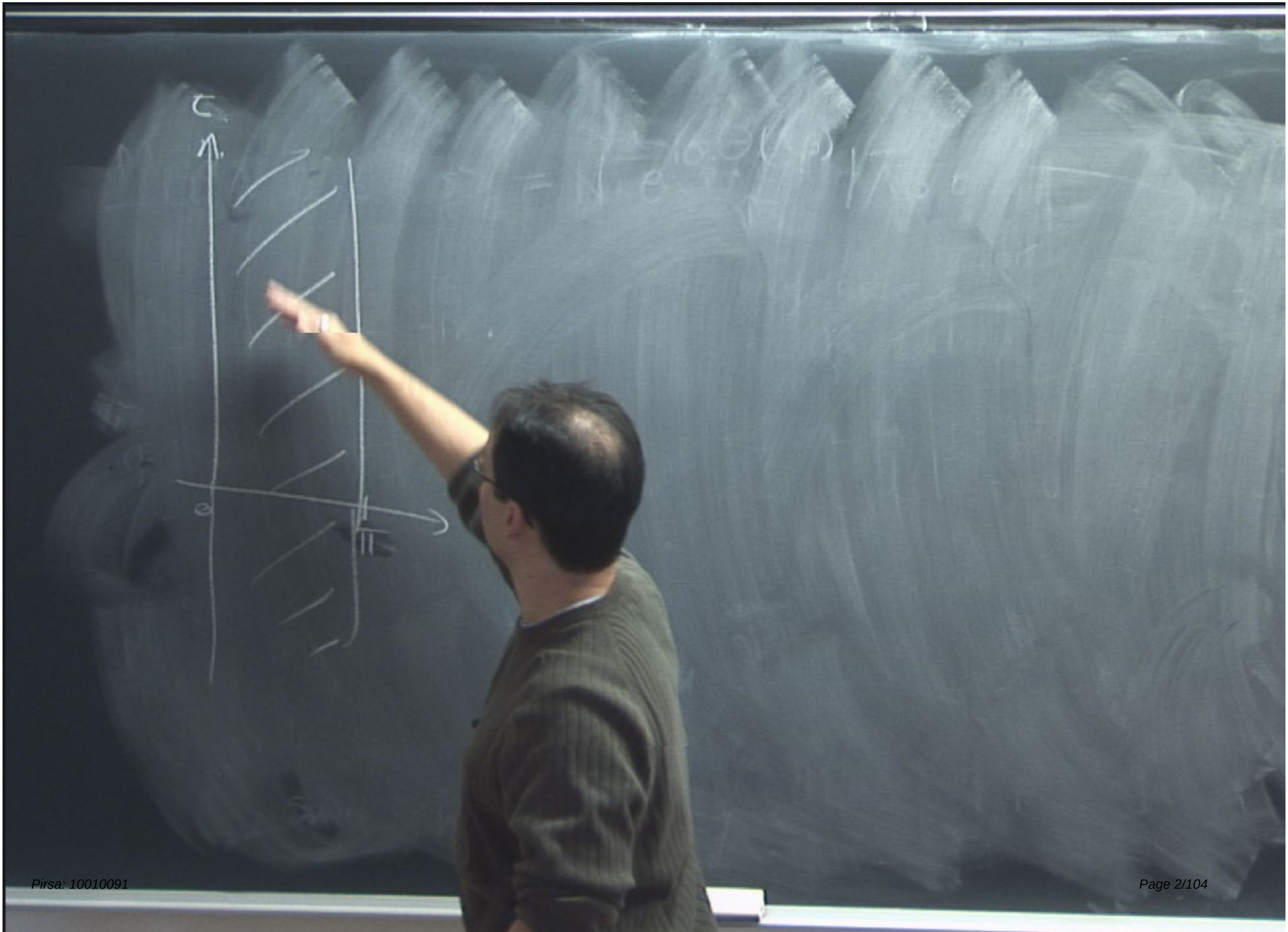
Title: String Theory - Review (PHYS 623) - Lecture 3

Date: Jan 27, 2010 11:20 AM

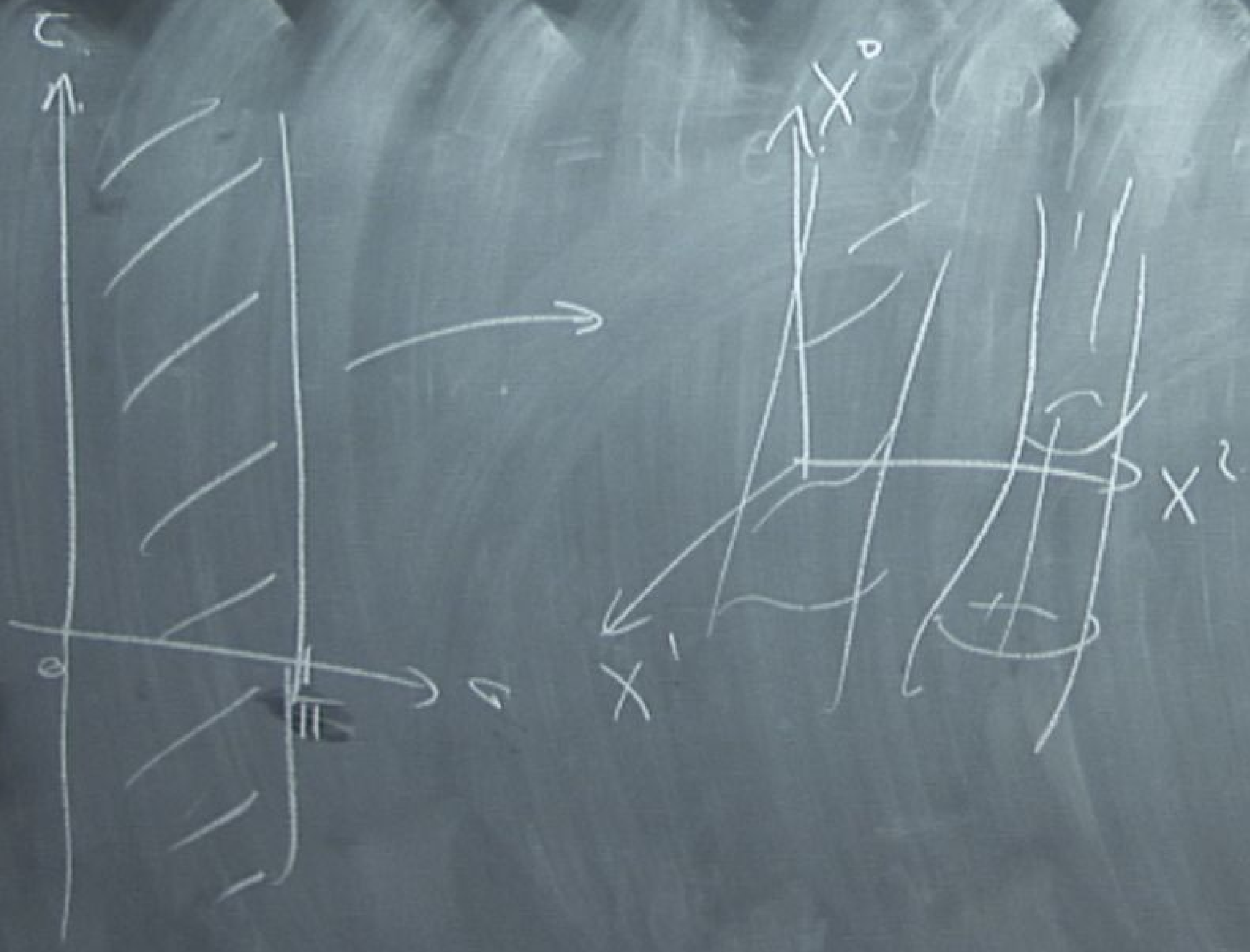
URL: http://pirsa.org/10010091

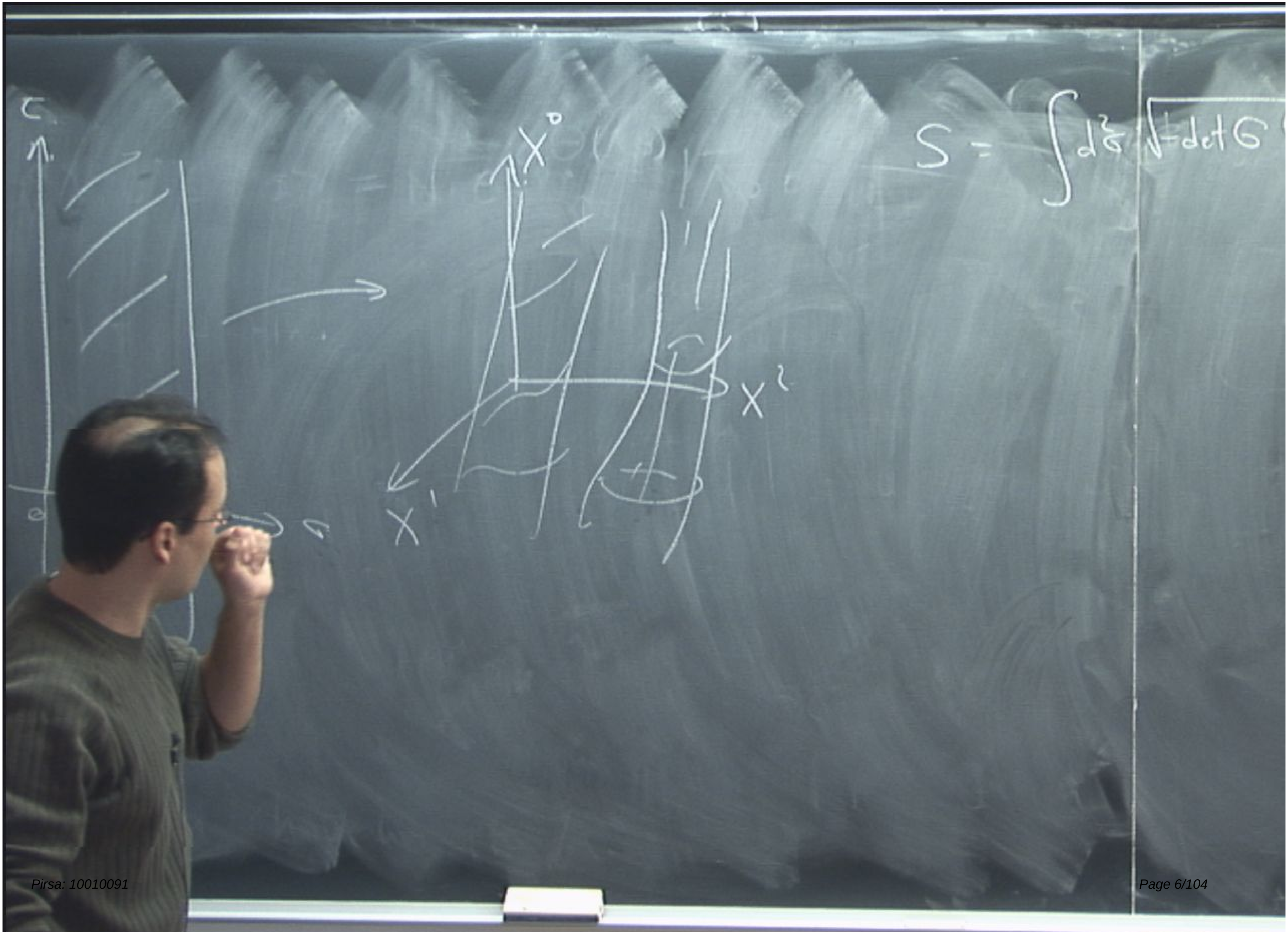
Abstract:


τ
0


τ
0
σ
X^0
X^2
X^1

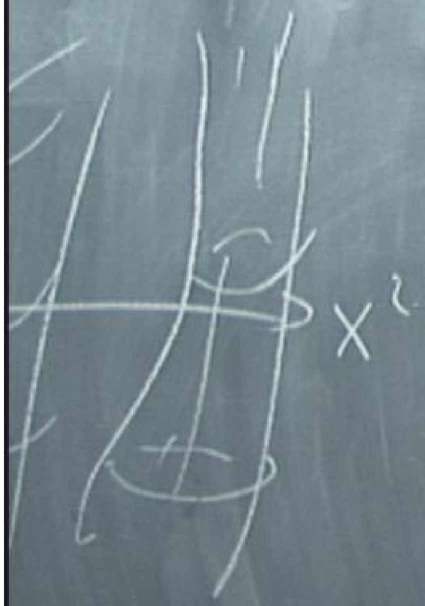

τ
0
π
σ
X^0
X^2
X^1

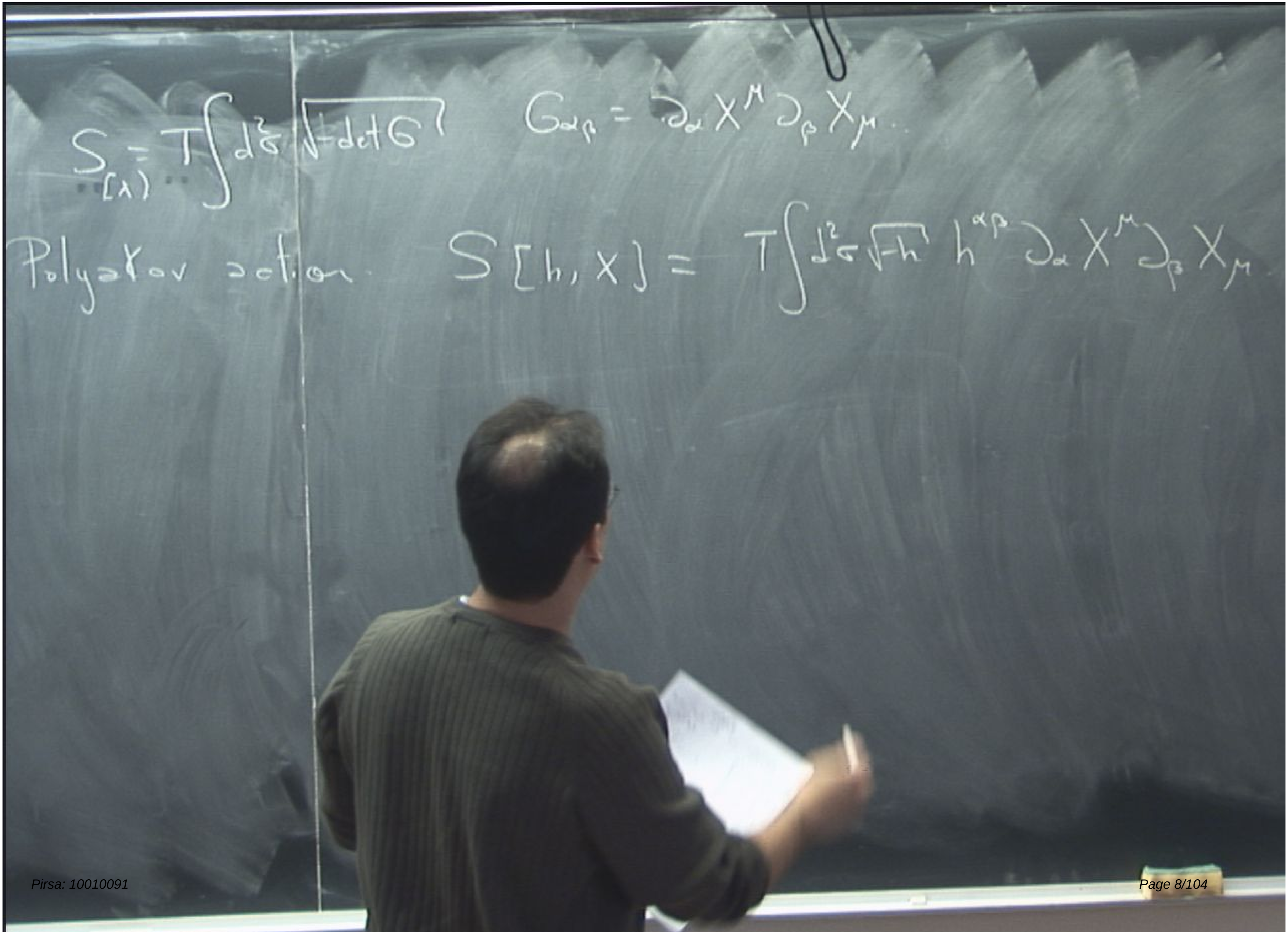

τ
0
σ
X^0
X^1
X^2



$$S = \int d^2\sigma \sqrt{-\det G}$$

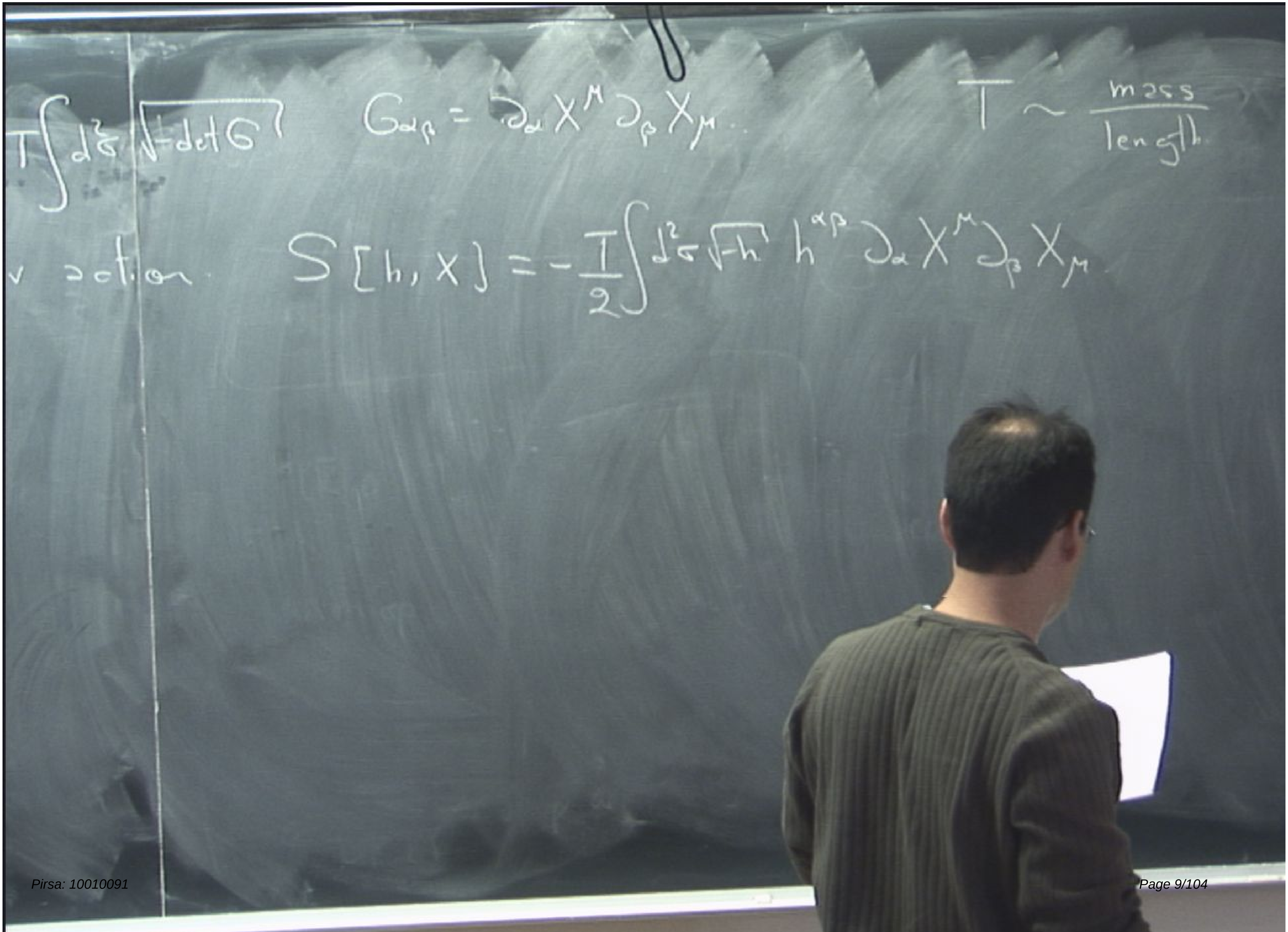
$$S = T\int d^2\sigma \sqrt{-\det G} \qquad G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$$





$$S_{[X]} = T\int d^2\sigma \sqrt{-\det G} \qquad G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$$

Polyakov action $S[h,X] = T\int d^2\sigma \sqrt{-h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu$



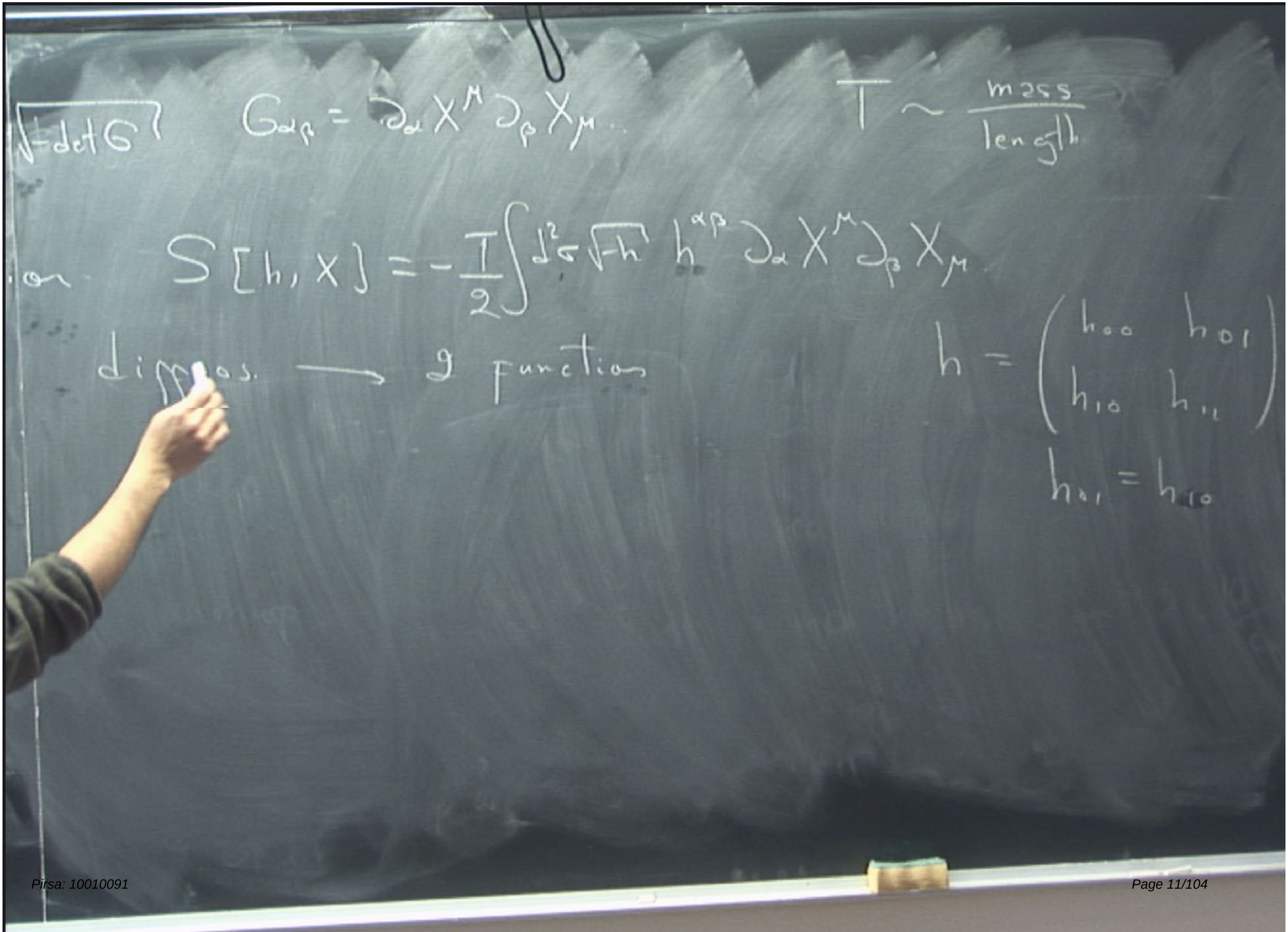
$T\int d^2\sigma \sqrt{-\det G}$ $\quad G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$ $\quad T \sim \frac{\text{mass}}{\text{length}}$

v action $\quad S[h,X] = -\frac{T}{2}\int d^2\sigma \sqrt{-h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu$

$T\int d^2\sigma \sqrt{-\det G}$ $\quad G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$ $\quad T \sim \frac{\text{mass}}{\text{length}}$

v action. $\quad S[h, X] = -\frac{T}{2}\int d^2\sigma \sqrt{-h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu$

diffeos. ⟶ 2 function



$\sqrt{-\det G}$ $G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$ ..

$T \sim \frac{\text{mass}}{\text{length}}$

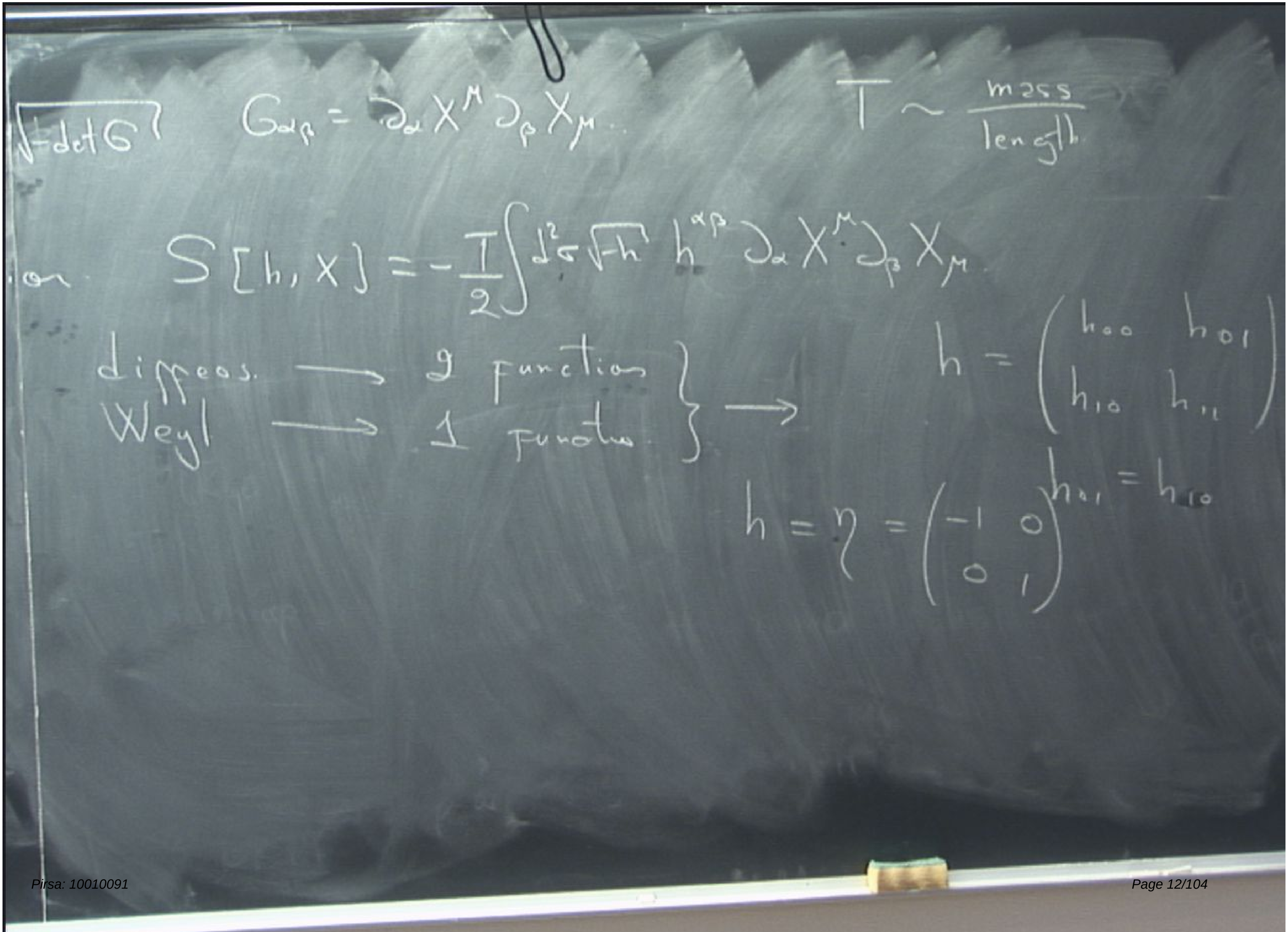
on

$$S[h, X] = -\frac{T}{2}\int d^2\sigma \sqrt{-h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu .$$

diffeos. $\longrightarrow$ 3 function

$$h = \begin{pmatrix} h_{00} & h_{01} \\ h_{10} & h_{11} \end{pmatrix}$$

$$h_{01} = h_{10}$$



$\sqrt{-\det G}$ $\quad G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$ $\quad T \sim \frac{\text{mass}}{\text{length}}$

on $\quad S[h,X] = -\frac{T}{2}\int d^2\sigma \sqrt{-h}\, h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu$

$$\left.\begin{array}{l} \text{diffeos.} \longrightarrow 2 \text{ function} \\ \text{Weyl} \longrightarrow 1 \text{ functio} \end{array}\right\} \longrightarrow \quad h = \begin{pmatrix} h_{00} & h_{01} \\ h_{10} & h_{11} \end{pmatrix}$$

$h_{01} = h_{10}$

$$h = \eta = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\sqrt{-\det G}$ $\quad G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$ $\quad T \sim \frac{\text{mass}}{\text{length}}$

$\eta_{\mu\nu}$

on $\quad S[h, X] = -\frac{T}{2}\int d^2\sigma \sqrt{-h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu$

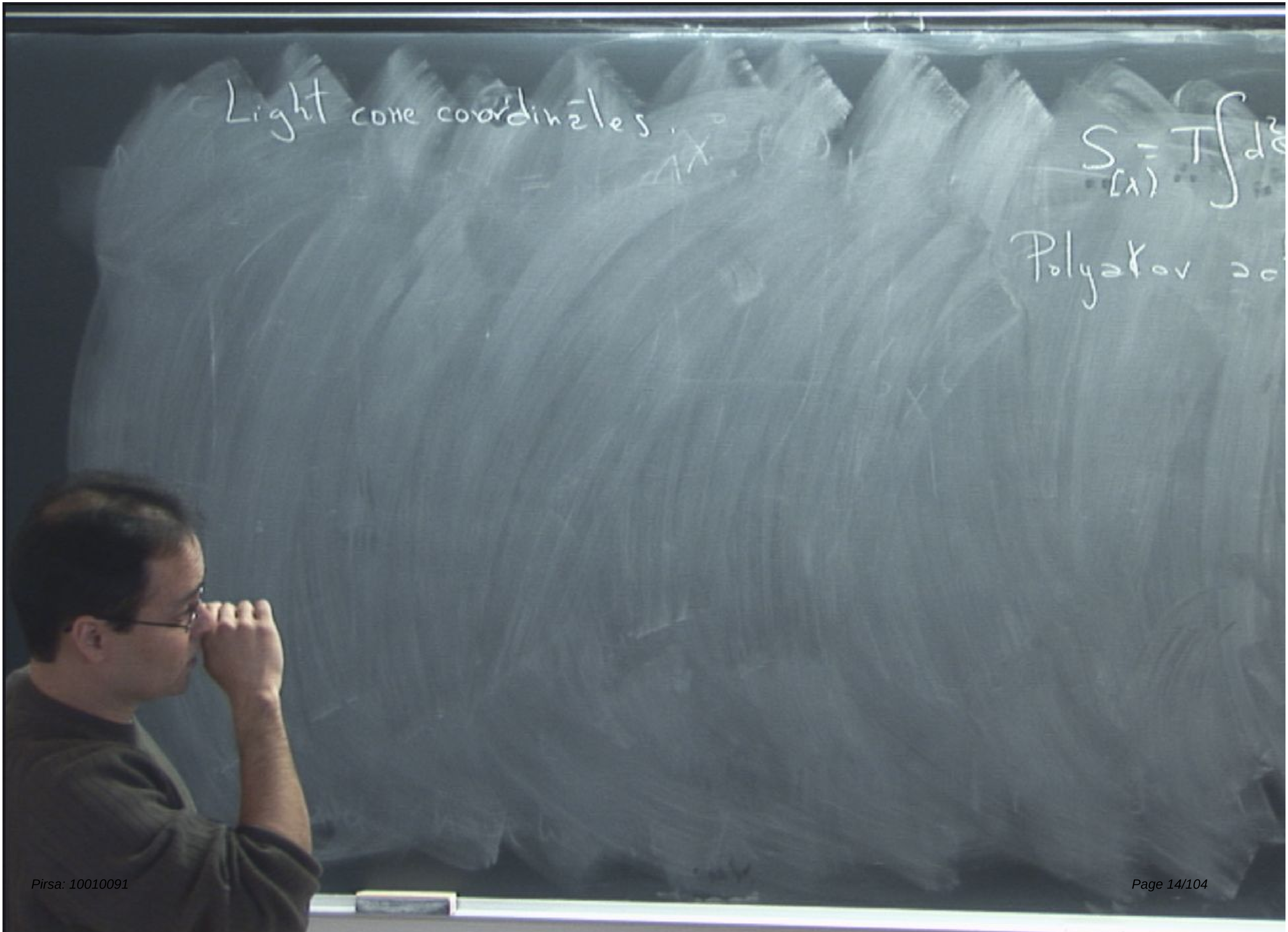
$h = \begin{pmatrix} h_{00} & h_{01} \\ h_{10} & h_{11} \end{pmatrix}$

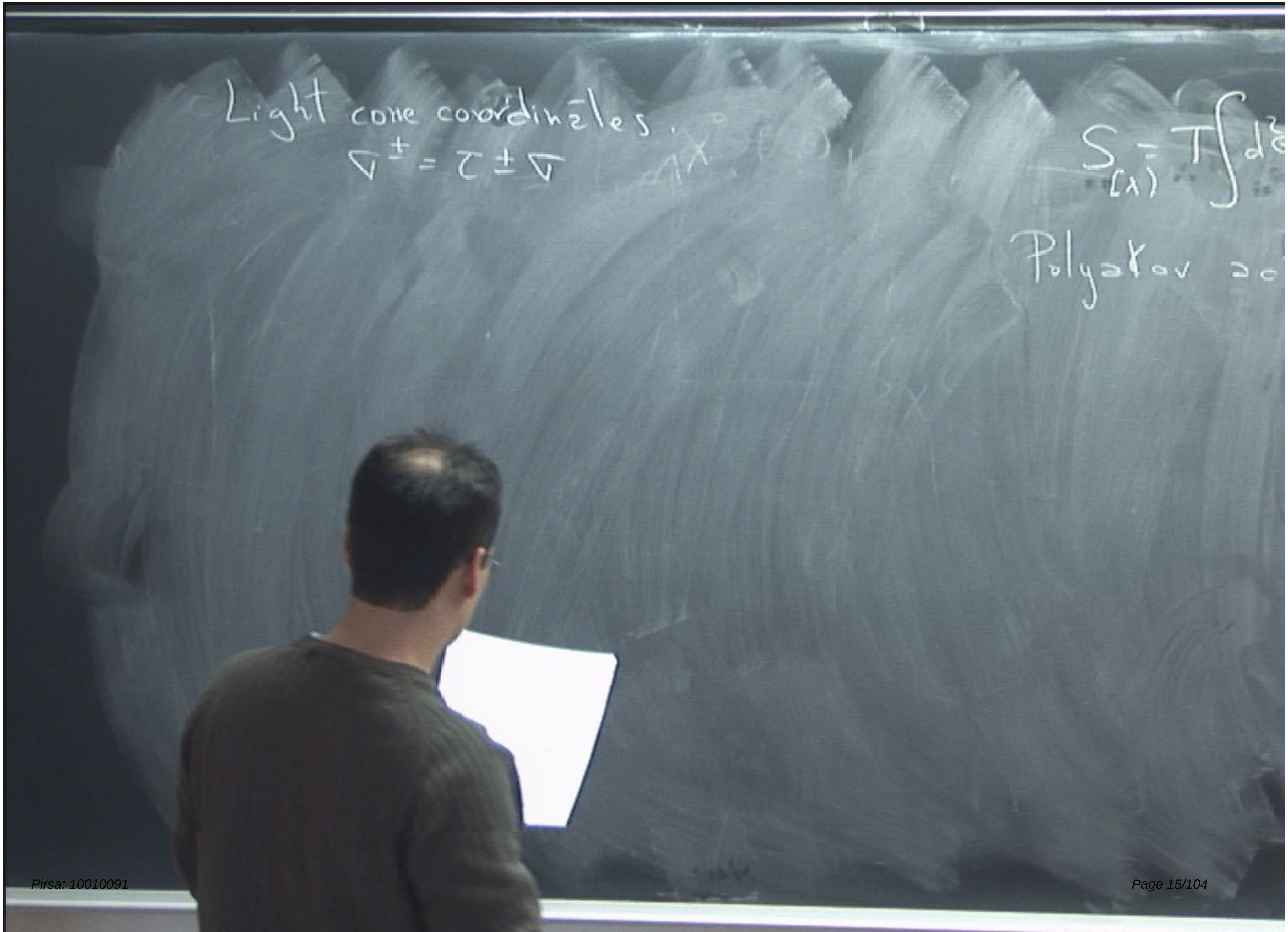
$$\left.\begin{array}{l} \text{diffeos.} \longrightarrow 2 \text{ function} \\ \text{Weyl} \longrightarrow 1 \text{ functio} \end{array}\right\} \longrightarrow$$

$h_{01} = h_{10}$

$h = \eta = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$\eta_{\alpha\beta}$


Light cone coordinates.
$S_{(\lambda)} = T\int d^2$
Polyakov ac



Light cone coordinates.

