Title: Gravitational Physics - Review (PHYS 636) - Lecture 8

Date: Jan 12, 2010 10:00 AM

URL: http://pirsa.org/10010038

Abstract:

KERR

$$ds^2 = dt^2 - \frac{\rho^2}{\Delta}\left(dr^2 + \Delta d\theta^2\right) - (r^2+a^2)\sin^2\theta\, d\varphi^2$$

$$- \frac{2GMr}{\rho^2}\left(dt - a\sin^2\theta\, d\varphi\right)^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta.$$

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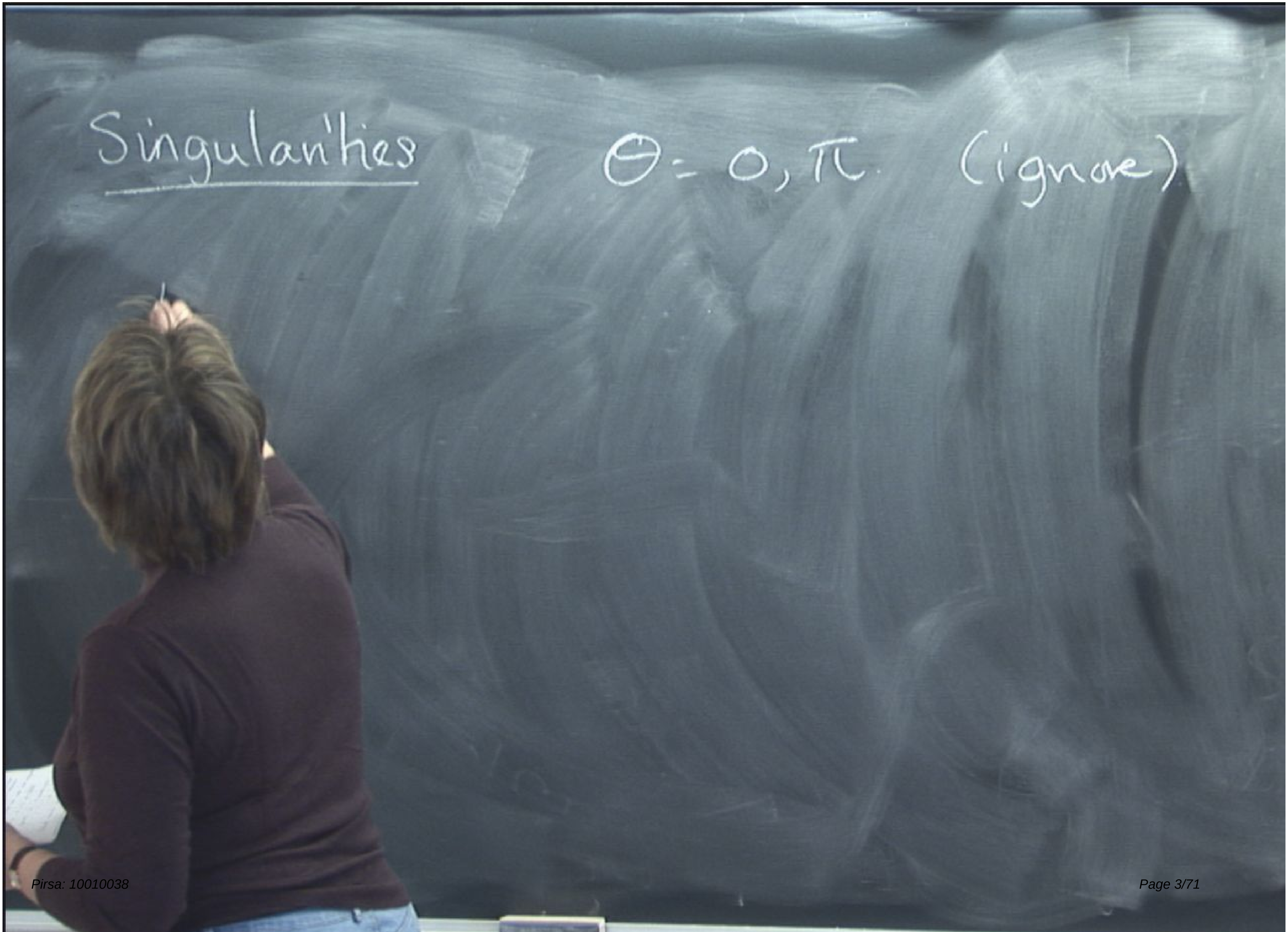
