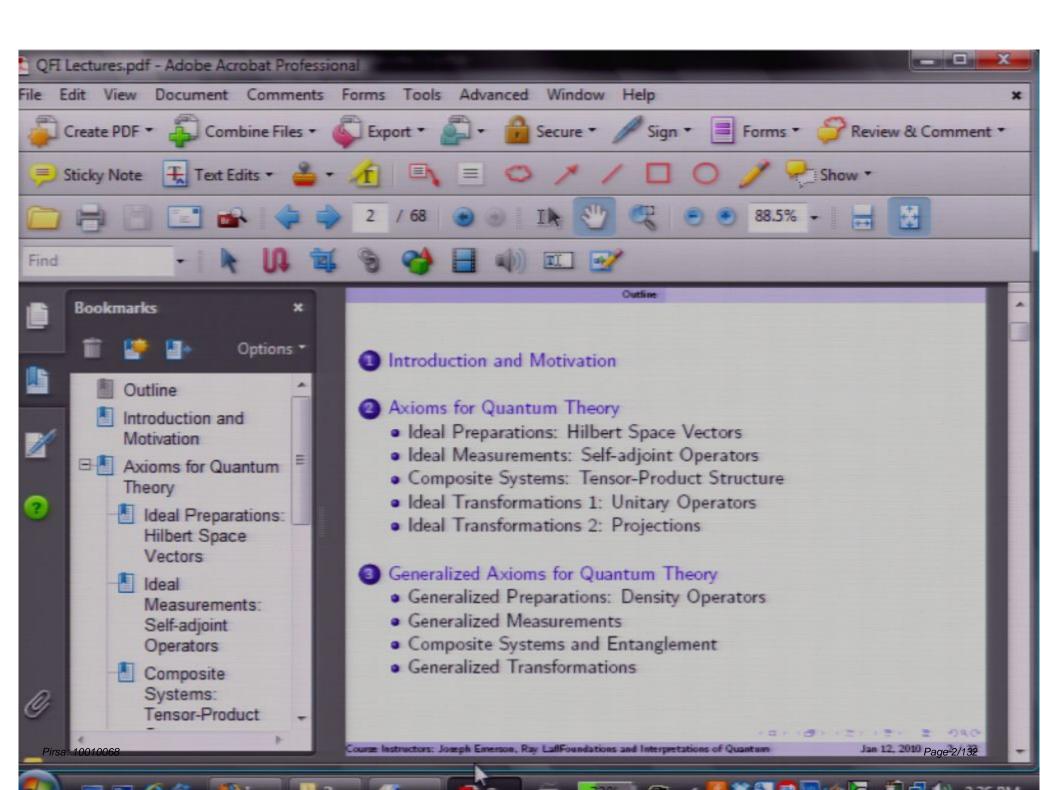
Title: Foundations and Interpretation of Quantum Theory - Lecture 1

Date: Jan 12, 2010 02:30 PM

URL: http://pirsa.org/10010068

Abstract: <span>After a review of the axiomatic formulation of quantum theory, the generalized operational structure of the theory will be introduced (including POVM measurements, sequential measurements, and CP maps). There will be an introduction to the orthodox (sometimes called Copenhagen) interpretation of quantum mechanics and the historical problems/issues/debates regarding that interpretation, in particular, the measurement problem and the EPR paradox, and a discussion of contemporary views on these topics. The majority of the course lectures will consist of guest lectures from international experts covering the various approaches to the interpretation of quantum theory (in particular, many-worlds, de Broglie-Bohm, consistent/decoherent histories, and statistical/epistemic interpretations, as time permits) and fundamental properties and tests of quantum theory (such as entanglement and experimental tests of Bell inequalities, contextuality, macroscopic quantum phenomena, and the problem of quantum gravity, as time permits).

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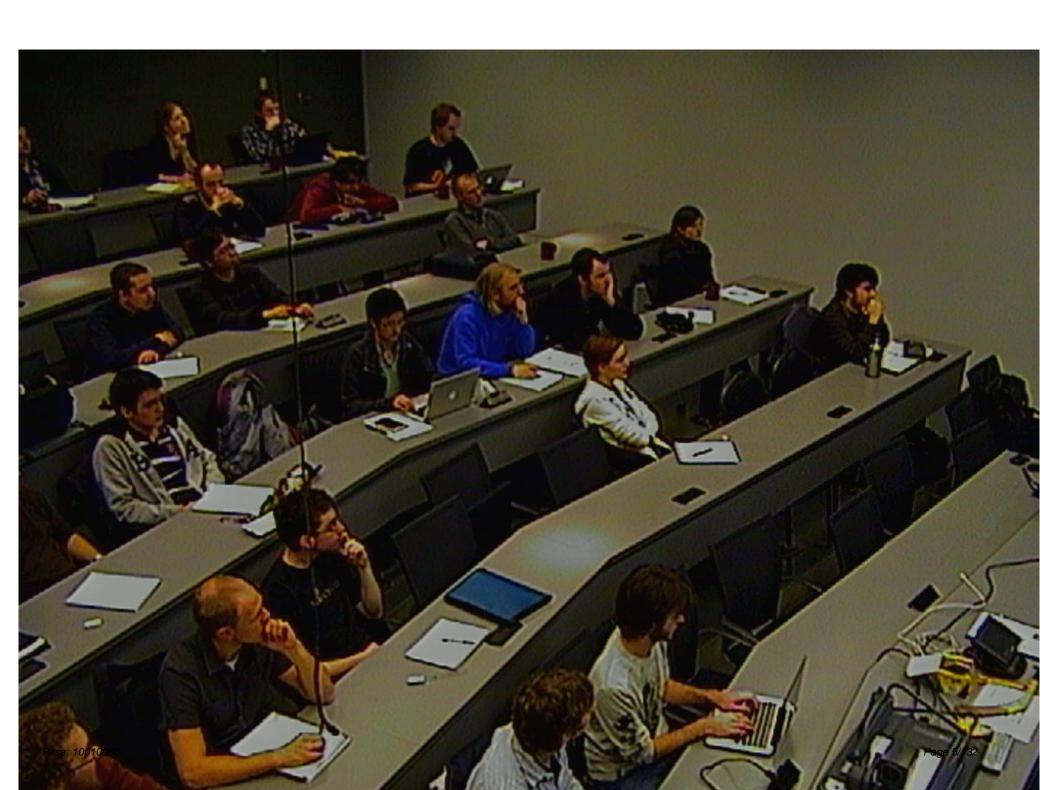


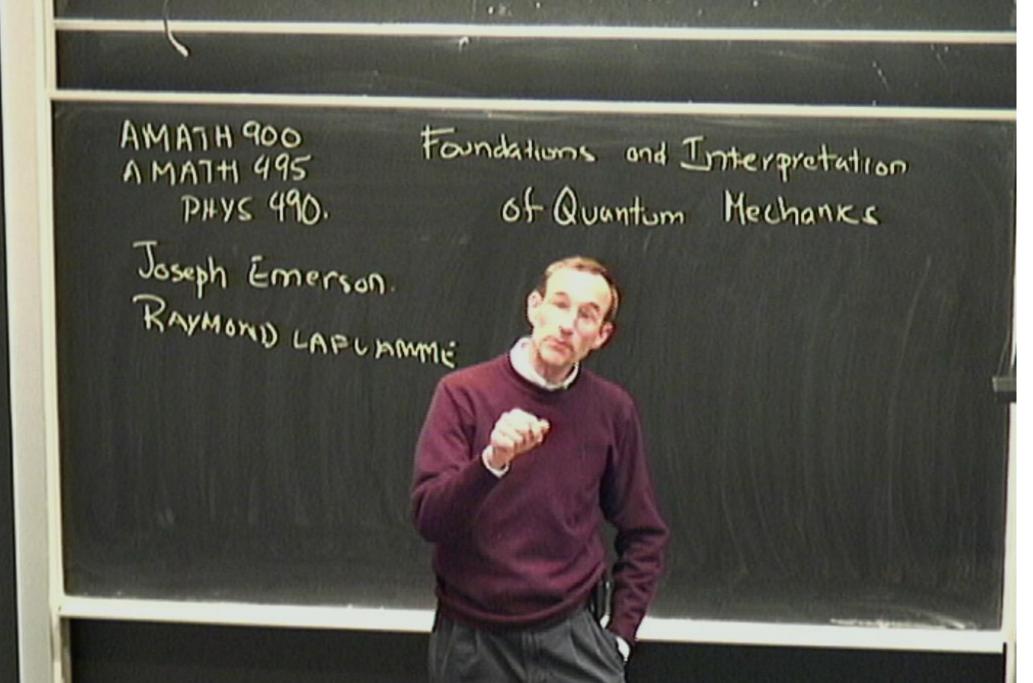
AMATH 900 AMATH 495 PHYS 490. Foundations and Interpretation of Quantum Mechanics Joseph Emerson.
RAYMOND LAPLAMME

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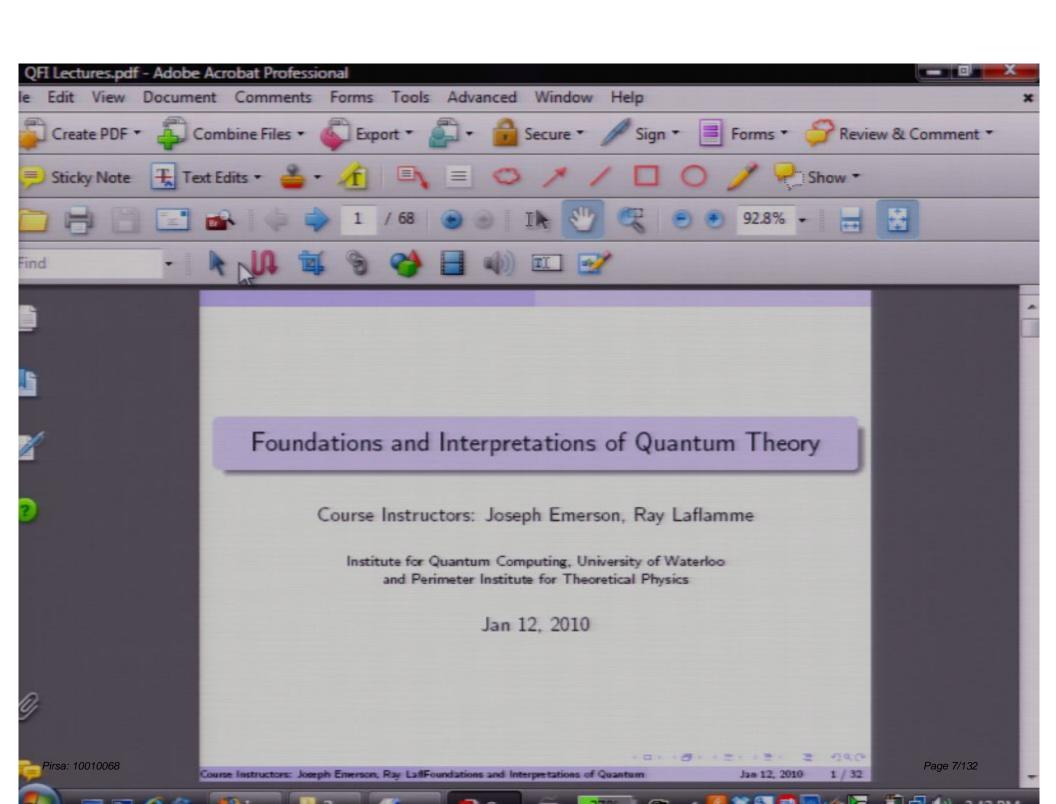
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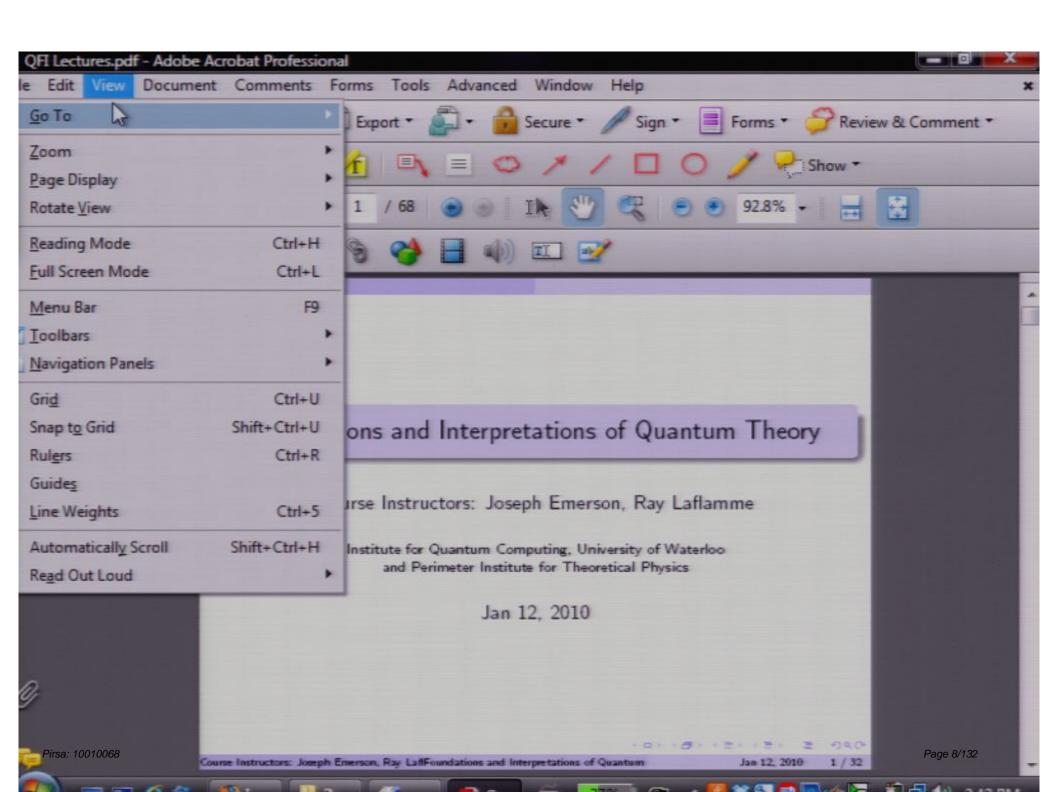


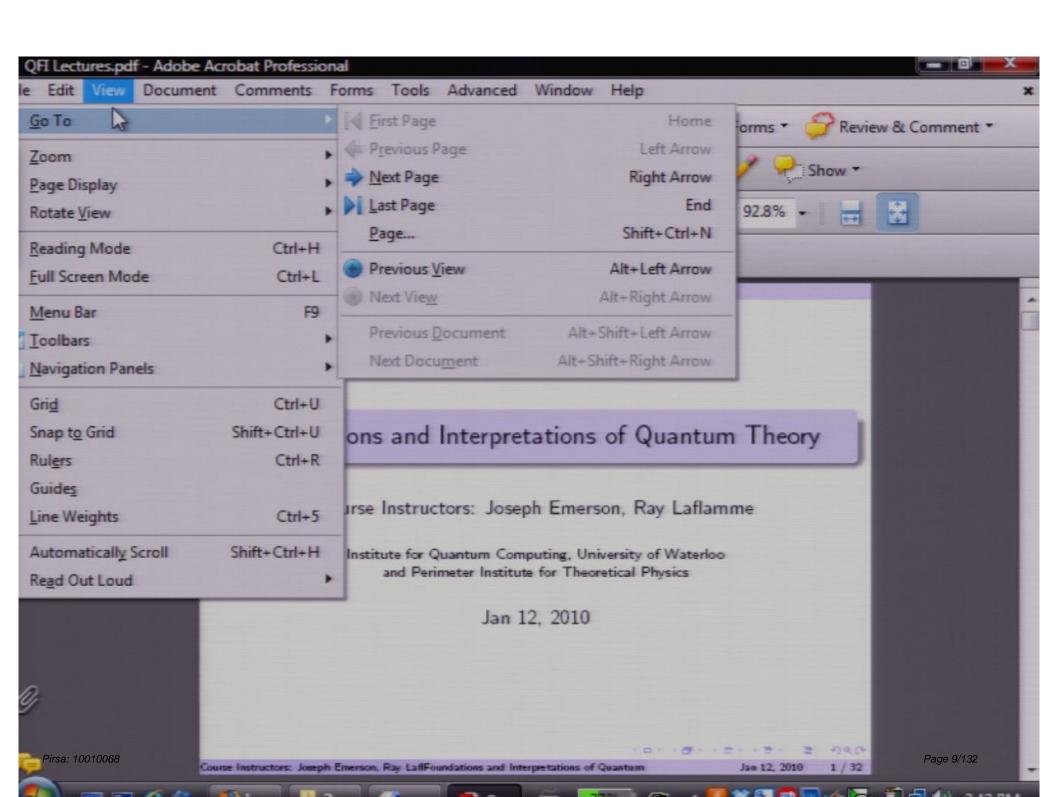




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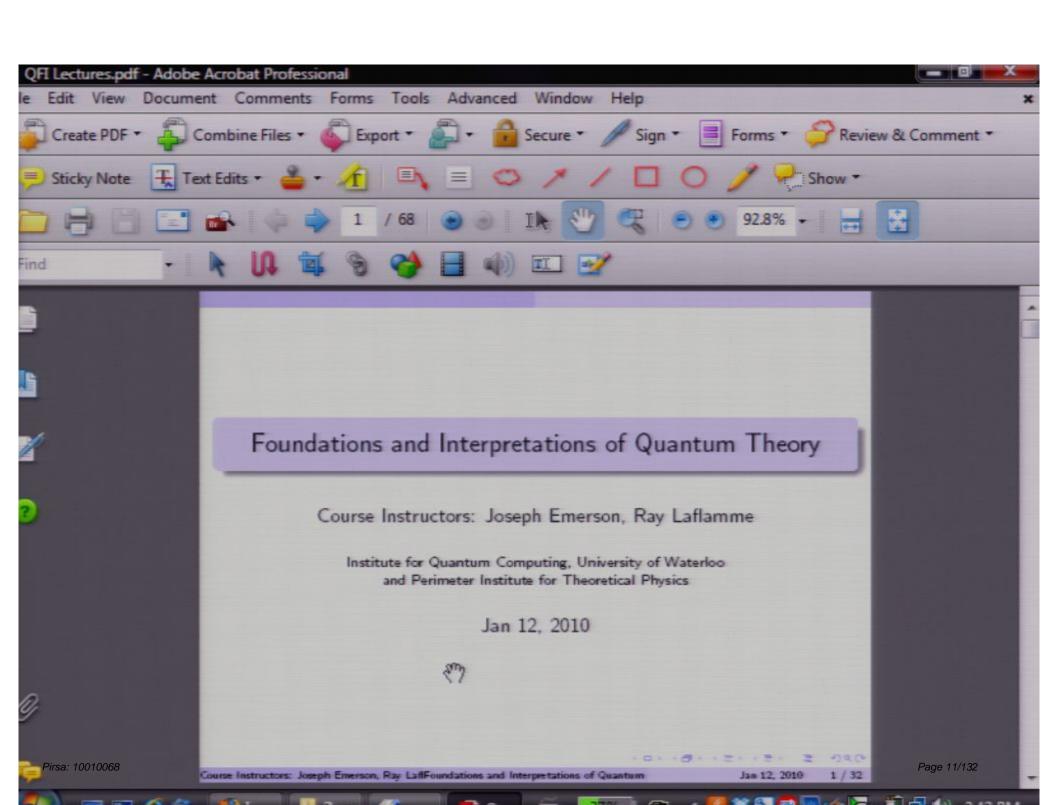
# Foundations and Interpretations of Quantum Theory

Course Instructors: Joseph Emerson, Ray Laflamme

Institute for Quantum Computing, University of Waterloo and Perimeter Institute for Theoretical Physics

Jan 12, 2010

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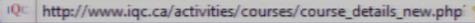
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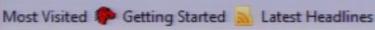














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#### Street Address

475 Wes Graham Way Waterloo, ON

#### Contact

200 University Ave. W. Waterloo, ON N2L 3G1

+1 (519) 888-4021

Send Email...



superconductivity and micro-circuitry. More recently, we have seen the coherence and entanglement of single quantum systems veri ed routinely in todays labs and these distinctive quantum phenomena are now being directly exploited as the basis for emerging quantum technologies. And yet, in spite of these successes, there are questions and controversy surrounding very basic issues about the physical nature of the theory. While such questions are sometimes dismissed as mere philosophy, the study of these foundational issues has played a critical role in conceptual breakthroughs in areas ranging from quantum computation and quantum cryptography to the nature of quantum chaos and the quantum-classical transition.

