Title: Gravitational Physics - Review (PHYS 636) - Lecture 13

Date: Jan 20, 2010 10:00 AM

URL: http://pirsa.org/10010046

Abstract:

Fraction of

The KK monopole (Gross, Perry, Sorkin)

The KK monopole (Gross, Perry, Sorkin)

Look for $F \sim Q \sin\theta \, d\theta \wedge d\varphi$

$$A_\varphi^{(N)} = Q(1-\cos\theta)$$

$$A_\varphi^{(S)} = -Q(1+\cos\theta)$$

The KK monopole (Gross, Perry, Sorkin)

Look for $F \sim Q \sin\theta \, d\theta \wedge d\varphi$

$$A_\varphi^{(N)} = Q(1-\cos\theta)$$

$$A_\varphi^{(S)} = -Q(1+\cos\theta)$$

In this case $2Q\,d\varphi$

The KK monopole (Gross, Perry, Sorkin)

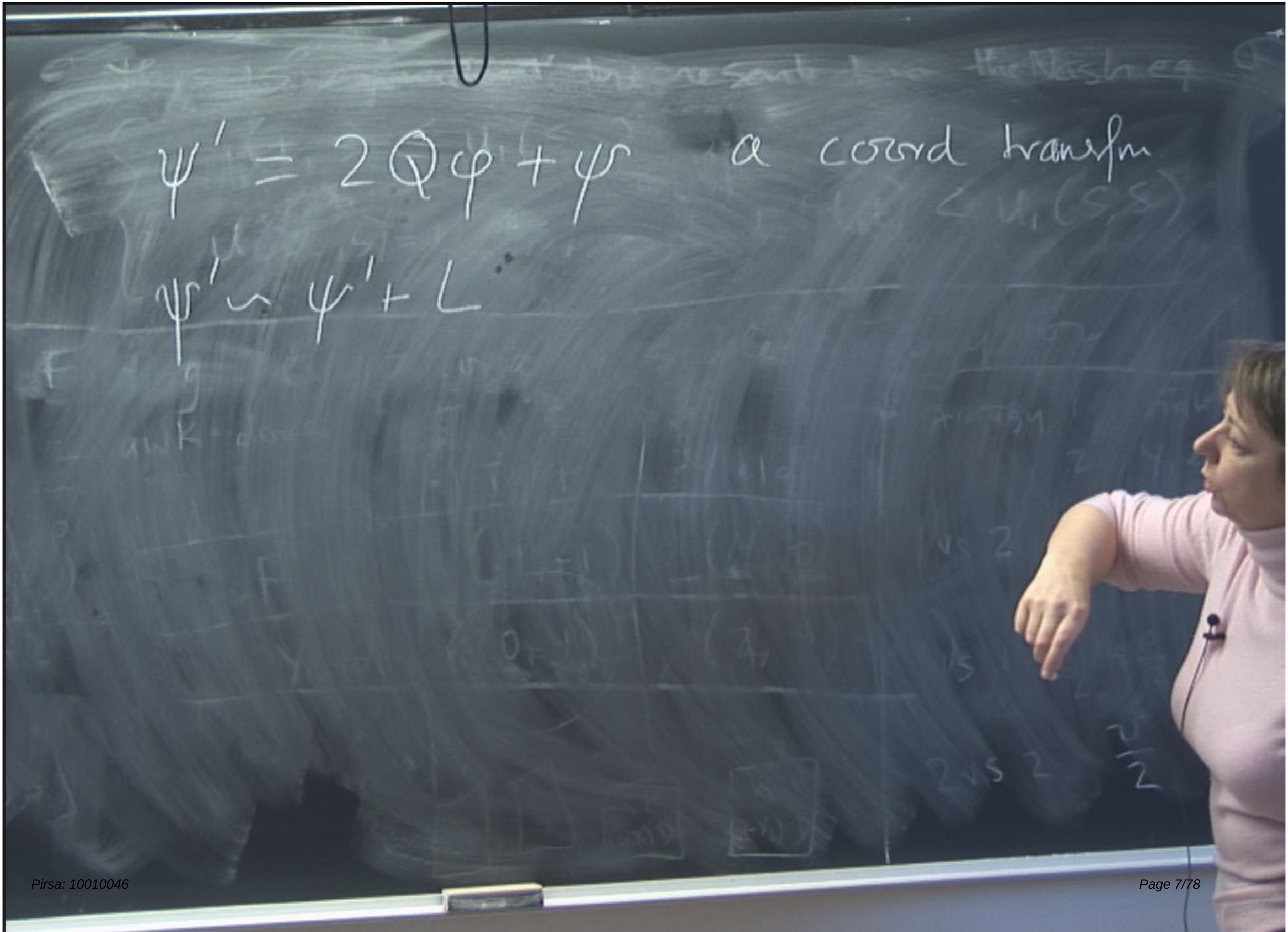
Look for $F \sim Q \sin\theta \, d\theta \wedge d\varphi$

$$A^{(N)}_{\varphi} = Q(1-\cos\theta)$$

$$A^{(S)}_{\varphi} = -Q(1+\cos\theta)$$

In this case $2Q\,d\varphi$ must be an allowable co-ord transfm.

$\psi' = 2Q\varphi + \psi$ a coord transfm


ψ' = 2Qφ + ψ
a coord transfm
ψ' ~ ψ' + L

$\psi' = 2Q\varphi + \psi$ a coord transfm

$\psi' \sim \psi' + L$ so $4\pi Q = nL$ $n \in \mathbb{Z}$

$\psi' = 2Q\varphi + \psi$ a coord transfm

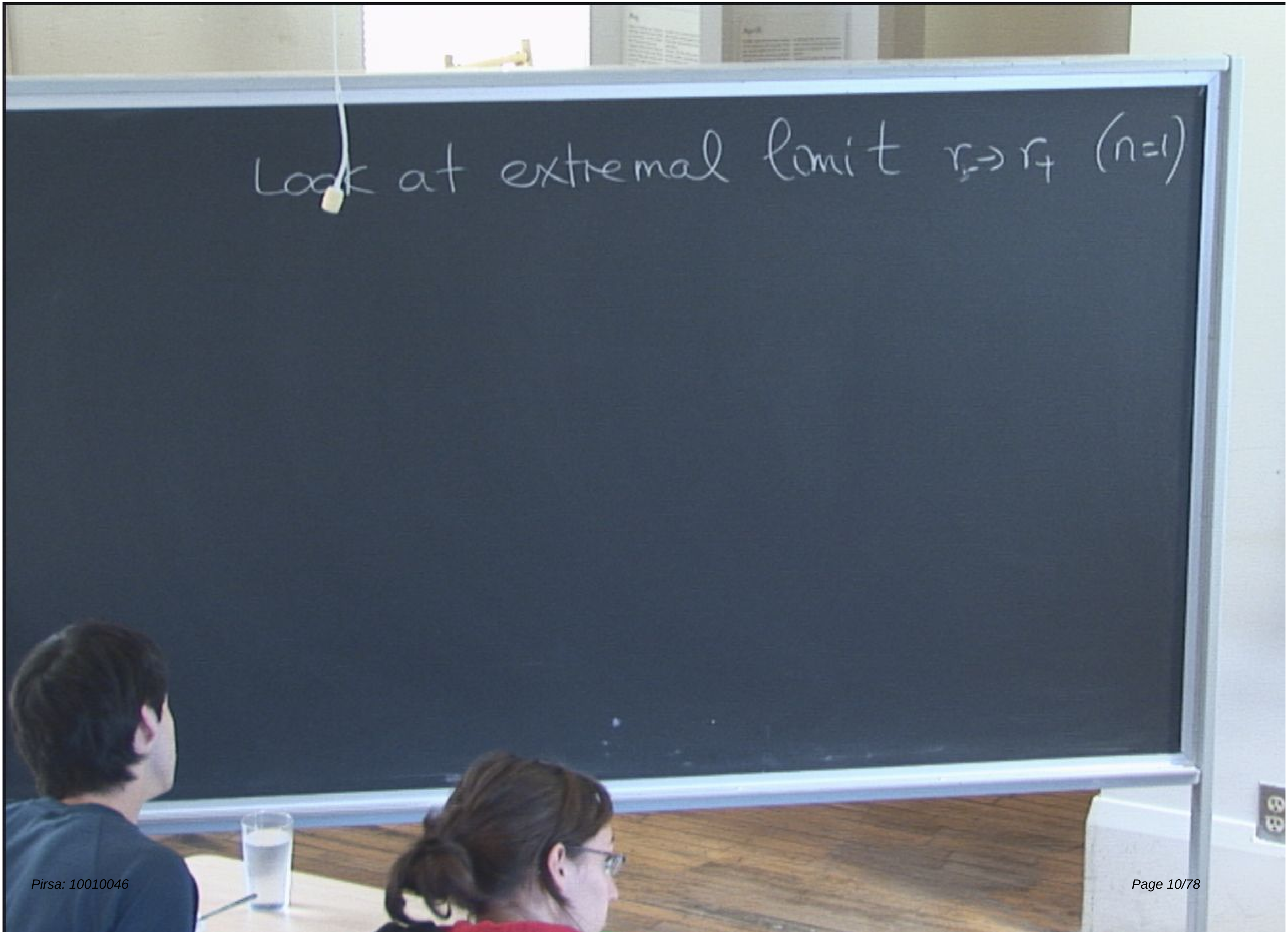
$\psi' \sim \psi' + L$ so $4\pi Q = nL$ $n \in \mathbb{Z}$

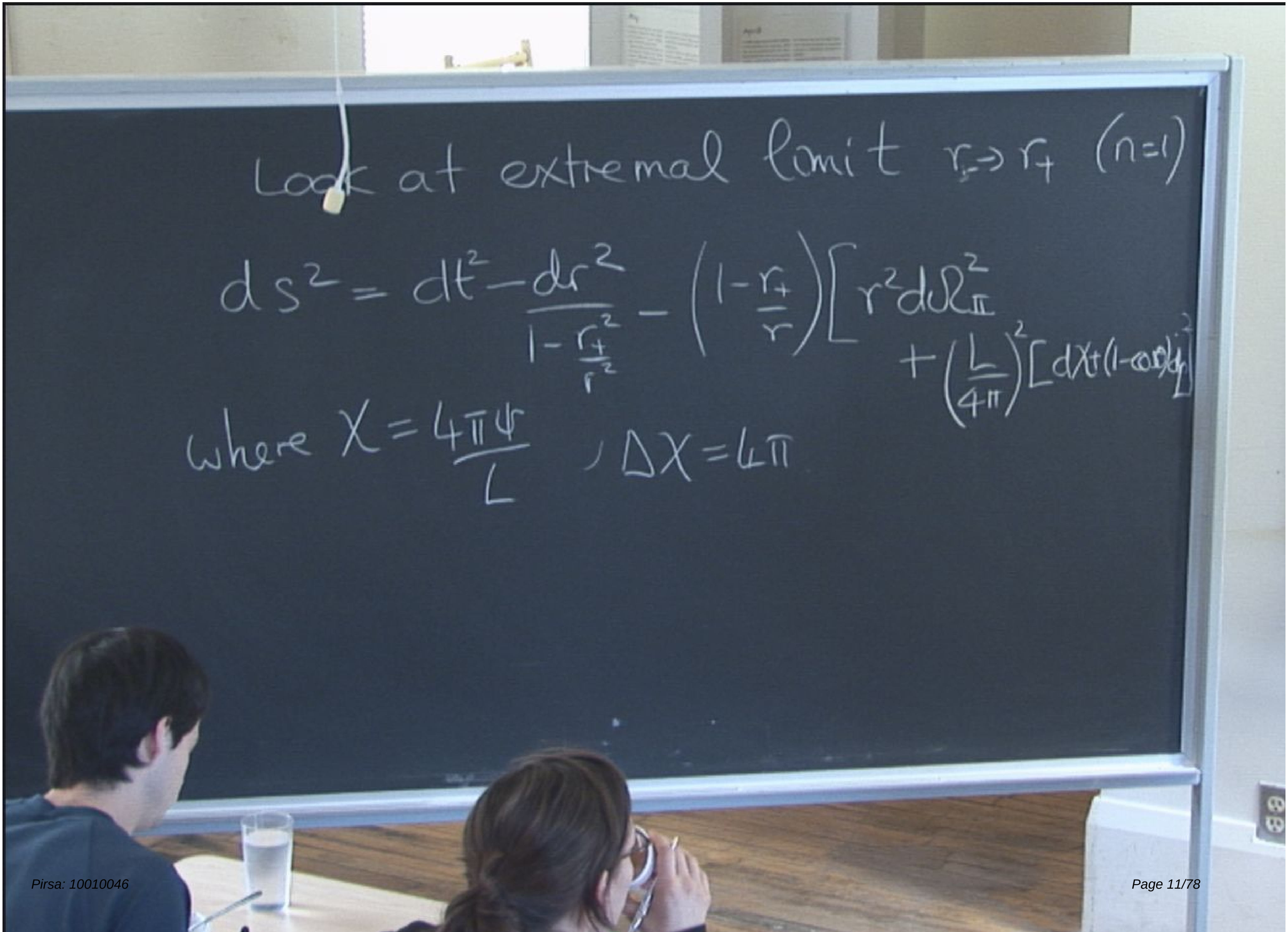
The black hole solns are:

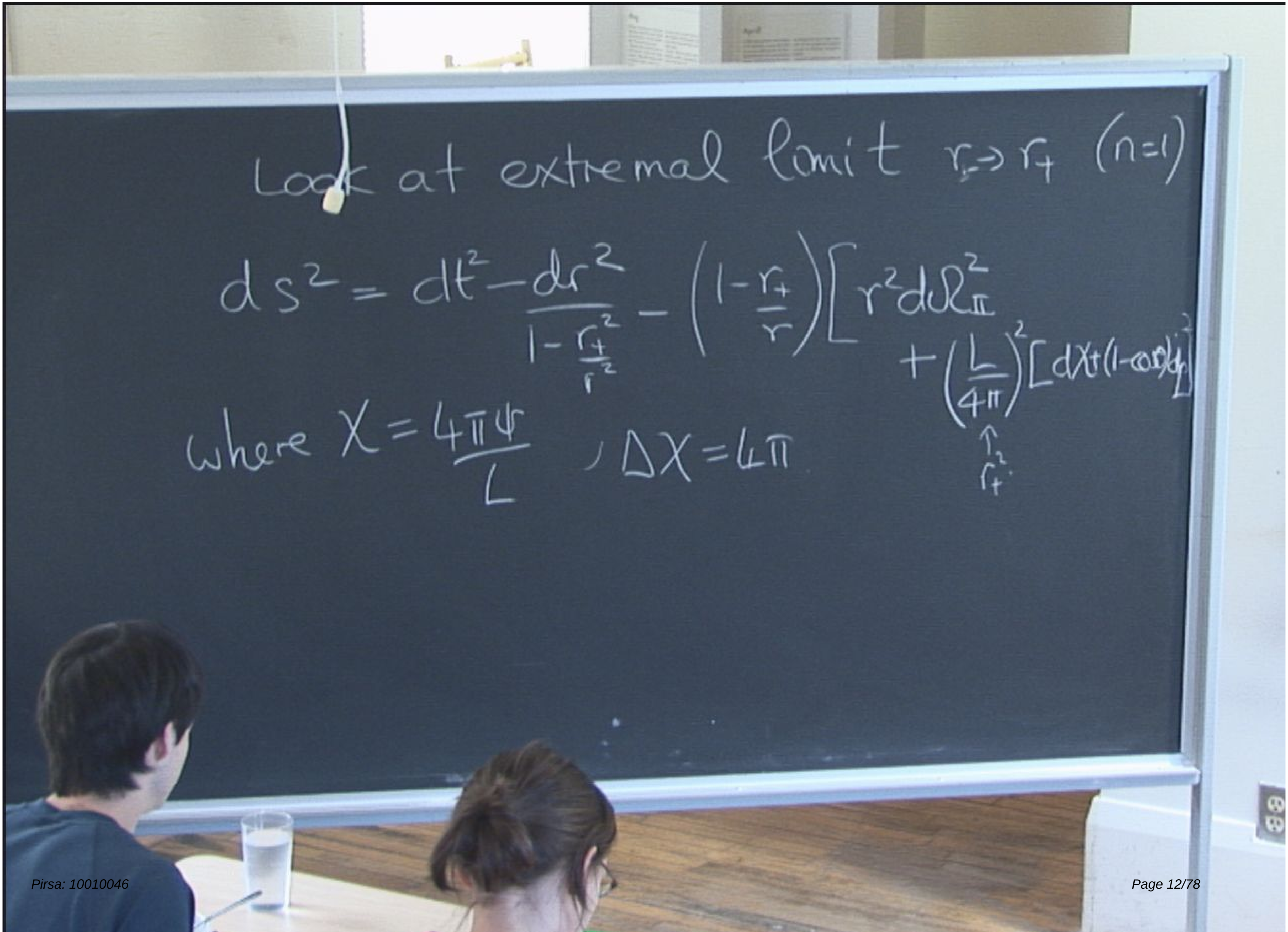
$$ds^2 = \left(\frac{r-r_+}{r-r_-}\right)dt^2 - \frac{dr^2}{\left(1-\frac{r_+}{r}\right)} - r(r-r_-)\,d\Omega_{II}^2$$

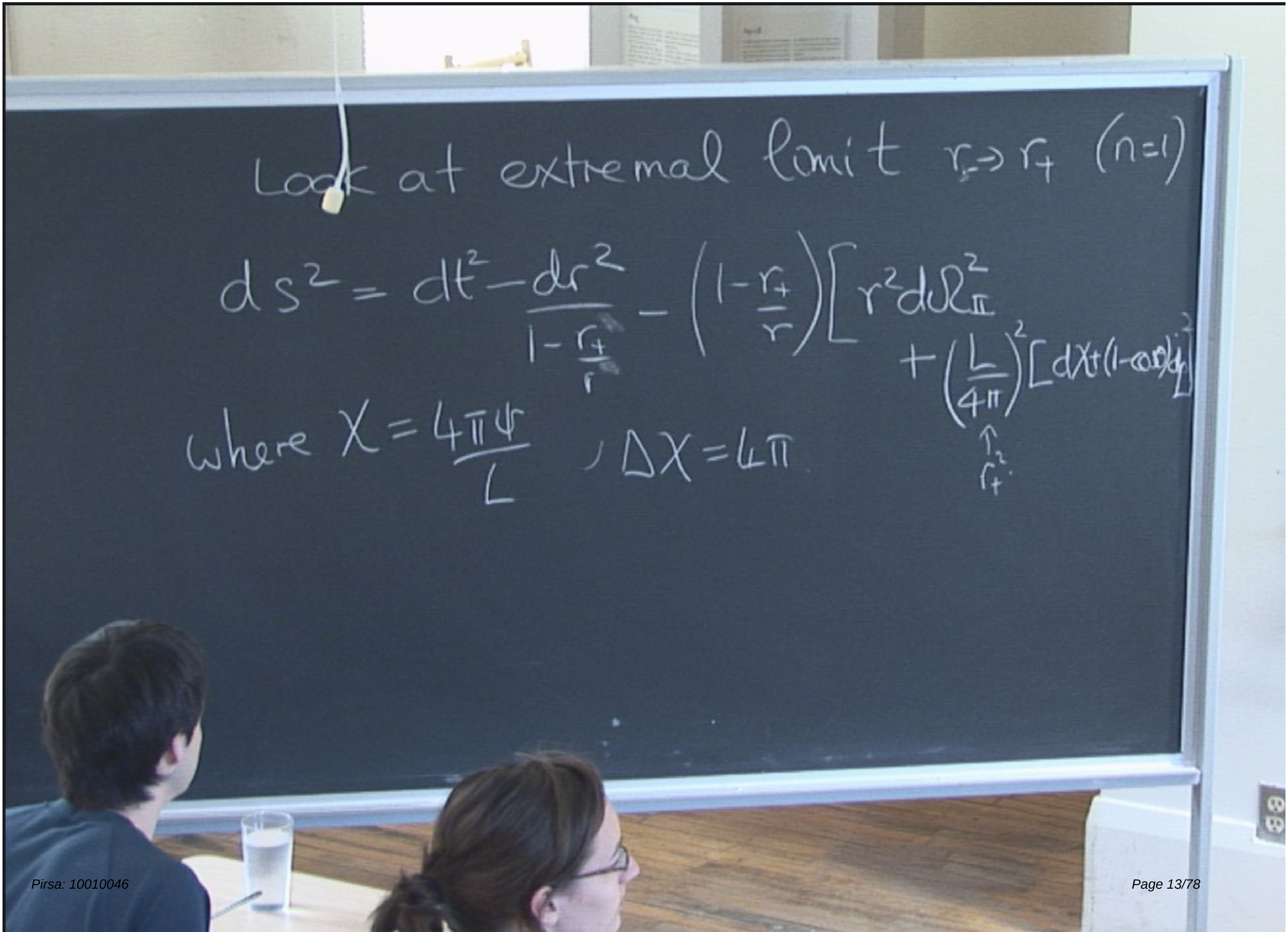
$$- \left(1-\frac{r_-}{r}\right)\left[d\psi + \sqrt{r_+ r_-}\,(1-\cos\theta)\,d\varphi\right]^2$$

$\frac{\upsilon}{2}$


Look at extremal limit $r_- \to r_+$ $(n=1)$


Look at extremal limit $r_- \to r_+$ $(n=1)$
$ds^2 = dt^2 - \frac{dr^2}{1-\frac{r_+^2}{r^2}} - \left(1-\frac{r_+}{r}\right)\left[r^2 d\Omega_{II}^2 + \left(\frac{L}{4\pi}\right)^2 \left[d\chi + (1-\cos\theta)d\phi\right]^2\right]$
where $\chi = \frac{4\pi\psi}{L}$, $\Delta\chi = 4\pi$

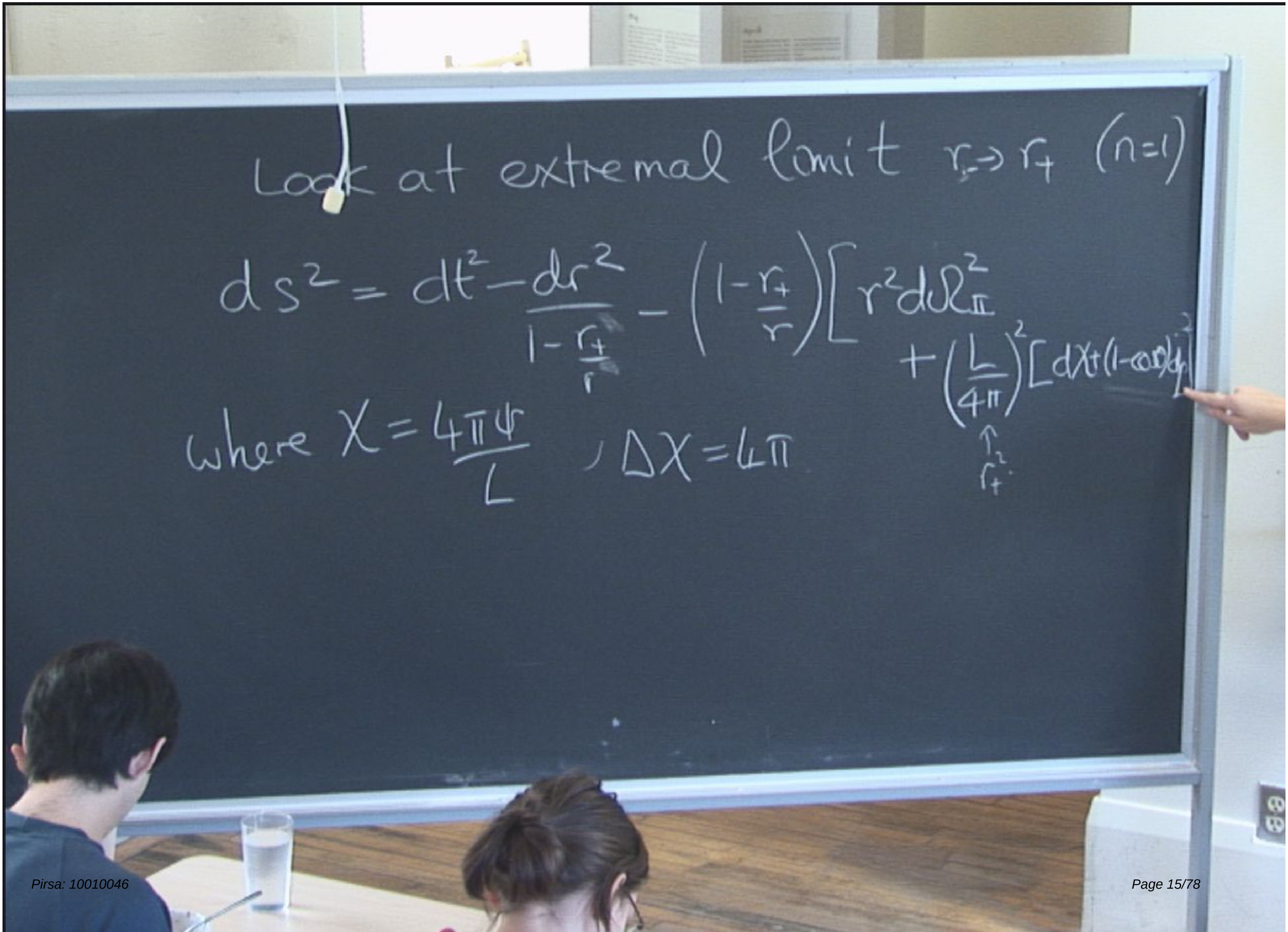

Look at extremal limit $r_- \to r_+$ (n=1)
$ds^2 = dt^2 - \frac{dr^2}{1-\frac{r_+^2}{r^2}} - \left(1-\frac{r_+}{r}\right)\left[r^2 d\Omega_{II}^2 + \left(\frac{L}{4\pi}\right)^2 [d\chi + (1-\cos\theta)d\phi]^2\right]$
r_+^2
where $\chi = \frac{4\pi\psi}{L}$, $\Delta\chi = 4\pi$


Look at extremal limit $r_- \to r_+$ $(n=1)$
$ds^2 = dt^2 - \frac{dr^2}{1-\frac{r_+}{r}} - \left(1-\frac{r_+}{r}\right)\left[r^2 d\Omega_{II}^2 + \left(\frac{L}{4\pi}\right)^2 \left[d\chi + (1-\cos\theta)d\phi\right]^2\right]$
r_+^2
where $\chi = \frac{4\pi\psi}{L}$, $\Delta\chi = 4\pi$

Look at extremal limit $r_- \to r_+$ $(n=1)$

$$ds^2 = dt^2 - \frac{dr^2}{1-\frac{r_+}{r}} - \left(1-\frac{r_+}{r}\right)\left[r^2 d\Omega_{II}^2 + \underbrace{\left(\frac{L}{4\pi}\right)^2}_{r_+^2}\left[d\chi + (1-\cos\theta)d\phi\right]^2\right]$$

where $\chi = \frac{4\pi\psi}{L}$, $\Delta\chi = 4\pi$


Look at extremal limit $r_- \to r_+$ $(n=1)$
$ds^2 = dt^2 - \frac{dr^2}{1-\frac{r_+}{r}} - \left(1-\frac{r_+}{r}\right)\left[r^2 d\Omega_{II}^2 + \left(\frac{L}{4\pi}\right)^2 [d\chi + (1-\cos\theta)d\phi]^2\right]$
r_+^2
where $\chi = \frac{4\pi\psi}{L}$, $\Delta\chi = 4\pi$

$r \to r_+$, let $\rho = 2\sqrt{r_+}\sqrt{r - r_+}$

$$ds^2 \sim dt^2 - d\rho^2 - \frac{\rho^2}{4}\underbrace{\left[d\theta^2 + \sin^2\theta\, d\varphi^2 + (d\chi + (1-\cos\theta)d\varphi)^2\right]}_{S^3 \text{ in Euler angles}}$$

$r \to r_+$, let $\rho = 2\sqrt{r_+}\sqrt{r - r_+}$

$$ds^2 \simeq dt^2 - d\rho^2 - \frac{\rho^2}{4}\underbrace{\left[d\theta^2 + \sin^2\theta\, d\varphi^2 + \left(d\chi + (1-\cos\theta)\,d\varphi\right)^2 \right]}_{S^3 \text{ in Euler angles}}$$

$n > 1$ get lens space.

