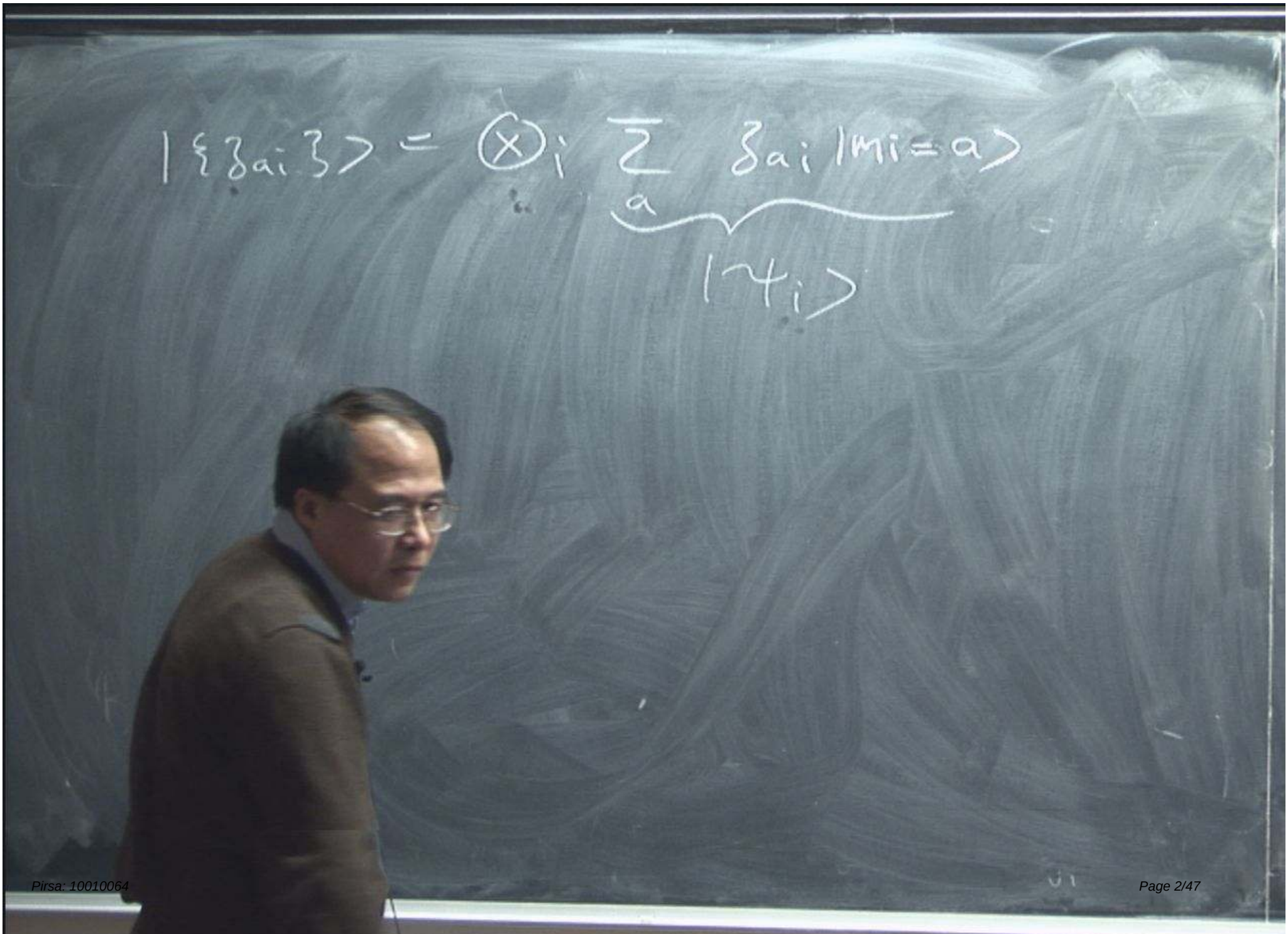
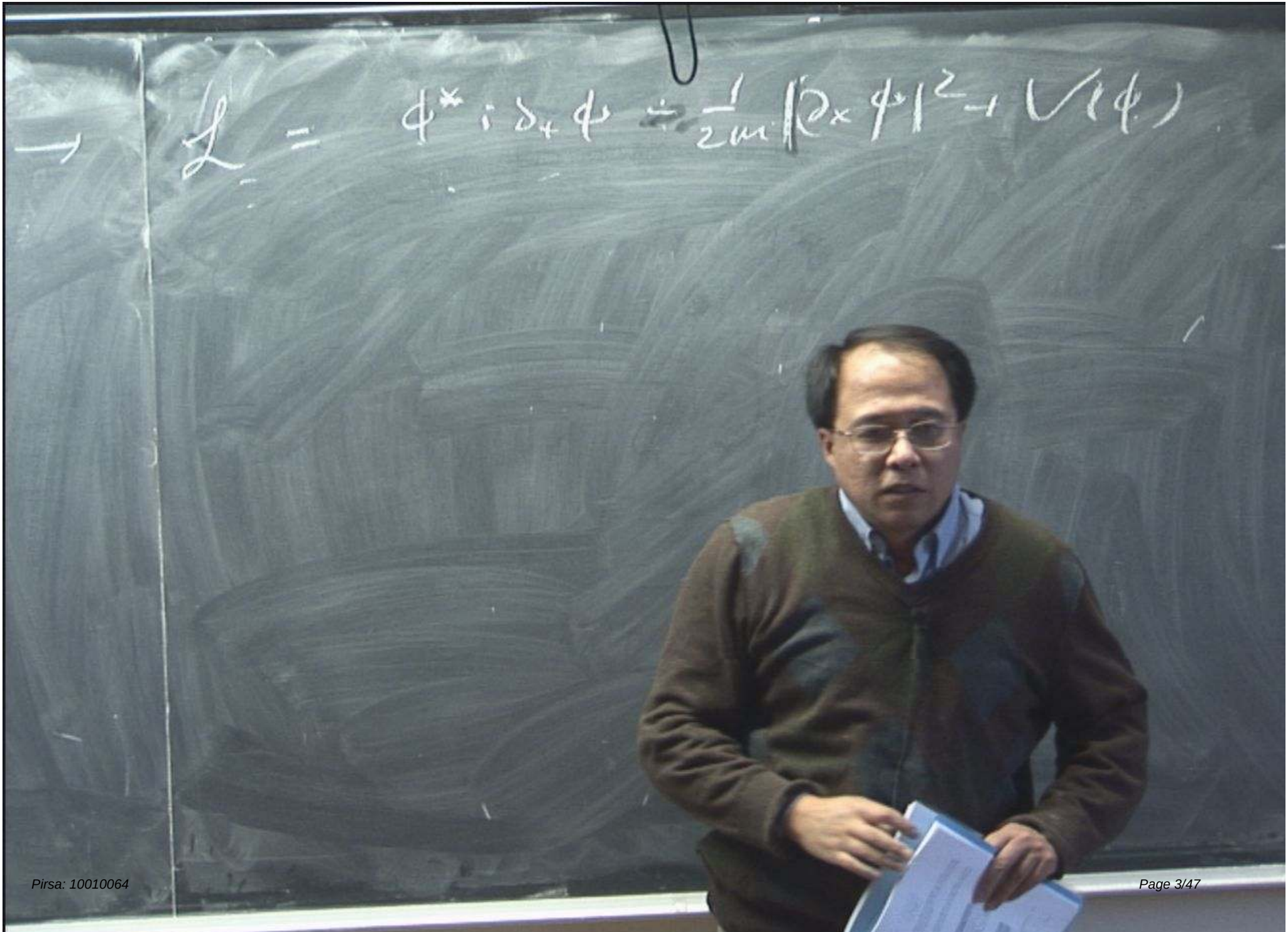
Title: Condensed Matter Review (PHYS 637) - Lecture 12

Date: Jan 19, 2010 11:00 AM

URL: http://pirsa.org/10010064

Abstract:

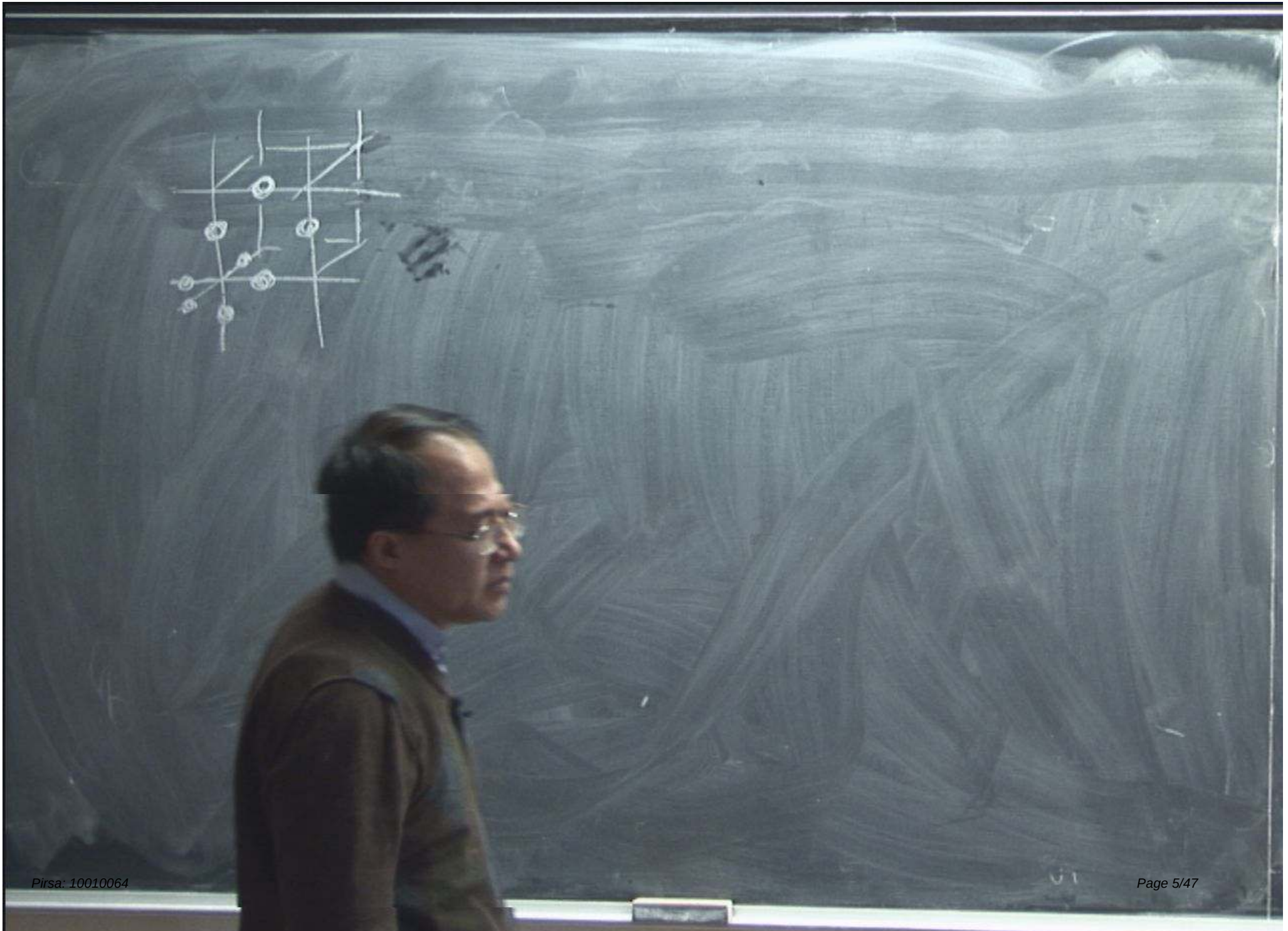

$|\{\delta_{ai}\}\rangle = \bigotimes_i \sum_a \delta_{ai} |m_i = a\rangle$
$|\psi_i\rangle$

