

Title: Foundations and Interpretation of Quantum Theory - Lecture 3

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Abstract: <span>After a review of the axiomatic formulation of quantum theory, the generalized operational structure of the theory will be introduced (including POVM measurements, sequential measurements, and CP maps). There will be an introduction to the orthodox (sometimes called Copenhagen) interpretation of quantum mechanics and the historical problems/issues/debates regarding that interpretation, in particular, the measurement problem and the EPR paradox, and a discussion of contemporary views on these topics. The majority of the course lectures will consist of guest lectures from international experts covering the various approaches to the interpretation of quantum theory (in particular, many-worlds, de Broglie-Bohm, consistent/decoherent histories, and statistical/epistemic interpretations, as time permits) and fundamental properties and tests of quantum theory (such as entanglement and experimental tests of Bell inequalities, contextuality, macroscopic quantum phenomena, and the problem of quantum gravity, as time permits).</span>

# Lecture 3: Historical Perspectives on Interpretation

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*Course:* AMATH 900/AMATH 495/PHYS 490  
Foundations and Interpretations of Quantum Theory

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*Hosted by:* Institute for Quantum Computing, University of Waterloo,  
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# What are some of the challenges to interpreting quantum theory?

Let's start by considering some of the historical issues that have motivated much heated debate about interpretation.

- **The indeterminism and uncertainty of quantum predictions.**
  - ▶ Indeterminism of outcomes
  - ▶ Heisenberg uncertainty principle
  - ▶ Robertson inequality



# What are some of the challenges to interpreting quantum theory?

Let's start by considering some of the historical issues that have motivated much heated debate about interpretation.

- **The indeterminism and uncertainty of quantum predictions.**
  - ▶ Indeterminism of outcomes
  - ▶ Heisenberg uncertainty principle
  - ▶ Robertson inequality
- **Coherent superposition and the wave-particle duality.**
- **The special role of measurement.**

# The indeterminism and uncertainty of quantum predictions

## Indeterminism of outcomes

- Given a measurement of an observable,  $A$ , unless the preparation lies within an (possibly degenerate) eigenspace of the observable, the outcome of the measurement (i.e. the observed eigenvalue) is not determined by the theory.

# The indeterminism and uncertainty of quantum predictions

## Heisenberg Uncertainty Principle

The *uncertainty principle* due to Heisenberg (1925) states that it is impossible to measure the position and momentum of a particle to arbitrary precision *simultaneously*. The best one can do is

$$\delta x \delta p \simeq \hbar$$

where  $\delta x$  denotes the resolution for determining  $x$  for an individual system in a single experimental trial and likewise for  $p$ .





# The indeterminism and uncertainty of quantum predictions

## Heisenberg Uncertainty Principle

- The principle is that, while there is no limitation to how small the resolution can be for determining either  $x$  or  $p$ , there is a limitation on determining **both** within a single experiment.
- The principle is inferred (!) from the *Heisenberg microscope* example, which shows that simultaneous measurement of position and momentum for a single particle has limited precision.
- The Heisenberg uncertainty principle is a direct result of *Einstein's realization* (!) that electromagnetic energy is quantized,

$$E = nh\nu$$

for  $n = \{1, 2, 3 \dots\}$ . Hence, even when using light as a probe, there is an unavoidable disturbance that is inversely proportional to the resolution of the measurement  $\lambda = c/\nu$ .



# The indeterminism and uncertainty of quantum predictions

## Heisenberg Uncertainty Principle

- The particle nature of light places an inverse proportionality between measurement resolution and system disturbance - this differs from the classical picture where the energy of the light probe is proportional to intensity of the wave and there is no fundamental trade-off.
- The Heisenberg uncertainty principle influenced the idea of complementarity developed by Bohr (which we will discuss later).

# The indeterminism and uncertainty of quantum predictions

## Robertson Inequality

Heisenberg's uncertainty principle is often confused with Robertson's inequality (1929).

- Let

$$\Delta\hat{A}^2 = \langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2$$

be the variance over an ensemble of measurement outcomes for the observable  $A$  and likewise for  $B$ . Then,

$$\Delta\hat{A}\Delta\hat{B} \geq \frac{1}{2}|\langle[\hat{A}, \hat{B}]\rangle|,$$

where the product of the variances is non-zero if the two observables do not commute.

