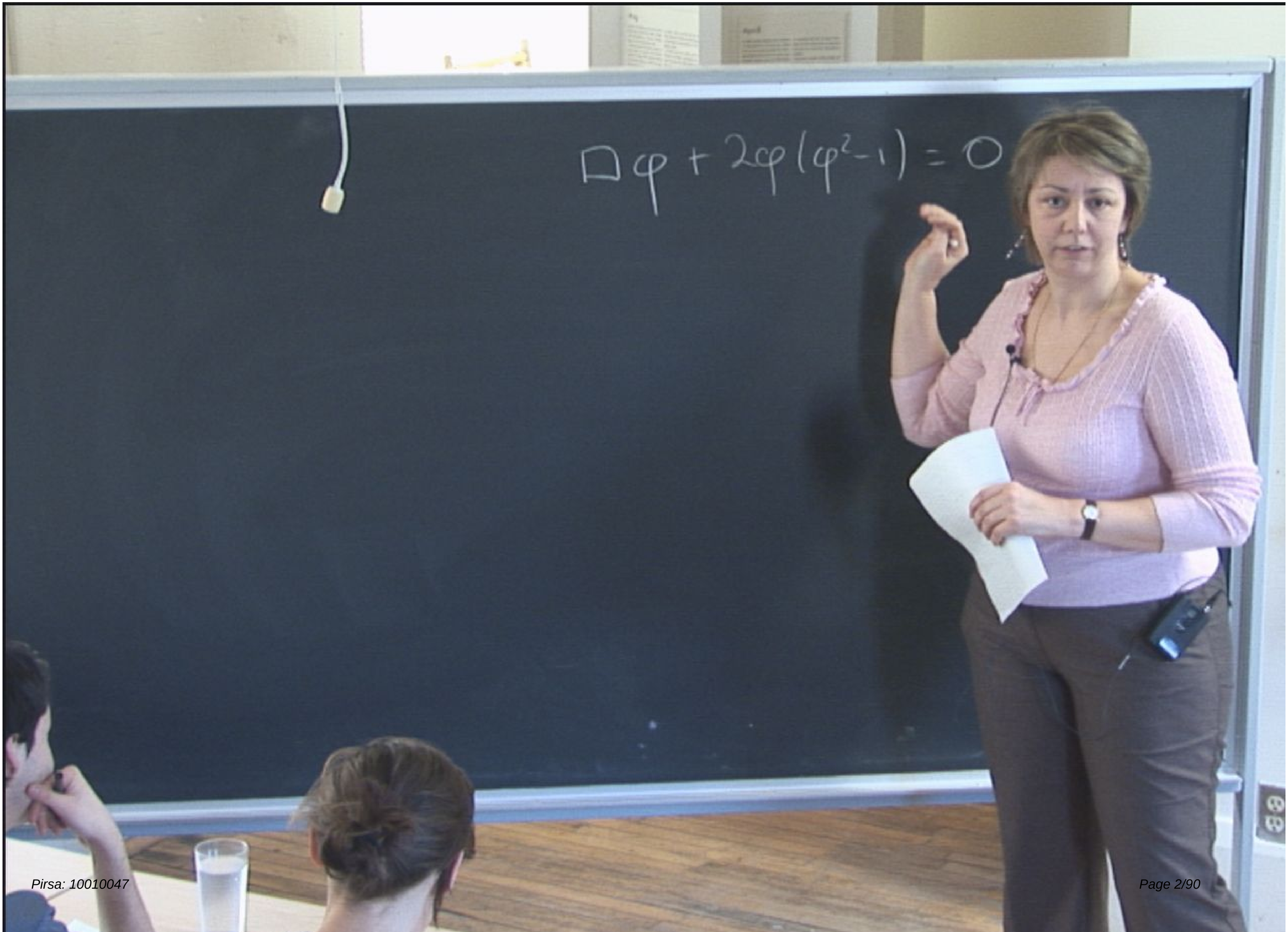
Title: Gravitational Physics - Review (PHYS 636) - Lecture 14

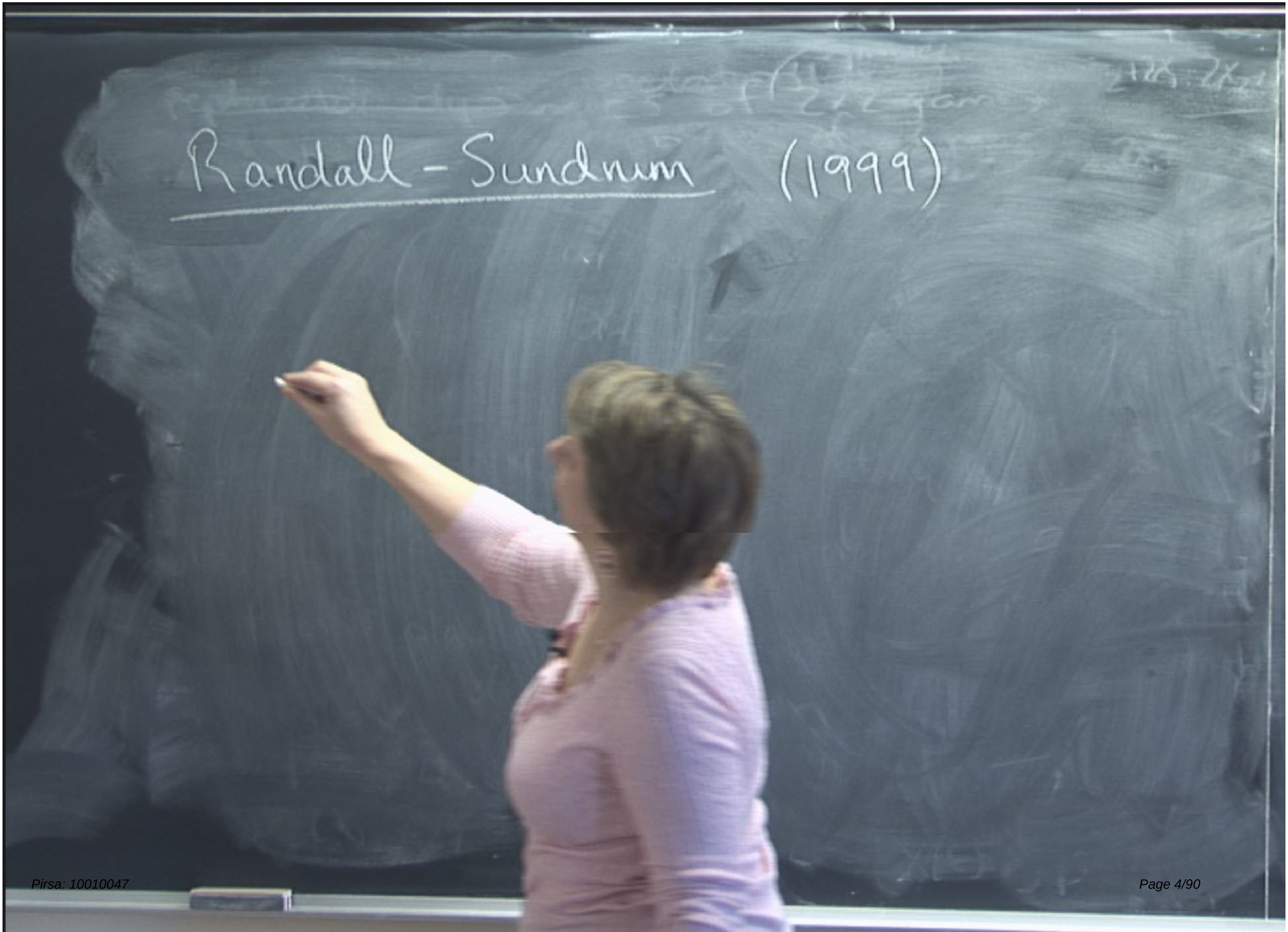
Date: Jan 21, 2010 10:00 AM

URL: http://pirsa.org/10010047

Abstract:


$\Box\varphi + 2\varphi(\varphi^2-1) = 0$

Randall - Sundrum


Randall - Sundrum (1999)

Randall - Sundrum (1999)

Domain wall in adS spacetime

Randall - Sundrum (1999)

Domain wall in adS spacetime

$$ds^2 = e^{-2k|z|} \eta_{\mu\nu} dx^\mu dx^\nu - dz^2$$

Randall - Sundrum (1999)

Domain wall in adS spacetime

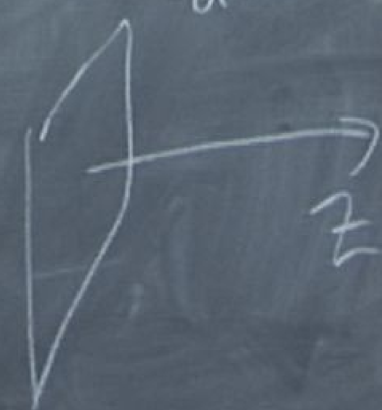
$$ds^2 = e^{-2k|z|} \underbrace{\eta_{\mu\nu}\, dx^\mu dx^\nu}_{\text{braneworld.}} - \underset{\substack{\uparrow \\ \text{extra} \\ \text{dim.}}}{dz^2}$$

