Title: Condensed Matter Review (PHYS 637) - Lecture 15

Date: Jan 22, 2010 11:00 AM

URL: http://pirsa.org/10010067

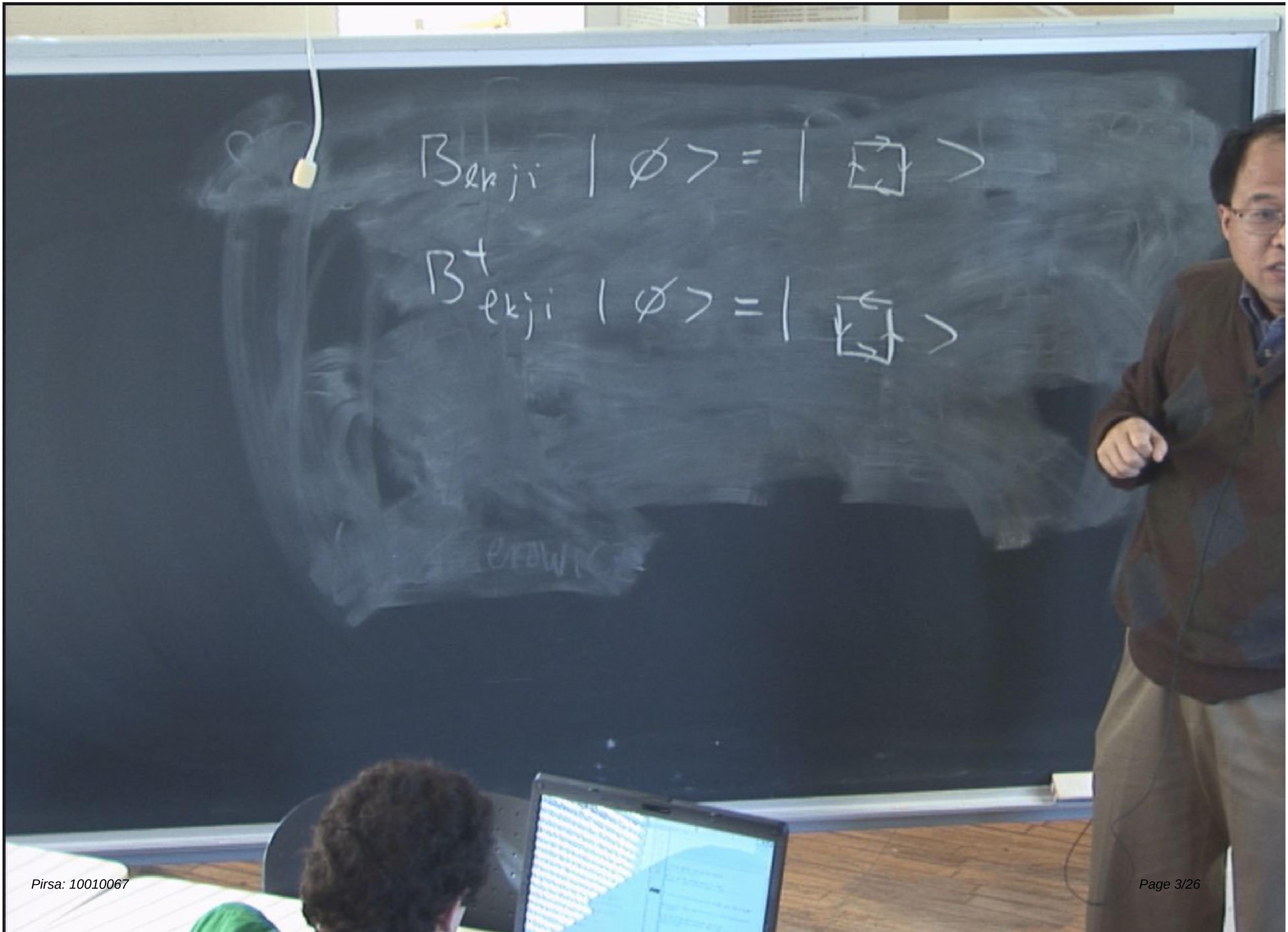
Abstract:

