Title: Quantum Information - Review (PHYS 635) - Lecture 3

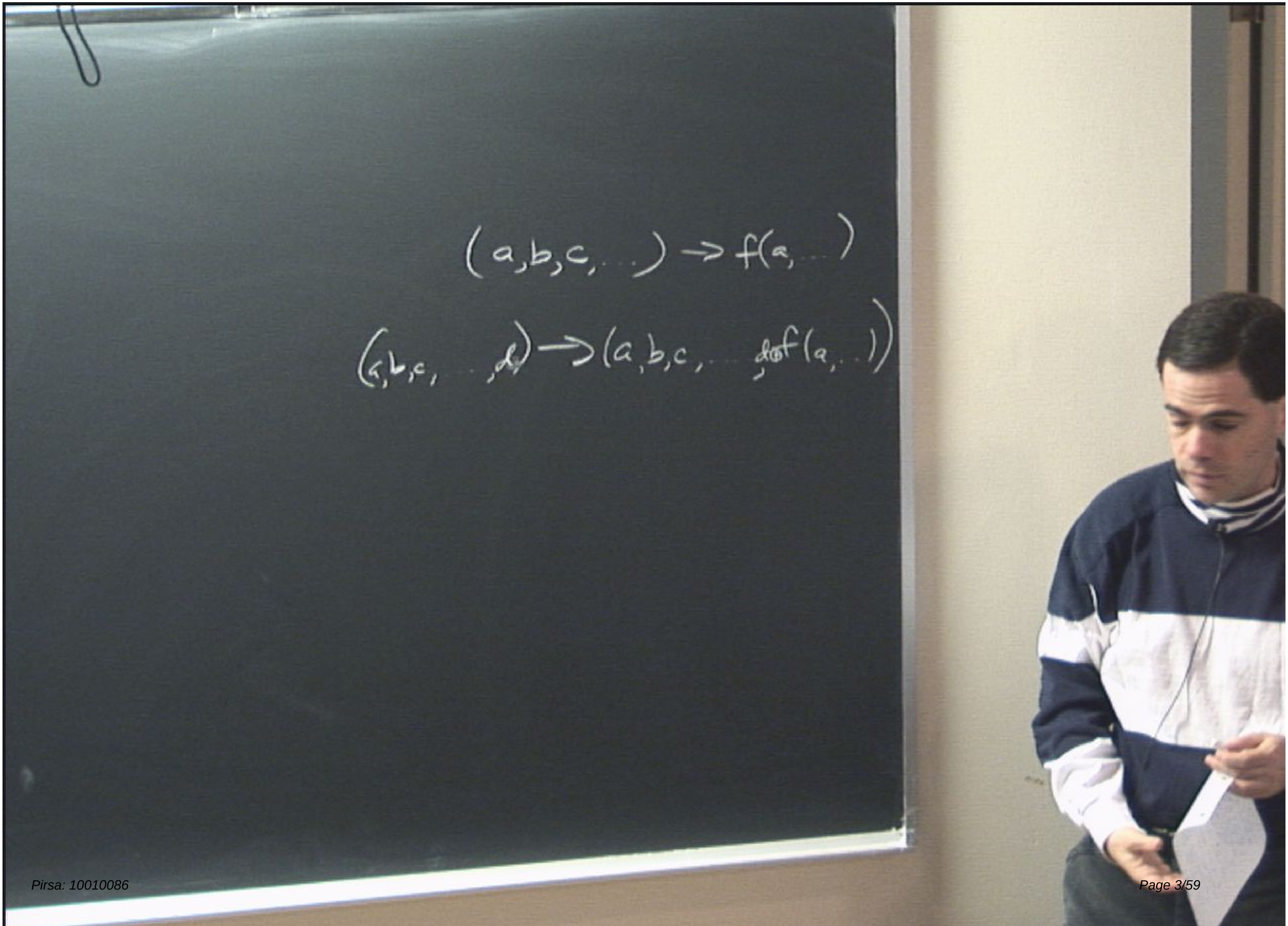
Date: Jan 27, 2010 09:00 AM

URL: http://pirsa.org/10010086

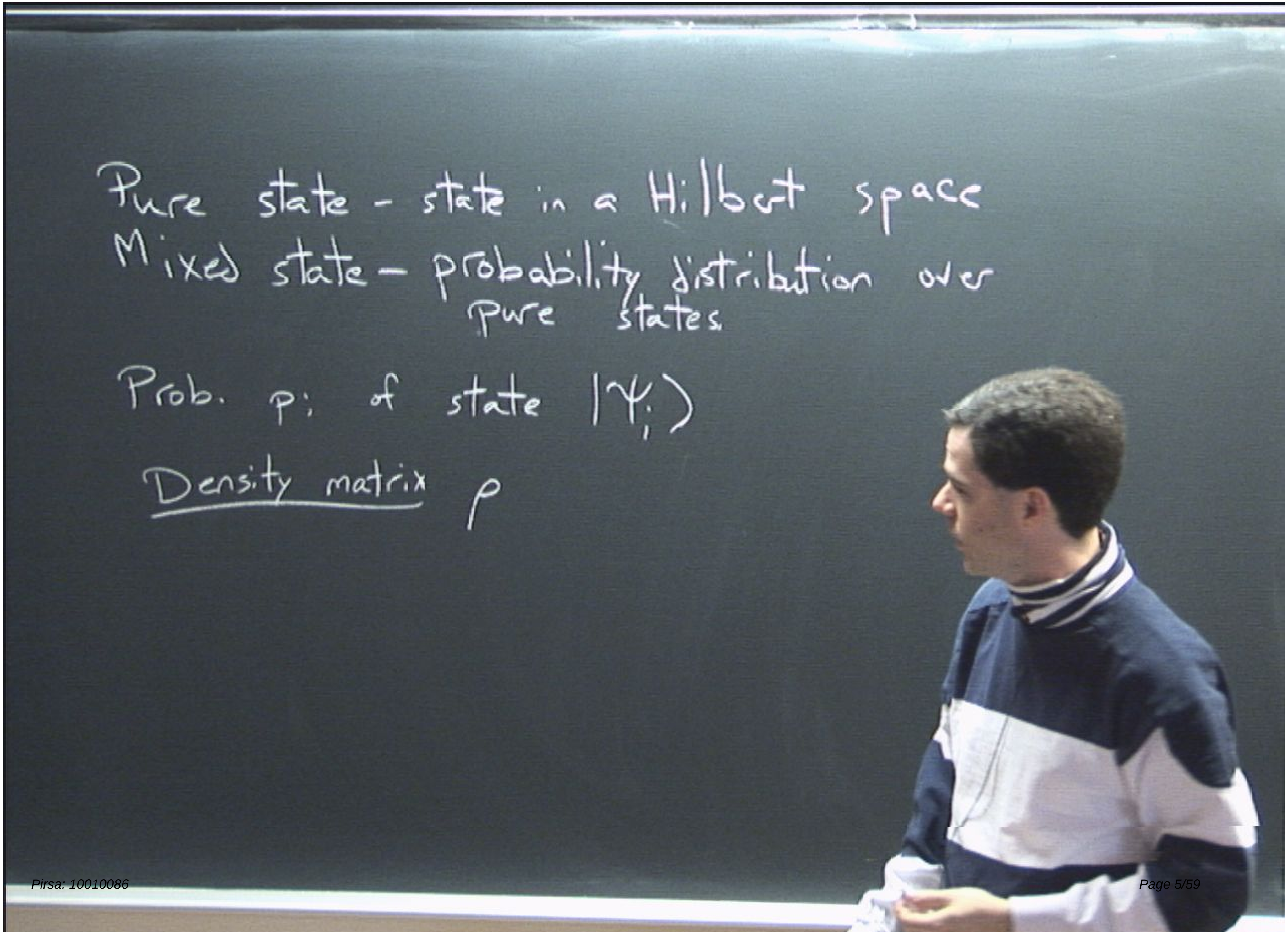
Abstract: <div id="Cleaner">Week 1: Basic topics (Qubits, quantum gates, quantum circuits, density matrices, quantum operations, entropy, entanglement)
<div id="Cleaner"><div id="Cleaner">Week 2: Algorithms and complexity (Languages, complexity classes, oracles, RSA, Deutsch-Jozsa algorithm, Shor's algorithm, Grover's algorithm)
<div id="Cleaner"><div id="Cleaner">Week 3: Information theory and implementations (Overview of implementations, quantum error correction, quantum cryptography, quantum information theory)

$$(a, b, c, \ldots) \to f(a, \ldots)$$

$$\to (a, b, c, \ldots, f(a, \ldots))$$


(a,b,c,...) → f(a,...)
(a,b,c,...,d) → (a,b,c,...,d⊕f(a,...))

Pure state - state in a Hilbert space
Mixed state - probability distribution over pure states.

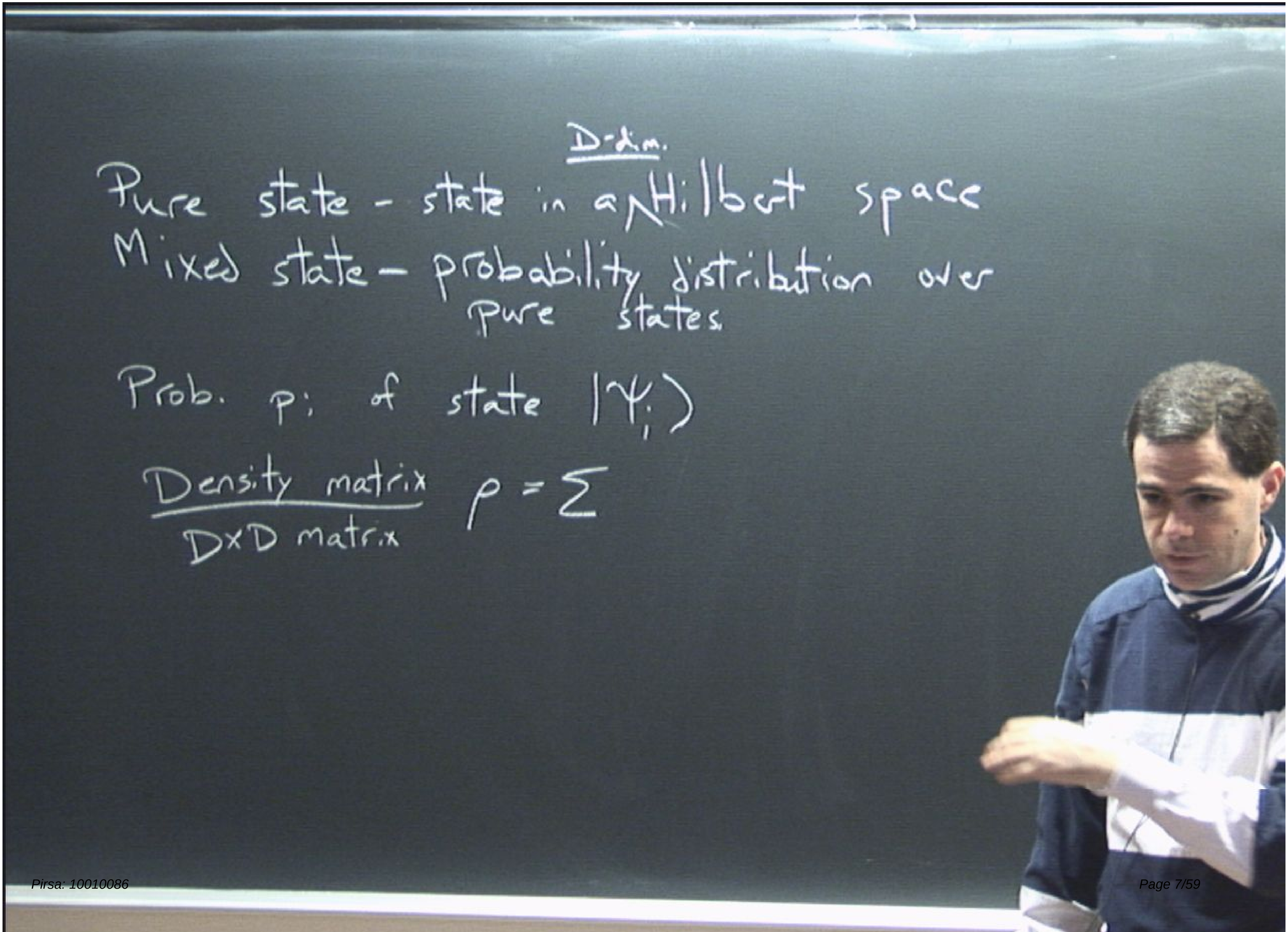

Pure state - state in a Hilbert space
Mixed state – probability distribution over pure states.
Prob. p_i of state $|\psi_i\rangle$
Density matrix ρ

Pure state - state in a Hilbert space

Mixed state - probability distribution over pure states.

Prob. p_i of state $|\psi_i\rangle$

<u>Density matrix</u> ρ


D-dim.
Pure state - state in a Hilbert space
Mixed state – probability distribution over pure states.
Prob. p_i of state $|\psi_i\rangle$
Density matrix
D×D matrix
$\rho = \Sigma$

D-dim.

Pure state - state in a D-dim. Hilbert space

Mixed state – probability distribution over pure states.

Prob. p_i of state $|\psi_i\rangle$

$$\underline{\text{Density matrix}} \quad \rho = \sum p_i |\psi_i\rangle\langle\psi_i|$$

$D \times D$ matrix

$$|\psi_i\rangle\underbrace{\langle\psi_i|\psi\rangle}$$

$|\psi_i\rangle\langle\psi_i|$ - projector on $|\psi_i\rangle$

D-dim.

Pure state - state in a Hilbert space

Mixed state - probability distribution over pure states.

Prob. p_i of state $|\psi_i\rangle$

<u>Density matrix</u> $\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$

D×D matrix

$|\psi_i\rangle\underbrace{\langle\psi_i|\psi\rangle}$

$|\psi_i\rangle\langle\psi_i|$ - projector on $|\psi_i\rangle$

Density for pure state $|\psi_i\rangle$

D-dim.

Pure state - state in a D-dim. Hilbert space

Mixed state - probability distribution over pure states.