$r \to r_+$, let $\rho = 2\sqrt{r_+}\sqrt{r - r_+}$

$$ds^2 \sim dt^2 - d\rho^2 - \frac{\rho^2}{4}\underbrace{\left[d\theta^2 + \sin^2\theta\, d\varphi^2 + (d\chi + (1-\cos\theta)d\varphi)^2\right]}_{S^3 \text{ in Euler angles}}$$

$n > 1$ get Lens space

Example of a Hopf fibration (of S^1 over S^2)

K is nonsingular

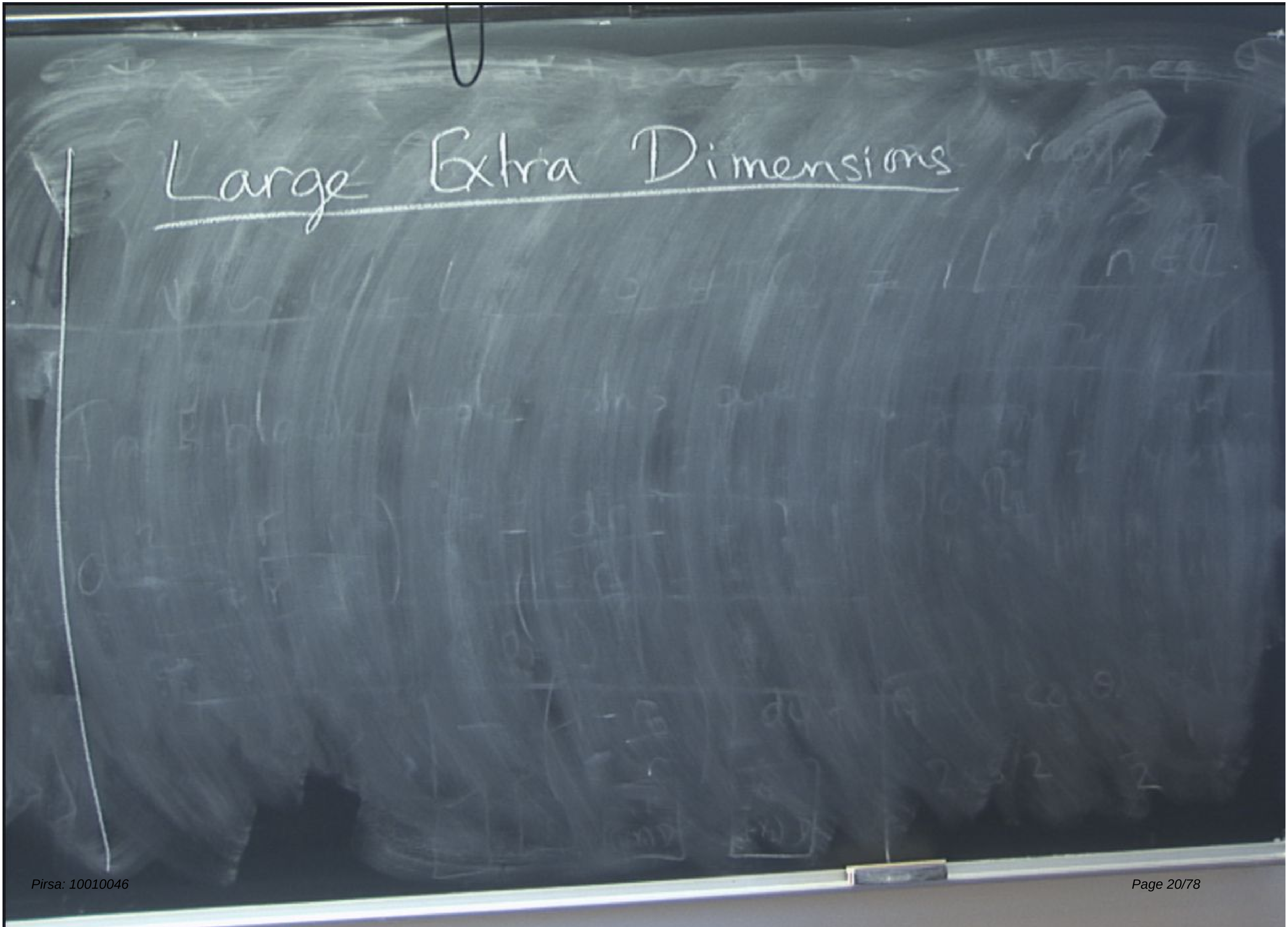
$r \to r_+$, let $\rho = 2\sqrt{r_+}\sqrt{r-r_+}$;

$$ds^2 = dt^2 - d\rho^2 - \frac{\rho^2}{4}\underbrace{\left[d\theta^2 + \sin^2\theta\, d\varphi^2 + (d\chi + (1-\cos\theta)d\varphi)^2\right]}_{S^3 \text{ in Euler angles}}$$

$n > 1$ get Lens space

Example of a Hopf fibration (of S^1 over S^2)

χ is nonsingular


Large Extra Dimensions

Large Extra Dimensions (LED's)

or Braneworlds

Large Extra Dimensions (LED's)

or Braneworlds

Notice that in KK

$$\frac{1}{16\pi G_5} \rightarrow \frac{L}{16\pi G_5} = \frac{1}{16\pi G_N}$$

Large Extra Dimensions (LED's)

or Braneworlds

Notice that in KK

$$\frac{1}{16\pi G_5} \rightarrow \frac{L}{16\pi G_5} = \frac{1}{16\pi G_N}$$

or. $\frac{M_5^3}{2}$

Large Extra Dimensions (LED's)

or Braneworlds

Notice that in KK

$$\frac{1}{16\pi G_5} \to \frac{L}{16\pi G_5} = \frac{1}{16\pi G_N}$$

or. $$\frac{M_5^3}{2} \to \frac{M_5^3 L}{2} \Rightarrow M_p^2 = M_5^3 L$$

4D Planck mass derived from M_s & L

4D Planck mass derived from M_s & L

More generally:

$$M_p^2 = M_D^{D-2} L^{D-2} \quad \text{or} \quad M_{4+n}^{n+2} L^n$$

4D Planck mass derived from M_s & L

More generally:

$$M_p^2 = M_D^{D-2} L^{D-2} \quad \text{or} \quad M_{4+n}^{n+2} L^n$$

$$L_{KK} \sim \underbrace{\left(\frac{M_p}{M_D}\right)}_{10}^{1+2/n} M_p^{-1}$$

4D Planck mass derived from M_s & L

More generally:

$$M_p^2 = M_D^{D-2} L^{D-2} \text{ or } M_{4+n}^{n+2} L^n$$

$$L_{KK} \sim \underbrace{\left(\frac{M_p}{M_D}\right)^{1+2/n}}_{\left(\frac{10^{16}\,\mathrm{TeV}}{M_D}\right)^{1+2/n}} M_p^{-1} \quad L_p.$$

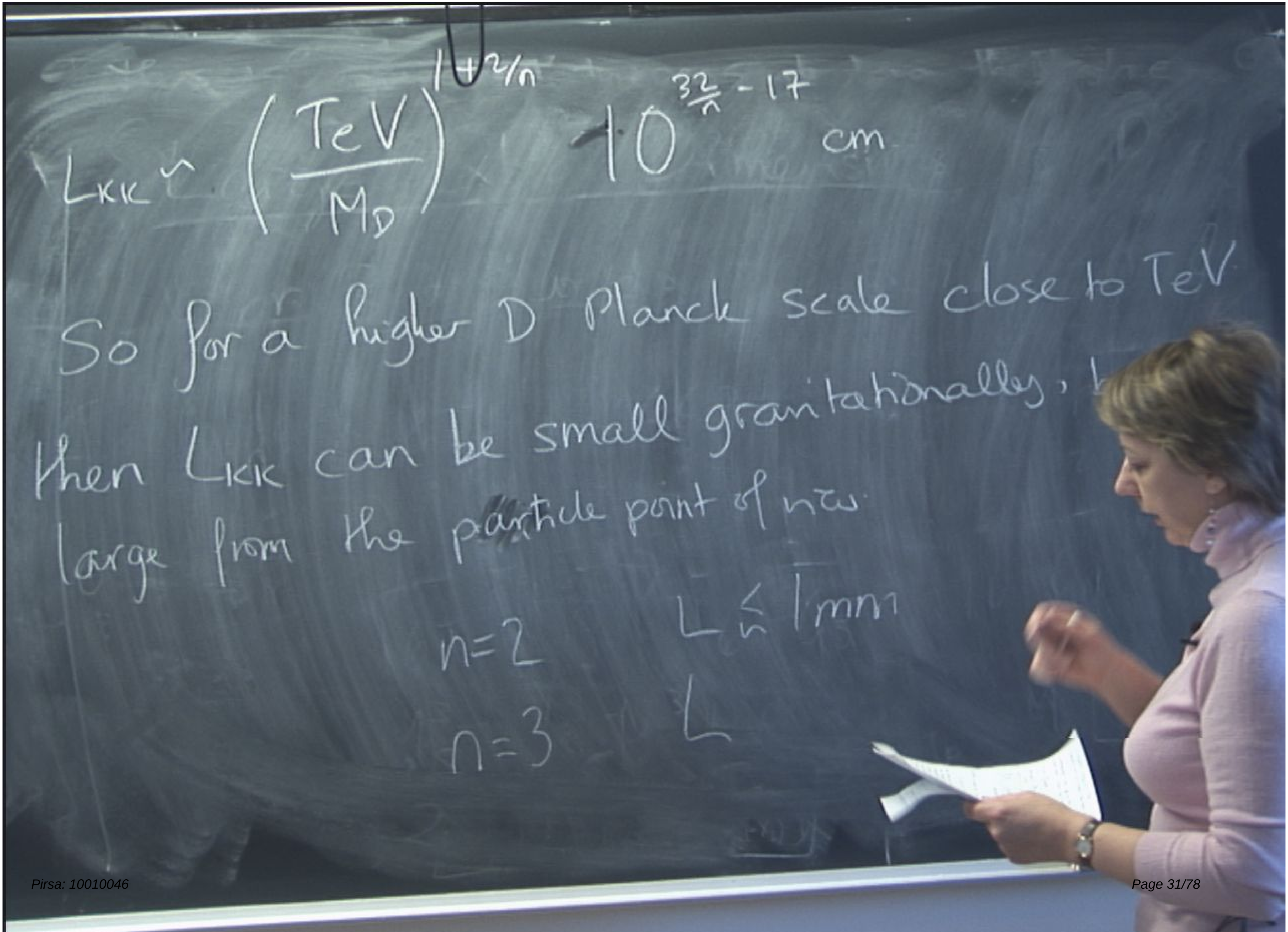
$$L_{KK} \sim \left(\frac{TeV}{M_D}\right)^{1+2/n} 10^{\frac{32}{n}-17} \text{ cm}$$

4D Planck mass derived from M_s & L

More generally:

$$M_p^2 = M_D^{D-2} L^{D-4} \text{ or } M_{4+n}^{n+2} L^n$$

$$L_{kk} \sim \underbrace{\left(\frac{M_p}{M_D}\right)^{1+2/n}}_{\left(\frac{10^{16}\,\mathrm{TeV}}{M_D}\right)^{1+2/n}} M_p^{-1} \quad L_p.$$


$L_{KK} \sim \left(\frac{TeV}{M_D}\right)^{1+2/n} 10^{\frac{32}{n}-17}$ cm
So for a higher D Planck scale close to TeV
then L_{KK} can be small gravitationally, but
large from the particle point of new
$n=2$ $L \lesssim 1$ mm
$n=3$ L

$$L_{KK} \sim \left(\frac{TeV}{M_D}\right)^{1+2/n} 10^{\frac{32}{n}-17} \text{ cm}$$

So for a higher D Planck scale close to TeV
then L_{KK} can be small gravitationally, but
large from the particle point of view.