String-net condensation and new states of matter

Michael Levin and Xiao-Gang Wen

http://dao.mit.edu/~wen

Artificial light and quantum orders ..., PRB **68** 115413 (2003)
Fermions, strings, and gauge fields ..., PRB **67** 245316 (2003)
Strings-net condensation ..., cond-mat/0404001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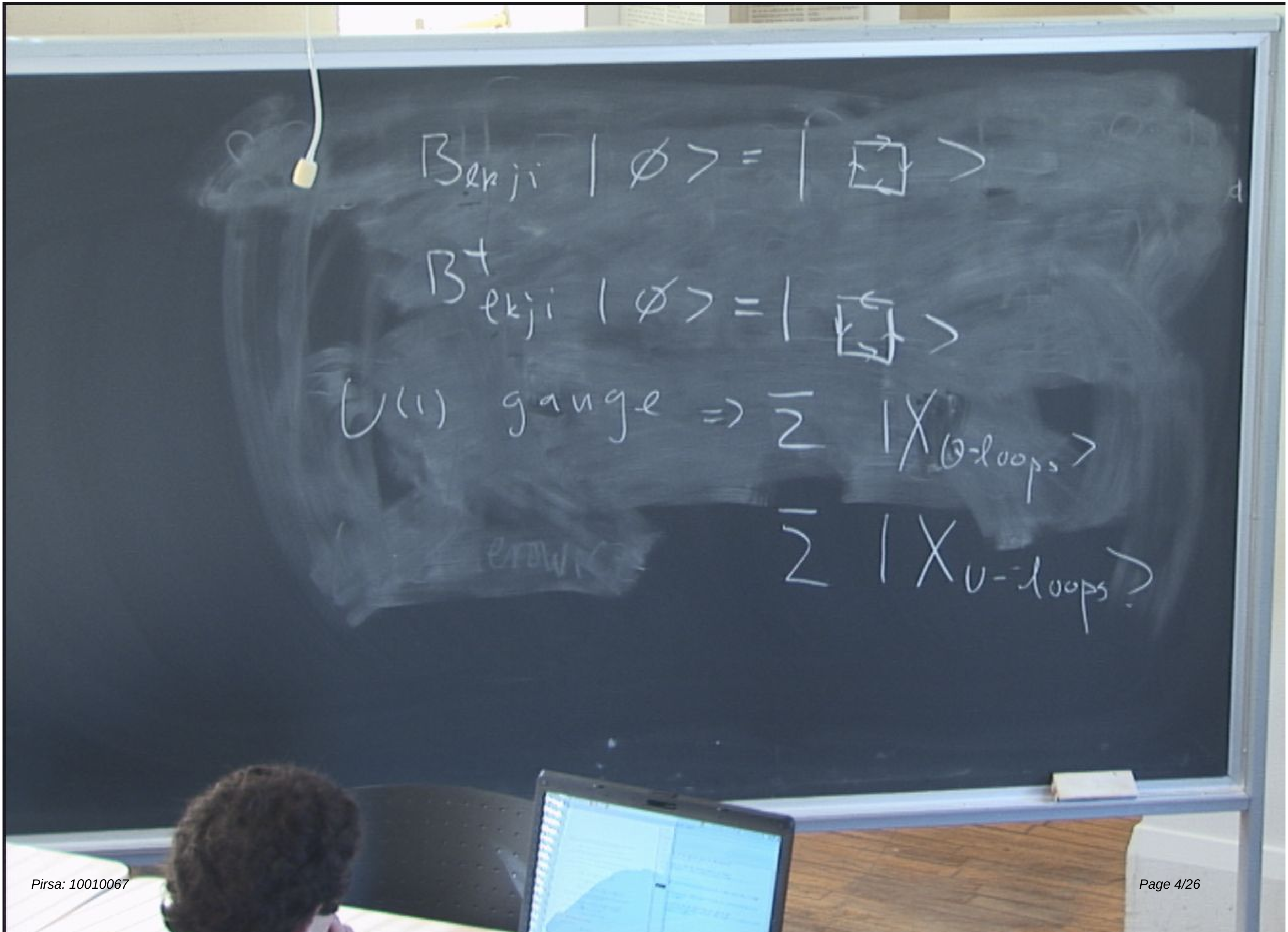
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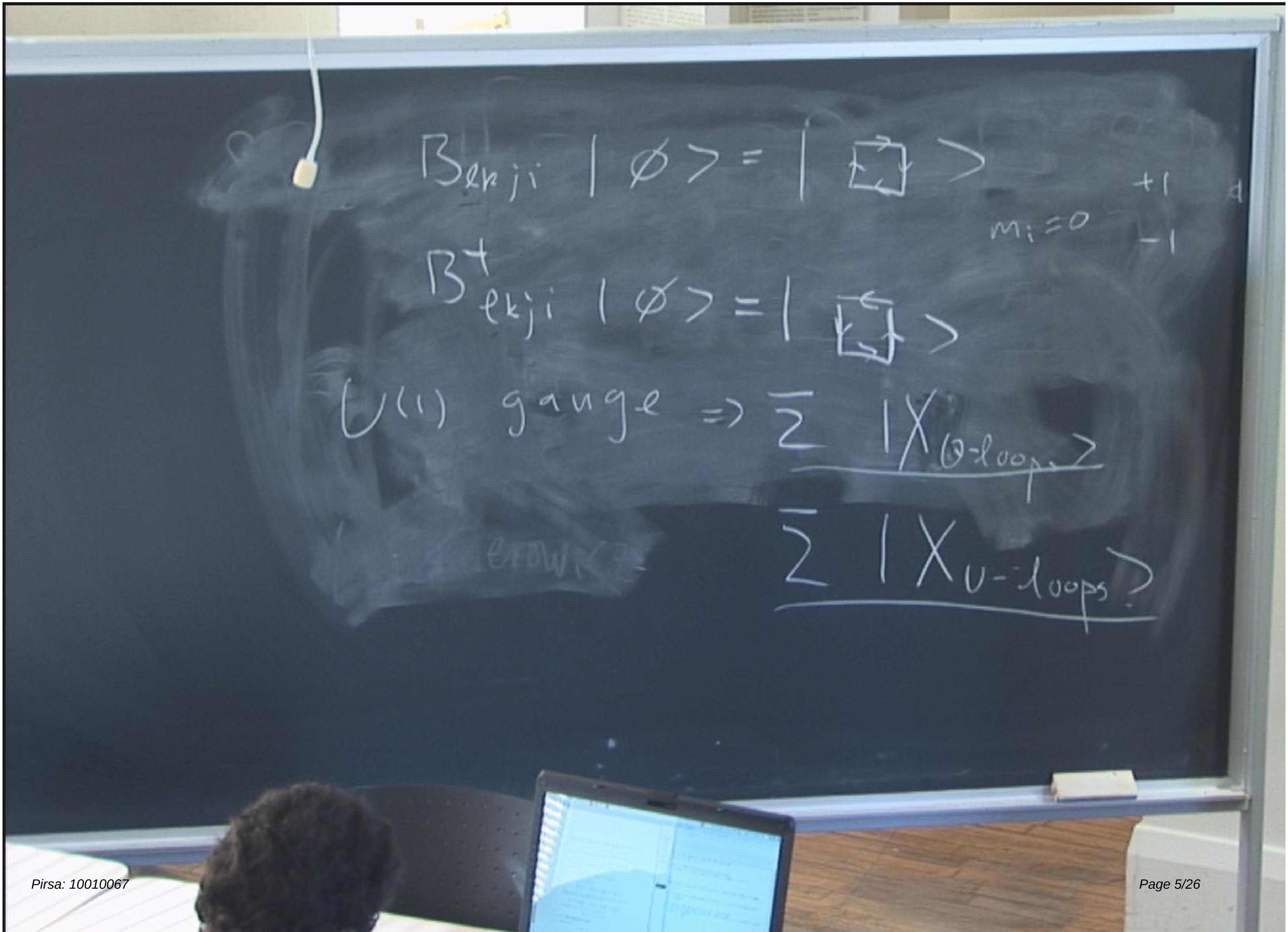
U(1) gauge $\Rightarrow \sum | X_{(O\text{-loops})} \rangle$
$\sum | X_{U\text{-loops}} \rangle$?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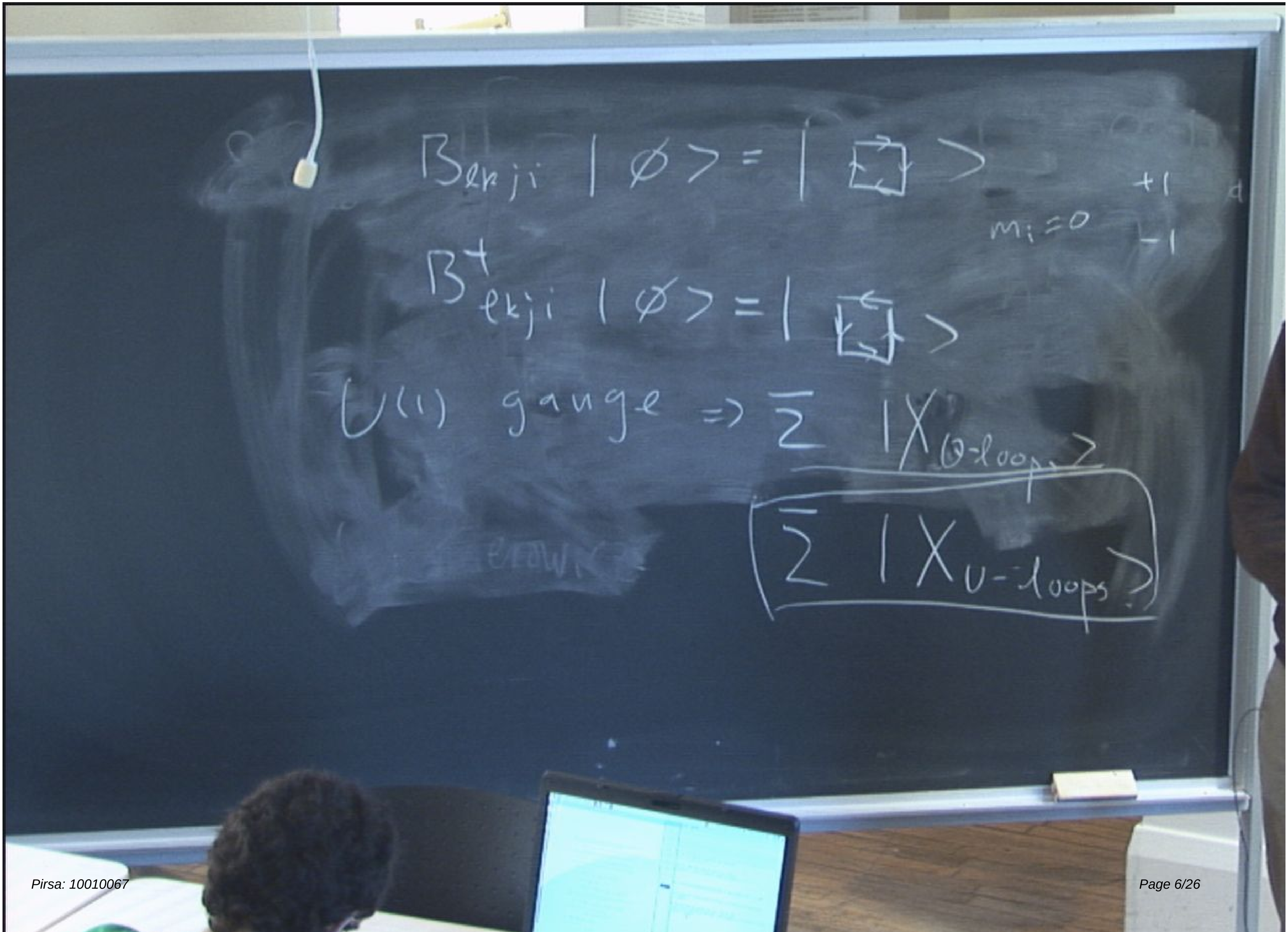
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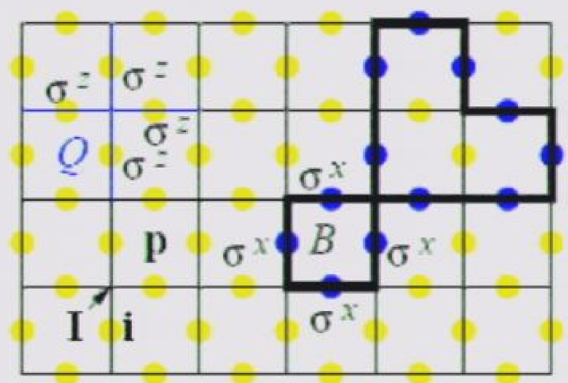
An exactly soluble Z_2 gauge theory Kitaev 03