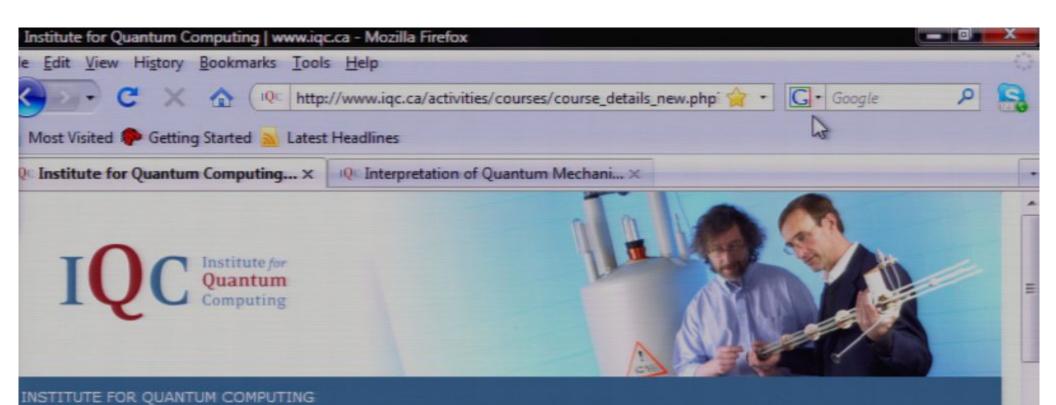
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#### Schedule:

Lecturer	Tentative Lecture Title	Date
Joseph Emerson	Axioms for quantum mechanics	Week of January 11, 2009
Joseph Emerson	Basic problems of interpretation	Week of January 18, 2009
Joseph Emerson	Constraints on hidden variable models	Week of January 25,

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#### Foundations and Interpretation of Quantum Theory (Winter 2010)

Semester/Year Offered: Winter 2010 Code: AMATH 900/AMATH 495/PHYS 490

Instructors: Joseph Emerson and Raymond Laflamme

Location: PI and RAC Time: Tuesdays and Thursdays, 2:30-3:50

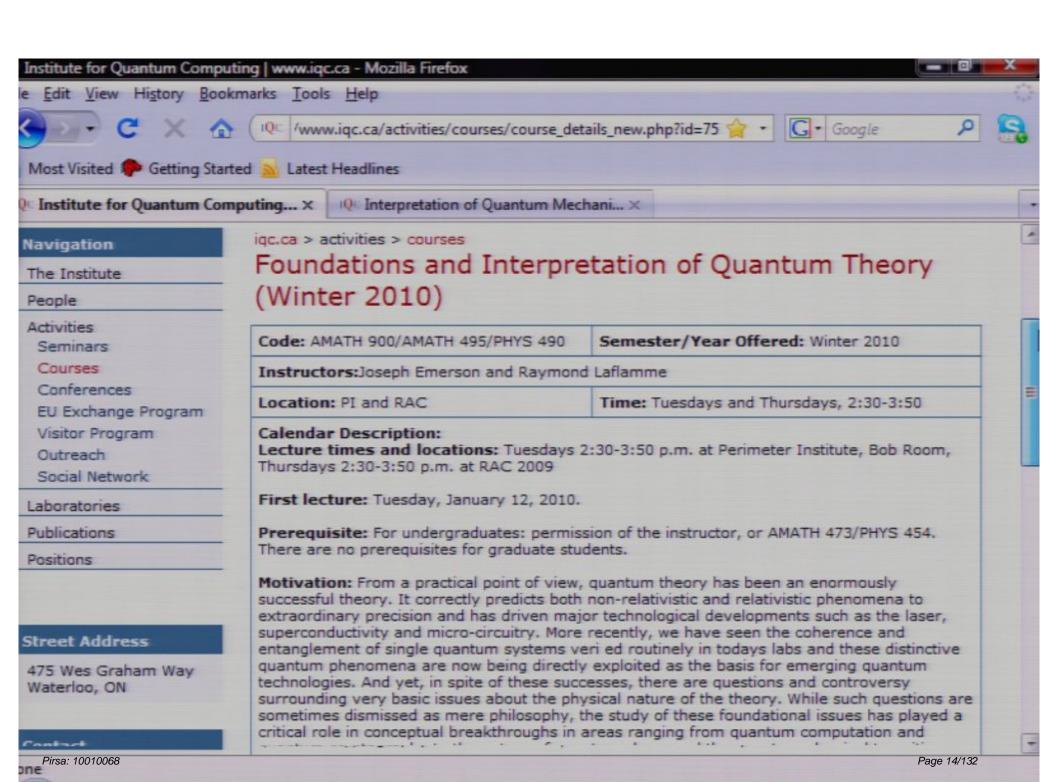
Calendar Description:

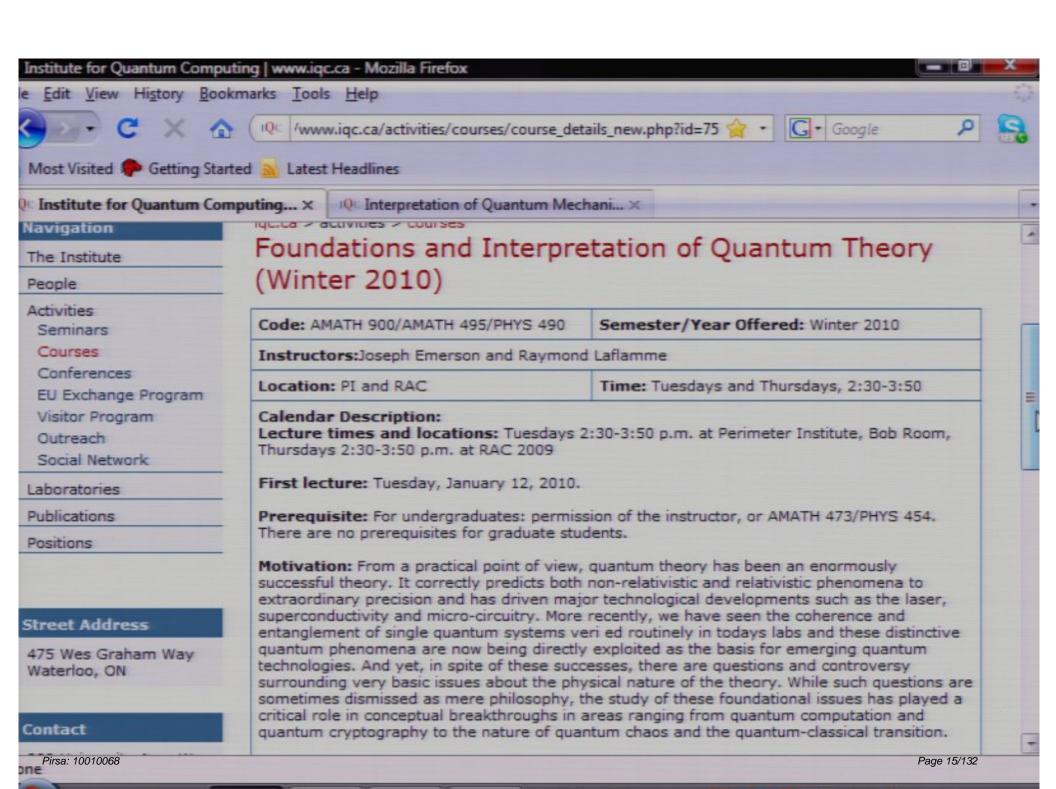
Lecture times and locations: Tuesdays 2:30-3:50 p.m. at Perimeter Institute, Bob Room,

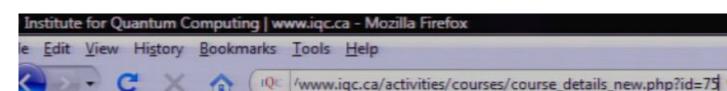
Thursdays 2:30-3:50 p.m. at RAC 2009

First lecture: Tuesday, January 12, 2010.

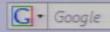
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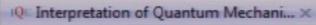




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the problem of quantum gravity, as time permits).

#### Schedule:

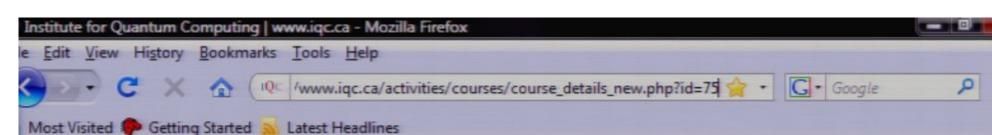
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Joseph Emerson	Basic problems of interpretation	Week of January 18, 2009
Joseph Emerson	Constraints on hidden variable models	Week of January 25, 2009
Robin Blume- Kohout	Probability and its interpretation	Week of February 1, 2009
Gregor Weihs	Experimental tests of Bell inequality	Week of February 8, 2009
Alex Wilce	Convex sets framework for probabilistic theories	Week of February 22, 2009
Roderich Tumulka	deBroglie-Bohm interpretation	Week of March 1, 2009
Chris Fuchs	Quantum Bayesian view	Week of March 8, 2009
TBA	TBA	Week of March 15, 2009
Tony Leggett	Fundamental tests of quantum mechanics	Week of March 22, 2009
Michel Devoret	Macroscopic quantum coherence	Week of March 29, 2009

Transportation: Shuttle transportation to/from PI and RAC is available as follows. Priority will be given to students registered in the course. Pick up at EIT will be at the entrace facing DC.

Tuesdays

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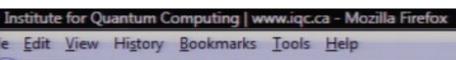
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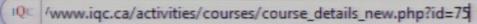
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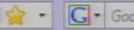
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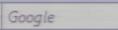
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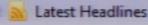








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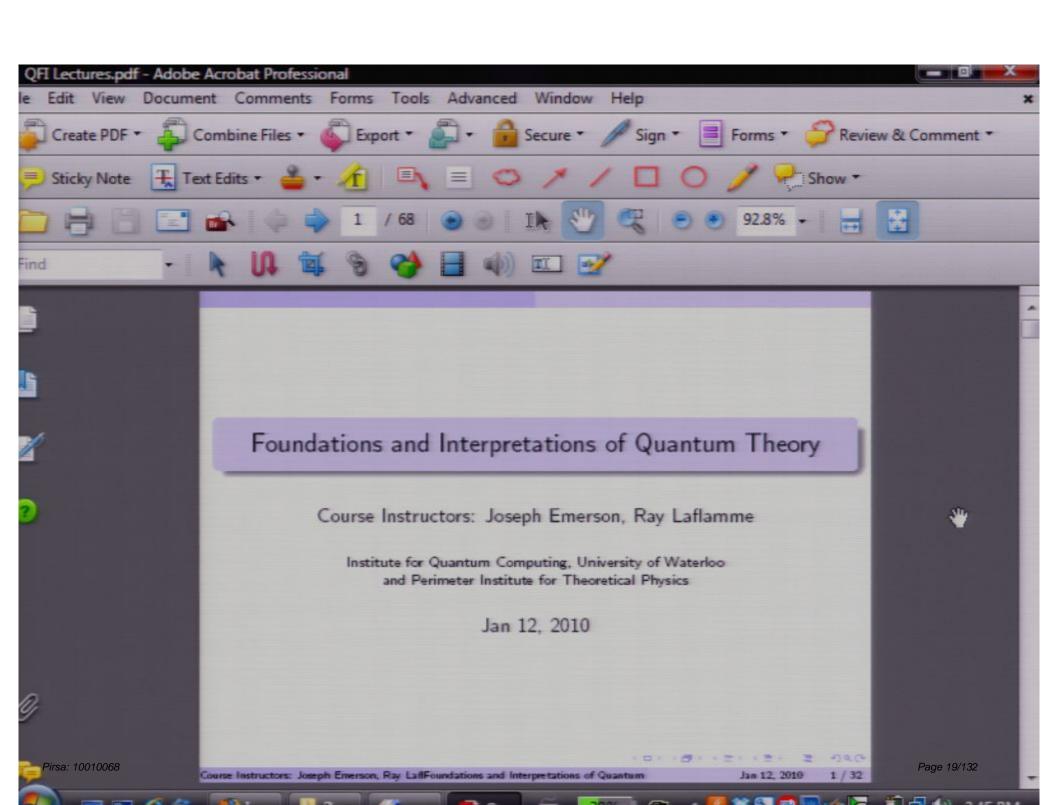
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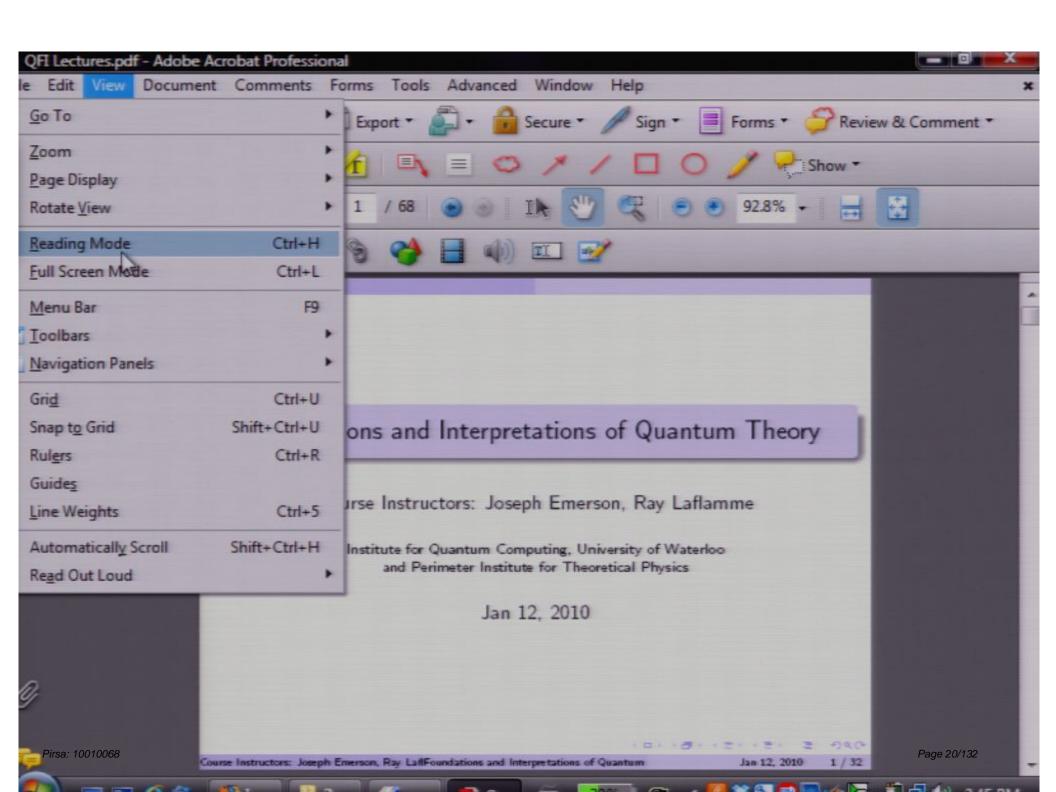
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Tuesdays

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Institute for Quantum Computing, University of Waterloo and Perimeter Institute for Theoretical Physics

Jan 12, 2010

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- Introduction and Motivation
- 2 Axioms for Quantum Theory
  - Ideal Preparations: Hilbert Space Vectors
  - Ideal Measurements: Self-adjoint Operators
  - Composite Systems: Tensor-Product Structure
  - Ideal Transformations 1: Unitary Operators
  - Ideal Transformations 2: Projections
- 3 Generalized Axioms for Quantum Theory
  - Generalized Preparations: Density Operators
  - Generalized Measurements
  - Composite Systems and Entanglement
  - Generalized Transformations

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 The purpose of this course is to gain a deeper understanding of what kind of theory quantum theory is, and to learn what it tells us about the world.

