Title: Gravitational Physics - Review (PHYS 636) - Lecture 2

Date: Jan 04, 2010 11:00 AM

URL: http://pirsa.org/10010049

Abstract:

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Lie Derivative

Uses a vector field to take derivative of other vectors (tensors).

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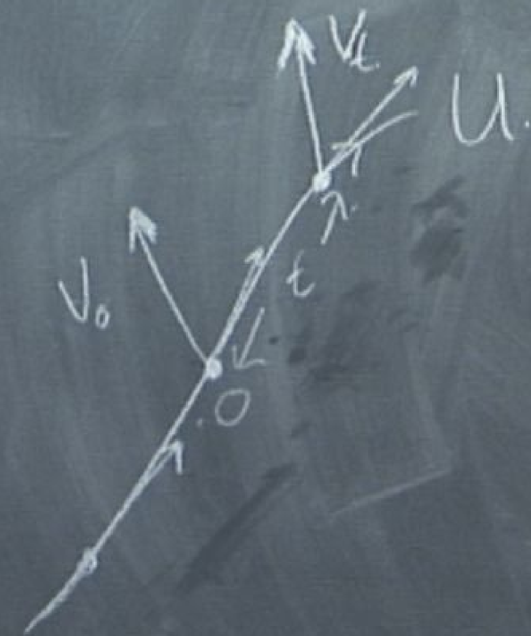
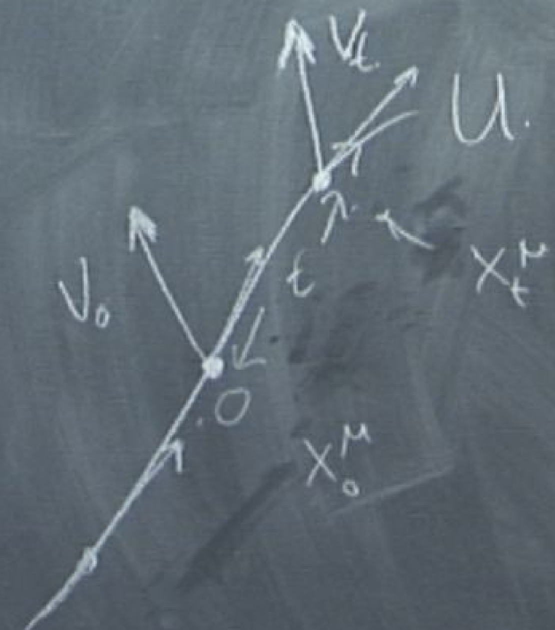


Lie Derivative

Uses a vector field to take derivative of other vectors (tensors).

$$X^\mu_t = X^\mu_0 + t\,U^\mu + O(t^2)$$



$$\left(\mathcal{L}_u V\right)^{\mu} = \lim_{t \to 0} \frac{1}{t} \left[V^{\mu}\left(x_0^{\alpha} + t u^{\alpha}\right) - \frac{\partial x_t^{\mu}}{\partial x_0^{\nu}} V_0^{\nu} \right]$$

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$$= u^\nu V^\mu{}_{,\nu} - u^\mu{}_{,\nu} V^\nu$$

For co-vector: $(\mathcal{L}_V \omega)_\alpha = \omega_{\alpha,\beta} V^\beta + \omega_\beta V^\beta{}_{,}$

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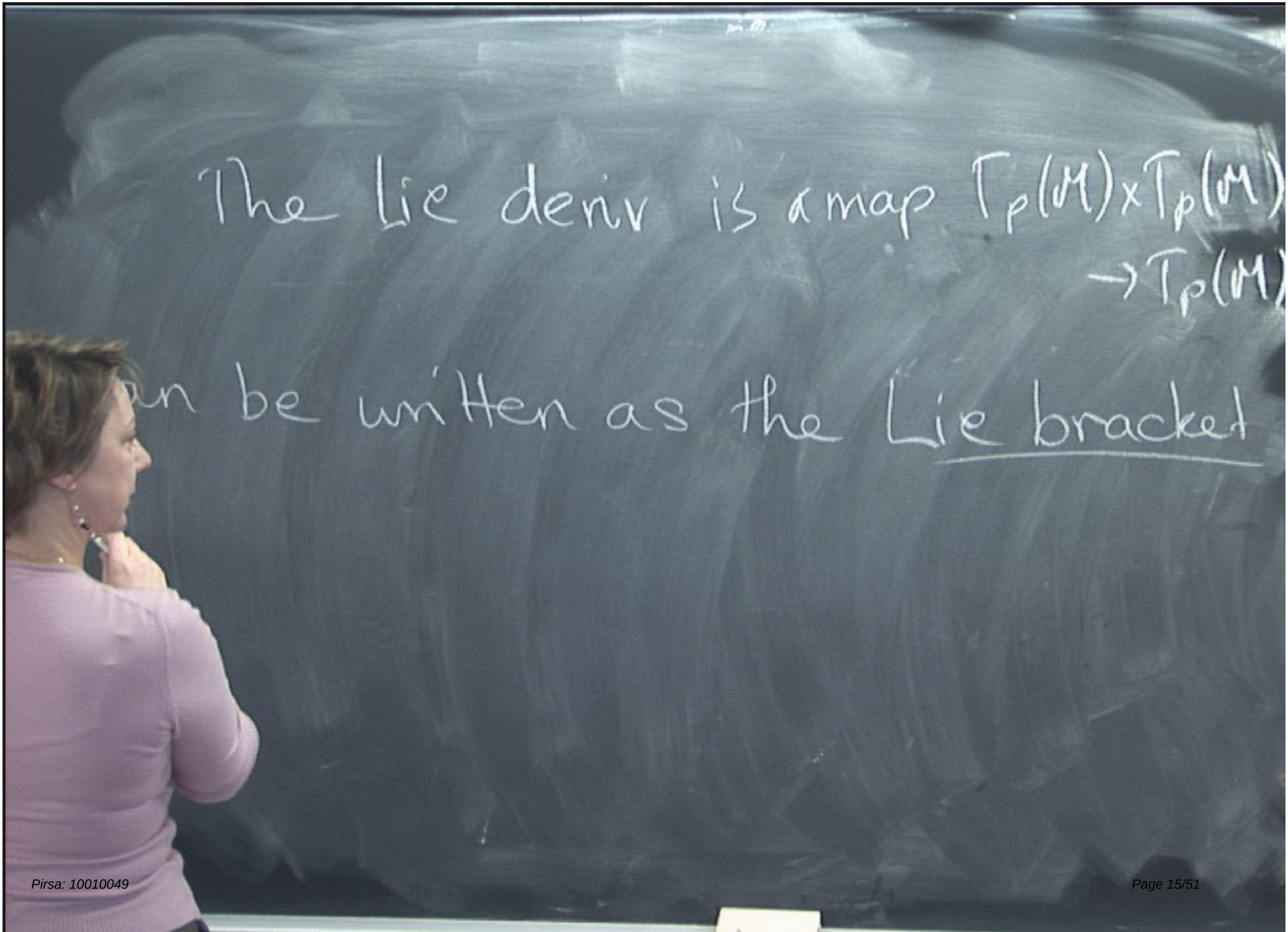
$+$ natural extension for tensors