$$H_{Z_2} = -U\sum_{\mathbf{I}} Q_{\mathbf{I}} - g\sum_{\mathbf{p}} B_{\mathbf{p}} + J\sum_{\mathbf{i}} \sigma^z_{\mathbf{i}}, \quad B_{\mathbf{p}} \equiv \prod_{\text{edges of } \mathbf{p}} \sigma^x_{\mathbf{i}}, \quad Q_{\mathbf{I}} \equiv \prod_{\text{legs of } \mathbf{I}}$$



- Down spin state = state with no string. String = line of up-spins
- U-term → closed strings as low energy states
- g-term → string fluctuations
- J-term → string tension. We will set $J = 0$ from now on.
- $[Q_{\mathbf{I}}, B_{\mathbf{p}}] = 0$ → exact eigenstates = common eigenstates of $Q_{\mathbf{I}} = \pm 1$ and $B_{\mathbf{p}} = \pm 1$. Energy = $-U\sum_{\mathbf{I}} q_{\mathbf{I}} - g\sum_{\mathbf{p}} b_{\mathbf{p}}$
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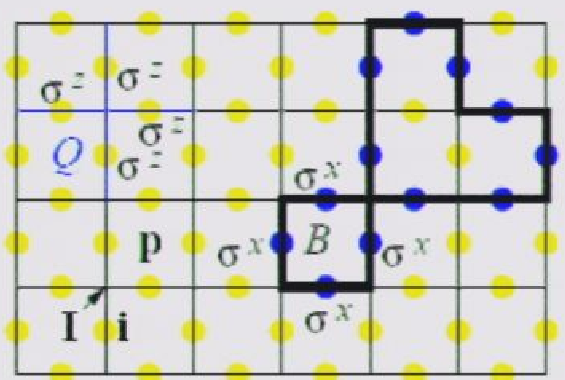

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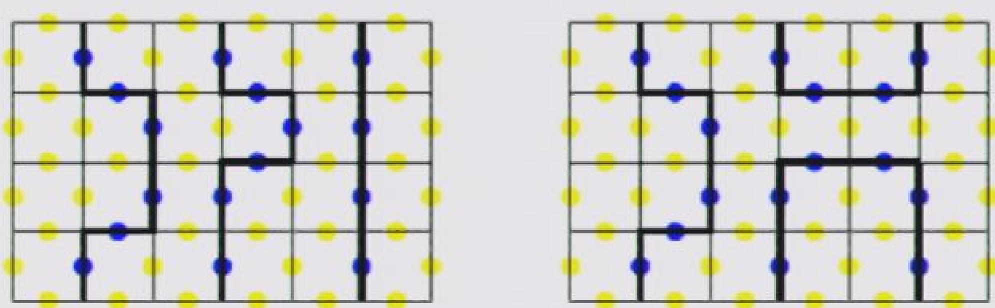
Properties of H_{Z_2}

- The ground state of H_{Z_2} has four fold topological degeneracy on torus, characterized by even/odd number of loops which go all the way around the torus in x- or y-direction.



- On genus g surface $\rightarrow$ 4^g degenerate ground states.
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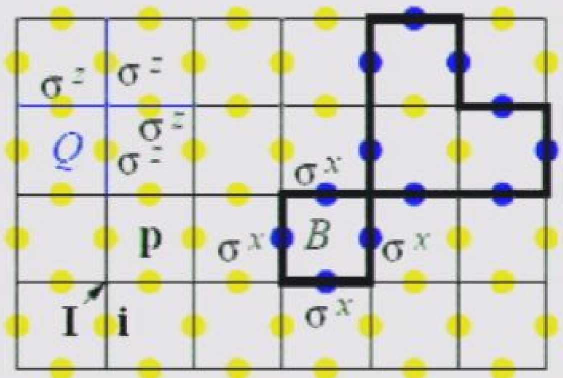

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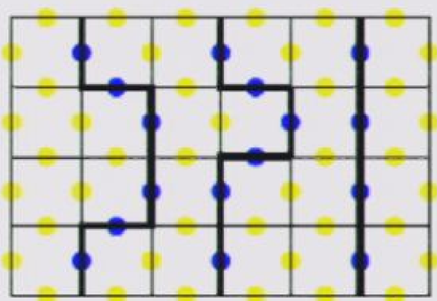

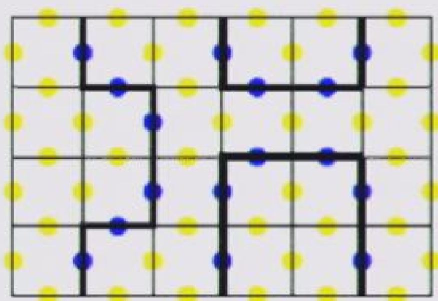

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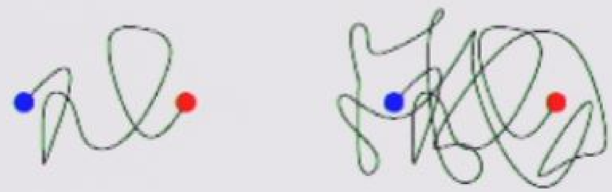
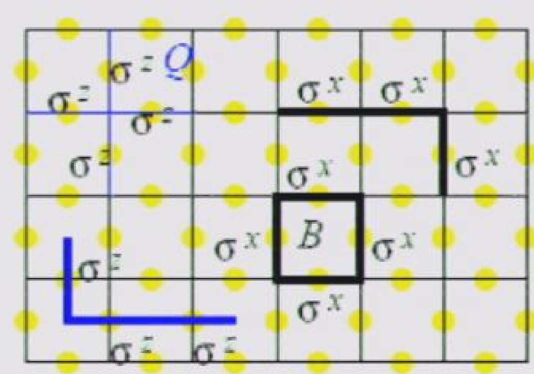
Two kinds of quasiparticles

- Strings are unobservable in string condensed state. Ends of strings behave like independent particles.



