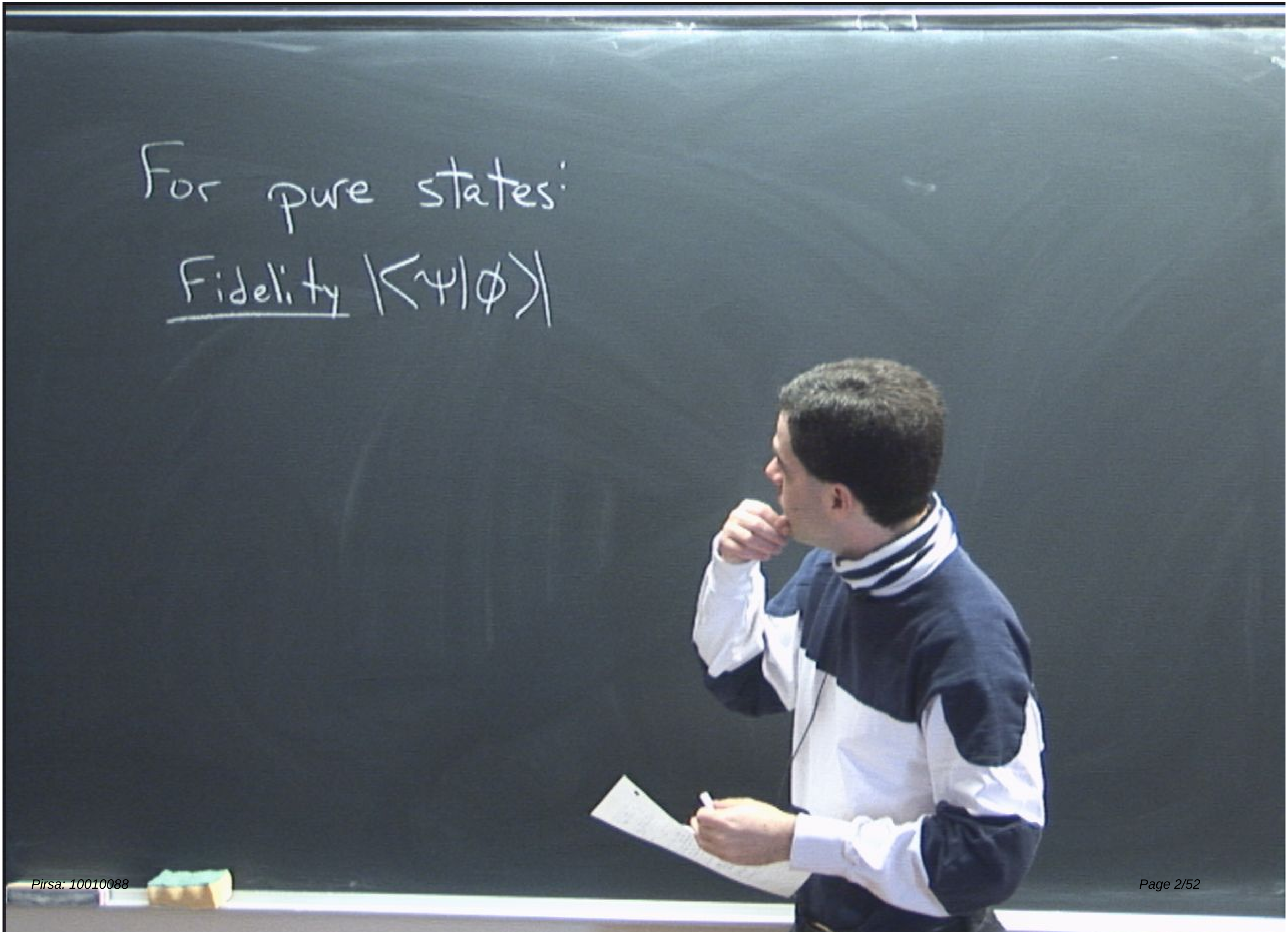
Title: Quantum Information - Review (PHYS 635) - Lecture 5

Date: Jan 29, 2010 09:00 AM

URL: http://pirsa.org/10010088

Abstract: <div id="Cleaner">Week 1: Basic topics (Qubits, quantum gates, quantum circuits, density matrices, quantum operations, entropy, entanglement)<div id="Cleaner"><div id="Cleaner">Week 2: Algorithms and complexity (Languages, complexity classes, oracles, RSA, Deutsch-Jozsa algorithm, Shor's algorithm, Grover's algorithm)<div id="Cleaner"><div id="Cleaner">Week 3: Information theory and implementations (Overview of implementations, quantum error correction, quantum cryptography, quantum information theory)


For pure states:
Fidelity $|\langle\psi|\phi\rangle|$

For pure states:

Fidelity $|\langle\psi|\phi\rangle|$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|$

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|, \; I - |\psi\rangle\langle\psi|$

If state is actually $|\phi\rangle$,

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|, \ I - |\psi\rangle\langle\psi|$

If state is actually $|\phi\rangle$, then prob. of $|\psi\rangle\langle\psi|$ is

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|, \ I - |\psi\rangle\langle\psi|$

If state is actually $|\phi\rangle$, then prob. of $|\psi\rangle\langle\psi|$ is

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|,\ I - |\psi\rangle\langle\psi|$

If state is actually $|\phi\rangle$, then prob. of $|\psi\rangle\langle\psi|$ is F^2

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|,\ I - |\psi\rangle\langle\psi|$

If state is actually $|\phi\rangle$, then prob. of $|\psi\rangle\langle\psi|$ is F^2

Fidelity between $|\psi\rangle$ & ρ.

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|, \ I - |\psi\rangle\langle\psi|$

If state is actually $|\phi\rangle$, then prob. of $|\psi\rangle\langle\psi|$ is F^2

Fidelity between $|\psi\rangle$ & ρ.

$\langle\psi|\rho|\psi\rangle = F^2$

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|, \; I - |\psi\rangle\langle\psi|$

If state is actually $|\phi\rangle$, then prob. of $|\psi\rangle\langle\psi|$ is F^2

Fidelity between $|\psi\rangle$ & ρ.

$\langle\psi|\rho|\psi\rangle = F^2$

$= tr(\rho\, |\psi\rangle\langle\psi|)$

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|, I - |\psi\rangle\langle\psi|$

If state is actually $|\phi\rangle$, then prob. of $|\psi\rangle\langle\psi|$ is F^2

Fidelity between $|\psi\rangle$ & ρ:

$$\langle\psi|\rho|\psi\rangle = F^2$$
$$= \mathrm{tr}(\rho |\psi\rangle\langle\psi|)$$

Fidelity between ρ & σ:

$$F(\rho,\sigma) = \mathrm{tr}\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$$

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|,\ I - |\psi\rangle\langle\psi|$

If state is actually $|\phi\rangle$, then prob. of $|\psi\rangle\langle\psi|$ is F^2

Fidelity between $|\psi\rangle$ & ρ:

$$\langle\psi|\rho|\psi\rangle = F^2$$

$$= tr(\rho\,|\psi\rangle\langle\psi|)$$

Fidelity between ρ & σ:

$$F(\rho,\sigma) = tr\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$$

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|,\ I - |\psi\rangle\langle\psi|$

If state is actually $|\phi\rangle$, then prob. of $|\psi\rangle\langle\psi|$ is F^2

Fidelity between $|\psi\rangle$ & ρ.

$$\langle\psi|\rho|\psi\rangle = F^2$$
$$= \mathrm{tr}(\rho\,|\psi\rangle\langle\psi|)$$

Fidelity between ρ & σ:

$$F(\rho,\sigma) = \mathrm{tr}\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$$

Properties of Fidelity:

① $\rho = \sigma \Leftrightarrow F(\rho,\sigma) = 1$

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|, I - |\psi\rangle\langle\psi|$

If state is actually $|\phi\rangle$, then prob. of $|\psi\rangle\langle\psi|$ is F^2

Fidelity between $|\psi\rangle$ & ρ.

$$\langle\psi|\rho|\psi\rangle = F^2$$
$$= tr(\rho|\psi\rangle\langle\psi|)$$

Fidelity between ρ & σ:

$$F(\rho,\sigma) = tr\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$$

Properties of Fidelity:

1. $\rho = \sigma \Leftrightarrow F(\rho,\sigma) = 1$

$F(\rho,\sigma) = 0 \Leftrightarrow \rho$ & σ have orthogonal support

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|,\ I - |\psi\rangle\langle\psi|$

If state is actually $|\phi\rangle$, then prob. of $|\psi\rangle\langle\psi|$ is F^2

Fidelity between $|\psi\rangle$ & ρ.

$$\langle\psi|\rho|\psi\rangle = F^2$$
$$= tr(\rho|\psi\rangle\langle\psi|)$$

Fidelity between ρ & σ:

$$F(\rho,\sigma) = tr\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$$

Properties of Fidelity:

① $\rho = \sigma \Leftrightarrow F(\rho,\sigma) = 1$

$F(\rho,\sigma) = 0 \Leftrightarrow \rho$ & σ have orthogonal support

② $F(\rho,\sigma) = F(\sigma,\rho)$

For pure states:

Fidelity $|\langle\psi|\phi\rangle| = F$

0 means $|\psi\rangle, |\phi\rangle$ are $\perp$

1 means $|\psi\rangle = |\phi\rangle$ (up to phase)

If we want to know if state is $|\psi\rangle$, make measurement

$|\psi\rangle\langle\psi|, \ I - |\psi\rangle\langle\psi|$

If state is actually $|\phi\rangle$, then prob. of $|\psi\rangle\langle\psi|$ is F^2

Fidelity between $|\psi\rangle$ & ρ.

$$\langle\psi|\rho|\psi\rangle = F^2$$
$$= tr(\rho|\psi\rangle\langle\psi|)$$

Fidelity between ρ & σ:

$$F(\rho,\sigma) = tr\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$$

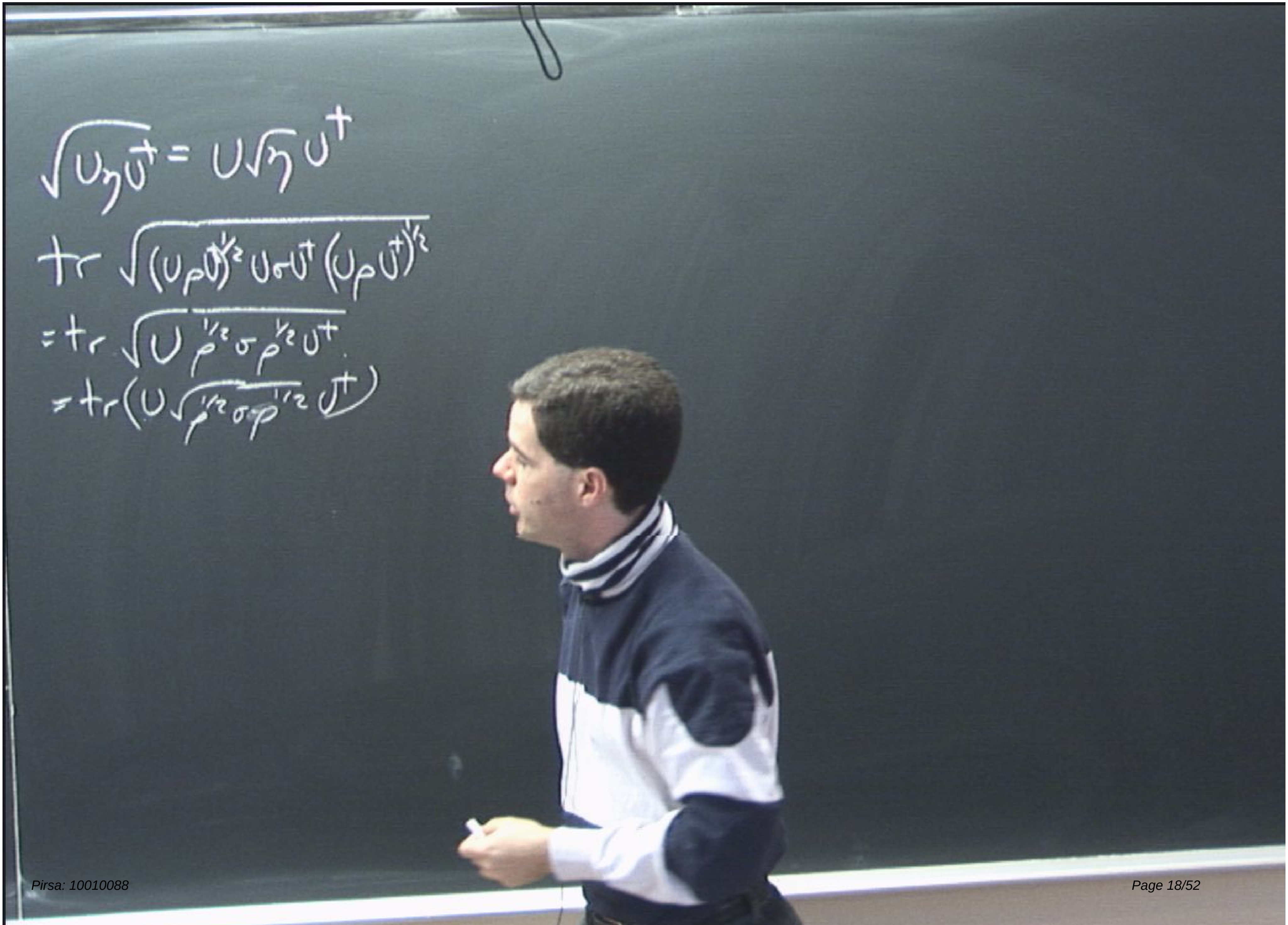
Properties of Fidelity:

