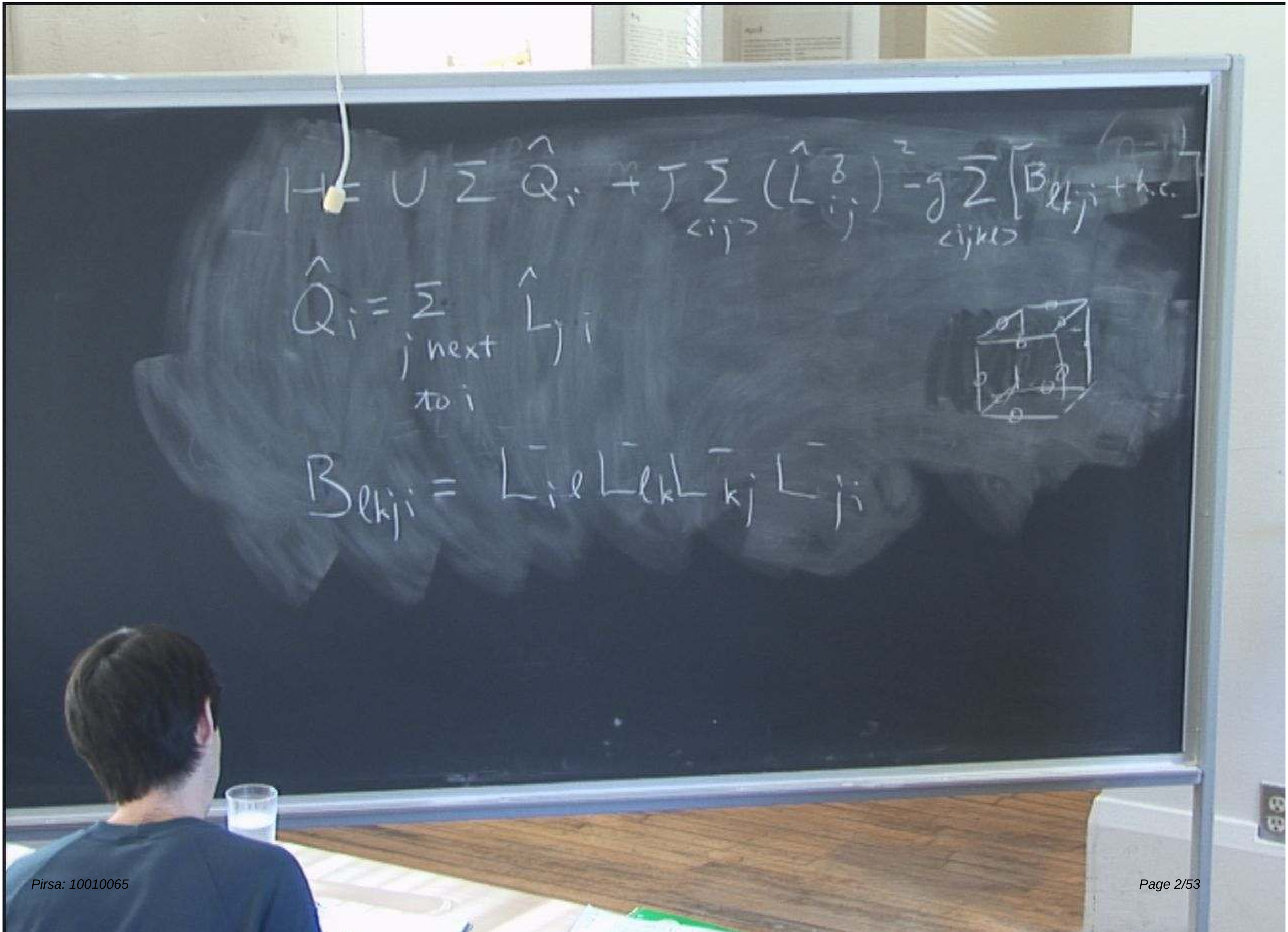
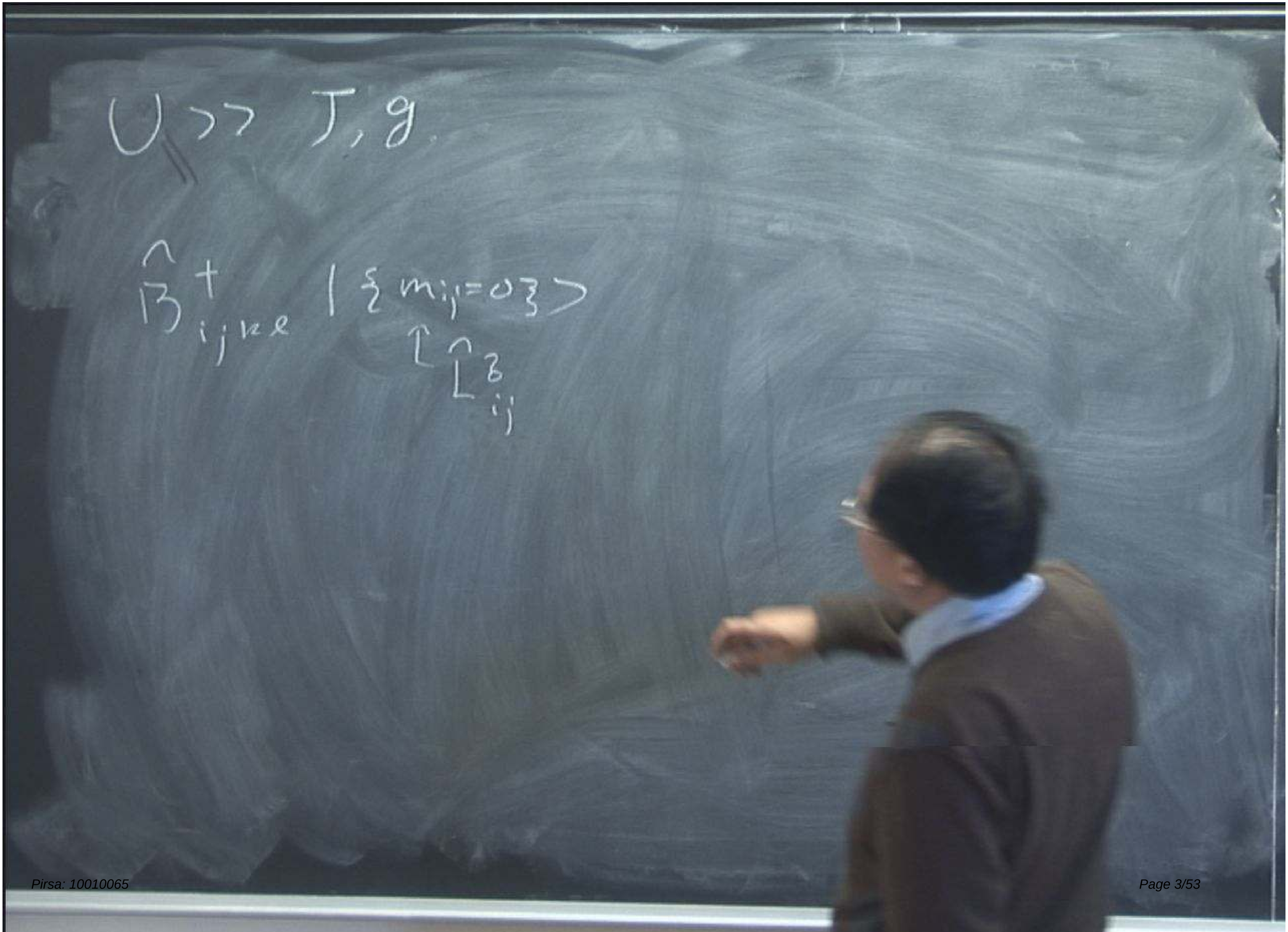
Title: Condensed Matter Review (PHYS 637) - Lecture 13

Date: Jan 20, 2010 11:00 AM

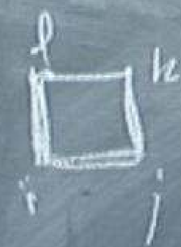
URL: http://pirsa.org/10010065

Abstract:


$H = U \sum \hat{Q}_i + J \sum_{\langle ij \rangle} (\hat{L}^z_{ij})^2 - g \sum_{\langle ijkl \rangle} [B_{lkji} + h.c.]$
$\hat{Q}_i = \sum_{j \text{ next to } i} \hat{L}_{ji}$
$B_{lkji} = \bar{L}_{il} \bar{L}_{lk} \bar{L}_{kj} \bar{L}_{ji}$

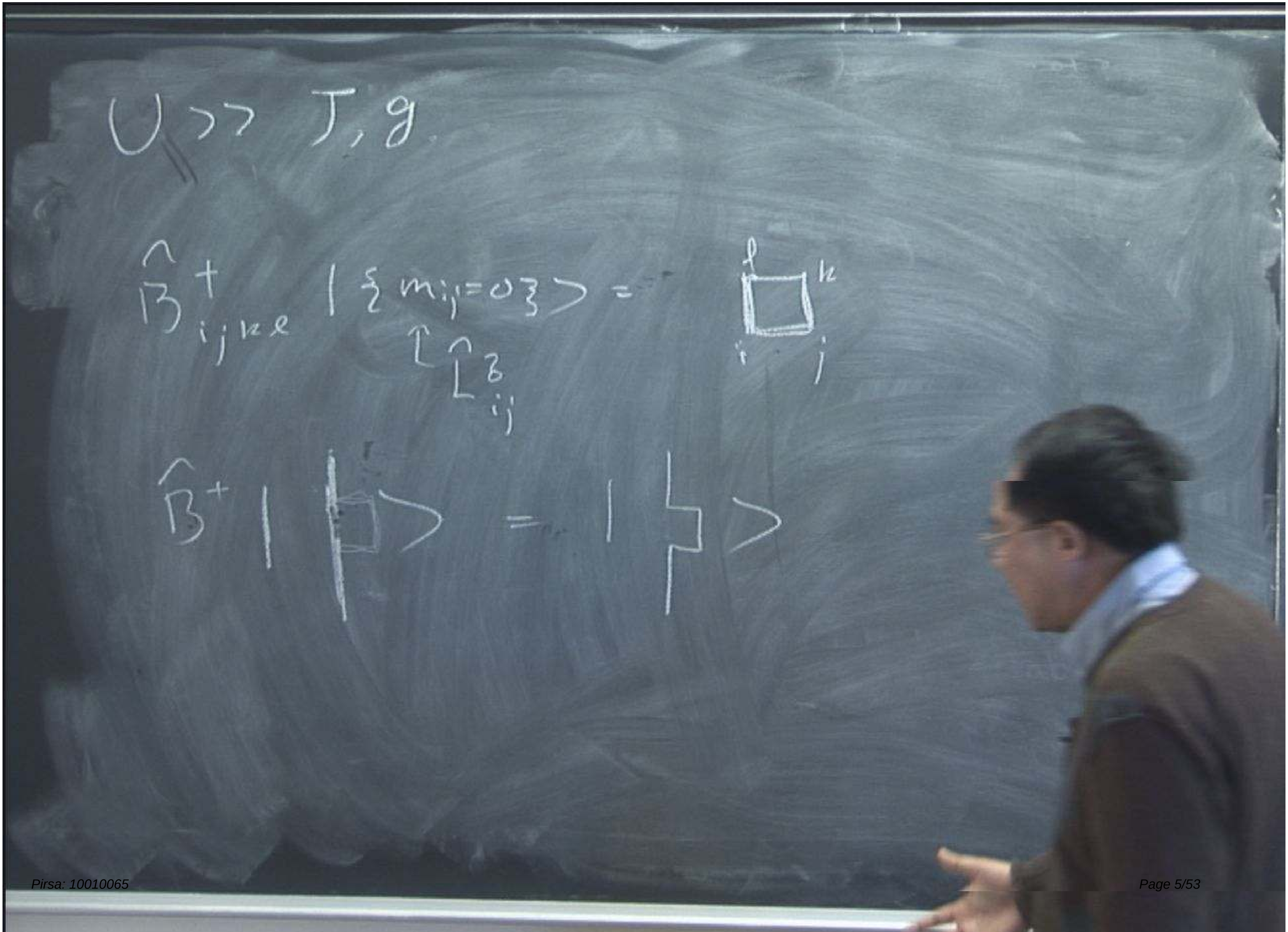

U >> J, g.
B̂†ijkl | {mij = 0} >
L̂zij

$U \gg J, g$

$\hat{B}^\dagger_{ijkl} \, | \{ m_{ij} = 0 \} \rangle =$ 

$\hat{L} \quad \hat{L}^z_{ij}$

$\hat{B}^+ \, |$  $\rangle = \, |$  $\rangle$


U >> J, g
B̂†ijkl |{mij = 0}> =
l k i j
L̂zij
B̂† | > = | >

$U \gg J, g.$

$g \gg J$:

$J = 0 \quad |\text{ground}\rangle = \sum_{X_{\text{loops}}} |X_{\text{loops}}\rangle$

$$U \gg J, g.$$

$$g \gg J \; :$$

$$J = 0 \quad |\text{ground}\rangle = \sum_{X_{loops}} |X_{loops}\rangle$$

$$[\hat{Q}_i , \hat{B}_{ijkl}] = 0.$$

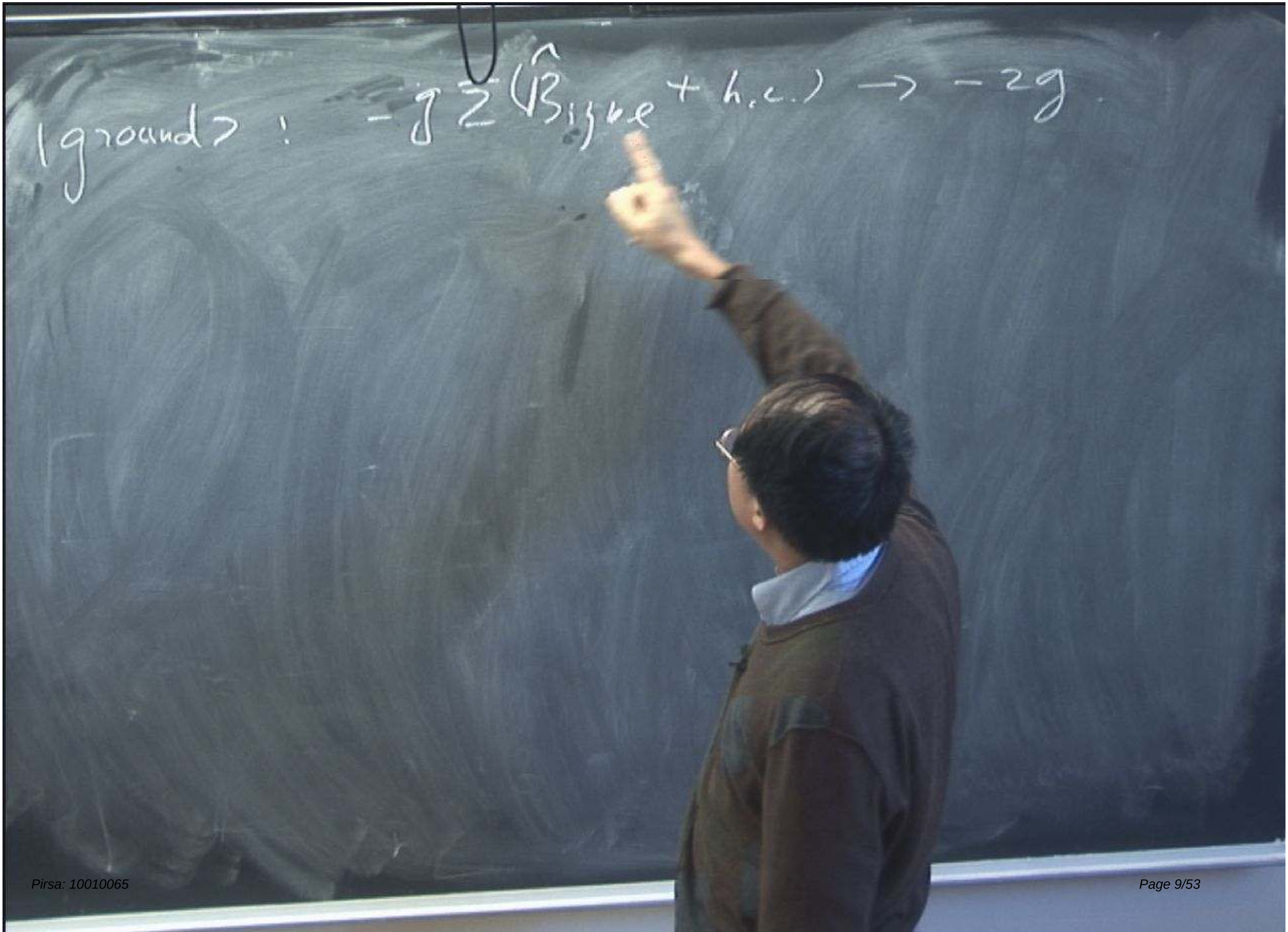
$U \gg J, g$

$g \gg J$:

$J = 0 \quad |\text{ground}\rangle = \sum_{X_{loops}} |X_{loops}\rangle$

$[\hat{Q}_i, \hat{B}_{ijkl}] = 0$

$\hat{B}_{ijkl} \sum_{X_{loops}} |X_{loops}\rangle = \sum_{X_{loops}} |X_{loops}\rangle$

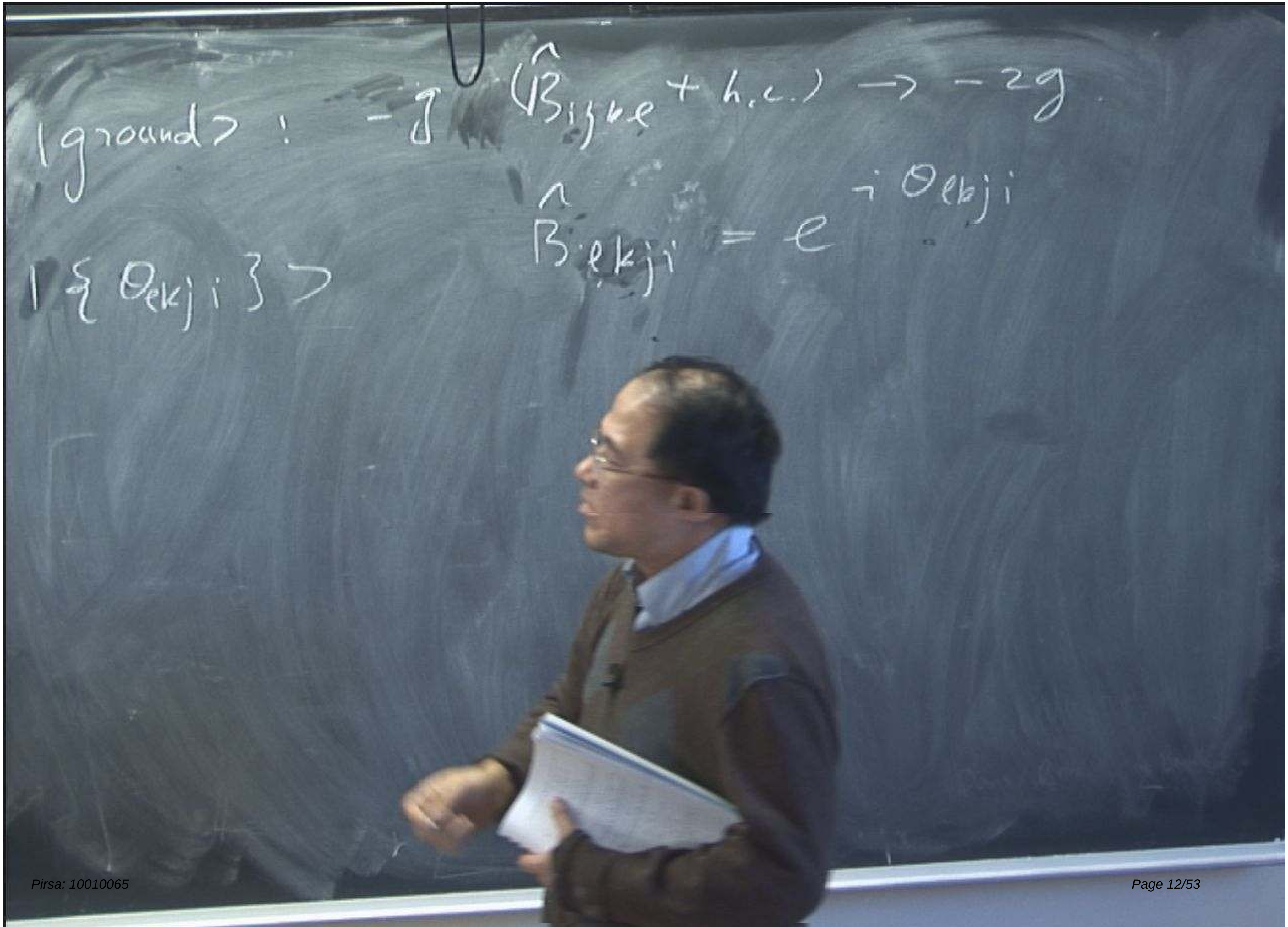

|ground> : $-g \sum (\hat{B}_{ijke} + h.c.) \to -2g$

$|\text{ground}\rangle$: $-g\,(\hat{B}_{ijke} + h.c.) \to -2g$

$|\text{ground}\rangle : \quad -g\,(\hat{B}_{ijke} + h.c.) \longrightarrow -2g$

$\hat{B}_{\ell kji} = e^{i\theta_{\ell kji}}$

|


$|\text{ground}\rangle$: $-g\,(\hat{B}_{ijkl} + h.c.) \to -2g$
$|\{\theta_{lkji}\}\rangle$
$\hat{B}_{lkji} = e^{i\theta_{lkji}}$