- In the case of position and momentum, this implies

$$\Delta\hat{x}\Delta\hat{p} \geq \frac{\hbar}{2}.$$

# The indeterminism and uncertainty of quantum predictions

## Robertson Inequality

- Robertson's result is a rigorous inequality, whereas Heisenberg's result is a heuristic argument.
- Moreover, conceptually the two are quite distinct: one refers to simultaneous measurements on a single system, whereas the other refers to variances of statistics for ensembles of measurements where only one operator is measured on each individual system.
- Note that different experimental measurement set-ups are required for measuring the non-commuting observables in the Robertson inequality, whereas only one experimental measurement set-up is relevant to the Heisenberg uncertainty principle.



# Coherent superposition and the wave-particle duality

**The wave-particle duality** of light and matter is another challenging concept.

- Einstein's 1905 analysis of the photoelectric effect suggested the wave-particle duality for light.
- Experiments with electrons, such as scattering of electrons from a crystal lattice, later suggested a wave-particle duality also for particles.
- Of course, the Schrodinger equation is a wave-equation describing particles, but the paradigmatic illustration of wave-particle duality is the *double-slit experiment*.
- In the double-slit experiment, if the particle passes the slits in a *coherent superposition*, then a wave-like interference pattern is observed on the screen (under an ensemble of single-particle experiments).

# Coherent superposition and the wave-particle duality

## Coherent superposition

- If the coherent superposition is compromised, e.g., by measurement of which slit the particle passes through, or decoherence from the environment, then a sum of classical probability distributions is observed at the screen.
- We will see explicitly later exactly how the standard quantum formalism for analyzing “measurement” and/or environmental decoherence confirms/predicts this experimental fact.



# Coherent superposition and the wave-particle duality

Notice the there are two seemingly inconsistent narratives that are offered as “explanation” of the double-slit experiment: the wave view vs “superposed reality” view.

- The wave view is to say that the particle acts like a “wave” until it is observed (recall that upon observation only one localized point on the screen is illuminated after the particle passes through the set-up.)
- The superposed reality explanation is that the particle remains a particle but somehow passes through both slits at the same time, in a “superposed reality”.
- So what *is* the story? Is the particle a wave or is it a particle that is in two places at once? Or is neither story satisfactory?
- We will see later that the deBroglie-Bohm interpretation tells an entirely different story.



$$\frac{| \psi_1 \rangle + | \psi_2 \rangle}{\sqrt{2}}$$

$$\frac{| \psi_1 \rangle \langle \psi_1 | + | \psi_2 \rangle \langle \psi_2 |}{2}$$

# The special role of measurement

**Measurement** has an unusual role in quantum theory.

Consider the process whereby an intervention produces a definite (though perhaps random) outcome.

- The *usual* quantum description of this process refers to an “observer” or “classical apparatus” that lies outside the theory.
- This special role for measurement is explicit in the case of von Neumann’s projection postulate and Bohr’s insistence that the measurement apparatus must be described “classically”. In either of these approaches the consequence is an awkward dualism at the very foundations of the theory.
- Can quantum theory and its interpretation be formulated without a special role for the observer and without the necessity of distinguishing certain aspects of an experiment as “classical”?



**The Copenhagen interpretation** is understood differently by different historical commentators.

- The interpretation is usually attributed to Niels Bohr, and sometimes various elements of Heisenberg's ideas are included.
- There was no consensus between them. Bohr was critical of Heisenberg's realist tendencies, for example, in his description of the Heisenberg microscope.