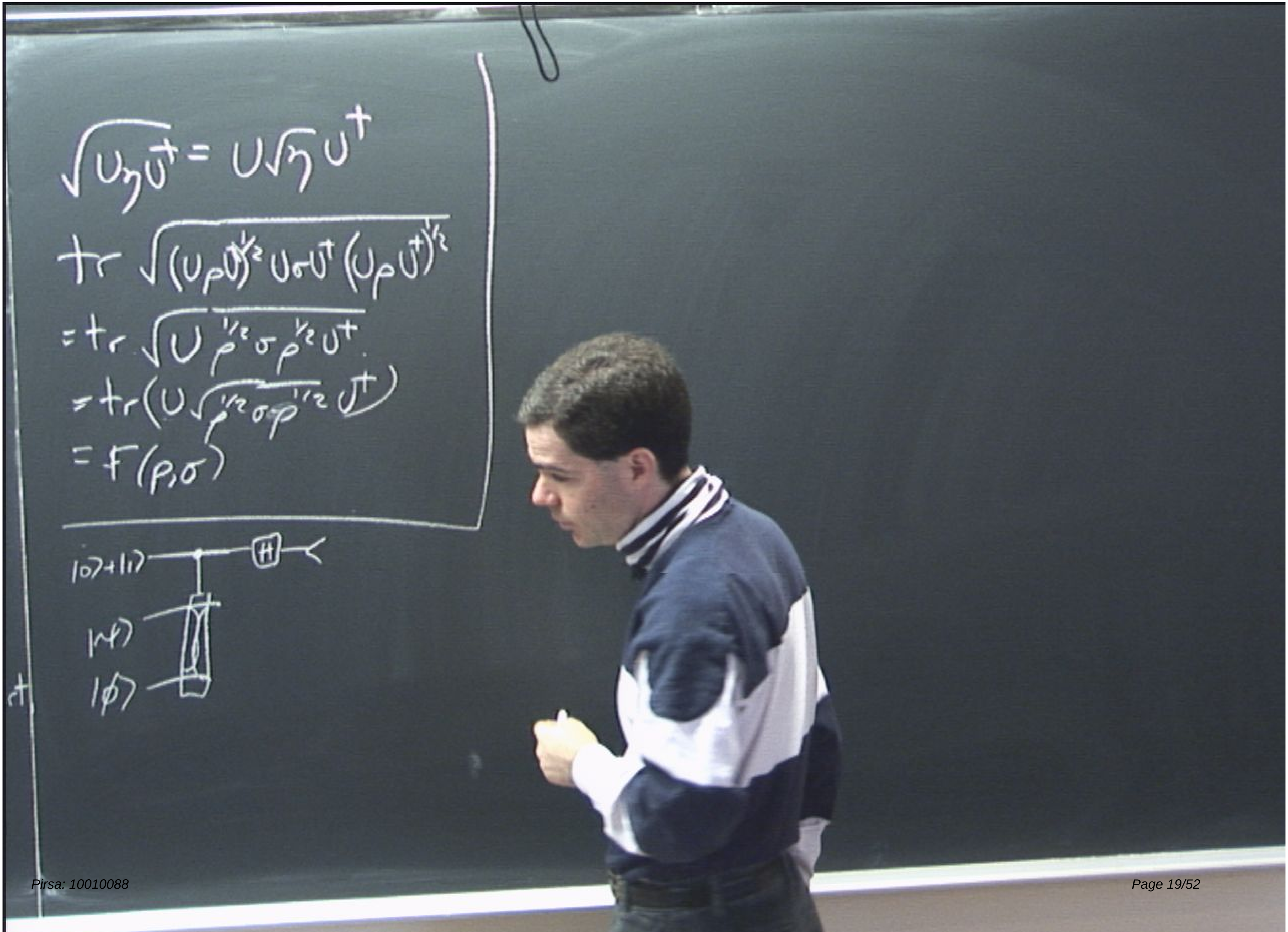
① $\rho = \sigma \Leftrightarrow F(\rho,\sigma) = 1$

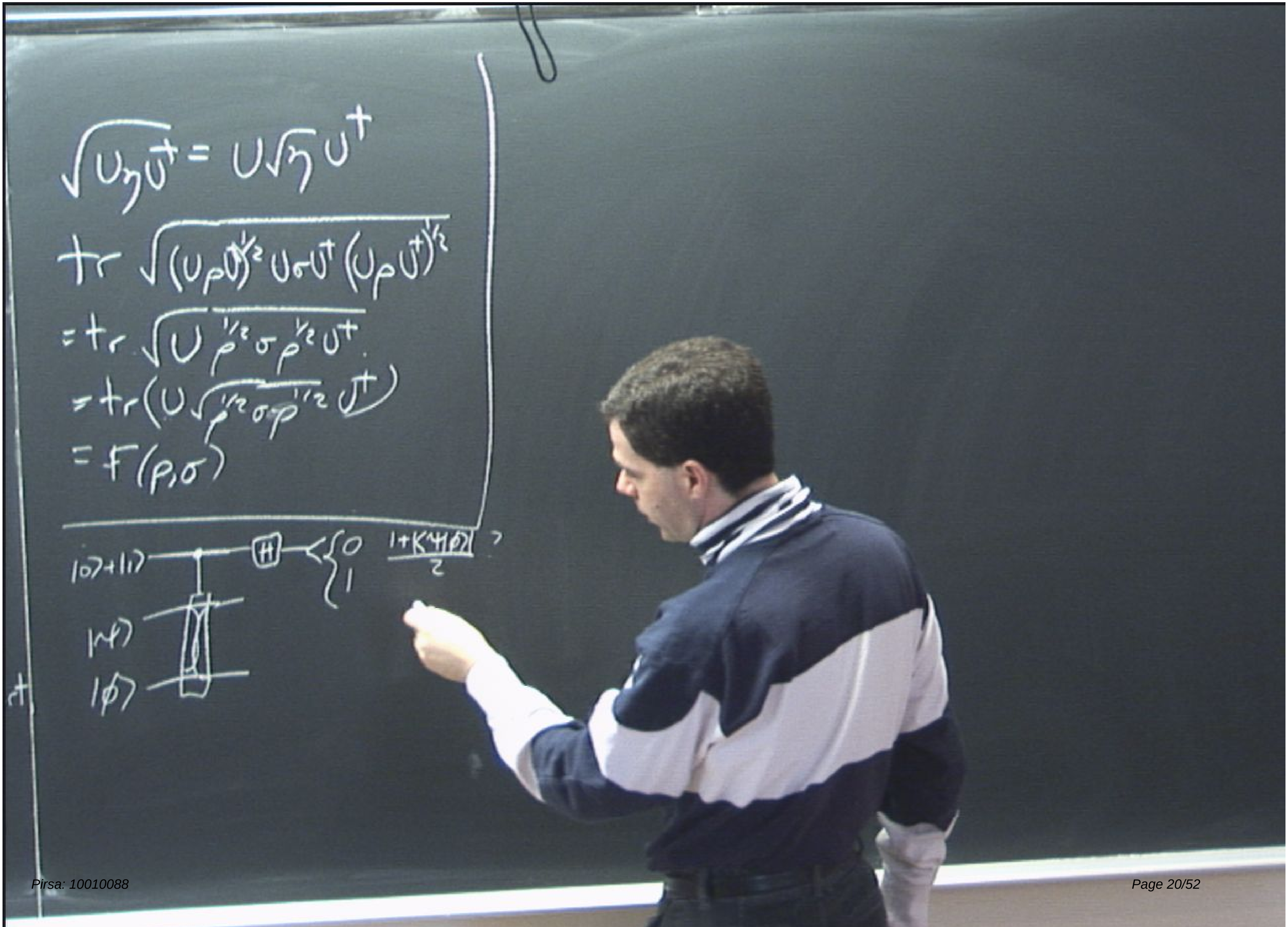
$F(\rho,\sigma) = 0 \Leftrightarrow \rho$ & σ have orthogonal support

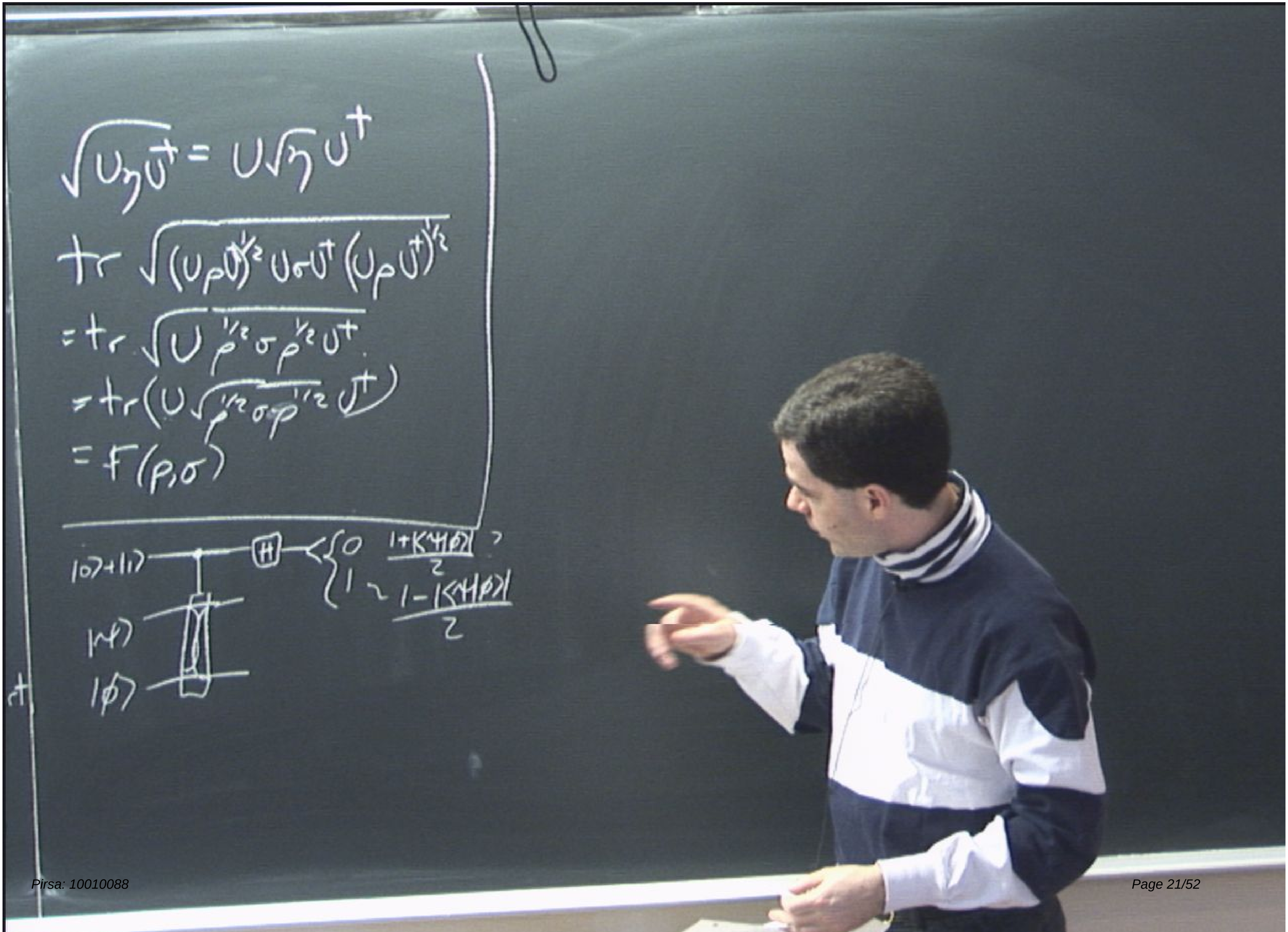
② $F(\rho,\sigma) = F(\sigma,\rho)$

③ $F(U\rho U^\dagger, U\sigma U^\dagger) = F(\rho,\sigma)$


$\sqrt{U\eta U^\dagger} = U\sqrt{\eta}\,U^\dagger$
$\mathrm{tr}\sqrt{(U\rho U^\dagger)^{1/2}\,U\sigma U^\dagger\,(U\rho U^\dagger)^{1/2}}$
$= \mathrm{tr}\sqrt{U\rho^{1/2}\sigma\rho^{1/2}U^\dagger}$
$= \mathrm{tr}\left(U\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\,U^\dagger\right)$