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Page 24/132

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Page 25/132

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   Throughout this course you will hear a surprising variety of answers to this simply question.
- A related goal is to understand what we may or may not deduce about "reality", or, to use a more philosophical term, about the fundamental *ontology*, in light of quantum theory.
- Of course, one option is to deny that there is any reality at all, and another is to say that there are infinitely many.
- Somewhat amazingly, we will see that, if you accept that there is "something really going on", ie some unique reality, then irrespective of what ontology you believe in, it must satisfy certain constraints, in

Pirsa: 100 poarticular, non-locality and contextuality.

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  - Not surprisingly, "users" with a poor understanding of these interpretational issues will often be led to erroneous conclusions about what is, and is not, possible to achieve with quantum theory. There are many historical examples of this.

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  - Not surprisingly, "users" with a poor understanding of these interpretational issues will often be led to erroneous conclusions about what is, and is not, possible to achieve with quantum theory. There are many historical examples of this.
  - On the flip side, major advances in the application of quantum theory, such as quantum information technology, were born out of concerns about the unusual ontological implications of quantum phenomena such as superposition and entanglement.
    Page 31/132

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- Finally, after almost a century of efforts, no one has been able to understand how to combine quantum theory and general relativity to construct a single theoretical framework, capable of describing physical phenomena on all scales and with all known forces involved.
  - Is this because too many researchers have neglected answering carefully the simple question: "What is a quantum state?"
- Before we get to these issues, we first we have to be clear that we know how to be practical quantum "users" ...

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#### Ideal Preparations: Hilbert Space Vectors

Axiom 1. An ideal preparation procedure is described by a Hilbert space vector  $\psi \in \mathcal{H}$ .

• Ideal preparations are often called "pure states".

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- In finite dimensions  $\mathcal{H} = \mathbb{C}^d$ .
- Normalization implies  $||\psi|| = 1$ , which prescribes a hypersphere  $S^{2d-1}$  in a 2d-dimensional real vector space.
- Because state vectors have a complex phase which is physically insignificant, distinct preparations are in one-to-one correspondence with elements of the complex projective space CP<sup>d-1</sup>.

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 Let be a self-adjoint operator with discrete eigenvalues a<sub>I</sub> and eigenvectors {|a<sub>I</sub>, m<sub>I</sub>⟩}, where m<sub>I</sub> indexes an orthogonal set of vectors spanning any degenerate subspaces.

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Axiom 2. An ideal measurement procedure is represented by a self-adjoint operator  $\hat{A}$ .

- (a) The set of observable outcomes is given by the eigenvalues {a<sub>I</sub>} of Â.
- (b) The probability of finding outcome  $a_l$ , given preparation  $|\psi\rangle$ , is  $\Pr(a_l) = \operatorname{Tr}(|\psi\rangle\langle\psi|P_{a_l})$ .

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  eigenvectors {|a<sub>I</sub>, m<sub>I</sub>⟩}, where m<sub>I</sub> indexes an orthogonal set of vectors
  spanning any degenerate subspaces.
- An important representation of a self-adjoint operator is its spectral decomposition. In the case of a discrete spectrum we have

$$\hat{A} = \sum_{I} a_{I} \hat{P}_{a_{I}},$$

where we introduce projectors onto the (possibly degenerate) eigenspaces associated with distinct eigenvalues  $P_{a_l} = \sum_{m_l} |a_l, m_l\rangle\langle a_l, m_l|$ .

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Suppose we have two systems, A and B, with state spaces  $\mathcal{H}_A$  and  $\mathcal{H}_B$ , both of which are separable Hilbert spaces and hence possess orthonormal bases  $\{|a_k\rangle\}$  and  $\{|b_\ell\rangle\}$  respectively. We wish to describe these systems jointly by a Hilbert space  $\mathcal{H}_{AB}$ . How is this Hilbert space related to the Hilbert spaces of the subsystems?

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Axiom 3. The Hilbert space of a composite system is given by the tensor product of the subsystem Hilbert spaces,

$$\mathcal{H}_{AB} = \mathcal{H}_{A} \otimes \mathcal{H}_{B}$$
.

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Remark that H<sub>AB</sub> is spanned by {|a<sub>k</sub>⟩ ⊗ |b<sub>ℓ</sub>⟩}, ∀ k, ℓ, and dim(H<sub>A</sub> ⊗ H<sub>B</sub>) = dim(H<sub>A</sub>)dim(H<sub>B</sub>). Hence an arbitrary state vector |ψ<sub>AB</sub>⟩ ∈ H<sub>AB</sub> of the joint system can be obtained from linear combinations of the joint-basis states,

$$|\psi_{AB}\rangle = \sum_{k,\ell} \psi_{k\ell} |a_k\rangle \otimes |b_\ell\rangle, \quad \psi_{k\ell} = (\langle a_k | \otimes \langle b_\ell |) |\psi_{AB}\rangle \in \mathbb{C}.$$

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$$|\psi_{AB}\rangle = \sum_{k,\ell} \psi_{k\ell} |a_k\rangle \otimes |b_\ell\rangle, \quad \psi_{k\ell} = (\langle a_k | \otimes \langle b_\ell |) |\psi_{AB}\rangle \in \mathbb{C}.$$

• In the finite-dimensional case, where  $\mathcal{H}_A=\mathbb{C}^M$  and  $\mathcal{H}_B=\mathbb{C}^N$ , we have

$$\mathcal{H}_{AB} = \mathcal{H}_{A} \otimes \mathcal{H}_{B} = \mathbb{C}^{MN}$$
.

 Terminology: The state associated with a composition of two subsystems is called bipartite; similarly the state associated with a composition of three subsystems is called tripartite, and so on. The state space of each subsystem is called a factor space of the full

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#### Ideal Transformations

During the time-interval between the preparation procedure and the measurement procedure the system evolves under a dynamical transformation:

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#### Ideal Transformations

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Axiom 4. Ideal dynamical transformations are generated by a linear operator U,

$$|\psi(t_2)\rangle = \hat{U}(t_2,t_1)|\psi(t_1)\rangle$$

satisfying,

$$i\hbar \frac{\partial \hat{U}(t_2, t_1)}{\partial t_2} = \hat{H}(t_2)\hat{U}(t_2, t_1),$$

where  $\hat{H}(t)$  is a self-adjoint operator representing the system energy function and  $t \in \mathbb{R}$  is time, subject to the initial condition  $\hat{U}(t_1,t_1)=1$ .

#### Ideal Transformations

From Axiom 4 we can deduce Schrödinger's equation,

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H}(t) |\psi(t)\rangle.$$



# What is the state after an ideal filtering measurement?

 In order to understand how to describe the state of a quantum system after an ideal filtering measurement, von Neumann considered the Compton experiment where photons are scattered off electrons that are initially at rest.



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AMATH 900 AMATH 495 PHYS 490.

Foundations and Interpretation of Quantum Mechanics

Joseph Emerson.
RAYMOND LAPLAMME



# What is the state after an ideal filtering measurement?

- von Neumann then imagined a scenario where there is a finite time difference Δt between the measurement of the electron and of the photon, i.e., a time-delay between the detection times of these particles.
  - Forward reference: note the similarity to the EPR and Bell-type arguments involving entangled states.
- Because, after measurement of the electron, the photon's momentum can be predicted with certainty (within some finite precision) this means that, after the electron momentum has been observed, the state describing the photon must be somehow "updated" (to within the same finite precision) to a new state that is consistent with the observed outcome for the electron momentum.

#### Another kind of Transformation

 The transformation taking the quantum state accorded just before measurement of the electron to that just after measurement is not consistent with the usual unitary (Schrodinger) evolution. We will prove this later, but for now consider that:

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  - After measurement, the state update must be discontinuous an instantaneous update of the photon's state is required.

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  - After measurement, the state update must be indeterministic it is conditional on the random outcome of the measurement of the electron momentum.
  - After measurement, the state update must be discontinuous an instantaneous update of the photon's state is required.
- This practical consideration of the ideal filtering type measurements that were possible with "entangled" systems forced von Neumann to formally introduce a second kind of dynamical transformation into quantum theory: the projection postulate.

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### Evolution under Sequential Measurements

 von Neumann realized from his analysis of sequential measurements that two kinds of transformation were required in quantum mechanics (von Neumann, 1932):

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# Evolution under Sequential Measurements

- von Neumann realized from his analysis of sequential measurements that two kinds of transformation were required in quantum mechanics (von Neumann, 1932):
  - Process 1. After observation/measurement of an outcome a<sub>k</sub>, the system is left in the eigenstate |a<sub>k</sub>⟩ associated with the detected eigenvalue a<sub>k</sub>. We have the map,

$$|\psi\rangle = \sum_{k} c_{k} |a_{k}\rangle \rightarrow |a_{k}\rangle.$$

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# Projection Postulate

Axiom 5. After observation/measurement of an outcome  $a_k$ , the system is left in the eigenstate  $|a_k\rangle$  associated with the detected eigenvalue  $a_k$ , that is,

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 Terminology: the projection postulate is also known as the "reduction of the wavepacket" and "the collapse of the wavefunction".

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- Terminology: the projection postulate is also known as the "reduction of the wavepacket" and "the collapse of the wavefunction".
- von Neumann imagined "Process 1" as an essential randomness in nature, and he considered it grounds for abandoning the "principle of sufficient cause", which I take to mean "causal determinism."

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### General states as mixtures of pure states

- Suppose we want to describe a quantum system which is prepared according to one procedure, represented by state  $|\psi_1\rangle$ , with probability  $p_1$  and according to a distinct procedure, represented by state  $|\psi_2\rangle$ , with probability  $p_2$ . How can we do this?
- If we are measuring the operator  $A = \sum_a a \hat{P}_a$  which possesses non-degenerate eigenvalues  $a \in \mathbb{R}$  associated with orthogonal eigenspaces  $\hat{P}_a$ , then the probability of obtaining outcome a given preparation  $\psi_1$  is

$$\Pr(a|\psi_1) = \operatorname{Tr}(\hat{P}_a|\psi_1\rangle\langle\psi_1|),$$

and similarly for preparation 2.

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and similarly for preparation 2.

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 If we do not know which preparation took place then the net probability of finding outcome a is simply

$$Pr(a) = p_1 Pr(a|\psi_1) + p_2 Pr(a|\psi_2).$$

By linearity of the trace we deduce that,

$$Pr(a) = Tr(\hat{P}_{a\rho})$$

where

$$\rho = p_1 |\psi_1\rangle \langle \psi_1| + p_2 |\psi_2\rangle \langle \psi_2|$$

is non-negative operator called a *density operator* satisfying the normalization condition  $\mathrm{Tr}(\rho)=1$  (which ensures that probabilities are conserved).

# General states as mixtures of pure states

- In this way we can construct general quantum states from probabilistic mixtures (convex combinations) of pure states as follows:
- (i) Discrete case:  $\rho = \sum_i p_i |\psi_i\rangle \langle \psi_i|$  with  $\sum_i p_i = 1$  and  $p_i \geq 0$ .
- (ii) Continuous case:  $\rho = \int d\lambda p(\lambda) |\psi(\lambda)\rangle \langle \psi(\lambda)|$  for  $\lambda \in \mathbb{R}$ , with  $\int d\lambda p(\lambda) = 1$  and  $p(\lambda) \geq 0$ .

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# General states from the partial trace

- Suppose we have a quantum state (density operator)  $\rho = \rho_{AB}$  on a composite Hilbert space  $\mathcal{H}_{AB}$ , where in general  $\rho$  need not correspond to a pure state  $\rho = |\psi\rangle\langle\psi|$  but may be a probabilistic mixture of pure states. How does one describe the state of subsystem A alone (with a state  $\rho_A$ ) or B alone (with a state  $\rho_B$ )?
- The relationship between ρ<sub>A</sub> and ρ<sub>AB</sub> is generated by the partial trace operation:

$$\rho_{\mathsf{A}} = \mathrm{Tr}_{\mathsf{B}}(\rho_{\mathsf{A}\mathsf{B}}).$$

• The state  $\rho_A$  is called the *reduced state* associated with  $\rho_{AB}$ .

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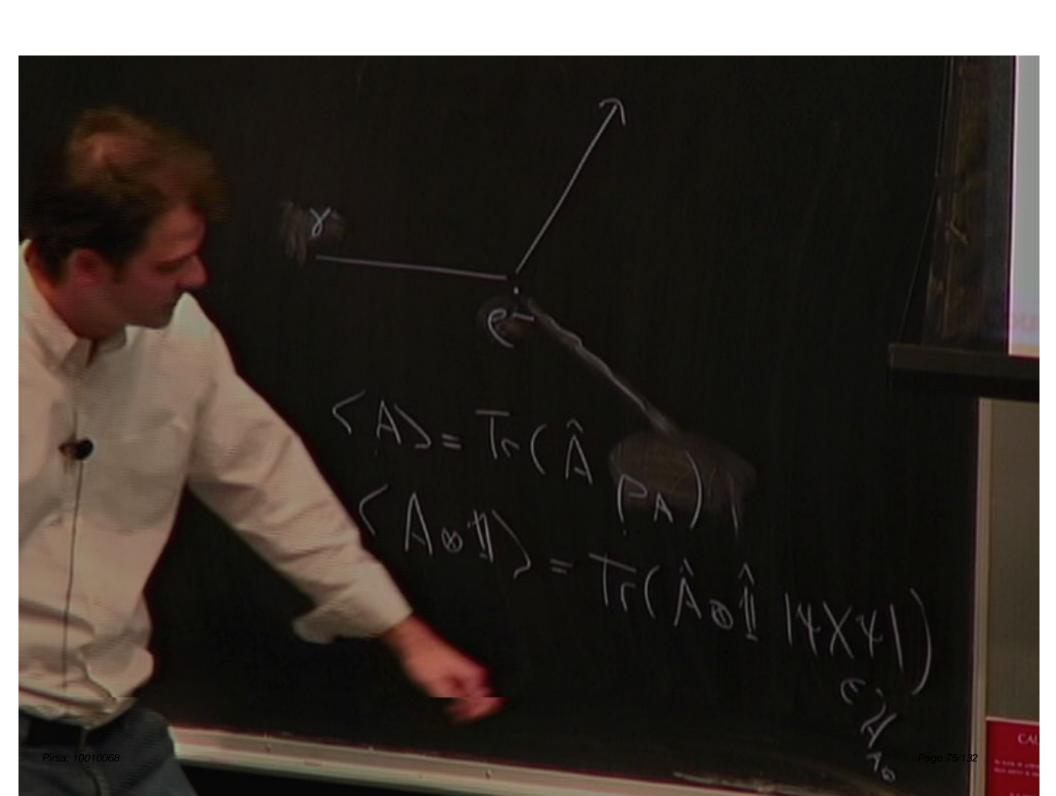
 This relationship can be deduced from physical consistency of demanding that

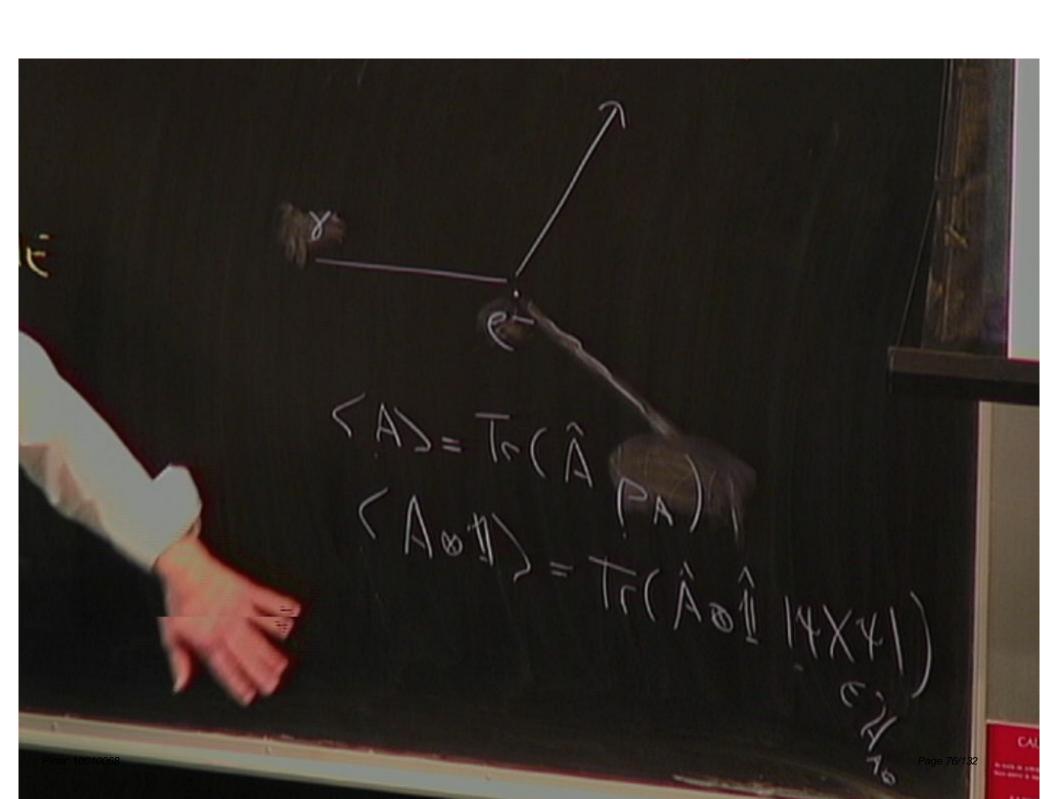
$$\langle \hat{A} \otimes \mathbb{1}_{B} \rangle = \langle \hat{A} \rangle$$