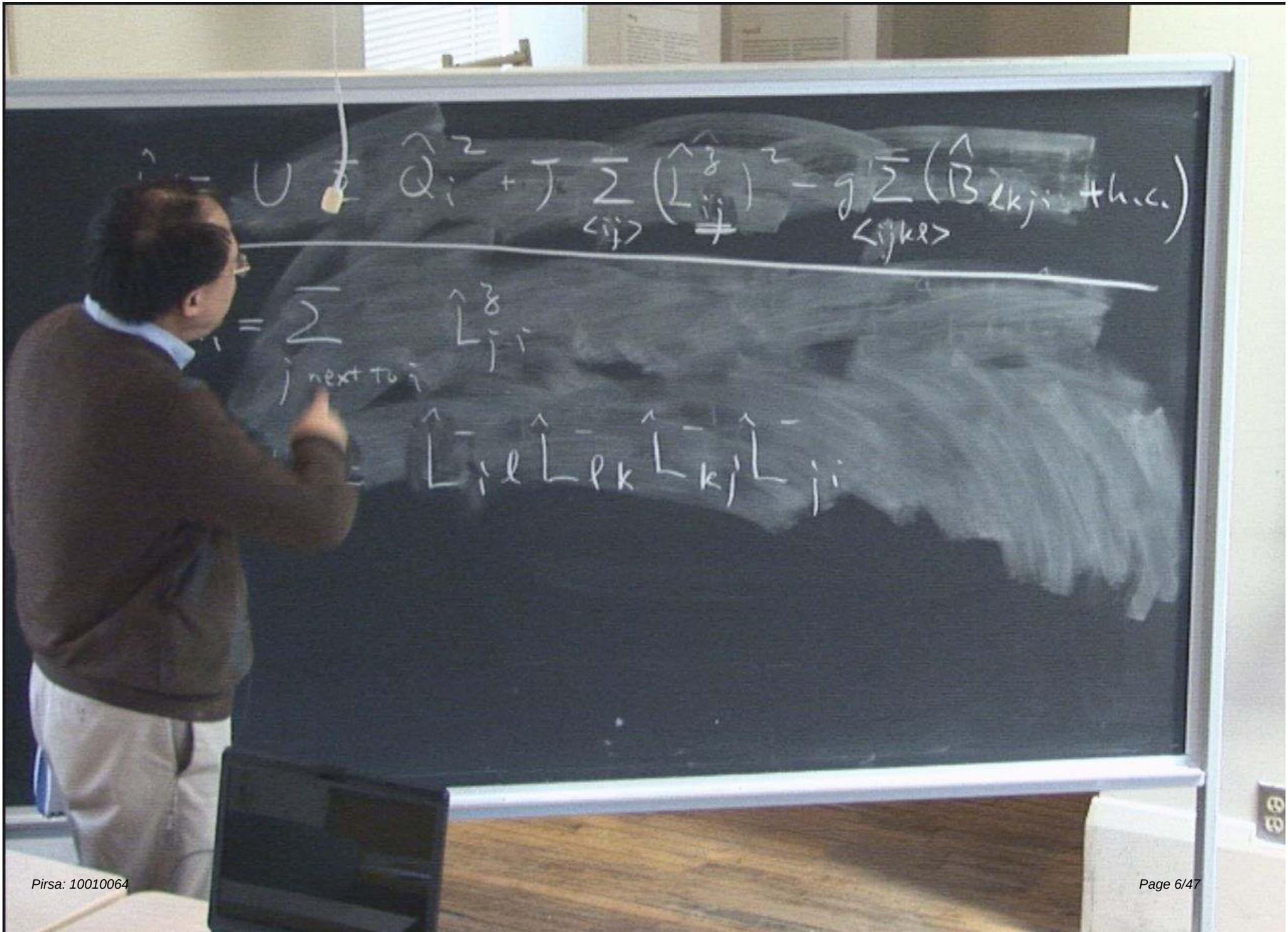

$\mathcal{L} = \phi^* i\partial_t \phi - \frac{1}{2m}|\nabla\phi|^2 - V(\phi)$

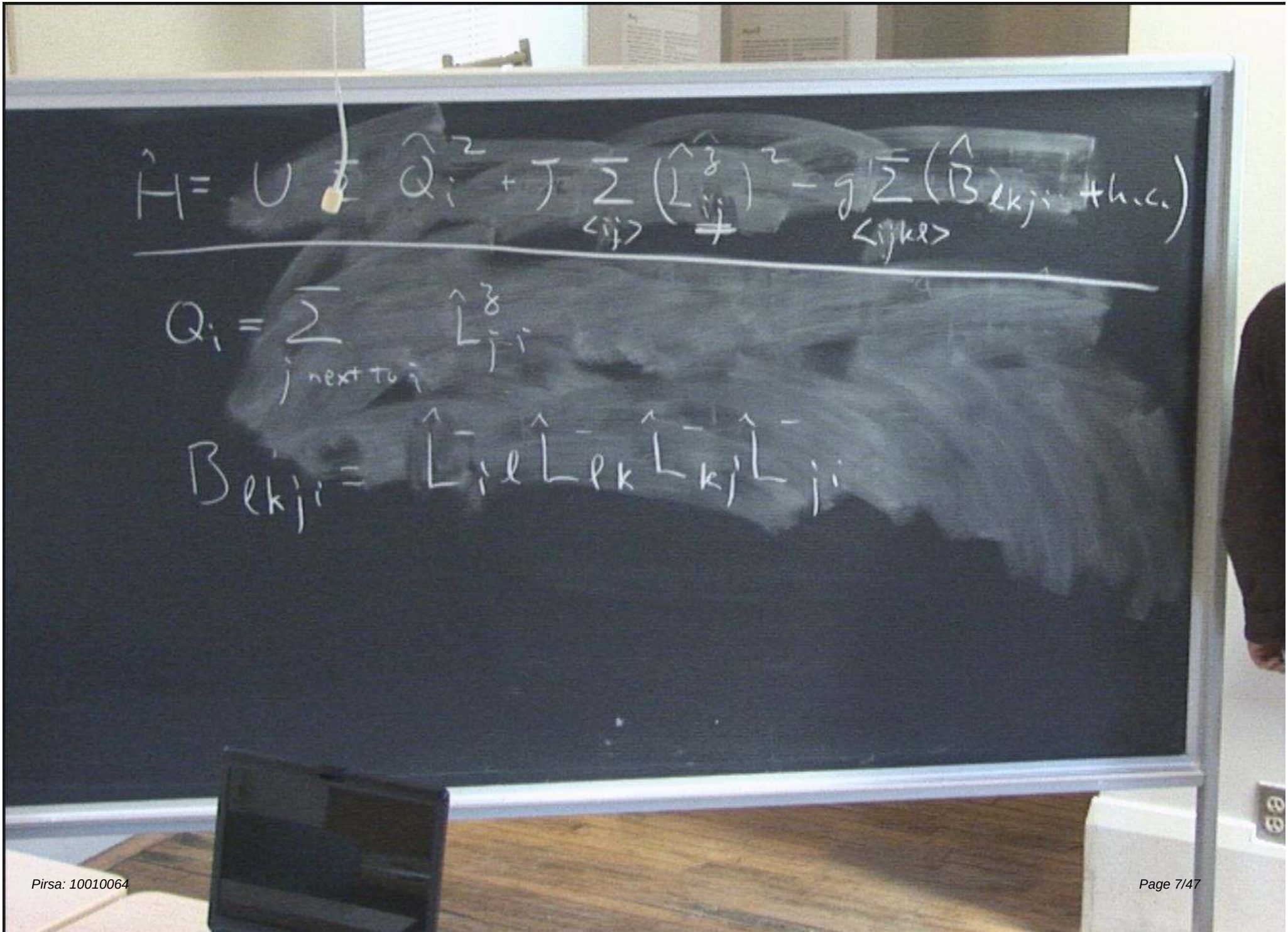
$$\hat{H} = U \sum_i \hat{Q}_i^2 + J \sum_{\langle ij \rangle} \left(\hat{L}^z_{ij}\right)^2 - g \sum_{\langle ijkl \rangle} \left(\hat{B}_{lkji} + \text{h.c.}\right)$$

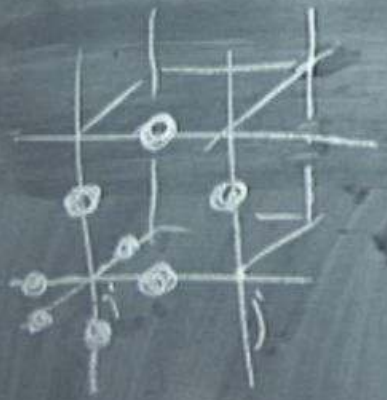
$$Q_i = \sum_{j \text{ next to } i} L^z_{j,i}$$

$$B_{lkji} = L^-_{il} L^-_{lk} L^-_{kj} L^-_{ji}$$

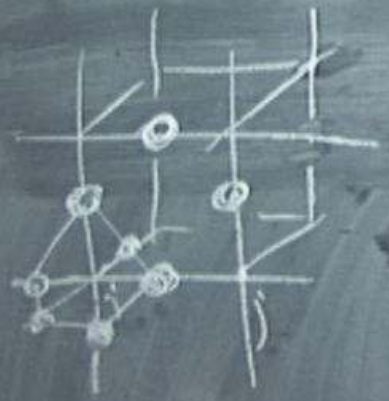



$\hat{Q}_i^2$
$\langle ij \rangle$
$\langle ijkl \rangle$
h.c.
$\hat{L}^z_{ji}$
j next to i
$\hat{L}^-_{il}\hat{L}^-_{lk}\hat{L}^-_{kj}\hat{L}^-_{ji}$

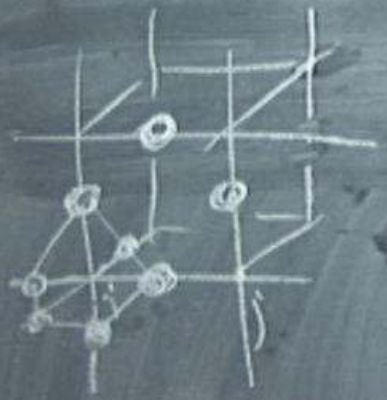

$\hat{H} = U \sum \hat{Q}_i^2 + J \sum_{\langle ij \rangle} (\hat{L}^z_{ij})^2 - g \sum_{\langle ijkl \rangle} (\hat{B}_{lkji} + h.c.)$
$Q_i = \sum_{j \text{ next to } i} \hat{L}^z_{j-i}$
$B_{lkji} = \hat{L}^-_{il} \hat{L}^-_{lk} \hat{L}^-_{kj} \hat{L}^-_{ji}$



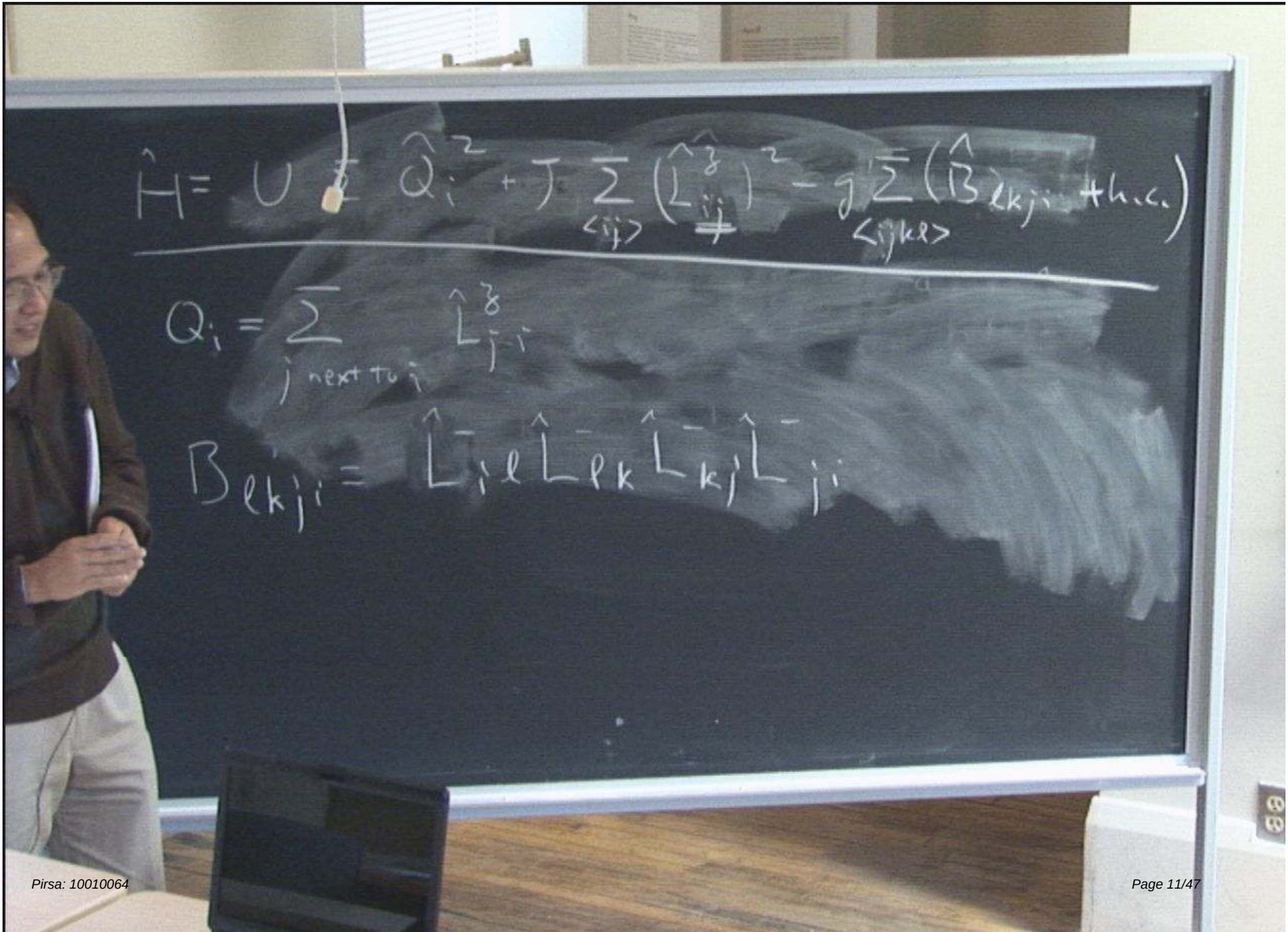
$Q_{ij}^{2} \longrightarrow L_{ij}^{3} \; L_{i'j'}^{3}$