Prob. p_i of state $|\psi_i\rangle$

Density matrix $\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$

D×D matrix

$|\psi_i\rangle\underbrace{\langle\psi_i|\psi\rangle}$

$|\psi_i\rangle\langle\psi_i|$ - projector on $|\psi_i\rangle$

Density for pure state $|\psi_i\rangle$

$p_1 = \frac{1}{2}$ $|\psi_1\rangle = |0\rangle$

$p_2 = \frac{1}{4}$

$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

D-dim.

Pure state - state in a Hilbert space

Mixed state - probability distribution over pure states.

Prob. p_i of state $|\psi_i\rangle$

Density matrix $\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$

D×D matrix

$|\psi_i\rangle\underbrace{\langle\psi_i|\psi\rangle}$

$|\psi_i\rangle\langle\psi_i|$ - projector on $|\psi_i\rangle$

Density for pure state $|\psi_i\rangle$

$p_1 = \frac{1}{2}$ $|\psi_1\rangle = |0\rangle$

$p_2 = \frac{1}{4}$

$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$p_3 = \frac{1}{4}$ $|\psi_3\rangle = \frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle$

D-dim.

Pure state - state in a D-dim. Hilbert space

Mixed state – probability distribution over pure states.

Prob. p_i of state $|\psi_i\rangle$

Density matrix $\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$

D×D matrix

$|\psi_i\rangle\underbrace{\langle\psi_i|\psi\rangle}$

$|\psi_i\rangle\langle\psi_i|$ – projector on $|\psi_i\rangle$

Density for pure state $|\psi_i\rangle$

$p_1 = \frac{1}{2}$ $|\psi_1\rangle = |0\rangle$

$p_2 = \frac{1}{4}$

$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$p_3 = \frac{1}{4}$ $|\psi_3\rangle = \frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle$

$$\rho = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{4}\left(\frac{1}{2}\right)\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|\right) + \frac{1}{4}\left(\frac{9}{25}|0\rangle\langle 0| - \frac{12}{25}|0\rangle\langle 1| - \frac{12}{25}|1\rangle\langle 0| + \frac{16}{25}|1\rangle\langle 1|\right)$$

pace

over

$p_1 = \frac{1}{2}$ $|\psi_1\rangle = |0\rangle$

$p_2 = \frac{1}{4}$

$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$p_3 = \frac{1}{4}$ $|\psi_3\rangle = \frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle$

$$\rho = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{4}\left(\frac{1}{2}\right)\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|\right)$$

$$+ \frac{1}{4}\left(\frac{9}{25}|0\rangle\langle 0| - \frac{12}{25}|0\rangle\langle 1| - \frac{12}{25}|1\rangle\langle 0| + \frac{16}{25}|1\rangle\langle 1|\right)$$

$$\rho = \frac{143}{200}|0\rangle\langle 0| + \frac{1}{200}|0\rangle\langle 1| + \frac{1}{200}|1\rangle\langle 0| + \frac{57}{200}|1\rangle\langle 1|$$

pace

over

$p_1 = \frac{1}{2} \quad |\psi_1\rangle = |0\rangle$

$p_2 = \frac{1}{4}$

$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$p_3 = \frac{1}{4} \quad |\psi_3\rangle = \frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle$

$$\rho = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{4}\left(\frac{1}{2}\right)\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|\right)$$

$$+ \frac{1}{4}\left(\frac{9}{25}|0\rangle\langle 0| - \frac{12}{25}|0\rangle\langle 1| - \frac{12}{25}|1\rangle\langle 0| + \frac{16}{25}|1\rangle\langle 1|\right)$$

$$\rho = \frac{143}{200}|0\rangle\langle 0| + \frac{1}{200}|0\rangle\langle 1| + \frac{1}{200}|1\rangle\langle 0| + \frac{57}{200}|1\rangle\langle 1|$$

$$= \frac{1}{200}\begin{pmatrix} 143 & 1 \\ 1 & 57 \end{pmatrix}$$

pace

over

$p_1 = \frac{1}{2}$ $|\psi_1\rangle = |0\rangle$

$p_2 = \frac{1}{4}$

$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$p_3 = \frac{1}{4}$ $|\psi_3\rangle = \frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle$

$$\rho = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{4}\left(\frac{1}{2}\right)\Big(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|\Big)$$

$$+ \frac{1}{4}\left(\frac{9}{25}|0\rangle\langle 0| - \frac{12}{25}|0\rangle\langle 1| - \frac{12}{25}|1\rangle\langle 0| + \frac{16}{25}|1\rangle\langle 1|\right)$$

$$\rho = \frac{143}{200}|0\rangle\langle 0| + \frac{1}{200}|0\rangle\langle 1| + \frac{1}{200}|1\rangle\langle 0| + \frac{57}{200}|1\rangle\langle 1|$$

$$= \frac{1}{200}\begin{pmatrix} 143 & 1 \\ 1 & 57 \end{pmatrix}$$

Properties of density matrix

pace

over

$p_1 = \frac{1}{2}$ $|\psi_1\rangle = |0\rangle$

$p_2 = \frac{1}{4}$

$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$p_3 = \frac{1}{4}$ $|\psi_3\rangle = \frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle$

$$\rho = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{4}\left(\frac{1}{2}\right)\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|\right)$$

$$+ \frac{1}{4}\left(\frac{9}{25}|0\rangle\langle 0| - \frac{12}{25}|0\rangle\langle 1| - \frac{12}{25}|1\rangle\langle 0| + \frac{16}{25}|1\rangle\langle 1|\right)$$

$$\rho = \frac{143}{200}|0\rangle\langle 0| + \frac{1}{200}|0\rangle\langle 1| + \frac{1}{200}|1\rangle\langle 0| + \frac{57}{200}|1\rangle\langle 1|$$

$$= \frac{1}{200}\begin{pmatrix} 143 & 1 \\ 1 & 57 \end{pmatrix}$$

Properties of density matrix

- Has trace 1

$$\rho = \frac{143}{200}|0\rangle\langle 0| + \frac{1}{200}|0\rangle\langle 1| + \frac{1}{200}|1\rangle\langle 0| + \frac{57}{200}|1\rangle\langle 1|$$

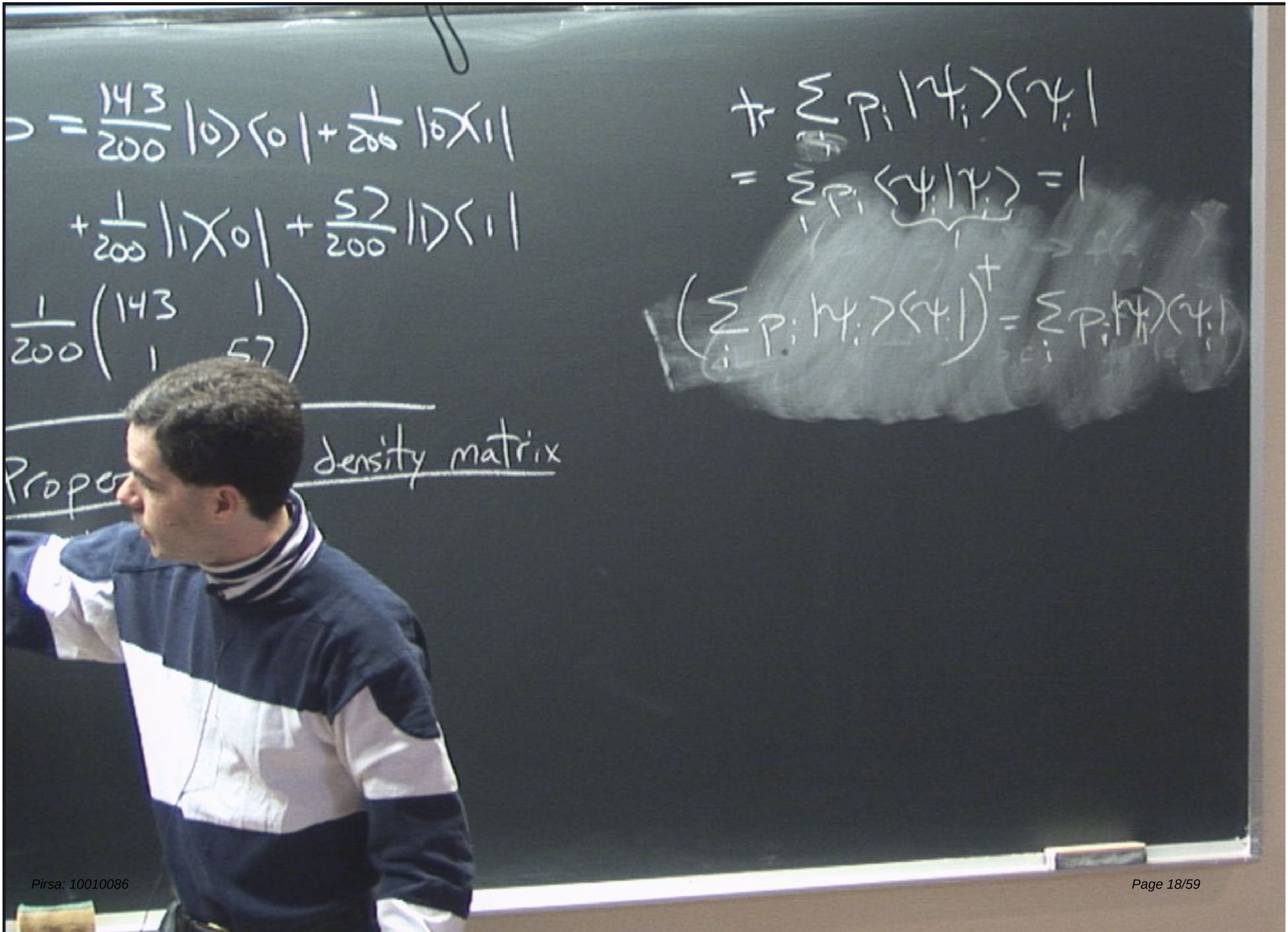
$$= \frac{1}{200}\begin{pmatrix} 143 & 1 \\ 1 & 57 \end{pmatrix}$$

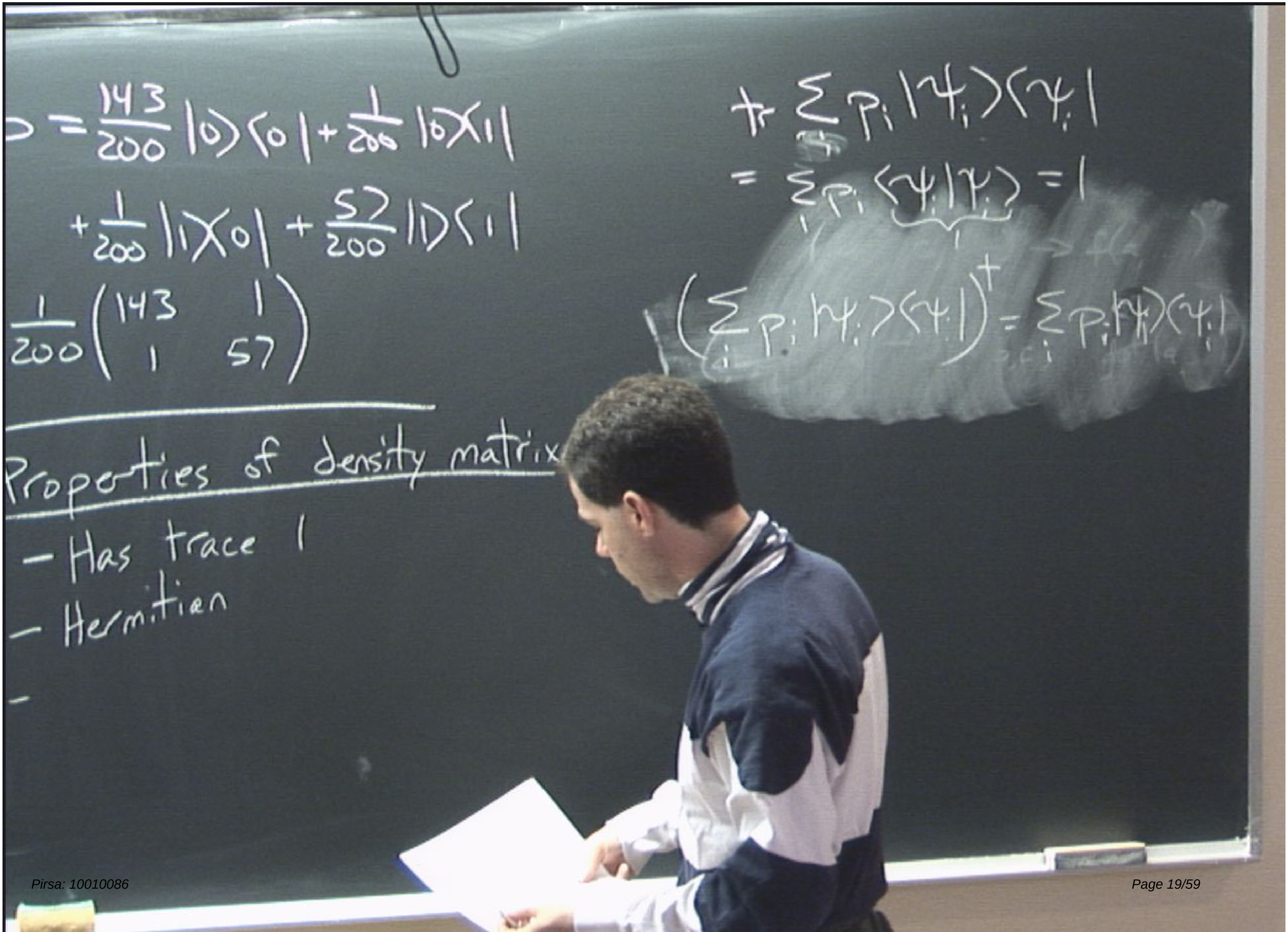
$$\mathrm{tr} \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