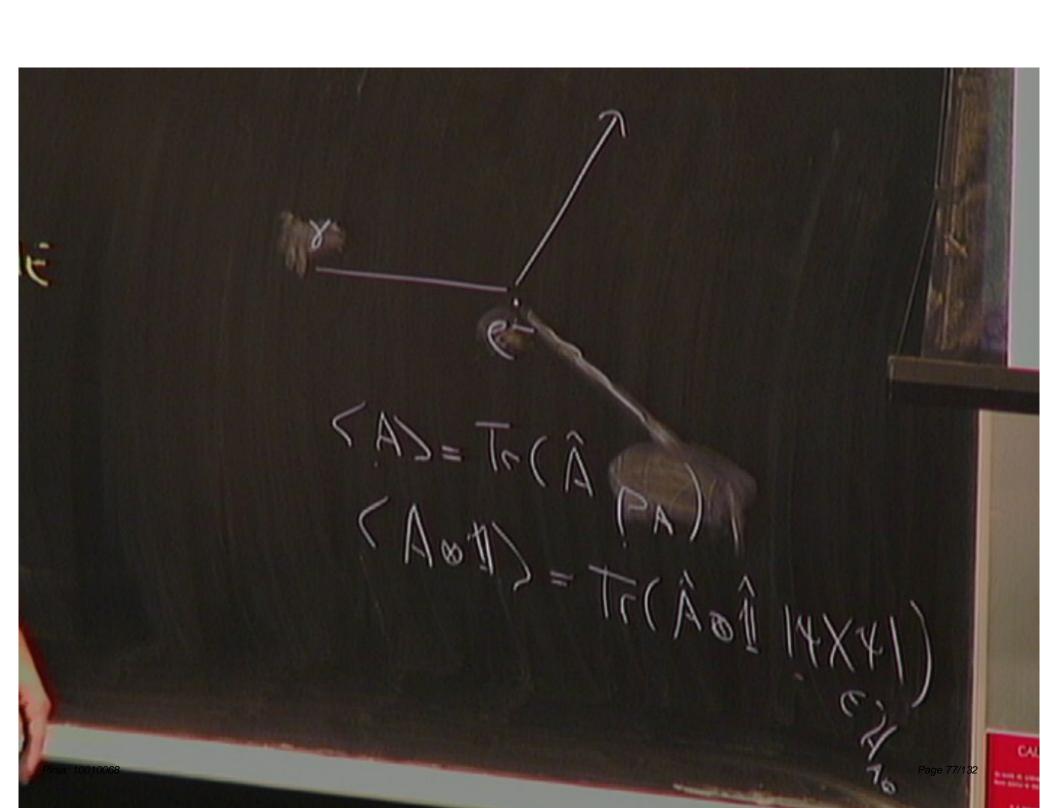
for all Hermitian operators  $\hat{A}$  and states  $\hat{\rho}_{AB}$ . Hence,

$$(\rho_A)_{\ell\ell'} = \sum_{k} (\rho_{AB})_{\ell k\ell' k},$$

which gives us an explicit matrix representation of  $\hat{\rho}_A$  in terms of the matrix elements of  $\rho_{AB}$  via the partial trace.







# General states from the partial trace

**Definition:** The partial trace over a subsystem B of an operator O acting on the composite space  $\mathcal{H}_{AB}$ ,

$$\hat{O}_A = \operatorname{Tr}_B[\hat{O}_{AB}],$$

can be defined in terms of the matrix representation,

$$(\hat{O}_A)_{\ell\ell'} = \langle \ell | \hat{O}_A | \ell' \rangle = \sum_{k} \langle \ell | \otimes \langle k | \hat{O}_{AB} | \ell' \rangle \otimes | k \rangle.$$

 It should be understood that the operation Tr<sub>B</sub>(·) takes as input any linear operator on H<sub>AB</sub> (not necessarily a density operator) and generates a linear operator on H<sub>A</sub>.

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## Generalized states

Generalized Axiom 1: The physical configuration of a system positive semidefinite operator  $\rho$  subject to the normalization constraint  ${\rm Tr}(\rho)=1$ 

- An operator P is positive semi-definite iff it is self-adjoint and satisfies  $\langle u|P|u\rangle \geq 0$  for every vector u in the Hilbert space.
- A positive semidefinite operator is sometimes just called a positive operator or a non-negative operator.

- For a state operator  $\hat{\rho}$  subject to the normalization condition  $\mathrm{Tr}(\hat{\rho})=1$  there are three equivalent definitions of *purity*:
  - i)  $\hat{\rho}^2 = \hat{\rho}$ , which means that  $\rho$  is projector.
  - ii)  $\operatorname{Tr}(\hat{\rho}^2) = 1$ .
  - iii)  $\hat{\rho}=|\psi\rangle\langle\psi|$ , defining a projector onto a one-dimensional subspace of  ${\cal H}$  .

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**Definition:** If  $\rho$  can not be expressed in the form  $\rho = |\psi\rangle\langle\psi|$  for any  $\psi \in \mathcal{H}$ , i.e., if  $\rho$  is not a *pure state*, then it is called a *mixed state*.

General states obtained via partial trace are sometimes called *improper mixtures*, whereas the term *proper mixtures* refers to general states obtained from probabilistic mixing of pure states. These two conceptual classes of mixed states are mathematically equivalent. The justification for this claim comes from the mixed state purification theorem we will prove later as well the following theorem.

## Theorem

Any general state operator can be expressed as a convex combination of pure states.

# Purification of Mixed States

We have already seen how any mixed state can always be decomposed as a infinite number of different convex combination of pure states. It is also the case that:

## Theorem

Any mixed state can be realized as the reduced state associated with a pure state on an extended Hilbert space.

General states obtained via partial trace are sometimes called *improper mixtures*, whereas the term *proper mixtures* refers to general states obtained from probabilistic mixing of pure states. These two conceptual classes of mixed states are mathematically equivalent. The justification for this claim comes from the mixed state purification theorem we will prove later as well the following theorem.

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## Theorem

Any mixed state can be realized as the reduced state associated with a pure state on an extended Hilbert space.

The pure state on the extended Hilbert space is called the *purification* of  $\rho_A$ .

If the reduced state has finite rank m, then clearly  $m \leq M = \dim(\mathcal{H}_A)$ . From the proof we see that the ancilla factor space  $\mathcal{H}_B$  must have dimension greater than or equal to m.

This purification of a mixed state is never unique. Indeed any state  $|\psi'\rangle=(1\!\!1\otimes U)|\psi\rangle$ , where U is an arbitrary unitary operator acting on  $\mathcal{H}_B$ , provides a valid purification of  $\rho_A$ .

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# Generalized Measurements

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# Generalized Measurements

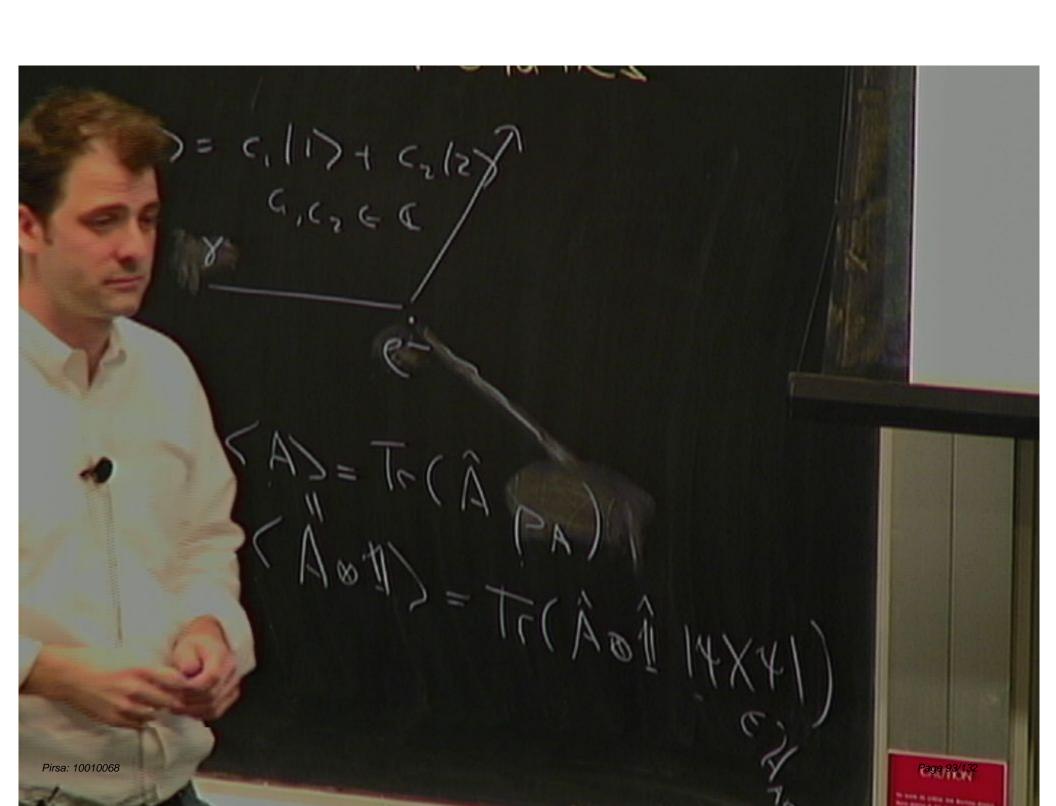
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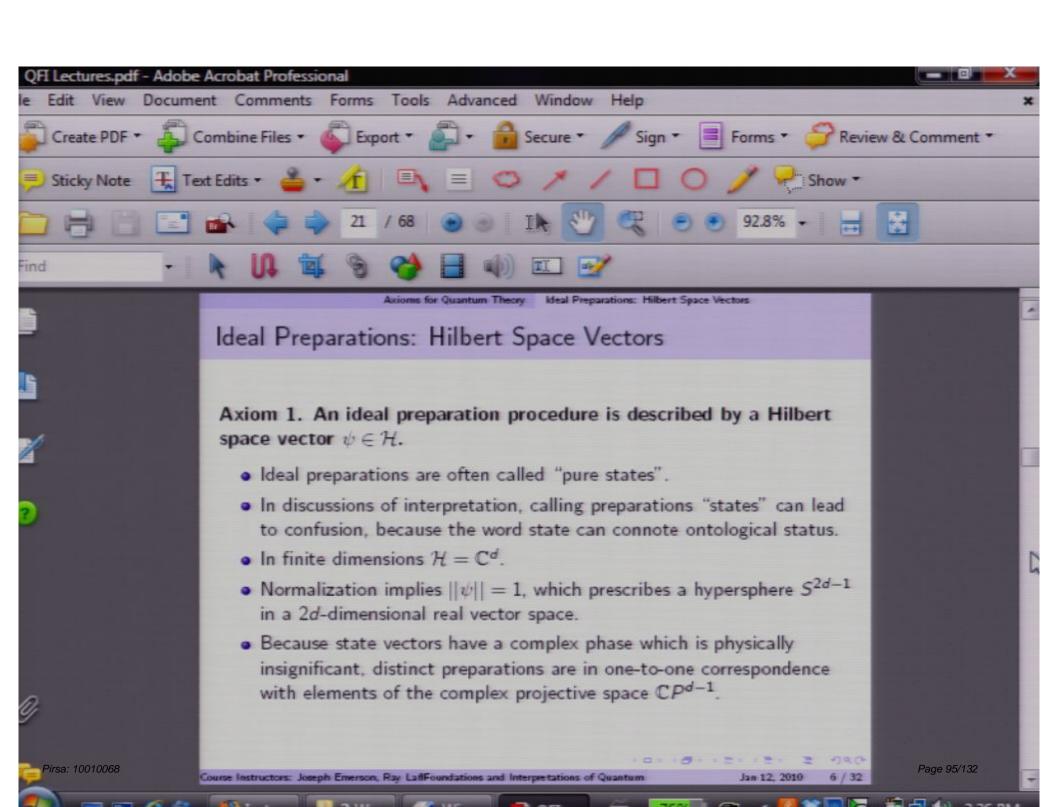
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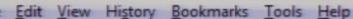
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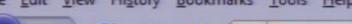


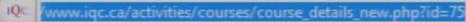
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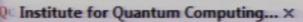






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experimental tests of Bell inequalities, contextuality, macroscopic quantum phenomena, and the problem of quantum gravity, as time permits).

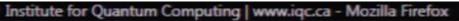
#### Schedule:

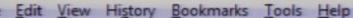
Lecturer	Tentative Lecture Title	Date
Joseph Emerson	Axioms for quantum mechanics	Week of January 11, 2009
Joseph Emerson	Basic problems of interpretation	Week of January 18, 2009
Joseph Emerson	Constraints on hidden variable models	Week of January 25, 2009
Robin Blume- Kohout	Probability and its interpretation	Week of February 1, 2009
Gregor Weihs	Experimental tests of Bell inequality	Week of February 8, 2009
Alex Wilce	Convex sets framework for probabilistic theories	Week of February 22, 2009
Roderich Tumulka	deBroglie-Bohm interpretation	Week of March 1, 2009
Chris Fuchs	Quantum Bayesian view	Week of March 8, 2009
TBA	TBA	Week of March 15, 2009
Tony Leggett	Fundamental tests of quantum mechanics	Week of March 22, 2009
Michel Devoret	Macroscopic quantum coherence	Week of March 29, 2009

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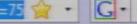
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permits) and fundamental properties and tests of quantum theory (such as entanglement and experimental tests of Bell inequalities, contextuality, macroscopic quantum phenomena, and the problem of quantum gravity, as time permits).

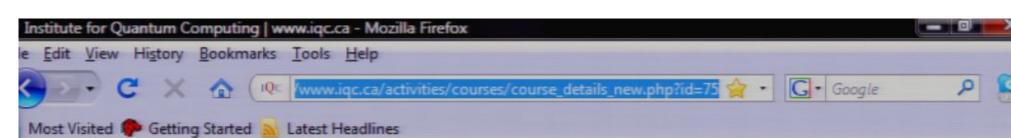
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### Tuesdays

2:05 EIT to PI

2:25 EIT to PI (this trip is reserved for those who have class on main campus until 2:20)

3:55 PI to EIT

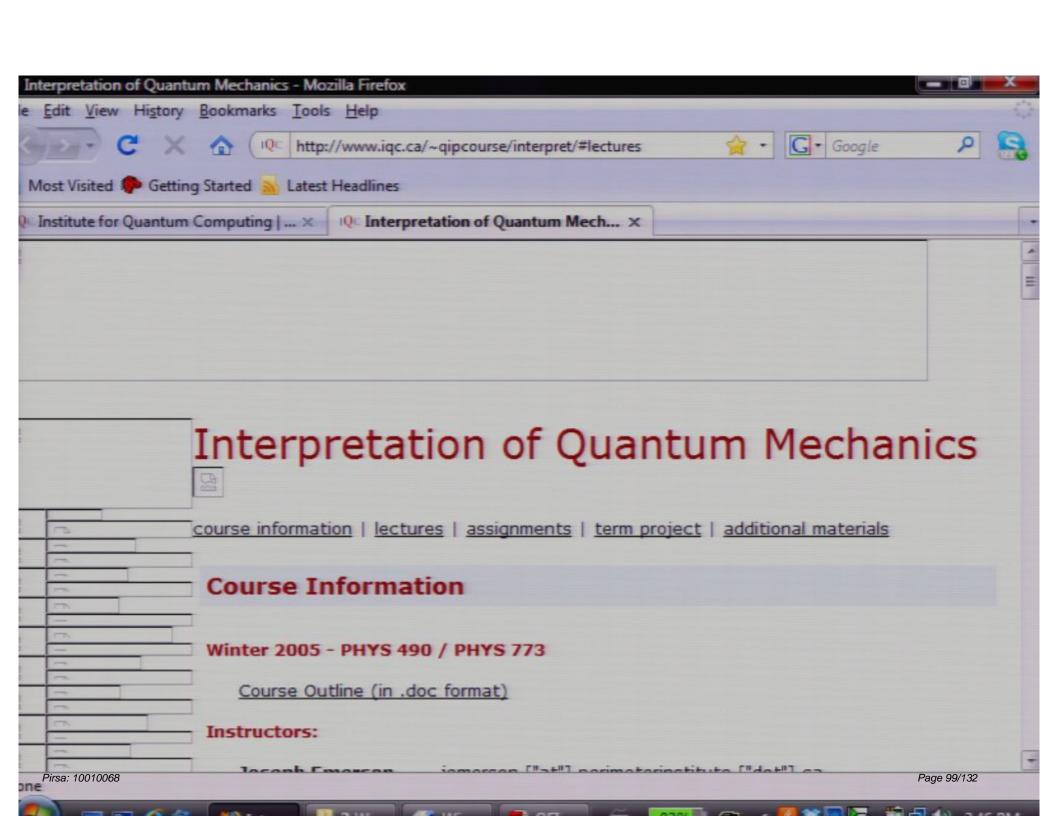
### Thursdays

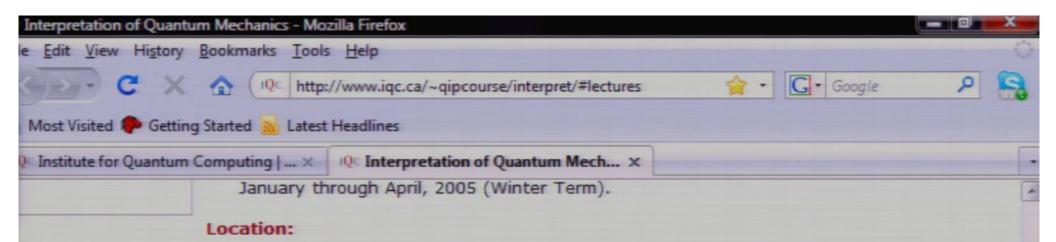
2:05 EIT to RAC

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3:55 RAC to EIT

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Lectures

Perimeter Institute, Room 405.

Click on the image to view the lecture in Mediasite Live format, which requires Microsoft Internet Explorer (for <u>Windows</u> or <u>Mac</u>) and <u>Windows Media Player</u>. <u>Macromedia Flash</u> versions of the lectures are also available here.

