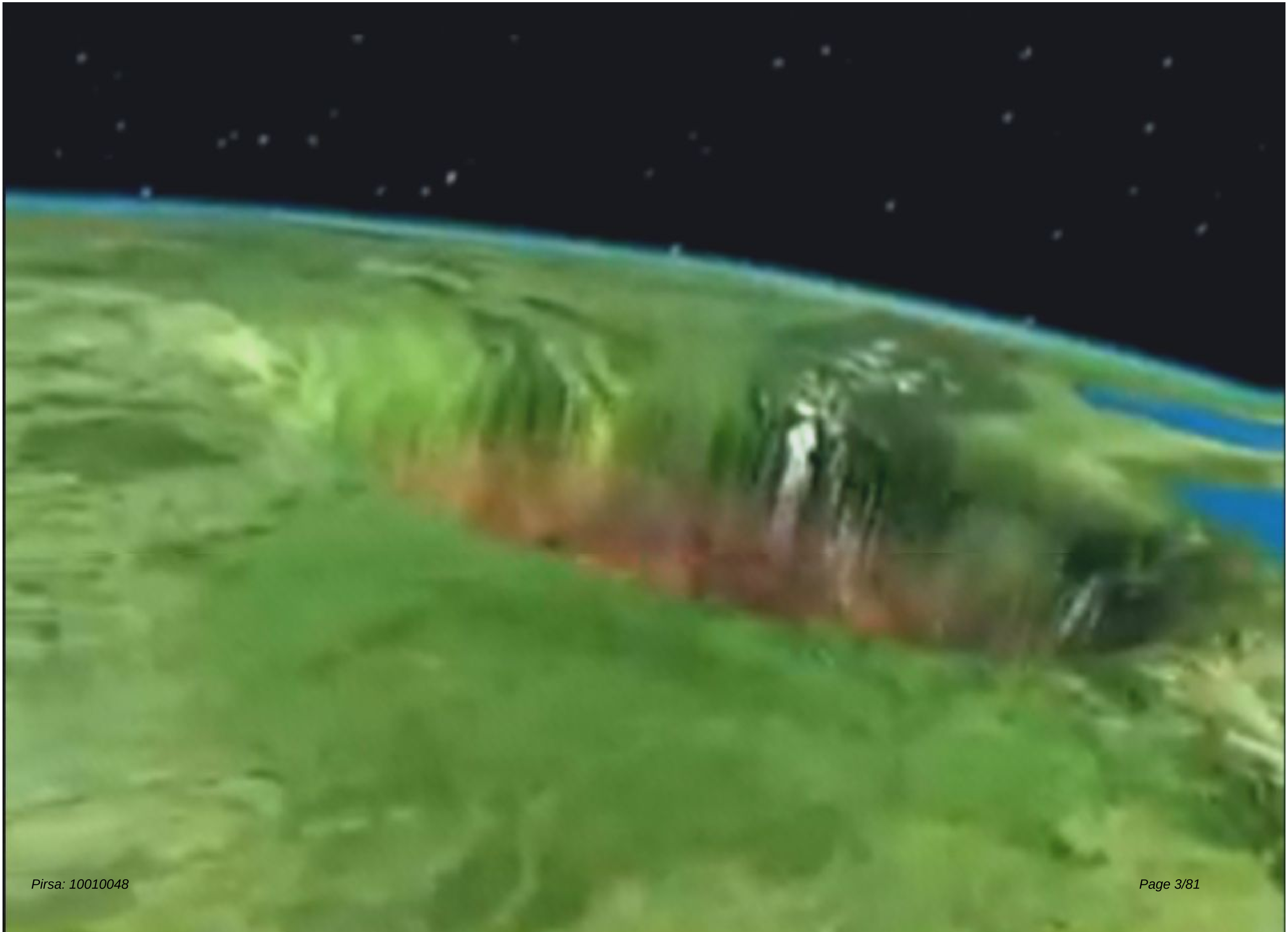
Title: Gravitational Physics - Review (PHYS 636) - Lecture 15

Date: Jan 22, 2010 10:00 AM

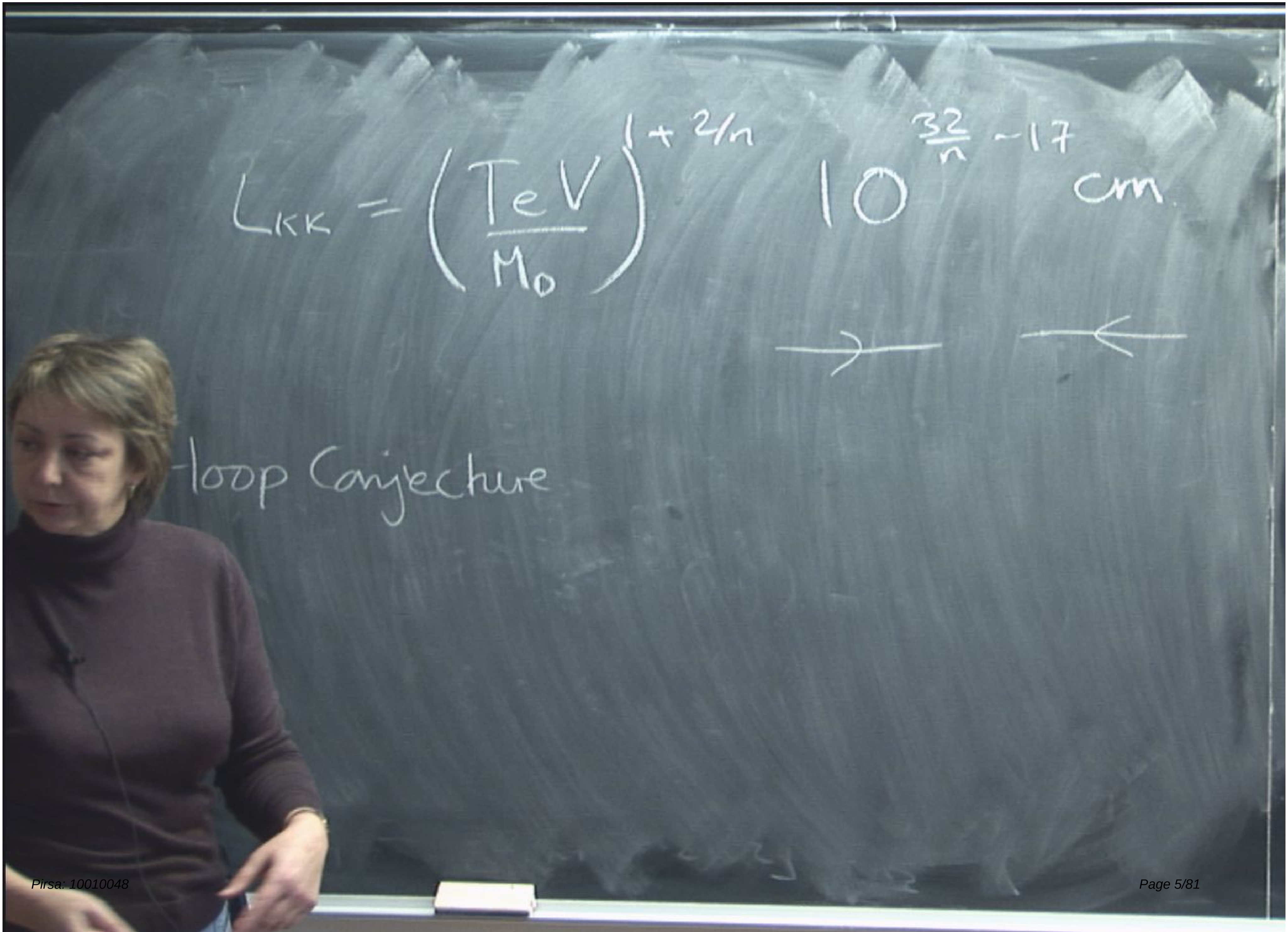
URL: http://pirsa.org/10010048

Abstract:

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS



$$L_{KK} = \left(\frac{\mathrm{TeV}}{M_D}\right)^{1+2/n} \quad 10^{\frac{32}{n}-17} \mathrm{cm}$$



$L_{KK} = \left(\frac{TeV}{M_0}\right)^{1+2/n}$
$10^{\frac{32}{n}-17}$ cm
-loop Conjecture

$$L_{KK} = \left(\frac{\mathrm{TeV}}{M_D}\right)^{1+2/n} 10^{\frac{32}{n}-17}\ \mathrm{cm}$$

$\rightarrow$ —— —— $\leftarrow$

Hoop Conjecture

If mass is localized to within $2GM$, a black hole forms.

$$L_{KK} = \left(\frac{TeV}{M_0}\right)^{1+2/n} \quad 10^{\frac{32}{n}-17} \text{ cm}$$

Hoop Conjecture

If mass is localized to within $2GM$, a black hole forms.

$$L_{KK} = \left(\frac{\text{TeV}}{M_0}\right)^{1+2/n} \qquad 10^{\frac{32}{n}-17}\ \text{cm}$$

Hoop Conjecture

If mass is localized to within $2GM$, a black hole forms.

$$L_{KK} = \left(\frac{\mathrm{TeV}}{M_D}\right)^{1+2/n} 10^{\frac{32}{n}-17}\ \mathrm{cm}$$

<u>Hoop Conjecture</u>

? $v \to c$

If mass is localized to within $2GM$, a black hole forms.

