Title: Gravitational Physics - Review (PHYS 636) - Lecture 10

Date: Jan 14, 2010 10:00 AM

URL: http://pirsa.org/10010040

Abstract:

The connection on Σ is defined via

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$$\beta_{aij} = n_{ib} \nabla_a n_j{}^b$$

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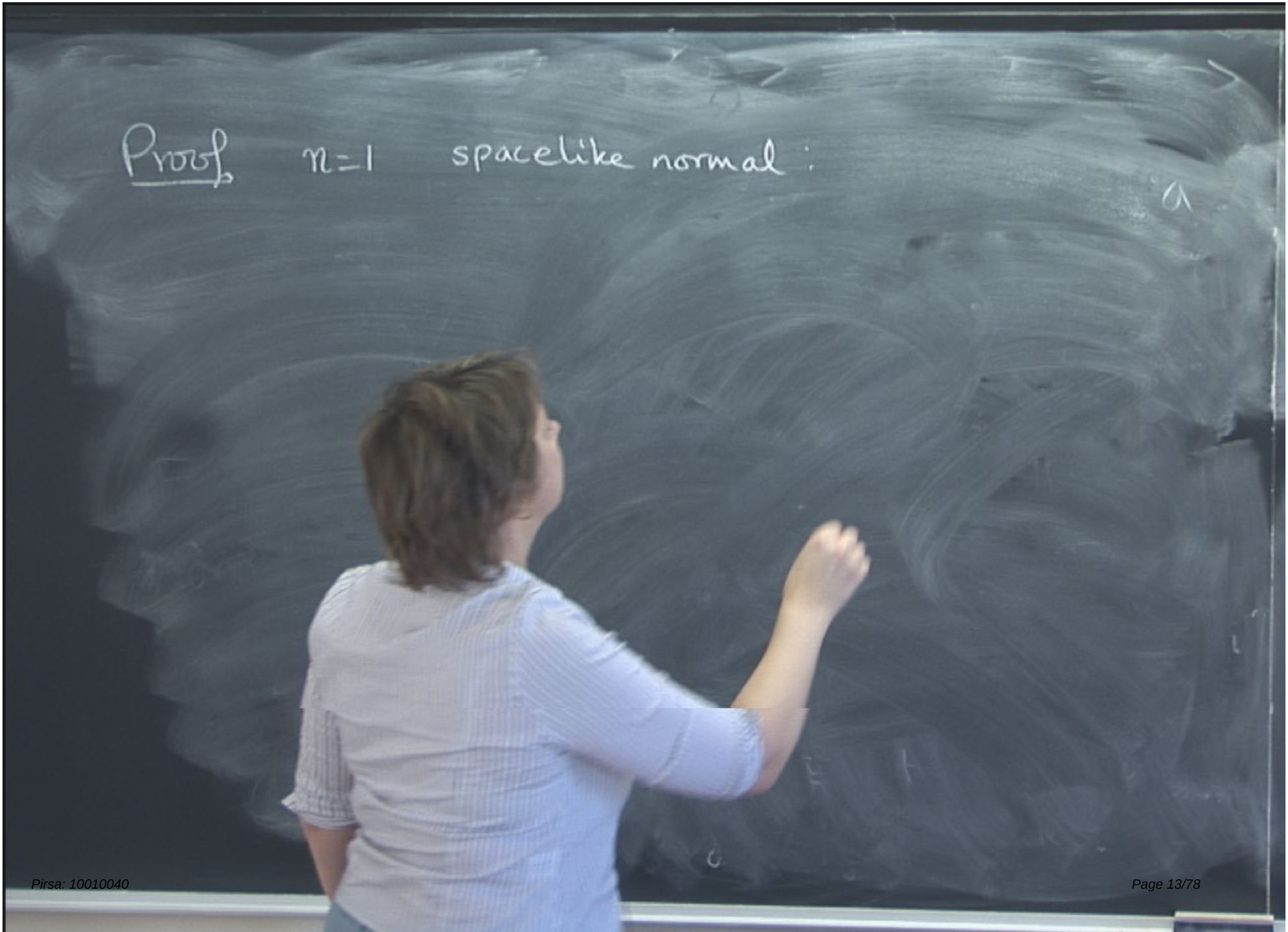
Proof n=1 spacelike normal :

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$=$

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$$ = h^e_c \, h^f_d \, h^a_g \, \nabla_e \, {}^{(d)}\nabla_f V^g - c \leftrightarrow d. $$

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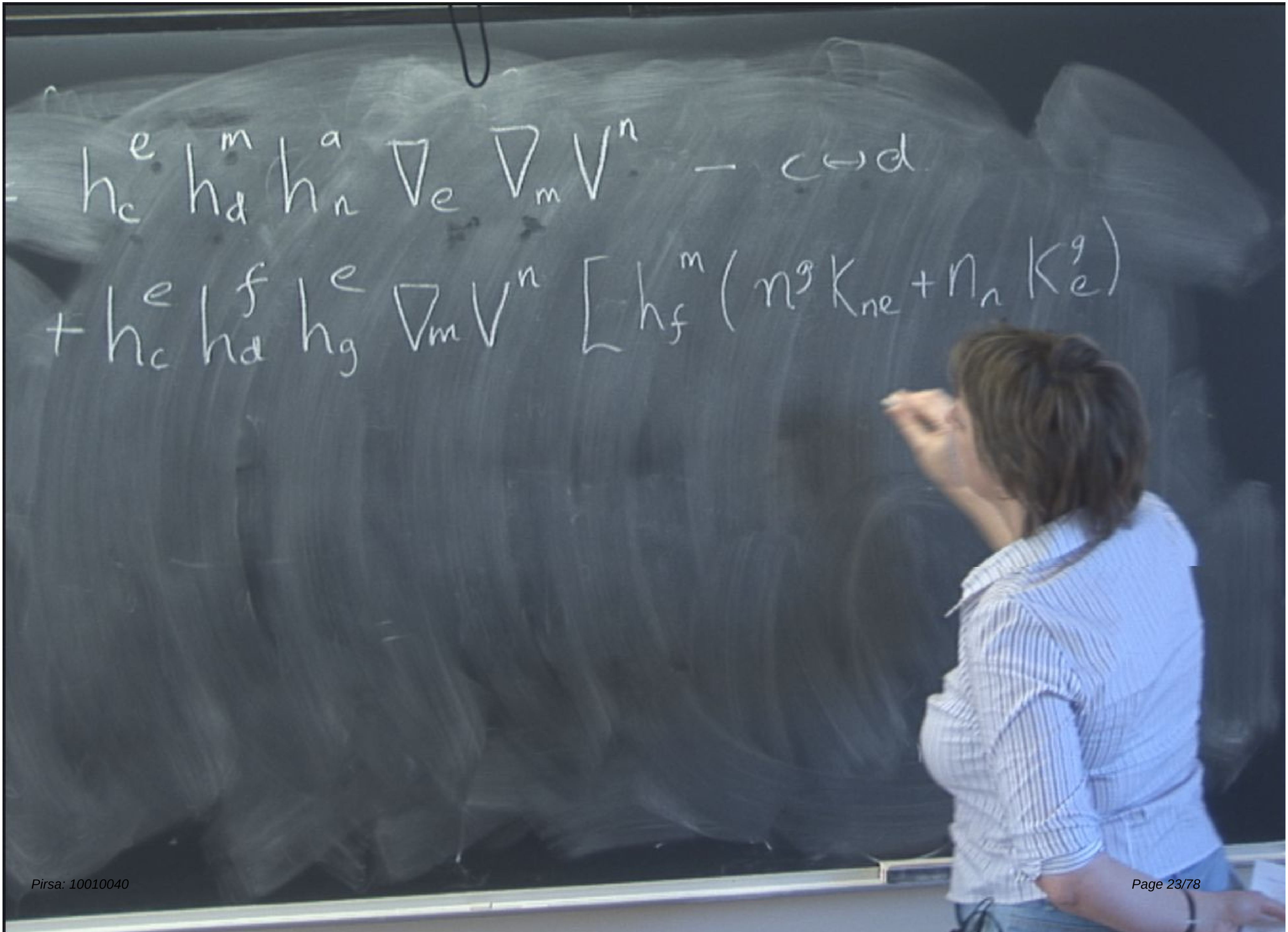
$$h_c{}^e \, h_d{}^m \, h_n{}^a \, \nabla_e \nabla_m V^n \; - \; c \leftrightarrow d$$