$Q_{ij}^{2} \longrightarrow L_{ij}^{3}\, L_{i'j'}^{3}$



$Q_i^2 \longrightarrow L^3_{ij}\, L^3_{i'j'}$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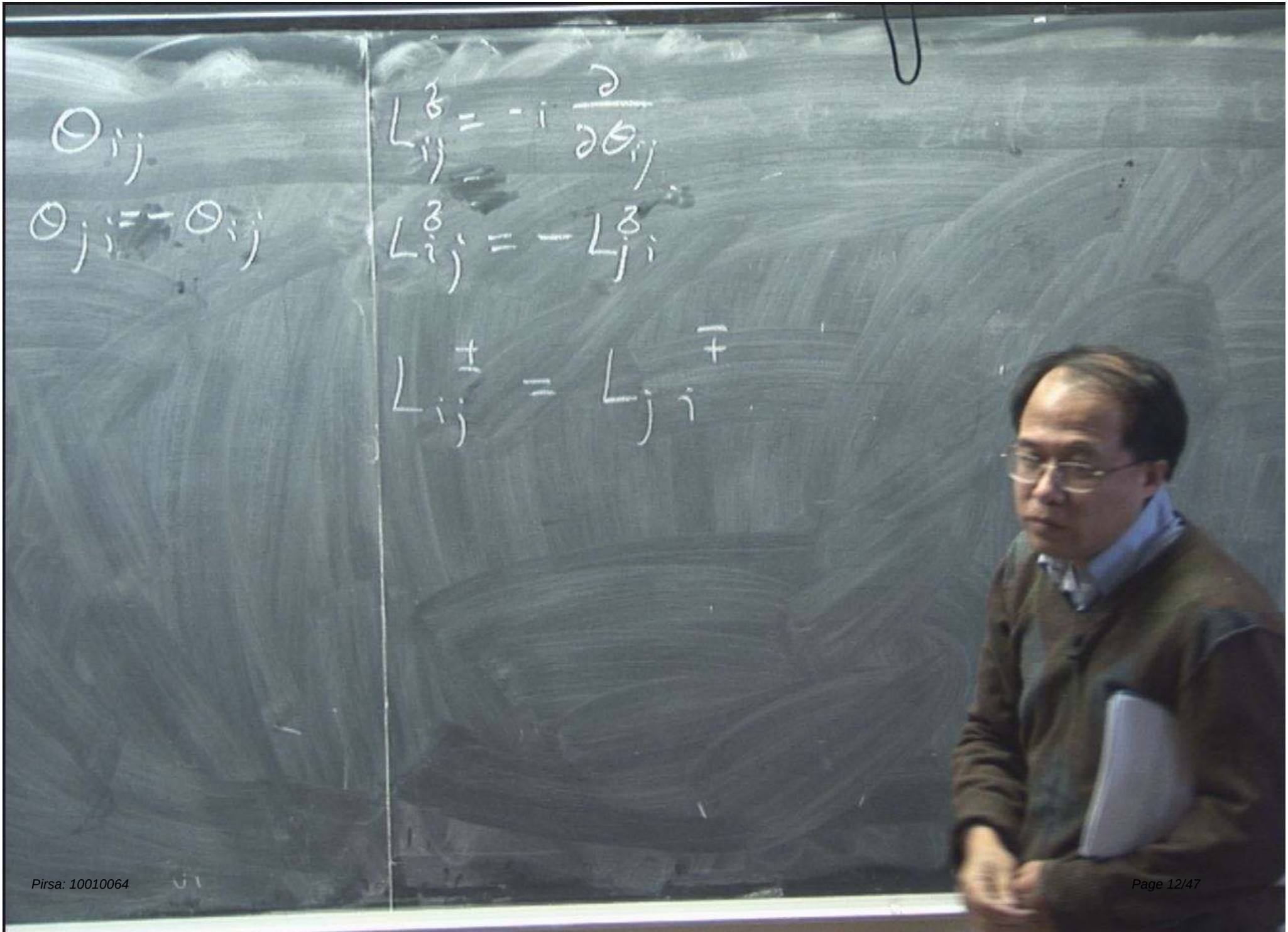
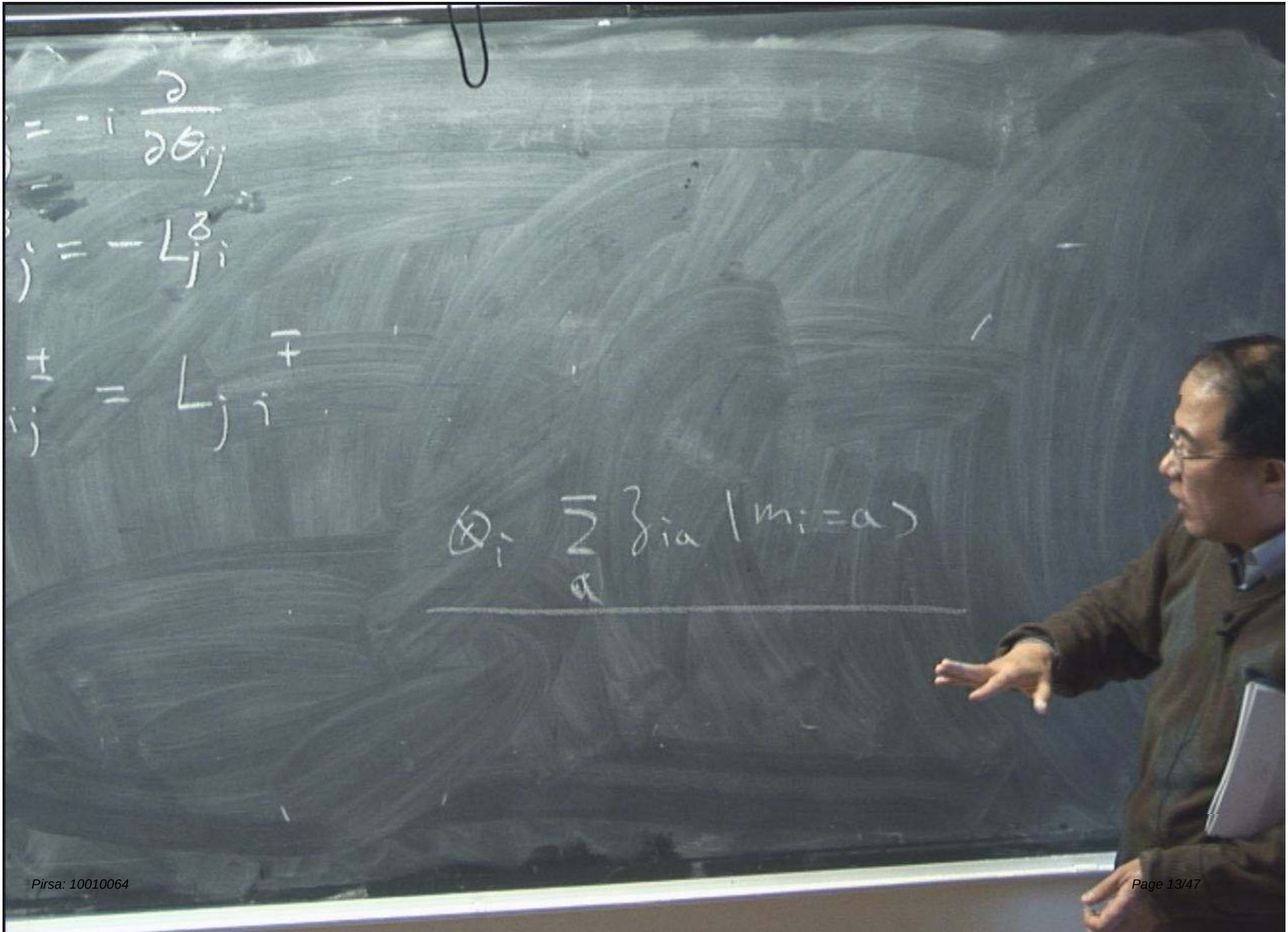
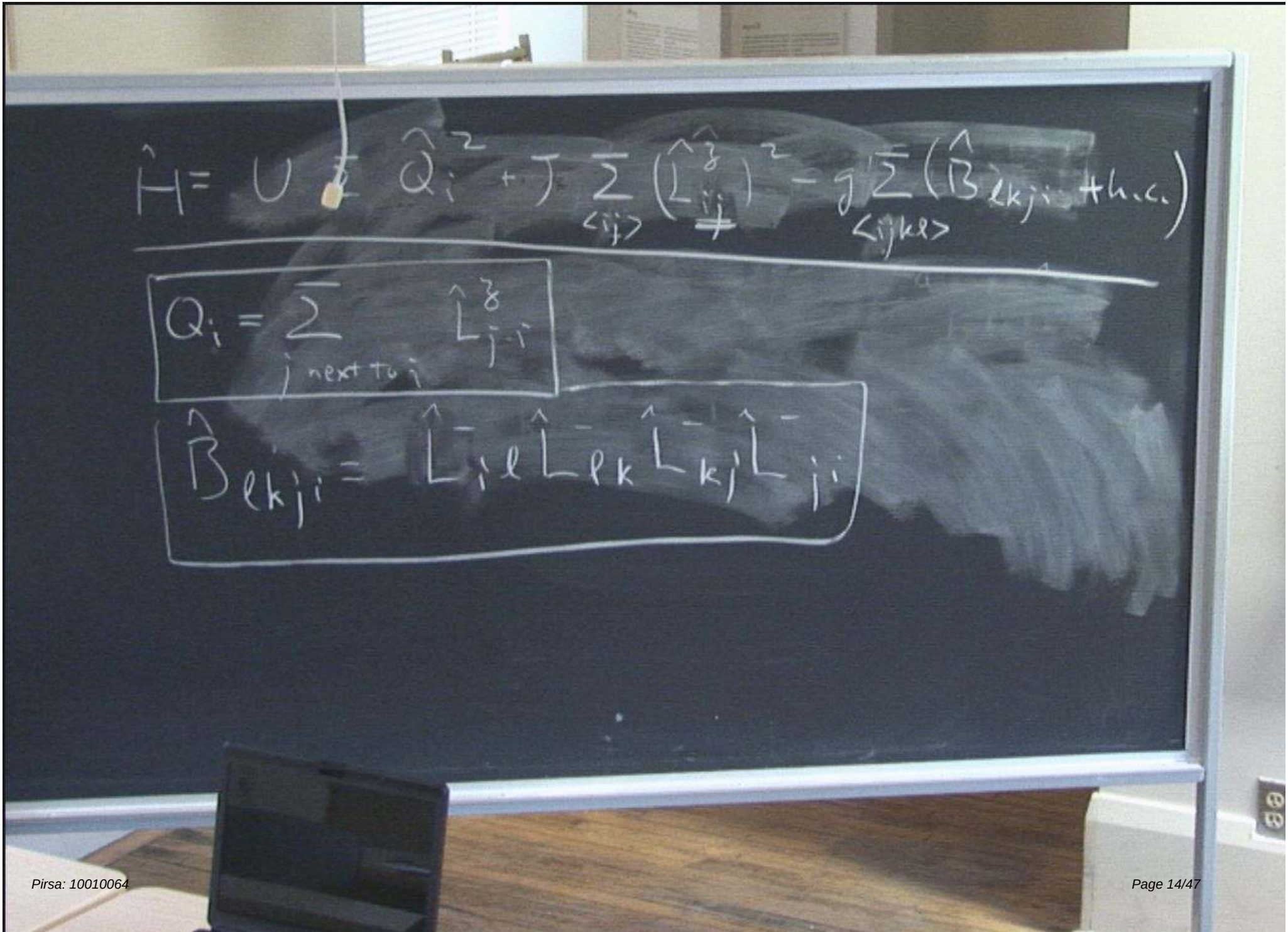
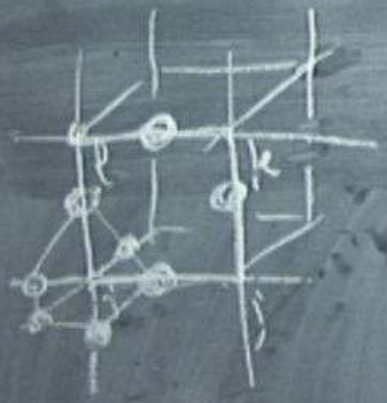
$B_{lkji} = \hat{L}^-_{il} \hat{L}^-_{lk} \hat{L}^-_{kj} \hat{L}^-_{ji}$


