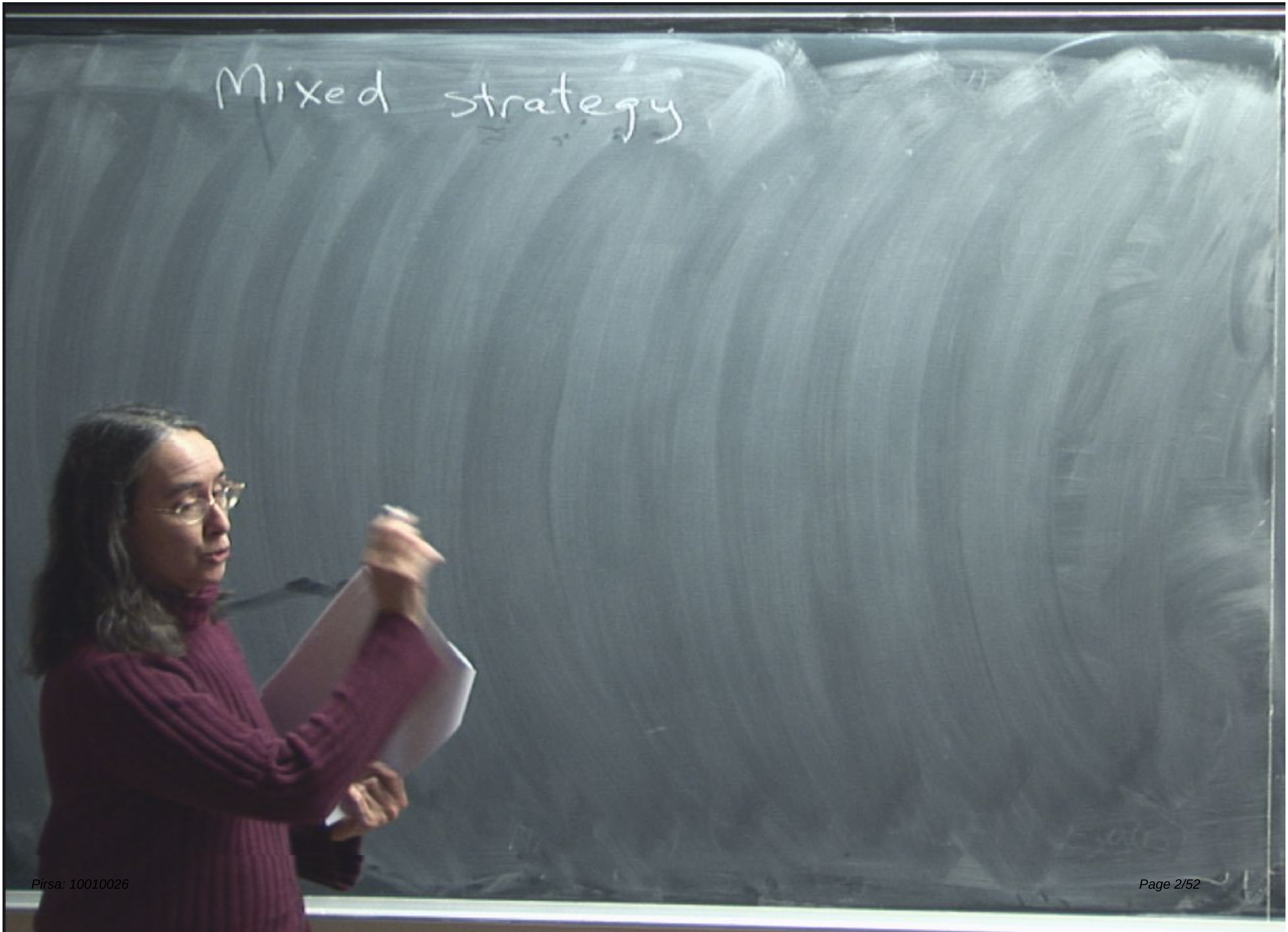
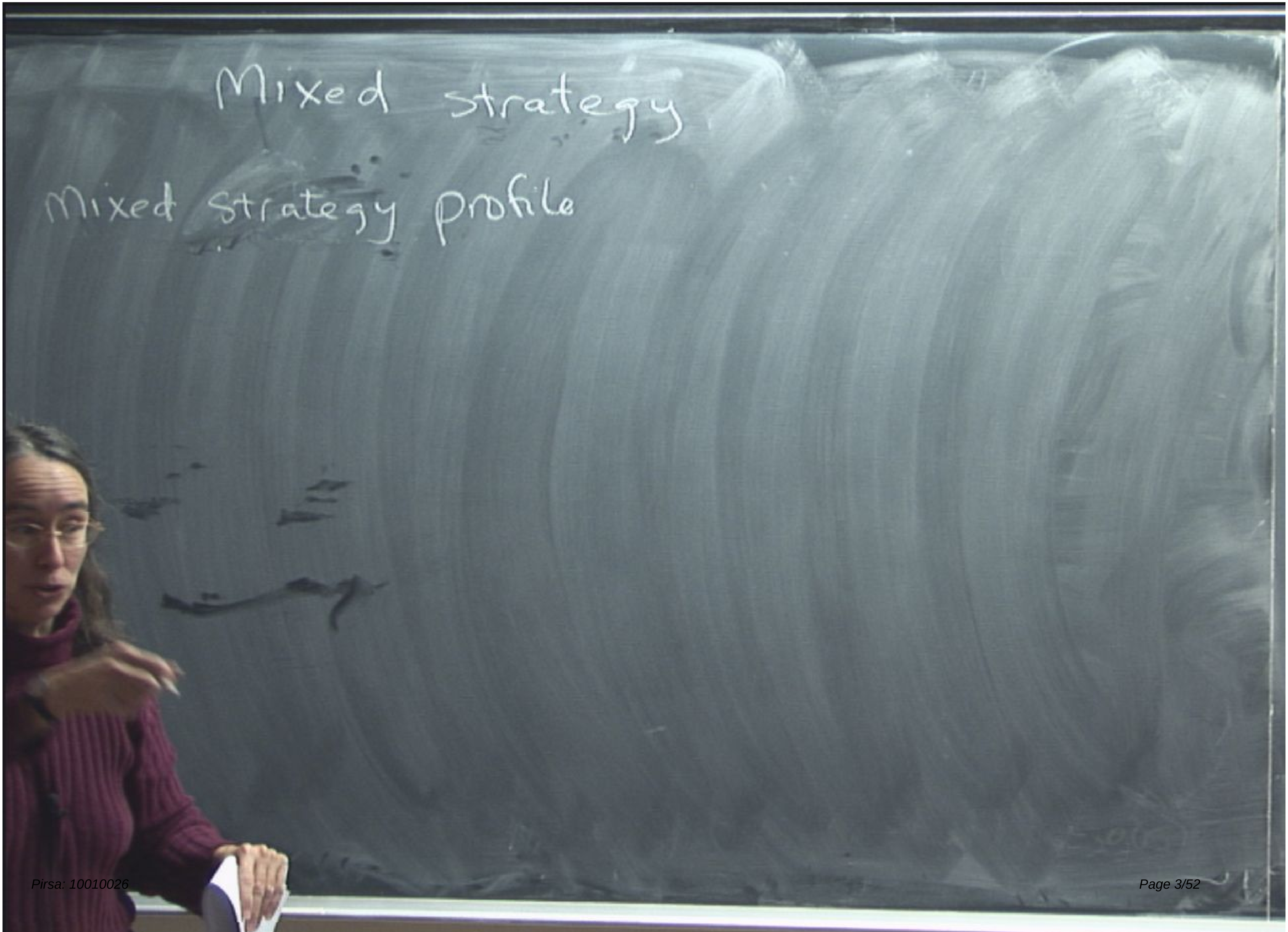