Perturbation Theory in GR

Perturbation Theory in GR

V. useful physically – Cosmological P.T.
Black holes (not mergers)

Perturbation Theory in GR

V. useful physically – Cosmological P.T.
Black holes (not mergers)
"gravitons"

Perturbation Theory in GR

V. useful physically – Cosmological P.T.
Black holes (not mergers)
"gravitons"

We want a more general soln which may not be precisely symmetric.

$$g_{ab} = g_{0ab} + h_{ab}.$$

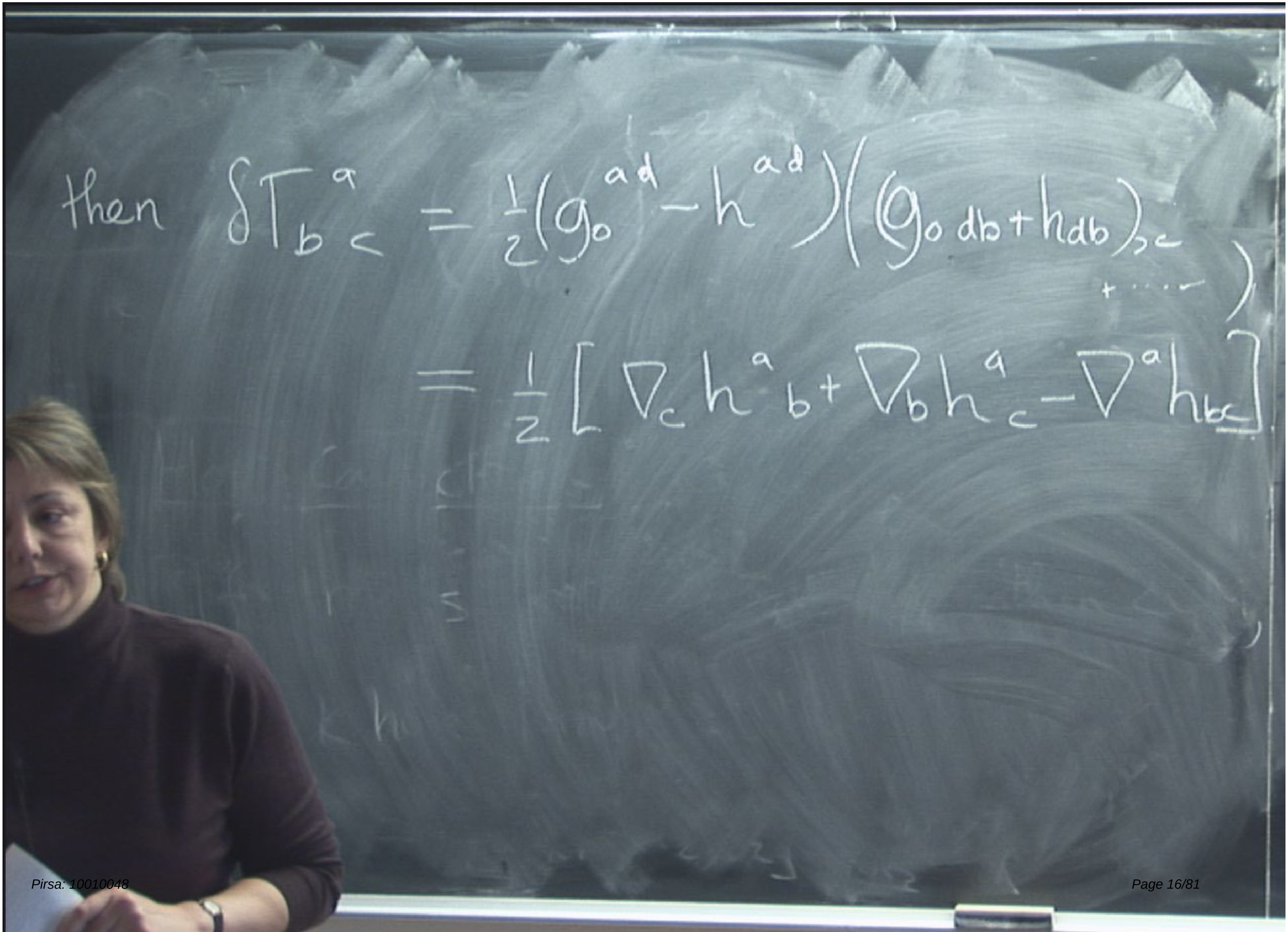
Perturbation Theory in GR

V. useful physically – Cosmological P.T.
Black holes (not mergers)
"gravitons"

We want a more general soln which may not be precisely symmetric,

$$g_{ab} = g_{0ab} + h_{ab}.$$

then $\delta T_b{}^a{}_c = \frac{1}{2}(g_0{}^{ad} - h^{ad})\big((g_{0\,db} + h_{db})_{,c} + \cdots\big)$

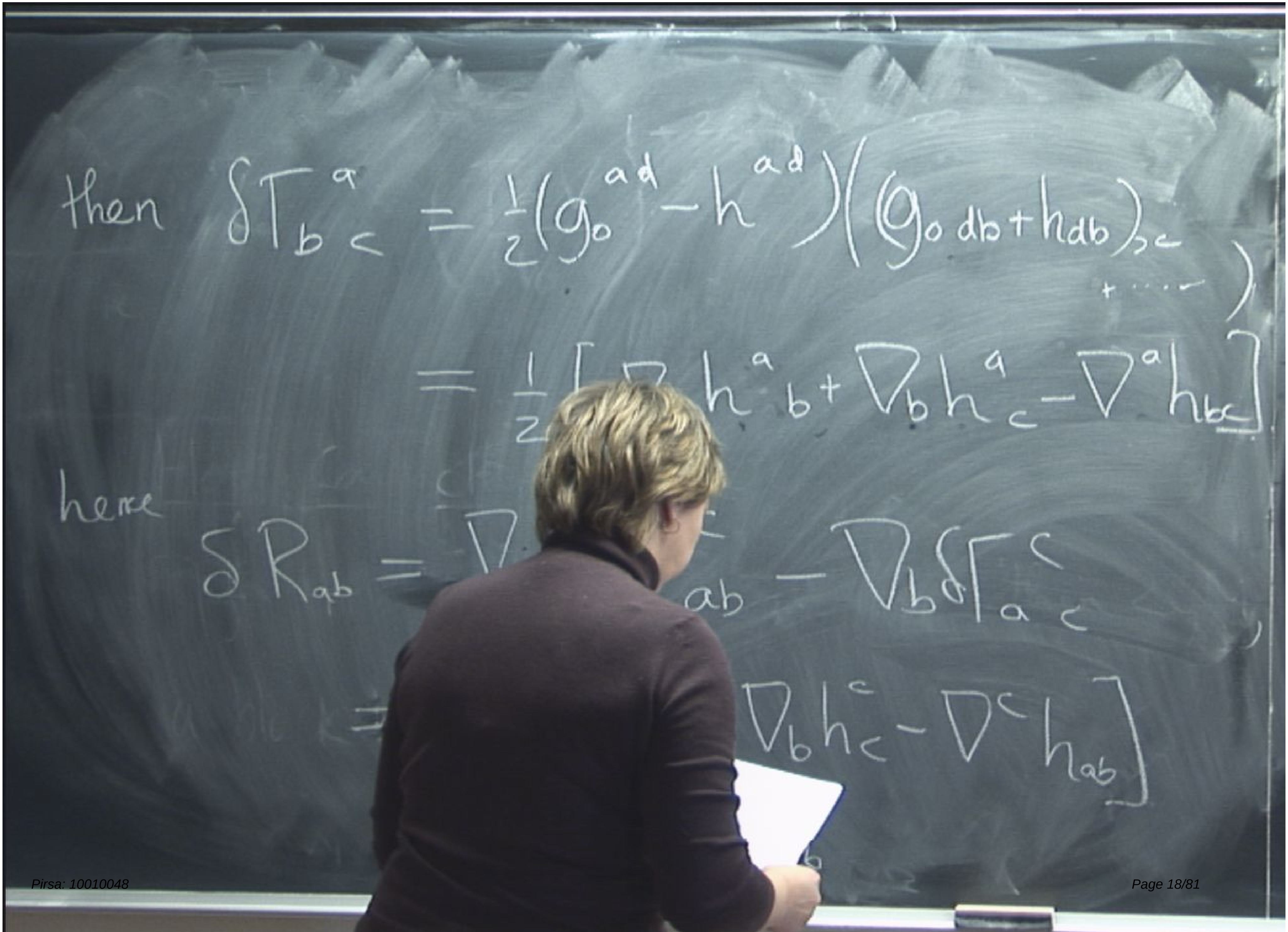

then $\delta \Gamma^{a}_{b\,c} = \frac{1}{2}(g_0^{ad} - h^{ad})\big((g_{0\,db} + h_{db})_{,c} + \cdots\big)$
$= \frac{1}{2}\left[\nabla_c h^{a}{}_{b} + \nabla_b h^{a}{}_{c} - \nabla^{a} h_{bc}\right]$

