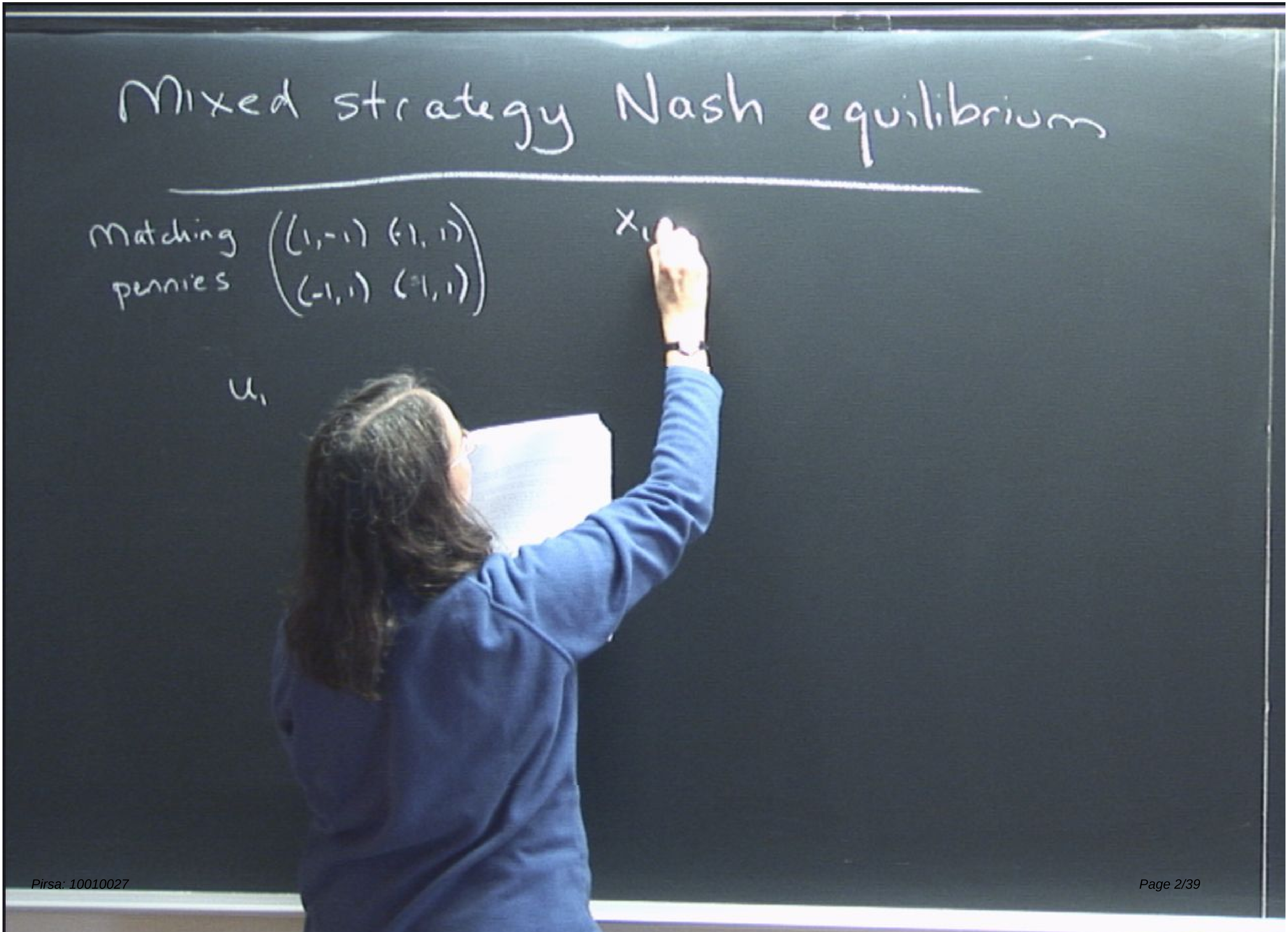
Title: Dynamical Systems - Review (PHYS 607) - Lecture 13

Date: Jan 20, 2010 09:00 AM

URL: http://pirsa.org/10010027

Abstract:


Mixed strategy Nash equilibrium
Matching pennies
(1,-1) (-1, 1)
x_1
u_1

Mixed strategy Nash equilibrium

Matching pennies
$$\begin{array}{ccc} & H & T \\ \begin{array}{c} H \\ T \end{array} & \multicolumn{2}{c}{\begin{pmatrix} (1,-1) & (-1,1) \\ (-1,1) & (-1,1) \end{pmatrix}} \end{array}$$