- Ground state $\Phi(X) = 1$, Vortex state $\Phi_v(X) = (-)^{W_{\mathbf{x}}(X)}$
  $W_{\mathbf{x}}(X)$ number of closed strings that wind around $\mathbf{x}$.

- Two kind of open-string operators create two kinds of particles-like excitations at their ends:



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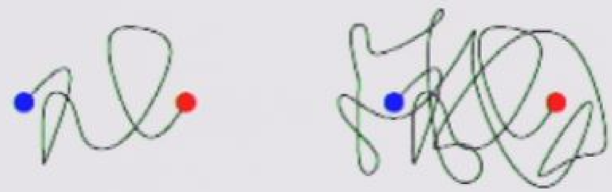

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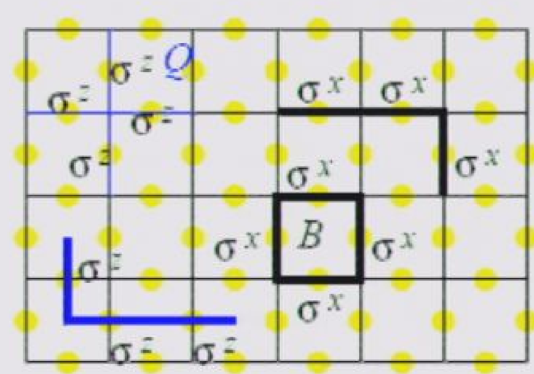

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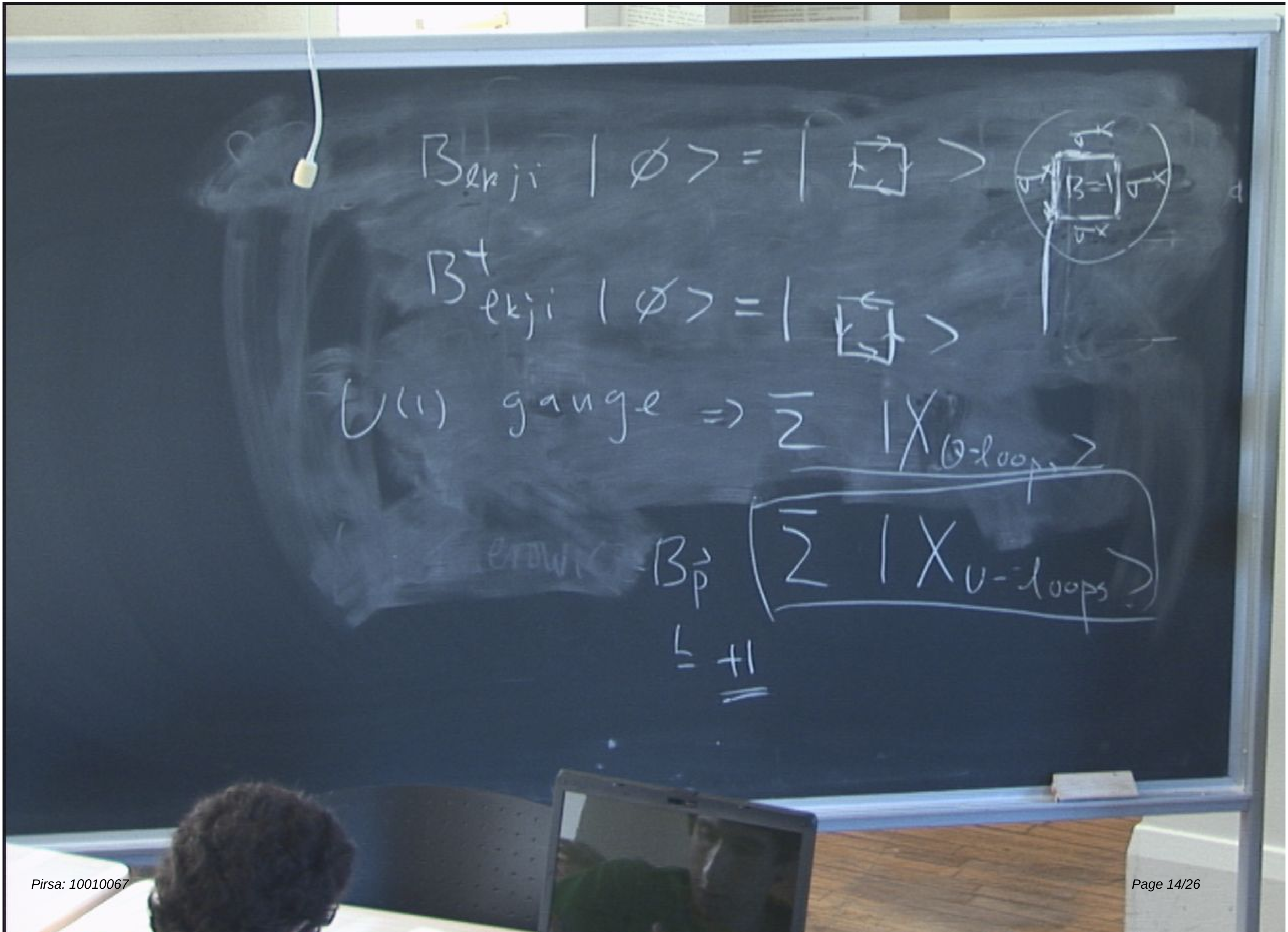
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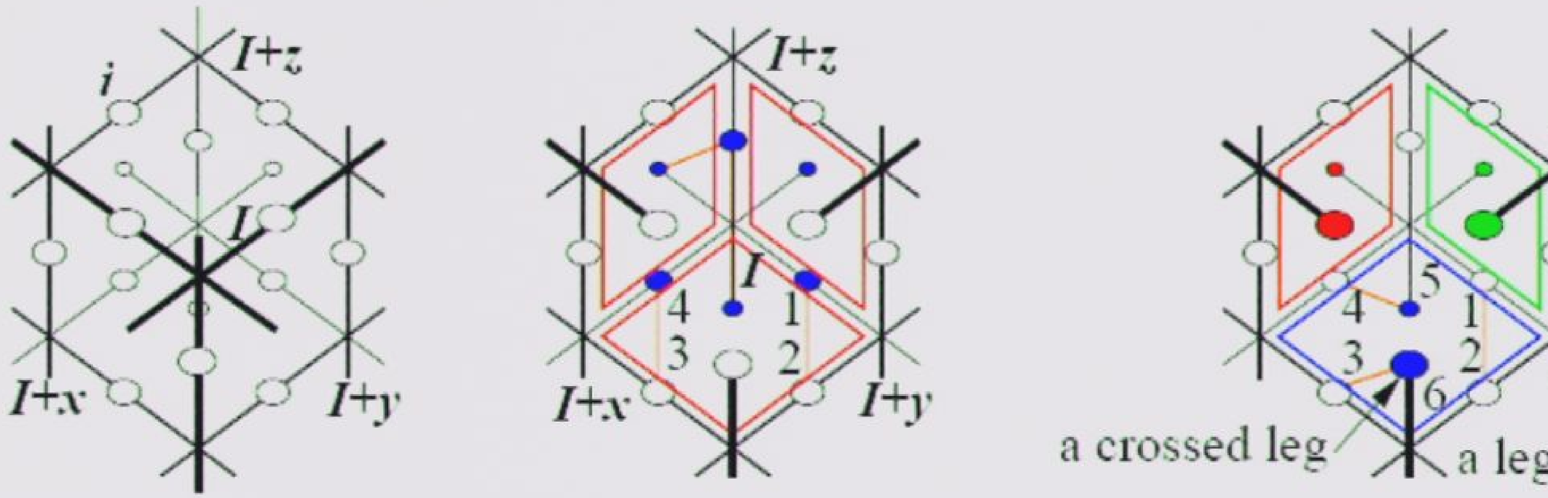
3D topological order on Cubic lattice