$\sqrt{U\eta U^\dagger} = U\sqrt{\eta}\,U^\dagger$
$\mathrm{tr}\sqrt{(U\rho U^\dagger)^{1/2}\,U\sigma U^\dagger\,(U\rho U^\dagger)^{1/2}}$
$= \mathrm{tr}\sqrt{U\rho^{1/2}\sigma\rho^{1/2}U^\dagger}$
$= \mathrm{tr}\left(U\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\,U^\dagger\right)$
$= F(\rho,\sigma)$
$|0\rangle+|1\rangle$
H
$|\psi\rangle$
$|\phi\rangle$


$\sqrt{U\eta U^\dagger} = U\sqrt{\eta}\,U^\dagger$
$\mathrm{tr}\sqrt{(U\rho U^\dagger)^{1/2}\,U\sigma U^\dagger\,(U\rho U^\dagger)^{1/2}}$
$= \mathrm{tr}\sqrt{U\rho^{1/2}\sigma\rho^{1/2}U^\dagger}$
$= \mathrm{tr}(U\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\,U^\dagger)$
$= F(\rho,\sigma)$
$|0\rangle+|1\rangle$
H
0
1
$\frac{1+|\langle\psi|\phi\rangle|^2}{2}$?
$|\psi\rangle$
$|\phi\rangle$


$\sqrt{U\eta U^\dagger} = U\sqrt{\eta}\,U^\dagger$
$\mathrm{tr}\sqrt{(U\rho U^\dagger)^{1/2}\,U\sigma U^\dagger\,(U\rho U^\dagger)^{1/2}}$
$= \mathrm{tr}\sqrt{U\rho^{1/2}\sigma\rho^{1/2}U^\dagger}$
$= \mathrm{tr}\left(U\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\,U^\dagger\right)$
$= F(\rho,\sigma)$
$|0\rangle+|1\rangle$
H
$|\psi\rangle$
$|\phi\rangle$
$0 \quad \frac{1+|\langle\psi|\phi\rangle|}{2}$?
$1 \sim \frac{1-|\langle\psi|\phi\rangle|}{2}$

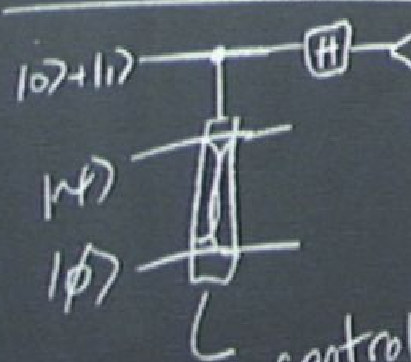
$$\sqrt{U\eta U^\dagger} = U\sqrt{\eta}\,U^\dagger$$

$$\mathrm{tr}\sqrt{(U\rho U^\dagger)^{1/2}\,U\sigma U^\dagger\,(U\rho U^\dagger)^{1/2}}$$

$$= \mathrm{tr}\sqrt{U\rho^{1/2}\sigma\rho^{1/2}U^\dagger}$$

$$= \mathrm{tr}\left(U\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\,U^\dagger\right)$$

$$= F(\rho,\sigma)$$



$$\begin{cases} 0 & \frac{1+|\langle\psi|\phi\rangle|}{2} \\ 1 & \frac{1-|\langle\psi|\phi\rangle|}{2} \end{cases}$$

controlled-SWAP: If control 0, do nothing
If control 1, SWAP qubits.

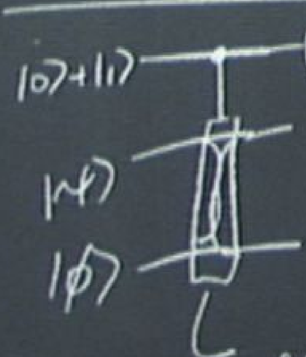
$$\sqrt{U\eta U^\dagger} = U\sqrt{\eta}\,U^\dagger$$

$$\mathrm{tr}\sqrt{(U\rho U^\dagger)^{1/2}\,U\sigma U^\dagger\,(U\rho U^\dagger)^{1/2}}$$

$$= \mathrm{tr}\sqrt{U\rho^{1/2}\sigma\rho^{1/2}U^\dagger}$$

$$= \mathrm{tr}\left(U\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\,U^\dagger\right)$$

$$= F(\rho,\sigma)$$



$$\begin{cases} 0 & \dfrac{1+|\langle\psi|\phi\rangle|}{2} \\ 1 \sim & \dfrac{1-|\langle\psi|\phi\rangle|}{2} \end{cases}$$

controlled-SWAP: If control 0, do nothing. If control 1, SWAP qubits.

Thm. (Uhlmann's thm.):

$$F(\rho,\sigma) = \max_{|\psi\rangle|\phi\rangle}$$

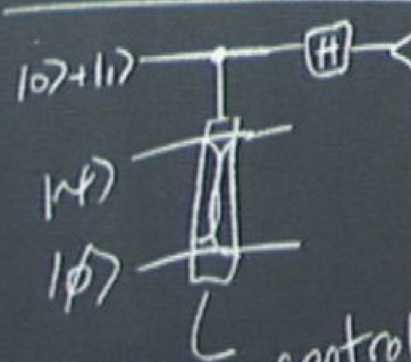
$$\sqrt{U\eta U^\dagger} = U\sqrt{\eta}\,U^\dagger$$

$$\operatorname{tr}\sqrt{(U\rho U^\dagger)^{1/2}\,U\sigma U^\dagger\,(U\rho U^\dagger)^{1/2}}$$

$$= \operatorname{tr}\sqrt{U\rho^{1/2}\sigma\rho^{1/2}U^\dagger}$$

$$= \operatorname{tr}\left(U\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\,U^\dagger\right)$$

$$= F(\rho,\sigma)$$



$$\begin{cases} 0 & \frac{1+|\langle\psi|\phi\rangle|}{2} \\ 1 & \frac{1-|\langle\psi|\phi\rangle|}{2} \end{cases}$$

controlled-SWAP: If control 0, do nothing
If control 1, SWAP qubits.

Thm. (Uhlmann's thm.):

$$F(\rho,\sigma) = \max_{|\psi\rangle|\phi\rangle} |\langle\psi|\phi\rangle|$$

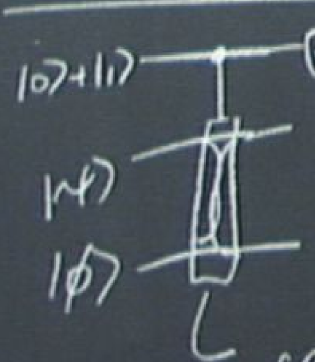
$$\sqrt{U\eta U^\dagger} = U\sqrt{\eta}\,U^\dagger$$

$$\mathrm{tr}\sqrt{(U\rho U^\dagger)^{1/2}\, U\sigma U^\dagger\, (U\rho U^\dagger)^{1/2}}$$

$$= \mathrm{tr}\sqrt{U\rho^{1/2}\sigma\rho^{1/2}U^\dagger}$$

$$= \mathrm{tr}\left(U\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\,U^\dagger\right)$$

$$= F(\rho,\sigma)$$



$$\begin{cases} 0 & \dfrac{1+|\langle\psi|\phi\rangle|}{2} \; ? \\ 1 & \sim \dfrac{1-|\langle\psi|\phi\rangle|}{2} \end{cases}$$

controlled-SWAP: If control 0, do nothing
If control 1, SWAP qubits.

Thm. (Uhlmann's thm.):

$$F(\rho,\sigma) = \max_{|\psi\rangle|\phi\rangle\in} |\langle\psi|\phi\rangle|$$

$\mathrm{tr}\ |\psi\rangle\langle\psi|$

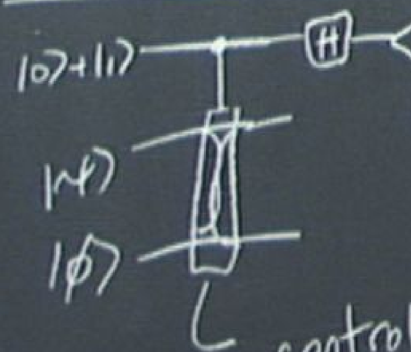
$$\sqrt{U\eta U^\dagger} = U\sqrt{\eta}\,U^\dagger$$

$$\mathrm{tr}\sqrt{(U\rho U^\dagger)^{1/2}\, U\sigma U^\dagger\, (U\rho U^\dagger)^{1/2}}$$
$$= \mathrm{tr}\sqrt{U\rho^{1/2}\sigma\rho^{1/2}U^\dagger}$$
$$= \mathrm{tr}\left(U\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\,U^\dagger\right)$$
$$= F(\rho,\sigma)$$

Thm. (Uhlmann's thm):

$$F(\rho,\sigma) = \max |\langle\psi|\phi\rangle|$$

where ρ, σ act on system Q, $|\psi\rangle, |\phi\rangle$ are states on $R\otimes Q$, $\mathrm{tr}_R |\psi\rangle\langle\psi| = \rho$, $\mathrm{tr}_R |\phi\rangle\langle\phi| = \sigma$



$$\begin{cases} 0 & \frac{1+|\langle\psi|\phi\rangle|}{2} \\ 1 & \sim \frac{1-|\langle\psi|\phi\rangle|}{2} \end{cases}$$

controlled-SWAP: If control 0, do nothing
If control 1, SWAP qubits.

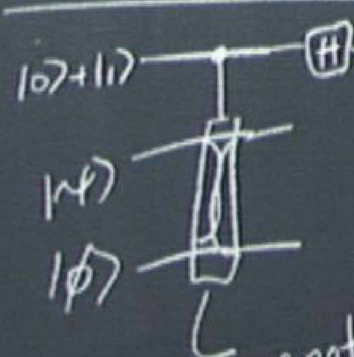
$$\sqrt{U\eta U^\dagger} = U\sqrt{\eta}\,U^\dagger$$

$$\mathrm{tr}\sqrt{(U\rho U^\dagger)^{1/2}\,U\sigma U^\dagger\,(U\rho U^\dagger)^{1/2}}$$

$$= \mathrm{tr}\sqrt{U\rho^{1/2}\sigma\rho^{1/2}U^\dagger}$$

$$= \mathrm{tr}\left(U\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\,U^\dagger\right)$$

$$= F(\rho,\sigma)$$



$$\left\{\begin{array}{ll} 0 & \frac{1+|\langle\psi|\phi\rangle|}{2} \\ 1 & \frac{1-|\langle\psi|\phi\rangle|}{2} \end{array}\right.$$

controlled-SWAP: If control 0, do nothing
If control 1, SWAP qubits.

Thm. (Uhlmann's thm):

$$F(\rho,\sigma) = \max |\langle\psi|\phi\rangle|$$

where ρ,σ act on system Q, $|\psi\rangle, |\phi\rangle$ are states on $R\otimes Q$, $\mathrm{tr}_R|\psi\rangle\langle\psi| = \rho$, $\mathrm{tr}_R|\phi\rangle\langle\phi| = \sigma$.

Fidelity between ρ & σ is maximum achievable fidelity between purificatio[ns] of ρ & σ.

