Title: Quantum Field Theory for Cosmology - Lecture 6

Date: Jan 28, 2010 05:00 PM

URL: http://pirsa.org/10010076

Abstract: This course begins with a thorough introduction to quantum field theory. Unlike the usual quantum field theory courses which aim at applications to particle physics, this course then focuses on those quantum field theoretic techniques that are important in the presence of gravity. In particular, this course introduces the properties of quantum fluctuations of fields and how they are affected by curvature and by gravitational horizons. We will cover the highly successful inflationary explanation of the fluctuation spectrum of the cosmic microwave background - and therefore the modern understanding of the quantum origin of all inhomogeneities in the universe (see these amazing visualizations from the data of the Sloan Digital Sky Survey. They display the inhomogeneous distribution of galaxies several billion light years into the universe: Sloan Digital Sky Survey).

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QFT for Cosmology, Achim Kempf, Winter 2010, Lecture 6

Recall:

- There are two basic mechanisms to increase the amplitudes of oscillators, i.e., also to excite a field's mode oscillators, i.e. to create particles:

  a.) A time-varying driving force $J(t)$

  b.) A time-varying spring "constant" $\omega(t)$

- We are presently considering case a) :

$$\hat{H}(t) = \frac{1}{2}\hat{p}(t)^2 + \frac{\omega^2}{2}\hat{q}(t)^2 - J(t)\,\hat{q}(t)$$

with a temporary force: $J(t) = 0$ for all $t \notin [0,T]$

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Examples: 1. Temporary emission from antenna, 2. Brief interaction (scattering) of particles.

- We defined a convenient variable $a(t)$,

$$a(t) := \sqrt{\frac{\omega}{2}}\,\hat{q}(t) + i\frac{1}{\sqrt{2\omega}}\,\hat{p}(t)$$

so that: $\hat{H}(t) = \omega\left(a^+(t)\,a(t) + \frac{1}{2}\right) - \frac{1}{\sqrt{2\omega}}J(t)\left(a^+(t) + a(t)\right)$

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- This meant that we had to solve the simpler problem:

$*$ $\quad i\dot{a}(t) = \omega\, a(t) - \frac{1}{\sqrt{2\omega}} J(t) \qquad$ (EOM)

$*$ $\quad [a(t), a^+(t)] = 1$ for all $t \qquad$ (CCR)

- We solved (EOM) with the arbitrary initial condition:

$$a(0) = a_{in}$$

(an operator on Hilbert space that we still have to choose.)

and obtained:

$$a(t) = a_{in}\,e^{-i\omega t} + \frac{i}{\cdots}\int^{t} J(t')\,e^{i\omega(t'-t)}\,dt'$$

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Automatic Updates

Updating your computer is almost complete. Your computer needs to be restarted for the updates to take effect. Windows will restart your computer automatically in 14 ...utes.

Do you want to restart your computer now?

Restart Now Restart Later

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Automatic Updates

Updating your computer is almost complete. Your computer needs to be restarted for the updates to take effect. Windows will restart your computer automatically in 14:42 minutes.

Do you want to restart your computer now?

Restart Now   Restart Later

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We can solve **(CCR)** for all times t by finding an operator a_{in} which obeys:

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Structure of the solution

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Structure of the solution

□ Recall that the solution (Sol) can also be written as

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- Recall that the solution (Sol) can also be written as:

$$a(t) = \left(a_{in} + 1 \frac{i}{\sqrt{2\omega}} \int_0^t J(t') e^{i\omega t'} dt'\right) e^{-i\omega t}$$

- Since the force vanishes, $J(t) = 0$, when $t \notin [0,T]$ we noticed that:

$$a(t) = \begin{cases} a_{in} e^{-i\omega t} & \text{for } t<0 \\ \text{see above} & \text{for } 0<t<T \end{cases}$$