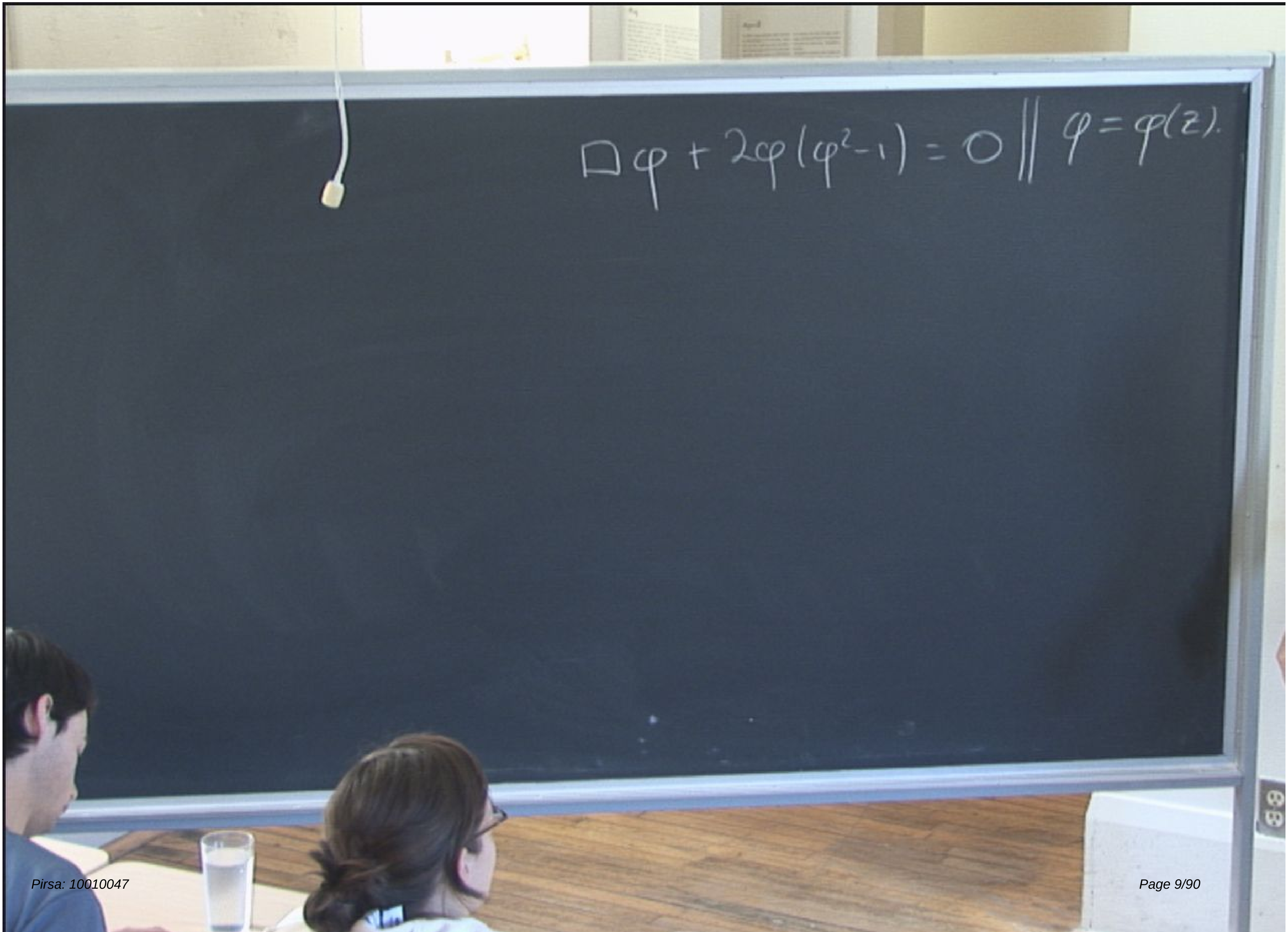
Randall - Sundrum (1999)

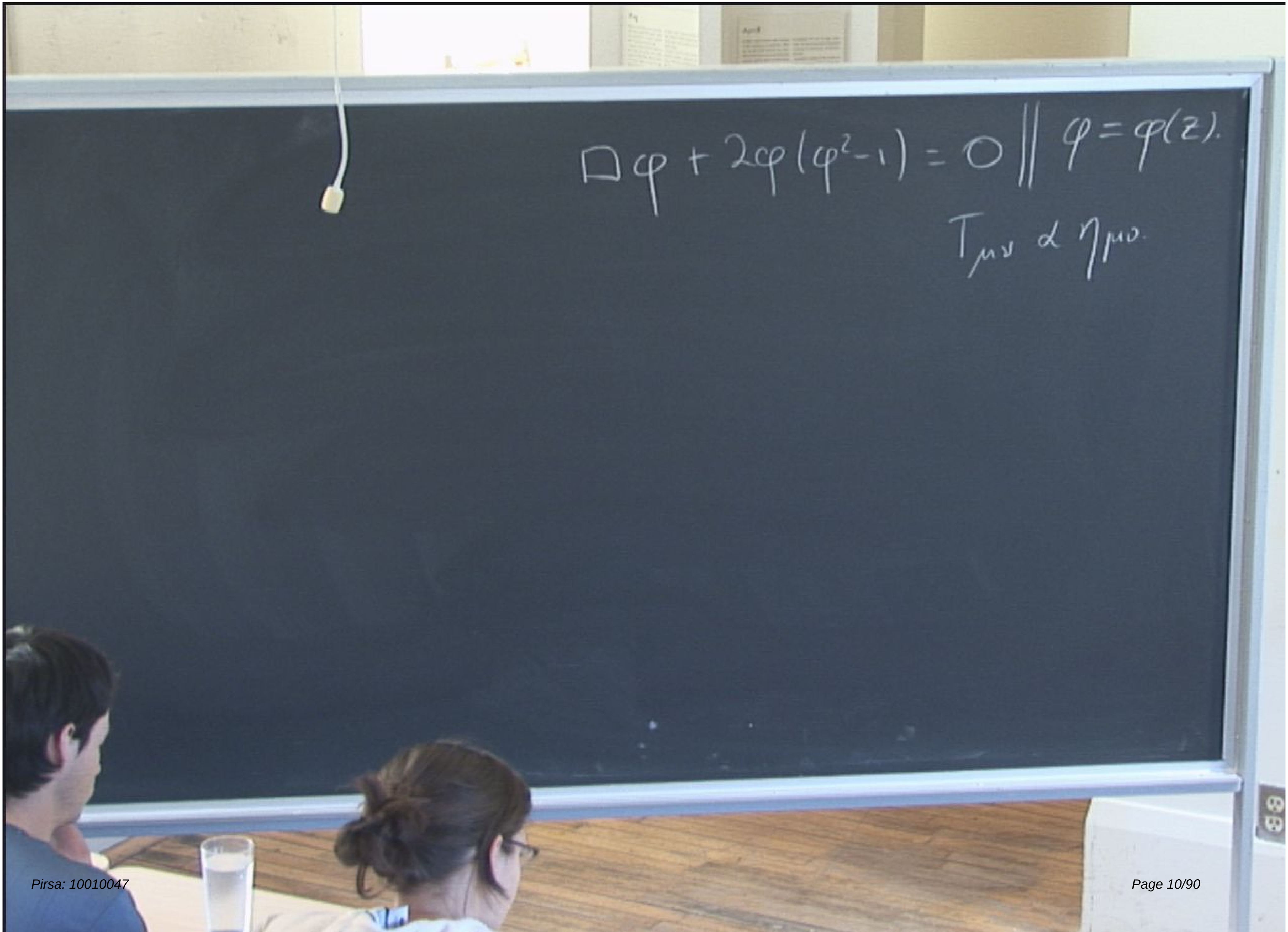
Domain wall in adS spacetime

$$ds^2 = e^{-2k|z|}\underbrace{\eta_{\mu\nu}\,dx^\mu dx^\nu}_{\text{braneworld.}} - \underset{\substack{\uparrow\\ \text{extra}\\ \text{dim.}}}{dz^2}$$

$$S_{\mu\nu} = \frac{6k}{8\pi G}\,\eta_{\mu\nu}.$$




$\Box\varphi + 2\varphi(\varphi^2-1) = 0 \quad || \quad \varphi = \varphi(z).$


$\Box\varphi + 2\varphi(\varphi^2 - 1) = 0 \quad \| \quad \varphi = \varphi(z).$
$T_{\mu\nu} \propto \eta_{\mu\nu}.$

Randall - Sundrum (1999)

Domain wall in adS spacetime

$$ds^2 = e^{-2k|z|} \underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{\text{braneworld.}} - \underset{\substack{\uparrow \\ \text{extra dim.}}}{dz^2}$$

$$S_{\mu\nu} = \frac{6k}{8\pi G} \eta_{\mu\nu} \, ; \quad \Lambda = -6k^2$$

Randall - Sundrum (1999)

Domain wall in adS spacetime

0,1,2,3

$$ds^2 = e^{-2k|z|} \underbrace{\eta_{\mu\nu}\, dx^\mu dx^\nu}_{\text{braneworld.}} - \underbrace{dz^2}_{\text{extra dim.}}$$

$$S_{\mu\nu} = \frac{6k}{8\pi G}\,\eta_{\mu\nu}\,; \quad \Lambda = -6k^2$$

Randall - Sundrum (1999)

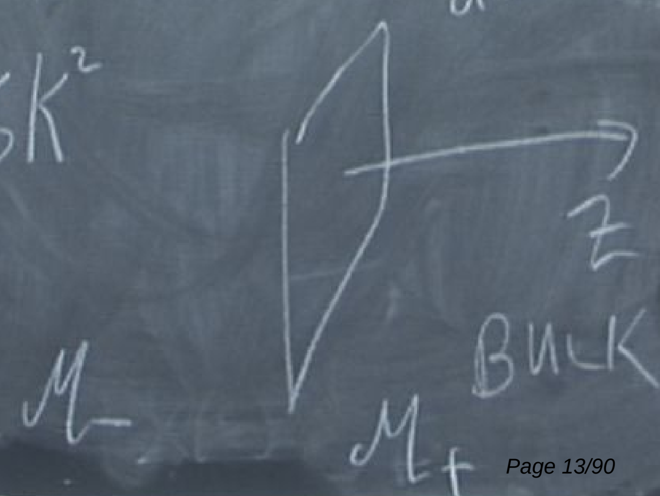
Domain wall in adS spacetime

0,1,2,3

$$ds^2 = e^{-2k|z|}\underbrace{\eta_{\mu\nu}dx^\mu dx^\nu}_{\text{braneworld}} - dz^2$$

extra dim.

$$S_{\mu\nu} = \frac{6k}{8\pi G}\eta_{\mu\nu}\,;\quad \Lambda = -6k^2$$



$$\mathcal{M}_+ : \quad n = \frac{\partial}{\partial z} \qquad \text{so} \quad n_z = -1$$

$\mathcal{M}_+ : \quad n = \frac{\partial}{\partial z} \qquad \text{so } n_z = -1$

$$K_{+\mu\nu} = X^a_{\ ,(\mu} X^b_{\ ,\nu)} \nabla_a n_{+b}$$

$\mathcal{M}_+ : \quad n = \frac{\partial}{\partial z} \qquad \text{so} \quad n_z = -1$

$$K_{+\mu\nu} = X^a{}_{,(\mu} X^b{}_{,\nu)} D_a n_{+b} = \Gamma^z_{\mu\nu}$$

$\mathcal{M}_+ : \quad n = \frac{\partial}{\partial z} \quad$ so $\quad n_z = -1$

$$K_{+\mu\nu} = X^a_{,(\mu} \underline{X^b_{,\nu)}} \nabla_a n_{+b} = \quad \Gamma^z_{\mu\nu} = \frac{1}{2} g_{\mu\nu,z}$$