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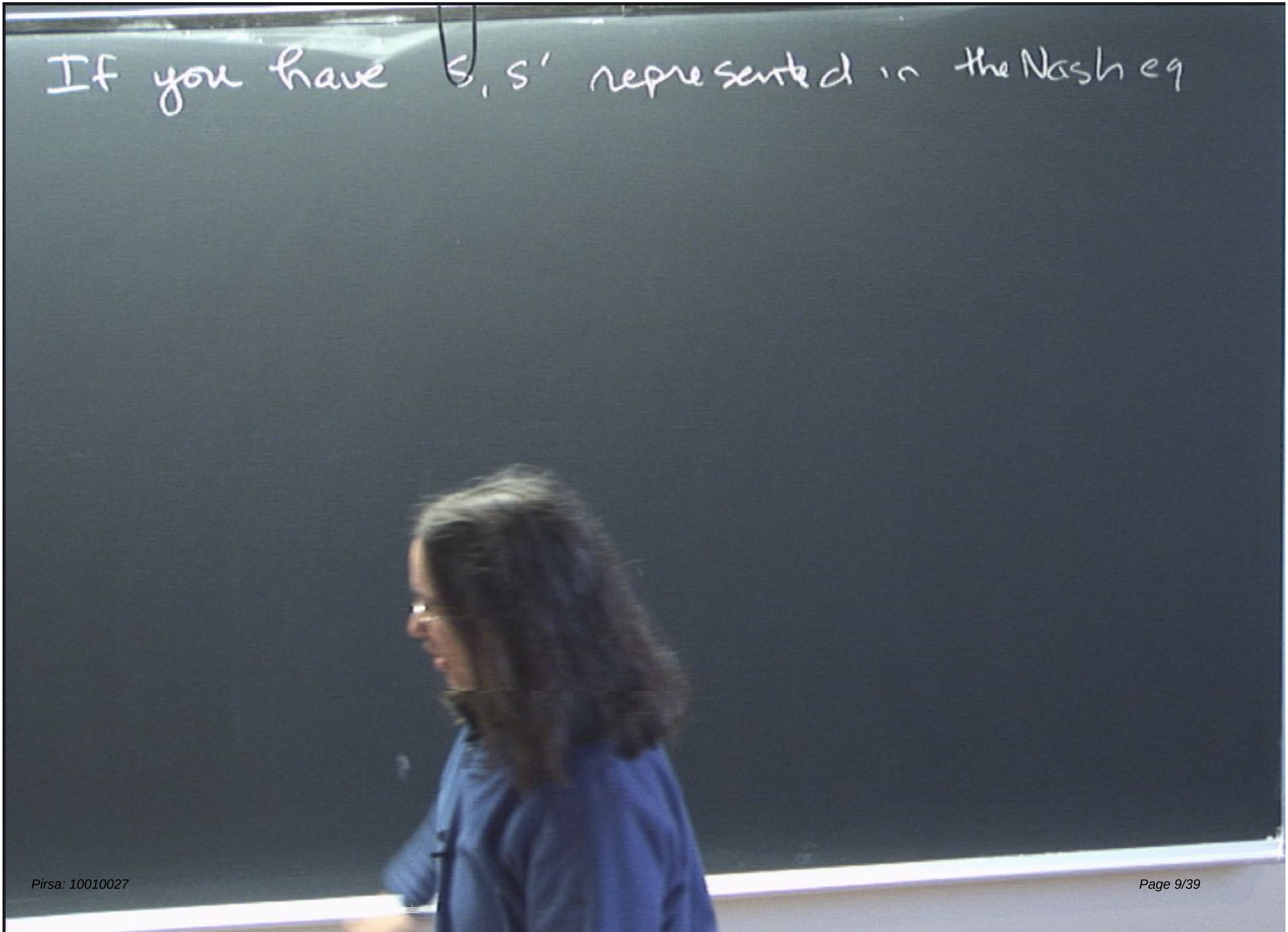
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has same payoff as

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payoff for tails $= -x_2 + (1-x_2)$

