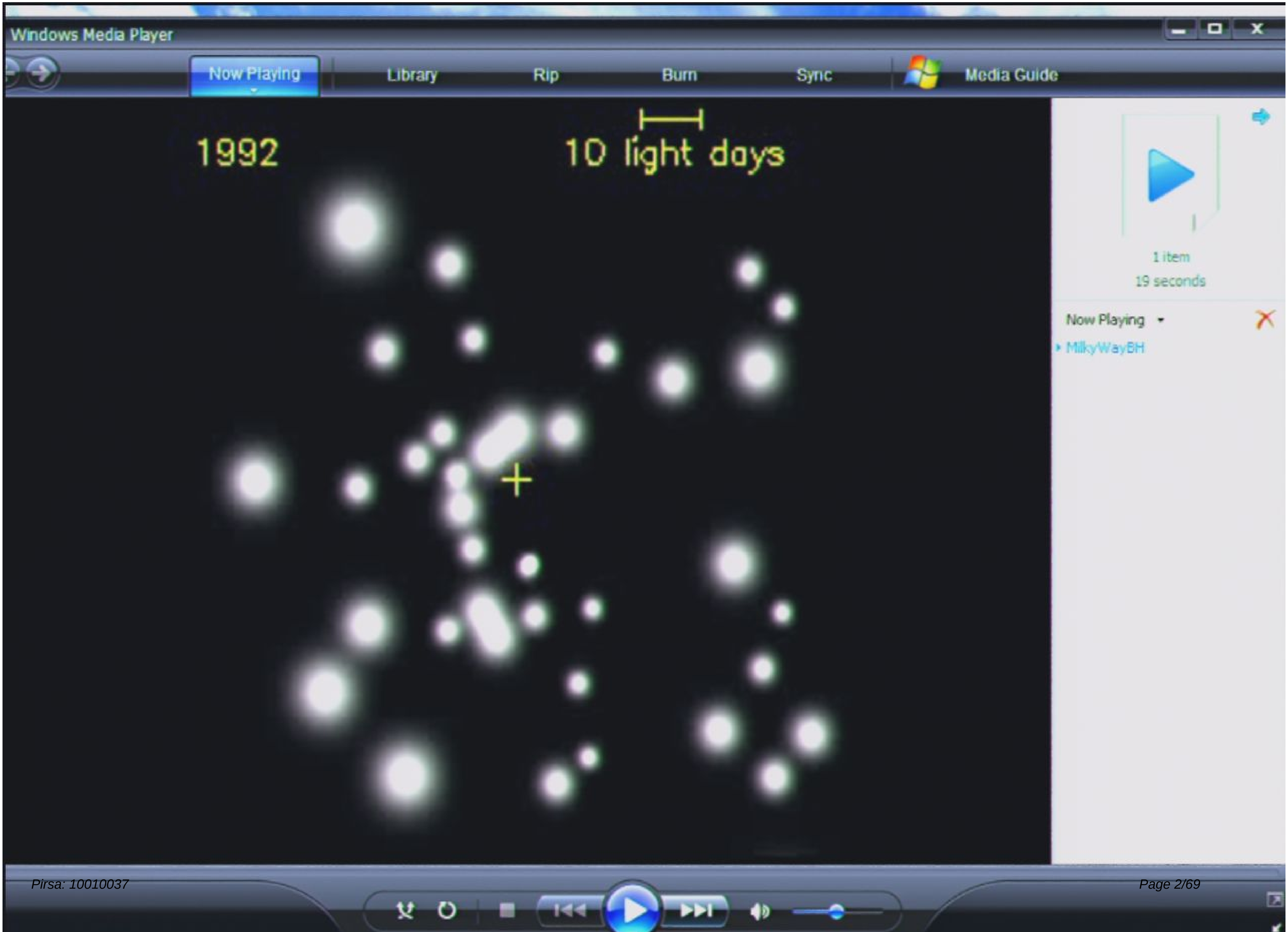
Title: Gravitational Physics - Review (PHYS 636) - Lecture 7