$\mathcal{M}_+ : \quad n = \frac{\partial}{\partial z} \qquad \text{so} \quad n_z = -1$

$$K_{+\mu\nu} = X^a_{,(\mu} X^b_{,\nu)} \nabla_a n_{+b} = \Gamma^z_{\mu\nu} = \frac{1}{2} g_{\mu\nu,z}$$

$$= -k\, g_{+\mu\nu}$$

$\mathcal{M}_+ : \quad n = \frac{\partial}{\partial z} \quad$ so $\quad n_z = -1$

$$K_{+\mu\nu} = X^a_{,(\mu} X^b_{,\nu)} \nabla_a n_{+b} = \Gamma^z_{\mu\nu} = \frac{1}{2} g_{\mu\nu,z}$$

$$= -k\, g_{+\mu\nu} \qquad (K_{-\mu\nu} = k\, g_{-\mu\nu})$$

$\mathcal{M}_+ :\quad n = \frac{\partial}{\partial z} \qquad \text{so } n_z = -1$

$$K_{+\mu\nu} = X^a{}_{,(\mu} X^b{}_{,\nu)} \nabla_a n_{+b} = \Gamma^z_{\mu\nu} = \frac{1}{2} g_{\mu\nu,z}$$

$$= -k\, g_{+\mu\nu} \qquad (K_{-\mu\nu} = k\, g_{-\mu\nu})$$

ISRAEL EQNS $\quad \Delta K_{ab} - \Delta K\, h_{ab} = 8\pi G\, S_{ab}$

$\mathcal{M}_+$: $n = \frac{\partial}{\partial z}$ so $n_z = -1$

$$K_{+\mu\nu} = X^a_{,(\mu} X^b_{,\nu)} \nabla_a n_{+b} = \Gamma^z_{\mu\nu} = \frac{1}{2} g_{\mu\nu,z}$$

$$= -k\, g_{+\mu\nu} \qquad (K_{-\mu\nu} = k\, g_{-\mu\nu})$$

ISRAEL EQNS $\Delta K_{ab} - \Delta K\, h_{ab} = 8\pi G S_{ab}$

rearrange: $\Delta K - 4 \Delta K = 8\pi G S$

$\mathcal{M}_+ : \quad n = \frac{\partial}{\partial z} \qquad$ so $\quad n_z = -1$

$$K_{+\mu\nu} = X^a_{\ ,(\mu} \underline{X^b_{\ ,\nu)}} \nabla_a n_{+b} = \Gamma^z_{\mu\nu} = \frac{1}{2} g_{\mu\nu,z}$$

$$= -k\, g_{+\mu\nu} \qquad (K_{-\mu\nu} = k\, g_{-\mu\nu})$$

ISRAEL EQNS $\quad \Delta K_{ab} - \Delta K\, h_{ab} = 8\pi G\, S_{ab}$

$$\Delta K - 4 \Delta K = 8\pi G\, S$$

rearrange:

$$\Rightarrow \quad \Delta K_{ab} = 8\pi G \,(S_{ab}$$

$\mathcal{M}_+ : \quad n = \frac{\partial}{\partial z} \quad$ so $\quad n_z = -1$

$$K_{+\mu\nu} = X^a_{,(\mu} X^b_{,\nu)} \nabla_a n_{+b} = \Gamma^z_{\mu\nu} = \frac{1}{2} g_{\mu\nu,z}$$

$$= -k\, g_{+\mu\nu} \qquad (K_{-\mu\nu} = k\, g_{-\mu\nu})$$

ISRAEL EQNS $\quad \Delta K_{ab} - \Delta K\, h_{ab} = 8\pi G\, S_{ab}$

$$\Delta K - 4\Delta K = 8\pi G\, S$$

rearrange:

$$\Rightarrow \quad \Delta K_{ab} = 8\pi G \left(S_{ab} - \frac{1}{3} S\, h_{ab} \right)$$

$\mathcal{M}_+ :\quad n = \frac{\partial}{\partial z} \qquad \text{so} \quad n_z = -1$

$$K_{+\mu\nu} = X^a{}_{,(\mu} X^b{}_{,\nu)} \nabla_a n_{+b} = \Gamma^z_{\mu\nu} = \frac{1}{2} g_{\mu\nu,z}$$

$$= -k\, g_{+\mu\nu} \qquad (K_{-\mu\nu} = k\, g_{-\mu\nu})$$

ISRAEL EQNS $\quad \Delta K_{ab} - \Delta K\, h_{ab} = 8\pi G\, S_{ab}$

$$\Delta K - 4\Delta K = 8\pi G\, S$$

rearrange:

$$\Rightarrow \quad \Delta K_{ab} = 8\pi G \left(S_{ab} - \frac{1}{3} S\, h_{ab} \right)$$

$\mathcal{M}_+ : \quad n = \frac{\partial}{\partial z} \qquad \text{so} \quad n_z = -1$

$$K_{+\mu\nu} = X^a{}_{,(\mu} X^b{}_{,\nu)} \nabla_a n_{+b} = \Gamma^z_{\mu\nu} = \frac{1}{2} g_{\mu\nu,z}$$

$$= -k\, g_{+\mu\nu} \qquad (K_{-\mu\nu} = k\, g_{-\mu\nu})$$

ISRAEL EQNS $\quad \Delta K_{ab} - \Delta K\, h_{ab} = 8\pi G\, S_{ab}$

$$\Delta K - 4 \Delta K = 8\pi G\, S$$

rearrange:

$$\Delta K_{ab} = 8\pi G \left(S_{ab} - \frac{1}{3} S\, h_{ab} \right)$$

$$\Rightarrow \quad -2k\, \eta_{\mu\nu} = 8\pi G \left(\frac{6k}{8\pi G} \eta_{\mu\nu} - \frac{4}{3} \frac{6k}{8\pi G} \eta_{\mu\nu} \right)$$

$\mathcal{M}_+$: $n = \frac{\partial}{\partial z}$ so $n_z = -1$

$$K_{+\mu\nu} = X^a_{,(\mu} X^b_{,\nu)} \nabla_a n_{+b} = \Gamma^z_{\mu\nu} = \frac{1}{2} g_{\mu\nu,z}$$

$$= -k\, g_{+\mu\nu} \qquad (K_{-\mu\nu} = k\, g_{-\mu\nu})$$

ISRAEL EQNS $\Delta K_{ab} - \Delta K h_{ab} = 8\pi G S_{ab}$

$$\Delta K - 4\Delta K = 8\pi G S$$

rearrange:

$$\Delta K_{ab} = 8\pi G \left(S_{ab} - \frac{1}{3} S h_{ab} \right)$$

$$\Rightarrow \quad -2k\,\eta_{\mu\nu} = \cancel{8\pi G}\left(\frac{6k}{\cancel{8\pi G}} \eta_{\mu\nu} - \frac{4}{3} \frac{6k}{\cancel{8\pi G}} \eta_{\mu\nu} \right) \checkmark\checkmark$$

Randall - Sundrum (1999)

Domain wall in adS spacetime

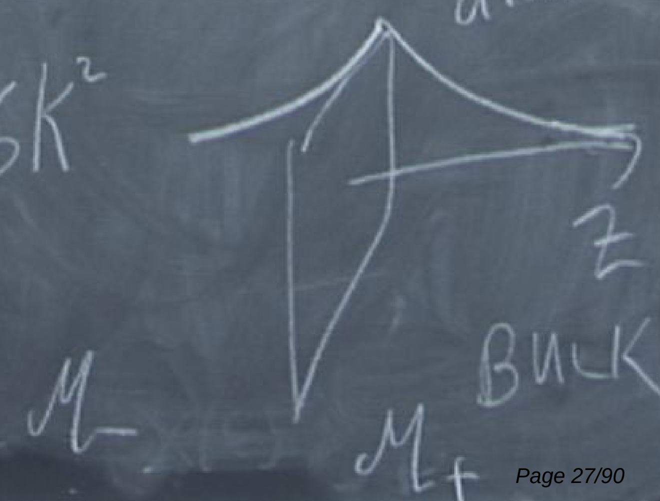
$$ds^2 = e^{-2k|z|}\, \underbrace{\eta_{\mu\nu}\, dx^\mu dx^\nu}_{\text{braneworld}} - dz^2$$

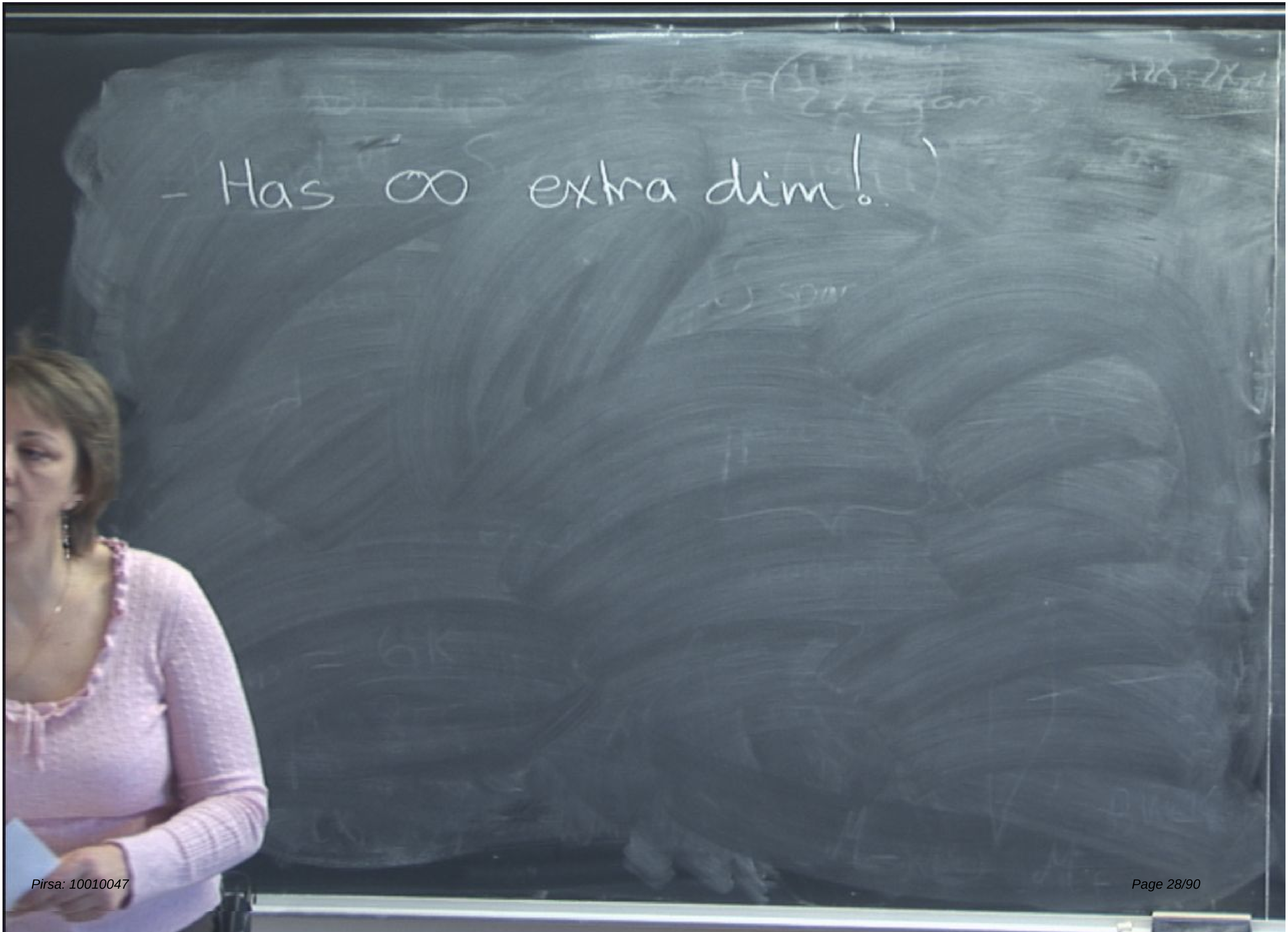
0,1,2,3

WARP FACTOR

extra dim.

$$S_{\mu\nu} = \frac{6k}{8\pi G}\eta_{\mu\nu}\,; \quad \Lambda = -6K^2$$




- Has ∞ extra dim!