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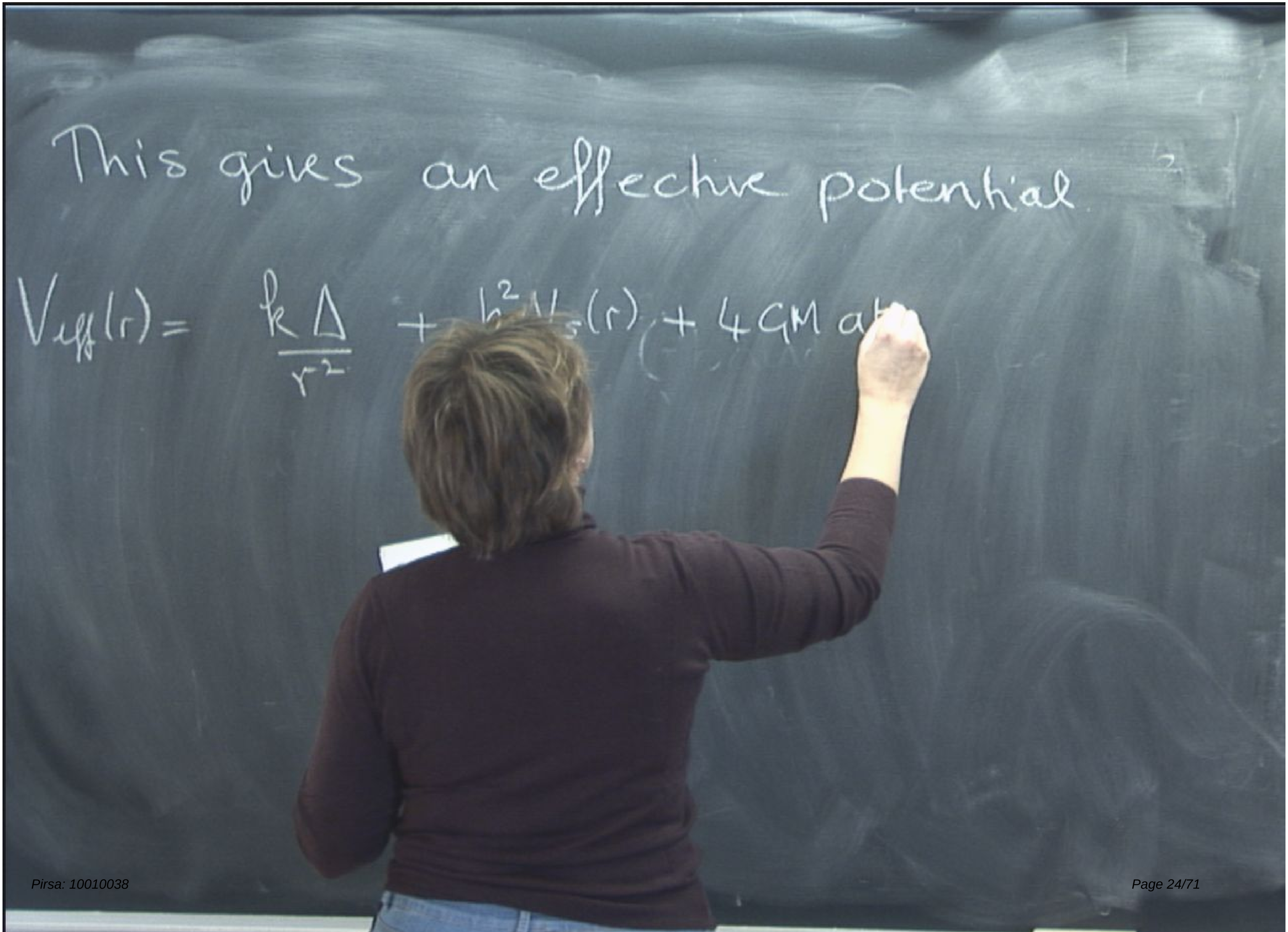
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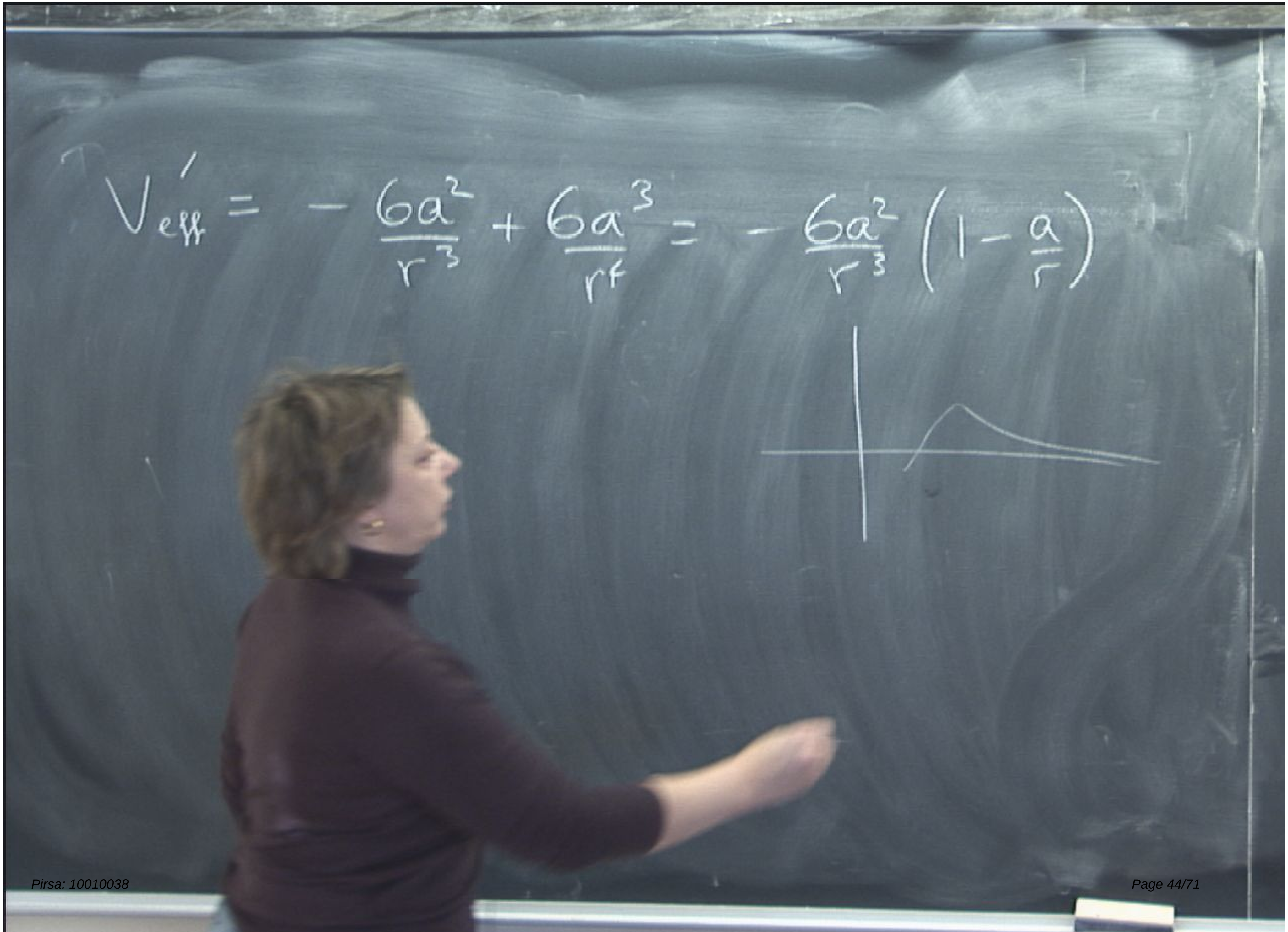
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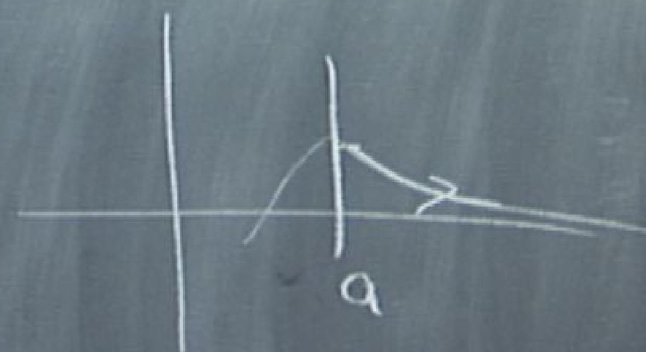

