Title: Gravitational Physics - Review (PHYS 636) - Lecture 4

Date: Jan 06, 2010 10:00 AM

URL: http://pirsa.org/10010032

Abstract:

perimeter scholars international

Lecture 4 More on curvature

Pull together techniques to attack a physical problem: spacetime around a star.

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Pull together techniques to attack a physical problem: spacetime around a star

Simplify the system

Lecture 4 More on curvature

Pull together techniques to attack a physical problem: spacetime around a star

Simplify the system	spherical symmetry
	static

- Killing vector $\frac{\partial}{\partial t}$

$SO(3)$ – Killing algebra

– Killing vector $\frac{\partial}{\partial t}$

$SO(3)$ Killing algebra   3 Killing vectors ack a

d a star

mmetry

– Killing vector $\frac{\partial}{\partial t}$

SO(3) Killing algebra   3 Killing vectors

∂_t a killing vector + static means $\dot{g}_{ab} = 0$

and we can choose the co-ord "t" such that

$g_{ti} = 0$ $(i = 1, 2, 3)$.

$$\xi_1 = \sin\phi\,\partial_\theta + \cot\theta\cos\phi\,\partial_\phi$$

$$\xi_2 = \cos\phi\,\partial_\theta - \cot\theta\sin\phi\,\partial_\phi$$

$$\xi_3 = \partial_\phi$$

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$$\xi_2 = \cos\phi\, \partial_\theta - \cot\theta \sin\phi\, \partial_\phi$$

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or can choose θ, ϕ co-ords

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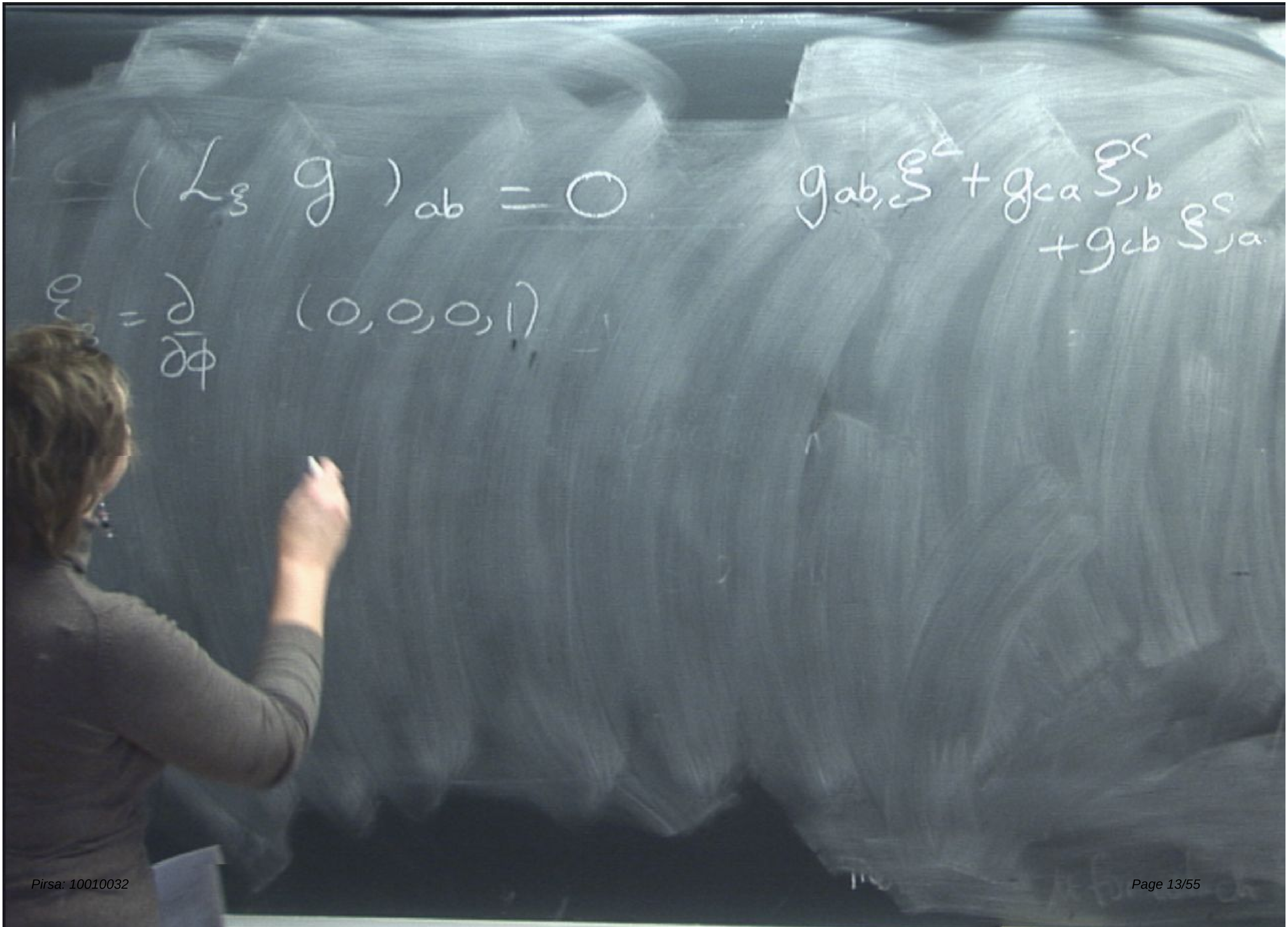
$$\xi_2 = \cos\phi\, \partial_\theta - \cot\theta \sin\phi\, \partial_\phi$$

$$\xi_3 = \partial_\phi$$

or can choose θ, ϕ co-ords, corresponding to the spherical symmetry.

$$(\mathcal{L}_\xi g)_{ab} = 0$$

$$\xi_3 = \frac{\partial}{\partial\phi} \quad (0,0,0,1)$$


$(\mathcal{L}_\xi g)_{ab} = 0$
$g_{ab,c}\xi^c + g_{ca}\xi^c_{,b} + g_{cb}\xi^c_{,a}$
$\xi = \frac{\partial}{\partial\phi}$
$(0,0,0,1)$

$$(\mathcal{L}_\xi g)_{ab} = 0 \qquad g_{ab,c}\xi^c + g_{ca}\xi^c_{,b} + g_{cb}\xi^c_{,a}$$

$$\xi_3 = \frac{\partial}{\partial\phi} \qquad (0,0,0,1)$$

$$\Rightarrow \quad g_{ab,\phi} = 0$$

$$\mathcal{L}_1 g_{ab} =$$

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$$\Rightarrow \quad g_{ab,\phi} = 0$$

$$\mathcal{L}_1 g_{ab} = \sin\phi\, g_{ab,\theta} + \xi^c_{1,a}\, g_{cb} + a \leftrightarrow b$$

$$= \sin\phi\, g_{ab,\theta} + \xi^\theta_{1,a}\, g_{\theta b} + \xi^\phi_{1,a}\, g_{\phi b} + a \leftrightarrow b$$

$$= \sin\phi\, g_{ab,\theta} + \cos\phi\, \delta^\phi_a\, g_{\theta b} + (-\csc^2\theta\cos\phi\,\delta^\theta_a - \cot\theta\sin\phi\,\delta^\phi_a)\, g_{\phi b} + a \leftrightarrow b$$

$$(\mathcal{L}_\xi g)_{ab} = 0 \qquad g_{ab,c}\xi^c + g_{ca}\xi^c_{,b} + g_{cb}\xi^c_{,a}$$

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taking $a,b = t$ or r gives $g_{ab,\theta} = 0$

$a = t/r$, $b = \phi$:

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$a = t/r,\ b = \phi$: $\sin\phi\, g_{t\phi,\theta} + \cos\phi\, g_{\theta t} - \cot\theta \sin\phi\, g_{\phi t} = 0$