$|\text{ground}\rangle$: $-g\,(\hat{B}_{ijkl} + h.c.) \longrightarrow -2g$

$\hat{B}_{lkji} = e^{-i\theta_{lkji}}$

$|\{\theta_{lkji}\}\rangle$

$\hookrightarrow E = -2g \sum_{\langle ijkl\rangle} \cos(\theta_{lkji})$

$|\text{ground}\rangle$: $-g\,(\hat{B}_{ijkl} + h.c.) \rightarrow -2g$

$|\{\theta_{lkji}\}\rangle$

$\hat{B}_{lkji} = e^{-i\theta_{lkji}}$

$\hookrightarrow E = -2g \sum_{\langle ijkl\rangle} \cos(\theta_{lkji})$

$|\text{ground}\rangle$: $-g\,(\hat{B}_{ijkl} + h.c.) \rightarrow -2g$

$|\{\theta_{lkji}\}\rangle$

$\hat{B}_{lkji} = e^{-i\theta_{lkji}}$

$$\hookrightarrow E = -2g \sum_{\langle ijkl\rangle} \cos(\theta_{lkji})$$

$$\Delta E = 2g\,(1 - \cos(\underline{\theta_{ijkl}}))$$

ground> : $-g\,(\hat{B}_{ijkl} + h.c.) \to -2g$

$\{\theta_{lkji}\}\rangle$

$$\hat{B}_{lkji} = e^{-i\theta_{lkji}}$$

$$\to E = -2g \sum_{\langle ijkl \rangle} \cos(\theta_{lkji})$$

$$\Delta E = 2g\,(1-\cos(\theta_{ijkl}))$$

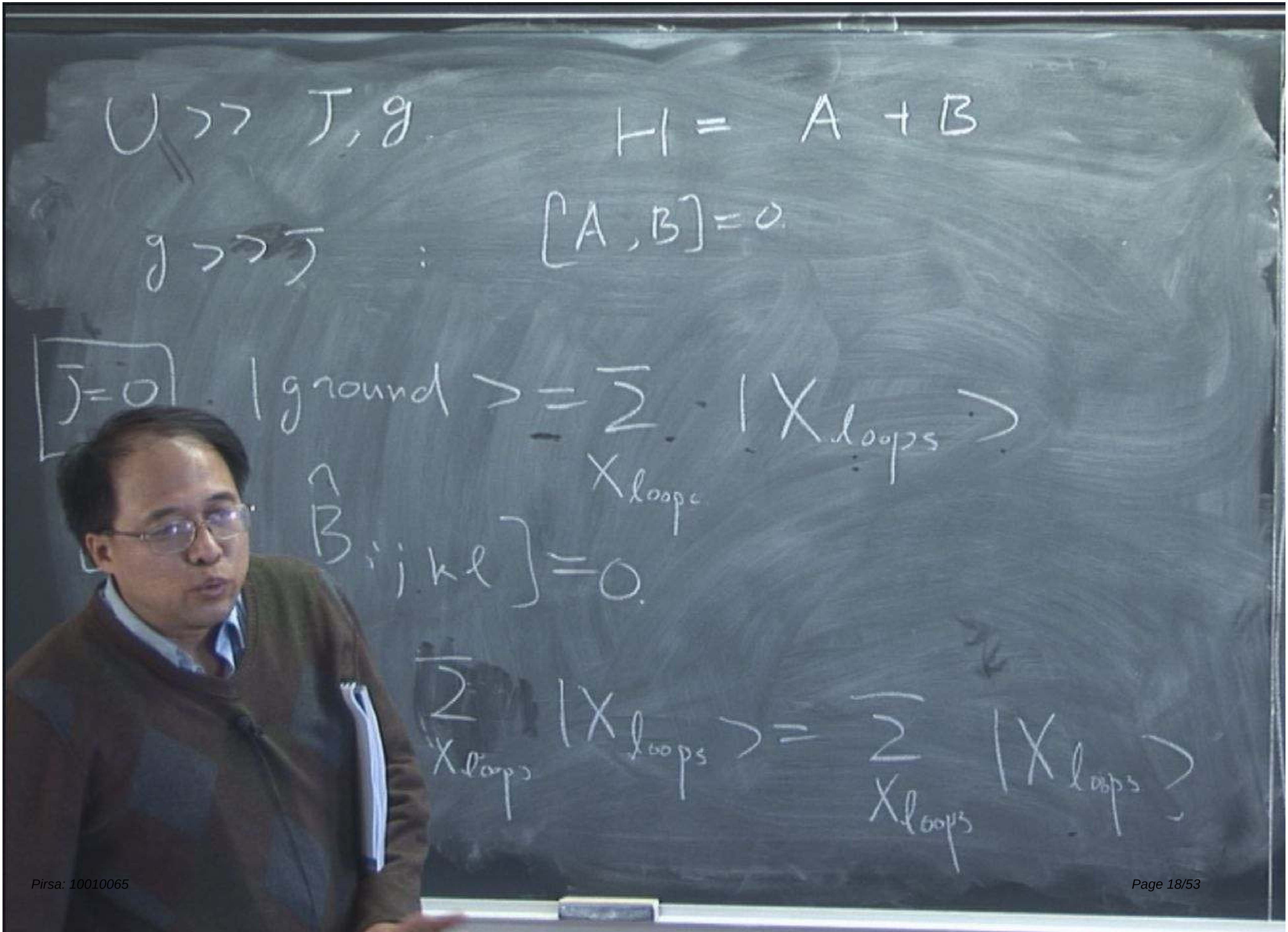
ground> : $-g\,(\hat{B}_{ijkl} + h.c.) \to -2g$

$\hat{B}_{lkji} = e^{i\theta_{lkji}}$

$\{\theta_{lkji}\}\rangle$

$\to E = -2g \sum_{\langle ijkl \rangle} \cos(\theta_{lkji})$

$\Delta E = 2g\,(1-\cos(\theta_{ijkl}))$


U >> J, g
H = A + B
[A, B] = 0
g >> J :
J = 0
|ground> = Σ |X_loops>
X_loops
B̂_ijkl] = 0
Σ |X_loops> = Σ |X_loops>
X_loops
X_loops

$U \gg J, g.$ $\qquad H = A + B$

$[A, B] = 0.$ $\qquad |a, b\rangle$

$g \gg J \; :$ $\qquad E(a, b)$

$$|\text{ground}\rangle = \sum_{X_{loops}} |X_{loops}\rangle$$

$$\hat{B}_{ijkl} \,] = 0.$$

$$\sum_{X_{loops}} |X_{loops}\rangle = \sum_{X_{loops}} |X_{loops}\rangle$$

$|\text{ground}\rangle$: $-g\,(\hat{B}_{ijkl}\,e + h.c.) \to -2g$

$\hat{B}_{lkji} = e^{-i\theta_{lkji}}$

if $J \neq 0$ $\quad \bar{J} \ll g$

$|\text{ground}\rangle$: $-g\,(\hat{B}_{ijkl} + h.c.) \longrightarrow -2g$

$$\hat{B}_{lkji} = e^{-i\theta_{lkji}}$$

if $J \neq 0$ $\quad J \ll g$

$|\{\theta_{ijkl}\}\rangle$

$|\text{ground}\rangle$: $\quad -g\,(\hat{B}_{ijkl} + h.c.) \to -2g$

$$\hat{B}_{lkji} = e^{-i\theta_{lkji}}$$

if $T \neq 0 \qquad T \ll g$

$|\{\theta_{ijkl}\}\rangle$

$|ground\rangle$: $-g\,(\hat{B}_{ijkl} + h.c.) \to -2g$

$$\hat{B}_{lkji} = e^{-i\theta_{lkji}}$$

if $T \neq 0$ $\quad T \ll g$

$|\{\theta_{ijkl}\}\rangle$

$|\text{ground}\rangle$: $-g\,(\hat{B}_{ijkl} + h.c.) \to -2g$

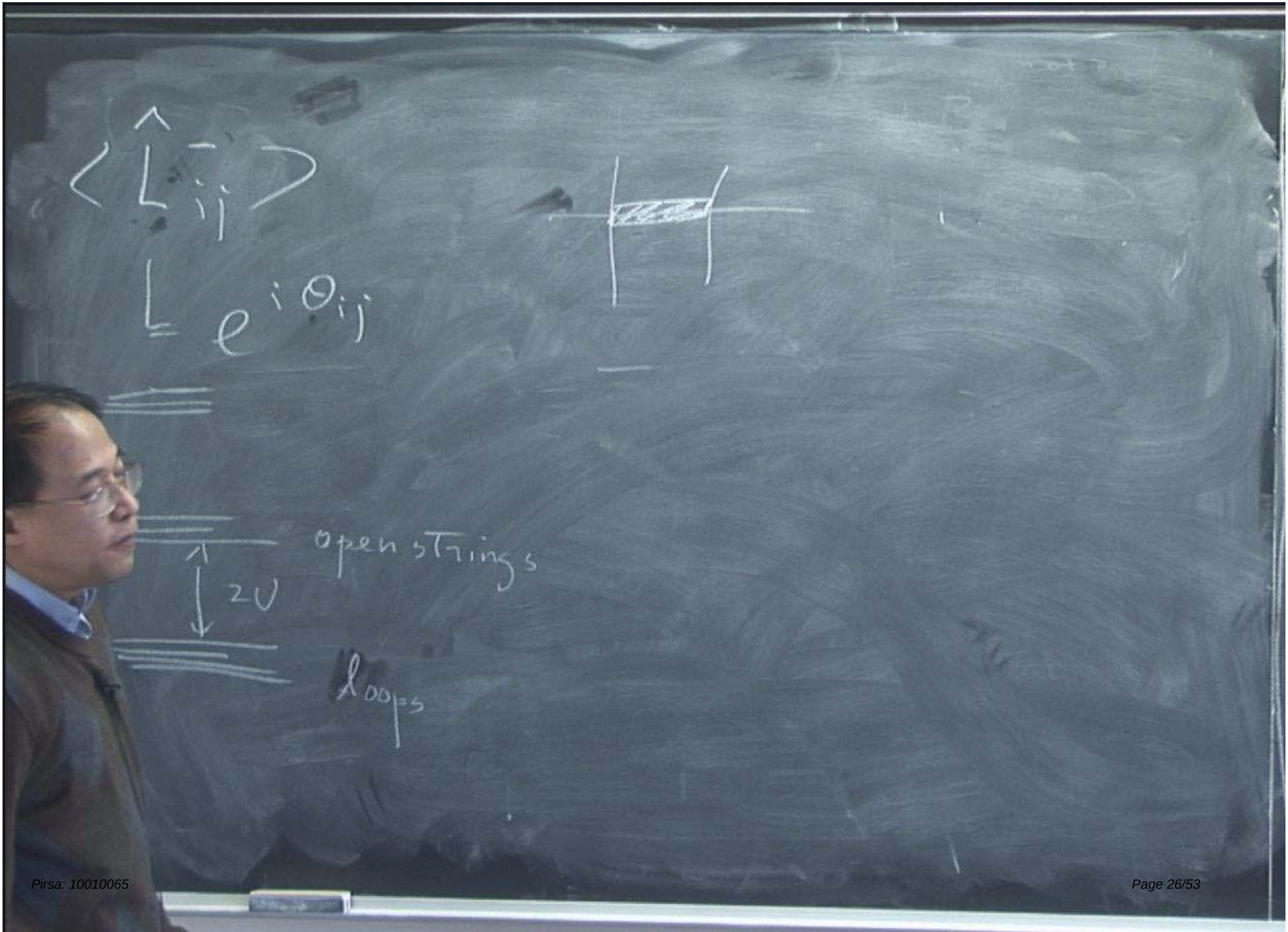
$$\hat{B}_{lkji} = e^{-i\theta_{lkji}}$$