- Untwisted-string model: $H = -U\sum_{\mathbf{I}} Q_{\mathbf{I}} - g\sum_{\mathbf{p}} B_{\mathbf{p}}$

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Can get 3D fermions for free (almost) Levin & Wen

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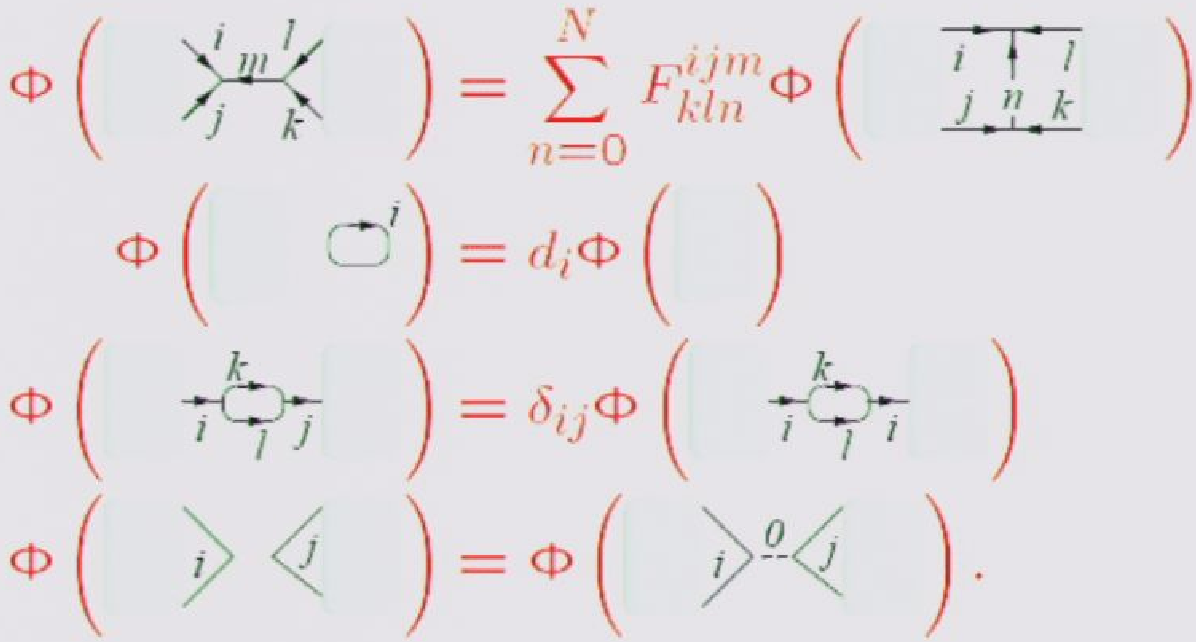
- Rebranching relation and 6j-symbol:



Topological string-net condensation is described by a set of data $(N, \delta_{ijk}, F^{ijm}_{kln})$

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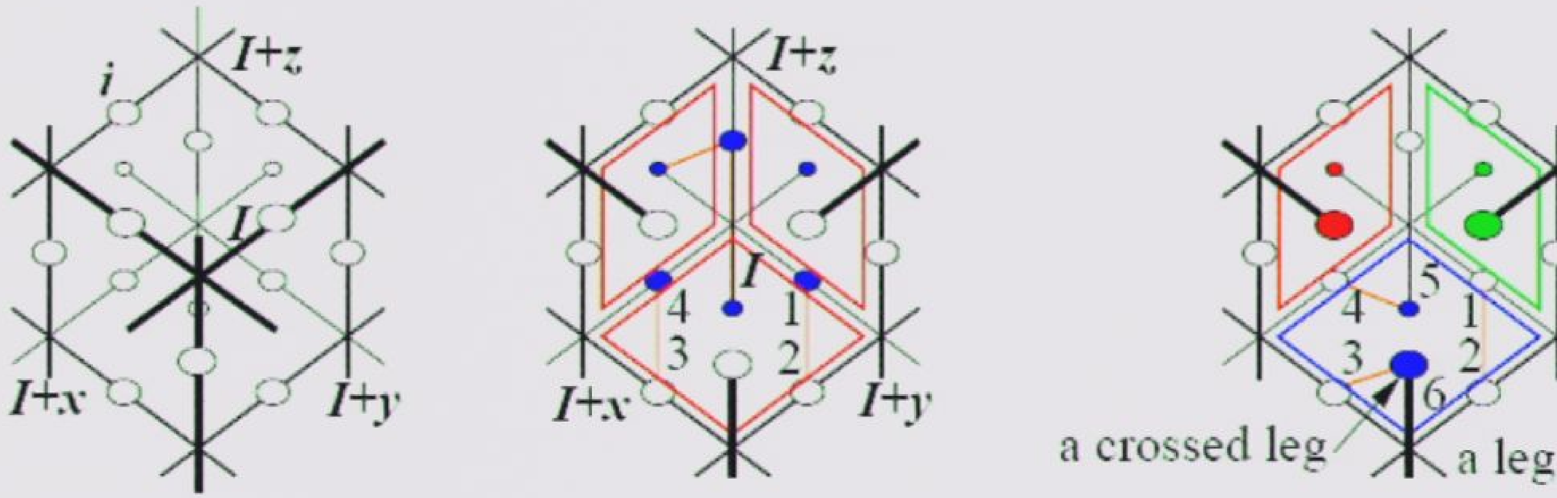