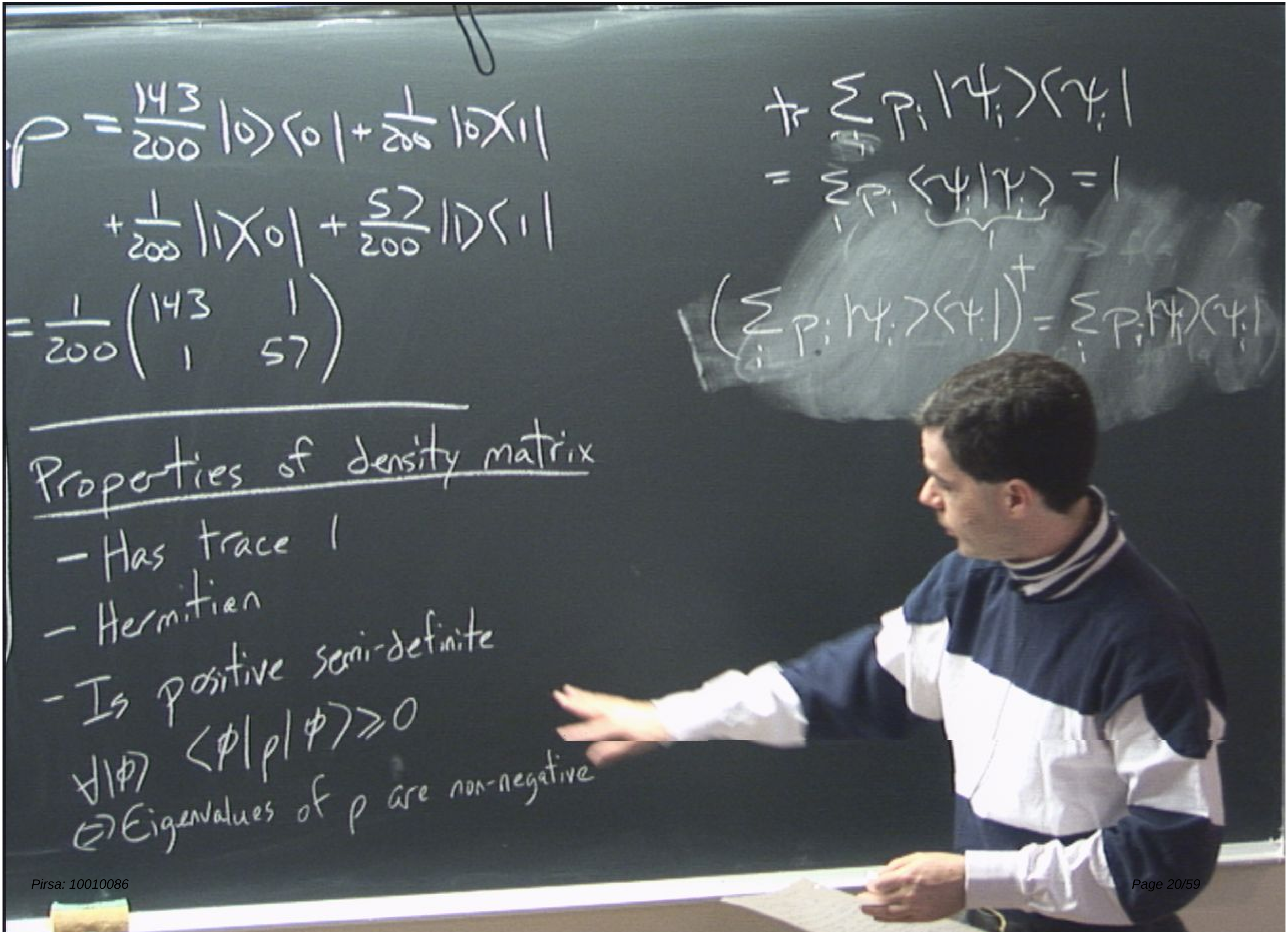
$$= \sum_i p_i \underbrace{\langle\psi_i|\psi_i\rangle}_{1} = 1$$

Properties of density matrix

- Has trace 1


= 143/200 |0⟩⟨0| + 1/200 |0⟩⟨1|
+ 1/200 |1⟩⟨0| + 57/200 |1⟩⟨1|
1/200 (143 1 ; 1 57)
Prope… density matrix
tr Σ p_i |ψ_i⟩⟨ψ_i|
= Σ_i p_i ⟨ψ_i|ψ_i⟩ = 1
(Σ_i p_i |ψ_i⟩⟨ψ_i|)† = Σ_i p_i |ψ_i⟩⟨ψ_i|


$\rho = \frac{143}{200}|0\rangle\langle 0| + \frac{1}{200}|0\rangle\langle 1|$
$+ \frac{1}{200}|1\rangle\langle 0| + \frac{57}{200}|1\rangle\langle 1|$
$\frac{1}{200}\begin{pmatrix} 143 & 1 \\ 1 & 57 \end{pmatrix}$
Properties of density matrix
– Has trace 1
– Hermitian
–
$\mathrm{tr} \sum_i p_i |\psi_i\rangle\langle\psi_i|$
$= \sum_i p_i \underbrace{\langle\psi_i|\psi_i\rangle}_{1} = 1$
$\left(\sum_i p_i |\psi_i\rangle\langle\psi_i|\right)^\dagger = \sum_i p_i |\psi_i\rangle\langle\psi_i|$


ρ = 143/200 |0⟩⟨0| + 1/200 |0⟩⟨1|
+ 1/200 |1⟩⟨0| + 57/200 |1⟩⟨1|
= 1/200 (143 1; 1 57)
Properties of density matrix
- Has trace 1
- Hermitian
- Is positive semi-definite
⟨φ|ρ|φ⟩ ≥ 0 ∀|φ⟩
⟺ Eigenvalues of ρ are non-negative
tr Σ pᵢ |ψᵢ⟩⟨ψᵢ|
= Σ pᵢ ⟨ψᵢ|ψᵢ⟩ = 1
(Σ pᵢ |ψᵢ⟩⟨ψᵢ|)† = Σ pᵢ |ψᵢ⟩⟨ψᵢ|

$$\rho = \frac{143}{200}|0\rangle\langle 0| + \frac{1}{200}|0\rangle\langle 1| + \frac{1}{200}|1\rangle\langle 0| + \frac{57}{200}|1\rangle\langle 1|$$

$$= \frac{1}{200}\begin{pmatrix} 143 & 1 \\ 1 & 57 \end{pmatrix}$$

Properties of density matrix

- Has trace 1
- Hermitian
- Is positive semi-definite

$\forall |\phi\rangle \quad \langle\phi|\rho|\phi\rangle \geq 0$

$\Leftrightarrow$ Eigenvalues of ρ are non-negative

$$\mathrm{tr} \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$= \sum_i p_i \underbrace{\langle\psi_i|\psi_i\rangle}_{1} = 1$$

$$\left(\sum_i p_i |\psi_i\rangle\langle\psi_i|\right)^\dagger = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$\langle\phi|\rho|\phi\rangle = \sum_i p_i \underbrace{\langle\phi|\psi_i\rangle\langle\psi_i|\phi\rangle}_{|\langle\phi|\psi_i\rangle|^2}$$

$$\rho = \frac{143}{200}|0\rangle\langle 0| + \frac{1}{200}|0\rangle\langle 1| + \frac{1}{200}|1\rangle\langle 0| + \frac{57}{200}|1\rangle\langle 1|$$

$$= \frac{1}{200}\begin{pmatrix} 143 & 1 \\ 1 & 57 \end{pmatrix}$$

Properties of density matrix

- Has trace 1
- Hermitian
- Is positive semi-definite

$\forall |\phi\rangle \quad \langle\phi|\rho|\phi\rangle \geq 0$

$\Leftrightarrow$ Eigenvalues of ρ are non-negative

$$\operatorname{tr} \sum_i p_i |\psi_i\rangle\langle\psi_i| = \sum_i p_i \underbrace{\langle\psi_i|\psi_i\rangle}_{1} = 1$$

$$\left(\sum_i p_i |\psi_i\rangle\langle\psi_i|\right)^\dagger = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$\langle\phi|\rho|\phi\rangle = \sum_i p_i \underbrace{\langle\phi|\psi_i\rangle\langle\psi_i|\phi\rangle}_{|\langle\phi|\psi_i\rangle|^2 \geq 0} \geq 0$$

$$\rho = \frac{143}{200}|0\rangle\langle 0| + \frac{1}{200}|0\rangle\langle 1| + \frac{1}{200}|1\rangle\langle 0| + \frac{57}{200}|1\rangle\langle 1|$$

$$= \frac{1}{200}\begin{pmatrix} 143 & 1 \\ 1 & 57 \end{pmatrix}$$

Properties of density matrix

a) Has trace 1

b) Hermitian

c) Is positive semi-definite

$\forall |\phi\rangle \quad \langle\phi|\rho|\phi\rangle \geq 0$

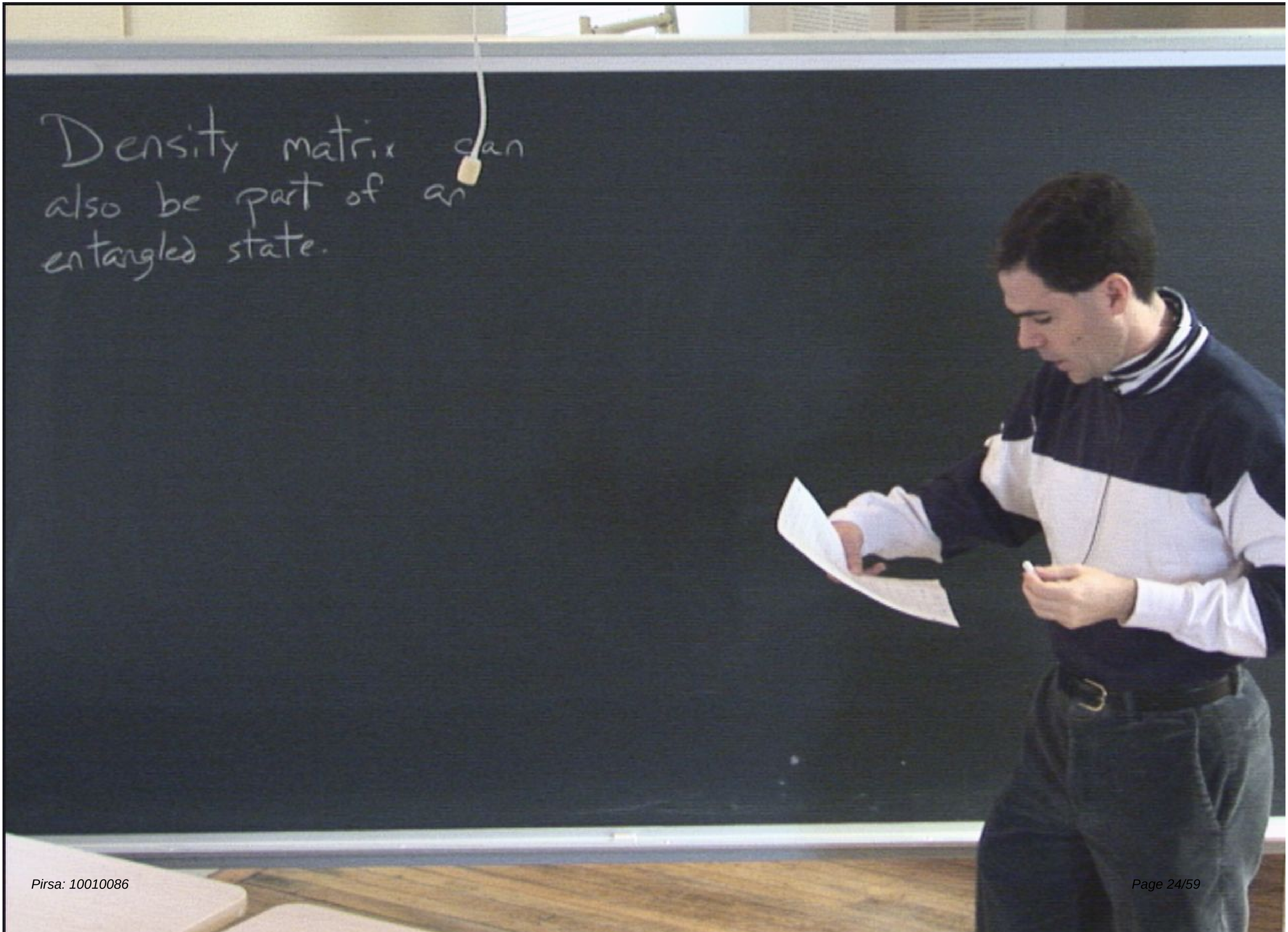
$\Leftrightarrow$ Eigenvalues of ρ are non-negative

