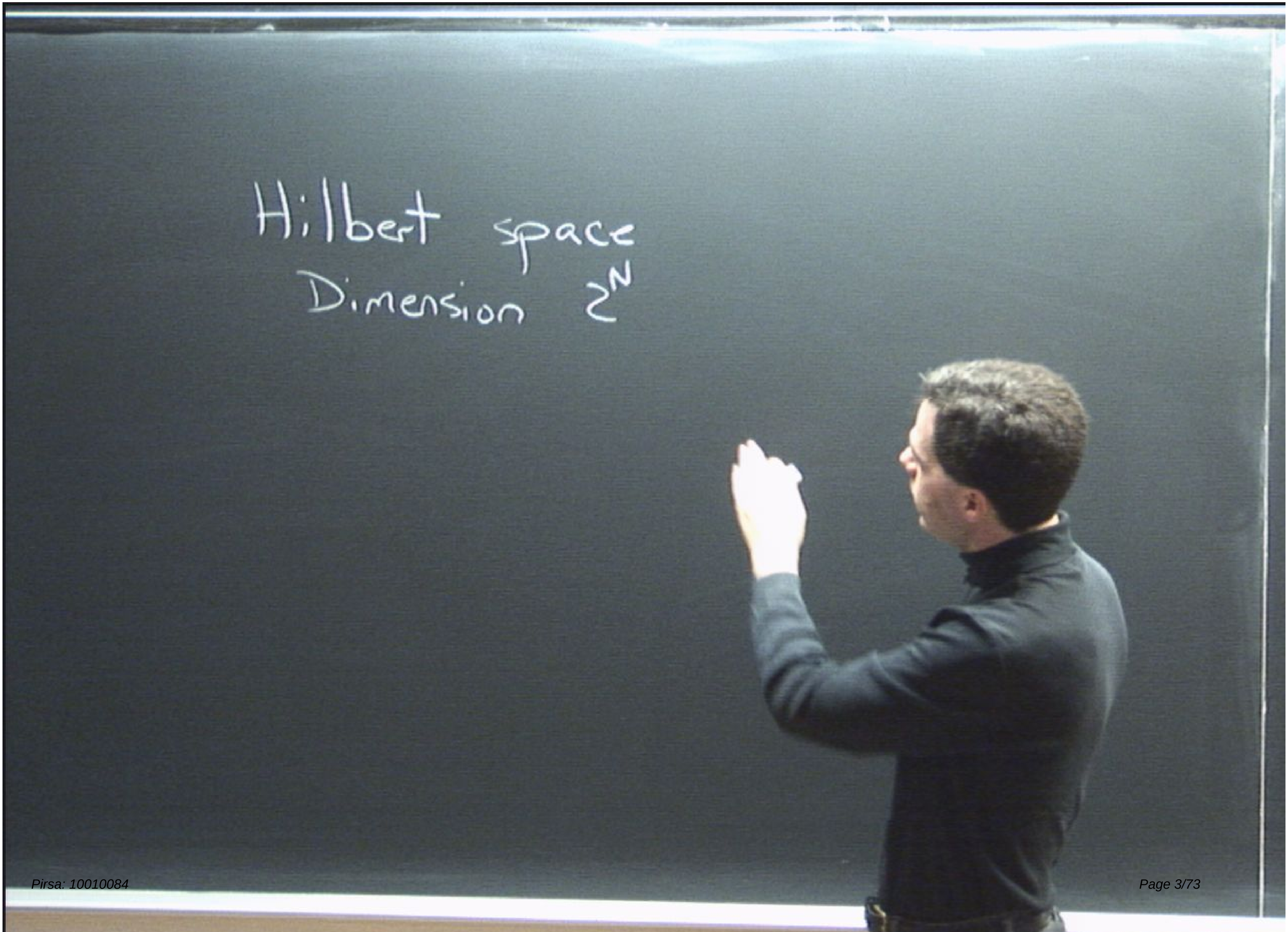
Title: Quantum Information - Review (PHYS 635) - Lecture 1

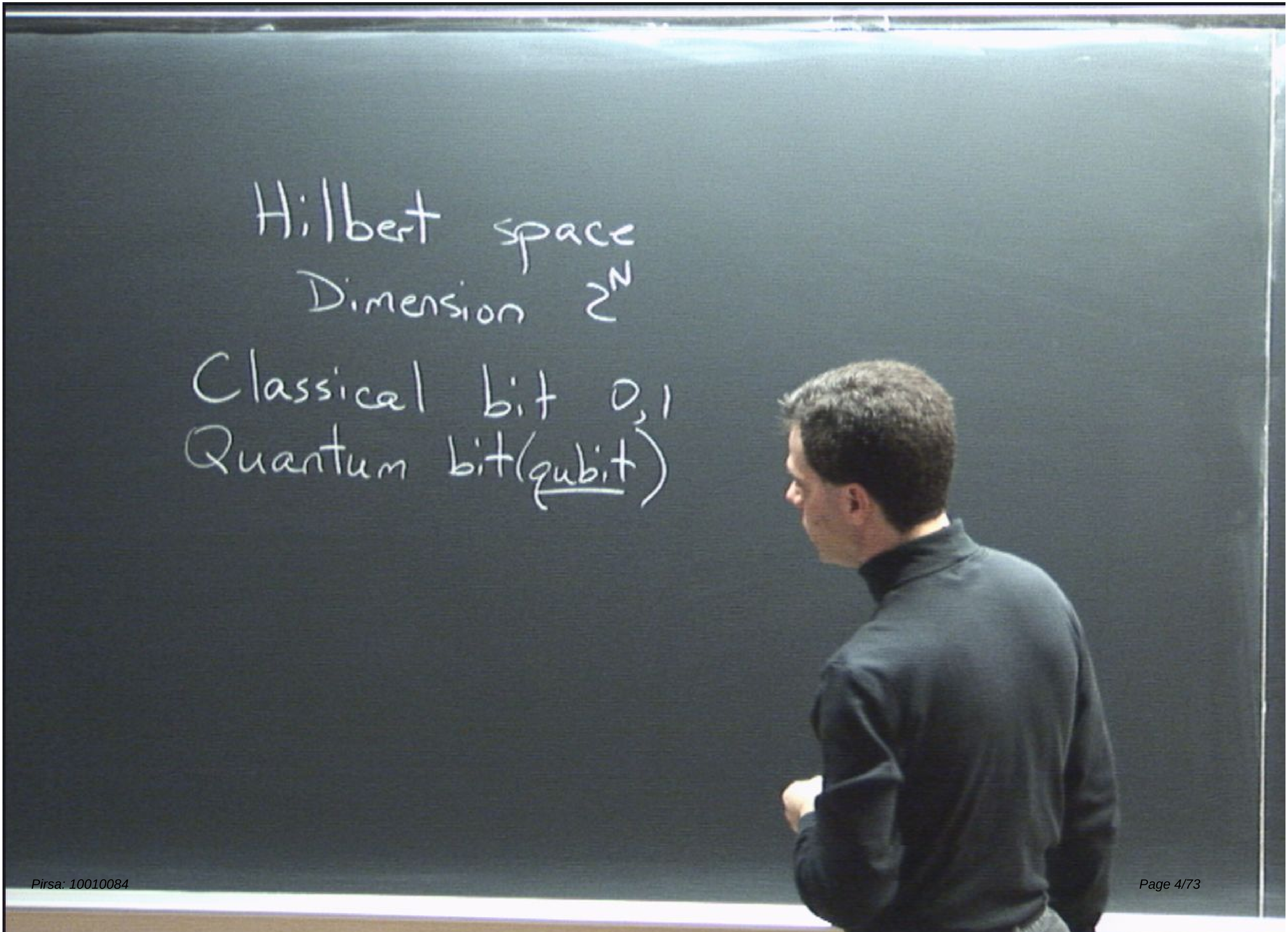
Date: Jan 25, 2010 09:00 AM

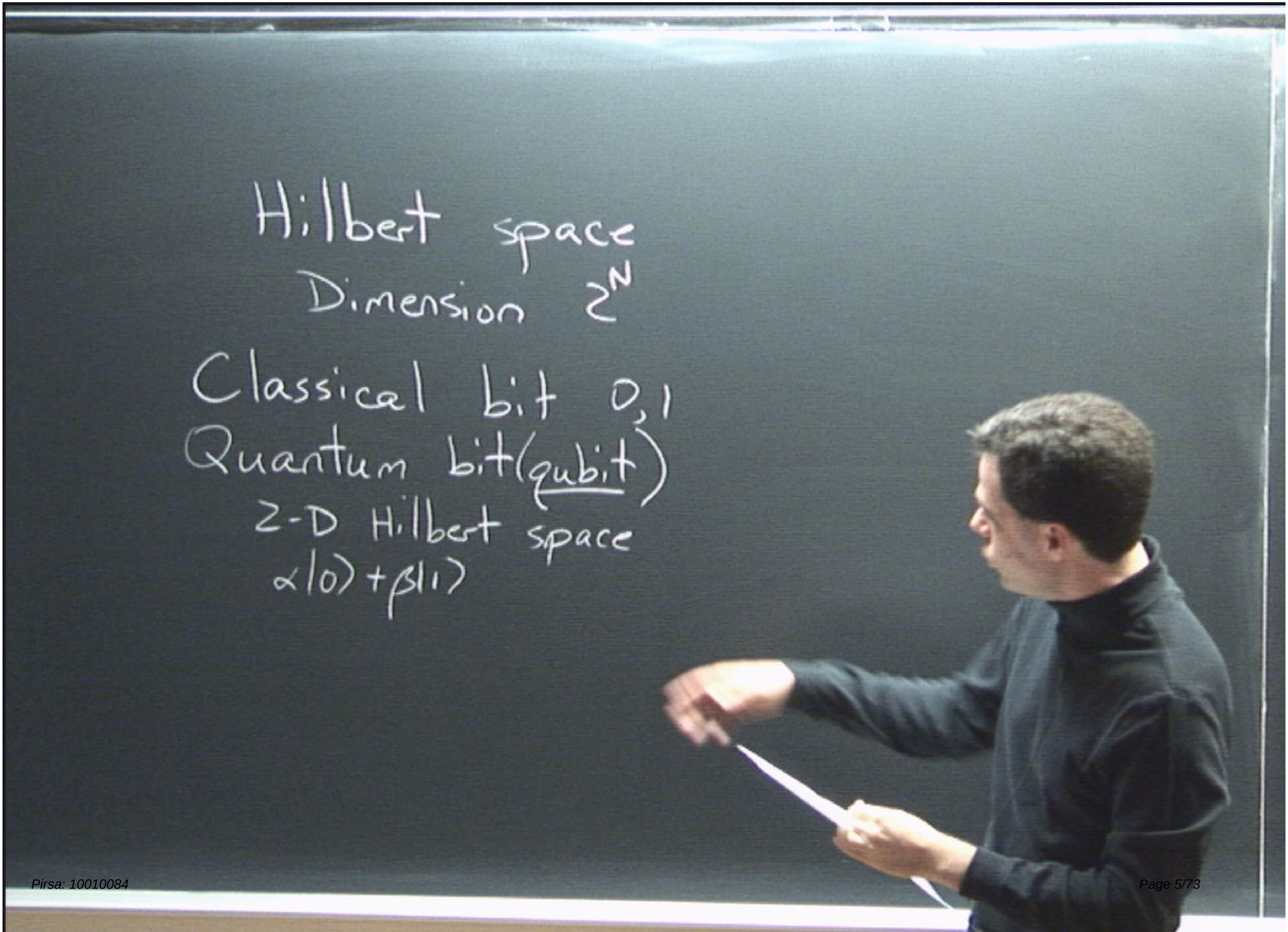
URL: http://pirsa.org/10010084

Abstract: <div id="Cleaner">Week 1: Basic topics (Qubits, quantum gates, quantum circuits, density matrices, quantum operations, entropy, entanglement)<div id="Cleaner"><div id="Cleaner">Week 2: Algorithms and complexity (Languages, complexity classes, oracles, RSA, Deutsch-Jozsa algorithm, Shor's algorithm, Grover's algorithm)<div id="Cleaner"><div id="Cleaner">Week 3: Information theory and implementations (Overview of implementations, quantum error correction, quantum cryptography, quantum information theory)

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS


Hilbert space
Dimension 2^N


Hilbert space
Dimension 2^N
Classical bit 0,1
Quantum bit (qubit)


Hilbert space
Dimension 2^N
Classical bit 0,1
Quantum bit (qubit)
2-D Hilbert space
$\alpha|0\rangle + \beta|1\rangle$

Hilbert space

Dimension 2^N

Classical bit 0,1

Quantum bit (qubit)

2-D Hilbert space

$\alpha|0\rangle + \beta|1\rangle$

N qubits – 2^N-D Hilbert space

Basis states – classical bit strings

2 qubits: $\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$

Hilbert space

Dimension 2^N

Classical bit 0,1

Quantum bit (qubit)

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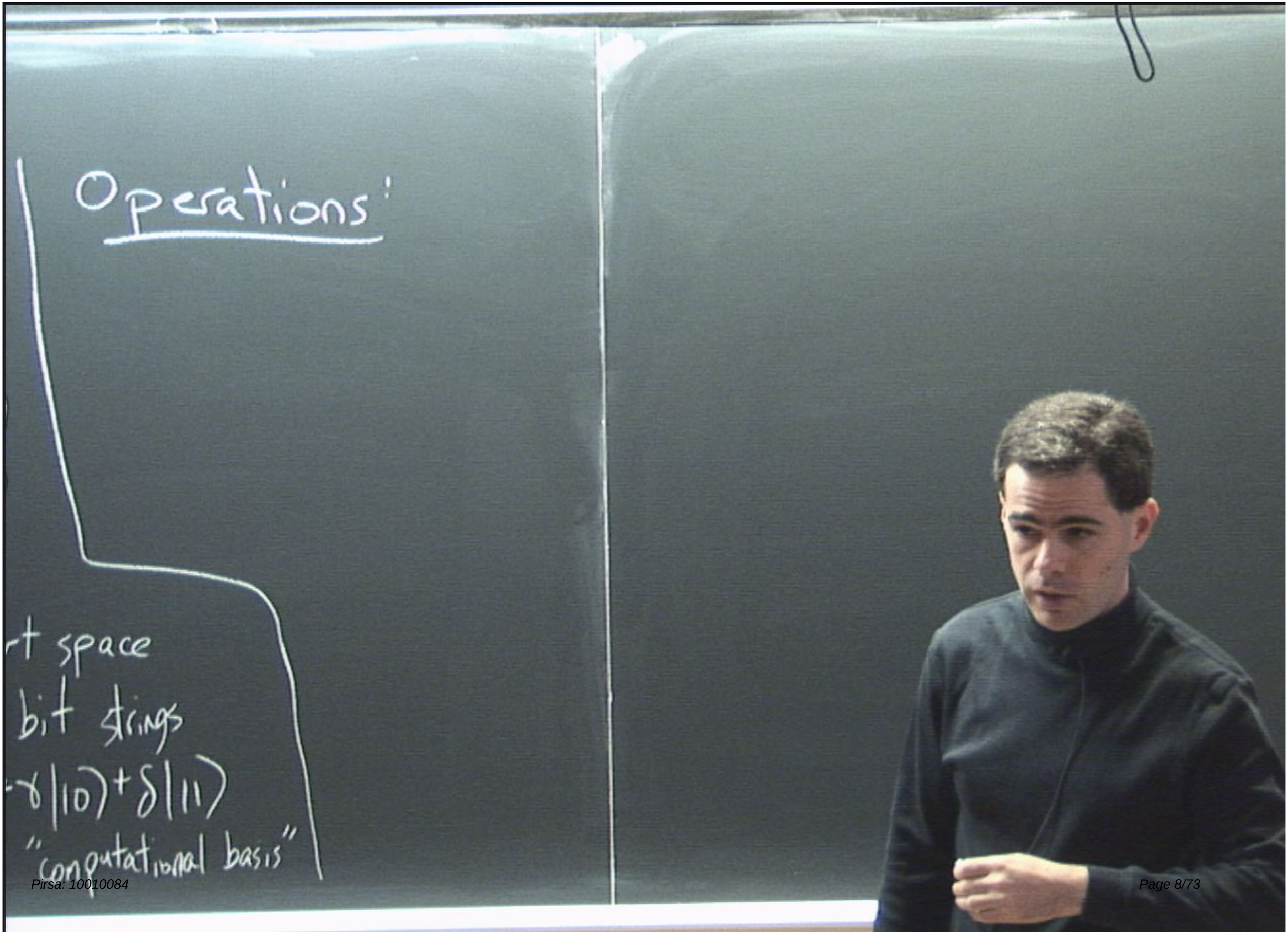
N qubits - 2^N-D Hilbert space

Basis states - classical bit strings

2 qubits: $\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$

"standard basis" "computational basis"


Operations:
rt space
bit strings
$+\gamma|10\rangle+\delta|11\rangle$
"computational basis"

Operations:

Unitary operators

$$|\psi\rangle \to U|\psi\rangle$$

$$|\phi\rangle \to U|\phi\rangle$$

$$\langle\psi|\phi\rangle \to \langle\psi|U^\dagger U|\phi\rangle$$

$$= \langle\psi|\phi\rangle$$

-t space

bit strings

$-\gamma|10\rangle+\delta|11\rangle$

"computational basis"

Operations:

Unitary operators

$|\psi\rangle \rightarrow U|\psi\rangle$

$|\phi\rangle \rightarrow U|\phi\rangle$

$\langle\psi|\phi\rangle \rightarrow \langle\psi|U^{\dagger}U|\phi\rangle$

$= \langle\psi|\phi\rangle$

Unitary operators are reversible

-rt space

bit strings

$\gamma|10\rangle + \delta|11\rangle$

"computational basis"

Reversible computation

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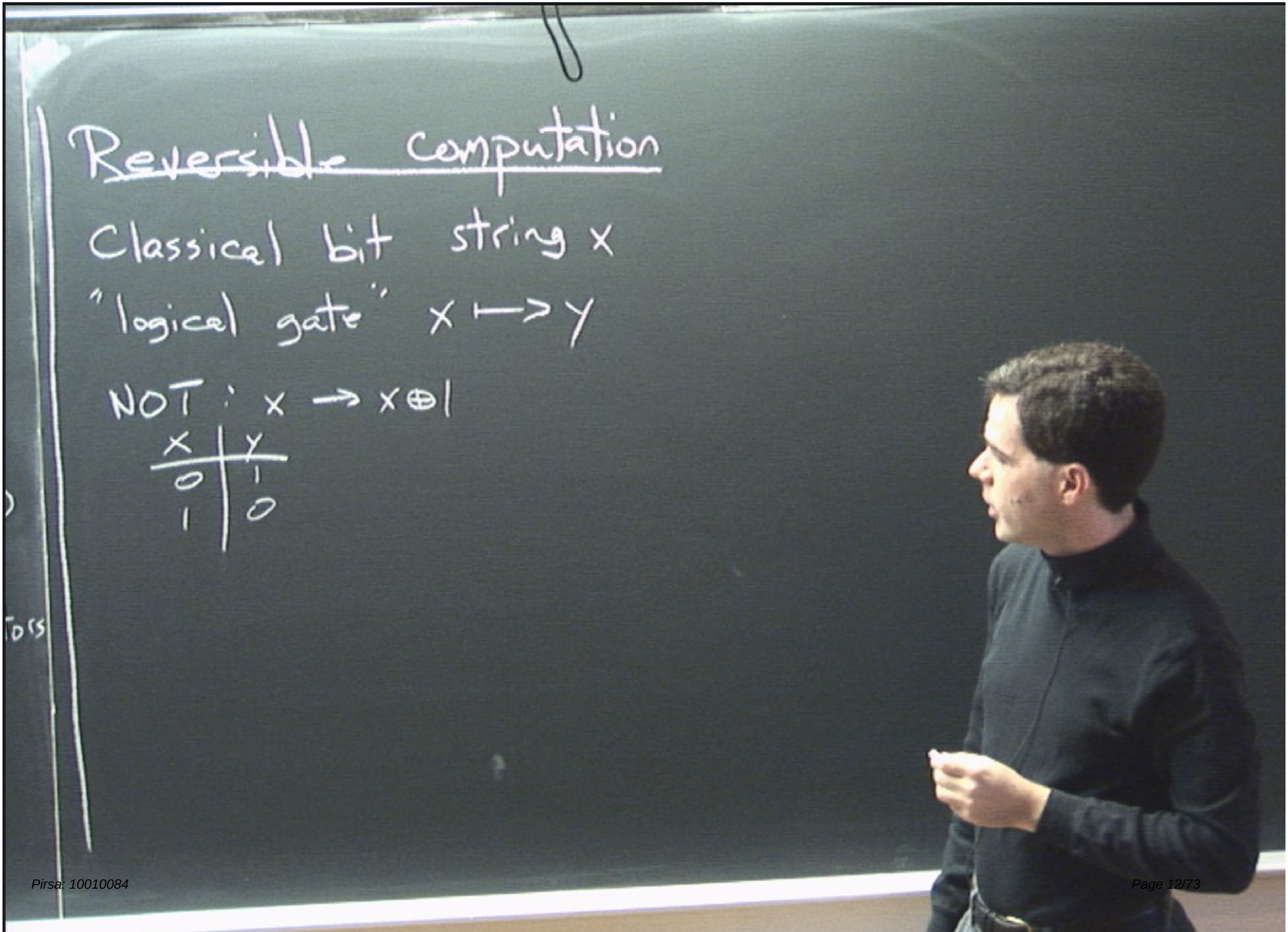
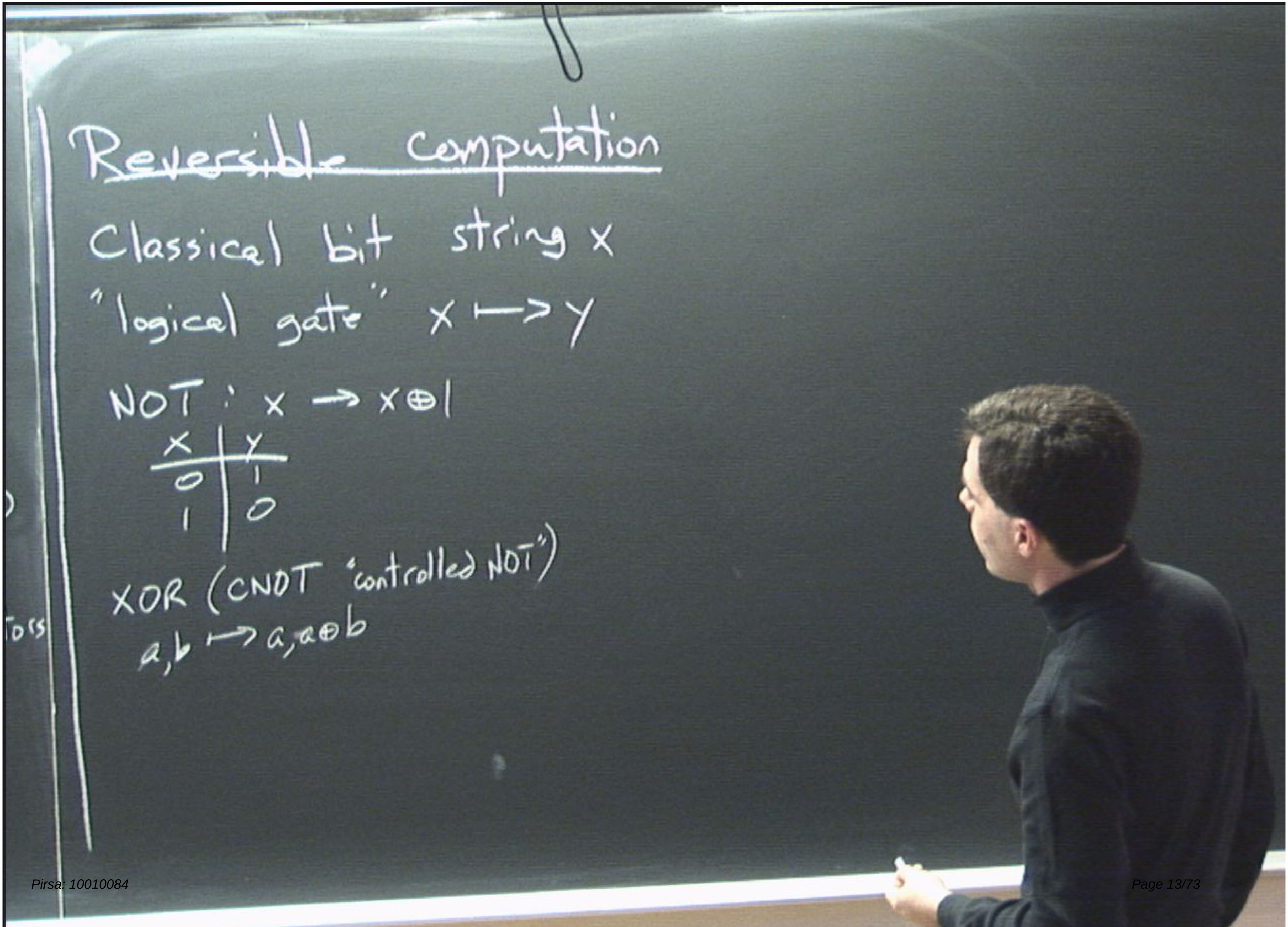

