Title: Gravitational Physics - Review (PHYS 636) - Lecture 12

Date: Jan 19, 2010 10:00 AM

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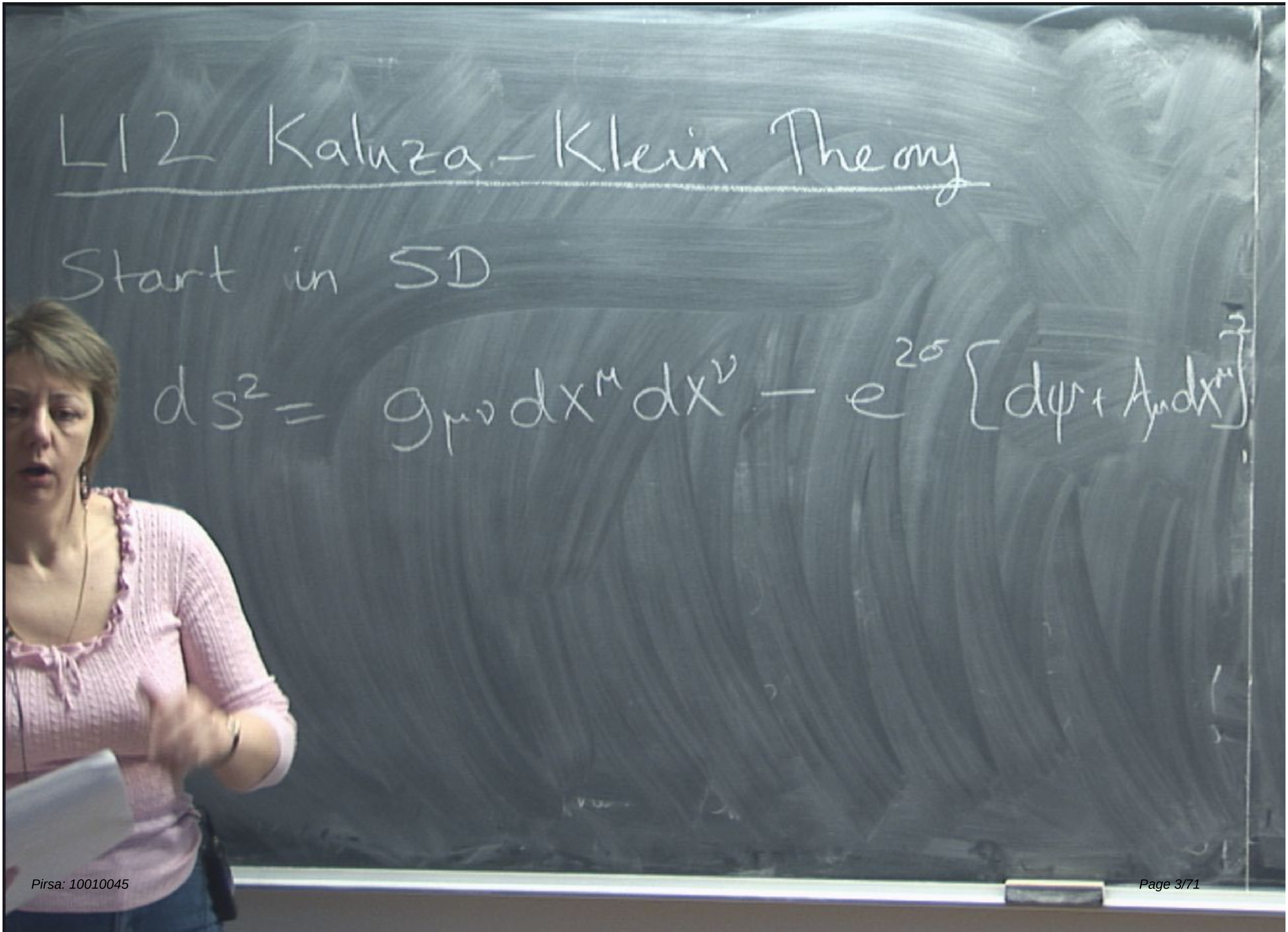
Abstract:

L12 Kaluza-Klein Theory


L12 Kaluza-Klein Theory
Start in 5D
$ds^2 = g_{\mu\nu} dx^\mu dx^\nu - e^{2\sigma}\left[d\psi + A_\mu dx^\mu\right]^2$

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$$ds^2 = g_{0\mu\nu}\, dx^\mu dx^\nu - e^{2\sigma}\left[d\psi + A_\mu dx^\mu\right]^2$$

In this form, $\det g_5 = \det g_0 \cdot e^{2\sigma}$.