Date: Jan 11, 2010 10:00 AM

URL: http://pirsa.org/10010037

Abstract: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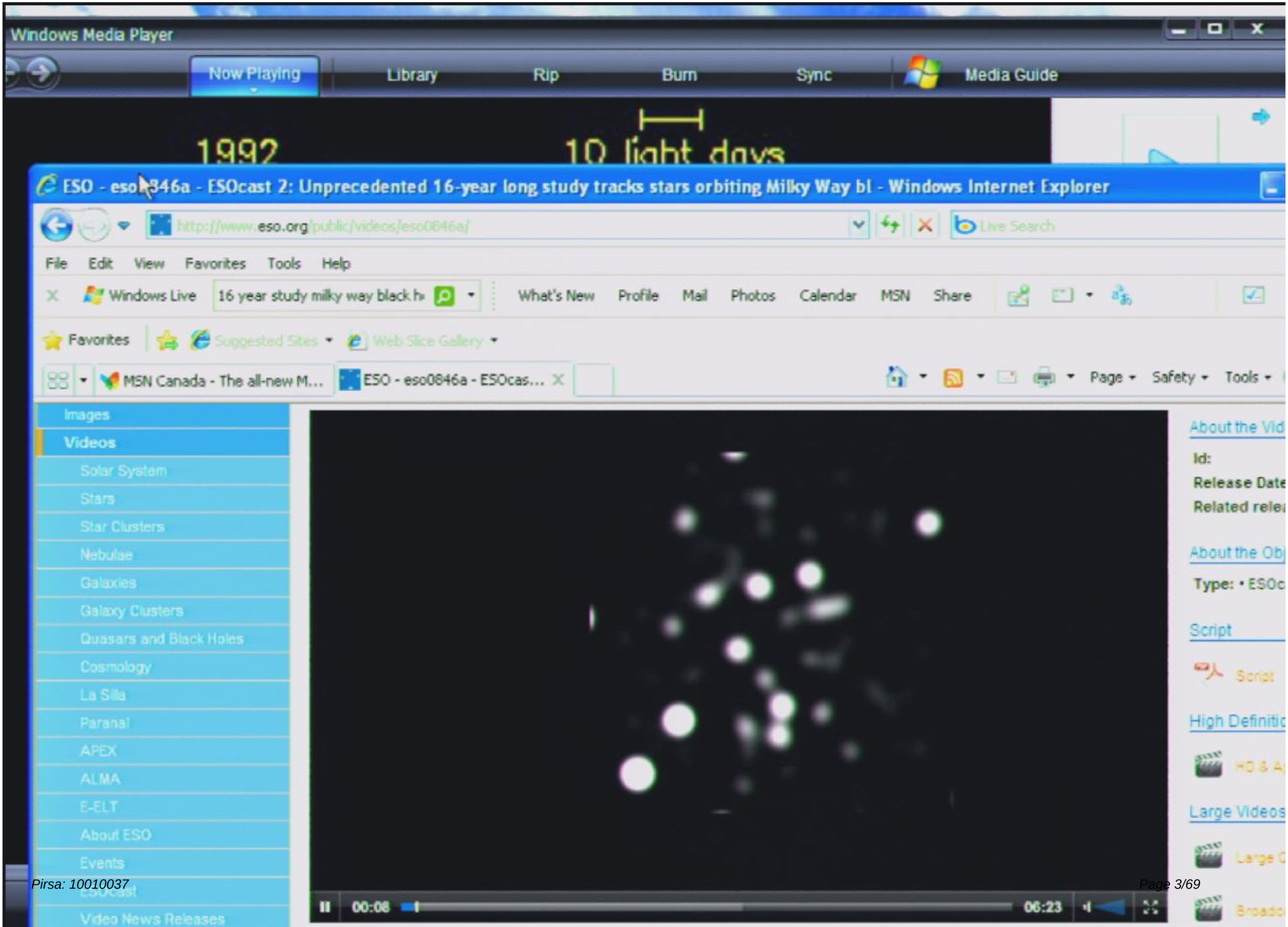
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Galaxy Clusters
Quasars and Black Holes
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$$M(r) = \int 4\pi r^2 T^0_{\ 0}(r)\, dr$$

r-r eqn

$$8\pi G \rho = \frac{2A'}{AB^2 r} + \frac{1}{r^2}\left(B^{-2}-1\right)$$

r-r eqn

$$8\pi G p = \frac{2A'}{AB^2 r} + \frac{1}{r^2}\left(B^{-2} - 1\right)$$

$$\Rightarrow \frac{(A^2)'}{A^2} = \frac{2GM(r) + 8\pi G r^3 p}{r\left(r - 2GM(r)\right)}$$

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solved by:

$$p = \rho_0 \left[\frac{\sqrt{1-\frac{2GM}{R}} - \sqrt{1-\frac{2GMr^2}{R^3}}}{\sqrt{1-\frac{2GMr^2}{R^3}} - 3\sqrt{1-\frac{2GM}{R}}} \right]$$

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– limit to mass of star – Chandrasekhar