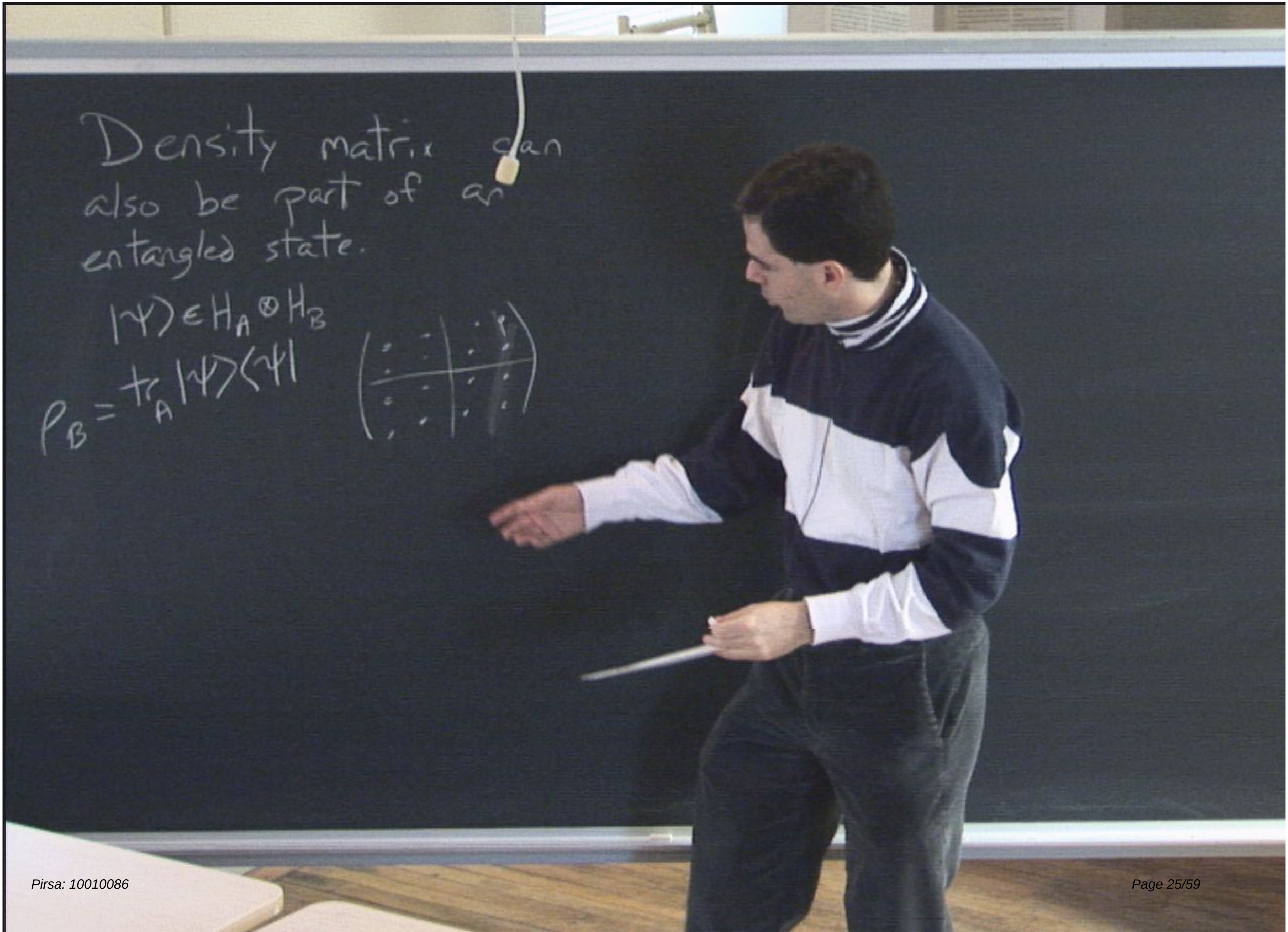
a) $\mathrm{tr} \sum_i p_i |\psi_i\rangle\langle\psi_i|$

$= \sum_i p_i \underbrace{\langle\psi_i|\psi_i\rangle}_{1} = 1$

b) $\left(\sum_i p_i |\psi_i\rangle\langle\psi_i|\right)^\dagger = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

c) $\langle\phi|\rho|\phi\rangle = \sum_i p_i \underbrace{\langle\phi|\psi_i\rangle\langle\psi_i|\phi\rangle}_{|\langle\phi|\psi_i\rangle|^2 \geq 0} \geq 0$

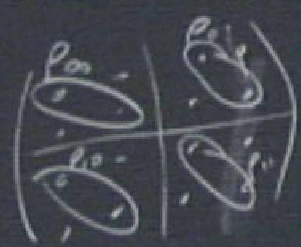

Density matrix can
also be part of an
entangled state.


Density matrix can
also be part of an
entangled state.
$|\psi\rangle \in H_A \otimes H_B$
$\rho_B = tr_A |\psi\rangle\langle\psi|$

Density matrix can also be part of an entangled state.

$|\psi\rangle \in H_A \otimes H_B$

$\rho_B = \mathrm{tr}_A |\psi\rangle\langle\psi|$



$\longrightarrow \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$

Density matrix can also be part of an entangled state.

$$|\psi\rangle \in H_A \otimes H_B$$

$$\rho_B = tr_A |\psi\rangle\langle\psi| \qquad \left(\begin{array}{c|c} \rho_{00} & \rho_{01} \\ \hline \rho_{10} & \rho_{11} \end{array}\right) \longrightarrow \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$$

$$\rho_A = tr_B |\psi\rangle\langle\psi|$$

...sity matrix can
...e part of an
...ed state.

$H_A \otimes H_B$

$|\psi\rangle\langle\psi|$

$|\psi\rangle\langle\psi|$

$$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \longrightarrow \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$$

Any density matrix can
be purified:

ρ = density matrix on H_A

$\exists\ |\psi\rangle \in H_A \otimes H_B$ s.t. $tr_B\, |\psi\rangle\langle\psi| = \rho$

sity matrix can

be part of an

ed state.

$H_A \otimes H_B$

$|\psi\rangle\langle\psi|$

$|\psi\rangle\langle\psi|$

$\rho_{00} \quad \rho_{01} \quad \rho_{10} \quad \rho_{11} \longrightarrow \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$

Any density matrix can <u>be purified</u>

ρ = density matrix on H_A

$\exists\, |\psi\rangle \in H_A \otimes H_B$ s.t. $\mathrm{tr}_B |\psi\rangle\langle\psi| = \rho$

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$|\psi\rangle = \sum_i \sqrt{p_i}\, |\psi_i\rangle_A \otimes |i\rangle_B$$

...sity matrix can
...be part of an
...ed state.

$H_A \otimes H_B$

$|\psi\rangle\langle\psi|$

$|\psi\rangle\langle\psi|$

ρ_{00} ρ_{01} ρ_{10} ρ_{11} $\rightarrow \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$

Any density matrix can <u>be purified</u>

ρ = density matrix on H_A

$\exists\, |\psi\rangle \in H_A \otimes H_B$ s.t. $\mathrm{tr}_B |\psi\rangle\langle\psi| = \rho$

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$|\psi\rangle = \sum_i \sqrt{p_i}\, |\psi_i\rangle_A \otimes |i\rangle_B$$

$$\mathrm{tr}_B |\psi\rangle\langle\psi| = \rho$$

...sity matrix can
...be part of an
...ed state.

$H_A \otimes H_B$

$|\psi\rangle\langle\psi|$

$|\psi\rangle\langle\psi|$

$\longrightarrow \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$

Any density matrix can
be purified

ρ = density matrix on H_A

$\exists\ |\psi\rangle \in H_A \otimes H_B$ s.t. $\mathrm{tr}_B |\psi\rangle\langle\psi| = \rho$

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$|\psi\rangle = \sum_i \sqrt{p_i}\, |\psi_i\rangle_A \otimes |i\rangle_B$$

$$\mathrm{tr}_B |\psi\rangle\langle\psi| = \rho$$

sity matrix can
be part of an
ed state.

$H_A \otimes H_B$

$|\psi\rangle\langle\psi|$

$|\psi\rangle\langle\psi|$

$$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \longrightarrow \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$$

Any density matrix can be purified

ρ = density matrix on H_A

$\exists\, |\psi\rangle \in H_A \otimes H_B$ s.t. $tr_B |\psi\rangle\langle\psi| = \rho$

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$|\psi\rangle = \sum_i \sqrt{p_i}\, |\psi_i\rangle_A \otimes |i\rangle_B$$

$$tr_B |\psi\rangle\langle\psi| = \rho$$

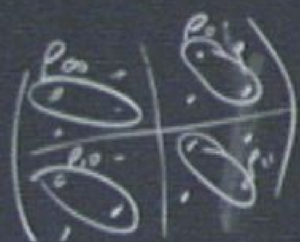

sity matrix can
be part of an
d state.
Any density matrix
be purified
ρ = density matrix on H_A
∃ |ψ⟩ ∈ H_A ⊗ H_B s.t.
ρ_AB on H_A ⊗ H_B
state on A ρ_A = tr_B ρ_AB
ρ = Σ_i p_i |ψ_i⟩⟨ψ_i|
|ψ⟩ = Σ_i √p_i |ψ_i⟩
tr_B |ψ⟩

...sity matrix can
...e part of an
...d state.

$H_A \otimes H_B$

$|\psi\rangle\langle\psi|$

$|\psi\rangle\langle\psi|$



$\longrightarrow \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$

ρ_{AB} on $H_A \otimes H_B$

State on A $\rho_A = tr_B \, \rho_{AB}$

Any density matrix can <u>be purified</u>

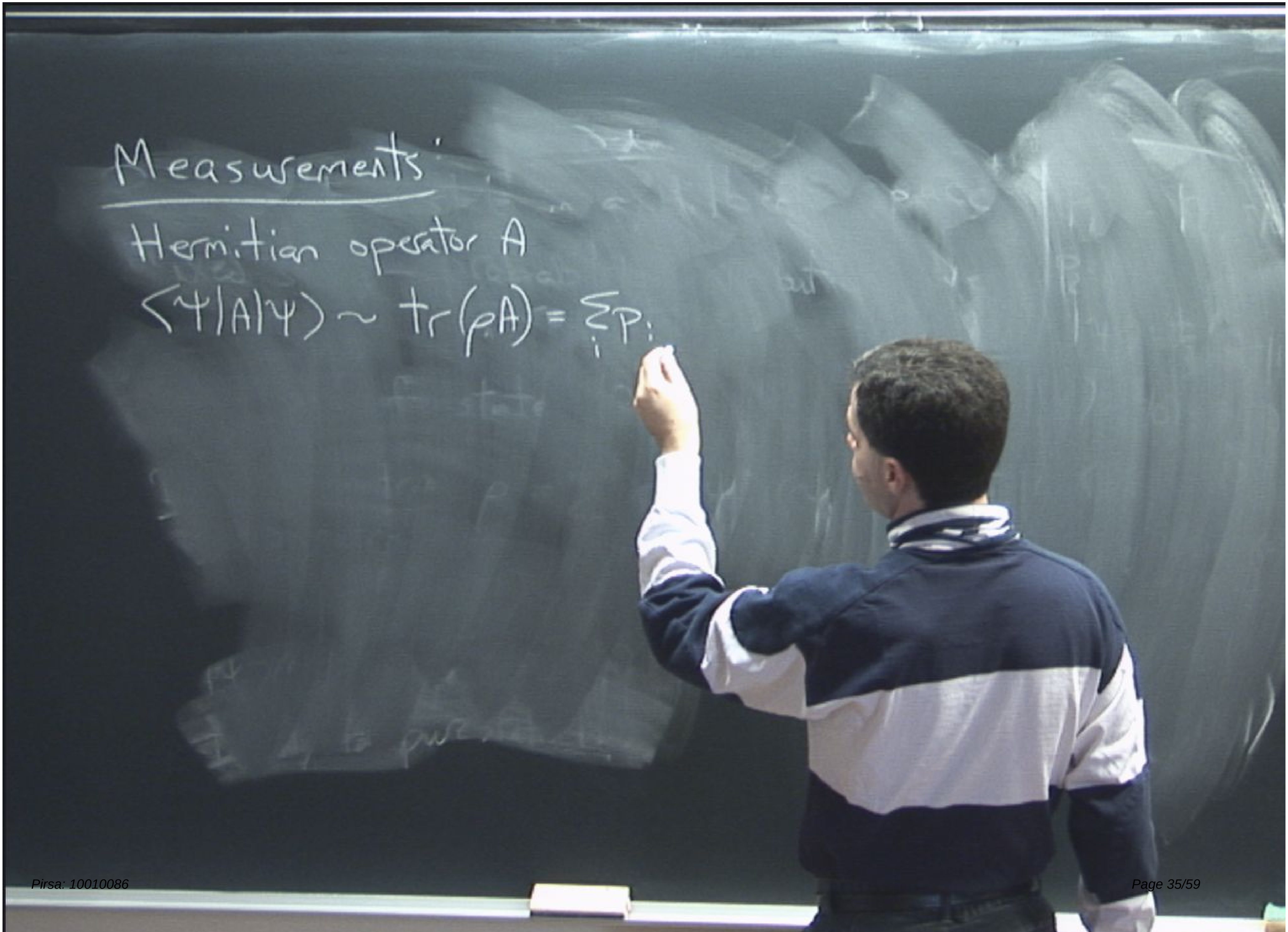
ρ = density matrix on H_A

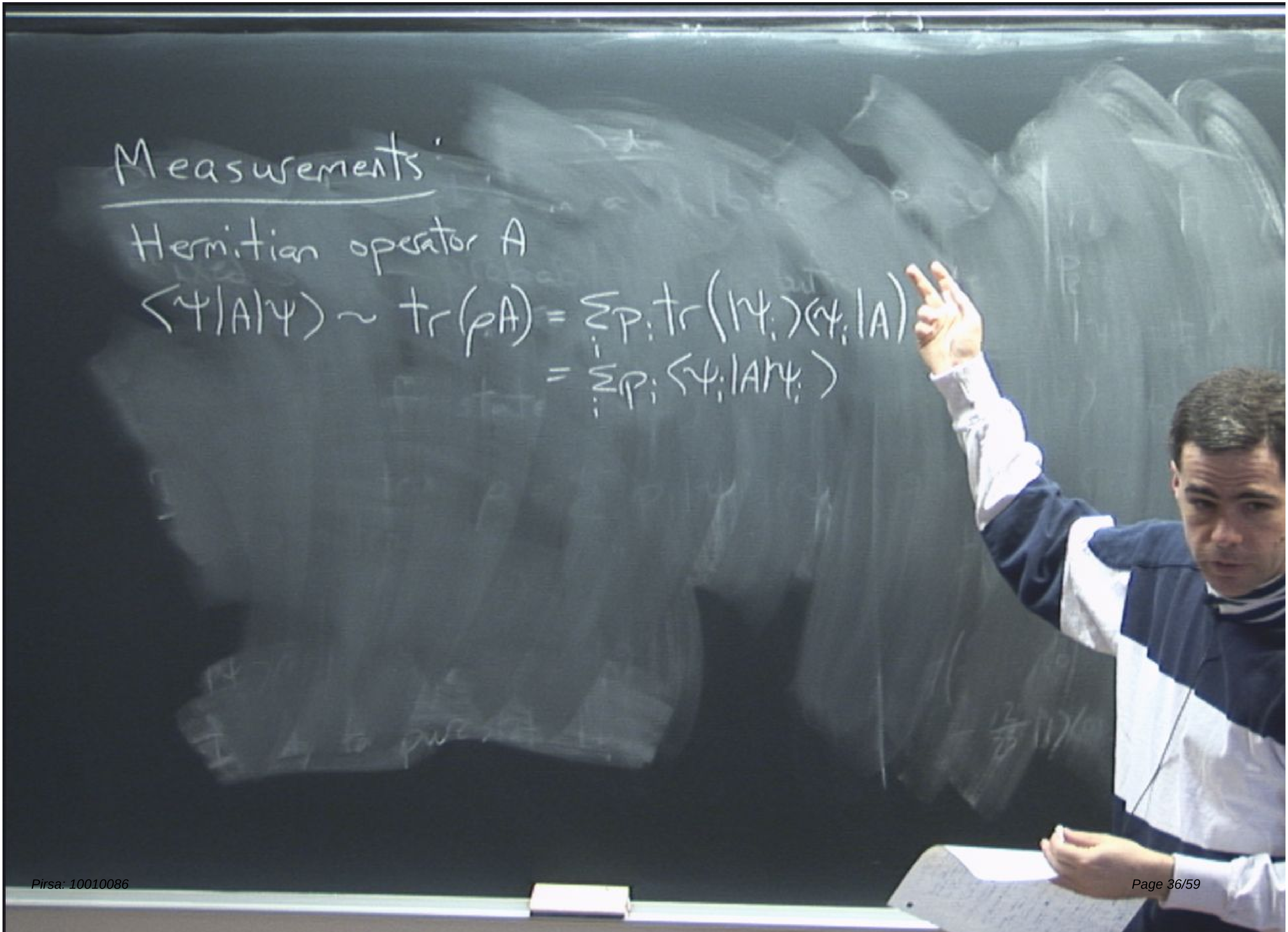
$\exists \, |\psi\rangle \in H_A \otimes H_B$ s.t. $tr_B |\psi\rangle\langle\psi| = \rho$