$\mu, \nu = 0, 1, 2, 3.$ co-ord

$a, b =$ " o/n.

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o|n basis: $\omega^a = e^a{}_\mu dx^\mu$ (o|n for $g_{0\mu\nu}$)

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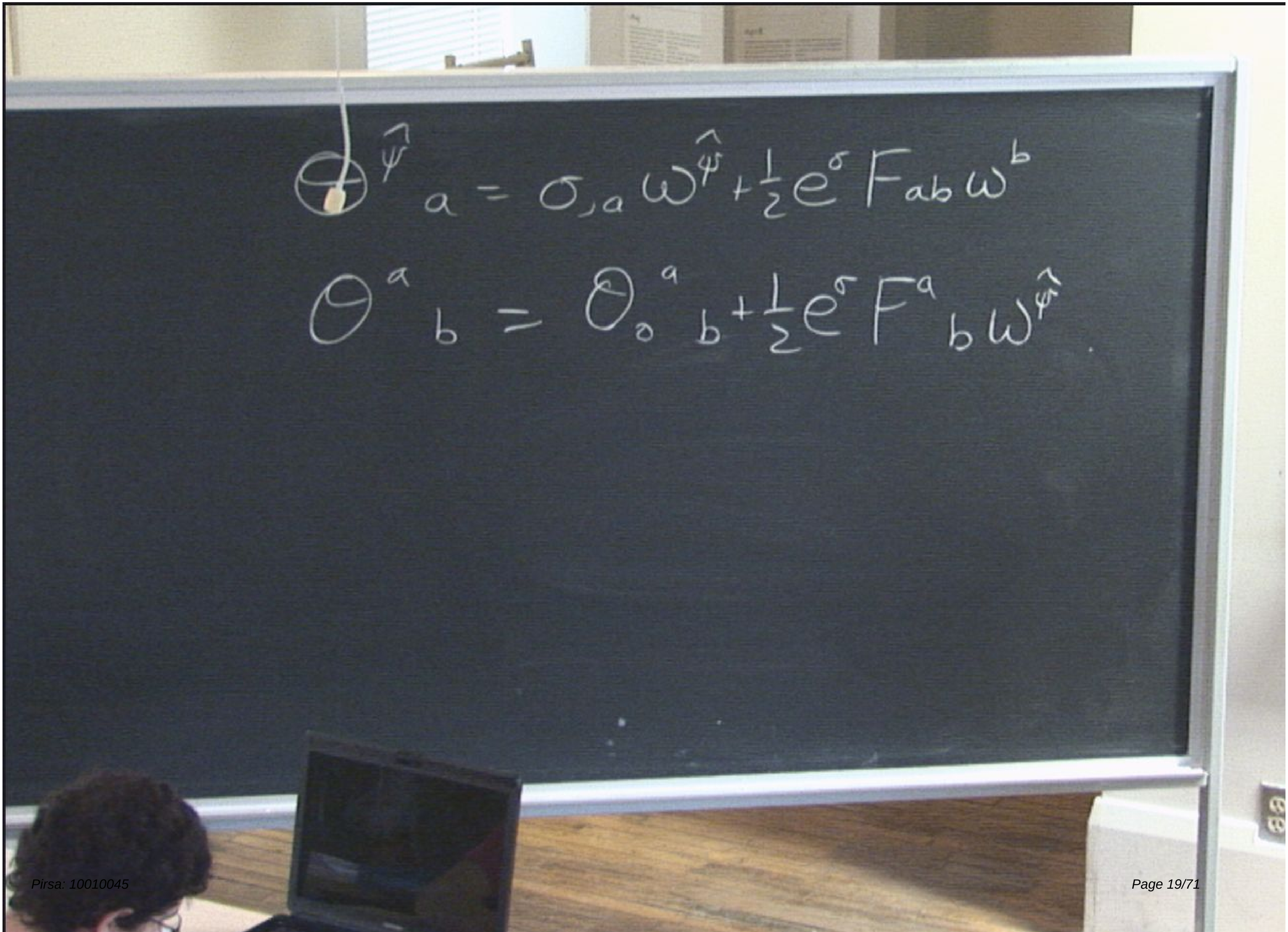
② $R^a{}_b = d\Theta^a{}_b + \Theta^a{}_c \wedge \Theta^c{}_b + \Theta^a{}_{\hat\varphi} \wedge \Theta^{\phi}{}_b$