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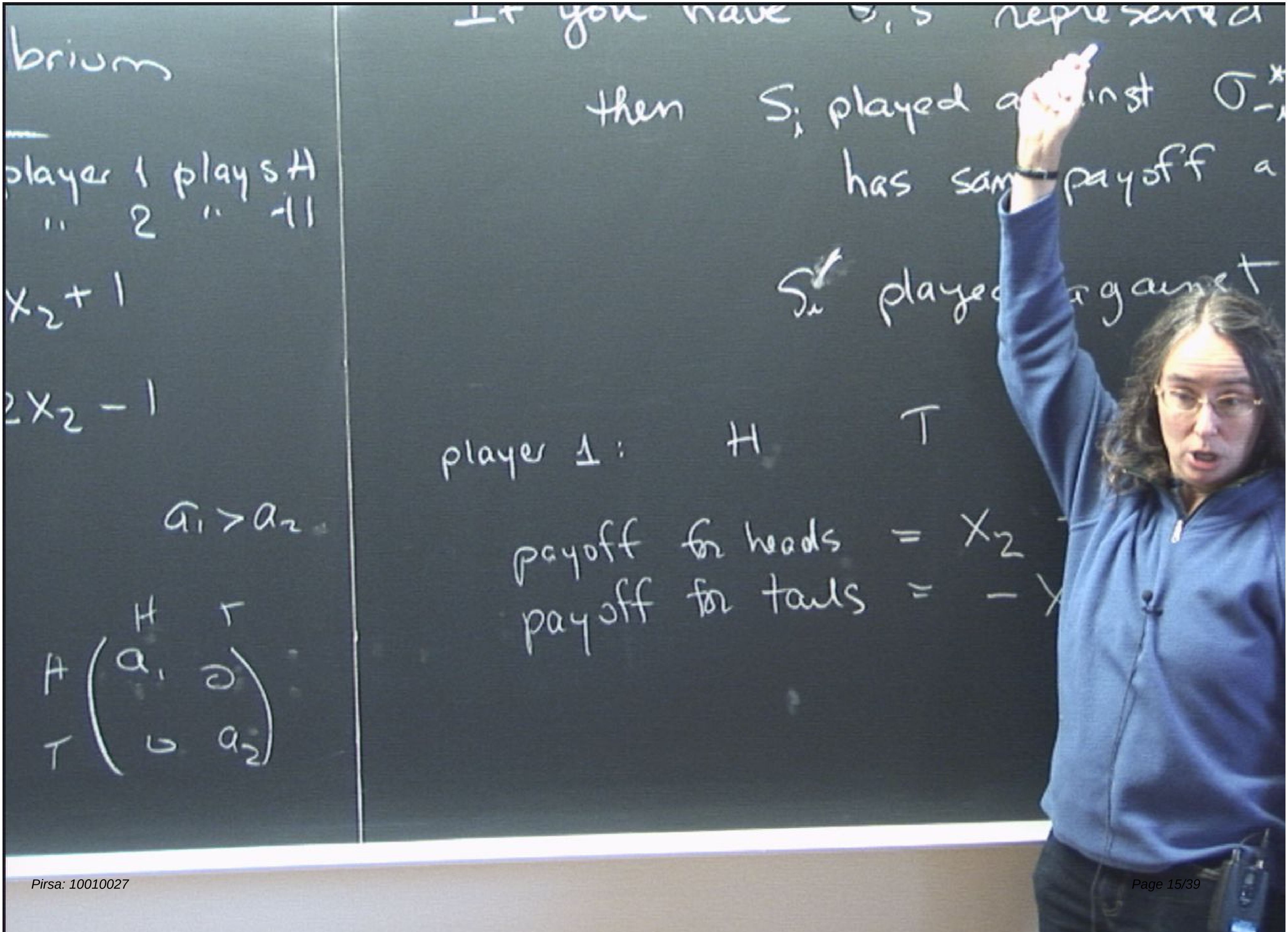
$$\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$$



brium
player 1 plays H
" 2 " -1|
x2 + 1
2x2 - 1
a1 > a2
H T
H (a1 0)
T (0 a2)
If you have σi's represented
then Si played against σ*-i
has same payoff a
Si' played against
player 1: H T
payoff for heads = X2
payoff for tails = -X

Evolutionary Game theory.