$n=2 \qquad L \lesssim 1\,\text{mm}$

$n=3 \qquad L \lesssim 10^{-7}\,\text{m}$

Gives a hierarchy between gravity & Standard Model, but for it to work, particles cannot "see" extra dims - they must be confined

Gives a hierarchy between gravity & Standard Model, but for it to work, particles cannot "see" extra dims - they must be confined to a 4D submanifold / brane

Gives a hierarchy between gravity & Standard Model, but for it to work, particles cannot "see" extra dims – they must be confined to a 4D submanifold / brane

e.g. Rubakov - Shaposhikov

Gives a hierarchy between gravity &
Standard Model, but for it to work, particles
cannot "see" extra dims - they must be
confined to a 4D submanifold / brane

e.g. Rubakov - Shaposhikov

"Our Universe as a domain wall"

5D spacetime:

$$\mathcal{L}_\varphi(\varphi) = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\varphi^2 - \eta^2)^2$$

5D spacetime:

$$\mathcal{L}_\varphi(\varphi) = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\varphi^2-\eta^2)^2$$

5D spacetime:

$$\mathcal{L}_\varphi(\varphi) = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\varphi^2-\eta^2)^2$$

$$\Rightarrow \Box\varphi + 2\lambda(\varphi^2-\eta^2)^2 = 0$$

5D spacetime:

$$\mathcal{L}_\varphi(\varphi) = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\varphi^2-\eta^2)^2$$

$$\Rightarrow \Box\varphi + 2\lambda(\varphi^2-\eta^2)^2 = 0$$

has soln

$$\phi = \eta \tanh z/\sqrt{\lambda}\eta$$

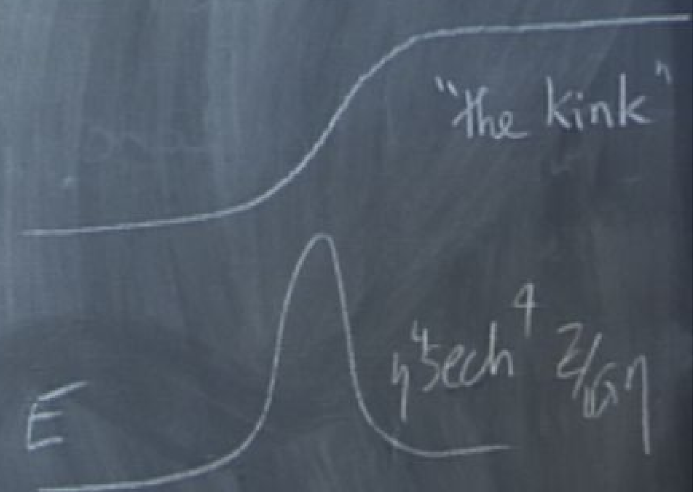
5D spacetime:

$$\mathcal{L}(\varphi) = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\varphi^2-\eta^2)^2$$

$$\Rightarrow \Box\varphi + 2\lambda(\varphi^2-\eta^2)^2 = 0$$

has soln

$$\phi = \eta \tanh z/\sqrt{\lambda}\eta$$



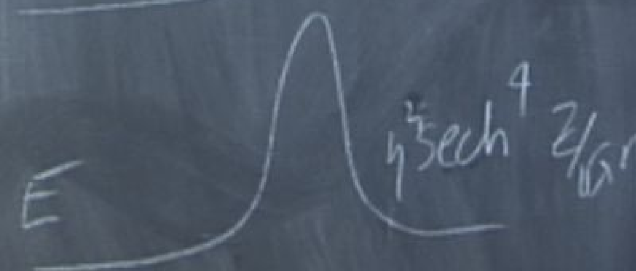
5D spacetime:

$$\mathcal{L}_\varphi(\varphi) = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\varphi^2-\eta^2)^2$$

$$\Rightarrow \Box\varphi + 2\lambda(\varphi^2-\eta^2)^2 = 0$$

has soln

$$\phi = \eta \tanh z/\sqrt{\lambda}\eta$$



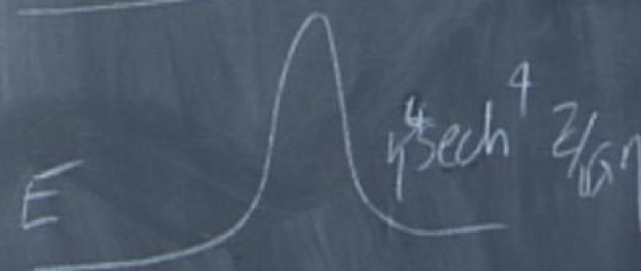
5D spacetime:

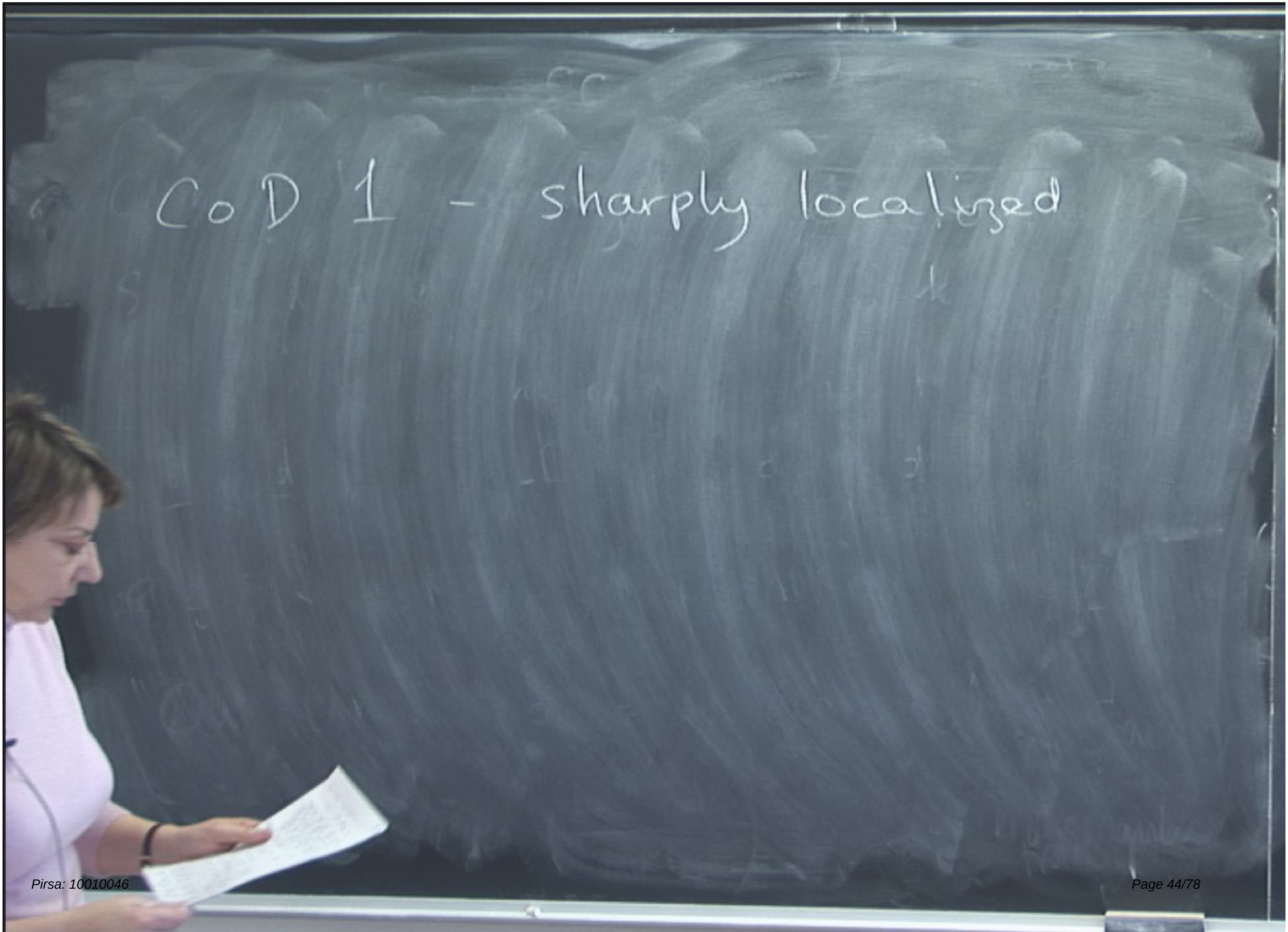
$$\mathcal{L}_\varphi(\varphi) = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\varphi^2-\eta^2)^2$$

$$\Rightarrow \Box\varphi + 2\lambda(\varphi^2-\eta^2)^2 = 0$$

has soln

$$\phi = \eta \tanh z/\sqrt{\lambda}\eta$$




CoD 1 - sharply localized

CoD 1 - sharply localized

Couple to fermion

$$\mathcal{L}_{\Psi} = i \bar{\Psi} \Gamma^a \nabla_a \Psi - g \phi \bar{\Psi} \Psi$$