then $\delta \Gamma_{b}{}^{a}{}_{c} = \frac{1}{2}(g_0{}^{ad} - h^{ad})\left((g_{0\,db} + h_{db})_{,c} + \cdots\right)$

$$= \frac{1}{2}\left[\nabla_c h^a{}_b + \nabla_b h^a{}_c - \nabla^a h_{bc}\right]$$

hence

$$\delta R_{ab} = \nabla_c \delta\Gamma_{ab}^{c} - \nabla_b \delta\Gamma_{a}{}^{c}{}_{c}$$


then $\delta \Gamma_b{}^a{}_c = \frac{1}{2}(g_0{}^{ad} - h^{ad})\big((g_{0\,db} + h_{db})_{,c} + \cdots\big)$
$= \frac{1}{2}\big[\nabla \; h^a{}_b + \nabla_b h^a{}_c - \nabla^a h_{bc}\big]$
hence
$\delta R_{ab} = \nabla \; {}_{ab} - \nabla_b \delta \Gamma_a{}^c{}_c$
$= \; \nabla_b h^c{}_c - \nabla^c h_{ab}\big]$

then $\delta\Gamma_b{}^a{}_c = \frac{1}{2}(g_0{}^{ad} - h^{ad})\left((g_{0\,db} + h_{db})_{,c} + \cdots\right)$

$$= \frac{1}{2}\left[\nabla_c h^a{}_b + \nabla_b h^a{}_c - \nabla^a h_{bc}\right]$$

hence

$$\delta R_{ab} = \nabla_c \delta\Gamma^c_{ab} - \nabla_b \delta\Gamma_a{}^c{}_c$$

$$= \frac{1}{2}\nabla_c\left[\nabla_a h^c{}_b + \nabla_b h^c{}_a - \nabla^c h_{ab}\right] - \frac{1}{2}\nabla_b \nabla_a h^c_c$$

$$= \frac{1}{2}\left(\nabla_c \nabla_a h^c{}_b + \nabla_c \nabla_b h^c{}_a - \Box h_{ab} - \nabla_a \nabla_b h \right)$$

$$= \frac{1}{2}\left(\nabla_c \nabla_a h^c{}_b + \nabla_c \nabla_b h^c{}_a - \Box h_{ab} - \nabla_a \nabla_b h \right)$$

using the Riemann identity:

$$= \frac{1}{2}\left(\nabla_c \nabla_a h^c{}_b + \nabla_c \nabla_b h^c{}_a - \Box h_{ab} - \nabla_a \nabla_b h \right)$$

using the Riemann identity:

$$= \frac{1}{2}\left(\nabla_a \nabla_c h^c{}_b + R^c{}_{dca} h^d{}_b + R_{bdca} h^{cd} \right.$$

$$= \frac{1}{2}\left(\nabla_c \nabla_a h^c{}_b + \nabla_c \nabla_b h^c{}_a - \Box h_{ab} - \nabla_a \nabla_b h \right)$$

using the Riemann identity:

$$= \frac{1}{2}\Big(\nabla_a \nabla_c h^c{}_b + R^c{}_{dca} h^d{}_b + R_{bdca} h^{cd} + a \leftrightarrow b.$$
$$- \Box h_{ab} - \nabla_a \nabla_b h \Big)$$

$$= \frac{1}{2}\left(\nabla_c \nabla_a h^c{}_b + \nabla_c \nabla_b h^c{}_a - \Box h_{ab} - \nabla_a \nabla_b h\right)$$

using the Riemann identity:

$$= \frac{1}{2}\left(\nabla_a \nabla_c h^c{}_b + R^c{}_{dca} h^d{}_b + R_{bdca} h^{cd} + a \leftrightarrow b \quad - \Box h_{ab} - \nabla_a \nabla_b h\right)$$

$$= \nabla_{(a} \nabla^c h_{b)c} + R_{d(a} h^d{}_{b)}$$

$$= \frac{1}{2}\left(\nabla_c \nabla_a h^c{}_b + \nabla_c \nabla_b h^c{}_a - \Box h_{ab} - \nabla_a \nabla_b h \right)$$

using the Riemann identity:

$$= \frac{1}{2}\left(\nabla_a \nabla_c h^c{}_b + R^c{}_{dca} h^d{}_b + R_{bdca} h^{cd} + a \leftrightarrow b. - \Box h_{ab} - \nabla_a \nabla_b h \right)$$

$$= \nabla_{(a} \nabla^c h_{b)c} + R_{d(a} h^d{}_{b)} - R_{bdac} h^{cd}$$

$$= \frac{1}{2}\left(\nabla_c \nabla_a h^c{}_b + \nabla_c \nabla_b h^c{}_a - \Box h_{ab} - \nabla_a \nabla_b h \right)$$

using the Riemann identity:

$$= \frac{1}{2}\left(\nabla_a \nabla_c h^c{}_b + R^c{}_{dca} h^d{}_b + R_{bdca} h^{cd} + a \leftrightarrow b \right.$$
$$\left. - \Box h_{ab} - \nabla_a \nabla_b h \right)$$

$$= \nabla_{(a} \nabla^c h_{b)c} + R_{d(a} h^d{}_{b)} - R_{bdac} h^{cd} - \frac{1}{2} \Box h_{ab}.$$
$$- \frac{1}{2} \nabla_a \nabla_b h.$$

$$\delta R_{ab} = -\frac{1}{2}\left[\Box h_{ab} + 2R_{acbd}h^{cd} - 2R^{d}{}_{(a}h_{b)d} - 2\nabla_{(a}\nabla^{c}\bar{h}_{b)c}. \right]$$

$$\bar{h}_{ab} = h_{ab} - \frac{1}{2} h g_{ab}$$

$$\delta R_{ab} = -\frac{1}{2}\Big[\Box h_{ab} + 2R_{acbd}h^{cd} - 2R^{d}{}_{(a}h_{b)d} - 2\nabla_{(a}\nabla^{c}\bar{h}_{b)c}.\Big]$$

$$= -\frac{1}{2}\Delta_L h_{ab}$$

Lichnerowicz

$$\bar{h}_{ab} = h_{ab} - \frac{1}{2}h\,g_{ab}$$

Gauge choice always important in G.R

Gauge choice always important in G.R

$$X^a \mapsto X^a + \xi^a$$

Gauge choice always important in G.R

$$X^a \mapsto X^a + \xi^a$$

$$g_{ab} \longmapsto g_{ab} + \mathcal{L}_\xi g_{ab}$$

Gauge choice always important in G.R

$$X^a \mapsto X^a + \xi^a$$

$$g_{ab} \longmapsto g_{ab} + \mathcal{L}_\xi g_{ab}$$

$$g_{ab} + \nabla_a \xi_b + \nabla_b \xi_a$$

Gauge choice always important in G.R

$$X^a \mapsto X^a + \xi^a$$

$$g_{ab} \longmapsto g_{ab} + \mathcal{L}_\xi g_{ab}$$

$$g_{ab} + \nabla_a \xi_b + \nabla_b \xi_a$$

i.e. coord transform can change h_{ab}

$$h_{\xi ab} = \nabla_a \xi_b + \nabla_b \xi_a$$

$$\Rightarrow \quad \bar{h}_{\xi ab} = 2\nabla_{(a}\xi_{b)} - (\nabla \cdot \xi) g_{ab}$$

$$h_{\xi ab} = \nabla_a \xi_b + \nabla_b \xi_a$$

$$\Rightarrow \bar{h}_{\xi ab} = 2\nabla_{(a} \xi_{b)} - (\nabla \cdot \xi) g_{ab}$$

$$\Rightarrow \nabla^a \bar{h}_{\xi ab} = \Box \xi_b + \nabla_a \nabla_b \xi^a - (\nabla \cdot \xi)_{,b}$$

$$h_{\xi\, ab} = \nabla_a \xi_b + \nabla_b \xi_a$$

$$\Rightarrow \quad \bar{h}_{\xi\, ab} = 2\nabla_{(a} \xi_{b)} - (\nabla\cdot\xi)\, g_{ab}$$

$$\Rightarrow \quad \nabla^a \bar{h}_{\xi\, ab} = \Box \xi_b + \nabla_a \nabla_b \xi^a - (\nabla\cdot\xi)_{,b}$$

$$= \Box \xi_b + \nabla_b \nabla\cdot\xi + R^a{}_{c\, ab}\, \xi^c - (\nabla\cdot\xi)_{,b}$$

$$h_{\xi ab} = \nabla_a \xi_b + \nabla_b \xi_a$$

$$\Rightarrow \quad \bar{h}_{\xi ab} = 2\nabla_{(a}\xi_{b)} - (\nabla\cdot\xi)\, g_{ab}$$

$$\Rightarrow \quad \nabla^a \bar{h}_{\xi ab} = \Box \xi_b + \nabla_a \nabla_b \xi^a - (\nabla\cdot\xi)_{,b}$$

$$= \Box \xi_b + \cancel{\nabla_b \nabla\cdot\xi} + R^a{}_{cab}\xi^c - \cancel{(\nabla\cdot\xi)_{,b}}$$

$$= \Box \xi_b + R_{bc}\xi^c$$

$$h_{\xi ab} = \nabla_a \xi_b + \nabla_b \xi_a$$

$$\Rightarrow \quad \bar{h}_{\xi ab} = 2\nabla_{(a}\xi_{b)} - (\nabla\cdot\xi)\, g_{ab}$$