$h_c^{\;e} h_d^{\;m} h_n^{\;a} \nabla_e \nabla_m V^n - c \leftrightarrow d$
$+ h_c^{\;e} h_d^{\;f} h_g^{\;e} \nabla_m V^n \left[h_f^{\;m} (n^g K_{ne} + n_n K^g_{\;e}) \right.$

$$- h_c^{\;e} h_d^{\;m} h_n^{\;a} \nabla_e \nabla_m V^n - c \leftrightarrow d$$

$$+ h_c^{\;e} h_d^{\;f} h_g^{\;e} \nabla_m V^n \left[h_f^{\;m} \left(n^g K_{ne} + n_n K^g_{\;e} \right) + h_n^{\;g} \left(n_f K^m_{\;\;e} + n^m K_{fe} \right) \right]$$

$$- c \leftrightarrow d$$

$$h_c^{\ e} h_d^{\ m} h_n^{\ a} \nabla_e \nabla_m V^n - c \leftrightarrow d$$

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$$= h_c^{\ e} h_d^{\ m} h_n^{\ a} \, {}^{(N)}R^n{}_{fem} V^f$$

$$= h_c^{\ e} h_d^{\ m} h^a_{\ n} \nabla_e \nabla_m V^n - c \leftrightarrow d$$

$$+ h_c^{\ e} h_d^{\ f} h^a_{\ g} \nabla_m V^n \Big[h_f^{\ m} \big(\cancel{n^g} K_{ne} + n_n K^g_{\ e} \big) + h_n^{\ g} \big(\cancel{n_f} K^m{}_e + \underline{n^m K_{fe}} \big) \Big] - c \leftrightarrow d$$

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$$+ h_c^{\ e} h_d^{\ m} K^a{}_e \underbrace{n_n \nabla_m V^n}_{-V^n \nabla_m n_n = V^n K_{mn}} - c \leftrightarrow d$$

$$= h^{e}{}_{c}\, h^{m}{}_{d}\, h^{a}{}_{n}\, h^{b}{}_{f}\, R^{n}{}_{bem}\, V^{f}$$

$$= h^e{}_c h^m{}_d h^a{}_n h^b{}_f R^n{}_{bem} V^f$$

$$- h^m{}_d K^a{}_c K_{mf} V^f$$

$$+ h^m{}_c K^a{}_d K_{mf} V^f$$

$$= h^{e}{}_{c}\, h^{m}{}_{d}\, h^{a}{}_{n}\, h^{b}{}_{f}\, R^{n}{}_{bem}\, V^{f}$$

$$- K^{a}{}_{c}\, K_{df}\, V^{f}$$

$$+ K^{a}{}_{d}\, K_{cf}\, V^{f}$$

$$= h^e{}_c h^m{}_d h^a{}_n h^b{}_f R^n{}_{bem} V^f$$

$$- K^a{}_c K_{df} V^f$$

$$+ K^a{}_d K_{cf} V^f$$

hence result

Gibbons – Hawking boundary term

Recall that

$$\delta S_{EH} = -\frac{1}{16\pi G}\int d^4x\, \sqrt{-g}\, \delta g^{ab}\left(R_{ab} - \frac{1}{2} R g_{ab}\right)$$

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$$-\frac{1}{16\pi G}\int_{\partial M} d^3x \sqrt{-g}\left(n^b \nabla_b \delta g^c_c - n_a \nabla_b \delta g^{ab}\right)$$

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Look at δK

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But $n_a n_b \delta g^{ab} = n_a n_b \delta(h^{ab} - n^a n^b) = 2 n_a \delta n^a$

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$$= -\frac{1}{2} K n_c n_d \delta g^{cd} - \frac{1}{2} n_c n_d n^a \nabla_a \delta g^{cd} - \frac{1}{2} \nabla_n \delta g^c_c$$

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Hence

$$\delta\left(K\sqrt{-h}\right) = \frac{1}{2}\left(K_{ab} - K g_{ab}\right)\delta g^{ab}$$

$$- \frac{1}{2}\,{}^{(3)}\nabla_d\left(n_c\,\delta g^{cd}\right)$$

$$+ \frac{1}{2}\, n_b \nabla_a \delta g^{ab} - \frac{1}{2}\nabla_n \delta g$$

Hence $$\delta\left(S_{EH} - \underbrace{\frac{1}{8\pi G}\int_{\partial M} K\sqrt{h}}_{S_{GH}}\right) = 0$$