(Note: Thursday, 3 March, the lecture will be at UW in BFG 2125 at the usual time.)

### WEEK 1: Postulates of Quantum Theory Joseph Emerson

- 4 January Lecture 1: Postulates of Quantum Theory I
- 6 January Lecture 2: Postulates of Quantum Theory II

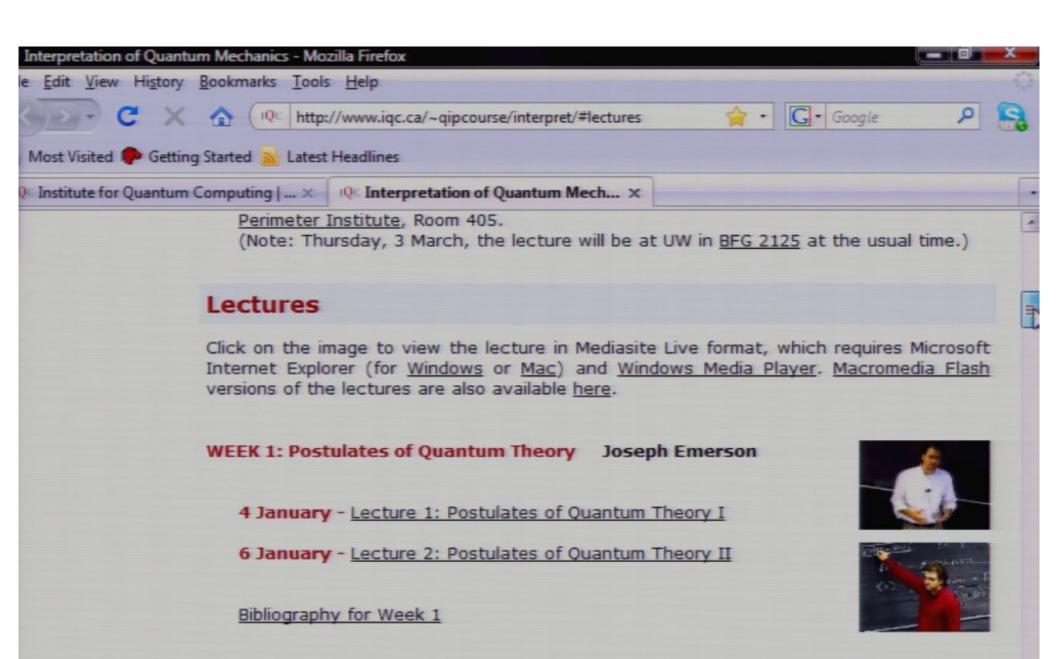
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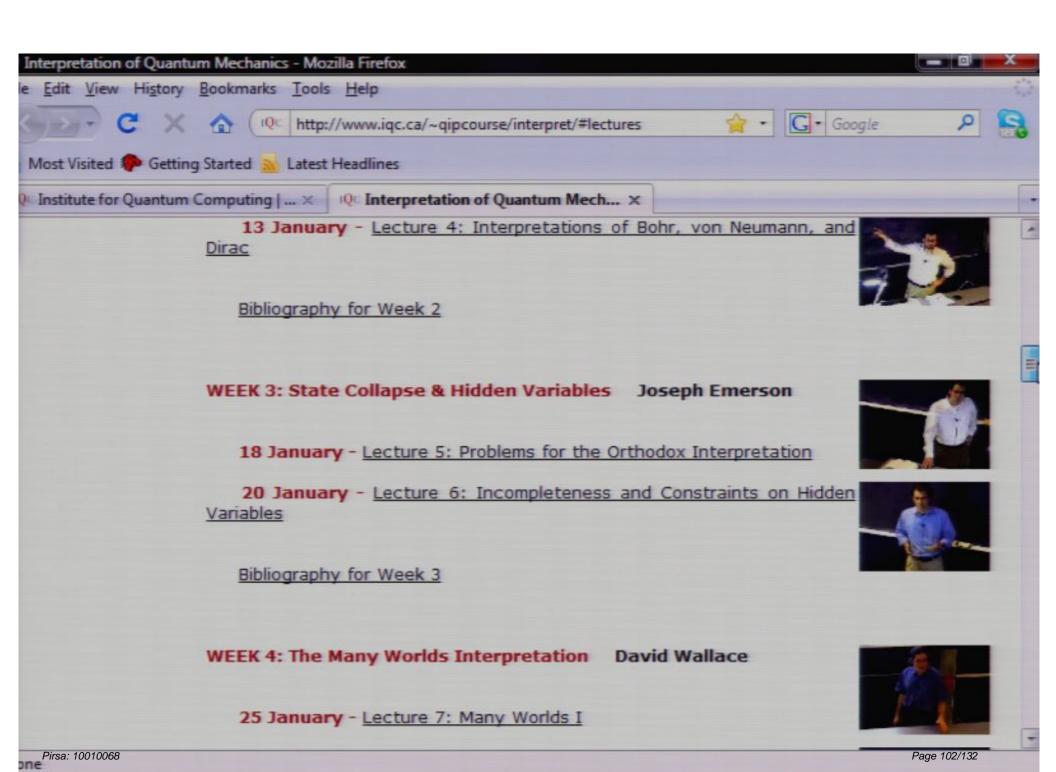
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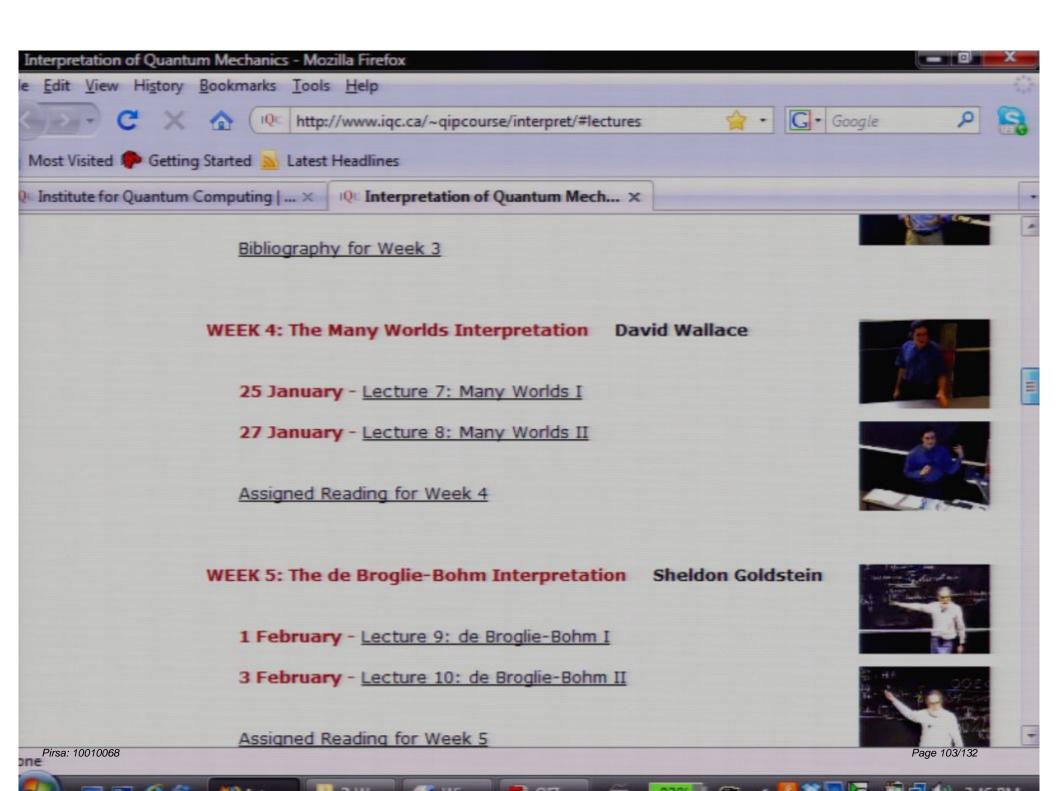
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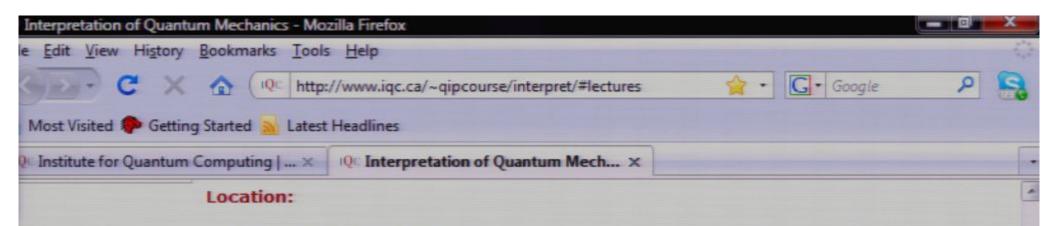




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## Lectures

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4 January - Lecture 1: Postulates of Quantum Theory I

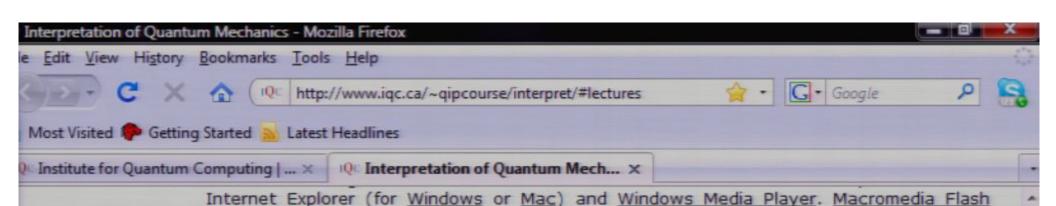
6 January - Lecture 2: Postulates of Quantum Theory II

Bibliography for Week 1

Perimeter Institute, Room 405.







4 January - Lecture 1: Postulates of Quantum Theory I

WEEK 1: Postulates of Quantum Theory Joseph Emerson

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6 January - Lecture 2: Postulates of Quantum Theory II

Bibliography for Week 1





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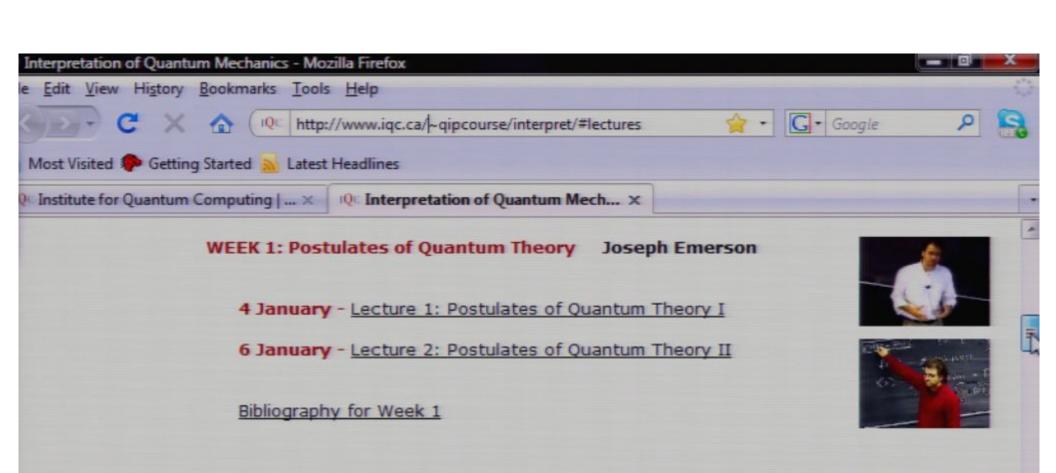
11 January - Lecture 3: Generalized States, Measurements,
Transformations

13 January - Lecture 4: Interpretations of Bohr, von Neumann, and Dirac



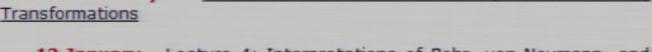


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11 January - Lecture 3: Generalized States, Measurements Transformations

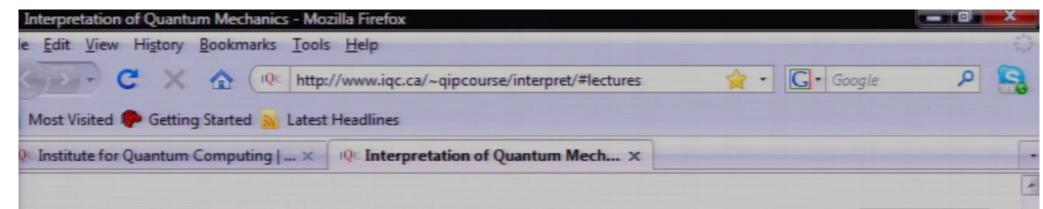


13 January - Lecture 4: Interpretations of Bohr, von Neumann, and Dirac



Bibliography for Week 2

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### WEEK 1: Postulates of Quantum Theory Joseph Emerson

- 4 January Lecture 1: Postulates of Quantum Theory I
- 6 January Lecture 2: Postulates of Quantum Theory II

Bibliography for Week 1





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11 January - Lecture 3: Generalized States, Measurements, Transformations

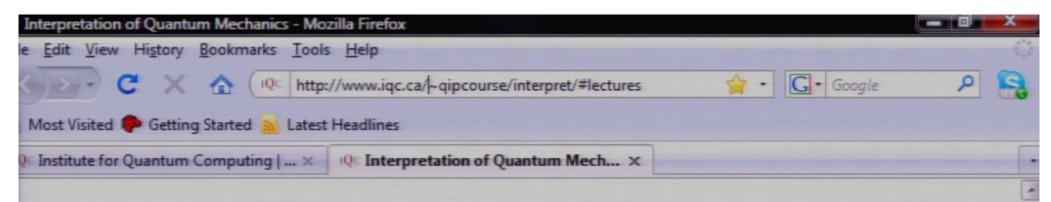


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Bibliography for Week 2

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### WEEK 1: Postulates of Quantum Theory Joseph Emerson

4 January - Lecture 1: Postulates of Quantum Theory I

6 January - Lecture 2: Postulates of Quantum Theory II

Bibliography for Week 1





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11 January - Lecture 3: Generalized States, Measurements, Transformations

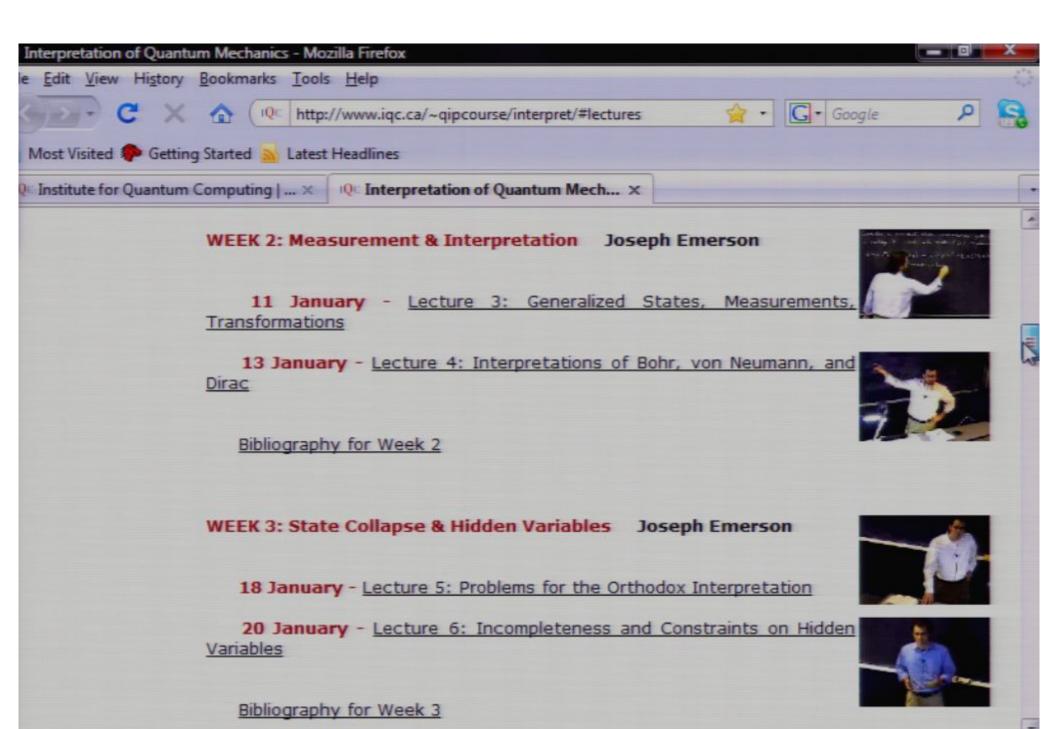
13 January - Lecture 4: Interpretations of Bohr, von Neumann, and <u>Dirac</u>