Θ_{ij}
$\Theta_{ji} = -\Theta_{ij}$
$L^3_{ij} = -i \frac{\partial}{\partial \Theta_{ij}}$
$L^3_{ij} = -L^3_{ji}$
$L^{\pm}_{ij} = L^{\mp}_{ji}$

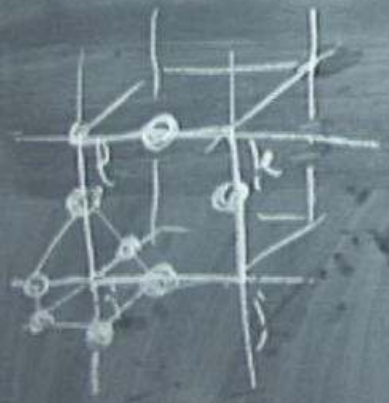

$= -i \frac{\partial}{\partial \theta_{ij}}$
$j = -L^{\delta}_{ji}$
$\pm \quad \mp$
$= L_{ji}$
$\otimes_i \sum_a \zeta_{ia} |m_i = a\rangle$


$\hat{H} = U \sum \hat{Q}_i^2 + J \sum_{\langle ij \rangle} (\hat{L}^z_{ij})^2 - g \sum_{\langle ijkl \rangle} (\hat{B}_{lkji} + h.c.)$
$Q_i = \sum_{j \text{ next to } i} \hat{L}^z_{ji}$
$\hat{B}_{lkji} = \hat{L}^-_{il} \hat{L}^-_{lk} \hat{L}^-_{kj} \hat{L}^-_{ji}$



$g = 0$ $\quad U \gg J > 0$

$|\{m_i\}\rangle$

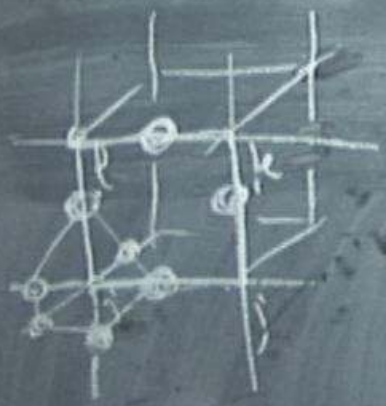


$g = 0 \qquad U \gg J > 0$:

$|\{m_i\}\rangle$ eigenstate

ground state $|\{m_i = 0\}\rangle$

$J = 0$



$g = 0$ $\quad$ $U \gg J > 0$:

$|\{m_i\}\rangle$ eigenstate

ground state $|\{m_i = 0\}\rangle$

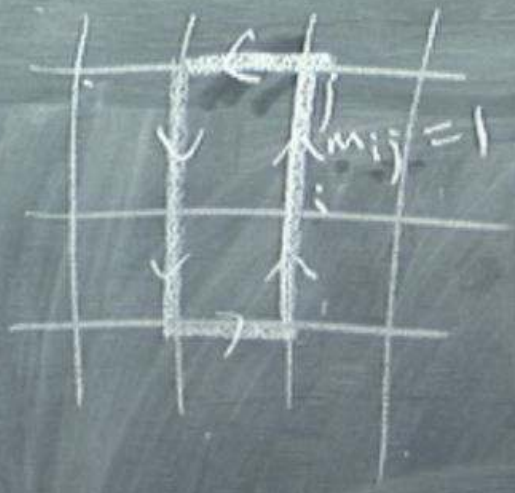
$J = 0$: $|string\rangle$, $E_{str} = 0$

$U \gg J > 0$:

$\}>$ eigenstate

(state $|\{m_i = 0\}>$

$|string>$, $E_{str} = 0$

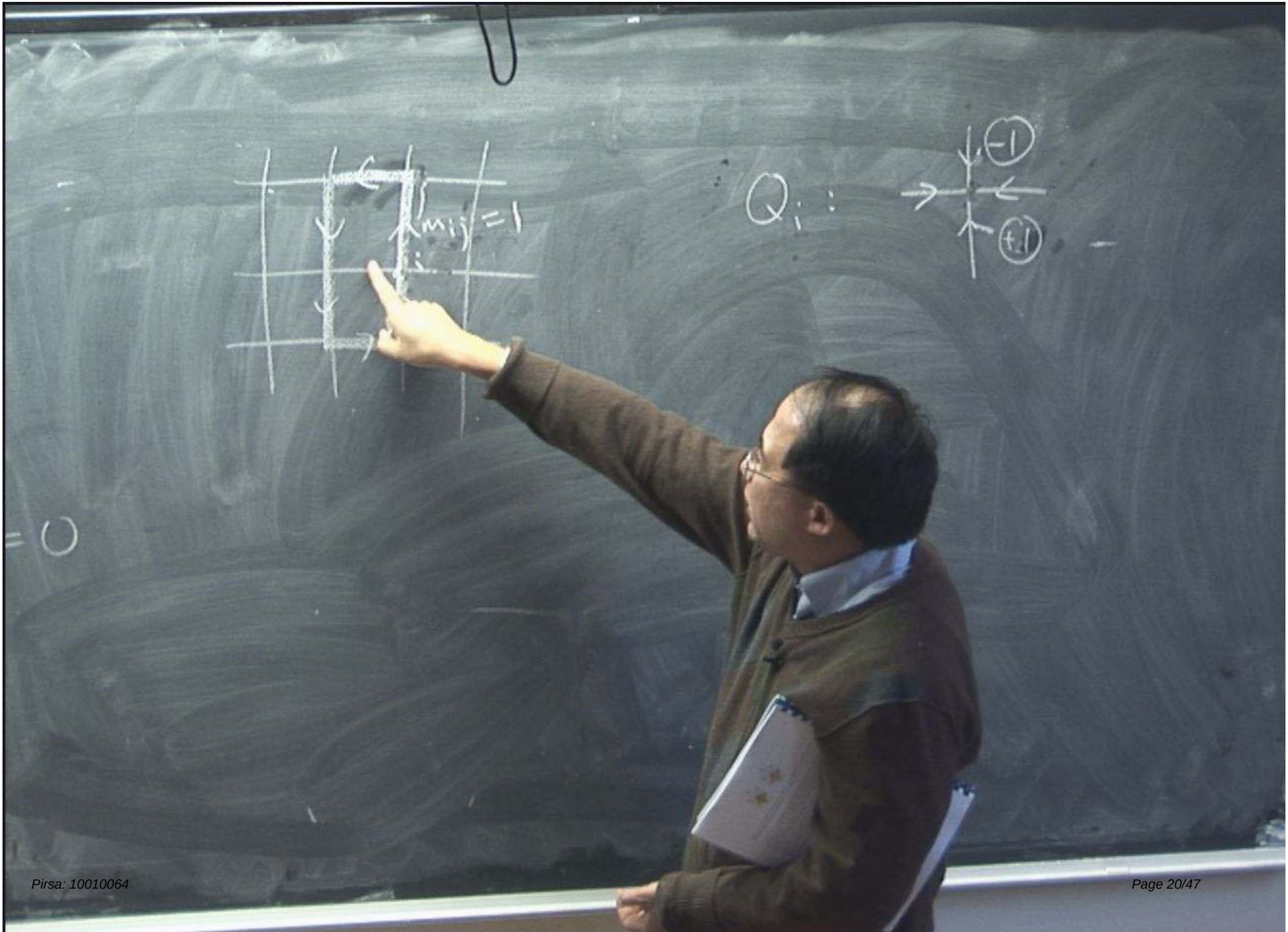


$U \gg J > 0$:

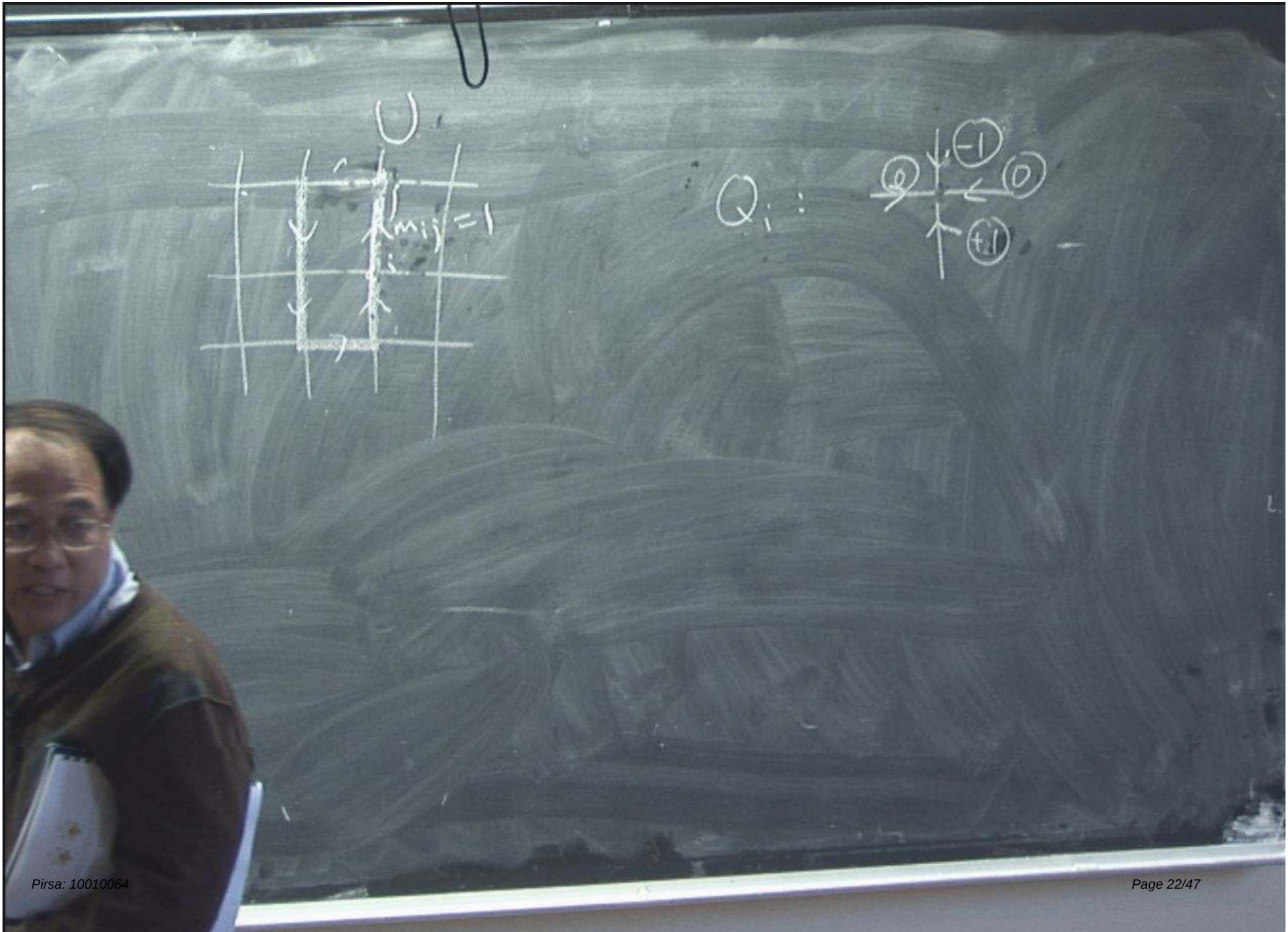
$|\{m_i = 0\}\rangle$ eigenstate

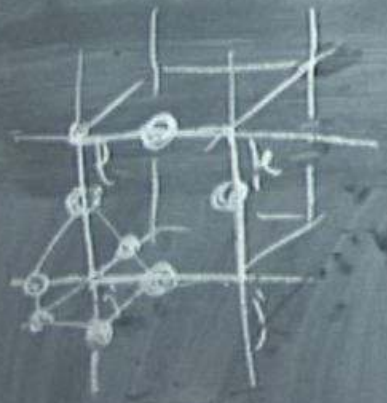
ground state $|\{m_i = 0\}\rangle$

: $|\text{string}\rangle$, $E_{\text{string}} = 0$


$m_{ij}=1$
Q_i :
-1
+1
= 0

$|m_{ij}| = 1$
Q_i :
-1
+1
$= 0$


U
$(m_{ij})=1$
Q_i :
-1
0
+1



$g = 0 \qquad U \gg J > 0$:

