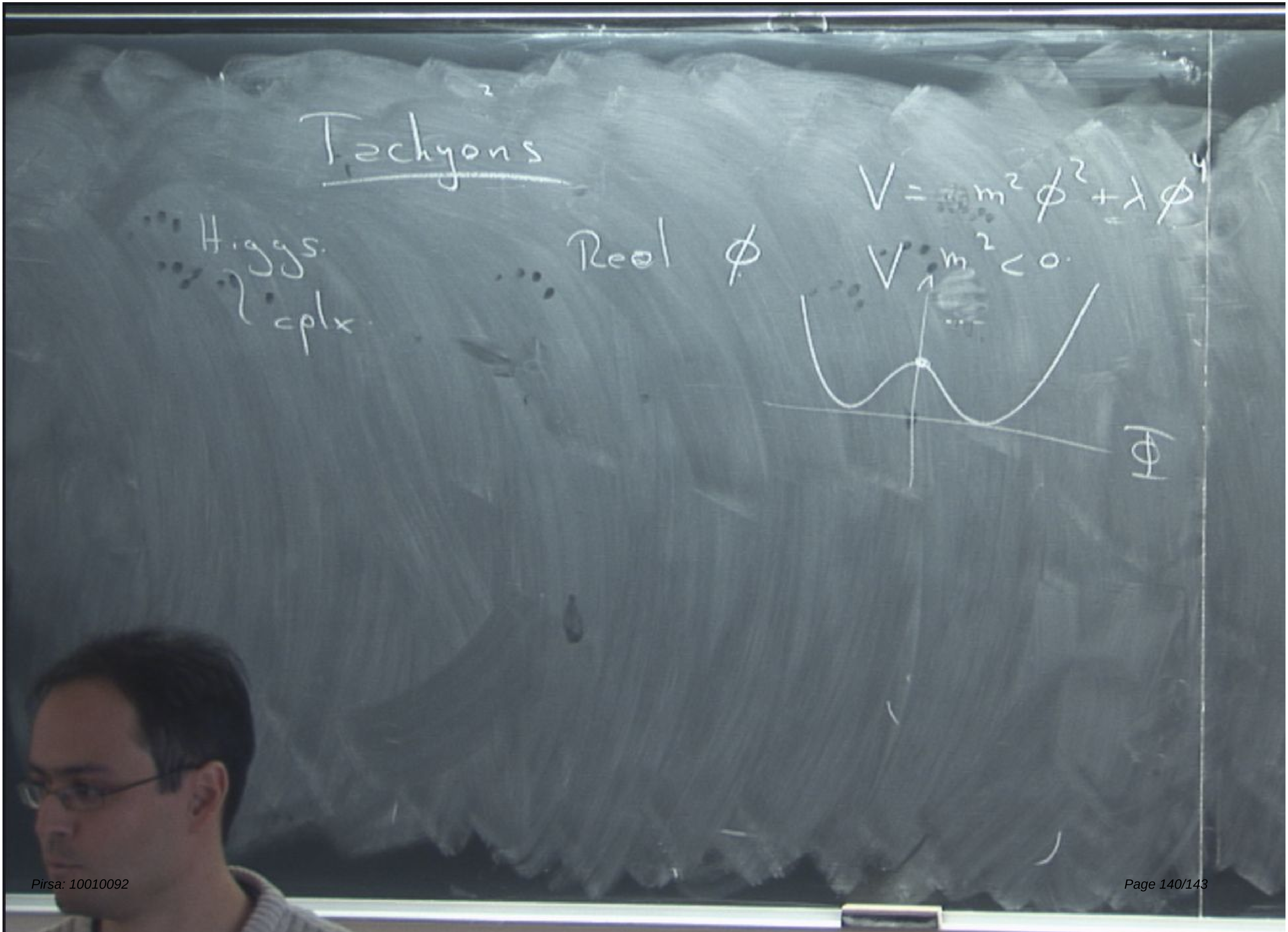
24^2 states

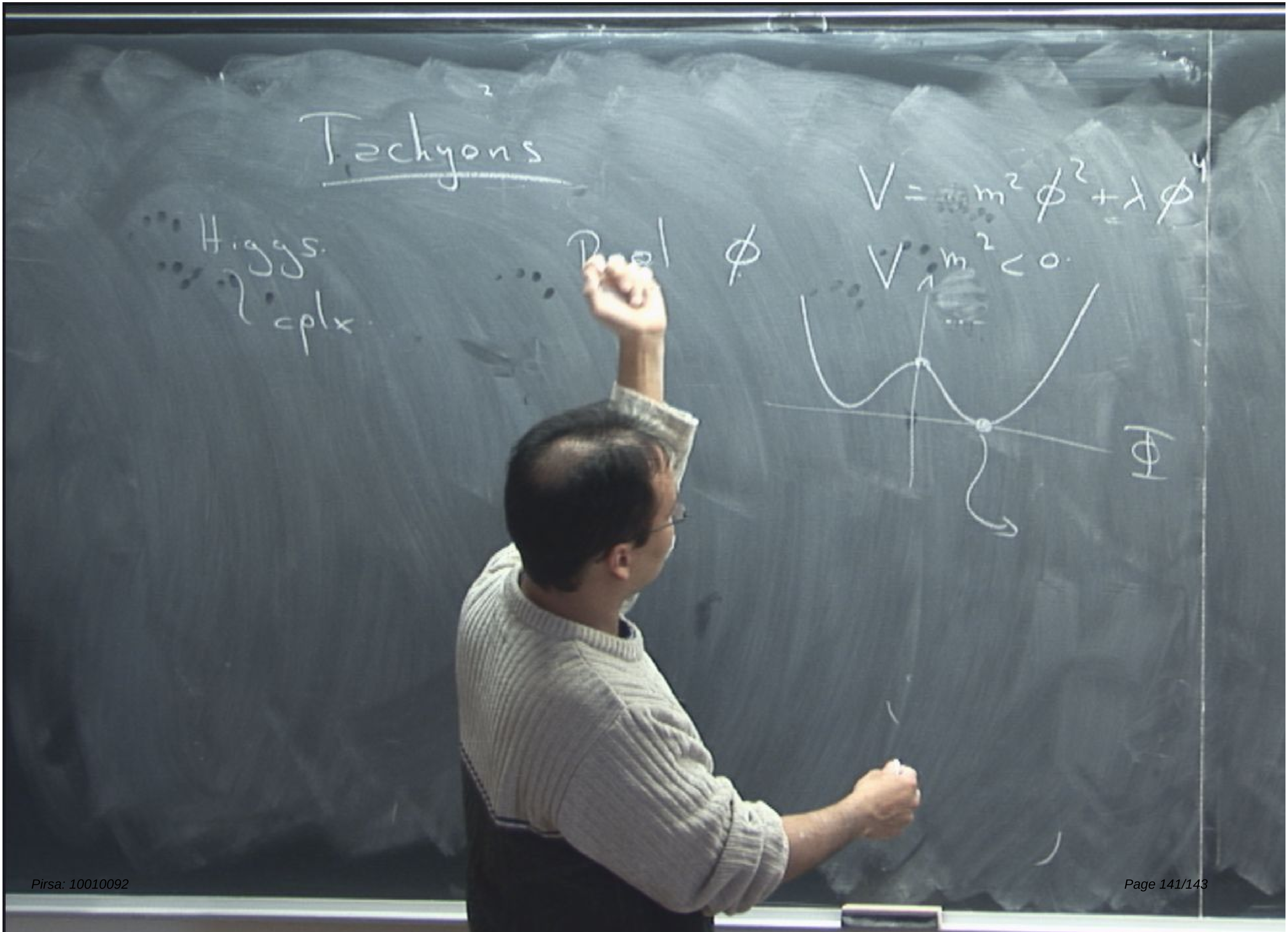
$$24^2 = \underbrace{\frac{24\times 23}{2}}_{\text{Antisym. rank 2 tensor}} + \underbrace{\left(\frac{24\times 25}{2} - 1\right)}_{SO(24)} + \underbrace{1}_{\text{Scalar Dilaton } \phi}$$