Reversible computation
Classical bit string x
"logical gate" x ⟼ y
NOT : x → x ⊕ 1
x | y
0 | 1
1 | 0


Reversible computation
Classical bit string x
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NOT : x → x⊕1
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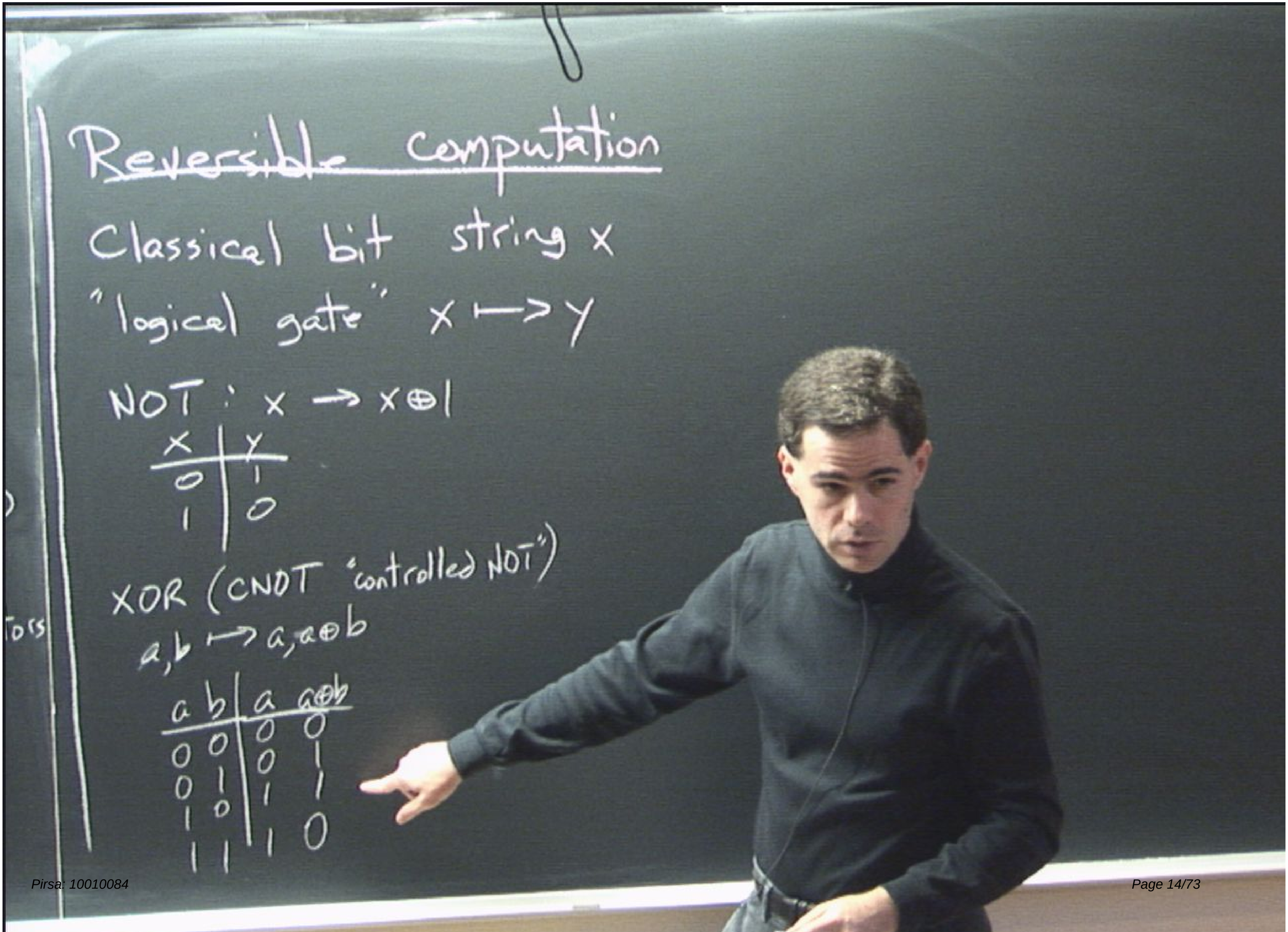
0 | 1
1 | 0
XOR (CNOT "controlled NOT")
a,b ↦ a,a⊕b


Reversible computation
Classical bit string x
"logical gate" x ⟼ y
NOT : x → x⊕1
x | y
0 | 1
1 | 0
XOR (CNOT "controlled NOT")
a,b ⟼ a, a⊕b
a b | a a⊕b
0 0 | 0 0
0 1 | 0 1
1 0 | 1 1
1 1 | 1 0

eversible computation

lassical bit string x

ogical gate" $x \longmapsto y$

OT : $x \to x \oplus 1$

x	y
0	1
1	0

OR (CNOT "controlled NOT")

$a,b \mapsto a, a\oplus b$

a	b	a	$a\oplus b$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

AND: $a,b \to a\wedge b = (a \text{ AND } b)$

a	b	$a\wedge b$

eversible computation

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0	1	
1	0	
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0	1	0	1
1	0	1	1
1	1	1	0

AND: $a,b \to a\wedge b = (a \text{ AND } b)$

a	b	$a\wedge b = ab \mod 2$
0	0	0
0	1	0
1	0	0
1	1	1

...le computation

bit string x

...gate" $x \longmapsto y$

$\rightarrow x \oplus 1$

...T "controlled NOT")

...$a \oplus b$

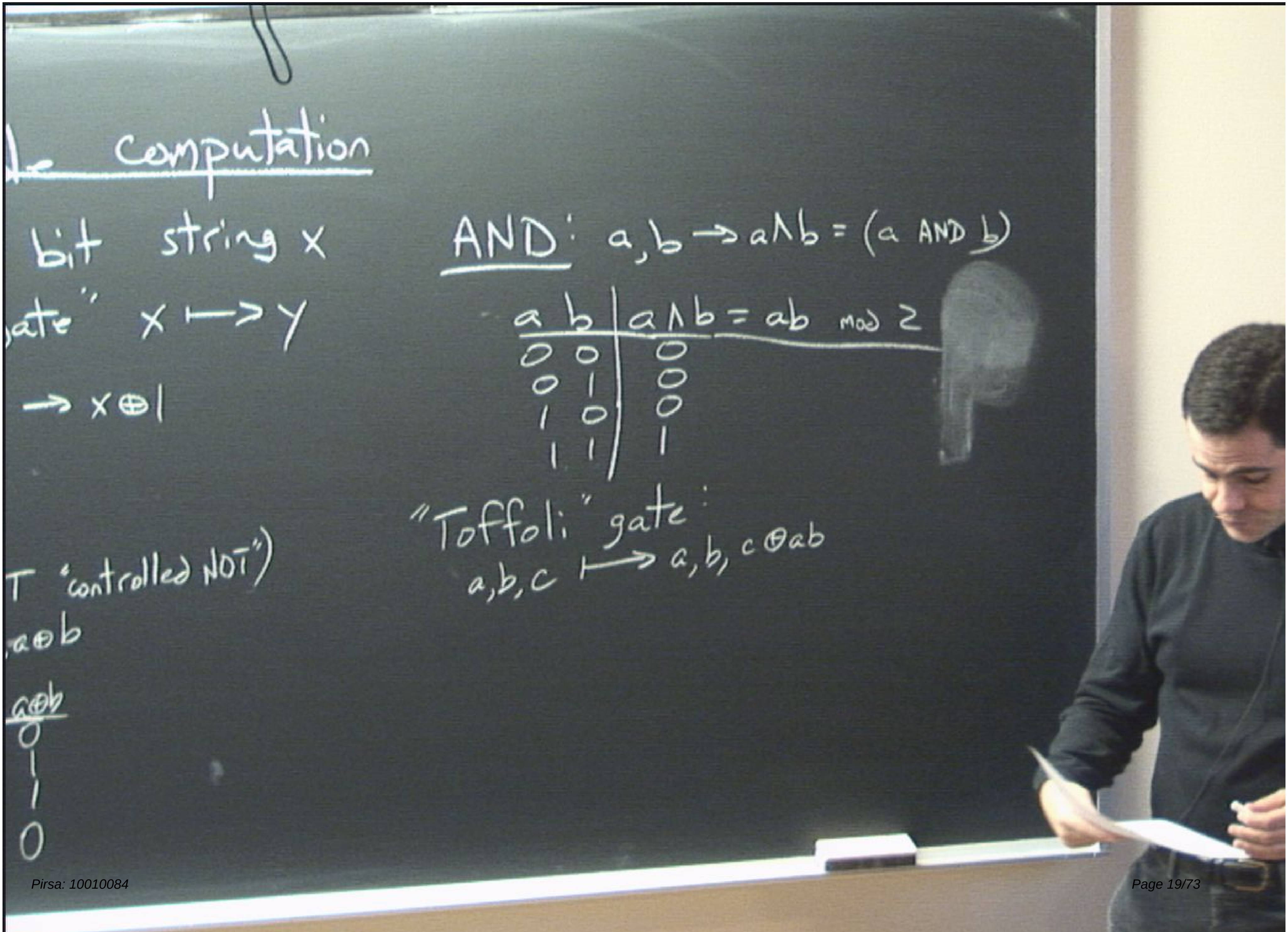
$a \oplus b$
0
1
1
0

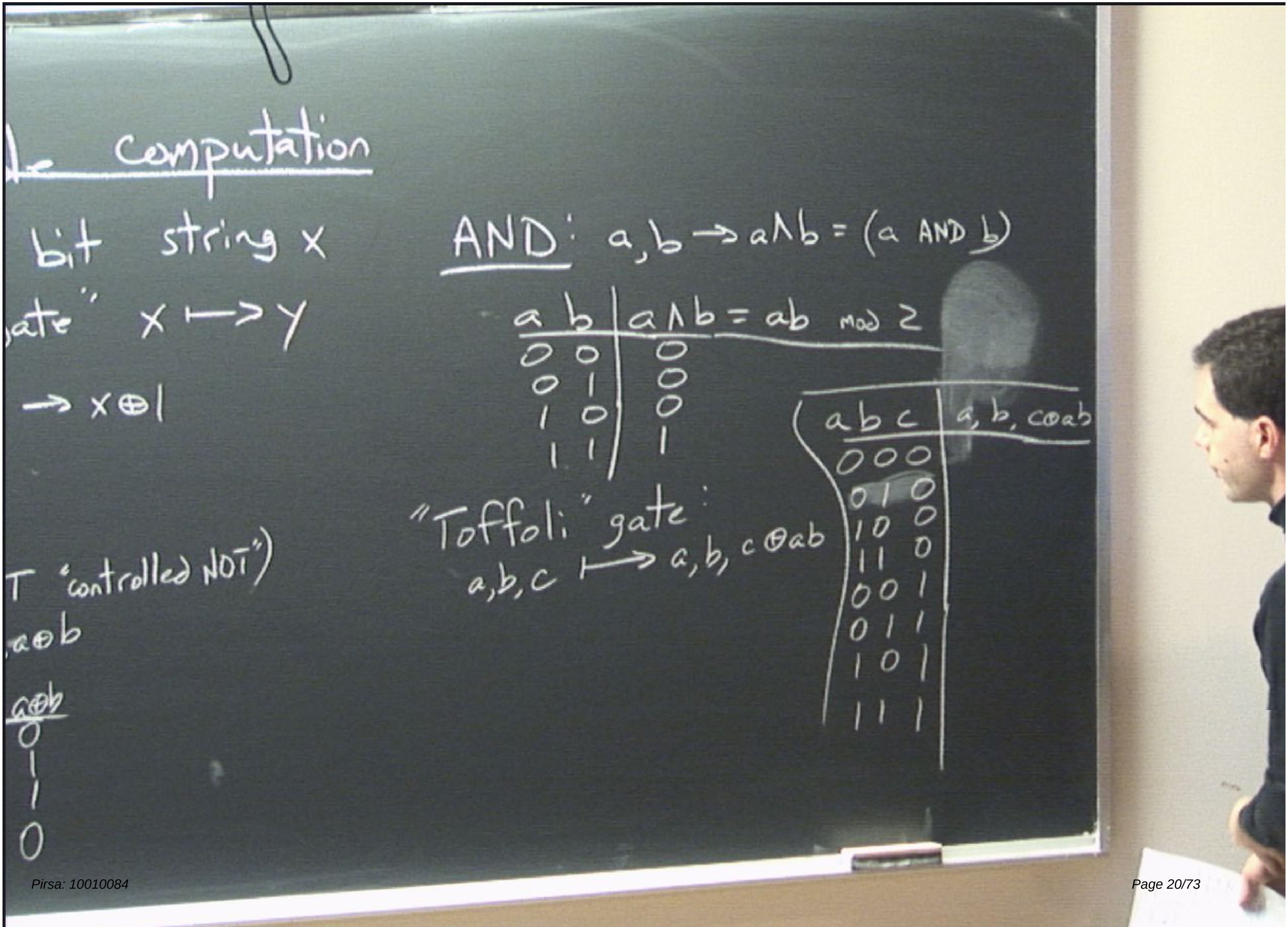
AND: $a, b \rightarrow a \wedge b = (a \text{ AND } b)$

a	b	$a \wedge b = ab \mod 2$
0	0	0
0	1	0
1	0	0
1	1	1

"Toffoli" gate:

$a, b, c \longmapsto a, b, c \oplus ab$


le computation
bit string x
gate" x ⟼ y
→ x⊕1
T "controlled NOT")
a⊕b
a⊕b
0
1
1
0
AND: a,b → a∧b = (a AND b)
a b | a∧b = ab mod 2
0 0 | 0
0 1 | 0
1 0 | 0
1 1 | 1
"Toffoli" gate:
a,b,c ⟼ a,b, c⊕ab



...le computation
bit string x
gate" x ⟼ y
→ x ⊕ 1
T "controlled NOT")
a⊕b
a⊕b
0
1
1
0
AND: a,b → a∧b = (a AND b)
a b | a∧b = ab mod 2
0 0 | 0
0 1 | 0
1 0 | 0
1 1 | 1
"Toffoli" gate:
a,b,c ⟼ a,b, c⊕ab
a b c | a, b, c⊕ab
0 0 0 | 0
0 1 0 | 0
1 0 0 | 0
1 1 0 | 0
0 0 1 | 1
0 1 1 | 1
1 0 1 | 1
1 1 1 | 1

...le computation

bit string x

"gate" $x \longmapsto y$

$\rightarrow x \oplus 1$

...T "controlled NOT")

$,a \oplus b$

$a \oplus b$
0
1
1
0

AND: $a, b \rightarrow a \wedge b = (a \text{ AND } b)$

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0	0	0
0	1	0
1	0	0
1	1	1

"Toffoli" gate:

$a, b, c \longmapsto a, b, c \oplus ab$

a	b	c	a	b	$c \oplus ab$
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	0	1	1	1
0	0	1	0	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0

computation

string x

$x \longmapsto y$

$x \oplus 1$

ntrolled NOT")

AND: $a, b \to a \wedge b = (a \text{ AND } b)$

a	b	$a \wedge b = ab \bmod 2$
0	0	0
0	1	0
1	0	0
1	1	1

"Toffoli" gate:

$a, b, c \longmapsto a, b, c \oplus ab$

$(\mathrm{Tof})^2 : a, b, c \mapsto a, b, c \oplus ab$

$\mapsto a, b, (c \oplus ab) \oplus ab$

a	b	c	a,	b,	$c \oplus ab$
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	0	1	1	1
0	0	1	0	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0

Computation

string x

$x \longmapsto y$

$x \oplus 1$

ntrolled NOT")

AND: $a, b \rightarrow a \wedge b = (a \text{ AND } b)$

a	b	$a \wedge b = ab \mod 2$
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0	1	0
1	0	0
1	1	1

"Toffoli" gate:

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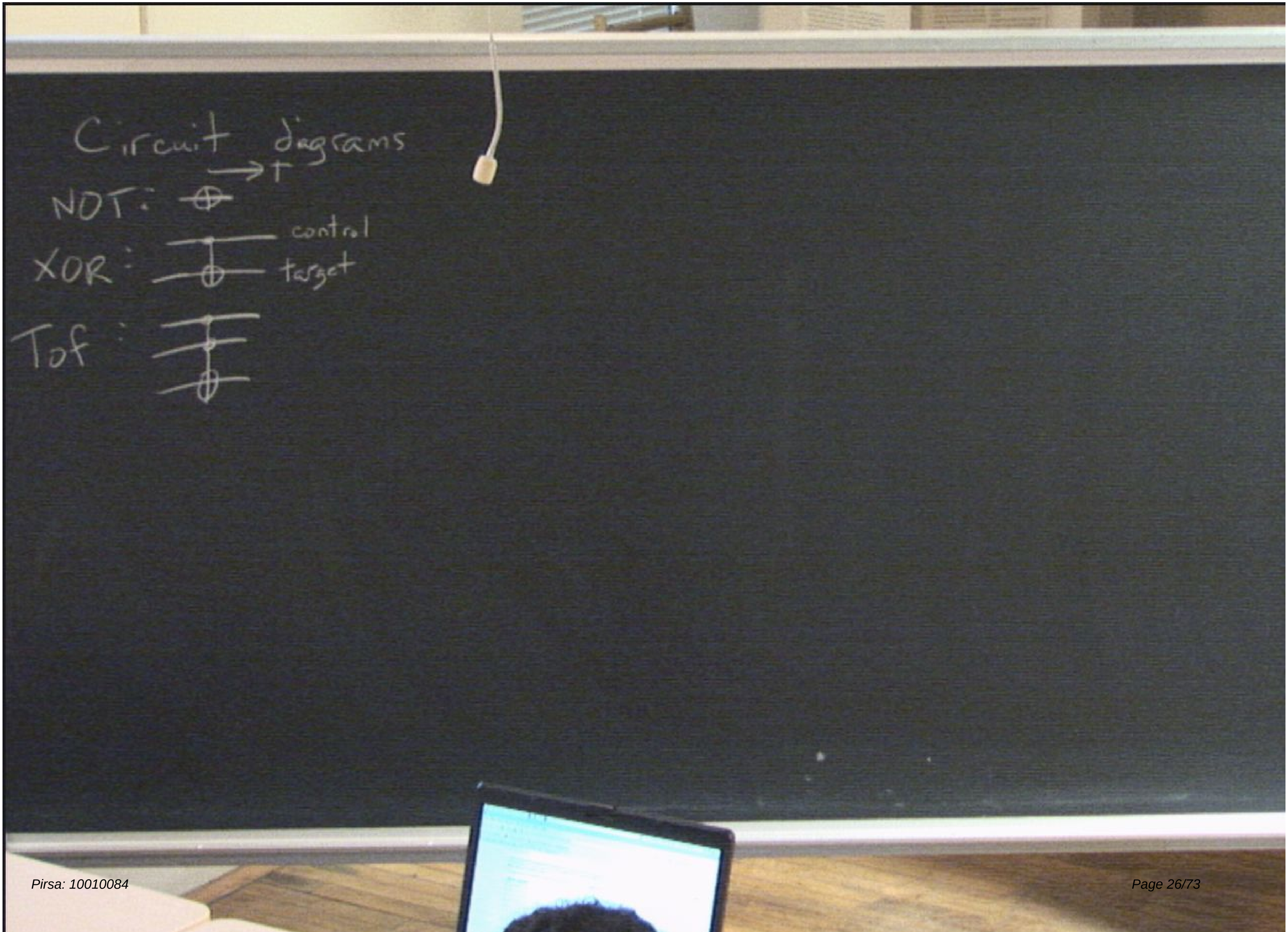
$(Tof)^2: a, b, c \mapsto a, b, c \oplus ab$