$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

$|\psi\rangle = \sum_i \sqrt{p_i}\, |\psi_i\rangle_A \otimes |i\rangle_B$

$tr_B |\psi\rangle\langle\psi| = \rho$


Measurements:
Hermitian operator A
⟨ψ|A|ψ⟩ ~ tr(ρA) = Σ_i p_i


Measurements:
Hermitian operator A
$\langle\psi|A|\psi\rangle \sim \mathrm{tr}(\rho A) = \sum_i p_i \mathrm{tr}(|\psi_i\rangle\langle\psi_i|A)$
$= \sum_i p_i \langle\psi_i|A|\psi_i\rangle$

Measurements:

Hermitian operator A

$$\langle\psi|A|\psi\rangle \sim \mathrm{tr}(\rho A) = \sum_i p_i \,\mathrm{tr}\left(|\psi_i\rangle\langle\psi_i| A\right)$$

$$= \sum_i p_i \langle\psi_i|A|\psi_i\rangle$$

Projective measurement $\Pi,\ I-\Pi$

$\mathrm{tr}(\rho\Pi)$ is prob. of outcome Π

Measurements:

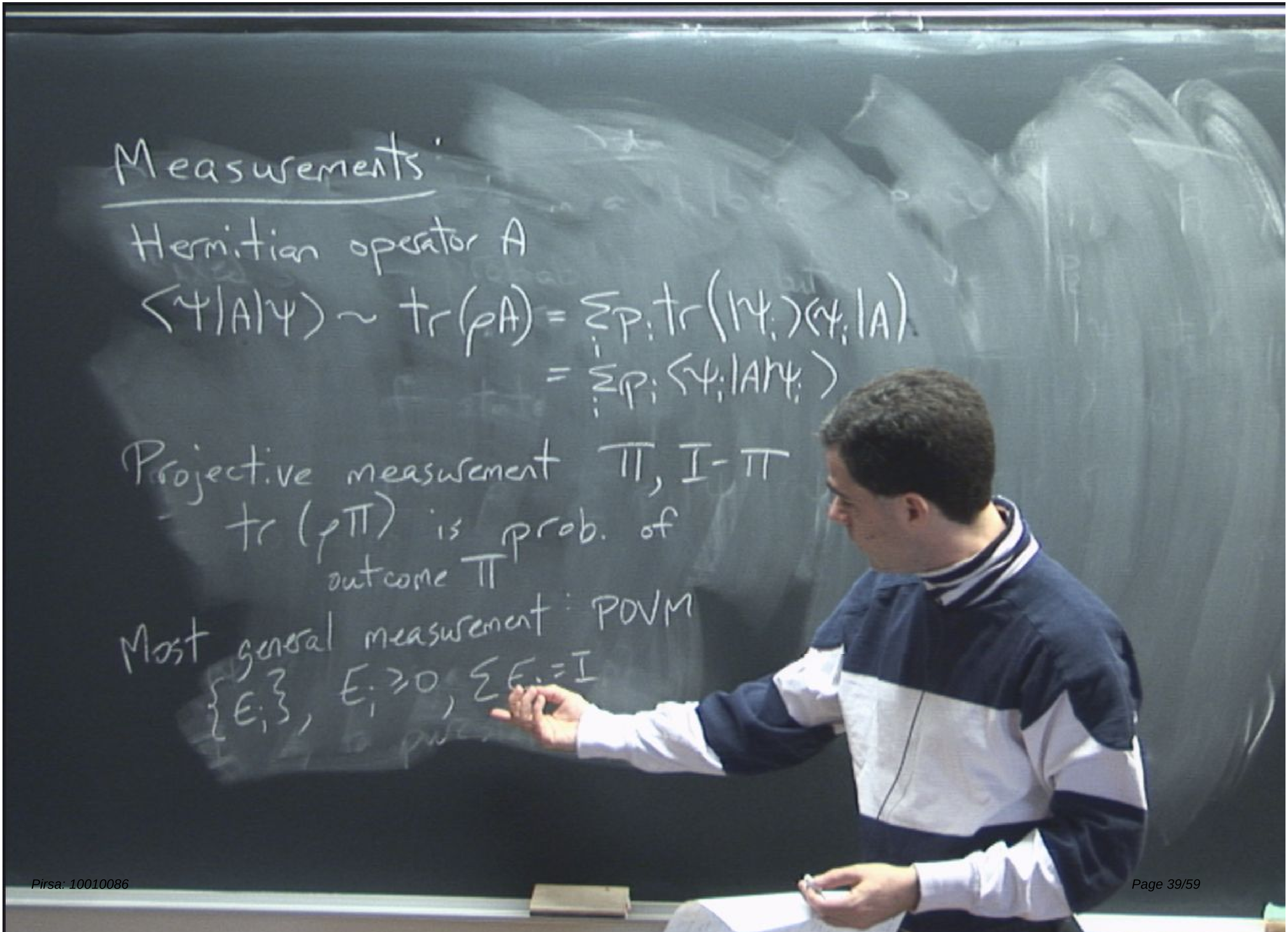
Hermitian operator A

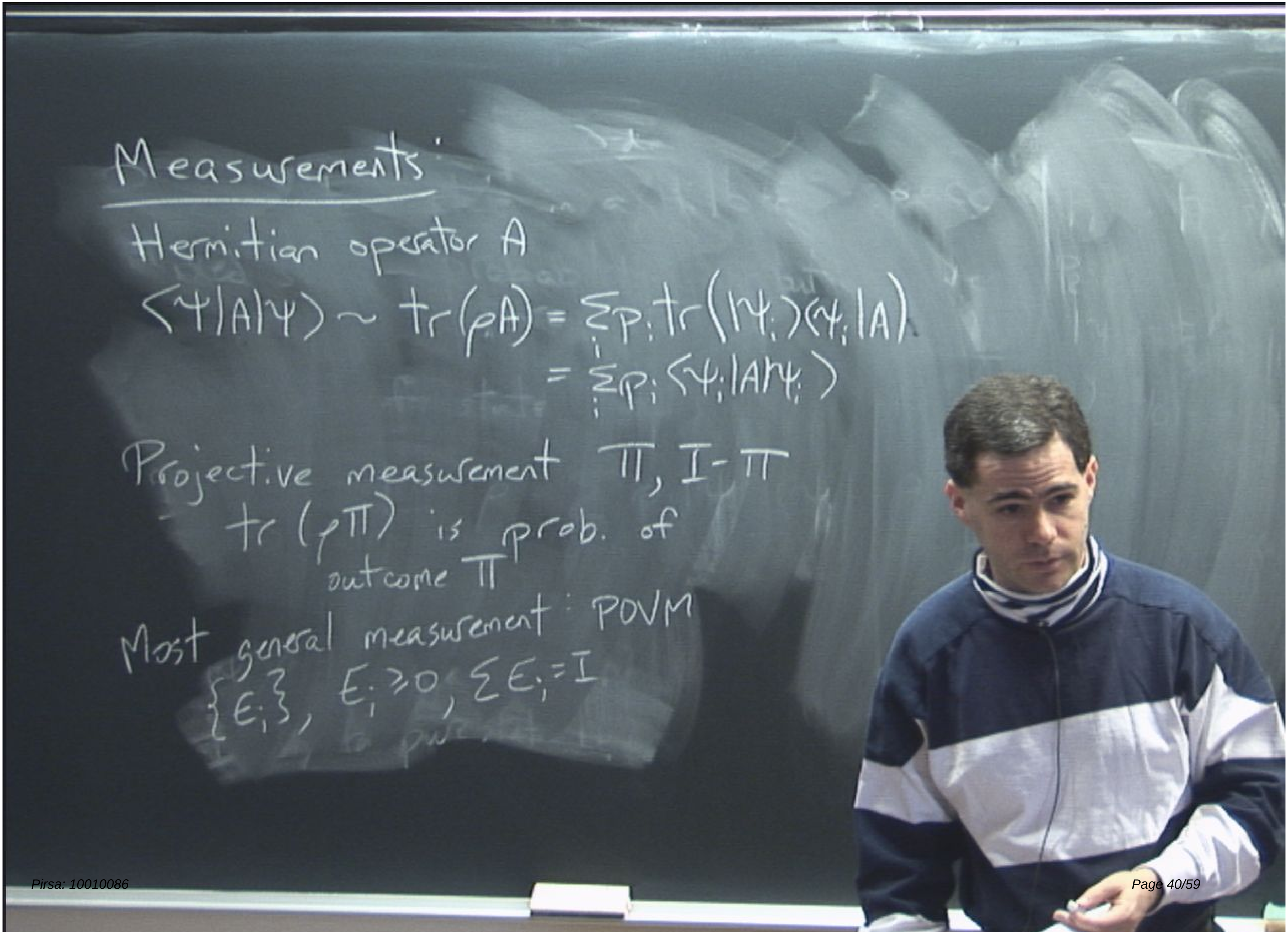
$$\langle\psi|A|\psi\rangle \sim \mathrm{tr}(\rho A) = \sum_i p_i \,\mathrm{tr}\left(|\psi_i\rangle\langle\psi_i| A\right)$$
$$= \sum_i p_i \langle\psi_i|A|\psi_i\rangle$$

Projective measurement $\Pi, I-\Pi$

$\mathrm{tr}(\rho\Pi)$ is prob. of outcome Π

Most general measurement: POVM


Measurements:
Hermitian operator A
$\langle\psi|A|\psi\rangle \sim \mathrm{tr}(\rho A) = \sum_i p_i \mathrm{tr}(|\psi_i\rangle\langle\psi_i|A)$
$= \sum_i p_i \langle\psi_i|A|\psi_i\rangle$
Projective measurement $\Pi, I-\Pi$
$\mathrm{tr}(\rho\Pi)$ is prob. of outcome Π
Most general measurement: POVM
$\{E_i\}, E_i \geq 0, \sum E_i = I$


Measurements:
Hermitian operator A
$\langle\psi|A|\psi\rangle \sim tr(\rho A) = \sum_i p_i tr(|\psi_i\rangle\langle\psi_i| A)$
$= \sum_i p_i \langle\psi_i|A|\psi_i\rangle$
Projective measurement $\Pi, I-\Pi$
$tr(\rho\Pi)$ is prob. of outcome Π
Most general measurement: POVM
$\{E_i\}, E_i \geq 0, \sum E_i = I$

Measurements:

Hermitian operator A

$$\langle\psi|A|\psi\rangle \sim tr(\rho A) = \sum_i p_i \, tr(|\psi_i\rangle\langle\psi_i| A)$$

$$= \sum_i p_i \langle\psi_i|A|\psi_i\rangle$$

Projective measurement $\Pi, I-\Pi$

$tr(\rho\Pi)$ is prob. of outcome Π

Most general measurement: POVM

$\{E_i\}$, $E_i \geq 0$, $\sum E_i = I$

Prob. of outcome i is $tr(\rho E_i)$

Measurements:

Hermitian operator A

$$\langle\psi|A|\psi\rangle \sim \mathrm{tr}(\rho A) = \sum_i p_i \, \mathrm{tr}(|\psi_i\rangle\langle\psi_i|A)$$
$$= \sum_i p_i \langle\psi_i|A|\psi_i\rangle$$