$\sigma^{\pm} = \tau \pm \sigma$

$S_{[X]} = T\int d^2$

Polyakov ac

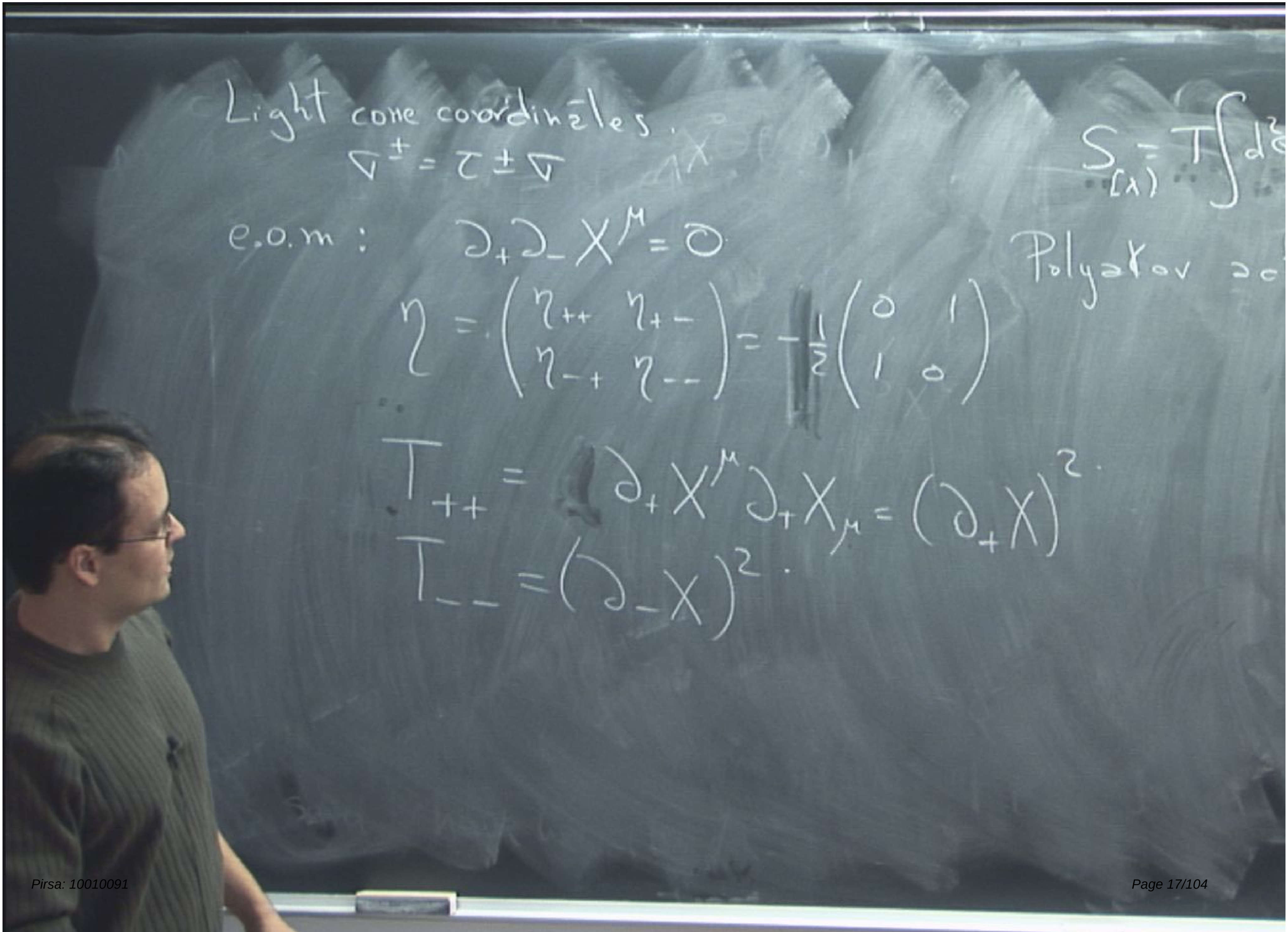
Light cone coordinates:

$\sigma^{\pm} = \tau \pm \sigma$

e.o.m: $\partial_+ \partial_- X^\mu = 0$

$S_{[\lambda]} = T \int d^2$

Polyakov ac



Light cone coordinates.

$$\sigma^{\pm} = \tau \pm \sigma$$

e.o.m : $\partial_+ \partial_- X^\mu = 0$

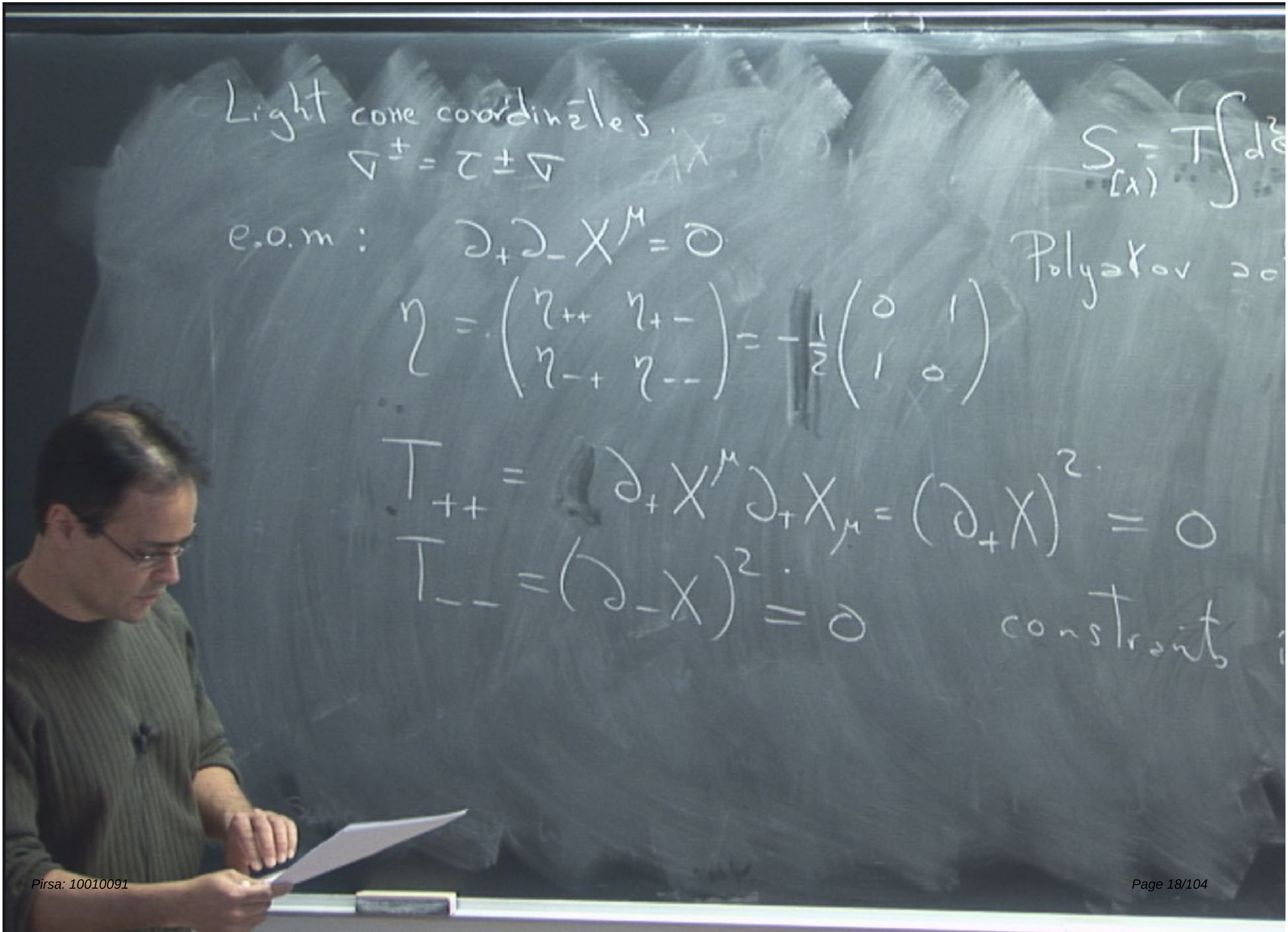
$$\eta = \begin{pmatrix} \eta_{++} & \eta_{+-} \\ \eta_{-+} & \eta_{--} \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$T_{++} = \partial_+ X^\mu \partial_+ X_\mu = (\partial_+ X)^2 .$$

$$T_{--} = (\partial_- X)^2 .$$

$S_{(\lambda)} = T \int d^2$

Polyakov ac



Light cone coordinates

$$\sigma^{\pm} = \tau \pm \sigma$$

e.o.m : $\partial_+ \partial_- X^\mu = 0$

$$\eta = \begin{pmatrix} \eta_{++} & \eta_{+-} \\ \eta_{-+} & \eta_{--} \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$T_{++} = \partial_+ X^\mu \partial_+ X_\mu = (\partial_+ X)^2 = 0$$

$$T_{--} = (\partial_- X)^2 = 0$$

$S_{[X]} = T \int d$

Polyakov a

constraints

Light cone coordinates.

$$\sigma^{\pm} = \tau \pm \sigma$$

$$S_{[X]} = T\int d^2$$

e.o.m: $\partial_+ \partial_- X^\mu = 0$

Polyakov ac

$$\eta = \begin{pmatrix} \eta_{++} & \eta_{+-} \\ \eta_{-+} & \eta_{--} \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$T_{++} = \partial_+ X^\mu \partial_+ X_\mu = (\partial_+ X)^2 = 0$$

$$T_{--} = (\partial_- X)^2 = 0$$

constraints

tG?

Solution: $X^{\mu}(\sigma,\tau) = X^{\mu}_{L}(\sigma^{+}) + X^{\mu}_{R}(\sigma^{-})$

pose.

Light cone coordinates.

$\sigma^{\pm} = \tau \pm \sigma$

$S_{[\lambda]} = T\int d$

e.o.m: $\partial_+\partial_- X^\mu = 0$

Polyakov ac

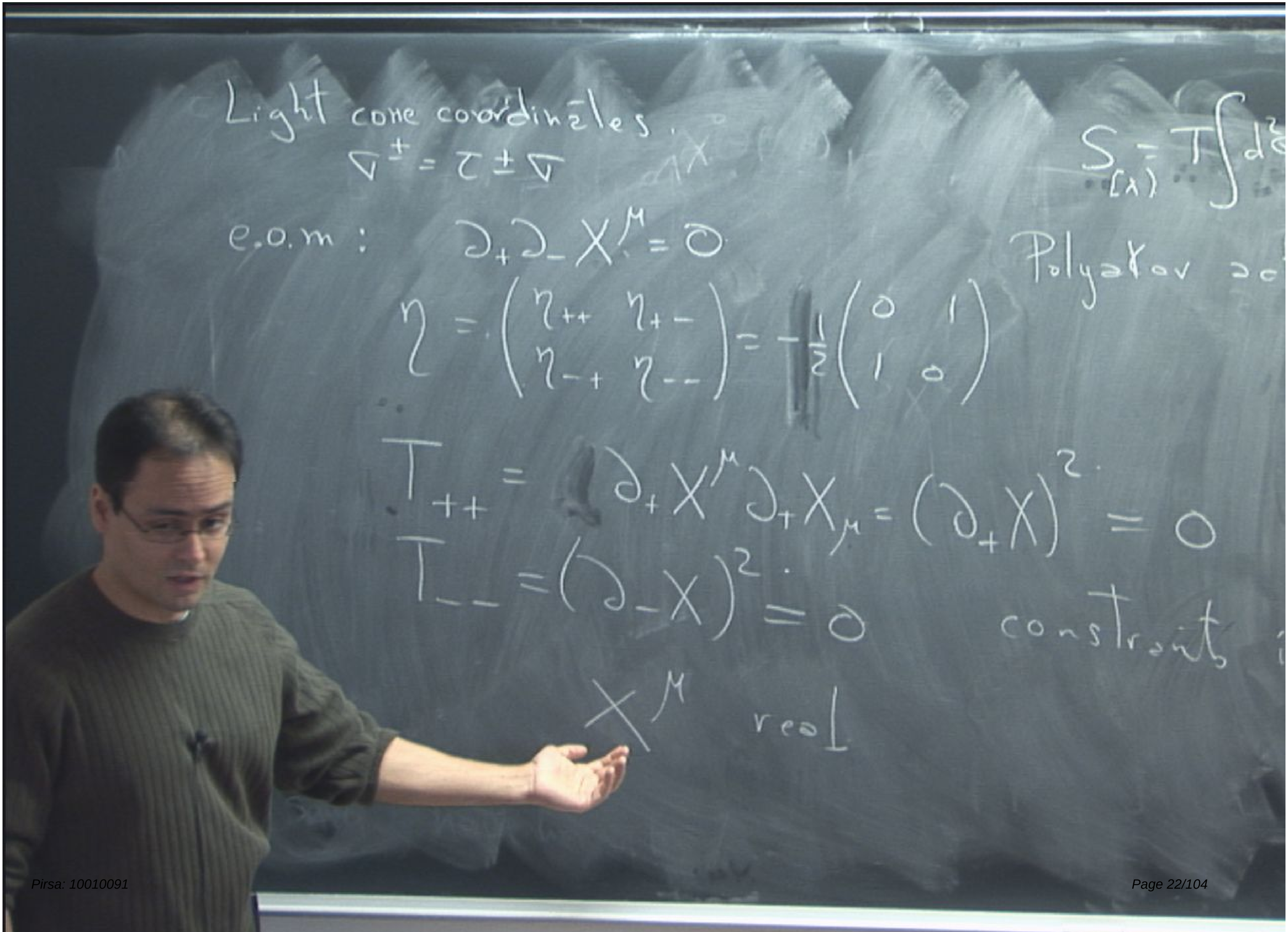
$$\eta = \begin{pmatrix} \eta_{++} & \eta_{+-} \\ \eta_{-+} & \eta_{--} \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$T_{++} = \partial_+ X^\mu \partial_+ X_\mu = (\partial_+ X)^2 = 0$$

$$T_{--} = (\partial_- X)^2 = 0$$

constraints

X^μ real



Light cone coordinates.

$\sigma^{\pm} = \tau \pm \sigma$

$S_{[X]} = T\int d^2$

e.o.m : $\partial_+\partial_- X^\mu = 0$

Polyakov ac

$$\eta = \begin{pmatrix} \eta_{++} & \eta_{+-} \\ \eta_{-+} & \eta_{--} \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$T_{++} = \partial_+ X^\mu \partial_+ X_\mu = (\partial_+ X)^2 = 0$$

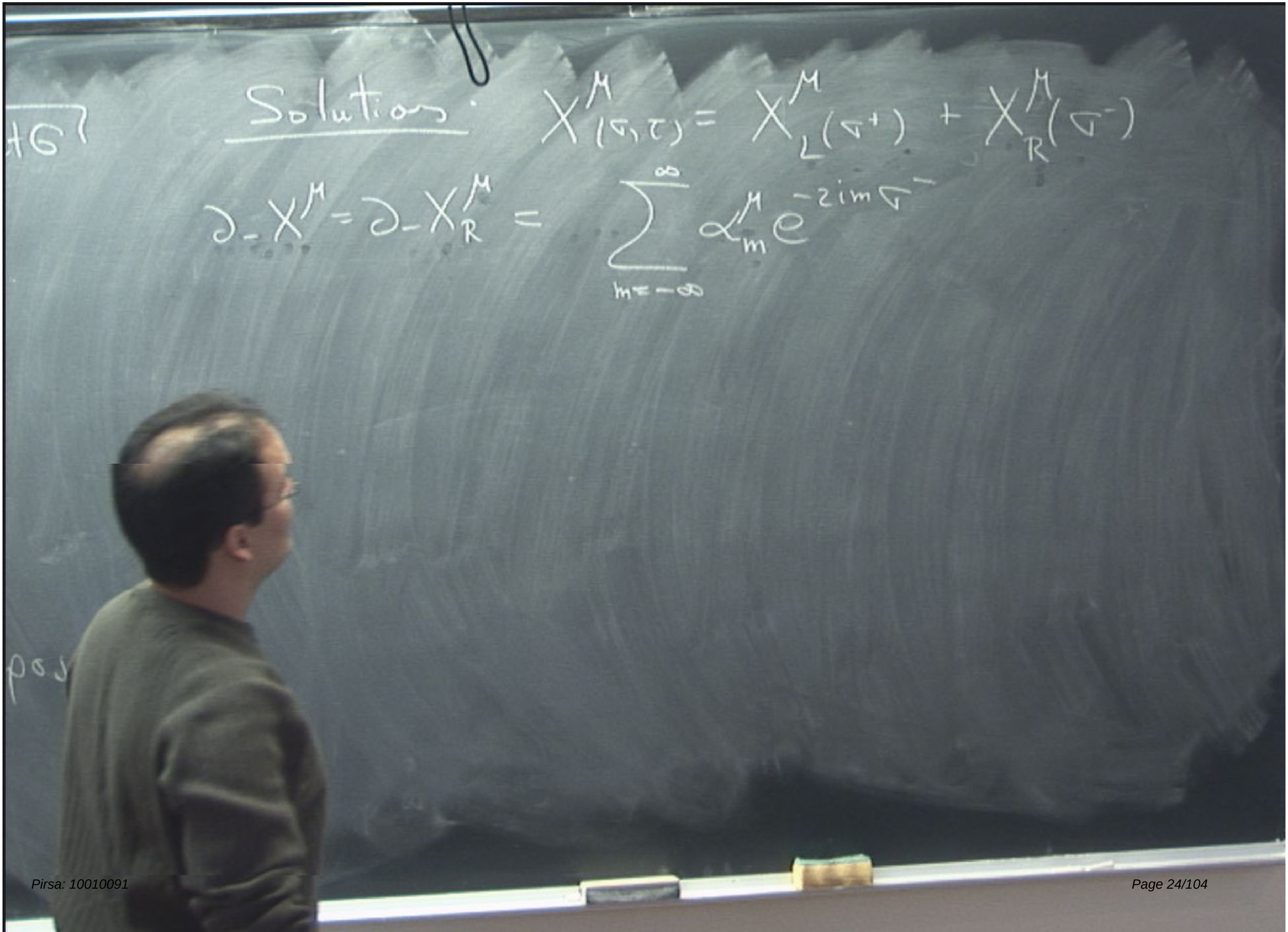
$$T_{--} = (\partial_- X)^2 = 0$$

constraints i

X^μ real

Solution: $X^{\mu}(\sigma,\tau) = X^{\mu}_{L}(\sigma^{+}) + X^{\mu}_{R}(\sigma^{-})$

$\partial_{-} X^{\mu} = \partial_{-} X^{\mu}_{R} =$



Solutions: $X^{\mu}(\sigma,\tau) = X^{\mu}_{L}(\sigma^{+}) + X^{\mu}_{R}(\sigma^{-})$

$$\partial_{-} X^{\mu} = \partial_{-} X^{\mu}_{R} = \sum_{m=-\infty}^{\infty} \alpha^{\mu}_{m} e^{-2im\sigma^{-}}$$

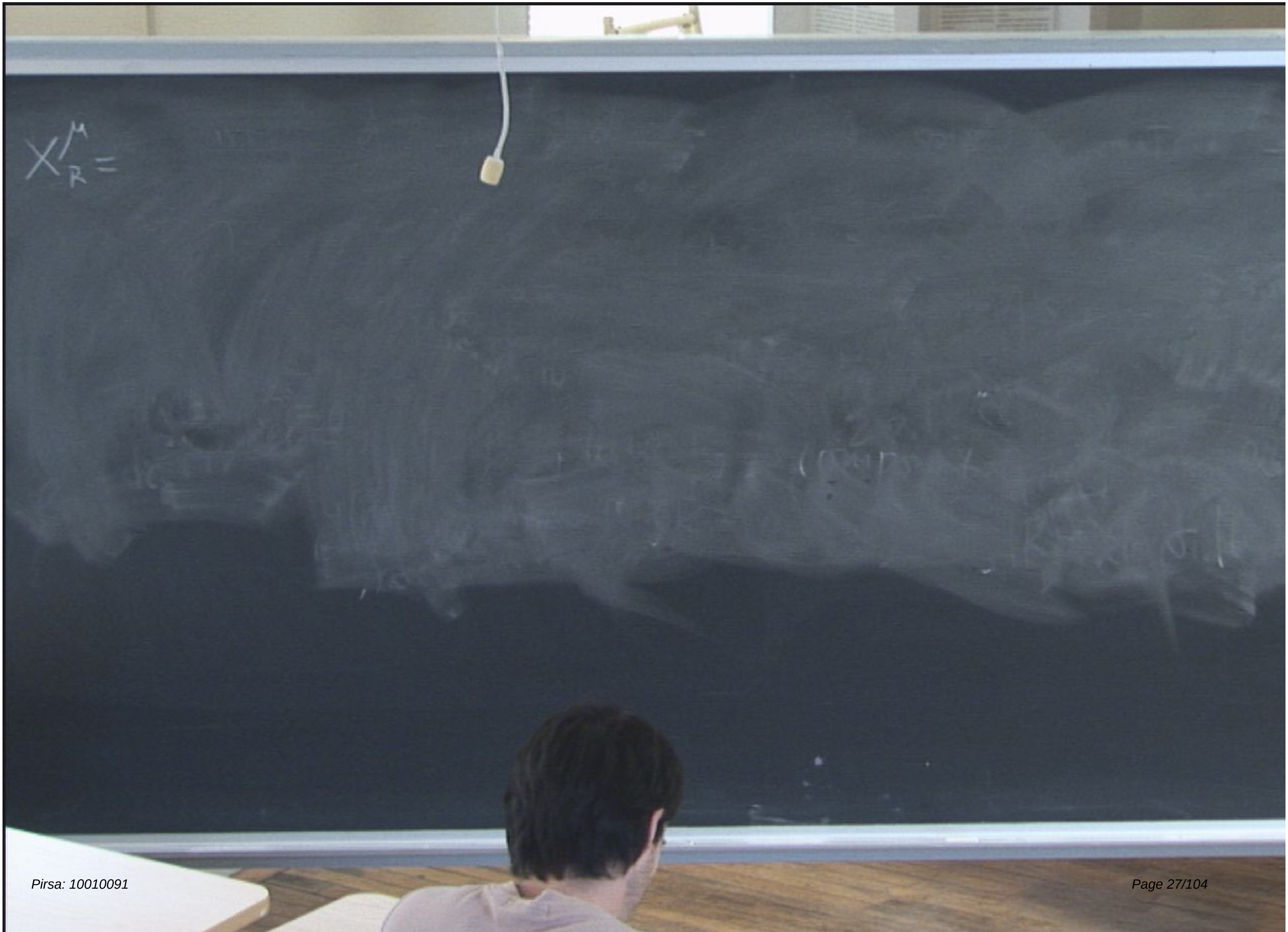
Solutions: $X^{\mu}(\sigma,\tau) = X^{\mu}_{L}(\sigma^{+}) + X^{\mu}_{R}(\sigma^{-})$

$$\partial_{-}X^{\mu} = \partial_{-}X^{\mu}_{R} = l_s \sum_{m=-\infty}^{\infty} \alpha^{\mu}_{m} e^{-2im\sigma^{-}}$$

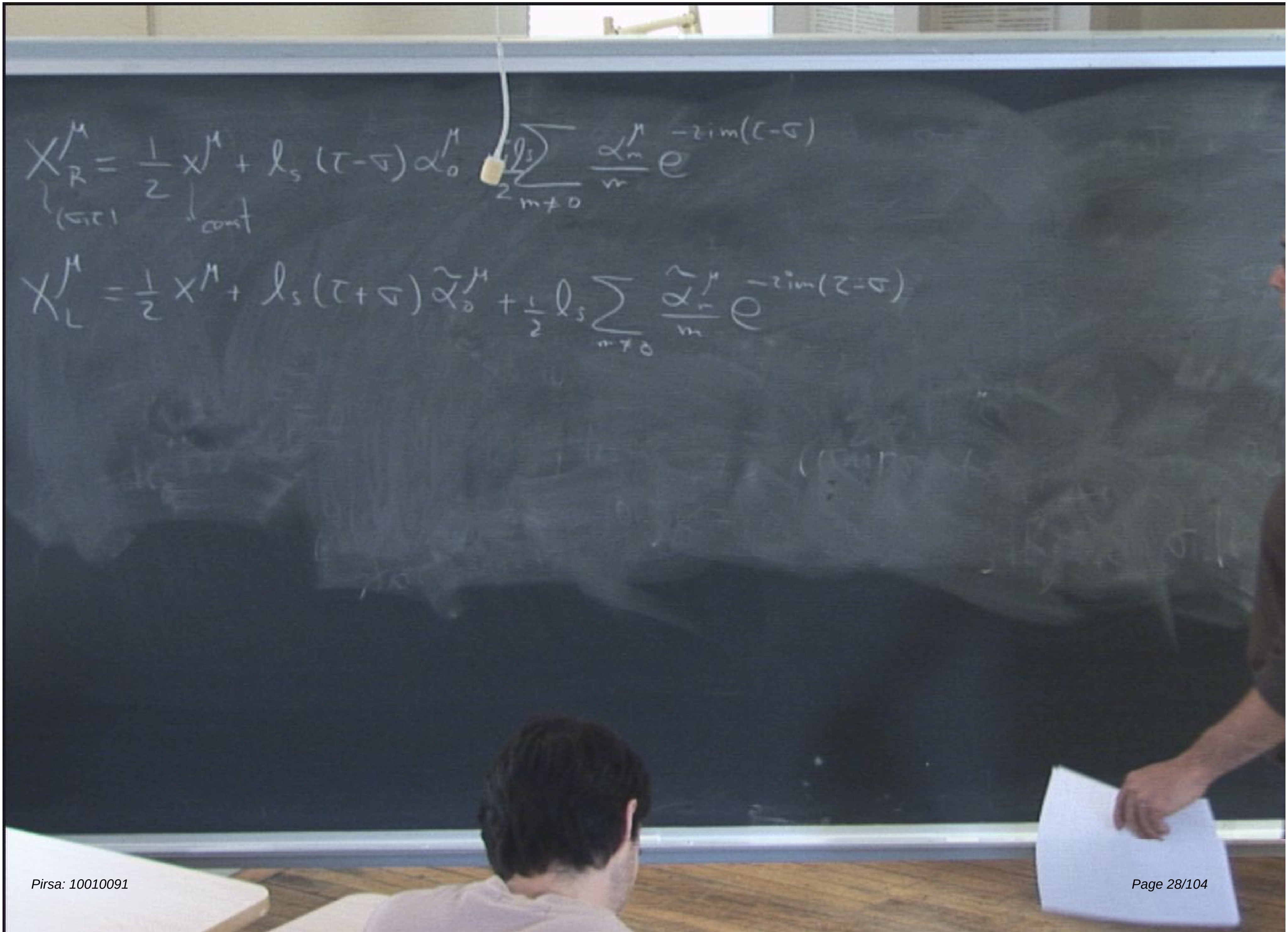
Solutions: $X^{\mu}(\sigma,\tau) = X^{\mu}_{L}(\sigma^{+}) + X^{\mu}_{R}(\sigma^{-})$

$$\partial_{-} X^{\mu} = \partial_{-} X^{\mu}_{R} = l_s \sum_{m=-\infty}^{\infty} \alpha^{\mu}_{m} e^{-2im\sigma^{-}}$$

$$\partial_{+} X^{\mu}_{L} = l_s \sum_{m=-\infty}^{\infty} \tilde{\alpha}^{\mu}_{m} e^{-2im\sigma^{+}}$$



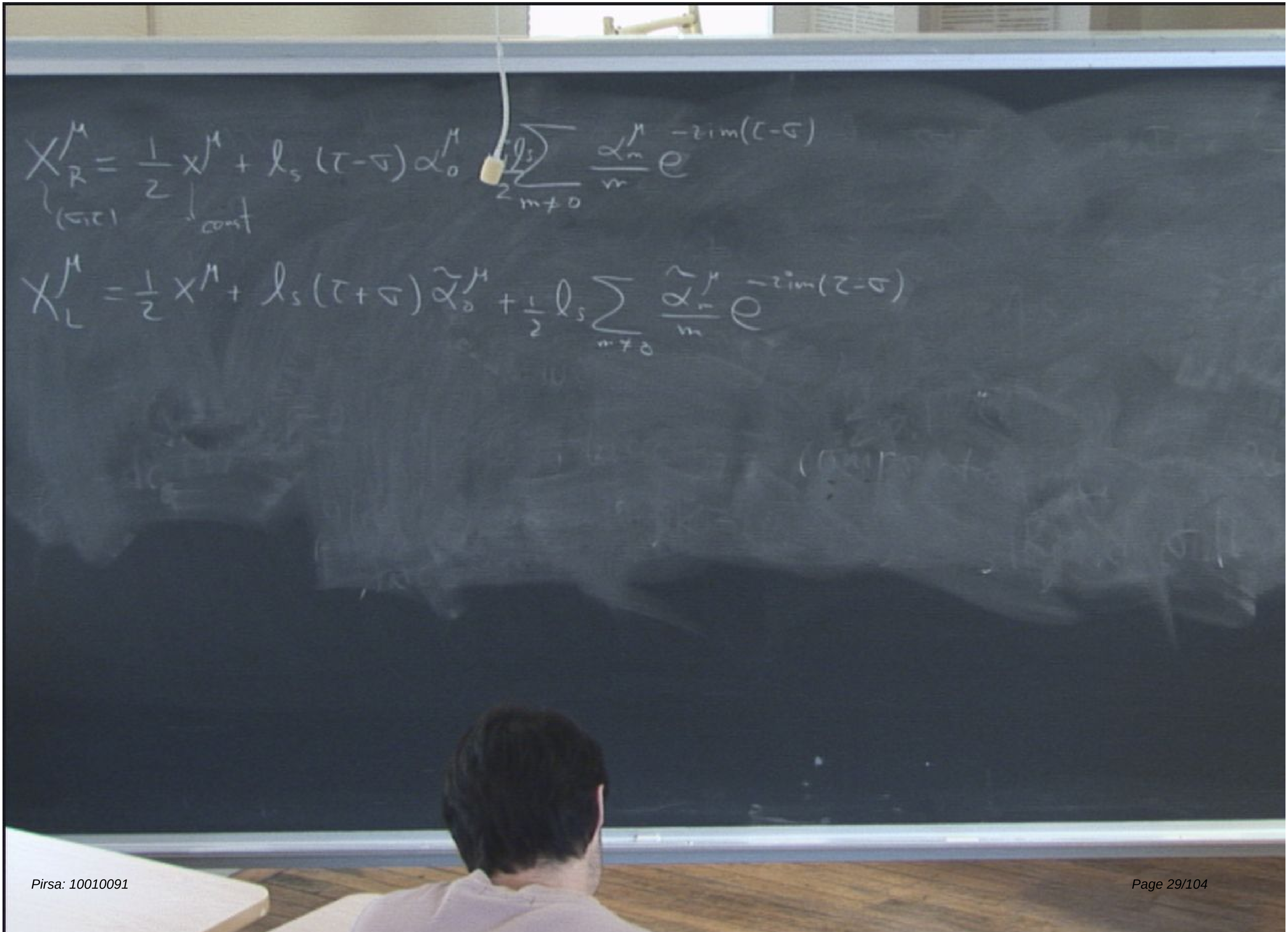
$X^\mu_R =$



$$X_R^\mu = \frac{1}{2} x^\mu + l_s (\tau - \sigma) \alpha_0^\mu + \frac{i l_s}{2} \sum_{m \neq 0} \frac{\alpha_m^\mu}{m} e^{-2im(\tau-\sigma)}$$