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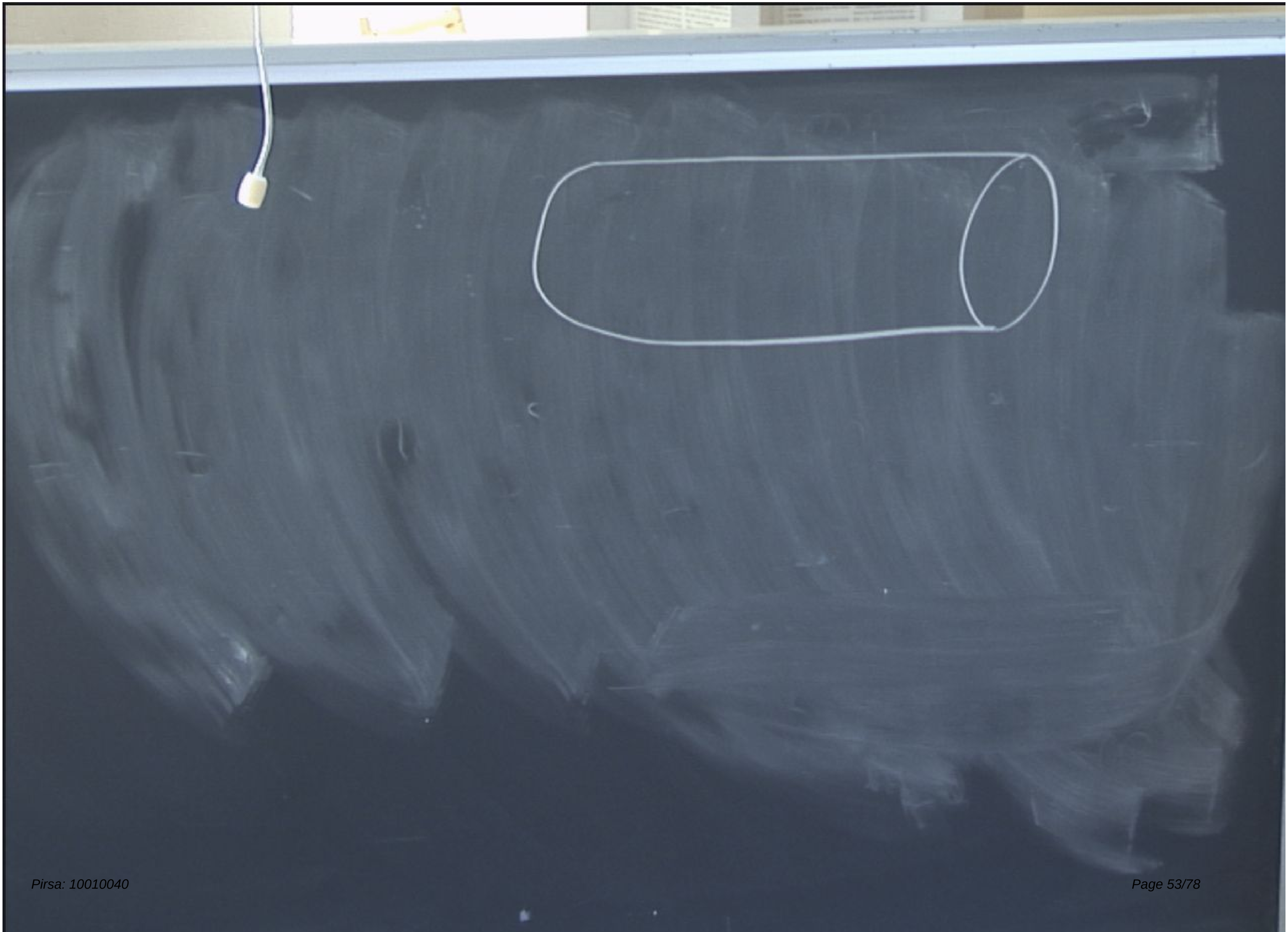