$|\{m_i\}\rangle$ eigenstate

ground state $|\{m_i = 0\}\rangle$

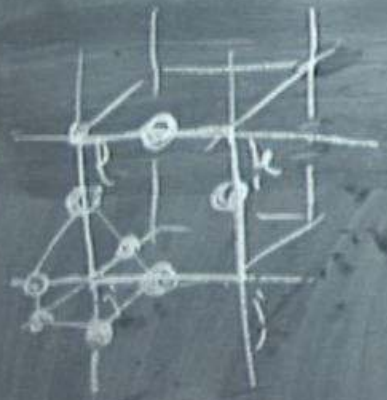
$J = g = 0$

$J = 0$: $|\text{loop}\rangle$, E_{loop}

open strings

U

loops



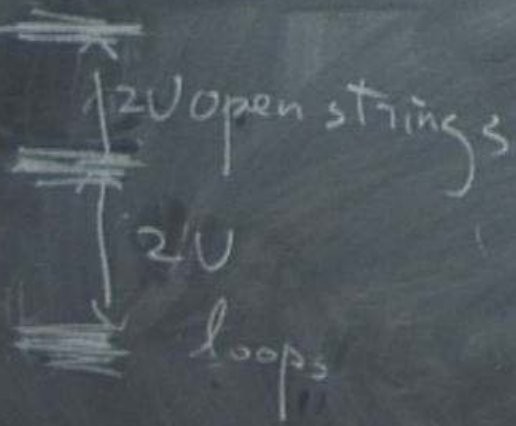
$g = 0 \qquad U \gg J > 0$:

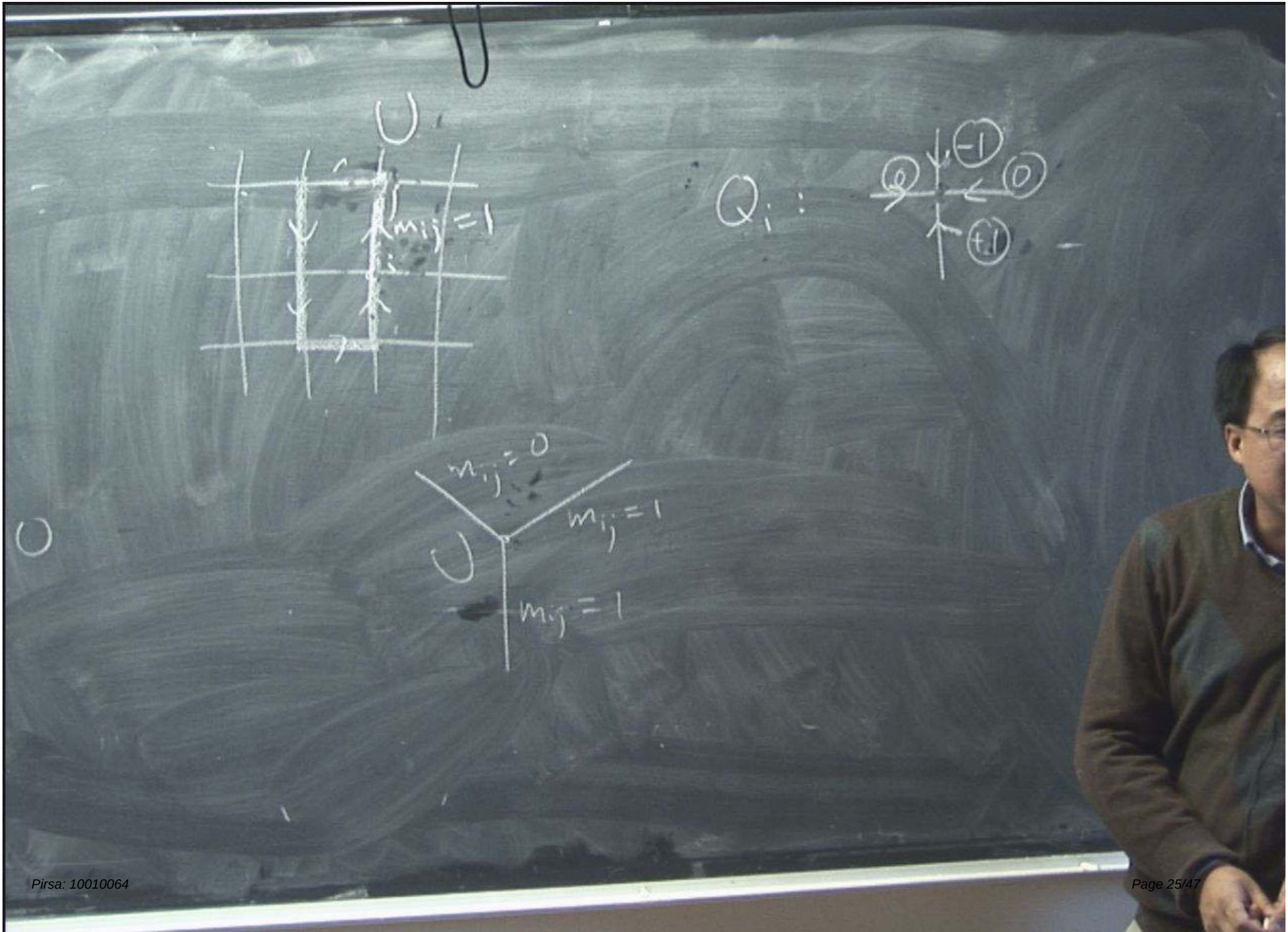
$|\{m_i\}\rangle$ eigenstate

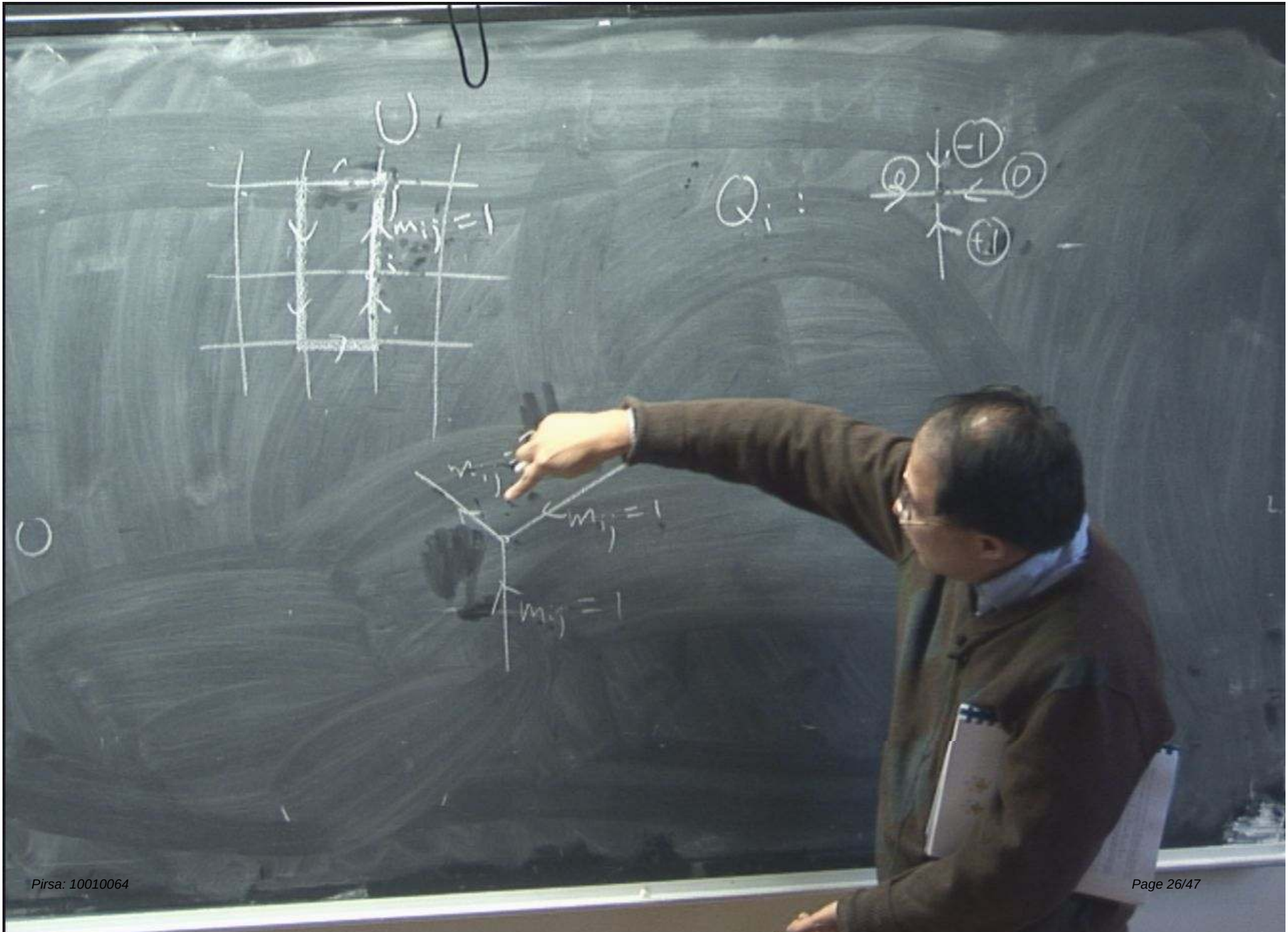
ground state $|\{m_i = 0\}\rangle$

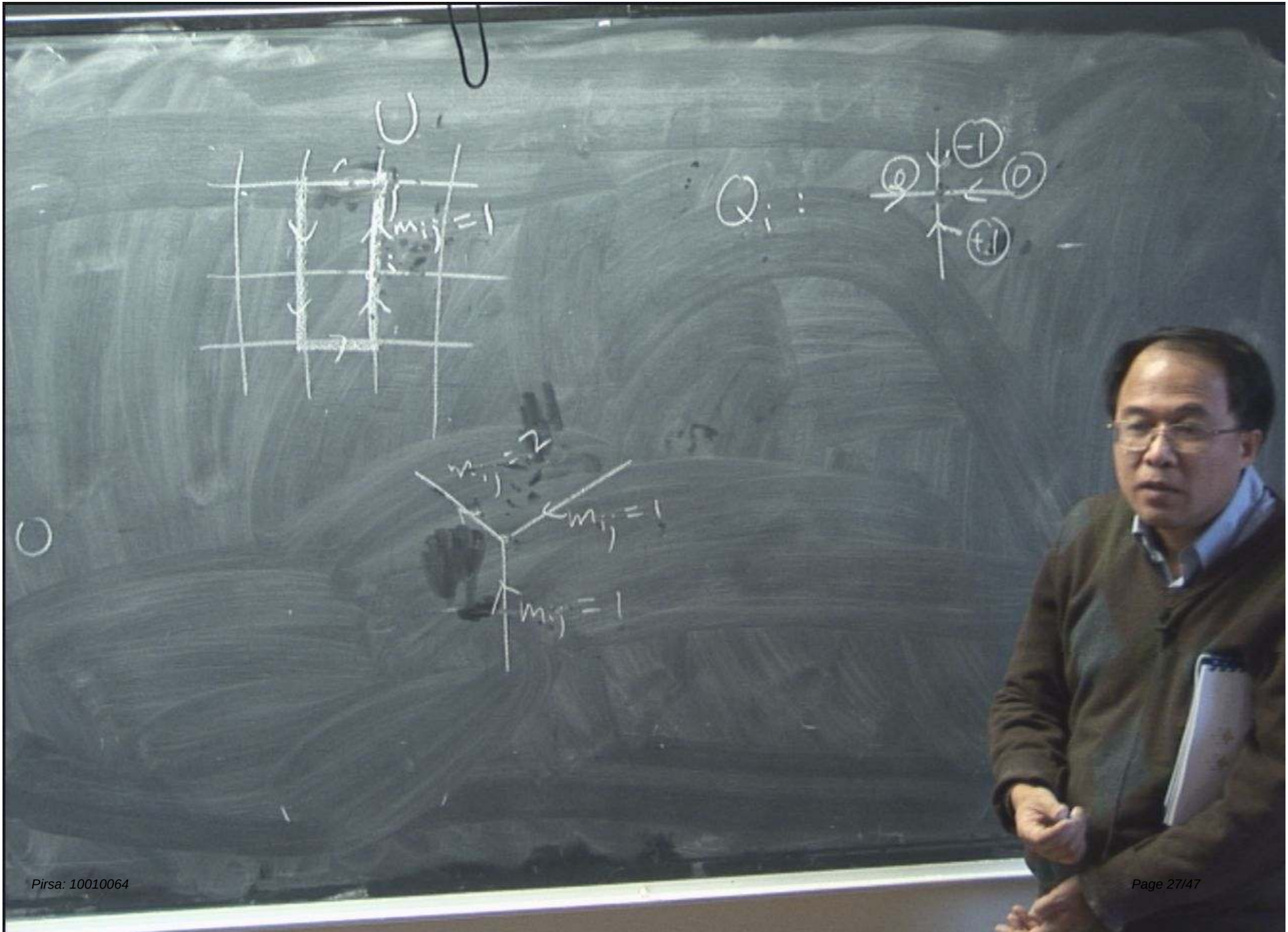
$J = g = 0$

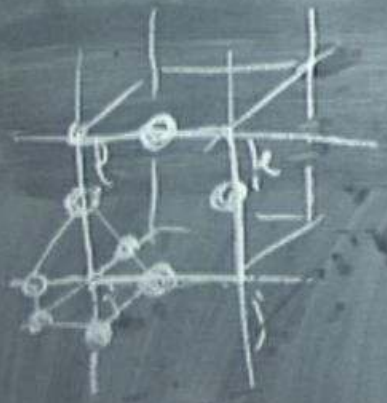
$\bar{J} = 0$: $|\text{loop}\rangle$, E_{loop}