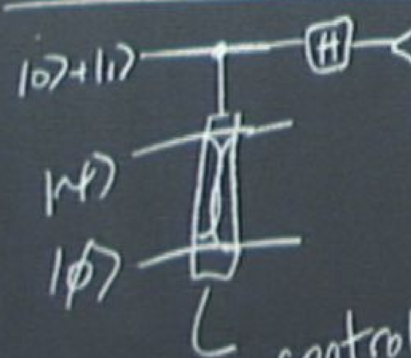
$$\sqrt{U\eta U^\dagger} = U\sqrt{\eta}\,U^\dagger$$

$$\mathrm{tr}\sqrt{(U\rho U^\dagger)^{1/2}\,U\sigma U^\dagger\,(U\rho U^\dagger)^{1/2}}$$

$$= \mathrm{tr}\sqrt{U\rho^{1/2}\sigma\rho^{1/2}U^\dagger}$$

$$= \mathrm{tr}\left(U\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\,U^\dagger\right)$$

$$= F(\rho,\sigma)$$



$$\begin{cases} 0 & \frac{1+|\langle\psi|\phi\rangle|}{2} \\ 1 & \frac{1-|\langle\psi|\phi\rangle|}{2} \end{cases}$$

controlled-SWAP: If control 0, do nothing
If control 1, SWAP qubits.

Thm. (Uhlmann's thm.):

$$F(\rho,\sigma) = \max |\langle\psi|\phi\rangle|$$

where ρ,σ act on system Q, $|\psi\rangle, |\phi\rangle$ are states on $R\otimes Q$, $\mathrm{tr}_R|\psi\rangle\langle\psi| = \rho$, $\mathrm{tr}_R|\phi\rangle\langle\phi| = \sigma$.

Fidelity between ρ & σ is maximum achievable fidelity between purificatio[ns] of ρ & σ.

$$\sqrt{U\eta U^\dagger} = U\sqrt{\eta}\,U^\dagger$$

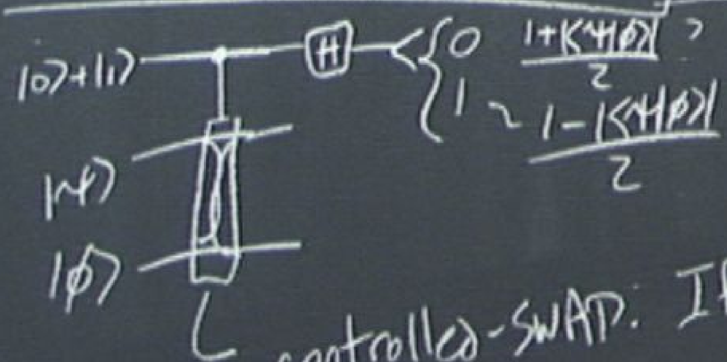
$$\mathrm{tr}\sqrt{(U\rho U^\dagger)^{1/2}\, U\sigma U^\dagger\, (U\rho U^\dagger)^{1/2}}$$
$$= \mathrm{tr}\sqrt{U\rho^{1/2}\sigma\rho^{1/2}U^\dagger}$$
$$= \mathrm{tr}\left(U\sqrt{\rho^{1/2}\sigma\rho^{1/2}}\,U^\dagger\right)$$
$$= F(\rho,\sigma)$$

Thm. (Uhlmann's thm):

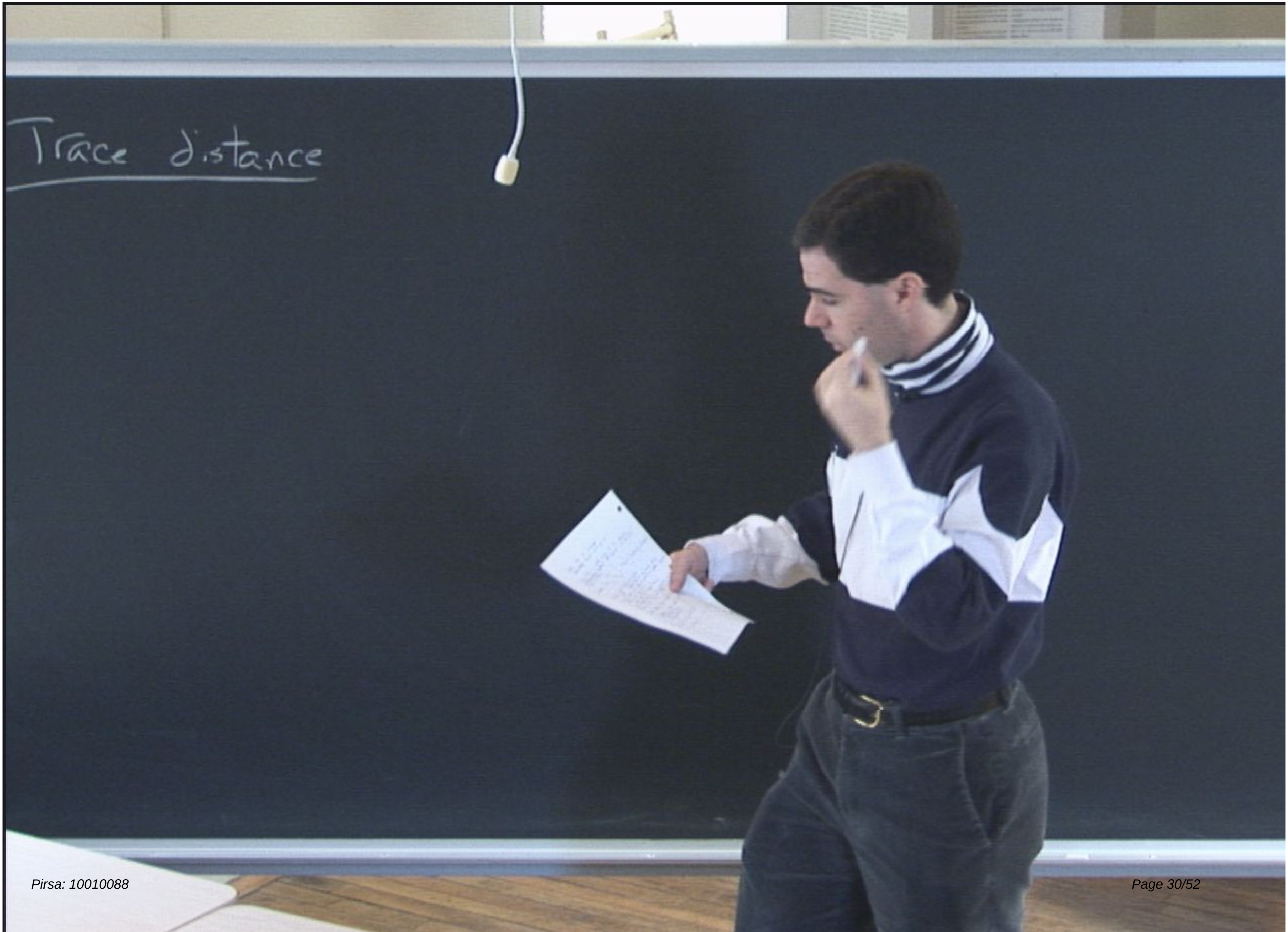
$$F(\rho,\sigma) = \max |\langle\psi|\phi\rangle|$$

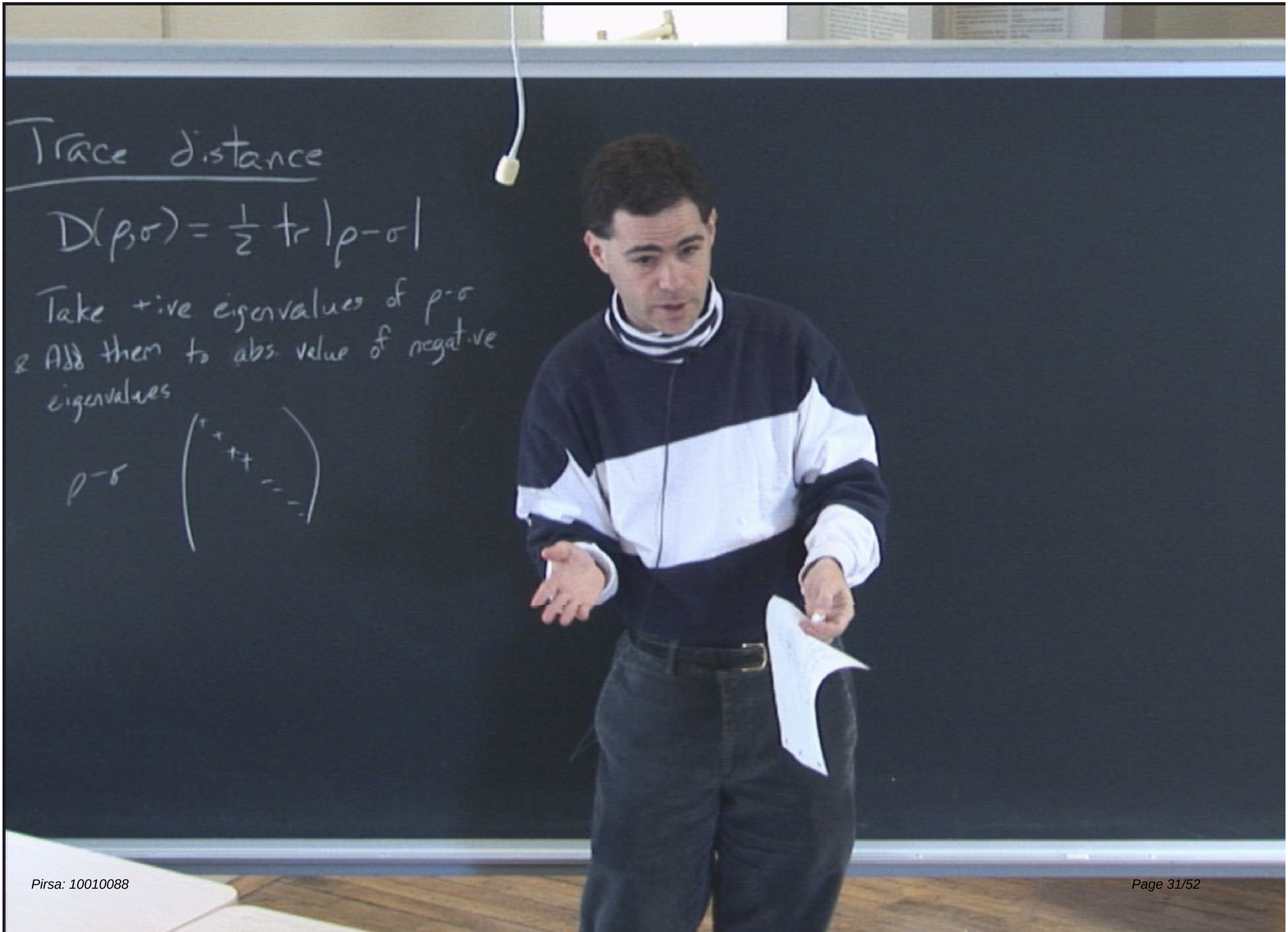
where ρ,σ act on system Q, $|\psi\rangle, |\phi\rangle$ are states on $R\otimes Q$, $\mathrm{tr}_R|\psi\rangle\langle\psi| = \rho$, $\mathrm{tr}_R|\phi\rangle\langle\phi| = \sigma$.