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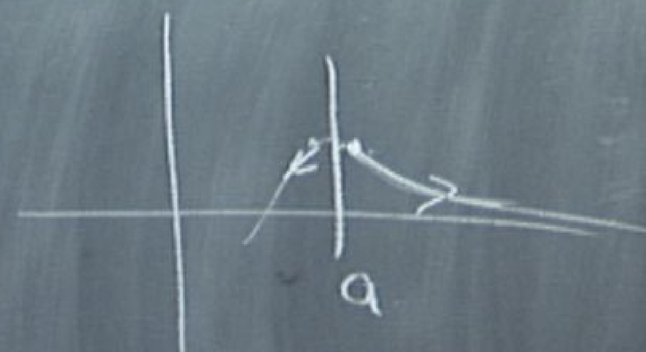

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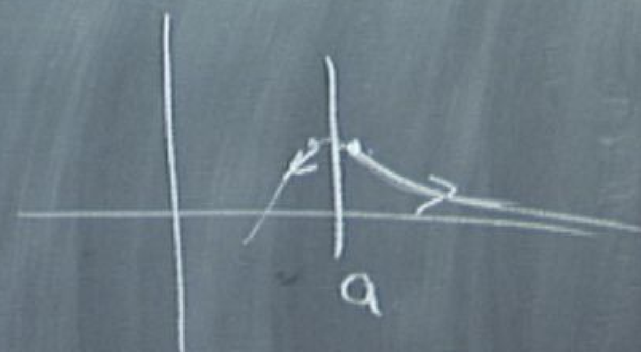
This shows that null geodesics with $r > a$ escape to infinity.



So $r = a$ is the event horizon.