Projective measurement $\Pi, I-\Pi$

$\mathrm{tr}(\rho\Pi)$ is prob. of outcome Π

Most general measurement: POVM

$\{E_i\}$, $E_i \geq 0$, $\sum E_i = I$

Prob. of outcome i is $\mathrm{tr}(\rho E_i)$

A

$) = \sum_i p_i \, tr(|\psi_i\rangle\langle\psi_i| A)$

$= \sum_i p_i \langle\psi_i|A|\psi_i\rangle$

nt $\Pi, I-\Pi$

prob. of

nt: POVM

$\sum_i E_i = I$

outcome i is $tr(\rho E_i)$

for any POVM $\{E_i\}$, $\exists$ circuit

$\rho = \frac{143}{200}|0\rangle\langle 0| + \frac{1}{200}$

$+ \frac{1}{200}|1\rangle\langle 0| + \frac{57}{200}$

$= \frac{1}{200}\begin{pmatrix} 143 & 1 \\ 1 & 57 \end{pmatrix}$

Properties of densit

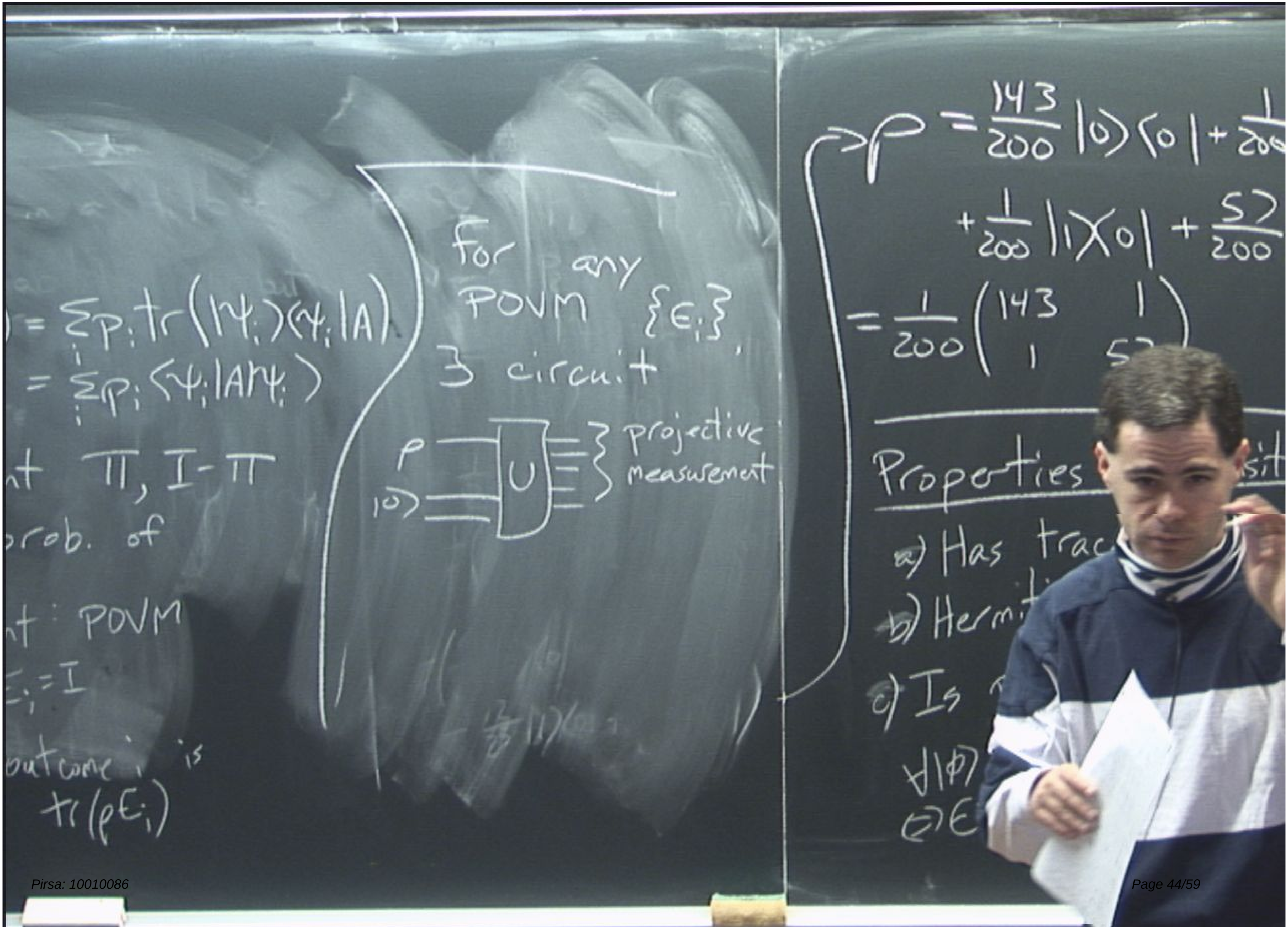
Has trace 1

...itian

...ositive semi-defi

$\langle\phi|\rho|\phi\rangle \geq 0$

...values of ρ are



= Σp_i tr(|ψ_i⟩⟨ψ_i|A)
= Σp_i ⟨ψ_i|A|ψ_i⟩
nt Π, I-Π
prob. of
nt: POVM
E_i = I
outcome i is
tr(ρE_i)
For any POVM {E_i},
∃ circuit
ρ
|0⟩
U
projective measurement
ρ = 143/200 |0⟩⟨0| + 1/200
+ 1/200 |1⟩⟨0| + 57/200
= 1/200 (143 1 ; 1 57)
Properties
a) Has trac
b) Hermi
c) Is
∀|φ⟩
e) E

$= \sum_i p_i \,\mathrm{tr}(|\psi_i\rangle\langle\psi_i| A)$

$= \sum_i p_i \langle\psi_i|A|\psi_i\rangle$

nt $\Pi, I-\Pi$

prob. of

nt: POVM

$\sum_i E_i = I$

outcome i is

$\mathrm{tr}(\rho E_i)$

for any POVM $\{E_i\}$, $\exists$ circuit

$$\rho \longrightarrow \boxed{U} \longrightarrow \left.\right\} \text{projective measurement}$$

$$|0\rangle \longrightarrow$$

that implements $\{E_i\}$

$\rho = \frac{143}{200}|0\rangle\langle 0| + \frac{1}{200}\ldots$

$+ \frac{1}{200}|1\rangle\langle 0| + \frac{57}{200}\ldots$

$= \frac{1}{200}\begin{pmatrix} 143 & 1 \\ 1 & 57 \end{pmatrix}$

Properties of densit...