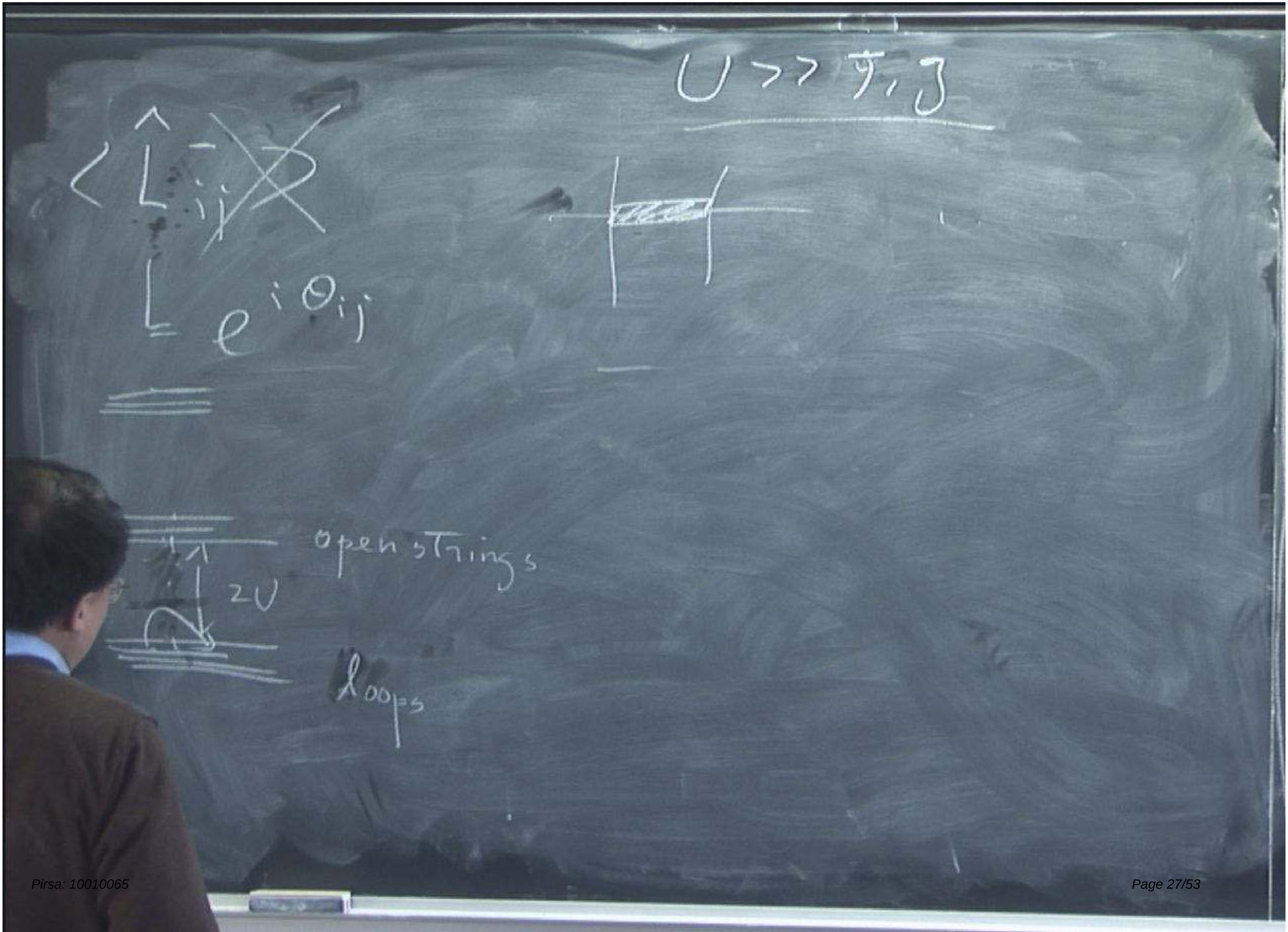
if $J \neq 0$ $\quad \boxed{J \ll g}$

$|\{\theta_{ijkl}\}\rangle$

$$\langle \hat{L}_{ij} \rangle$$

$$L\, e^{i\theta_{ij}}$$


$\langle \hat{L}_{ij} \rangle$
$L \; e^{i\theta_{ij}}$
open strings
2U
loops


$U >> t, J$
$\langle \hat{L}_{ij} \rangle$
$e^{i\theta_{ij}}$
open strings
$2U$
loops

$U \gg t, J$

$\partial_t \langle \hat{B}_{43;21} \rangle = i \langle [\hat{H},$

$$\partial_t \langle \hat{B}_{4321} \rangle = i \langle [\hat{H} , \hat{B}_{4321}] \rangle$$

$$= -iJ \langle (L^3_{21} + L^3_{32} + L^3_{43} + L^3_{14}$$

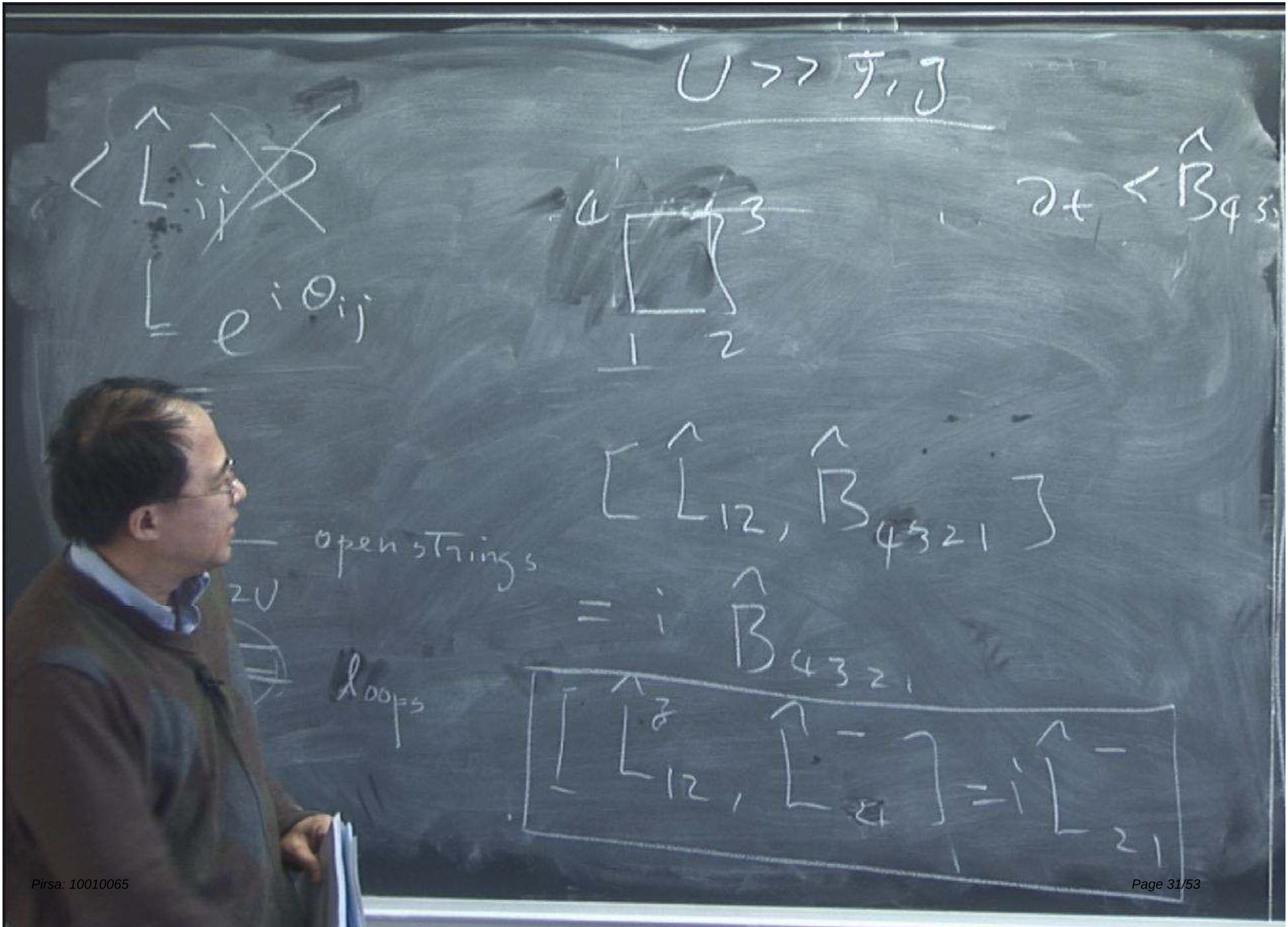
$$+ B (L \quad L^3$$

$$\dot{S}' = i \langle [\hat{H}, \hat{B}_{4321}] \rangle$$

$3,21$

$$= -iJ \langle (L^{z}_{21} + L^{z}_{32} + L^{z}_{43} + L^{z}_{14}) B_{4321}$$

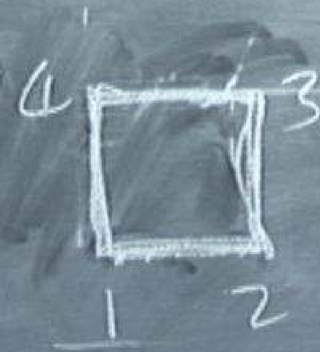
$$+ B (L^{z} + L^{z} + L^{z} + L^{z}) \rangle$$


$U >> \Psi, g$
$\langle \hat{L}_{ij}$
$e^{i\theta_{ij}}$
4 3
1 2
$\partial_t \langle \hat{B}_{43}$
$[\hat{L}_{12}, \hat{B}_{4321}]$
$= i \hat{B}_{4321}$
open strings
2U
loops
$[\hat{L}_{12}, \hat{L}_{21}] = i\hat{L}_{21}$