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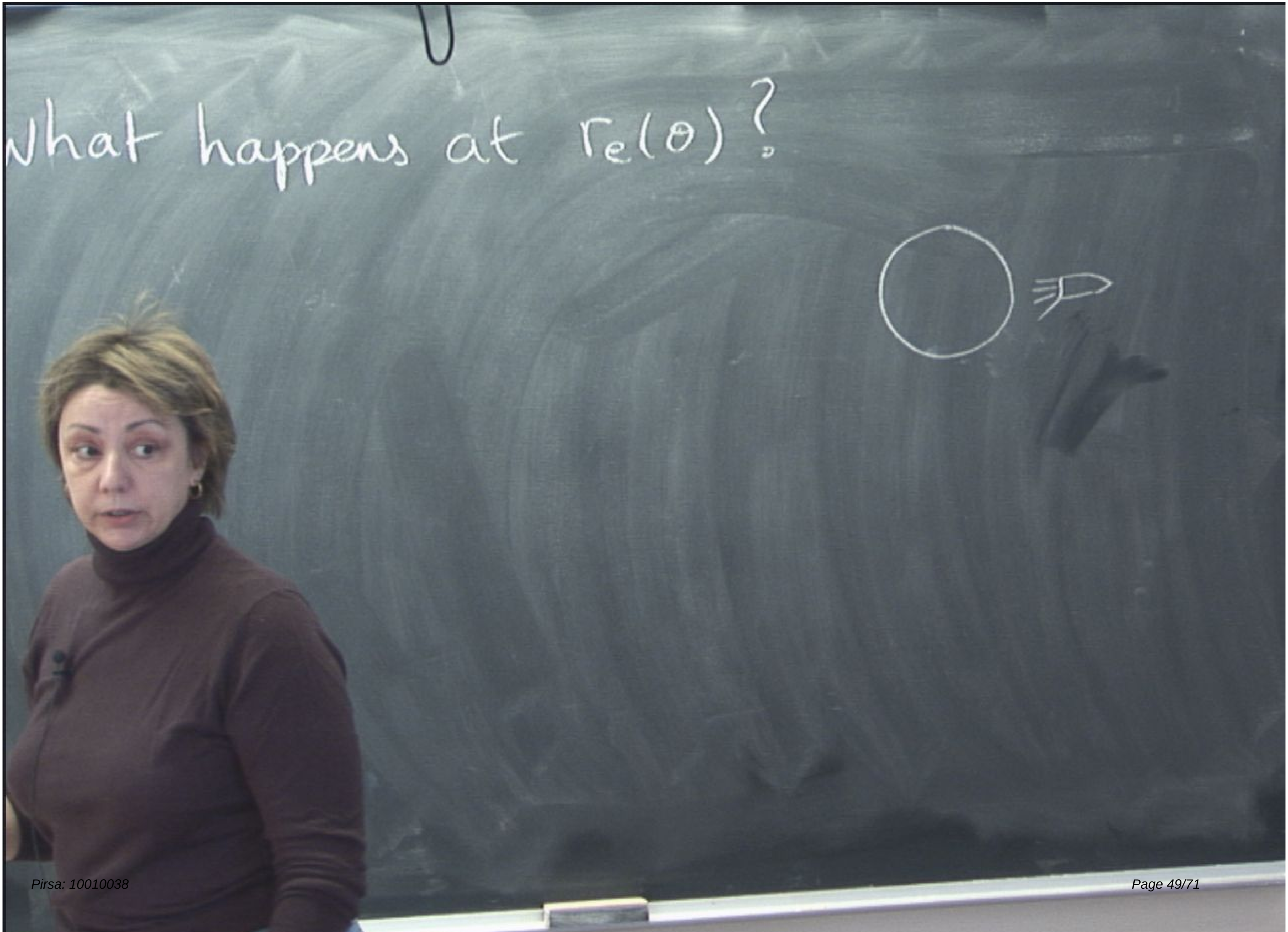
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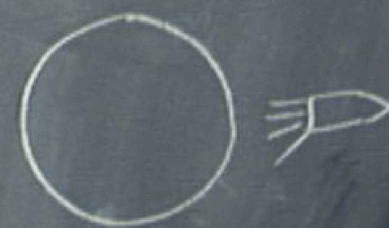

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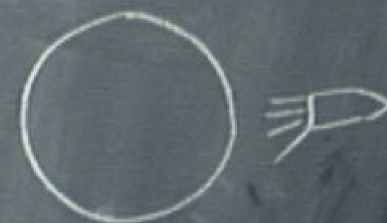