- Has ∞ extra dim!. How do we get 4D gravity?

- Has ∞ extra dim! How do we get 4D gravity?

Consider a small perturbation

$$S_{\mu\nu} \to \frac{6K}{8\pi G}\eta_{\mu\nu} + T_{\mu\nu}$$

$$\eta_{\mu\nu} \to g_{\mu\nu}$$

Ignore T^2 terms.

ignore bulk perturbations

$$R_{abcd} = K^2 \left(g_{ac} g_{bd} - g_{ad} g_{bc} \right)$$

* ignore bulk perturbations, keep $\mathbb{Z}_2$ symm.

$$R_{abcd} = K^2 (g_{ac} g_{bd} - g_{ad} g_{bc})$$

Israel $\Rightarrow$ $\Delta K_{ab} = 2 K_{+ab} = 8\pi G (S_{ab} - \frac{1}{3} S h_{ab})$

ignore bulk perturbations, keep $\mathbb{Z}_2$ symm.

$$R_{abcd} = K^2 (g_{ac} g_{bd} - g_{ad} g_{bc})$$

Israel $\Rightarrow$ $$\Delta K_{ab} = 2 K_{+ab} = 8\pi G (S_{ab} - \frac{1}{3} S h_{ab})$$

Gauss eqn:

$$^{(4)}R_{ab} = {}^{(5)}R^{c}{}_{a'db'} h^{d}_{c} h^{a'}_{a} h^{b'}_{b} - K_{+} K_{+ab} + K_{+ac} K_{+}{}^{c}{}_{b}$$

ignore bulk perturbations, keep $\mathbb{Z}_2$ symm.

$$R_{abcd} = K^2 (g_{ac} g_{bd} - g_{ad} g_{bc})$$

Israel $\Rightarrow$ $$\Delta K_{ab} = 2 K_{+ab} = 8\pi G (S_{ab} - \tfrac{1}{3} S h_{ab})$$

Gauss eqn:

$$^{(4)}R_{ab} = {}^{(5)}R^{c}{}_{a'db'} h^{d}_{c} h^{a'}_{a} h^{b'}_{b} - K_{+} K_{+ab} + K_{+ac} K_{+}{}^{c}{}_{b}$$

$$= {}^{(5)}R_{a'b'} h^{a'}_{b} h^{b'}_{b} + n^c n^d R_{acbd} + \frac{(4\pi G)^2}{3} S \left[S_{ab} - \frac{S}{3} h_{ab} \right]$$
$$+ (4\pi G)^2 \left[S_{ac} - \frac{S}{3} h_{ac} \right] \left[S^{c}{}_{b} - \frac{S}{3} h^{c}{}_{b} \right]$$

ignore bulk perturbations, keep $\mathbb{Z}_2$ symm.

$$R_{abcd} = K^2 (g_{ac} g_{bd} - g_{ad} g_{bc})$$

Israel $\Rightarrow$ $\Delta K_{ab} = 2 K_{+ab} = 8\pi G (S_{ab} - \frac{1}{3} S h_{ab})$

Gauss eqn:

$${}^{(4)}R_{ab} = {}^{(5)}R^{c}{}_{a'db'} h^{d}_{c} h^{a'}_{a} h^{b'}_{b} - K_{+} K_{+ab} + K_{+ac} \ldots$$

$$= {}^{(5)}R_{a'b'} h^{a'}_{b} h^{b'}_{b} + n^c n^d\, {}^{(5)}R_{acbd} \ldots (4\pi G)^2 S [S \ldots$$

$$+ (4\pi G)^2 [S_{ac} - \frac{S}{3} h_{ac}] [S^{c}{}_{b} - \ldots$$

ignore bulk perturbations, keep $\mathbb{Z}_2$ symm.

$$R_{abcd} = K^2 (g_{ac} g_{bd} - g_{ad} g_{bc})$$

Israel $\Rightarrow$ $\Delta K_{ab} = 2 K_{+ab} = 8\pi G (S_{ab} - \frac{1}{3} S h_{ab})$

Gauss eqn:

$${}^{(4)}R_{ab} = {}^{(5)}R^{c}{}_{a'db'} h^{d}_{c} h^{a'}_{a} h^{b'}_{b} - K_{+} K_{+ab} + K_{+ac} K_{+}{}^{c}{}_{b}$$

$$= {}^{(5)}R_{a'b'} h^{a'}_{b} h^{b'}_{b} + n^c n^d \, {}^{(5)}R_{acbd} + \frac{(4\pi G)^2}{3} S \left[S_{ab} - \frac{S}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[S_{ac} - \frac{S}{3} h_{ac} \right] \left[S^{c}{}_{b} - \frac{S}{3} h^{c}{}_{b} \right]$$

ignore bulk perturbations, keep $\mathbb{Z}_2$ symm.

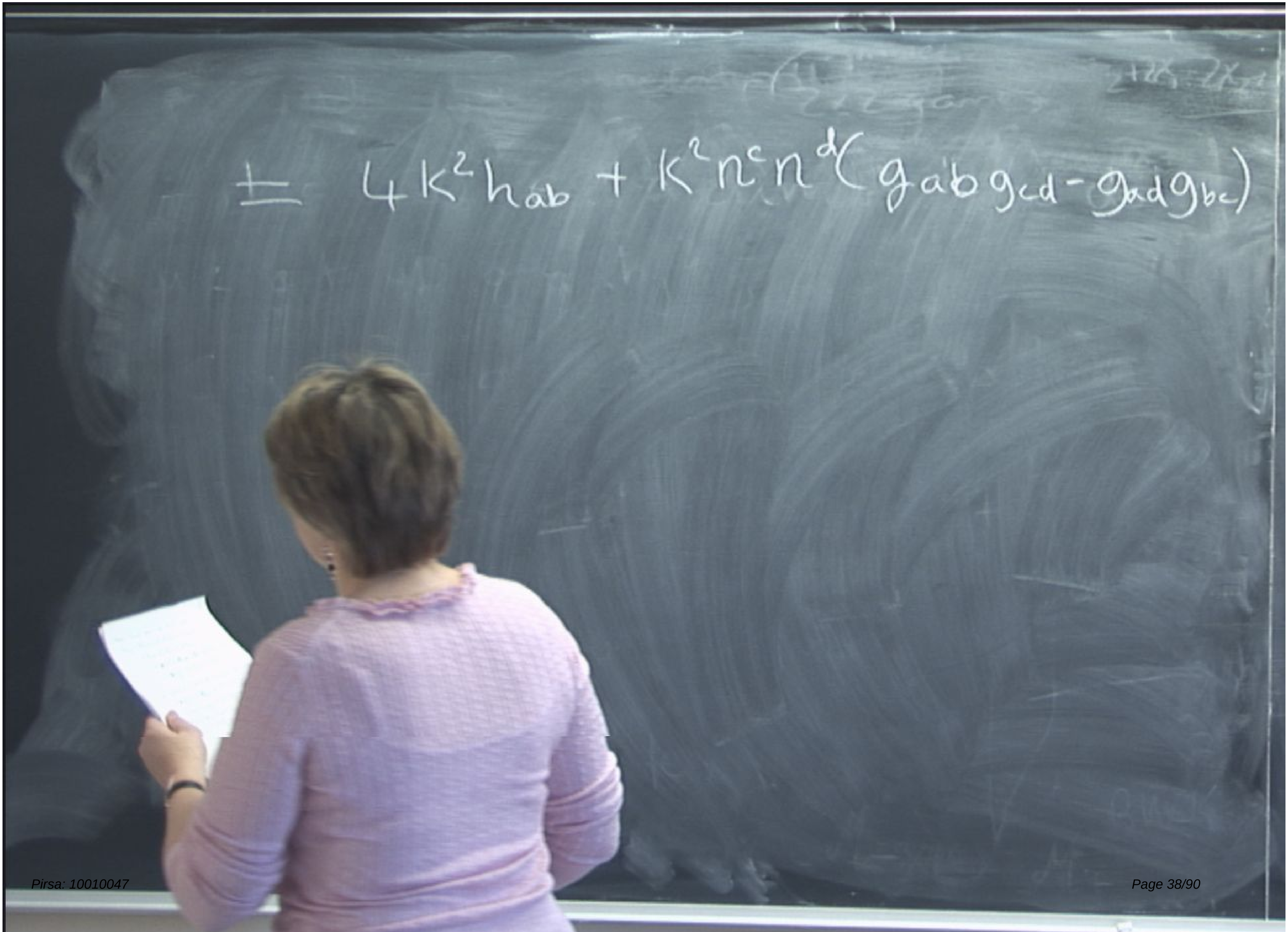
$$R_{abcd} = K^2 (g_{ac} g_{bd} - g_{ad} g_{bc})$$