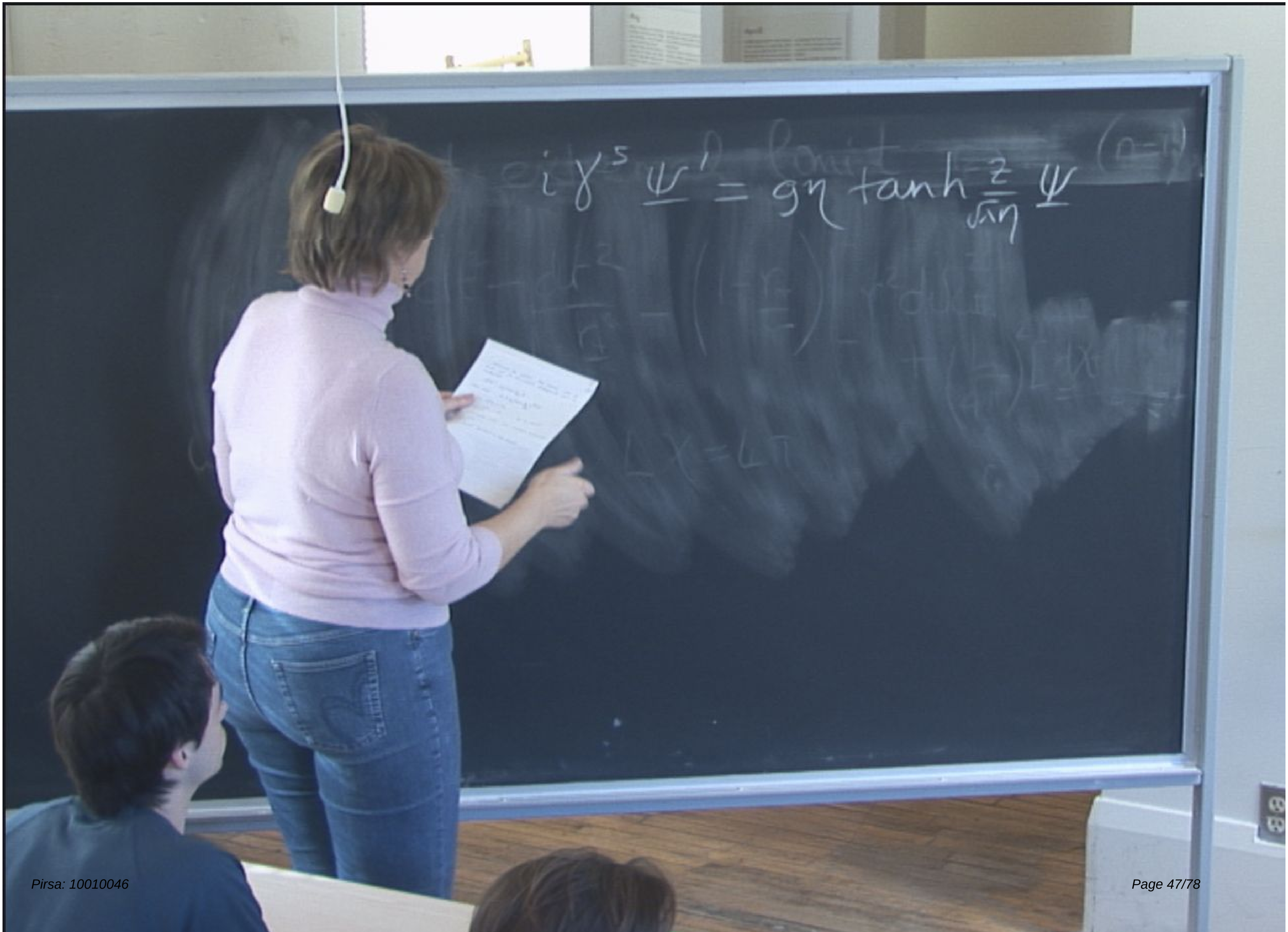
CoD 1 - sharply localized

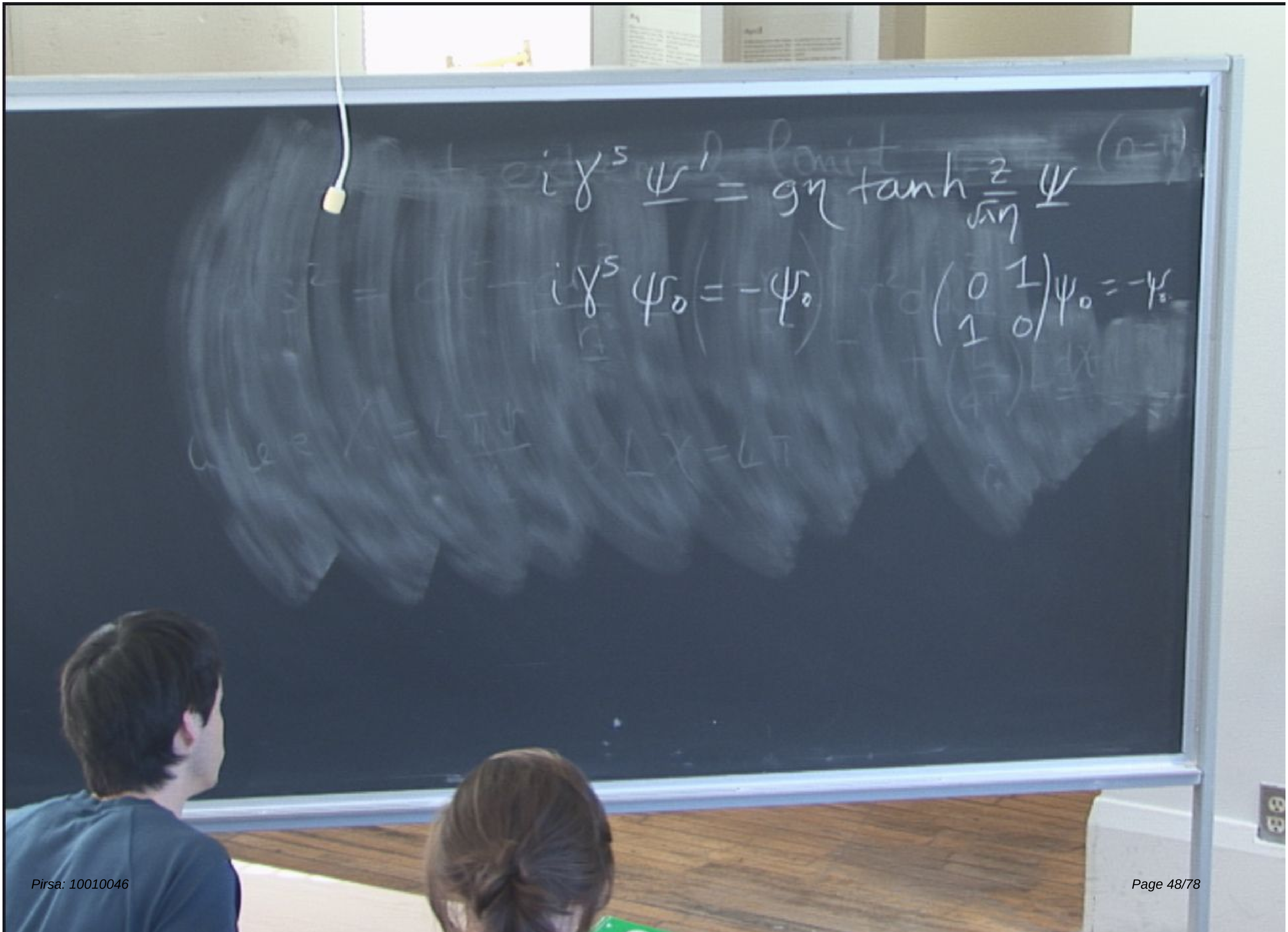
Couple to fermion

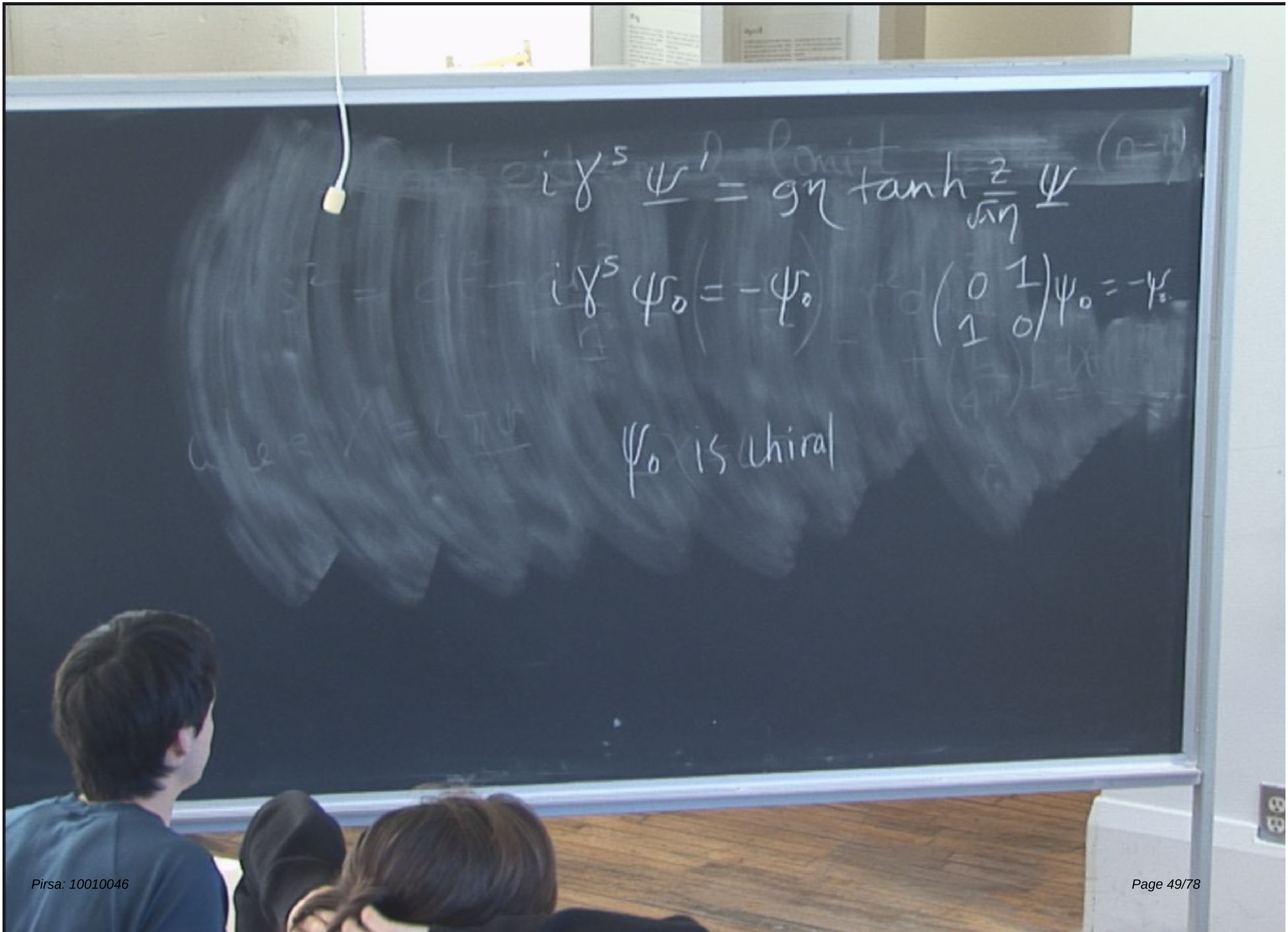
$$\mathcal{L}_\Psi = i\bar{\Psi}\Gamma^a\nabla_a\Psi - g\phi\bar{\Psi}\Psi$$

- in this case the lowest energy soln has $\Psi \neq 0$

$$\Psi \propto \psi_0 \left(\operatorname{sech} z/\sqrt{\lambda}\eta\right)^{\sqrt{\lambda}\eta^2 g}$$


$i\gamma^5 \underline{\psi}' = g\eta \tanh \frac{z}{\sqrt{\lambda}\eta} \underline{\psi}$


$i\gamma^5 \underline{\psi}' = g\eta \tanh \frac{z}{\sqrt{\lambda}\eta} \underline{\psi}$
$i\gamma^5 \psi_0 = -\psi_0$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \psi_0 = -\psi_0$


$i\gamma^5 \underline{\psi}' = g\eta \tanh \frac{z}{\sqrt{\lambda}\eta} \underline{\psi}$
$i\gamma^5 \psi_0 = -\psi_0$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \psi_0 = -\psi_0$
ψ_0 is chiral

CoD 1 - sharply localized

Couple to fermion

$$\mathcal{L}_\Psi = i \bar{\Psi} \Gamma^a \nabla_a \Psi - g \phi \bar{\Psi} \Psi$$

→ in this case the lowest energy soln has $\Psi \neq 0$

$$\Psi \propto \psi_0(x) \left(\operatorname{sech} z/\sqrt{\lambda}\eta \right)^{\sqrt{\lambda}\eta^2 g}$$

If $\not{\partial}\psi = 0$, then still solves

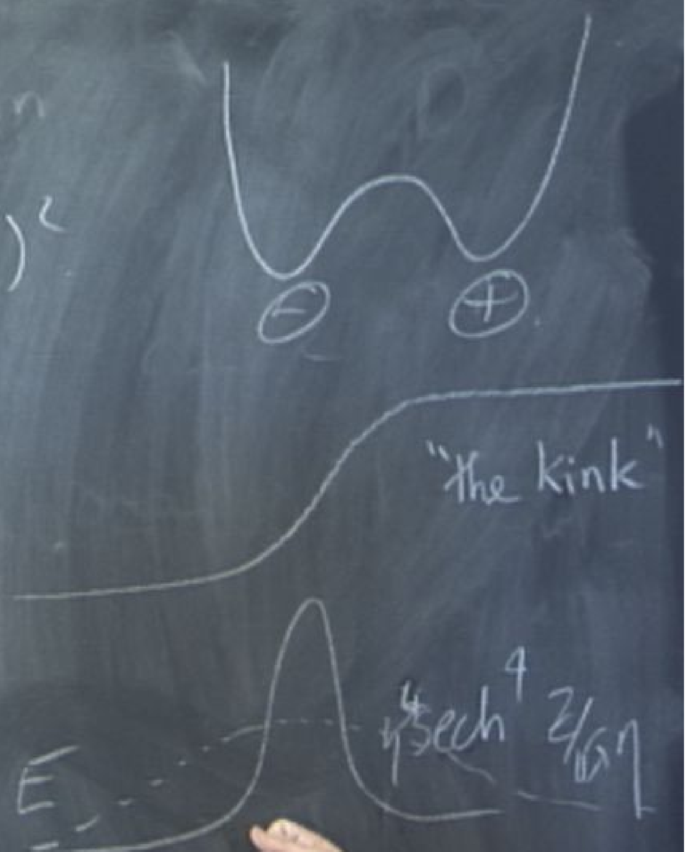
5D spacetime:

$$\mathcal{L}_\phi(\varphi) = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\varphi^2-\eta^2)^2$$

$$\Rightarrow \Box\varphi + 2\lambda(\varphi^2-\eta^2)^2 = 0$$

has soln

$$\phi = \eta \tanh z/\sqrt{\lambda}\eta$$



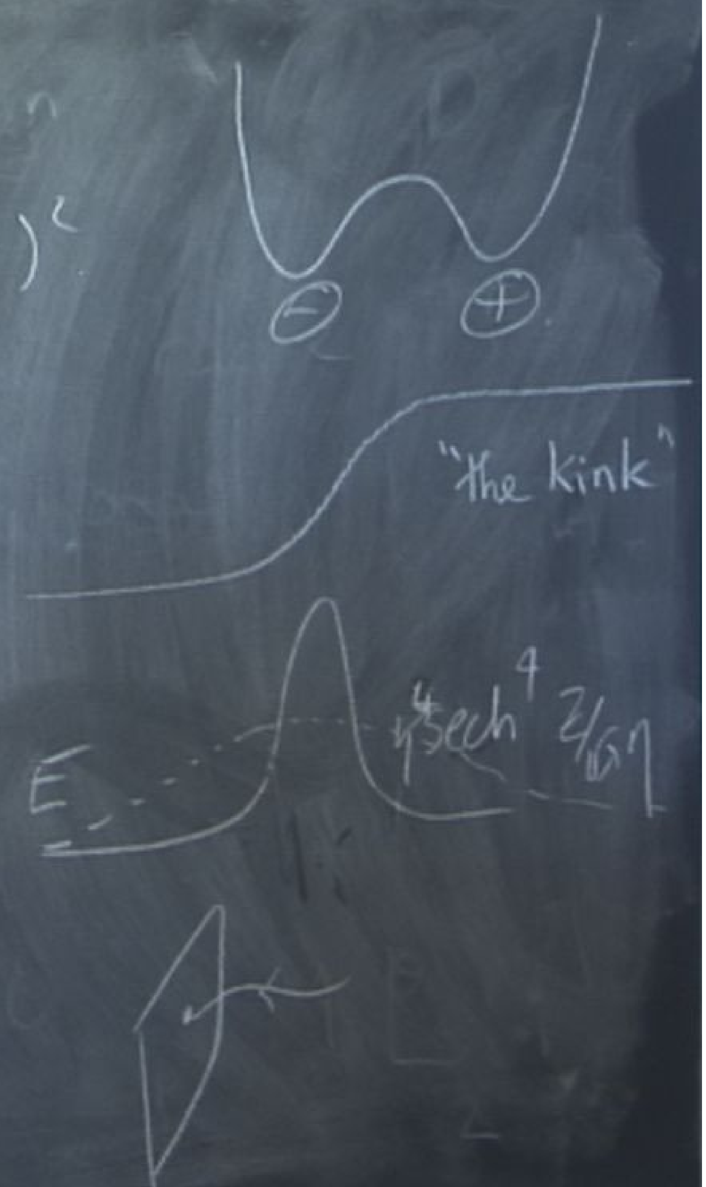
5D spacetime:

$$\mathcal{L}_\varphi(\varphi) = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\varphi^2-\eta^2)^2$$

$$\Rightarrow \Box\varphi + 2\lambda(\varphi^2-\eta^2)^2 = 0$$

has soln

$$\phi = \eta \tanh z/\sqrt{\lambda}\eta$$



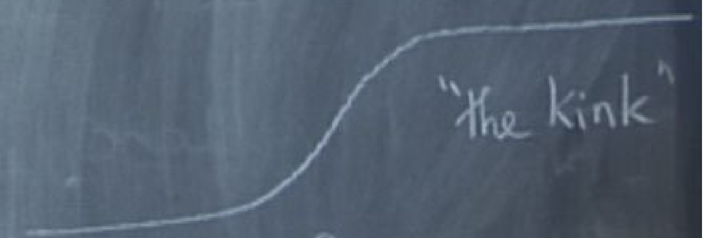
5D spacetime:

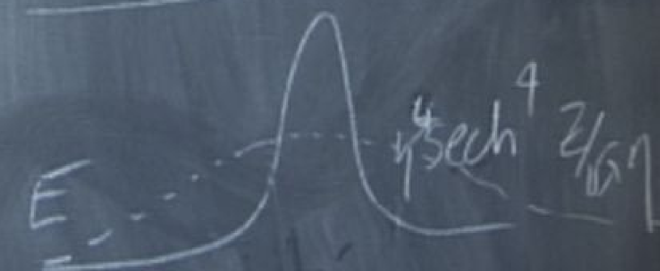
$$\mathcal{L}_\varphi(\varphi) = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\varphi^2-\eta^2)^2$$