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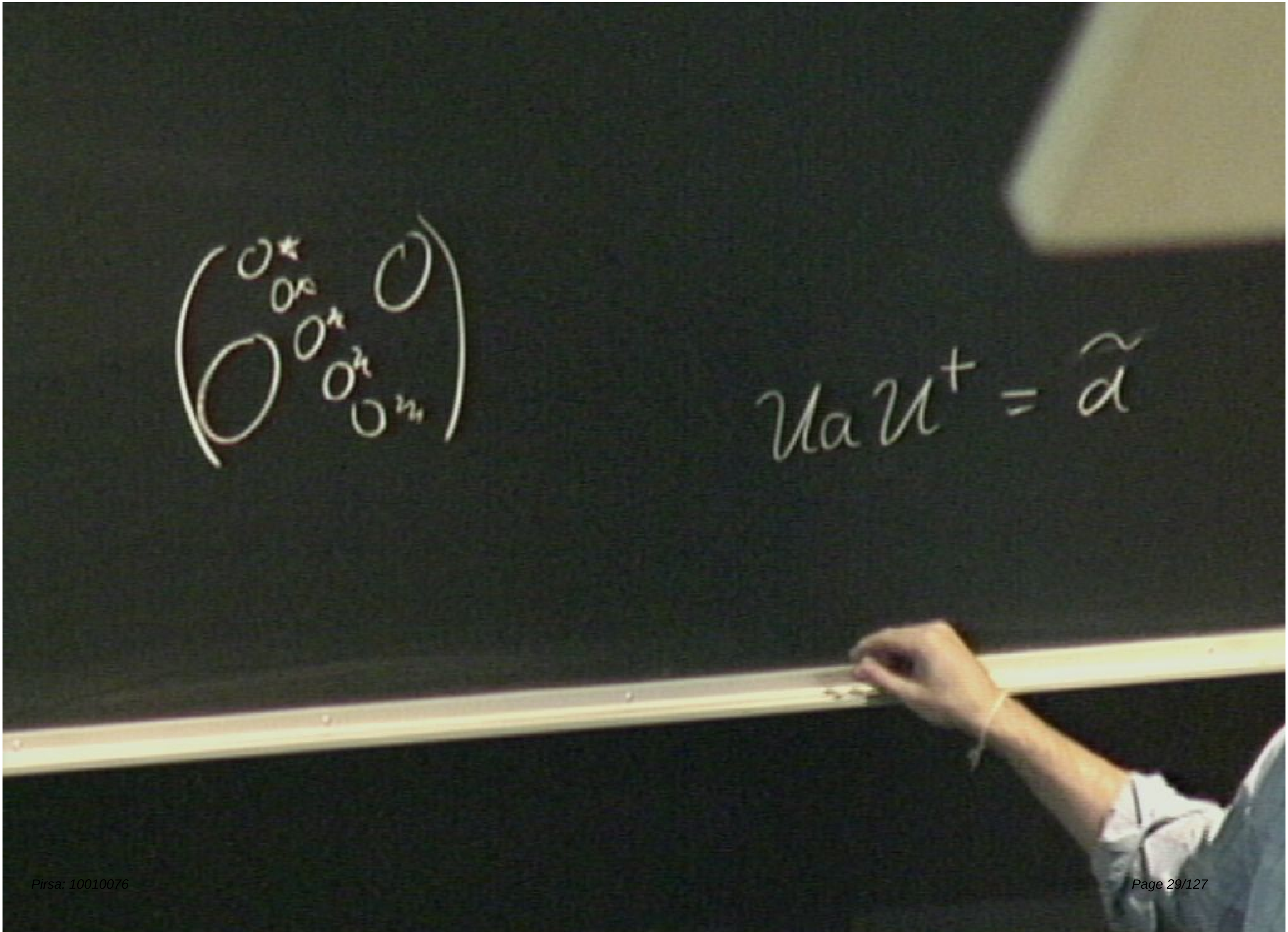
$\mathcal{U} a \mathcal{U}^{+} = \tilde{a}$

$$\begin{pmatrix} 0^{*} & & & & \\ & 0^{\infty} & & \mathcal{O} & \\ & & 0^{*} & & \\ & \mathcal{O} & & 0^{\lambda} & \\ & & & & 0^{\lambda_n} \end{pmatrix}$$

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number number

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$$= \left(\omega a_m^+ (a_m^+ \cancel{a_m} + 1) + \frac{\omega}{2} a_m^+\right)|0_m\rangle$$

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$$= \left(\omega\, a^+_{\vec{m}}\left(a^+_{\vec{m}}\cancel{a_{\vec{m}}} + 1\right) + \frac{\omega}{2}\,a^+_{\vec{m}}\right)|0_{\vec{m}}\rangle$$

$$= \omega\,\frac{3}{2}\,a^+|0_{\vec{m}}\rangle$$

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$\Rightarrow$ The state $|1_{\vec{m}}\rangle$ is eigenstate of $\hat{H}$ with eigenvalue $\frac{3}{2}\omega$. So it must be the 1^{st} excited state.