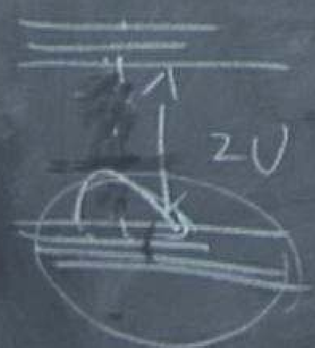
$U \gg t, g$

$\cancel{\langle \hat{L}^-_{ij}}$

$L\, e^{i\theta_{ij}}$



$\partial_t \langle \hat{B}_{43\ldots}$



open strings

loops

$$[\hat{L}^{z\,2}_{12}, \hat{B}_{4321}]$$

$$= i(\hat{L}^z \hat{B}_{4321} + B\,L)$$

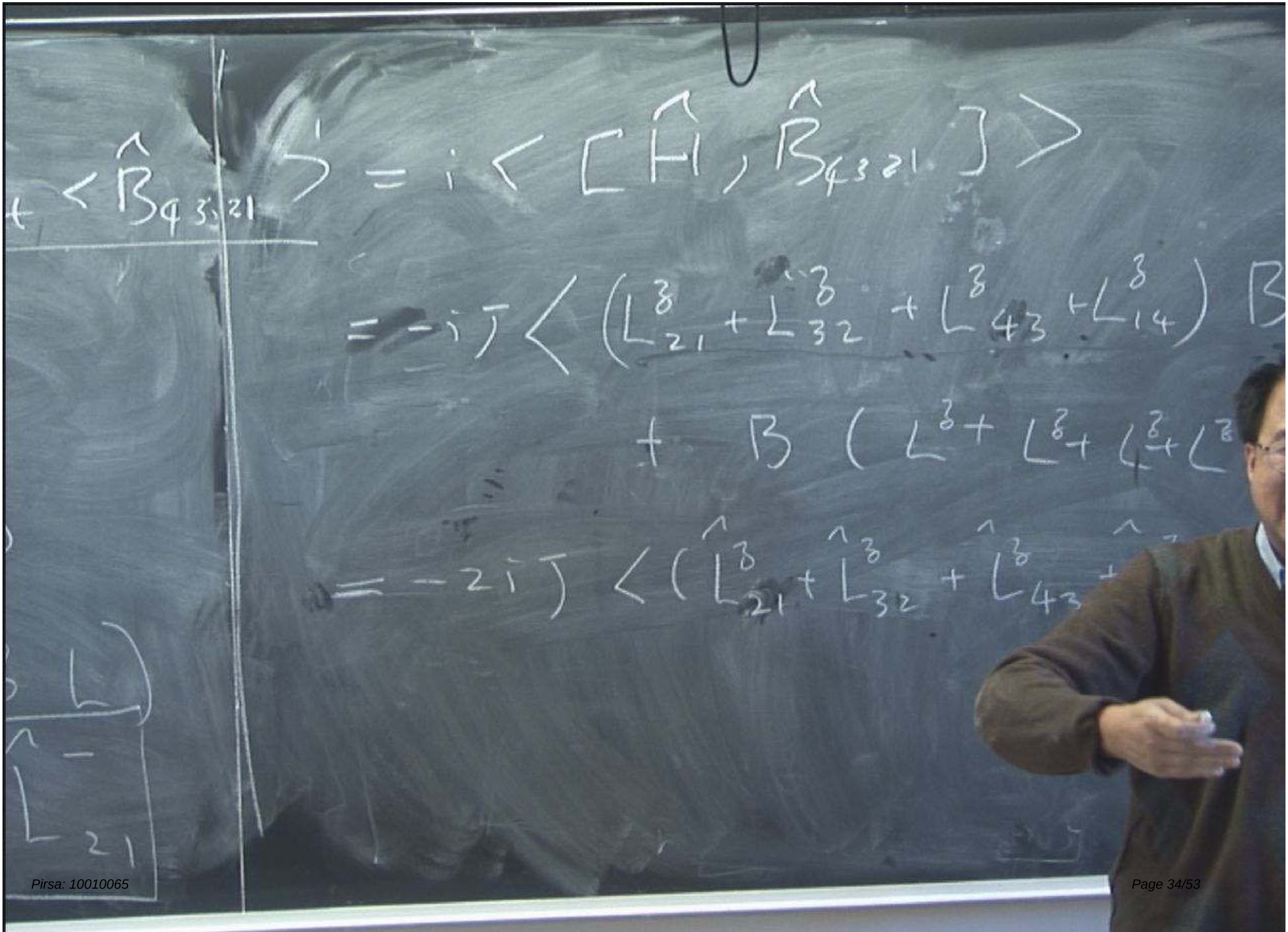
$$\boxed{[\hat{L}^z_{12}, \hat{L}^-_{21}] = i\hat{L}^-_{21}}$$

$$\dot{\ } = i\langle [\hat{H}, \hat{B}_{4321}]\rangle$$

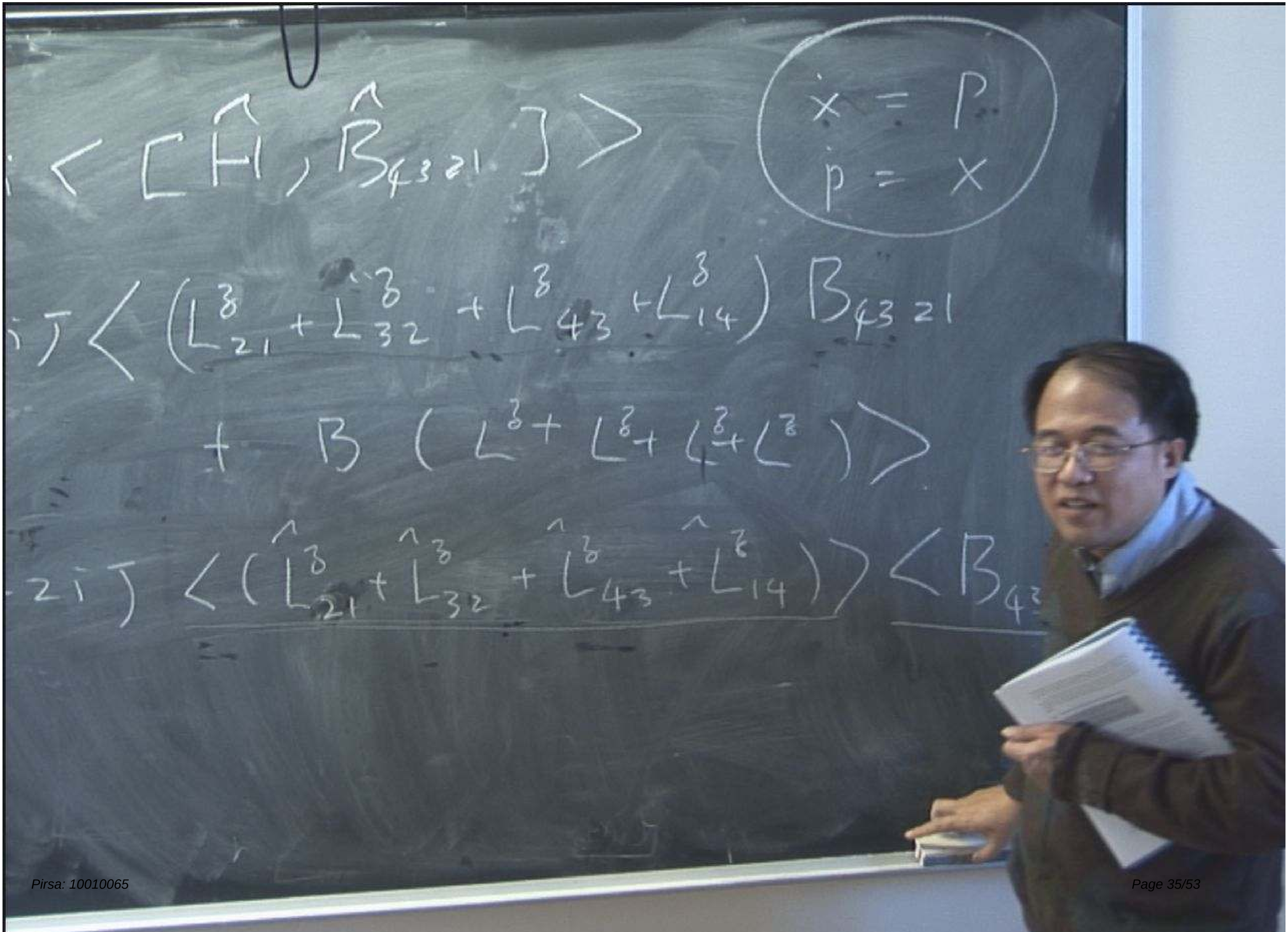
$$= -iJ\langle (L^z_{21} + L^z_{32} + L^z_{43} + L^z_{14}) B_{4321}$$

$$+ B\,(L^z + L^z + L^z + L^z)\rangle$$

$$= -2iJ\langle(\hat{L}^z_{21} + \hat{L}^z_{32} + \hat{L}^z_{43} + \hat{L}^z_{14})\rangle\langle B_{4321}\rangle$$



$+ \langle \hat{B}_{4321}$
$\rangle = i \langle [\hat{H}, \hat{B}_{4321}] \rangle$
$= -iJ \langle (L^z_{21} + L^z_{32} + L^z_{43} + L^z_{14}) B$
$+ B (L^z + L^z + L^z + L^z$
$= -2iJ \langle (\hat{L}^z_{21} + \hat{L}^z_{32} + \hat{L}^z_{43} +$
$L)$
$\hat{L}_{21}$


$i < [\hat{H}, \hat{B}_{4321}] >$
$\dot{x} = p$
$\dot{p} = x$
$iJ < (L^3_{21} + L^3_{32} + L^3_{43} + L^3_{14}) B_{4321}$
$+ B (L^3 + L^3 + L^3 + L^3) >$
$2iJ < (\hat{L}^3_{21} + \hat{L}^3_{32} + \hat{L}^3_{43} + \hat{L}^3_{14}) > < B_{43}$

$$\dot{\ } = i \langle [\hat{H}, \hat{B}_{4321}] \rangle$$

$$\dot{x} = p$$
$$\dot{p} = x$$

$$= -iJ \langle (L^{3}_{21} + \hat{L}^{3}_{32} + L^{3}_{43} + L^{3}_{14}) B_{4321}$$

$$+ B (L^{3} + L^{3} + L^{3} + L^{3}) \rangle$$

$$= -2iJ \underline{\langle (\hat{L}^{3}_{21} + \hat{L}^{3}_{32} + \hat{L}^{3}_{43} + \hat{L}^{3}_{14}) \rangle} \underline{\langle B_{4321} \rangle}$$

$U >> \Psi, g$

$\partial_t \langle L^3_{2,1} \rangle = i \langle [\hat{H}, \hat{L}^3_{2,1}] \rangle$