U
$m_{ij}=1$
Q_i :
-1
0
0
+1
$m_{ij}=0$
$m_{ij}=1$
U
$m_{ij}=1$
U


U
$m_{ij}=1$
Q_i :
-1
+1
$m_{ij}=1$
$m_{ij}=1$
$m_{ij}=1$


U
$m_{ij}=1$
Q_i :
-1
0
0
+1
$m_{ij}=2$
$m_{ij}=1$
$m_{ij}=1$
O



$g = 0 \qquad U \gg J > 0$;

$|\{m_i\}\rangle$ eigenstate

ground state $|\{m_i = 0\}\rangle$

$H_{J=g=0}$

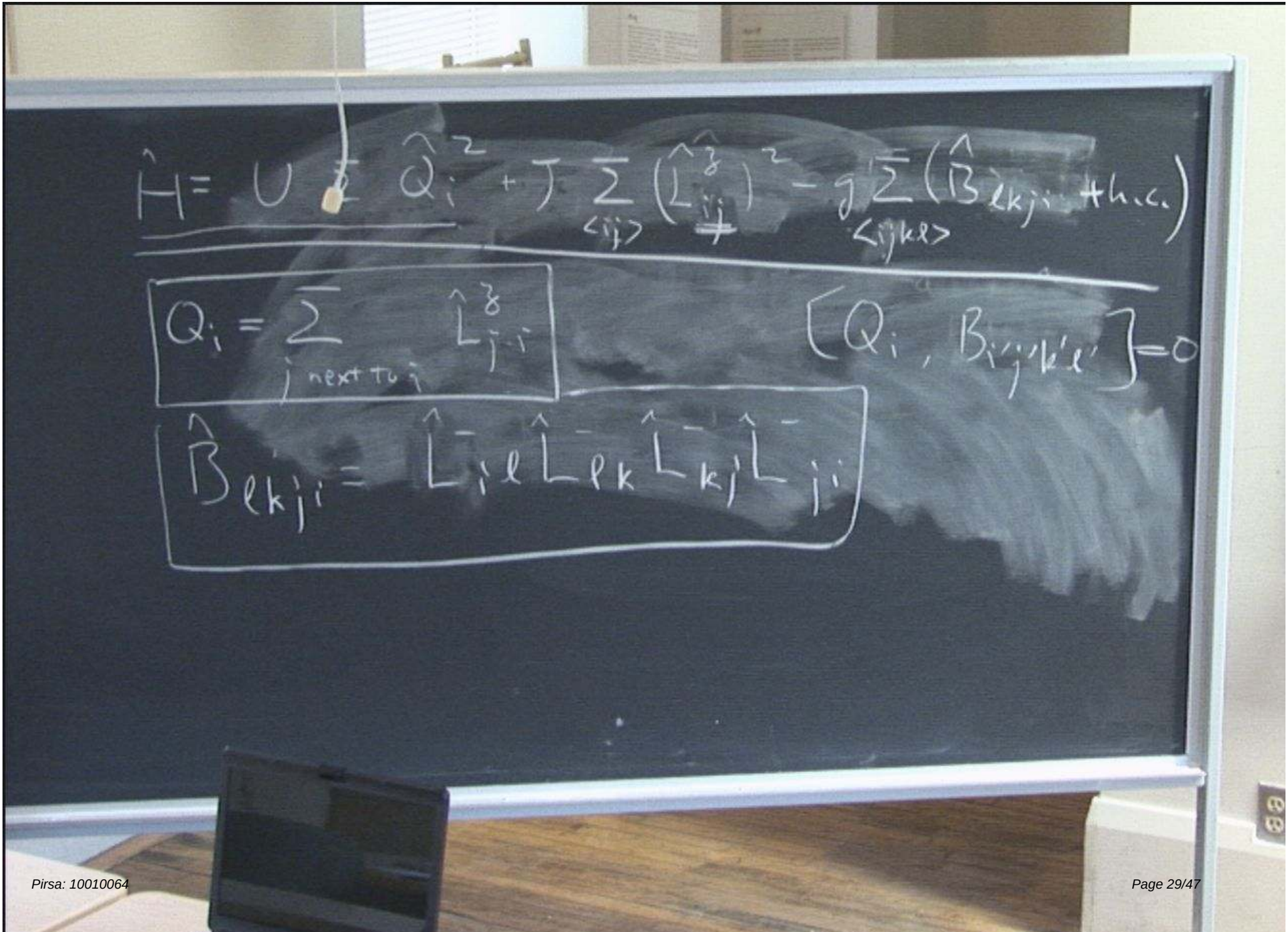
$\bar{J} = 0$: $|\text{loop}\rangle$, E_0

2U open strings

2U

loops

$\bar{J} > 0$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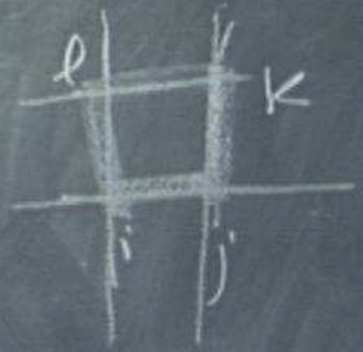
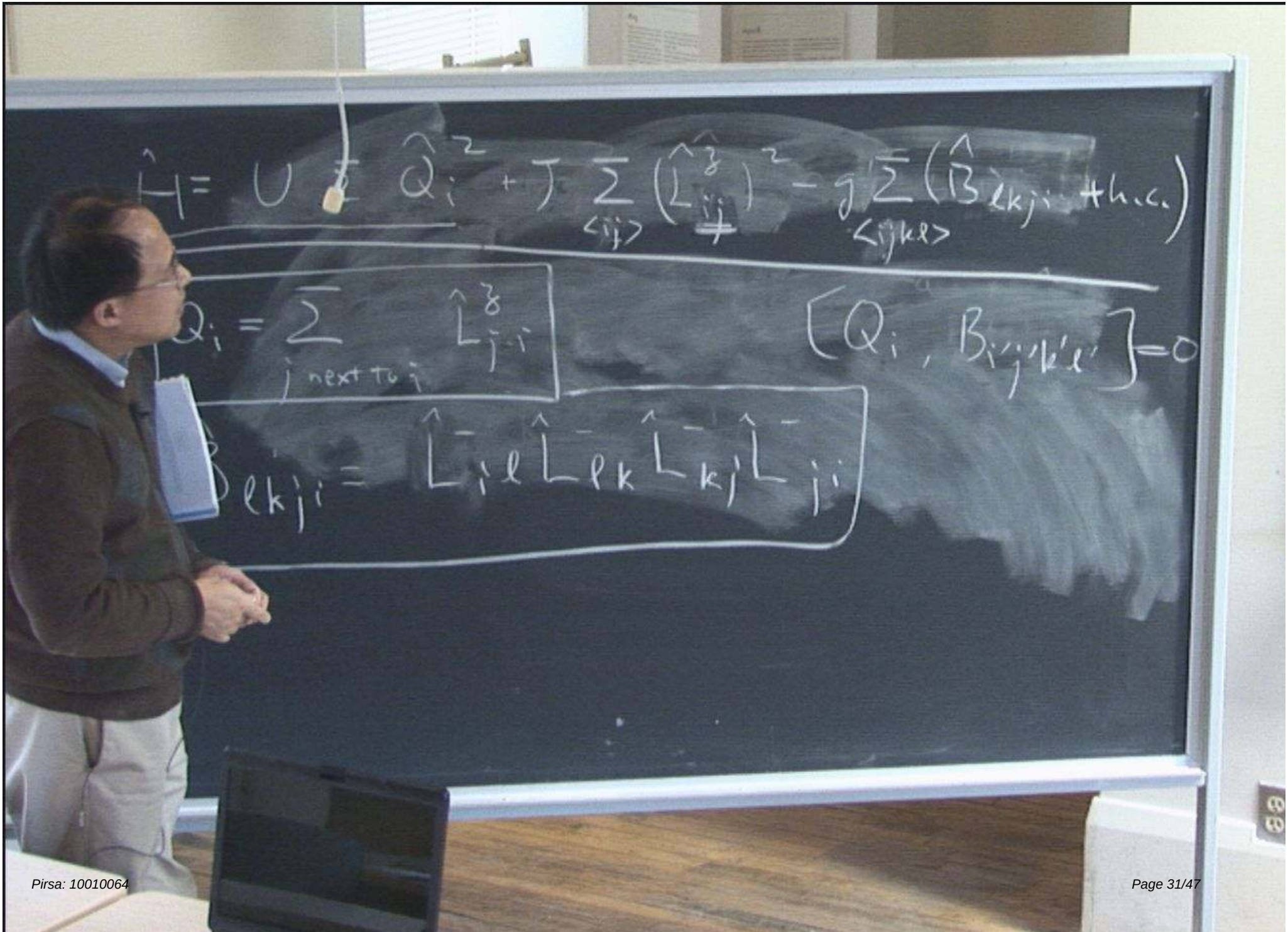
$Q_i = \sum_{j \text{ next to } i} \hat{L}^z_{ji}$
$[Q_i, B_{ijkl}] = 0$
$\hat{B}_{lkji} = \hat{L}^-_{il} \hat{L}^-_{lk} \hat{L}^-_{kj} \hat{L}^-_{ji}$

$J=0 \quad \partial \neq 0$

$B^{\dagger}_{\ell k j i}$ create a loop

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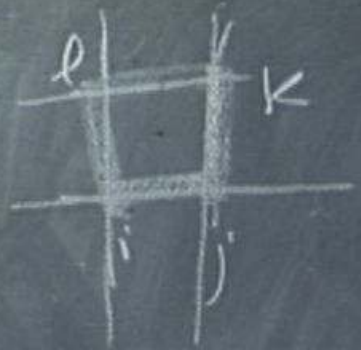
$Q_i = \sum_{j \text{ next to } i} L^z_{ji}$
$[Q_i, B_{i'j'k'l'}] = 0$
$B_{lkji} = L^-_{il} L^-_{lk} L^-_{kj} L^-_{ji}$

$J=0 \quad \mathcal{J}\neq 0$

$B^{\dagger}_{\ell k j}$ create a loop



$B_{ijk\ell}|X_{loop}>$

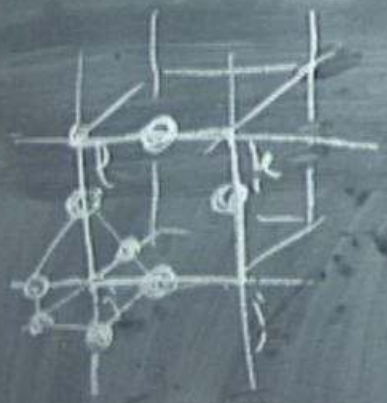
$J=0 \quad \partial \neq 0$

$B^{\dagger}_{\ell k j i}$ create a loop

$B_{ijk\ell}|X_{loop}\rangle = |X'_{loop}\rangle$

$X_{loop} = \phi$ $X'_{loop} =$

$X_{loop} =$ $X'_{loop} =$



$g = 0 \qquad U \gg J > 0$;