$$\Rightarrow \quad \nabla^a \bar{h}_{\xi ab} = \Box \xi_b + \nabla_a \nabla_b \xi^a - (\nabla\cdot\xi)_{,b}$$

$$= \Box \xi_b + \cancel{\nabla_b \nabla\cdot\xi} + R^a{}_{cab}\,\xi^c - \cancel{(\nabla\cdot\xi)_{,b}}$$

$$= \Box \xi_b + R_{bc}\,\xi^c \quad \|$$

$$h_{\xi ab} = \nabla_a \xi_b + \nabla_b \xi_a$$

$$\Rightarrow \quad \bar{h}_{\xi ab} = 2\nabla_{(a}\xi_{b)} - (\nabla\cdot\xi)\, g_{ab}$$

$$\Rightarrow \quad \nabla^a \bar{h}_{\xi ab} = \Box \xi_b + \nabla_a \nabla_b \xi^a - (\nabla\cdot\xi)_{,b}$$

$$= \Box \xi_b + \cancel{\nabla_b \nabla\cdot\xi} + R^a{}_{c\,ab}\,\xi^c - \cancel{(\nabla\cdot\xi)_{,b}}$$

$$= \Box \xi_b + R_{bc}\,\xi^c$$

Can always choose gauge s.t. $\nabla_a \bar{h}^a{}_b = 0$

Remaining gauge freedom?

Remaining gauge freedom?

$$X^a \longrightarrow X^a + \chi^a$$

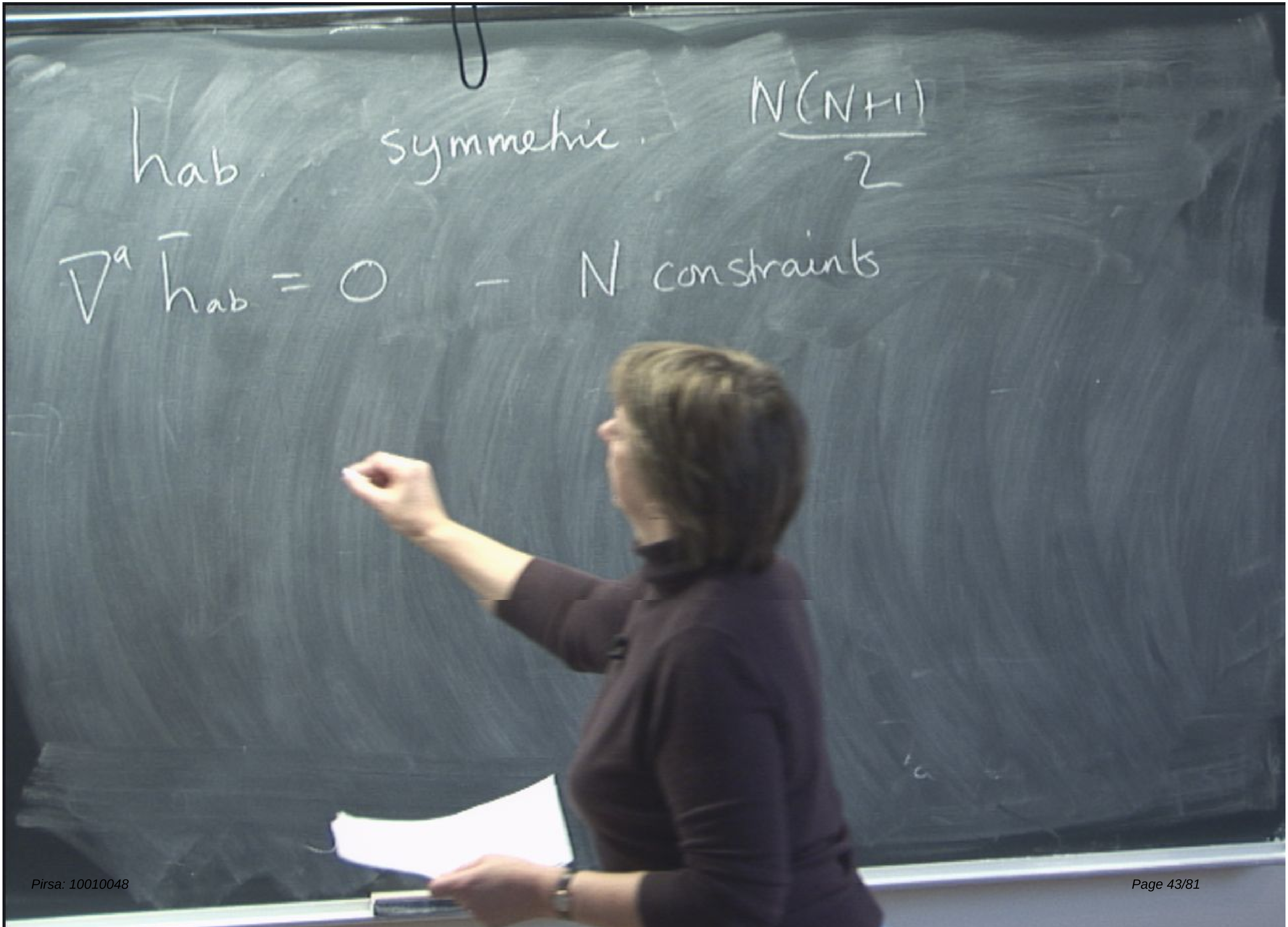
with $\Box \chi^a + R^a{}_b \chi^b = 0$

Remaining gauge freedom?

$$X^a \longrightarrow X^a + \chi^a$$

with $\quad \Box \chi^a + R^a{}_b \chi^b = 0$

$\checkmark$ N solns to this eqn


h_{ab} symmetric. $\frac{N(N+1)}{2}$
$\nabla^a \bar{h}_{ab} = 0$ – N constraints

h_{ab} symmetric. $\frac{N(N+1)}{2}$

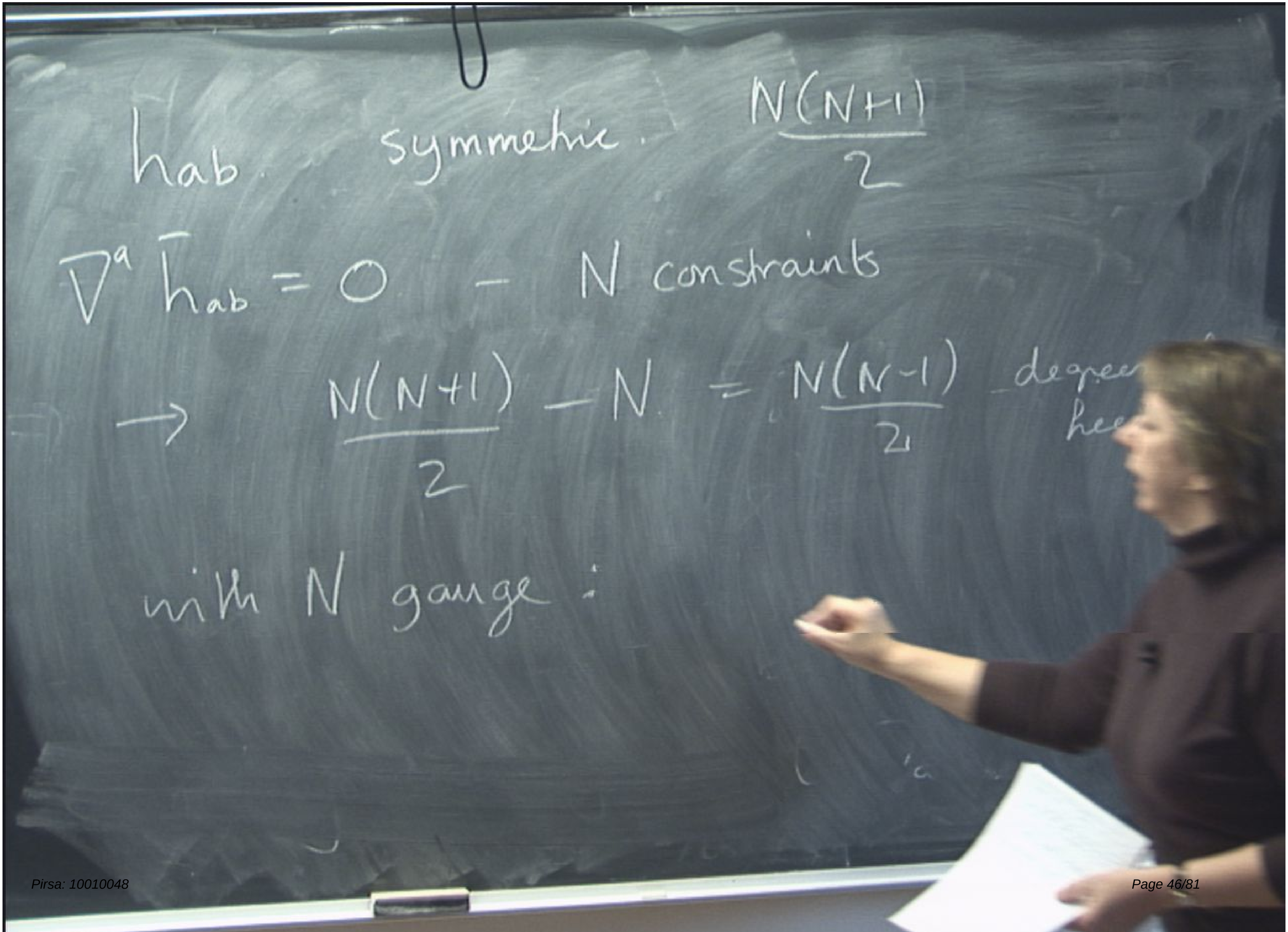
$\nabla^a \bar{h}_{ab} = 0$ — N constraints