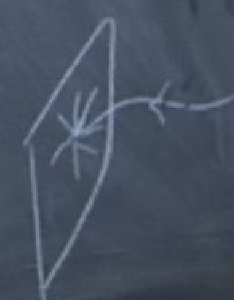
$$\Rightarrow \Box\varphi + 2\lambda(\varphi^2-\eta^2)^2 = 0$$

has soln

$$\phi = \eta \tanh z/\sqrt{\lambda}\eta$$







The Israel Eqns

The Israel Eqns

Recall the gravitational action $S_{EH} + S_{GH}$

has variation:

$$\delta S = -\frac{1}{8\pi G}$$

The Israel Eqns

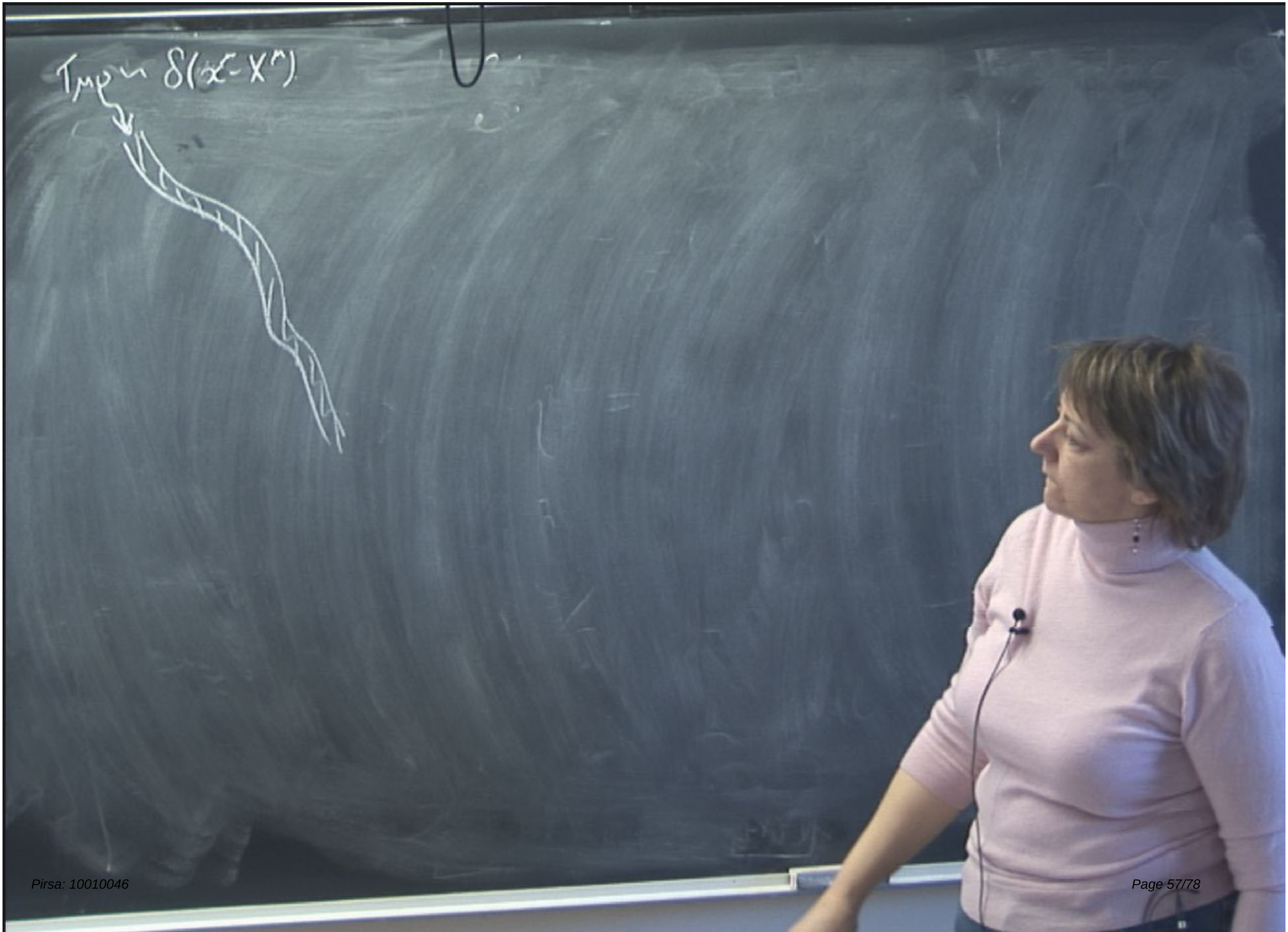
Recall the gravitational action $S_{EH} + S_{GH}$

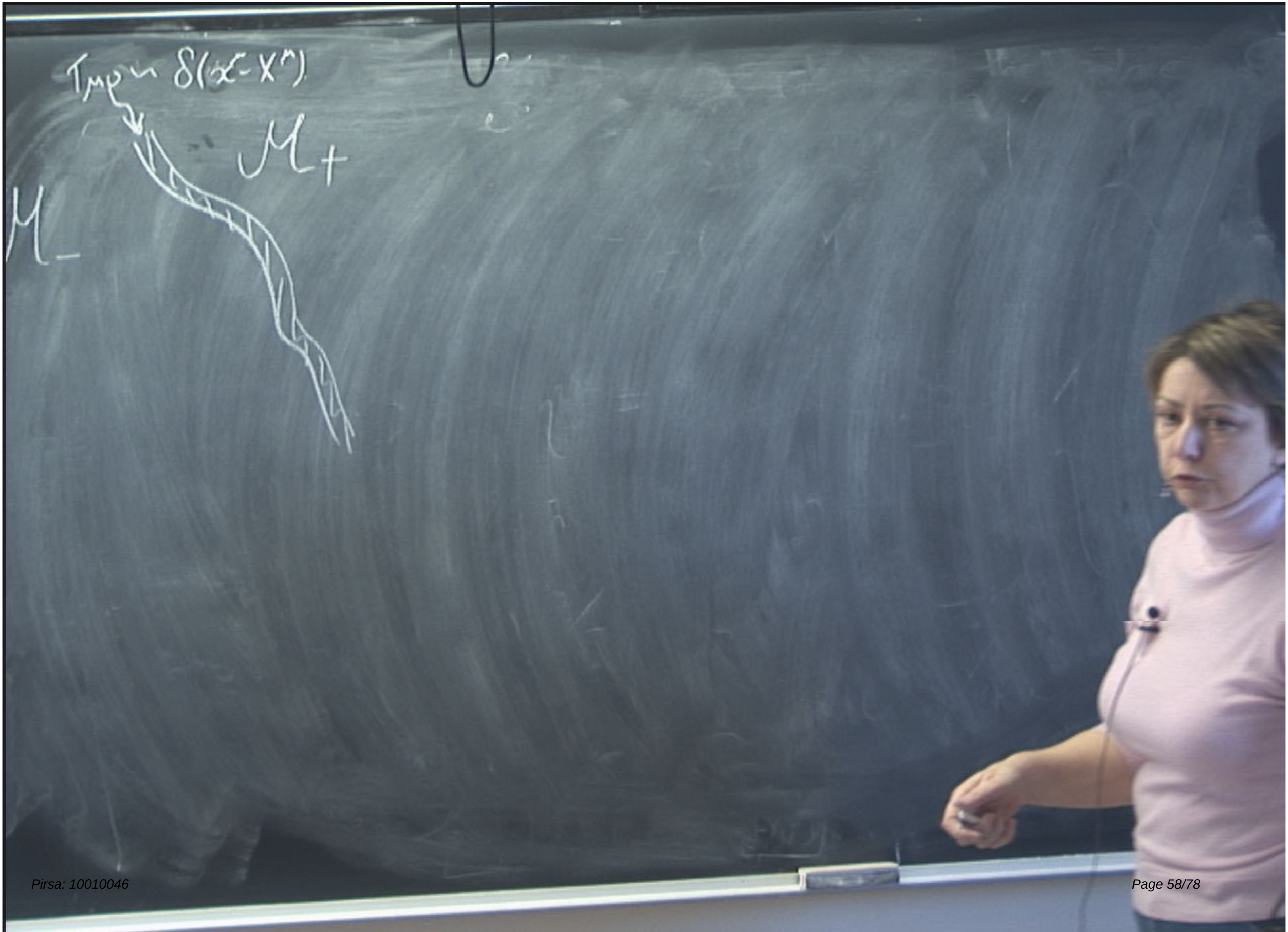
has variation:

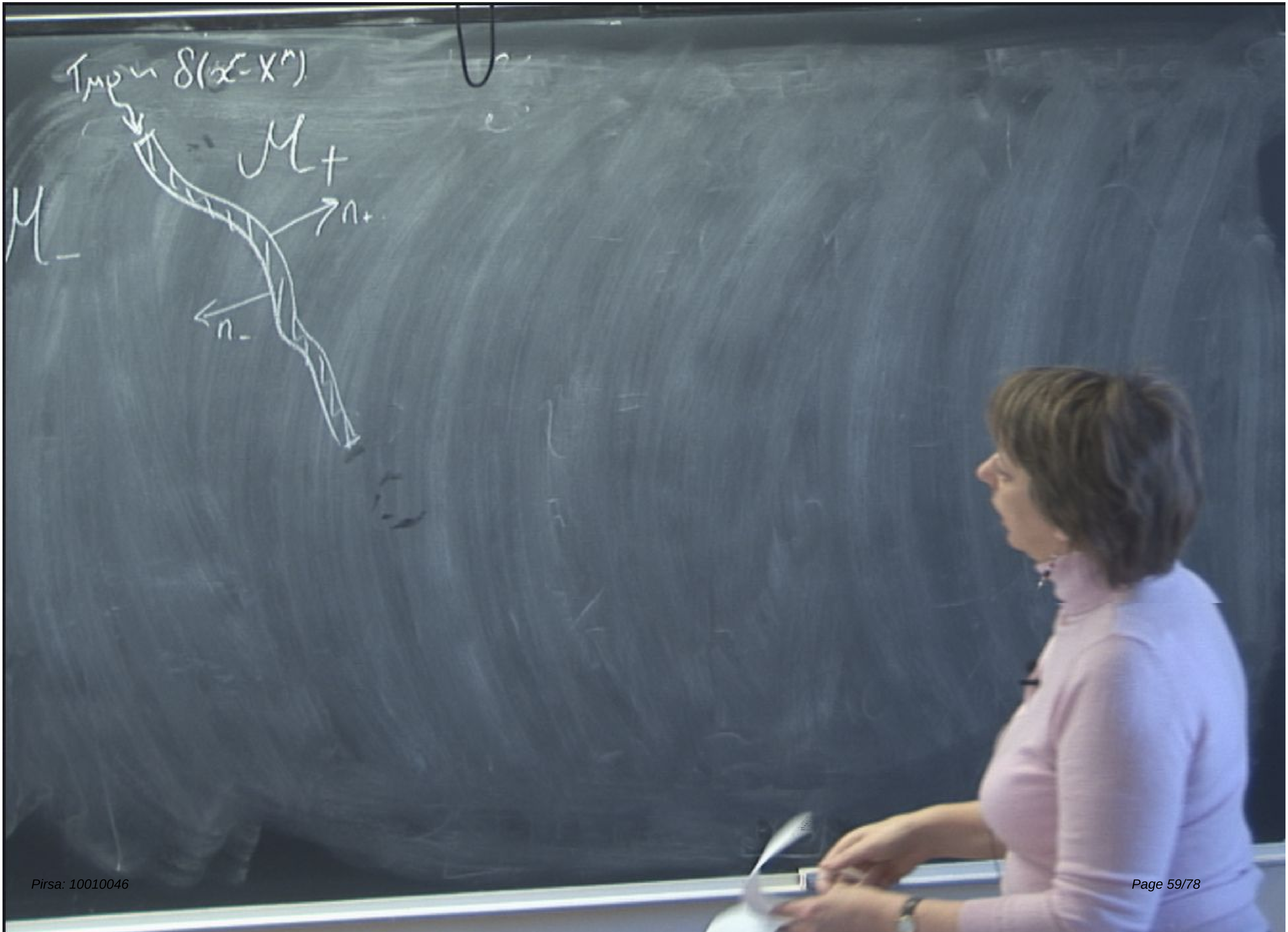
$$\delta S = -\frac{1}{8\pi G}\int_{\partial M} d^{N-1}x\sqrt{g}\;\frac{1}{2}\left(K_{ab} - K g_{ab}\right)\delta g^{ab}$$

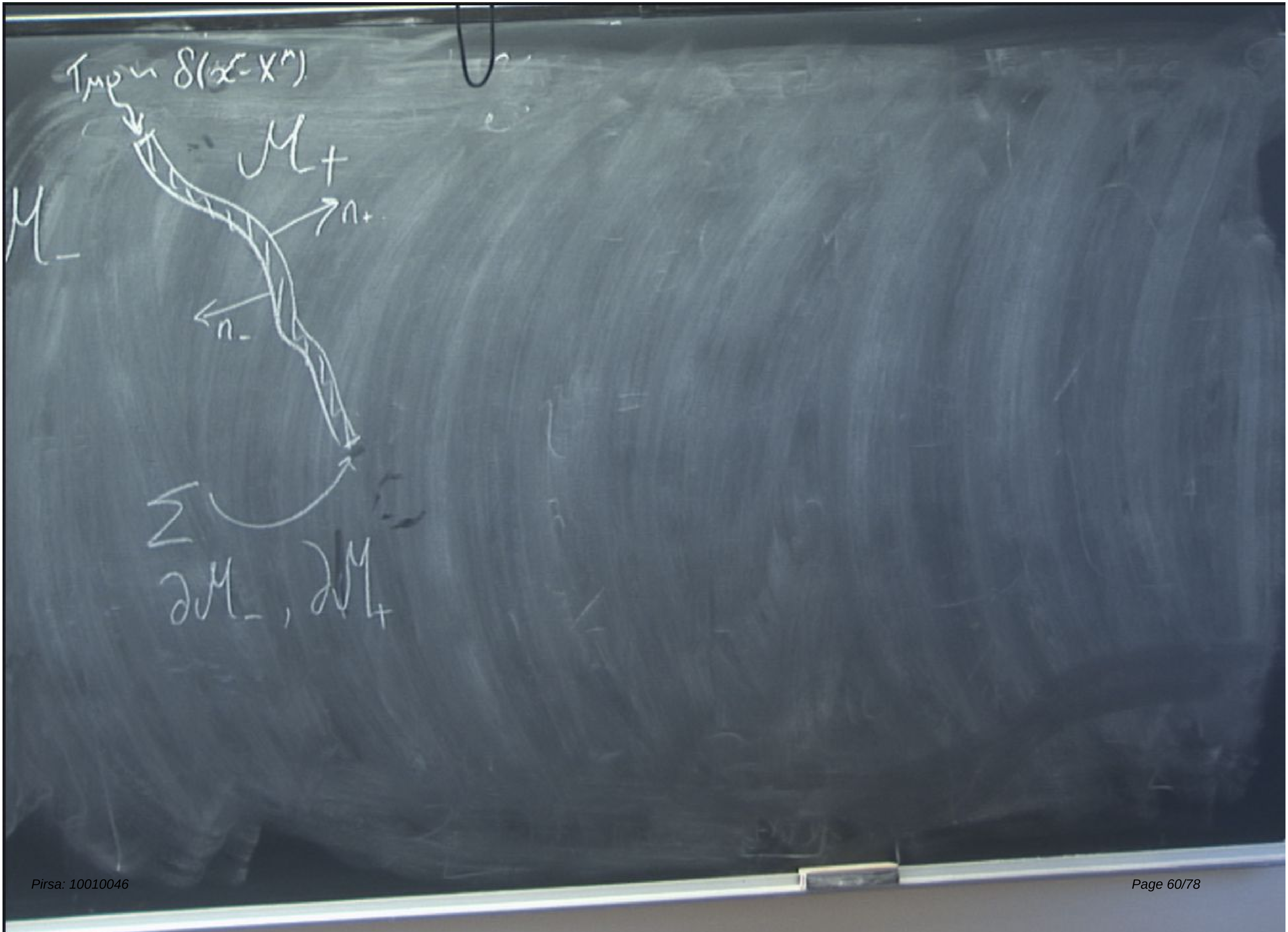
$$\cancel{-\frac{1}{2}\,{}^{(N-1)}\nabla_d\left(n_c\,\delta g^{cd}\right)}$$