The Lie deriv is a map $T_p(M) \times T_p(M) \to T_p(M)$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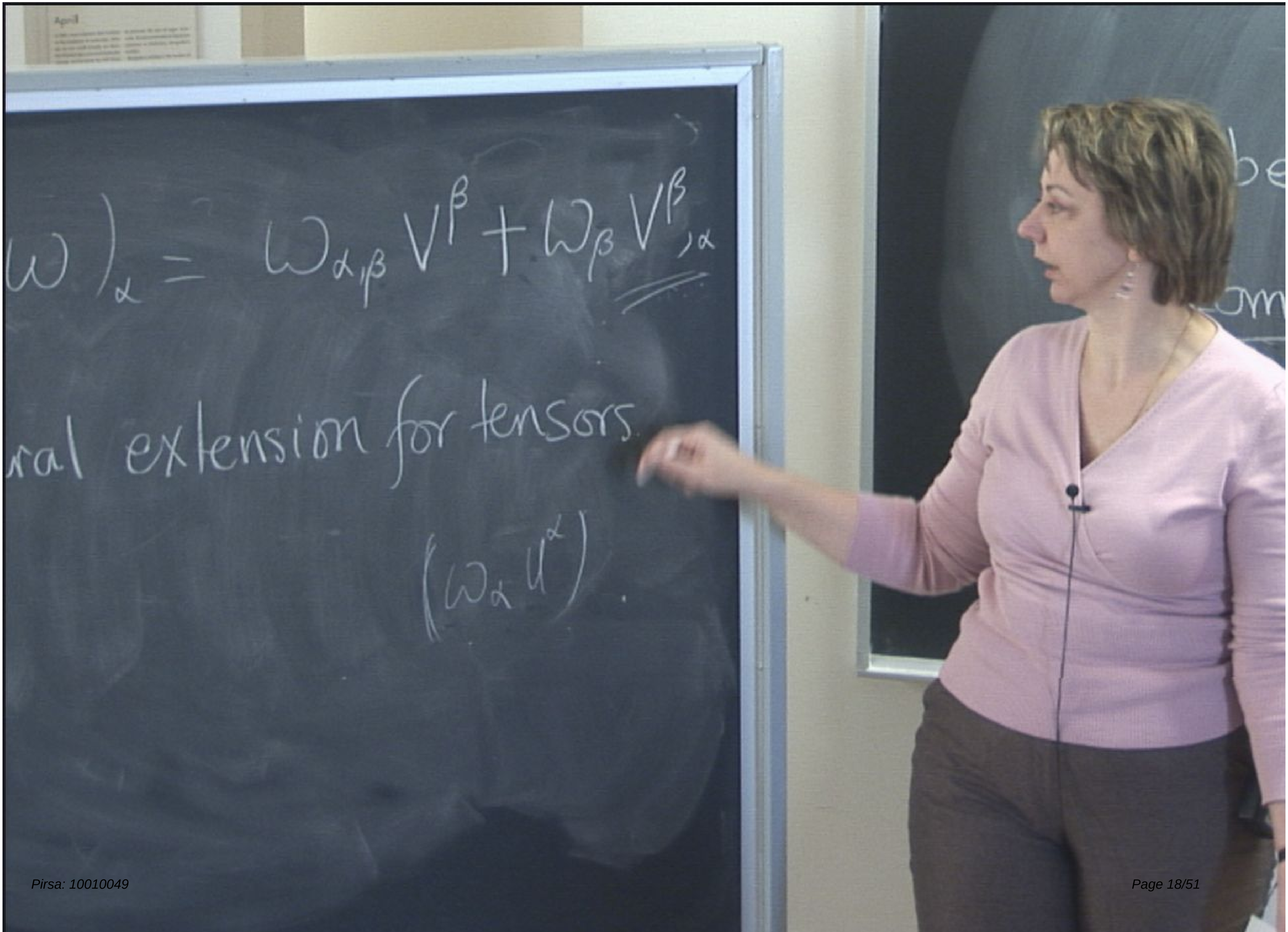
$\omega)_{\alpha} = \omega_{\alpha,\beta} V^{\beta} + \omega_{\beta} V^{\beta}{}_{,\alpha}$
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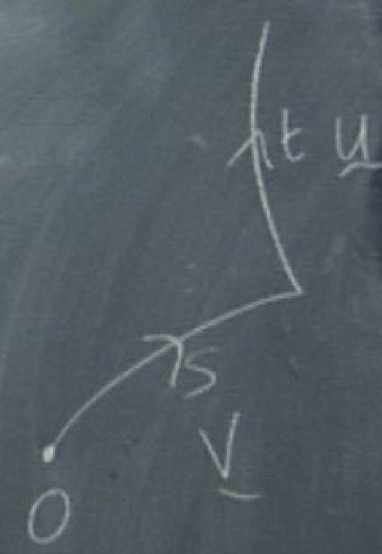