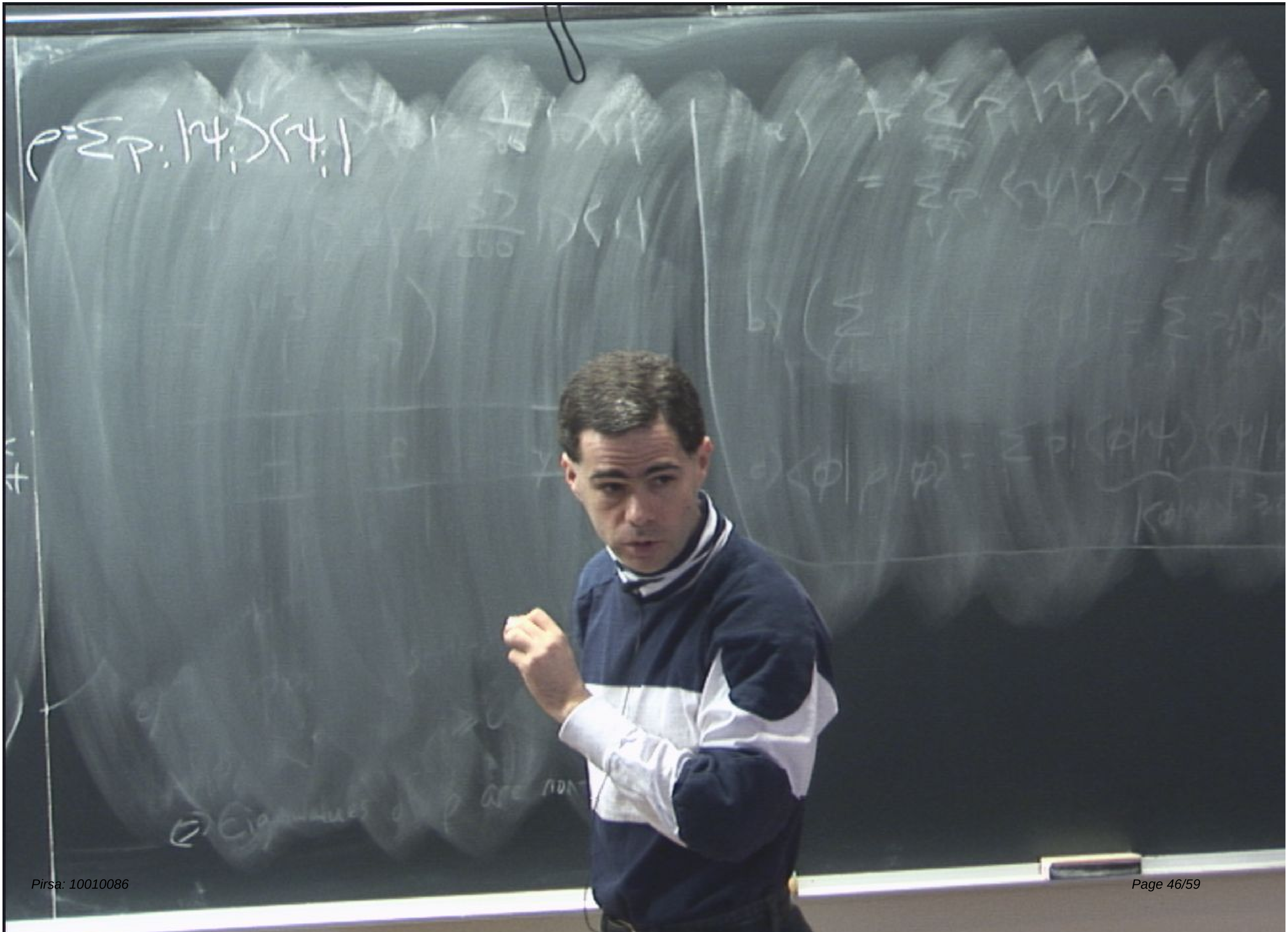
a) Has trace 1

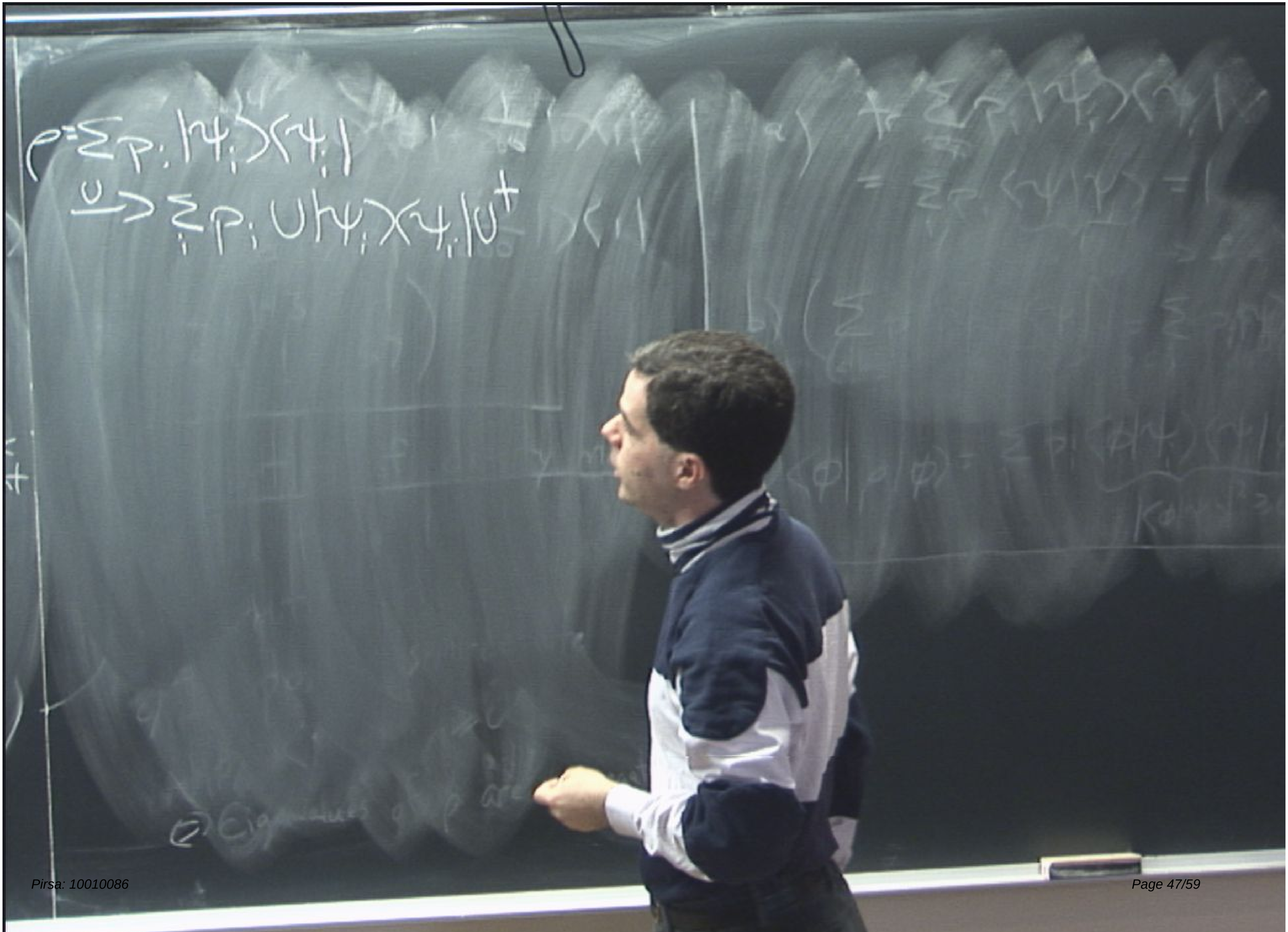
b) Hermitian

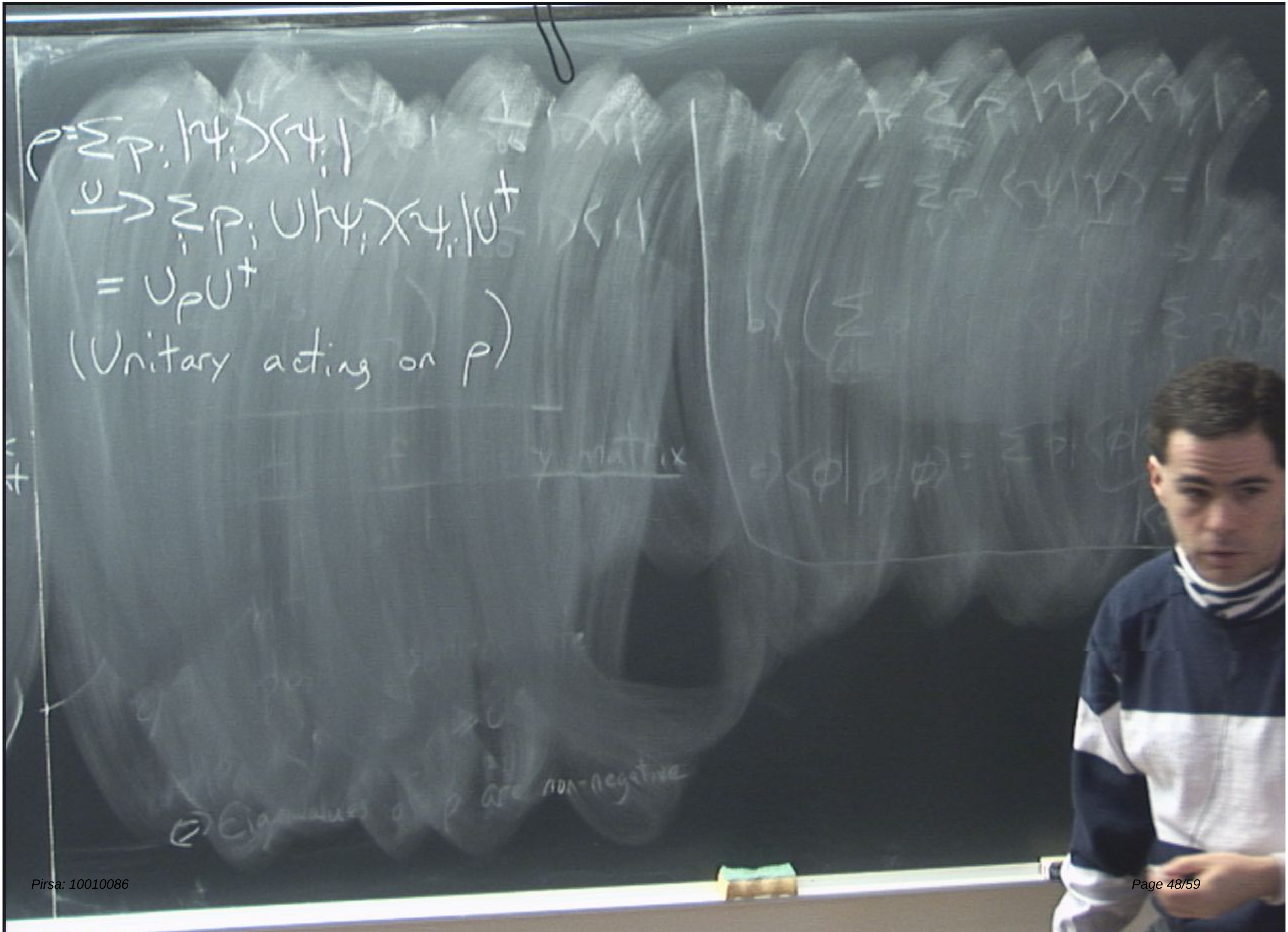
c) Is positive semi-def...

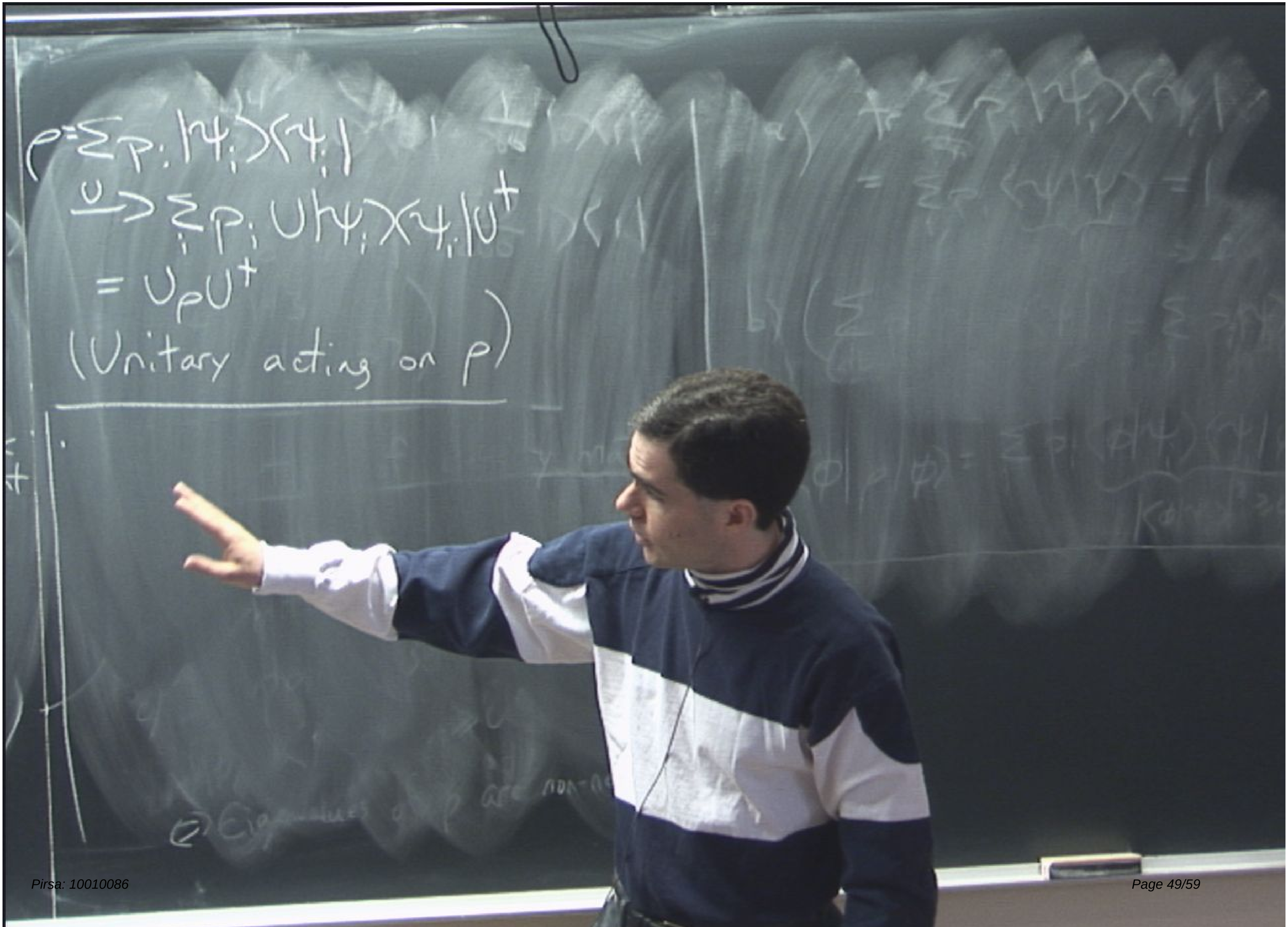
$\forall|\phi\rangle \quad \langle\phi|\rho|\phi\rangle \geq 0$

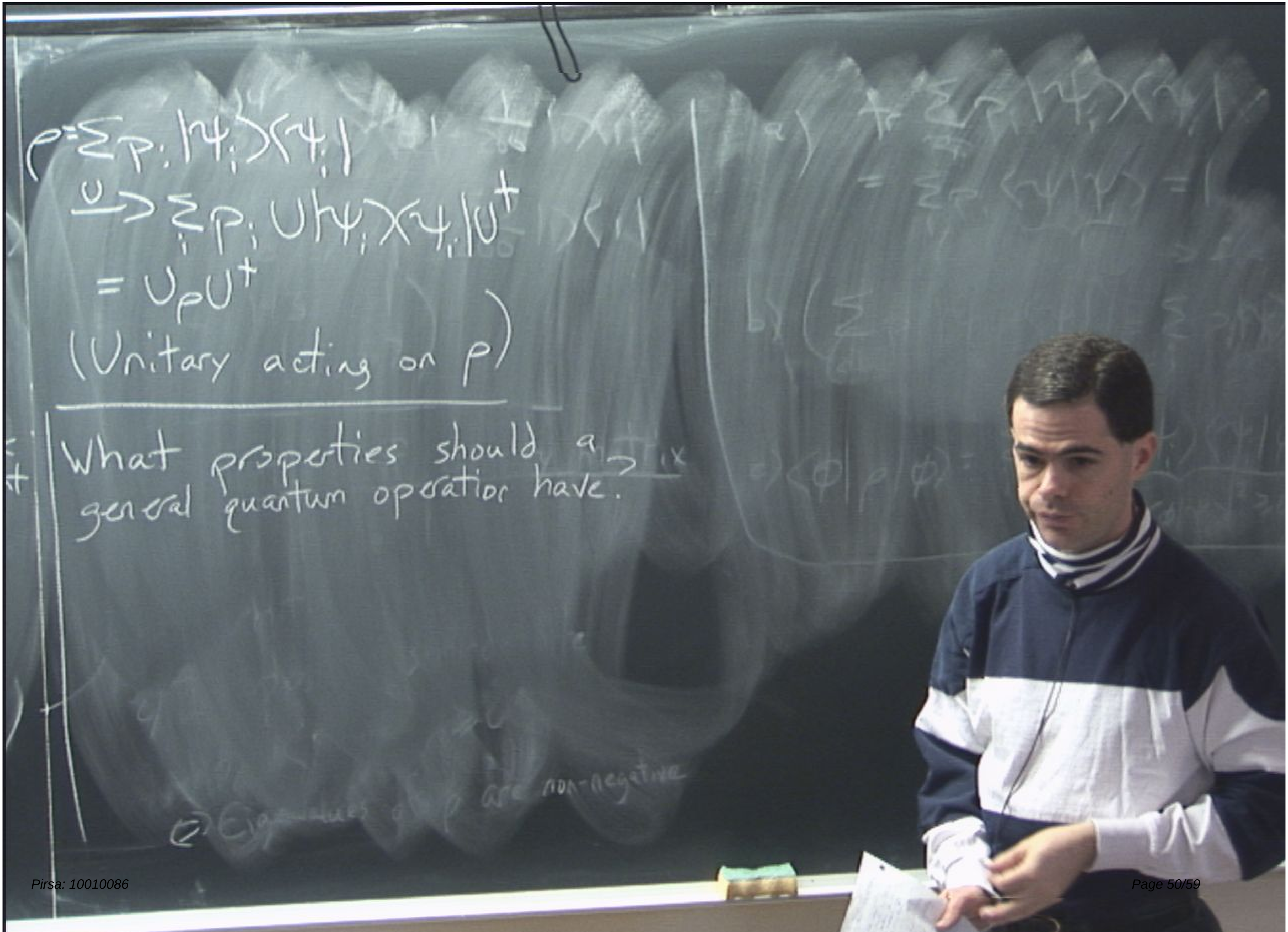
$\Leftrightarrow$ Eigenvalues of ρ are...