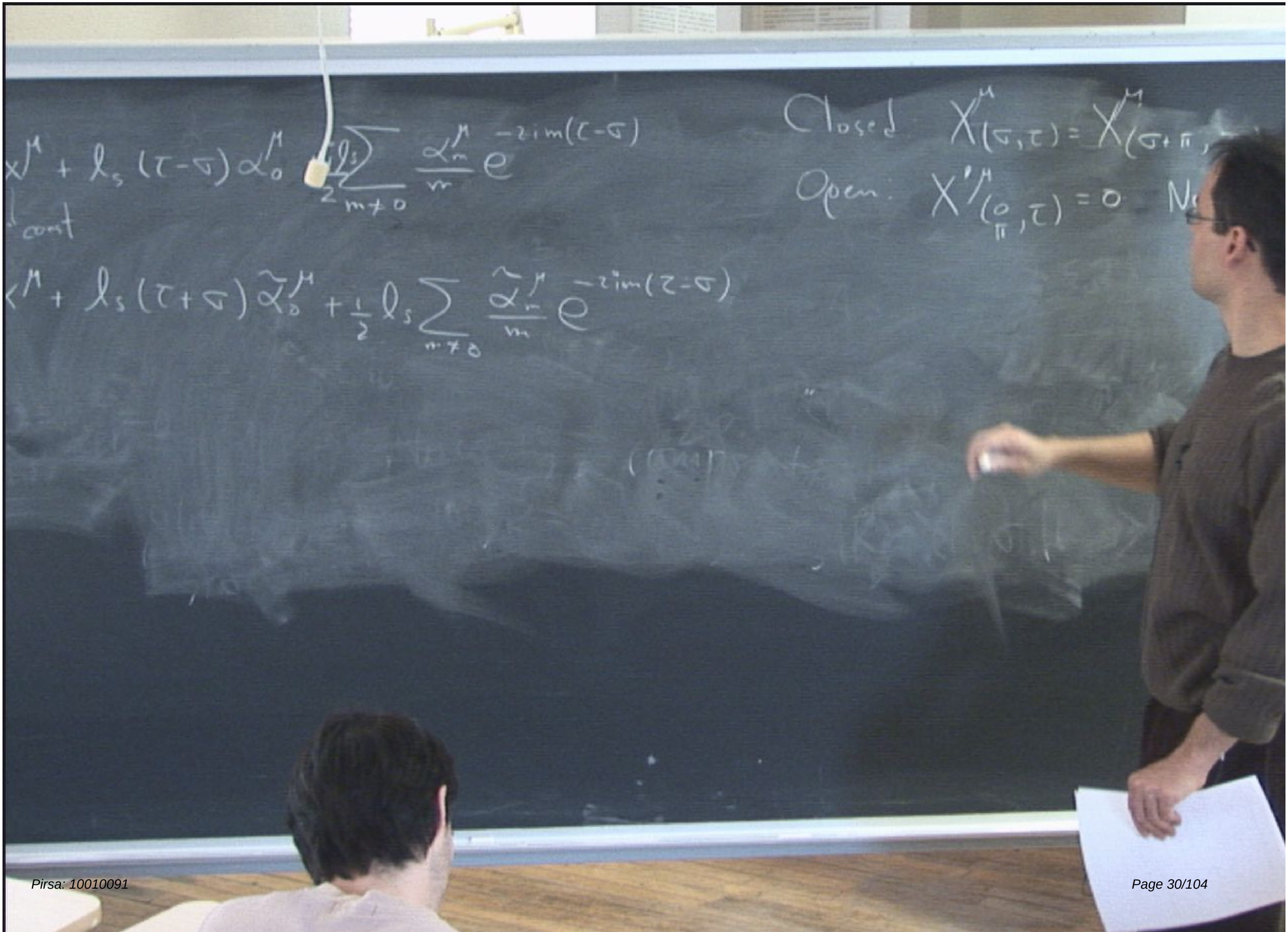
(σ, τ) const

$$X_L^\mu = \frac{1}{2} x^\mu + l_s (\tau + \sigma) \tilde{\alpha}_0^\mu + \frac{i}{2} l_s \sum_{m \neq 0} \frac{\tilde{\alpha}_m^\mu}{m} e^{-2im(\tau-\sigma)}$$



$$\underset{(\sigma,\tau)}{X_R^\mu} = \frac{1}{2}\underset{\text{const}}{x^\mu} + l_s(\tau-\sigma)\alpha_0^\mu \; \frac{i}{2} l_s \sum_{m\neq 0} \frac{\alpha_m^\mu}{m} e^{-2im(\tau-\sigma)}$$

$$X_L^\mu = \frac{1}{2}x^\mu + l_s(\tau+\sigma)\tilde{\alpha}_0^\mu + \frac{1}{2} l_s \sum_{m\neq 0} \frac{\tilde{\alpha}_m^\mu}{m} e^{-2im(\tau-\sigma)}$$



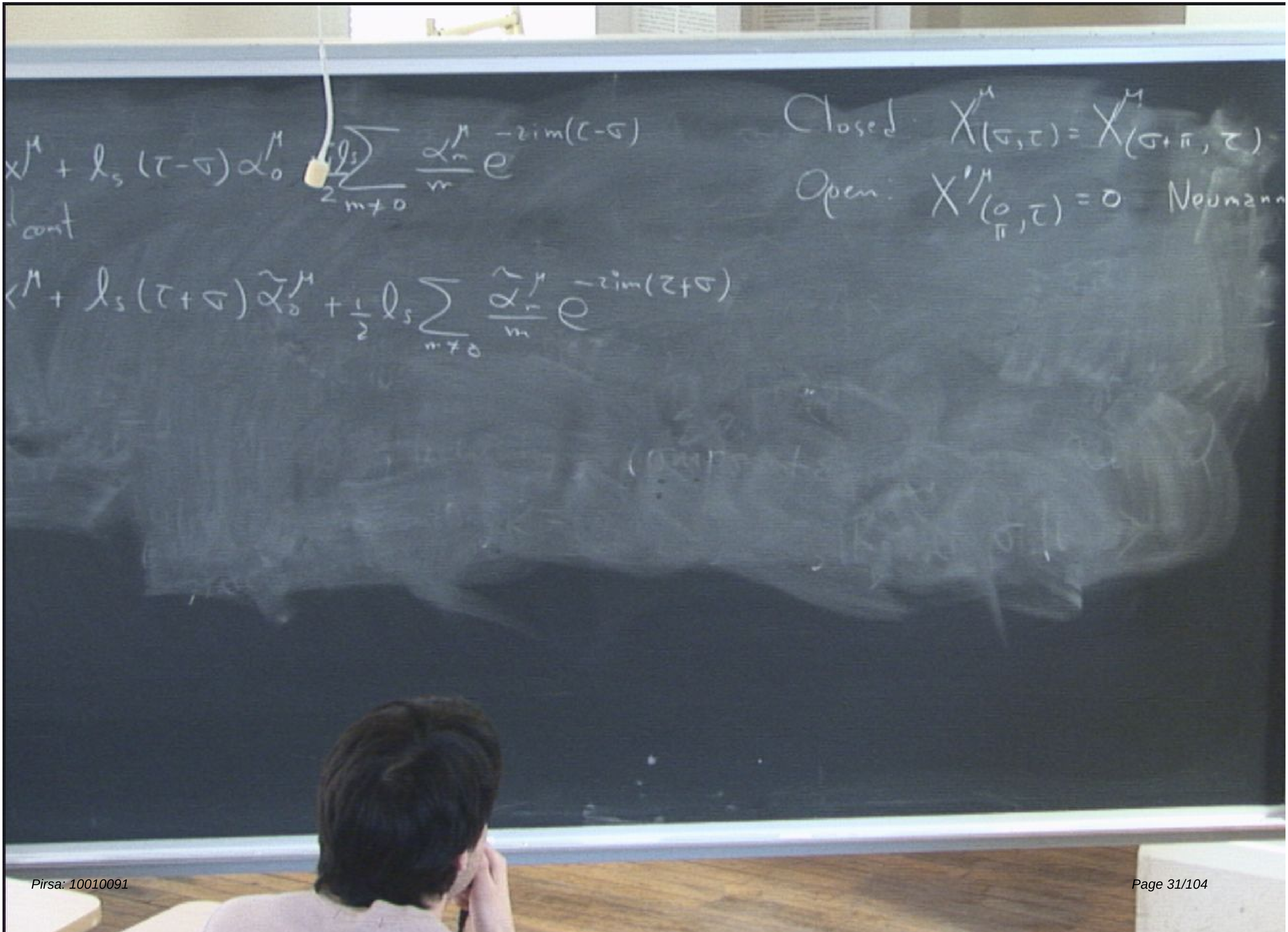
$x^\mu + l_s(\tau-\sigma)\alpha_0^\mu \quad \frac{il_s}{2}\sum_{m\neq 0}\frac{\alpha_m^\mu}{m}e^{-2im(\tau-\sigma)}$

const

$x^\mu + l_s(\tau+\sigma)\tilde{\alpha}_0^\mu + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}_m^\mu}{m}e^{-2im(\tau-\sigma)}$

Closed: $X^\mu(\sigma,\tau) = X^\mu(\sigma+\pi,$

Open: $X'^\mu(\overset{0}{\pi},\tau) = 0$ N



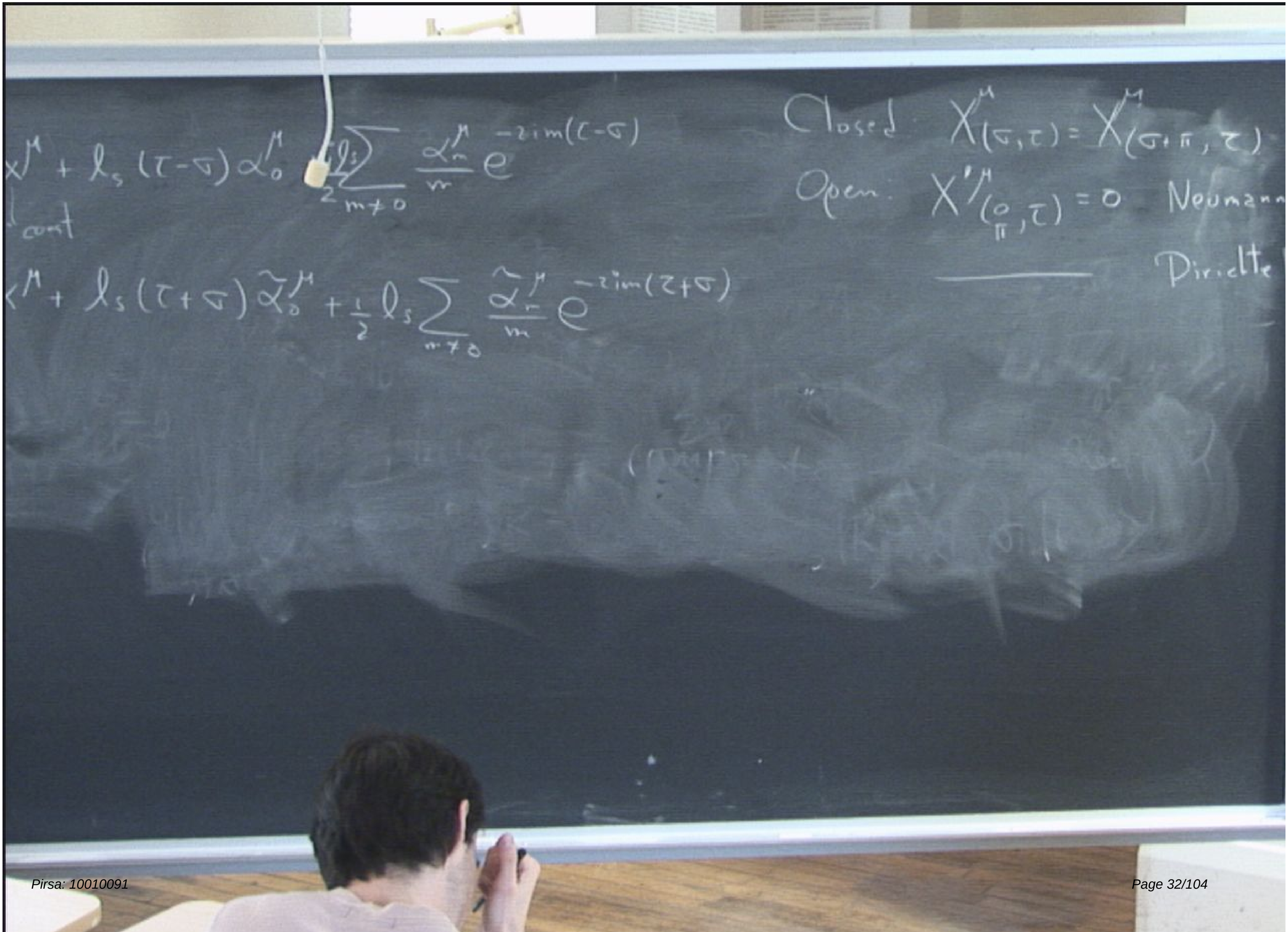
$$x^{\mu} + l_s(\tau-\sigma)\alpha_0^{\mu} + \frac{i l_s}{2}\sum_{m\neq 0}\frac{\alpha_m^{\mu}}{m}e^{-2im(\tau-\sigma)}$$

$$l_{cont}$$

$$x^{\mu} + l_s(\tau+\sigma)\tilde{\alpha}_0^{\mu} + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}_m^{\mu}}{m}e^{-2im(\tau+\sigma)}$$

Closed: $X^{\mu}(\sigma,\tau) = X^{\mu}(\sigma+\pi,\tau)$

Open: $X'^{\mu}(\overset{0}{\pi},\tau) = 0$ Neumann



$$x^\mu + l_s(\tau-\sigma)\alpha_0^\mu \;\; \frac{l_s}{2}\sum_{m\neq 0}\frac{\alpha_m^\mu}{m}e^{-2im(\tau-\sigma)}$$

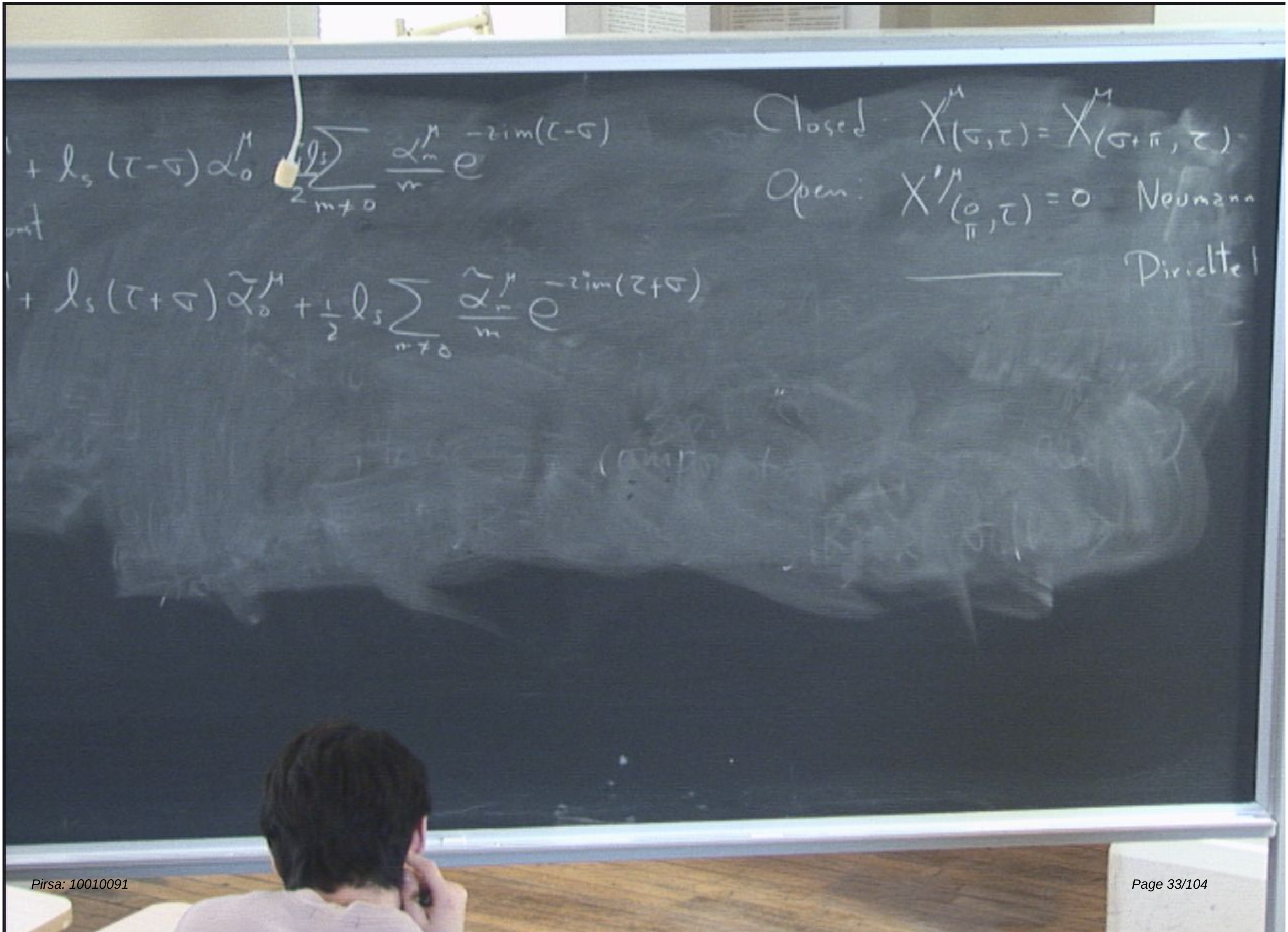
const

$$x^\mu + l_s(\tau+\sigma)\tilde{\alpha}_0^\mu + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}_m^\mu}{m}e^{-2im(\tau+\sigma)}$$

Closed: $X^\mu(\sigma,\tau) = X^\mu(\sigma+\pi,\tau)$

Open: $X'^\mu(\substack{0\\ \pi},\tau) = 0$ Neumann

Diriclte



$$+ l_s(\tau-\sigma)\alpha_0^\mu \; \frac{i l_s}{2}\sum_{m\neq 0}\frac{\alpha_m^\mu}{m}e^{-2im(\tau-\sigma)}$$

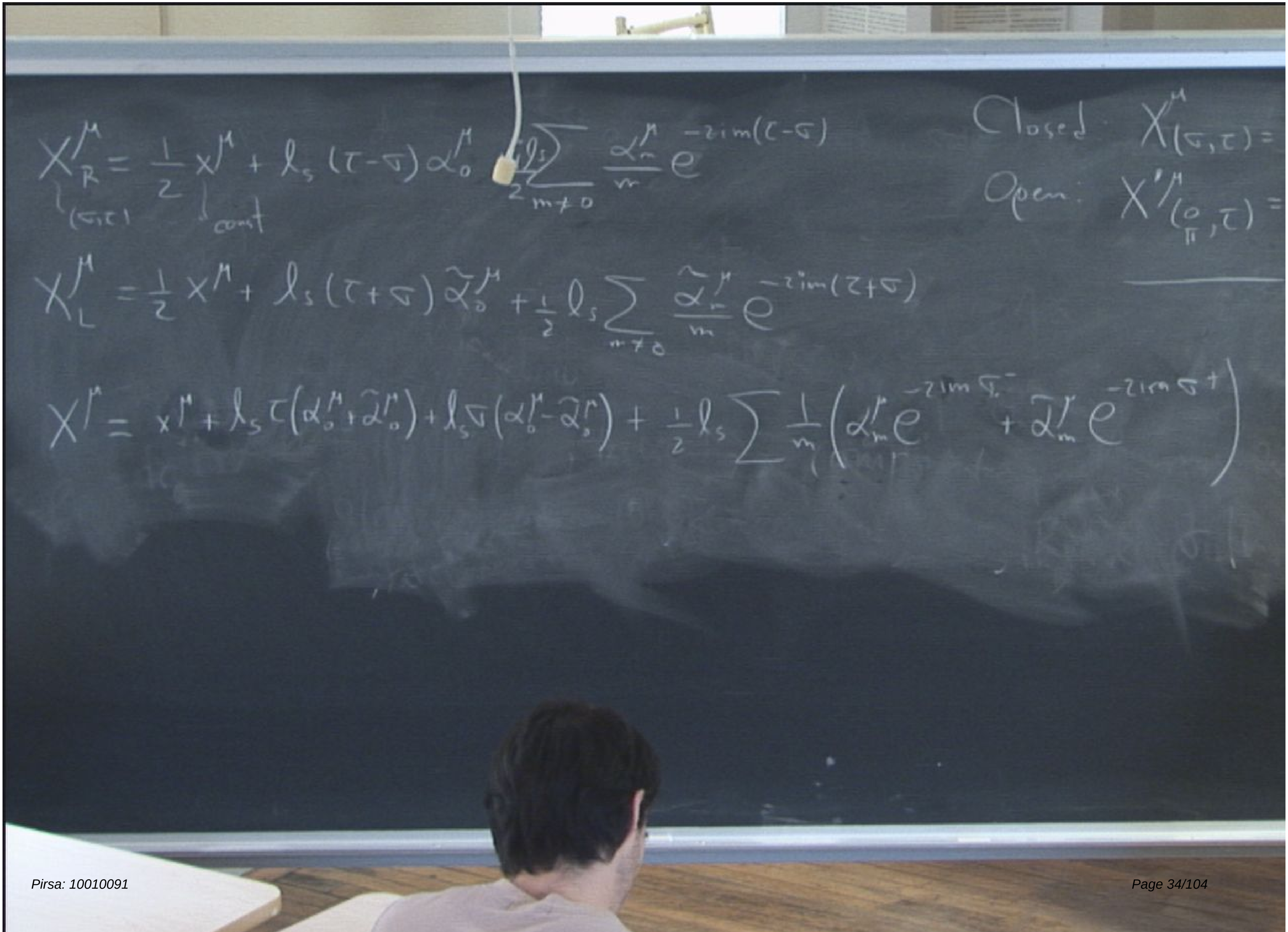
nst

$$+ l_s(\tau+\sigma)\tilde{\alpha}_0^\mu + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}_m^\mu}{m}e^{-2im(\tau+\sigma)}$$

Closed: $X^\mu(\sigma,\tau) = X^\mu(\sigma+\pi,\tau)$

Open: $X'^\mu\left(\begin{smallmatrix}0\\ \pi\end{smallmatrix},\tau\right) = 0$ Neumann

Dirichlet



$$X^\mu_R(\sigma,\tau) = \frac{1}{2}x^\mu + l_s(\tau-\sigma)\alpha_0^\mu \; \frac{i l_s}{2}\sum_{m\neq 0}\frac{\alpha_m^\mu}{m}e^{-2im(\tau-\sigma)}$$

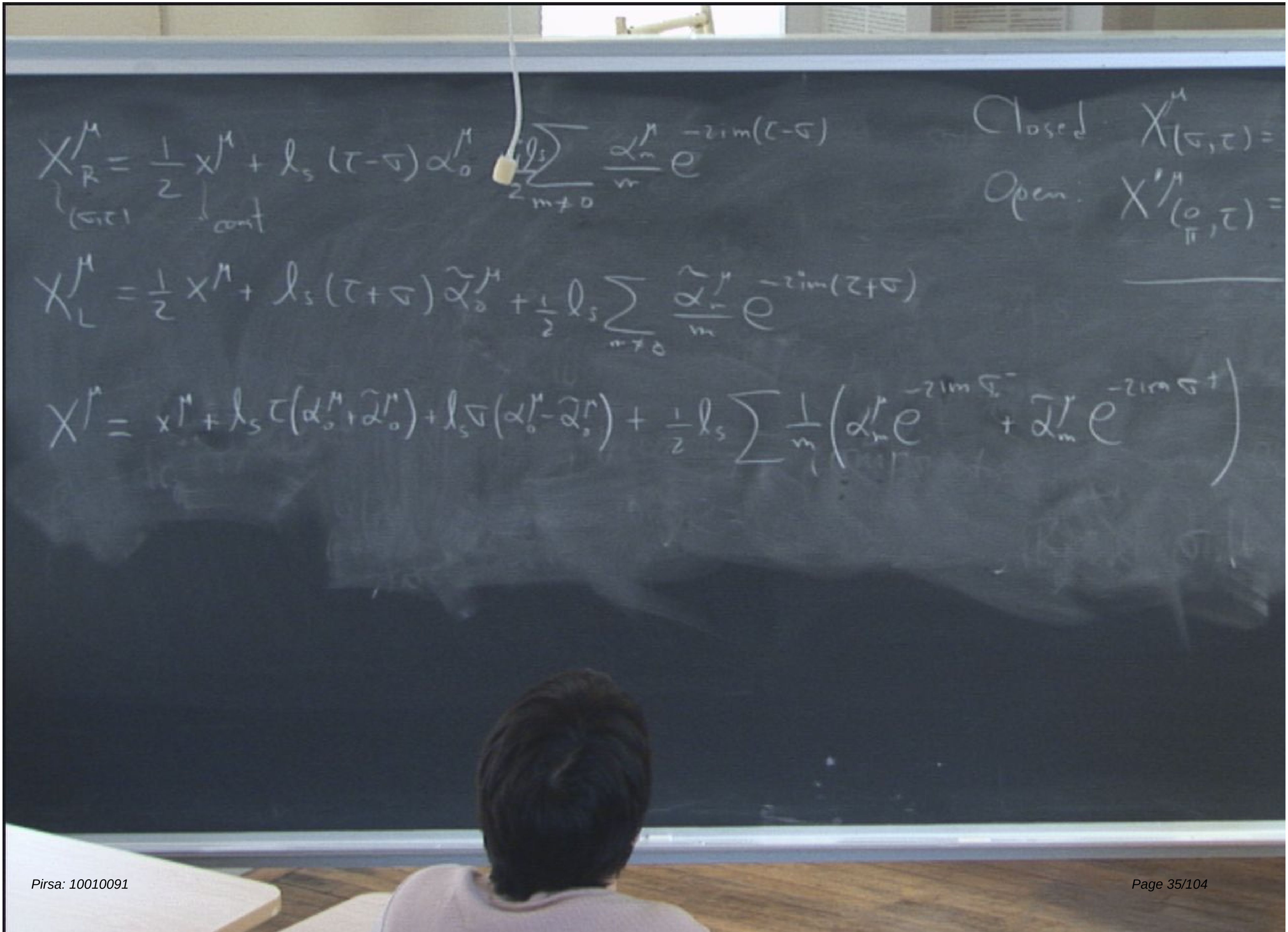
x^μ: const

$$X^\mu_L = \frac{1}{2}x^\mu + l_s(\tau+\sigma)\tilde{\alpha}_0^\mu + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}_m^\mu}{m}e^{-2im(\tau+\sigma)}$$

$$X^\mu = x^\mu + l_s\tau(\alpha_0^\mu+\tilde{\alpha}_0^\mu) + l_s\sigma(\alpha_0^\mu-\tilde{\alpha}_0^\mu) + \frac{1}{2}l_s\sum\frac{1}{m}\left(\alpha_m^\mu e^{-2im\sigma^-} + \tilde{\alpha}_m^\mu e^{-2im\sigma^+}\right)$$

Closed: $X^\mu(\sigma,\tau) =$

Open: $X'^\mu(^{0}_{\pi},\tau) =$



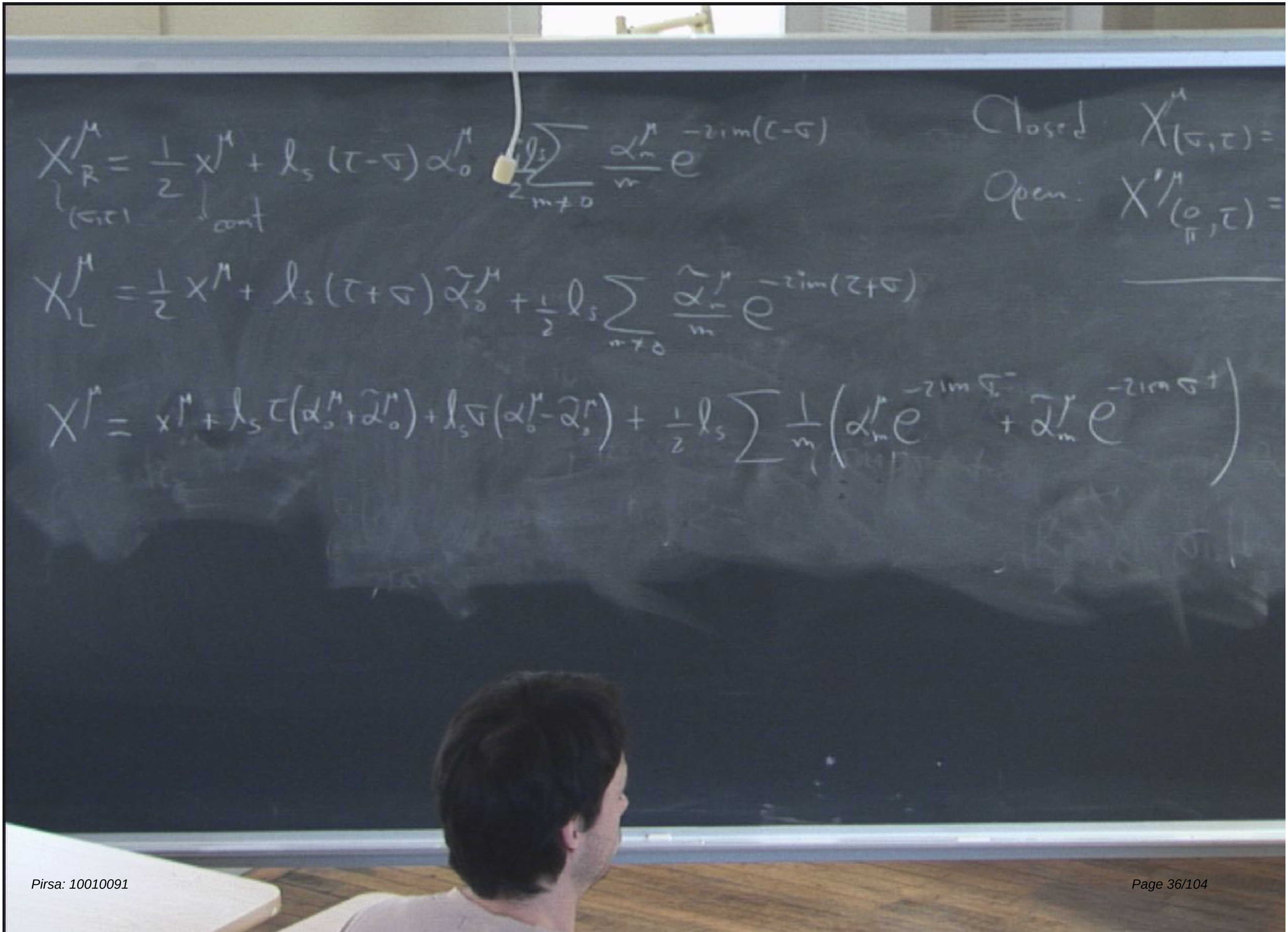
$$\underset{(\sigma,\tau)}{X_R^\mu} = \underset{\text{const}}{\frac{1}{2}x^\mu} + l_s(\tau-\sigma)\alpha_0^\mu + \frac{i}{2}l_s\sum_{m\neq 0}\frac{\alpha_m^\mu}{m}e^{-2im(\tau-\sigma)}$$

Closed $X^\mu_{(\sigma,\tau)} =$

Open. $X'^\mu_{(\substack{0\\ \pi},\tau)} =$

$$X_L^\mu = \frac{1}{2}x^\mu + l_s(\tau+\sigma)\tilde{\alpha}_0^\mu + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}_m^\mu}{m}e^{-2im(\tau+\sigma)}$$

$$X^\mu = x^\mu + l_s\tau(\alpha_0^\mu + \tilde{\alpha}_0^\mu) + l_s\sigma(\alpha_0^\mu - \tilde{\alpha}_0^\mu) + \frac{1}{2}l_s\sum\frac{1}{m}\left(\alpha_m^\mu e^{-2im\sigma^-} + \tilde{\alpha}_m^\mu e^{-2im\sigma^+}\right)$$