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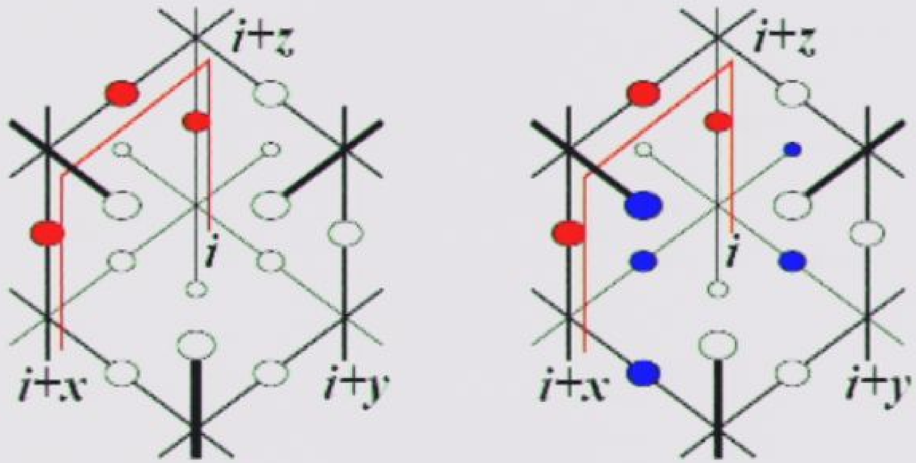
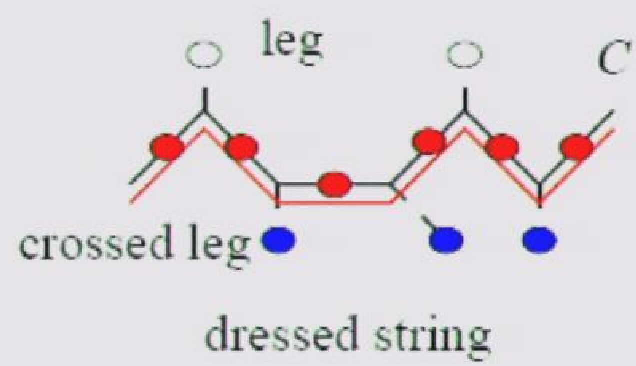
String operators and artificial charges Levin & Wen

- A pair of artificial charges is created by an open string operator which commute with the Hamiltonian except at its two ends. Strings cost no energy and is unobservable.





- In untwisted-string model – untwisted-string operator

$$\sigma^x_{i_1}\sigma^x_{i_2}\sigma^x_{i_3}\sigma^x_{i_4}\cdots$$

- In twisted-string model – twisted-string operator

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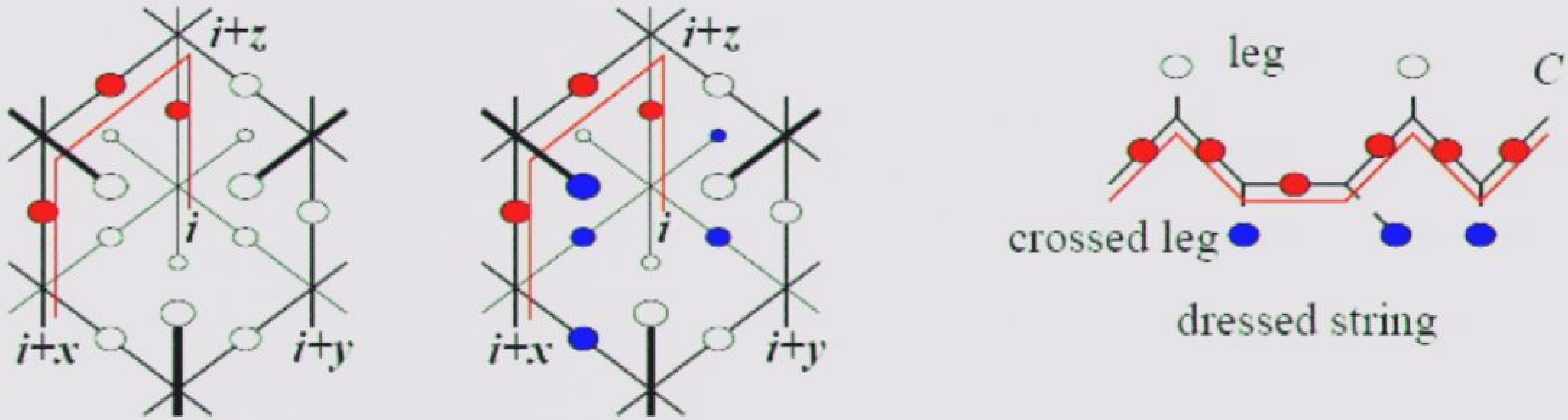

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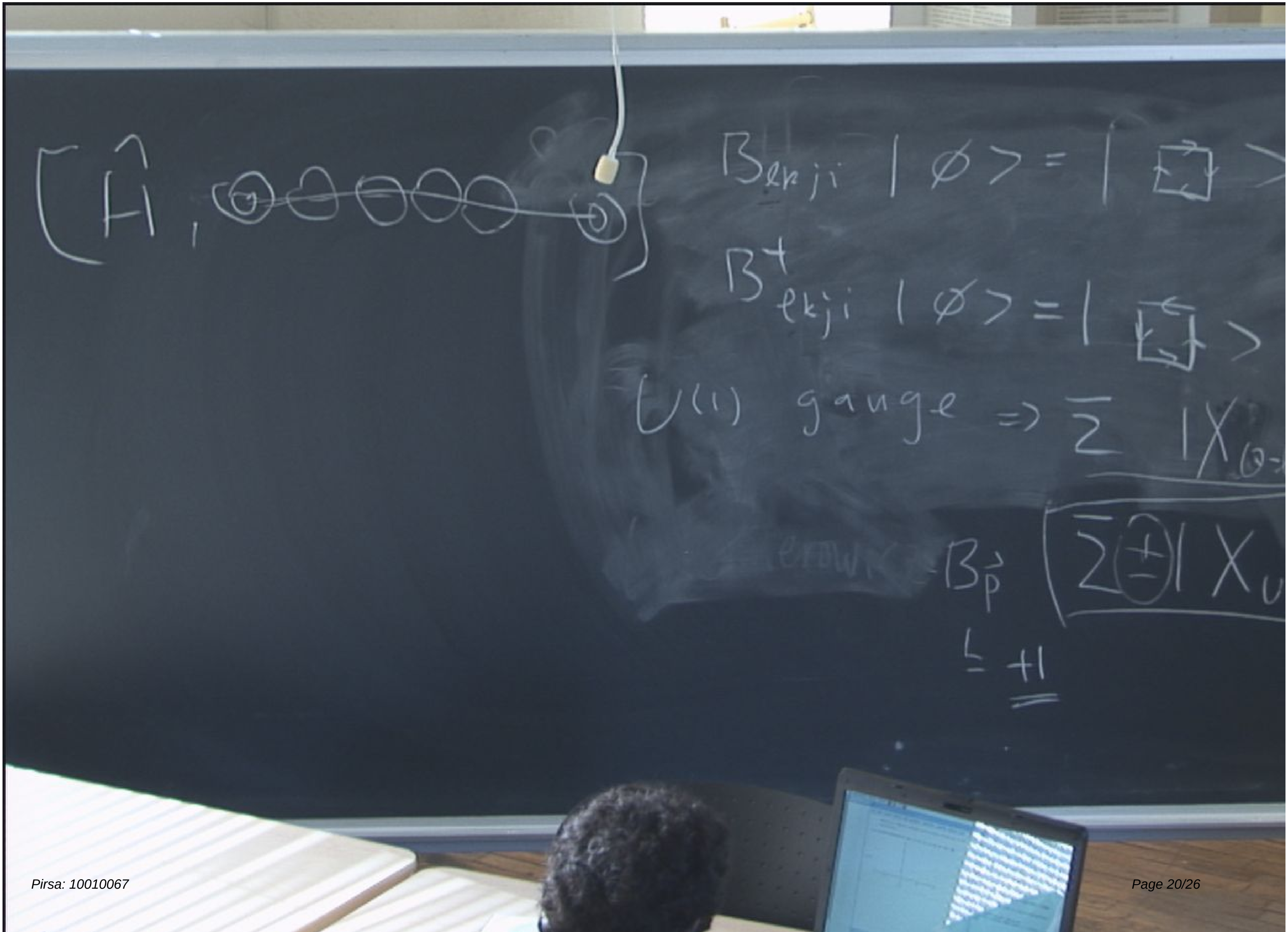
[Ĥ ,
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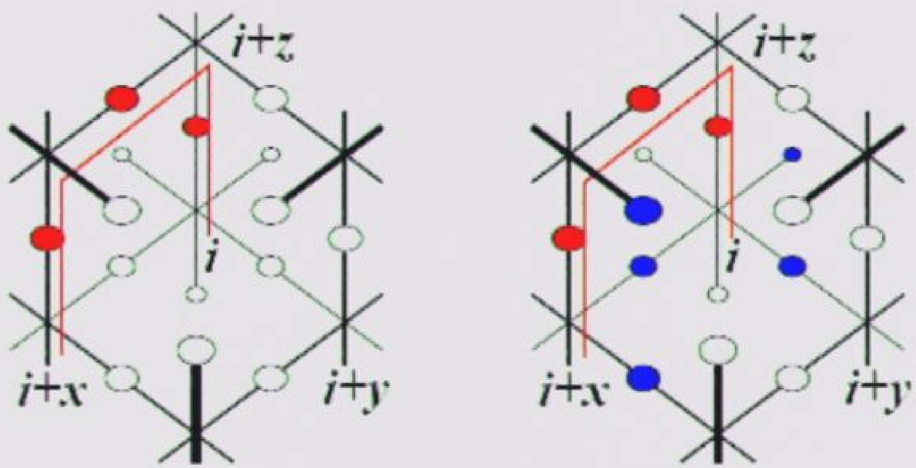