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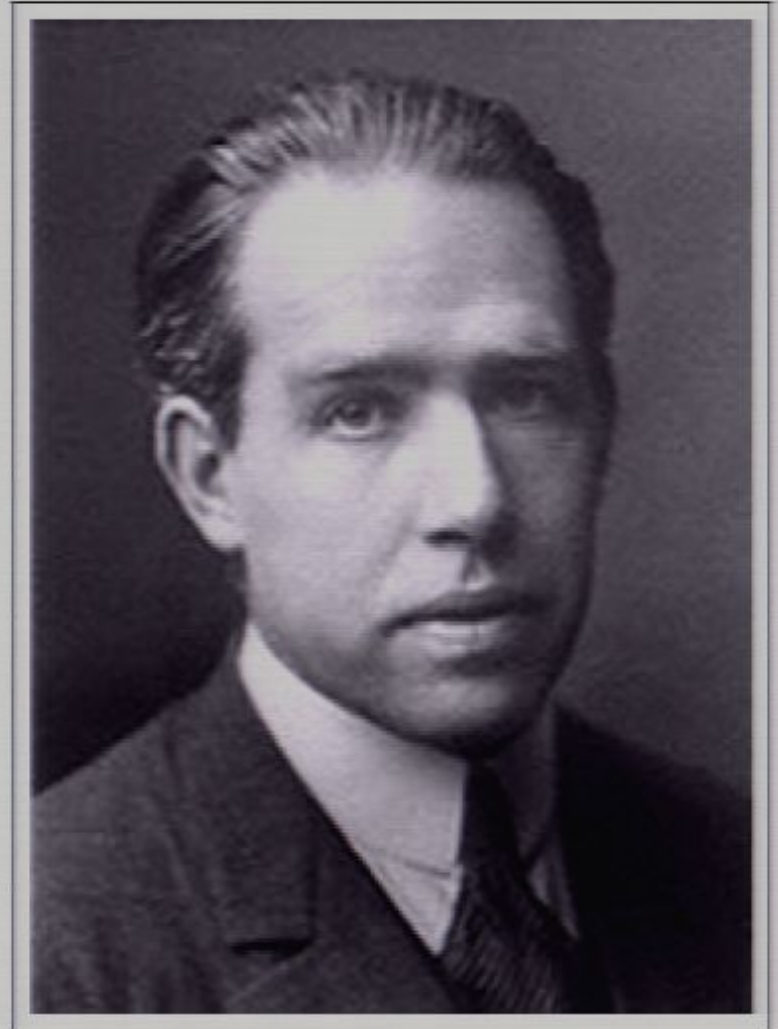
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# What is the Copenhagen Interpretation?

- The label “Copenhagen Interpretation” was never adopted by either Bohr or Heisenberg. Rather, it is a historical construct.
- Some of the difficulty in understanding the interpretation arises in part due to the fact that Bohr’s ideas are difficult to parse and assimilate into a coherent interpretation.
- So rather than attempt to synthesize a particular “close reading” of Bohr I will let you judge yourselves from a selection of direct quotes.

# The views of Bohr

**For Bohr, complementarity is a central principle of interpretation:**

*“Complementarity: any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena.”*

Bohr (1934)



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Bohr (1934)

- Huh?
- *My guess:* Bohr is saying that, when you are measuring, eg, the position of a particle then you can't talk about its momentum, but he is trying hard to avoid realist language that would suppose that the particle has some definite position *and* momentum prior to measurement.

## The views of Bohr

### Objective physical reality must be rejected:

“An independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation.”

Bohr (1928)

“... a subsequent measurement to a certain degree deprives the information given by a previous measurement of its significance for predicting the future course of the phenomena. Obviously, these facts not only set a limit to the extent of the information obtainable by measurements, but they also set a limit to the meaning which we may attribute to such information.”

Bohr (1934)



## The views of Bohr

**Another central concept for Bohr is the necessity of a boundary between quantum and classical.**

Bohr holds firm to the fact that although “atomic phenomena” must be described by quantum mechanics, our measuring devices must be described using classical physics:

“The experimental conditions can be varied in many ways but the point is that in each case we must be able to communicate to others what we have done and what we have learned, and that therefore the functioning of the measuring instruments must be described within the framework of classical physical ideas.”

Bohr (1934)



# The views of Bohr

**But by what criterion can we determine where the boundary lies?**

Bohr concedes there is none:

There is “the impossibility of any sharp separation between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear.”

Bohr (1949)

# The Copenhagen Interpretation

## Bohr's views became textbook dogma:

"It is in principle impossible . . . to formulate the basic concepts of quantum mechanics without using classical mechanics."

"By measurement, in quantum mechanics, we understand any process of interaction between quantum and classical objects, occurring apart from and independently of any observer. The importance of the concept of measurement in quantum mechanics was elucidated by N. Bohr... Thus quantum mechanics occupies a very unusual place among physical theories: it contains classical mechanics as a limiting case, yet at the same time requires this limiting case for its own formulation."

Landau and Lifschitz (Quantum Mechanics, pp. 2-3 of 3rd edition (1977))

# Criticism of the Copenhagen Interpretation

The necessity of postulating a classical world for the formulation of Bohr's interpretation is unsatisfactory: if quantum mechanics is the fundamental theory then the requirement of a 'classical apparatus' to interpret the theory is *ad hoc*.