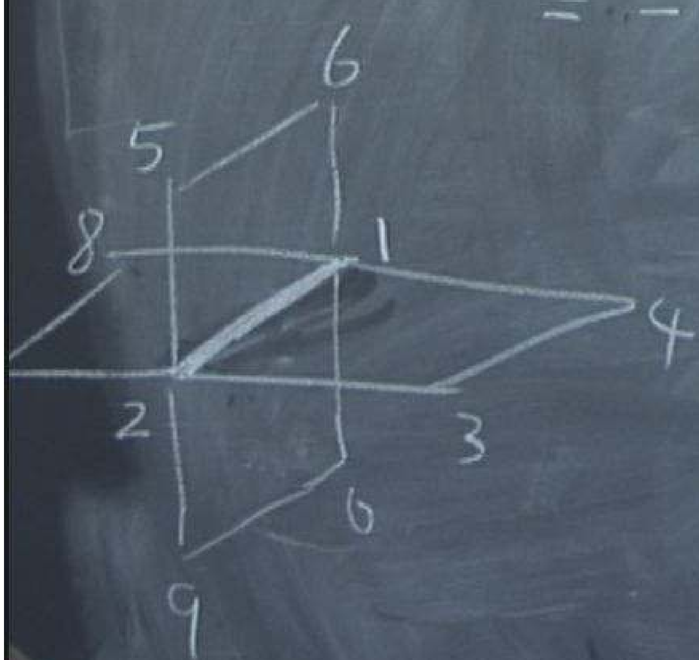
$\partial_t \langle \hat{B}_{4\,3}$

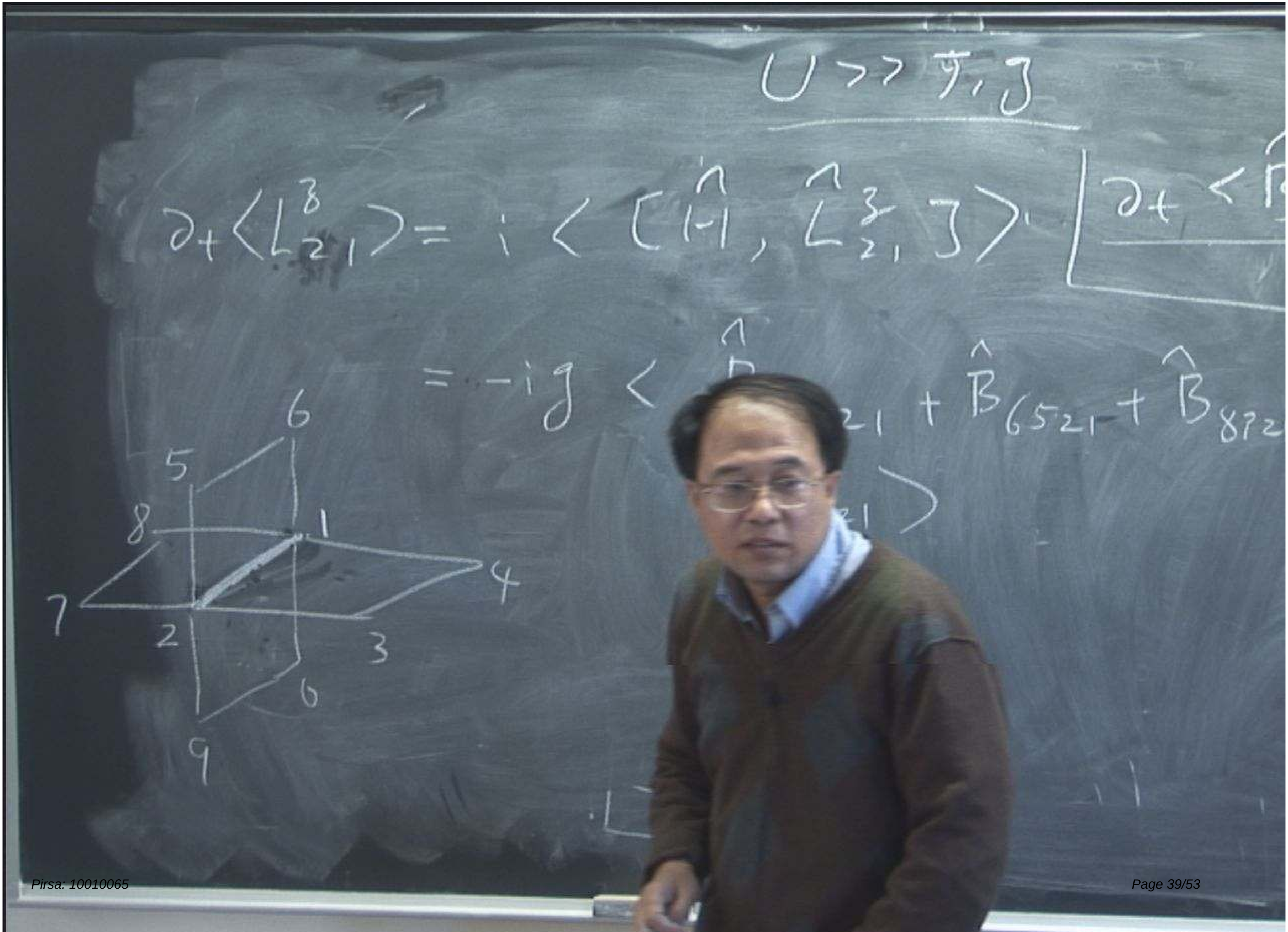
$$U \gg \bar{t}, g$$

$$\partial_t \langle L^{8}_{21} \rangle = i \langle [\hat{H}, \hat{L}^{3}_{21}] \rangle$$

$$\partial_t \langle \hat{B}_{43\ldots}$$

$$= -ig \langle \hat{B}_{4321} + \hat{B}_{6521} + \hat{B}_{8721} + \hat{B}_{0921} \rangle$$

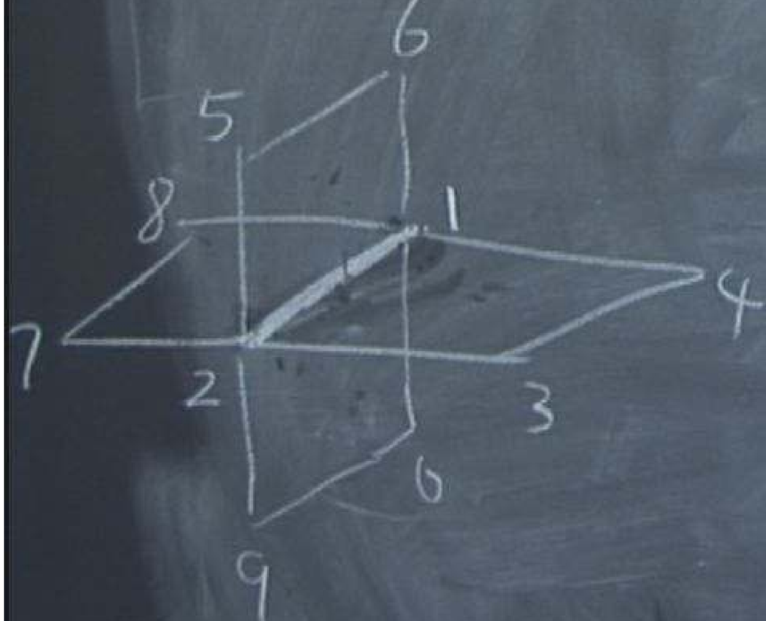



U >> t, J
∂t⟨L³₂₁⟩ = i ⟨[Ĥ, L̂³₂₁]⟩
= −ig ⟨ B̂
+ B̂6521 + B̂
6 5 8 1 4 7 2 3 6 9

$$U \gg J, g$$

$$\partial_t \langle L^3_{21} \rangle = i \langle [\hat{H}, \hat{L}^3_{21}] \rangle \qquad \partial_t \langle \hat{B}_4$$

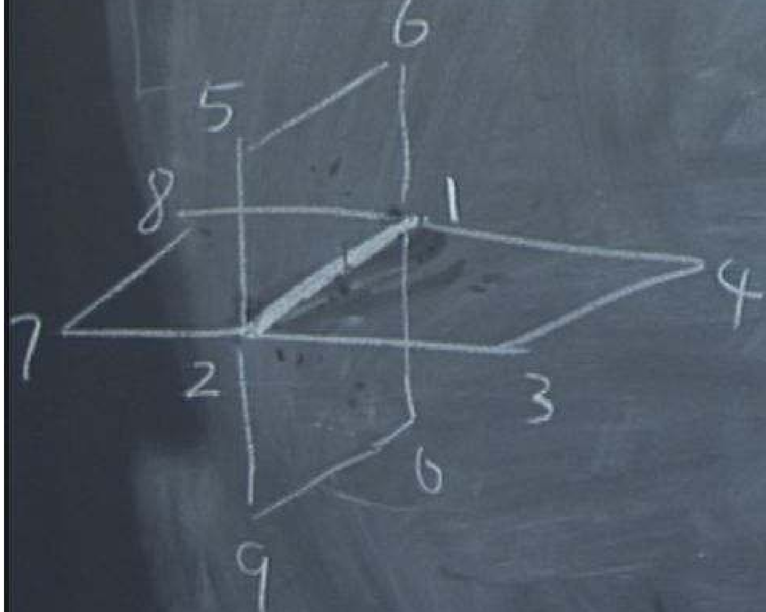
$$= -ig \langle \underline{\hat{B}_{4321}} + \hat{B}_{6521} + \hat{B}_{8721} + \hat{B}_{0921} \rangle$$



$$\boxed{\phi^E_{ij} = 2\pi \langle L^3_{ij} \rangle} \qquad \boxed{\langle \hat{B}_{\ell k j i} \rangle = e^{i\phi^B_{\ell k j i}}}$$

$$\partial_t \langle L^3_{21} \rangle = i \langle [\hat{H}, \hat{L}^3_{21}] \rangle \qquad \partial_t \langle \hat{B}_{\ell}$$

$$= -ig \langle \underline{\hat{B}_{4321}} + \hat{B}_{6521} + \hat{B}_{8721} + \hat{B}_{0921} \rangle$$

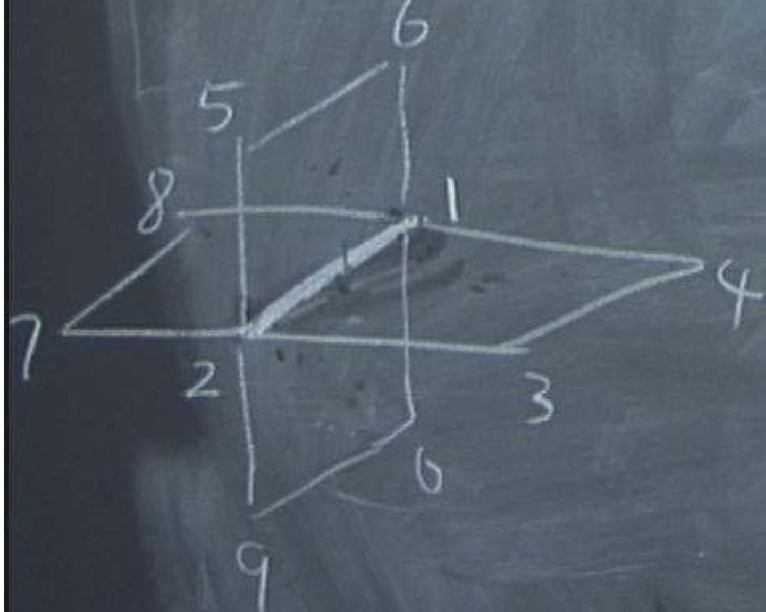


$$\boxed{\phi^{E}_{ij} = 2\pi \langle L^{B}_{ij} \rangle} \qquad \boxed{\langle \hat{B}_{\ell kji} \rangle = e^{i\phi^{B}_{\ell kji}}} \qquad \langle \hat{B}$$

$$\partial_t \langle L^{B}_{21} \rangle = i \langle [\hat{H}, \hat{L}^{B}_{21}] \rangle$$

$$= -ig \langle \hat{B}_{4321} + \hat{B}_{6521} + \hat{B}_{8}$$

$$+ \hat{B}_{0921} \rangle$$

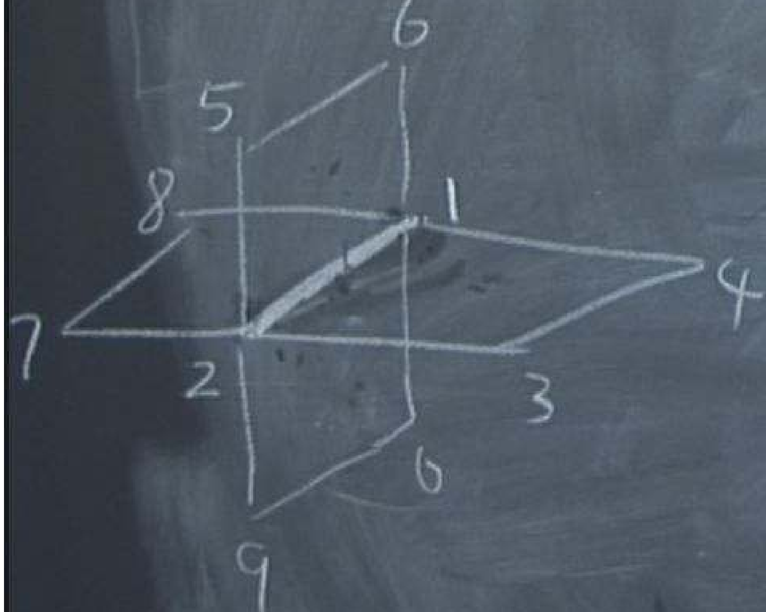


$$\boxed{\phi^E_{ij} = 2\pi \langle L^B_{ij} \rangle}$$

$$\boxed{\langle \hat{B}_{\ell kji} \rangle = e^{i\phi^B_{\ell kji}}}$$

$$\partial_t \langle L^B_{21} \rangle = i \langle [\hat{H}, \hat{L}^B_{21}] \rangle$$

$$\partial_t < \hat{B}_{\ell}$$



$$= -ig \langle \hat{B}_{4321} + \hat{B}_{6521} + \hat{B}_{8721} + \hat{B}_{0921} \rangle$$

$$\partial_t \phi^E_{21} = 4\pi g \left(\phi^B_{4321} + \phi^B_{6521} + \phi \ldots + \phi^B_{0921} \right)$$