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$$- \cot\theta \sin\phi\, g_{\phi t} = 0$$

since g is indep of ϕ $\quad g_{\theta t/r} = 0$

a

taking $a,b = t$ or r gives $g_{ab,\theta} = 0$

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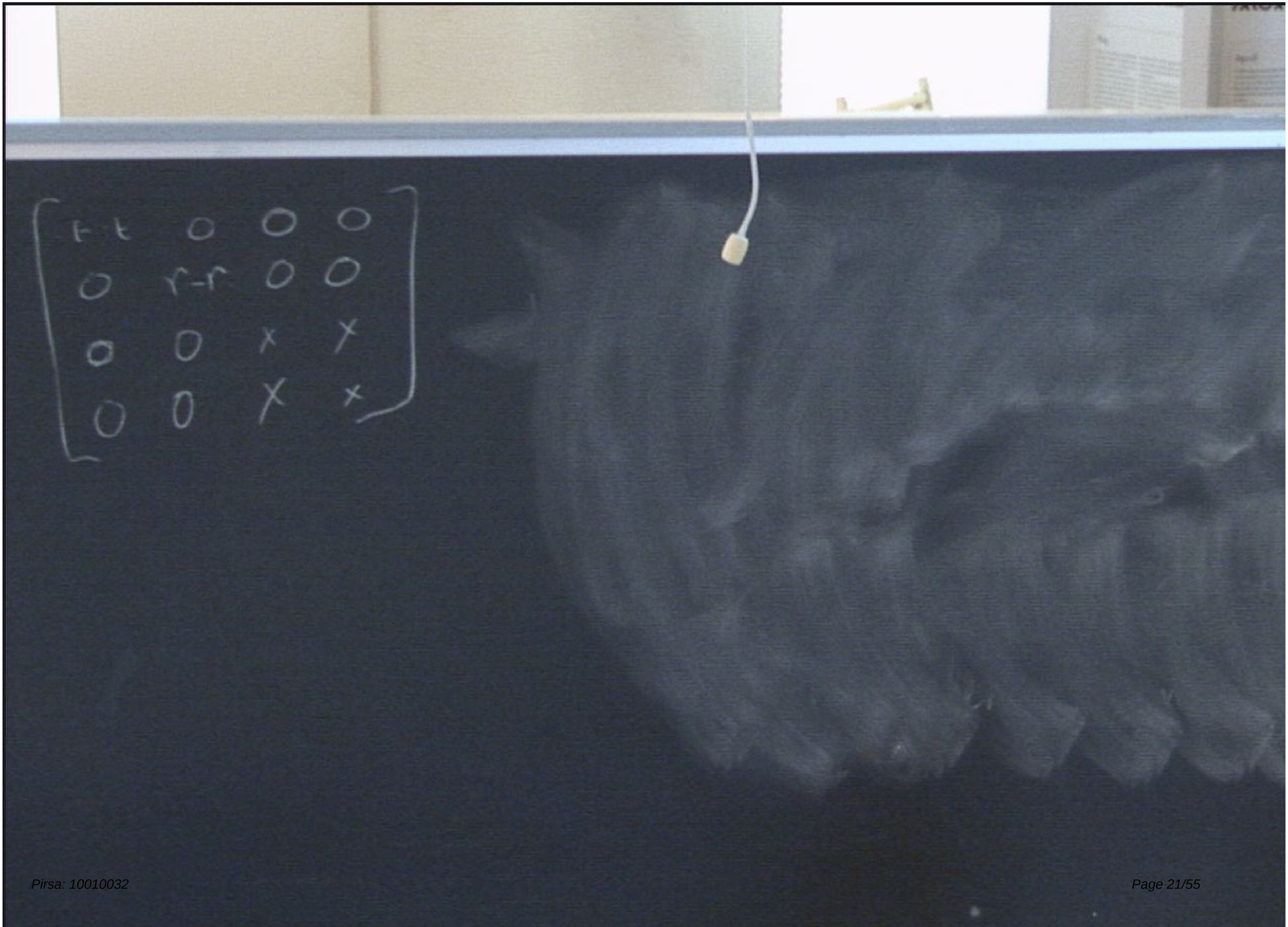
since g is indep of ϕ $\quad g_{\theta t/r} = 0$

$a = t/r,\ b = \theta$: $g_{t\theta,\theta} + \csc^2\theta \cos\phi\, g_{\phi t} = 0 \Rightarrow g_{\phi t/r}$

$+ a_c g_b$

$g_{\phi b}$


t-t 0 0 0
0 r-r 0 0
0 0 x x
0 0 x x

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$a = t/r,\ b = \theta$: $g_{t\theta,\theta} - \csc^2\theta \cos\phi\, g_{\phi t} = 0$

$a = \theta\ \ b = \theta$: $\sin\phi\, g_{\theta\theta,\theta} - \csc^2\theta \cos\phi\, g_{\phi\theta} = 0$

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Since g is indep of ϕ $\quad g_{\theta t/r} = 0$

$= t/r,\ b=\theta$: $g_{t\theta,\theta} - \csc^2\theta \cos\phi\, g_{\phi t} = 0 \Rightarrow g_{\phi t/r} = 0$

$a=\theta\ b=\theta$: $\sin\phi\, g_{\theta\theta,\theta} - \csc^2\theta\cos\phi\, g_{\phi\theta} = 0$

$a=\theta\ b=\phi$: $\sin\phi\, g_{\phi\theta,\theta} + \cos\phi\, g_{\theta\theta} - \csc^2\theta \cos\phi$

$a=\phi\ b=\phi$: $\sin\phi\, g_{\phi\phi,\theta} + 2\cos\phi\, g_{\phi\theta} - 2\cot\theta \sin\phi\, g_{\phi\phi} = 0$

taking a,b = t or r gives $g_{ab,\theta} = 0$

$a = t/r$, $b = \phi$: $\sin\phi\, g_{t\phi,\theta} + \cos\phi\, g_{\theta t} - \cot\theta\sin\phi\, g_{\phi t} = 0$

Since g is indep of ϕ $\quad g_{\theta t/r} = 0$

$a = t/r$, $b = \theta$: $g_{t\theta,\theta} - \csc^2\theta\cos\phi\, g_{\phi t} = 0 \Rightarrow$

$a = \theta \quad b = \theta$: $\sin\phi\, g_{\theta\theta,\theta} - \csc^2\theta\cos\phi\, g_{\phi\theta} = 0$

$a = \theta \quad b = \phi$: $\sin\phi\, g_{\phi\theta,\theta} + \cos\phi\, g_{\theta\theta} - \csc^2\theta\cos\phi\, g_{\phi\phi} - \cot\theta$

$a = \phi \quad b = \phi$: $\sin\phi\, g_{\phi\phi,\theta} + 2\cos\phi\, g_{\phi\theta} - 2\cot\theta\sin\phi\, g_{\phi\phi} =$

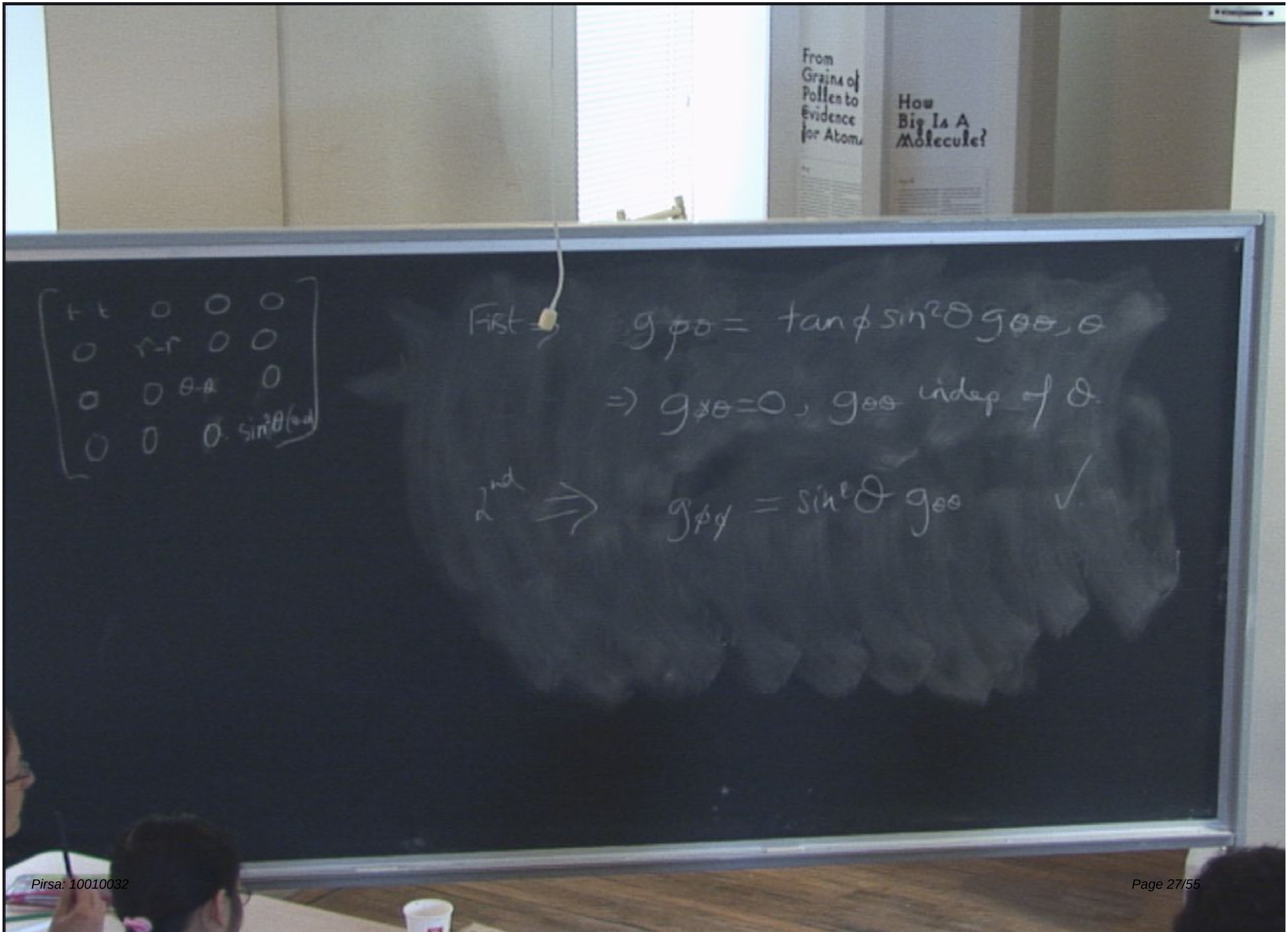
First $\Rightarrow$ $g_{\phi\theta} = \tan\phi \sin^2\theta \, g_{\theta\theta,\theta}$

$\Rightarrow g_{\phi\theta} = 0, \; g_{\theta\theta}$ indep of θ.