$|\{m_i\}\rangle$ eigenstate

ground state $|\{m_i = 0\}\rangle$

$J = g = 0$

$J = 0 \qquad g = 0$

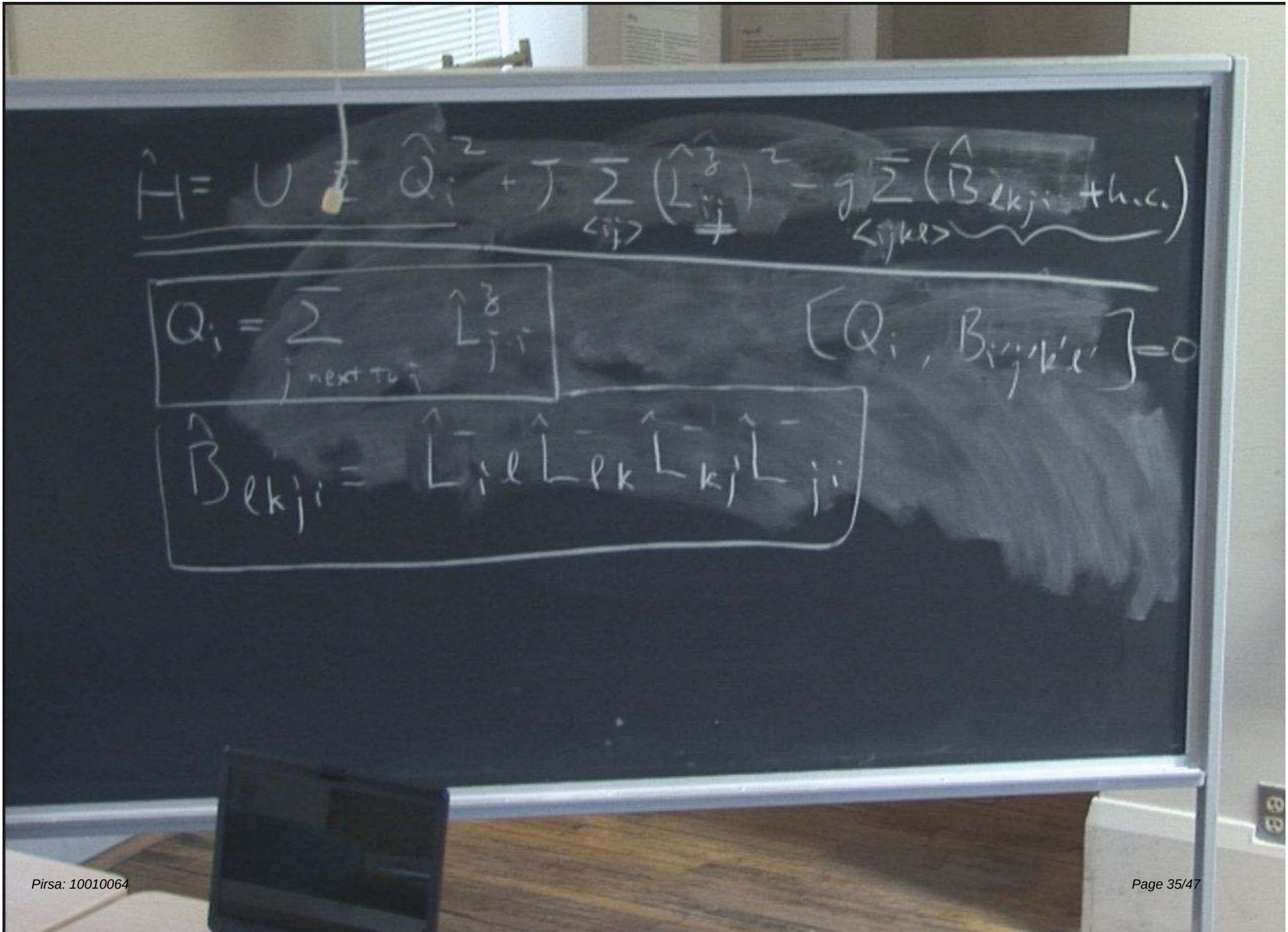
12U open strings

2U

loops

ground stat = $\sum_{X_{loop}} |X_{loop}\rangle$

$B_{ijkl} = 1$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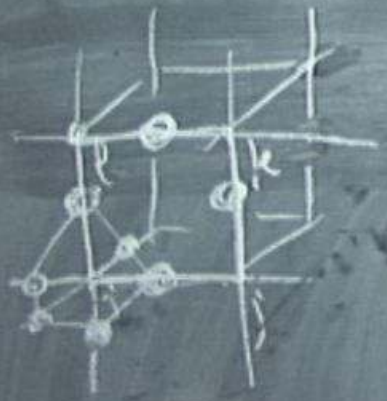
$[Q_i, B_{ijkl}] = 0$
$\hat{B}_{lkji} = \hat{L}^-_{il} \hat{L}^-_{lk} \hat{L}^-_{kj} \hat{L}^-_{ji}$



$g = 0 \quad \wedge \quad U \gg J > 0$:

$|\{m_i\}\rangle$ eigenstate

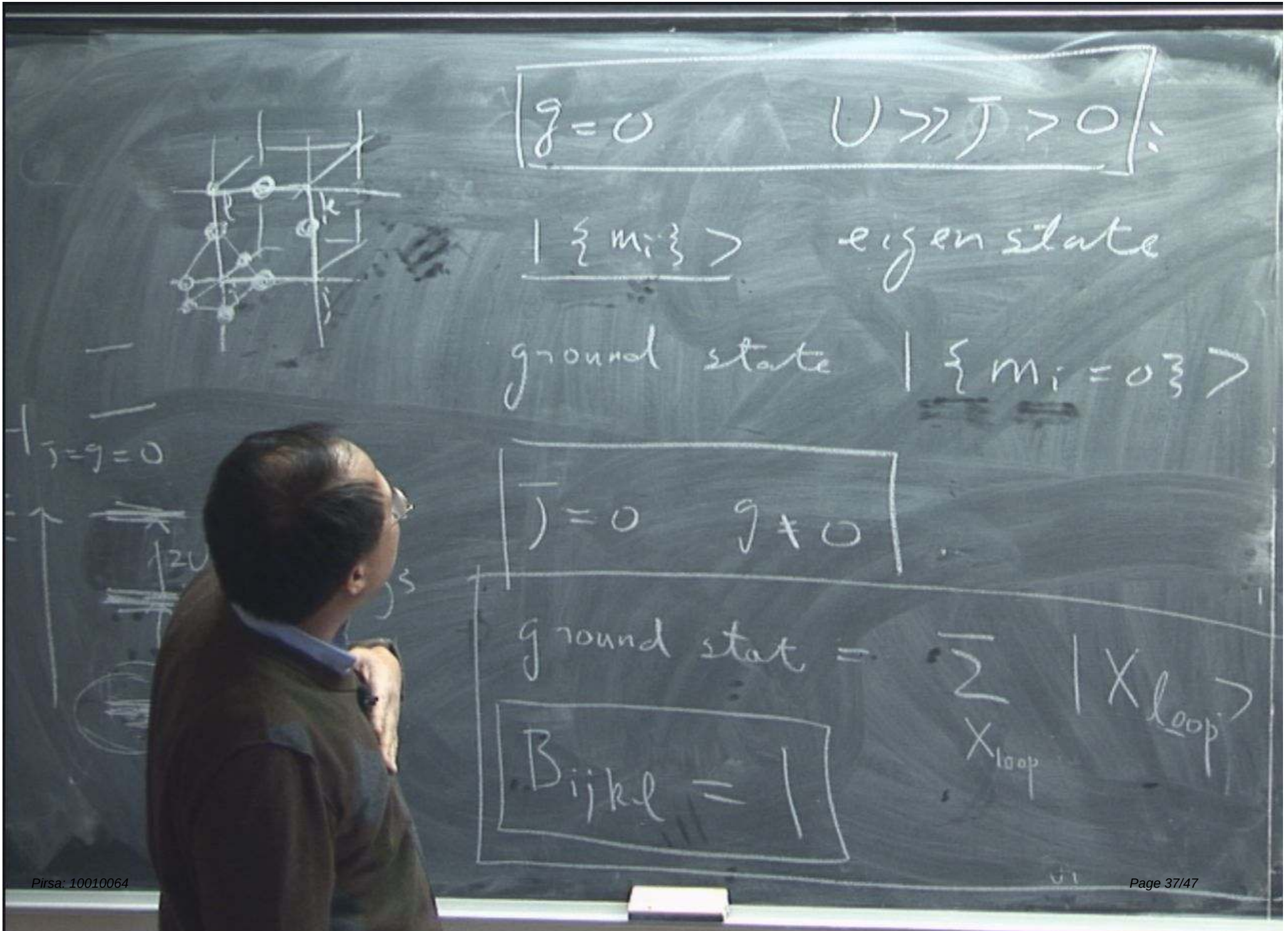
ground state $|\{m_i = 0\}\rangle$

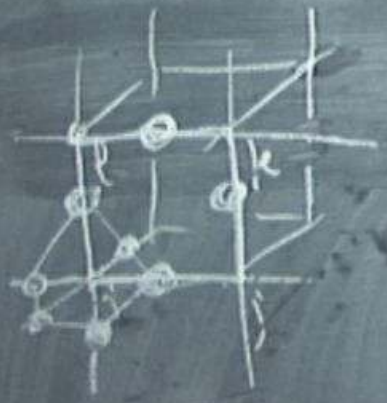
$H_{J=g=0}$ =

$J = 0 \quad g \neq 0$

ground stat = $\sum_{X_{loop}} |X_{loop}\rangle$

$B_{ijkl} = 1$


$g=0 \quad U \gg J > 0$
$|\{m_i\}\rangle$ eigenstate
ground state $|\{m_i=0\}\rangle$
$J=g=0$
$J=0 \quad g \neq 0$
ground stat = $\sum_{X_{loop}} |X_{loop}\rangle$
$B_{ijkl} = 1$