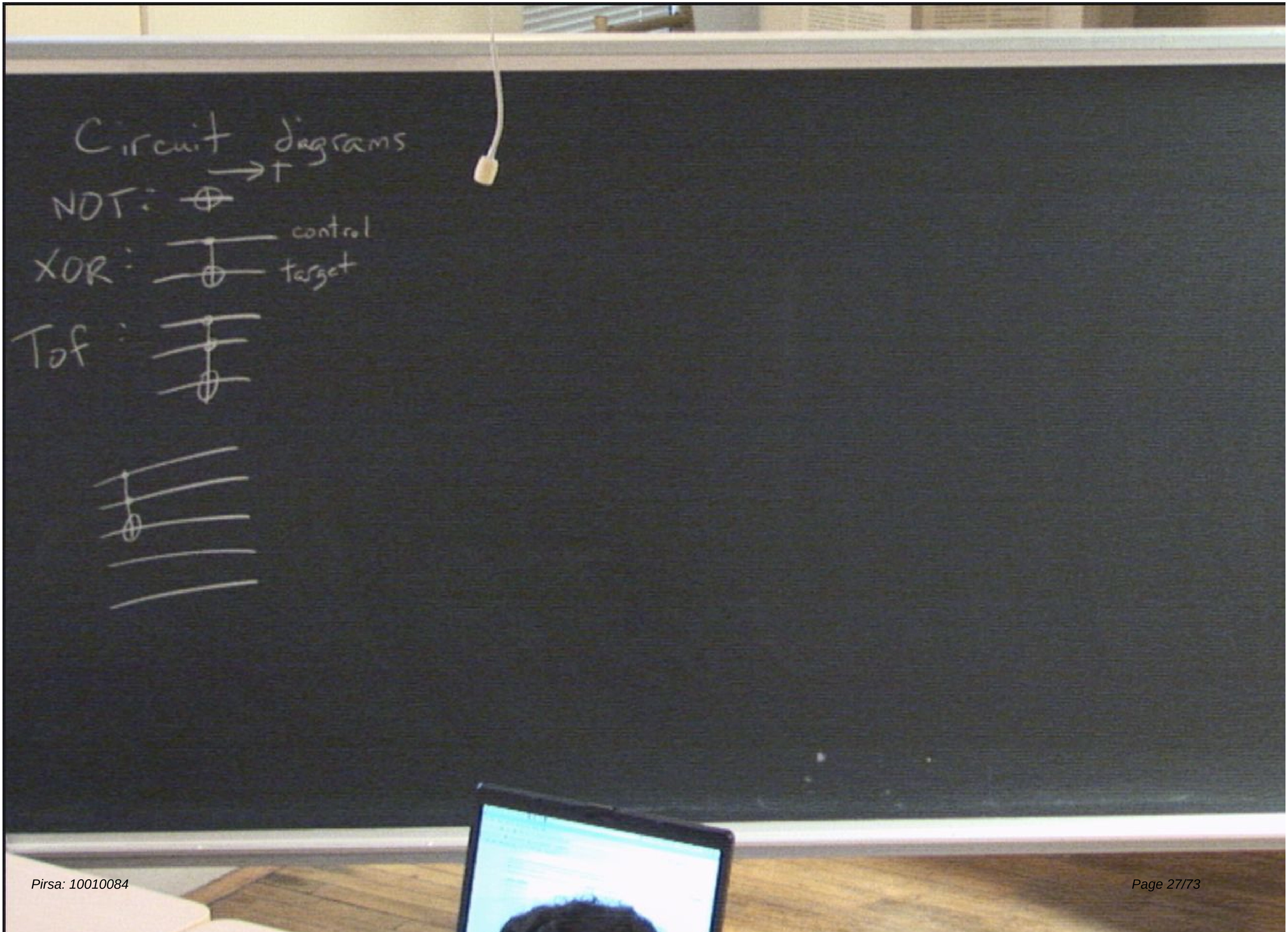
$\mapsto a, b, (c \oplus ab) \oplus ab$

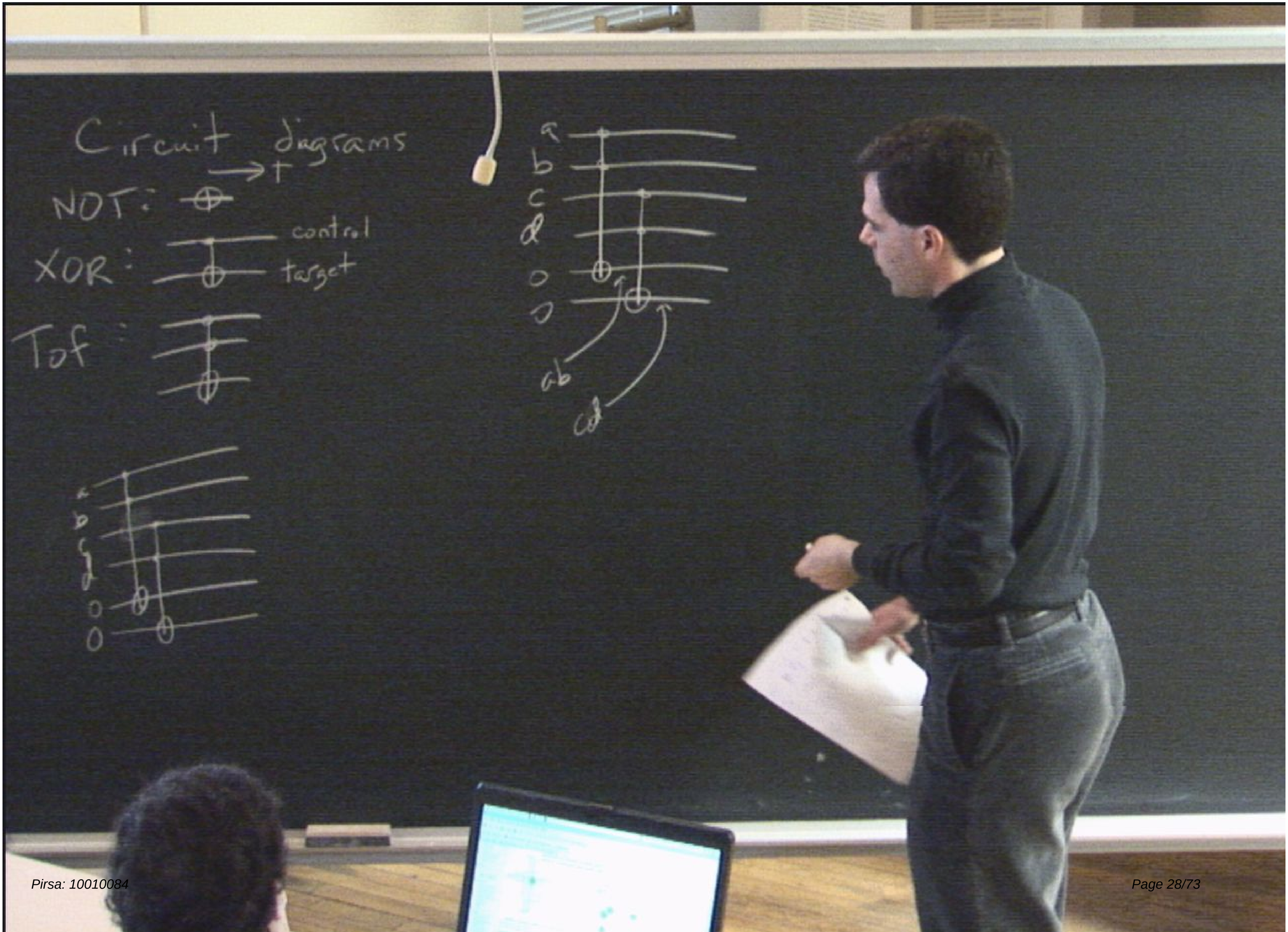
a	b	c	a,	b,	$c \oplus ab$
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	0	1	1	1
0	0	1	0	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0


Circuit diagrams
NOT:

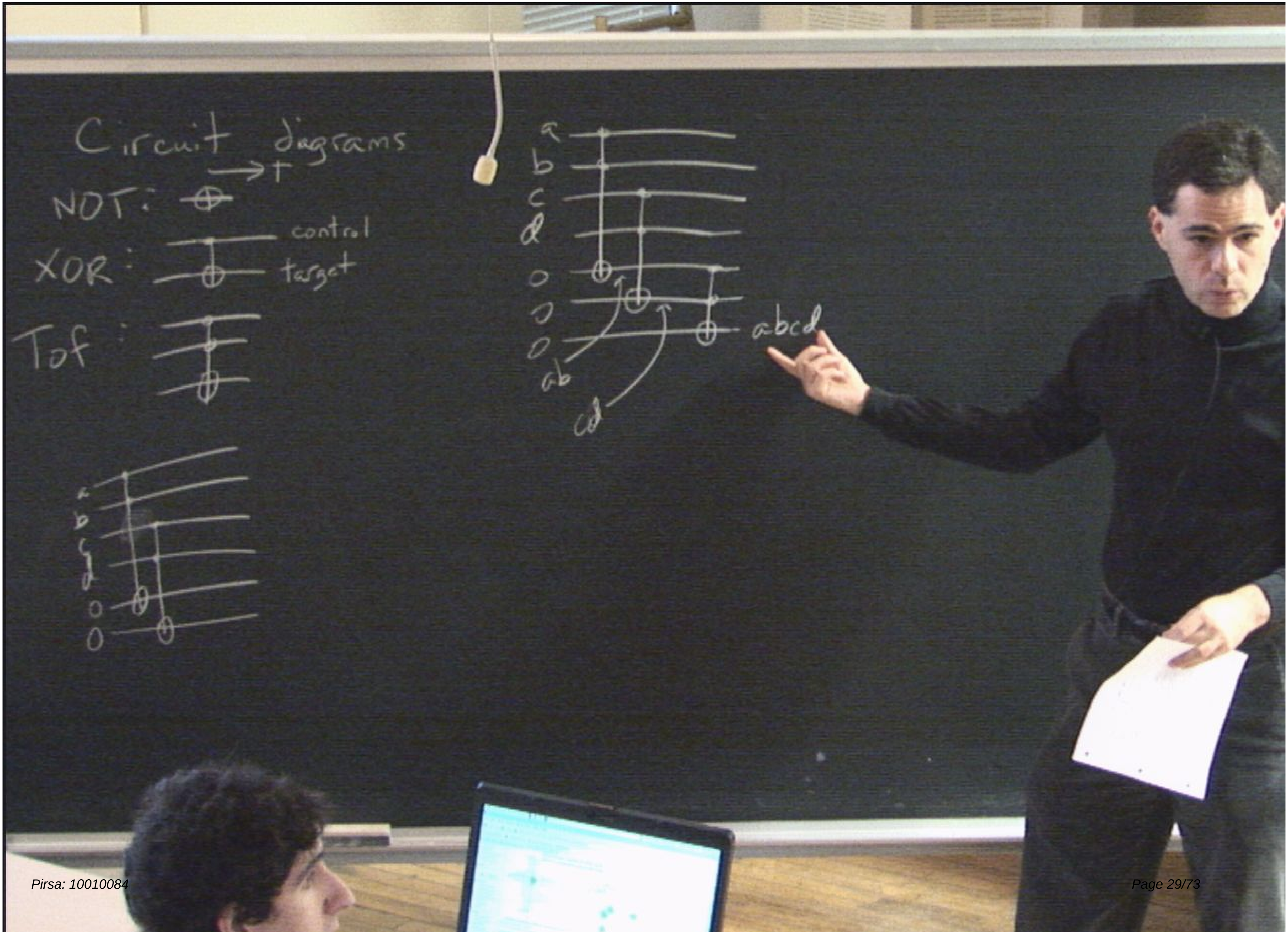

Circuit diagrams
NOT:
XOR:

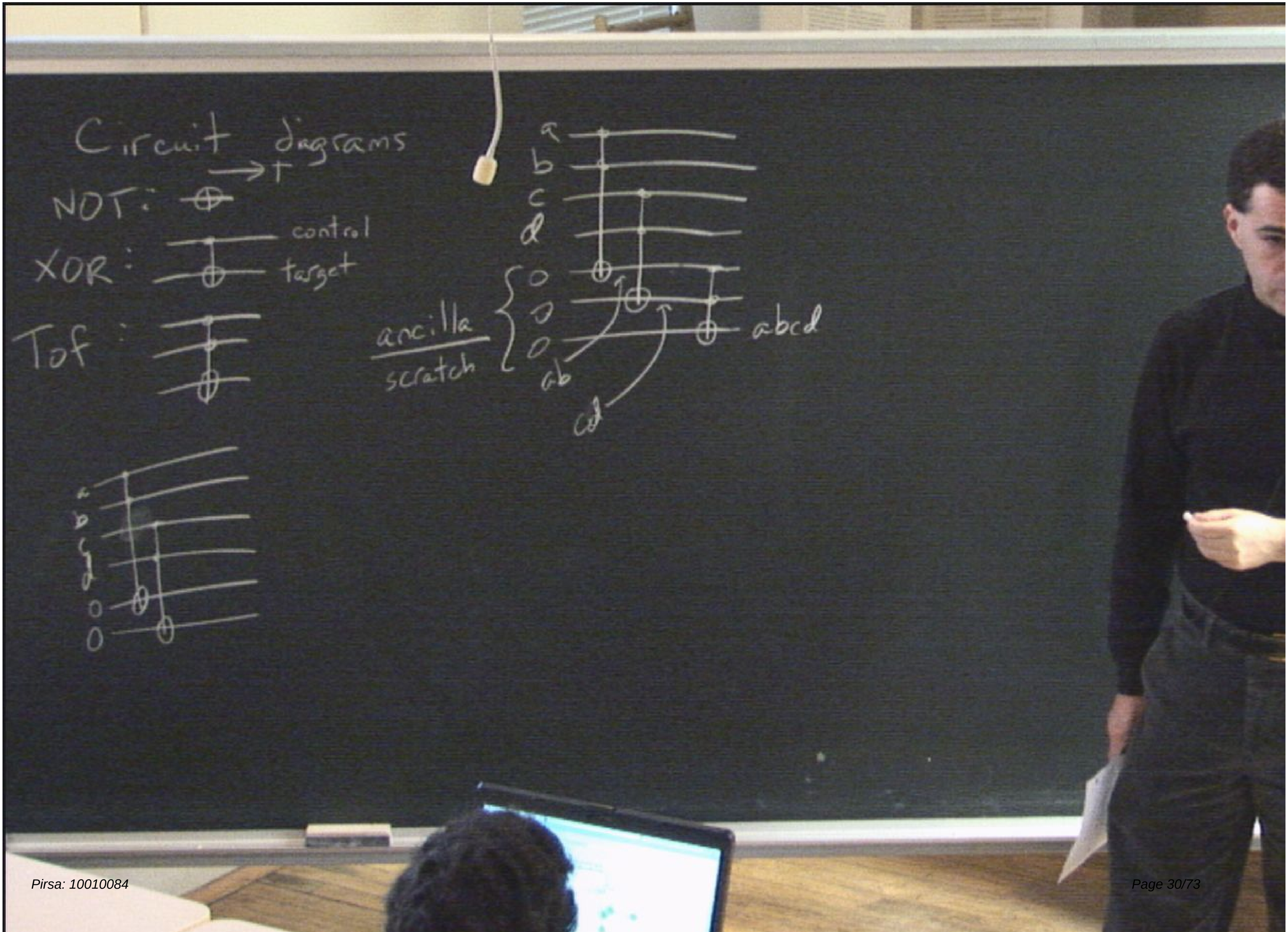

Circuit diagrams
NOT:
control
XOR:
target
Tof:

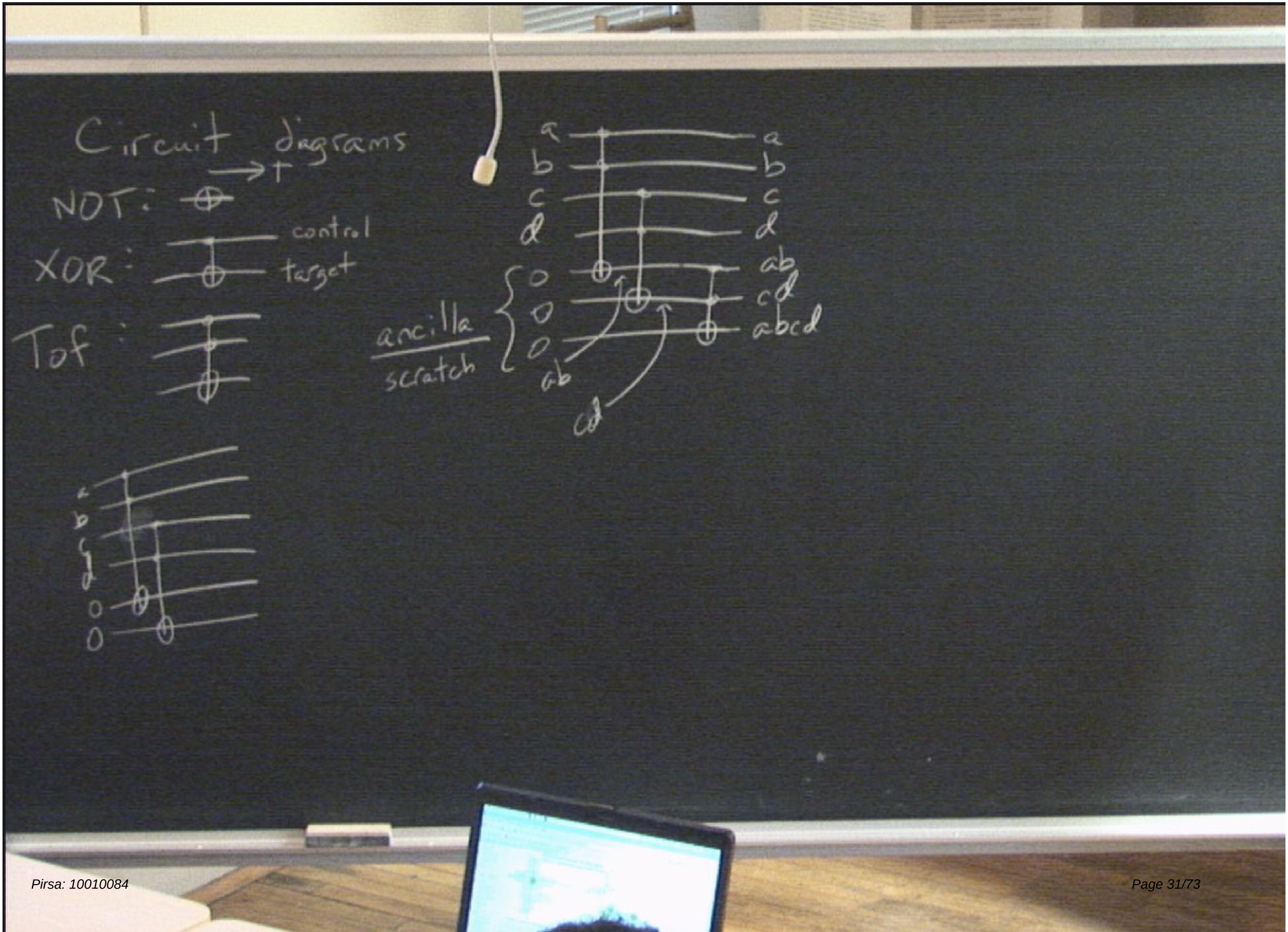

Circuit diagrams
t
NOT:
XOR:
control
target
Tof:



Circuit diagrams
NOT:
XOR:
control
target
Tof:
a
b
c
d
0
0
ab
cd


Circuit diagrams
NOT:
XOR:
control
target
Tof:
a
b
c
d
0
0
0
abcd
ab
cd


Circuit diagrams
t
NOT:
XOR:
control
target
Tof:
ancilla
scratch
a
b
c
d
0
0
0
ab
cd
abcd
a
b
c
d
0
0