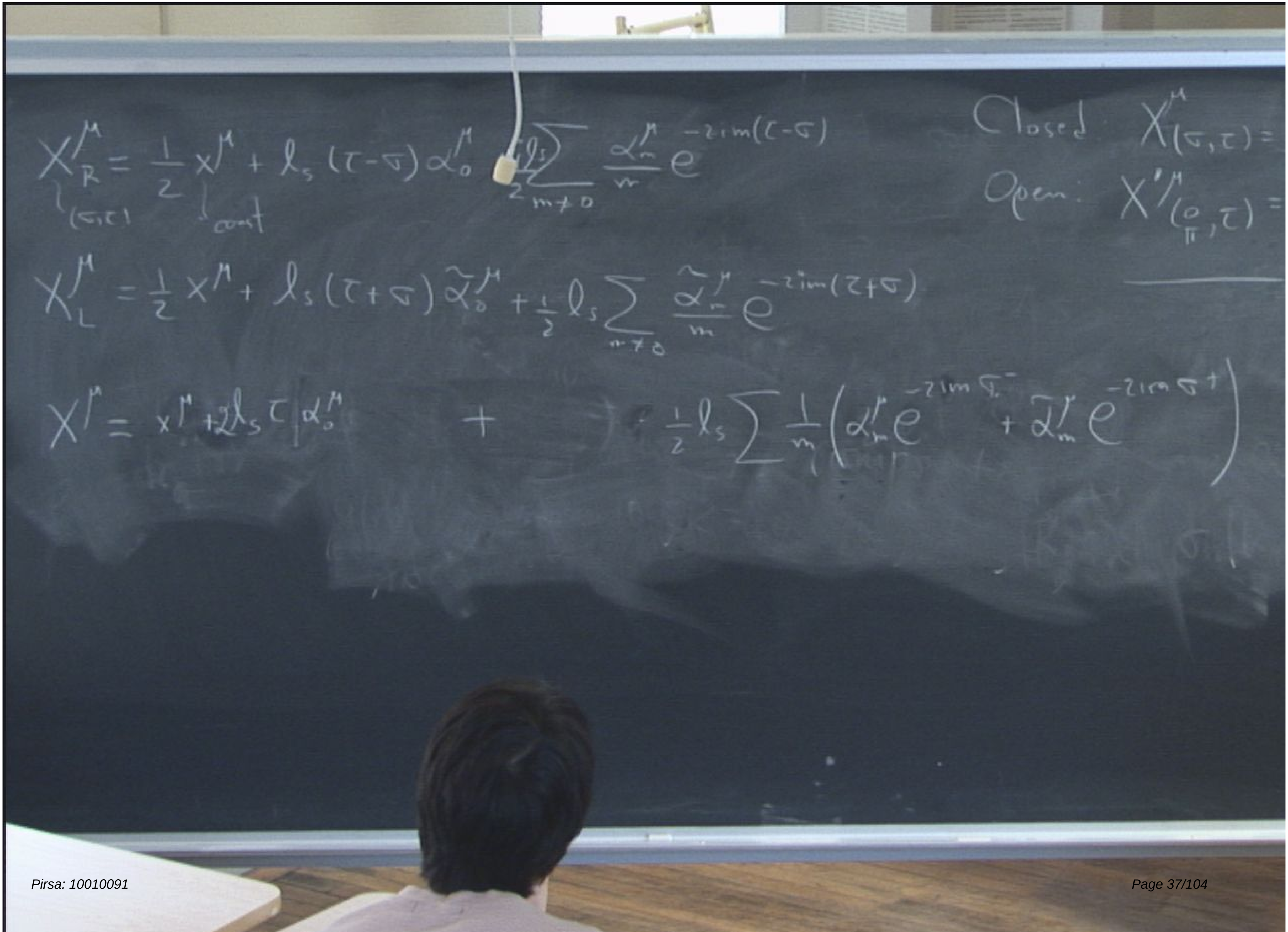
$$\underset{(\sigma,\tau)}{X_R^{\mu}} = \frac{1}{2}\underset{\text{const}}{x^{\mu}} + l_s(\tau-\sigma)\alpha_0^{\mu} + \frac{i l_s}{2}\sum_{m\neq 0}\frac{\alpha_m^{\mu}}{m}e^{-2im(\tau-\sigma)}$$

$$X_L^{\mu} = \frac{1}{2}x^{\mu} + l_s(\tau+\sigma)\tilde{\alpha}_0^{\mu} + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}_m^{\mu}}{m}e^{-2im(\tau+\sigma)}$$

$$X^{\mu} = x^{\mu} + l_s\tau(\alpha_0^{\mu}+\tilde{\alpha}_0^{\mu}) + l_s\sigma(\alpha_0^{\mu}-\tilde{\alpha}_0^{\mu}) + \frac{1}{2}l_s\sum\frac{1}{m}\left(\alpha_m^{\mu}e^{-2im\sigma^-} + \tilde{\alpha}_m^{\mu}e^{-2im\sigma^+}\right)$$

Closed: $X^{\mu}(\sigma,\tau) =$

Open: $X'^{\mu}(\overset{0}{\pi},\tau) =$



$$X^\mu_R = \frac{1}{2}x^\mu + l_s(\tau-\sigma)\alpha^\mu_0 \; \frac{i l_s}{2}\sum_{m\neq 0}\frac{\alpha^\mu_m}{m}e^{-2im(\tau-\sigma)}$$

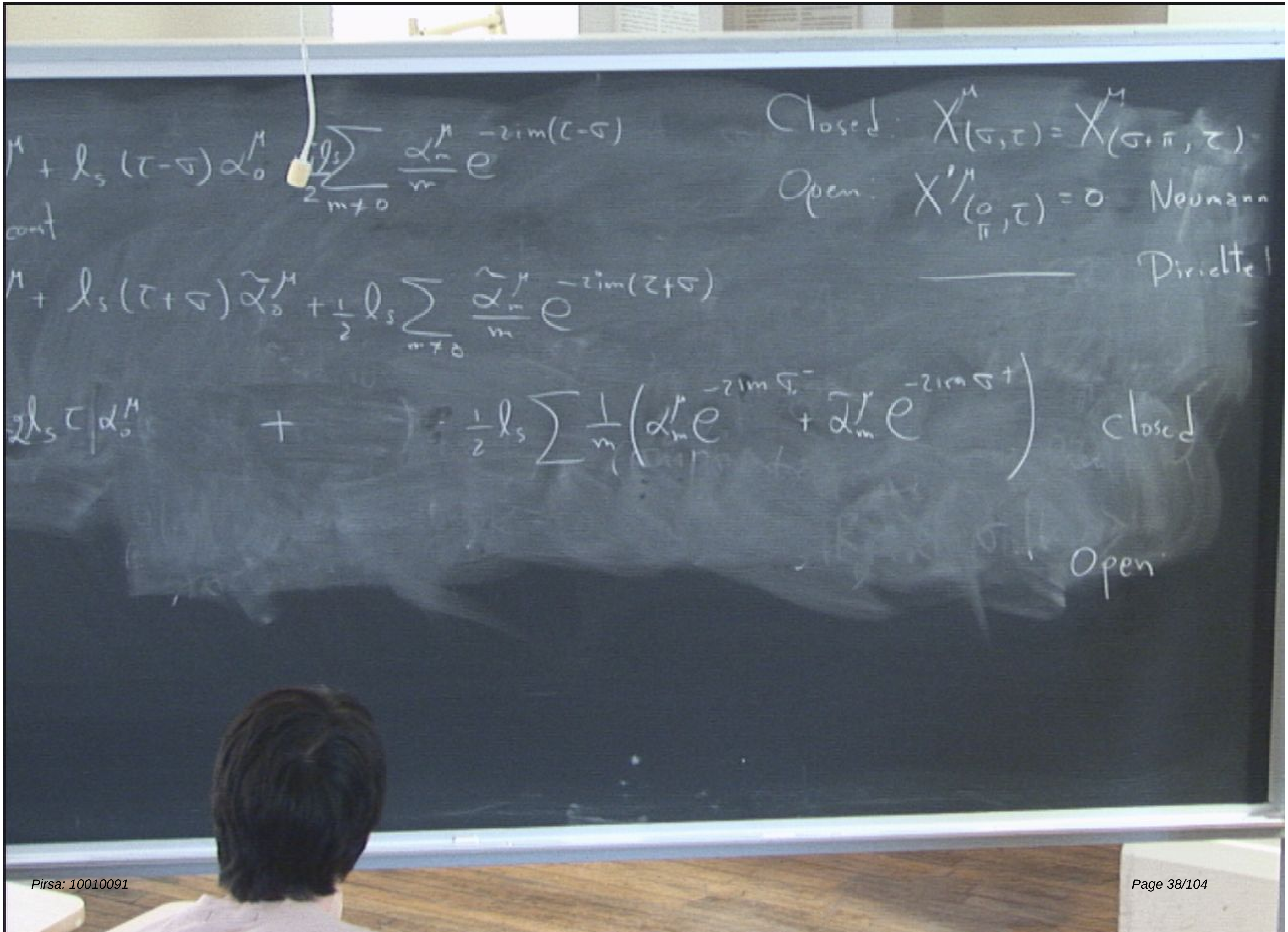
(σ,τ) const

$$X^\mu_L = \frac{1}{2}x^\mu + l_s(\tau+\sigma)\tilde{\alpha}^\mu_0 + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}^\mu_m}{m}e^{-2im(\tau+\sigma)}$$

$$X^\mu = x^\mu + 2l_s\tau\,\alpha^\mu_0 \quad + \quad \frac{1}{2}l_s\sum\frac{1}{m}\left(\alpha^\mu_m e^{-2im\sigma^-} + \tilde{\alpha}^\mu_m e^{-2im\sigma^+}\right)$$

Closed: $X'^\mu(\sigma,\tau) =$

Open: $X'^\mu(\begin{smallmatrix}0\\ \pi\end{smallmatrix},\tau) =$



Closed: $X^\mu(\sigma,\tau) = X^\mu(\sigma+\pi,\tau)$

Open: $X'^\mu(^0_\pi,\tau) = 0$ Neumann

Dirichle

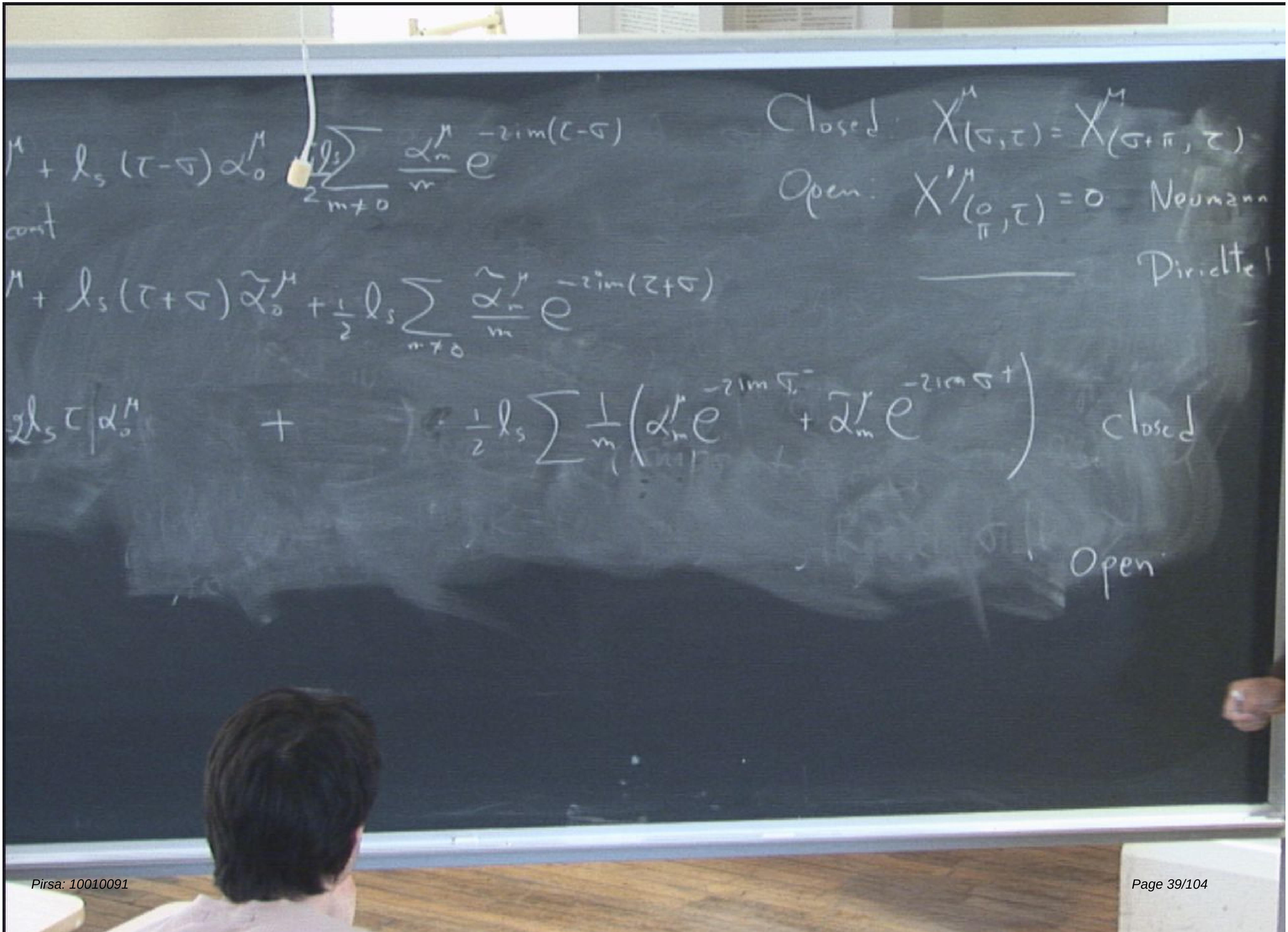
$$^\mu + l_s(\tau-\sigma)\alpha_0^\mu + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\alpha_m^\mu}{m}e^{-2im(\tau-\sigma)}$$

const

$$^\mu + l_s(\tau+\sigma)\tilde{\alpha}_0^\mu + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}_m^\mu}{m}e^{-2im(\tau+\sigma)}$$

$$2l_s\tau\,\alpha_0^\mu \quad + \quad \frac{1}{2}l_s\sum\frac{1}{m}\left(\alpha_m^\mu e^{-2im\sigma^-} + \tilde{\alpha}_m^\mu e^{-2im\sigma^+}\right)$$ closed

Open



$$^\mu + l_s(\tau-\sigma)\alpha_0^\mu + \frac{i l_s}{2}\sum_{m\neq 0}\frac{\alpha_m^\mu}{m}e^{-2im(\tau-\sigma)}$$

const

$$^\mu + l_s(\tau+\sigma)\tilde{\alpha}_0^\mu + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}_m^\mu}{m}e^{-2im(\tau+\sigma)}$$

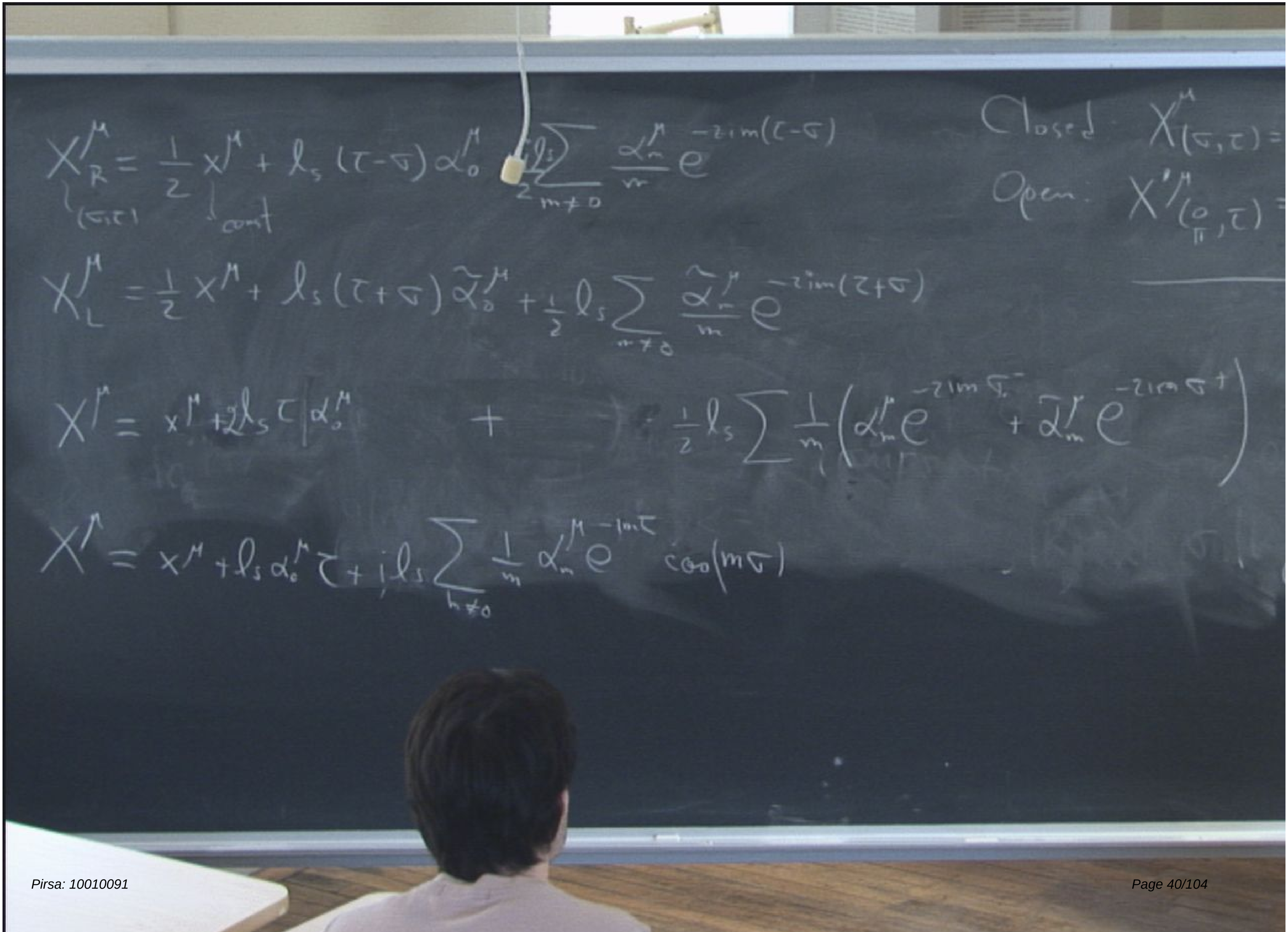
$$2l_s\tau\,\alpha_0^\mu \quad + \quad \frac{1}{2}l_s\sum\frac{1}{m}\left(\alpha_m^\mu e^{-2im\sigma^-} + \tilde{\alpha}_m^\mu e^{-2im\sigma^+}\right)$$ closed

Open

Closed: $X^\mu(\sigma,\tau) = X^\mu(\sigma+\pi,\tau)$

Open: $X'^\mu(\overset{0}{\pi},\tau) = 0$ Neumann

Diriclte



$$X^\mu_R = \frac{1}{2}x^\mu + l_s(\tau-\sigma)\alpha_0^\mu + \frac{i l_s}{2}\sum_{m\neq 0}\frac{\alpha_m^\mu}{m}e^{-2im(\tau-\sigma)}$$

(σ,τ) const

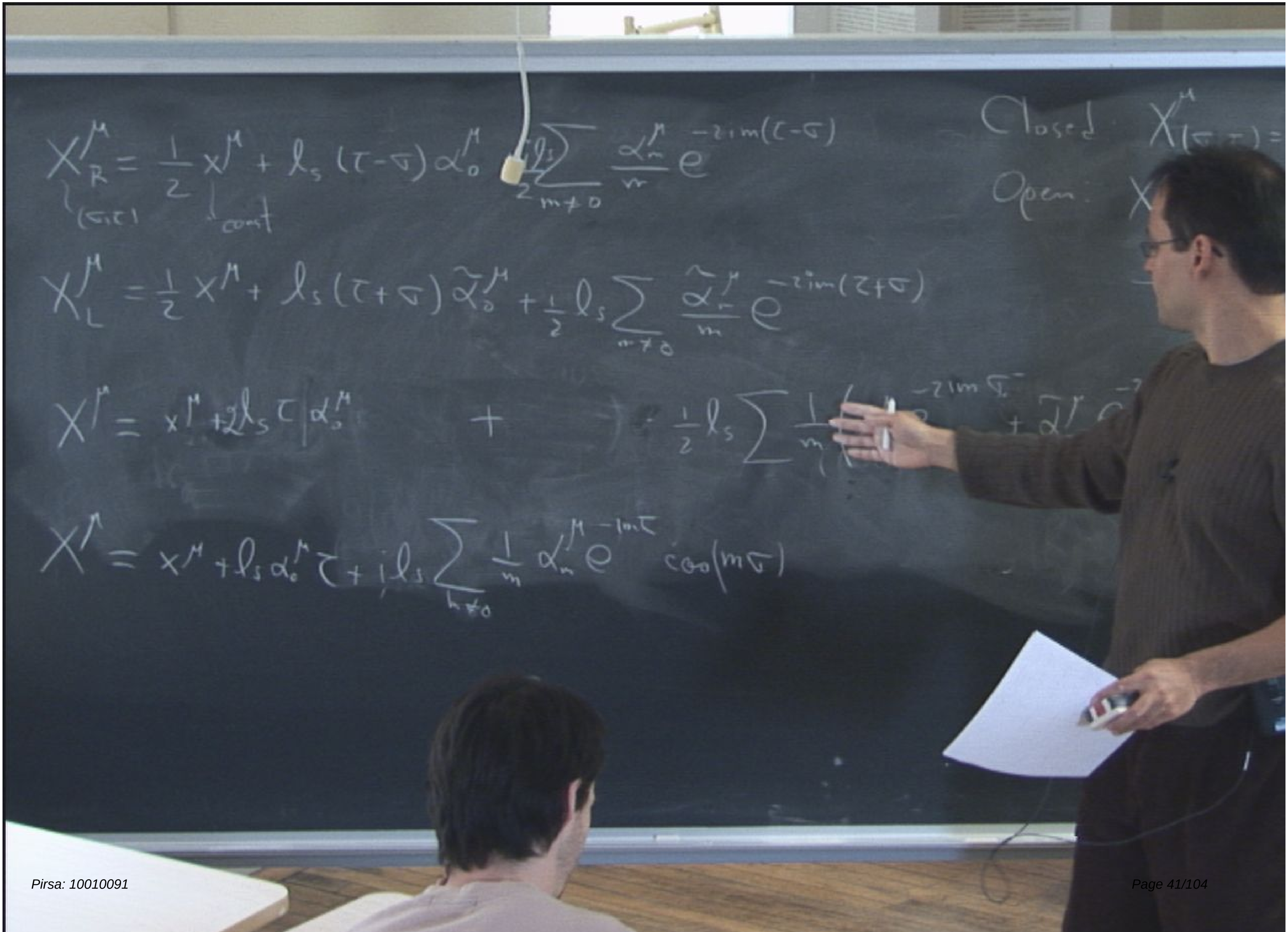
$$X^\mu_L = \frac{1}{2}x^\mu + l_s(\tau+\sigma)\tilde{\alpha}_0^\mu + \frac{i}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}_m^\mu}{m}e^{-2im(\tau+\sigma)}$$

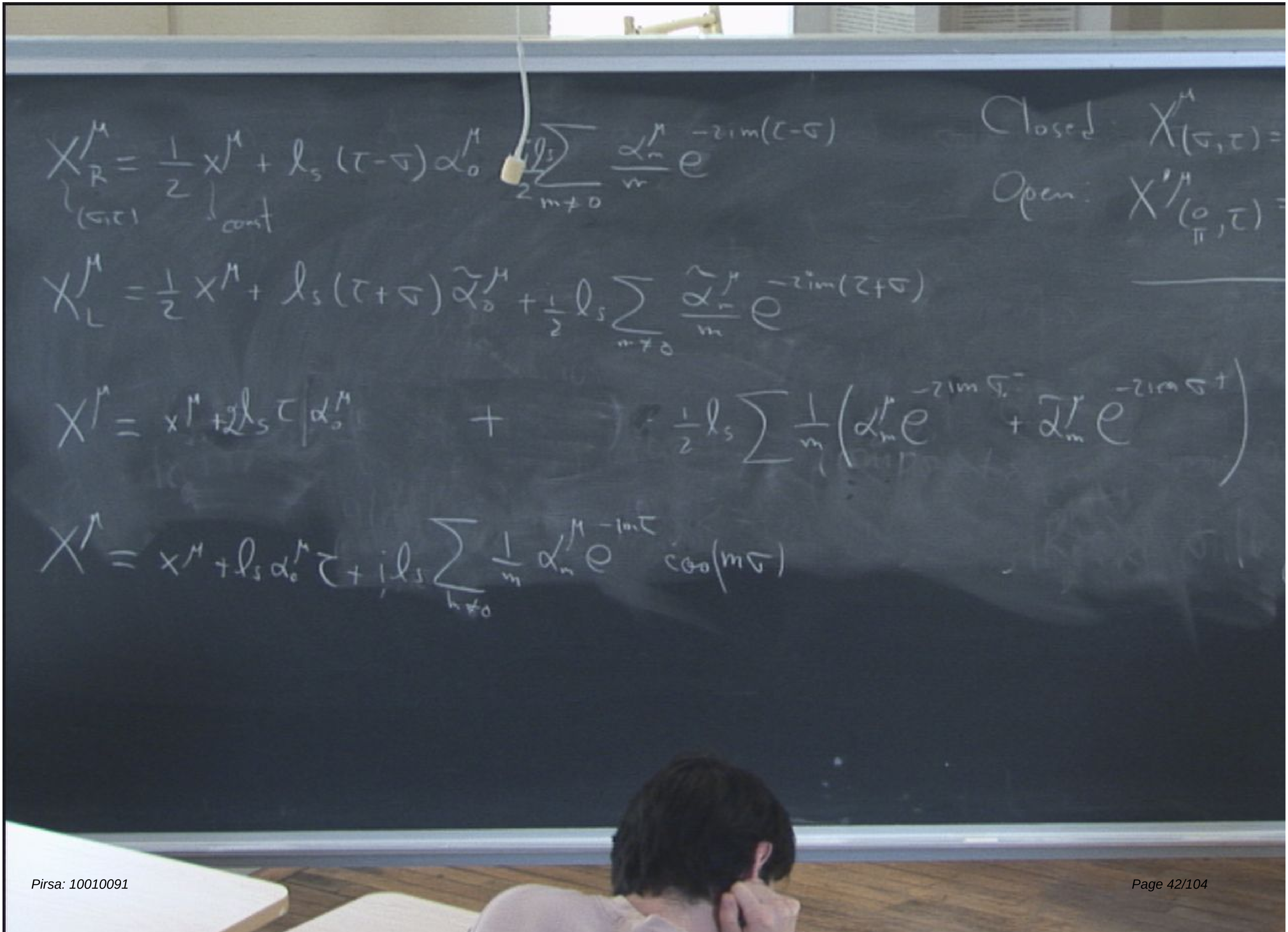
$$X^\mu = x^\mu + 2l_s\tau\,\alpha_0^\mu \quad + \quad \frac{i}{2}l_s\sum\frac{1}{m}\left(\alpha_m^\mu e^{-2im\sigma^-} + \tilde{\alpha}_m^\mu e^{-2im\sigma^+}\right)$$

$$X^\mu = x^\mu + l_s\alpha_0^\mu\tau + i l_s\sum_{n\neq 0}\frac{1}{m}\alpha_m^\mu e^{-im\tau}\cos(m\sigma)$$

Closed: $X^\mu(\sigma,\tau) =$

Open: $X'^\mu(^0_\pi,\tau) =$





$$\underset{(\sigma,\tau)}{X^\mu_R} = \underset{\text{const}}{\frac{1}{2}x^\mu} + l_s(\tau-\sigma)\alpha^\mu_0 \quad \frac{i l_s}{2}\sum_{m\neq 0}\frac{\alpha^\mu_m}{m}e^{-2im(\tau-\sigma)}$$

Closed: $X^\mu_{(\sigma,\tau)} =$

Open: $X'^\mu_{(\overset{0}{\pi},\tau)} =$

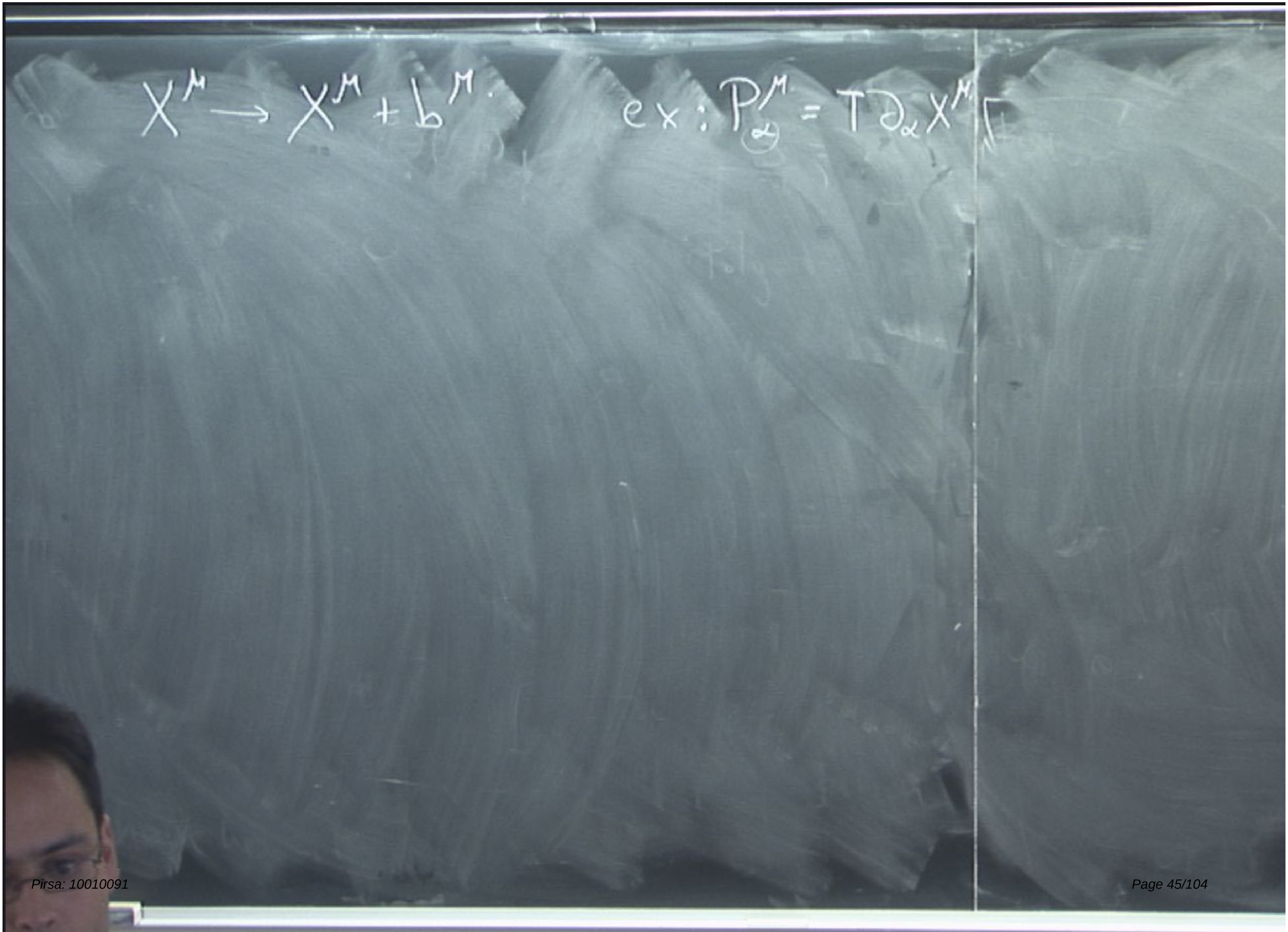
$$X^\mu_L = \frac{1}{2}x^\mu + l_s(\tau+\sigma)\tilde{\alpha}^\mu_0 + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}^\mu_m}{m}e^{-2im(\tau+\sigma)}$$

$$X^\mu = x^\mu + 2l_s\tau\,\alpha^\mu_0 \quad + \quad \frac{1}{2}l_s\sum\frac{1}{m}\left(\alpha^\mu_m e^{-2im\sigma^-} + \tilde{\alpha}^\mu_m e^{-2im\sigma^+}\right)$$

$$X^\mu = x^\mu + l_s\alpha^\mu_0\tau + i l_s\sum_{n\neq 0}\frac{1}{m}\alpha^\mu_m e^{-im\tau}\cos(m\sigma)$$

$$X^\mu \longrightarrow X^\mu + b^\mu .$$

$X^\mu \to X^\mu + b^\mu$. ex:



$X^{\mu} \longrightarrow X^{\mu} + b^{\mu}.$ ex: $P^{\mu}_{\textcircled{\alpha}} = T\partial_{\alpha}X^{\mu}$

$$X^\mu \longrightarrow X^\mu + b^\mu . \qquad \text{ex}: P^\mu_{\textcircled{α}} = T\partial_\alpha X^\mu$$

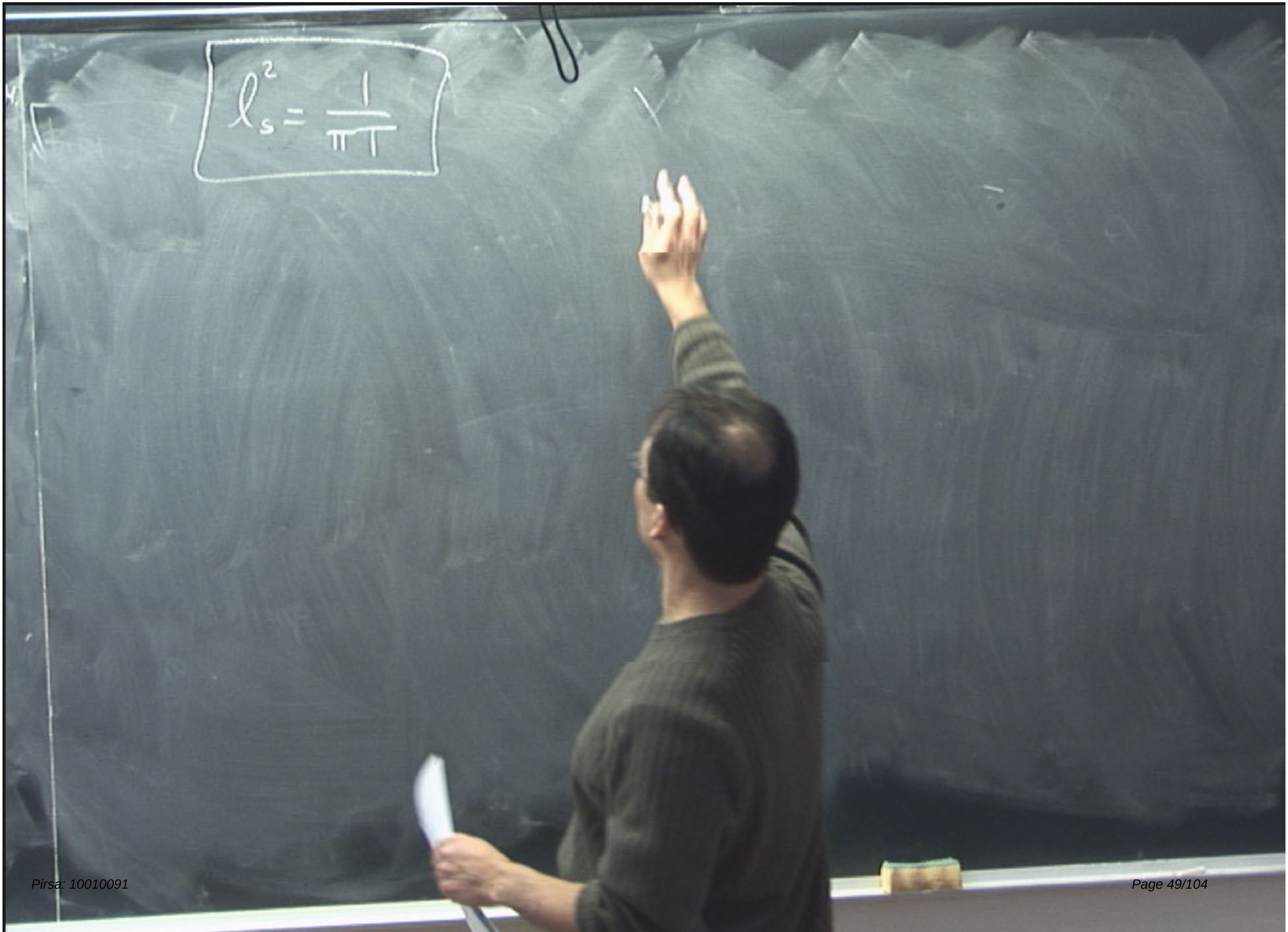
$$P^\mu = \int_0^\pi d\sigma\, T\, \partial_\tau X^\mu$$