$\rightarrow \frac{N(N+1)}{2} - N$.

h_{ab} symmetric. $\dfrac{N(N+1)}{2}$

$\nabla^a \bar{h}_{ab} = 0$ — N constraints

$\rightarrow \dfrac{N(N+1)}{2} - N = \dfrac{N(N-1)}{2}$ degrees of freedom



h_{ab} symmetric. $\frac{N(N+1)}{2}$
$\nabla^a \bar{h}_{ab} = 0$ – N constraints
$\rightarrow \frac{N(N+1)}{2} - N = \frac{N(N-1)}{2}$ degree
with N gauge :

h_{ab} symmetric. $\frac{N(N+1)}{2}$

$\nabla^a \bar{h}_{ab} = 0$ — N constraints

$\rightarrow \frac{N(N+1)}{2} - N = \frac{N(N-1)}{2}$ — degrees of freedom

with N gauge: $\frac{N(N-3)}{2}$ physical d.o.f

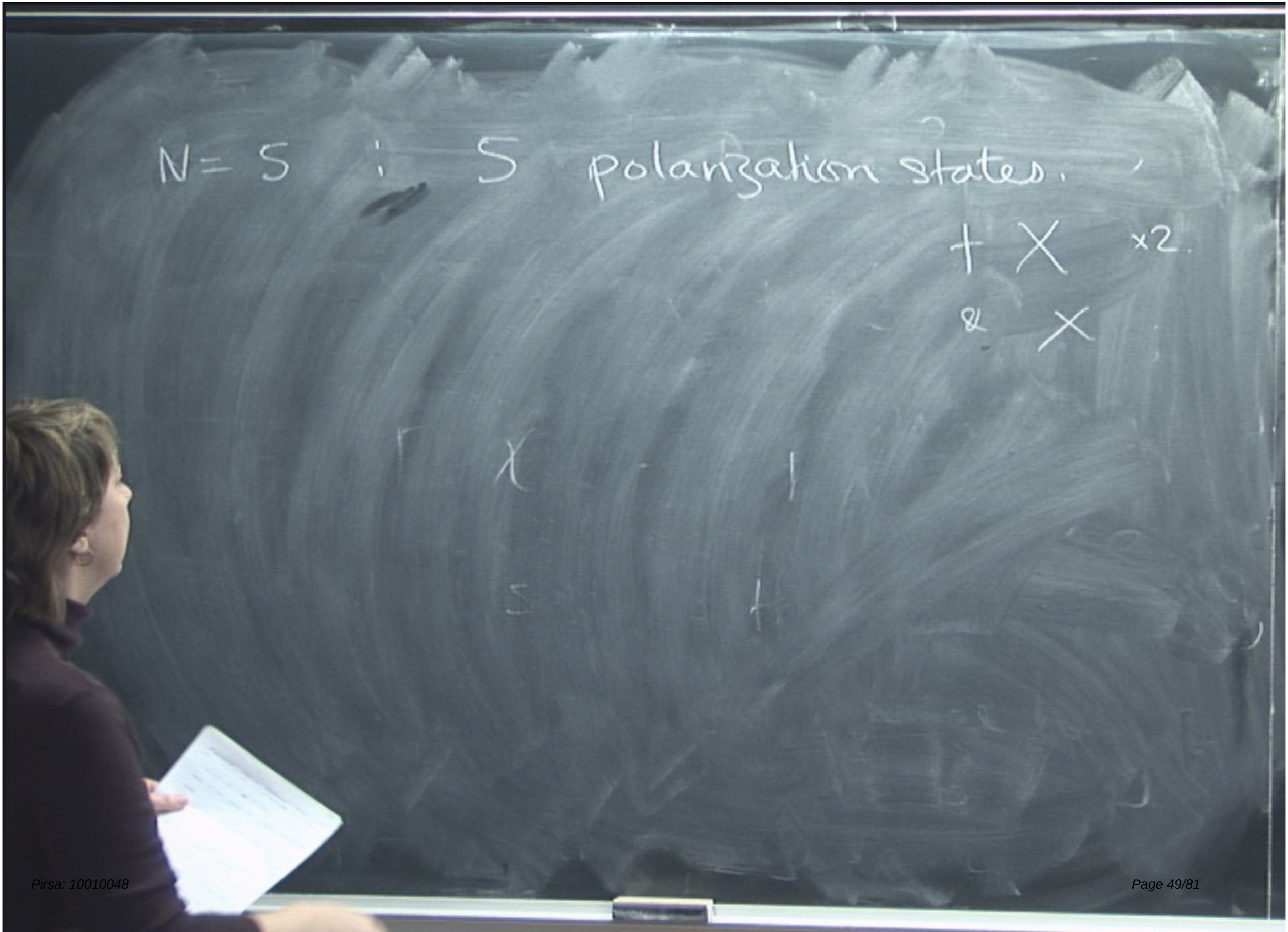
h_{ab} symmetric. $\frac{N(N+1)}{2}$

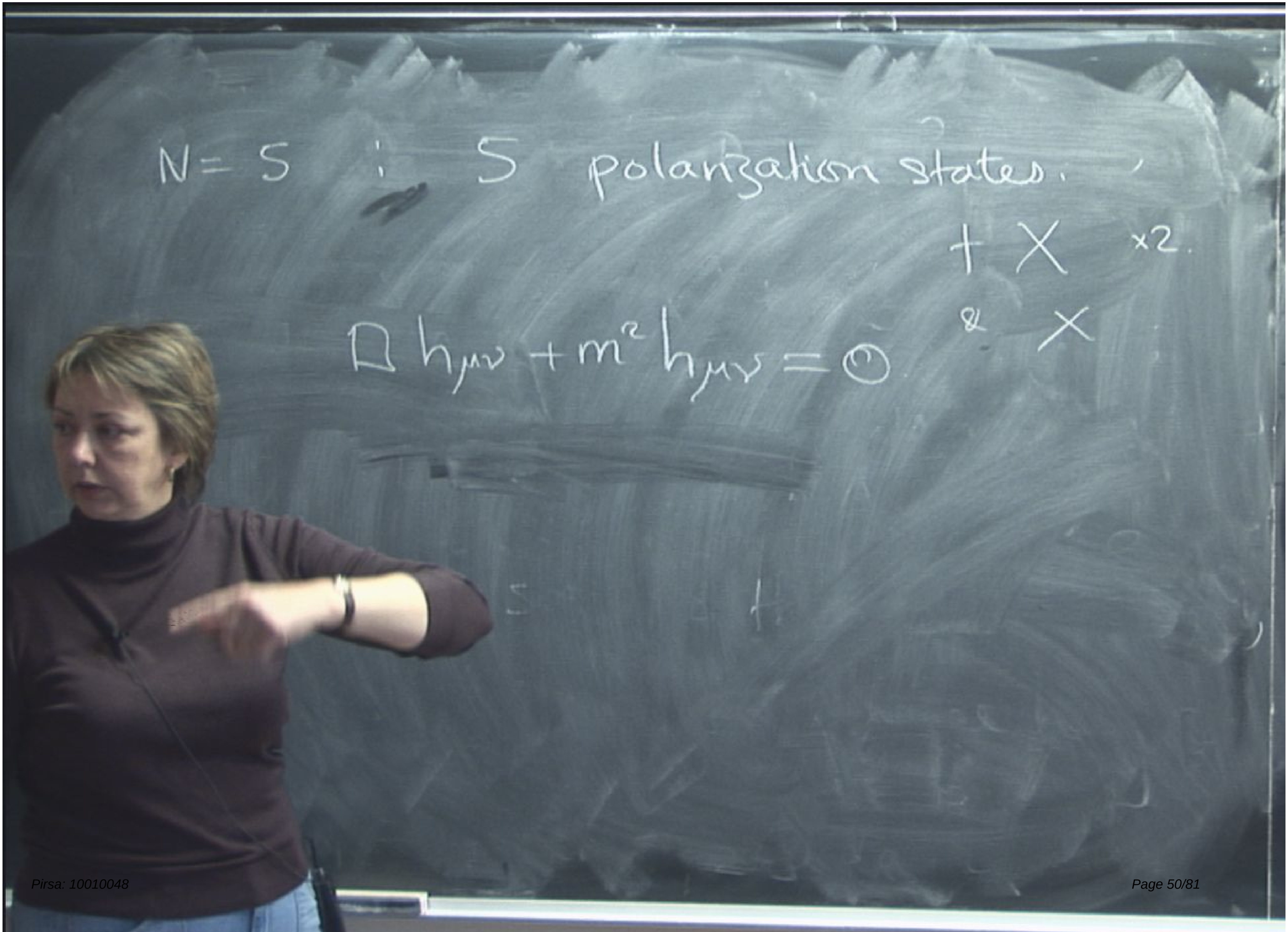
$\nabla^a \bar{h}_{ab} = 0$ — N constraints