Circuit diagrams
NOT:
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Tof:
ancilla
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a
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ab
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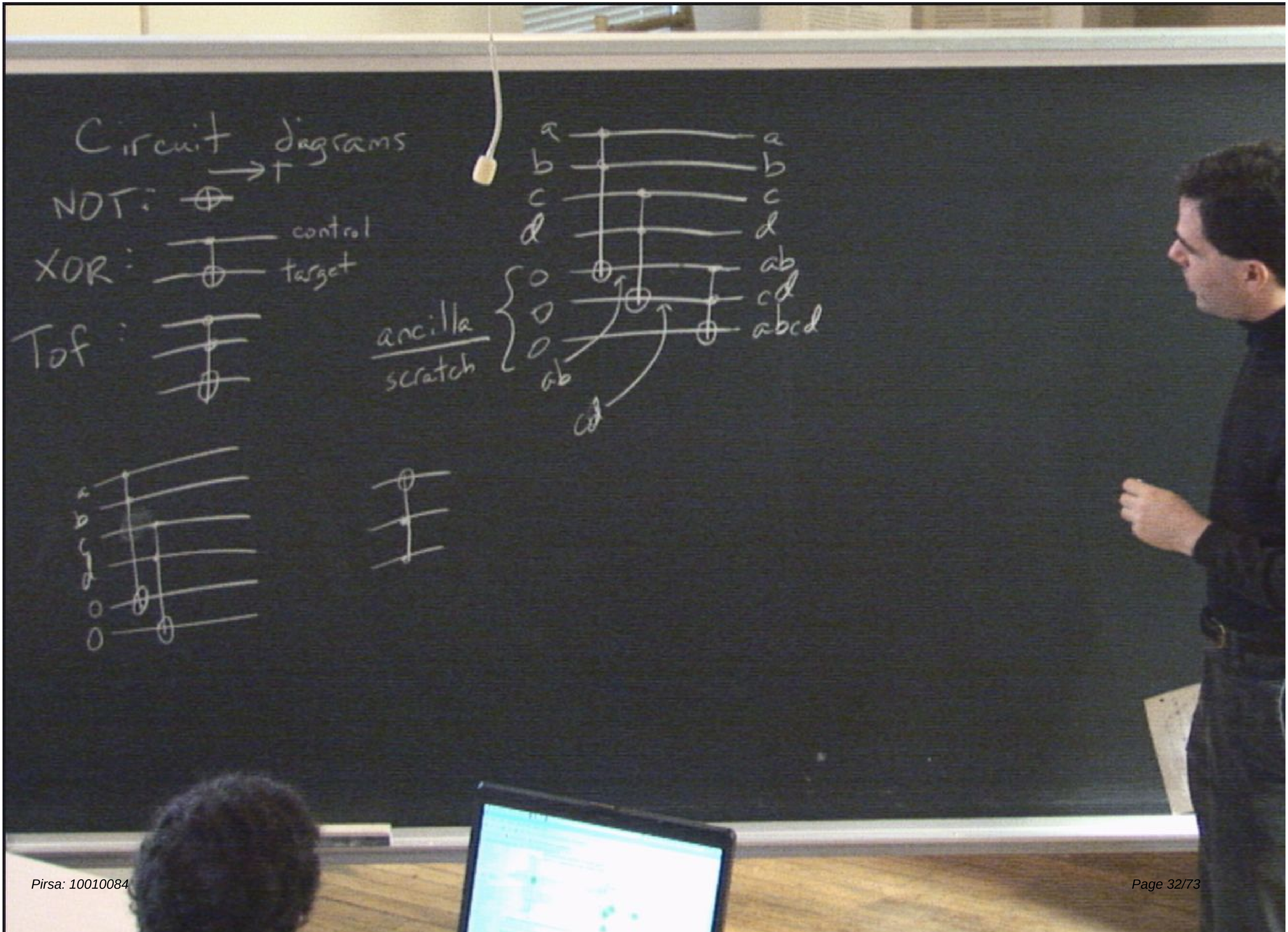
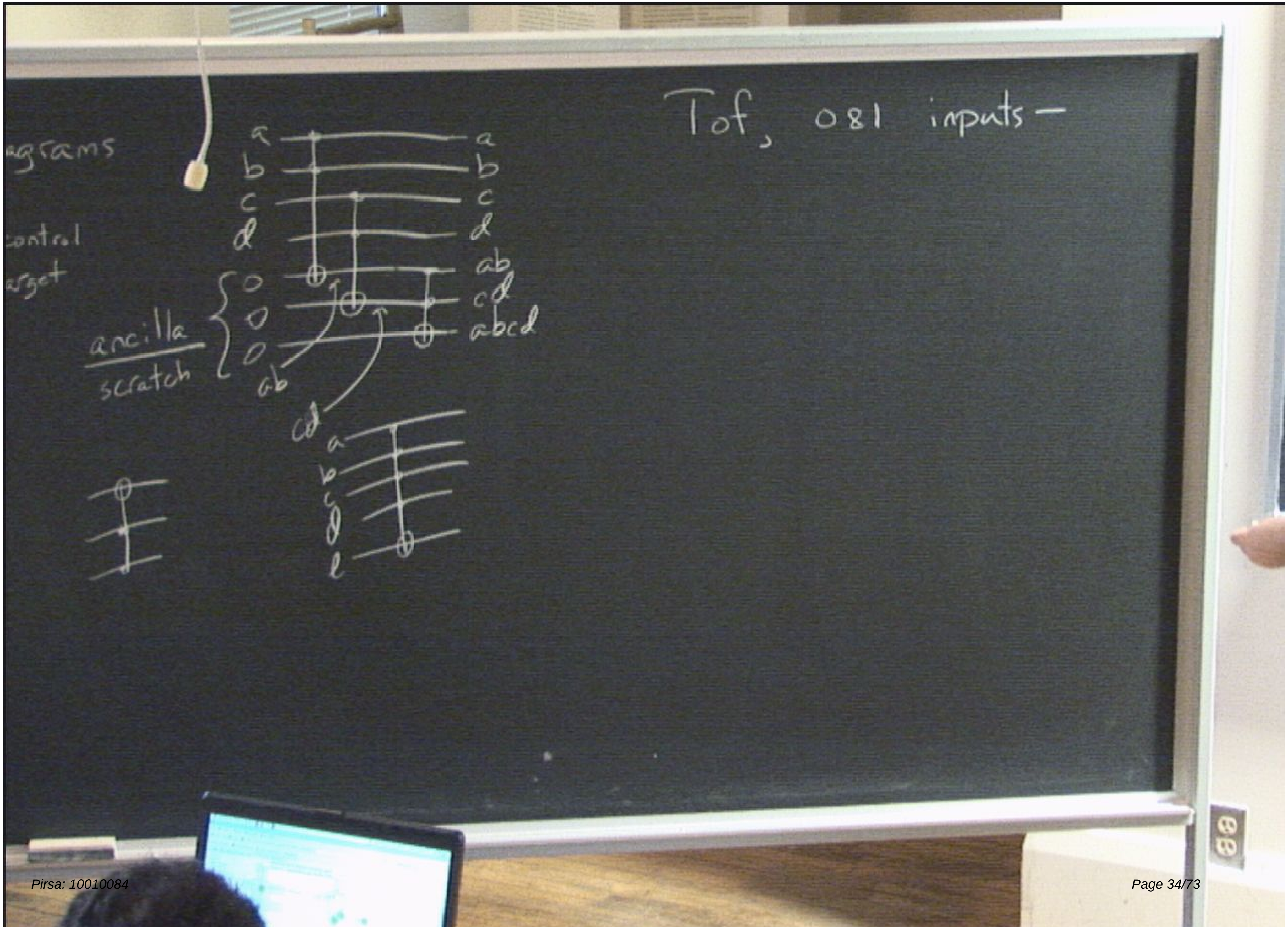
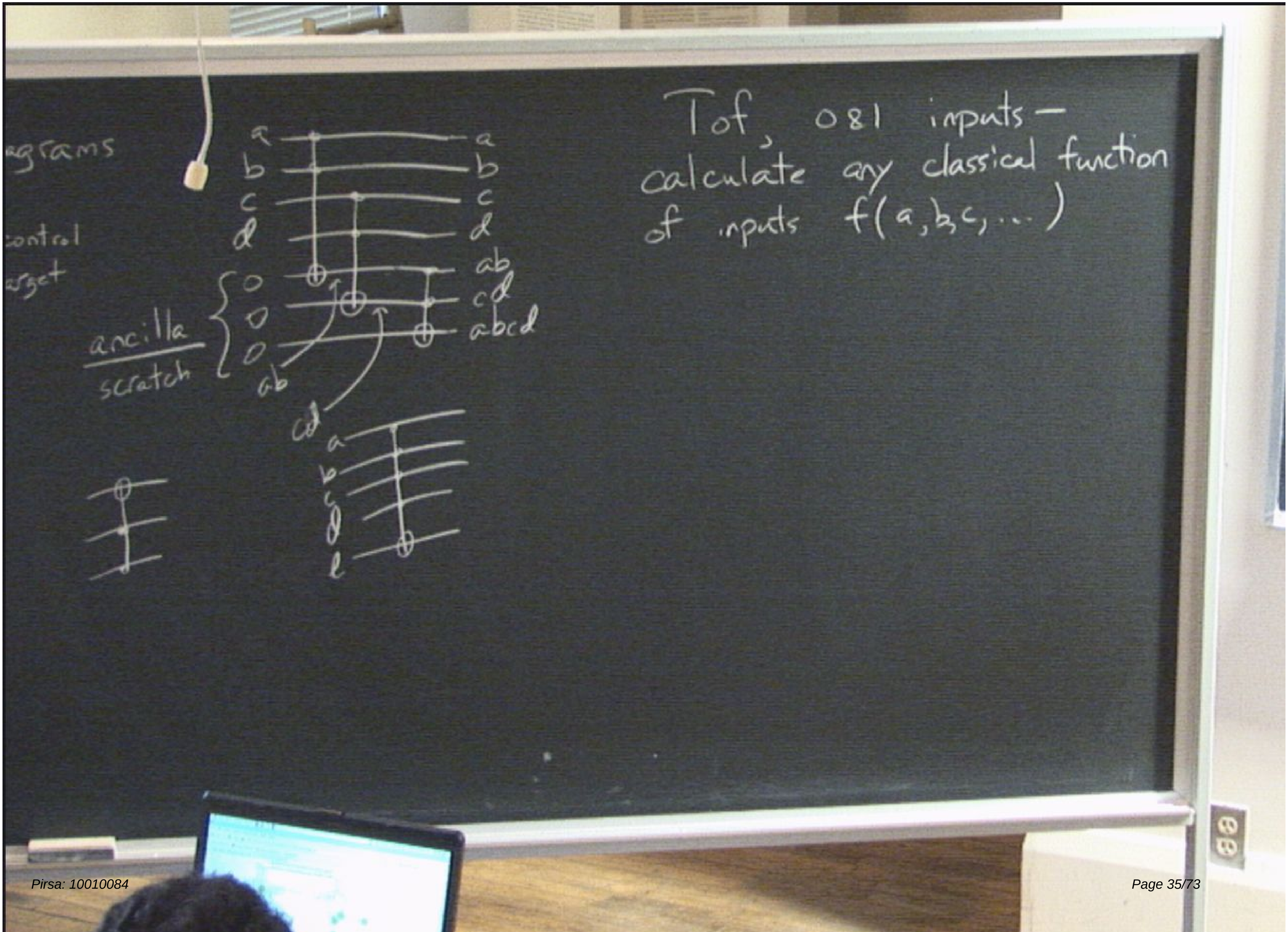
abcd


Circuit diagrams
NOT:
XOR:
control
target
Tof:
ancilla
scratch
a
b
c
d
ab
cd
abcd


Circuit diagrams
NOT:
XOR:
control
target
Tof:
ancilla
scratch
a
b
c
d
0
0
0
ab
cd
abcd


Tof, 0&1 inputs —
ancilla
scratch
a
b
c
d
ab
cd
abcd


Tof, 0&1 inputs —
calculate any classical function
of inputs f(a,b,c, ...)
ancilla
scratch
control
a
b
c
d
ab
cd
abcd
e

Reversible computation

Classical bit string x

"logical gate": $x \mapsto y$

NOT: $x \to x \oplus 1$

x	y
0	1
1	0

XOR (CNOT "controlled NOT")

$a, b \mapsto a, a \oplus b$

a	b	a	$a \oplus b$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Product of reversible gates is reversible

AND: $a, b \to a \wedge b = (a \text{ AND } b)$

a	b	$a \wedge b = ab \mod 2$
0	0	0
0	1	0
1	0	0
1	1	1

"Toffoli" gate:

$a, b, c \mapsto a, b, c \oplus ab$

$(\mathrm{Tof})^2 : a, b, c \mapsto a, b, c \oplus ab$

$\mapsto a, b, \underbrace{(c \oplus ab) \oplus ab}_{c}$

a	b	c	a,	b,	$c \oplus ab$
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	0	1	1	1
0	0	1	0	0	1
0	1	1	0	1	[illegible]
1	0	1	1	0	[illegible]
1	1	1	1	1	[illegible]

Reversible computation

Classical bit string x

"logical gate" $x \longmapsto y$

NOT : $x \to x \oplus 1$

x	y
0	1
1	0

XOR (CNOT "controlled NOT")

$a,b \longmapsto a, a\oplus b$

a	b	a	$a\oplus b$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Product of reversible gates is reversible

$(BA)^{-1} = A^{-1}B^{-1}$

AND: $a,b \to a\wedge b = (a \text{ AND } b)$

a	b	$a\wedge b = ab \bmod 2$
0	0	0
0	1	0
1	0	0
1	1	1

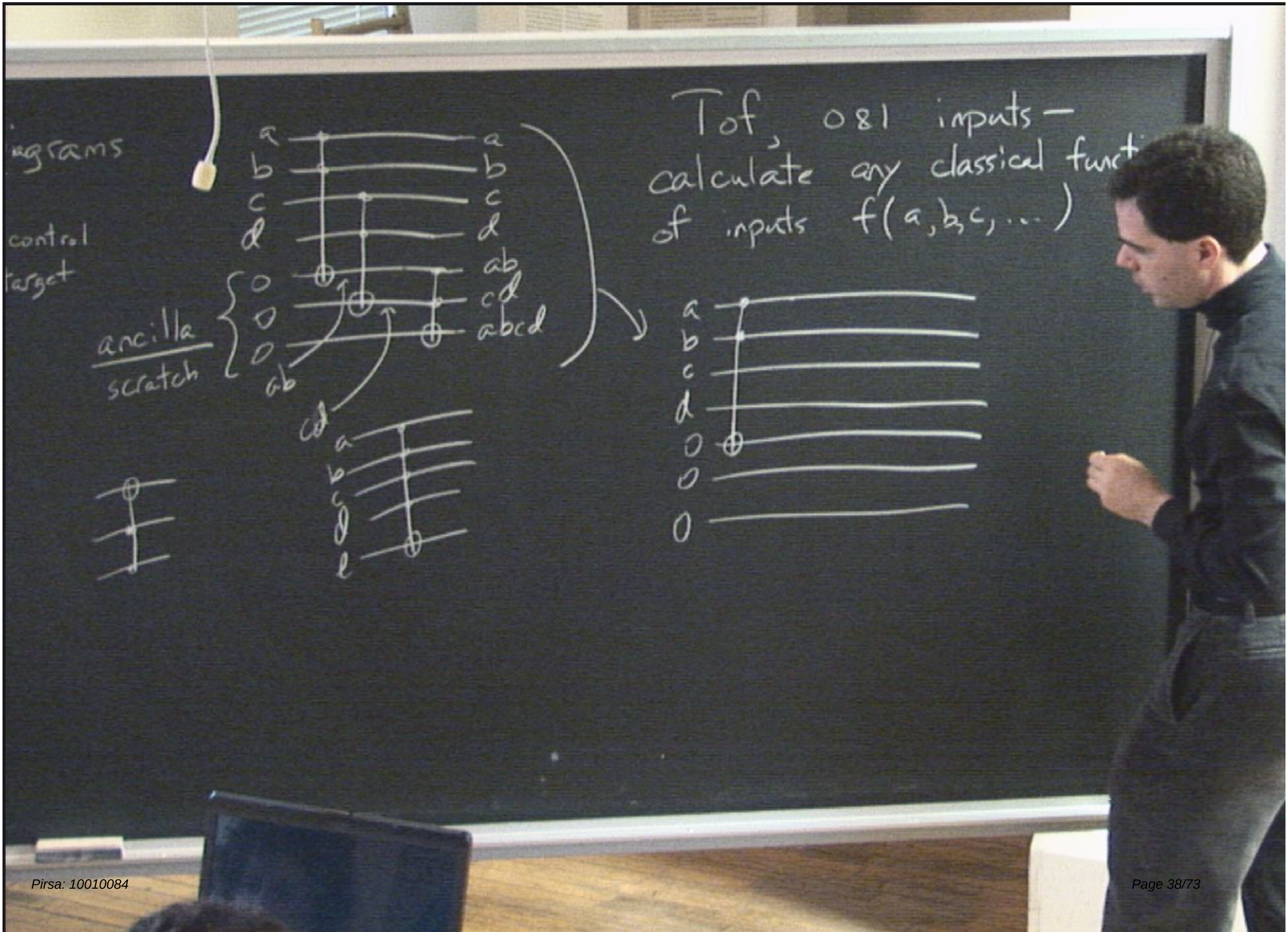
"Toffoli" gate:

$a,b,c \longmapsto a,b,c\oplus ab$

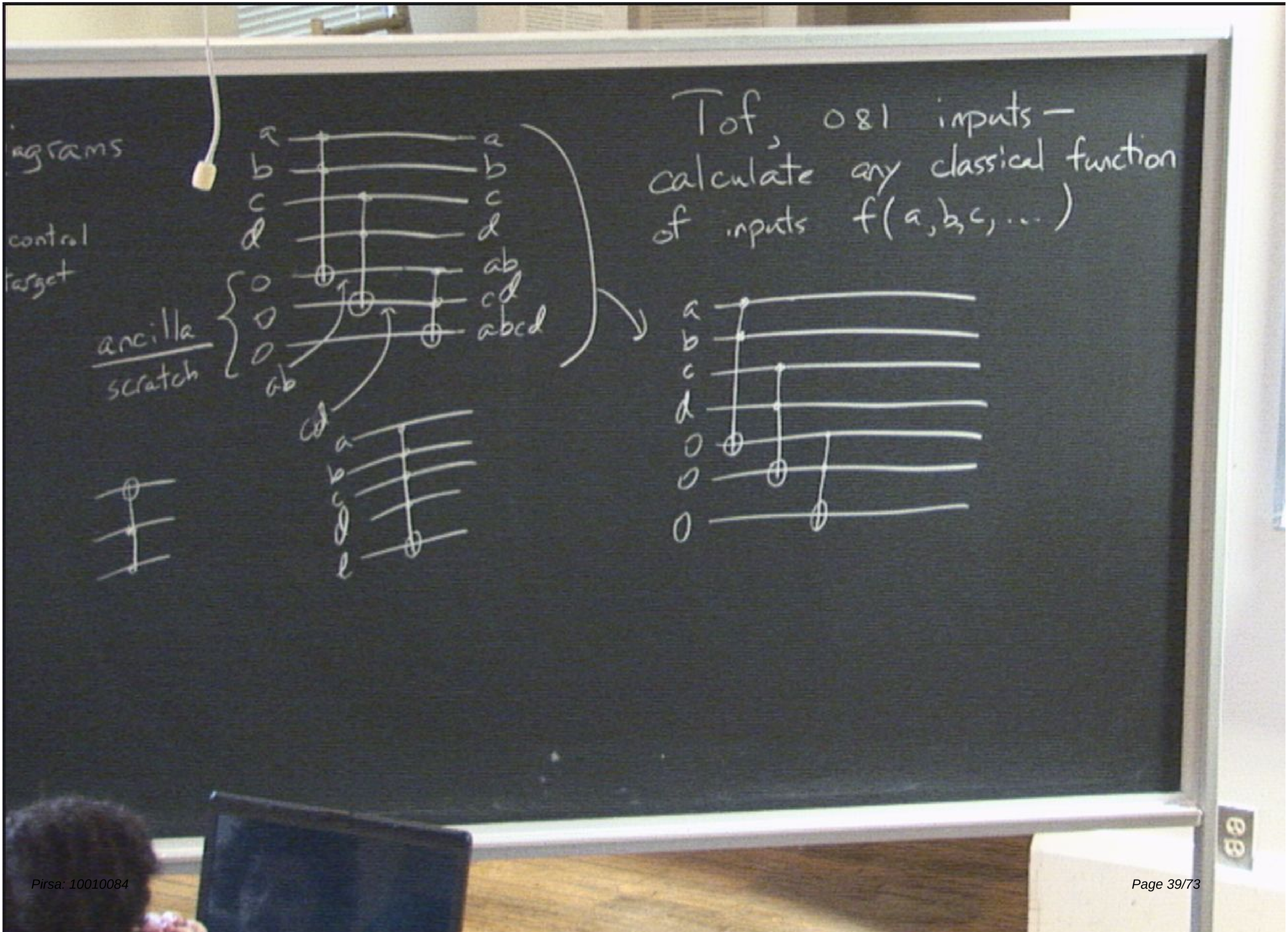
$(Tof)^2 : a,b,c \longmapsto a,b,c\oplus ab$

$\longmapsto a,b,\underbrace{(c\oplus ab)\oplus ab}_{c}$

a	b	c	a,	b,	$c\oplus ab$
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	0	1	1	1
0	0	1	0	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0