First $\Rightarrow$ $g_{\phi\theta} = \tan\phi \sin^2\theta \, g_{\theta\theta,\theta}$

$\Rightarrow$ $g_{\phi\theta} = 0$, $g_{\theta\theta}$ indep of θ.

2nd $\Rightarrow$ $g_{\phi\phi} = \sin^2\theta \, g_{\theta\theta}$


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
First ⇒ gφθ = tanφ sin²θ gθθ,θ
⇒ gφθ=0, gθθ indep of θ.
2nd ⇒ gφφ = sin²θ gθθ ✓

Hence we may write the metric as:

$$ds^2 = A^2(r)\,dt^2 - B^2(r)\,dr^2 - C^2(r)\underbrace{\left\{d\theta^2 + \sin^2\theta\, d\right.}_{d\Omega_{II}^2}$$

Now use

Hence we may write the metric as:

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Now use Cartan formalism

$$\underline{\omega}^{\hat{t}} = A\,\underline{d}t \qquad \underline{\omega}^{\hat{\theta}} = C\,\underline{d}\theta$$

$$\underline{\omega}^{\hat{r}} = B\,\underline{d}r \qquad \underline{\omega}^{\hat{\phi}} = C\sin\theta\,\underline{d}\phi$$

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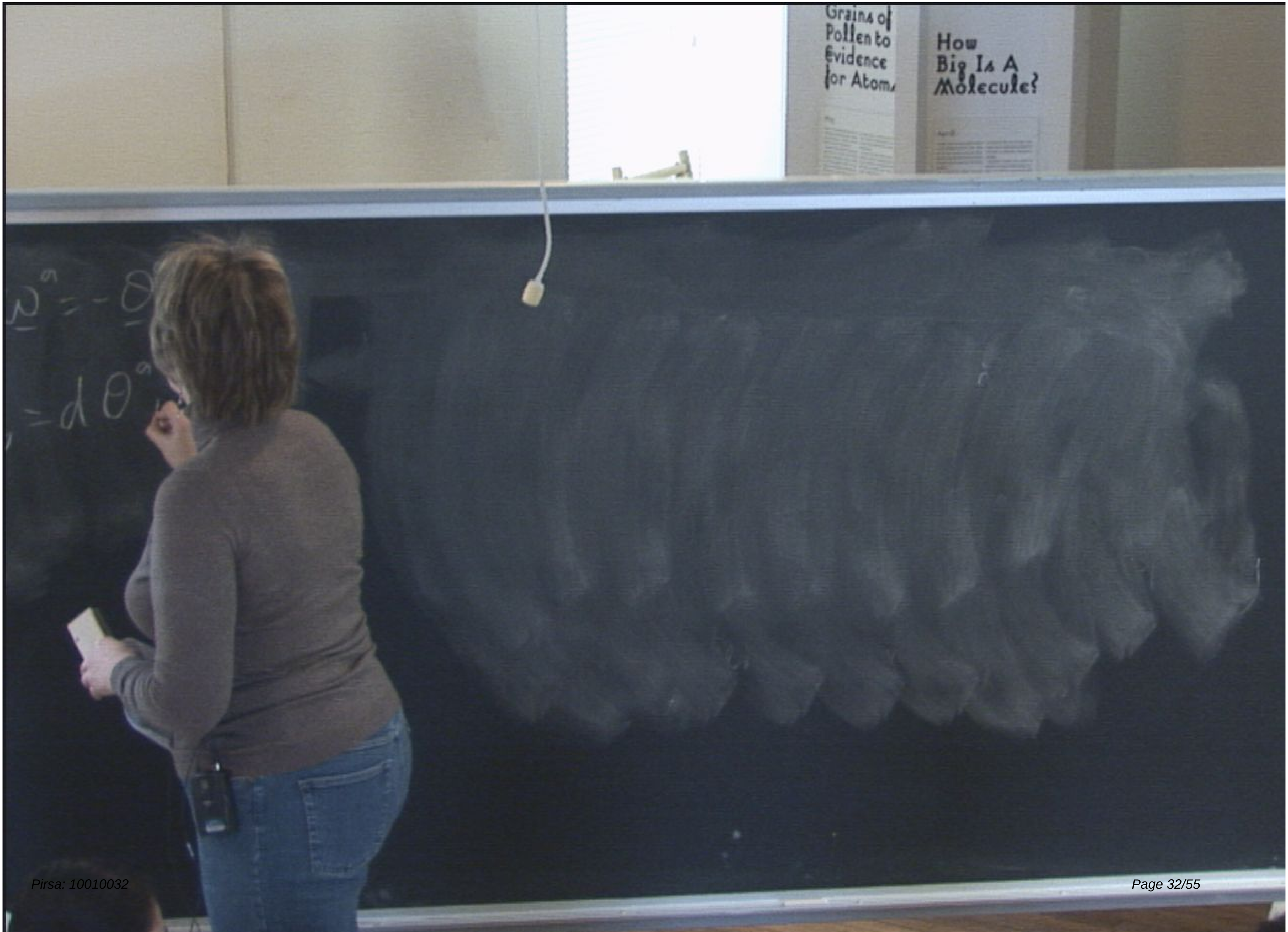
Then: $d\omega^{\hat{t}} = A' \, dr \wedge dt = -\frac{A'}{AB} \, \omega^{\hat{t}} \wedge \omega^{\hat{r}}$

$d\omega^{\hat{r}} = 0$

$d\omega^{\hat{\theta}} = C' \, dr \wedge d\theta = -\frac{C'}{CB} \, \omega^{\hat{\theta}} \wedge \omega^{\hat{r}}$

$d\omega^{\hat{\varphi}} = C' \, dr \wedge \sin\theta \, d\varphi + C \cos\theta \, d\theta \wedge d\varphi$

$= -\frac{C'}{CB} \, \omega^{\hat{\varphi}} \wedge \omega^{\hat{r}} - \frac{\cot\theta}{C} \, \omega^{\hat{\varphi}} \wedge \omega^{\hat{\theta}}$