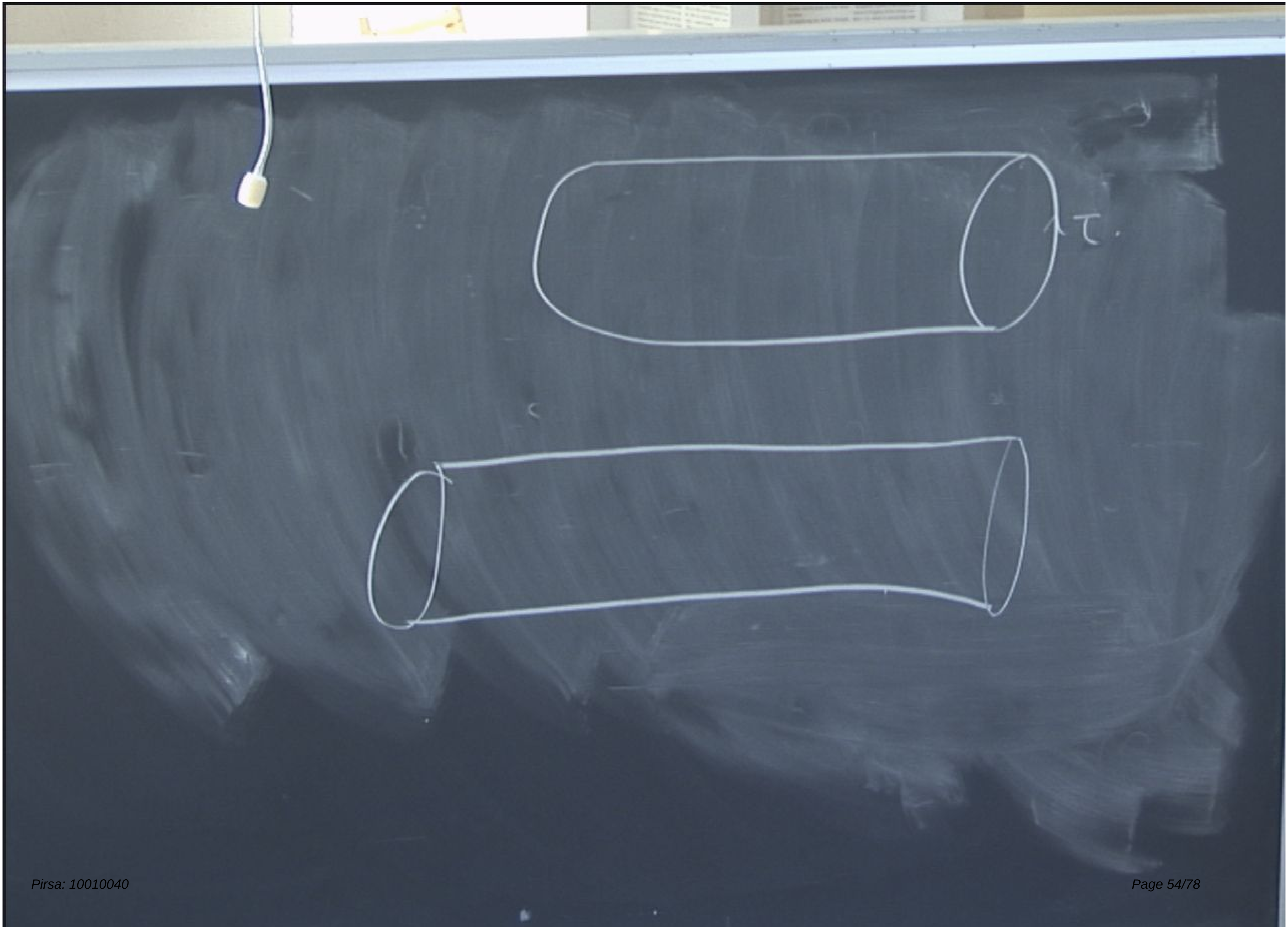
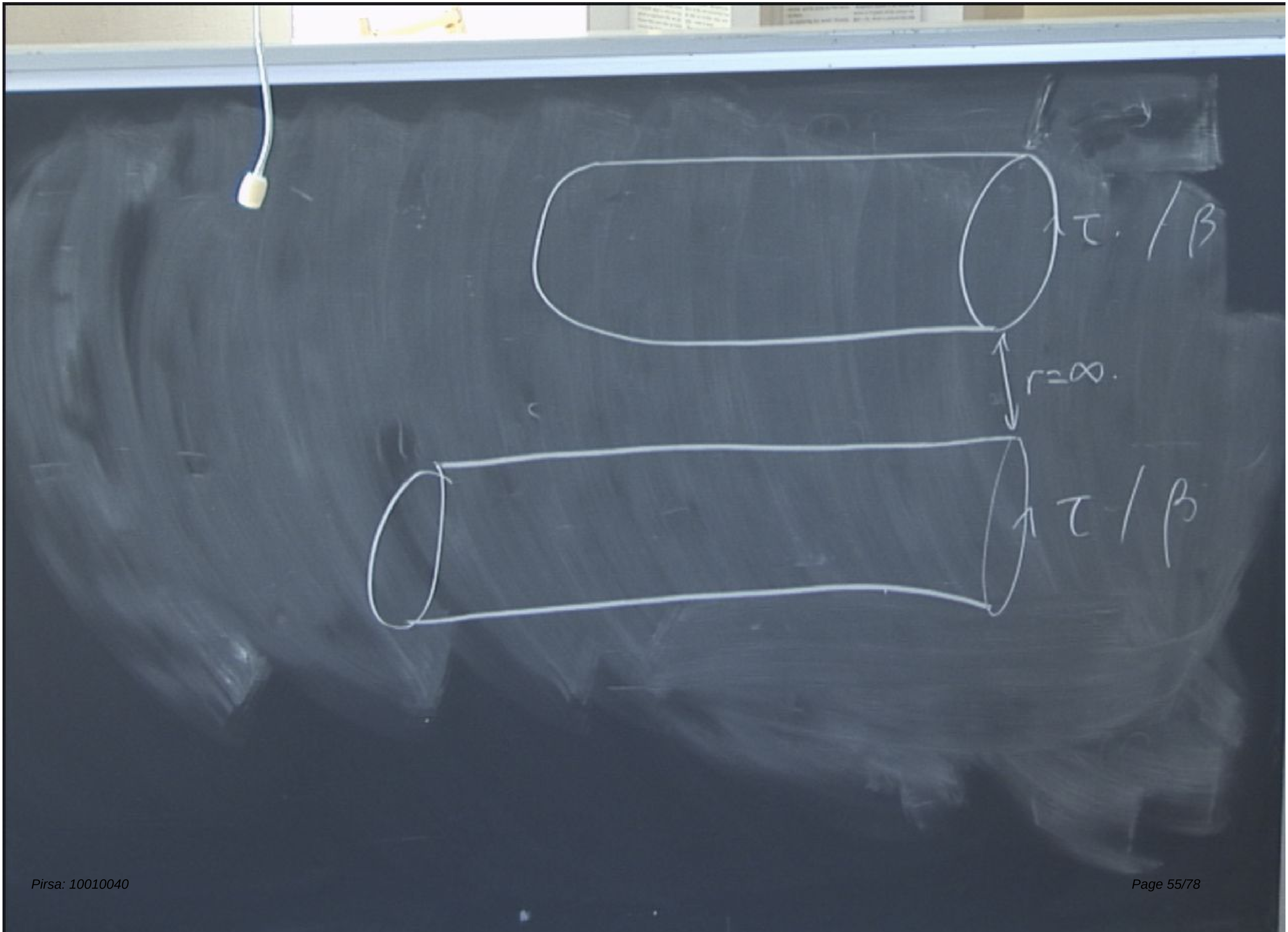
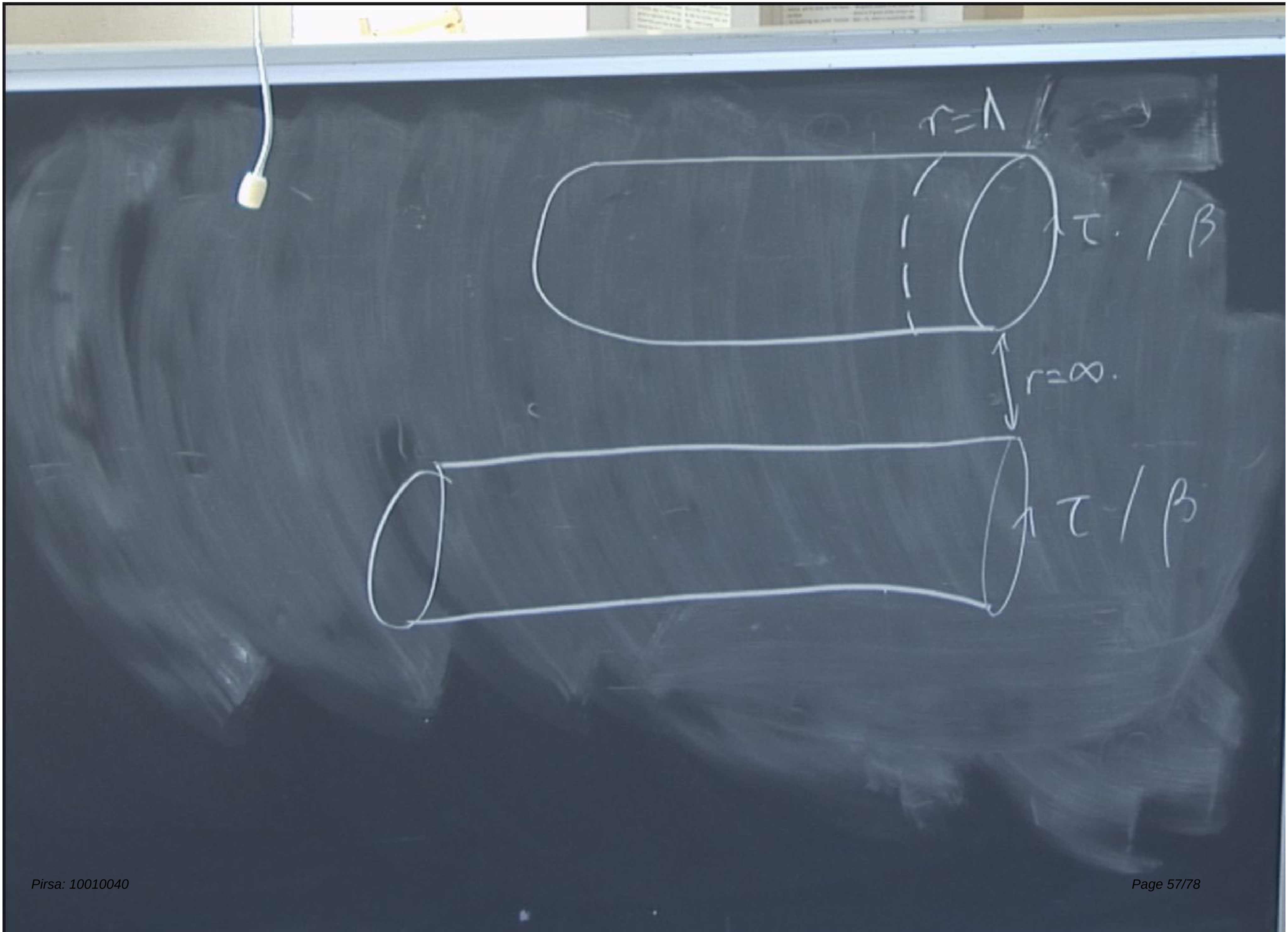