Maynard Smith

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Evolutionary Stability

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Evolutionary Stability

Stronger condition than Nash equilibrium

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2-person "game"

fitness $u_j[s]$

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fitness of j^{th} $u_j[\vec{s}]$

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$\vec{s}$ is evolutionarily stable if

$u_j[s, \epsilon t + (1-\epsilon)s] > u_1[t, \epsilon t + (1-\epsilon$

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this is equivalent to represent the Nash eq

u_1

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$$
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\end{cases}
$$

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Finding evolutionarily stable strategy for hawk-dove

	Fight	Yield
Fight	(-1, -1)	(4, 0)
Yield	(0, 4)	(2, 2)

s]

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1 vs 1 $\frac{v}{2} - \frac{c}{2}$

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Strategy 1 : Fight
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1 vs 1 $\frac{v}{2} - \frac{c}{2}$

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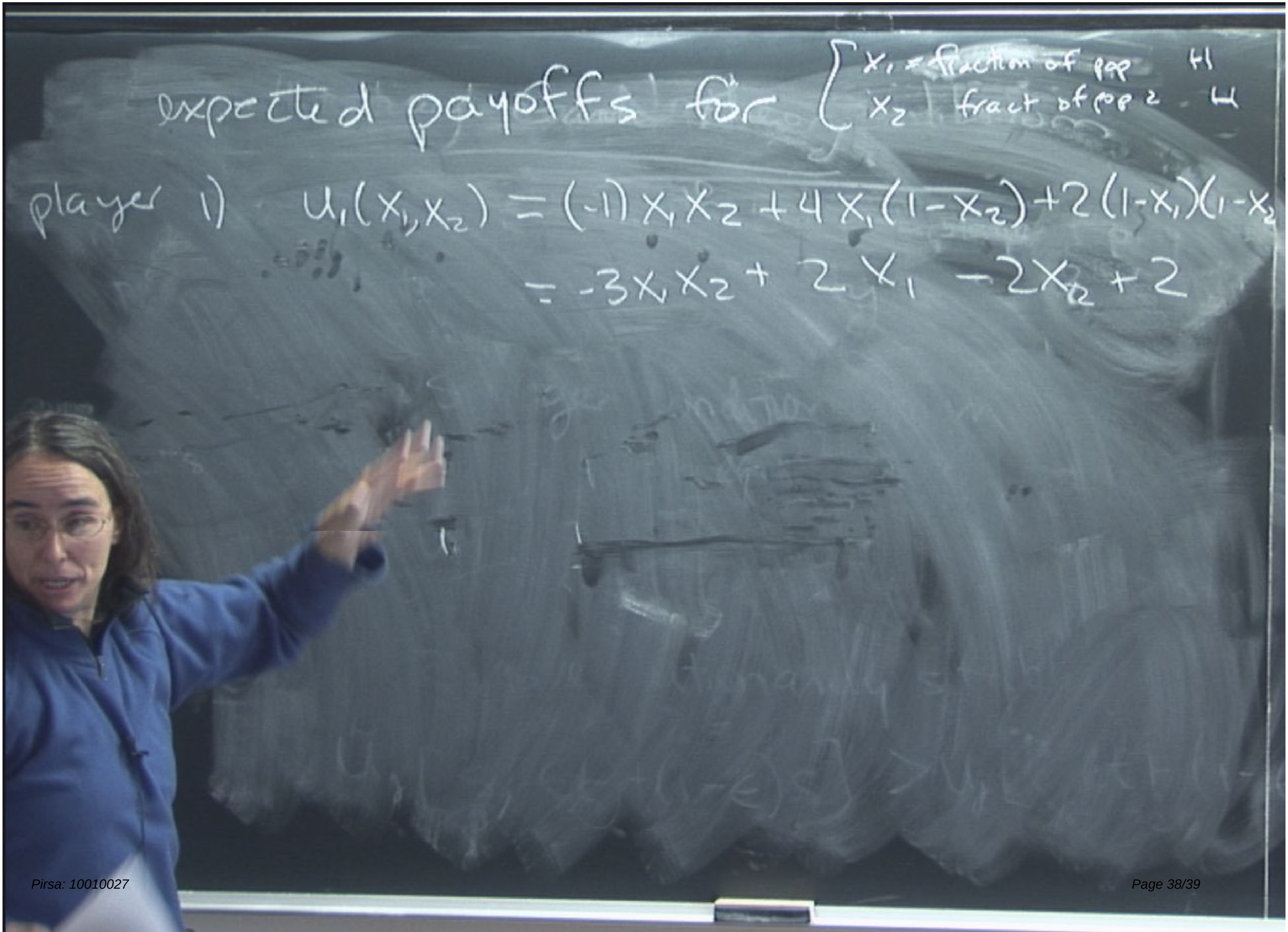
expected payoffs for $\left\{ \begin{array}{l} x_1 = \text{fraction of pop} \quad 1 \\ x_2 \ \text{fract of pop 2} \quad 1 \end{array} \right.$