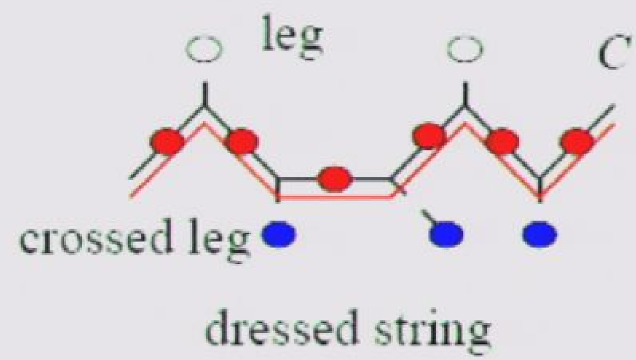

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Statistics of ends of strings

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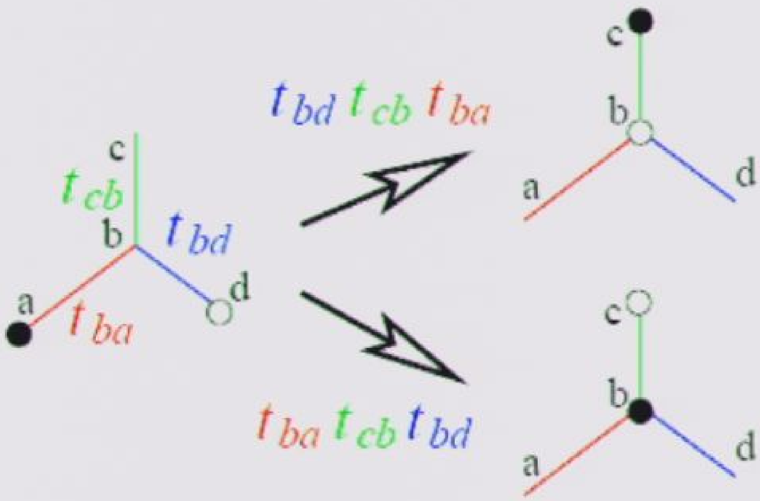

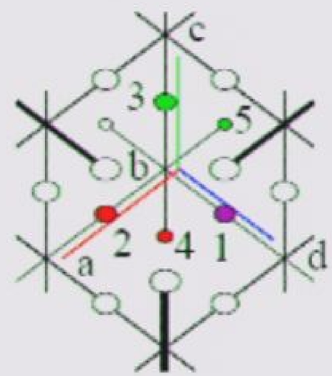

- An open string operator is a hopping operator of the artificial charges. Open string operator determine the statistics.
- For untwisted-string model: $t_{ba} = \sigma_2^x$, $t_{cb} = \sigma_3^x$, $t_{bd} = \sigma_1^x$
  We find $t_{bd}t_{cb}t_{ba} = t_{ba}t_{cb}t_{bd}$
  The ends of untwisted-string are bosons
- For twisted-string model: $t_{ba} = \sigma_4^z\sigma_1^z\sigma_2^x$, $t_{cb} = \sigma_5^z\sigma_3^x$, $t_{bd} = \sigma_1^x$
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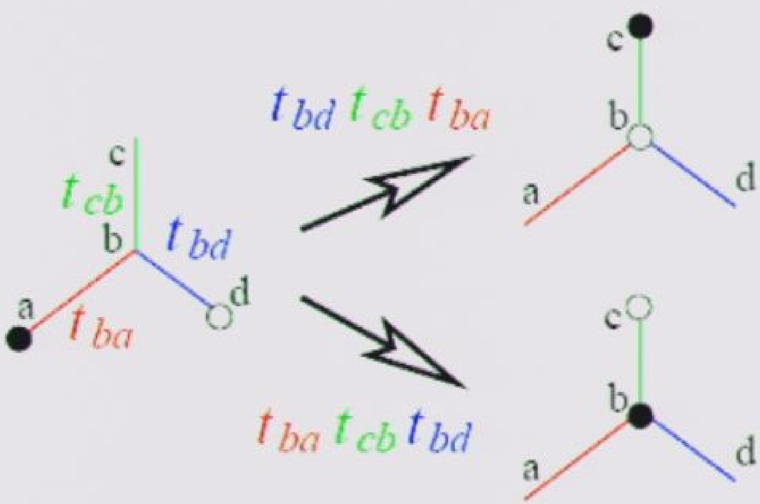

