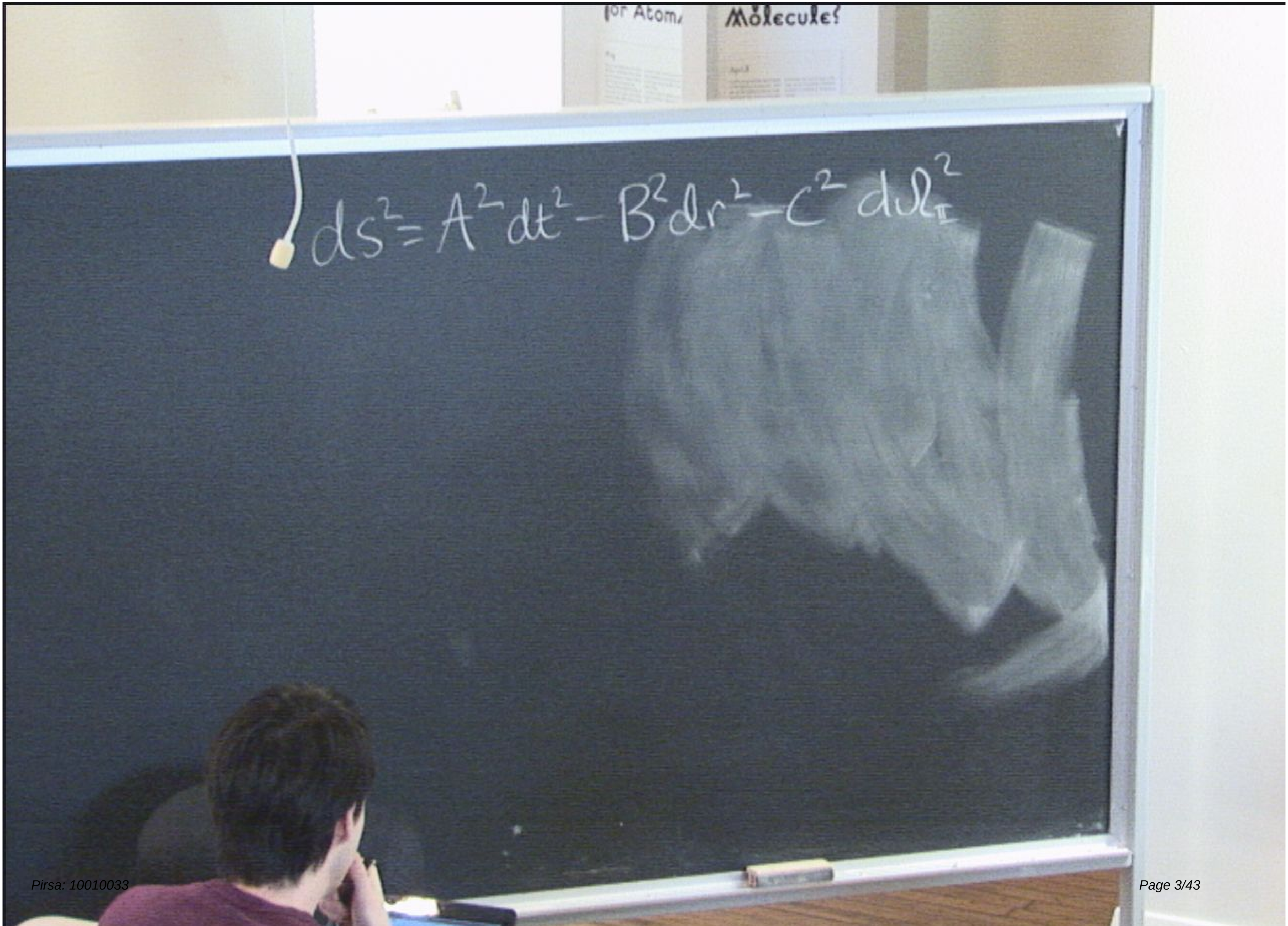
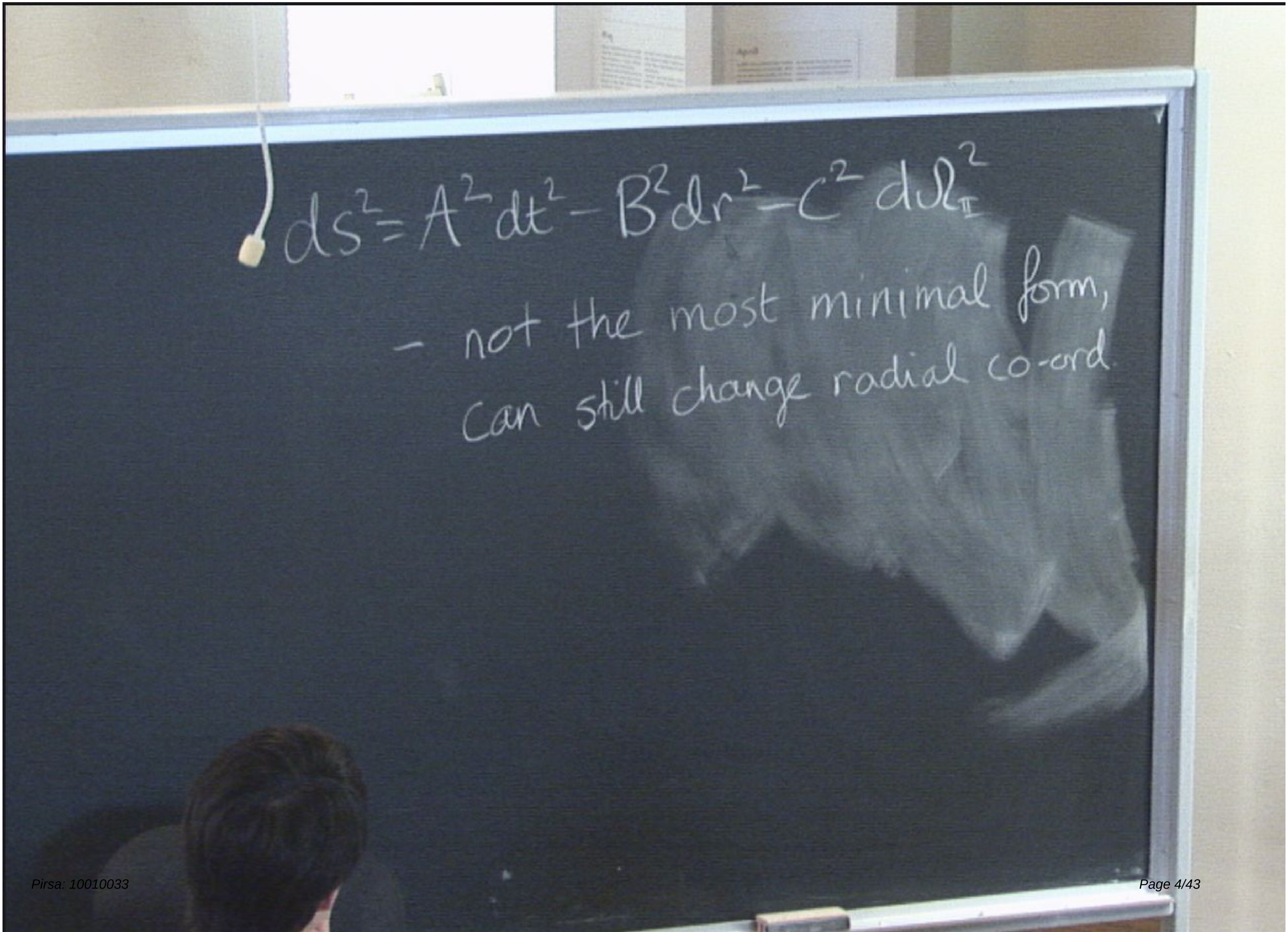
Title: Gravitational Physics - Review (PHYS 636) - Lecture 5

