Title: Gravitational Physics - Review (PHYS 636) - Lecture 9

Date: Jan 13, 2010 10:00 AM

URL: http://pirsa.org/10010039

Abstract:

$$Z \sim \operatorname{tr} e^{-H/T}$$

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-at.
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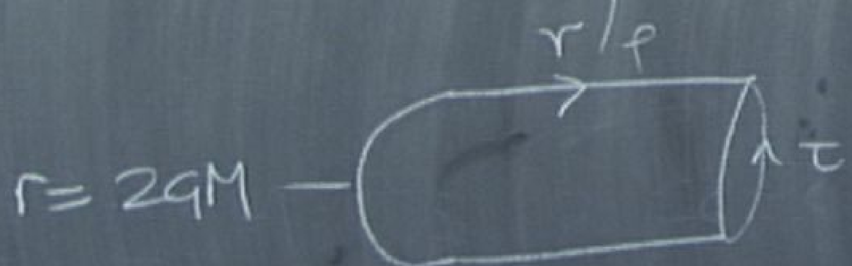

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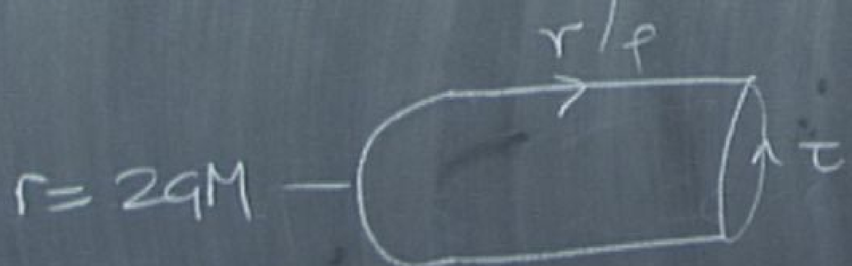

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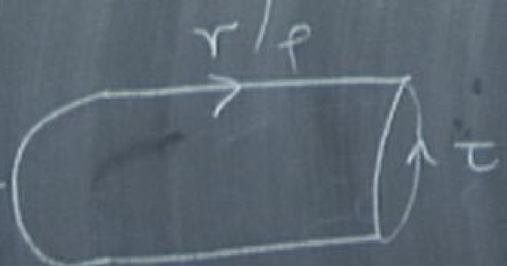

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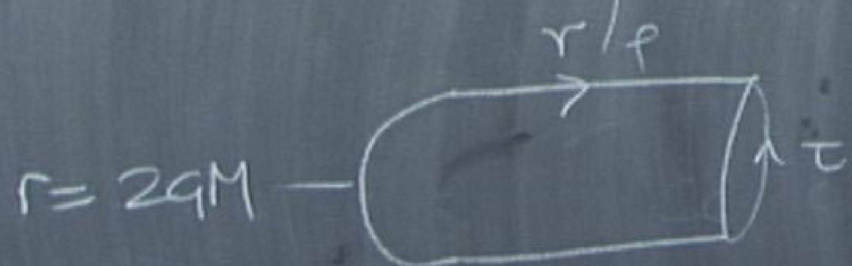

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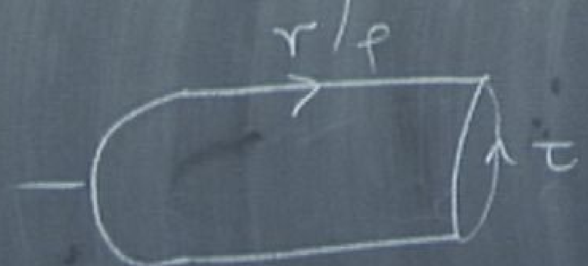

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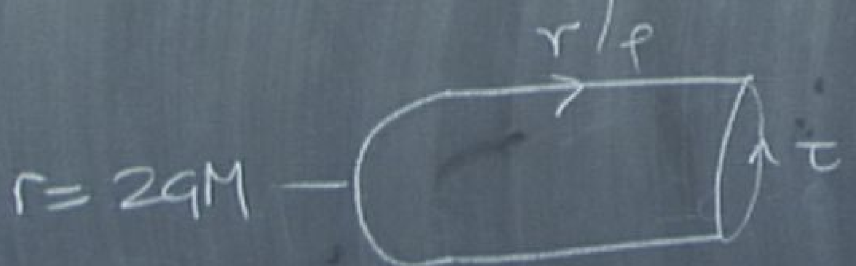