Tof, 0 & 1 inputs —
calculate any classical funct
of inputs f(a,b,c, ...)
grams
control
target
ancilla
scratch
a b c d
0 0 0
a b c d ab cd abcd
ab
cd
a b c d e
a b c d 0 0 0



agrams
control
target
ancilla
scratch
Tof, 0 & 1 inputs —
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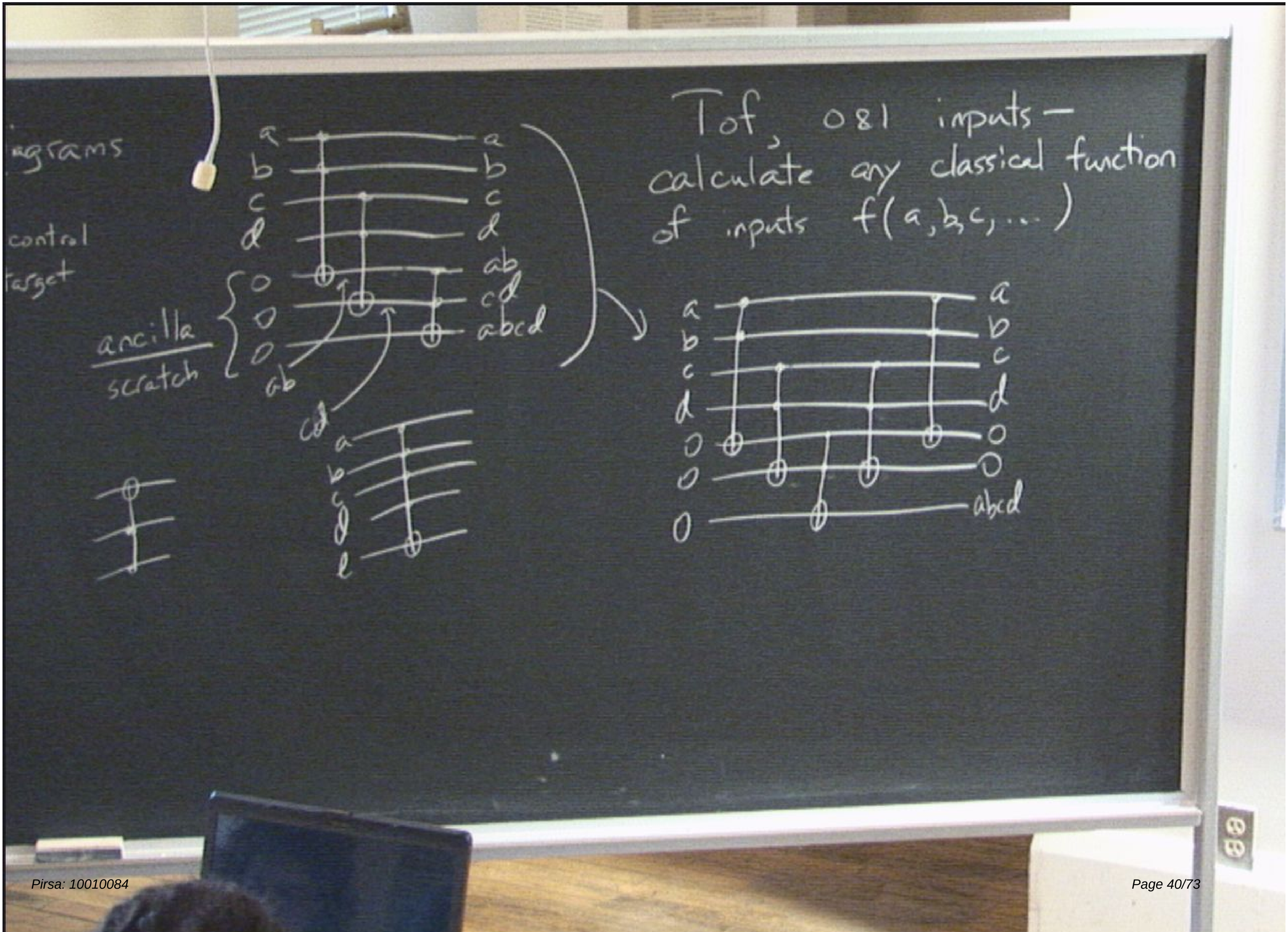
of inputs f(a,b,c,...)
a
b
c
d
0
0
0
ab
cd
ab
cd
abcd
e


Tof, 0 & 1 inputs —
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grams
control
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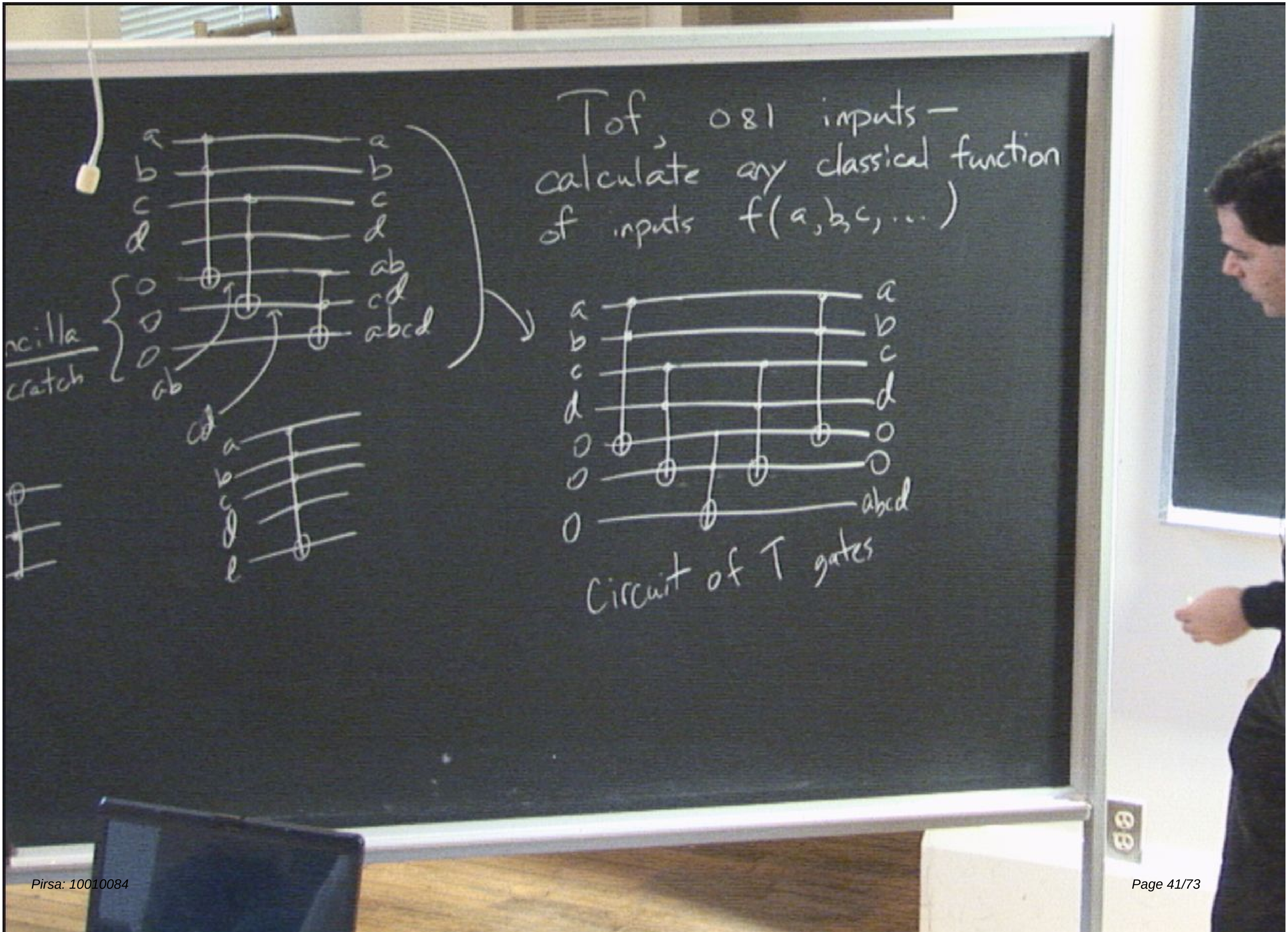
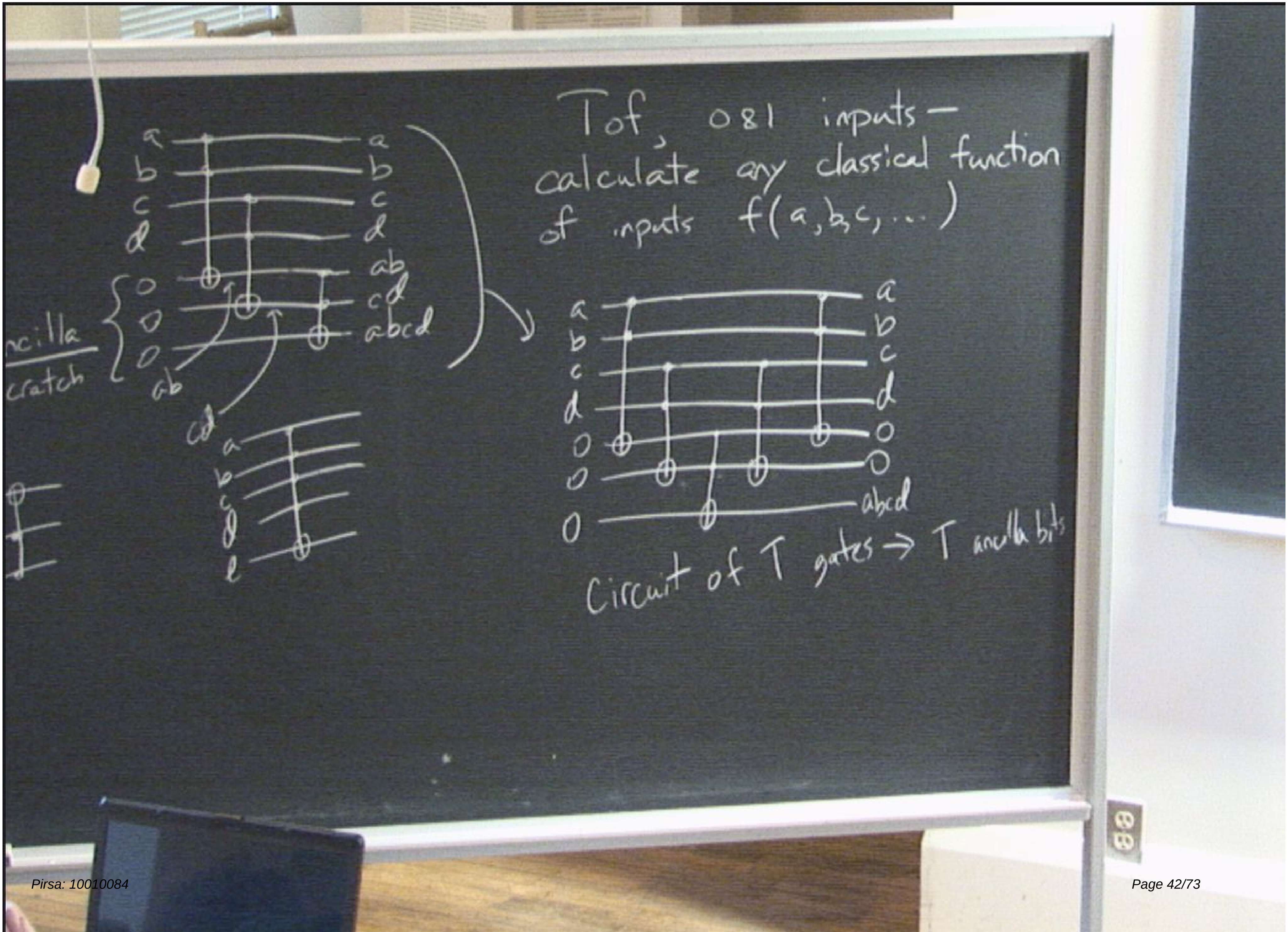
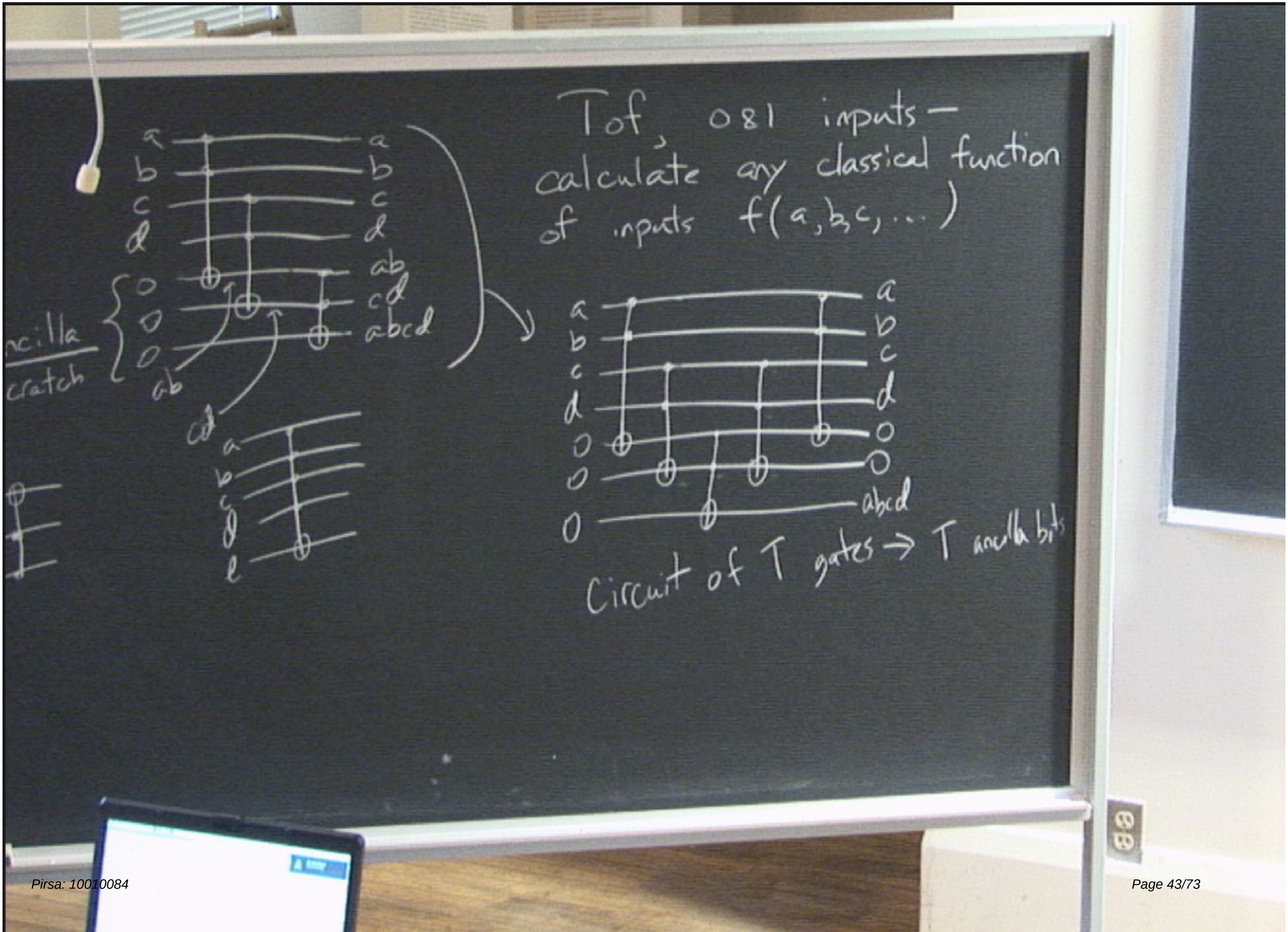
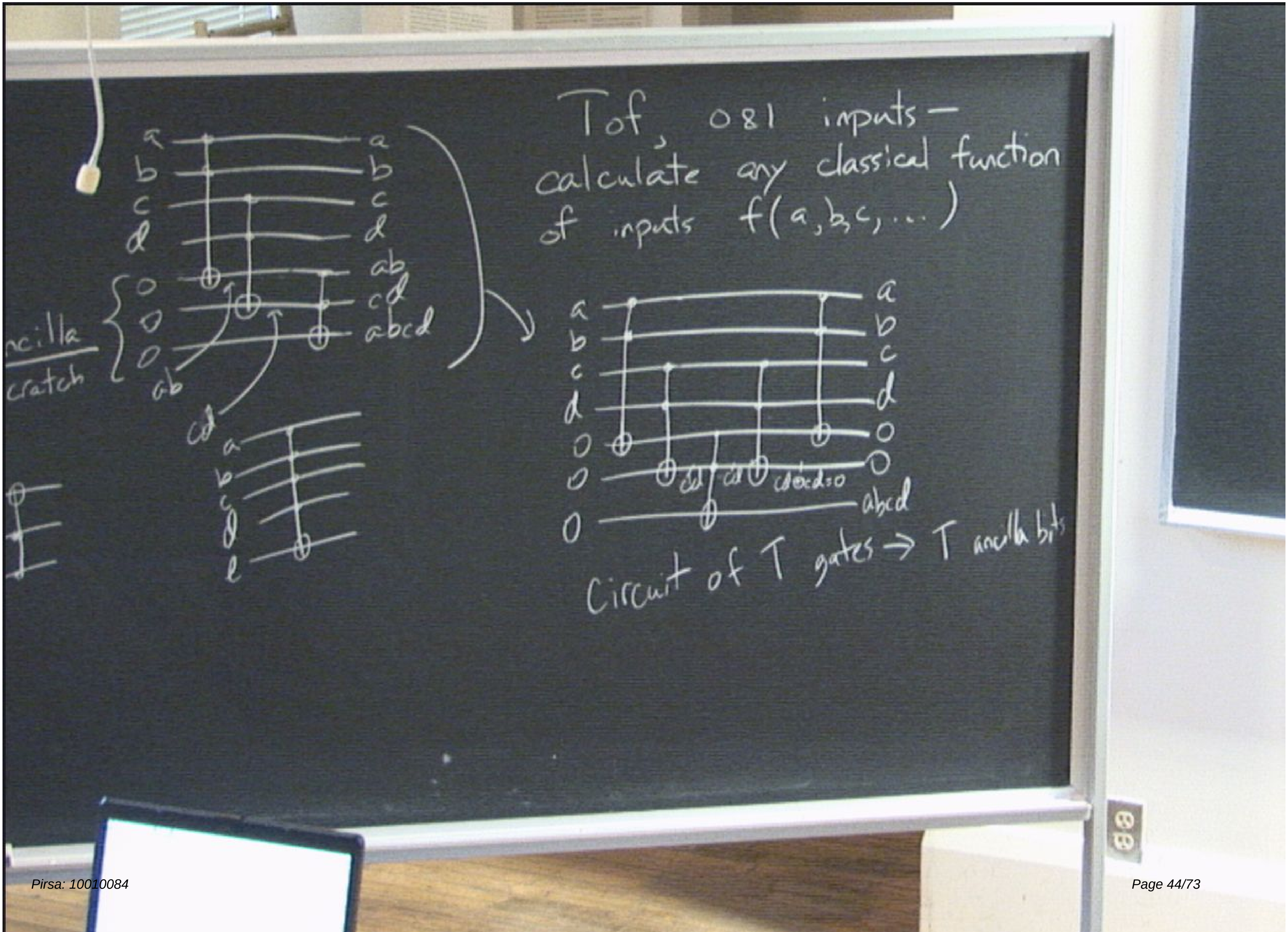
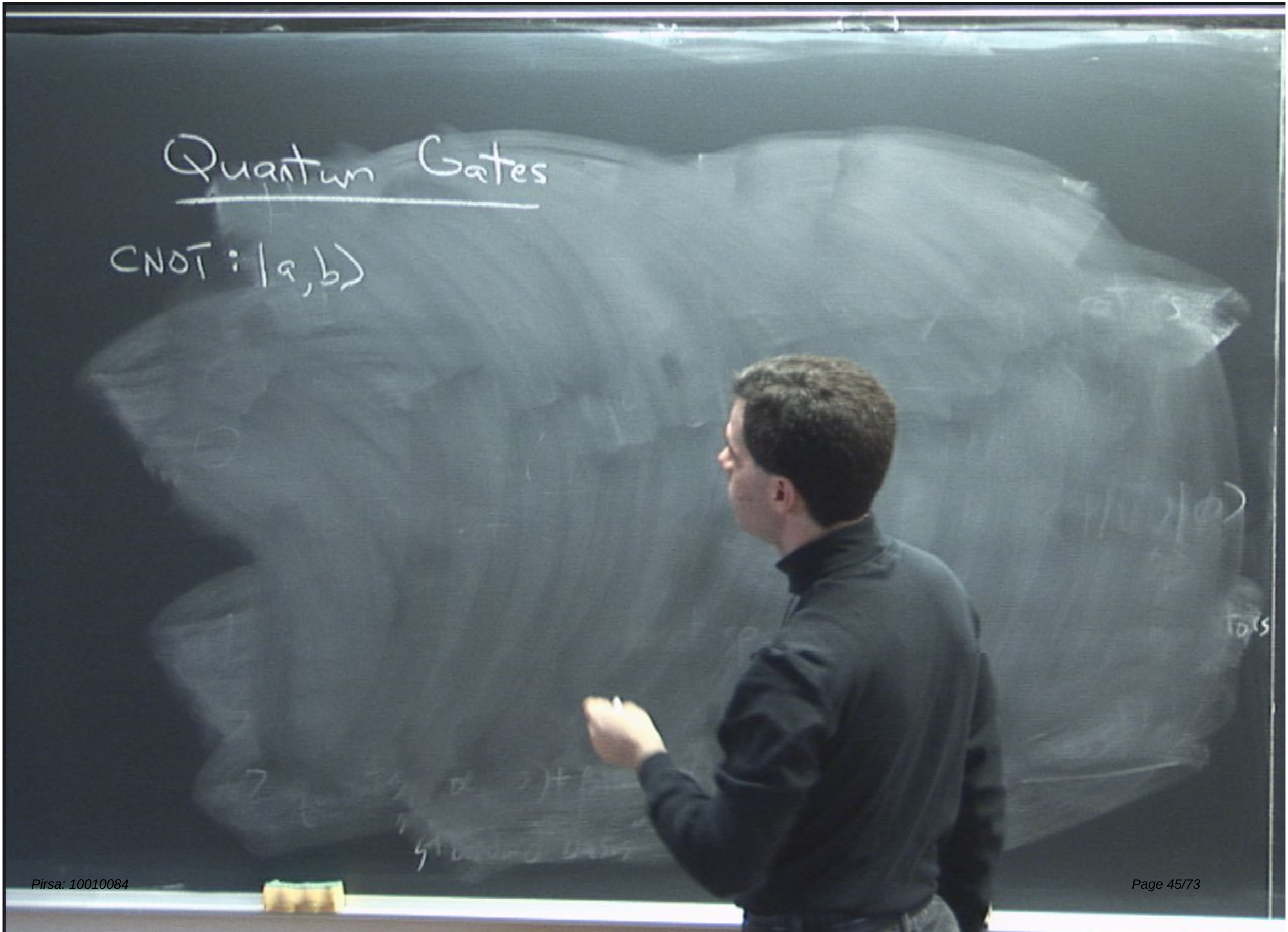
ancilla
scratch
a b c d ab cd abcd
a b c d 0 abcd