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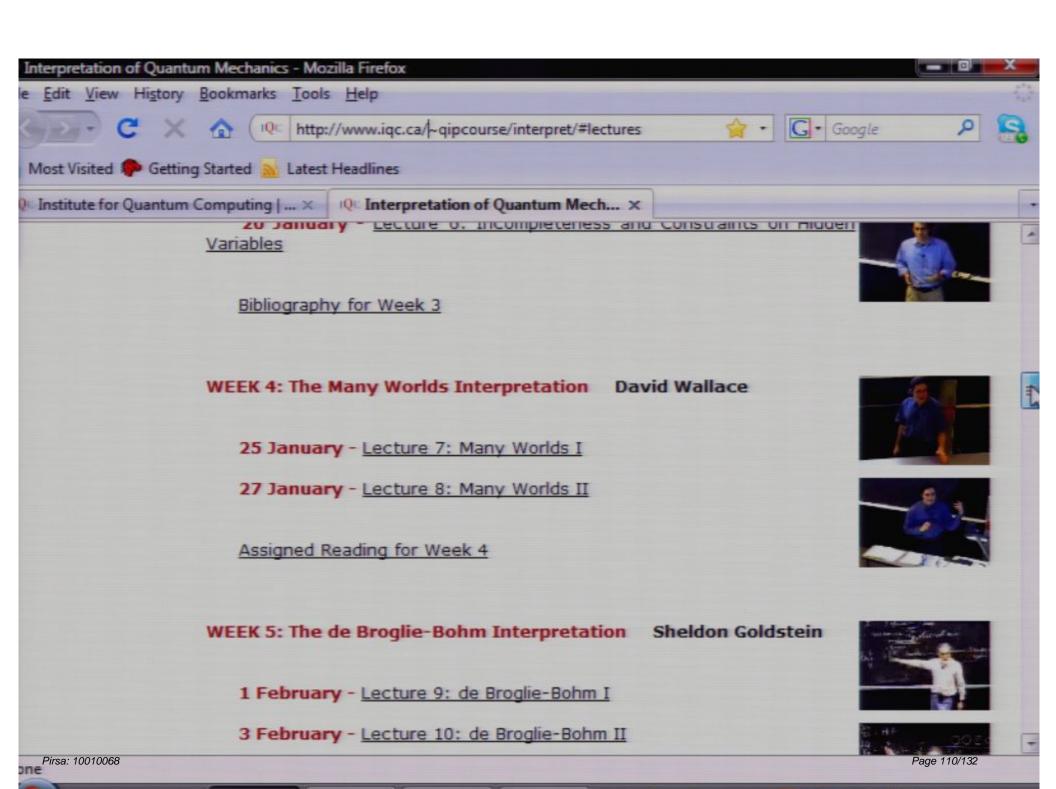
Bibliography for Week 2

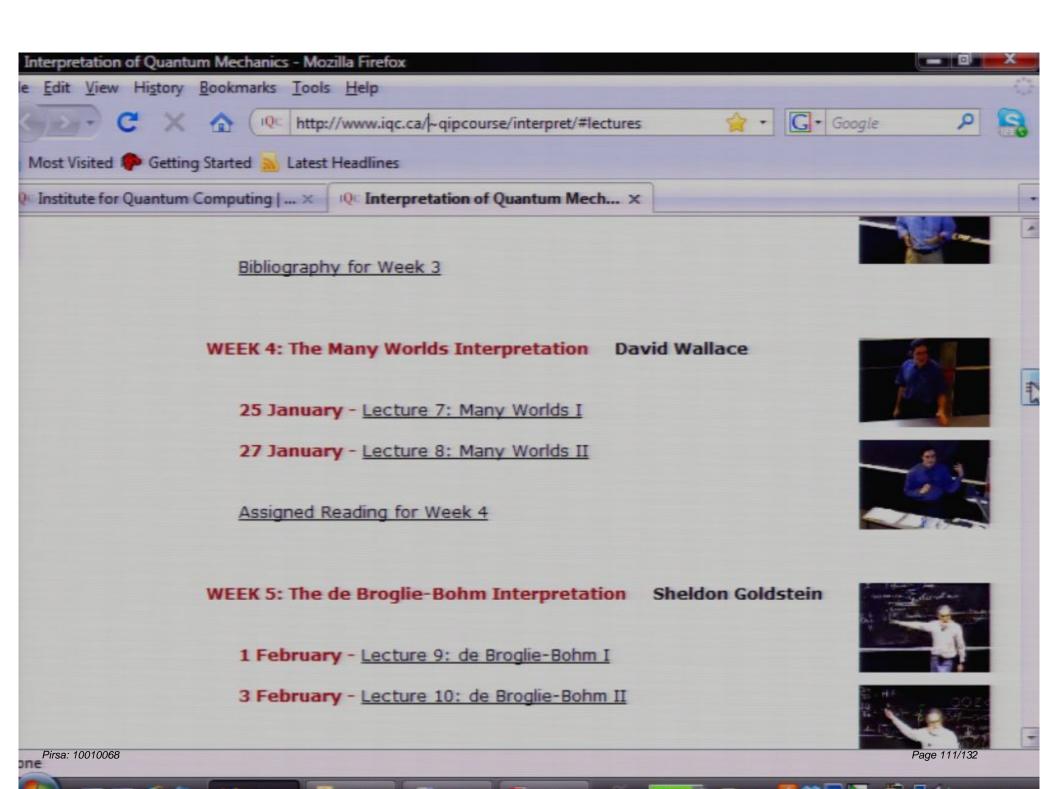
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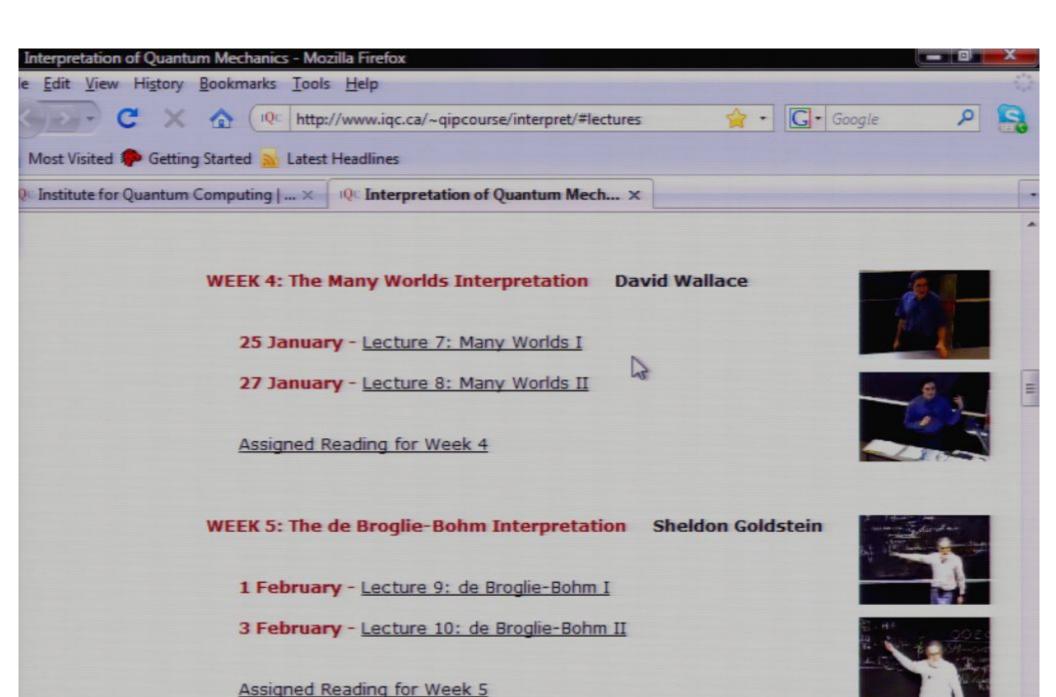


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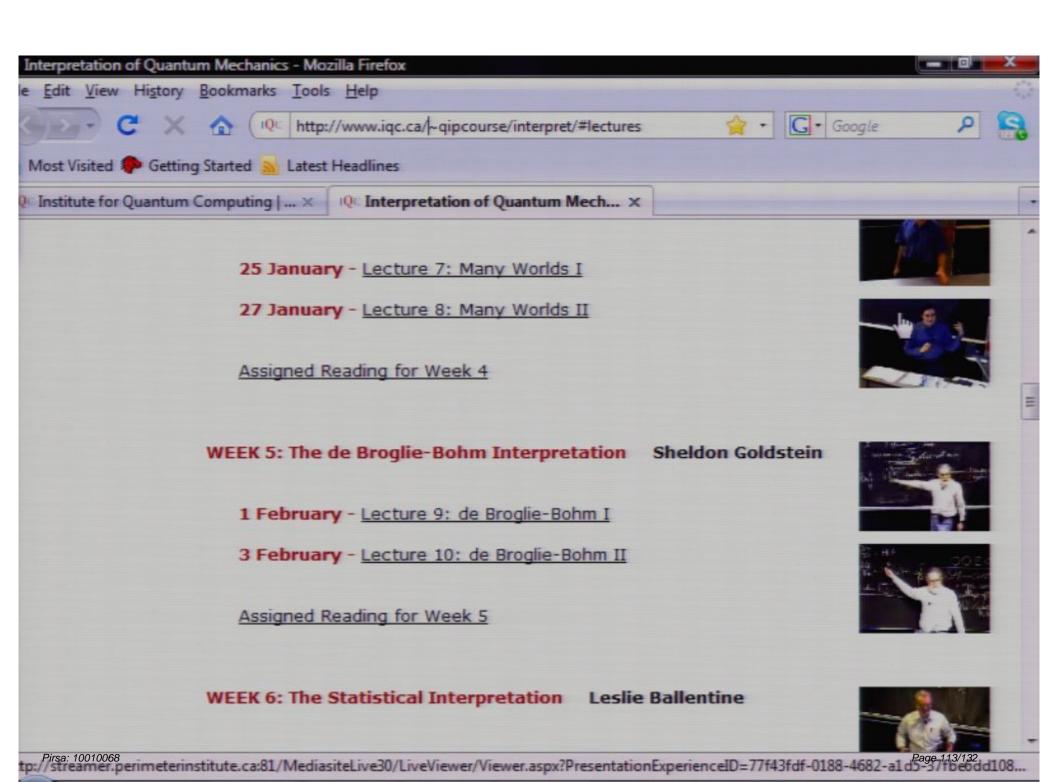
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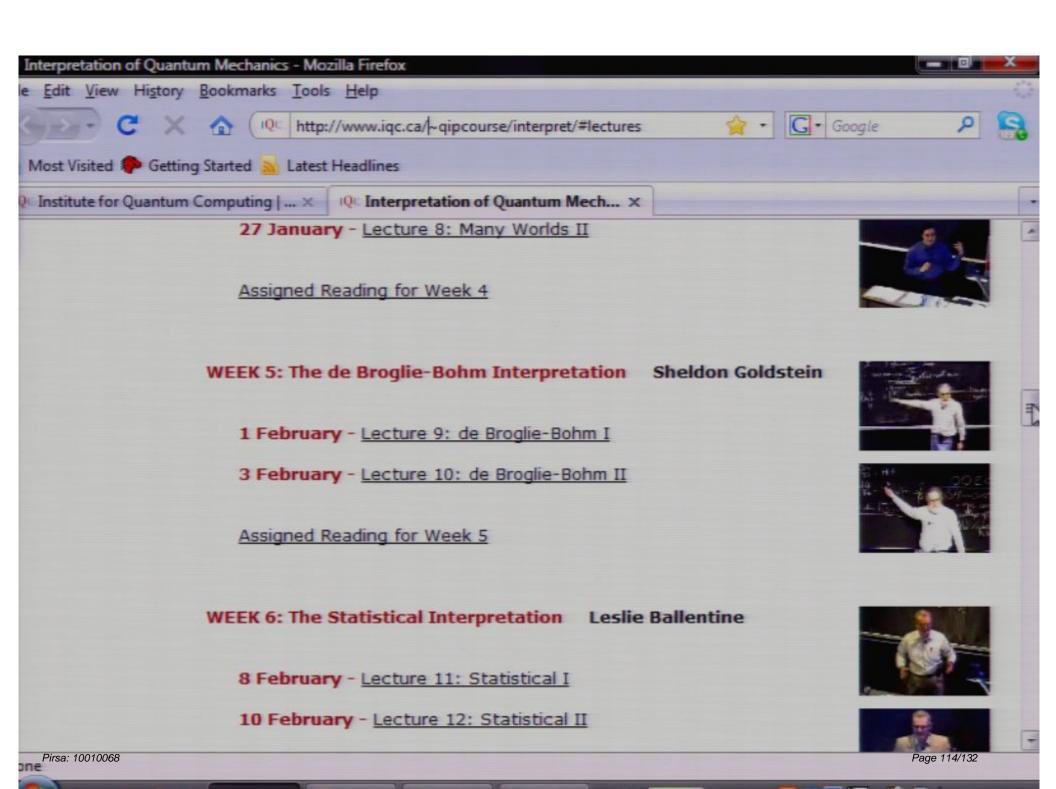


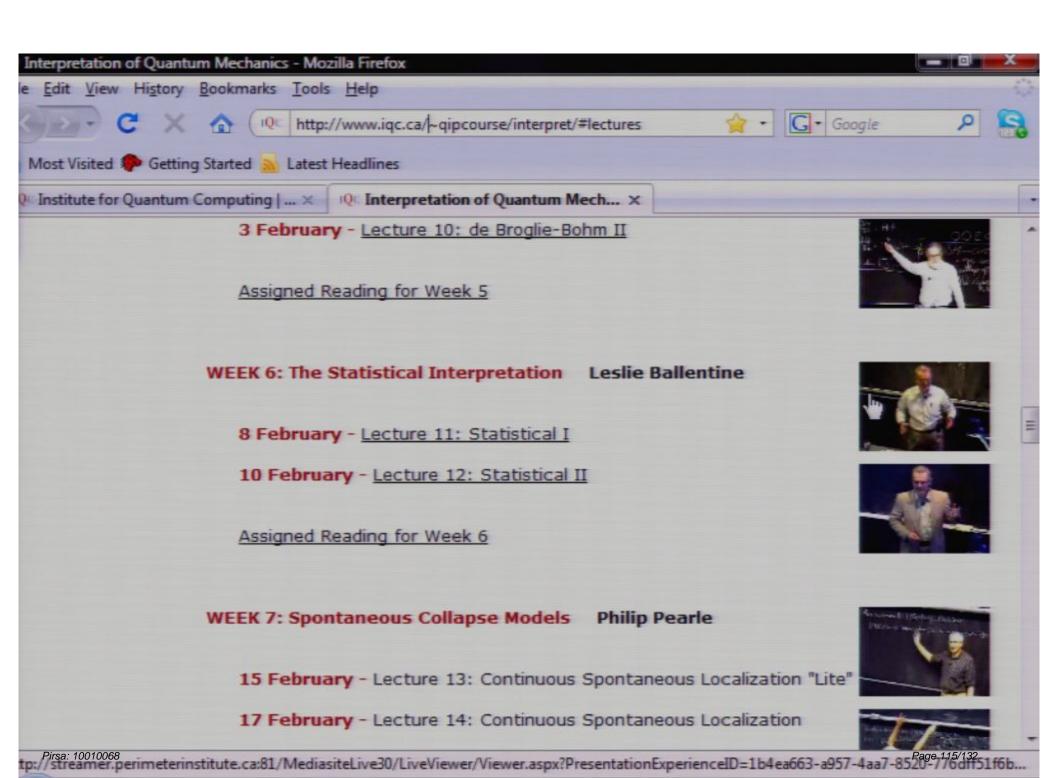


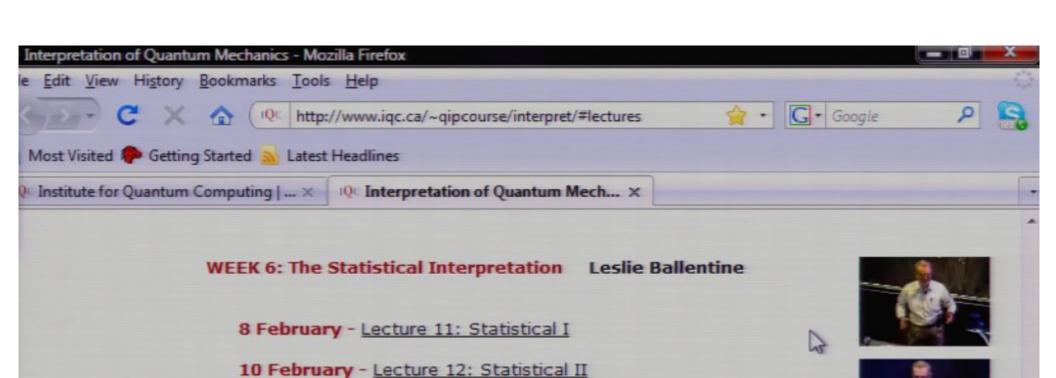


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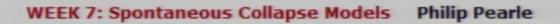








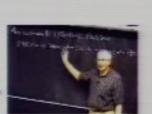
Assigned Reading for Week 6

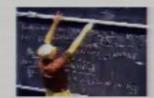


15 February - Lecture 13: Continuous Spontaneous Localization "Lite"

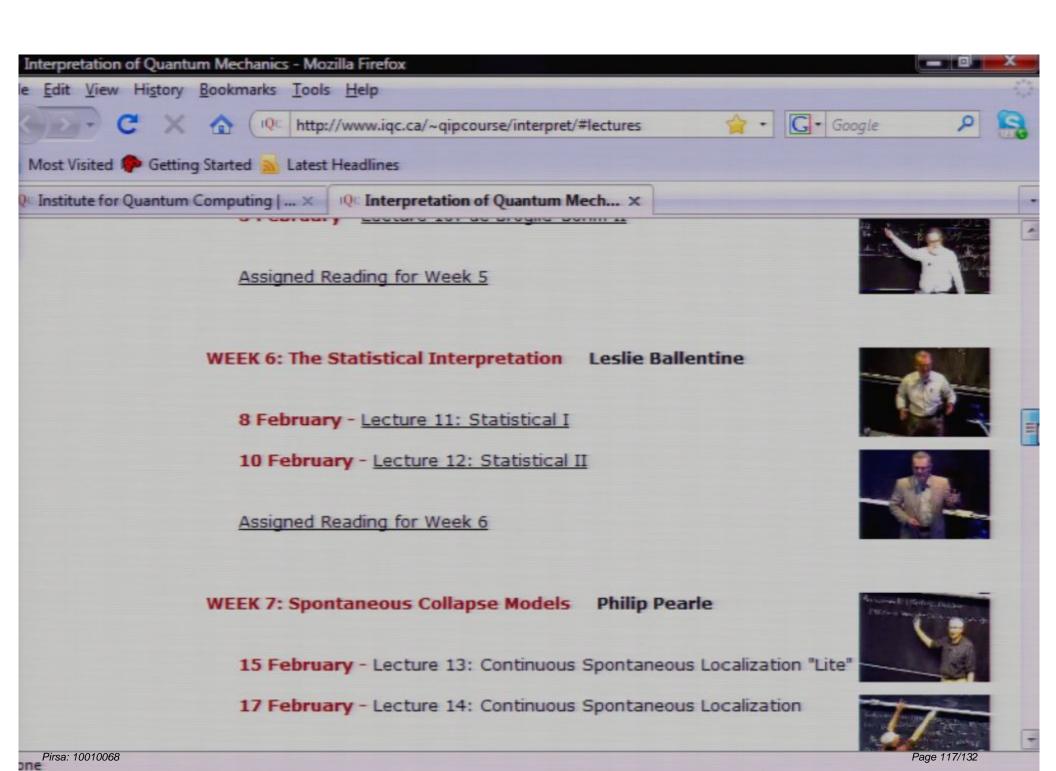
17 February - Lecture 14: Continuous Spontaneous Localization