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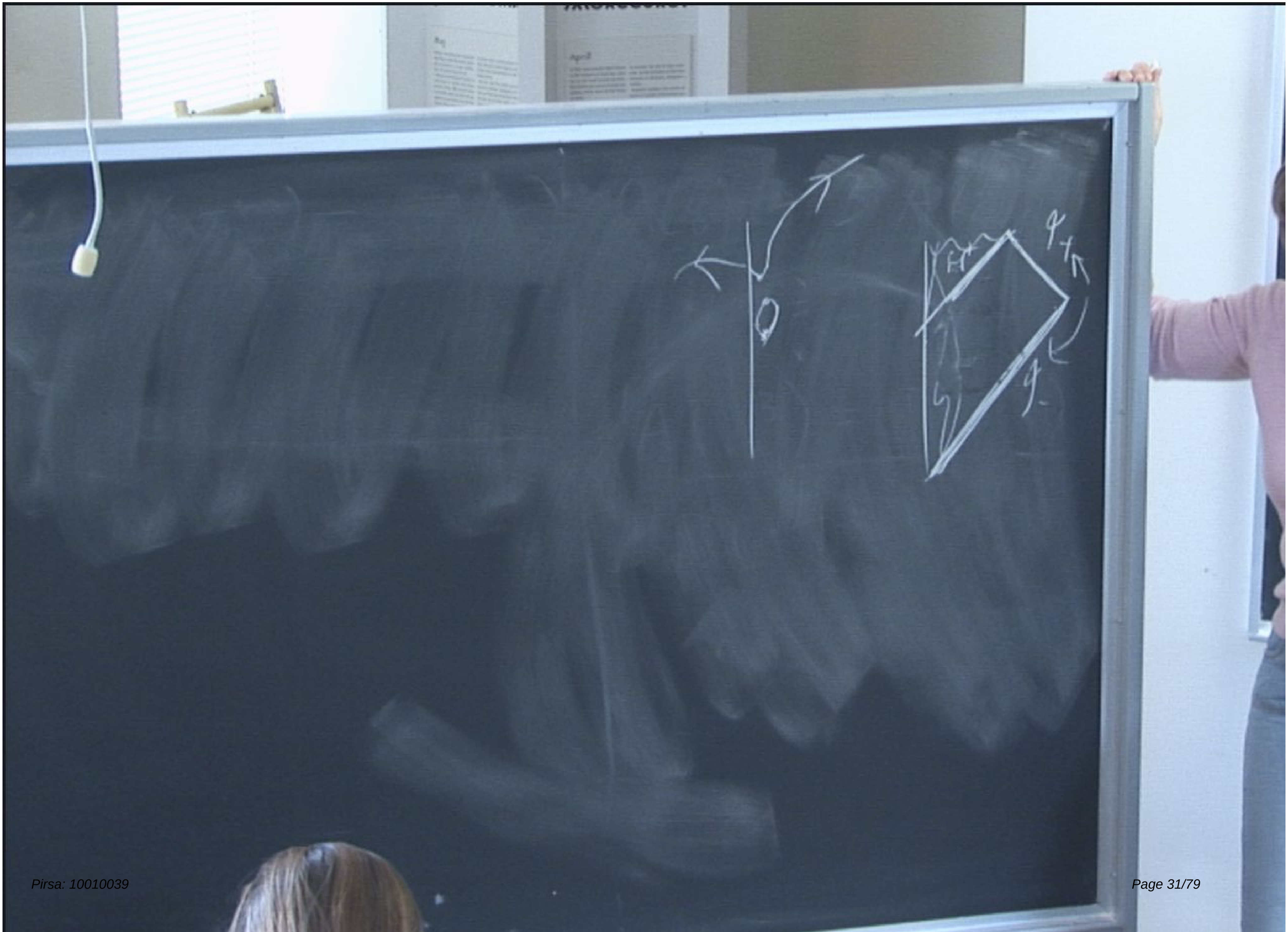