$X^{\mu} \longrightarrow X^{\mu} + b^{\mu}$. ex: $P^{\mu}_{\enclose{circle}{\alpha}} = T\partial_{\alpha}X^{\mu}$

$$P^{\mu} = \int_0^{\pi} d\sigma T \partial_{\tau} X^{\mu} = 2\pi l_s \alpha_0^{\mu} T$$

$$X^{\mu} \longrightarrow X^{\mu} + b^{\mu} \qquad \text{ex}: P^{\mu}_{\textcircled{\alpha}} = T\,\partial_{\alpha} X^{\mu}$$

$$P^{\mu} = \int_{0}^{\pi} d\sigma\, T\, \partial_{\tau} X^{\mu} = 2\pi \ell_s \alpha_0^{\mu}\, T = p^{\mu}$$

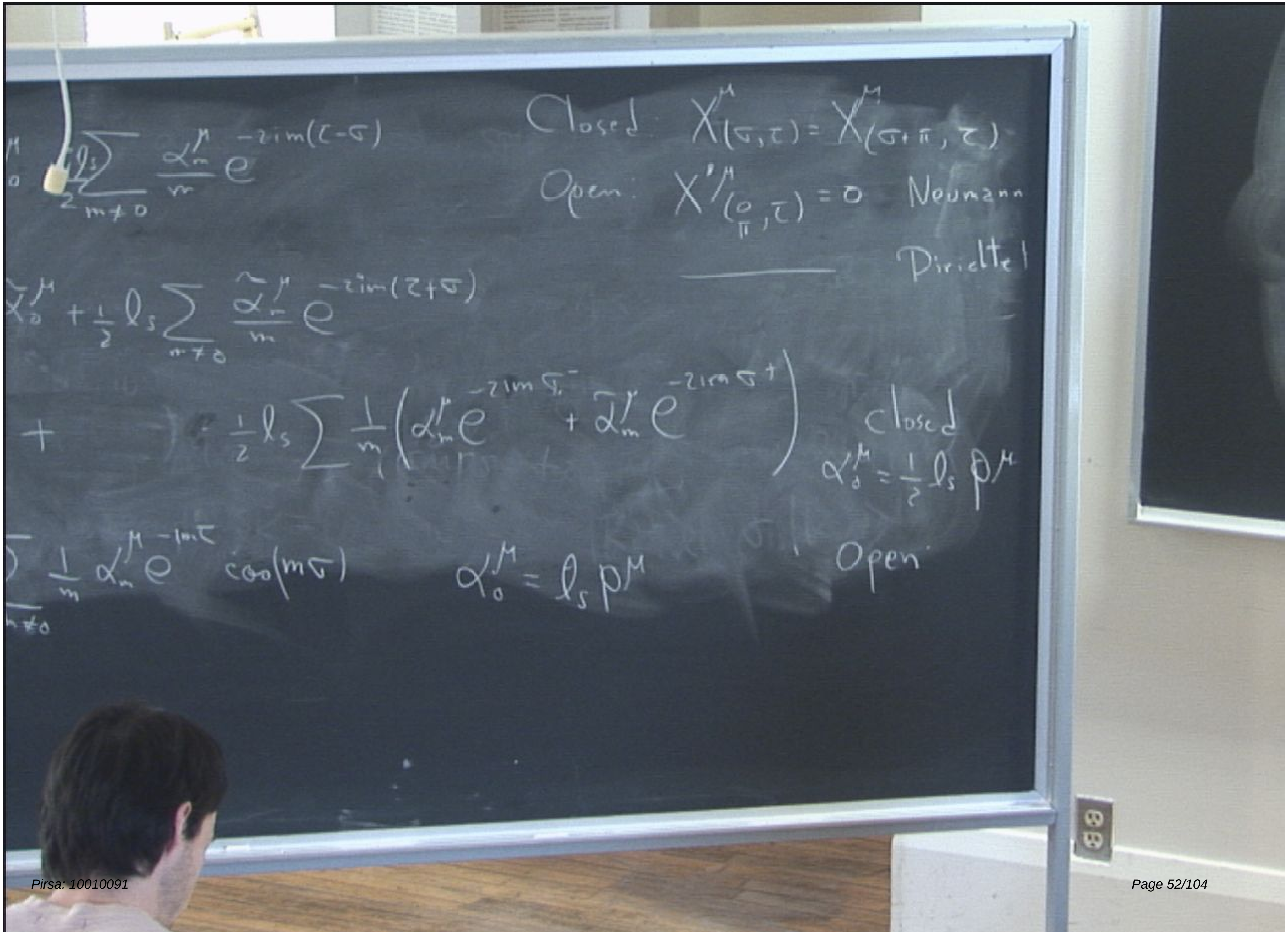


$$\boxed{\ell_s^2 = \frac{1}{\pi T}}$$

$$\boxed{l_s^2 = \frac{1}{\pi T}} \Rightarrow \alpha_0^\mu = \frac{1}{2\pi T l_s} p^\mu \Rightarrow \boxed{\alpha_0^\mu = \frac{l_s}{2} p^\mu}$$

$$\boxed{l_s^2 = \frac{1}{\pi T}} \Rightarrow \alpha_0^\mu = \frac{1}{2\pi T l_s} p^\mu \Rightarrow \boxed{\alpha_0^\mu = \frac{l_s}{2} p^\mu}$$

Closed



$$\overset{\mu}{{}_{0}} \; \frac{l_s}{2}\sum_{m\neq 0} \frac{\alpha^\mu_m}{m} e^{-2im(\tau-\sigma)}$$

Closed: $X^\mu(\sigma,\tau) = X^\mu(\sigma+\pi,\tau)$

Open: $X'^\mu\left({}^{0}_{\pi},\tau\right) = 0$ Neumann

Dirichlet

$$\tilde{x}^\mu_0 + \frac{1}{2} l_s \sum_{m\neq 0} \frac{\tilde{\alpha}^\mu_m}{m} e^{-2im(\tau+\sigma)}$$

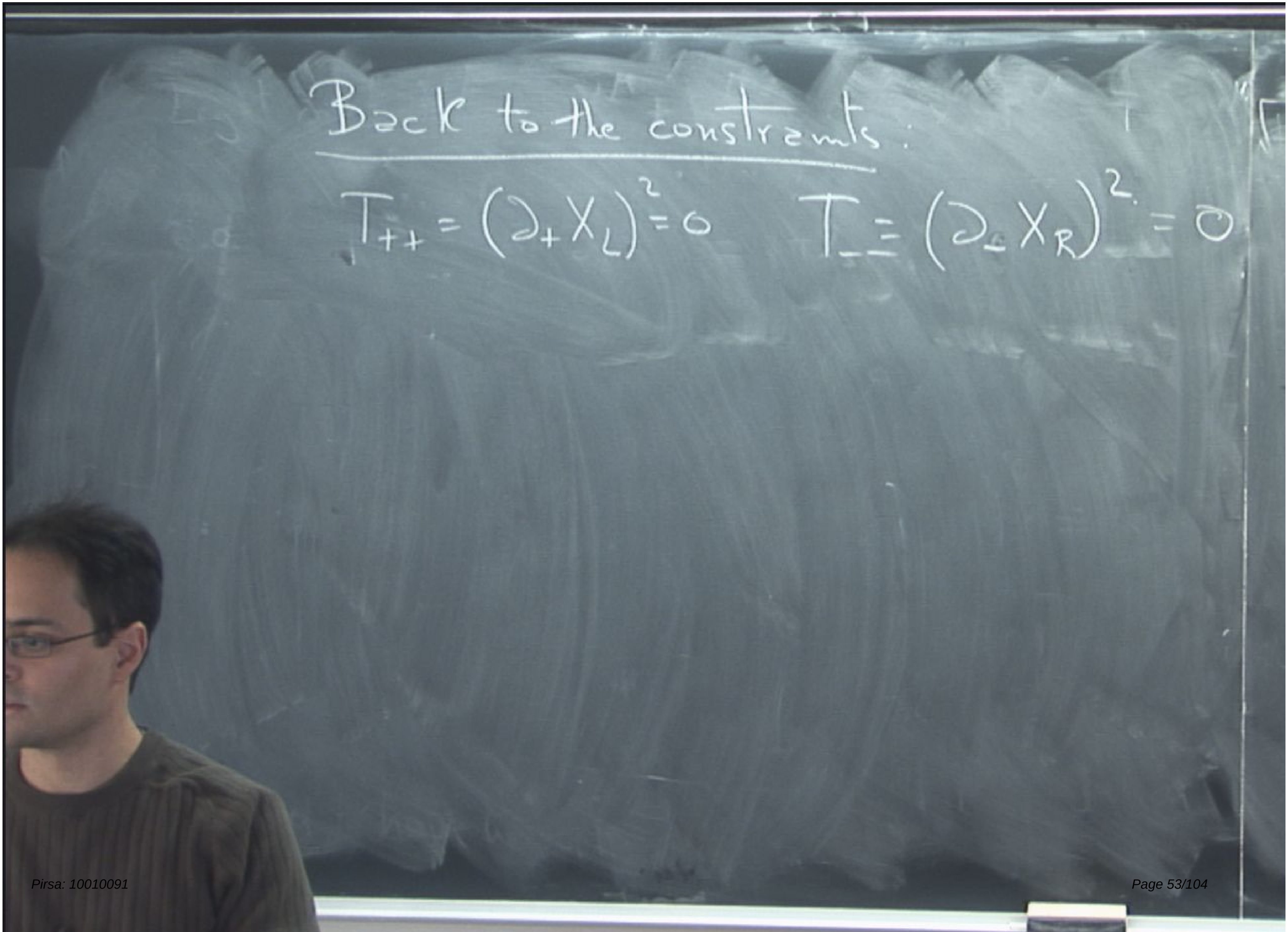
$$+ \quad \frac{1}{2} l_s \sum \frac{1}{m}\left(\alpha^\mu_m e^{-2im\sigma^-} + \tilde{\alpha}^\mu_m e^{-2im\sigma^+}\right)$$

closed

$\alpha^\mu_0 = \frac{1}{2} l_s p^\mu$

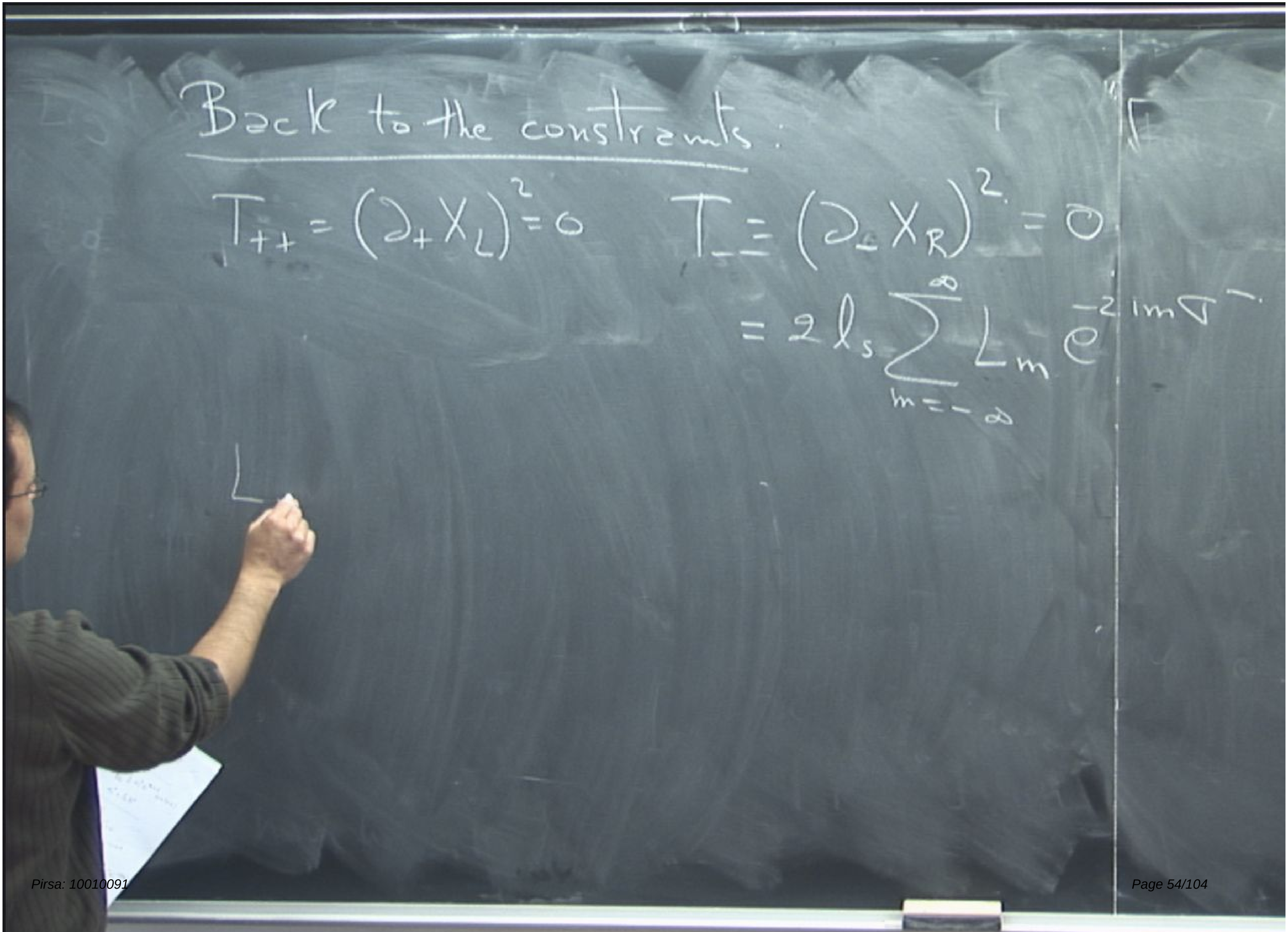
$$\sum_{m\neq 0} \frac{1}{m} \alpha^\mu_m e^{-im\tau} \cos(m\sigma) \qquad \alpha^\mu_0 = l_s p^\mu$$

Open



Back to the constraints:

$$T_{++} = (\partial_+ X_L)^2 = 0 \qquad T_{--} = (\partial_- X_R)^2 = 0$$



Back to the constraints:

$$T_{++} = (\partial_+ X_L)^2 = 0 \qquad T_{--} = (\partial_- X_R)^2 = 0$$

$$= 2 l_s \sum_{m=-\infty}^{\infty} L_m e^{-2im\sigma^-}$$

L

Back to the constraints:

$$T_{++} = (\partial_+ X_L)^2 = 0 \qquad T_{--} = (\partial_- X_R)^2 = 0$$

$$\tau = 0 \qquad = 2 l_s \sum_{m=-\infty}^{\infty} L_m e^{-2im\sigma^-}$$

$$L_m \sim \int_0^\pi d\sigma \, e^{2im\sigma} T_{--}$$

→

Back to the constraints:

$$T_{++} = (\partial_+ X_L)^2 = 0 \qquad T_{--} = (\partial_- X_R)^2 = 0$$

$$\tau = 0$$

$$= 2 l_s \sum_{m=-\infty}^{\infty} L_m e^{-2im\sigma}$$

$$L_m \sim \int_0^{\pi} d\sigma\, e^{2im\sigma}\, T_{--}$$

$$\longrightarrow \left(l_s \sum_{n=-\infty}^{\infty} \alpha_n^{\mu} e^{-2in\sigma} \right) \left(l \right.$$

onstraints:

$=0$

$$T_{--} = (\partial_- X_R)^2 = 0$$

$$= 2 l_s \sum_{m=-\infty}^{\infty} L_m e^{-2im\sigma^-}$$

$$\boxed{l_s^2 = \frac{1}{\pi T}} \Rightarrow \alpha_0^\mu =$$

$im\sigma$

$$T_{--} \longrightarrow \left(l_s \sum_{n=-\infty}^{\infty} \alpha_n^\mu e^{-2in\sigma}\right)\left(l_s \sum_{q=-\infty}^{\infty} \alpha_q^\nu e^{-2iq\sigma}\right)\eta_{\mu\nu}$$

Back to the constraints:

$$T_{++} = (\partial_+ X_L)^2 = 0 \qquad T_{--} = (\partial_- X_R)^2 = 0$$

$$\tau = 0 \qquad = 2 l_s \sum_{m=-\infty}^{\infty} L_m e^{-2im\sigma}$$

$$L_m \sim \int_0^{\pi} d\sigma\, e^{2im\sigma}\, T_{--}$$

$$T_{--} \longrightarrow \left(l_s \sum_{n=-\infty}^{\infty} \alpha_n^{\mu} e^{-2in\sigma} \right) \Big($$

$$l_s^2 \sum_{n=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \alpha_m^{\mu} \alpha_n \int_0^{\pi} d\sigma\, e^{2i(m-n-q)\sigma}$$

Back to the constraints:

$$T_{++} = (\partial_+ X_L)^2 = 0 \qquad T_{--} = (\partial_- X_R)^2 = 0$$

$$\tau = 0 \qquad = 2 l_s \sum_{m=-\infty}^{\infty} L_m e^{-2im\sigma}$$

$$L_m \sim \int_0^{\pi} d\sigma \, e^{2im\sigma} \, T_{--} \longrightarrow \Big(l_s \sum_{n=-\infty}^{\infty} \alpha_n^{\mu} e^{-2}$$

$$l_s^2 \sum_{n=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \alpha_n \cdot \alpha_q \underbrace{\int_0^{\pi} d\sigma \, e^{2i(m-n-q)\sigma}}_{\delta_{m-n-q,0}} \qquad q$$

Back to the constraints:

$T_{++} = (\partial_+ X_L)^2 = 0 \qquad T_{--} = (\partial_- X_R)^2 = 0$

$\tau = 0 \qquad = 2 l_s \sum_{m=-\infty}^{\infty} L_m e^{-2im\sigma^-}$

$$L_m \sim \int_0^\pi d\sigma\, e^{2im\sigma}\, T_{--} \longrightarrow \left(l_s \sum_{n=-\infty}^{\infty} \alpha_n^\mu e^{-2in\sigma}\right)\left(l_s \sum_{q=-}^{\infty}\right.$$

$$l_s^2 \sum_{n=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \alpha_n \alpha_q \underbrace{\int_0^\pi d\sigma\, e^{2i(m-n-q)\sigma}}_{\delta_{m-n-q,0}} \qquad q = m-n$$

$\Big[\, l_s^2$

s :

$$= (\partial_- X_R)^2 = 0$$

$$= 2 l_s^2 \sum_{m=-\infty}^{\infty} L_m e^{-2im\sigma^-}$$

$$\boxed{l_s^2 = \frac{1}{\pi T}} \Rightarrow \alpha_0^\mu = \frac{1}{2\pi T l_s} p^\mu =$$

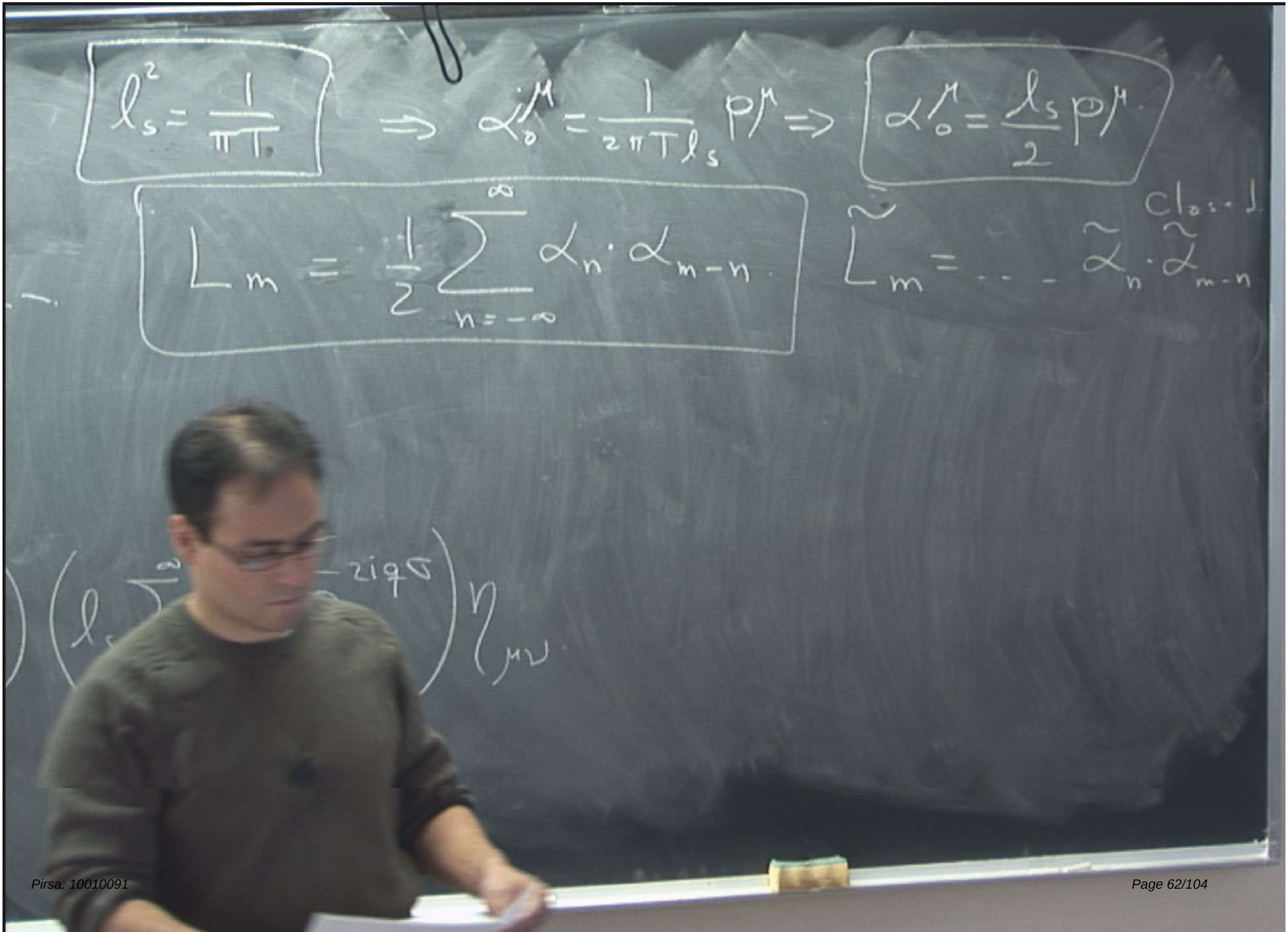
$$\boxed{L_m = \frac{1}{2} \sum_{n=-\infty}^{\infty} \alpha_n \cdot \alpha_{m-n}}$$

$$\longrightarrow \left(l_s \sum_{n=-\infty}^{\infty} \alpha_n^\mu e^{-2in\sigma} \right) \left(l_s \sum_{q=-\infty}^{\infty} \alpha_q^\nu e^{-2iq\sigma} \right) \eta_{\mu\nu}$$

$n - q)\sigma$

$q = m - n$

$m - n - q, 0$



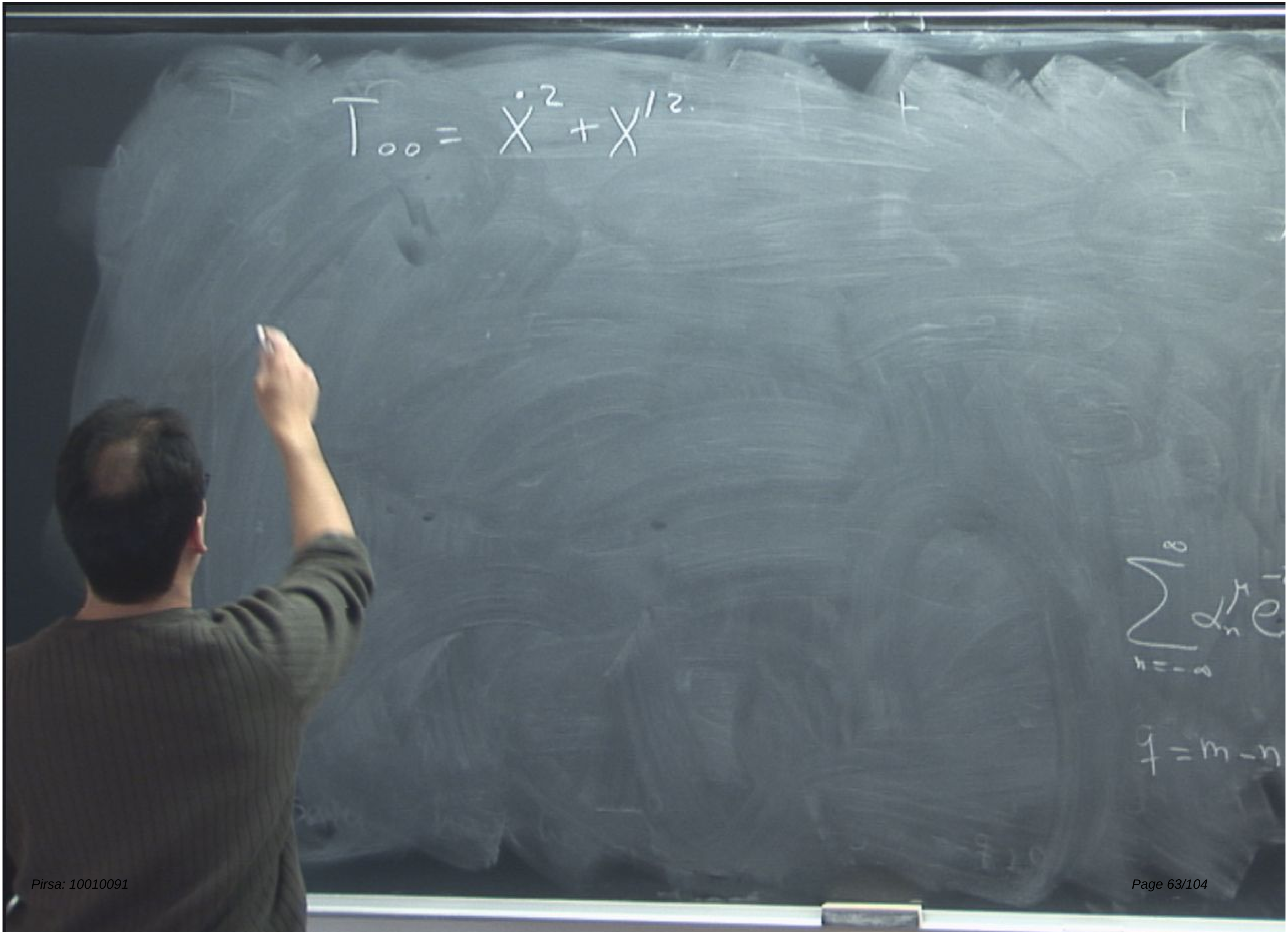
$$\boxed{\ell_s^2 = \frac{1}{\pi T}} \Rightarrow \alpha_0^\mu = \frac{1}{2\pi T \ell_s} p^\mu \Rightarrow \boxed{\alpha_0^\mu = \frac{\ell_s}{2} p^\mu}$$

$$\boxed{L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_n \cdot \alpha_{m-n}}$$

closed

$$\tilde{L}_m = \ldots \tilde{\alpha}_n \cdot \tilde{\alpha}_{m-n}$$

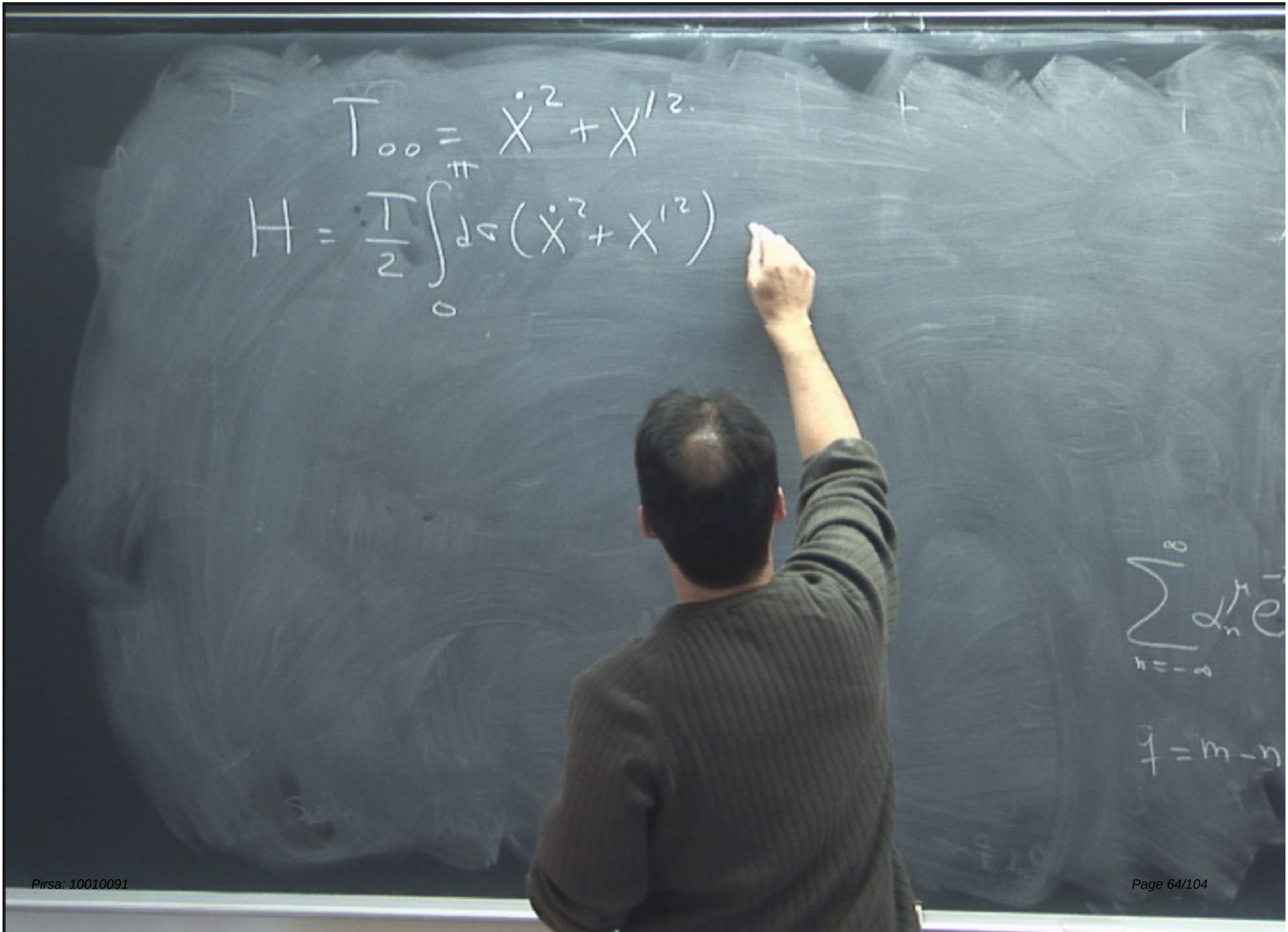
$$\Big)\Big(\ell_s \sum^{\infty} \quad {}^{-2iq\sigma}\Big)\eta_{\mu\nu}$$