* Is the vector $|1_{\vec{m}}\rangle$ normalized?

$$\langle 1_{\vec{m}}|1_{\vec{m}}\rangle = \langle 0|a_{\vec{m}}\,a^+_{\vec{m}}|0\rangle = \langle 0|\,a^+_{\vec{m}}a_{\vec{m}} + 1\,|0\rangle = \langle 0_{\vec{m}}|0_{\vec{m}}\rangle = 1$$

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* Consider now the state $|1_{\vec{m}}\rangle := a^+_{\vec{m}}\,|0_{\vec{m}}\rangle$:

$$\hat{H}_{(\vec{m})}|1_{\vec{m}}\rangle = \hat{H}_{(\vec{m})}\,a^+_{\vec{m}}\,|0_{\vec{m}}\rangle = \omega\left(a^+_{\vec{m}}a_{\vec{m}} + \frac{1}{2}\right)a^+_{\vec{m}}\,|0_{\vec{m}}\rangle$$

$$= \left(\omega\, a^+_{\vec{m}}\left(a^+_{\vec{m}}\cancel{a_{\vec{m}}} + 1\right) + \frac{\omega}{2}a^+_{\vec{m}}\right)|0_{\vec{m}}\rangle$$

$$= \omega\,\frac{3}{2}\,a^+|0_{\vec{m}}\rangle$$

$$= \frac{3}{2}\omega\,|1_{\vec{m}}\rangle$$

$\Rightarrow$ The state $|1_{\vec{m}}\rangle$ is eigenstate of $\hat{H}$ with eigenvalue $\frac{3}{2}\omega$. So it must be the 1^{st} excited state.

* Is the vector $|1_{\vec{m}}\rangle$ normalized?

$$\langle 1_{\vec{m}}|1_{\vec{m}}\rangle = \langle 0|a_{\vec{m}}\,a^+_{\vec{m}}|0\rangle = \langle 0|a^+_{\vec{m}}a_{\vec{m}} + 1|0\rangle = \langle 0_{\vec{m}}|0_{\vec{m}}\rangle = 1$$

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$$\hat{H}_{(t)} |1_{\vec{m}}\rangle = \hat{H}_{(t)} a^{+}_{\vec{m}} |0_{\vec{m}}\rangle = \omega \left(a^{+}_{\vec{m}} a_{\vec{m}} + \frac{1}{2} \right) a^{+}_{\vec{m}} |0_{\vec{m}}\rangle$$

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$$= \omega \frac{3}{2} a^{+} |0_{\vec{m}}\rangle$$

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$\Rightarrow$ The state $|1_{in}\rangle$ is eigenstate of $\hat{H}$ with eigenvalue $\frac{3}{2}\omega$. So it must be the 1^{st} excited state.

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$$\langle 1_{in}|1_{in}\rangle = \langle 0| a_{in} a^+_{in} |0\rangle = \langle 0| a^+_{in} a_{in} + 1 |0\rangle = \langle 0_{in}|0_{in}\rangle = 1$$

✓

* **Proposition:**

The set of vectors $\{|n_{in}\rangle\}_{n=0}^{\infty}$ defined through

$$|n_{in}\rangle := \frac{1}{\sqrt{n!}} (a^+)^n |0_{in}\rangle$$

is orthonormal, i.e., $\langle n|n'\rangle = \delta_{n,n'}$. Exercise: verify

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The set of vectors $\{|n_-\rangle\}_{n=0}^{\infty}$ defined through

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The vectors $|n_-\rangle$ are eigenvectors of $\hat{H}_{t<0}$:

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$$E_n = \omega\left(n + \frac{1}{2}\right)$$

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The vectors $|n_{in}\rangle$ are eigenvectors of $\hat{H}_{t<0}$:

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⇝ **Summary re choice of basis for $t<0$:**

- The Hamiltonian $\hat{H}(t)$ is constant for $t<0$.
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$$|\psi\rangle = |5_{in}\rangle$$

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Recall: o But $|\psi\rangle = |5_{in}\rangle$ generally ceases to be eigenvector of $\hat{H}(t)$ for t...