Date: Jan 07, 2010 10:00 AM

URL: http://pirsa.org/10010033

Abstract:


$ds^2 = A^2 dt^2 - B^2 dr^2 - C^2 d\Omega_{II}^2$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can still change radial co-ord.

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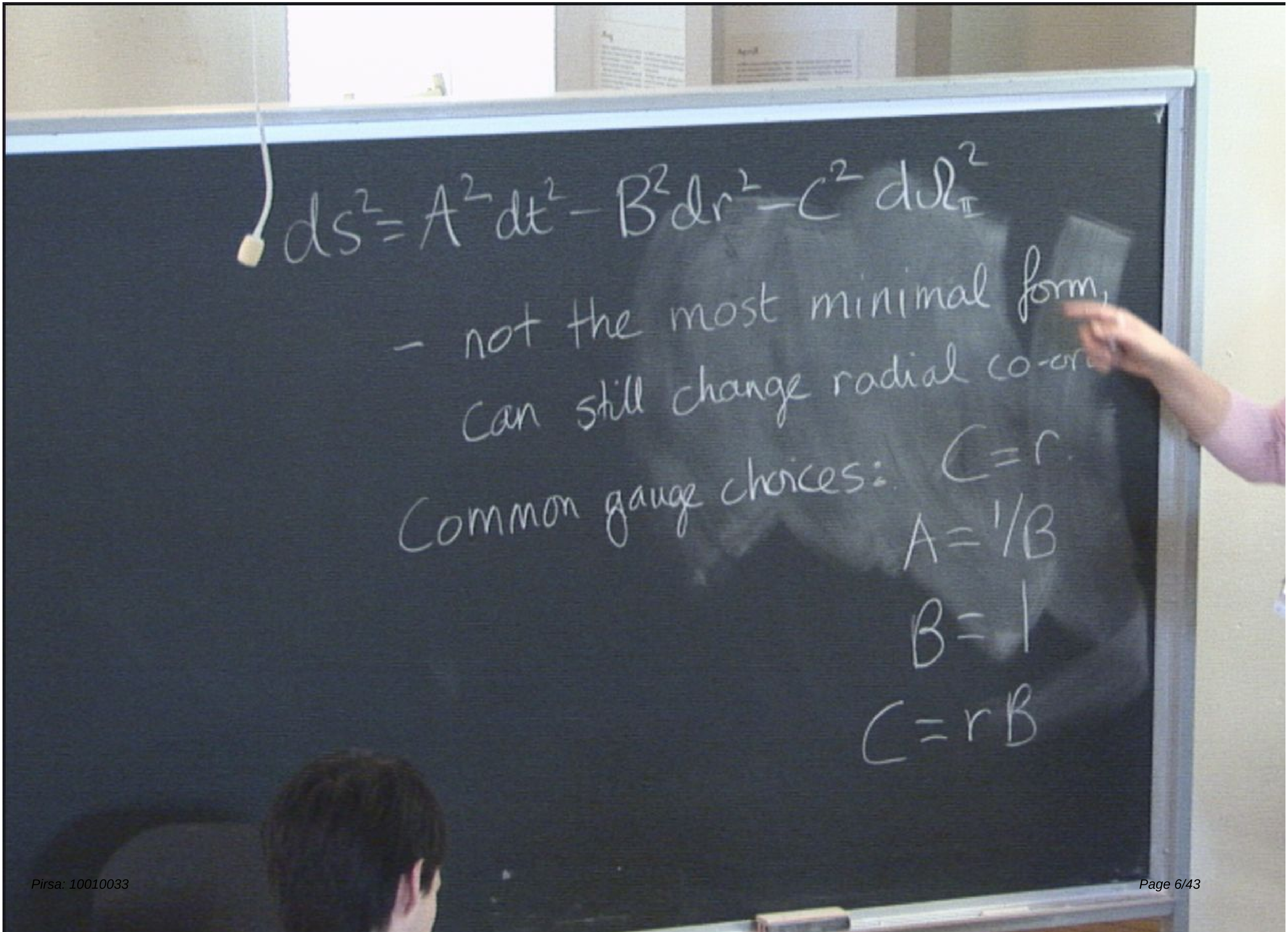
Common gauge choices:


$ds^2 = A^2 dt^2 - B^2 dr^2 - C^2 d\Omega_{II}^2$
- not the most minimal form,
can still change radial co-or
Common gauge choices: $C = r$
$A = 1/B$
$B = 1$
$C = rB$

$$\underline{\underline{R}}^{\hat{t}}{}_{\hat{r}} = -\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)\underline{\omega}^{\hat{t}} \wedge \underline{\omega}^{\hat{r}}$$

$$\underline{\underline{R}}^{\hat{t}}{}_{\hat{\theta}/\hat{\phi}} = -\frac{A'C'}{AB^2C}\,\underline{\omega}^{\hat{t}} \wedge \underline{\omega}^{\hat{\theta}/\hat{\phi}}$$

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$$\underset{\approx}{R}^{a}{}_{b} = \frac{1}{2} R^{a}{}_{bcd}\, \underset{\sim}{\omega}^{c} \wedge \underset{\sim}{\omega}^{d}$$