$\Theta^{\hat{\psi}}{}_{a} = \sigma_{,a}\,\omega^{\hat{\psi}} + \frac{1}{2}e^{\sigma}F_{ab}\,\omega^{b}$
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② $R^a{}_b = d\Theta^a{}_b + \Theta^a{}_c \wedge \Theta^c{}_b + \Theta^a{}_{\hat\psi} \wedge \Theta^{\hat\psi}{}_b$

$= d\Theta_0{}^a{}_b + \frac{1}{2} e^{\sigma} \sigma_{,c} \omega^c \wedge \omega^{\hat\psi} F^a{}_b$

$+ \frac{1}{2} e^{\sigma} F^a{}_{b;c} \omega^c \wedge \omega^{\hat\psi} + \frac{1}{2} e^{\sigma} F^a{}_b (\sigma_{,c} \omega^c \wedge \omega^{\hat\psi}$

$+ e^{\sigma} F)$

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$$+ \left(\Theta_0{}^a{}_c + \frac{1}{2} e^\sigma F^a{}_c \omega^{\hat{\psi}}\right) \wedge \left(\Theta_0{}^c{}_b + \frac{1}{2} e^\sigma F^c{}_b \omega^{\hat{\psi}}\right)$$

$$+$$

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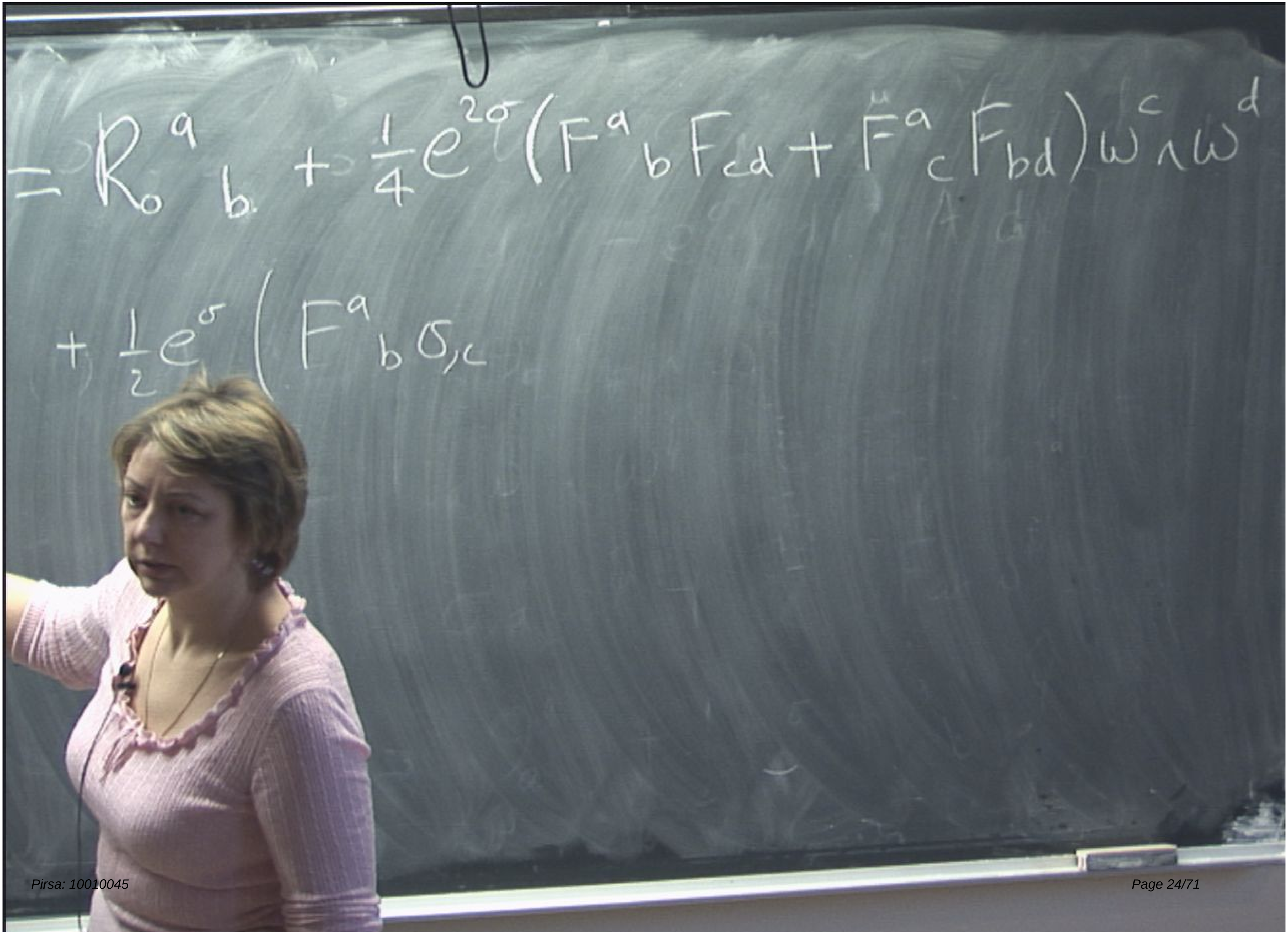
$$+ \left(\Theta_0{}^a{}_c + \frac{1}{2} e^\sigma F^a{}_c\, \omega^{\hat\psi}\right) \wedge \left(\Theta_0{}^c{}_b + \frac{1}{2} e^\sigma F^c{}_b\, \omega^{\hat\psi}\right)$$