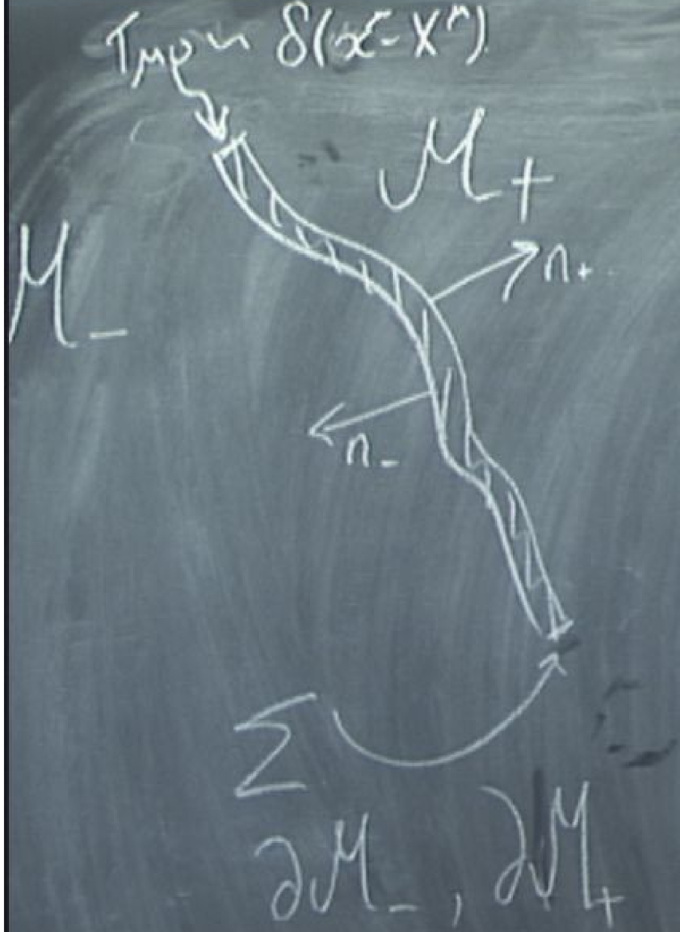
$\partial\partial M = 0$


$T_{\mu\nu} \sim \delta(x^\mu - X^\mu)$


$T_{\mu\nu} \sim \delta(x^\mu - X^\mu)$
M_+
M_-

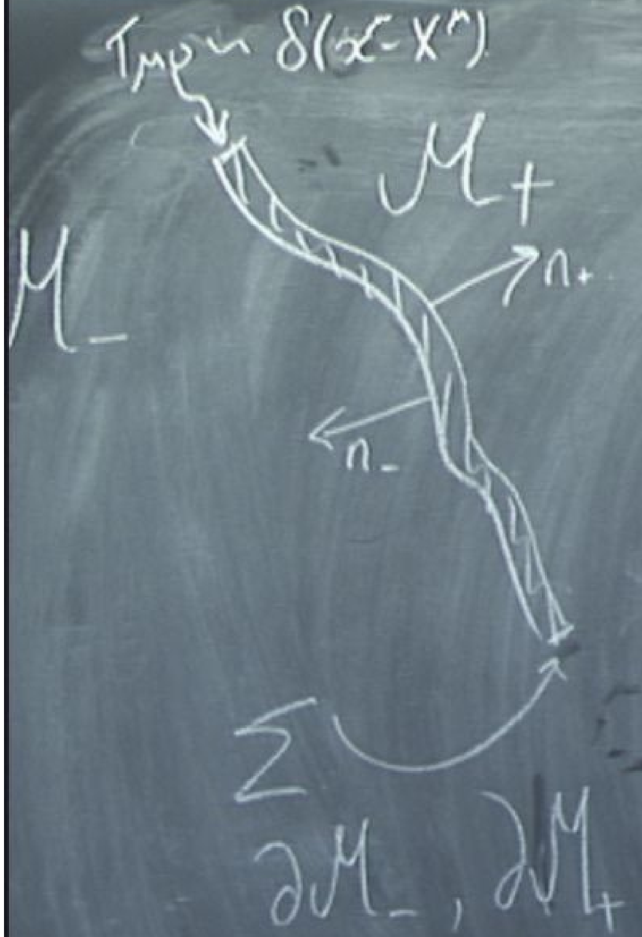

Tμν ~ δ(x^μ − X^μ)
M+
M−
n+
n−


$T_{\mu\nu} \sim \delta(x^\mu - X^\mu)$
$\mathcal{M}_+$
$\mathcal{M}_-$
n_+
n_-
Σ
$\partial\mathcal{M}_-, \partial\mathcal{M}_+$

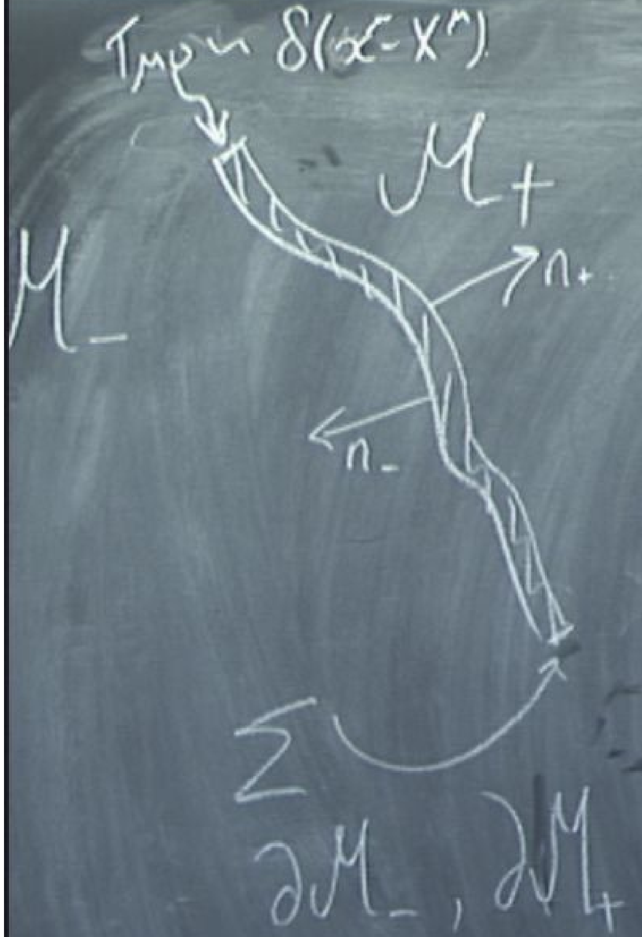


For the purpose of the Israel eqns, reverse the sign of n_-

so that "$\frac{\partial}{\partial z}$"



For the purpose of the Israel eqns, reverse the sign of n_- so that "$\frac{\partial}{\partial z}$" $\sim n$ where $\pm z$ is the distance from the wall in $\mathcal{M}_\pm$



For the purpose of the Israel eqns, reverse the sign of n_- so that "$\frac{\partial}{\partial z}$" $\sim n$ where $\pm z$ is the distance from the wall in $\mathcal{M}_\pm$

$$h_{+ab} = g_{+ab} + n_{+a} n_{+b}.$$

$$h_{-ab} = g_{-ab} + n_{-a} n_{-b}$$

$$h_{+ab} = g_{+ab} + n_{+a} n_{+b}$$

$$h_{-ab} = g_{-ab} + n_{-a} n_{-b}$$

equal, as
$\partial M_+ = \partial M_-$

$$h_{+ab} = g_{+ab} + n_{+a} n_{+b}$$

$$h_{-ab} = g_{-ab} + n_{-a} n_{-b}$$

equal, as $\partial M_+ = \partial M_-$

N.B. This does not require $g_{+ab} = g_{-ab}$

$$h_{+ab} = g_{+ab} + n_{+a} n_{+b}$$

$$h_{-ab} = g_{-ab} + n_{-a} n_{-b}$$

equal, as $\partial M_+ = \partial M_-$

N.B. This does not require $g_{+ab} = g_{-ab}$

$$\frac{\delta S_{grav}}{\delta g^{ab}} = -\frac{1}{8\pi G} \times \frac{1}{2} \left(K_{+ab} - K_+ h_{ab} - K_{-ab} + K_- h_{ab} \right)$$

$$h_{+ab} = g_{+ab} + n_{+a} n_{+b}$$

$$h_{-ab} = g_{-ab} + n_{-a} n_{-b}$$

equal, as $\partial M_+ = \partial M_-$

N.B. This does not require $g_{+ab} = g_{-ab}$

$$\frac{\delta S_{grav}}{\delta g^{ab}} = -\frac{1}{8\pi G} \times \frac{1}{2} \begin{pmatrix} K_{+ab} - K_+ h_{ab} \\ - K_{-ab} + K_- h_{ab} \end{pmatrix}$$

$$= -\frac{1}{8\pi G} \frac{1}{2} \left(\Delta K_{ab} - \Delta K h_{ab} \right)$$