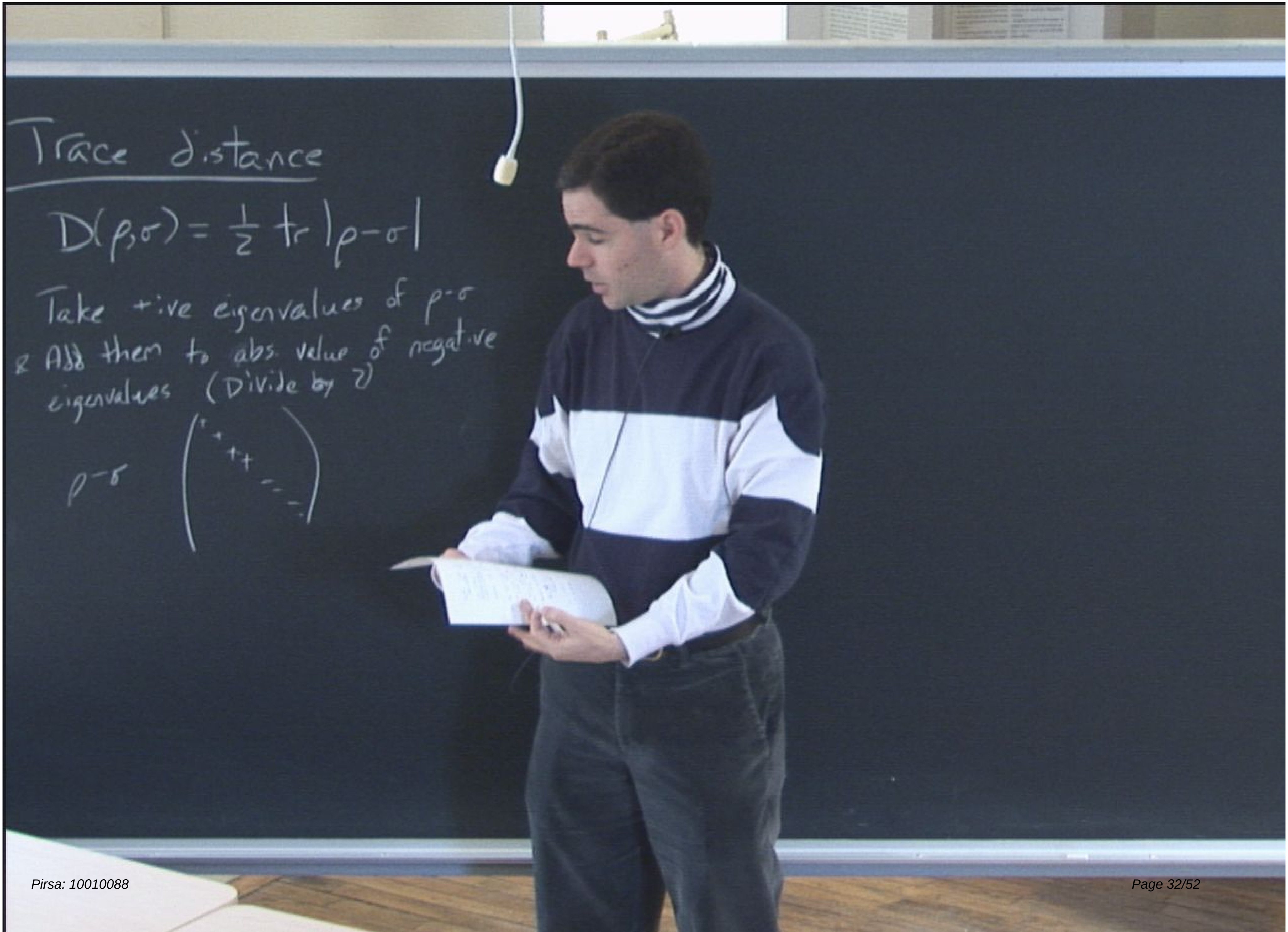
Fidelity between ρ & σ is maximum achievable fidelity between purificatio[ns] of ρ & σ.



controlled-SWAP: If control 0, do nothing
If control 1, SWAP qubits.


Trace distance


Trace distance
$D(\rho,\sigma) = \frac{1}{2} \mathrm{tr} |\rho - \sigma|$
Take +ive eigenvalues of ρ-σ
& Add them to abs value of negative
eigenvalues
$\rho - \sigma$


Trace distance
$D(\rho,\sigma) = \frac{1}{2} \mathrm{tr} |\rho-\sigma|$
Take +ive eigenvalues of $\rho-\sigma$
& Add them to abs. value of negative
eigenvalues (Divide by 2)
$\rho-\sigma$

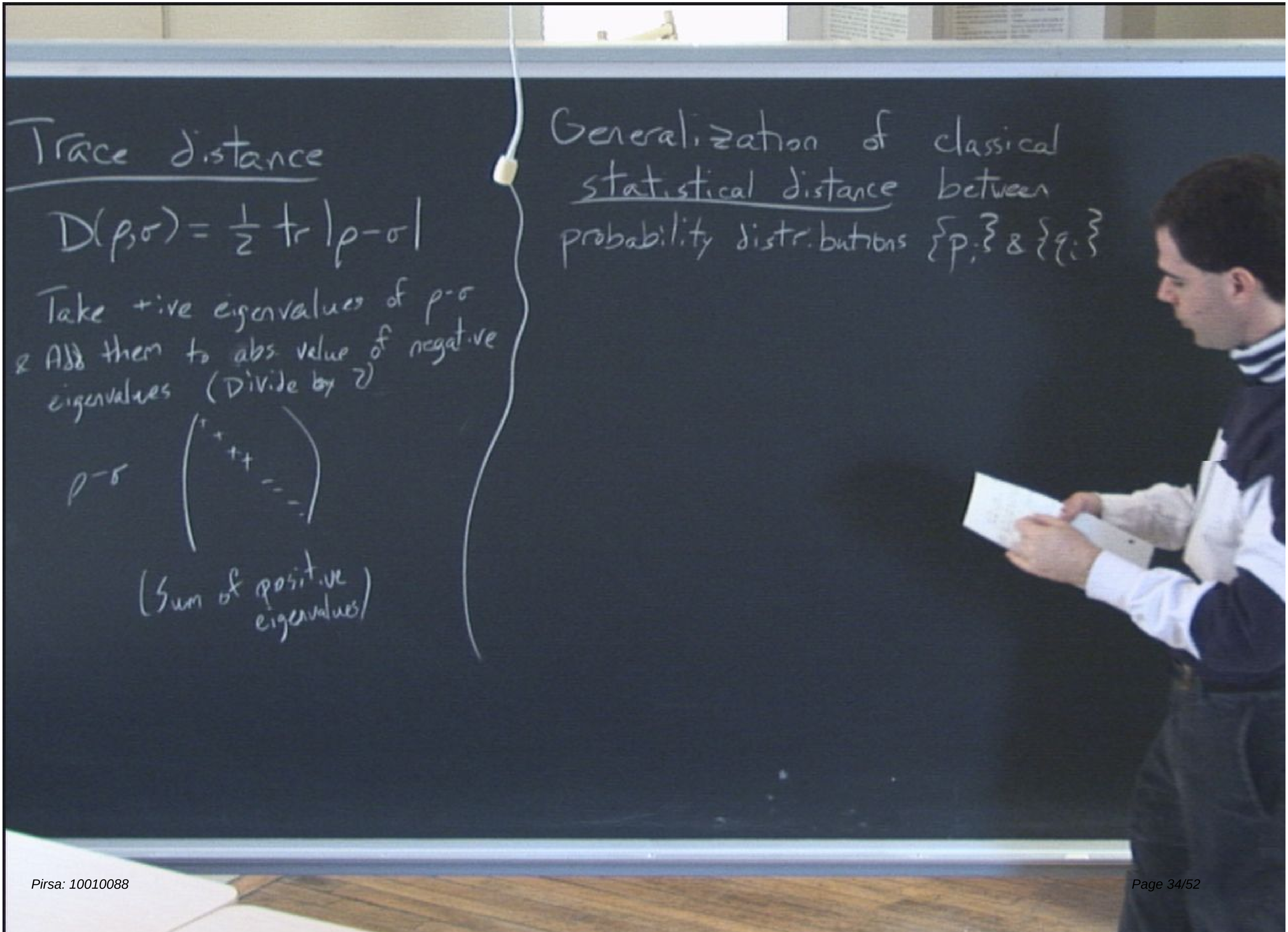
Trace distance

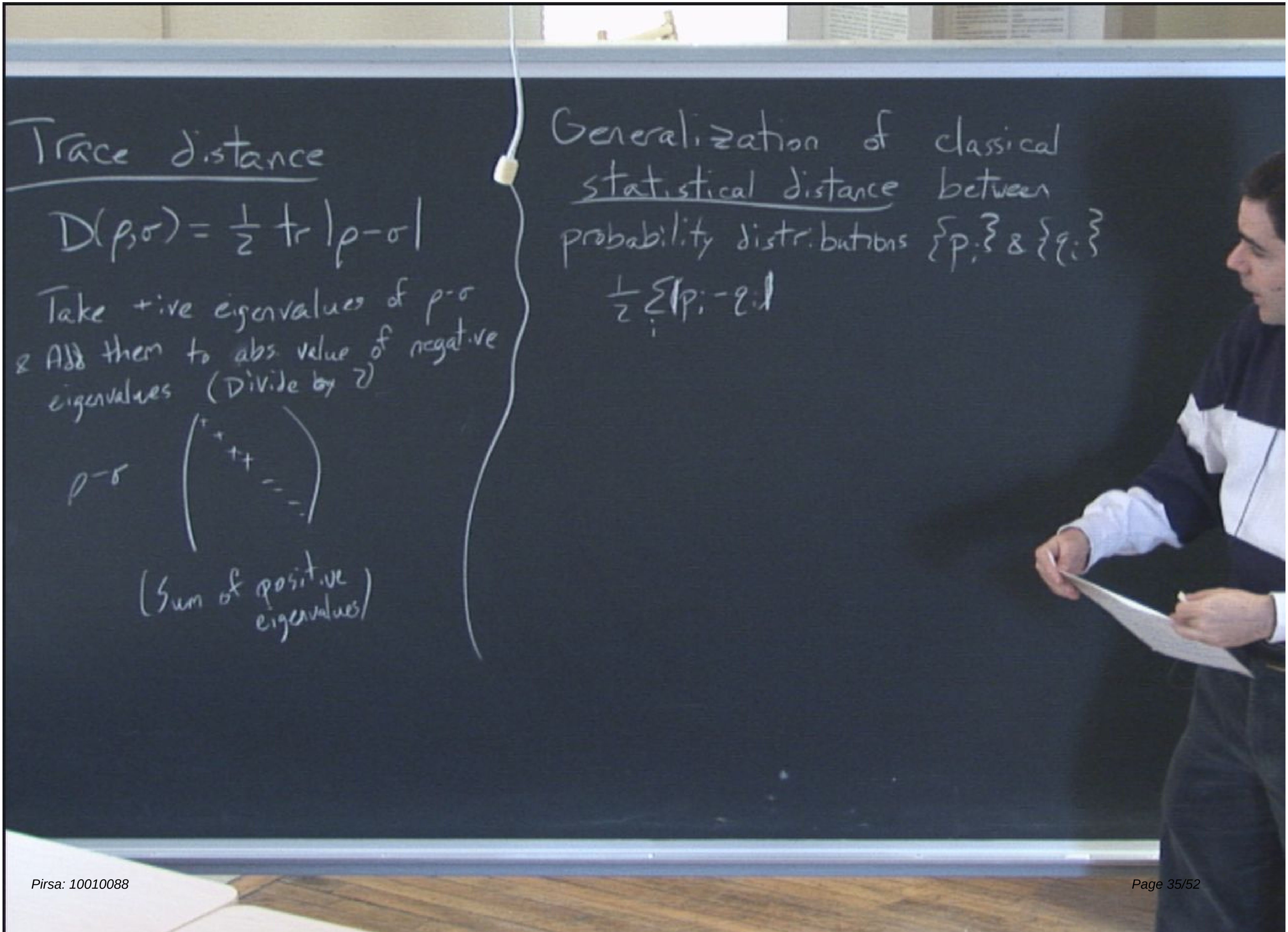
$$D(\rho,\sigma) = \frac{1}{2} \operatorname{tr} |\rho - \sigma|$$

Take +ive eigenvalues of $\rho-\sigma$
& Add them to abs. value of negative eigenvalues (Divide by 2)

$$\rho-\sigma \quad \begin{pmatrix} + & & & & & & & \\ & + & & & & & & \\ & & + & & & & & \\ & & & + & & & & \\ & & & & - & & & \\ & & & & & - & & \\ & & & & & & - & \\ & & & & & & & - \end{pmatrix}$$

(Sum of positive eigenvalues)


Trace distance
D(ρ,σ) = 1/2 tr |ρ−σ|
Take +ive eigenvalues of ρ-σ
& Add them to abs value of negative
eigenvalues (Divide by 2)
ρ−σ
(Sum of positive eigenvalues)
Generalization of classical
statistical distance between
probability distributions {p_i} & {q_i}