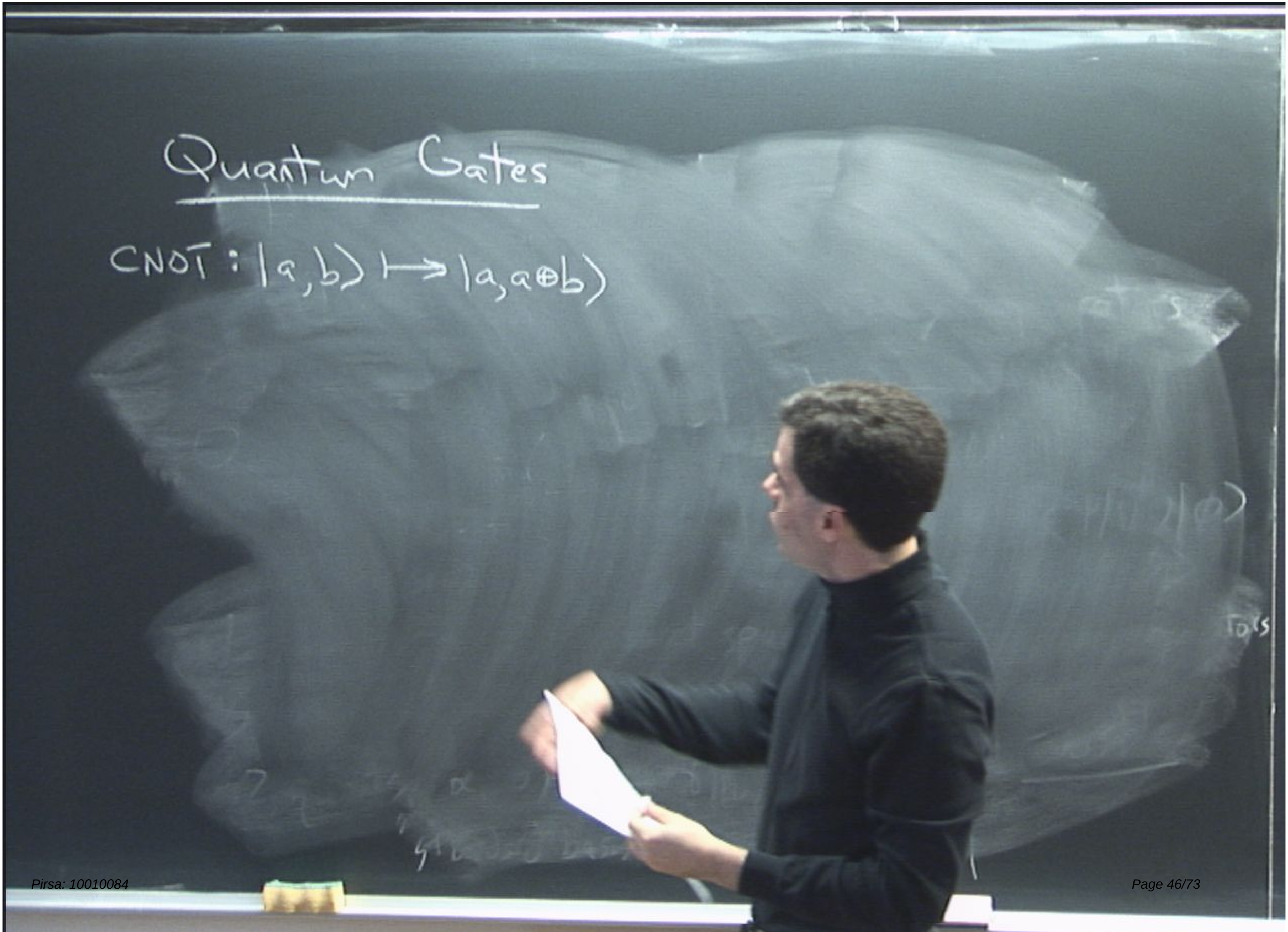

Tof, 0 & 1 inputs —
calculate any classical function
of inputs f(a,b,c,...)
Circuit of T gates
ab
cd
abcd
ncilla
cratch

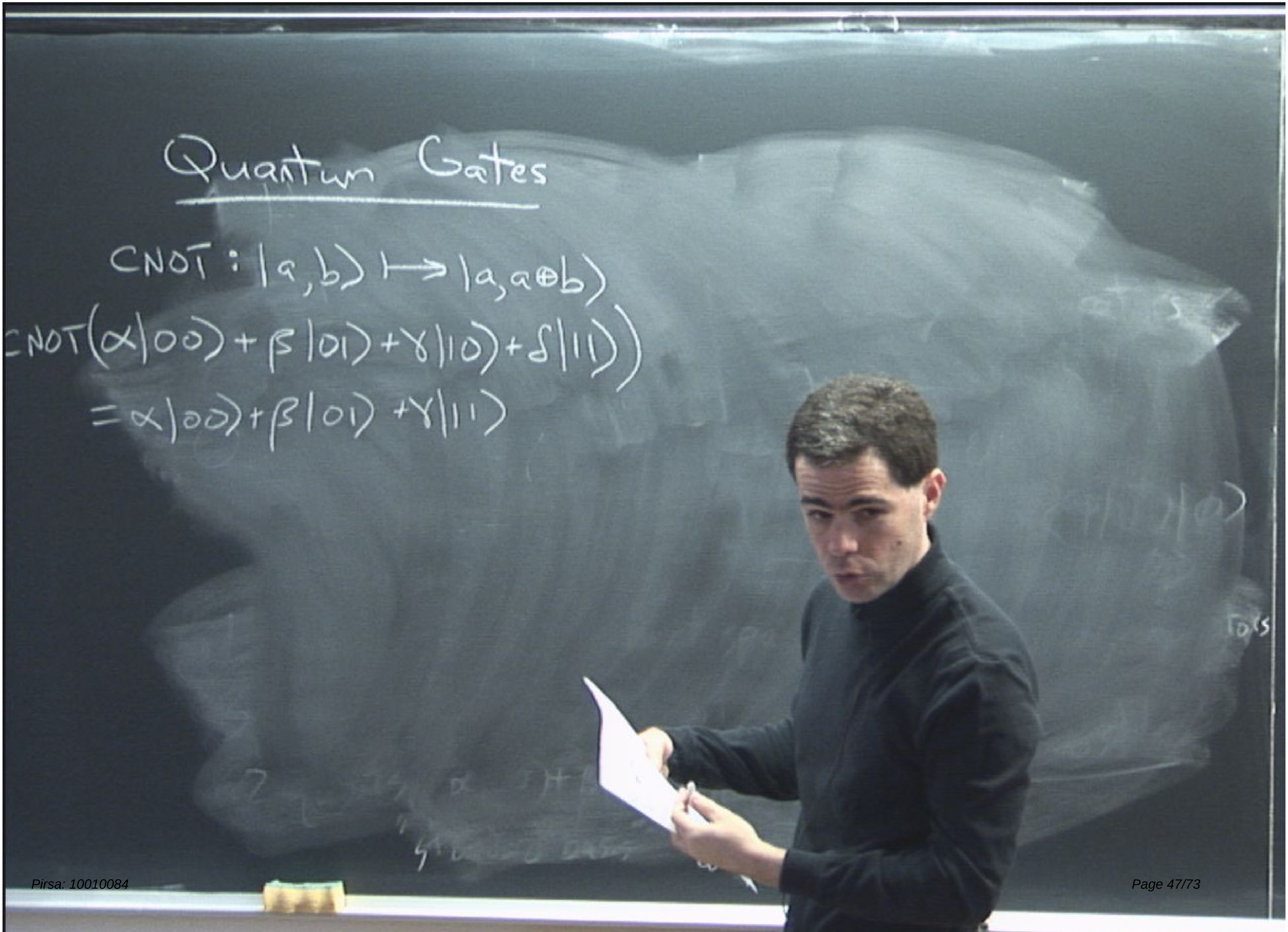

Tof, 0&1 inputs —
calculate any classical function
of inputs f(a,b,c,...)
ancilla
scratch
a b c d ab cd abcd
Circuit of T gates ⇒ T ancilla bits

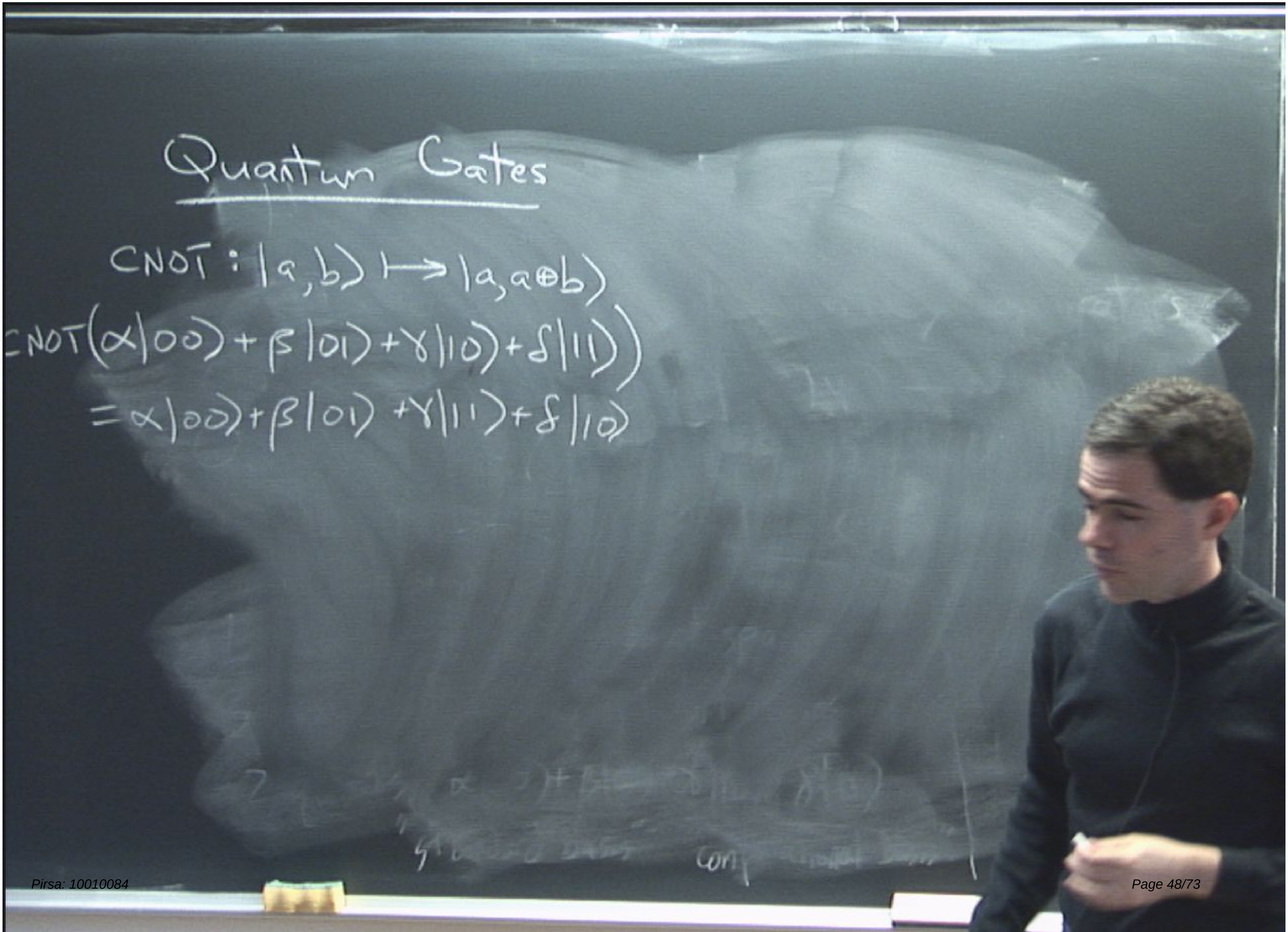

Tof, 0&1 inputs —
calculate any classical function
of inputs f(a,b,c,...)
Circuit of T gates → T ancilla bits
ncilla
cratch

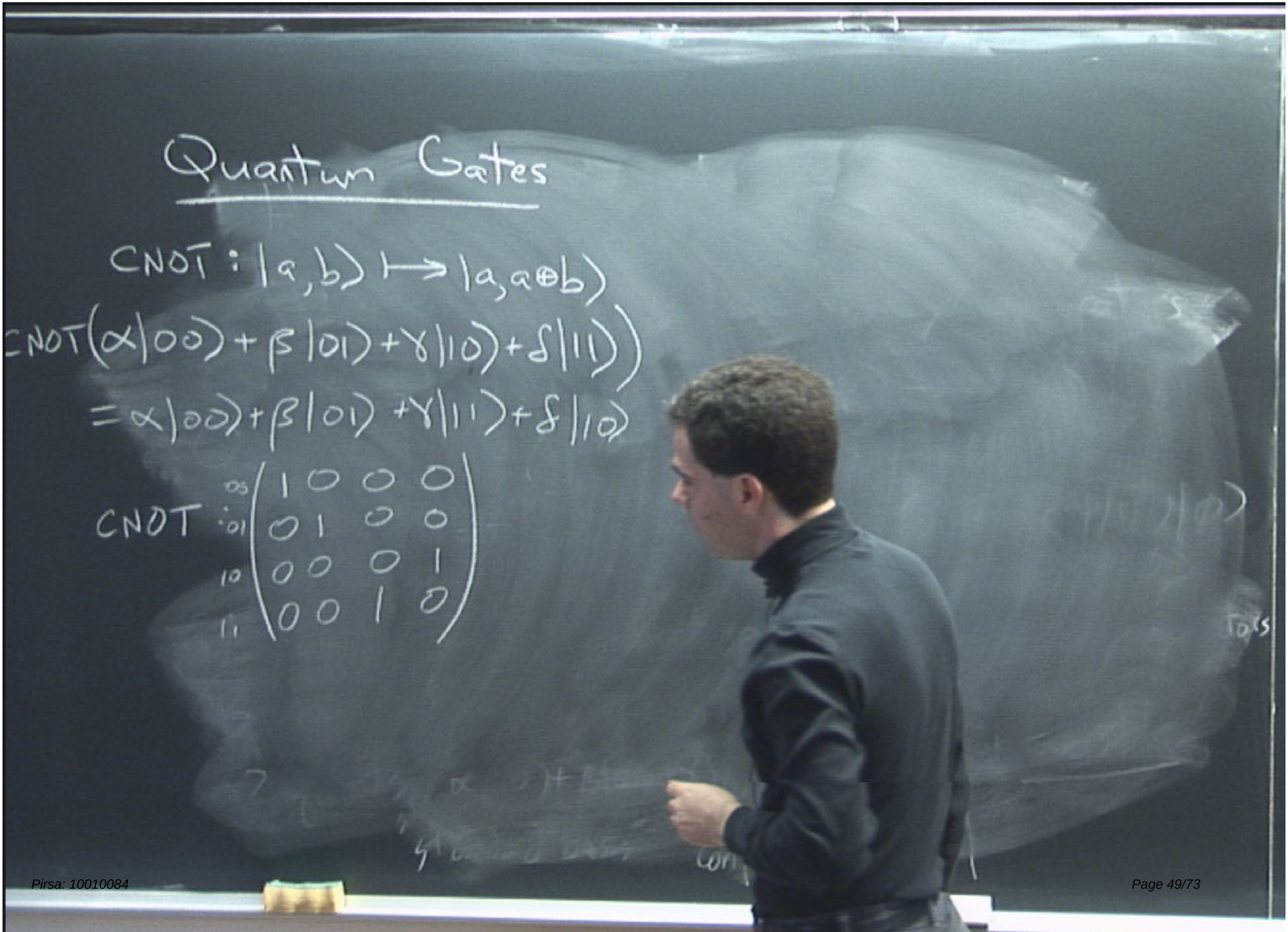

Tof, 0&1 inputs —
calculate any classical function
of inputs f(a,b,c,...)
ancilla
scratch
Circuit of T gates ⇒ T ancilla bits


Quantum Gates
CNOT : |a,b⟩


Quantum Gates
CNOT : $|a,b\rangle \mapsto |a,a\oplus b\rangle$


Quantum Gates
CNOT : $|a,b\rangle \mapsto |a,a\oplus b\rangle$
$\mathrm{CNOT}(\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle)$
$=\alpha|00\rangle+\beta|01\rangle+\gamma|11\rangle$


Quantum Gates
CNOT : |a,b⟩ ⟼ |a,a⊕b⟩
CNOT(α|00⟩+β|01⟩+γ|10⟩+δ|11⟩)
=α|00⟩+β|01⟩+γ|11⟩+δ|10⟩


Quantum Gates
CNOT : $|a,b\rangle \mapsto |a, a\oplus b\rangle$
CNOT$(\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle)$
$= \alpha|00\rangle + \beta|01\rangle + \gamma|11\rangle + \delta|10\rangle$
CNOT : $\begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

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$$+1 \bmod 3:\ \begin{pmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0 \end{pmatrix}$$

Quantum Gates

Tof. : $|a,b,c\rangle \to |a,b,c\oplus ab\rangle$

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Tof. : $|a,b,c\rangle \to |a,b,c\oplus ab\rangle$

Hadamard:

$$H|0\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)$$

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Quantum Gates

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...um Gates

$|a,b\rangle \mapsto |a, a\oplus b\rangle$

$\ldots|01\rangle + \gamma|10\rangle + \delta|11\rangle)\big)$

$\ldots|01\rangle + \gamma|11\rangle + \delta|10\rangle$

$$\begin{pmatrix} \ldots 0 & 0 & 0 \\ \ldots 1 & 0 & 0 \\ \ldots 0 & 0 & 1 \\ \ldots 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Tof.: $|a,b,c\rangle \to |a,b,c\oplus ab\rangle$

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$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

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$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad H^2 = \underline{I}$$

Reversible...

Classical b...

"logical gat...

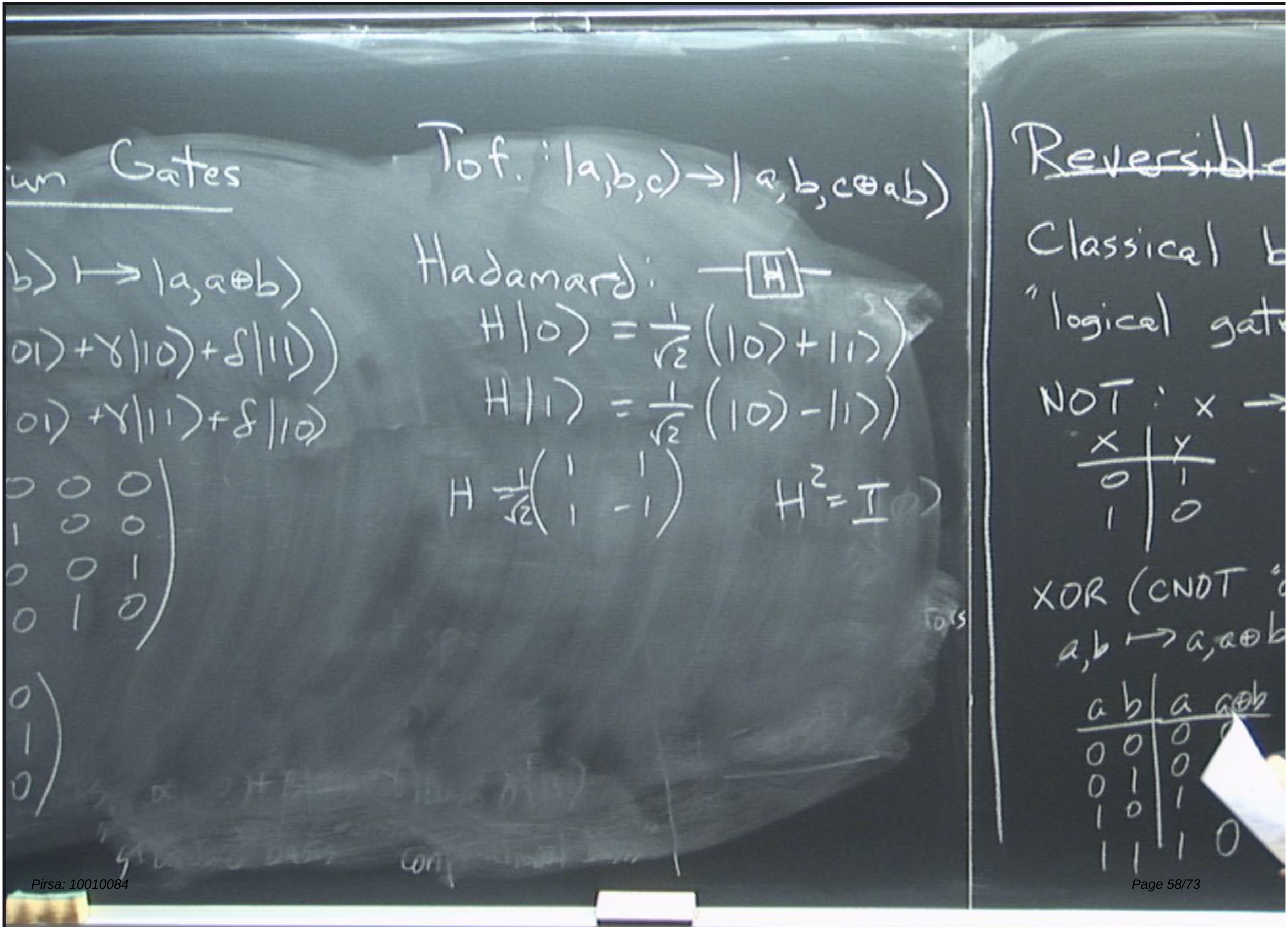
NOT: $x \to$ ...