τ



τ. / β
r=∞.
τ / β


τ / β
r=∞.
τ / β


r=Λ
τ / β
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E-Sch

Action is pure boundary term

$r = \Lambda$ normal is $\left(1 - \frac{2GM}{\Lambda}\right)^{1/2} \frac{\partial}{\partial r}$

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E-Sch

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$$\int d\tau\, d\theta\, d\varphi \left(1-\frac{2GM}{\Lambda}\right)^{1/2} \Lambda^2 \sin^2\theta\, K_\Lambda$$

E-Sch   Action is pure boundary term

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$$= \left(1-\frac{2GM}{\Lambda}\right)^{1/2}\left[\frac{2}{\Lambda}\right.$$

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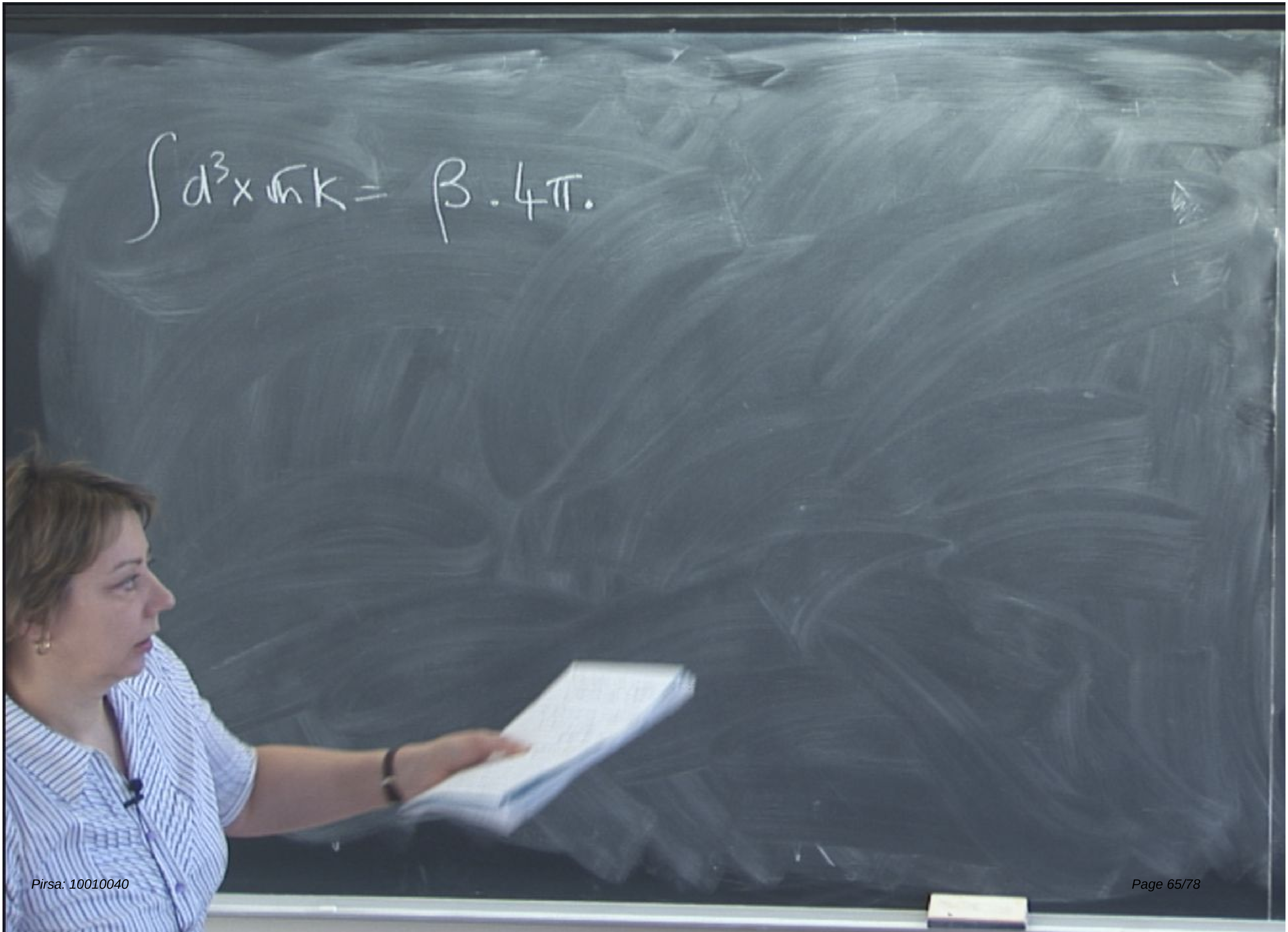
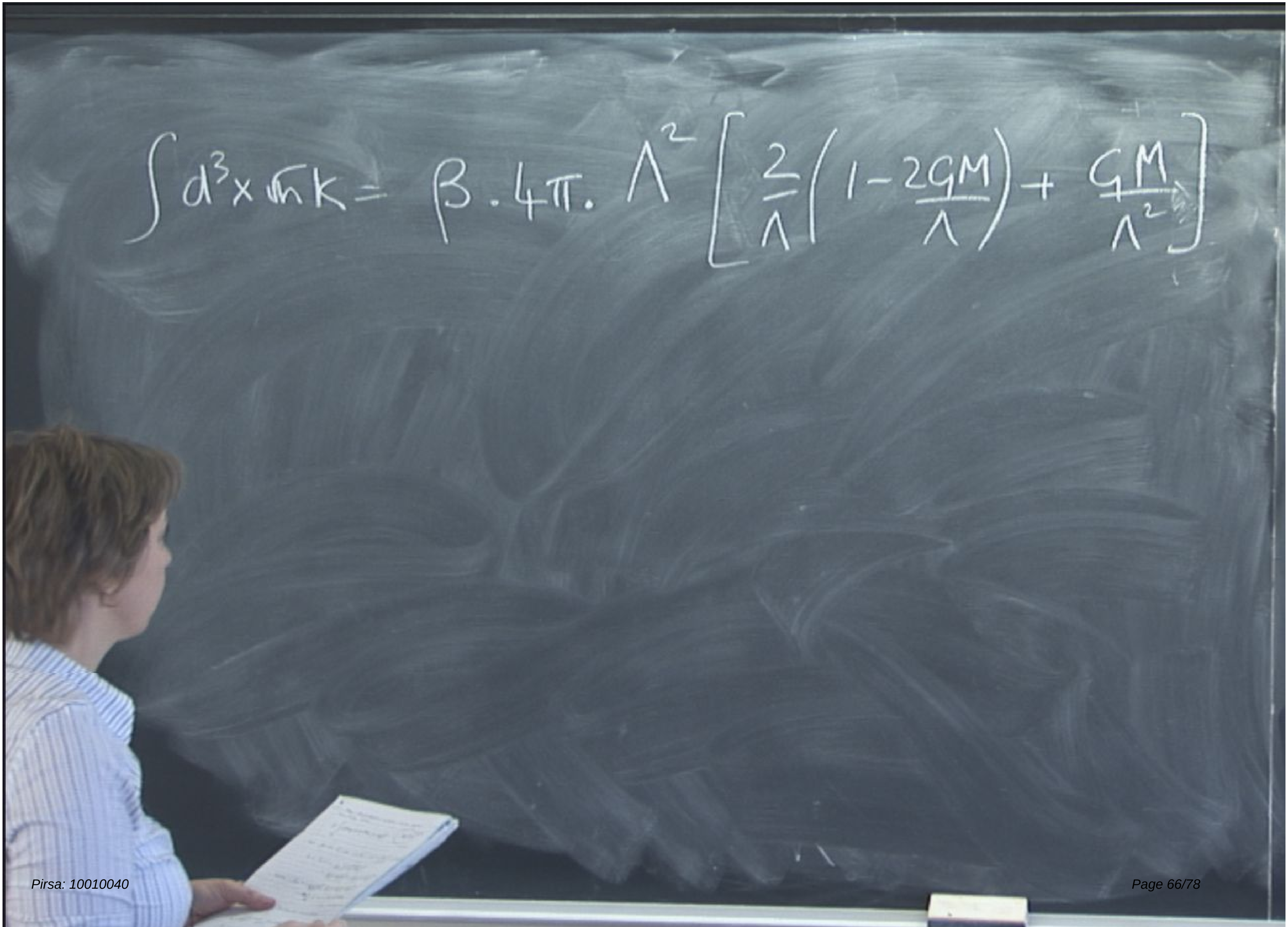
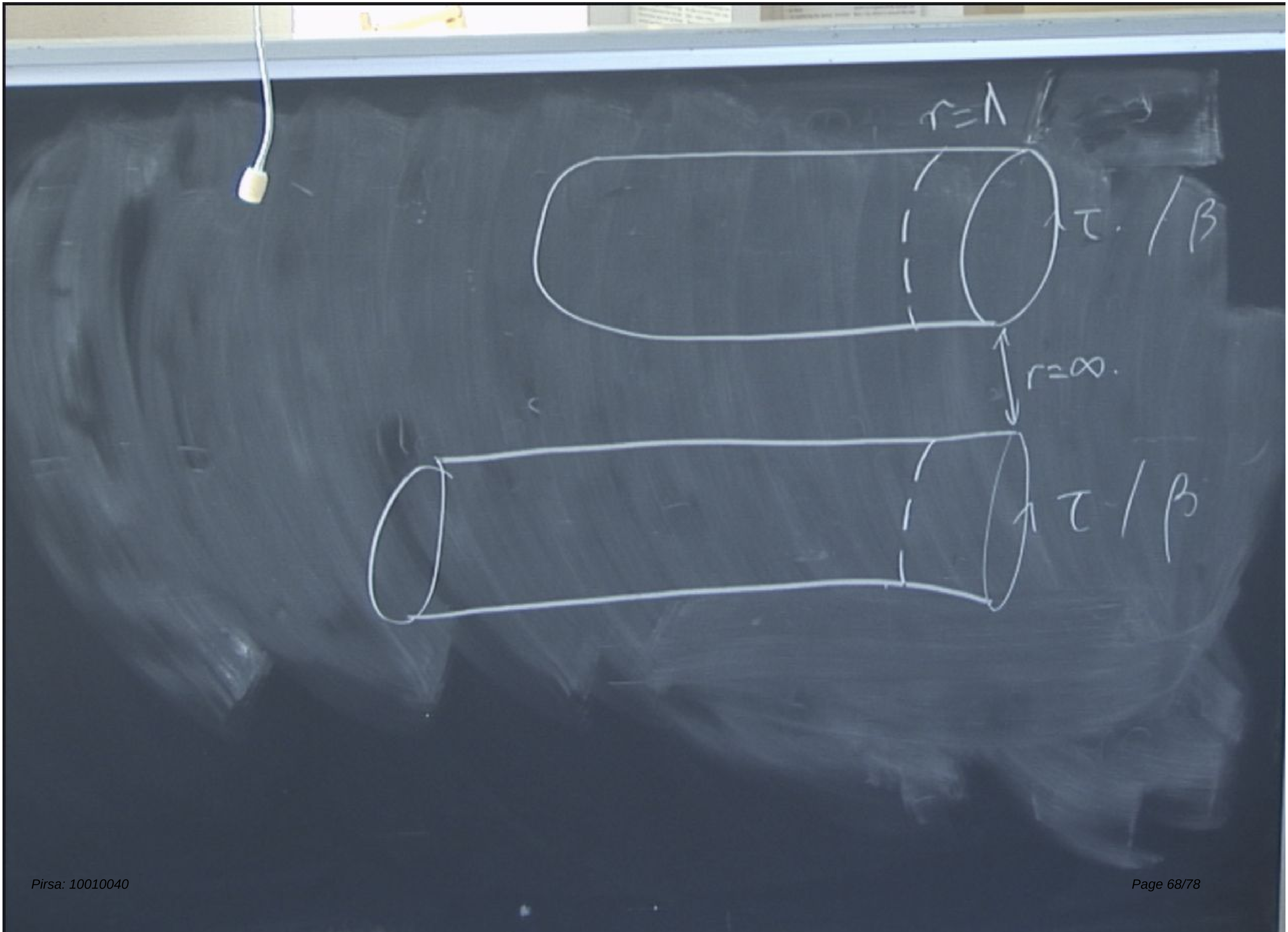
$$= \left(1-\frac{2GM}{\Lambda}\right)^{1/2}\left[\frac{2}{\Lambda} + \frac{GM}{\Lambda^2}\left(1-\frac{2GM}{\Lambda}\right)^{-1}\right]$$