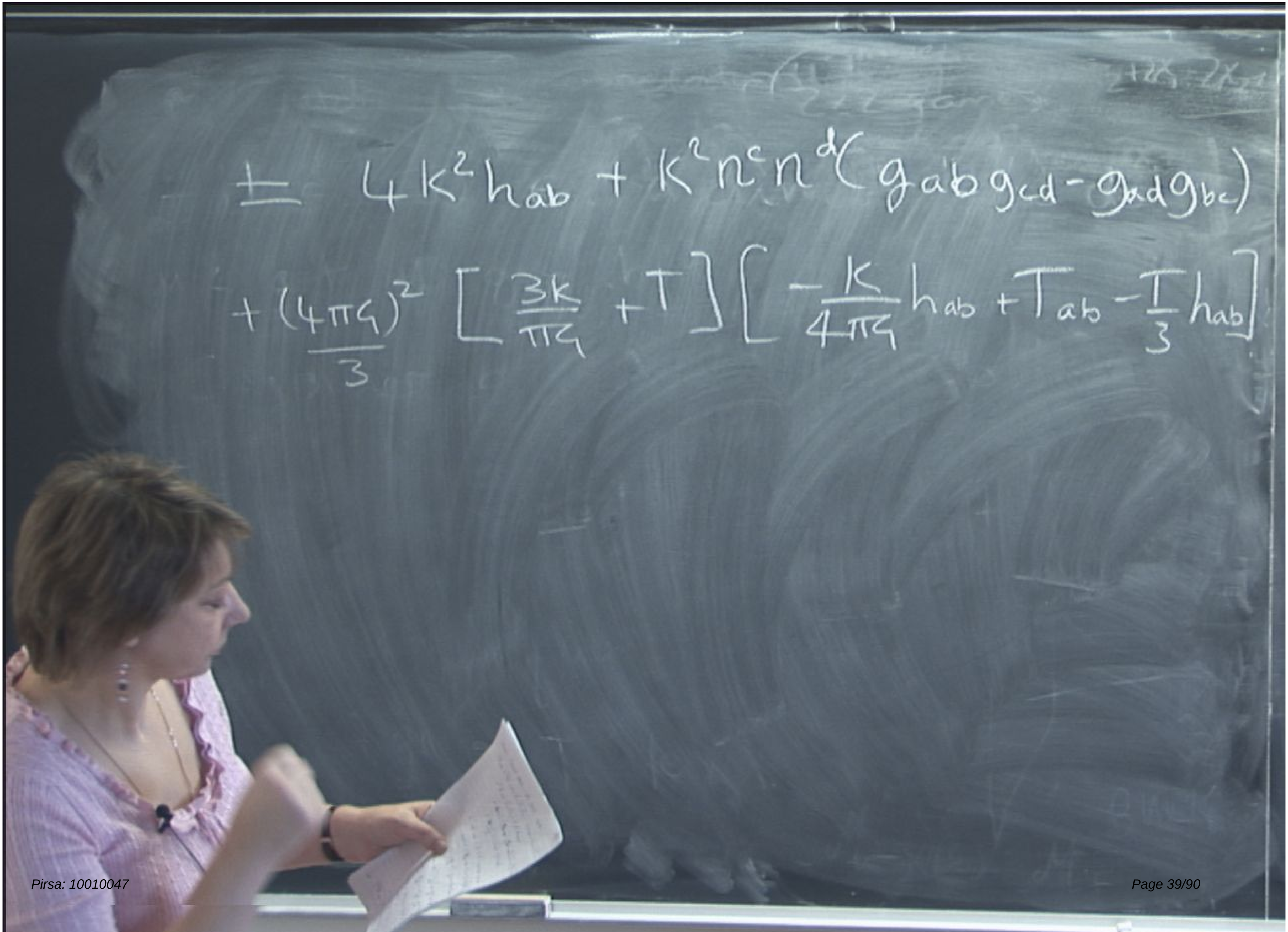
srael $\Rightarrow$ $\Delta K_{ab} = 2 K_{+ab} = 8\pi G (S_{ab} - \frac{1}{3} S h_{ab})$

Gauss eqn:

$$ {}^{(4)}R_{ab} = {}^{(5)}R^{c}{}_{a'db'} h^{d}_{c} h^{a'}_{a} h^{b'}_{b} - K_{+} K_{+ab} + K_{+ac} K_{+}{}^{c}{}_{b}$$

$$= {}^{(5)}R_{a'b'} h^{a'}_{b} h^{b'}_{b} + n^{c} n^{d}\,{}^{(5)}R_{acbd} + \frac{(4\pi G)^2}{3} S \left[S_{ab} - \frac{S}{3} h_{ab} \right] + (4\pi G)^2 \left[S_{ac} - \frac{S}{3} h_{ac} \right] \left[S^{c}{}_{b} - \frac{S}{3} h^{c}{}_{b} \right]$$


$\pm \quad 4k^2 h_{ab} + k^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})$


$\pm \; 4k^2 h_{ab} + k^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})$
$+ \frac{(4\pi G)^2}{3} \left[\frac{3k}{\pi G} + T \right] \left[-\frac{k}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$

$$\pm \quad 4K^2 h_{ab} + K^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3K}{\pi G} + T \right] \left[-\frac{K}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2K}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{K}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$\pm \; 4K^2 h_{ab} + K^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3K}{\pi G} + T \right] \left[-\frac{K}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2K}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{K}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$=$$

$$\pm \quad 4K^2 h_{ab} + K^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3K}{\pi G} + T \right] \left[-\frac{K}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2K}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{K}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$=$$

$$\pm \; 4K^2 h_{ab} + K^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})\Big|$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3K}{\pi G} + T \right] \left[-\frac{K}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2K}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{K}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$= 3K^2 h_{ab}.$$

$$\pm \quad 4K^2 h_{ab} + K^2 n^c n^d \left(g_{ab} g_{cd} - g_{ad} g_{bc} \right)$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3K}{\pi G} + T \right] \left[-\frac{K}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2K}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{K}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$= 3K^2 h_{ab} \qquad + (4\pi G)^2 \Big[$$

$$\pm \; 4k^2 h_{ab} + k^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc}) \Big|$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3k}{\pi G} + T \right] \left[-\frac{k}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2k}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{k}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$= 3k^2 h_{ab} \quad + (4\pi G)^2 \Big[$$

$$\pm \; 4k^2 h_{ab} + k^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3k}{\pi G} + T \right] \left[-\frac{k}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2k}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{k}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$= 3k^2 h_{ab} \qquad + (4\pi G)^2 \left[-\frac{4k^2}{(4\pi G)^2} \right.$$

$$\pm \; 4k^2 h_{ab} + k^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3k}{\pi G} + T \right] \left[-\frac{k}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2k}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{k}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$= 3k^2 h_{ab} \qquad + (4\pi G)^2 \left[-\frac{4k^2}{(4\pi G)^2} h_{ab} \right.$$

$$\pm \; 4k^2 h_{ab} + k^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})\Big|$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3k}{\pi G} + T \right] \left[-\frac{k}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2k}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{k}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$= 3k^2 h_{ab}. \quad + (4\pi G)^2 \left[-\frac{4k^2}{(4\pi G)^2} h_{ab} + \frac{k^2}{(4\pi G)^2} h_{ab}. \right.$$

$$\pm \quad 4K^2 h_{ab} + K^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3K}{\pi G} + T \right] \left[-\frac{K}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2K}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{K}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$h_{ab} \quad + (4\pi G)^2 \left[-\frac{4K^2}{(4\pi G)^2} h_{ab} + \frac{K^2}{(4\pi G)^2} h_{ab} \right.$$

$$\pm \quad 4K^2 h_{ab} + K^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc}) \Big|$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3K}{\pi G} + T \right] \left[-\frac{K}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2K}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{K}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$= 3K^2 h_{ab}. \quad + (4\pi G)^2 \left[-\frac{4K^2}{(4\pi G)^2} h_{ab} + \frac{K^2}{(4\pi G)^2} h_{ab} \right.$$

$$\pm \quad 4K^2 h_{ab} + K^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3K}{\pi G} + T \right] \left[-\frac{K}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2K}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{K}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$= 3K^2 h_{ab}. \quad + (4\pi G)^2 \left[-\frac{4K^2}{(4\pi G)^2} h_{ab} + \frac{K^2}{(4\pi G)^2} h_{ab} \right]$$

$$\ldots \pi G)^2 \left[\frac{-KT}{3 \times 4\pi G} h_{ab} \right.$$

$$\pm \; 4K^2 h_{ab} + K^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})\Big|$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3K}{\pi G} + T \right] \left[-\frac{K}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2K}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{K}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$= 3K^2 h_{ab}. \quad + (4\pi G)^2 \left[-\frac{4K^2}{(4\pi G)^2} h_{ab} + \frac{K^2}{(4\pi G)^2} h_{ab}. \right]$$

$$+ (4\pi G)^2 \left[\frac{-KT}{3 \times 4\pi G} h_{ab} + \frac{4K}{4\pi G} \left(T_{ab} - \frac{T}{3} h_{ab} \right) \right.$$

$$\pm \; 4K^2 h_{ab} + K^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3K}{\pi G} + T \right] \left[-\frac{K}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2K}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{K}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$= 3K^2 h_{ab} \quad + (4\pi G)^2 \left[-\frac{4K^2}{(4\pi G)^2} h_{ab} + \frac{K^2}{(4\pi G)^2} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[\frac{-KT}{3 \times 4\pi G} h_{ab} + \frac{4K}{4\pi G} \left(T_{ab} - \frac{T}{3} h_{ab} \right) \right.$$

$$\pm \quad 4K^2 h_{ab} + K^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})\Big|$$

$$+ \frac{(4\pi G)^2}{3}\left[\frac{3K}{\pi G} + T\right]\left[-\frac{K}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab}\right]$$

$$+ (4\pi G)^2 \left[-\frac{2K}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac}\right]$$

$$\times \left[-\frac{K}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b\right]$$

$$= 3K^2 h_{ab}. \quad + (4\pi G)^2 \left[-\frac{4K^2}{(4\pi G)^2} h_{ab} + \frac{K^2}{(4\pi G)^2} h_{ab}\right]$$

$$+ (4\pi G)^2 \left[\frac{-KT}{3 \times 4\pi G} h_{ab} + \frac{4K - K}{4\pi G}\left(T_{ab} - \frac{T}{3} h_{ab}\right)\right]$$

$$\pm \quad 4K^2 h_{ab} + K^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc}) \Big|$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3K}{\pi G} + T \right] \left[-\frac{K}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2K}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{K}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$= 3K^2 h_{ab} \qquad + (4\pi G)^2 \left[-\frac{4K^2}{(4\pi G)^2} h_{ab} + \frac{K^2}{(4\pi G)^2} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[\frac{-KT}{3 \times 4\pi G} h_{ab} + \frac{4K - 2K}{4\pi G} \left(T_{ab} - \frac{T}{3} h_{ab} \right) \right].$$