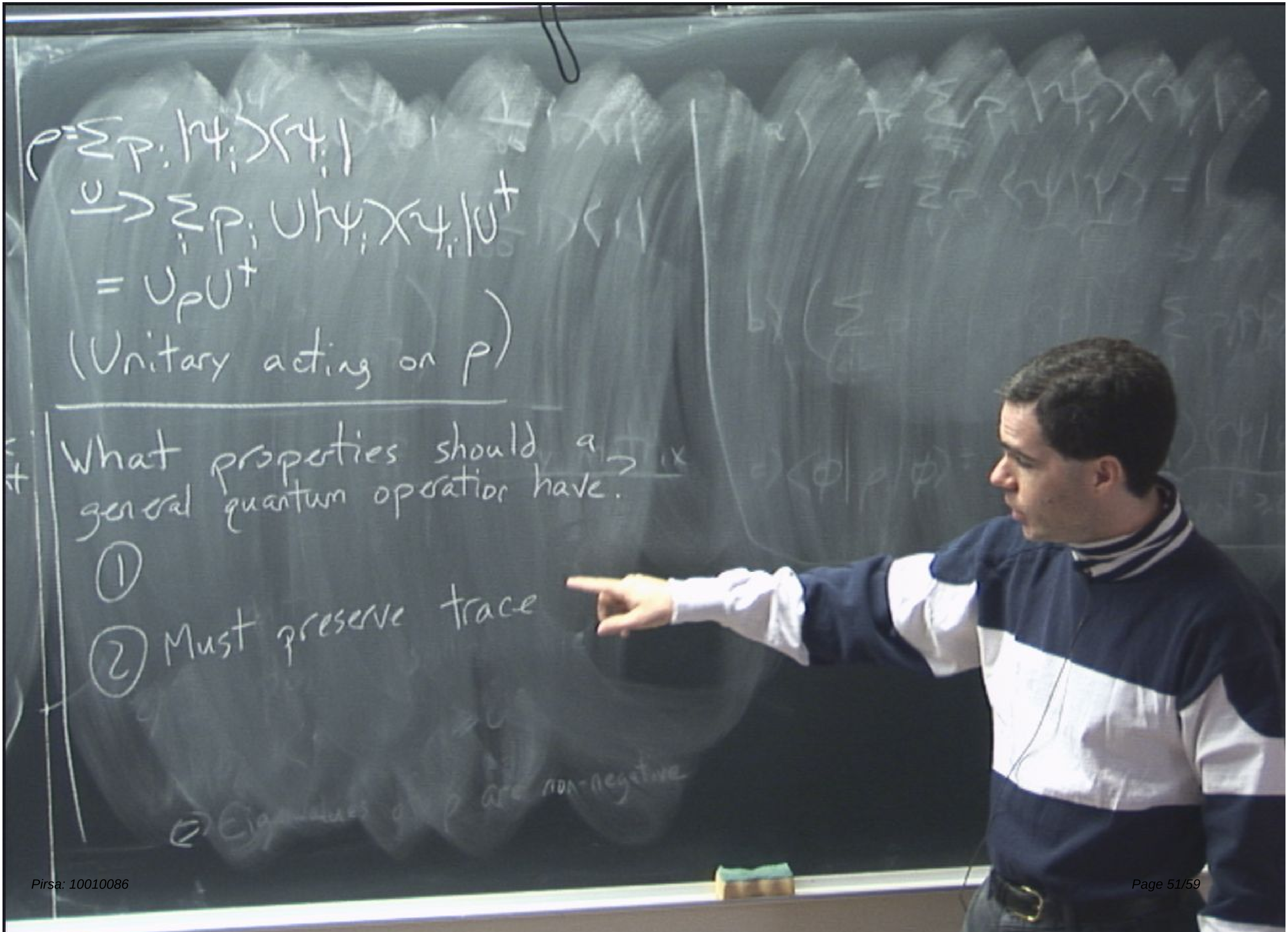

ρ = Σ p_i |ψ_i⟩⟨ψ_i|

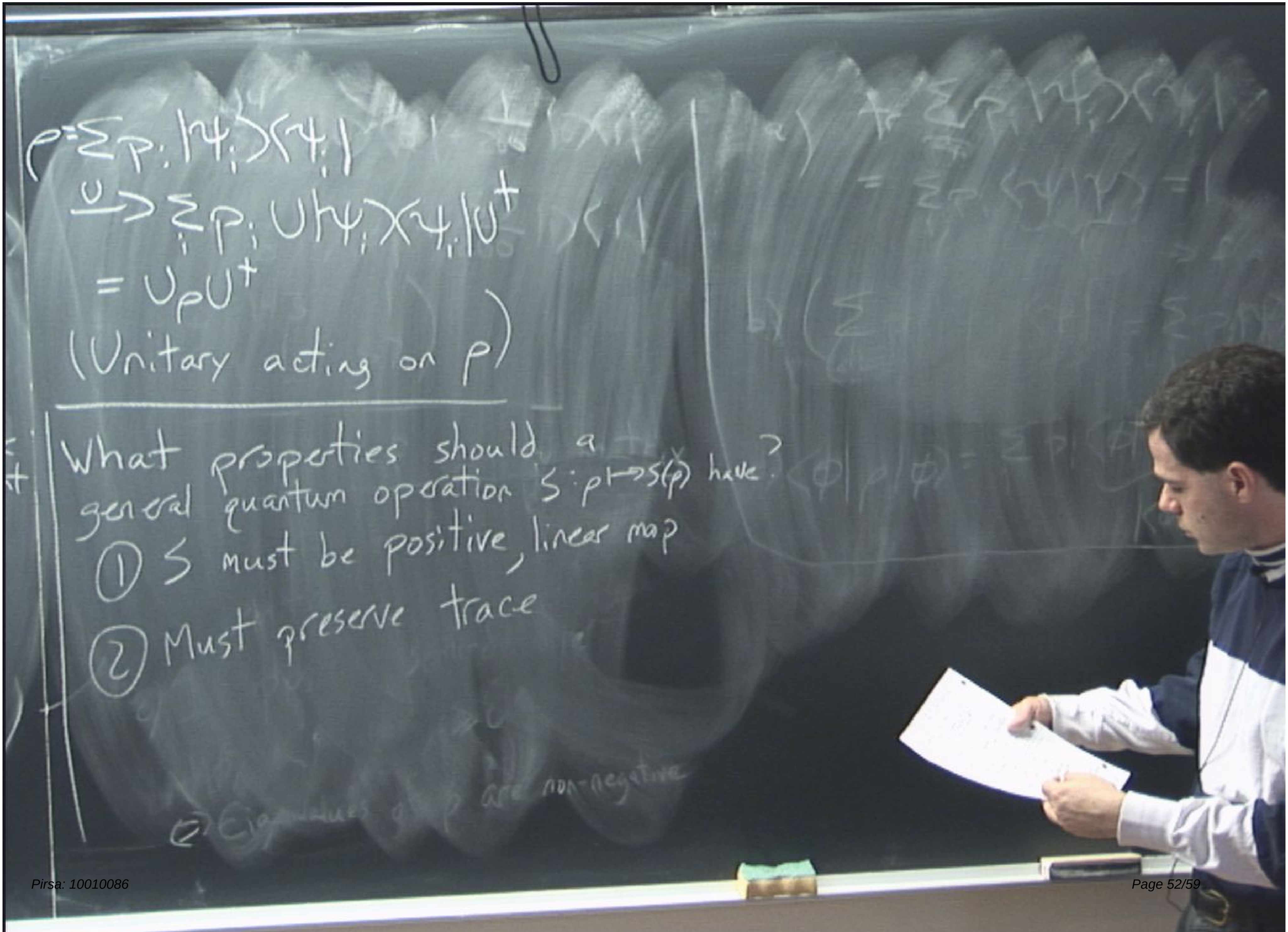

$\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$
$\xrightarrow{U} \sum_i p_i U|\psi_i\rangle\langle\psi_i|U^\dagger$


$\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$
$\xrightarrow{U} \sum_i p_i U|\psi_i\rangle\langle\psi_i|U^\dagger$
$= U\rho U^\dagger$
(Unitary acting on ρ)


$\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$
$\xrightarrow{U} \sum_i p_i U|\psi_i\rangle\langle\psi_i|U^\dagger$
$= U\rho U^\dagger$
(Unitary acting on ρ)


$\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$
$\xrightarrow{U} \sum_i p_i U|\psi_i\rangle\langle\psi_i|U^\dagger$
$= U\rho U^\dagger$
(Unitary acting on ρ)
What properties should a general quantum operation have?


ρ=Σp_i |ψ_i⟩⟨ψ_i|
U→ Σ_i p_i U|ψ_i⟩⟨ψ_i|U†
= UρU†
(Unitary acting on ρ)
What properties should a general quantum operation have?
1
2 Must preserve trace


$\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$
$\xrightarrow{U} \sum_i p_i U|\psi_i\rangle\langle\psi_i|U^\dagger$
$= U\rho U^\dagger$
(Unitary acting on ρ)
What properties should a general quantum operation $S: \rho \mapsto S(\rho)$ have?
① S must be positive, linear map
② Must preserve trace

$$\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$$
$$\xrightarrow{U} \sum_i p_i U|\psi_i\rangle\langle\psi_i|U^\dagger$$
$$= U\rho U^\dagger$$
(Unitary acting on ρ)

What properties should a general quantum operation $S: \rho \mapsto S(\rho)$ have?

1. S must be positive, linear map
2. Must preserve trace
3. Must be "completely positive" $\iff I \otimes S$ is positive

$$\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$$

$$\xrightarrow{U} \sum_i p_i U|\psi_i\rangle\langle\psi_i|U^\dagger$$

$$= U\rho U^\dagger$$

(Unitary acting on ρ)

What properties should a general quantum operation $S: \rho \mapsto S(\rho)$ have?

① S must be positive, linear map

② Must preserve trace

③ Must be "completely positive"

$\Leftrightarrow I \otimes S$ is positive

$\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$

$\xrightarrow{U} \sum_i p_i U|\psi_i\rangle\langle\psi_i|U^\dagger$

$= U\rho U^\dagger$

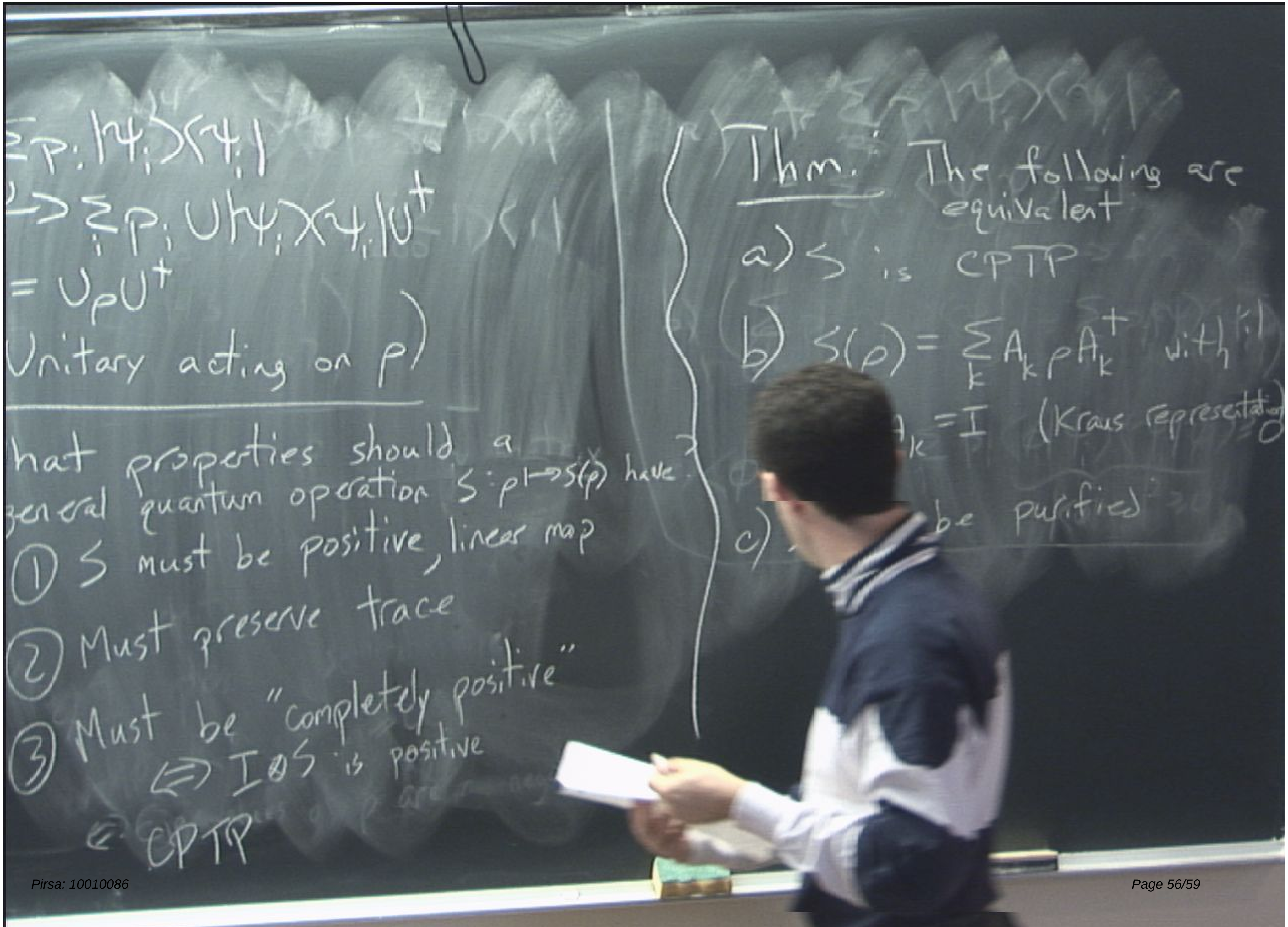
(Unitary acting on ρ)

What properties should a general quantum operation $S : \rho \mapsto S(\rho)$ have?

1. S must be positive, linear map
2. Must preserve trace
3. Must be "completely positive"

$\Leftrightarrow I \otimes S$ is positive

$\Leftarrow$ CPTP


Thm: The following are equivalent
a) S is CPTP
b) S(ρ) = Σ_k A_k ρ A_k^† with
(Kraus representation)
be purified
c)
Σ p_i |ψ_i⟩⟨ψ_i|
⟶ Σ p_i U|ψ_i⟩⟨ψ_i|U^†
= UρU^†
(Unitary acting on ρ)
What properties should a general quantum operation S: ρ ↦ S(ρ) have?
① S must be positive, linear map
② Must preserve trace
③ Must be "completely positive"
⟺ I⊗S is positive
⟸ CPTP

$\sum p_i |\psi_i\rangle\langle\psi_i|$

$\mapsto \sum_i p_i U|\psi_i\rangle\langle\psi_i|U^\dagger$

$= U\rho U^\dagger$

Unitary acting on ρ)

hat properties should a
eneral quantum operation $S: \rho \mapsto S(\rho)$ have?

(1) S must be positive, linear map

(2) Must preserve trace

(3) Must be "completely positive"

$\Leftrightarrow I \otimes S$ is positive

$\Leftarrow$ CPTP

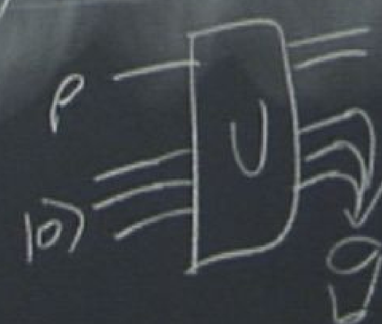
Thm: The following are equivalent

a) S is CPTP

b) $S(\rho) = \sum_k A_k \rho A_k^\dagger$ with

$\sum_k A_k^\dagger A_k = I$ (Kraus representation)

c) S can be purified



$p_i |\psi_i\rangle\langle\psi_i|$

$\to \sum_i p_i U|\psi_i\rangle\langle\psi_i|U^\dagger$

$= U\rho U^\dagger$

(unitary acting on ρ)

What properties should a general quantum operation $S: \rho \mapsto S(\rho)$ have?

1) S must be positive, linear map

2) Must preserve trace

3) Must be "completely positive"

$\Longleftrightarrow I \otimes S$ is positive

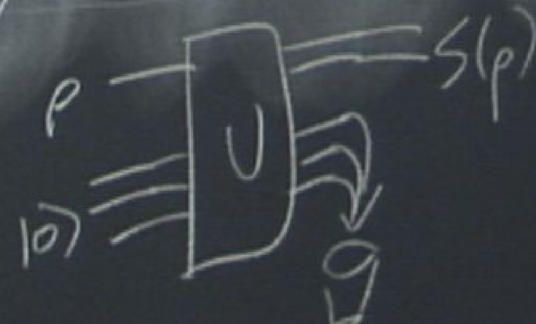
$\Leftarrow$ CPTP

Thm. The following are equivalent

a) S is CPTP

b) $S(\rho) = \sum_k A_k \rho A_k^\dagger$ with $\sum_k A_k^\dagger A_k = I$ (Kraus representation)

c) S can be purified



$p_i |\psi_i\rangle\langle\psi_i|$

$\Rightarrow \sum_i p_i U|\psi_i\rangle\langle\psi_i|U^\dagger$

$= U\rho U^\dagger$

(unitary acting on ρ)

What properties should a general quantum operation $S: \rho \mapsto S(\rho)$ have?

1) S must be positive, linear map

2) Must preserve trace

3) Must be "completely positive"

$\Leftrightarrow I \otimes S$ is positive

$\Leftarrow$ CPTP

Thm. The following are equivalent

a) S is CPTP

b) $S(\rho) = \sum_k A_k \rho A_k^\dagger$ with $\sum_k A_k^\dagger A_k = I$ (Kraus representation)

c) S can be purified