$$h_{+ab} = g_{+ab} + n_{+a} n_{+b}$$

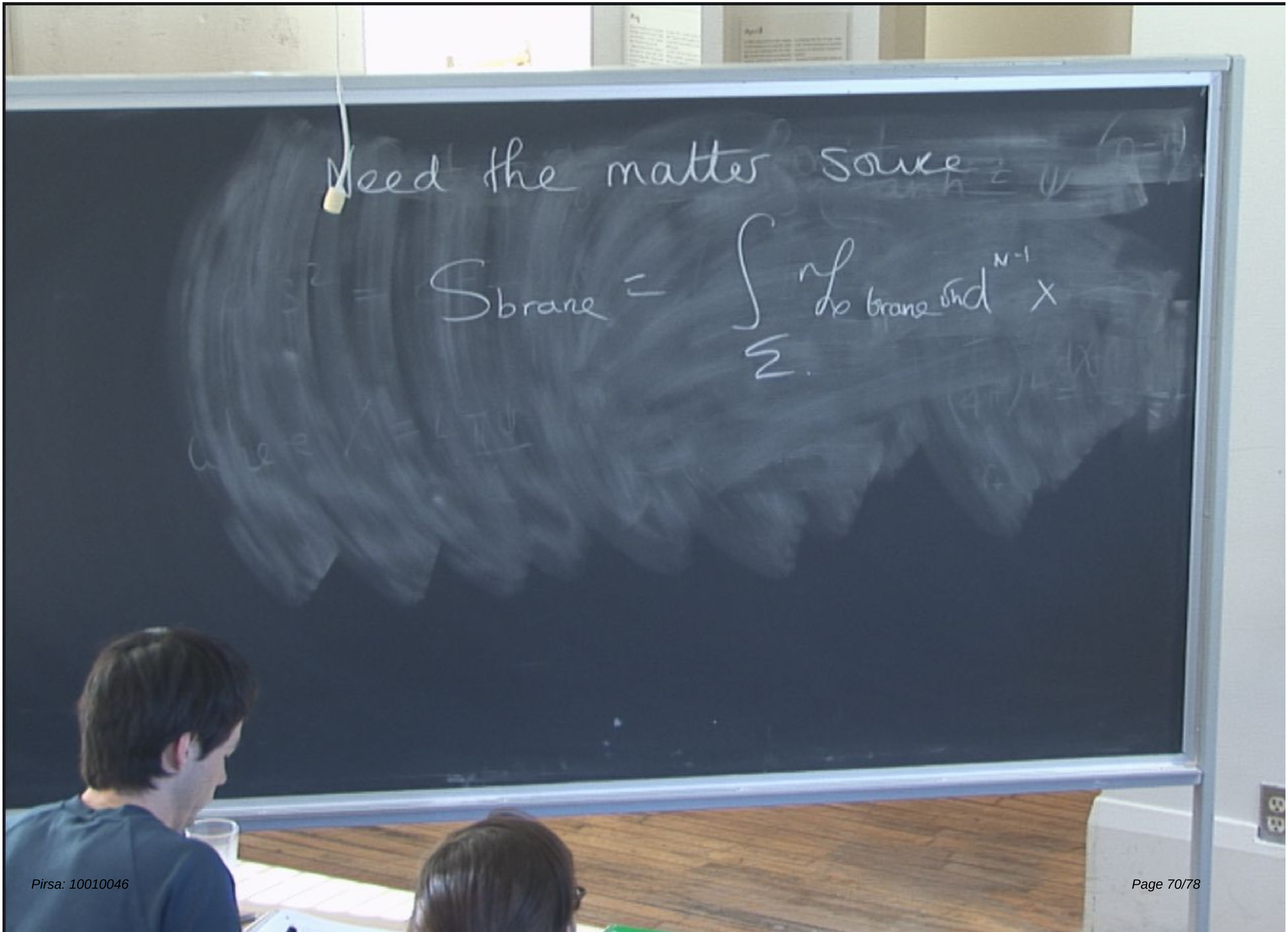
$$h_{-ab} = g_{-ab} + n_{-a} n_{-b}$$

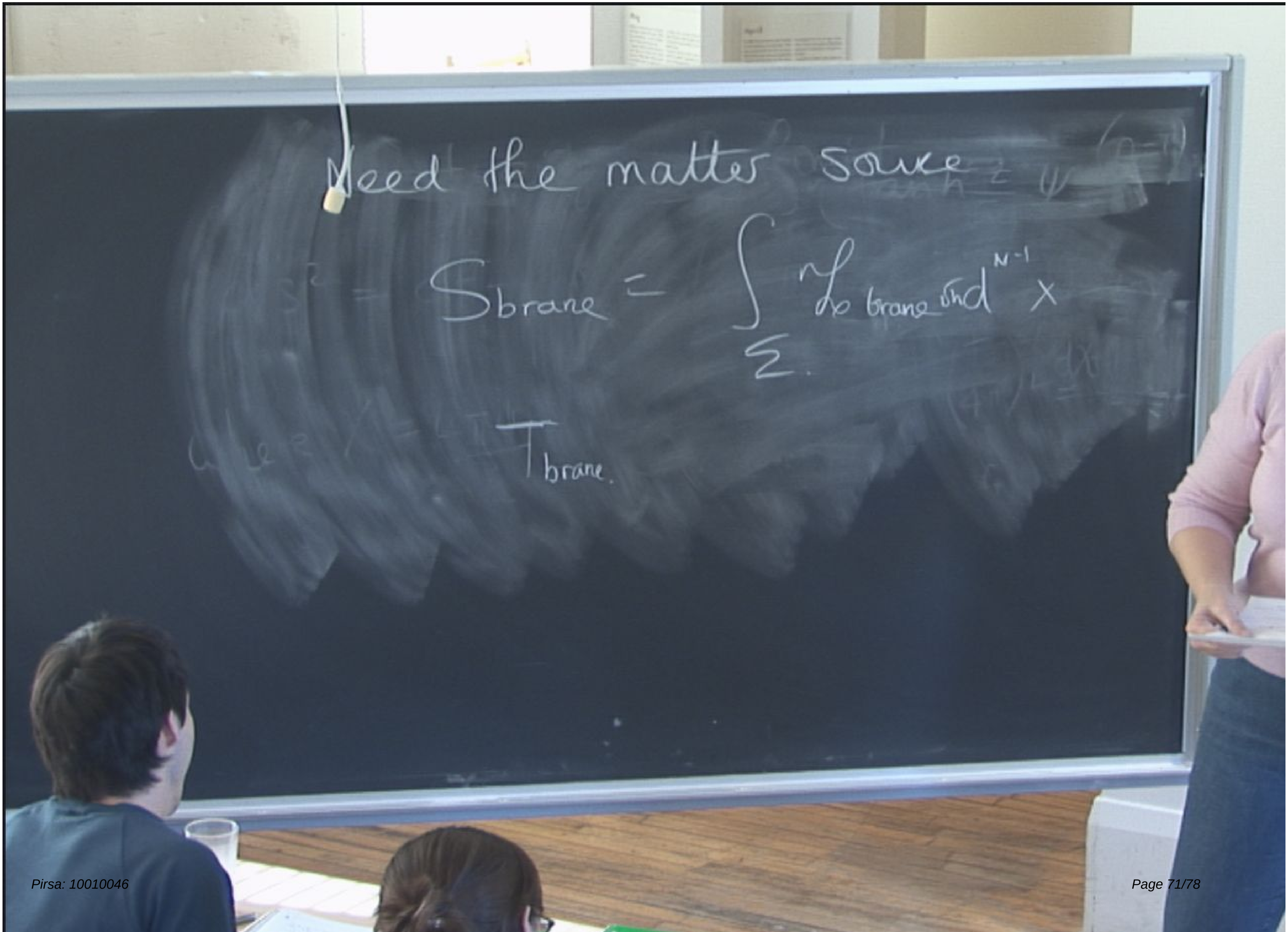
equal, as $\partial M_+ = \partial M_-$

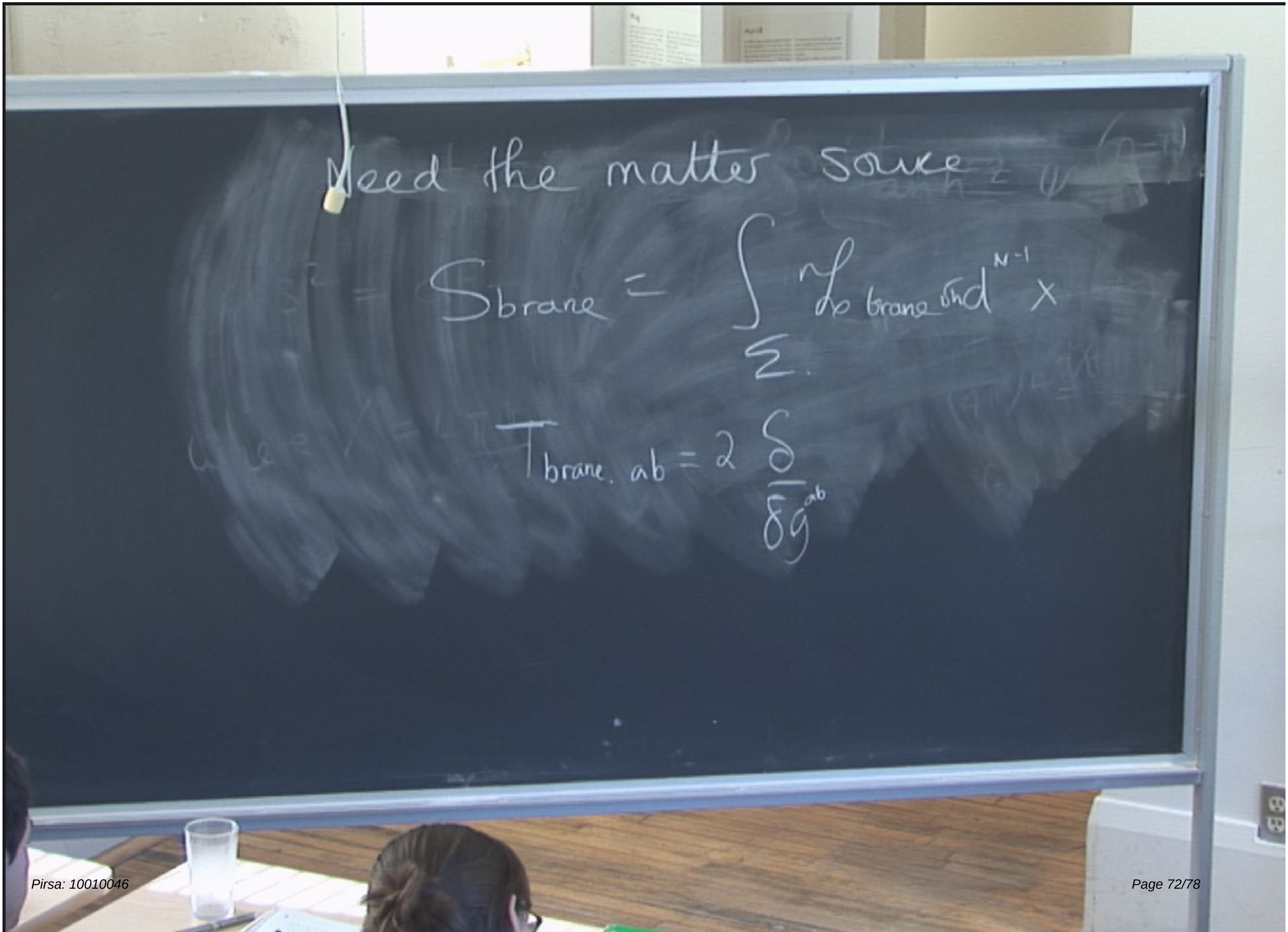
N.B. This does not require $g_{+ab} = g_{-ab}$

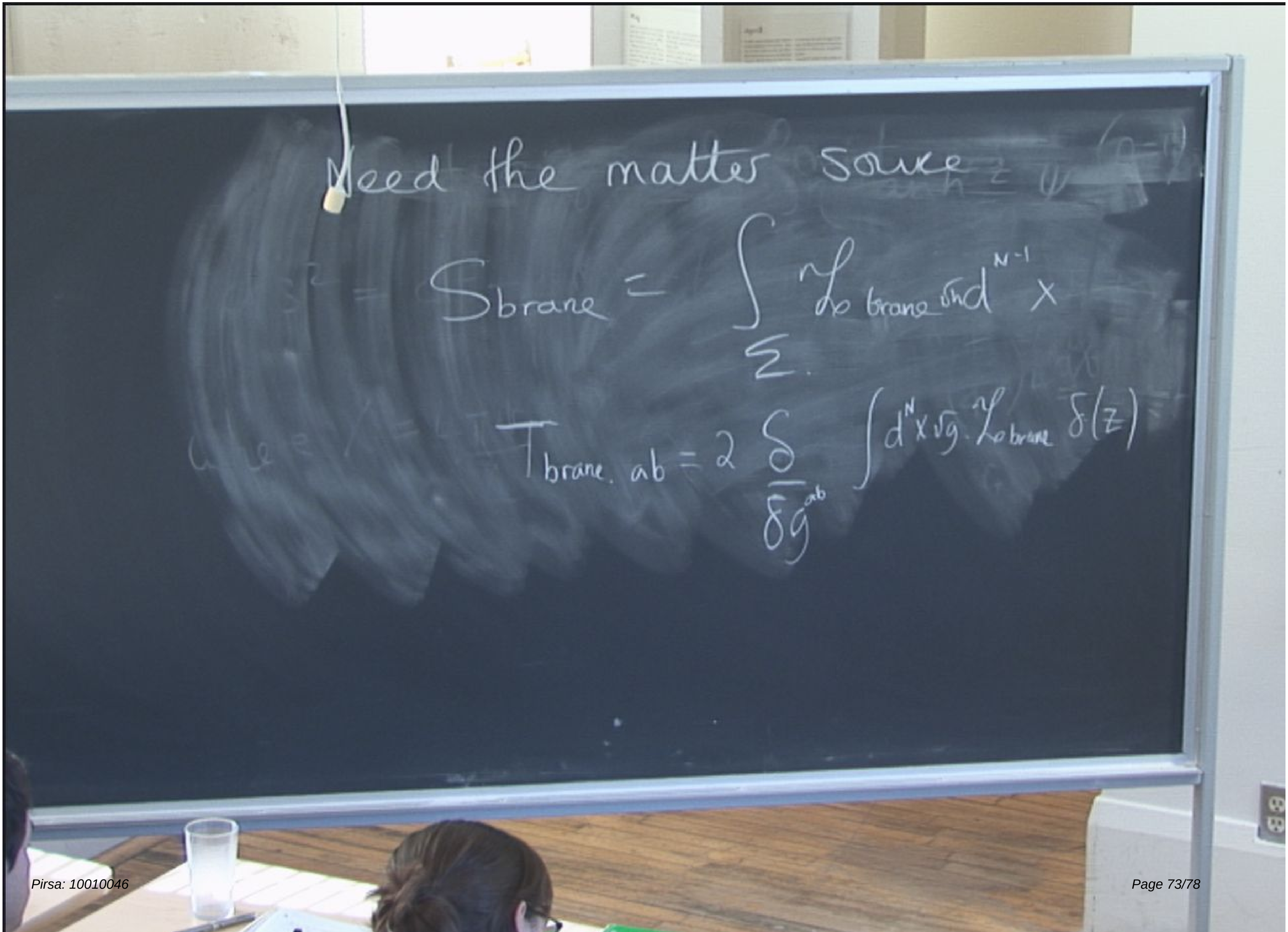
$$\frac{\delta S_{grav}}{\delta g^{ab}} = -\frac{1}{8\pi G} \times \frac{1}{2} \begin{pmatrix} K_{+ab} - K_+ h_{ab} \\ - K_{-ab} + K_- h_{ab} \end{pmatrix}$$

$$= -\frac{1}{8\pi G} \frac{1}{2} \left(\Delta K_{ab} - \Delta K h_{ab} \right)$$


Need the matter source
$S_{brane} = \int_{\Sigma} \mathcal{L}_{brane} \sqrt{h}\, d^{N-1}x$


Need the matter source
$S_{brane} = \int_{\Sigma} \mathcal{L}_{brane} \sqrt{h}\, d^{N-1}x$
T_{brane}.


Need the matter source
$S_{brane} = \int_{\Sigma} \mathcal{L}_{brane} \sqrt{h}\, d^{N-1}x$
$T_{brane\ ab} = 2 \frac{\delta}{\delta g^{ab}}$


Need the matter source
$S_{brane} = \int_{\Sigma} \mathcal{L}_{brane} \sqrt{h}\, d^{N-1}x$
$T_{brane,\, ab} = 2 \frac{\delta}{\delta g^{ab}} \int d^N x \sqrt{g}\, \mathcal{L}_{brane}\, \delta(z)$

$$T_{br\,ab} = S_{ab}\,\delta(z)$$

$$S_{ab} = 2\,\frac{\delta S_{brane}}{\delta h^{ab}}$$

$$T_{br\,ab} = S_{ab}\,\delta(z) \Big]$$

M_-) (M_+.

$$S_{ab} = 2\,\frac{\delta S_{brane}}{\delta h^{ab}}$$

$$T_{br\,ab} = S_{ab}\,\delta(z)$$

$$S_{ab} = 2\,\frac{\delta S_{brane}}{\delta h^{ab}}$$

Can't glue while remaining flat → energy momentum

$$T_{br\,ab} = S_{ab}\,\delta(z)$$

$$S_{ab} = 2\,\frac{\delta S_{brane}}{\delta h^{ab}}$$

Can't glue while remaining flat → energy momentum

$$T_{br\,ab} = S_{ab}\,\delta(z)$$

$$S_{ab} = 2\,\frac{\delta S_{brane}}{\delta h^{ab}}$$

Can't glue while remaining flat → energy momentum

$$\boxed{\Delta K_{ab} - \Delta K h_{ab} = 8\pi G\, S_{ab}}$$

ISRAEL EQNS