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B. Resonance: best $J(t)$? (consider e.g. antenna)

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Exercise: verify

(Remark: same as classical $q(t)$ due to Ehrenfest theorem)

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B. Resonance:

* The amplitude of the excited motion of the oscillator is determined by J_0, as equation (*) shows.

* We expect that the driving force $J(t)$ is most efficient at creating a large J_0 if it oscillates at roughly the oscillator's natural frequency ω.

* Indeed: J_0 is the Fourier component of $J(t)$ for the frequency ω on the interval $[0,T]$:

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c. Energy expectation

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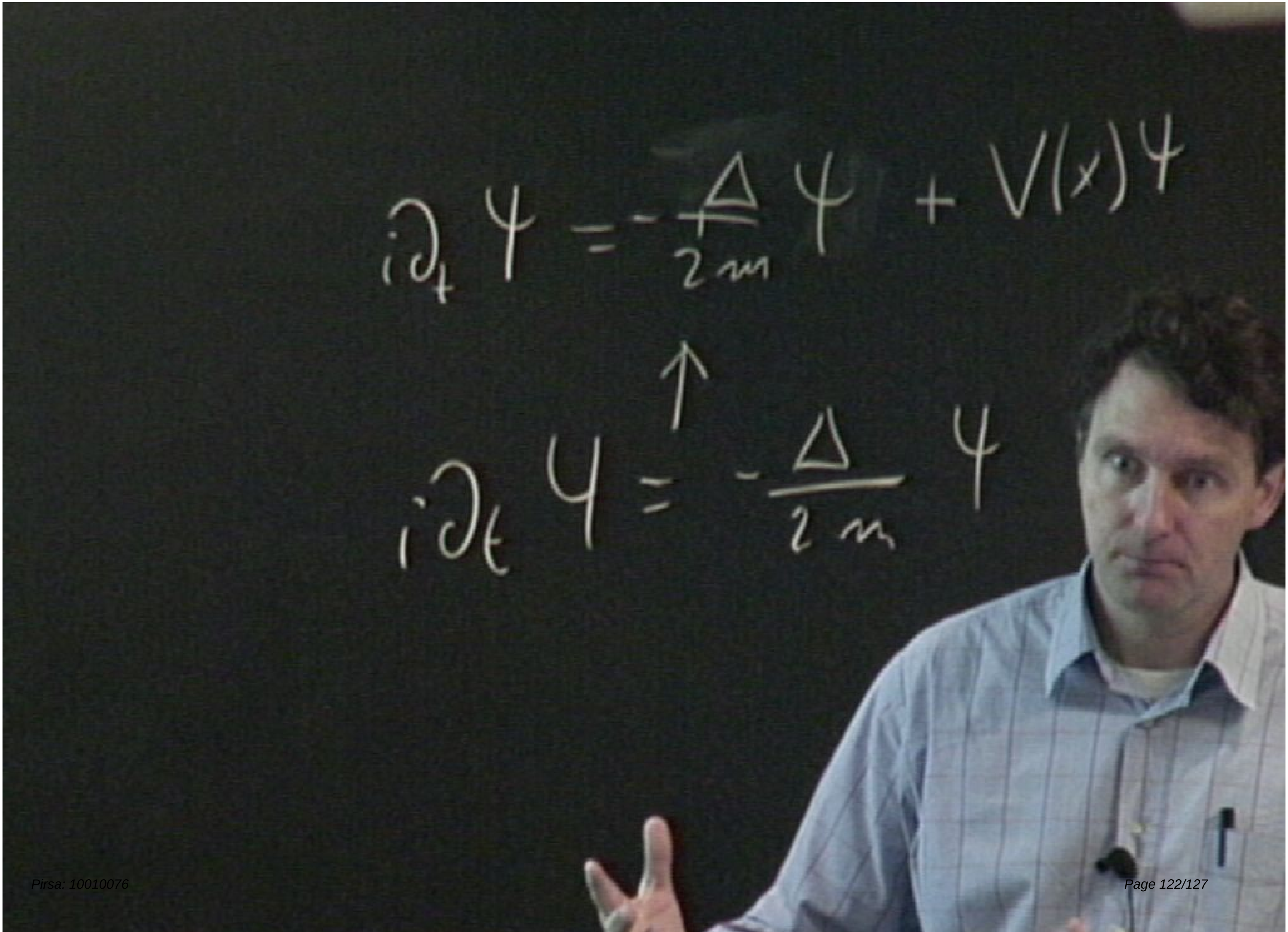
$i\partial_t \psi = -\frac{\Delta}{2m}\psi + V(x)\psi$
$\uparrow$
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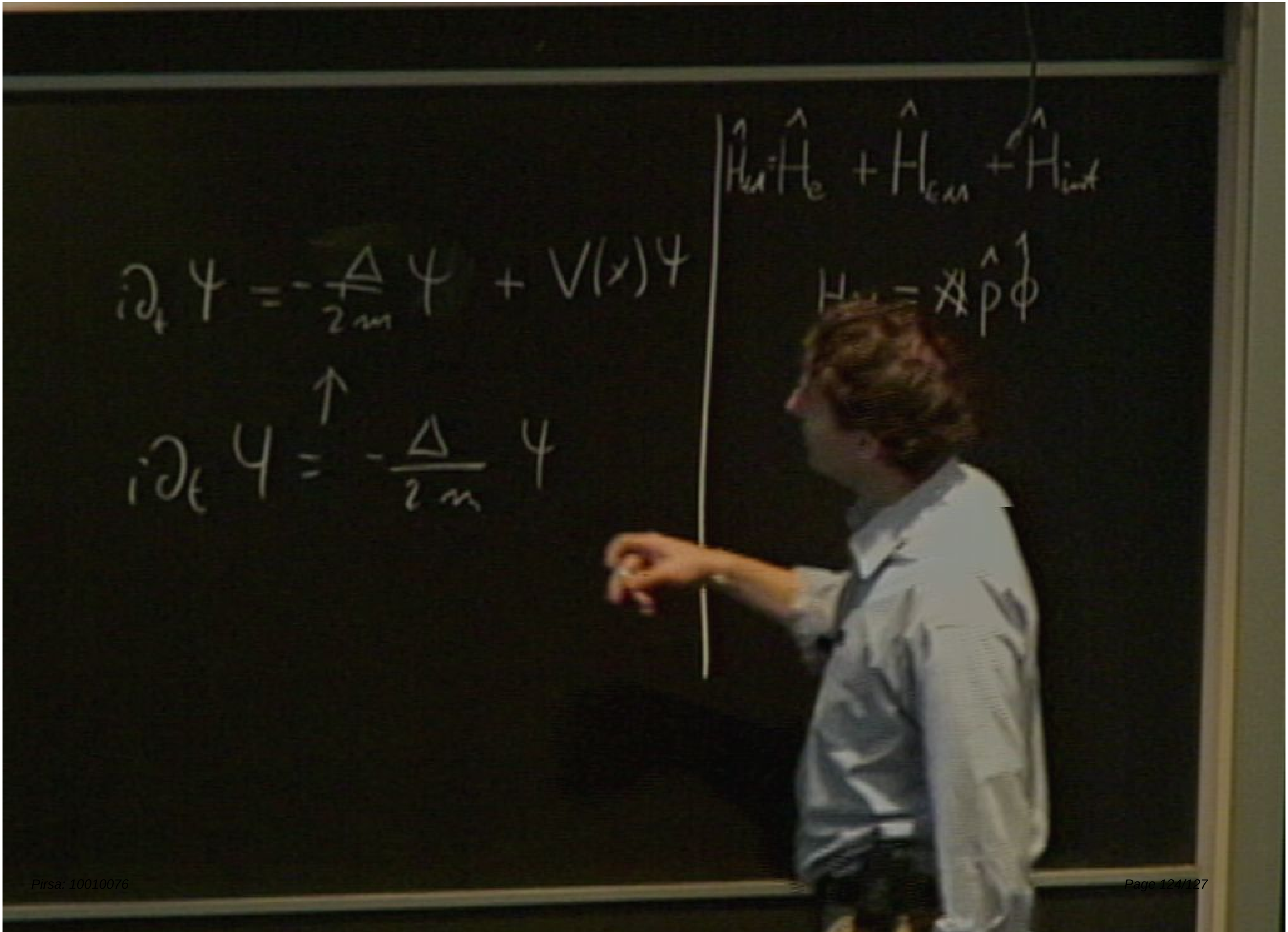