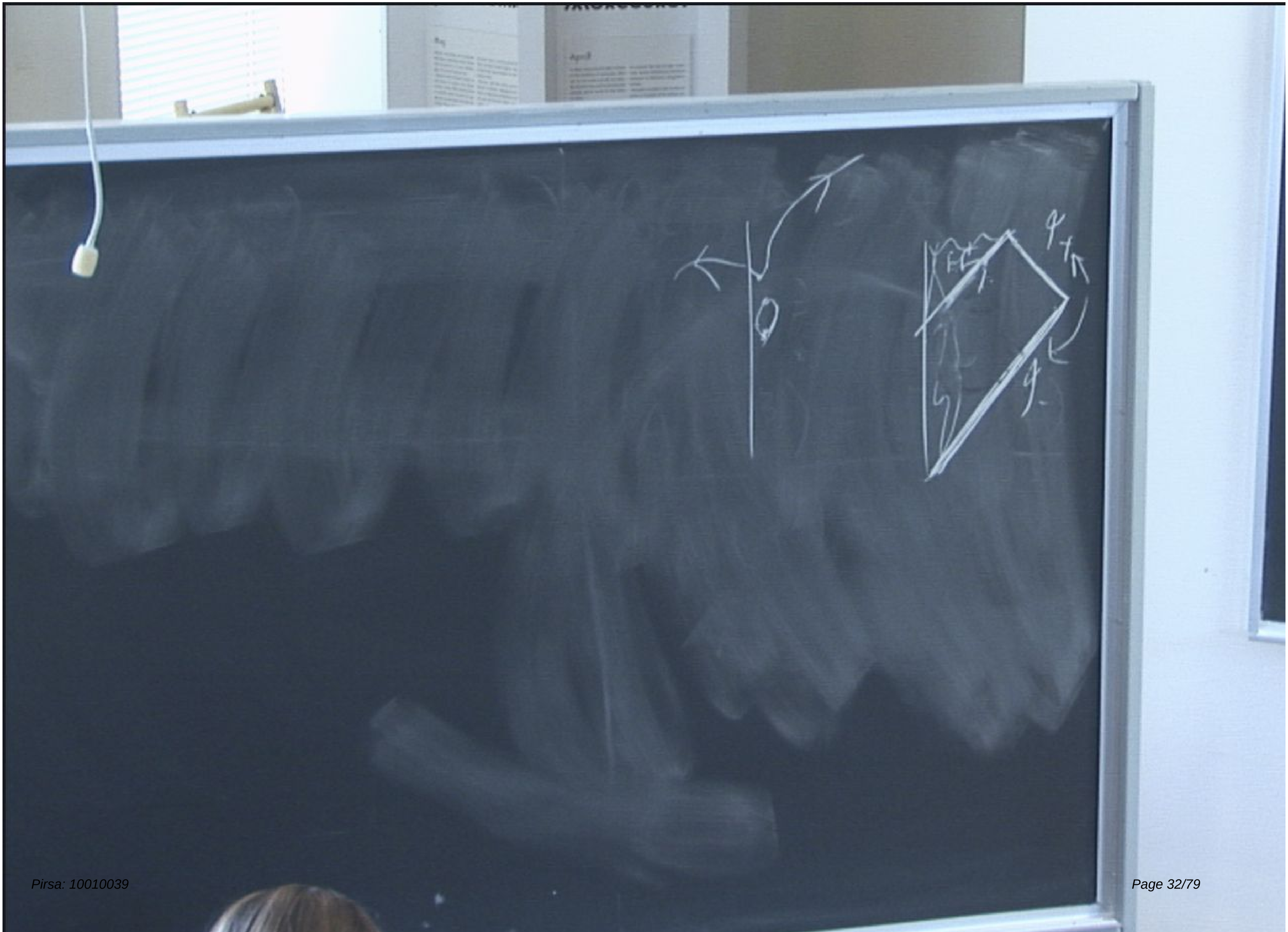