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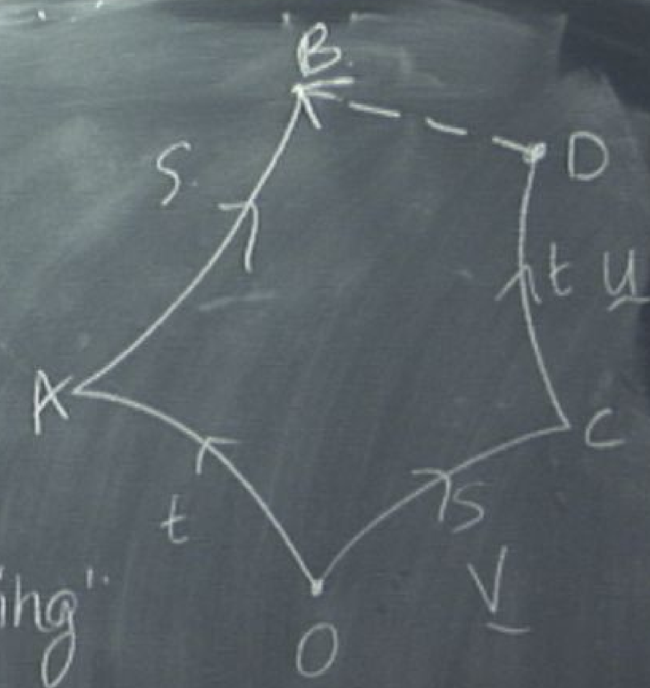

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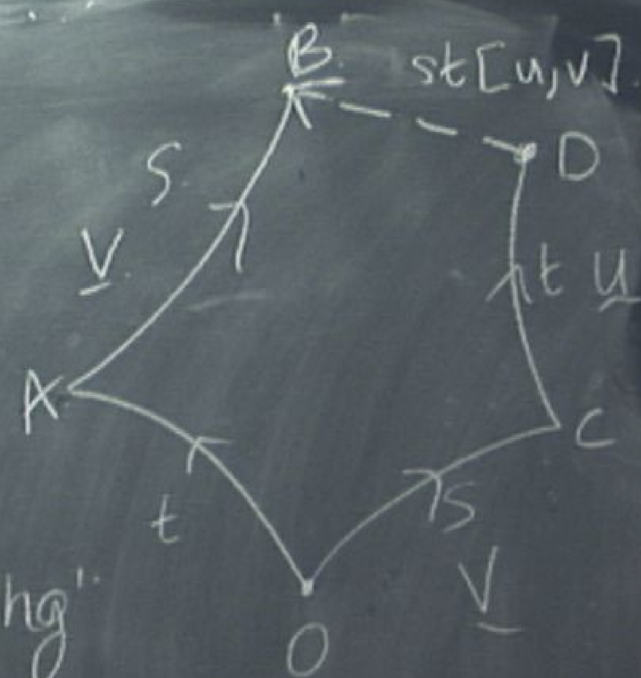

$$X^\alpha_B = X^\alpha_A + s V^\alpha_A + \frac{1}{2} s^2 V^\alpha{}_{,\beta} V^\beta \Big|_A$$

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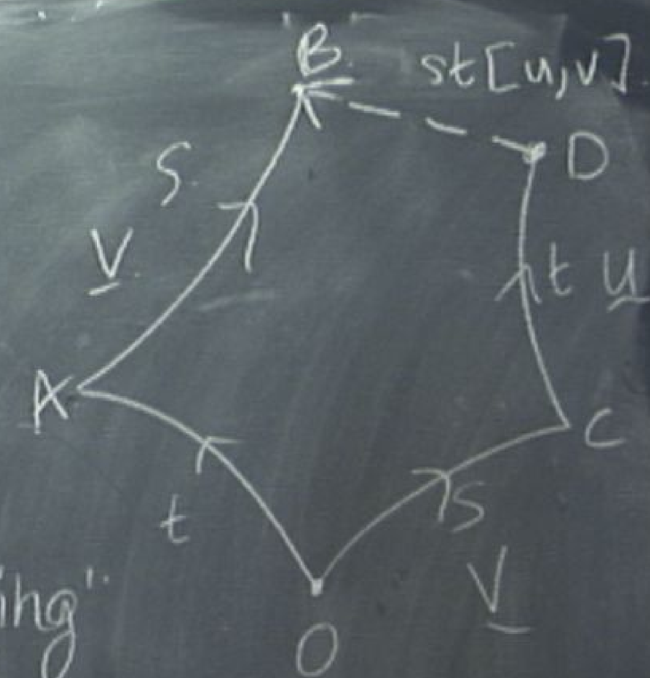