$\int d^3x \sqrt{h}\, K = \beta \cdot 4\pi.$


$\int d^3x \sqrt{h}\, K = \beta \cdot 4\pi \cdot \Lambda^2 \left[\frac{2}{\Lambda}\left(1 - \frac{2GM}{\Lambda}\right) + \frac{GM}{\Lambda^2} \right]$

$$\int d^3x \sqrt{h}\, k = \beta . 4\pi . \Lambda^2 \left[\frac{2}{\Lambda}\left(1 - \frac{2GM}{\Lambda}\right) + \frac{GM}{\Lambda^2} \right]$$

$$= 4\pi\beta \left[2\Lambda - 3GM + O\left(\frac{1}{\Lambda}\right) \right]$$



r=Λ
τ. / β
r=∞.
τ / β

$$\int d^3x\,\sqrt{h}\,K = \beta \cdot 4\pi \cdot \Lambda^2 \left[\frac{2}{\Lambda}\left(1 - \frac{2GM}{\Lambda}\right) + \frac{GM}{\Lambda^2} \right]$$

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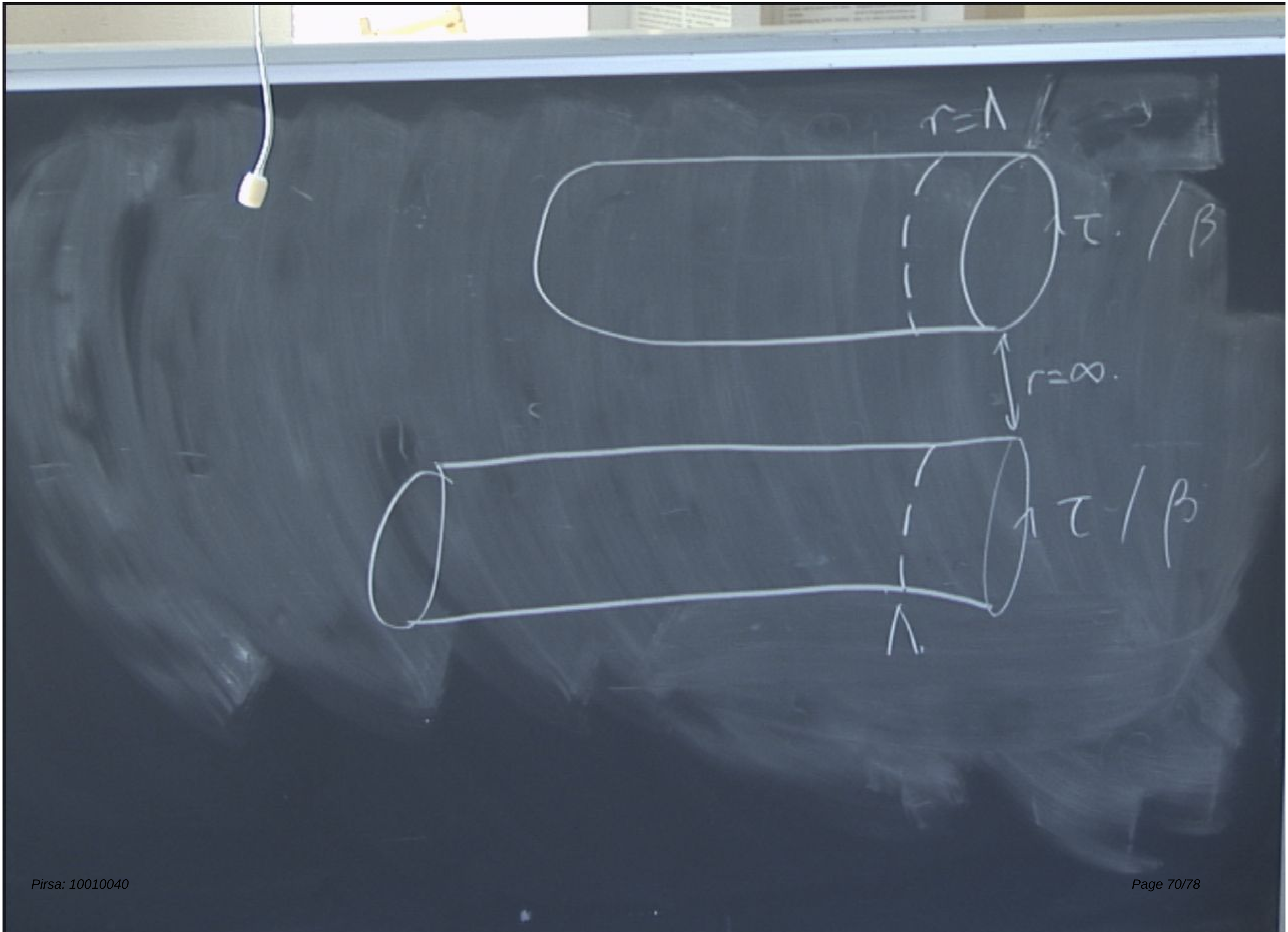
Need to subtract off the "flat space" part



r=Λ
τ. / β
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τ / β
Λ.

$$\int d^3x \sqrt{h}\, K = \beta \cdot 4\pi \cdot \Lambda^2 \left[\frac{2}{\Lambda}\left(1 - \frac{2GM}{\Lambda}\right) + \frac{GM}{\Lambda^2} \right]$$

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Need to subtract off the "flat space" part