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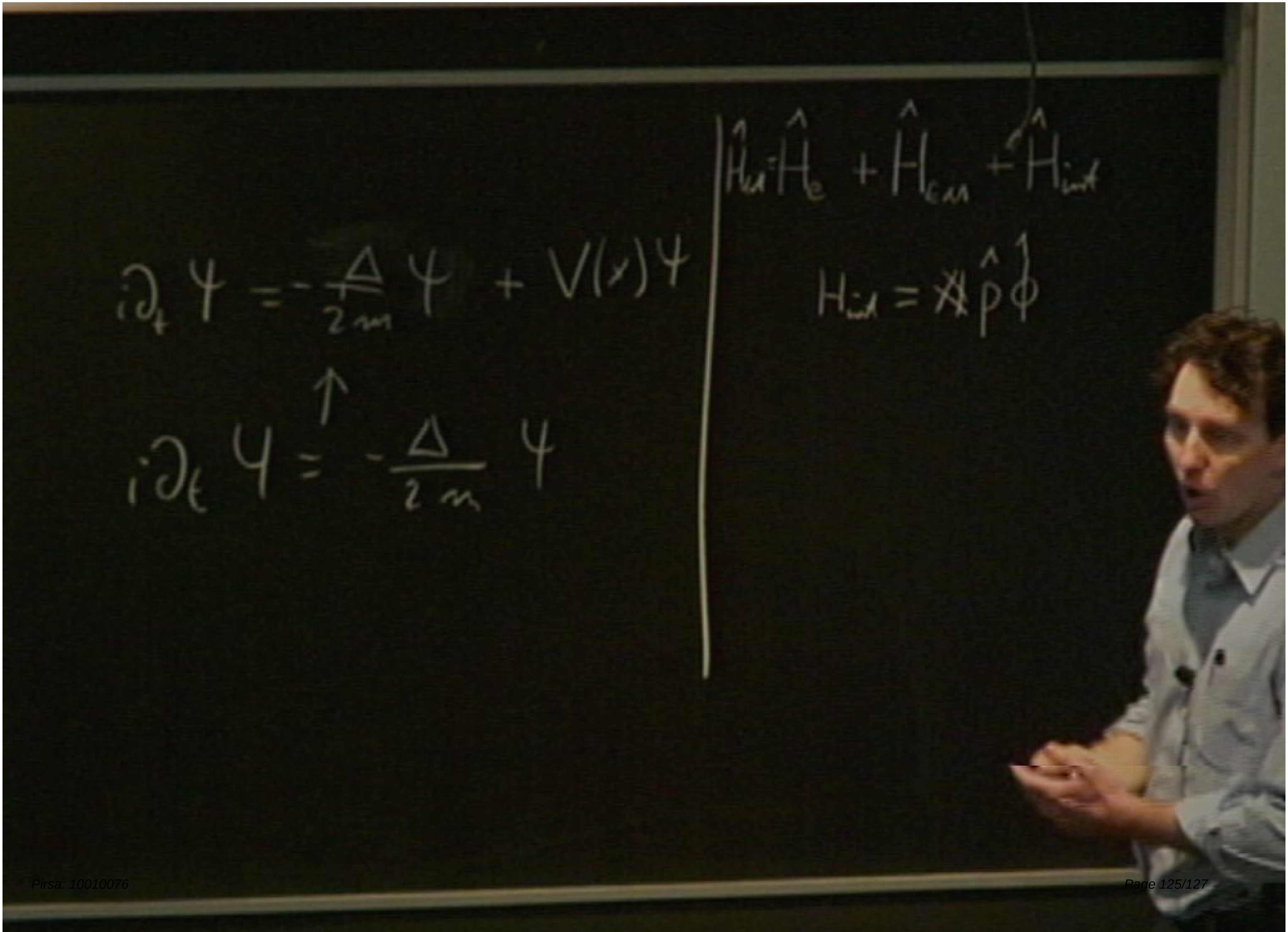
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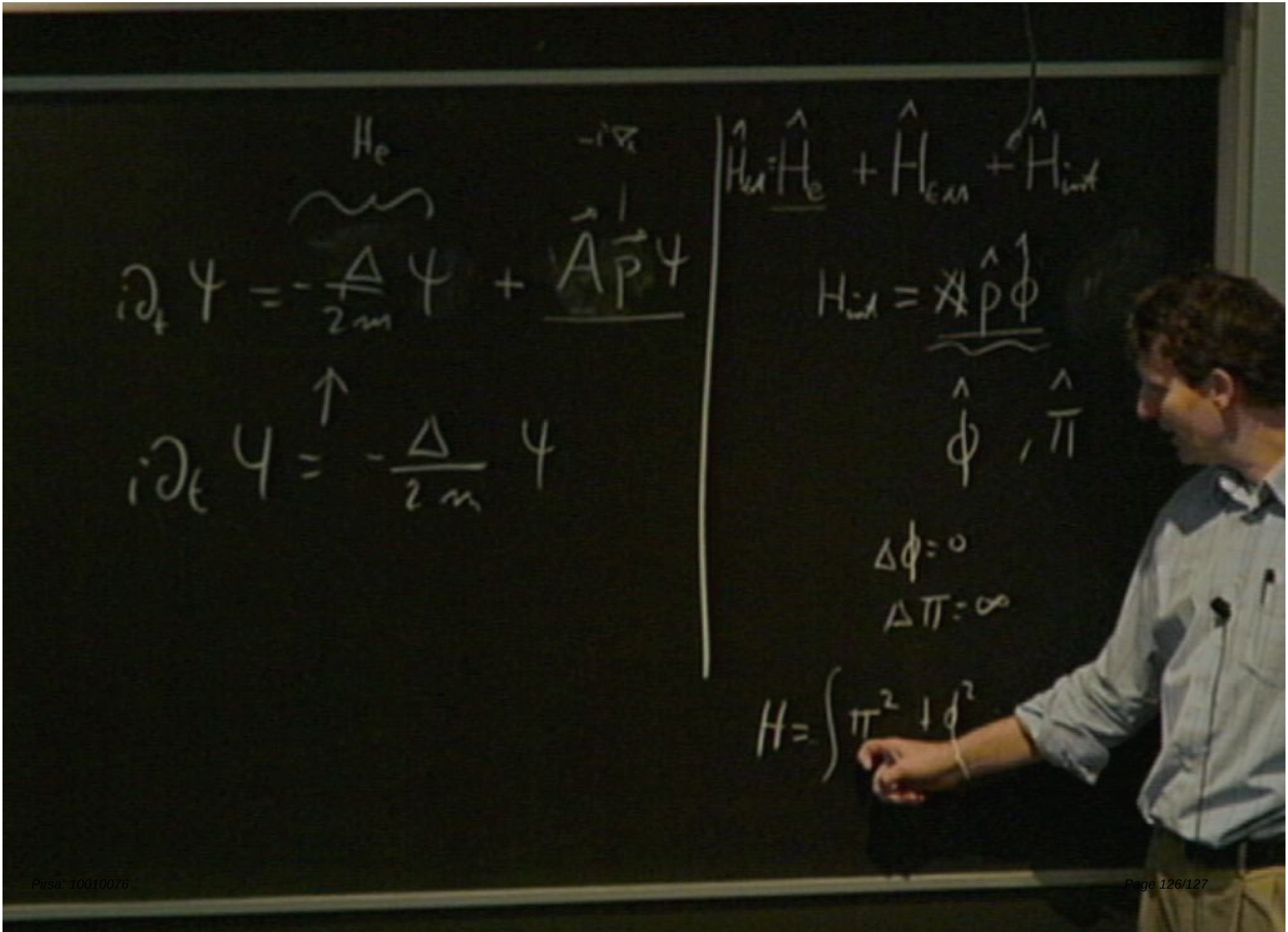
H_e
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$\Delta\pi = \infty$
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