$$X^\alpha_B = X^\alpha_A + s V^\alpha_A + \frac{1}{2} s^2 V^\alpha{}_{,\beta} V^\beta \Big|_A$$

$$= X^\alpha_O + t U^\alpha_O + \frac{1}{2} t^2 U^\alpha{}_{,\beta} U^\beta \Big|_O$$

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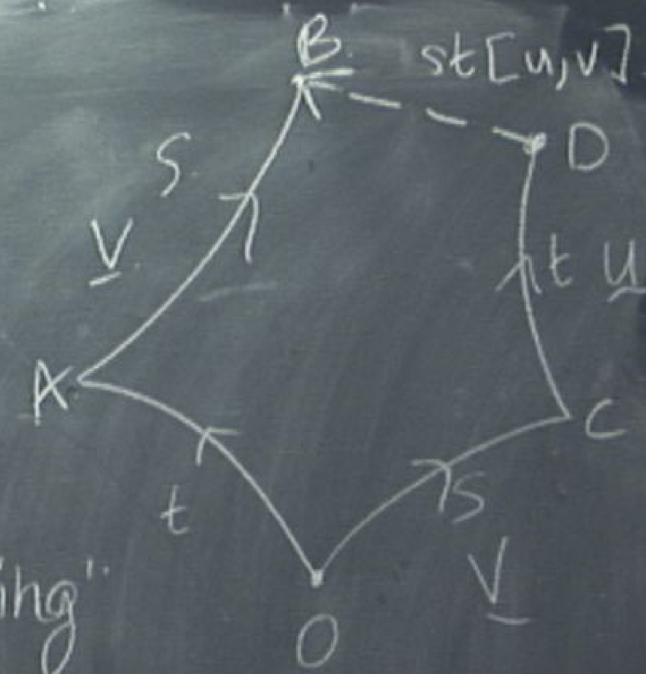
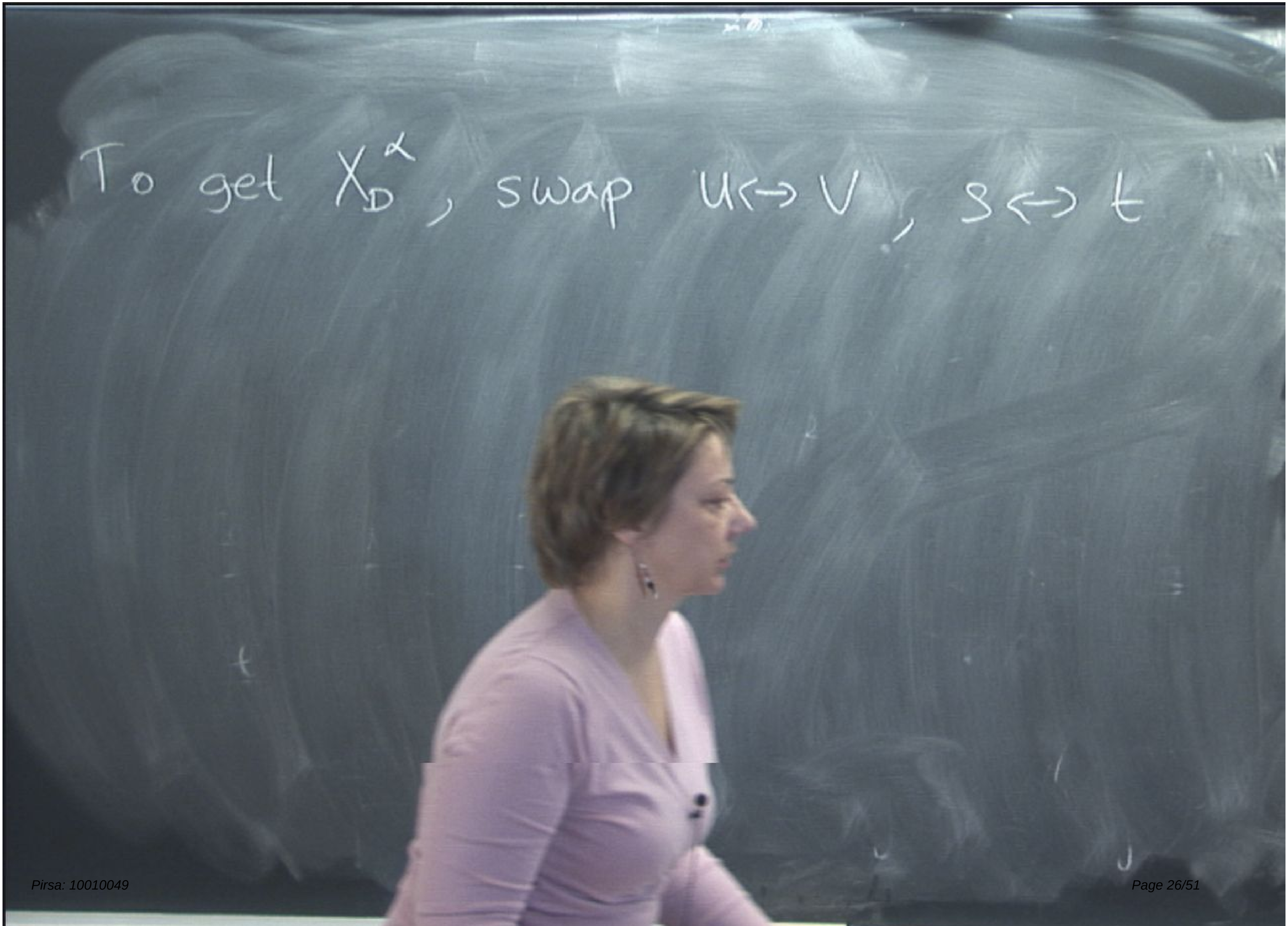