$$T_{00} = \dot{X}^2 + X'^2$$

$$\sum_{n=-\infty}^{\infty} \alpha_n^{\mu} e$$

$q = m - n$

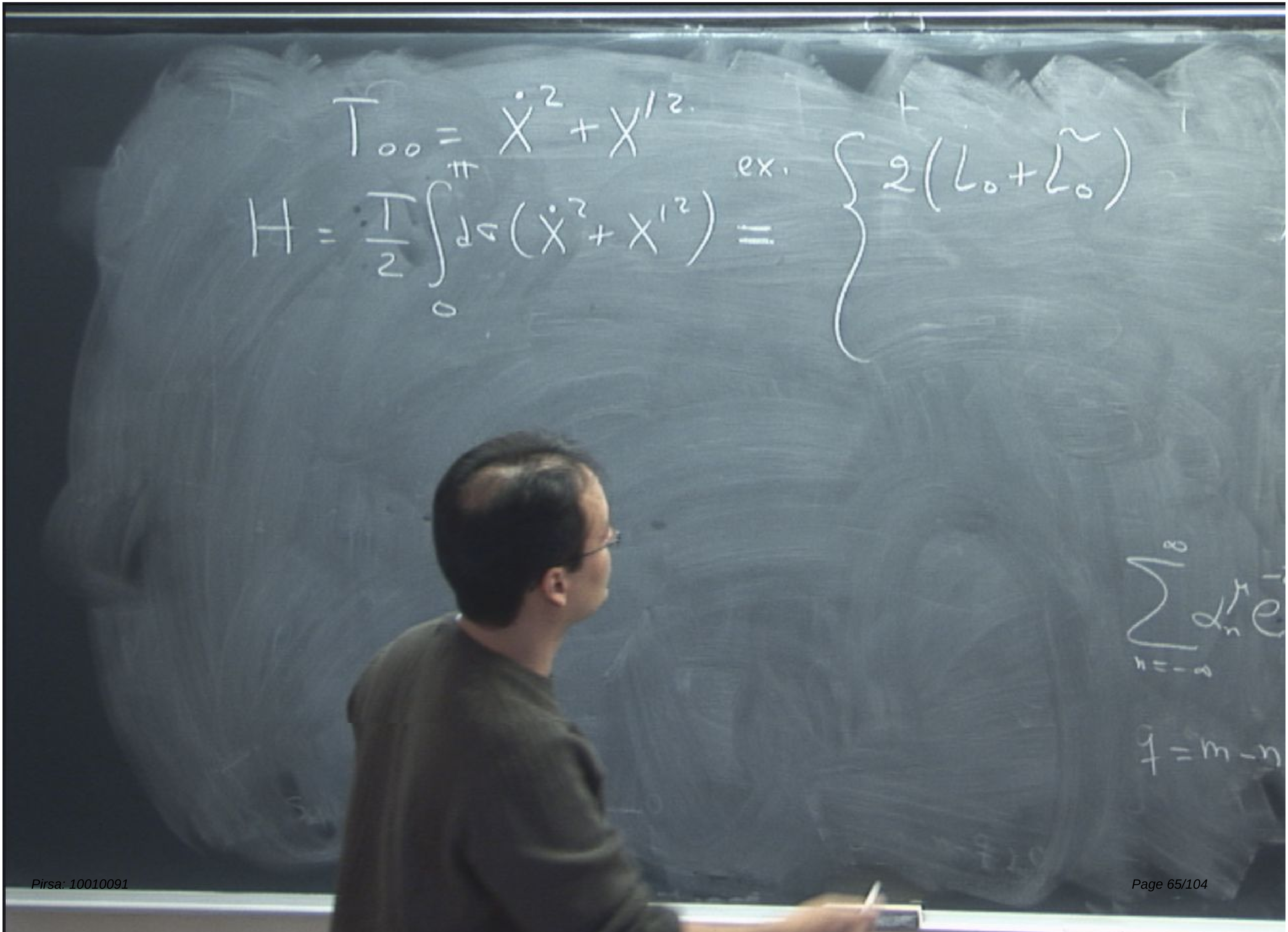


$$T_{00} = \dot{X}^2 + X'^2$$

$$H = \frac{T}{2}\int_0^{\pi} d\sigma \left(\dot{X}^2 + X'^2\right)$$

$$\sum_{n=-\infty}^{\infty} \alpha_n^{\mu} e$$

$$q = m - n$$

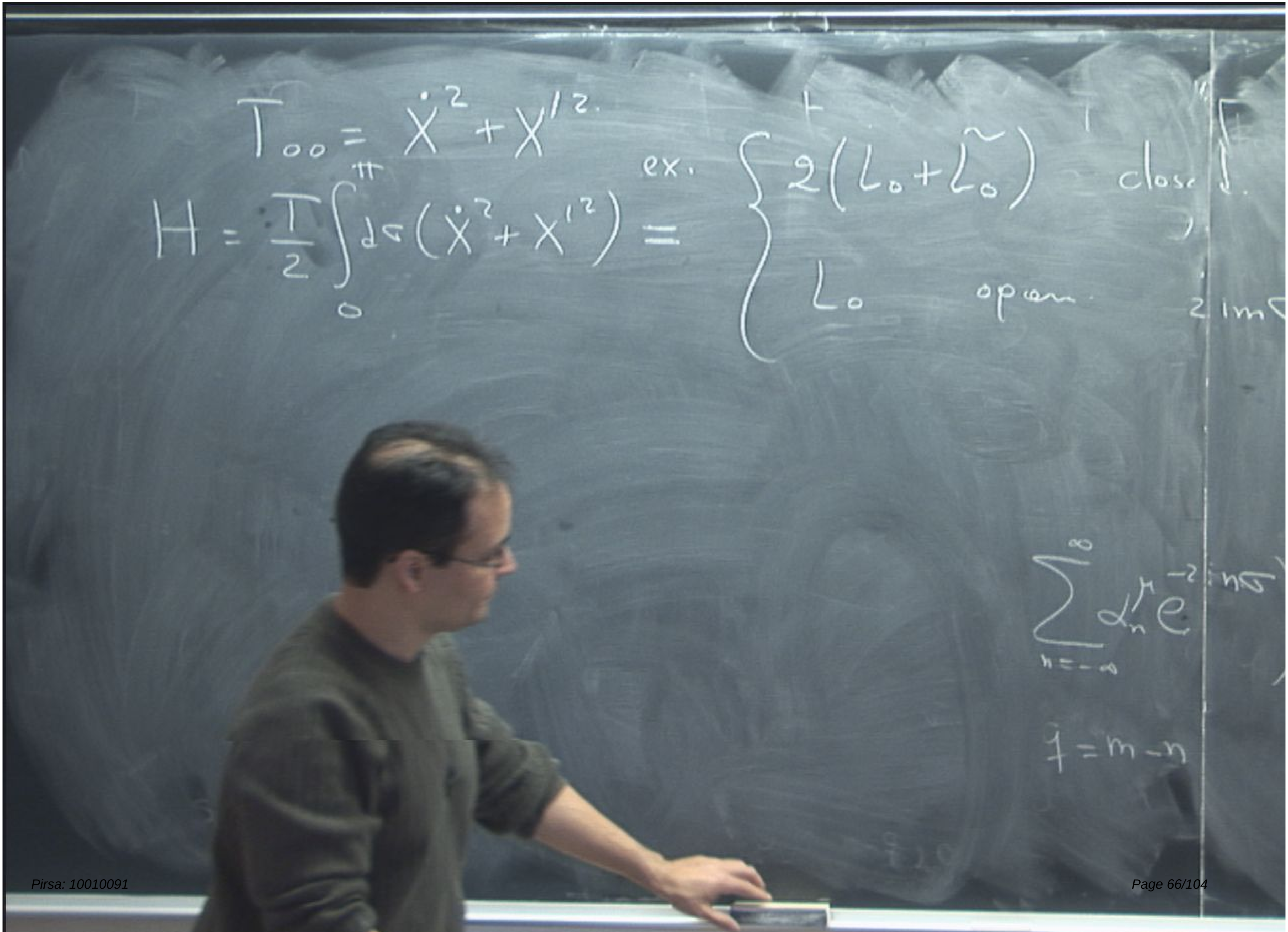


$$T_{00} = \dot{X}^2 + X'^2$$

$$H = \frac{T}{2}\int_0^{\pi} d\sigma \left(\dot{X}^2 + X'^2\right) = \quad \text{ex.} \left\{ 2\left(L_0 + \tilde{L}_0\right) \right)$$

$$\sum_{n=-\infty}^{\infty} \alpha_n^\mu e$$

$$q = m - n$$



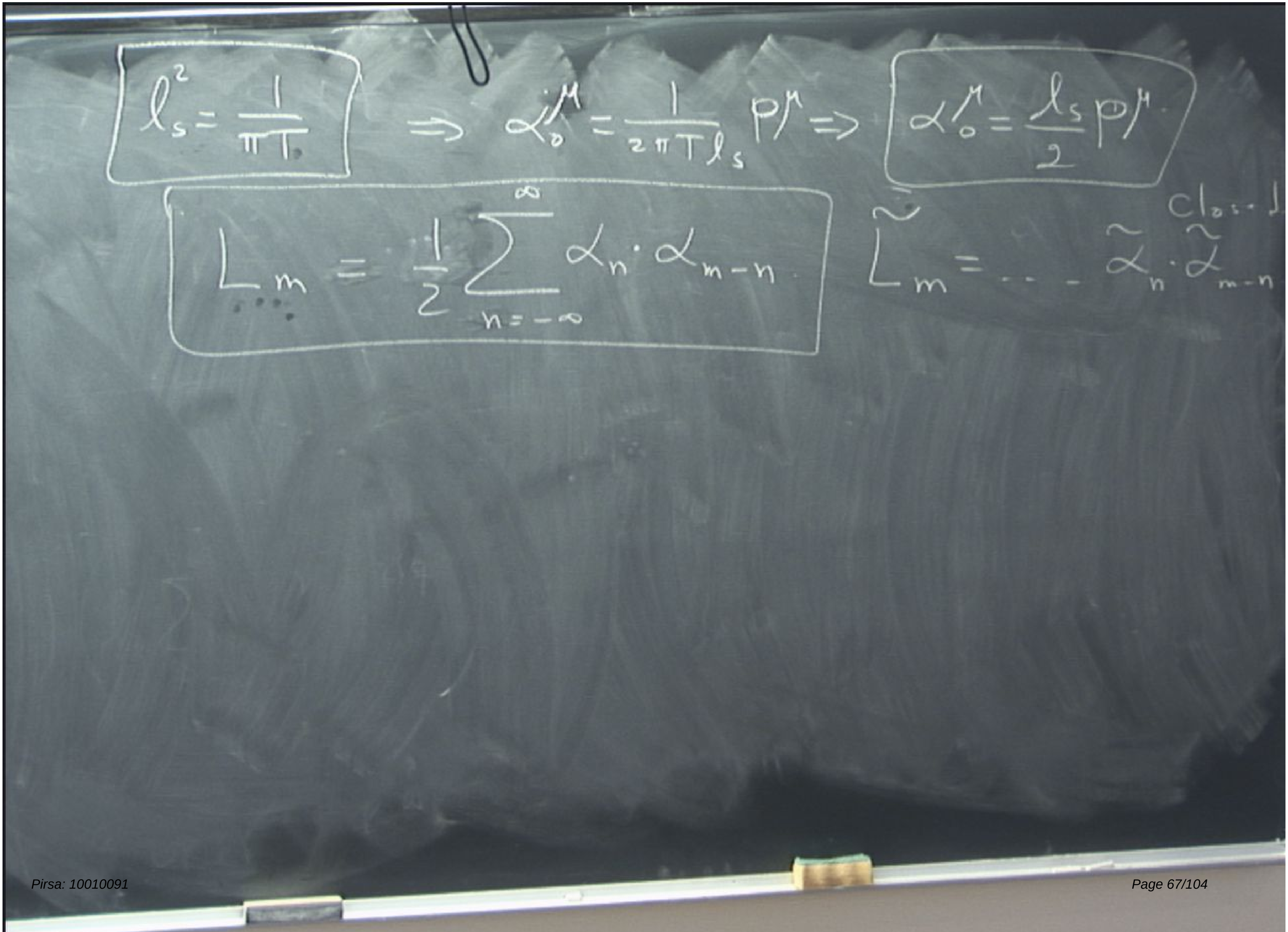
$$T_{00} = \dot{X}^2 + X'^2 .$$

$$H = \frac{T}{2}\int_0^{\pi} d\sigma\,(\dot{X}^2 + X'^2) \overset{\text{ex.}}{=} \begin{cases} 2(L_0 + \tilde{L}_0) & \text{close} \\ L_0 & \text{open.} \end{cases}$$

2 im

$$\sum_{n=-\infty}^{\infty} \alpha_n^{\mu} e^{-2in\sigma}$$

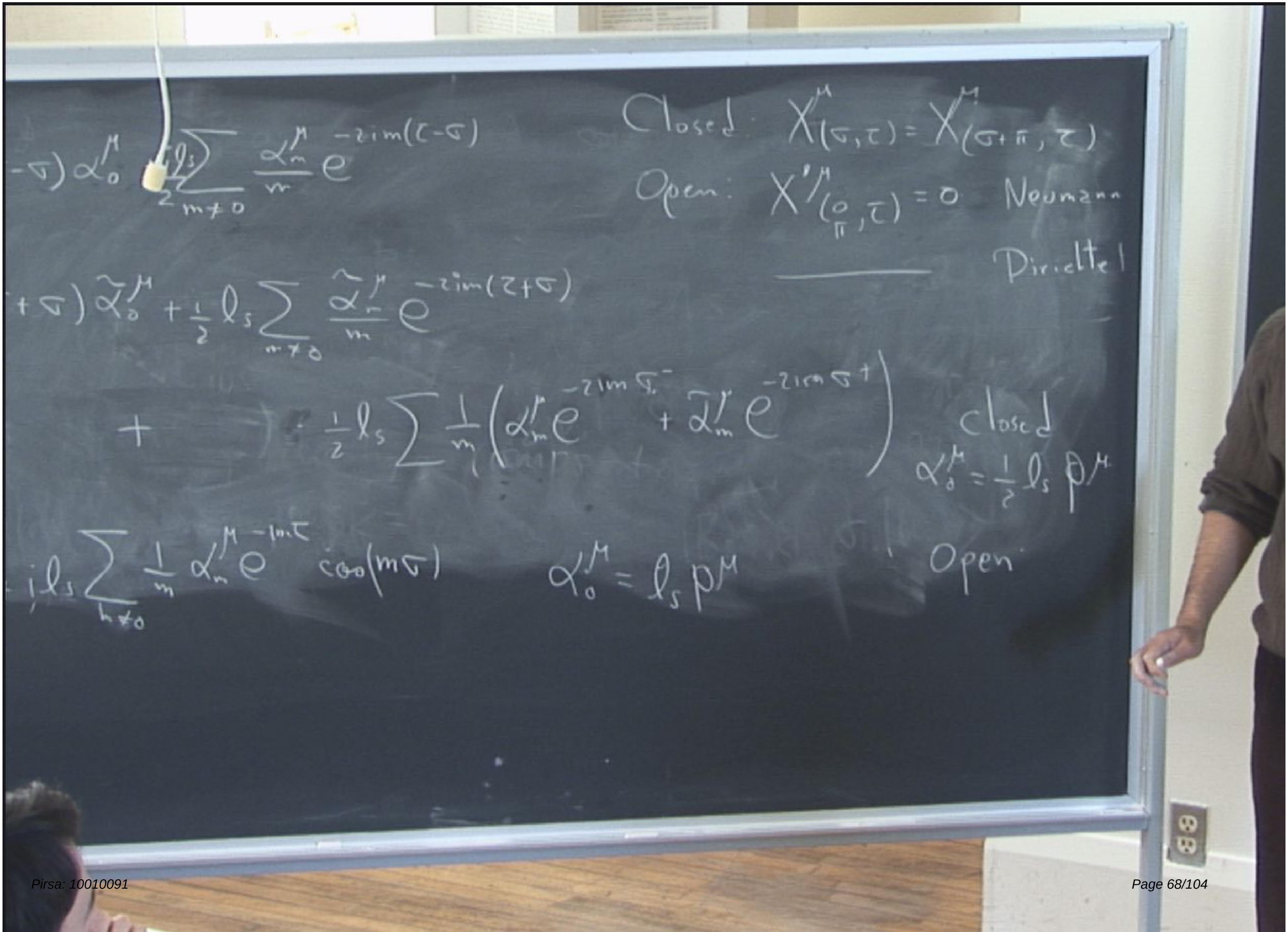
$$j = m - n$$



$$\boxed{l_s^2 = \frac{1}{\pi T}} \Rightarrow \alpha_0^\mu = \frac{1}{2\pi T l_s} p^\mu \Rightarrow \boxed{\alpha_0^\mu = \frac{l_s p^\mu}{2}}$$

Closed

$$\boxed{L_m = \frac{1}{2} \sum_{n=-\infty}^{\infty} \alpha_n \cdot \alpha_{m-n}} \qquad \tilde{L}_m = \ldots \tilde{\alpha}_n \cdot \tilde{\alpha}_{m-n}$$



$$-\sigma)\alpha_0^\mu \; l_s\sum_{m\neq 0}\frac{\alpha_m^\mu}{m}e^{-2im(\tau-\sigma)}$$

$$+\sigma)\tilde{\alpha}_0^\mu + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}_m^\mu}{m}e^{-2im(\tau+\sigma)}$$

$$+ \quad \frac{1}{2}l_s\sum\frac{1}{m}\left(\alpha_m^\mu e^{-2im\sigma^-} + \tilde{\alpha}_m^\mu e^{-2im\sigma^+}\right)$$

closed

$$\alpha_0^\mu = \frac{1}{2}l_s p^\mu$$

$$i l_s\sum_{n\neq 0}\frac{1}{m}\alpha_m^\mu e^{-im\tau}\cos(m\sigma) \qquad \alpha_0^\mu = l_s p^\mu$$

Open

Closed: $X^\mu(\sigma,\tau) = X^\mu(\sigma+\pi,\tau)$

Open: $X'^\mu(\substack{0\\\pi},\tau) = 0$ Neumann

Dirichlet

$$T_{00} = \dot{X}^2 + X'^2$$

$$H = \frac{T}{2}\int_0^{\pi} d\sigma\,(\dot{X}^2 + X'^2) = \overset{\text{ex.}}{} \begin{cases} 2(L_0 + \tilde{L}_0) & \text{clos} \\ L_0 & \text{open} \end{cases}$$

$$M^2 = -p^2$$

$$\boxed{\ell_s^2 = \frac{1}{\pi T}} \Rightarrow \alpha_0^\mu = \frac{1}{2\pi T \ell_s} p^\mu \Rightarrow \boxed{\alpha_0^\mu = \frac{\ell_s}{2} p^\mu}$$

$$\boxed{L_m = \frac{1}{2} \sum_{n=-\infty}^{\infty} \alpha_n \cdot \alpha_{m-n}}$$

Closed

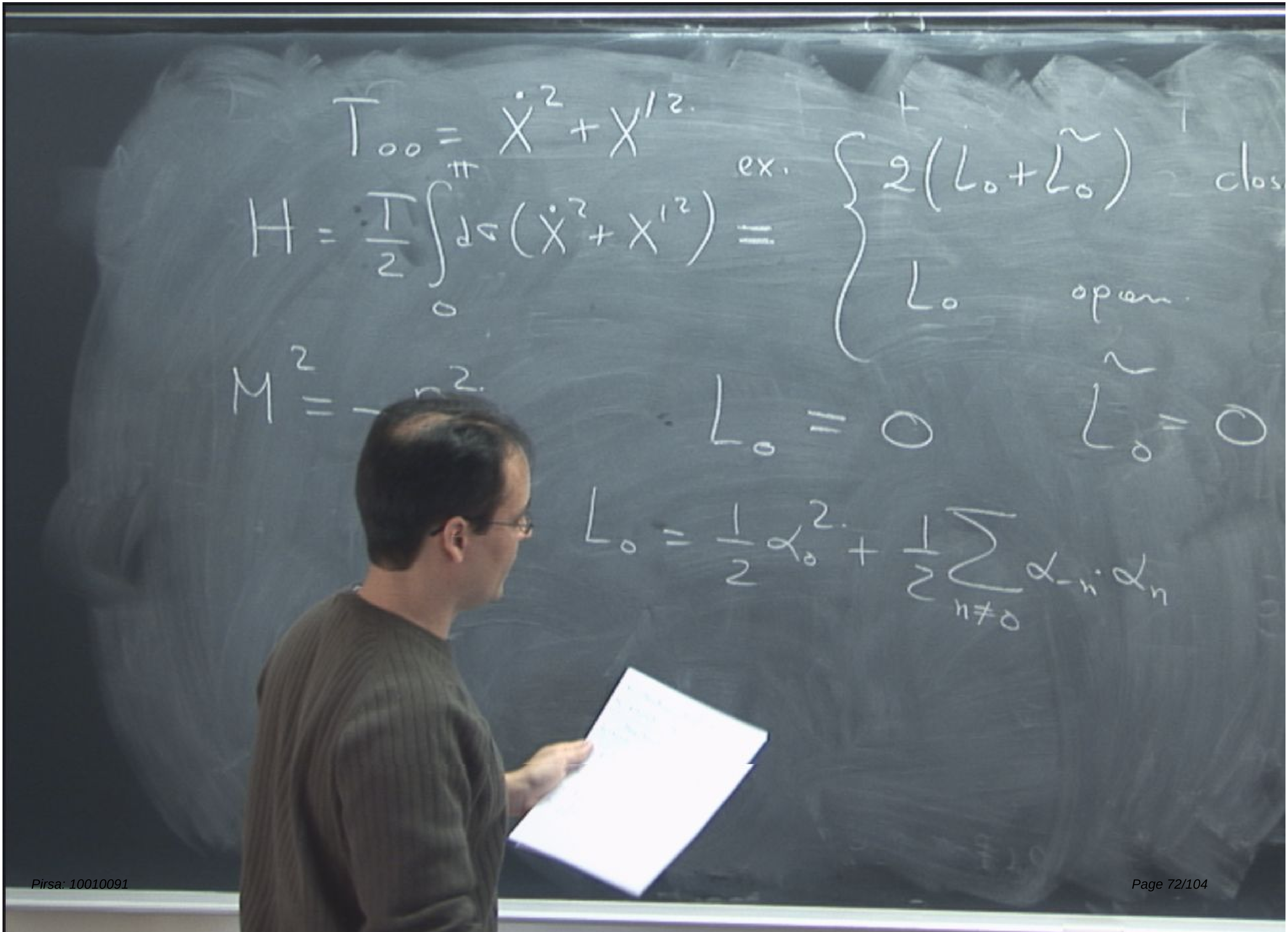
$$\tilde{L}_m = \ldots \tilde{\alpha}_n \cdot \tilde{\alpha}_{m-n}$$

$$T_{00} = \dot{X}^2 + X'^2$$

$$H = \frac{T}{2}\int_0^\pi d\sigma\,(\dot{X}^2 + X'^2) = \quad \text{ex.} \begin{cases} 2(L_0 + \tilde{L}_0) & \text{clos} \\ L_0 & \text{open} \end{cases}$$

$$M^2 = -p^2$$

$$L_0 = 0 \qquad \tilde{L}_0 = 0$$



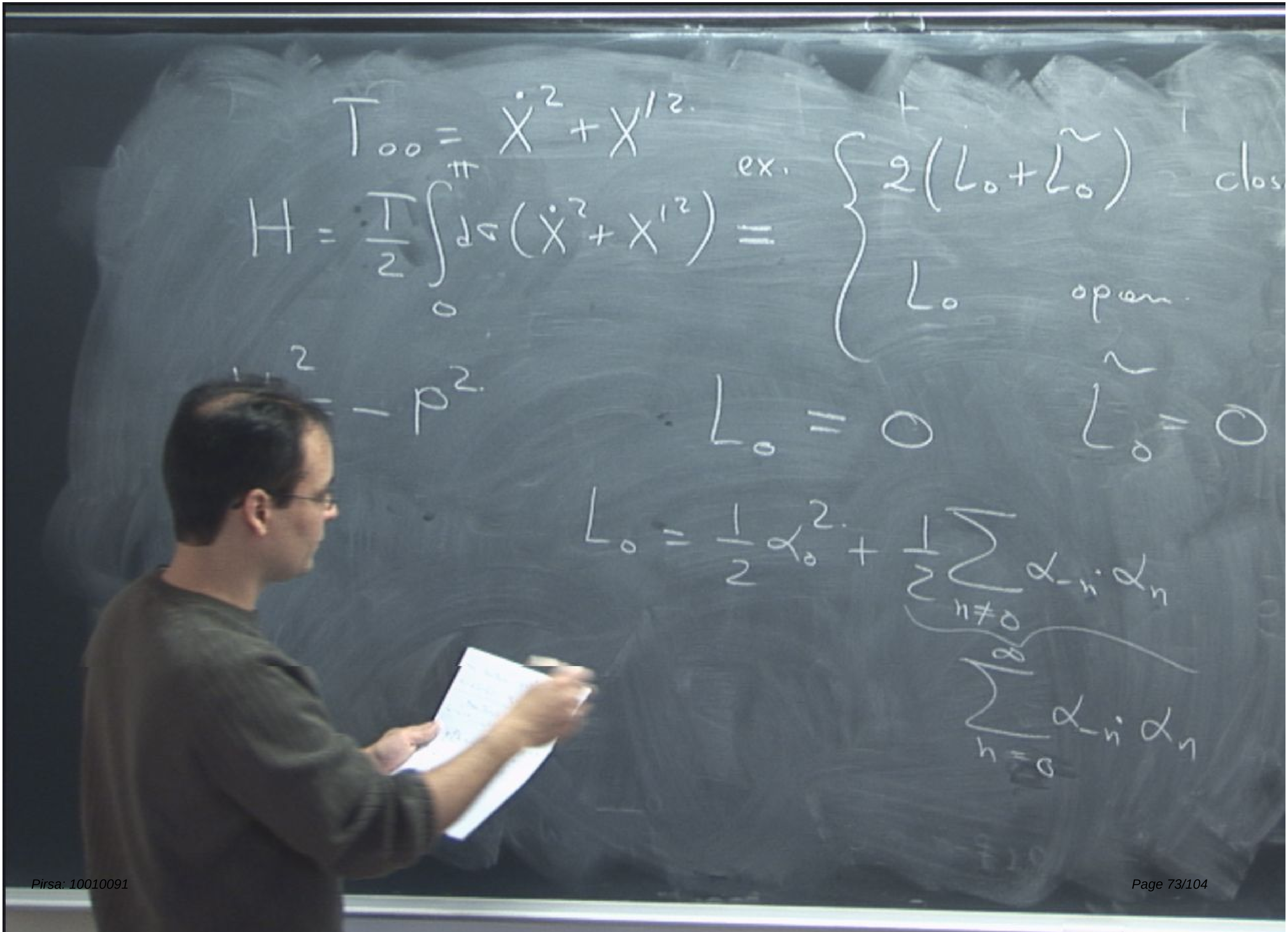
$$T_{00} = \dot{X}^2 + X'^2$$

$$H = \frac{T}{2}\int_0^\pi d\sigma\,(\dot{X}^2 + X'^2) = \text{ex.} \begin{cases} 2(L_0 + \tilde{L}_0) & \text{clos} \\ L_0 & \text{open.} \end{cases}$$

$$M^2 = -p^2.$$

$$L_0 = 0 \qquad \tilde{L}_0 = 0$$

$$L_0 = \frac{1}{2}\alpha_0^2 + \frac{1}{2}\sum_{n\neq 0}\alpha_{-n}\cdot\alpha_n$$



$$T_{00} = \dot{X}^2 + X'^2$$

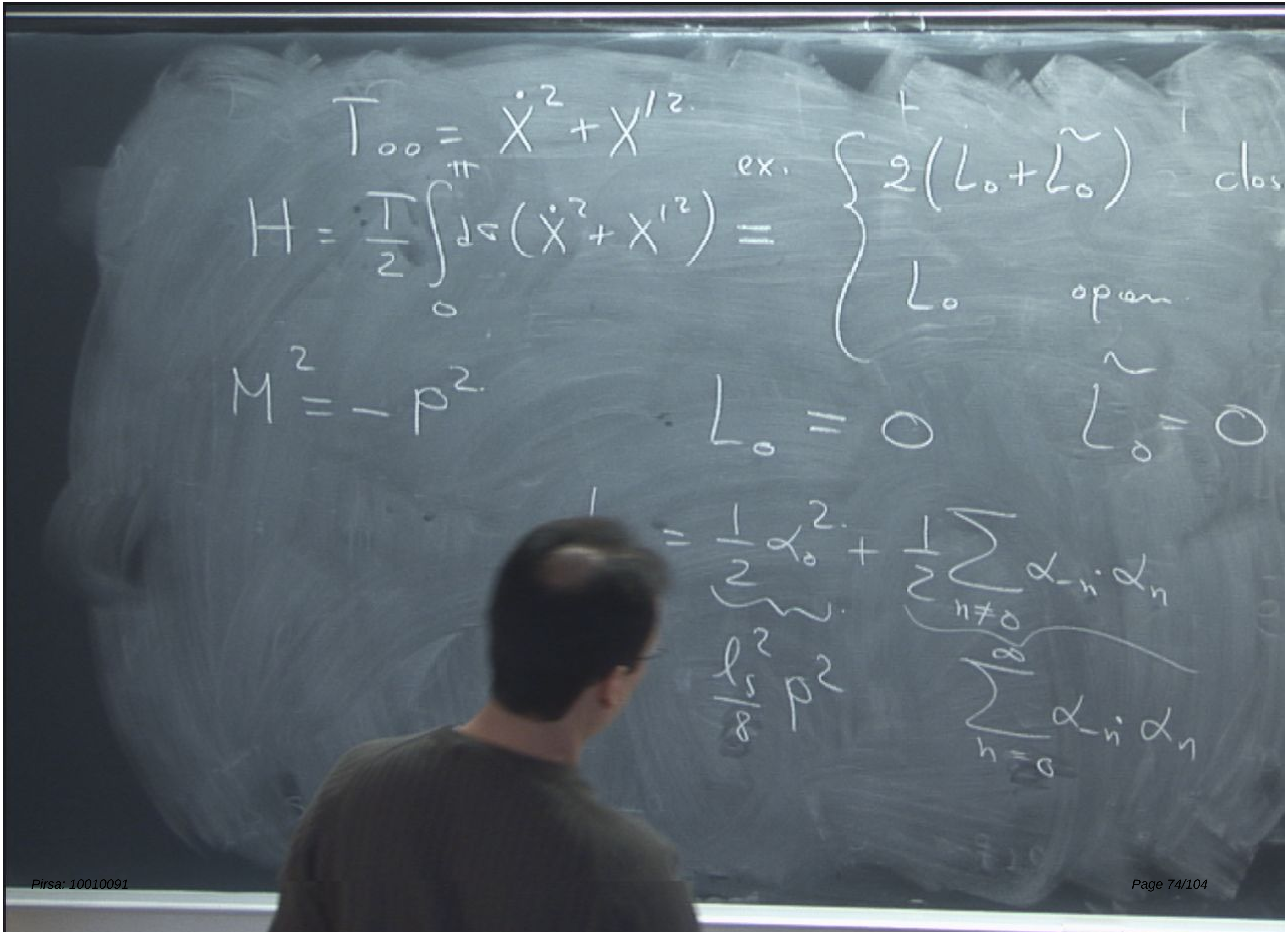
ex.

$$H = \frac{T}{2}\int_0^{\pi} d\sigma\,(\dot{X}^2 + X'^2) = \begin{cases} 2(L_0 + \tilde{L}_0) & \text{clos} \\ L_0 & \text{open} \end{cases}$$

$${}^{2} = -p^2$$

$$L_0 = 0 \qquad \tilde{L}_0 = 0$$

$$L_0 = \frac{1}{2}\alpha_0^2 + \underbrace{\frac{1}{2}\sum_{n\neq 0}\alpha_{-n}\cdot\alpha_n}_{\sum_{n=0}^{\infty}\alpha_{-n}\cdot\alpha_n}$$



$$T_{00} = \dot{X}^2 + X'^2$$

$$H = \frac{T}{2}\int_0^\pi d\sigma\,(\dot{X}^2 + X'^2) = \text{ex.} \begin{cases} 2(L_0 + \tilde{L}_0) & \text{clos} \\ L_0 & \text{open} \end{cases}$$

$$M^2 = -p^2 \qquad L_0 = 0 \qquad \tilde{L}_0 = 0$$

$$= \underbrace{\frac{1}{2}\alpha_0^2}_{\frac{l_s^2}{8}p^2} + \underbrace{\frac{1}{2}\sum_{n\neq 0}\alpha_{-n}\cdot\alpha_n}_{\sum_{n=0}^{\infty}\alpha_{-n}\cdot\alpha_n}$$

$$T_{00} = \dot{X}^2 + X'^2$$

$$H = \frac{T}{2}\int_0^{\pi} d\sigma \left(\dot{X}^2 + X'^2\right) = \quad \text{ex.} \begin{cases} 2(L_0 + \tilde{L}_0) & \text{clos} \\ L_0 & \text{open} \end{cases}$$

$$M^2 = -p^2$$

$$L_0 = 0 \qquad \tilde{L}_0 = 0$$

$$L_0 = \underbrace{\frac{1}{2}\alpha_0^2}_{\frac{l_s^2}{8}p^2} + \underbrace{\frac{1}{2}\sum_{n\neq 0}\alpha_{-n}\cdot\alpha_n}_{\sum_{n=1}^{\infty}\alpha_{-n}\cdot\alpha_n}$$

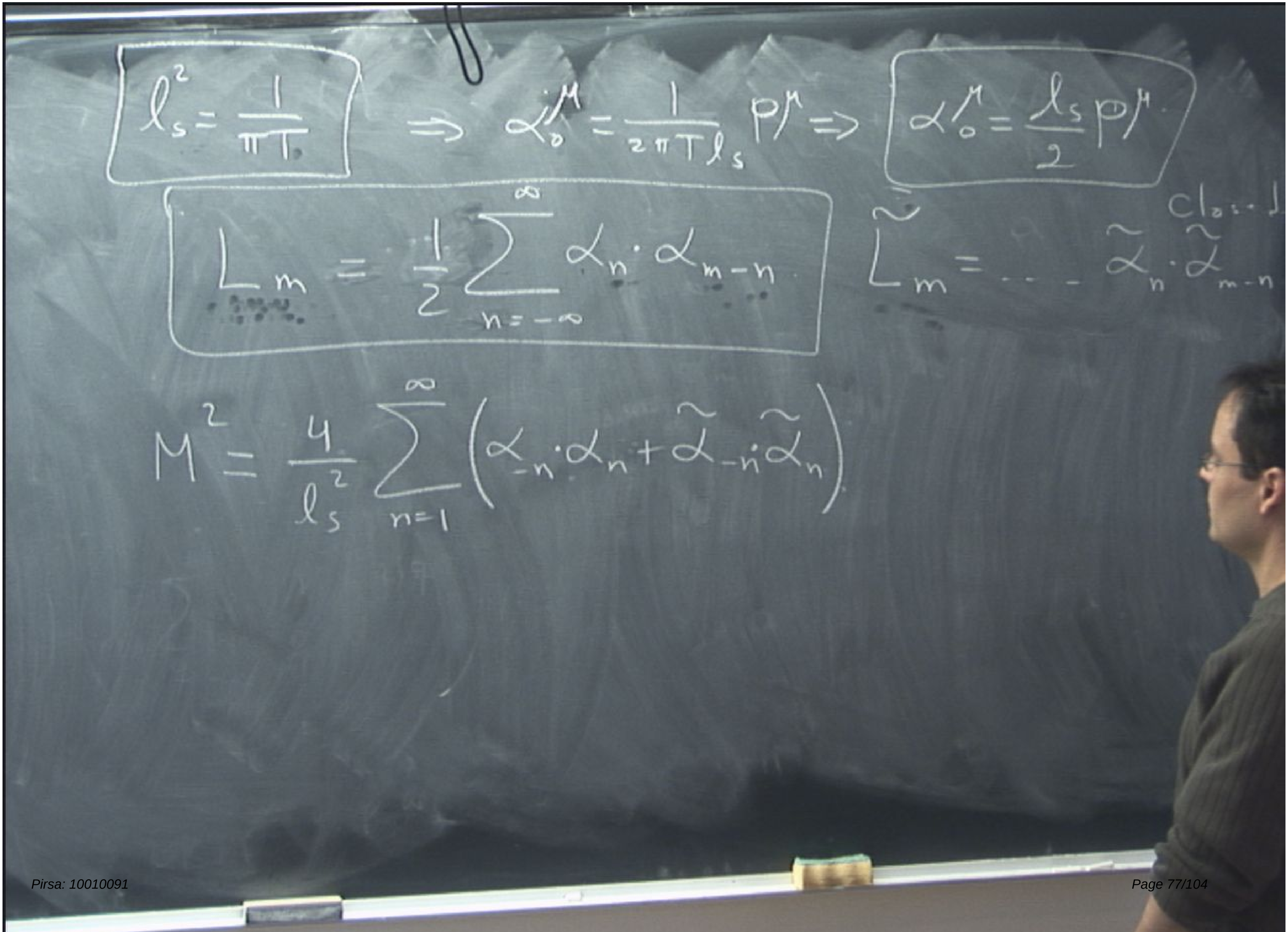
$$T_{00} = \dot{X}^2 + X'^2$$

$$H = \frac{T}{2}\int_0^\pi d\sigma\,(\dot{X}^2 + X'^2) = \overset{\text{ex.}}{} \begin{cases} 2(L_0 + \tilde{L}_0) & \text{clos} \\ L_0 & \text{open.} \end{cases}$$

$$M^2 = -p^2$$

$$L_0 = 0 \qquad \tilde{L}_0 = 0$$

$$L_0 = \underbrace{\frac{1}{2}\alpha_0^2}_{\frac{l_s^2}{8}p^2 \,\to\, -M^2} + \underbrace{\frac{1}{2}\sum_{n\neq 0}\alpha_{-n}\cdot\alpha_n}_{\sum_{n=1}^{\infty}\alpha_{-n}\cdot\alpha_n} = 0$$