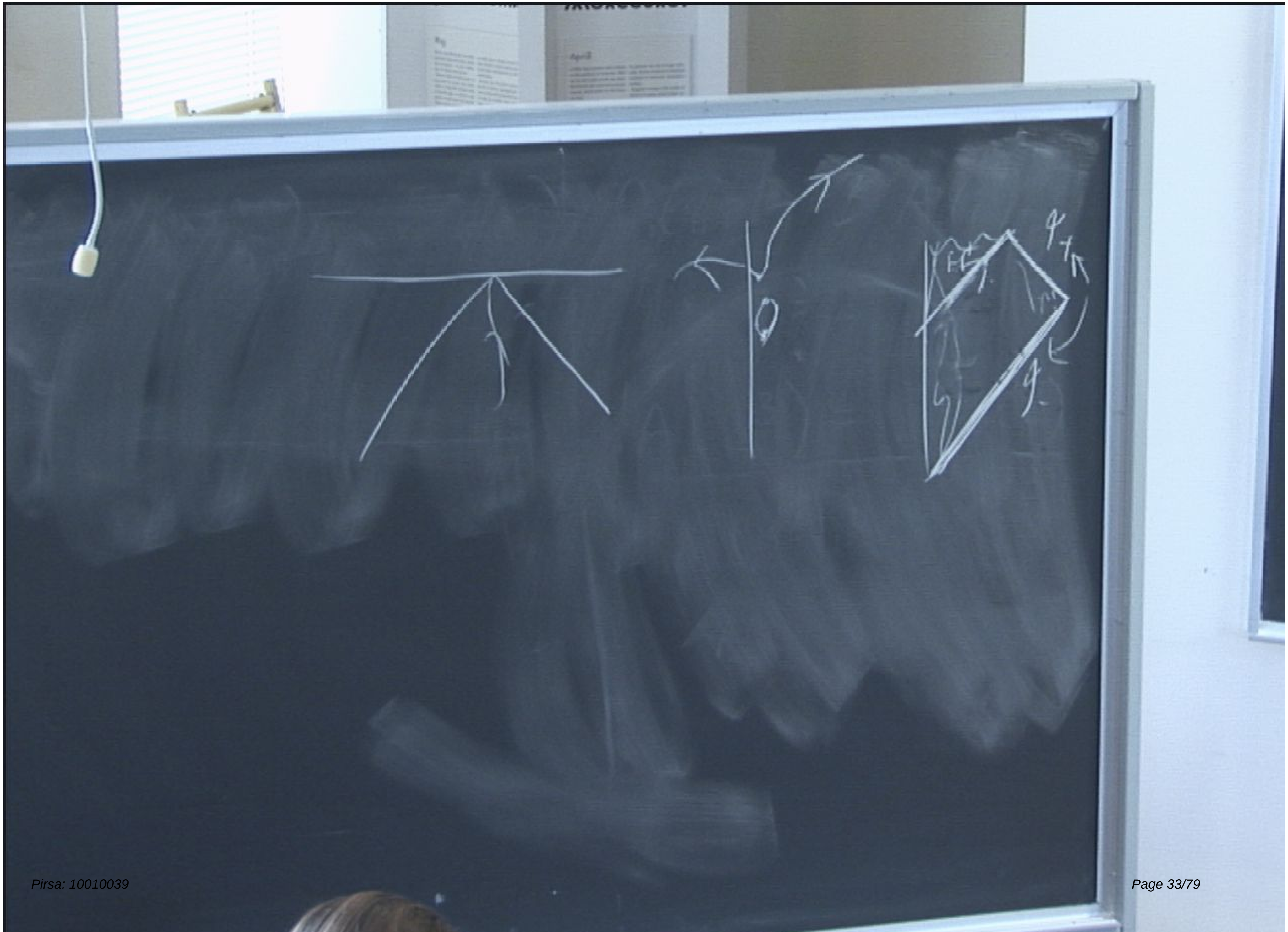
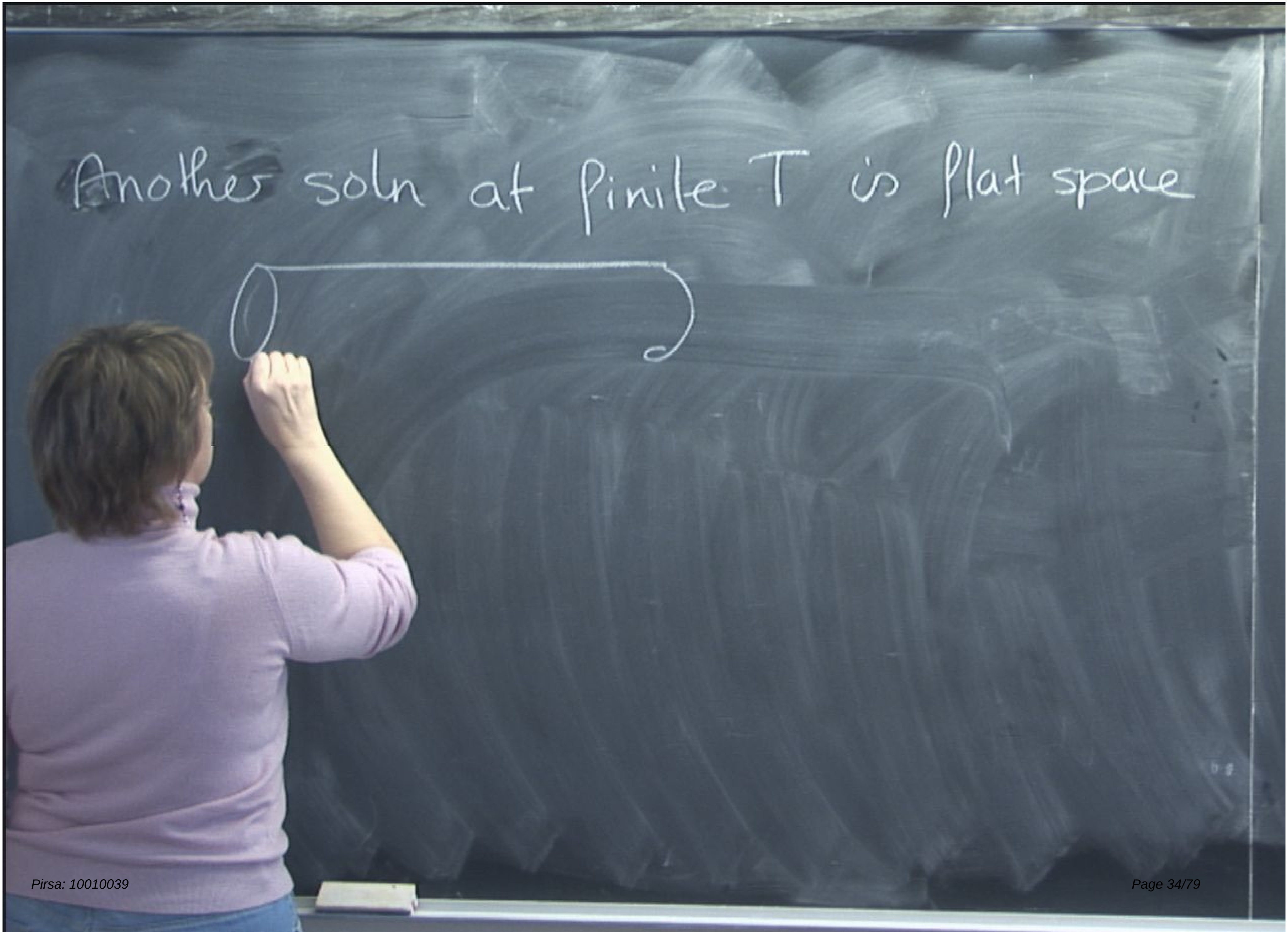
$$T = \frac{1}{8\pi GM} \to \frac{\hbar c^3}{8\pi GM k_B}$$ in any quantum field.