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- Perhaps this objection can be overcome by replacing the notion of 'classical apparatus' with the more modern and abstract notion of "information". Hence the 'classical apparatus' that serves to define the 'conditions of observation' is then just the '*input and output information*' that defines the task in question.

# Criticism of the Copenhagen Interpretation

Furthermore, the exact quantum-classical boundary is left unspecified, leading to a potential consistency problem in the interpretation.

- Specifically, one would expect to be able to describe any physical system, including a measurement device, using the usual unitary quantum mechanics, but Bohr's dualistic view of atomic-scale anti-realism and macro-scale realism implies the existence of some yet-to-be-determined and somewhat arbitrary quantum-classical boundary.

# Criticism of the Copenhagen Interpretation

Bohr's flat denial of any "atomic-scale" realism just doesn't seem to be supported by any convincing argument, but merely asserted as dogma.

- It is well-known that Einstein never accepted Bohr's anti-realism:

"To believe this [the absence of an atomic-scale reality] is logically possible without contradiction; but it is so very contrary to my scientific instinct that I cannot forego the search for a more complete description."

Einstein (1936)

It is unfortunate that the popular view of the celebrated Einstein-Bohr debates is that Bohr won and that Einstein did not understand quantum mechanics.



# Criticism of the Copenhagen Interpretation

**Were Bohr's views informed by quantum mechanics, or were they imposed upon quantum mechanics?**

"How closely the idea of complementarity was in accord with Bohr's older philosophical ideas became apparent through an episode which took place ... on a sailing trip from Copenhagen to Svendborg on the Island Fyn... Bohr was full of the new interpretation of quantum theory, and as the boat took us full sail southward ... there was plenty of time to reflect philosophically on the nature of atomic theory. ... Finally, one of Bohr's friends remarked drily, 'But Niels, this is not really new, you said exactly the same ten years ago.' "

Heisenberg (1967)

# Criticism of the Copenhagen Interpretation

**Einstein may have been aware of this:**

“The Heisenberg-Bohr tranquilizing philosophy - or religion? - is so delicately contrived that, for time being, it provides a gentle pillow for the true believer from which he cannot very easily be aroused. So let him lie there.”

A. Einstein (letter to Schrodinger, 1928)



# The Orthodox Interpretation of von Neumann and Dirac

In spite of Bohr's significant influence the view of quantum mechanics that formed the basis for textbook dogma is the view developed by von Neumann and Dirac, which we will call the **orthodox interpretation**. This view arises from the following considerations:

- Recall the von Neumann projection postulate for measurement of the (non-degenerate) observable  $\hat{A} = \sum_n a_n P_n$ , where  $P_n = |\psi_n\rangle\langle\psi_n|$ .
- The state after an ideal filtering-type measurement, when outcome  $a_k$  has been obtained, is given by  $\rho_k = \frac{P_k \rho P_k}{\text{Tr}(P_k \rho P_k)}$ .
- In other words, we have found the system to have property  $a_k$ , and furthermore, we can reliably predict that, if a subsequent measurement were performed, then the outcome  $a_k$  will again be obtained.



# The Orthodox Interpretation of von Neumann and Dirac

- One might be tempted to say that, following such an ideal filtering-type measurement, the system, which is in the eigenstate  $\psi_k$ , indeed “has” the property  $a_k$ .
- More generally one might say that whenever the quantum state is known to be some vector  $\psi_k$ , then any eigenvalue  $a_k$  in the spectral decomposition of an observable, for which  $\psi_k$  is an eigenstate, is a well-defined *objective* property of the system, and this holds **independently of whether an observation is actually made**.
- In more philosophical terminology, we say that under these conditions  $a_k$  is an *ontic* property of the system, or is an element of the *ontology*. That is, it **exists**.
- Arthur Fine calls this principle the **eigenvalue-eigenstate link** and credits the principle to Dirac and von Neumann.

# The literal realism of von Neumann and Dirac

“The expression that an observable ‘has a particular value’ for a particular state is permissible in quantum mechanics in the special case when a measurement of the observable is certain to lead to the particular value, so that the state is an eigenstate of the observable. In the general case we cannot speak of an observable having an value for a particular state, but we can speak of its having an average value for the state.”



Dirac (1958)



# The Orthodox Interpretation

- Because pure quantum states are always eigenvectors of some well-defined observable(s), pure quantum states are the source of these objective properties of the quantum systems.
- But the orthodox interpretation goes beyond **merely** asserting that the quantum state identifies **some** of the possible objective properties of the system through the eigenvalue-eigenstate link...