player 1) $u_1(x_1, x_2)$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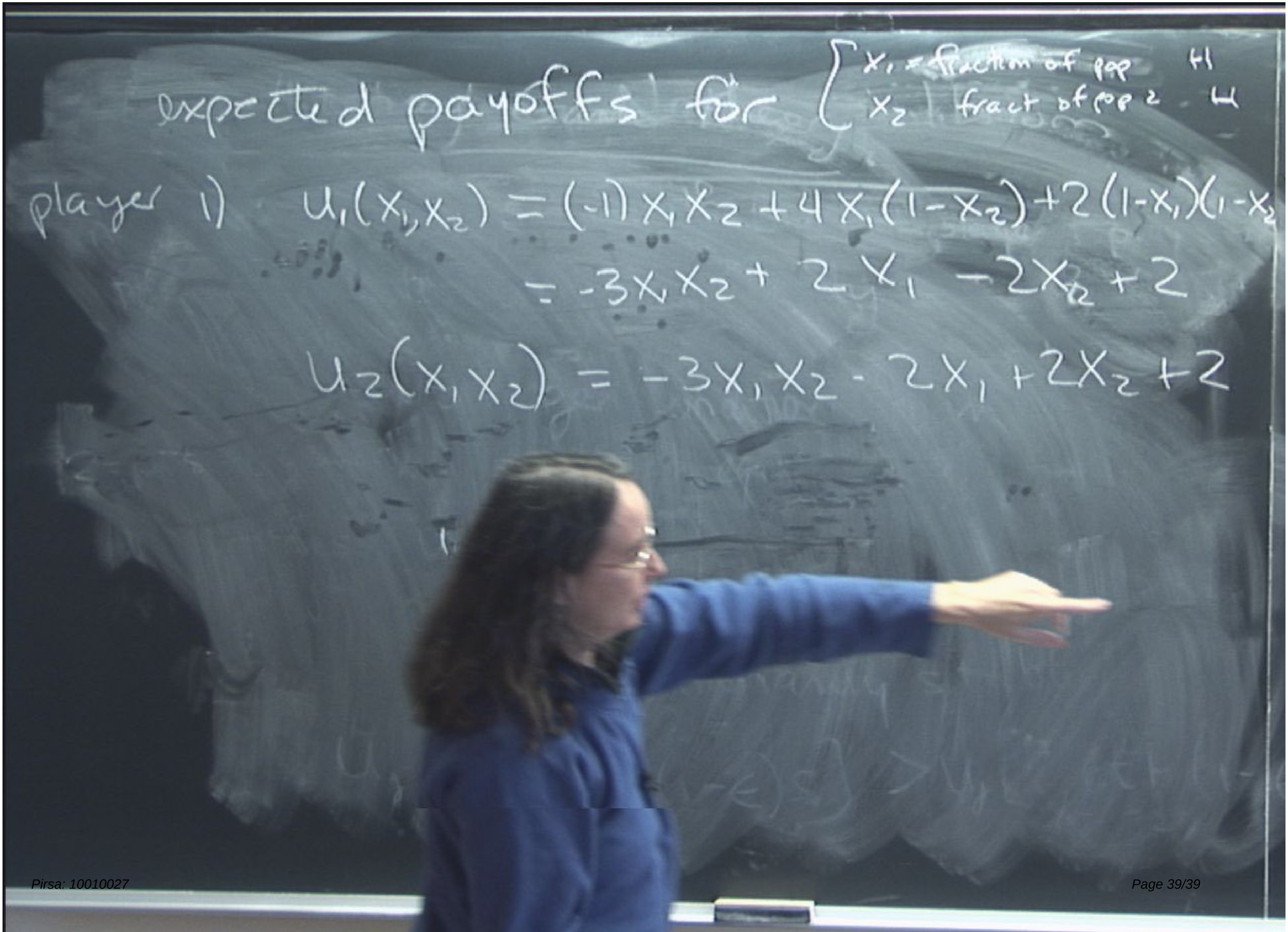
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u2(x1 x2) = -3 x1 x2 - 2 x1 + 2 x2 + 2