$$X^\alpha_B = X^\alpha_A + s V^\alpha_A + \frac{1}{2} s^2 V^\alpha{}_{,\beta} V^\beta \Big|_A$$

$$= X^\alpha_0 + t U^\alpha_0 + \frac{1}{2} t^2 U^\alpha{}_{,\beta} U^\beta \Big|_0 + s\left(V^\alpha_0 + t V^\alpha{}_{,\beta} U^\beta\right) + \frac{1}{2} s^2 V^\alpha{}_{,\beta} V^\beta \Big|_0$$


To get X_D^α, swap $u \leftrightarrow v$, $s \leftrightarrow t$

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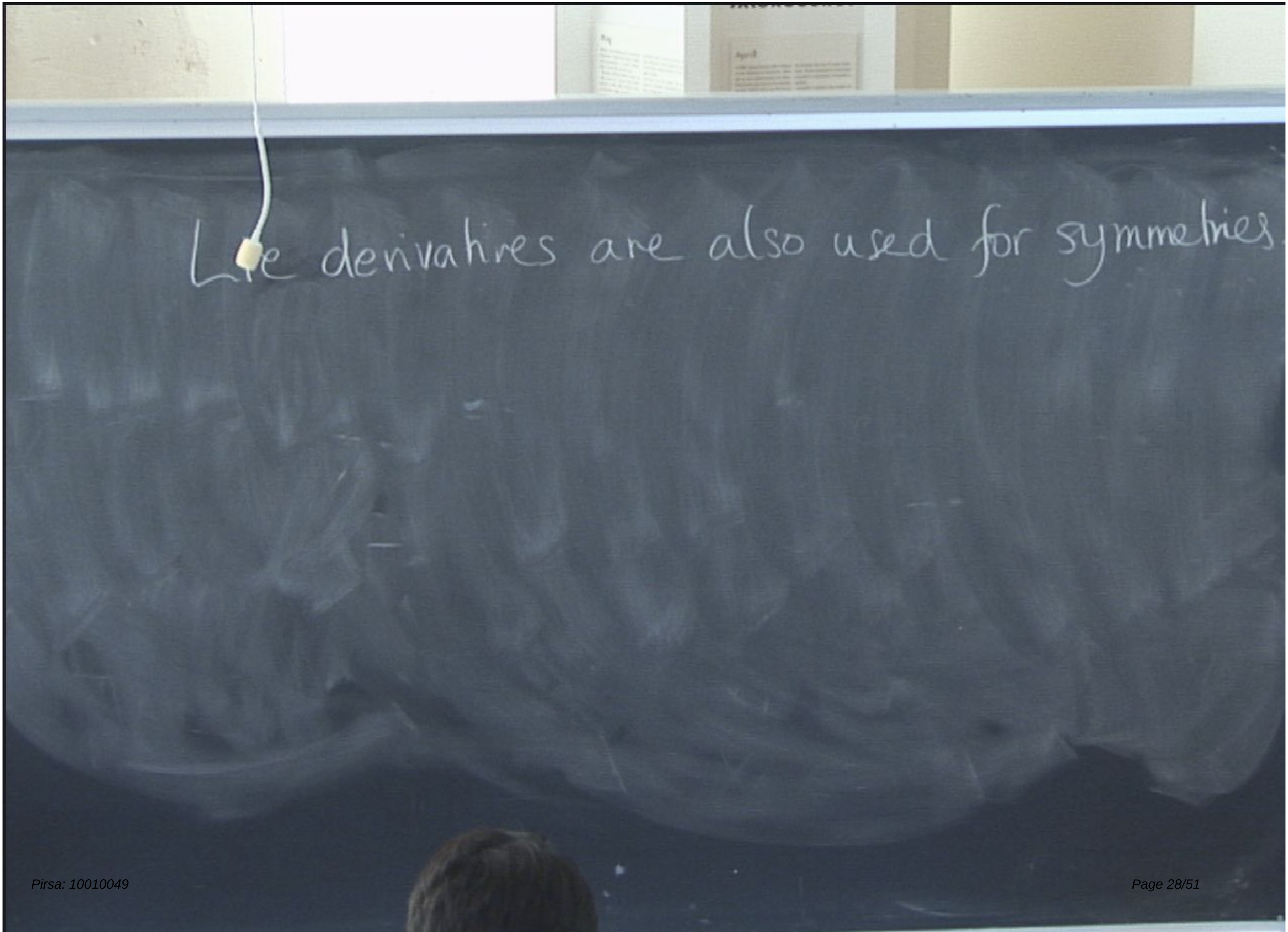
Then

$$X_B^\alpha - X_D^\alpha = st\left[V^\alpha{}_{,\beta} U^\beta - U^\alpha{}_{,\beta} V^\beta\right]$$

$$= st\,[u, v]^\alpha$$


Lie derivatives are also used for symmetries.