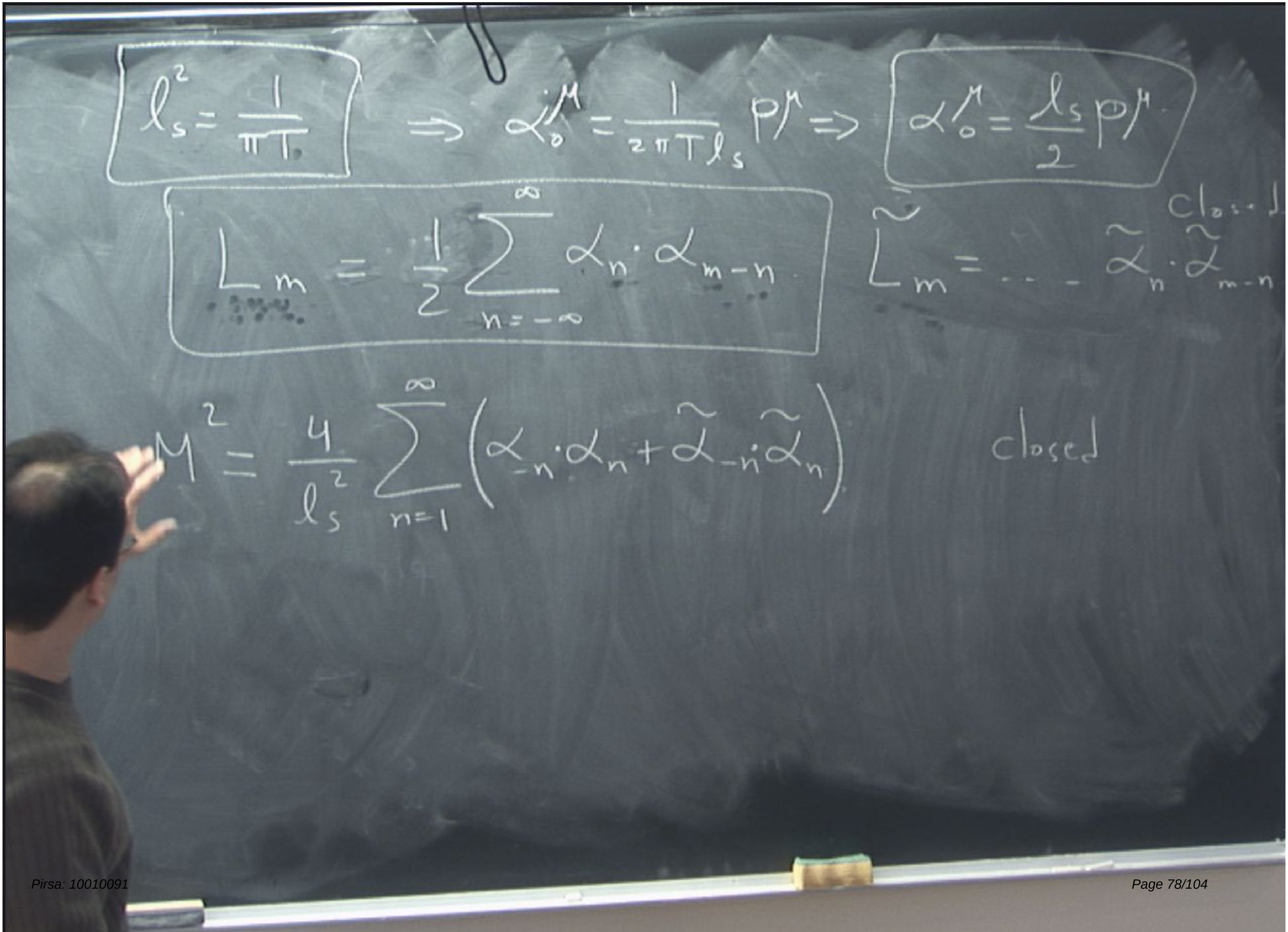


$$\boxed{l_s^2 = \frac{1}{\pi T}} \Rightarrow \alpha_0^\mu = \frac{1}{2\pi T l_s} p^\mu \Rightarrow \boxed{\alpha_0^\mu = \frac{l_s}{2} p^\mu}$$

$$\boxed{L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_n \cdot \alpha_{m-n}} \qquad \tilde{L}_m = \ldots \tilde{\alpha}_n \cdot \tilde{\alpha}_{m-n}$$

Closed

$$M^2 = \frac{4}{l_s^2}\sum_{n=1}^{\infty}\left(\alpha_{-n}\cdot\alpha_n + \tilde{\alpha}_{-n}\cdot\tilde{\alpha}_n\right)$$



$$\boxed{l_s^2 = \frac{1}{\pi T}} \Rightarrow \alpha_0^\mu = \frac{1}{2\pi T l_s} p^\mu \Rightarrow \boxed{\alpha_0^\mu = \frac{l_s p^\mu}{2}}$$

$$\boxed{L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_n \cdot \alpha_{m-n}} \qquad \tilde{L}_m = \ldots \tilde{\alpha}_n \cdot \tilde{\alpha}_{m-n}$$

Closed

$$M^2 = \frac{4}{l_s^2}\sum_{n=1}^{\infty}\left(\alpha_{-n}\cdot\alpha_n + \tilde{\alpha}_{-n}\cdot\tilde{\alpha}_n\right)$$

closed

$$\boxed{l_s^2 = \frac{1}{\pi T}} \Rightarrow \alpha_0^\mu = \frac{1}{2\pi T l_s} p^\mu \Rightarrow \boxed{\alpha_0^\mu = \frac{l_s p^\mu}{2}}$$

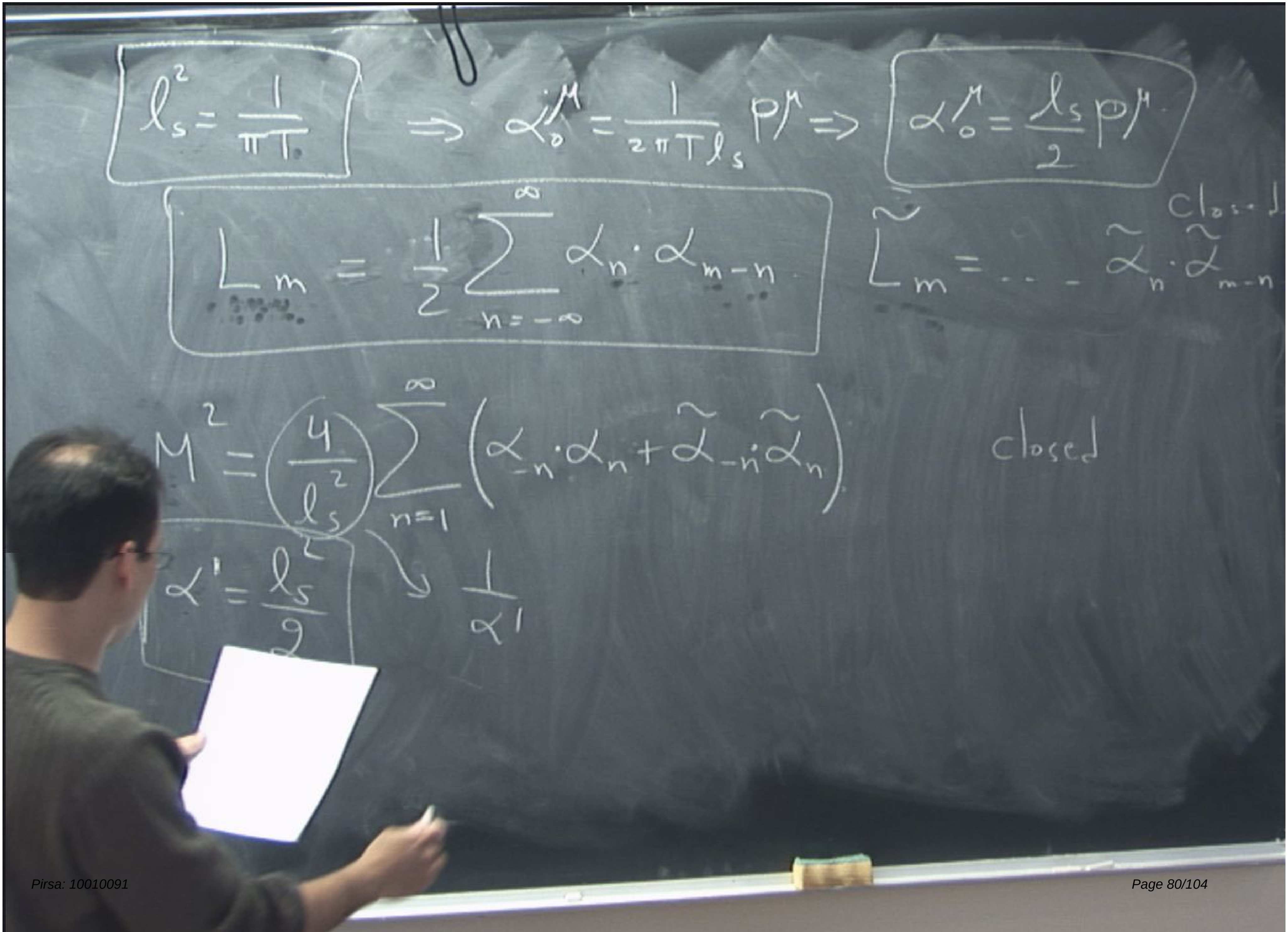
Closed

$$\boxed{L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_n \cdot \alpha_{m-n}} \qquad \tilde{L}_m = \ldots \tilde{\alpha}_n \cdot \tilde{\alpha}_{m-n}$$

$$M^2 = \frac{4}{l_s^2} \sum_{n=1}^{\infty} \left(\alpha_{-n} \cdot \alpha_n + \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n \right)$$

closed

$$\boxed{\alpha' = \frac{l_s^2}{2}}$$



$$\boxed{l_s^2 = \frac{1}{\pi T}} \Rightarrow \alpha_0^\mu = \frac{1}{2\pi T l_s} p^\mu \Rightarrow \boxed{\alpha_0^\mu = \frac{l_s p^\mu}{2}}$$

$$\boxed{L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_n \cdot \alpha_{m-n}}$$

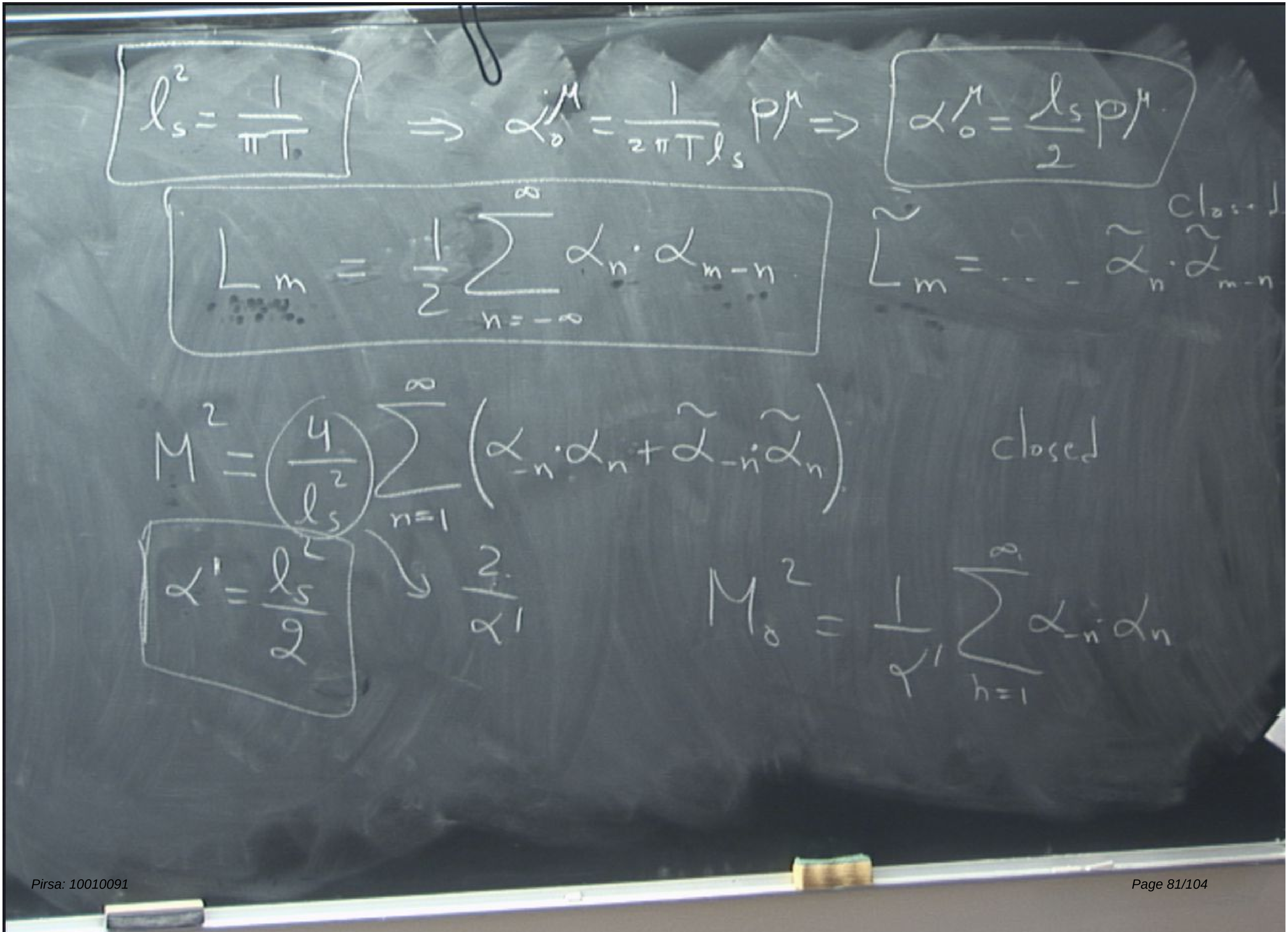
closed

$$\tilde{L}_m = \dots \tilde{\alpha}_n \cdot \tilde{\alpha}_{m-n}$$

$$M^2 = \frac{4}{l_s^2} \sum_{n=1}^{\infty} \left(\alpha_{-n} \cdot \alpha_n + \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n \right)$$

closed

$$\boxed{\alpha' = \frac{l_s^2}{2}} \quad \frac{4}{l_s^2} \rightarrow \frac{1}{\alpha'}$$



$$\boxed{l_s^2 = \frac{1}{\pi T}} \Rightarrow \alpha_0^\mu = \frac{1}{2\pi T l_s} p^\mu \Rightarrow \boxed{\alpha_0^\mu = \frac{l_s p^\mu}{2}}$$

$$\boxed{L_m = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_n\cdot\alpha_{m-n}} \qquad \tilde{L}_m = \ldots\ \tilde{\alpha}_n\cdot\tilde{\alpha}_{m-n}$$

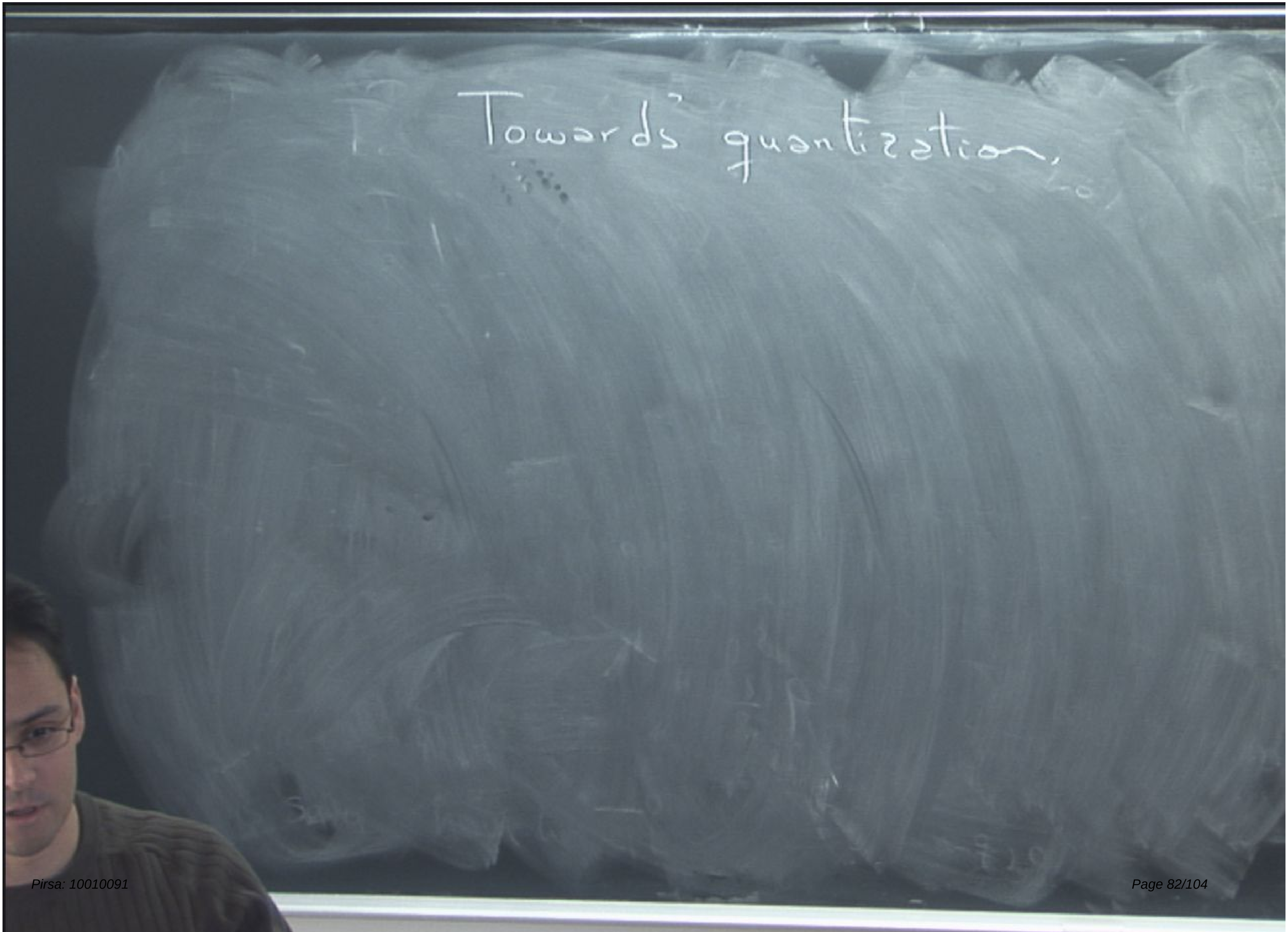
Closed

$$M^2 = \frac{4}{l_s^2}\sum_{n=1}^{\infty}\left(\alpha_{-n}\cdot\alpha_n + \tilde{\alpha}_{-n}\cdot\tilde{\alpha}_n\right)$$

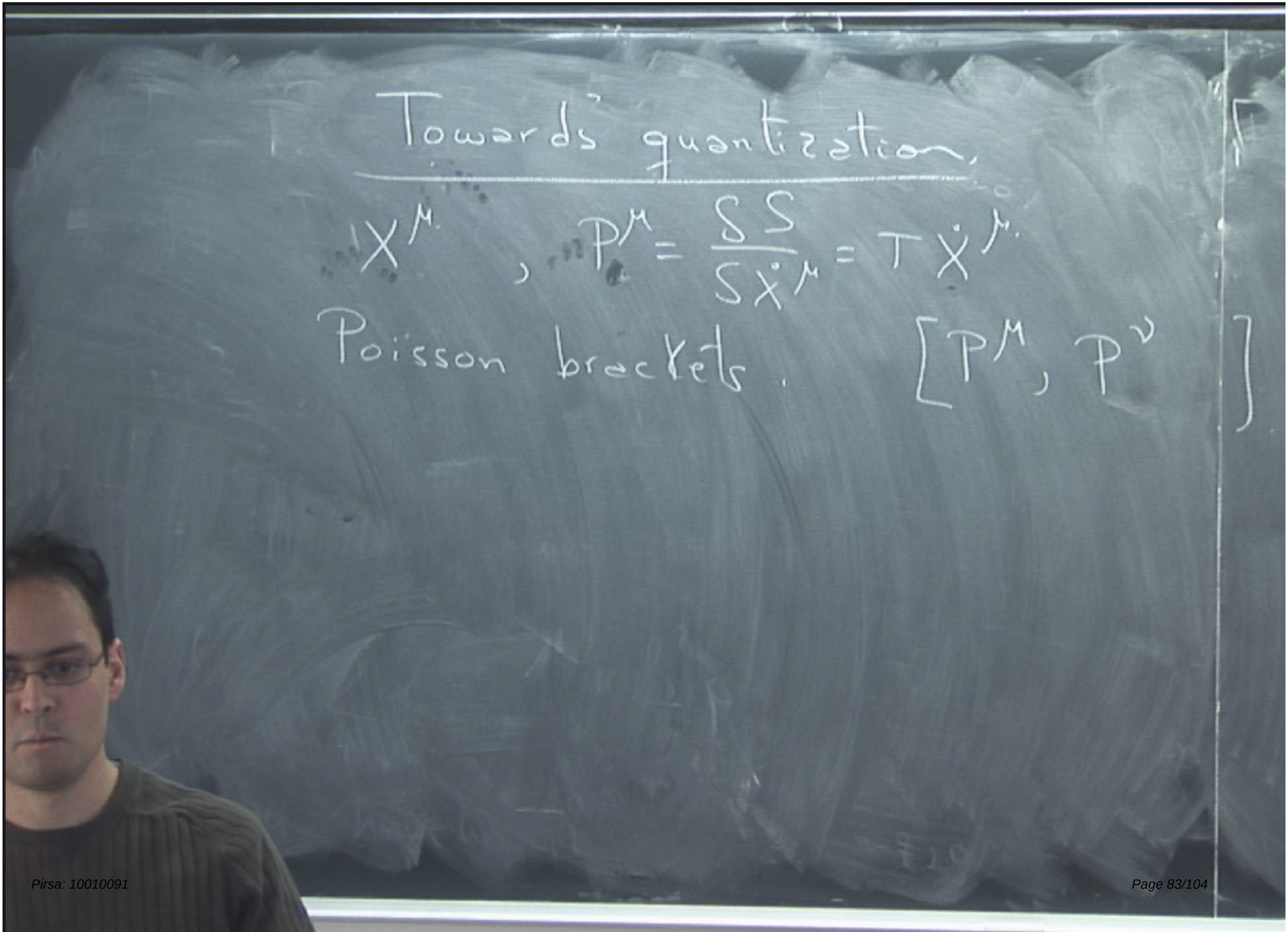
closed

$$\boxed{\alpha' = \frac{l_s^2}{2}} \qquad \frac{4}{l_s^2} \to \frac{2}{\alpha'}$$

$$M_o^2 = \frac{1}{\alpha'}\sum_{n=1}^{\infty}\alpha_{-n}\cdot\alpha_n$$



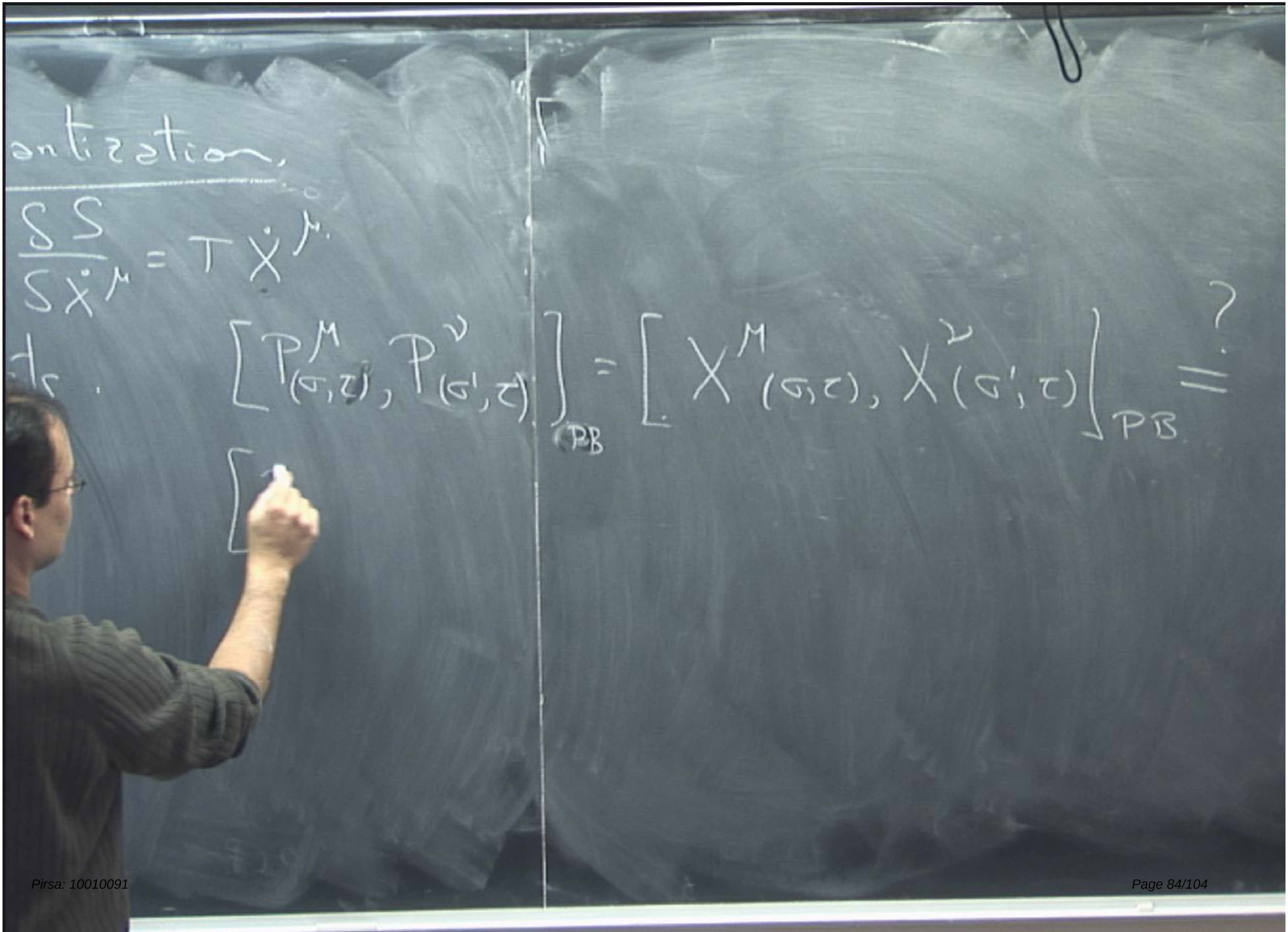
Towards quantization.



Towards quantization

X^μ, $P^\mu = \frac{\delta S}{\delta \dot{X}^\mu} = T \dot{X}^\mu$

Poisson brackets. $[P^\mu, P^\nu \quad]$



antization,

$$\frac{\delta S}{\delta \dot{X}^\mu} = T\dot{X}^\mu.$$

ds .

$$\left[P^\mu_{(\sigma,\tau)}, P^\nu_{(\sigma',\tau)}\right]_{PB} = \left[X^\mu_{(\sigma,\tau)}, X^\nu_{(\sigma',\tau)}\right]_{PB} \overset{?}{=}$$

$[$

ation,

$${}_{\mu} = T\dot{X}^{\mu}.$$

$$\left[P^{\mu}_{(\sigma,\tau)}, P^{\nu}_{(\sigma',\tau)}\right]_{PB} = \left[X^{\mu}_{(\sigma,\tau)}, X^{\nu}_{(\sigma',\tau)}\right]_{PB} \overset{?}{=} 0$$

$$\left[P^{\mu}_{(\sigma,\tau)}, X^{\nu}_{(\sigma',\tau)}\right]_{PB} = \eta^{\mu\nu}$$

ation.

$$_\mu = T\dot{X}^\mu.$$

$$[P^\mu_{(\sigma,\tau)}, P^\nu_{(\sigma',\tau)}]_{PB} = [X^\mu_{(\sigma,\tau)}, X^\nu_{(\sigma',\tau)}]_{PB} \overset{?}{=} 0$$

$$[P^\mu_{(\sigma,\tau)}, X^\nu_{(\sigma',\tau)}]_{PB} = \eta^{\mu\nu}\delta(\sigma-\sigma')$$

ation.

$= T\dot{X}^{\mu}.$

$$[P^{\mu}_{(\sigma,\tau)}, P^{\nu}_{(\sigma',\tau)}]_{PB} = [X^{\mu}_{(\sigma,\tau)}, X^{\nu}_{(\sigma',\tau)}]_{PB} \overset{?}{=} 0$$

$$[P^{\mu}_{(\sigma,\tau)}, X^{\nu}_{(\sigma',\tau)}]_{PB} = \eta^{\mu\nu}\,\delta(\sigma-\sigma')$$

ation,

$${}_\mu = T\dot{X}^\mu.$$

$$\left[P^\mu_{(\sigma,\tau)}, P^\nu_{(\sigma',\tau)}\right]_{PB} = \left[X^\mu_{(\sigma,\tau)}, X^\nu_{(\sigma',\tau)}\right]_{PB} \overset{?}{=} 0$$

$$\left[P^\mu_{(\sigma,\tau)}, X^\nu_{(\sigma',\tau)}\right]_{PB} = \eta^{\mu\nu}\,\delta(\sigma-\sigma')$$

$${}^\nu_n\Big]_{PB} = \left[\tilde{\alpha}^\mu_m, \tilde{\alpha}^\nu_n\right]_{PB} = i\,m\,\eta^{\mu\nu}\,\delta_{m+n,0}$$

uantization,

$$= \frac{\delta S}{\delta \dot{X}^{\mu}} = T \dot{X}^{\mu}$$

kets .

$$[P^{\mu}_{(\sigma,\tau)}, P^{\nu}_{(\sigma',\tau)}]_{PB} = [X^{\mu}_{(\sigma,\tau)}, X^{\nu}_{(\sigma',\tau)}]_{PB} \overset{?}{=}$$

$$[P^{\mu}_{(\sigma,\tau)}, X^{\nu}_{(\sigma',\tau)}]_{PB} = \eta^{\mu\nu} \delta(\sigma-\sigma')$$

$$\alpha^{\mu}_{m}, \alpha^{\nu}_{n}]_{PB} = [\tilde{\alpha}^{\mu}_{m}, \tilde{\alpha}^{\nu}_{n}]_{PB} = i m \eta^{\mu\nu} \delta_{m+n,0}$$

ards' quantization,

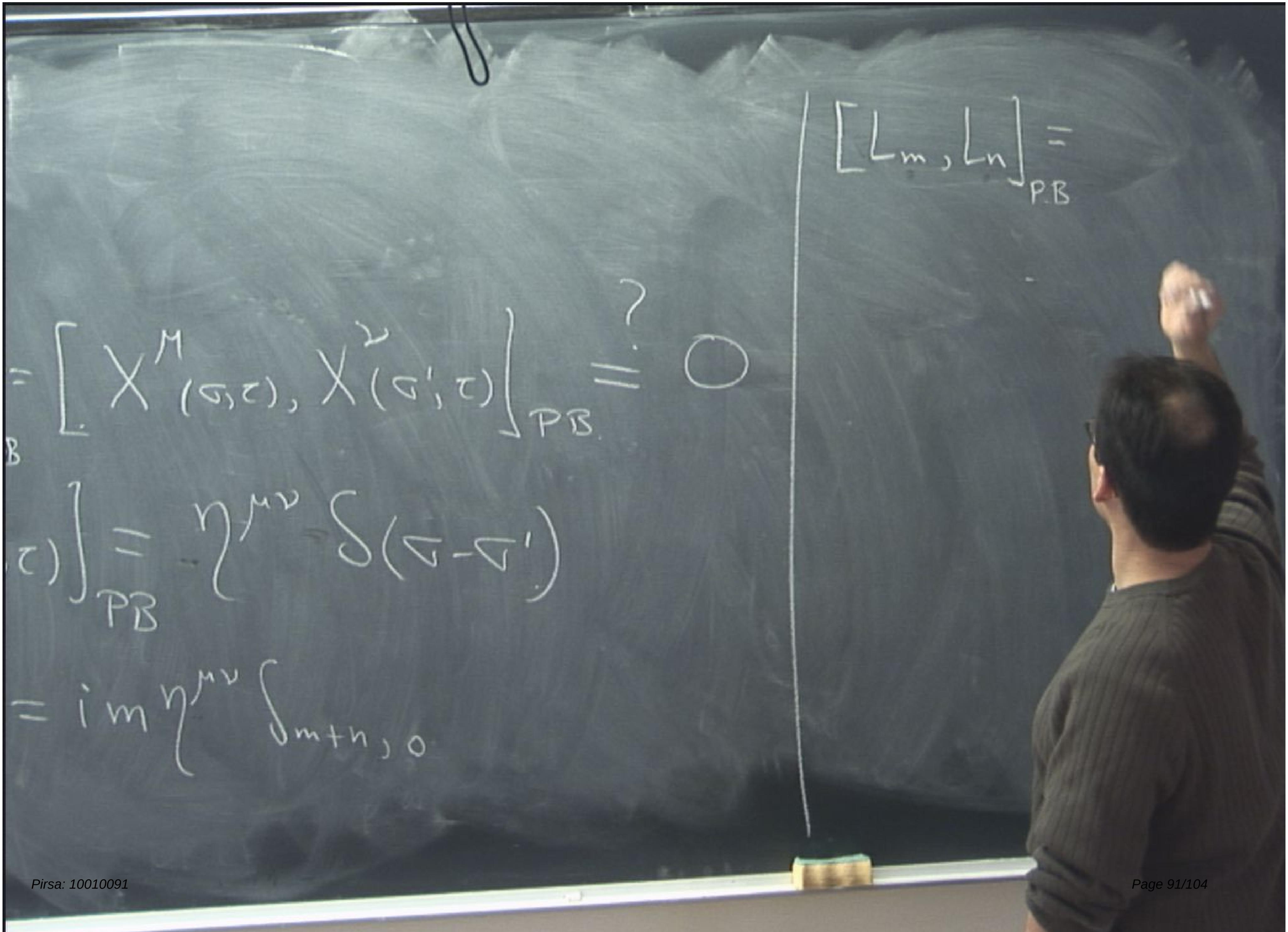
, $P^\mu = \frac{\delta S}{\delta \dot{X}^\mu} = T \dot{X}^\mu$

n brackets : $\left[P^\mu_{(\sigma,\tau)}, P^\nu_{(\sigma',\tau)}\right]_{PB} = \left[X^\mu_{(\sigma,\tau)}, X^\nu_{(\sigma',\tau}\right.$

ne(τ)