$$+ \left(\sigma^{,a}\, \omega^{\hat\psi} + \frac{1}{2} e^\sigma F^a{}_c\, \omega^c\right) \wedge \left(\sigma_{,b}\, \omega^{\hat\psi} + \frac{1}{2} e^\sigma F_{bd}\, \omega^d\right)$$

$$= R_0{}^a{}_b + \frac{1}{4} e^{2\sigma} \left(F^a{}_b F_{cd} + F^a{}_c F_{bd} \right) \omega^c \wedge \omega^d$$


$= R_0{}^a{}_b + \frac{1}{4}e^{2\sigma}(F^a{}_b F_{cd} + F^a{}_c F_{bd})\,\omega^c \wedge \omega^d$
$+ \frac{1}{2}e^{\sigma}\left(F^a{}_b\, \sigma_{,c}\right.$

$$= R_0{}^a{}_b + \frac{1}{4}e^{2\sigma}\left(F^a{}_b F_{cd} + F^a{}_c F_{bd}\right)\omega^c \wedge \omega^d$$

$$+ \frac{1}{2}e^{\sigma}\left(2F^a{}_b \sigma_{,c} + F^a{}_c \sigma_{,b} - \sigma^{,a} F_{bc}\right)\omega^c \wedge \omega^{\hat{4}}$$

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$$+ \frac{1}{2} e^{\sigma}\left(F^a{}_{b,c}\,\omega^c - F^a{}_c\, \Theta_0{}^c{}_b + \Theta_0{}^a{}_c\, F^c{}_b\right) \wedge \omega^{\psi}$$

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But $dF = 0 = \frac{1}{2} d \left(F_{ab} \omega^a \wedge \omega^b \right)$

$$= \frac{1}{2} \left[F_{ab,c} \omega^c \wedge \omega^a \wedge \omega^b - F_{ab} \theta^a{}_c \wedge \omega^c \wedge \omega^b + F_{ab} \omega^a \wedge \theta^b{}_c \wedge \omega^c \right].$$

$$= R_0{}^a{}_b + \frac{1}{4}e^{2\sigma}\left(F^a{}_b F_{cd} + F^a{}_c F_{bd}\right)\omega^c \wedge \omega^d$$