Grains of Pollen to Evidence for Atoms
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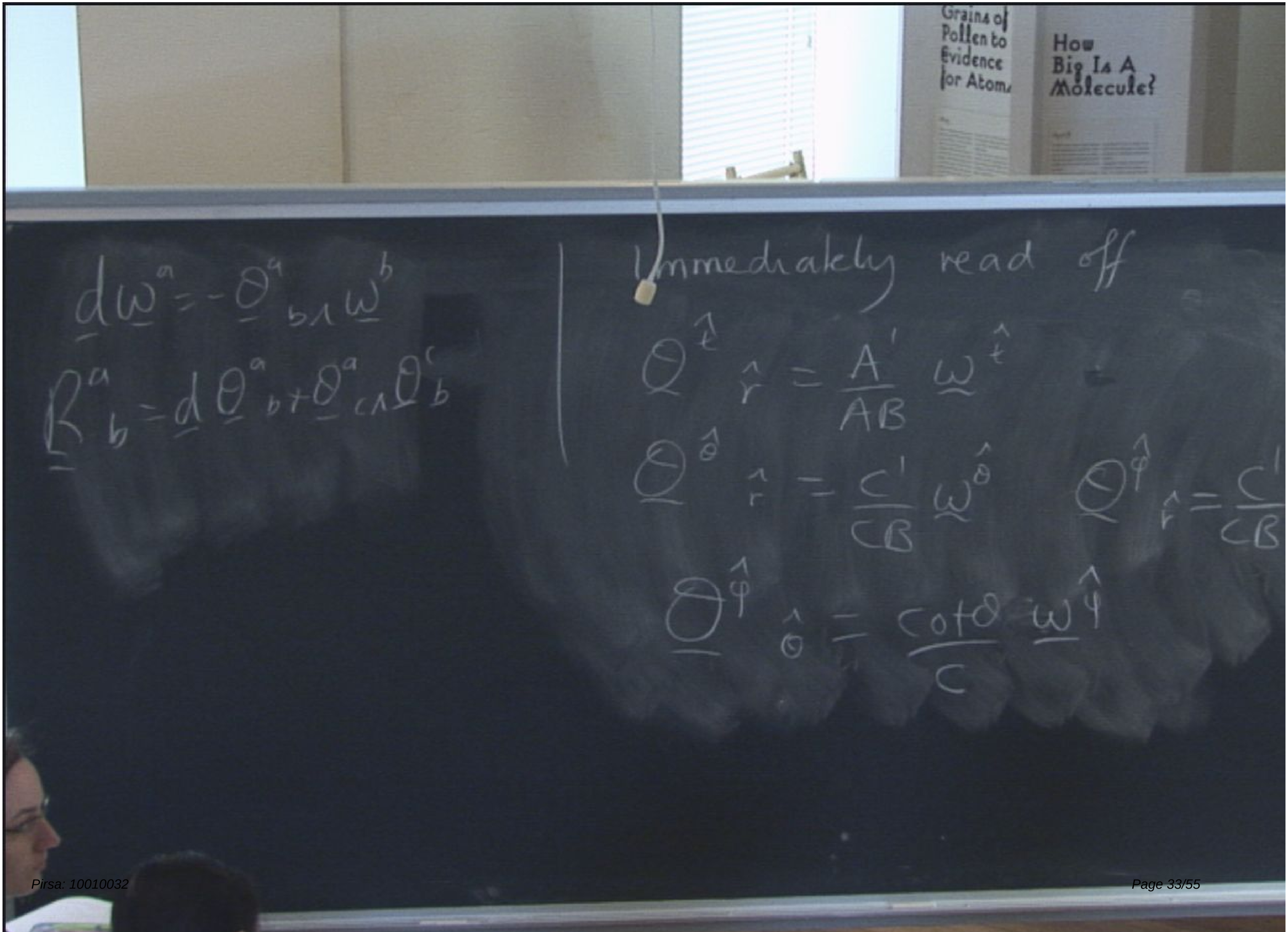

How Big Is A Molecule?
$d\omega^a = -\theta^a{}_b \wedge \omega^b$
$R^a{}_b = d\theta^a{}_b + \theta^a{}_c \wedge \theta^c{}_b$
Immediately read off
$\theta^{\hat t}{}_{\hat r} = \frac{A'}{AB}\,\omega^{\hat t}$
$\theta^{\hat\theta}{}_{\hat r} = \frac{C'}{CB}\,\omega^{\hat\theta}$
$\theta^{\hat\varphi}{}_{\hat r} = \frac{C'}{CB}$
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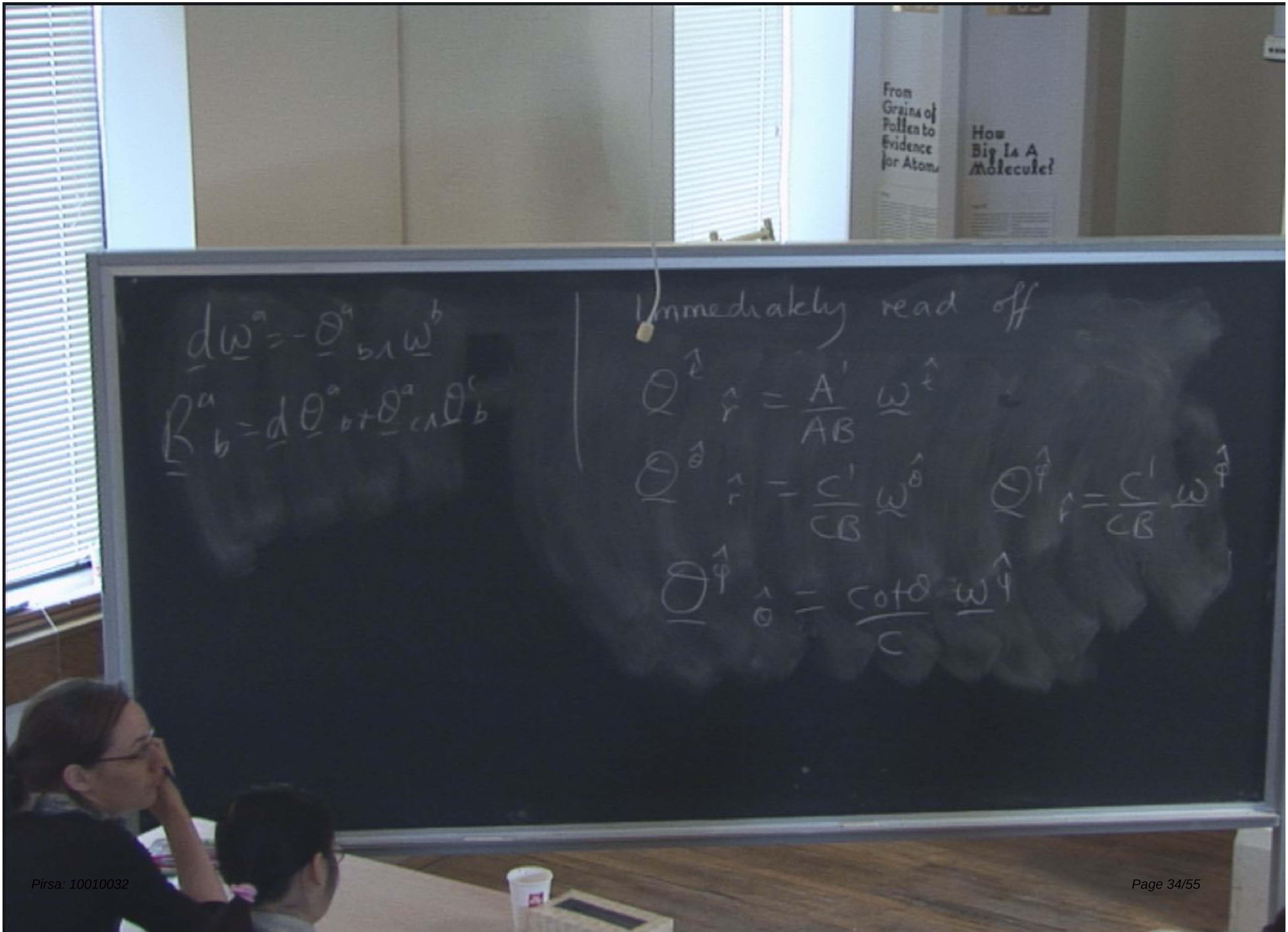
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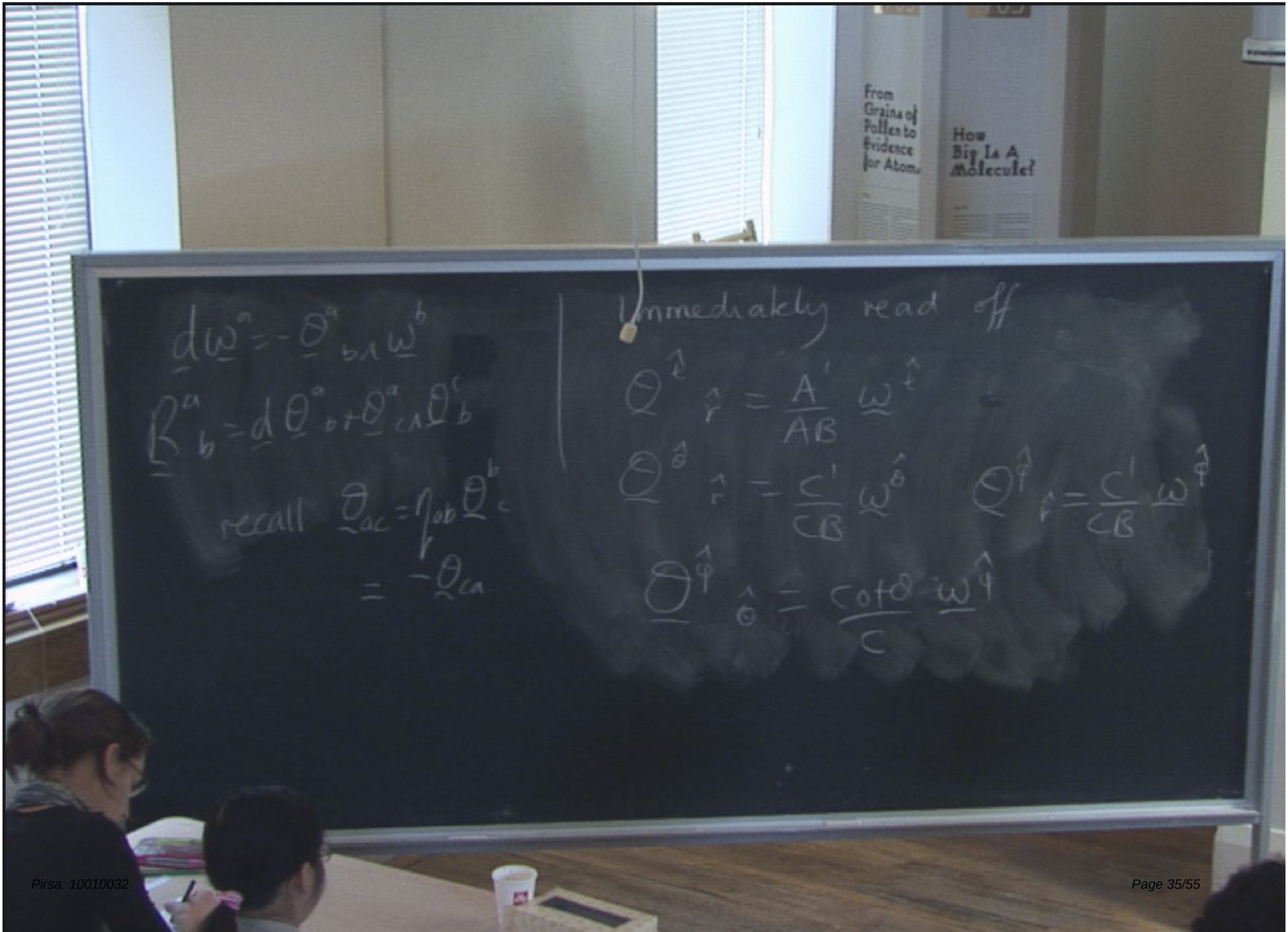