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Read off:

$$R^{\hat{t}}{}_{\hat{r}\hat{t}\hat{r}} = -\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)$$

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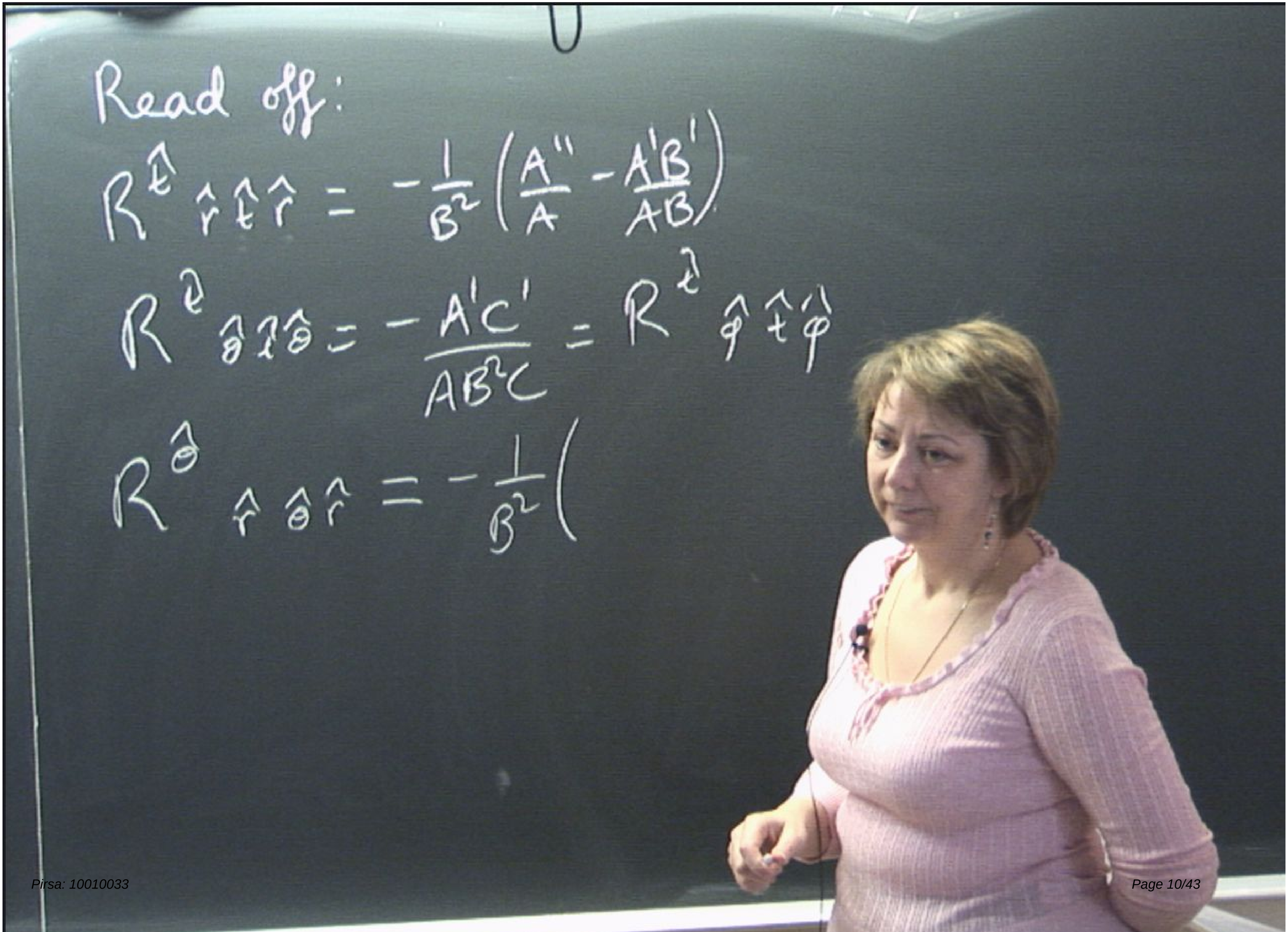
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$$R^{\hat t}{}_{\hat r \hat t \hat r} = -\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)$$

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all other cpts vanish

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These are written in an o/n basis, conventiona[l]

write in co-ord basis.