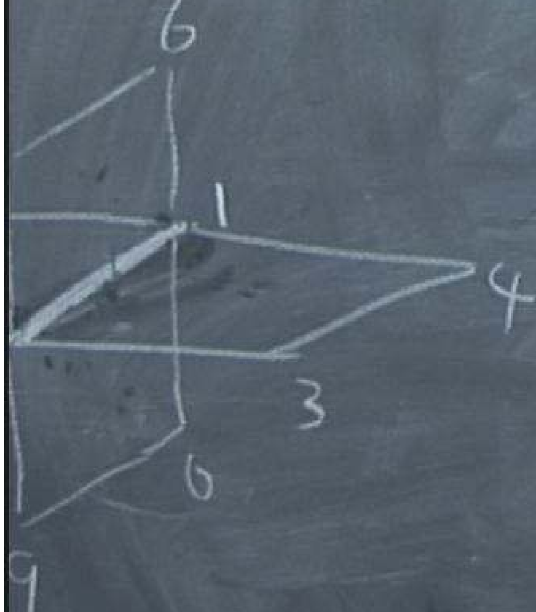
$$E_{ij} = 2\pi \langle L^B_{ij} \rangle$$

$$\langle \hat{B}_{lkji} \rangle = e^{i\phi^B_{lkji}}$$

$$\partial_t \langle \hat{B}_{4321} \rangle$$

$$t \langle L^B_{21} \rangle = i \langle [\hat{H}, \hat{L}^B_{21}] \rangle$$

$$= -ig \langle \hat{B}_{4321} + \hat{B}_{6521} + \hat{B}_{8721} + \hat{B}_{0921} - h.c. \rangle$$



$$\boxed{\partial_t \phi^E_{21} = 4\pi g \left(\phi^B_{4321} + \phi^B_{6521} + \phi^B_{8721} + \phi^B_{0921} \right)}$$

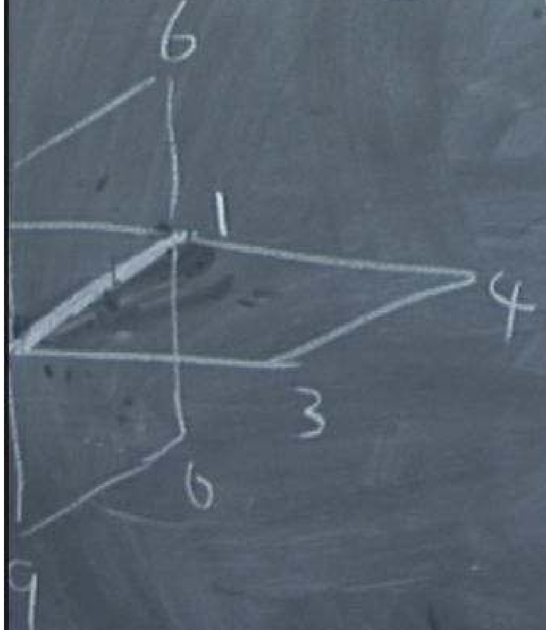
$E_{ij} = 2\pi \langle L^{B}_{ij} \rangle$

$\langle \hat{B}_{\ell k j i} \rangle = e^{i\phi^{B}_{\ell k j i}}$

$\partial_t \langle \hat{B}_{4321} \rangle$

$\partial_t \langle L^{B}_{21} \rangle = i \langle [\hat{H}, \hat{L}^{B}_{21}] \rangle$

$= -ig \langle \hat{B}_{4321} + \hat{B}_{6521} + \hat{B}_{8721} + \hat{B}_{0921} - h.c. \rangle$



$$\boxed{\partial_t \phi^{E}_{21} = 4\pi g \left(\phi^{B}_{4321} + \phi^{B}_{6521} + \phi^{B}_{8721} + \phi^{B}_{0921} \right)}$$


e^{i φ^B_{lkji}}
∂_t e^{iθ} = i∂_t θ (e^{iθ})
∂_t < B̂_{4321} > = i < [H, B̂_{4321}] >
= −iJ < (L^3_{21} + L^3_{32} + L^3_{43} + L^3_1
B (
B̂_{6521} + B̂_{8721}
= −2iJ < (L_{21}
φ^B_{4321} + φ^B_{6521} + φ^B_{8721}
+ φ^B_{09,21})

$$e^{i\theta} = i\partial_t \theta \, (e^{i\theta})$$

$$\boxed{\begin{aligned} \dot{x} &= p \\ \dot{p} &= x \end{aligned}}$$

$$= i \langle [H, \hat{B}_{4321}] \rangle$$

$$= -iJ \langle (L^z_{21} + \hat{L}^z_{32} + L^z_{43} + L^z_{14}) B_{4321}$$

$$+ B \, (L^z + L^z + L^z + L^z) \rangle$$

$$= -2iJ \langle (\hat{L}^z_{21} + \hat{L}^z_{32} + \hat{L}^z_{43} + \hat{L}^z_{14}) \rangle \langle B_{4321} \rangle$$

$$\boxed{\partial_t \phi^B_{4321} = -\frac{J}{\hbar} \left(\phi^E_{21} + \phi^E_{32} + \phi^E_{43} + \phi^B_{14} \right)}$$

$e^{i\theta} = i\partial_t\theta\, e^{i\theta}$

$\rangle = i\langle [H, \hat{B}_{4321}]\rangle$

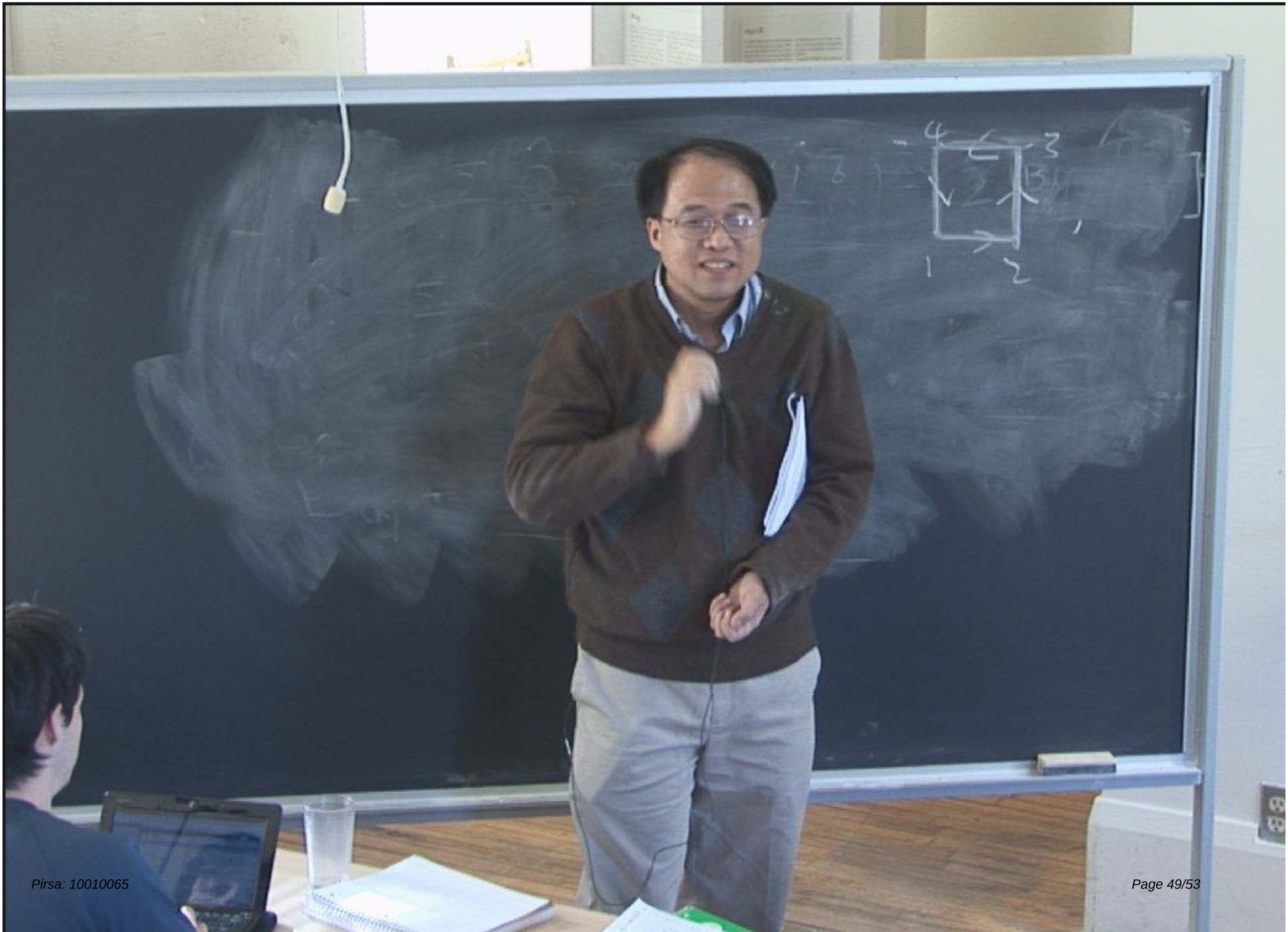
$$\begin{aligned}\dot{x} &= p\\ \dot{p} &= x\end{aligned}$$

$$= -iJ\langle (\hat{L}^z_{21} + \hat{L}^z_{32} + L^z_{43} + L^z_{14})\, B_{4321}$$

$$+\ B\,(L^z + L^z + L^z + L^z)\rangle$$

$$= -2iJ\,\langle(\hat{L}^z_{21} + \hat{L}^z_{32} + \hat{L}^z_{43} + \hat{L}^z_{14})\rangle\langle B_{4321}\rangle$$

$$\boxed{\partial_t \phi^B_{4321} = -\frac{J}{\hbar}\left(\phi^E_{21} + \phi^E_{32} + \phi^E_{43} + \phi^B_{14}\right)}$$



$$e^{i\theta} = i\partial_t \theta \, (e^{i\theta})$$

$$\rangle = i \langle [H, \hat{B}_{4321}] \rangle$$

$$\begin{aligned} \dot{x} &= p \\ \dot{p} &= x \end{aligned}$$

$$= -iJ \langle (L^z_{21} + L^z_{32} + L^z_{43} + L^z_{14}) B_{4321}$$

$$+ B \, (L^z + L^z + L^z + L^z) \rangle$$

$$= -2iJ \langle (\hat{L}^z_{21} + \hat{L}^z_{32} + \hat{L}^z_{43} + \hat{L}^z_{14}) \rangle \langle B_{4321} \rangle$$

$$\boxed{\partial_t \phi^B_{4321} = -\frac{J}{\hbar} \left(\phi^E_{21} + \phi^E_{32} + \phi^E_{43} + \phi^E_{14} \right)}$$