$\left[P^\mu_{(\sigma,\tau)}, X^\nu_{(\sigma',\tau)}\right]_{PB} = \eta^{\mu\nu} \delta(\sigma -$

: $\left[\alpha^\mu_m, \alpha^\nu_n\right]_{PB} = \left[\tilde{\alpha}^\mu_m, \tilde{\alpha}^\nu_n\right]_{PB} = i m \eta^{\mu\nu} \delta_{m+n,0}$



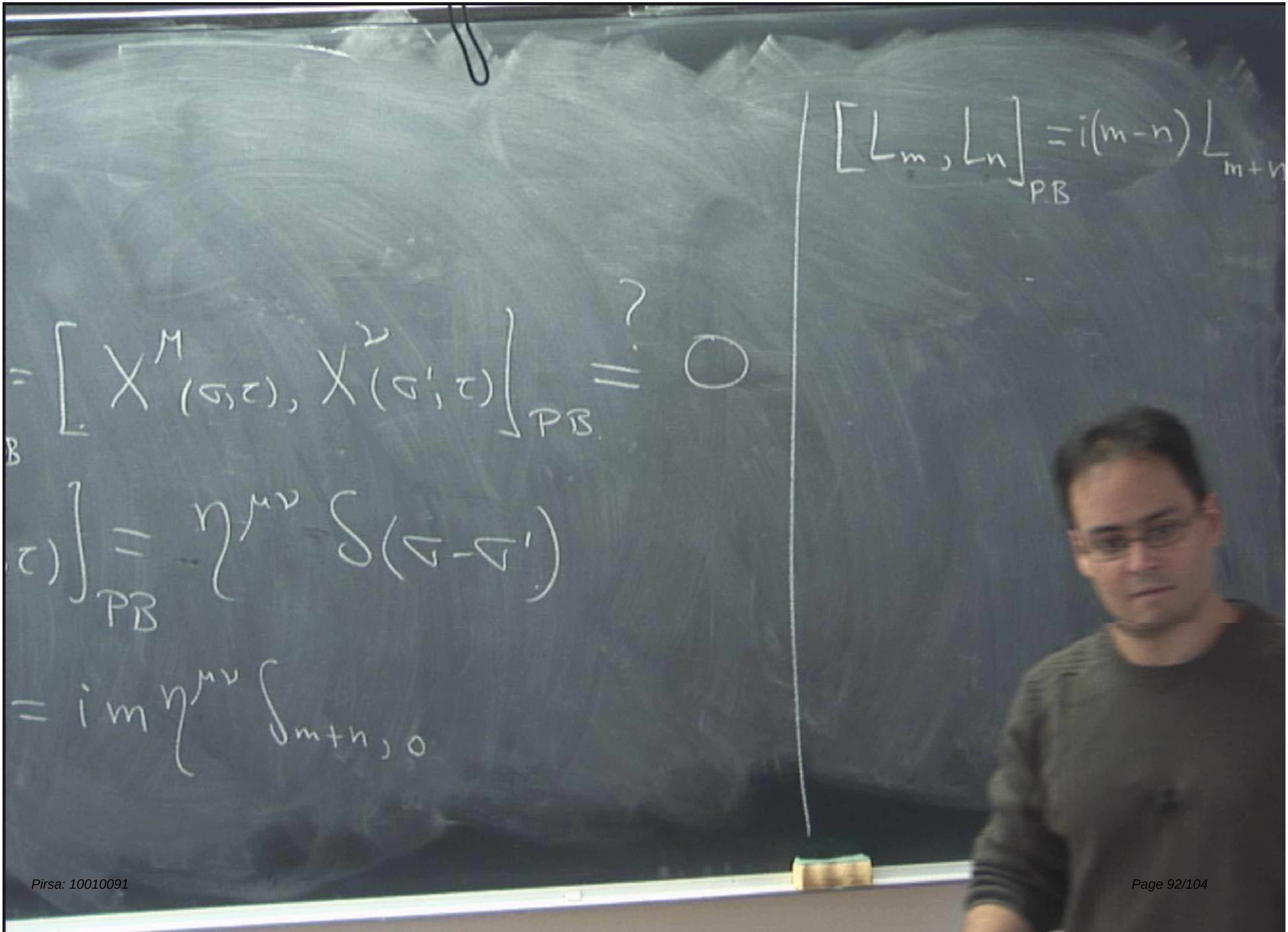
$$= \left[X^{\mu}(\sigma,\tau), X^{\nu}(\sigma',\tau) \right]_{PB} \overset{?}{=} 0$$

B

$$\tau) \Big]_{PB} = \eta^{\mu\nu} \delta(\sigma-\sigma')$$

$$= i m \eta^{\mu\nu} \delta_{m+n,0}$$

$$\left[L_m , L_n \right]_{P.B} =$$



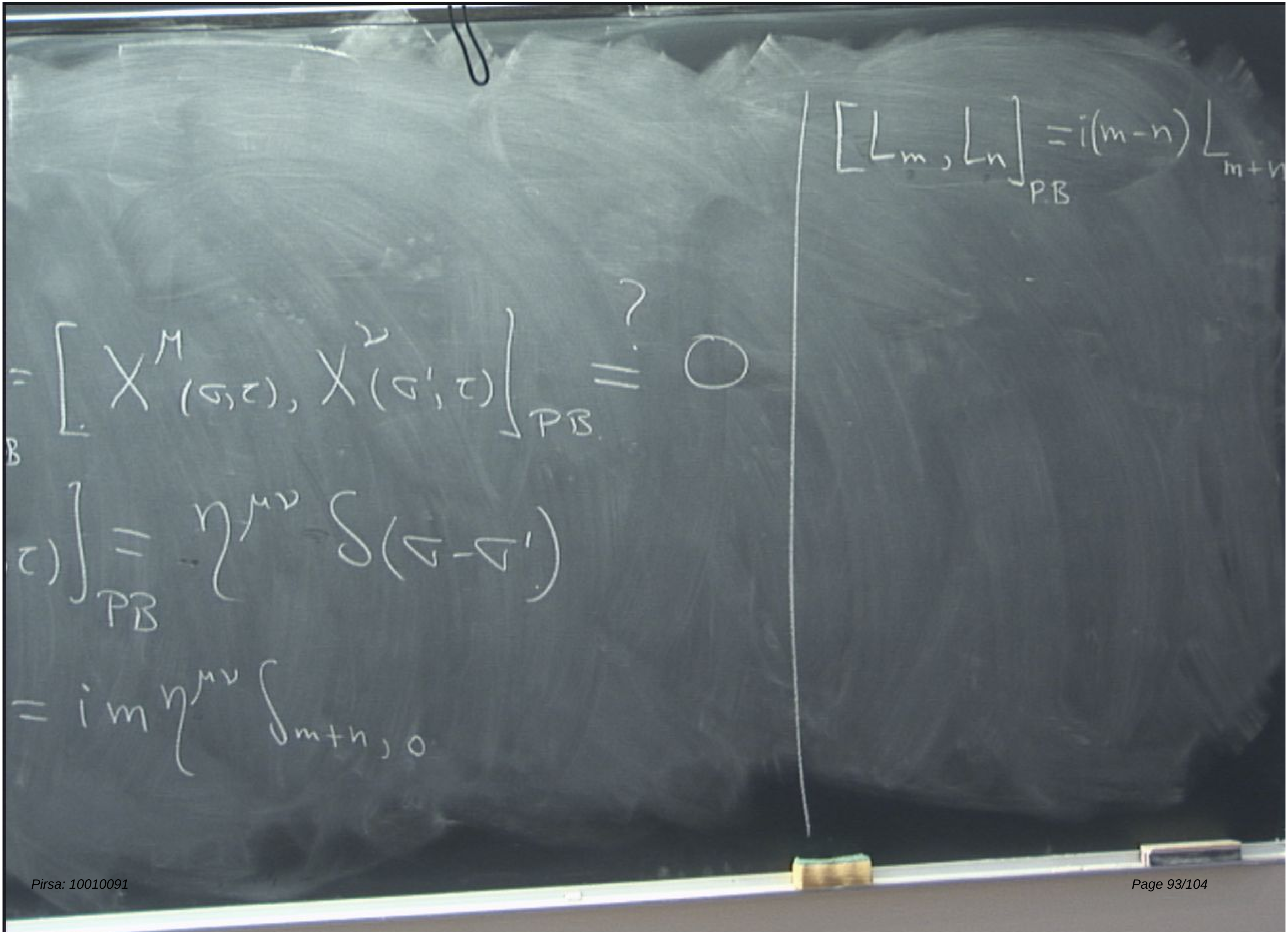
$$= \left[X^{\mu}(\sigma,\tau), X^{\nu}(\sigma',\tau) \right]_{PB} \overset{?}{=} 0$$

$$B$$

$$\tau) \Big]_{PB} = \eta^{\mu\nu} \, \delta(\sigma - \sigma')$$

$$= i m \eta^{\mu\nu} \delta_{m+n,0}$$

$$\left[L_m , L_n \right]_{P.B} = i(m-n) L_{m+n}$$



$$= \left[X^{\mu}(\sigma,\tau), X^{\nu}(\sigma',\tau) \right]_{PB} \overset{?}{=} 0$$

B

$$\tau) \Big]_{PB} = \eta^{\mu\nu} \delta(\sigma - \sigma')$$

$$= i m \eta^{\mu\nu} \delta_{m+n,0}$$

$$\left[L_m , L_n \right]_{P.B} = i(m-n) L_{m+n}$$

$$= \left[X^{\mu}(\sigma,\tau), X^{\nu}(\sigma',\tau) \right]_{PB} \overset{?}{=} 0$$

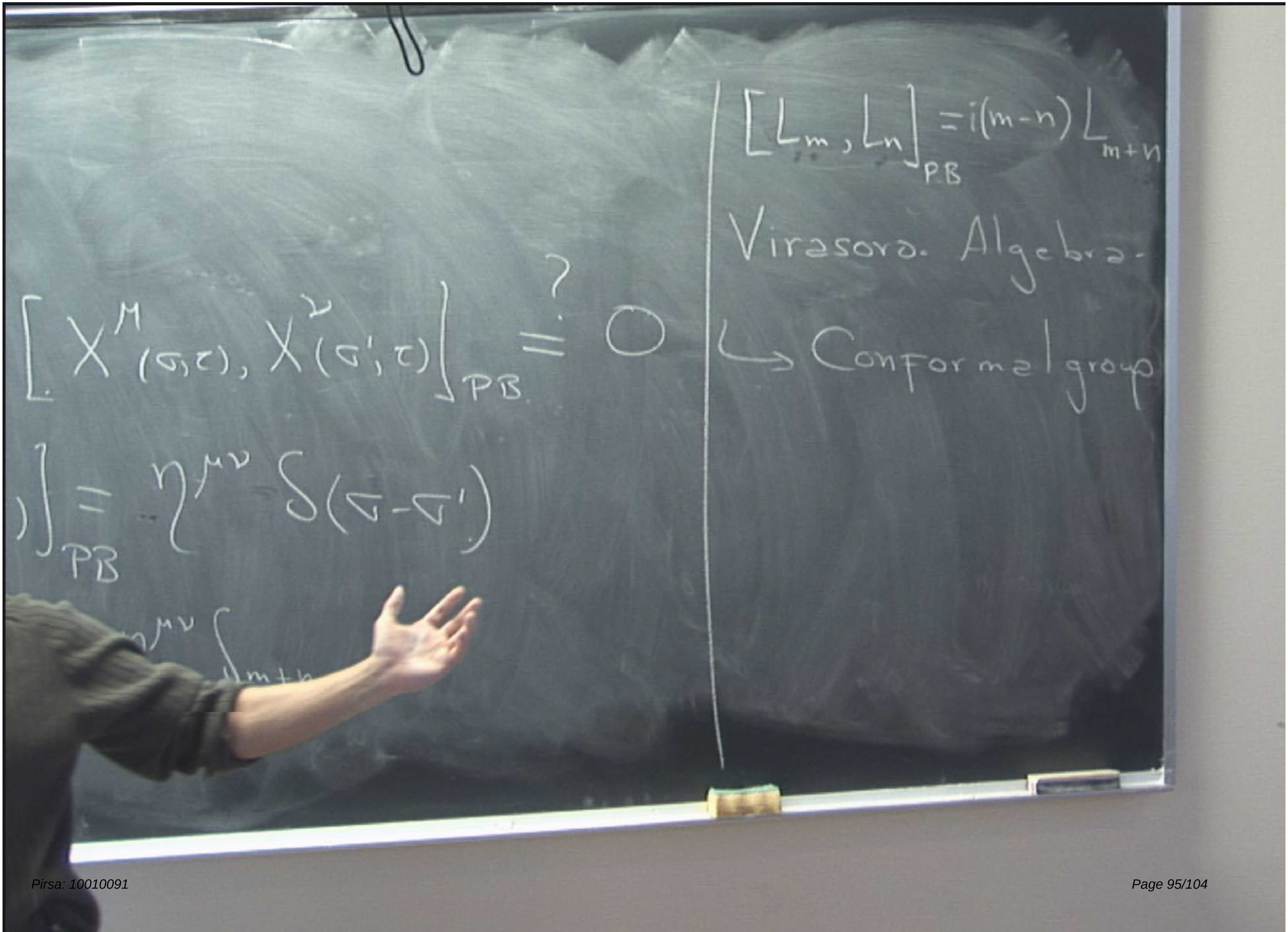
B

$$\tau) \Big]_{PB} = \eta^{\mu\nu} \delta(\sigma-\sigma')$$

$$= i m \eta^{\mu\nu} \delta_{m+n,0}$$

$$\left[L_m, L_n \right]_{P.B} = i(m-n) L_{m+n}$$

Virasoro Algebra.



$$[X^{\mu}(\sigma,\tau), X^{\nu}(\sigma',\tau)]_{PB} \overset{?}{=} 0$$

$$)]_{PB} = \eta^{\mu\nu}\delta(\sigma-\sigma')$$

$$\eta^{\mu\nu}\delta_{m+n}$$

$$[L_m, L_n]_{PB} = i(m-n)L_{m+n}$$

Virasoro Algebra.

↪ Conformal group

$$= \left[X^{\mu}(\sigma,\tau), X^{\nu}(\sigma',\tau) \right]_{PB} \overset{?}{=} 0$$

B

$$\tau) \Big]_{PB} = \eta^{\mu\nu} \delta(\sigma - \sigma')$$

$$= i m \eta^{\mu\nu} \delta_{m+n,0}$$

$$\left[L_m , L_n \right]_{P.B} = i(m-n) L_{m+n}$$

Virasoro Algebra

$\hookrightarrow$ Conformal group

$\hookrightarrow$ Infinite # of generators only in 2D

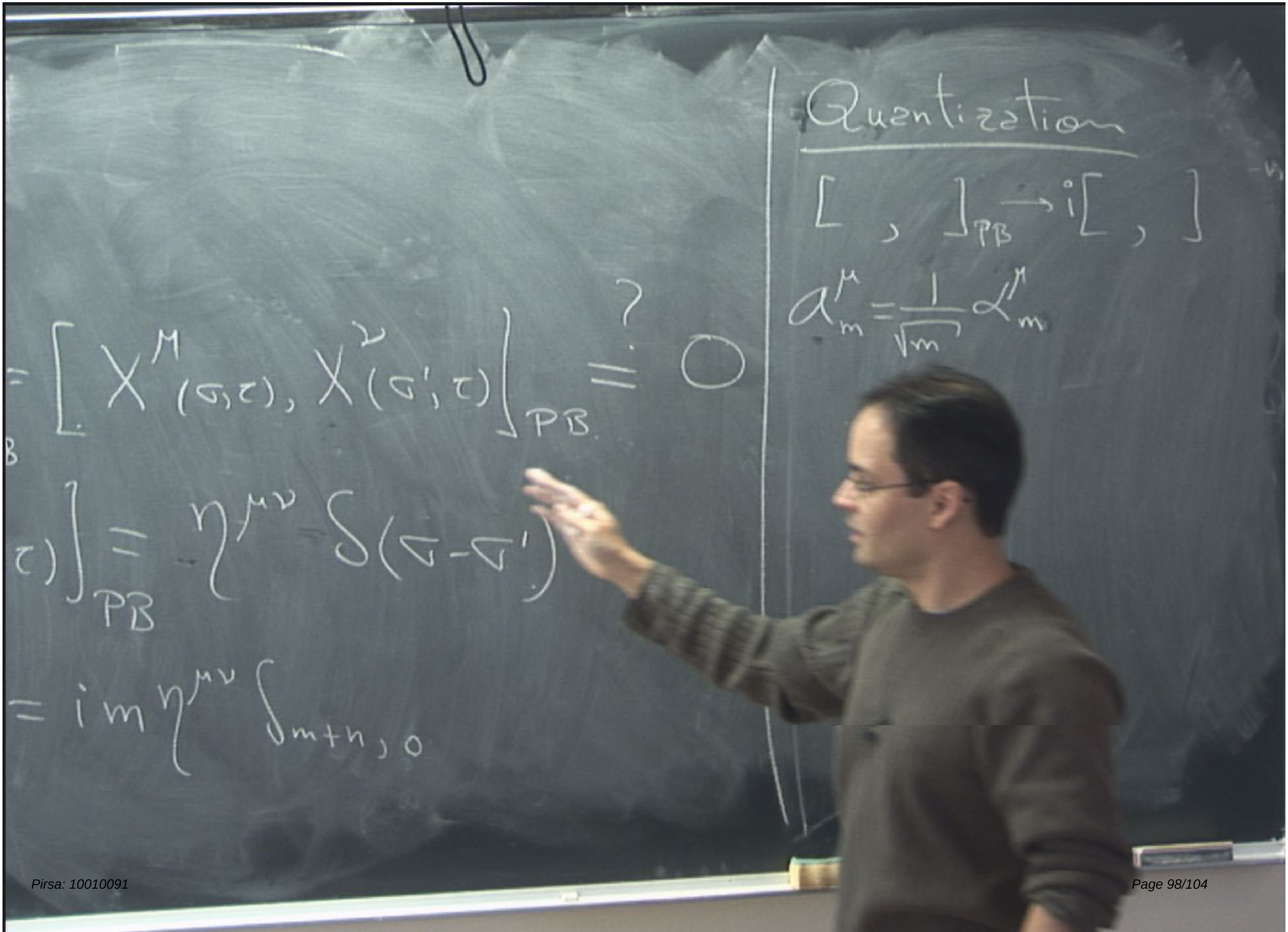
$$= \left[X^{\mu}(\sigma,\tau), X^{\nu}(\sigma',\tau) \right]_{PB} \overset{?}{=} 0$$

$$\tau) \Big]_{PB} = \eta^{\mu\nu} \delta(\sigma - \sigma')$$

$$= i m \eta^{\mu\nu} \delta_{m+n,0}$$

Quantization

$$[\ ,\ ]_{PB} \rightarrow i[\ ,\ ]$$



$$= \left[X^{\mu}(\sigma,\tau), X^{\nu}(\sigma',\tau) \right]_{PB} \overset{?}{=} 0$$

B

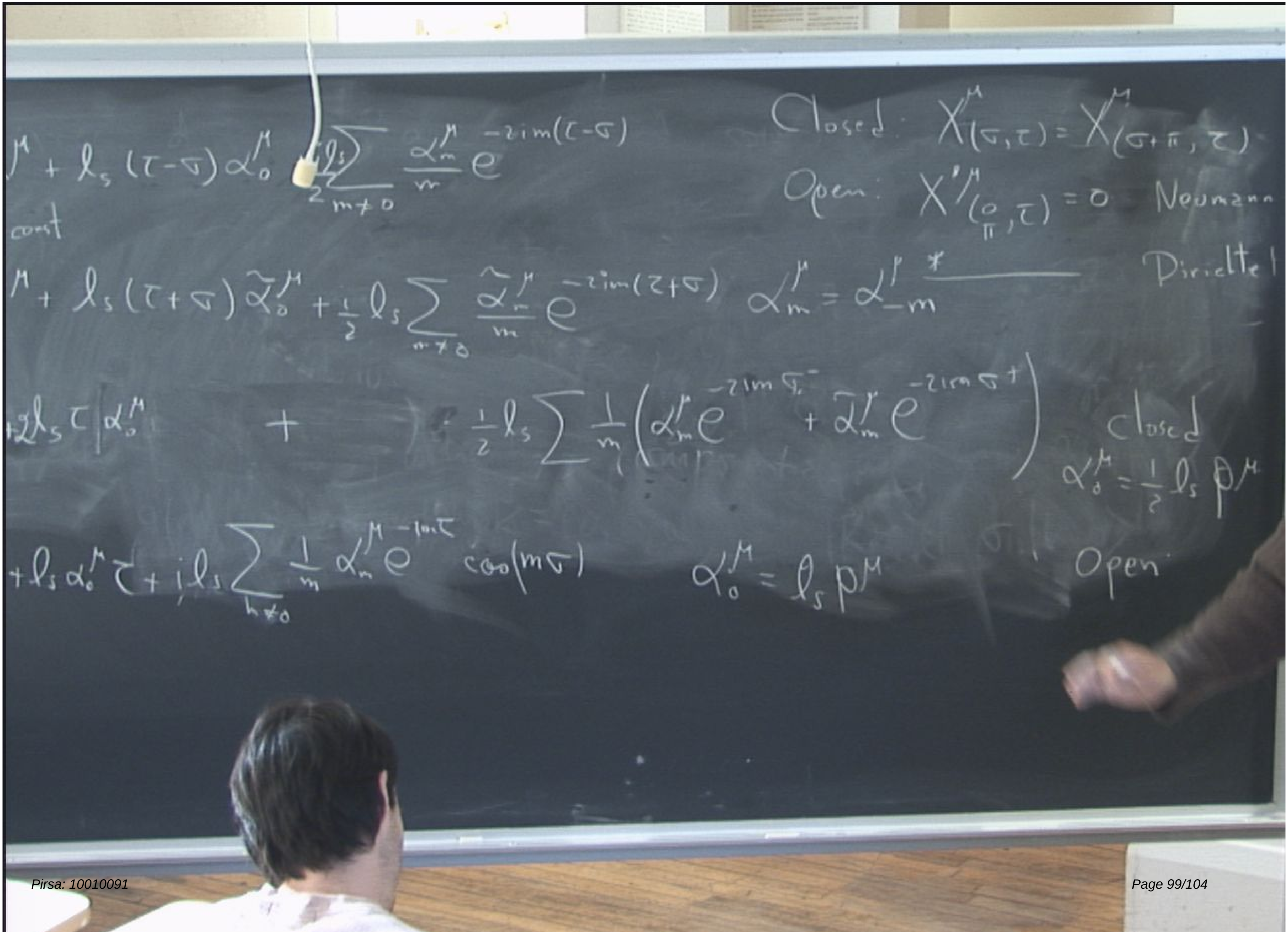
$$\tau) \Big]_{PB} = \eta^{\mu\nu} \delta(\sigma - \sigma')$$

$$= i m \eta^{\mu\nu} \delta_{m+n,0}$$

Quantization

$$[\ ,\ ]_{PB} \rightarrow i[\ ,\ ]$$

$$a^{\mu}_{m} = \frac{1}{\sqrt{m}} \alpha^{\mu}_{m}$$



$$^{\mu} + l_s(\tau-\sigma)\alpha_0^{\mu}\ \frac{i l_s}{2}\sum_{m\neq 0}\frac{\alpha_m^{\mu}}{m}e^{-2im(\tau-\sigma)}$$

const

$$^{\mu} + l_s(\tau+\sigma)\tilde{\alpha}_0^{\mu} + \frac{1}{2}l_s\sum_{m\neq 0}\frac{\tilde{\alpha}_m^{\mu}}{m}e^{-2im(\tau+\sigma)}$$

$$+2l_s\tau|\alpha_0^{\mu} \qquad + \qquad \frac{1}{2}l_s\sum\frac{1}{m}\left(\alpha_m^{\mu}e^{-2im\sigma^-} + \tilde{\alpha}_m^{\mu}e^{-2im\sigma^+}\right)$$

$$+l_s\alpha_0^{\mu}\tau + il_s\sum_{n\neq 0}\frac{1}{m}\alpha_m^{\mu}e^{-im\tau}\cos(m\sigma)$$

Closed: $X^{\mu}(\sigma,\tau) = X^{\mu}(\sigma+\pi,\tau)$

Open: $X'^{\mu}\left({}^{0}_{\pi},\tau\right) = 0$ Neumann

Dirichte

$\alpha_m^{\mu} = \alpha_{-m}^{\mu\,*}$

closed

$\alpha_0^{\mu} = \frac{1}{2}l_s p^{\mu}$

$\alpha_0^{\mu} = l_s p^{\mu}$ Open

$$= \left[X^{\mu}(\sigma,\tau), X^{\nu}(\sigma',\tau)\right]_{PB} \overset{?}{=} 0$$

B

$$\tau)\Big]_{PB} = \eta^{\mu\nu}\,\delta(\sigma-\sigma')$$

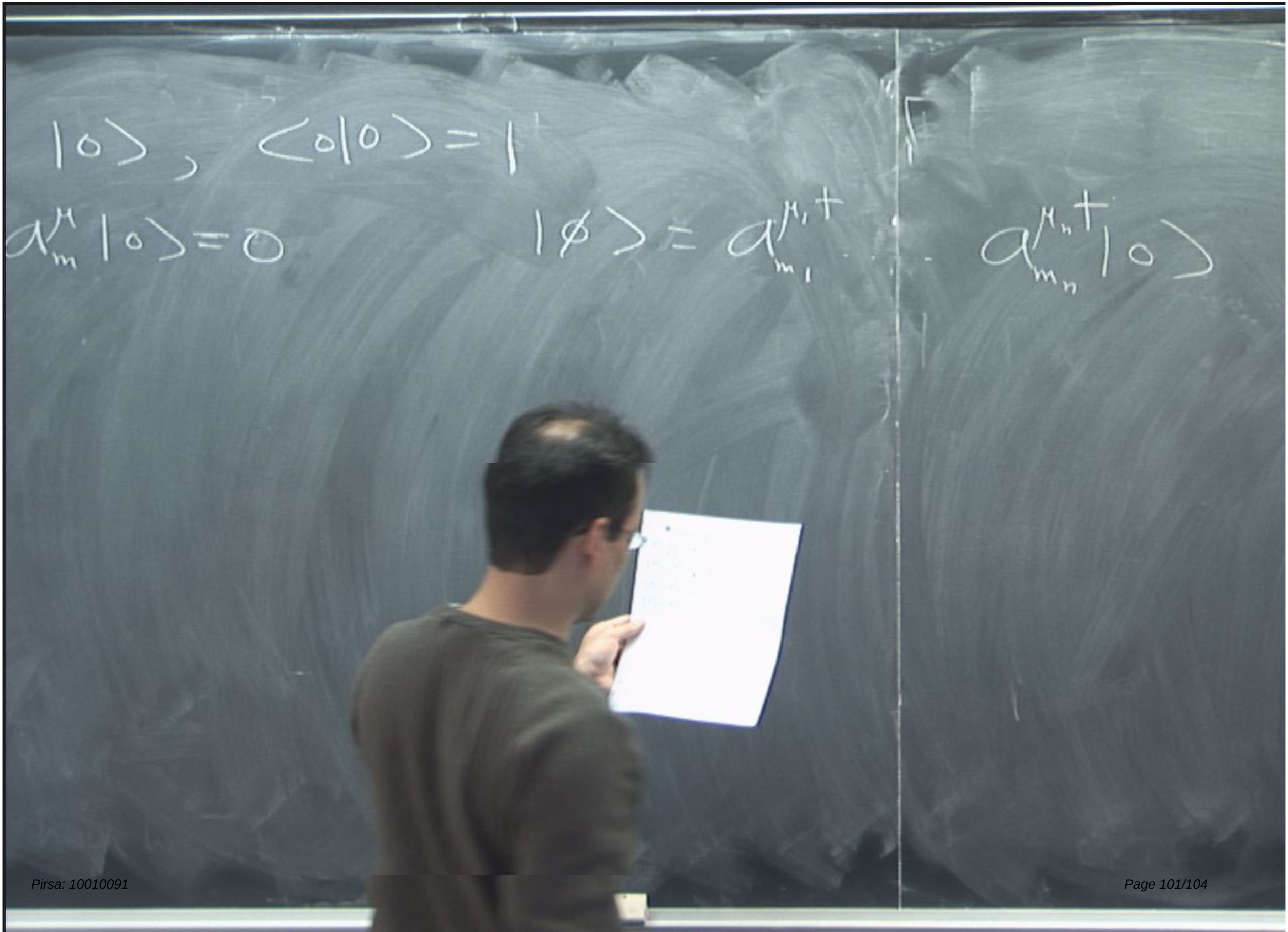
$$= i m \eta^{\mu\nu}\,\delta_{m+n,0}$$

Quantization

$$[\ ,\ ]_{PB} \longrightarrow i[\ ,\ ]$$

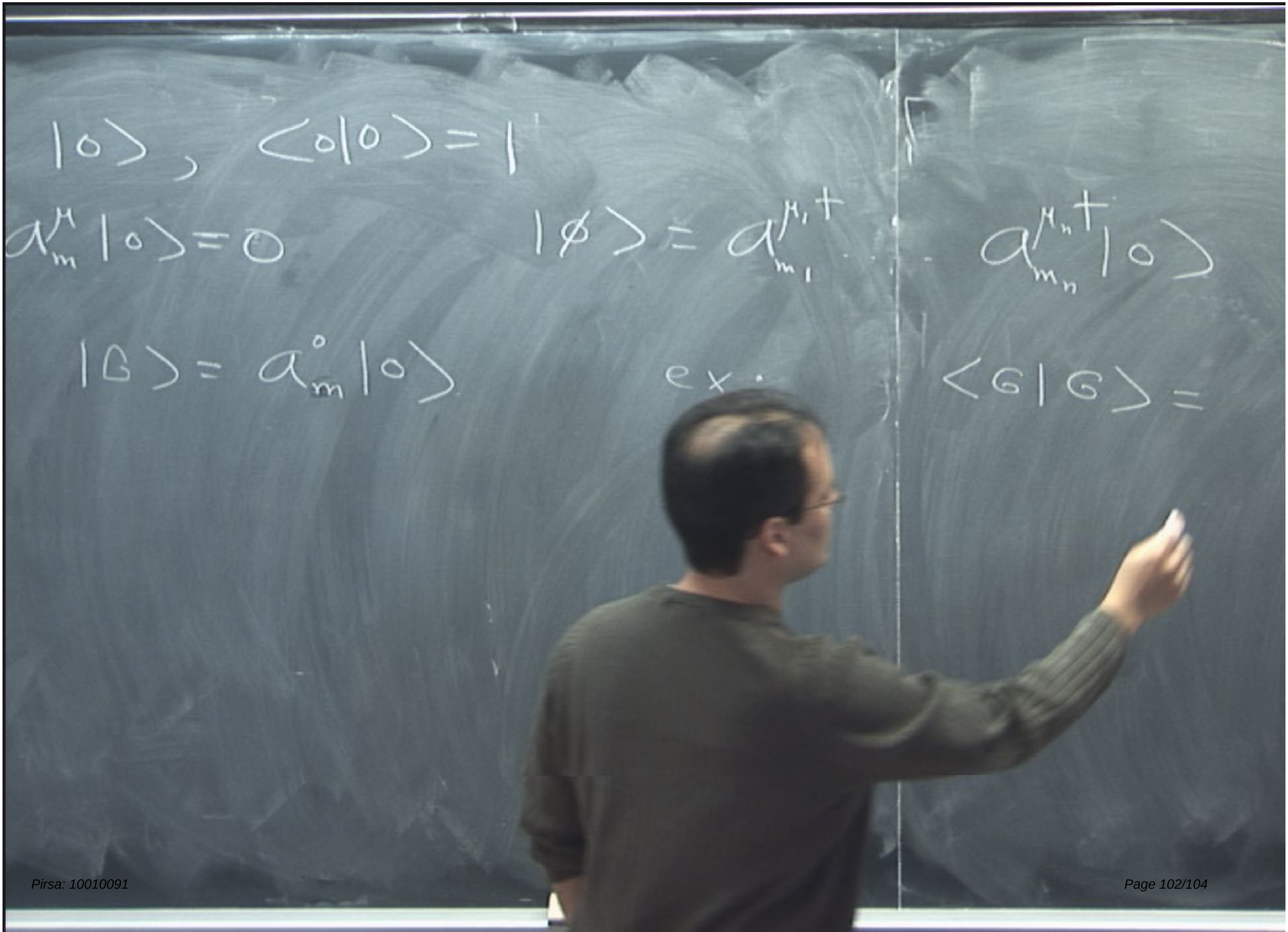
$$a^{\mu}_{m} = \frac{1}{\sqrt{m}}\alpha^{\mu}_{m}$$

$$a^{\mu\dagger}_{m} = \frac{1}{\sqrt{m}}\alpha^{\mu}_{-m} \qquad m>0$$



$$|0\rangle, \quad \langle 0|0\rangle = 1$$

$$a^{\mu}_{m}|0\rangle = 0 \qquad |\phi\rangle = a^{\mu_1\dagger}_{m_1} \cdots a^{\mu_n\dagger}_{m_n}|0\rangle$$



$|0\rangle$, $\langle 0|0\rangle = 1$

$a^{\mu}_{m}|0\rangle = 0$

$|\phi\rangle = a^{\mu_1 \dagger}_{m_1} \cdots a^{\mu_n \dagger}_{m_n} |0\rangle$

$|G\rangle = a^{0}_{m}|0\rangle$

ex.

$\langle G|G\rangle =$

$|0\rangle$, $\langle 0|0\rangle = 1$

$^{\mu}_{m}|0\rangle = 0$ $\qquad$ $|\phi\rangle = a^{\mu_1 \dagger}_{m_1} \cdots a^{\mu_n \dagger}_{m_n}|0\rangle$

$|G\rangle = a^{0}_{m}|0\rangle$ $\qquad$ ex: $\langle G|G\rangle = -1$

↳ Ghost

$|0\rangle$, $\langle 0|0\rangle = 1$

$a^{\mu}_{m}|0\rangle = 0$ $|\phi\rangle = a^{\mu_1 \dagger}_{m_1} \cdots a^{\mu_n \dagger}_{m_n}|0\rangle$

$|G\rangle = a^{0\dagger}_{m}|0\rangle$ ex: $\langle G|G\rangle = -1$

↳ Ghost