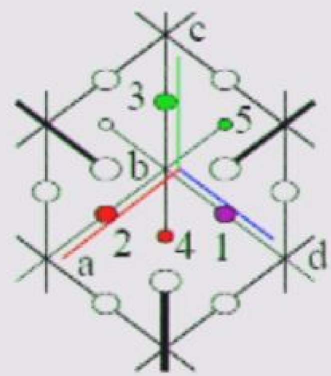

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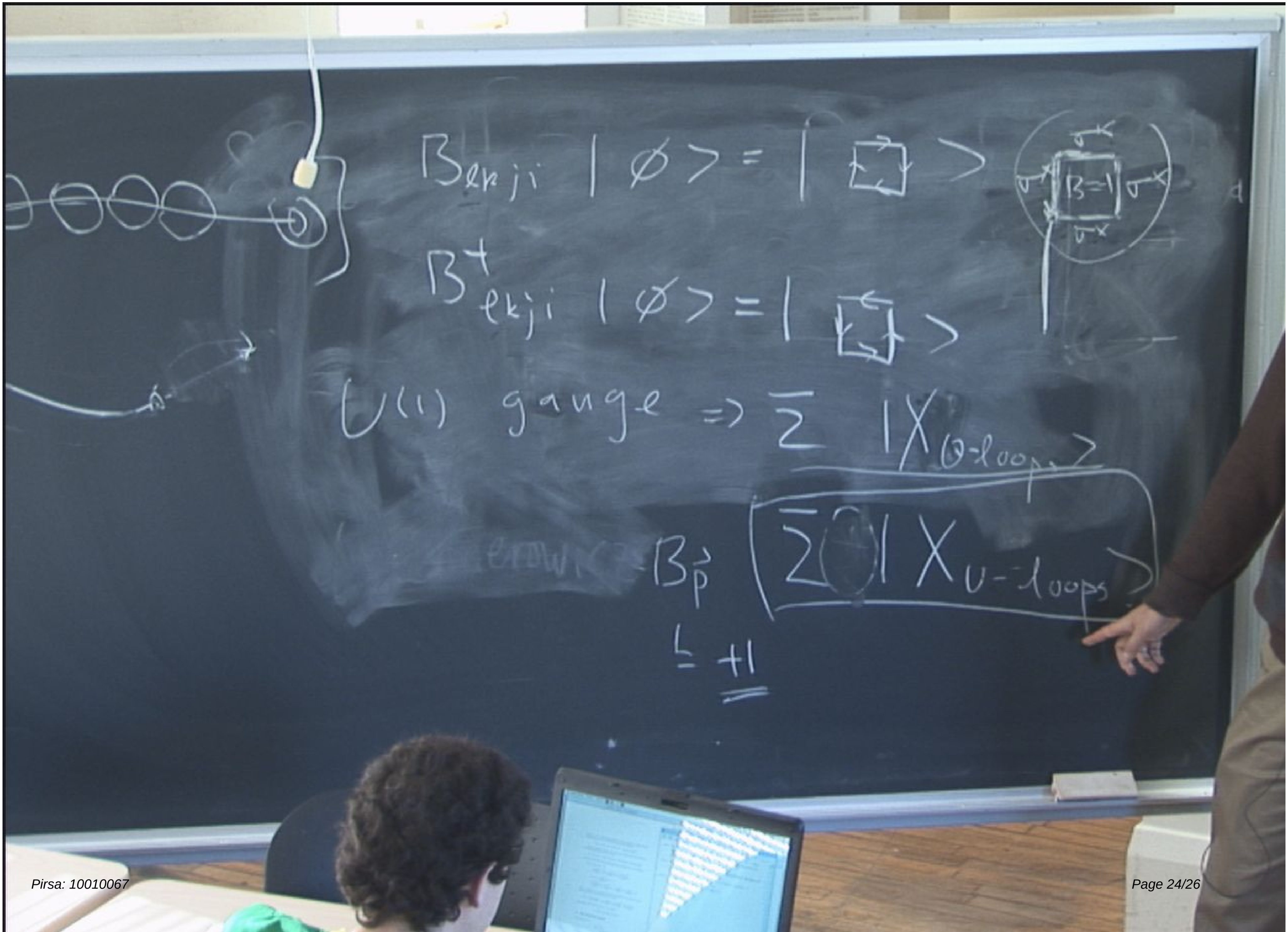
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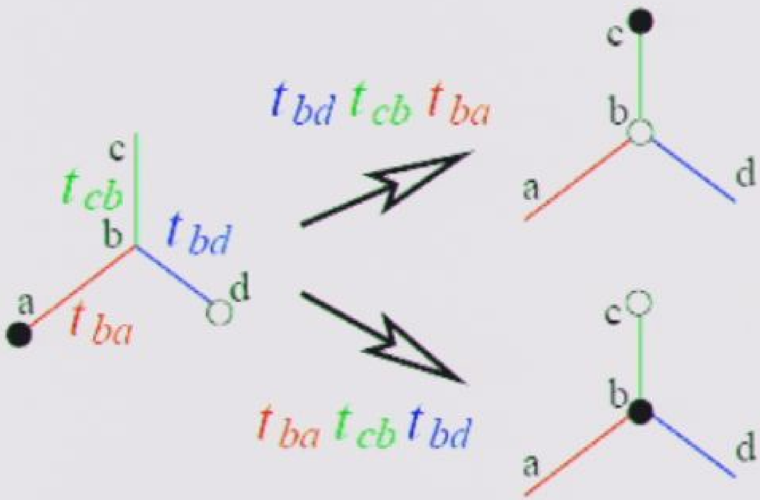

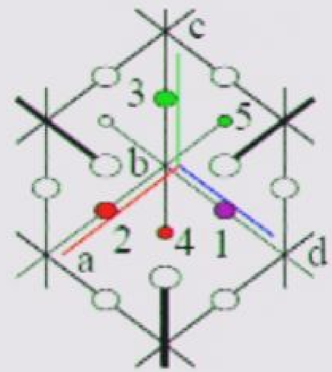

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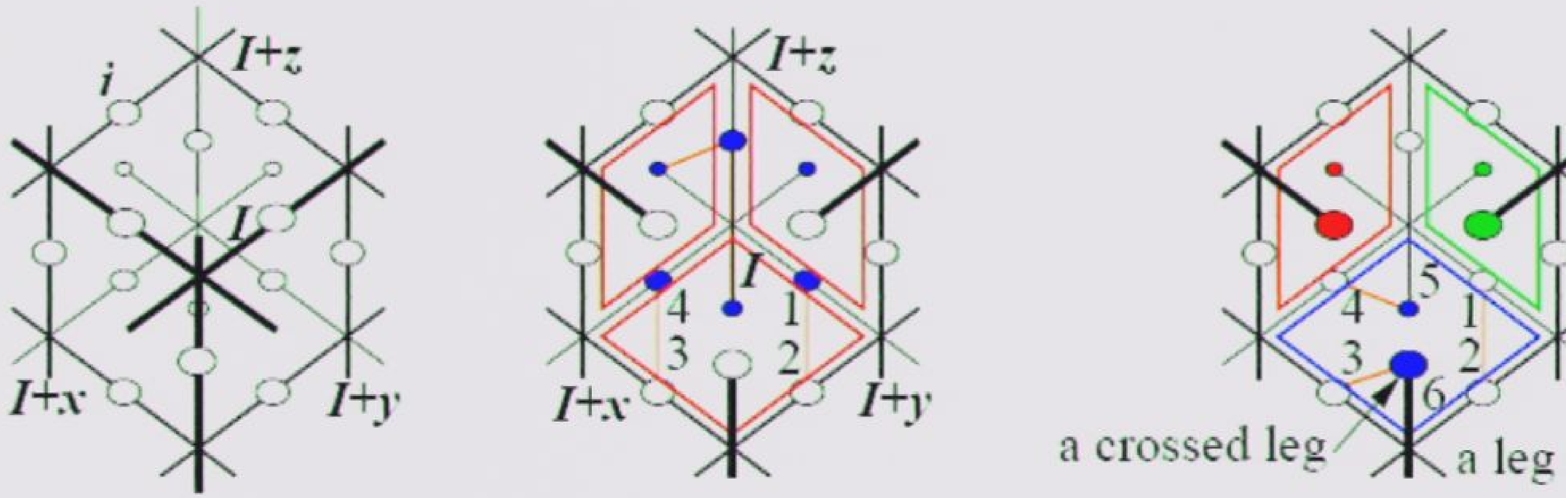
3D topological order on Cubic lattice



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