$d\omega^a = -\theta^a{}_b \wedge \omega^b$
$R^a{}_b = d\theta^a{}_b + \theta^a{}_c \wedge \theta^c{}_b$
recall $\theta_{ac} = \eta_{ab}\theta^b{}_c$
$= -\theta_{ca}$
Immediately read off
$\theta^{\hat t}{}_{\hat r} = \frac{A'}{AB}\,\omega^{\hat t}$
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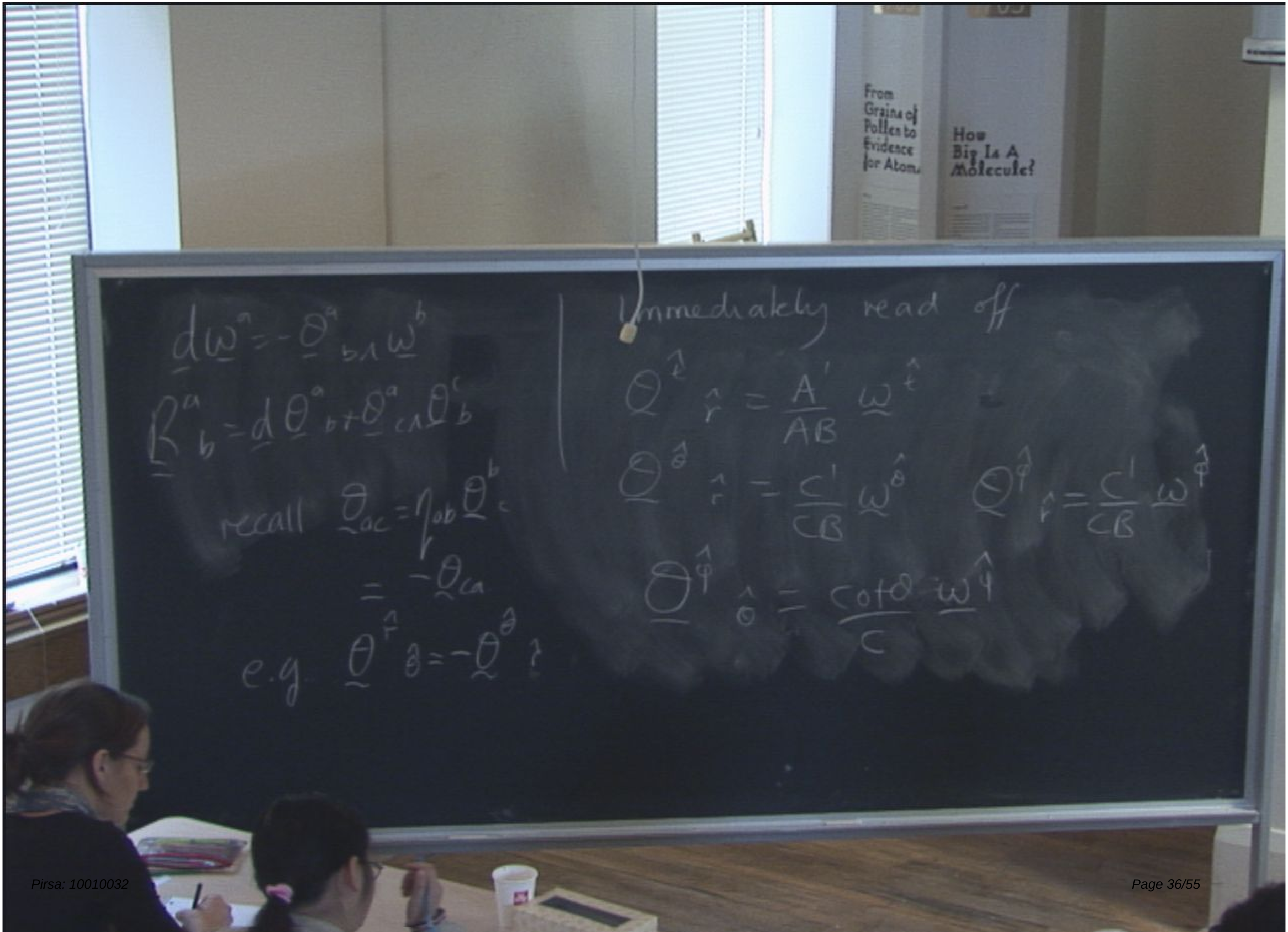
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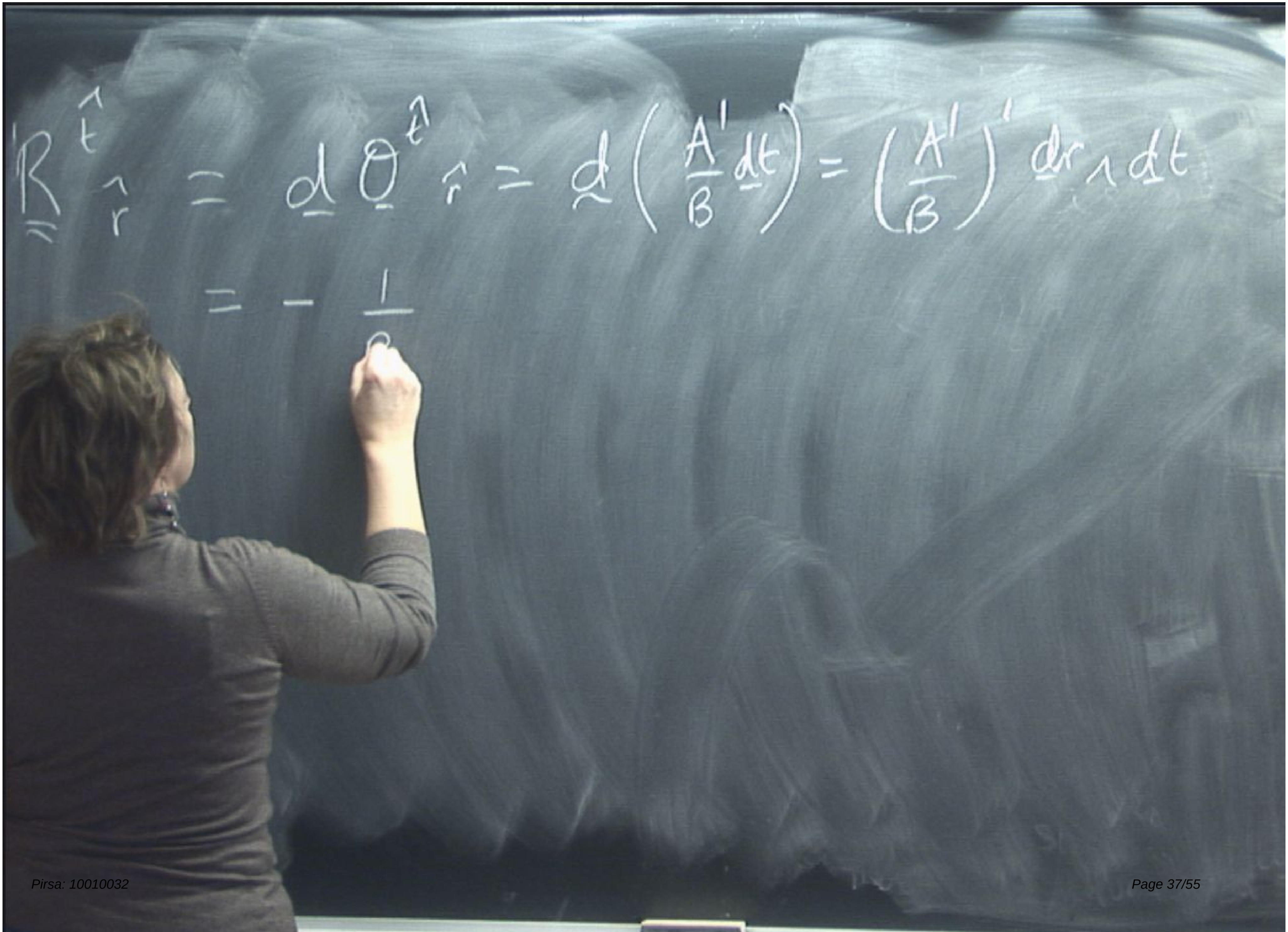
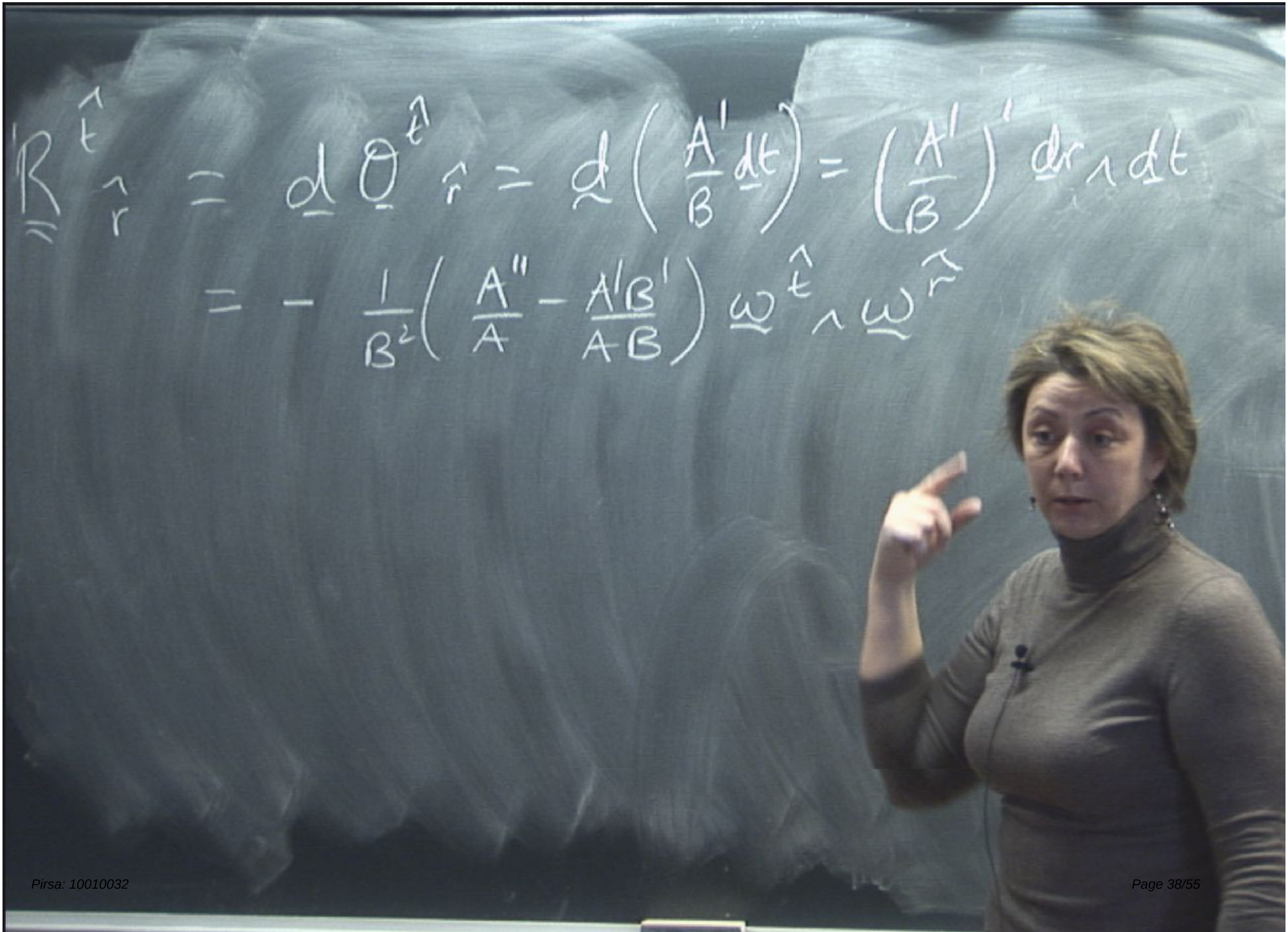
recall $\theta_{ac} = \eta_{ab}\theta^b{}_c$
$= -\theta_{ca}$
e.g. $\theta^{\hat r}{}_{\hat\theta} = -\theta^{\hat\theta}{}_{\hat r}$?
Immediately read off
$\theta^{\hat t}{}_{\hat r} = \frac{A'}{AB}\,\omega^{\hat t}$
$\theta^{\hat\theta}{}_{\hat r} = \frac{C'}{CB}\,\omega^{\hat\theta}$
$\theta^{\hat\varphi}{}_{\hat r} = \frac{C'}{CB}\,\omega^{\hat\varphi}$
$\theta^{\hat\varphi}{}_{\hat\theta} = \frac{\cot\theta}{C}\,\omega^{\hat\varphi}$