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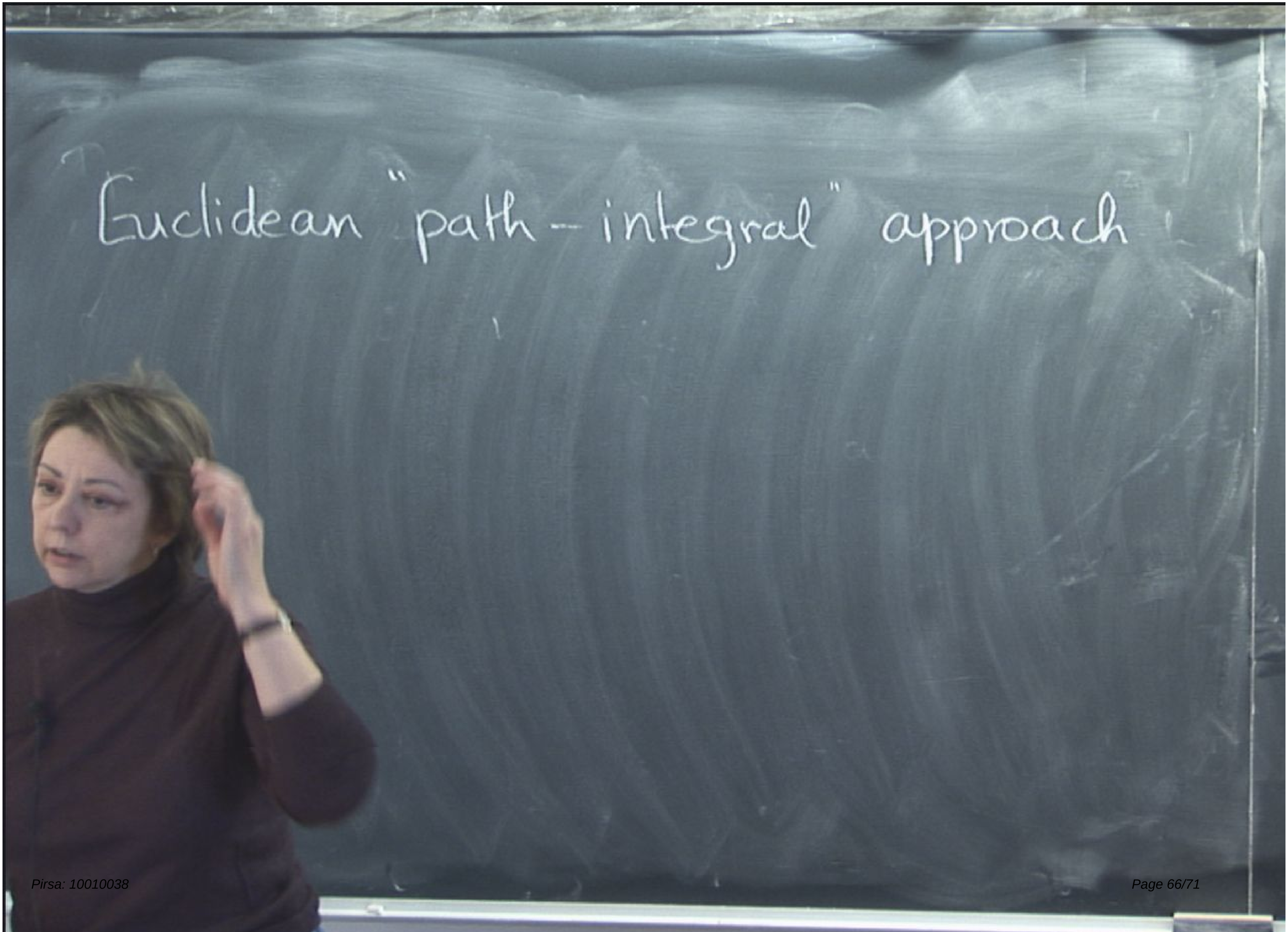
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Writing $\beta = 1/T$, β is the periodicity of Euclidean time

$t \to i\tau$

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