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For both flat space & E-Sch,

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For both flat space & E-Sch,

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Recall $g^{ab}\delta R_{ab} = -\nabla_a \nabla_b \delta g^{ab} + \Box(g_{ab}\delta g^{ab})$

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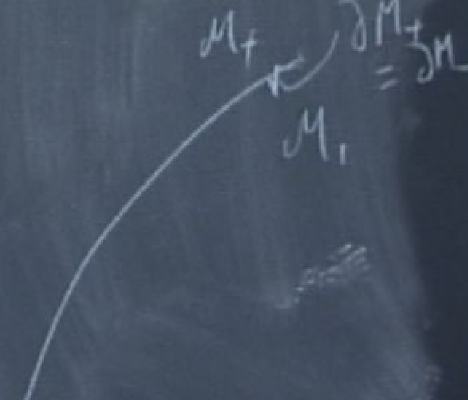

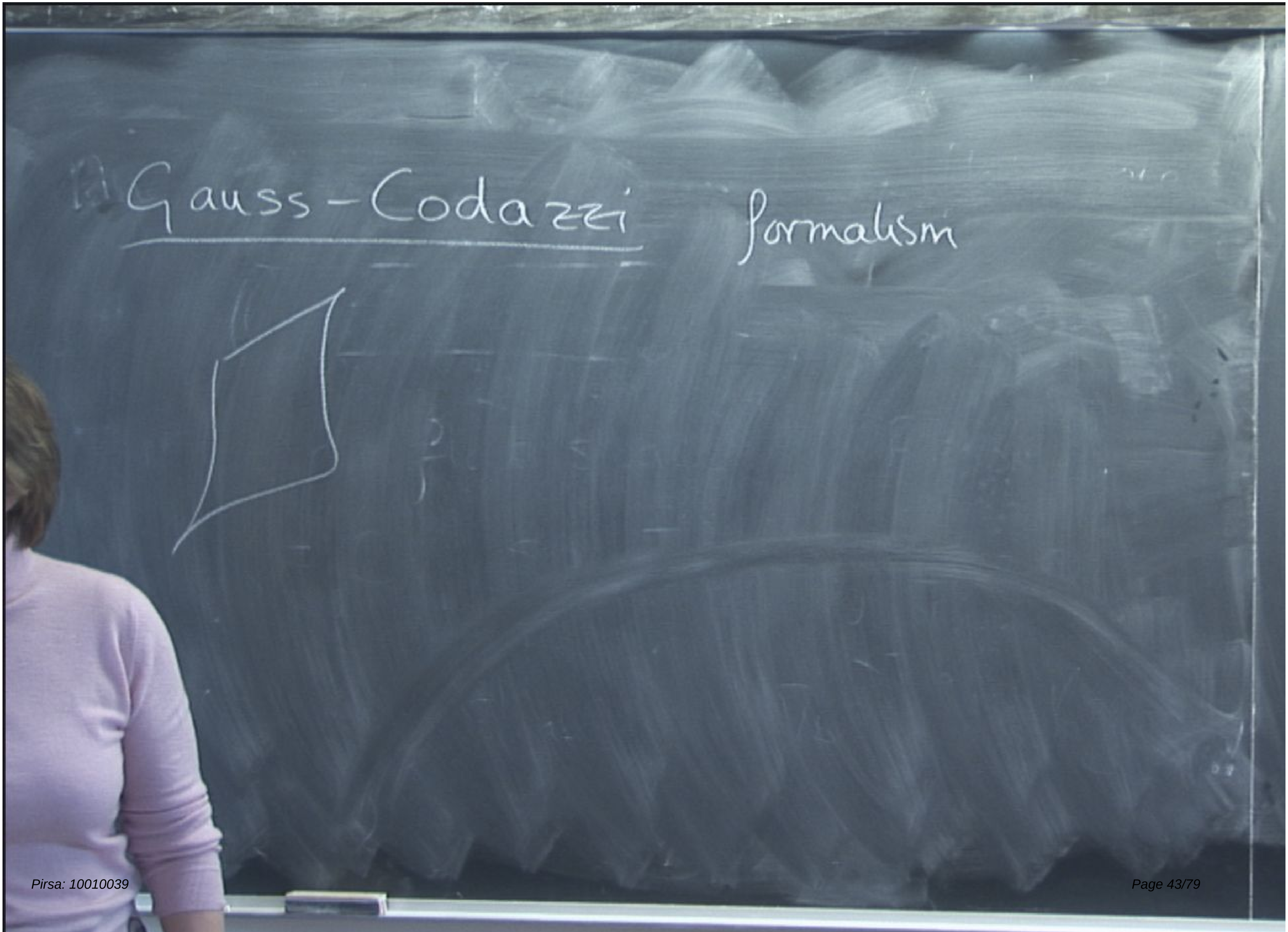
Gauss-Codazzi formalism

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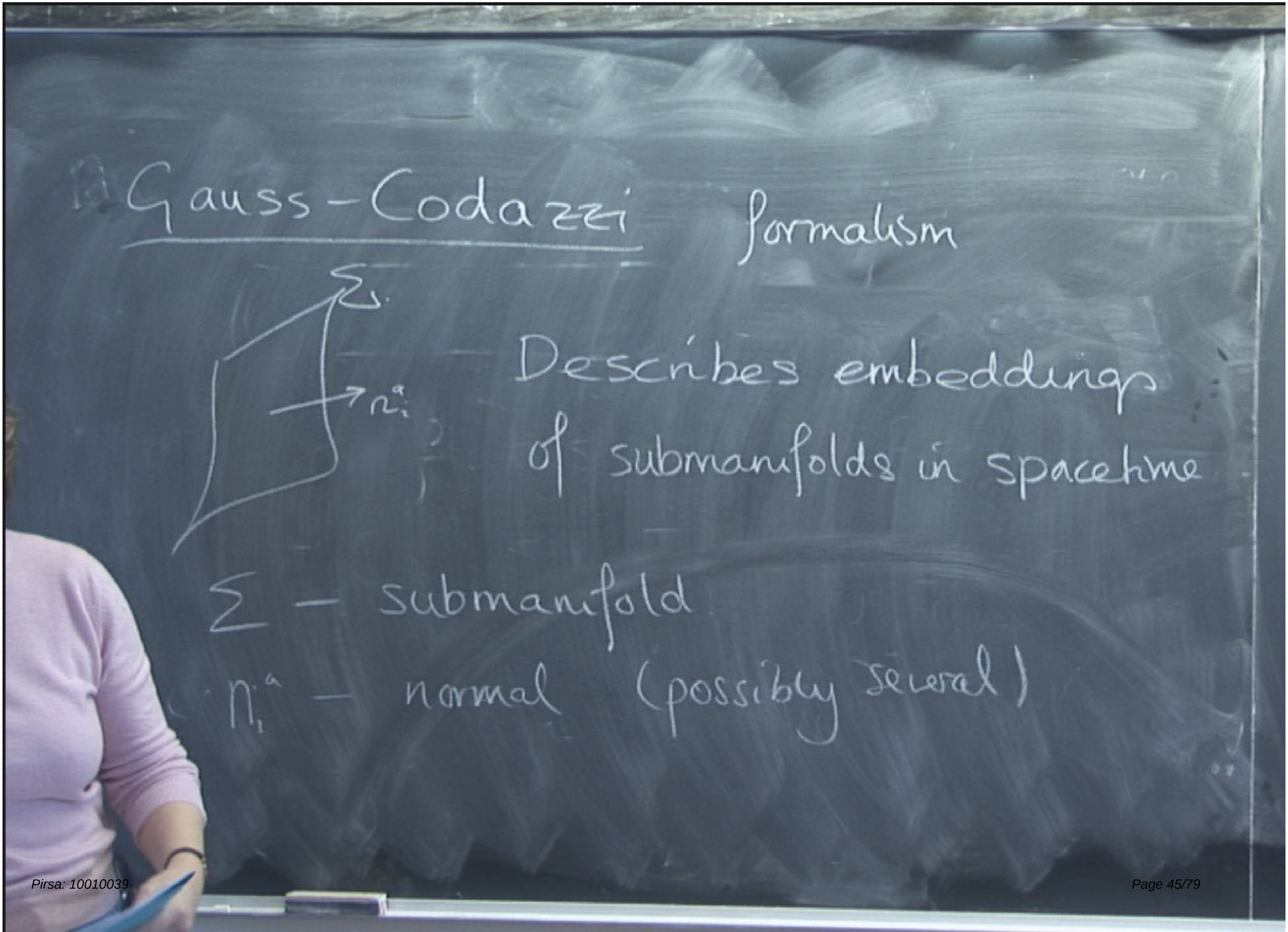
Describes embeddings of submanifolds in spacetime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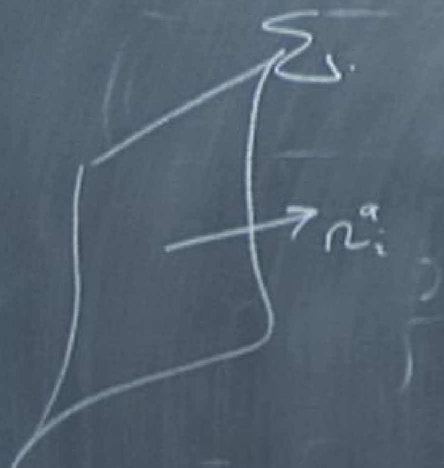