$R^{\hat{t}}{}_{\hat{r}} = d\theta^{\hat{t}}{}_{\hat{r}} = d\left(\frac{A'}{B}dt\right) = \left(\frac{A'}{B}\right)' dr \wedge dt$
$= - \frac{1}{B}$


$R^{\hat t}{}_{\hat r} = d\theta^{\hat t}{}_{\hat r} = d\left(\frac{A'}{B} dt\right) = \left(\frac{A'}{B}\right)' dr \wedge dt$
$= -\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right) \omega^{\hat t} \wedge \omega^{\hat r}$

$$\underset{\sim}{R}^{\hat{t}}{}_{\hat{r}} = \underset{\sim}{d}\,\underset{\sim}{\theta}^{\hat{t}}{}_{\hat{r}} = \underset{\sim}{d}\left(\frac{A'}{B}\underset{\sim}{dt}\right) = \left(\frac{A'}{B}\right)'\underset{\sim}{dr}\wedge\underset{\sim}{dt}$$

$$= -\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)\underset{\sim}{\omega}^{\hat{t}}\wedge\underset{\sim}{\omega}^{\hat{r}}$$

$$\underset{\approx}{R}^{\hat{t}}{}_{\hat{r}} = \underline{d}\,\underline{\theta}^{\hat{t}}{}_{\hat{r}} = \underset{\sim}{d}\left(\frac{A'}{B}\underline{d}t\right) = \left(\frac{A'}{B}\right)'\underline{d}r \wedge \underline{d}t$$

$$= -\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)\underset{\sim}{\omega}^{\hat{t}} \wedge \underset{\sim}{\omega}^{\hat{r}}$$

$$\underset{\approx}{R}^{\hat{t}}{}_{\hat{\theta}} = \underset{\sim}{\theta}^{\hat{t}}{}_{\hat{r}} \wedge \underset{\sim}{\theta}^{\hat{r}}{}_{\hat{\theta}}$$

$$\underset{\approx}{R}^{\hat{t}}{}_{\hat{r}} = \underline{d}\,\underline{\theta}^{\hat{t}}{}_{\hat{r}} = \underset{\sim}{d}\left(\frac{A'}{B}\underline{d}t\right) = \left(\frac{A'}{B}\right)'\underline{d}r \wedge \underline{d}t$$

$$= -\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)\underset{\sim}{\omega}^{\hat{t}} \wedge \underset{\sim}{\omega}^{\hat{r}}$$

$$\underset{\approx}{R}^{\hat{t}}{}_{\hat{\theta}} = \underset{\sim}{\theta}^{\hat{t}}{}_{\hat{r}} \wedge \underset{\sim}{\theta}^{\hat{r}}{}_{\hat{\theta}}$$