$\frac{\partial}{\partial}$

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$\frac{\partial}{\partial x^\mu}$, $\underline{d}x^\mu$

↑ vector basis

↑ covector.

$$\underline{\omega}^{\hat a} = e_\mu{}^{a}\,\underline{d}x^\mu$$

$$\underline{\omega}^{\hat t} = A\,\underline{d}t \Rightarrow e_t{}^{\hat t} = A.$$

$$\& \quad e_{\hat t}{}^{t} = 1/A.$$

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i.e. $\underline{e}_{\hat{t}} = \frac{1}{A}\frac{\partial}{\partial t}.$

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$$R^{\hat{t}}{}_{\hat{r}\hat{t}\hat{r}} = -\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right) \rightarrow R^{t}{}_{rtr} = -\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)$$

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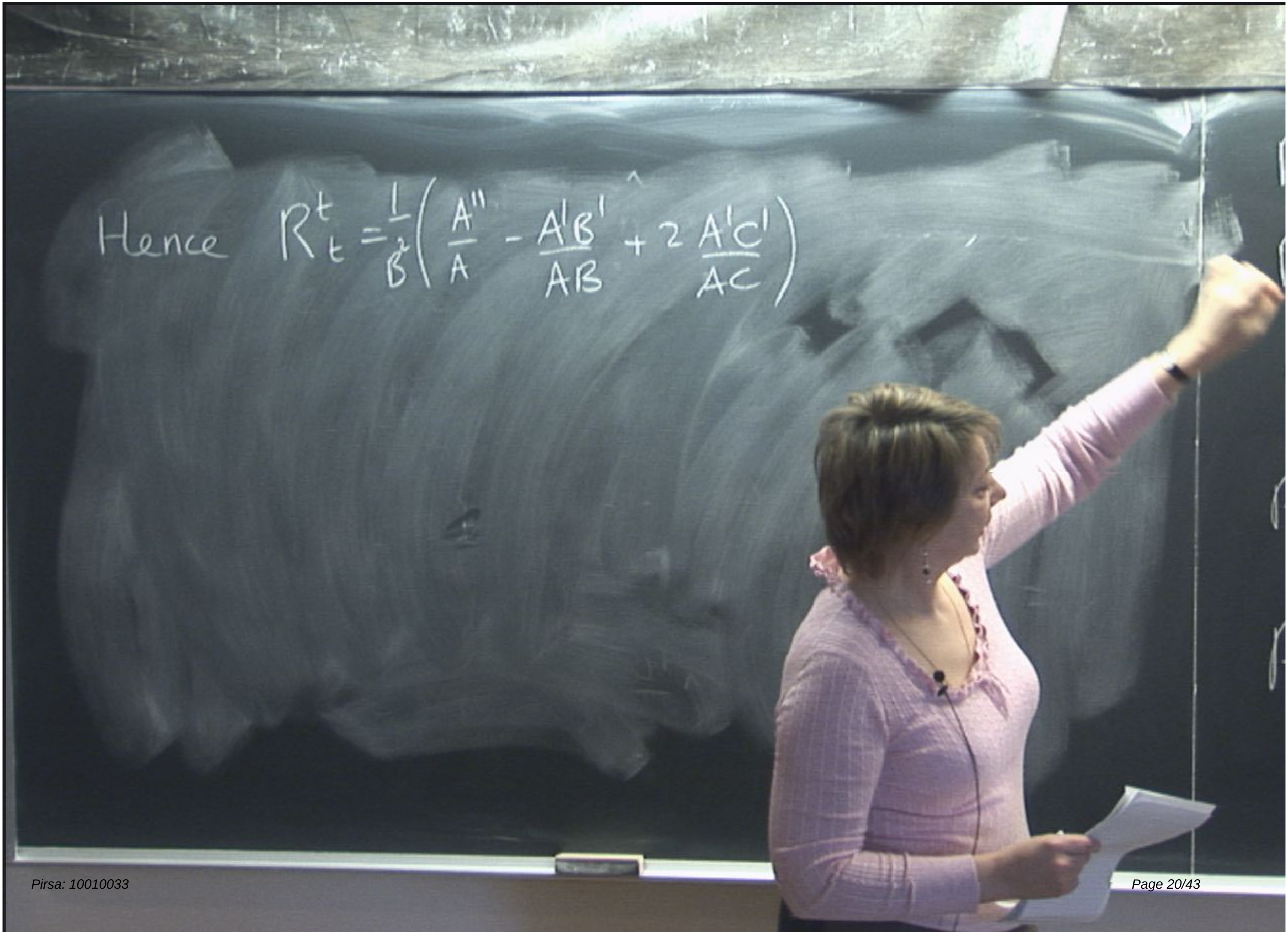
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and