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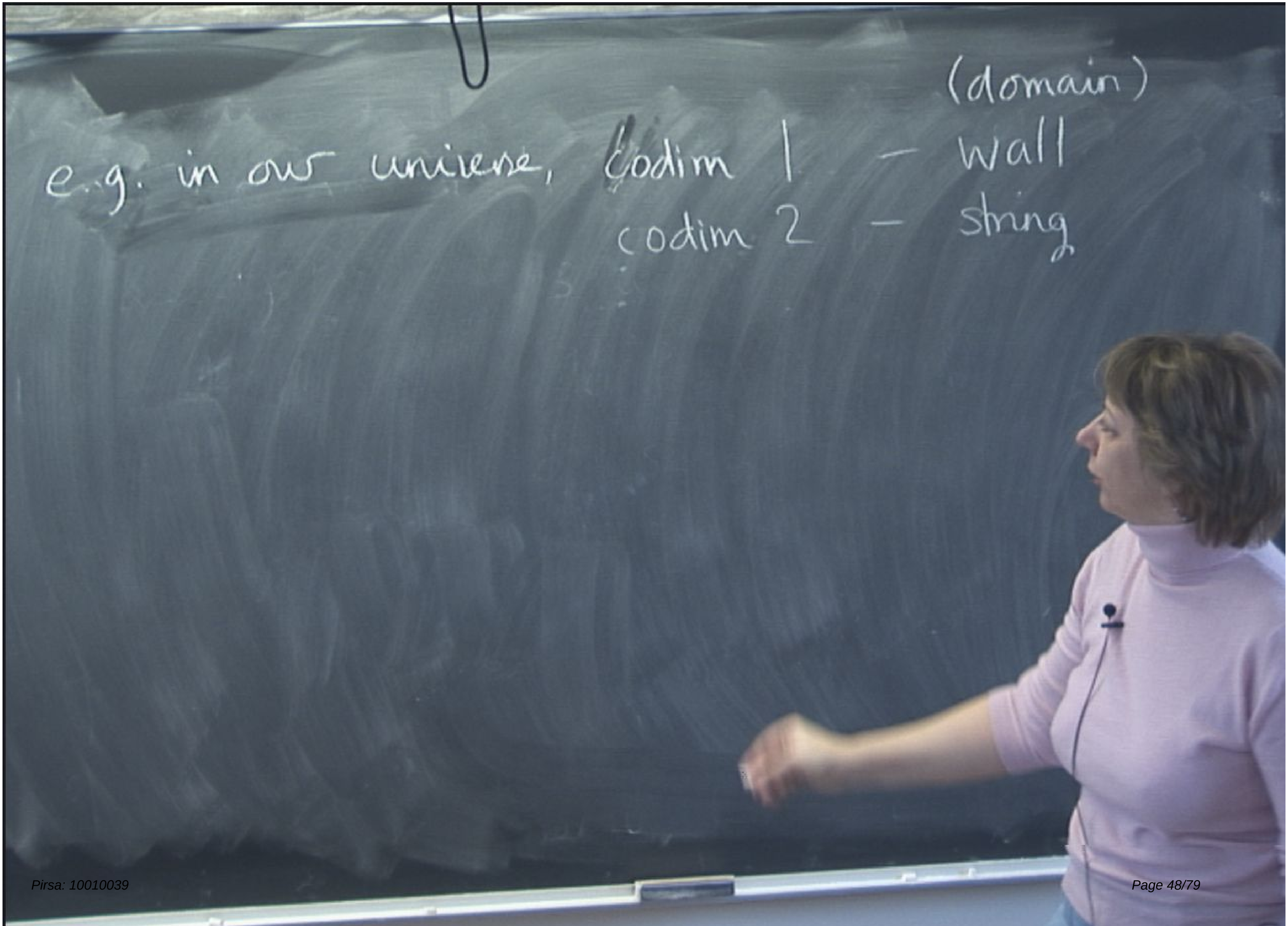
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(will take $n_i^{\mu} n_{j\mu} = 0 \quad i \neq j$)

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$$h_{ab} = g_{ab}$$

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$$g_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^A} \frac{\partial X^\nu}{\partial \sigma^B} = \gamma_{AB} \text{ — intrinsic metric}$$

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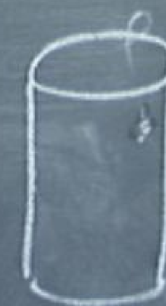

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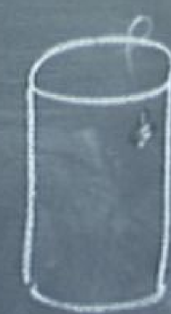

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$$K_{\theta\theta} = r = a$$

Taking (θ, z) coords on cylinder

$$K_{AB} = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \quad \gamma_{AB} = \begin{pmatrix} a^2 & 0 \\ 0 & 1 \end{pmatrix}$$