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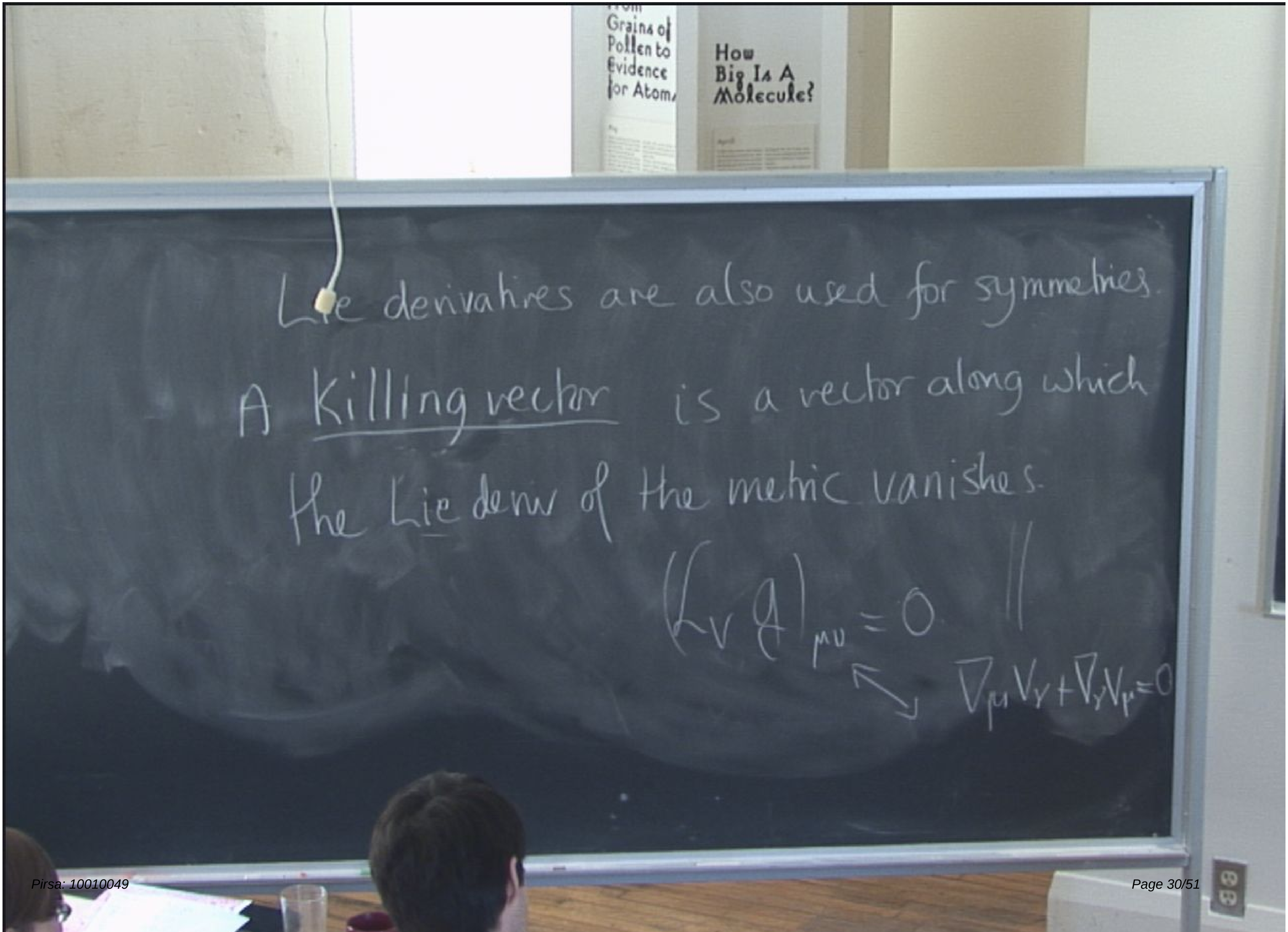
A Killing vector is a vector along which the Lie deriv of the metric vanishes.

$$(\mathcal{L}_V g)_{\mu\nu} = 0.$$


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
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$(\mathcal{L}_V g)_{\mu\nu} = 0$
$\nabla_\mu V_\nu + \nabla_\nu V_\mu = 0$

Forms and Exterior Derivative

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$\underset{\sim}{d}$: Acting on a fn, the <u>ext. deriv.</u> maps the function onto a covector, or 1-form $\underset{\sim}{d} f$.

$$\langle df | X \rangle = X f \quad \text{all } X \in T_p(M)$$

For a co-ord basis:

$$\langle df | \frac{\partial}{\partial x^\mu} \rangle = \frac{\partial f}{\partial x^\mu}$$

niv.

$T_p(M)$

niv.

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A p-form is an antisymmetric tensor of rank $(0,p)$ ("p indices "downstairs"")

$T_p(M)$

niv.