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$$+ \frac{1}{2}e^{\sigma}\left(F^a{}_{b,c}\,\omega^c + \overbrace{\cancel{F^a{}_c\,\theta_0{}^c{}_b}}^{0} + \theta_0{}^a{}_c F^c{}_b\right) \wedge \omega^{\psi}$$

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$$\Theta^{\hat{\psi}}{}_{a} = \sigma_{,a}\,\omega^{\hat{\psi}} + \frac{1}{2} e^{\sigma} F_{ab}\,\omega^{b}$$

$$\Theta^{a}{}_{b} = \Theta_{0}{}^{a}{}_{b} + \frac{1}{2} e^{\sigma} F^{a}{}_{b}\,\omega^{\hat{\psi}}.$$

$$R^{a}{}_{bcd} = R_{0}{}^{a}{}_{bcd} + \frac{1}{2} e^{2\sigma}\Big(F^{a}{}_{b} F_{cd} + \frac{1}{2} F^{a}{}_{c} F_{bd} - \frac{1}{2} F^{a}{}_{d} F_{bc}\Big)$$

$$② \quad R^{\hat\psi}{}_{a} = d\Theta^{\hat\psi}{}_{a} + \Theta^{\hat\psi}{}_{b} \wedge \Theta^{b}{}_{a}$$

$$②\quad R^{\hat\psi}{}_a = d\,\Theta^{\hat\psi}{}_a + \Theta^{\hat\psi}{}_b \wedge \Theta^b{}_a$$

$$= \sigma_{,ab}\,\omega^b \wedge \omega^{\hat\psi} + \sigma_{,a}\left(\sigma_{,b}\,\omega^b \wedge \omega^{\hat\psi} + e^{\sigma} F\right)$$

(2) $R^{\hat\psi}{}_a = d\theta^{\hat\psi}{}_a + \theta^{\hat\psi}{}_b \wedge \theta^b{}_a$

$$= \sigma_{,ab}\, \omega^b \wedge \omega^{\hat\psi} + \sigma_{,a}\left(\sigma_{,b}\, \omega^b \wedge \omega^{\hat\psi} + e^{\sigma} F\right)$$

$$+ \frac{1}{2} e^{\sigma} \sigma_{,c} F_{ab}\, \omega^c \wedge \omega^b + \frac{1}{2} e^{\sigma} F_{ab,c}\, \omega^c \wedge \omega^b$$

$$- \frac{1}{2} e^{\sigma} F_{ab}\, \theta_0{}^b{}_c \wedge \omega^c$$

(2) $R^{\hat\psi}{}_a = d\theta^{\hat\psi}{}_a + \theta^{\hat\psi}{}_b \wedge \theta^b{}_a$

$= \sigma_{,ab}\, \omega^b \wedge \omega^{\hat\psi} + \sigma_{,a}\left(\sigma_{,b}\, \omega^b \wedge \omega^{\hat\psi} + e^\sigma F\right)$

$+ \frac{1}{2} e^\sigma\, \sigma_{,c} F_{ab}\, \omega^c \wedge \omega^b + \frac{1}{2} e^\sigma F_{ab,c}\, \omega^c \wedge \omega^b$

$- \frac{1}{2} e^\sigma F_{ab}\, \theta_0{}^b{}_c \wedge \omega^c$

$+ \left(\sigma_{,b}\, \omega^{\hat\psi} + \frac{1}{2} e^\sigma F_{bc}\, \omega^c\right) \wedge \left(\theta_0{}^b{}_a + \frac{1}{2} e^\sigma F^b{}_a\, \omega^{\hat\psi}\right)$

$$= \sigma_{,ab}\, \omega^b \wedge \omega^{\hat\psi} + \sigma_{,a}\sigma_{,b}\, \omega^b \wedge \omega^{\hat\psi} - \sigma_{,b}\, \Theta_0{}^b{}_a \wedge \omega^{\hat\psi}$$

$$+ \frac{1}{4} e^{2\sigma} F_{cb} F^c{}_a\, \omega^b \wedge \omega^{\hat\psi}$$

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$$+ e^{\sigma}\left(\sigma_{,a} F + \frac{1}{2} F_{ab}\sigma_{,c}\,\omega^b \wedge \omega^c\right).$$