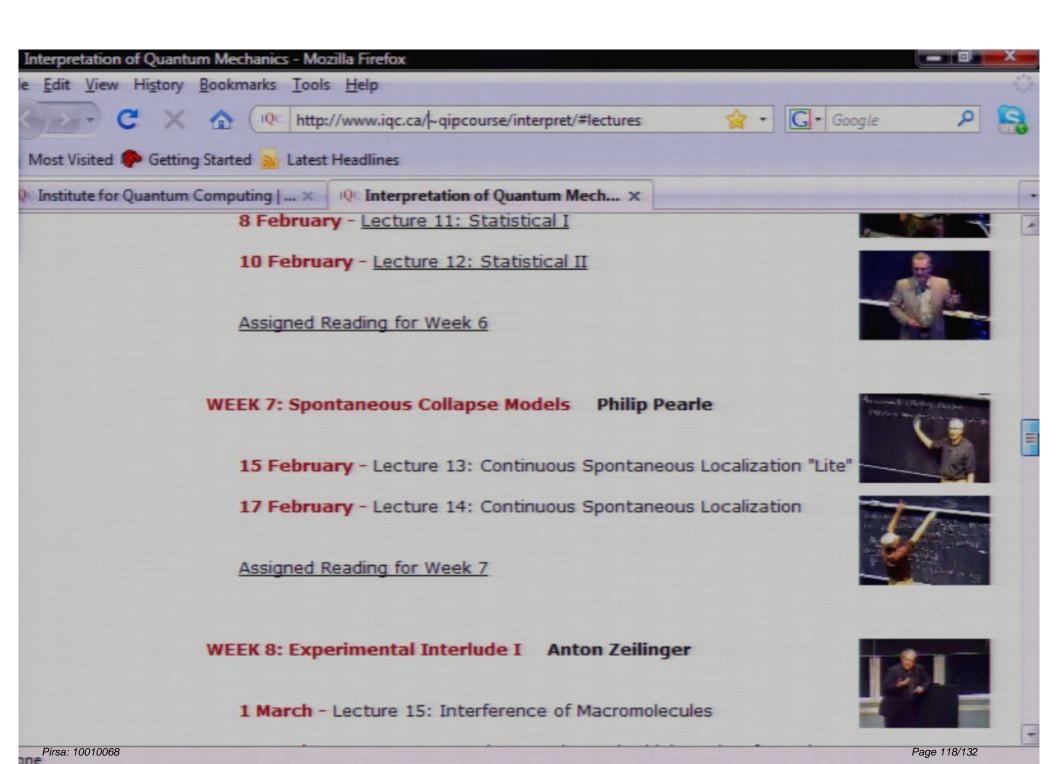
Assigned Reading for Week 7

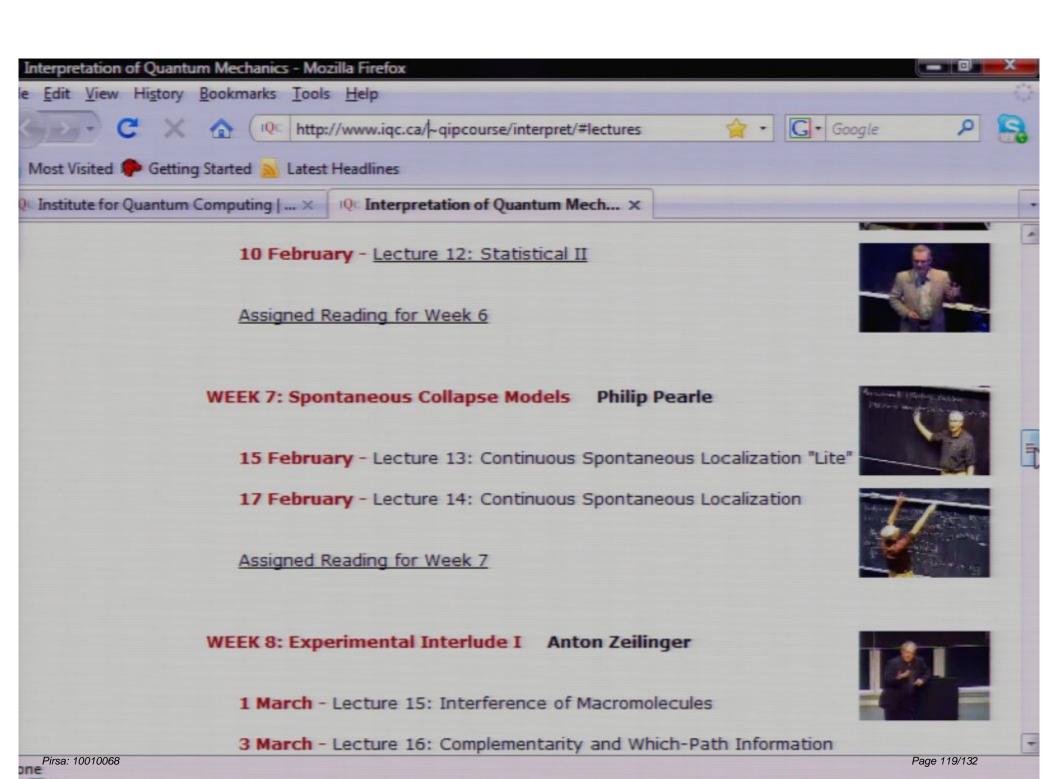


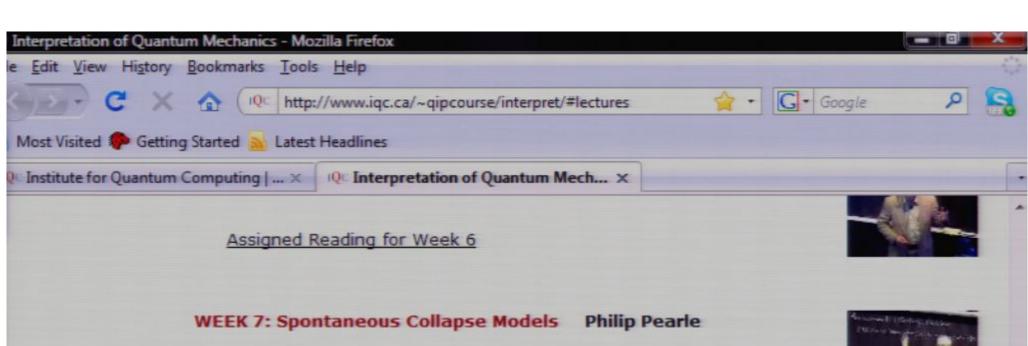


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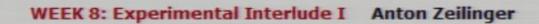




15 February - Lecture 13: Continuous Spontaneous Localization "Lite"

17 February - Lecture 14: Continuous Spontaneous Localization

Assigned Reading for Week 7



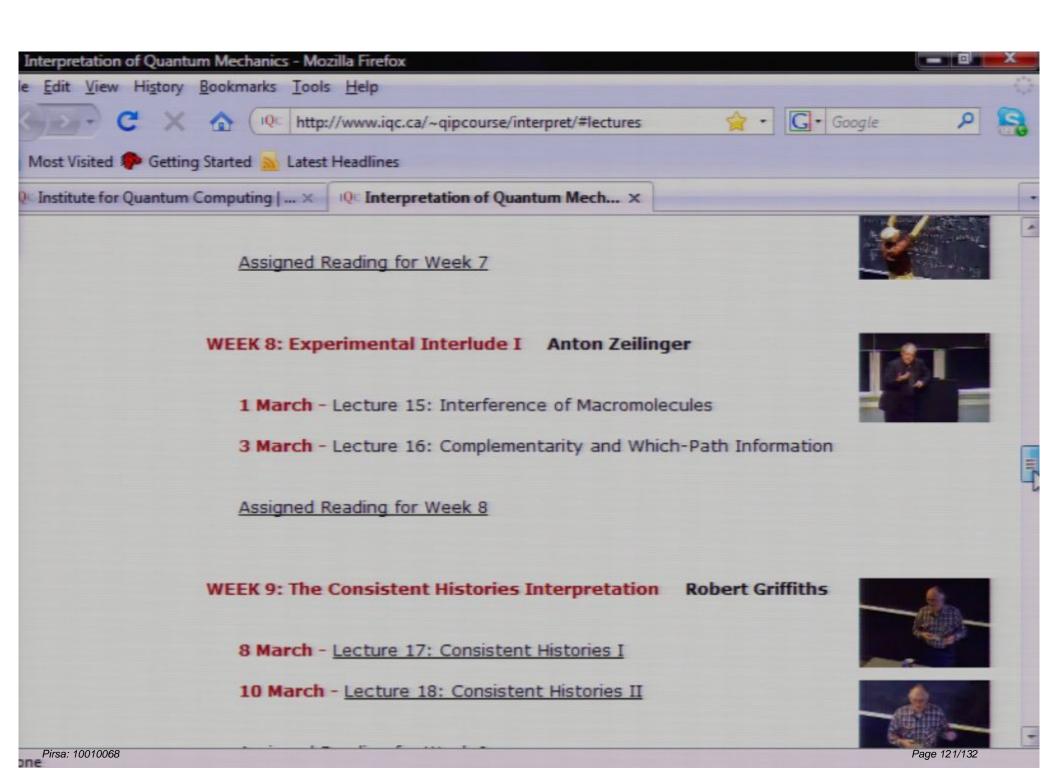
1 March - Lecture 15: Interference of Macromolecules

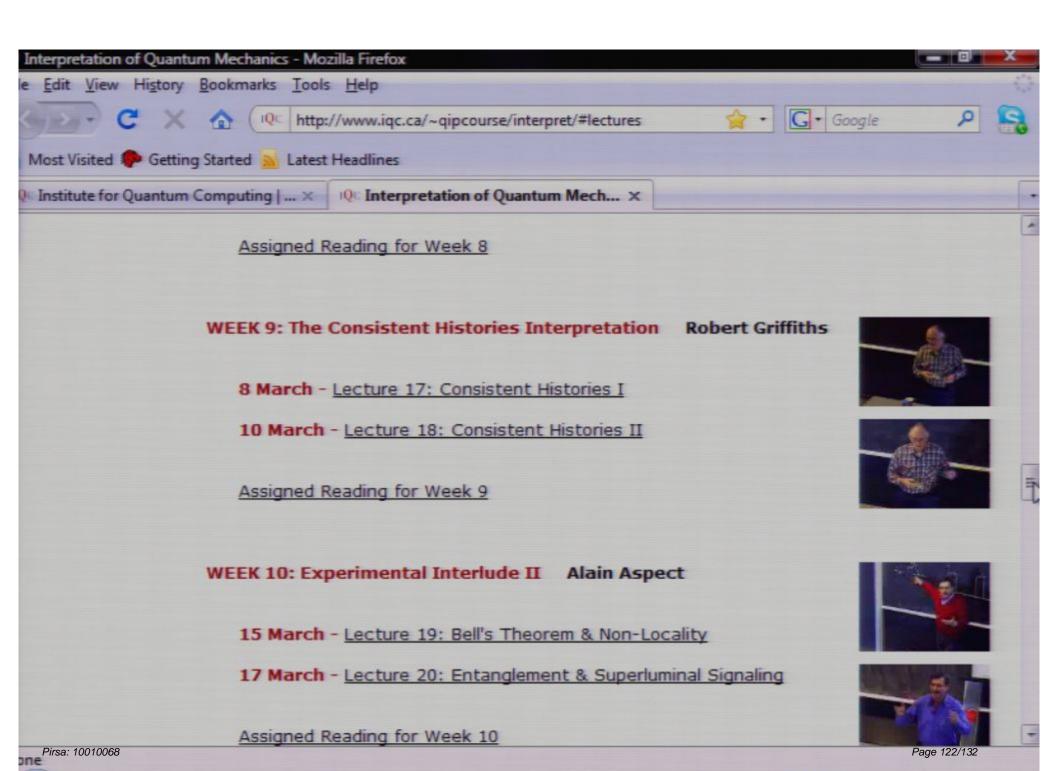
3 March - Lecture 16: Complementarity and Which-Path Information