Trace distance
$D(\rho,\sigma) = \frac{1}{2} \mathrm{tr} |\rho - \sigma|$
Take +ive eigenvalues of $\rho-\sigma$
& Add them to abs value of negative eigenvalues (Divide by 2)
$\rho - \sigma$
(Sum of positive eigenvalues)
Generalization of classical statistical distance between probability distributions $\{p_i\}$ & $\{q_i\}$
$\frac{1}{2} \sum_i |p_i - q_i|$

Trace distance

$$D(\rho,\sigma) = \frac{1}{2}\,\mathrm{tr}\,|\rho-\sigma|$$

Take +ive eigenvalues of $\rho-\sigma$ & Add them to abs. value of negative eigenvalues (Divide by 2)

$$\rho-\sigma \quad \begin{pmatrix} + & & & & & & \\ & + & & & & & \\ & & + & & & & \\ & & & + & & & \\ & & & & - & & \\ & & & & & - & \\ & & & & & & - \end{pmatrix}$$

(Sum of positive eigenvalues)

Generalization of classical statistical distance between probability distributions $\{p_i\}$ & $\{q_i\}$

$$\frac{1}{2}\sum_i |p_i - q_i|$$

L_1-distance

...nce

... $\operatorname{tr}|\rho-\sigma|$

...envalues of $\rho-\sigma$
...os. value of negative
...vide by 2

(positive eigenvalues)

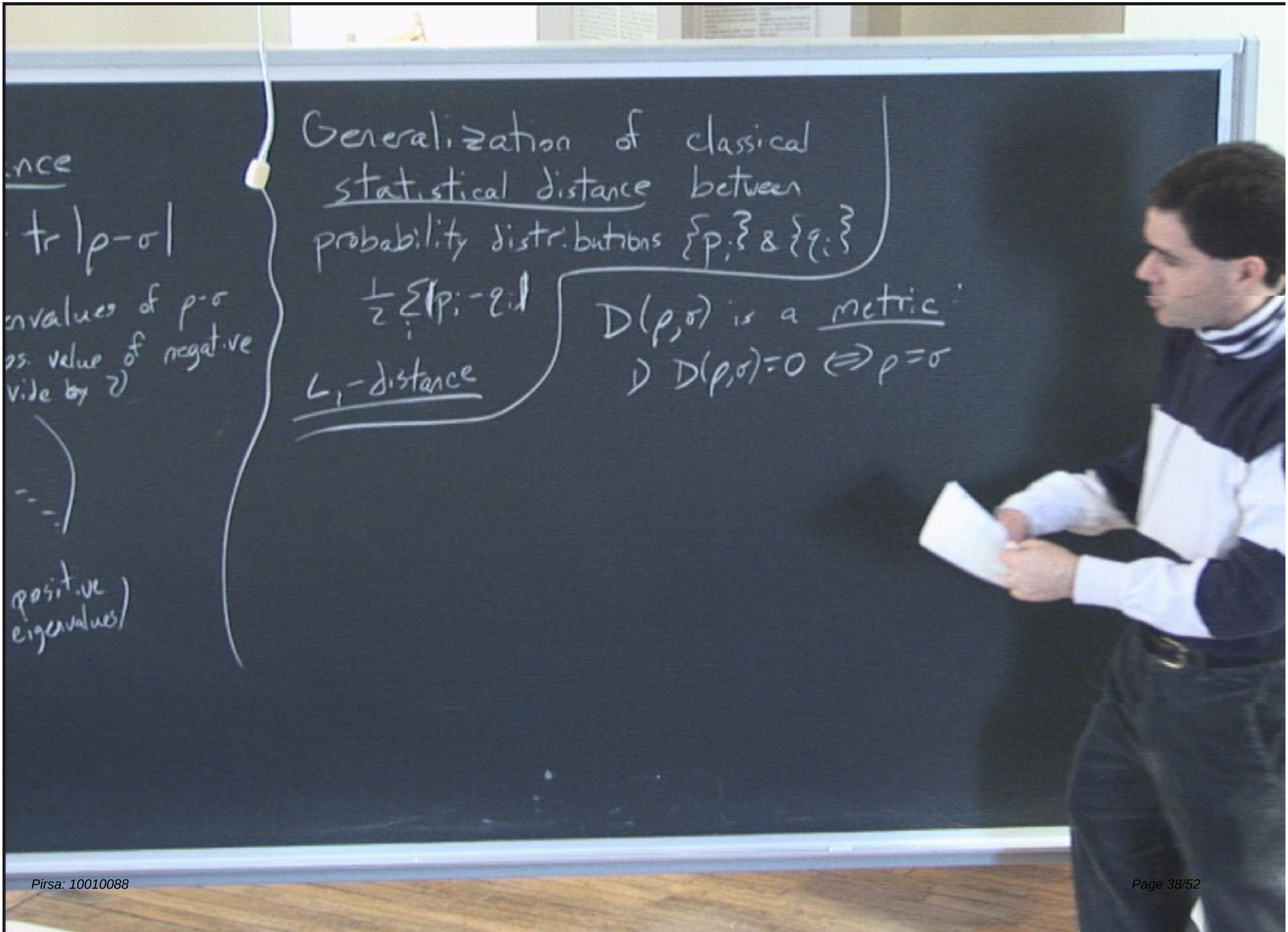
Generalization of classical statistical distance between probability distributions $\{p_i\}$ & $\{q_i\}$

$\frac{1}{2}\sum_i |p_i - q_i|$

L_1-distance

$D(\rho,\sigma)$ is a metric:

1) $D(\rho,\sigma)=0$


nce
tr $|\rho-\sigma|$
nvalues of $\rho-\sigma$
os. value of negative
vide by 2
positive eigenvalues
Generalization of classical
statistical distance between
probability distributions $\{p_i\}$ & $\{q_i\}$
$\frac{1}{2}\sum_i |p_i - q_i|$
L_1-distance
$D(\rho,\sigma)$ is a metric
1) $D(\rho,\sigma)=0 \iff \rho=\sigma$

nce

$\mathrm{tr}\,|\rho-\sigma|$

envalues of $\rho-\sigma$

s. value of negative

vide by 2

(positive eigenvalues)

Generalization of classical statistical distance between probability distributions $\{p_i\}$ & $\{q_i\}$

$$\frac{1}{2}\sum_i |p_i - q_i|$$

L_1-distance

$D(\rho,\sigma)$ is a metric:

1) $D(\rho,\sigma)=0 \iff \rho=\sigma$

2) $D(\rho,\sigma)=D(\sigma,\rho)$

...nce

... $\mathrm{tr}\,|\rho-\sigma|$

...nvalues of $\rho-\sigma$

...s. value of negative

...vide by 2

(positive eigenvalues)

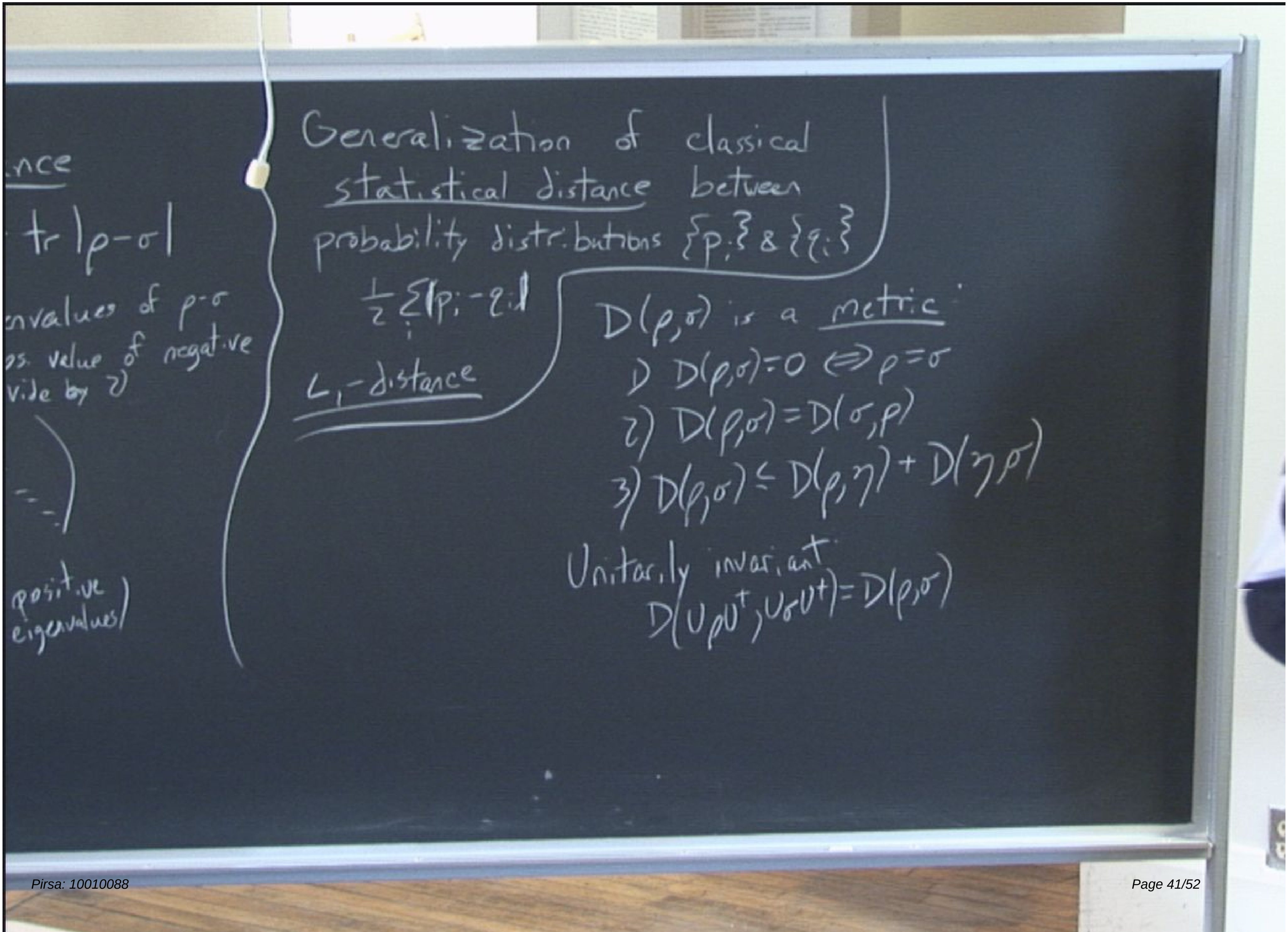
Generalization of classical <u>statistical distance</u> between probability distributions $\{p_i\}$ & $\{q_i\}$