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$$+ e^{\sigma}\left(\sigma_{,a} F + \frac{1}{2} F_{ab}\, \sigma_{,c}\, \omega^b \wedge \omega^c\right)$$

$$\hookrightarrow R^{\hat{\psi}}{}_{a \hat{\psi} b} = -\left(\nabla_a \nabla_b \sigma + \sigma_{,a}\sigma_{,b}\right) - \frac{1}{4} e^{2\sigma} F_{ac} F_b{}^c$$

$$\Rightarrow \; R^{\psi}{}_{\psi} = -\Box\sigma - (\nabla\sigma)^2 - \frac{1}{4} e^{2\sigma} F^2$$

$$R_{ab} = R_{0\,ab}$$

$$\Rightarrow \quad R^{\psi}{}_{\psi} = -\Box\sigma - (\nabla\sigma)^2 - \frac{1}{4} e^{2\sigma} F^2$$

$$R_{ab} = R_{0\,ab} + \frac{1}{2} e^{2\sigma} F_{ac} F_b{}^c - \nabla_a \nabla_b \sigma$$
$$- \sigma_{,a} \sigma_{,b}$$

$$= \sigma_{,ab}\, \omega^b \wedge \omega^{\hat\psi} + \sigma_{,a} \sigma_{,b}\, \omega^b \wedge \omega^{\hat\psi} - \sigma_{,b}\, \Theta_0{}^b{}_a \wedge \omega^{\hat\psi}$$

$$\Gamma_0{}^b{}_{c\,a}\, \omega^c$$

$$+ \frac{1}{4} e^{2\sigma} F_{cb} F^c{}_a\, \omega^b \wedge \omega^{\hat\psi}$$

$$+ e^{\sigma} \left(\sigma_{,a} F + \frac{1}{2} F_{ab} \sigma_{,c}\, \omega^b \wedge \omega^c \right)$$

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$$- \sigma_{,a} \sigma_{,b}$$

$$R = R_0 + \frac{1}{4} e^{2\sigma} F^2 \underbrace{- 2\Box\sigma - 2(\nabla\sigma)^2}_{-2e^{-\sigma}\Box e^{\sigma}}$$

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$$R_{ab} = R_{0\,ab} + \frac{1}{2} e^{2\sigma} F_{ac} F_b{}^c - \nabla_a \nabla_b \sigma - \sigma_{,a}\sigma_{,b}$$

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$$\& \quad \sqrt{g_5} = e^{\sigma}\sqrt{g_0}$$

$$\Rightarrow \quad R^{\psi}{}_{\psi} = -\Box\sigma - (\nabla\sigma)^2 - \frac{1}{4} e^{2\sigma} F^2$$

$$R_{ab} = R_{0\,ab} + \frac{1}{2} e^{2\sigma} F_{ac} F_b{}^c - \nabla_a \nabla_b \sigma$$

$$- \sigma_{,a} \sigma_{,b}$$

$$R_5 = R_0 + \frac{1}{4} e^{2\sigma} F^2 \underbrace{- 2\Box\sigma - 2(\nabla\sigma)^2}_{-2e^{-\sigma}\Box e^{\sigma}}$$

$$\& \quad \sqrt{g_5} = e^{\sigma} \sqrt{g_0}$$

$$\Rightarrow S_5 = -\frac{1}{16\pi G_5}\int \sqrt{g_5}\, R_5\, d^5x$$

$$= -\frac{1}{16\pi G_5}\int d^4x\, d\varphi\, e^{\sigma}\sqrt{g_0}\left[R_0 + \frac{1}{4}e^{2\sigma}F^2 + 2e^{-\sigma}\Box e^{\sigma}\right]$$

$$=$$

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$$= -\frac{L}{16\pi G_5}\int d^4x\, \sqrt{g_0}\left[e^{\sigma}R_0 + \frac{1}{4}e^{3\sigma}F^2\right]$$

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$$= -\frac{L}{16\pi G_5}\int d^4x \sqrt{g_0}\left[e^{\sigma}R_0 + \frac{1}{4}e^{3\sigma}F^2\right]$$