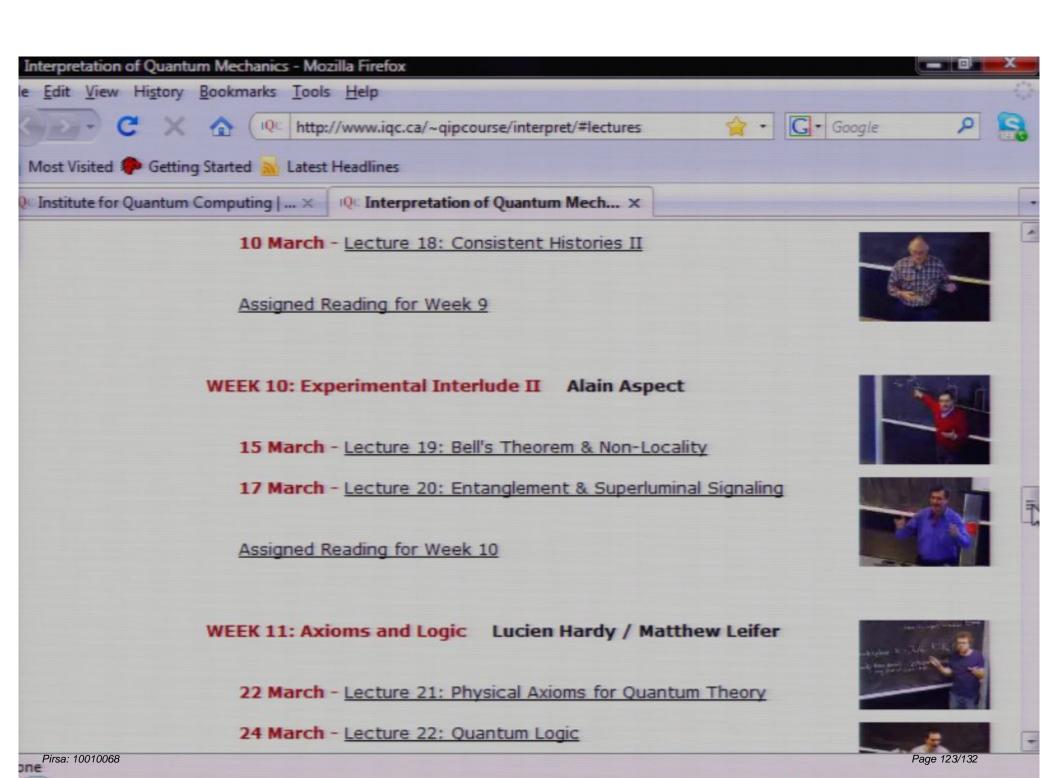


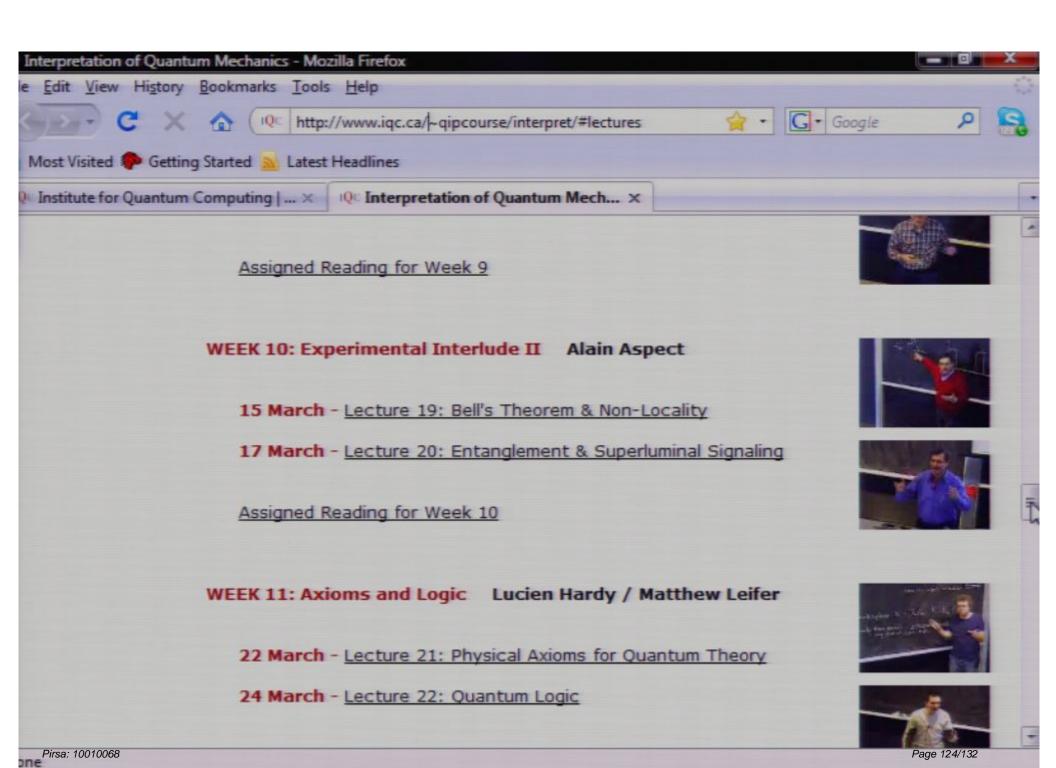


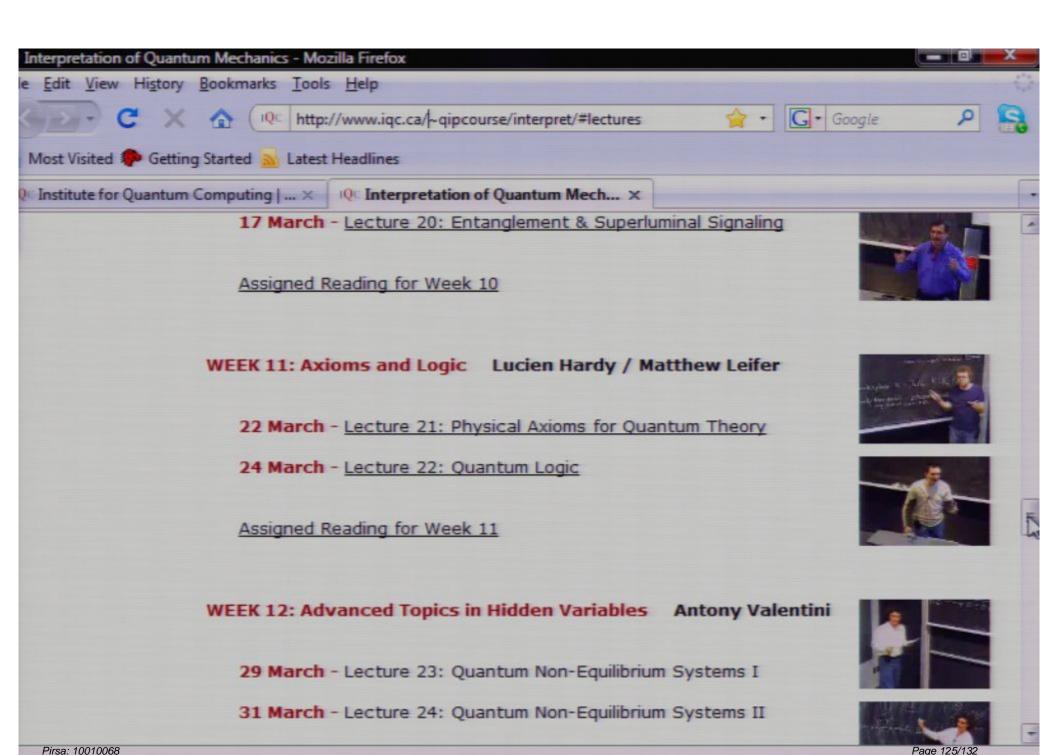


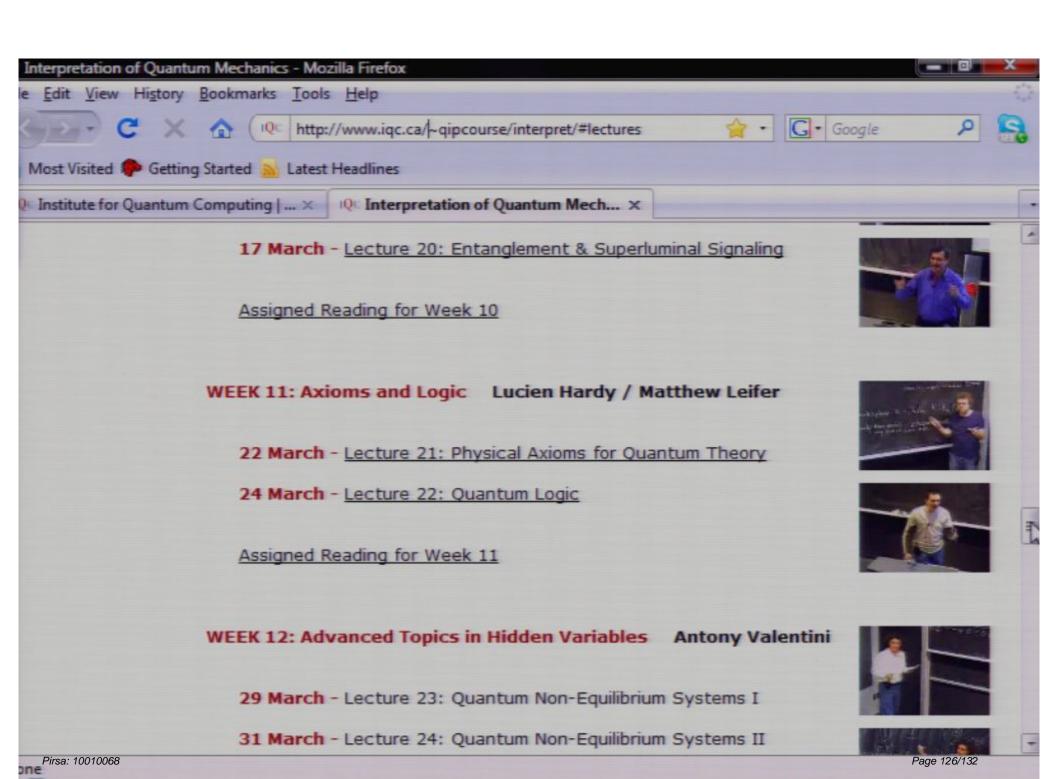


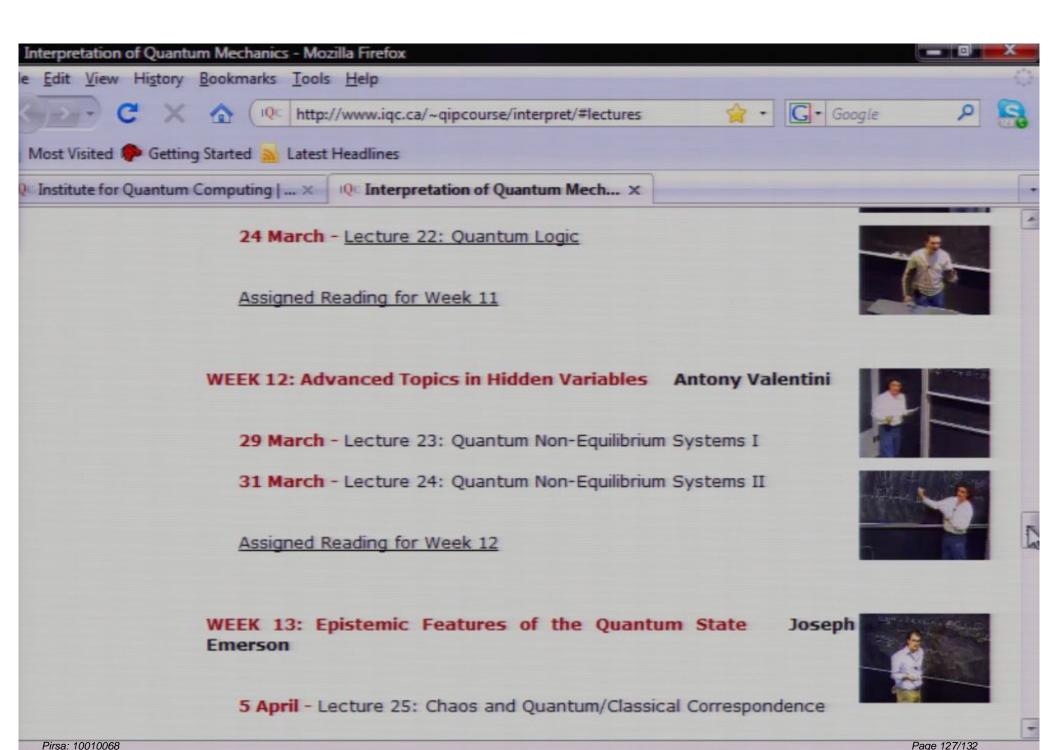




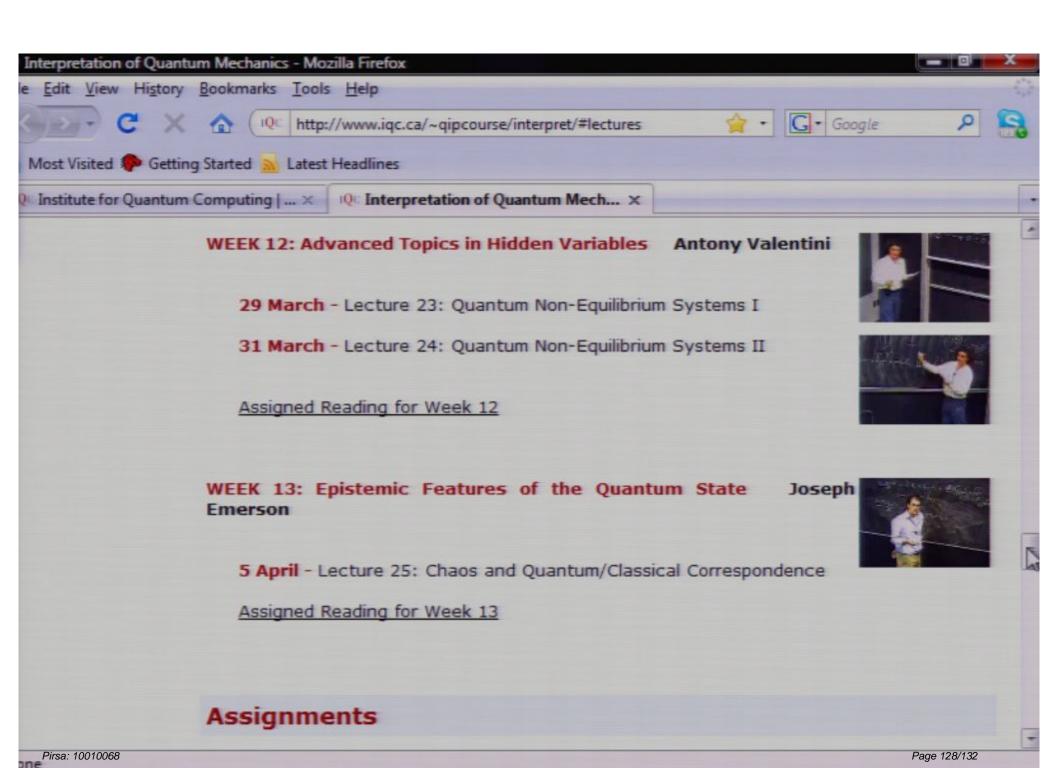


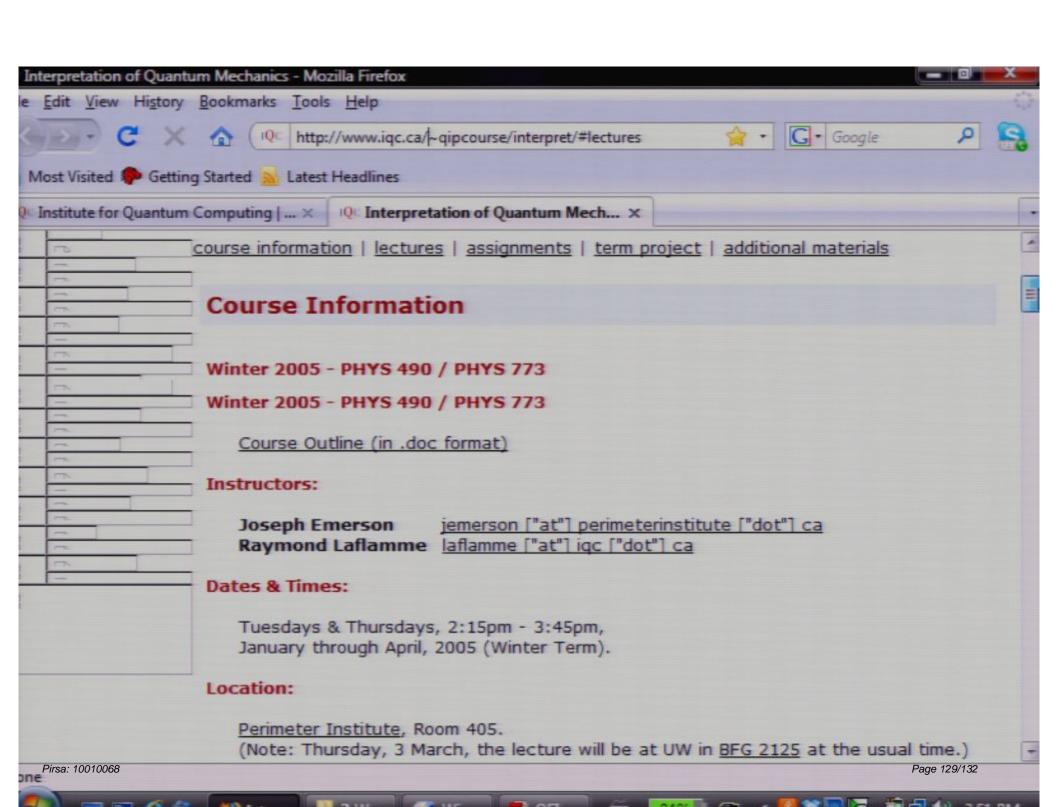


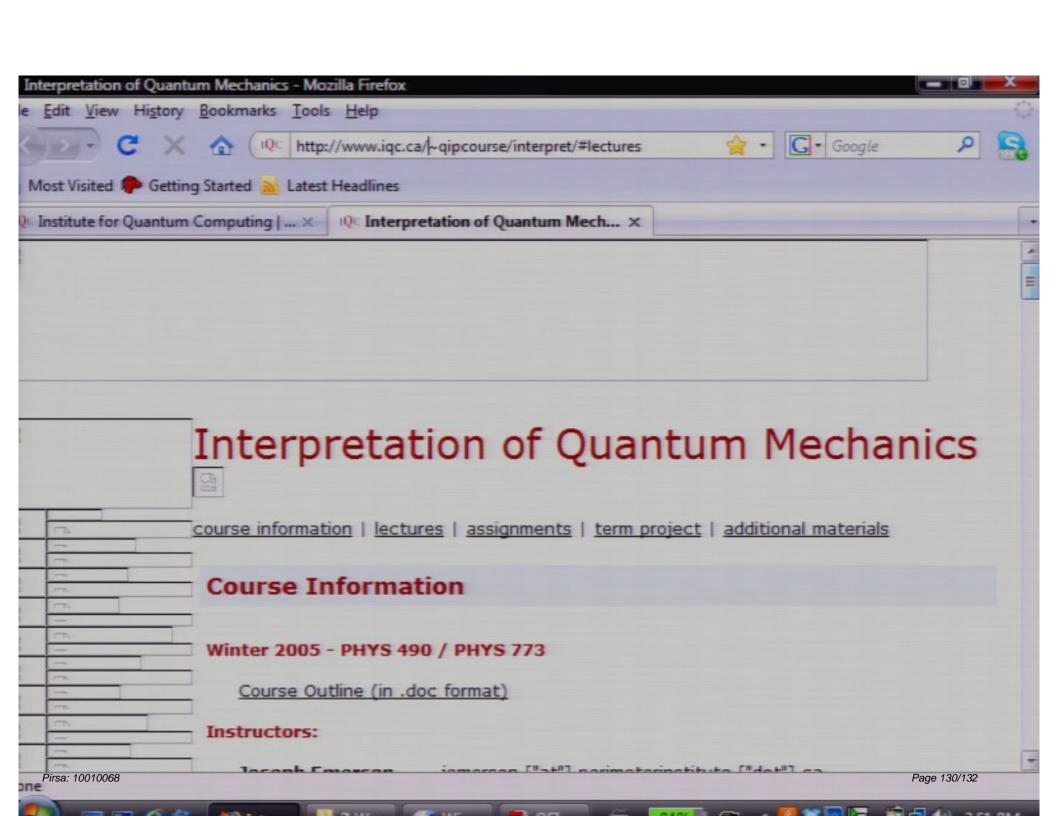


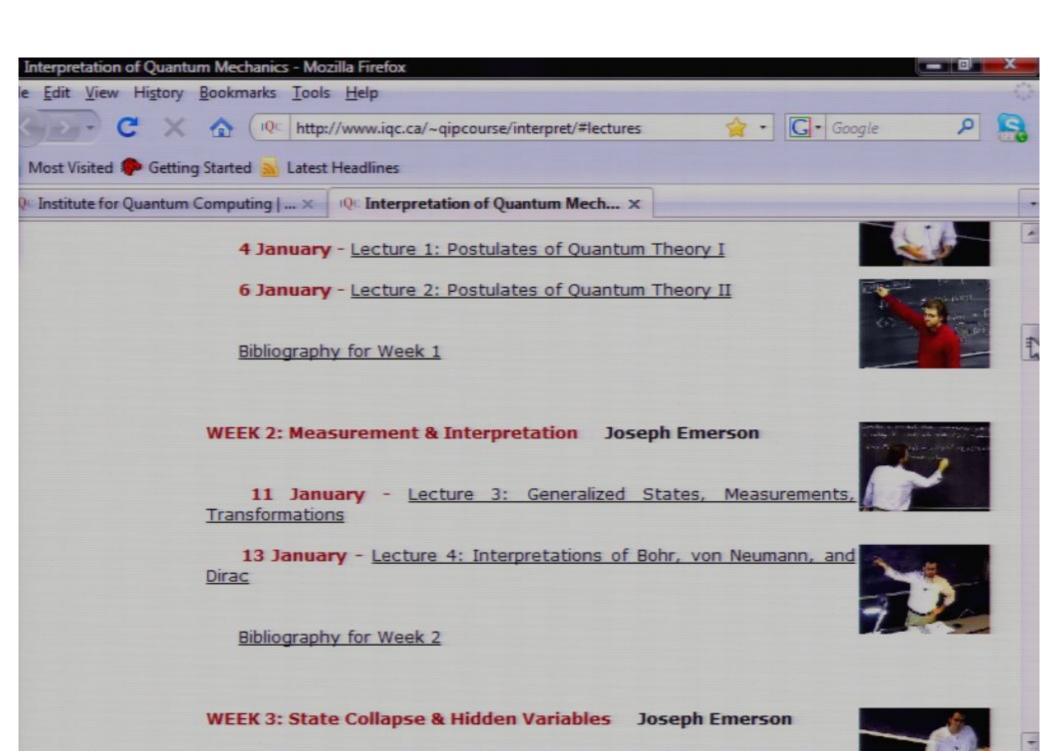


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