$\rightarrow \frac{N(N+1)}{2} - N = \frac{N(N-1)}{2}$ degrees of freedom

with N gauge: $\frac{N(N-3)}{2}$ physical d.o.f

$N=4$: 2 polarization states 


N = 5 : 5 polarization states.
+ X ×2
& X


N = 5 : 5 polarization states.
+ X ×2
$\Box h_{\mu\nu} + m^2 h_{\mu\nu} = 0$
& X

$N=5$: 5 polarization states:

$+\ \times$ ×2

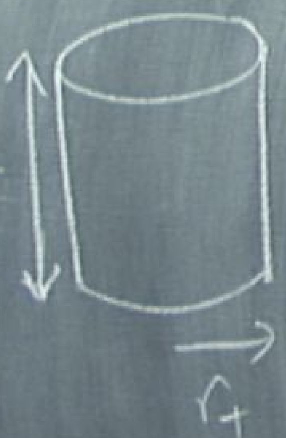
& $\times$

$$\Box h_{\mu\nu} + m^2 h_{\mu\nu} = 0$$

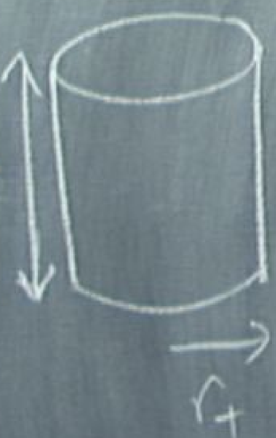
Example: Black string instability

$$ds^2 = \left(1-\frac{2GM}{r}\right)dt^2 - \frac{dr^2}{1-\frac{2GM}{r}} - r^2 d\Omega^2 - dz^2$$

Suppose we have KK theory; 5^{th} dim has length L

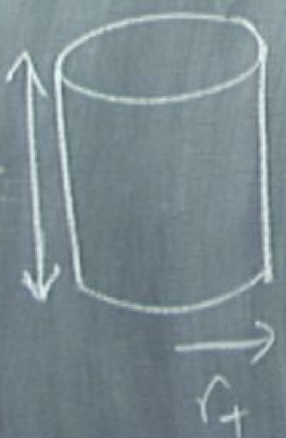


Suppose we have KK theory; 5^{th} dim has length L



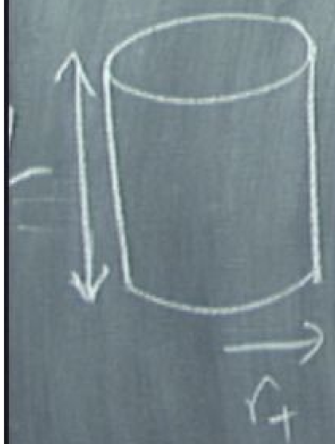
$$M = \frac{r_+ L}{2G_5}$$

Suppose we have KK theory; 5^{th} dim has length L



$$M = \frac{r_+ L}{2G_5} = \frac{2G_N M L}{2G_5} = M.$$

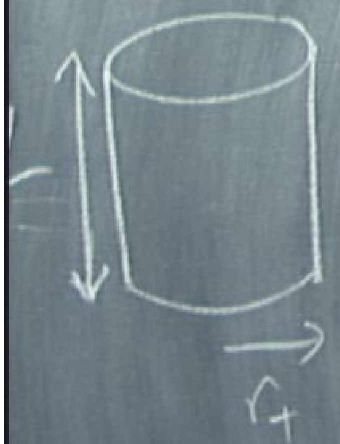
Suppose we have KK theory; 5^{th} dim has length L



$$M = \frac{r_+ L}{2G_5} = \frac{2G_N M L}{2G_5} = M.$$

$$S_4 = \frac{1}{4G_5}.$$

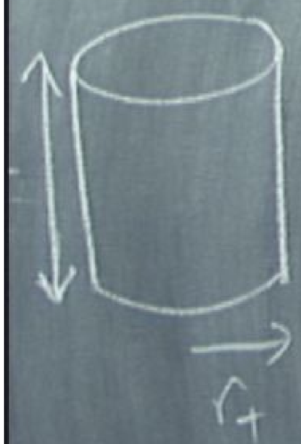
Suppose we have KK theory; 5^{th} dim has length L



$$M = \frac{r_+ L}{2G_5} = \frac{2G_N M L}{2G_5} = M.$$

$$S_4 = \frac{1}{4G_5} \times 4\pi r_+^2 L = 4\pi \frac{G_N^2 M^2 L}{G_5}$$

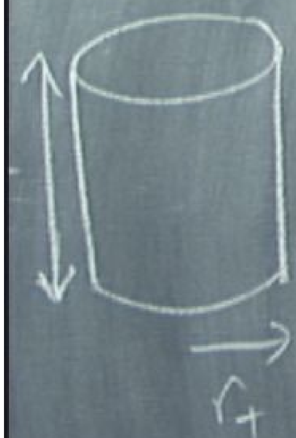
Suppose we have KK theory; 5^{th} dim has length L



$$M = \frac{r_+ L}{2G_5} = \frac{2 G_N M L}{2 G_5} = M$$

$$S_4 = \frac{1}{4G_5} \times 4\pi r_+^2 L = 4\pi \frac{G_N^2 M^2 L}{G_5} = \frac{4\pi G_5 M^2}{L}$$

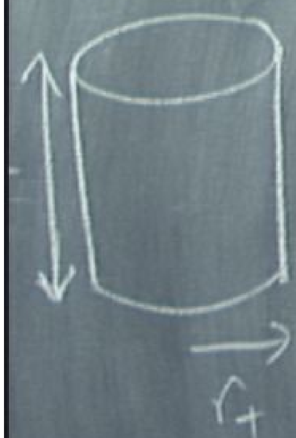
Suppose we have KK theory; 5^{th} dim has length L



$$M = \frac{r_+ L}{2G_5} = \frac{2 G_N M L}{2 G_5} = M$$

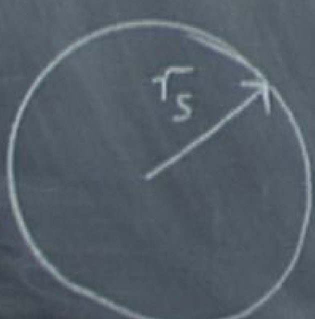
$$S_4 = \frac{1}{4G_5} \times 4\pi r_+^2 L = 4\pi \frac{G_N^2 M^2 L}{G_5} = \frac{4\pi G_5 M^2}{L}$$

Suppose we have KK theory; 5^{th} dim has length L



$$M = \frac{r_+ L}{2G_5} = \frac{2G_N M L}{2G_5} = M.$$

$$S_4 = \frac{1}{4G_5} \times 4\pi r_+^2 L = 4\pi \frac{G_N^2 M^2 L}{G_5} = \frac{4\pi G_5 M^2}{L}$$



$$M = \frac{3 \cdot 2\pi^2 r_5^2}{16\pi G_5} = \frac{3}{8}\frac{\pi}{G_5} r_5^2$$

$$S_5 = \frac{2\pi^2 r_s^3}{4G_5} =$$

$$S_5 = \frac{2\pi^2 r_s^3}{4G_5} = \frac{2\pi^2}{4G_5}\left(\frac{8G_5 M}{3\pi}\right)^{3/2}$$

$$S_5 = \frac{2\pi^2 r_s^3}{4G_5} = \frac{2\pi^2}{4G_5}\left(\frac{8G_5 M}{3\pi}\right)^{3/2}$$

$$= \sqrt{\frac{2\pi}{3}}\ \frac{8}{3}\ M^{3/2} G_5^{1/2}$$

$$S_5 = \frac{2\pi^2 r_s^3}{4G_5} = \frac{2\pi^2}{4G_5}\left(\frac{8G_5 M}{3\pi}\right)^{3/2}$$

$$= \sqrt{\frac{2\pi}{3}}\ \frac{8}{3}\ M^{3/2}\, G_5^{1/2}$$

The black hole would be preferred when.

$$S_5 = \frac{2\pi^2 r_s^3}{4G_5} = \frac{2\pi^2}{4G_5}\left(\frac{8G_5M}{3\pi}\right)^{3/2}$$

$$= \sqrt{\frac{2\pi}{3}}\,\frac{8}{3}\,M^{3/2}G_5^{1/2}$$

The black hole would be preferred when.

$$\frac{8}{3}\sqrt{\frac{2\pi}{3}}\,M^{3/2}G_5^{1/2} \geq \frac{4\pi G_5^{1/2} M^2}{L}$$

$$S_5 = \frac{2\pi^2 r_s^3}{4G_5} = \frac{2\pi^2}{4G_5}\left(\frac{8G_5 M}{3\pi}\right)^{3/2}$$

$$= \sqrt{\frac{2\pi}{3}}\;\frac{8}{3}\; M^{3/2} G_5^{1/2}$$

The black hole would be preferred when.