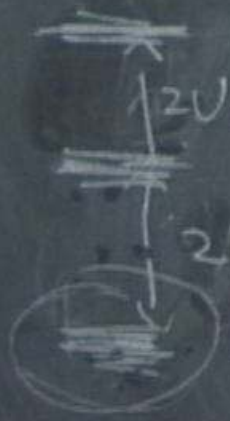


$g = 0 \qquad U \gg J > 0$

$|\{m_i\}\rangle$ eigenstate

ground state $|\{m_i = 0\}\rangle$

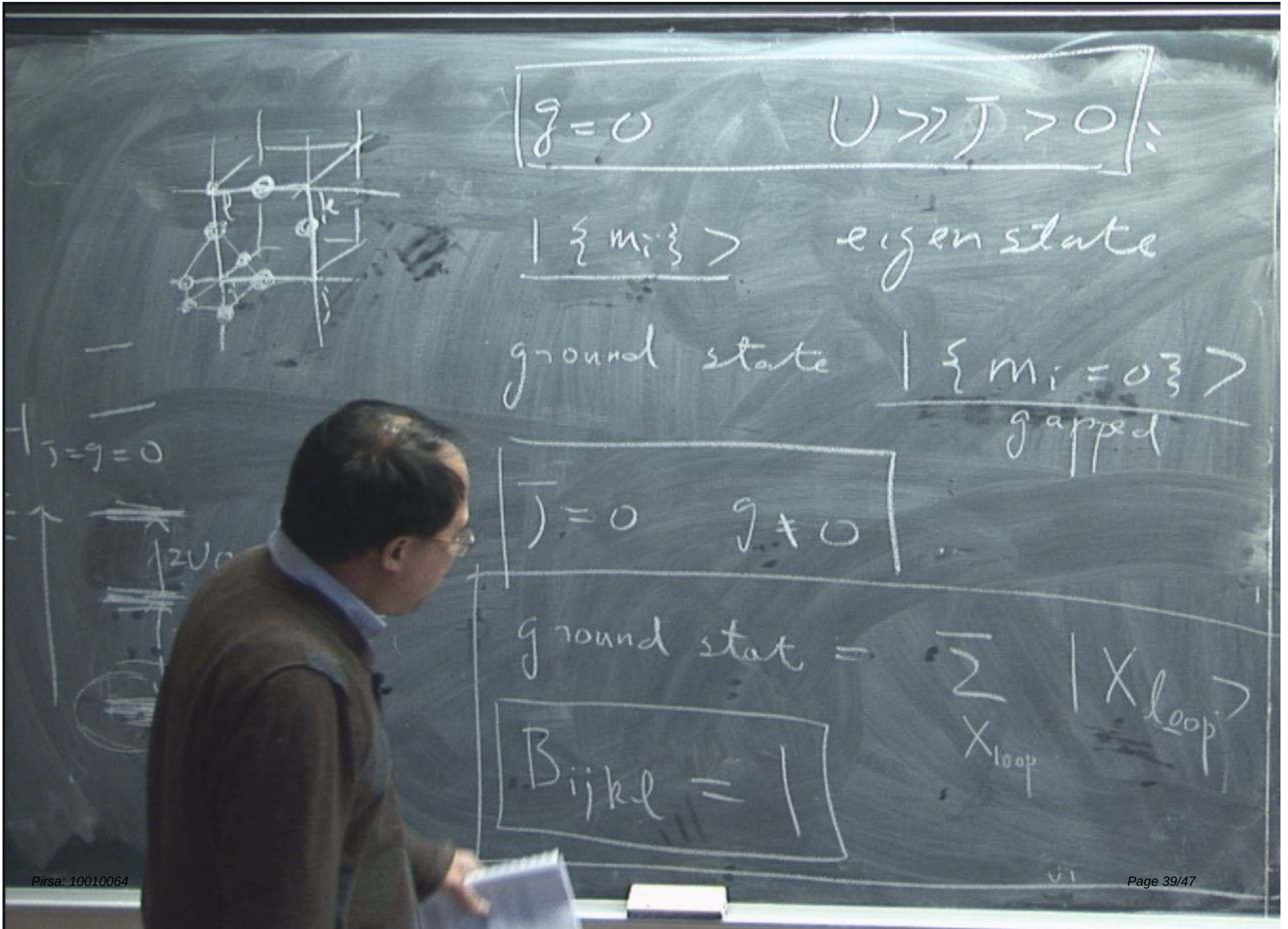
$J = g = 0$



$\bar{J} = 0 \qquad g \neq 0$

ground stat $= \sum_{X_{loop}} |X_{loop}\rangle$

$B_{ijkl} = 1$


$g=0 \quad U \gg J > 0$
$|\{m_i\}\rangle$ eigenstate
ground state $|\{m_i=0\}\rangle$
gapped
$J=0 \quad g \neq 0$
ground stat = $\sum_{X_{loop}} |X_{loop}\rangle$
$B_{ijkl} = 1$

$J=0 \quad \vec{J} \neq 0$

$\overline{Z} \quad |\{m_i\}\rangle$

$\{m_i\}$

$U \gg J > 0$;

$J = 0 \quad g \neq 0$

$> \quad$ eigenstate

state $|\{m_i = 0\}\rangle$

gapped

$\overline{\Sigma} \; |\{m_i\}\rangle$

$\{m_i\}$

$Q_i = 0$

$g \neq 0$ | $g \gg J$??

stat = $\overline{\Sigma} \; |X_{loop}\rangle$

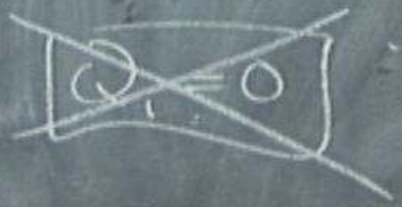
X_{loop}

$Q_i = 0$

$J=0 \quad J\neq 0$

$$\overline{Z} \quad |\{m_i\}\rangle = \otimes_i \left(\sum_{m_i} |m_i\rangle \right)$$

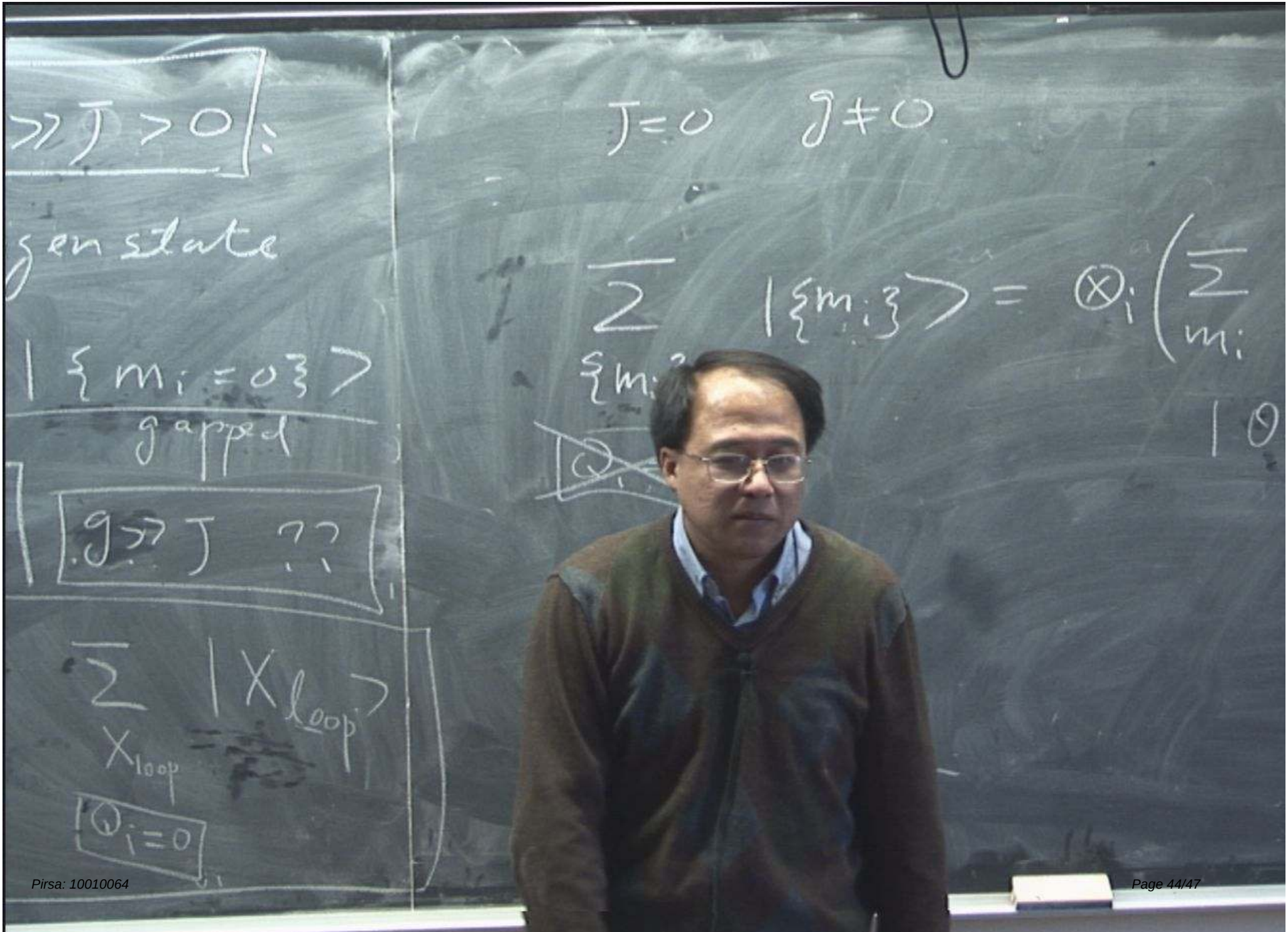
$\{m_i\}$

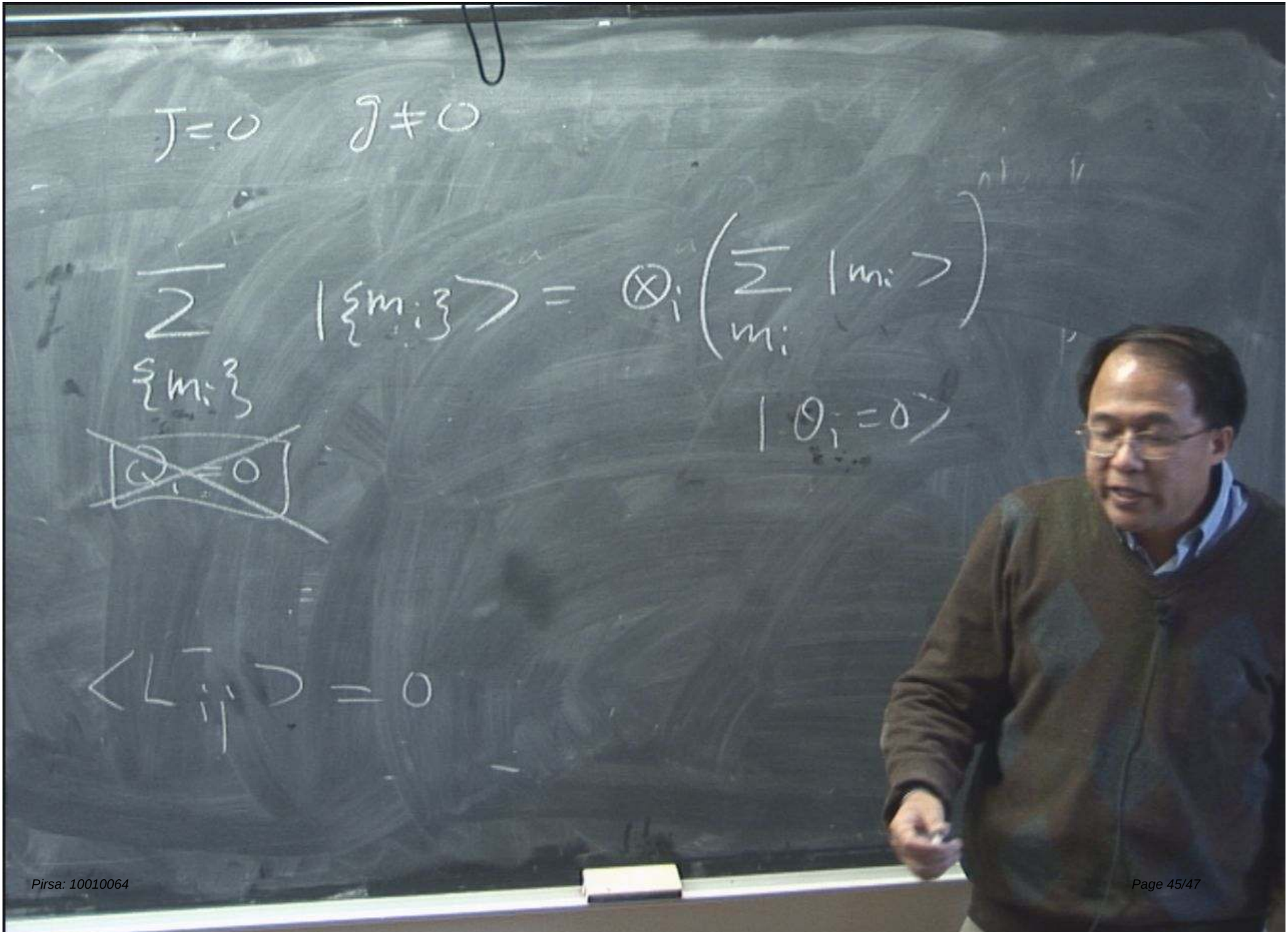


$J = 0 \quad \partial \neq 0$

$$\overline{\sum_{\{m_i\}}} \; |\{m_i\}\rangle = \otimes_i \left(\sum_{m_i} |m_i\rangle \right)$$

$$|\theta_i = 0\rangle$$


J > 0
J = 0 g ≠ 0
gen state
|{m_i = 0}>
gapped
g >> J ??
Σ |X_loop>
X_loop
Q_i = 0
Σ |{m_i}> = ⊗_i (Σ_{m_i}
|0

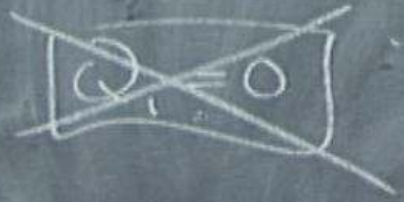

J=0 J≠0
Z
{m_i}
Q_i = 0
<L_ij> = 0

$J = 0 \qquad \partial \neq 0$

$$\overline{Z} \quad |\{m_i\}\rangle = \otimes_i \left(\sum_{m_i} |m_i\rangle \right)$$

$\{m_i\}$

$|\theta_i = 0\rangle$



$$\langle \bar{L}_{ij} \rangle = (\beta_1 + \beta_2^*) \neq 0$$

$J = 0 \qquad J \neq 0$

$$\overline{Z} \qquad |\{m_i\}\rangle = \otimes_i \left(\sum_{m_i} |m_i\rangle \right)$$

$\{m_i\}$

$|\theta_i = 0\rangle$

$\theta_i = 0$

$$\langle L_{ij} \rangle = (\beta_1 + \beta_2^*) \neq 0$$

$= 0$