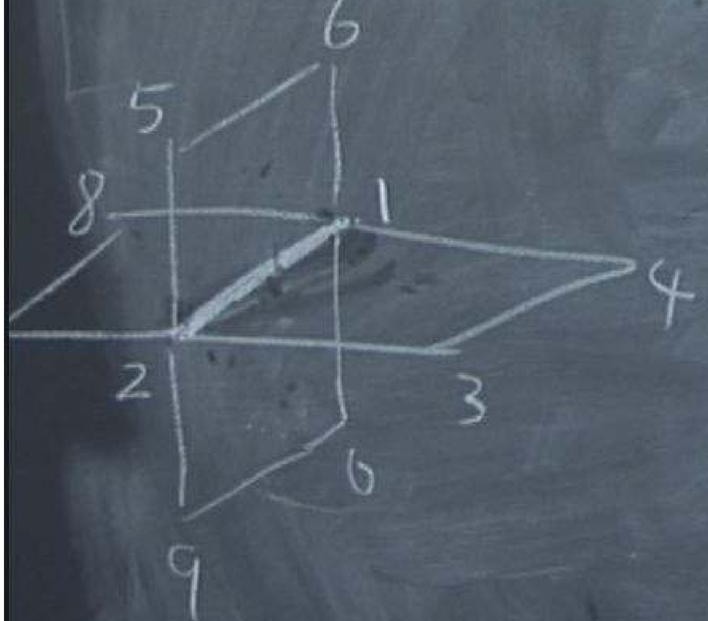
$$\boxed{\phi^E_{ij} = 2\pi \langle L^3_{ij} \rangle}$$

$$\boxed{\langle \hat{B}_{\ell k j i} \rangle = e^{i\phi^B_{\ell k j i}}}$$

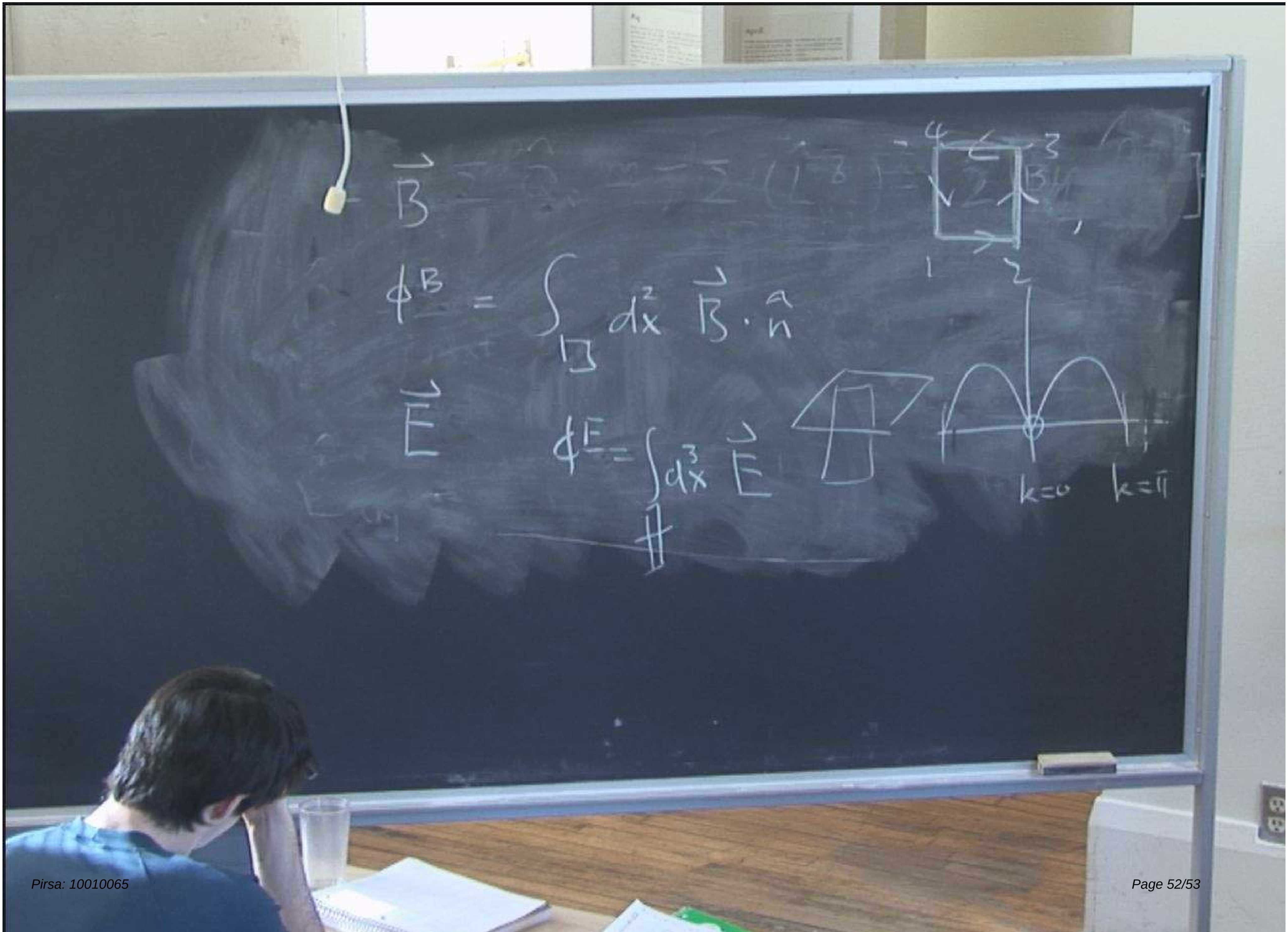
$$\partial_t \langle \hat{B}_{q3\ldots}$$

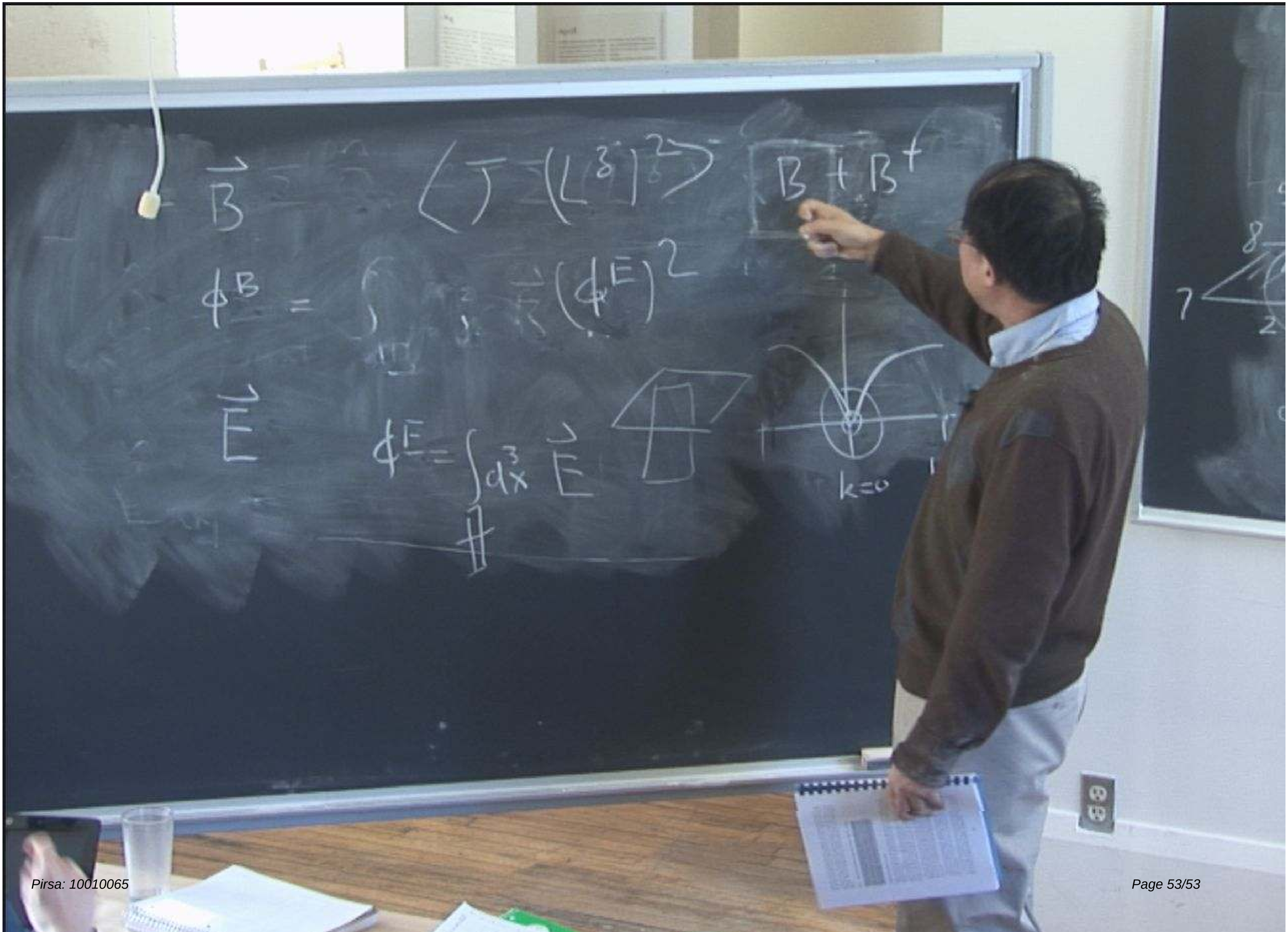
$$\partial_t \langle L^3_{21} \rangle = i \langle [\hat{H}, \hat{L}^3_{21}] \rangle$$

$$= -ig \langle \hat{B}_{4321} + \hat{B}_{6521} + \hat{B}_{8721} + \hat{B}_{0921} - h.c. \rangle$$



$$\boxed{\partial_t \phi^E_{21} = 4\pi g \left(\phi^B_{4321} + \phi^B_{6521} + \phi^B_{87\ldots} + \phi^B_{0921} \right)}$$


ϕB = ∫ d2x B · n
ϕE = ∫ d3x E
k=0
k=π


k=0