$\sim 1.4 M_\odot$

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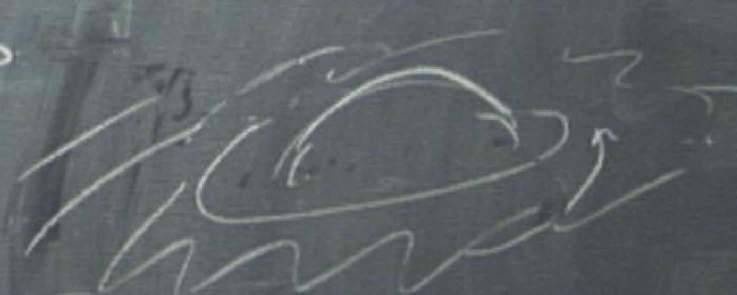

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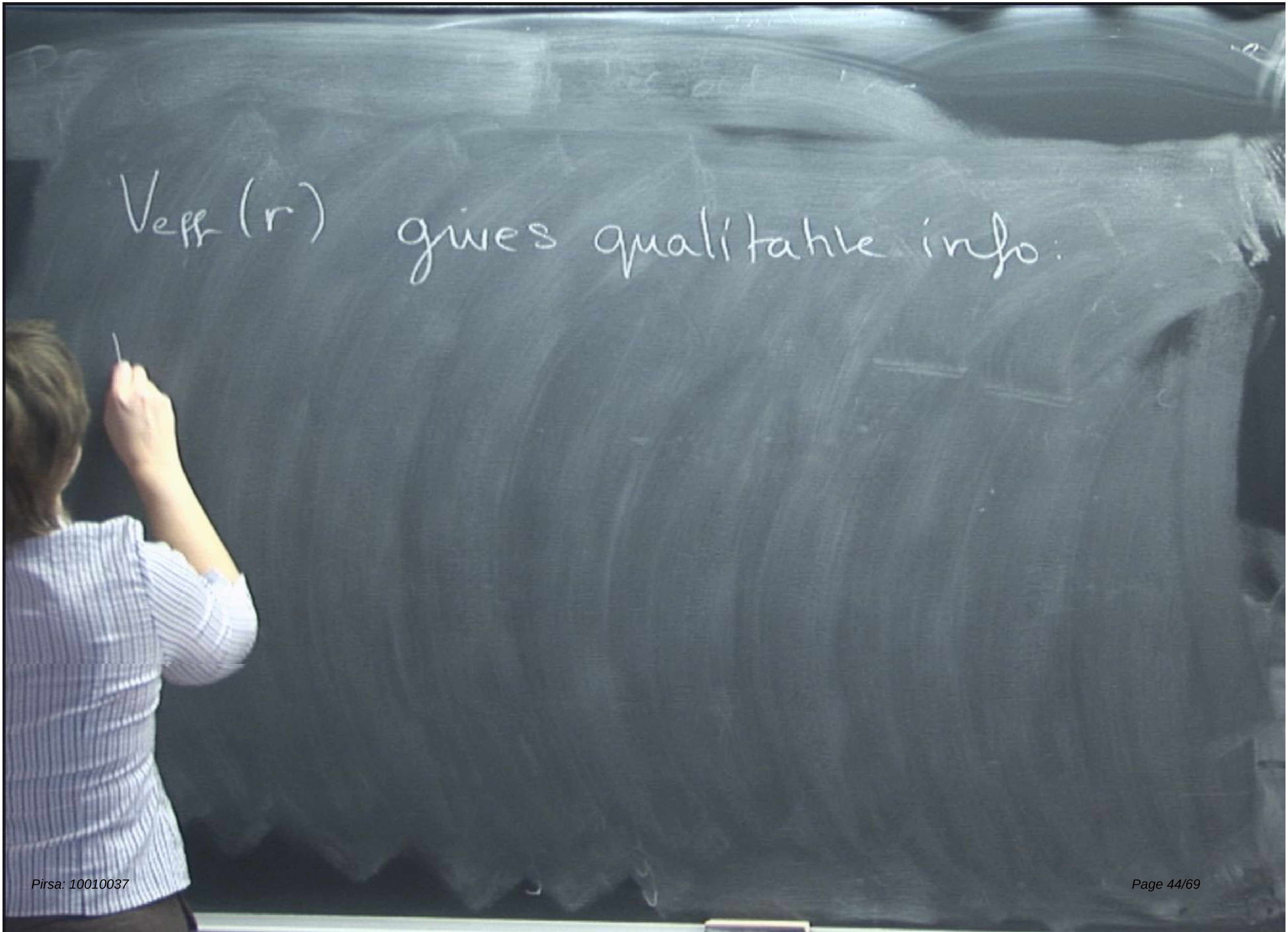
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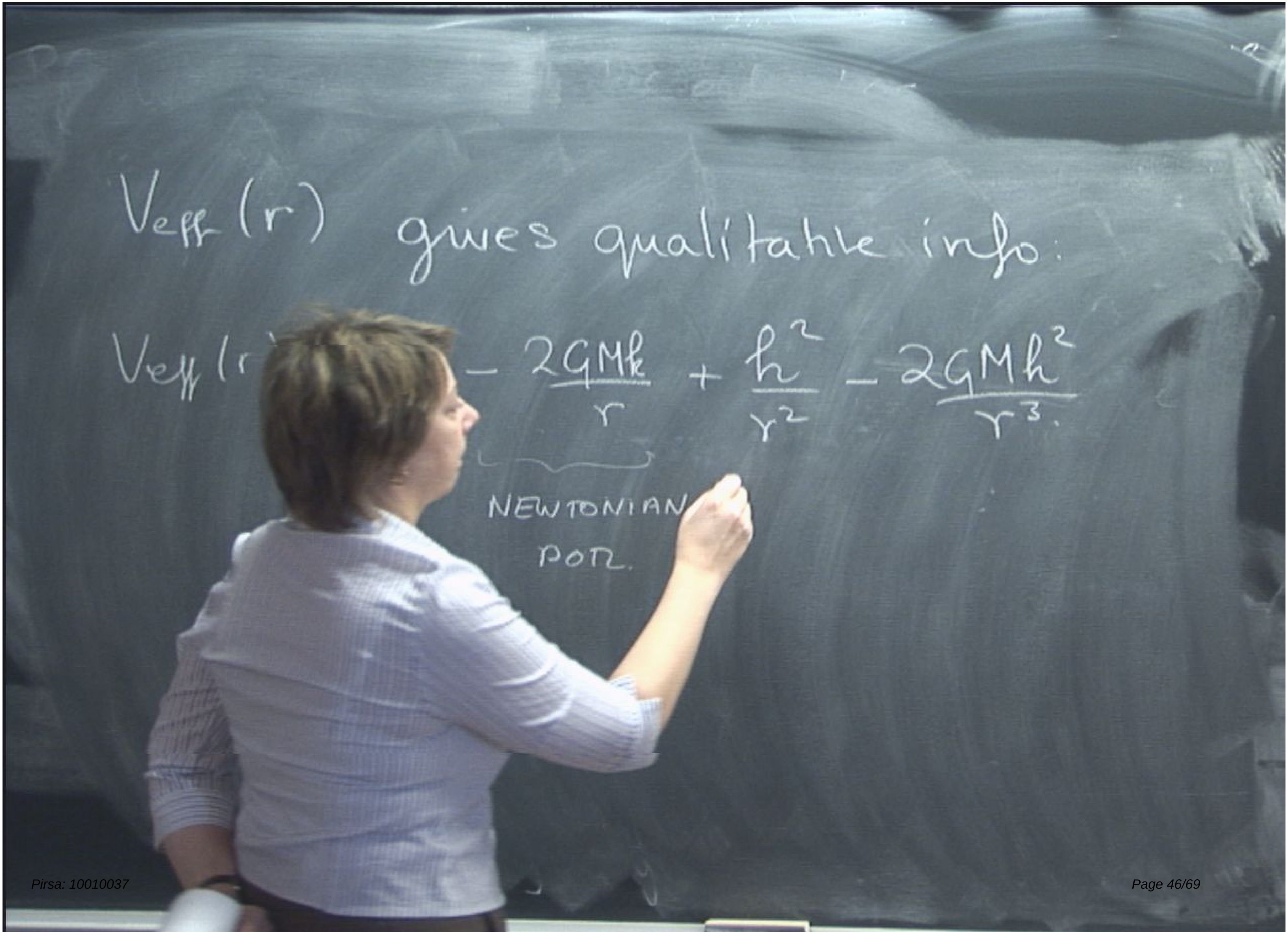
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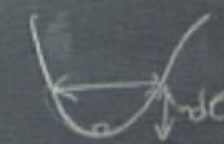

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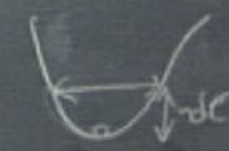

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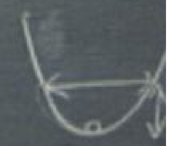

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