– nearly Einstein – Maxwell

Conventional to work in "Einstein frame" where there is no e^{σ} in front of R

Do this by a conformal transformation

$$g_{0\,\mu\nu} = \Omega^2(x)\, g_{\mu\nu}$$

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$$\cos\theta = \frac{\underline{u}\cdot\underline{v}}{|\underline{u}|\,|\underline{v}|}$$

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Do this by a conformal transformation

$$g_{0\,\mu\nu} = \Omega^2(x)\, g_{\mu\nu}$$

(conformal - preserves angles

$$\cos\theta = \frac{u \cdot v}{|u|\,|v|} \rightarrow \frac{g_{\mu\nu}\, u^\mu v^\nu}{|g_{\mu\nu}\, u^\mu u^\nu|^{1/2}|}$$

Conventional to work in "Einstein frame" where there is no e^{σ} in front of R

Do this by a conformal transformation

$$g_{0\mu\nu} = \Omega^2(x)\, g_{\mu\nu}$$

(Conformal - preserves angles

$$\cos\theta = \frac{u \cdot v}{|u||v|} \rightarrow \frac{g_{\mu\nu} u^\mu v^\nu}{|g_{\mu\nu} u^\mu u^\nu|^{1/2} |g_{\mu\nu} v^\mu v^\nu|^{1/2}}).$$

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$$= e^{\sigma} R + 3 e^{\sigma}\,\Box\sigma - \frac{3}{2} e^{\sigma}(\nabla\sigma)^2$$

Finally, write $\sigma = \sqrt{3}\,\varphi$ to get

$$S = \frac{L}{16\pi G_5}\int d^4x \sqrt{g}\left[-R + \frac{1}{2}(\nabla\varphi)^2 - \frac{1}{4}e^{\sqrt{3}\varphi}F^2\right]$$

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Black holes in KK theory

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Black holes in KK theory

Black string is a KK "black hole"

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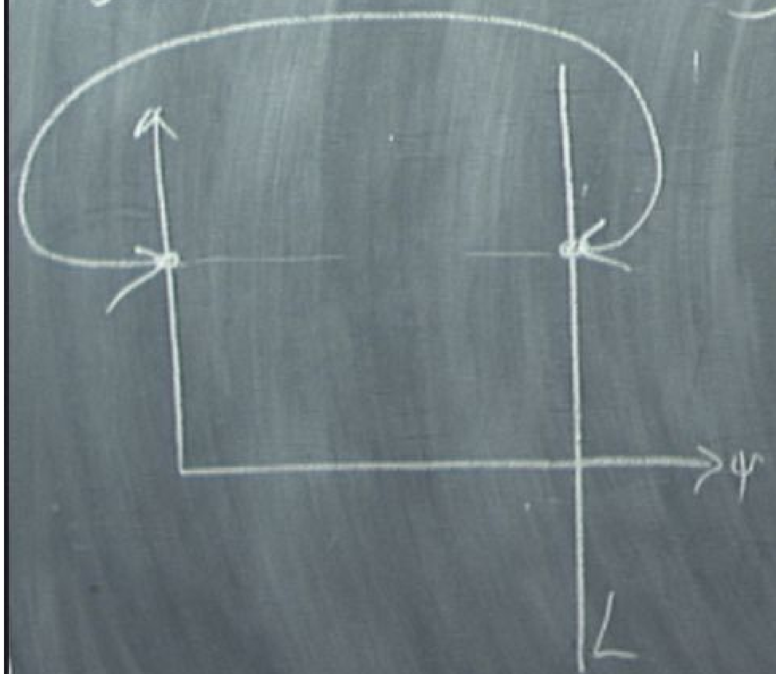