# The Orthodox Interpretation of von Neumann and Dirac

“In this method of description, it is evident that everything which can be said about the state of a system must be derived from its wave function.”

von Neumann (1932/1955)

- Note the implicit use of the term “state” to mean “ontic state”, for otherwise the statement is a tautology!



# The Orthodox Interpretation

- The orthodox interpretation asserts that there are **no other objective properties** of the system except those identified in this way.
- As a result, the wave function provides the **complete summary** of the ontic properties of the quantum system through the eigenvalue-eigenstate link.
- In summary, the orthodox interpretation suggests that the pure quantum state provides the fundamental ontology, ie, the **objective reality of the world is literally just the quantum wavefunction itself**.



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- Would you expect anything less from the **central mathematical object** in the **most fundamental theory of nature**?



The hypothesis that the quantum state is literally real, and moreover provides the complete ontology, leads to some difficult issues when we consider the measurement process.

- Consider the ideal measurement of an observable  $A = \sum_n a_n P_n$ , applied to a given preparation  $\rho$ , that yields the outcome  $a_m$ .
- The post-measurement state  $\rho_m$ , conditional upon the outcome  $a_m$ , is determined by *Luders' rule*,

$$\rho \rightarrow \rho_m = \frac{P_m \rho P_m}{\text{Tr}(P_m \rho)}.$$

- If the observed eigenvalue is non-degenerate, then  $P_m$  is rank-one, and the state  $\rho_m = P_m = |\psi_m\rangle\langle\psi_m|$  will be pure - in this special case Luders' rule reduces to the projection postulate, the "process 1" proposed by von Neumann.

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The projection postulate can not be deduced from the usual unitary evolution of the Schrodinger equation.

- Conceptually it is clear that unitary evolution takes any given initial state to a fixed final state: this is deterministic.
- In contrast, collapse is fundamentally stochastic: applying the same measurement to the same preparation produces different (apparently random) final states (depending on the outcome).
- Is it possible that the final state outcome is not random but dependent on the quantum state associated with some additional degrees of freedom, and the whole process may be described by a unitary transformation? **No.**



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- Consider an atom described by a pure state corresponding to a coherent superposition of moving along two distinct trajectories.
- We arrange so that both trajectories pass through a detector such that a macroscopic pointer is moved to the 'left' if the atom is on the 'up' trajectory and to the 'right' if the atom is on the 'down' trajectory.



- We want to model the measurement process with a unitary transformation and for complete generality we extend the quantum system to include additional degrees of freedom denoted by a state  $|\chi\rangle$ .
- If we demand *faithful measurements* this means that we must have, for any  $|\chi\rangle$ ,

$$\begin{aligned}U|\text{up}\rangle \otimes |\text{ready}\rangle \otimes |\chi\rangle &= |\text{up}\rangle \otimes |\text{left}\rangle \otimes |\chi'\rangle \\U|\text{down}\rangle \otimes |\text{ready}\rangle \otimes |\chi\rangle &= |\text{down}\rangle \otimes |\text{right}\rangle \otimes |\chi''\rangle\end{aligned}$$

where  $|\chi'\rangle$  and  $|\chi''\rangle$  are allowed to be independent of  $|\chi\rangle$ .

- Now if we prepare a coherent superposition over atomic trajectories, and allow for both possible outcomes, then by linearity it follows that, for any  $\chi$ ,

$$U(\alpha|\text{up}\rangle + \beta|\text{down}\rangle) \otimes |\text{ready}\rangle \otimes |\chi\rangle = \alpha|\text{up}\rangle \otimes |\text{left}\rangle \otimes |\chi'\rangle + \beta|\text{down}\rangle \otimes |\text{right}\rangle \otimes |\chi''\rangle$$

so it is impossible that after the interaction the state is driven to one or the other outcome.

- Hence the projection postulate can not be modeled by a unitary transformation.

Let's see what  
unitary predicts  
for the state of  
the pointer?

$$\begin{aligned}\hat{\rho}_a &= \text{Tr}_e \text{Tr}_{env} (| \Phi \rangle \langle \Phi |) \\ &= |L\rangle \langle L| |\alpha|^2 \\ &\quad + |R\rangle \langle R| |\beta|^2\end{aligned}$$



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