$$\frac{1}{2}\sum_i |p_i - q_i|$$

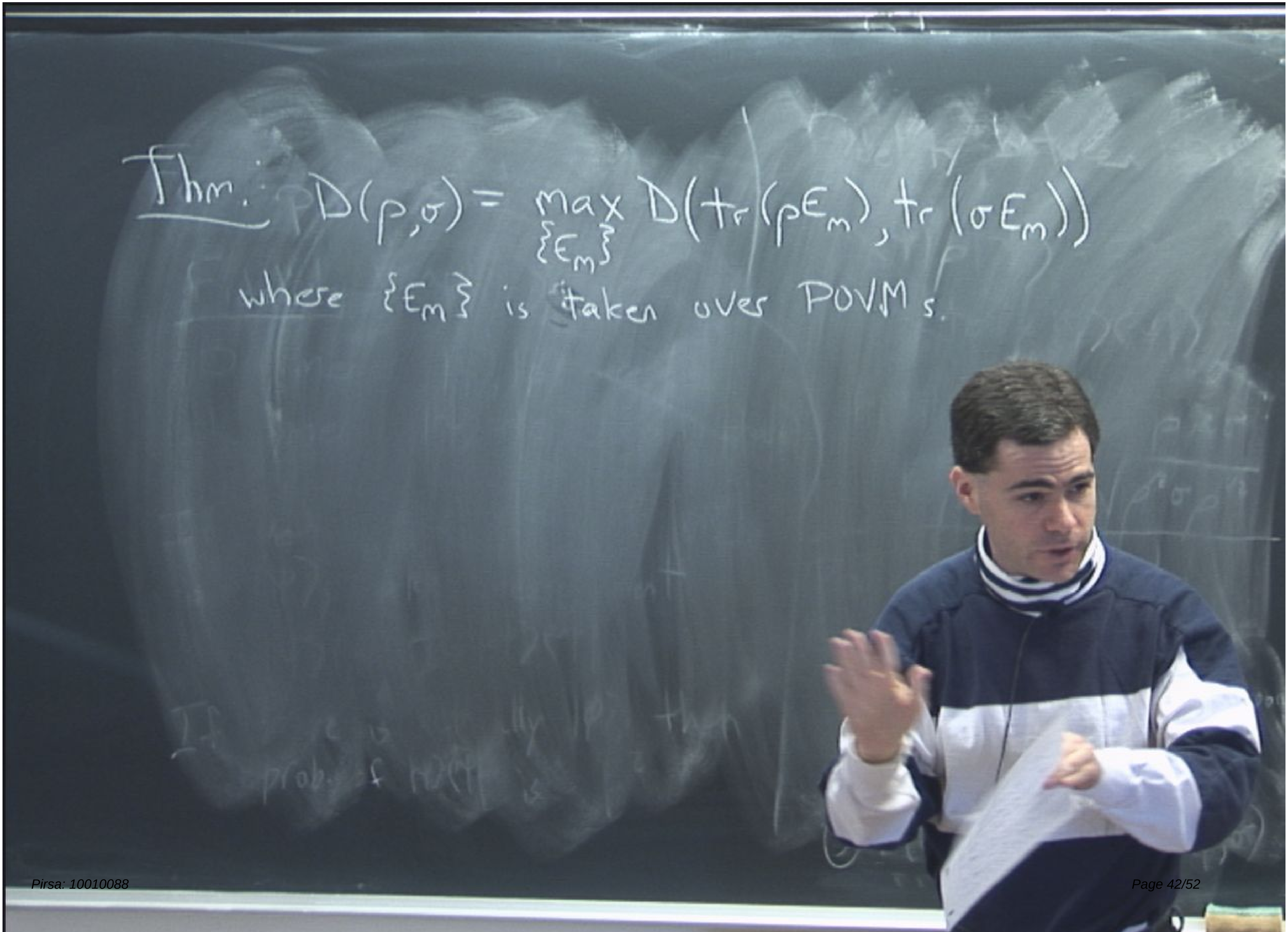
<u>L_1-distance</u>

$D(\rho,\sigma)$ is a <u>metric</u>:

1) $D(\rho,\sigma)=0 \Leftrightarrow \rho=\sigma$
2) $D(\rho,\sigma)=D(\sigma,\rho)$
3) $D(\rho,\sigma) \le D(\rho,\eta) + D(\eta,\rho)$



nce
tr $|\rho-\sigma|$
nvalues of $\rho-\sigma$
s. value of negative
vide by 2
positive eigenvalues)
Generalization of classical statistical distance between probability distributions $\{p_i\}$ & $\{q_i\}$
$\frac{1}{2}\sum_i |p_i - q_i|$
L_1-distance
$D(\rho,\sigma)$ is a metric:
1) $D(\rho,\sigma)=0 \iff \rho=\sigma$
2) $D(\rho,\sigma)=D(\sigma,\rho)$
3) $D(\rho,\sigma) \leq D(\rho,\eta) + D(\eta,\rho)$
Unitarily invariant:
$D(U\rho U^\dagger, U\sigma U^\dagger) = D(\rho,\sigma)$


Thm.: $D(\rho,\sigma) = \max_{\{E_m\}} D(\mathrm{tr}(\rho E_m), \mathrm{tr}(\sigma E_m))$
where $\{E_m\}$ is taken over POVMs.

Thm: $D(\rho,\sigma) = \max_{\{E_m\}} D(\mathrm{tr}(\rho E_m), \mathrm{tr}(\sigma E_m))$

where $\{E_m\}$ is taken over POVMs.

Trace distance is related to best chance to guess which of ρ & σ we have

Thm: $D(\rho,\sigma) = \max_{\{E_m\}} D(\mathrm{tr}(\rho E_m), \mathrm{tr}(\sigma E_m))$

where $\{E_m\}$ is taken over POVMs.

Trace distance is related to best chance to guess which of ρ & σ we have

$\mathrm{tr}(\rho E_m)$ & $\mathrm{tr}(\sigma E_m)$ are classical prob. distributions

$, \mathrm{tr}(\sigma E_m))$

Ms.

ce

b. distributions

Quantum operations decrease distinguishability

Thm: $\mathcal{E}$ is CPTP

Thm. (Uhlmann's

$F(\rho, \sigma) = m$

where ρ, σ a

$|\psi\rangle, |\phi\rangle$ are sta

$\mathrm{tr}_R |\psi\rangle\langle\psi|$

Fidel

achi

of

$\mathrm{tr}(\sigma E_m))$

POVMs.

ice

prob. distributions

Quantum operations decrease distinguishability

Thm: $\mathcal{E}$ is CPTP

$$D(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \le D(\rho,\sigma)$$

Thm. (Uhlmann's

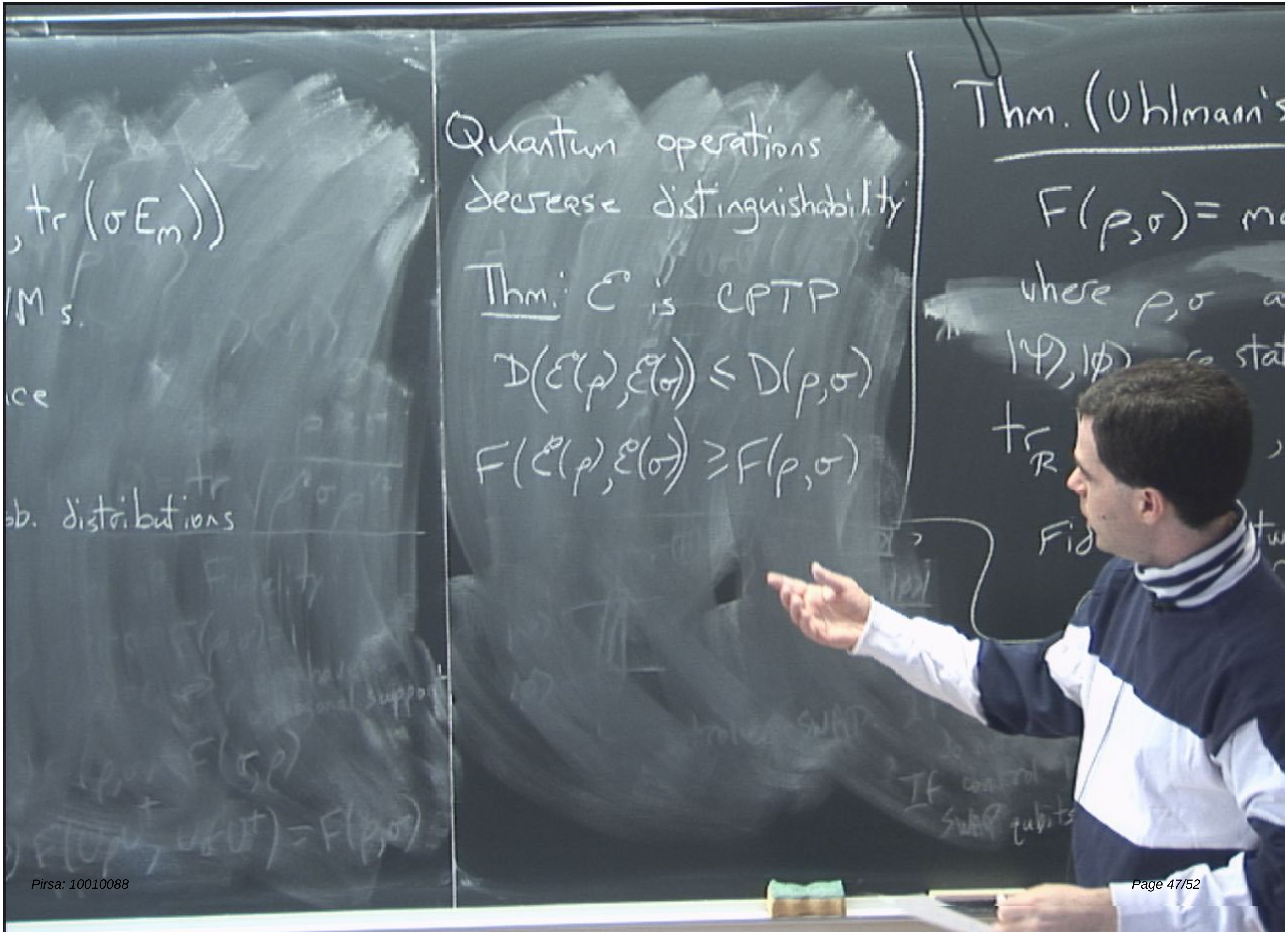
$F(\rho,\sigma) = m$

where ρ, σ

$|\psi\rangle\, |\phi\rangle$ are stat

$= \rho$,

betw


Quantum operations
decrease distinguishability
Thm: $\mathcal{E}$ is CPTP
$D(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \leq D(\rho,\sigma)$
$F(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \geq F(\rho,\sigma)$
Thm. (Uhlmann's
$\mathrm{tr}(\sigma E_m))$
prob. distributions

distance

$\ldots,\sigma) = \frac{1}{2}\,\mathrm{tr}\,|\rho-\sigma|$

+ive eigenvalues of $\rho-\sigma$
...em to abs. value of negative
...lues (Divide by 2)

$$\begin{pmatrix} + & & & & & & \\ & + & & & & & \\ & & + & & & & \\ & & & + & & & \\ & & & & - & & \\ & & & & & - & \\ & & & & & & - \end{pmatrix}$$

(Sum of positive eigenvalues)

Generalization of classical statistical distance between probability distributions $\{p_i\}$ & $\{q_i\}$