$$\pm \quad 4K^2 h_{ab} + K^2 n^c n^d (g_{ab} g_{cd} - g_{ad} g_{bc})$$

$$+ \frac{(4\pi G)^2}{3} \left[\frac{3K}{\pi G} + T \right] \left[-\frac{K}{4\pi G} h_{ab} + T_{ab} - \frac{T}{3} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[-\frac{2K}{8\pi G} h_{ac} + T_{ac} - \frac{1}{3} T h_{ac} \right]$$

$$\times \left[-\frac{K}{4\pi G} h^c{}_b + T^c{}_b - \frac{T}{3} h^c{}_b \right]$$

$$= \cancel{3K^2 h_{ab}} \quad + (4\pi G)^2 \left[-\cancel{\frac{4K^2}{(4\pi G)^2}} h_{ab} + \cancel{\frac{K^2}{(4\pi G)^2}} h_{ab} \right]$$

$$+ (4\pi G)^2 \left[\frac{-KT}{3 \times 4\pi G} h_{ab} + \frac{4K - 2K}{4\pi G} \left(T_{ab} - \frac{T}{3} h_{ab} \right) \right].$$

$$(4) \quad R_{ab}^{2} = 4\pi G\left[-KTh_{ab} + 2k\left(T_{ab} - \frac{T}{3}h_{ab}\right)\right].$$

$$(4)\quad R^{2}_{ab} = 4\pi G\left[-\frac{K}{3}T\,h_{ab} + 2k\left(T_{ab} - \frac{T}{3}h_{ab}\right)\right].$$

$$(4)\ {}^{3}R_{ab} = 4\pi G\left[-\frac{KT}{3} h_{ab} + 2k\left(T_{ab} - \frac{T}{3} h_{ab}\right)\right].$$

$$(4)\quad R^{2}_{ab} = 4\pi G\left[-\frac{K}{3}T\,h_{ab} + 2k\left(T_{ab} - \frac{T}{3}h_{ab}\right)\right].$$

$$= 8\pi G K\left[T_{ab} - \frac{1}{3}T\,h_{ab} - \frac{1}{6}T\,h_{ab}\right].$$

$$ {}^{(4)}R_{ab}{}^{2} = 4\pi G\left[-\frac{kT}{3}h_{ab} + 2k\left(T_{ab} - \frac{T}{3}h_{ab}\right)\right]. $$

$$ = 8\pi G k\left[T_{ab} \underbrace{- \frac{1}{3}T h_{ab} - \frac{1}{6}T h_{ab}}_{-\frac{1}{2}T h_{ab}.}\right]. $$

$$ {}^{(4)}R = 8\pi G k(-T) $$

$$ \rightarrow {}^{(4)}R_{\mu\nu} - \frac{1}{2}\,{}^{(4)}R\, g_{\mu\nu} = 8\pi \underbrace{Gk}_{G_N.}\, T_{\mu\nu} $$

$$ {}^{(4)}R_{ab} = 4\pi G\left[-\frac{k}{3}T\, h_{ab} + 2k\left(T_{ab} - \frac{T}{3}h_{ab}\right)\right]. $$

$$ = 8\pi G k\left[T_{ab} \underbrace{- \frac{1}{3}T\, h_{ab} - \frac{1}{6}T\, h_{ab}}_{-\frac{1}{2}T\, h_{ab}}\right]. $$

$$ {}^{(4)}R = 8\pi G k(-T) $$

4D

EINSTEIN.

$$ \rightarrow {}^{(4)}R_{\mu\nu} - \frac{1}{2}\,{}^{(4)}R\, g_{\mu\nu} = 8\pi \underbrace{Gk}_{G_N} T_{\mu\nu} $$

$$^{(4)}R_{ab} = 4\pi G\left[-\frac{k}{3}T\,h_{ab} + 2k\left(T_{ab} - \frac{T}{3}h_{ab}\right)\right]$$

$$= 8\pi G k\left[T_{ab} \underbrace{- \frac{1}{3}T\,h_{ab} - \frac{1}{6}T\,h_{ab}}_{-\boxed{\frac{1}{2}}T\,h_{ab}}\right]$$

$$^{(4)}R = 8\pi G k(-T)$$

$$\rightarrow {}^{(4)}R_{\mu\nu} - \frac{1}{2}\,{}^{(4)}R\,g_{\mu\nu} = 8\pi \underbrace{Gk}_{G_N}\,T_{\mu\nu}$$

4D EINSTEIN

Cosmological Braneworld

Cosmological Braneworld

Now consider a more general bulk:

$$ds^2 = V(r)\,dt^2 - \frac{dr^2}{V(r)} - r^2\, d\underline{x}^2_{\boxed{\text{III}}}$$

Cosmological Braneworld

Now consider a more general bulk:

$$ds^2 = V(r)\,dt^2 - \frac{dr^2}{V(r)} - r^2 d\underline{x}^2_{\cancel{\kappa}}$$

$$\kappa + k^2 r^2 - \frac{\mu}{r^2}$$

Cosmological Braneworld

Now consider a more general bulk:

$$ds^2 = V(r)\,dt^2 - \frac{dr^2}{V(r)} - r^2 d\underline{x}^2_{\kappa}$$

$$\kappa + k^2 r^2 - \frac{\mu}{r^2}$$

Cosmological Braneworld

Now consider a more general bulk:

$$ds^2 = V(r)\,dt^2 - \frac{dr^2}{V(r)} - r^2 d\underline{x}^2_{\cancel{\kappa}}$$

$K + k^2 r^2 - \frac{\mu}{r^2}$

spatial part of V.