$$\frac{8}{3}\sqrt{\frac{2\pi}{3}}\; M^{3/2} \cancel{G_5^{1/2}} \geq \frac{4\pi G_5^{1/2} M^2}{L}$$

$$L^{1/2} \geq$$

$$S_5 = \frac{2\pi^2 r_s^3}{4G_5} = \frac{2\pi^2}{4G_5}\left(\frac{8G_5 M}{3\pi}\right)^{3/2}$$

$$= \sqrt{\frac{2\pi}{3}}\,\frac{8}{3}\,M^{3/2}\,G_5^{1/2}$$

The black hole would be preferred when.

$$\frac{\overset{2}{\cancel{8}}}{3}\sqrt{\frac{2\pi}{3}}\,M^{3/2}\,\cancel{G_5^{1/2}} \geq \frac{\cancel{4}\pi G_5^{1/2} M^2}{L}$$

$$L^{1/2} >$$

$$S_5 = \frac{2\pi^2 r_s^3}{4G_5} = \frac{2\pi^2}{4G_5}\left(\frac{8G_5M}{3\pi}\right)^{3/2}$$

$$= \sqrt{\frac{2\pi}{3}}\,\frac{8}{3}\,M^{3/2}\,G_5^{1/2}$$

The black hole would be preferred when.

$$\frac{\cancel{8}^{\,2}}{3}\sqrt{\frac{2\pi}{3}}\,M^{3/2}\,\cancel{G_5}^{1/2} \geq \frac{\cancel{4}\pi\,G_5^{1/2}M^2}{L}$$

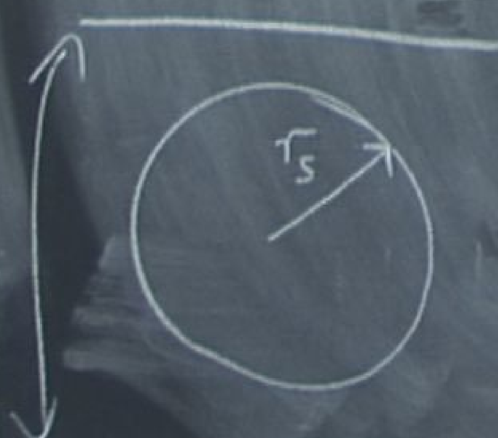
$$L^{1/2} \geq \frac{3\sqrt{3}}{2\sqrt{2\pi}}\,G_N\,M^{1/2}\left(\frac{M}{M}\right)^{3/2}$$

Suppose we have KK theory; 5^{th} dim has

length L

$$M = \frac{r_+ L}{2G_5} \doteq \frac{2G_N M L}{2G_5} = M.$$

$$S_4 = \frac{1}{4G_5} \times 4\pi r_+^2 L = 4\pi \frac{G_N^2 M^2 L}{G_5} = 4\pi \ldots$$



$$M = \frac{3 \cdot 2\pi^2 r_5^2}{16\pi G_5} = \frac{3}{8}\frac{\pi}{G_5} r_5^2$$

$$L \geq \frac{27}{8\pi} G_N M$$

$$L \geq \frac{27}{8\pi} G_N M$$

→ Instability

$$L \gtrsim \frac{27}{8\pi} G_N M$$

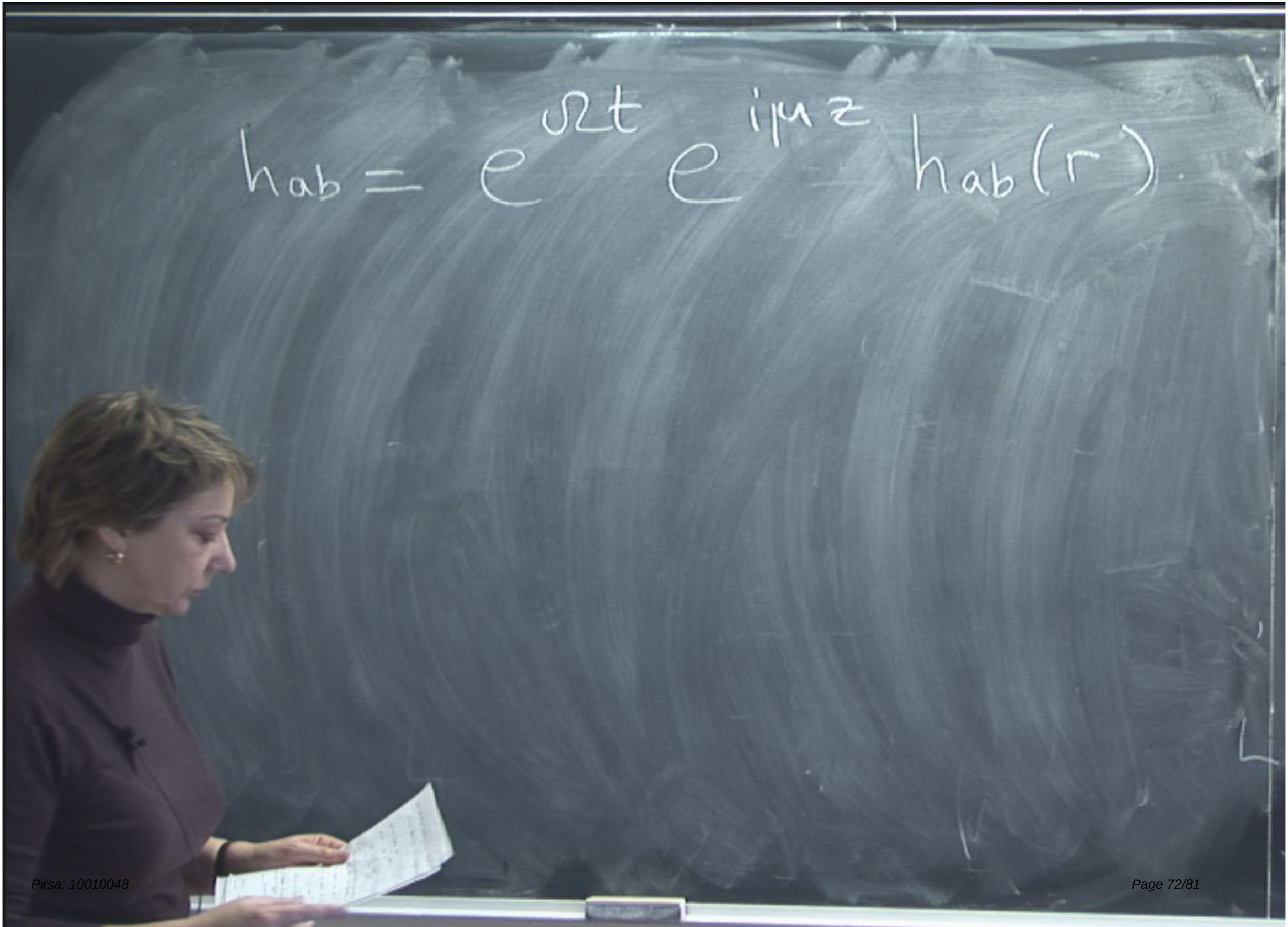
$\rightarrow$ Instability

Check by looking at $\Delta_L h_{ab}$

h_{ab} splits into irreducible KK modes

h_{55} – scalar $h_{\mu\nu}$ – tensor.