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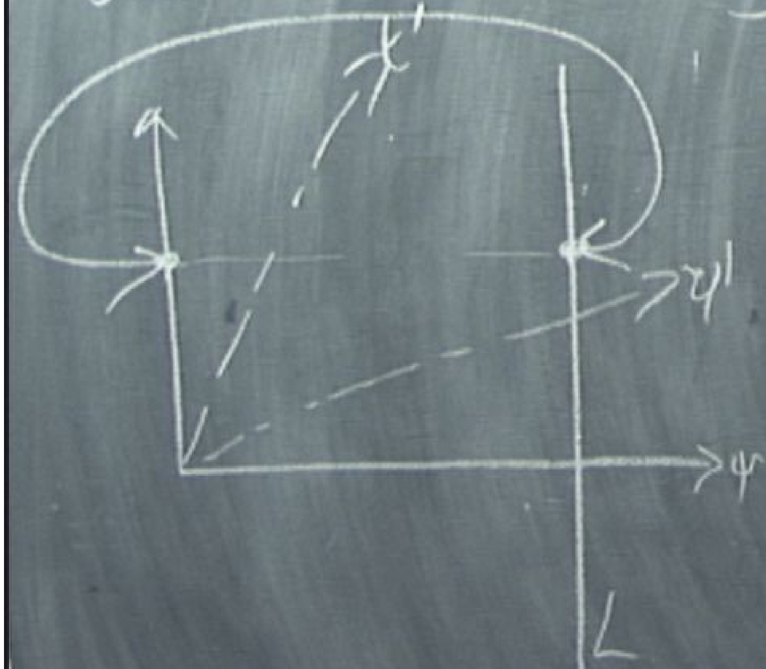

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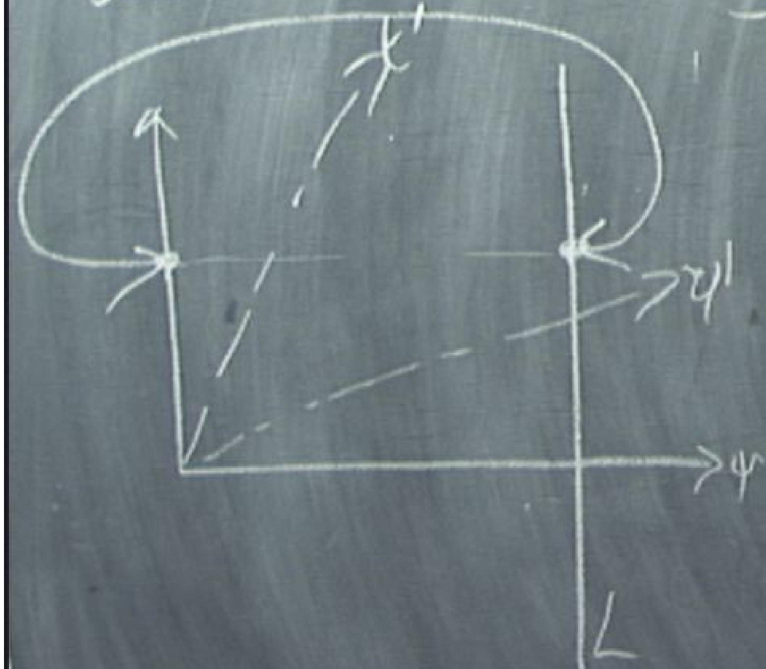

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Motion in ψ-dirn is equiv to charge

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$$= \left(1 - \frac{2GM}{(1-v^2)r + 2GMv^2}\right) dt^2 - \frac{dr^2}{\left(1-\frac{2GM}{r}\right)} - r^2 d\Omega_{II}^2$$

$$- \left(1 + \frac{2GMv^2}{(1-v^2)r}\right)\left(d\psi + \frac{2GMv}{(1-v^2)r + 2GMv^2}\, dt\right)^2$$

Writing $\hat{r} = r + \dfrac{2GMv^2}{1-v^2}$ $\quad q = \dfrac{2GMv}{1-v^2}$

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$$ds_4^2 = \left(1-\frac{2GM}{(1-v^2)\hat{r}}\right)\left(1-\frac{vq}{\hat{r}}\right)^{-1/2} dt^2 - \left(1-\frac{2GM}{(1-v^2)\hat{r}}\right)^{-1}\left(1-\frac{vq}{\hat{r}}\right)^{1/2} d\hat{r}^2$$

$$- \left(1-\frac{vq}{\hat{r}}\right)^{3/2} \hat{r}^2\, d\Omega^2$$

ext limit $v \to 1$
$M \to 0$