x	y
0	1
1	0

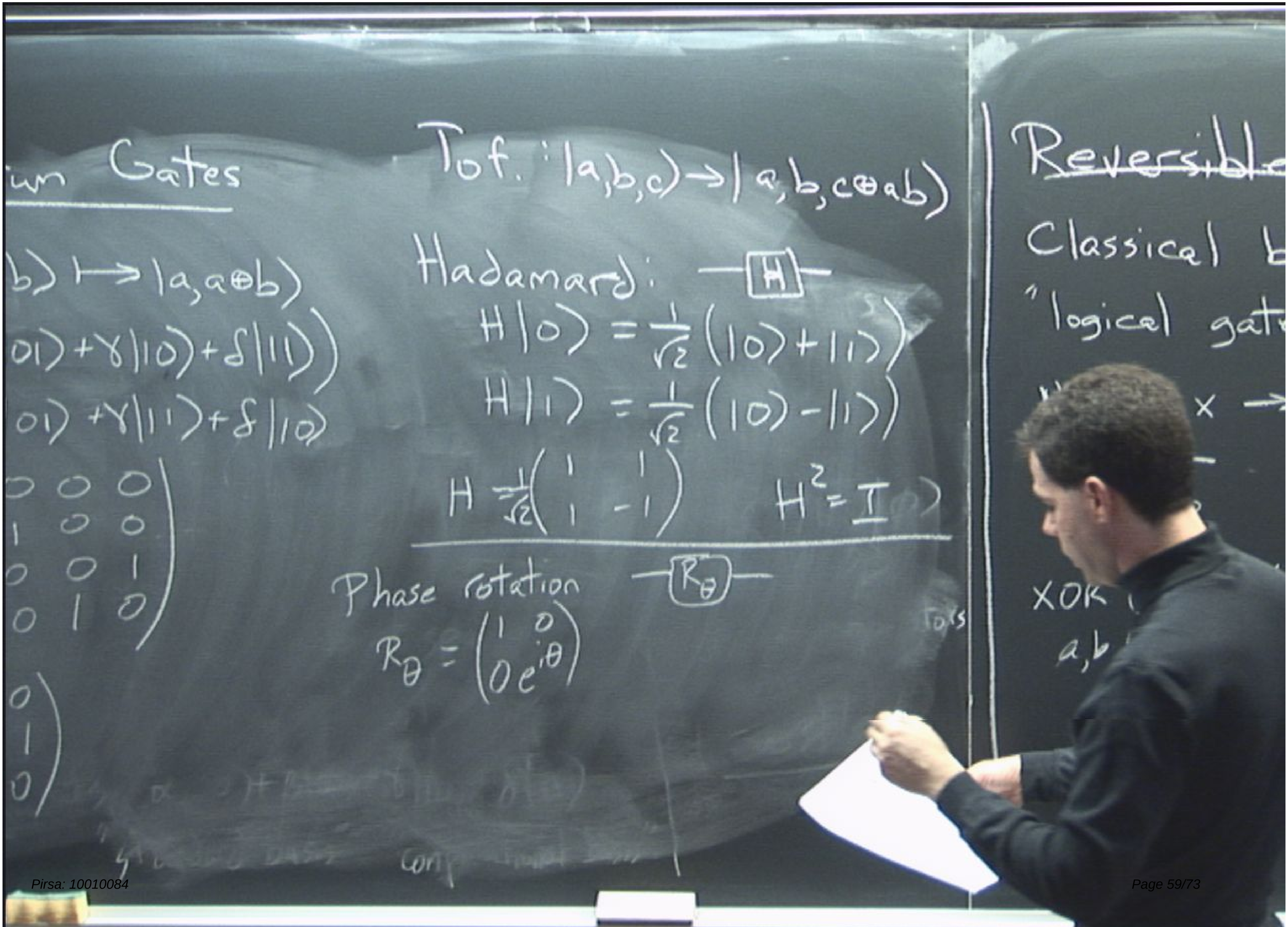
XOR (CNOT ...

$a,b \mapsto a, a\oplus b$

a	b	a	$a\oplus b$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0



...um Gates
b) ↦ |a, a⊕b)
01) + γ|10) + δ|11))
01) + γ|11) + δ|10)
Tof.: |a,b,c) → |a,b,c⊕ab)
Hadamard: —[H]—
H|0) = 1/√2 (|0) + |1))
H|1) = 1/√2 (|0) − |1))
H = 1/√2 (1 1 ; 1 −1)
H² = I
Reversible
Classical b
"logical gat
NOT: x →
XOR (CNOT
a,b ↦ a,a⊕b


Gates
Tof. : |a,b,c⟩ → |a,b,c⊕ab⟩
|a,a⊕b⟩
Hadamard:
H
H|0⟩ = 1/√2 (|0⟩+|1⟩)
H|1⟩ = 1/√2 (|0⟩−|1⟩)
H = 1/√2 (1 1; 1 −1)
H² = I
Phase rotation
Rθ
Rθ = (1 0; 0 e^{iθ})
Reversible
Classical
"logical
XOR
a,b

...um Gates

$|a,b\rangle \mapsto |a, a\oplus b\rangle$

$...01\rangle + \gamma|10\rangle + \delta|11\rangle)$

$...01\rangle + \gamma|11\rangle + \delta|10\rangle$

$$\begin{matrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}\Bigg)$$

$$\begin{matrix} 0 \\ 1 \\ 0 \end{matrix}\Bigg)$$

Tof.: $|a,b,c\rangle \to |a,b,c\oplus ab\rangle$

Hadamard: $-\boxed{H}-$

$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $\quad H^2 = \underline{I}$

Phase rotation $-\boxed{R_\theta}-$

$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$ $\quad |0\rangle \to |0\rangle$

$|1\rangle \to e^{i\theta}|1\rangle$

Reversible...

Classical b...

"logical gat...

NOT : $x \to$ ...

x	y
0	1
1	0

XOR (CNOT ...

$a,b \mapsto a, a\oplus b$

a b	a	$a\oplus b$
0 0	0	0
...	0	1

um Gates

$b\rangle \mapsto |a, a\oplus b\rangle$

$01\rangle + \gamma|10\rangle + \delta|11\rangle)\big)$

$01\rangle + \gamma|11\rangle + \delta|10\rangle$

$$\begin{matrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}\Bigg)$$

$$\begin{matrix} 0 \\ 1 \\ 0 \end{matrix}\Bigg)$$

Tof.: $|a,b,c\rangle \to |a,b,c\oplus ab\rangle$

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$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

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Phase rotation $-\boxed{R_\theta}-$

$$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \qquad |0\rangle \to |0\rangle, \quad |1\rangle \to e^{i\theta}|1\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \to \alpha|0\rangle + e^{i\theta}\beta|1\rangle$$

Reversible

Classical b

"logical gat

NOT : $x \to$

x	y
0	1
1	0

XOR (CNOT

$a,b \mapsto a, a\oplus b$

a	b	a	$a\oplus b$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

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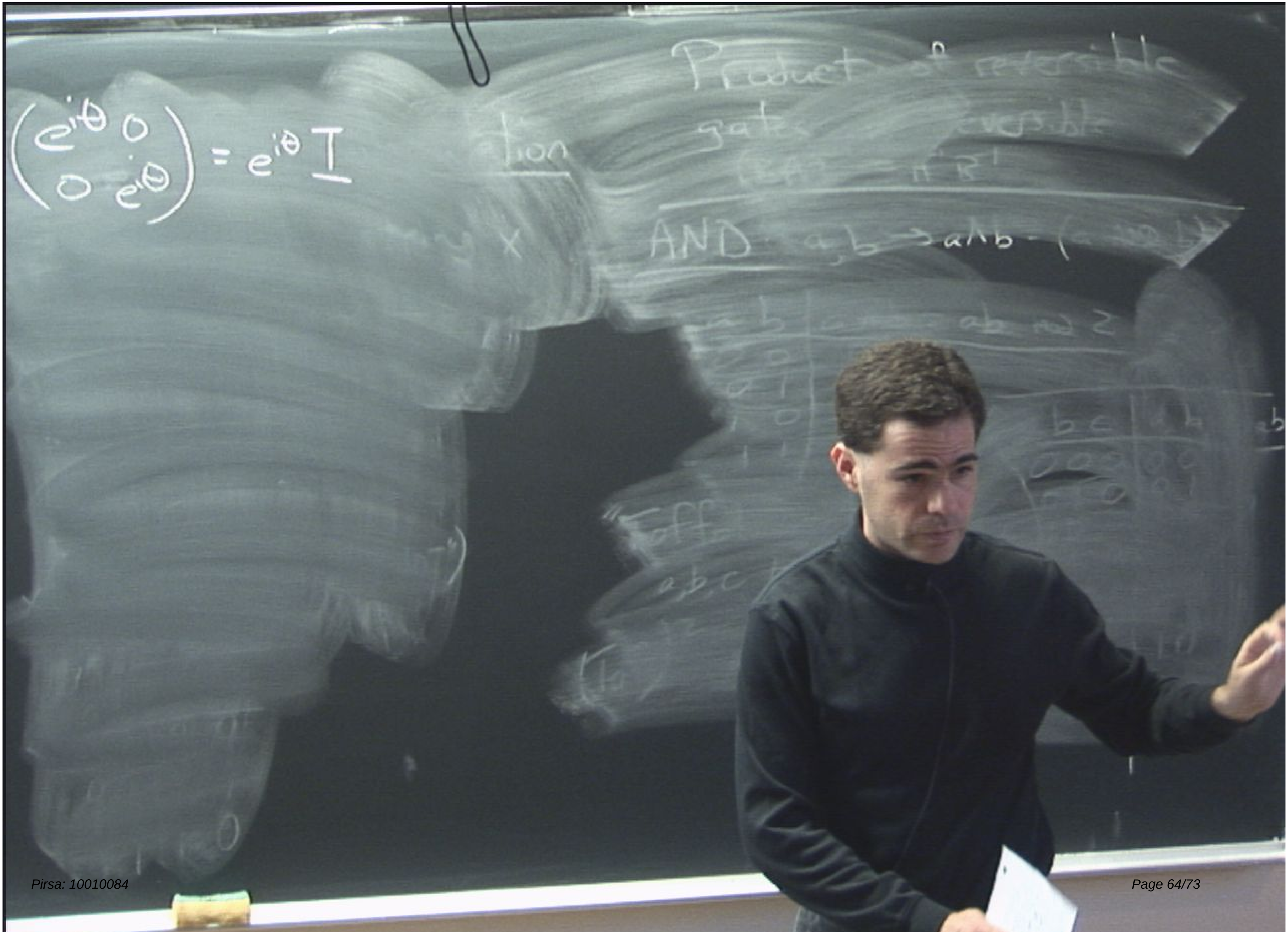
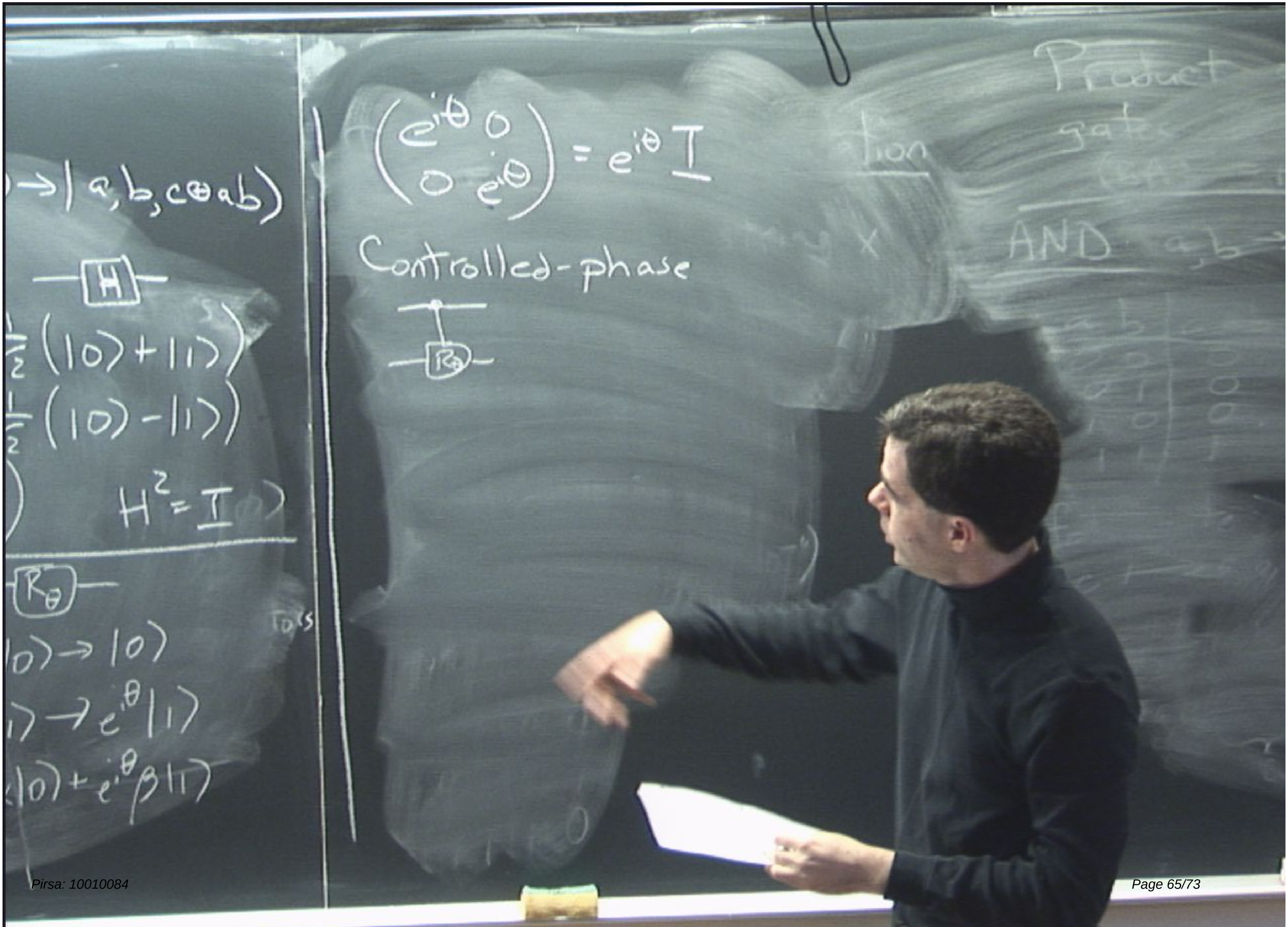
$$|1\rangle \rightarrow e^{i\theta}|1\rangle$$

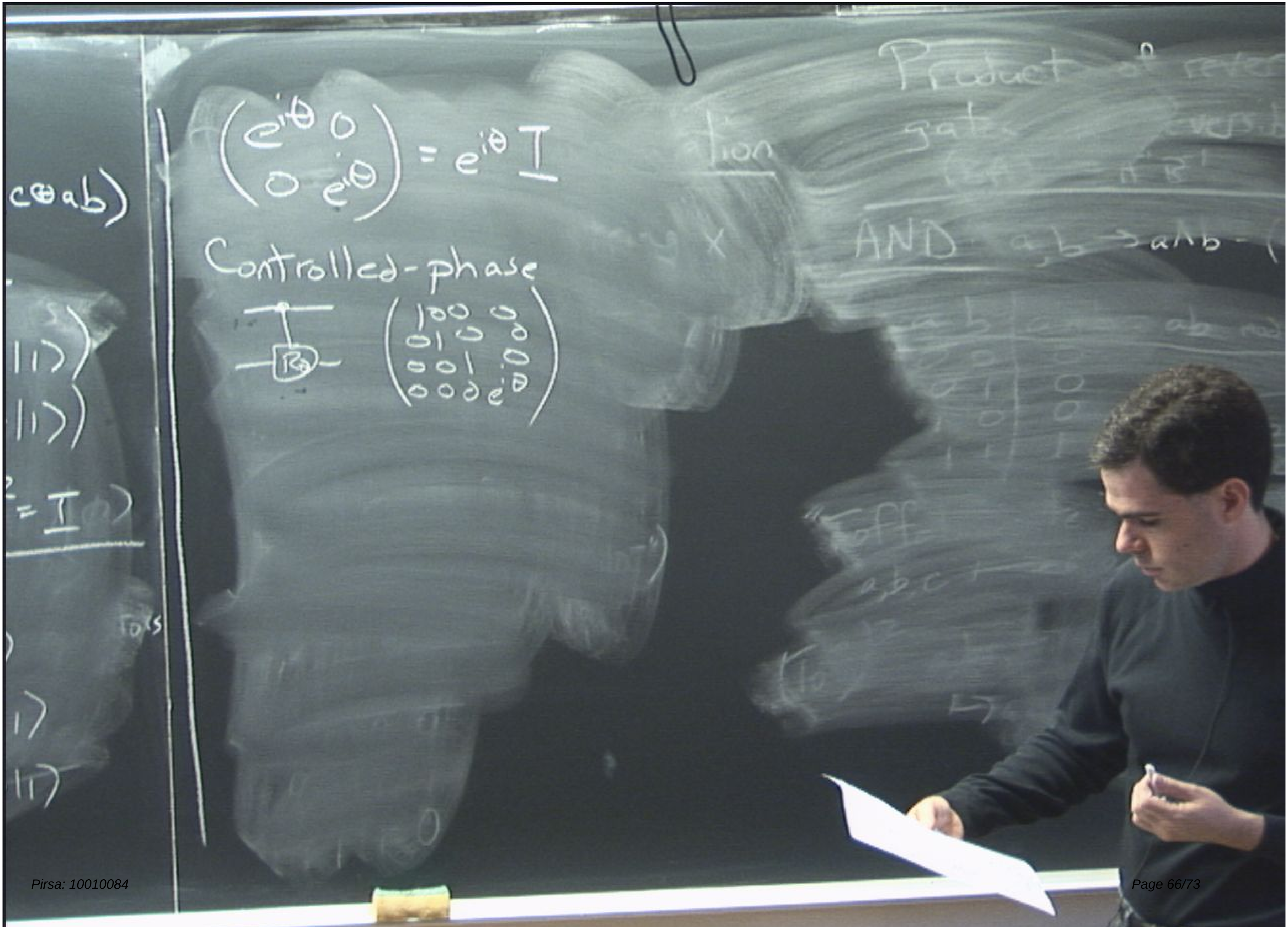
$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle + e^{i\theta}\beta|1\rangle$$