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For a co-ord basis:

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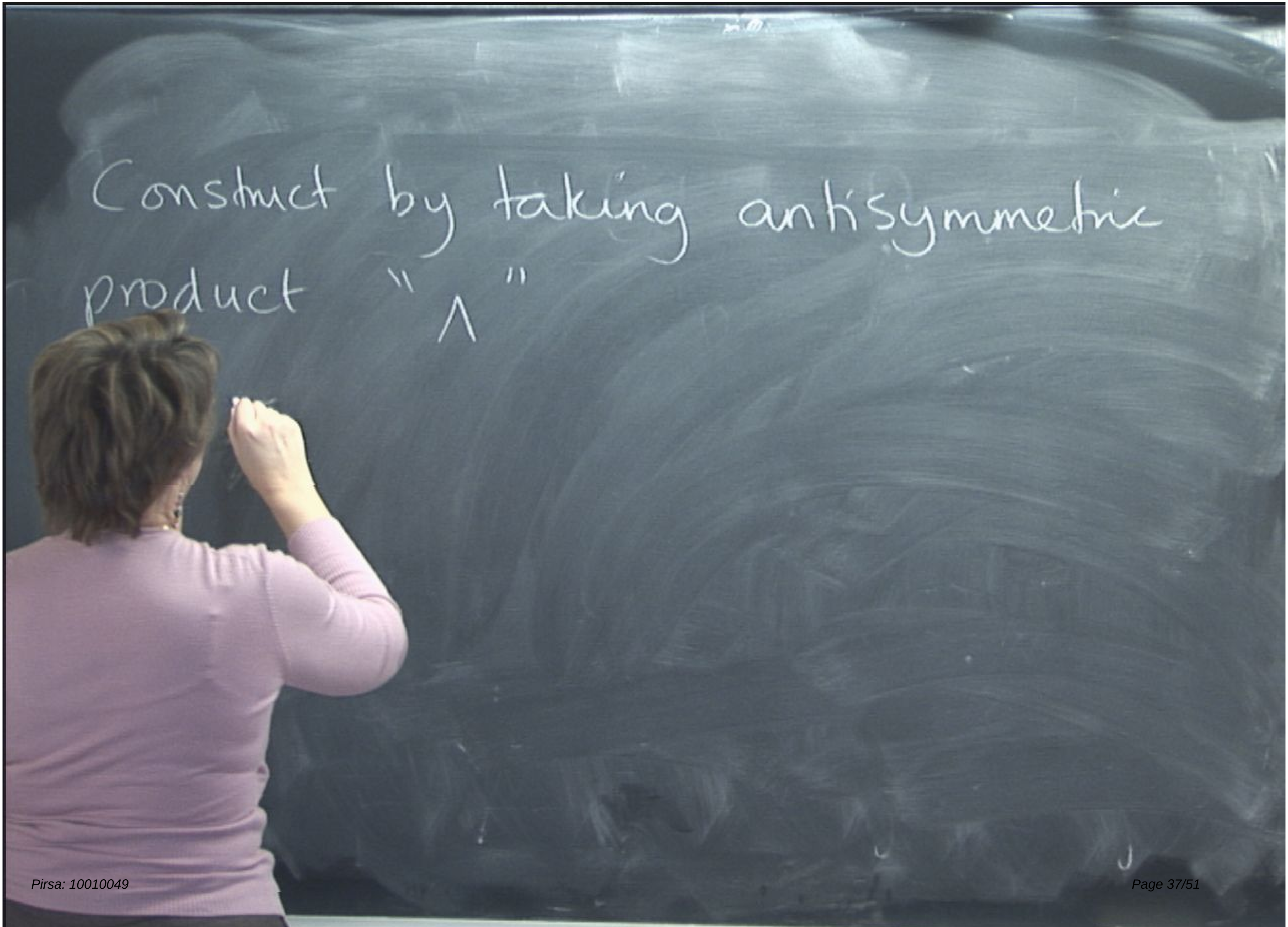
A p-form is an antisymmetric tensor of rank $(0,p)$ ("p indices "downstairs"), write the space of p-forms as $\bigwedge^p T_q^*(M)$


Construct by taking antisymmetric
product "∧"

Construct by taking antisymmetric product "$\wedge$"

$$A \wedge B = A \otimes B - B \otimes A.$$

$$(A \wedge B)_{\mu\nu} = A_\mu B_\nu - A_\nu B_\mu.$$

For a p-form and a q-form

$$\left(A^{(p)} \wedge B^{(q)}\right)_{a_1 \dots a_{p+q}} = \frac{(p+q)!}{p!\,q!} A_{[a_1 \dots a_p} B_{a_{p+}$$

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notation is taking all permutations of indices, weighted by sign of permutations

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notation is taking all permutations of indices, weighted by sign of permutations

$$Q_{[\mu\nu]} = \frac{1}{2}(Q_{\mu\nu} - Q_{\nu\mu})$$

$$Q_{(\mu\nu)} = \frac{1}{2}(Q_{\mu\nu} + Q_{\nu\mu})$$

"$\wedge$" is linear but not commutative.