$h_{5\mu}$ – vector

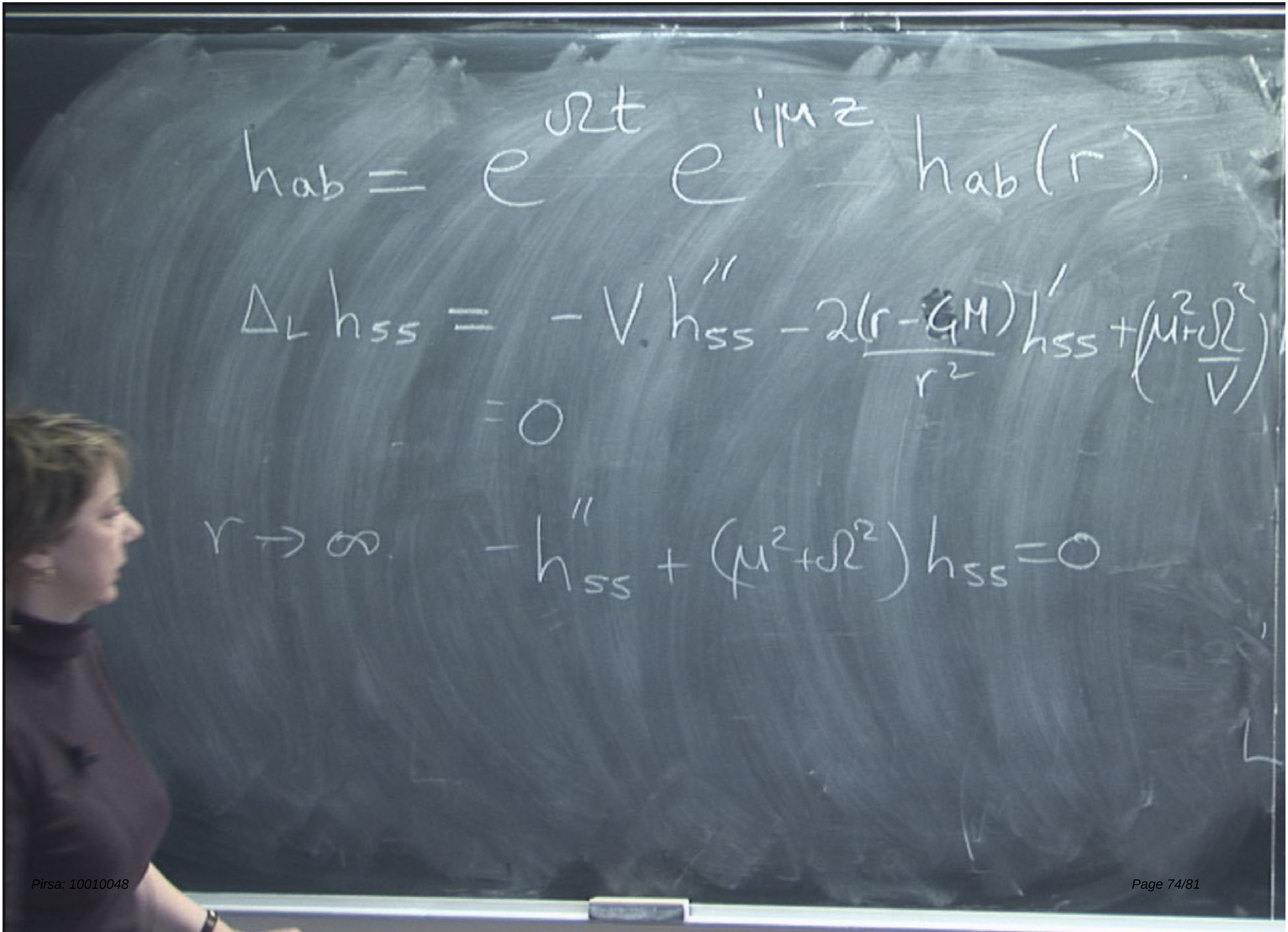

$h_{ab} = e^{\Omega t} e^{i\mu z} h_{ab}(r).$

$$h_{ab} = e^{\Omega t} e^{i\mu z} h_{ab}(r).$$

$$\Delta_L h_{ss} = -V\, h''_{ss} - 2\frac{(r-GM)}{r^2} h'_{ss} + \left(\mu^2 + \frac{\Omega^2}{V}\right)$$

$$= 0$$

r


$h_{ab} = e^{\Omega t} e^{i\mu z} h_{ab}(r)$
$\Delta_L h_{ss} = -V h''_{ss} - \frac{2(r-GM)}{r^2} h'_{ss} + \left(\mu^2 + \frac{\Omega^2}{V}\right)$
$= 0$
$r \to \infty \quad -h''_{ss} + (\mu^2 + \Omega^2) h_{ss} = 0$

$$h_{ab} = e^{\Omega t} e^{i\mu z} h_{ab}(r).$$

$$\Delta_L h_{ss} = -V h''_{ss} - \frac{2(r-GM)}{r^2} h'_{ss} + \left(\mu^2 + \frac{\Omega^2}{V}\right)$$

$$= 0$$

$r \to \infty$. $\quad -h''_{ss} + (\mu^2 + \Omega^2) h_{ss} = 0$

$$h_{ss} \sim e^{\pm\sqrt{\Omega^2+\mu^2}\, r}$$

$r \to 2GM$

$$h_{ab} = e^{\Omega t} e^{i\mu z} h_{ab}(r).$$

$$\Delta_L h_{ss} = -V h_{ss}'' - \frac{2(r-GM)}{r^2} h_{ss}' + \left(\mu^2 + \frac{\Omega^2}{V}\right) h_{ss}$$

$$= 0$$

$r \to \infty$. $\quad -h_{ss}'' + (\mu^2 + \Omega^2) h_{ss} = 0$

$$h_{ss} \sim e^{\pm\sqrt{\Omega^2+\mu^2}\, r_*}$$

$r \to 2GM \quad h'' + \dfrac{h'}{r-2GM} = \dfrac{(2GM\Omega)^2}{(r-2GM)^2} h$

$h \propto (r-2GM)^{\pm 2GM\Omega}$

If $\exists$ soln, then h has to be regular

$$h \propto \begin{cases} e^{-\sqrt{\Omega^2+\mu^2}\, r} & r \to \infty \\ (r-2GM)^{2GM\Omega} & r \to 2GM \end{cases}$$

$$h \propto (r-2GM)^{\pm 2GM\Omega}$$

If $\exists$ soln, then h has to be regular

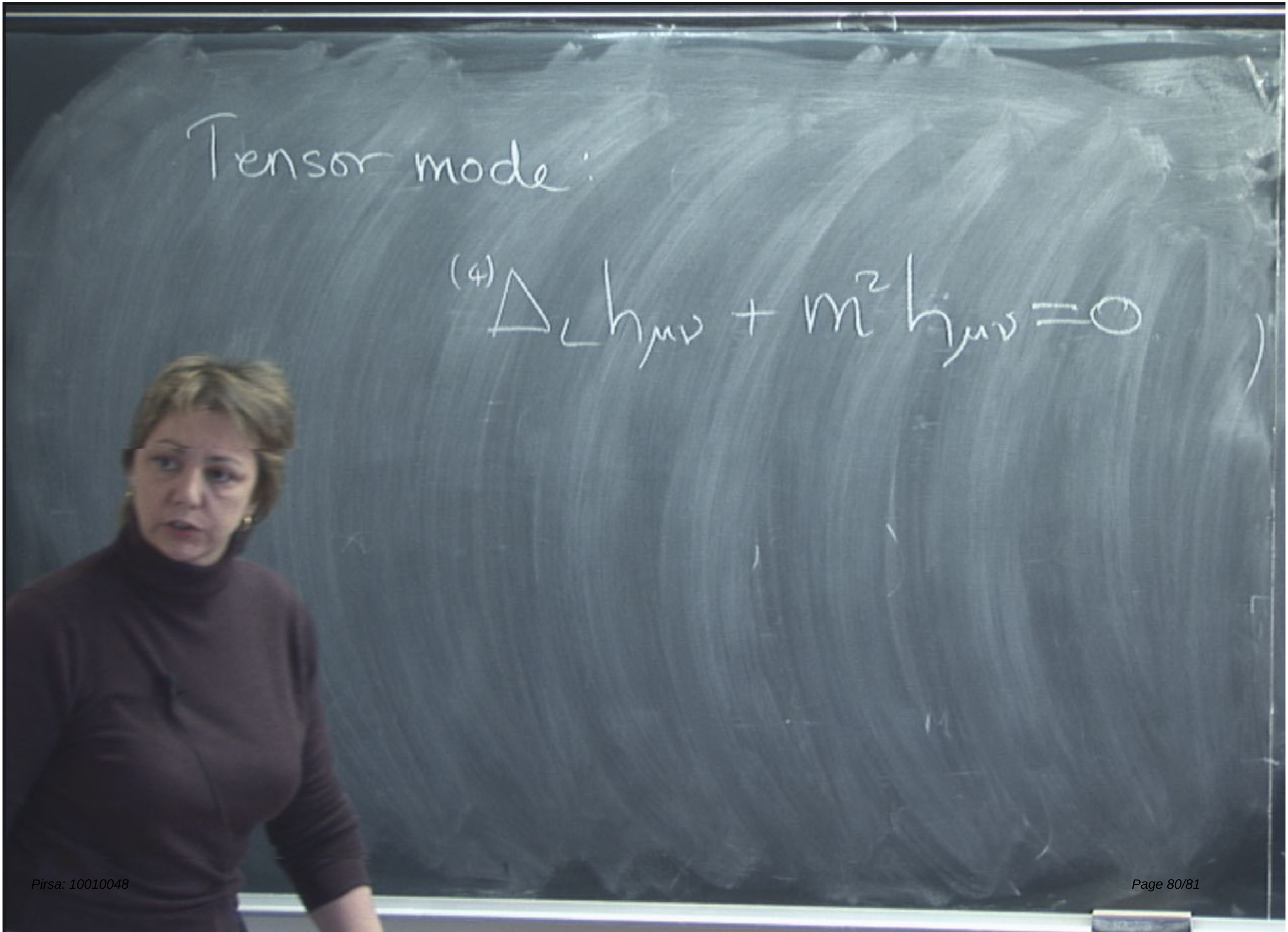
$$h \propto \begin{cases} e^{-\sqrt{\Omega^2+\mu^2}\, r} & r \to \infty \\ (r-2GM)^{2GM\Omega} & r \to 2GM \end{cases}$$

$$h \propto (r-2GM)^{\pm 2GM\Omega}$$

If $\exists$ soln, then h has to be regular

$$h \propto \begin{cases} e^{-\sqrt{\Omega^2+\mu^2}\, r} & r \to \infty \\ (r-2GM)^{2GM\Omega} & r \to 2GM \end{cases}$$

$\Rightarrow h' = 0$ at $r = r_0$. $h'' < 0$


Tensor mode:
$^{(4)}\Delta_L h_{\mu\nu} + m^2 h_{\mu\nu} = 0$

Tensor mode:

$$ {}^{(4)}\Delta_L h_{\mu\nu} + m^2 h_{\mu\nu} = 0 $$

All gauge solns massless, so any soln is physical.