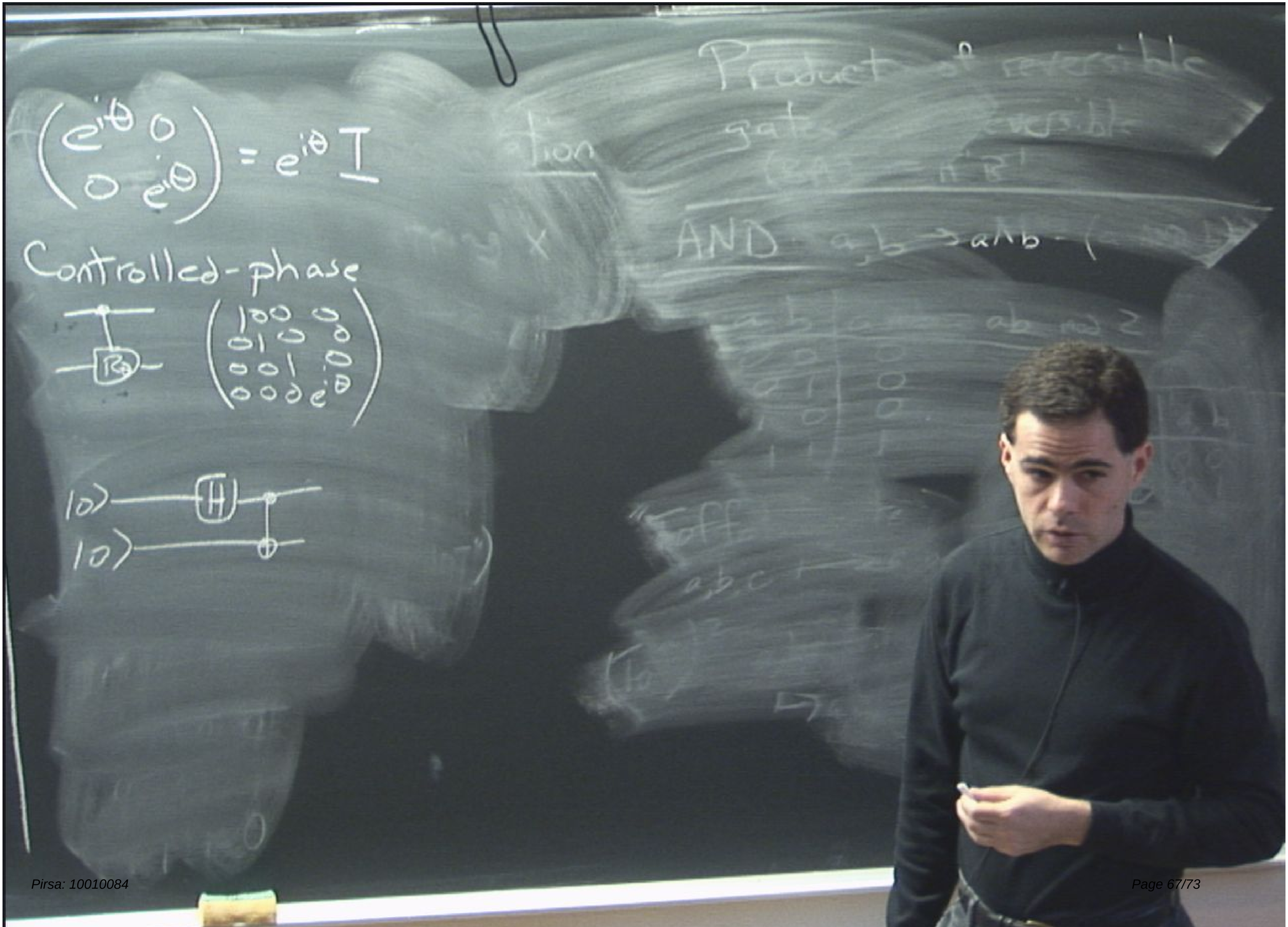

Product of reversible
AND

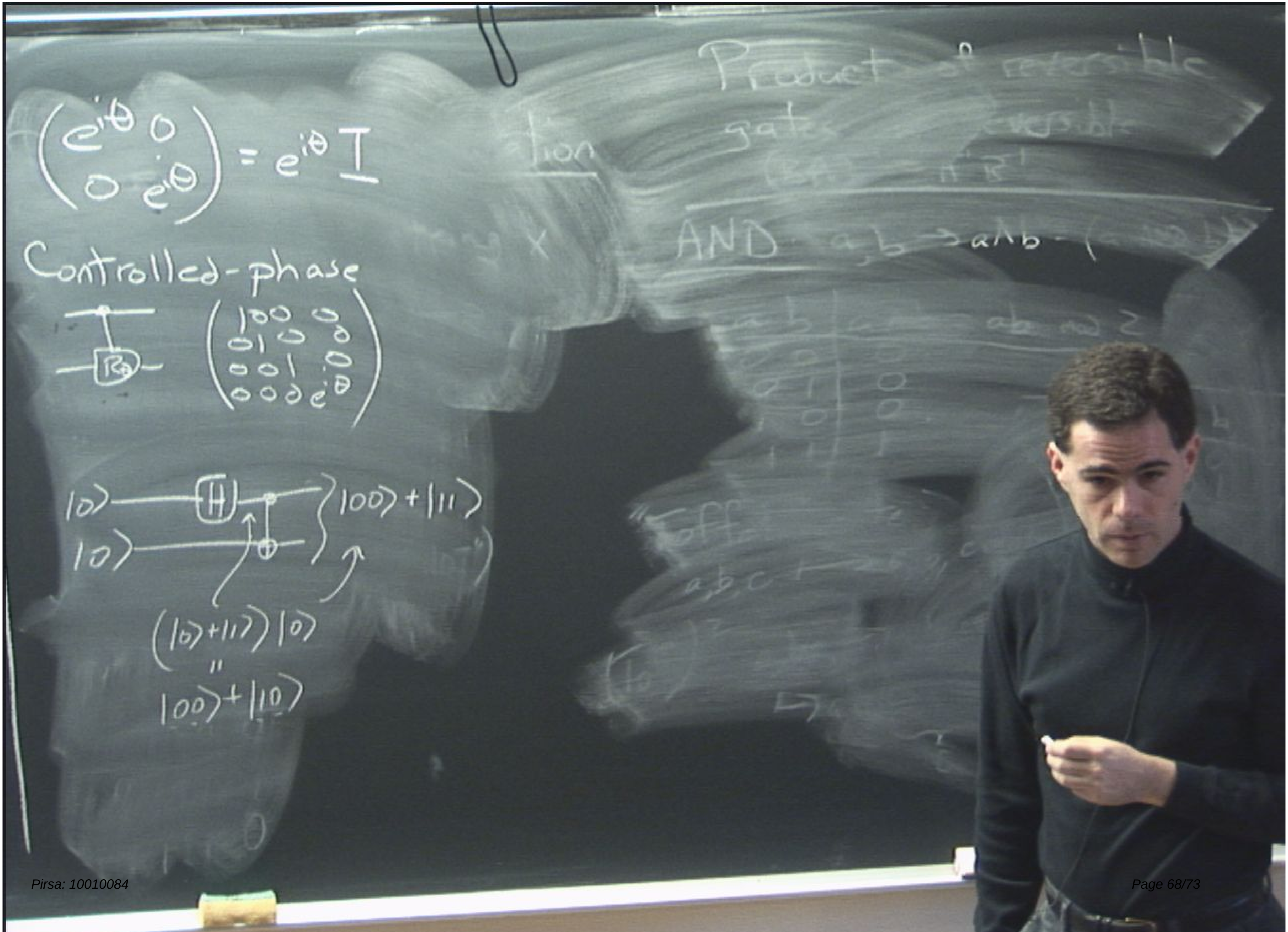

$\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta} I$
Product of reversible
AND

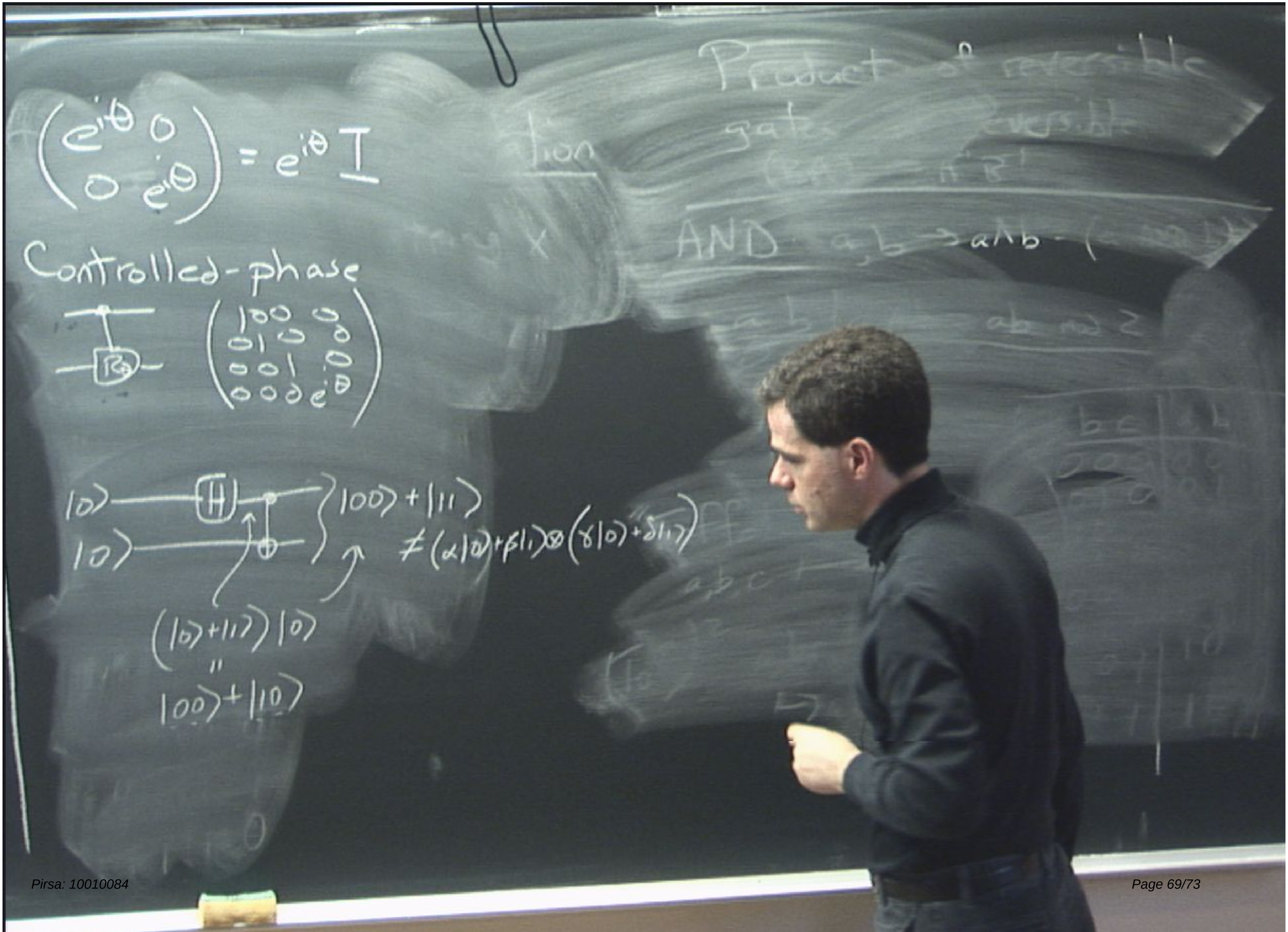


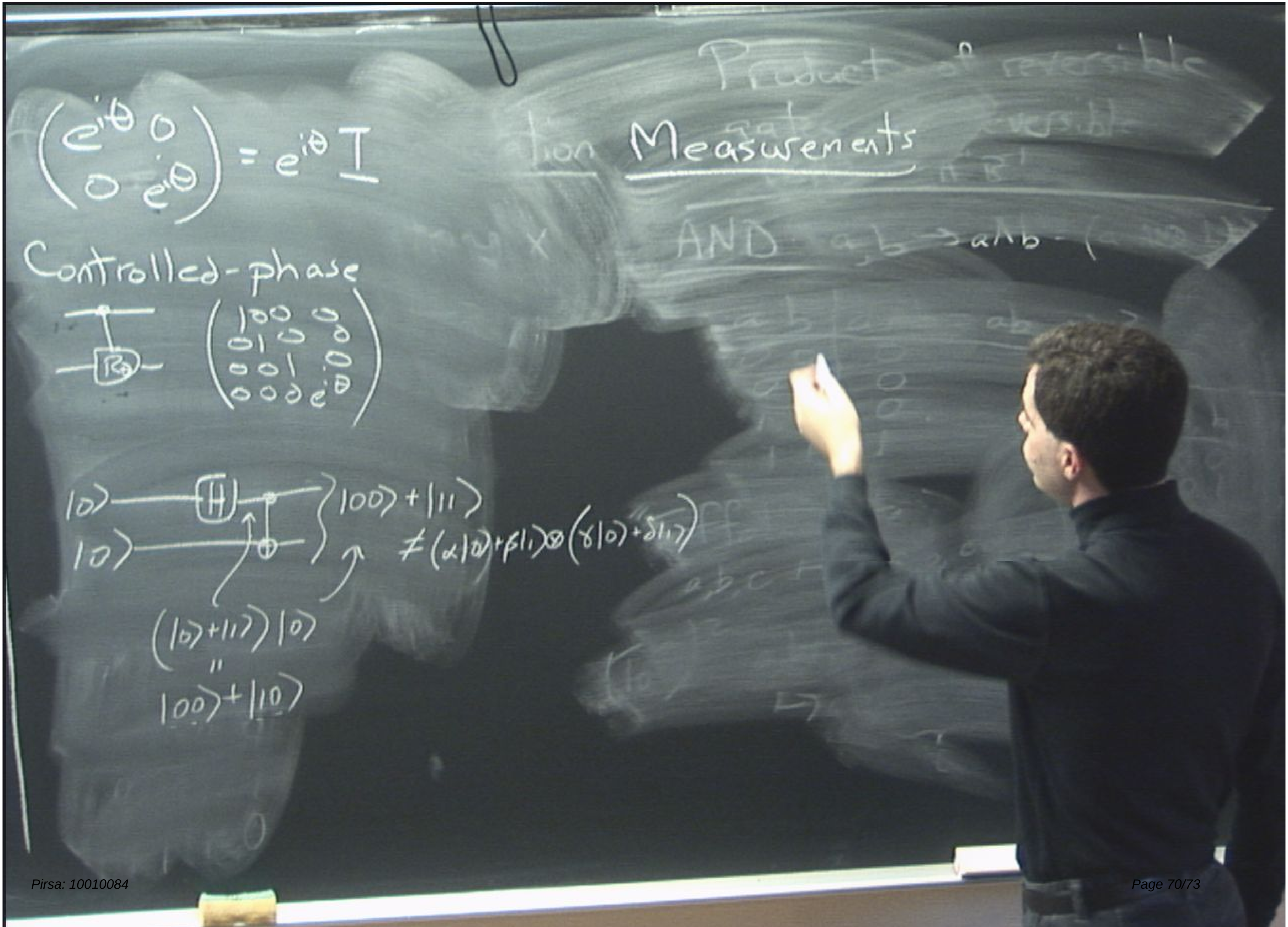
$\rightarrow |a, b, c \oplus ab\rangle$
$\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta} I$
Controlled-phase
R_θ
H
$(|0\rangle + |1\rangle)$
$(|0\rangle - |1\rangle)$
$H^2 = I$
$|0\rangle \rightarrow |0\rangle$
$|1\rangle \rightarrow e^{i\theta}|1\rangle$
$|0\rangle + e^{i\theta}\beta|1\rangle$
Product
AND

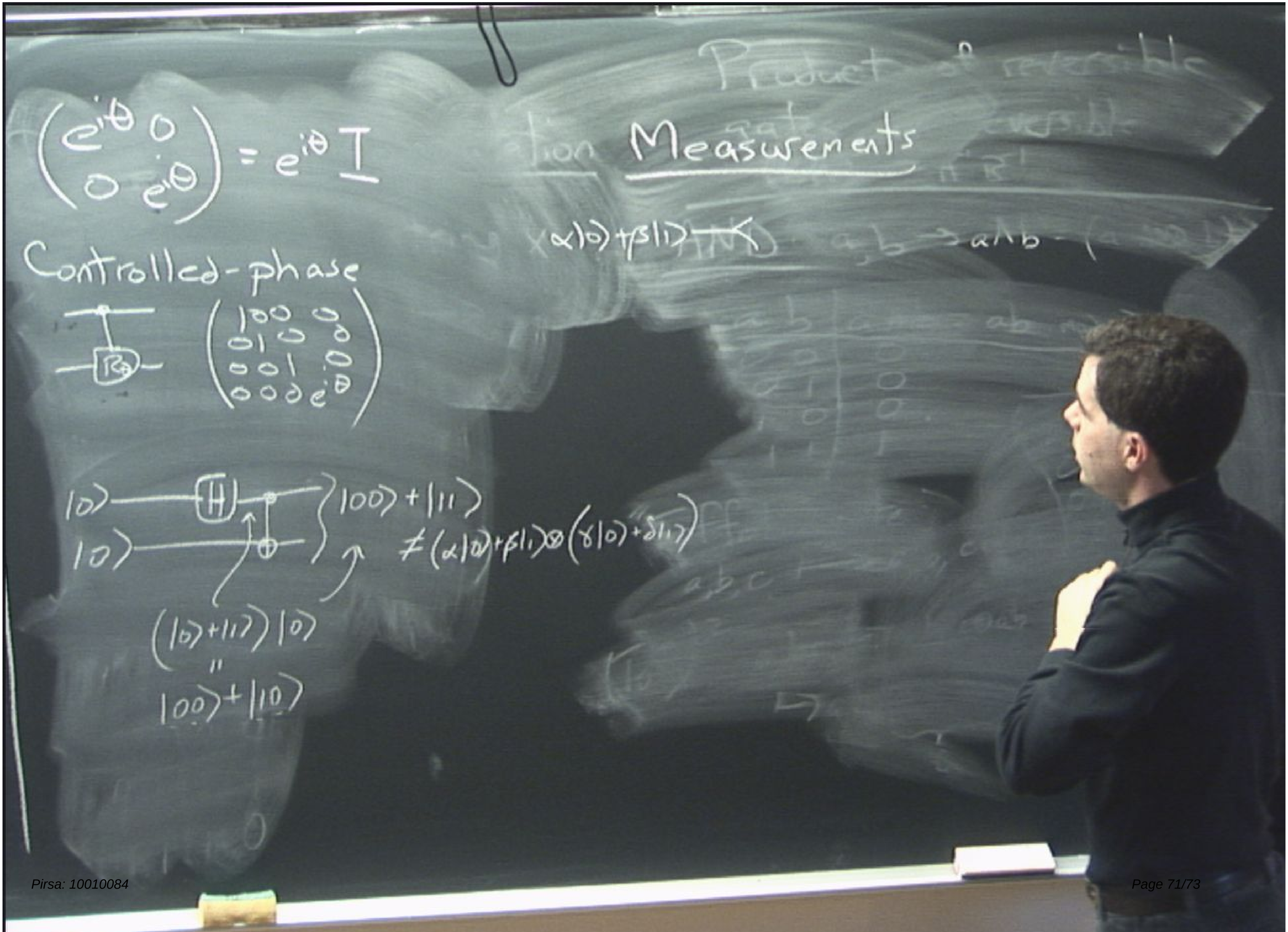

$\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta} \underline{I}$
Controlled-phase
$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\theta} \end{pmatrix}$
Product
AND


Product of reversible
AND
Controlled-phase


$\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta} I$
Controlled-phase
$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\theta} \end{pmatrix}$
$|0\rangle$
$|0\rangle$
H
$|00\rangle + |11\rangle$
$(|0\rangle + |1\rangle)|0\rangle$
$=$
$|00\rangle + |10\rangle$
Product of reversible
gates
AND


Product of reversible
= e^{iθ} I
Controlled-phase
AND
|0⟩
|0⟩
|00⟩ + |11⟩
≠ (α|0⟩+β|1⟩)⊗(γ|0⟩+δ|1⟩)
(|0⟩+|1⟩)|0⟩
|00⟩+|10⟩


Product of reversible
Measurements
$\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta} I$
Controlled-phase
AND
$|0\rangle$
$|0\rangle$
$|00\rangle + |11\rangle$
$\neq (\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle)$
$(|0\rangle + |1\rangle)|0\rangle$
$|00\rangle + |10\rangle$



Measurements
Controlled-phase
$\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta} I$
$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\theta} \end{pmatrix}$
$\alpha|0\rangle + \beta|1\rangle$
$|0\rangle$
$|0\rangle$
H
$|00\rangle + |11\rangle$
$\neq (\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle)$
$(|0\rangle + |1\rangle)|0\rangle$
$=$
$|00\rangle + |10\rangle$

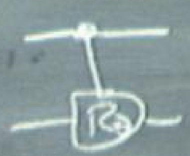
$$\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta} I$$

Measurements

$\alpha|0\rangle + \beta|1\rangle$ — 0 w/ prob. $|\alpha|^2$

1 w/ prob. $|\beta|^2$

Controlled-phase



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\theta} \end{pmatrix}$$

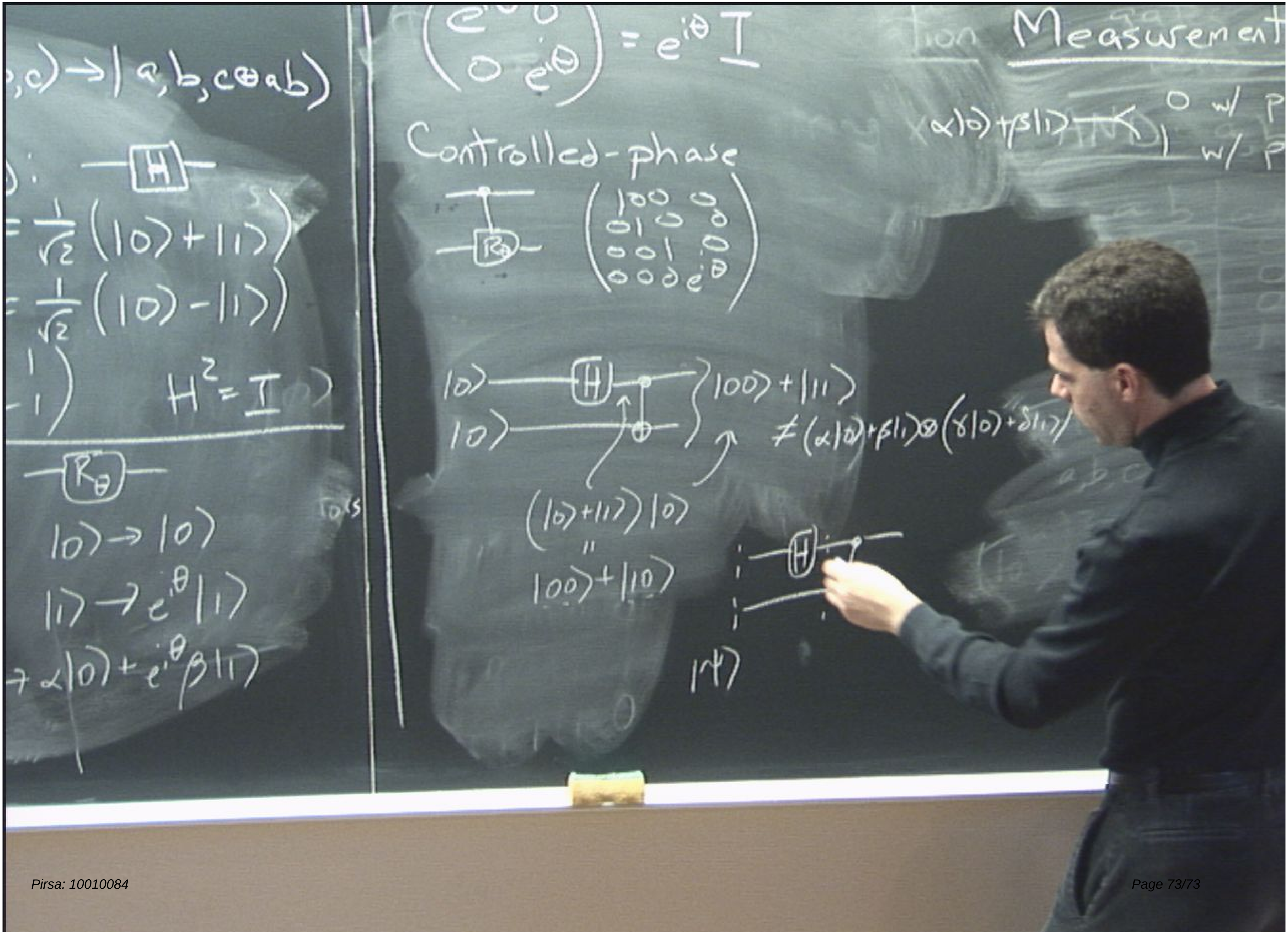
$|00\rangle + |11\rangle$

$\neq (\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle)$

$(|0\rangle + |1\rangle)|0\rangle$

$=$

$|00\rangle + |10\rangle$


Measurement
Controlled-phase
|a,b,c⊕ab⟩
= e^{iθ} I
|00⟩ + |11⟩
≠ (α|0⟩+β|1⟩)⊗(γ|0⟩+δ|1⟩)
(|0⟩+|1⟩)|0⟩
|00⟩+|10⟩
|0⟩ → |0⟩
|1⟩ → e^{iθ}|1⟩
α|0⟩ + e^{iθ}β|1⟩
H² = I
|ψ⟩
α|0⟩+β|1⟩
0 w/ p
1 w/ p