$\sin\theta\,\underline{d}\varphi$

$$= \underset{\sim}{d}\,\theta^{\hat{t}}{}_{\hat{r}} = \underset{\sim}{d}\left(\frac{A'}{B}\,\underset{\sim}{dt}\right) = \left(\frac{A'}{B}\right)' \underset{\sim}{dr} \wedge \underset{\sim}{dt}$$

$$= -\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right) \underset{\sim}{\omega}^{\hat{t}} \wedge \underset{\sim}{\omega}^{\hat{r}}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{\theta\phi} = \partial_\theta A_\phi = \cos\theta .$$

$$A_\mu = (0, 0, 0, \sin\theta)$$

$$\hat{\theta} = \underset{\sim}{\theta}^{\hat{t}}{}_{\hat{r}} \wedge \underset{\sim}{\theta}^{\hat{r}}{}_{\hat{\theta}}$$

$$\sin\theta\, \underset{\sim}{d}\varphi$$

$$\underset{\sim}{d}^2 = 0 .$$

$$= \underset{\sim}{d}\,\underset{\sim}{\Theta}^{\hat{t}}{}_{\hat{r}} = \underset{\sim}{d}\left(\frac{A'}{B}\underset{\sim}{d}t\right) = \left(\frac{A'}{B}\right)'\underset{\sim}{d}r \wedge \underset{\sim}{d}t$$

$$= -\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)\underset{\sim}{\omega}^{\hat{t}} \wedge \underset{\sim}{\omega}^{\hat{r}}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

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$$A_\mu = (0, 0, 0, \sin\theta)$$

$$\hat{\theta} = \underset{\sim}{\Theta}^{\hat{t}}{}_{\hat{r}} \wedge \underset{\sim}{\Theta}^{\hat{r}}{}_{\hat{\theta}}$$

$$\sin\theta\,\underset{\sim}{d}\varphi$$

$$\underset{\sim}{d}^2 = 0.$$

$$d(\sin\theta\,\underset{\sim}{d}\varphi).$$

$$= d(\sin\theta) \wedge \underset{\sim}{d}\varphi = $$

$$+ \sin\cancel{\theta\,d}(\underset{\sim}{d}\varphi)$$

$$\Theta^{\hat t}{}_{\hat r} = d\left(\frac{A'}{B}\,dt\right) = \left(\frac{A'}{B}\right)' dr \wedge dt$$

Then:

$$\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)\omega^{\hat t} \wedge \omega^{\hat r}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{\theta\phi} = \partial_\theta A_\phi = \cos\theta$$

$$A_\mu = (0, 0, 0, \sin\theta)$$

$$\Theta^{\hat t}$$

$$\hat r \wedge \Theta^{\hat r}{}_{\hat\theta}$$

$$\sin\theta\, d\varphi$$

$$d^2 = 0$$

$$d(\sin\theta\, d\varphi)$$

$$= d(\sin\theta) \wedge d\varphi = \cos\theta\, d\theta \wedge d\varphi$$

$$+ \cancel{\sin\theta\, d(d\varphi)}$$

$$d\omega^{\hat\varphi} =$$

$$\underset{\approx}{R}^{\hat{t}}{}_{\hat{r}} = \underline{d}\,\underline{\theta}^{\hat{t}}{}_{\hat{r}} = \underset{\sim}{d}\left(\frac{A'}{B}\underline{dt}\right) = \left(\frac{A'}{B}\right)'\underline{dr}\wedge\underline{dt}$$

$$= -\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)\underset{\sim}{\omega}^{\hat{t}}\wedge\underset{\sim}{\omega}^{\hat{r}}$$

$$\underset{\approx}{R}^{\hat{t}}{}_{\hat{\theta}} = \underset{\sim}{\theta}^{\hat{t}}{}_{\hat{r}}\wedge\underset{\sim}{\theta}^{\hat{r}}{}_{\hat{\theta}} = \frac{A'}{AB}\underset{\sim}{\omega}^{\hat{t}}\wedge\left(-\frac{C'}{CB}\right)\underset{\sim}{\omega}^{\hat{\theta}}$$

$$\underset{\sim}{R}^{\hat t}{}_{\hat r} = d\theta^{\hat t}{}_{\hat r} = d\left(\frac{A'}{B}\,dt\right) = \left(\frac{A'}{B}\right)' dr \wedge dt$$

$$= -\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right) \omega^{\hat t} \wedge \omega^{\hat r}$$

$$\underset{\sim}{R}^{\hat t}{}_{\hat \theta} = \underset{\sim}{\theta}^{\hat t}{}_{\hat r} \wedge \underset{\sim}{\theta}^{\hat r}{}_{\hat \theta} = \frac{A'}{AB}\,\omega^{\hat t} \wedge \left(-\frac{C'}{CB}\right) \omega^{\hat \theta}$$

$$\underset{\sim}{R}^{\hat t}{}_{\hat \varphi} = \underset{\sim}{\theta}^{\hat t}{}_{\hat r} \wedge \underset{\sim}{\theta}^{\hat r}{}_{\hat \varphi} = \frac{A'}{AB}\,\omega^{\hat t} \wedge$$

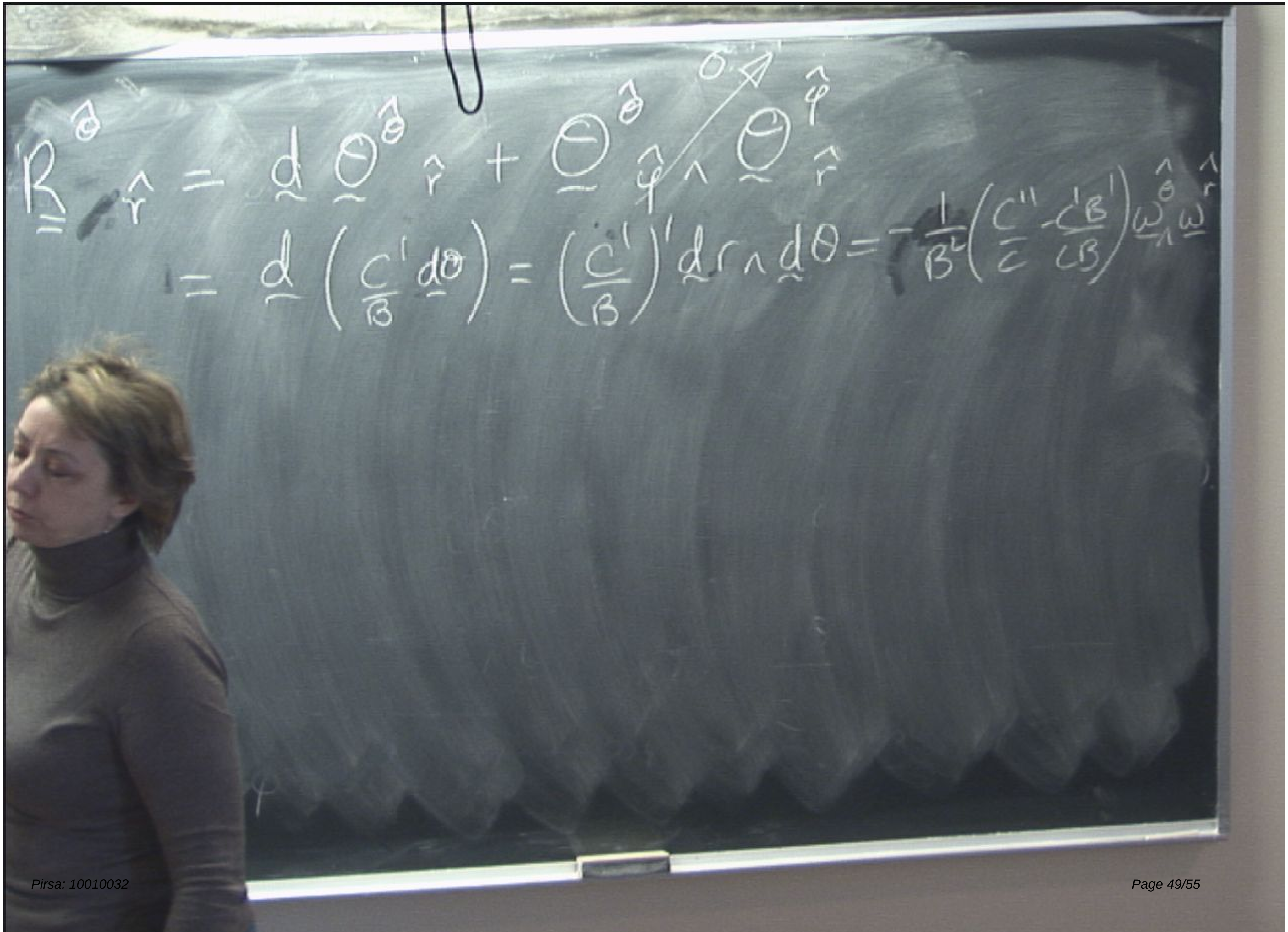
$$R^{\hat t}{}_{\hat r} = d\,\theta^{\hat t}{}_{\hat r} = d\left(\frac{A'}{B}\,dt\right) = \left(\frac{A'}{B}\right)' dr \wedge dt$$