$$R = \frac{2}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB} + \frac{2C''}{C} - \frac{2C'B'}{CB} + \frac{C'^2}{C^2} + 2\frac{A'C'}{AC}\right) - \frac{2}{C^2}$$

$$\Rightarrow \quad G^t_{\ t} = -\frac{1}{B^2}\left(2\frac{C''}{C} - 2\frac{C'B'}{CB} + \frac{C'^2}{C^2}\right) + \frac{1}{C^2}$$

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$$G^\theta_{\ \theta} = -\frac{1}{B^2}\left(\frac{A''}{A} + \frac{C''}{C} - \frac{A'B'}{AB} - \frac{C'B'}{CB} + \frac{A'C'}{AC}\right) = G^\varphi_{\ \varphi}$$

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Now choose a gauge. Looking at $G^0{}_0$ & $G^r{}_r$ suggests $C = r$ might be useful.

$$8\pi G T^0_0 = \frac{2B'}{B^3 r} - \frac{1}{B^2 r^2} + \frac{1}{r^2}$$

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$4\pi r^2 dr\, T^0_0$ = mass between r & $r+dr$.

$$\Rightarrow \quad \frac{r}{B^2} = r - 2GM(r) \rightarrow \frac{1}{B^2} = 1 - \frac{2GM(r)}{r}$$

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Next $\quad 8\pi G(T^0_0 - T^r_r) = \frac{2}{rB^2}\left(\frac{A'}{A} + \frac{B'}{B}\right)$

in vacuum, $T_{ab} = 0$ & we have $A \propto 1/B$.

Therefore, choosing $A \to 1$ at ∞, the metric outside a star is

$$ds^2 = \left(1 - \frac{2GM}{r}\right)dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\Omega_{II}^2.$$

- What happens more generally?

∘ Add a Λ $\quad 8\pi G T_{ab} = \Lambda g_{ab}$

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$$\frac{r}{B^2} = r - \int^r \Lambda r^2 \left(- 2GM(r) \right)$$

$$= r - 2GM - \frac{\Lambda}{3} r^3$$

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Sch-(a)dS.

If $\Lambda > 0$ then 2 horizons

$M \ll 1/\Lambda$ $\quad r_h \sim 2GM$ $\quad r_A \sim \sqrt{3/\Lambda}$

$\underbrace{r_h}$ event horizon of b.h.

$\underbrace{\sqrt{3/\Lambda}}$ acceleration horizon

star is

$$ds^2 = \left(1 - \frac{2GM}{r} - \frac{\Lambda}{3}r^2\right)dt^2 - \left(1 - \frac{2GM}{r} - \frac{\Lambda}{3}r^2\right)^{-1}dr^2 - r^2\,d\Omega_{II}^2.$$

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$M \ll 1/\Lambda$ $\underbrace{r_h \sim 2GM}_{\text{event horizon of b.h.}}$ $\underbrace{r_\Lambda \sim \sqrt{3/\Lambda}}_{\text{acceleration horizon}}$

If $\Lambda < 0$, just have b.h. horizon

anti de-Sitter, hyperbolic (negatively curved)

If we replace $d\Omega$ by dx^2+dy^2, or hyperbolic space.

$\frac{r}{\ }$

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If we replace $d\Omega$ by dx^2+dy^2, or hyperbolic space.

$$\left(\frac{r}{B^2}\right)' = K - 2G\, 4\pi r^2 T^0_0$$

$$\frac{r}{B^2} = K - \frac{2GM}{r} - \frac{\Lambda}{3} r^2$$

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$$\left(\frac{r}{B^2}\right)' = K - 2G\,4\pi r^2 T^0_0$$

$$\frac{1}{B^2} = K - \frac{2GM}{r} - \frac{\Lambda}{3} r^2$$

If $\Lambda \geq 0$ $\qquad \ominus \sim 0 \qquad \ominus \sim 0$

must have $K = 1$

For $\Lambda < 0$, can have $\kappa = 0$ or -1.

$\to$ adS space allows for different topol

of black hole