In flat space $K = \frac{2}{r}$

$$\int d^3x \sqrt{h}\, K = \beta \cdot 4\pi \cdot \Lambda^2 \left[\frac{2}{\Lambda}\left(1-\frac{2GM}{\Lambda}\right) + \frac{GM}{\Lambda^2} \right]$$

$$= 4\pi\beta \left[2\Lambda - 3GM + O\left(\frac{1}{\Lambda}\right) \right]$$

Need ... tract off the "flat space" part

... space $K = \frac{2}{r}$, but

...ng proper length of τ-direction

$$\ldots x \sqrt{h}\, K = 4\pi\beta\, \Lambda^2 \left(1-\frac{2GM}{\Lambda}\right)^{1/2} \frac{2}{\Lambda}$$

$$\int d^3x \sqrt{h} K = \beta \cdot 4\pi \cdot \Lambda^2 \left[\frac{2}{\Lambda}\left(1 - \frac{2GM}{\Lambda}\right) + \frac{GM}{\Lambda^2} \right]$$

$$= 4\pi\beta \left[2\Lambda - 3GM + O\left(\frac{1}{\Lambda}\right) \right]$$

Need to subtract off the "flat space" pa

In flat space $K = \frac{2}{r}$, but

matching proper length of τ-direction

gives $\int d^3x \sqrt{h} K = 4\pi\beta \Lambda^2 \left(1 - \frac{2GM}{\Lambda}\right)^{1/2} \frac{2}{\Lambda}$

$$\int d^3x\sqrt{h}\,K = \beta \cdot 4\pi \cdot \Lambda^2\left[\frac{2}{\Lambda}\left(1-\frac{2GM}{\Lambda}\right)+\frac{GM}{\Lambda^2}\right]$$

$$= 4\pi\beta\left[2\Lambda - 3GM + O\left(\frac{1}{\Lambda}\right)\right]$$

Need to subtract off the "flat space" part

In flat space $K = \frac{2}{r}$, but

matching proper length of τ-direction

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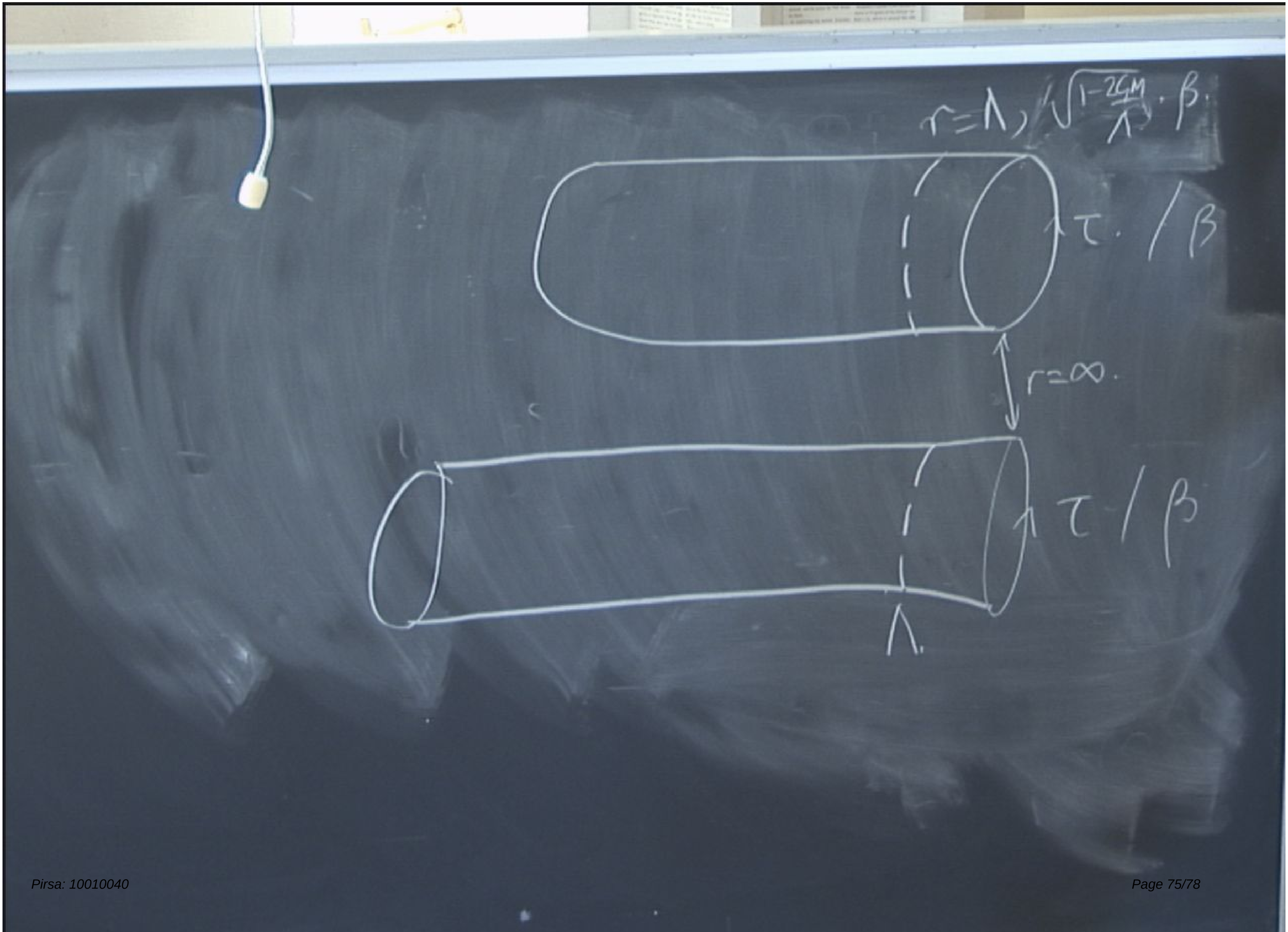

$r=\Lambda, \sqrt{1-\frac{2GM}{\Lambda}} \cdot \beta.$
$\tau.$ / β
$r=\infty.$
τ / β
$\Lambda.$

$$\sim \quad 4\pi\beta \; 2\Lambda \left(1 - \frac{GM}{\Lambda} + \cdots \right)$$

$$4\pi\beta \left[2\Lambda - 2GM \right]$$

$$\sim 4\pi\beta \, 2\Lambda \left(1 - \frac{GM}{\Lambda} + \cdots\right)$$

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$$I_E = -\frac{4\pi\beta}{8\pi G}\left[2\Lambda - 3GM\right] + \frac{4\pi\beta}{8\pi G}\left[2\Lambda - 2GM\right]$$

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