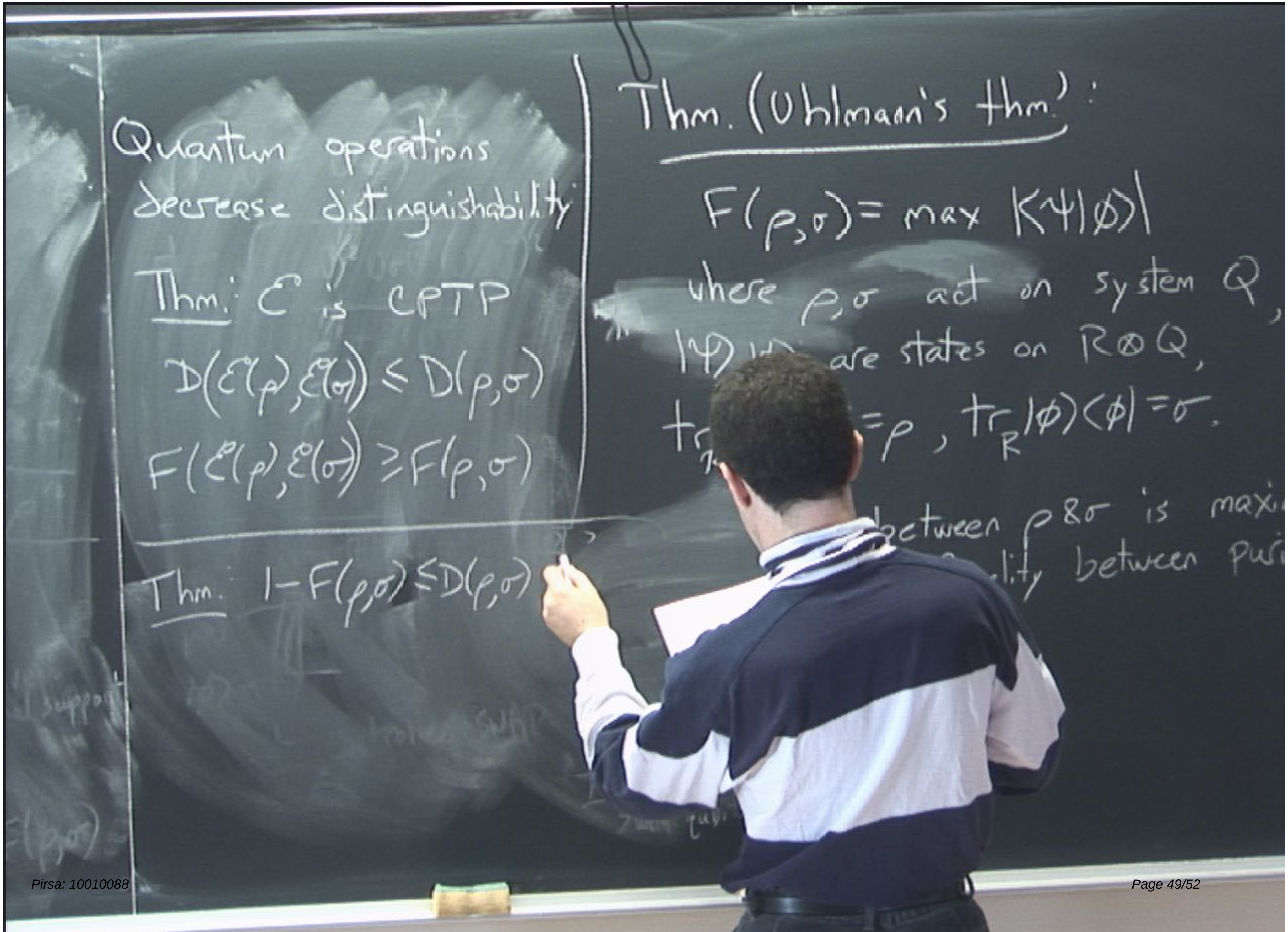
$\frac{1}{2}\sum_i |p_i - q_i|$

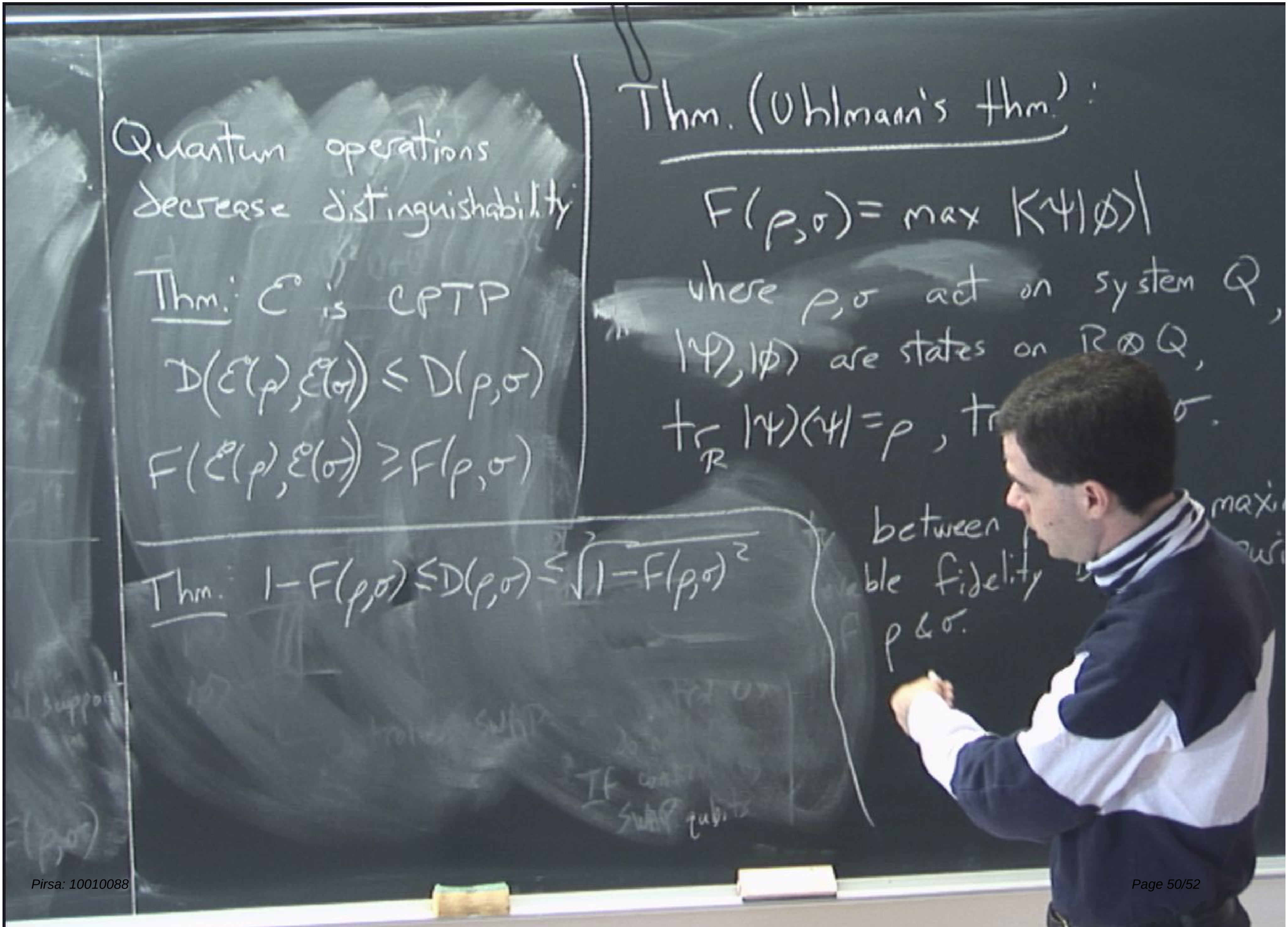
L_1-distance

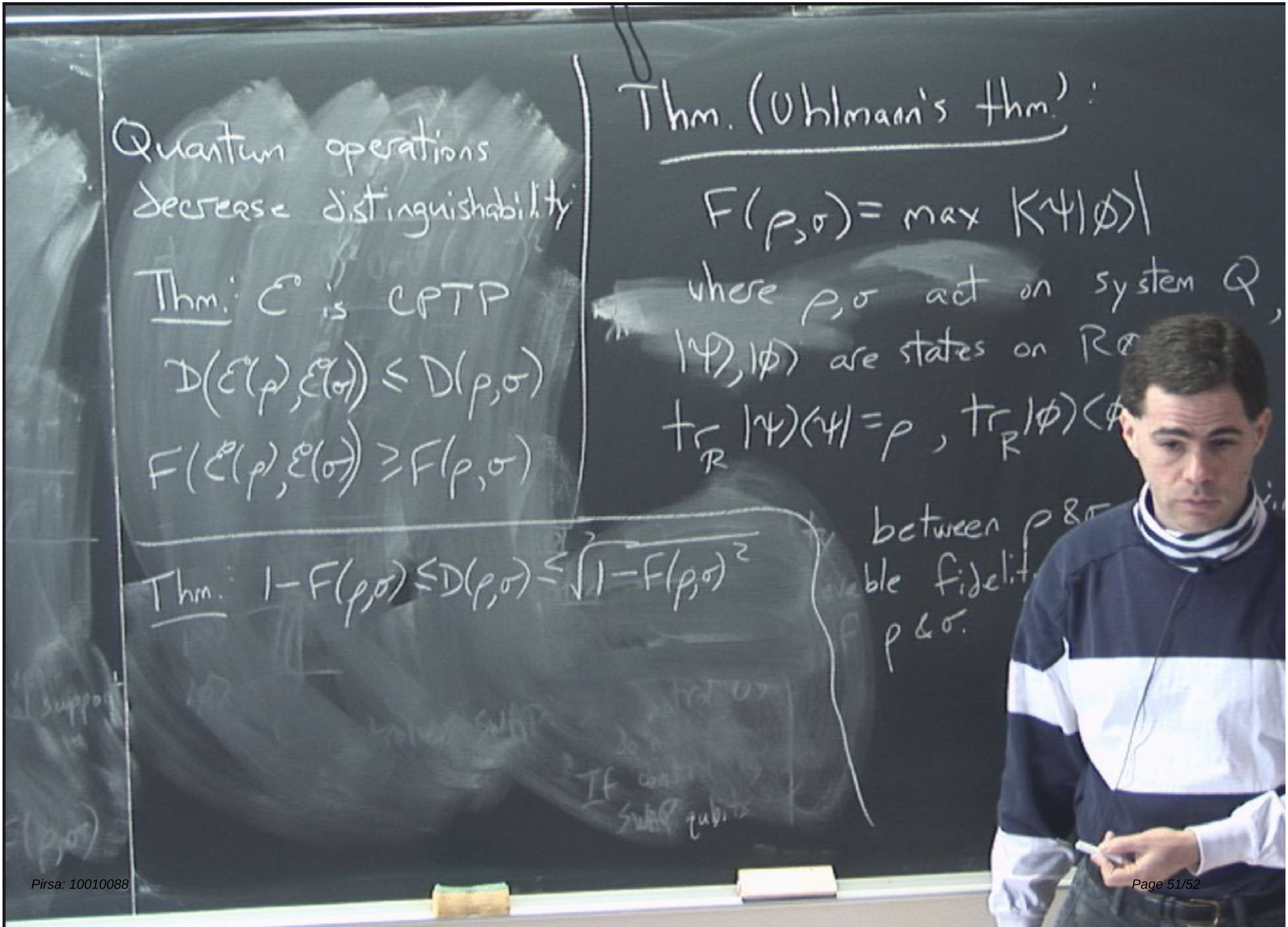
$D(\rho,\sigma)$ is a metric:

1) $D(\rho,\sigma)=0 \iff \rho=\sigma$
2) $D(\rho,\sigma)=D(\sigma,\rho)$
3) $D(\rho,\sigma) \leq D(\rho,\eta) + D(\eta,\sigma)$

Unitarily invariant:
$D(U\rho U^\dagger, U\sigma U^\dagger) = D(\rho,\sigma)$


Quantum operations
decrease distinguishability
Thm: $\mathcal{E}$ is CPTP
$D(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \leq D(\rho,\sigma)$
$F(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \geq F(\rho,\sigma)$
Thm: $1-F(\rho,\sigma) \leq D(\rho,\sigma)$
Thm. (Uhlmann's thm):
$F(\rho,\sigma) = \max |\langle\psi|\phi\rangle|$
where ρ,σ act on system Q,
are states on $R\otimes Q$,
$\mathrm{tr}_R|\phi\rangle\langle\phi| = \sigma$.
between ρ & σ is maxi
between puri


Quantum operations
decrease distinguishability
Thm: $\mathcal{E}$ is CPTP
$D(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \le D(\rho,\sigma)$
$F(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \ge F(\rho,\sigma)$
Thm: $1-F(\rho,\sigma) \le D(\rho,\sigma) \le \sqrt{1-F(\rho,\sigma)^2}$
Thm. (Uhlmann's thm):
$F(\rho,\sigma) = \max |\langle\psi|\phi\rangle|$
where ρ,σ act on system Q,
$|\psi\rangle, |\phi\rangle$ are states on $R\otimes Q$,
$\mathrm{tr}_R |\psi\rangle\langle\psi| = \rho$,
between
ble fidelity
ρ & σ.



Quantum operations decrease distinguishability
Thm: $\mathcal{E}$ is CPTP
$D(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \leq D(\rho,\sigma)$
$F(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \geq F(\rho,\sigma)$
Thm: $1-F(\rho,\sigma) \leq D(\rho,\sigma) \leq \sqrt{1-F(\rho,\sigma)^2}$
Thm. (Uhlmann's thm):
$F(\rho,\sigma) = \max |\langle\psi|\phi\rangle|$
where ρ,σ act on system Q,
$|\psi\rangle, |\phi\rangle$ are states on R
$\mathrm{tr}_R |\psi\rangle\langle\psi| = \rho$, $\mathrm{tr}_R |\phi\rangle\langle$
between ρ &
ble fidelit
ρ & σ.

n operations

e distinguishability

$\mathcal{E}$ is CPTP

$D(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \le D(\rho,\sigma)$

$F(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \ge F(\rho,\sigma)$

$1-F(\rho,\sigma) \le D(\rho,\sigma) \le \sqrt{1-F(\rho,\sigma)^2}$

Thm. (Uhlmann's thm):

$$F(\rho,\sigma) = \max |\langle\psi|\phi\rangle|$$

where ρ,σ act on system Q,

$|\psi\rangle, |\phi\rangle$ are states on $R\otimes Q$,

$\mathrm{tr}_R |\psi\rangle\langle\psi| = \rho$, $\mathrm{tr}_R |\phi\rangle\langle\phi| = \sigma$.

between ρ & σ is maximum
ble fidelity between purifications
ρ & σ.

$\sum_i \sqrt{\lambda_i}\,|i\rangle_R \otimes |i\rangle_Q$ $\quad \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \end{pmatrix}$ $\quad \begin{pmatrix} \eta_1 & & \\ & \eta_2 & \\ & & \ddots \end{pmatrix}$

$\sum_i \sqrt{\eta_i}\,|i\rangle_R \otimes |i\rangle_Q$