$$= -\frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)\omega^{\hat t} \wedge \omega^{\hat r}$$

$$R^{\hat t}{}_{\hat \theta} = \theta^{\hat t}{}_{\hat r} \wedge \theta^{\hat r}{}_{\hat \theta} = \frac{A'}{AB}\,\omega^{\hat t} \wedge \left(-\frac{C'}{CB}\right)\omega^{\hat \theta}$$

$$R^{\hat t}{}_{\hat \varphi} = \theta^{\hat t}{}_{\hat r} \wedge \theta^{\hat r}{}_{\hat \varphi} = \frac{A'}{AB}\,\omega^{\hat t} \wedge \left(-\frac{C'}{CB}\right)\omega^{\hat \varphi}$$

$$\underset{\approx}{R}^{\hat{\theta}}{}_{\hat{r}} = \underset{\sim}{d}\,\underset{\sim}{\Theta}^{\hat{\theta}}{}_{\hat{r}} + \underset{\sim}{\Theta}^{\hat{\theta}}{}_{\hat{\varphi}} \wedge \underset{\sim}{\Theta}^{\hat{\varphi}}{}_{\hat{r}}$$



$R^{\hat\theta}{}_{\hat r} = d\,\theta^{\hat\theta}{}_{\hat r} + \theta^{\hat\theta}{}_{\hat\varphi} \wedge \theta^{\hat\varphi}{}_{\hat r}$
0
$= d\left(\frac{C'}{B}\,d\theta\right) = \left(\frac{C'}{B}\right)' dr \wedge d\theta = -\frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right)\omega^{\hat\theta} \wedge \omega^{\hat r}$

$$R^{\hat\theta}{}_{\hat r} = d\,\Theta^{\hat\theta}{}_{\hat r} + \overset{0}{\cancel{\Theta^{\hat\theta}{}_{\hat\varphi}\wedge\Theta^{\hat\varphi}{}_{\hat r}}}$$

$$= d\left(\frac{C'}{B}\,d\theta\right) = \left(\frac{C'}{B}\right)' dr\wedge d\theta = -\frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right)\omega^{\hat\theta}\wedge\omega^{\hat r}$$

$$R^{\hat\varphi}{}_{\hat r} = d\,\Theta^{\hat\varphi}{}_{\hat r} + \Theta^{\hat\varphi}{}_{\hat\theta}\wedge\Theta^{\hat\theta}{}_{\hat r}$$

$$= d\left(\frac{C'}{B}\sin\theta\,d\varphi\right) + \frac{\cot\theta}{C}\,\omega^{\hat\varphi}\wedge\frac{C'}{CB}\,\omega^{\hat\theta}$$

$$\underset{\approx}{R}^{\hat\theta}{}_{\hat r} = \underset{\sim}{d}\,\underset{\sim}{\Theta}^{\hat\theta}{}_{\hat r} + \cancel{\underset{\sim}{\Theta}^{\hat\theta}{}_{\hat\varphi} \wedge \underset{\sim}{\Theta}^{\hat\varphi}{}_{\hat r}}^{\,0}$$

$$= \underset{\sim}{d}\left(\frac{C'}{B}\underset{\sim}{d}\theta\right) = \left(\frac{C'}{B}\right)'\underset{\sim}{d}r \wedge \underset{\sim}{d}\theta = -\frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right)\underset{\sim}{\omega}^{\hat\theta} \wedge \underset{\sim}{\omega}^{\hat r}$$

$$\underset{\approx}{R}^{\hat\varphi}{}_{\hat r} = \underset{\sim}{d}\,\underset{\sim}{\Theta}^{\hat\varphi}{}_{\hat r} + \underset{\sim}{\Theta}^{\hat\varphi}{}_{\hat\theta} \wedge \underset{\sim}{\Theta}^{\hat\theta}{}_{\hat r}$$

$$= \underset{\sim}{d}\left(\frac{C'}{B}\sin\theta\,\underset{\sim}{d}\varphi\right) + \frac{\cot\theta}{C}\underset{\sim}{\omega}^{\hat\varphi} \wedge \frac{C'}{CB}\underset{\sim}{\omega}^{\hat\theta}$$

$$= \left(\frac{C'}{B}\right)'\sin\theta\,\underset{\sim}{d}r \wedge \underset{\sim}{d}\varphi + \frac{C'}{B}\cos\theta\,\underset{\sim}{d}\theta \wedge \underset{\sim}{d}\varphi + \frac{C'}{B}\cancel{\cos\theta}\,\underset{\sim}{d}\varphi \wedge \underset{\sim}{d}\theta$$

$$= -\frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right)\underset{\sim}{\omega}^{\hat\varphi} \wedge \underset{\sim}{\omega}^{\hat r}$$

$$\underset{\approx}{R}^{\hat\theta}{}_{\hat r} = \underset{\sim}{d}\,\underset{\sim}{\Theta}^{\hat\theta}{}_{\hat r} + \cancelto{0}{\underset{\sim}{\Theta}^{\hat\theta}{}_{\hat\varphi} \wedge \underset{\sim}{\Theta}^{\hat\varphi}{}_{\hat r}}$$

$$= \underset{\sim}{d}\left(\frac{C'}{B}\,\underset{\sim}{d}\theta\right) = \left(\frac{C'}{B}\right)' \underset{\sim}{d}r \wedge \underset{\sim}{d}\theta = -\frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right)\underset{\sim}{\omega}^{\hat\theta} \wedge \underset{\sim}{\omega}^{\hat r}$$

$$\underset{\approx}{R}^{\hat\varphi}{}_{\hat r} = \underset{\sim}{d}\,\underset{\sim}{\Theta}^{\hat\varphi}{}_{\hat r} + \underset{\sim}{\Theta}^{\hat\varphi}{}_{\hat\theta} \wedge \underset{\sim}{\Theta}^{\hat\theta}{}_{\hat r}$$

$$= \underset{\sim}{d}\left(\frac{C'}{B}\sin\theta\,\underset{\sim}{d}\varphi\right) + \frac{\cot\theta}{C}\,\underset{\sim}{\omega}^{\hat\varphi} \wedge \frac{C'}{CB}\,\underset{\sim}{\omega}^{\hat\theta}$$

$$= \left(\frac{C'}{B}\right)'\sin\theta\,\underset{\sim}{d}r \wedge \underset{\sim}{d}\varphi + \cancel{\frac{C'}{B}\cos\theta\,\underset{\sim}{d}\theta \wedge \underset{\sim}{d}\varphi} + \cancel{\frac{C'}{B}\cos\theta\,\underset{\sim}{d}\varphi \wedge \underset{\sim}{d}\theta}$$

$$= -\frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right)\underset{\sim}{\omega}^{\hat\varphi} \wedge \underset{\sim}{\omega}^{\hat r}$$

$$\underline{\underline{R}}^{\hat{\varphi}}{}_{\hat{\theta}} = \underline{d}\,\underline{\Theta}^{\hat{\varphi}}{}_{\hat{\theta}} + \underline{\Theta}^{\hat{\varphi}} \wedge \underline{\Theta}_{\hat{\theta}}$$

$$\underline{\underline{R}}^{\hat{\varphi}}{}_{\hat{\theta}} = \underset{\sim}{d}\,\underline{\Theta}^{\hat{\varphi}}{}_{\hat{\theta}} + \underline{\Theta}^{\hat{\varphi}}{}_{\hat{r}} \wedge \underline{\Theta}^{\hat{r}}{}_{\hat{\theta}}$$

$$= \underset{\sim}{d}\left(\cos\theta\, \underset{\sim}{d}\varphi\right) + \frac{c'}{cB}\,\underline{\omega}^{\hat{\varphi}} \wedge \left(-\frac{c'}{cB}\,\underline{\omega}^{\hat{\theta}}\right)$$

$$R^{\hat\varphi}{}_{\hat\theta} = d\,\Theta^{\hat\varphi}{}_{\hat\theta} + \Theta^{\hat\varphi}{}_{\hat r} \wedge \Theta^{\hat r}{}_{\hat\theta}$$

$$= d\left(\cos\theta\, d\varphi\right) + \frac{C'}{CB}\,\omega^{\hat\varphi} \wedge \left(-\frac{C'}{CB}\,\omega^{\hat\theta}\right)$$

$$= -\sin\theta\, d\theta \wedge d\varphi - \frac{C'^2}{C^2B^2}\,\omega^{\varphi} \wedge \omega^{\theta}$$

$$= \left(\frac{1}{C^2} - \frac{C'^2}{C^2B^2}\right)\omega^{\varphi} \wedge \omega^{\theta}$$