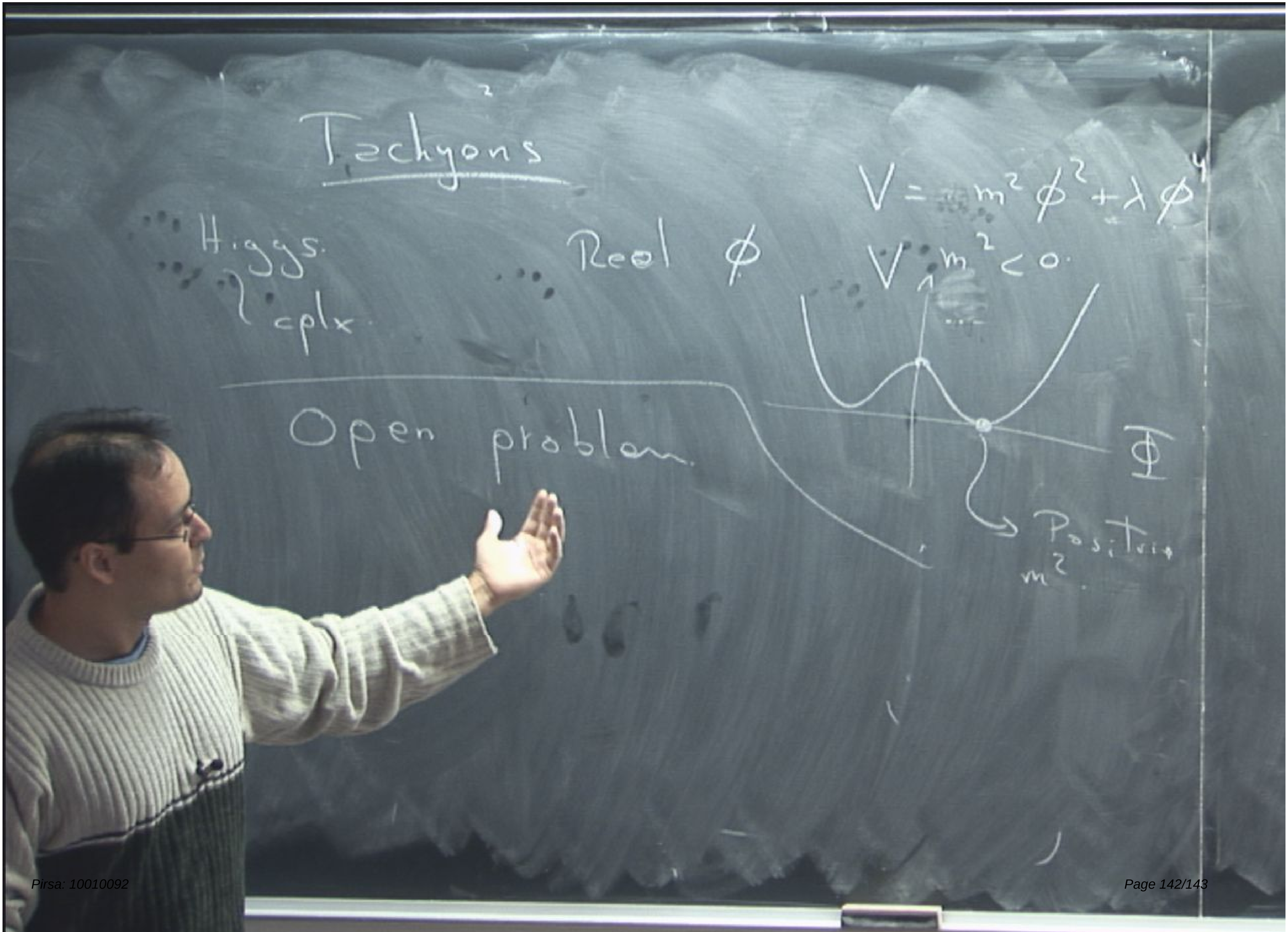
Antisym. rank 2 tensor $B_{\mu\nu}$.

symmetry rank 2 traceless $g_{\mu\nu} \to$ Graviton

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$


Tachyons
Higgs.
cplx
Real ϕ
$V = m^2\phi^2 + \lambda\phi^4$
V
$m^2 < 0.$
Φ


Tachyons
Higgs.
cplx
Real ϕ
V = m²ϕ² + λϕ⁴
V
m² < 0.
Φ


Tachyons
Higgs
cplx
Real ϕ
$V = m^2\phi^2 + \lambda\phi^4$
V $m^2 < 0$
Open problem
Φ
Positive
m^2

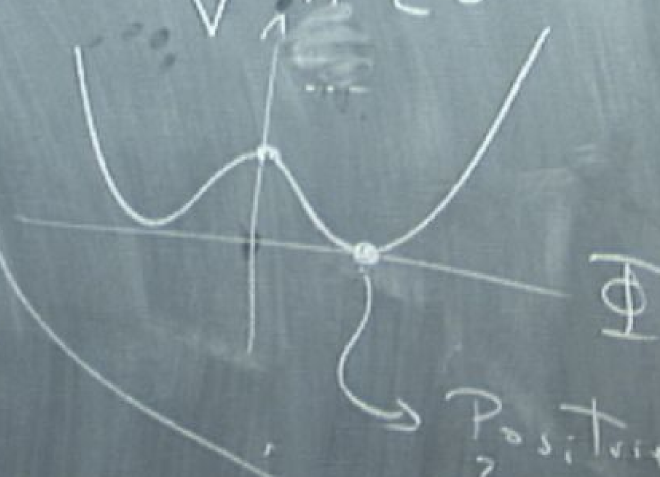
Tachyons

Higgs.
cplx.

Real ϕ

$V = m^2\phi^2 + \lambda\phi^4$

V, $m^2 < 0$.



Open problem.

* open string tachyon

$V(\Phi(x))$

* closed string tachyon ?