$$A^{(p)} \wedge B^{(q)} = (-)^{pq} B \wedge A$$

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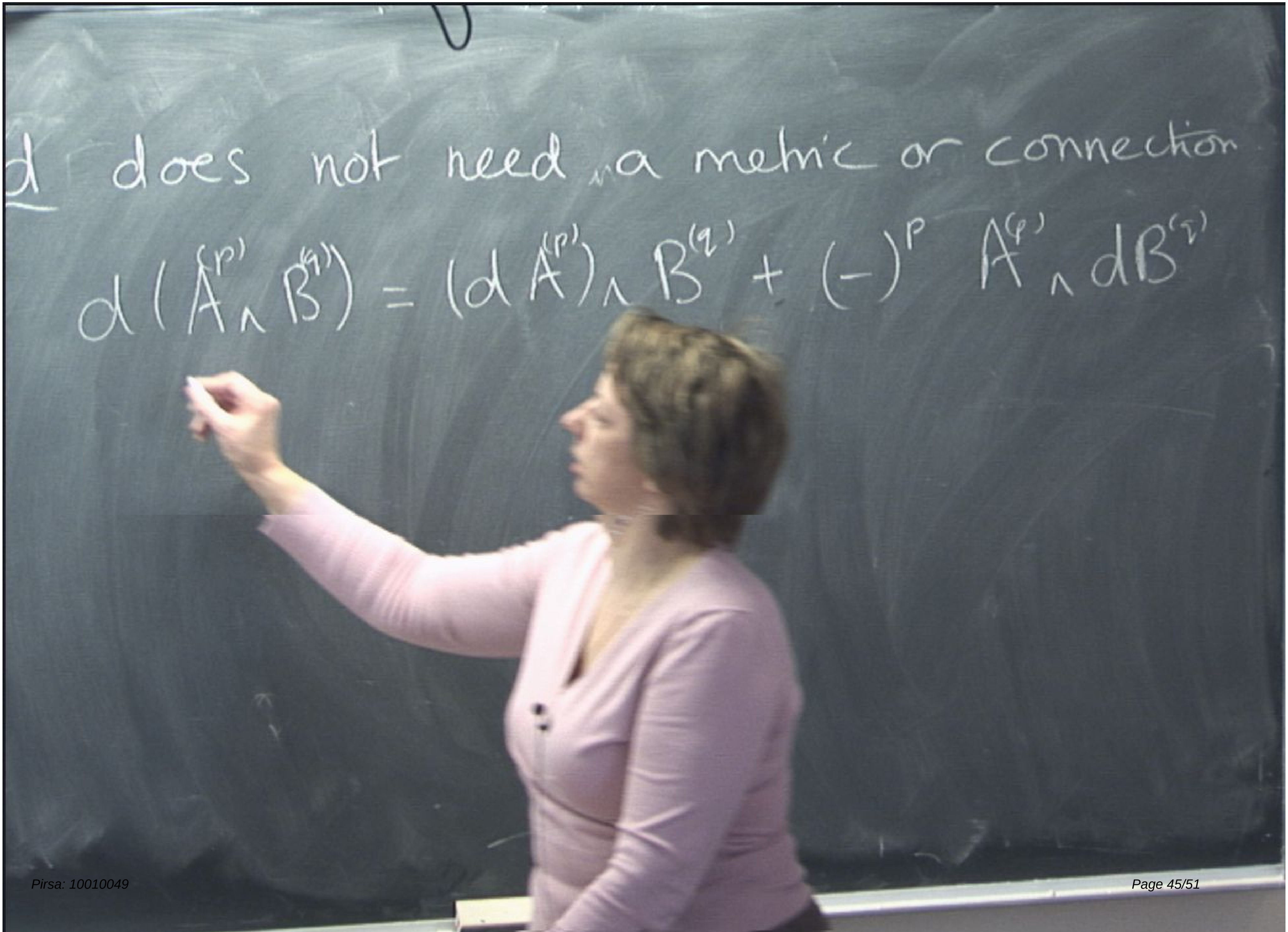
$$A_{a_1 \ldots a_p} \to (dA)_{a_1 \ldots a_{p+1}} = \frac{(p+1)!}{p!} \partial_{[a_1} A_{a_2 \ldots a_{p+1}]}$$


d does not need a metric or connection
$d(A^{(p)} \wedge B^{(q)}) = (dA^{(p)}) \wedge B^{(q)} + (-)^p A^{(p)} \wedge dB^{(q)}$

$\underline{d}$ does not need a metric or connection

$$\underline{d}\left(A^{(p)} \wedge B^{(q)}\right) = \left(\underline{d} A^{(p)}\right) \wedge B^{(q)} + (-)^{p} A^{(p)} \wedge \underline{d} B^{(q)}$$

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$$\underset{\sim}{d}\left(A^{(p)} \wedge B^{(q)}\right) = \left(\underset{\sim}{d} A^{(p)}\right) \wedge B^{(q)} + (-)^{p} A^{(p)} \wedge \underset{\sim}{d}$$

Relation between $\underset{\sim}{d}$ and $\mathcal{L}$:

$$\langle \underset{\sim}{d}\underset{\sim}{\omega} \mid \underset{\sim}{u}, \underset{\sim}{v} \rangle = \underset{\sim}{u} \langle \underset{\sim}{\omega} \mid \underset{\sim}{v} \rangle - \underset{\sim}{v} \langle \underset{\sim}{\omega} \mid \underset{\sim}{u} \rangle$$
$$- \langle \underset{\sim}{\omega} \mid [\underset{\sim}{u}, v$$

$(\phi, \underline{A}) \sim A_\mu \Leftrightarrow$ e-m gauge pot.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

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$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\left(\begin{array}{c|ccc} 0 & E_1 & E_2 & E_3 \\ \hline & 0 & B_3 & -B_2 \\ & & 0 & B_1 \\ & & & 0 \end{array}\right)$$

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