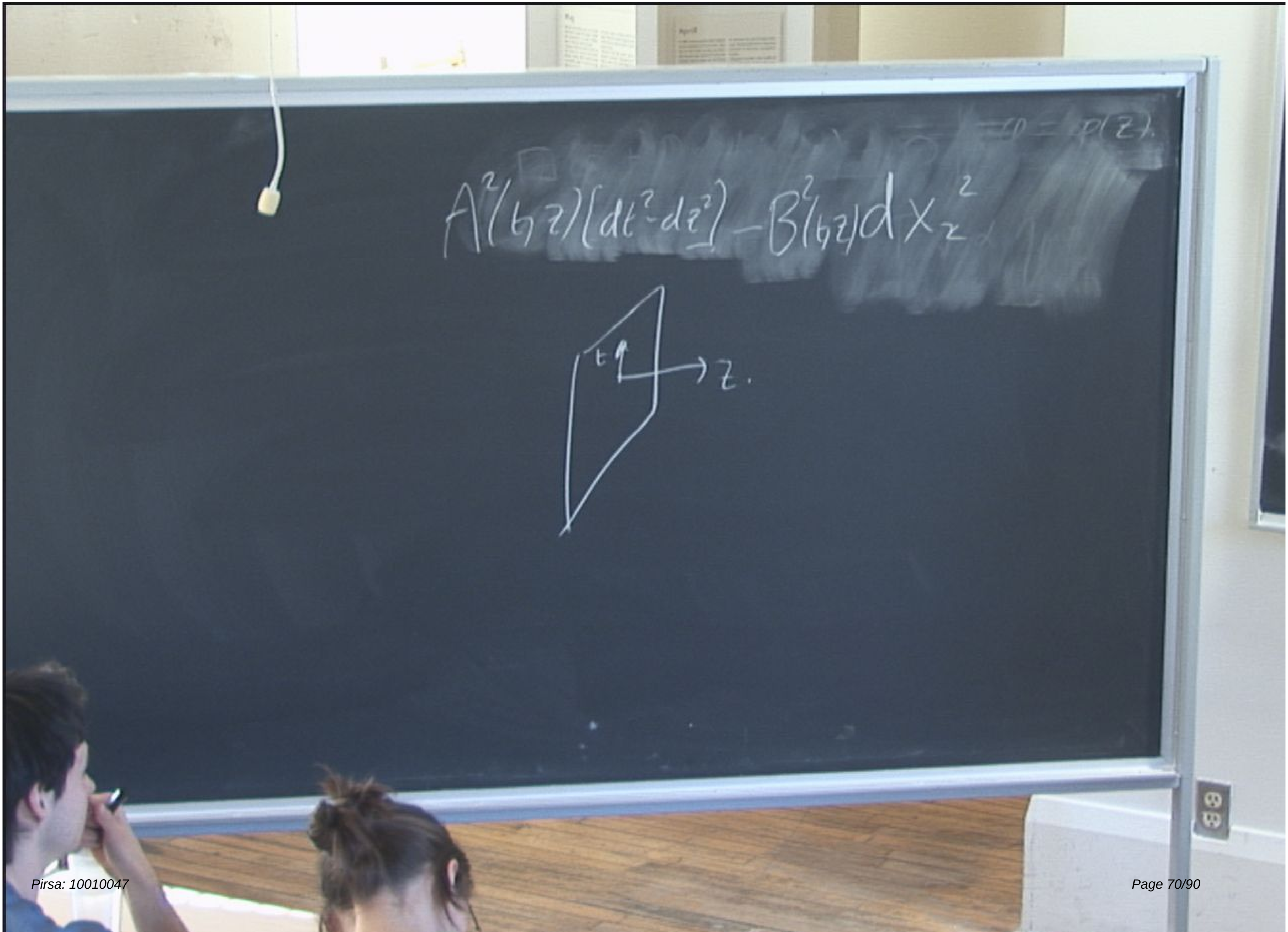
Cosmological Braneworld

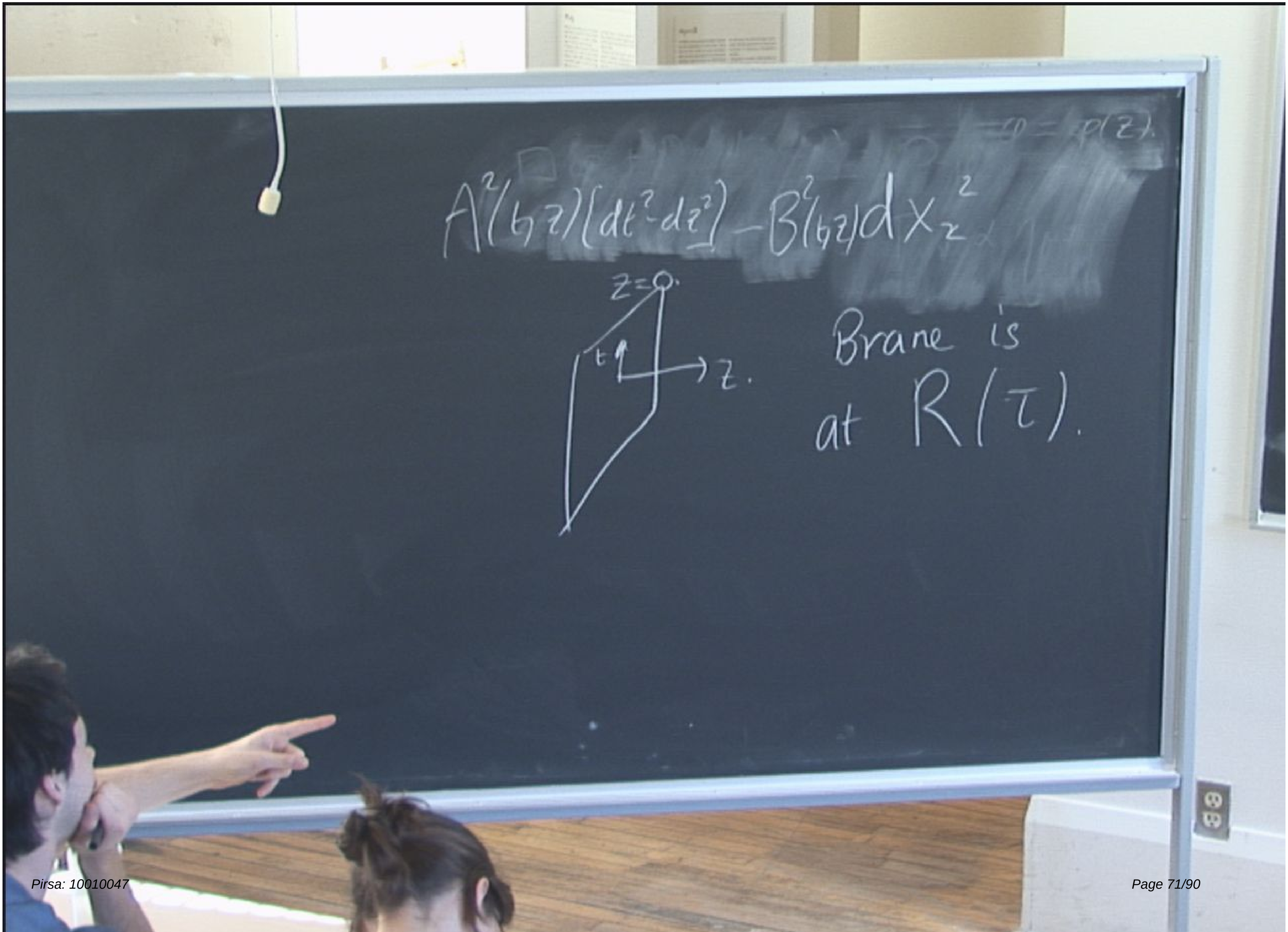
Now consider a more general bulk:

$$ds^2 = V(r)\,dt^2 - \frac{dr^2}{V(r)} - r^2 d\underline{X}^2_{\kappa}$$

$$\kappa + k^2 r^2 - \frac{\mu}{r^2}$$

spatial part of V.


$A^2(t,z)[dt^2-dz^2] - B^2(t,z)dx_\perp^2$
t
z.


A²(t,z)[dt²-dz²] - B²(t,z)dx_z²
z=0.
t
z.
Brane is
at R(τ).

$$X^a = \left(t(\tau),\ R(\tau),\ \underline{x} \right)$$

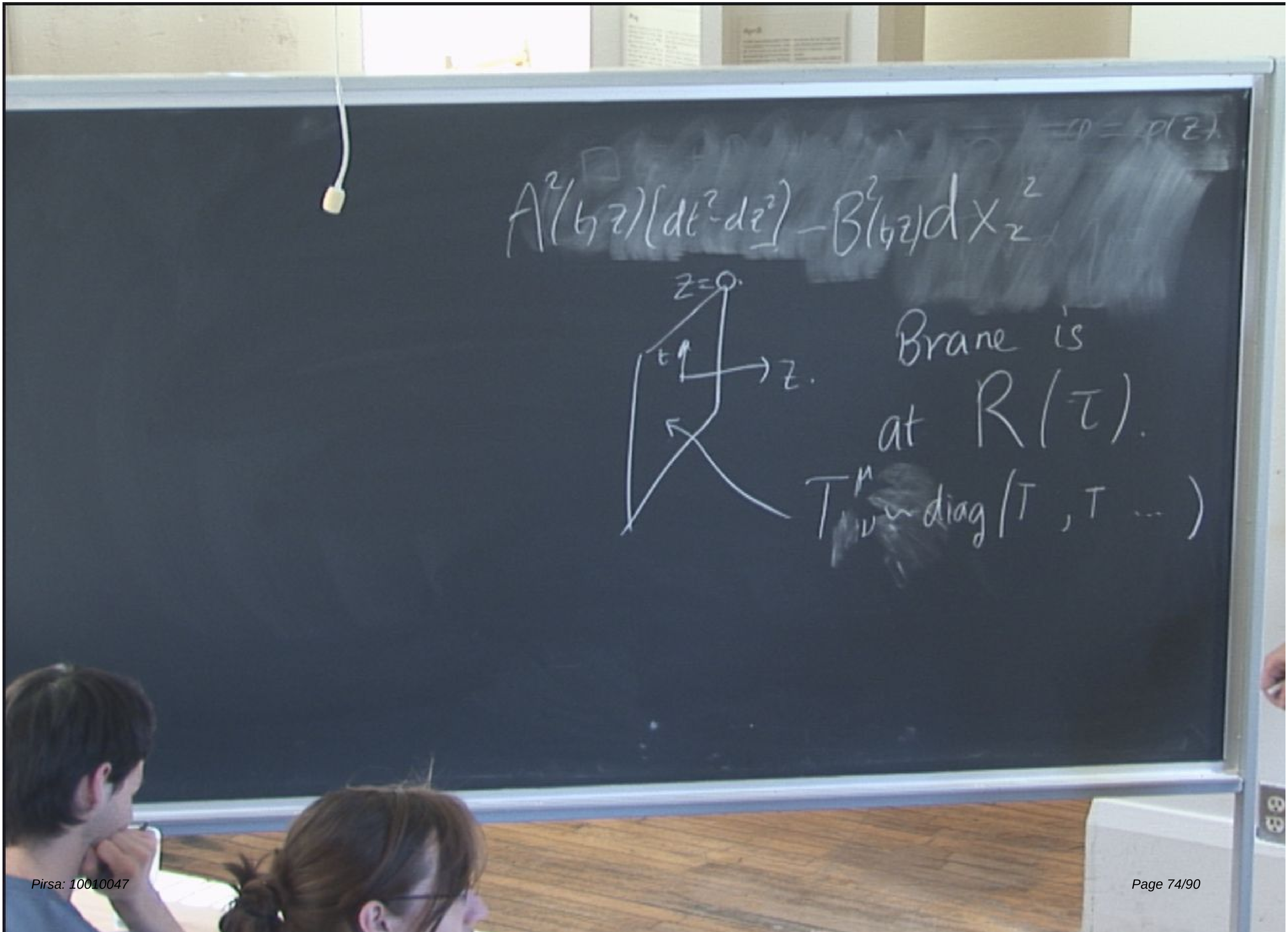
$$\dot{X}^a = u^a = \left(\dot{t},\ \dot{R},\ \underline{0} \right) \qquad V\dot{t}^2 - \frac{\dot{R}^2}{V} = 1$$

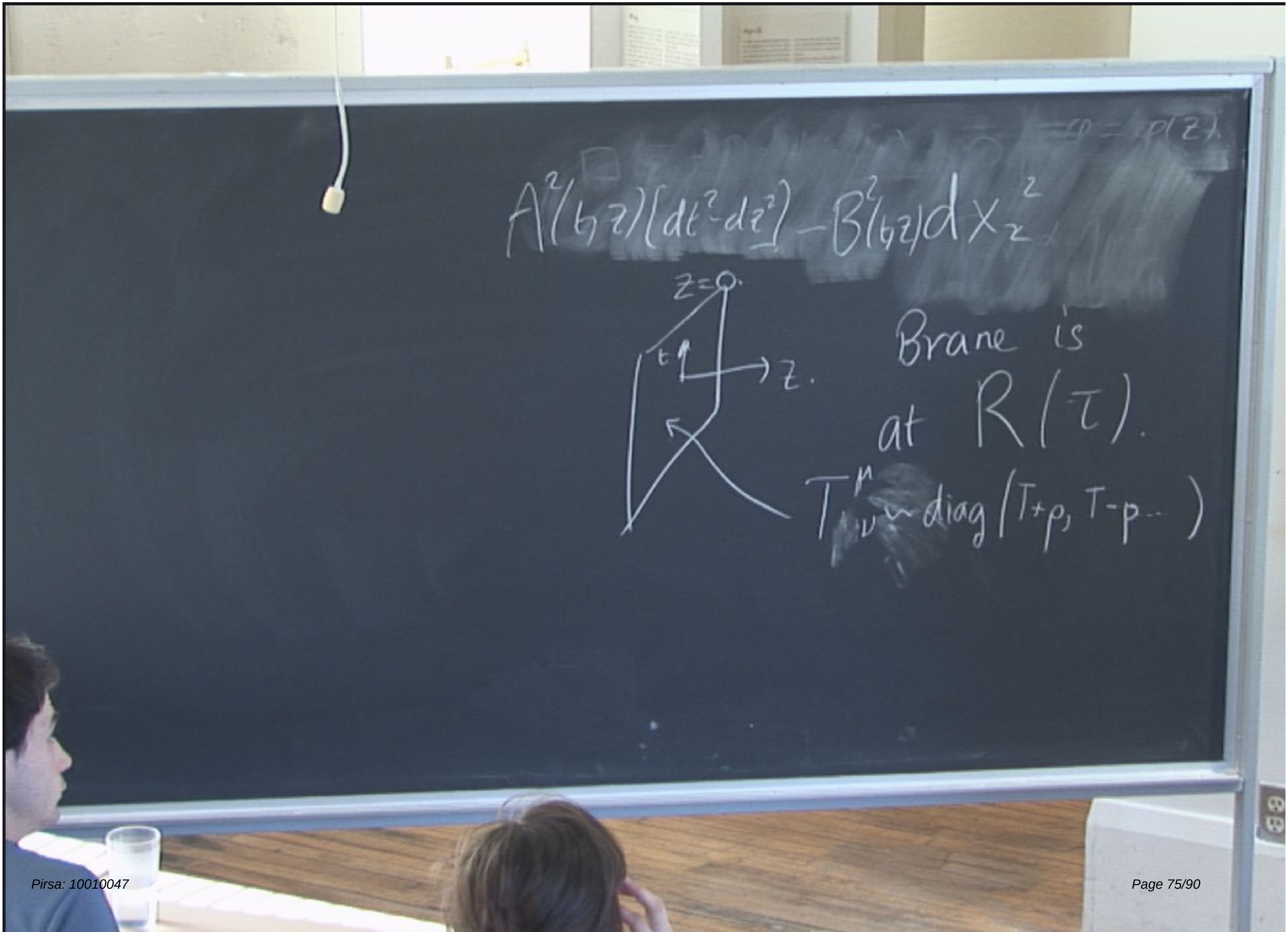
$$X^a = \left(t(\tau),\ R(\tau),\ \underline{x} \right)$$

$$\dot{X}^a = u^a = \left(\dot{t},\ \dot{R},\ \underline{0} \right) ; \quad V\dot{t}^2 - \frac{\dot{R}^2}{V} = 1$$

$$n_a = \left(-\dot{R},\ \dot{t},\ \underline{0} \right)$$

$$K_{ij} = -\Gamma^r_{ij}\, n_r = -\frac{1}{2} V\, g_{ij}'\, \dot{t} = -\frac{V\dot{t}}{2r}\, g_{ij}$$


A²(t,z)(dt²-dz²) - B²(t,z) dx_z²
z=0
t
z.
Brane is
at R(τ).
T^μ_ν ~ diag(T, T ...)


$\varphi = \varphi(z)$
$A^2(t,z)[dt^2-dz^2] - B^2(t,z)dx_z^2$
$z=0$
t
z.
Brane is
at $R(\tau)$.
$T^\mu_\nu \sim \mathrm{diag}(T+\rho, T-p \dots)$

Take $\mathbb{Z}_2$ symmetry, $\Delta K_{ab} = 2K_{+ab}$

$$S_{00} = \frac{6K}{8\pi G} + \rho$$

$$S_{ij} = g_{ij}\left(\frac{6K}{8\pi G} - p\right)$$

i.e. $S_{\mu\nu} = \frac{6K}{8\pi G} g_{\mu\nu} + \rho u_\mu u_\nu - p(g_{\mu\nu} - u_\mu u_\nu)$

Take $\mathbb{Z}_2$ symmetry, $\Delta K_{ab} = 2 K_{+ab}$

$$S_{00} = \frac{6K}{8\pi G} + \rho$$

$$S_{ij} = g_{ij}\left(\frac{6K}{8\pi G} - p\right)$$

i.e. $S_{\mu\nu} = \frac{6K}{8\pi G} g_{\mu\nu} + \rho u_\mu u_\nu - p(g_{\mu\nu} - u_\mu u_\nu)$

$$S = \frac{3K}{\pi G}$$

Take $\mathbb{Z}_2$ symmetry, $\Delta K_{ab} = 2 K_{+ab}$

$$S_{00} = \frac{6K}{8\pi G} + \rho$$

$$S_{ij} = g_{ij}\left(\frac{6K}{8\pi G} - p \right)$$

i.e. $S_{\mu\nu} = \frac{6K}{8\pi G} g_{\mu\nu} + \rho u_\mu u_\nu - p(g_{\mu\nu} - u_\mu u_\nu)$

$$S = \frac{3K}{\pi G} + \rho - 3p$$

$$\Rightarrow S_{ij} - \frac{1}{3} S\, g_{ij} = g_{ij}\left(\frac{6K}{8\pi G} - \cancel{p} - \frac{K}{\pi G} - \frac{p}{3} + \cancel{p}\right)$$

$$K_{ij} = -\Gamma^{r}_{ij}\, n_r = -\frac{1}{2} V\, g_{ij}'\, \dot{t} = -\frac{V\dot{t}}{2r}\, g_{ij}$$

$$\Rightarrow S_{ij} - \frac{1}{3} S g_{ij} = g_{ij}\left(\frac{6K}{8\pi G} - \cancel{p} - \frac{K}{\pi G} - \frac{\rho}{3} + \cancel{p}\right)$$

$$= \frac{-K}{4\pi G} g_{ij} - \frac{\rho}{3} g_{ij}$$

$$K_{ij} = -\Gamma^{r}_{ij} n_r = -\frac{1}{2} V g_{ij}' \dot{t} = \frac{-V\dot{t}}{2r} g_{ij}$$

$$\Rightarrow S_{ij} - \frac{1}{3} S g_{ij} = g_{ij}\left(\frac{6k}{8\pi G} - \cancel{\rho} - \frac{k}{\pi G} - \frac{\rho}{3} + \cancel{\rho}\right)$$

$$= \frac{-k}{4\pi G} g_{ij} - \frac{\rho}{3} g_{ij}$$

$$K_{ij} = -\Gamma^{r}_{ij} n_r = -\frac{1}{2} V g_{ij}' \dot{t} = -\frac{V\dot{t}}{2r} g_{ij}$$

$$\Rightarrow \; S_{ij} - \frac{1}{3} S g_{ij} = g_{ij}\left(\frac{6k}{8\pi G} - \cancel{\rho} - \frac{k}{\pi G} - \frac{\rho}{3} + \cancel{\rho}\right)$$

$$= -\frac{k}{4\pi G} g_{ij} - \frac{\rho}{3} g_{ij}$$

$$K_{ij} = -\Gamma^{r}_{ij} n_r = -\frac{1}{2} V g_{ij}' \dot{t} = -\frac{V\dot{t}}{2r} g_{ij}$$

$$\Rightarrow S_{ij} - \frac{1}{3} S g_{ij} = g_{ij}\left(\frac{6K}{8\pi G} - \cancel{p} - \frac{K}{\pi G} - \frac{\rho}{3} + \cancel{p}\right)$$

$$= \frac{-K}{4\pi G} g_{ij} - \frac{\rho}{3} g_{ij}$$

Israel eqns $\quad K_{ab} = 4\pi G\left(S_{ab} - \frac{1}{3} S g_{ab}\right)$

$$K_{ij} = -\Gamma^{r}_{ij} n_r = -\frac{1}{2} V g_{ij}' \dot{t} = -\frac{V\dot{t}}{2r} g_{ij}$$

$$\Rightarrow S_{ij} - \frac{1}{3} S g_{ij} = g_{ij}\left(\frac{6k}{8\pi G} - \cancel{p} - \frac{k}{\pi G} - \frac{\rho}{3} + \cancel{p}\right)$$

$$= -\frac{k}{4\pi G} g_{ij} - \frac{\rho}{3} g_{ij}$$

Israel eqns $\quad K_{ab} = 4\pi G\left(S_{ab} - \frac{1}{3} S g_{ab}\right)$

$$-\frac{V\dot{t}}{2R} = 4\pi G\left(-\frac{k}{4\pi G} - \frac{\rho}{3}\right)$$

$$\Rightarrow S_{ij} - \frac{1}{3} S g_{ij} = g_{ij}\left(\frac{6k}{8\pi G} - \cancel{p} - \frac{k}{\pi G} - \frac{\rho}{3} + \cancel{p}\right)$$

$$= -\frac{k}{4\pi G} g_{ij} - \frac{\rho}{3} g_{ij}$$

Israel eqns $\quad K_{ab} = 4\pi G\left(S_{ab} - \frac{1}{3} S g_{ab}\right)$

$$-\frac{V\dot{t}}{2R} = 4\pi G\left(-\frac{k}{4\pi G} - \frac{\rho}{3}\right)$$

But $\quad V\dot{t}^2 - \frac{\dot{R}^2}{V} = 1 \Rightarrow V\dot{t} = \sqrt{V + \dot{R}^2}$

$$\Rightarrow S_{ij} - \frac{1}{3} S g_{ij} = g_{ij}\left(\frac{6k}{8\pi G} - \cancel{p} - \frac{k}{\pi G} - \frac{\rho}{3} + \cancel{p}\right)$$

$$= \frac{-k}{4\pi G} g_{ij} - \frac{\rho}{3} g_{ij}$$

Israel eqns $\quad K_{ab} = 4\pi G\left(S_{ab} - \frac{1}{3} S g_{ab}\right)$

$$-\frac{V\dot{t}}{2R} = 4\pi G\left(\frac{-k}{4\pi G} - \frac{\rho}{3}\right)$$

But $\quad V\dot{t}^2 - \frac{\dot{R}^2}{V} = 1 \Rightarrow V\dot{t} = \sqrt{V + \dot{R}^2}$

$$\Rightarrow \sqrt{\frac{\dot{R}^2}{R^2} + \frac{\chi}{R^2} + k^2 - \frac{\mu}{R^4}} = k + \frac{4\pi G \rho}{3}.$$

$$\frac{K}{R^2} + \frac{\dot{R}^2}{R^2} = \frac{8\pi G k}{3}\rho + \left(\frac{4\pi G}{3}\right)^2 \rho^2 + \frac{\mu}{R^4}$$

$$\Rightarrow \sqrt{\frac{\dot{R}^2}{R^2} + \frac{\chi}{R^2} + k^2 - \frac{\mu}{R^4}} = k + \frac{4\pi G \rho}{3}.$$

$$\underbrace{\frac{K}{R^2} + \frac{\dot{R}^2}{R^2}}_{\text{LHS Friedmann eqn}} = \underbrace{\frac{8\pi G k}{3}\rho}_{\text{RHS of F. eqn}} + \left(\frac{4\pi G}{3}\right)^2 \rho^2 + \frac{\mu}{R^4}$$

$$\Rightarrow \sqrt{\frac{\dot{R}^2}{R^2} + \frac{\chi}{R^2} + k^2 - \frac{\mu}{R^4}} = k + \frac{4\pi G \rho}{3}.$$

$$\underbrace{\frac{k}{R^2} + \frac{\dot{R}^2}{R^2}}_{\text{LHS Friedmann eqn}} = \underbrace{\frac{8\pi G k}{3}\rho}_{\text{RHS of F. eqn}} + \underbrace{\left(\frac{4\pi G}{3}\right)^2 \rho^2}_{\text{NEW } \rho^2.} + \frac{\mu}{R^4}$$

$$\Rightarrow \sqrt{\frac{\dot{R}^2}{R^2} + \frac{\chi}{R^2} + k^2 - \frac{\mu}{R^4}} = k + \frac{4\pi G\rho}{3}.$$

$$\underbrace{\frac{k}{R^2} + \frac{\dot{R}^2}{R^2}}_{\text{LHS Friedmann eqn}} = \underbrace{\frac{8\pi G k}{3}\rho}_{\text{RHS of F. eqn}} + \underbrace{\left(\frac{4\pi G}{3}\right)^2 \rho^2}_{\text{NEW } \rho^2.} + \underbrace{\frac{\mu}{R^4}}_{\text{NEW c.f. radn.}}$$