Title: Dynamical Systems - Review (PHYS 607) - Lecture 12

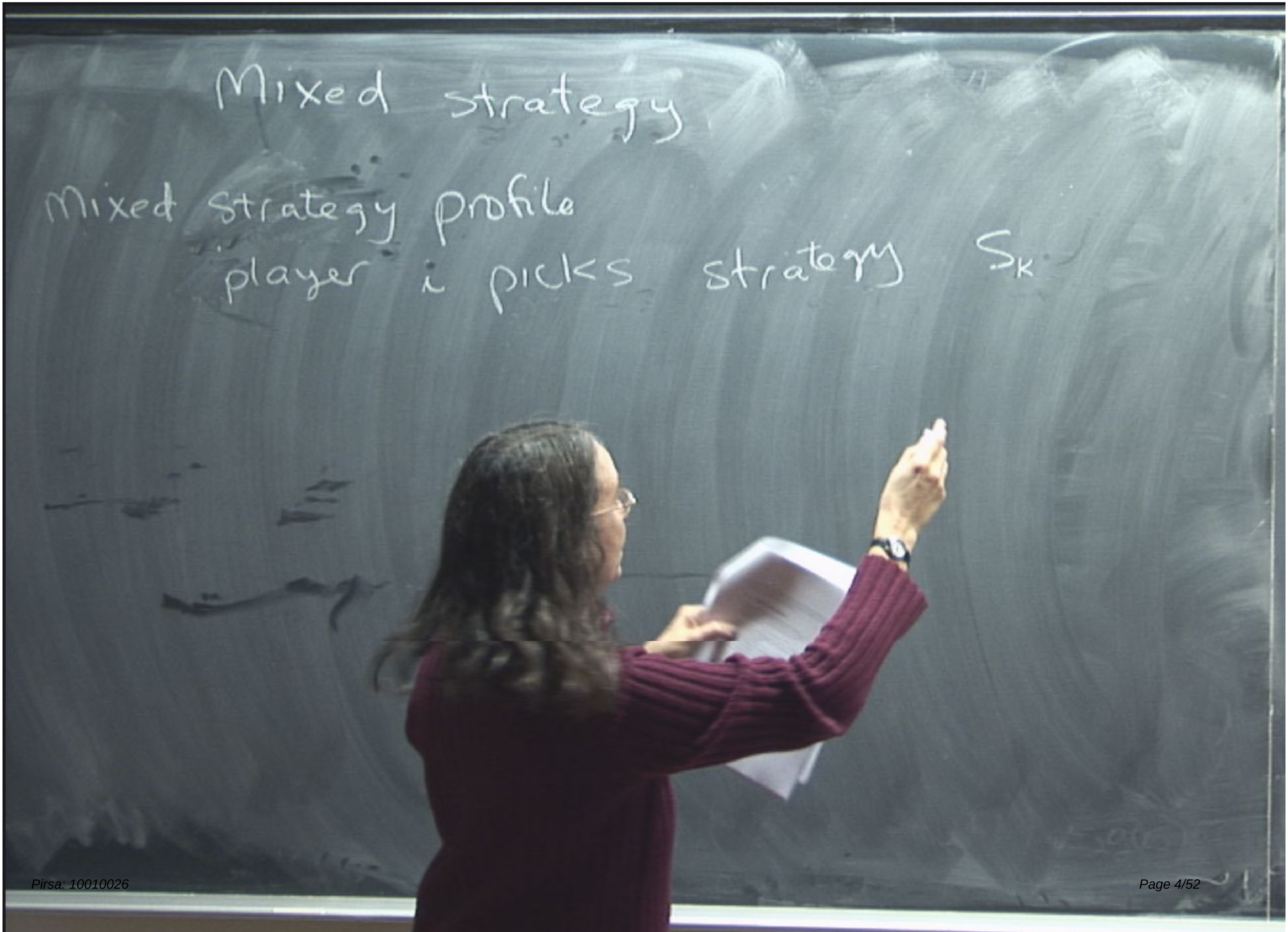
Date: Jan 19, 2010 09:00 AM

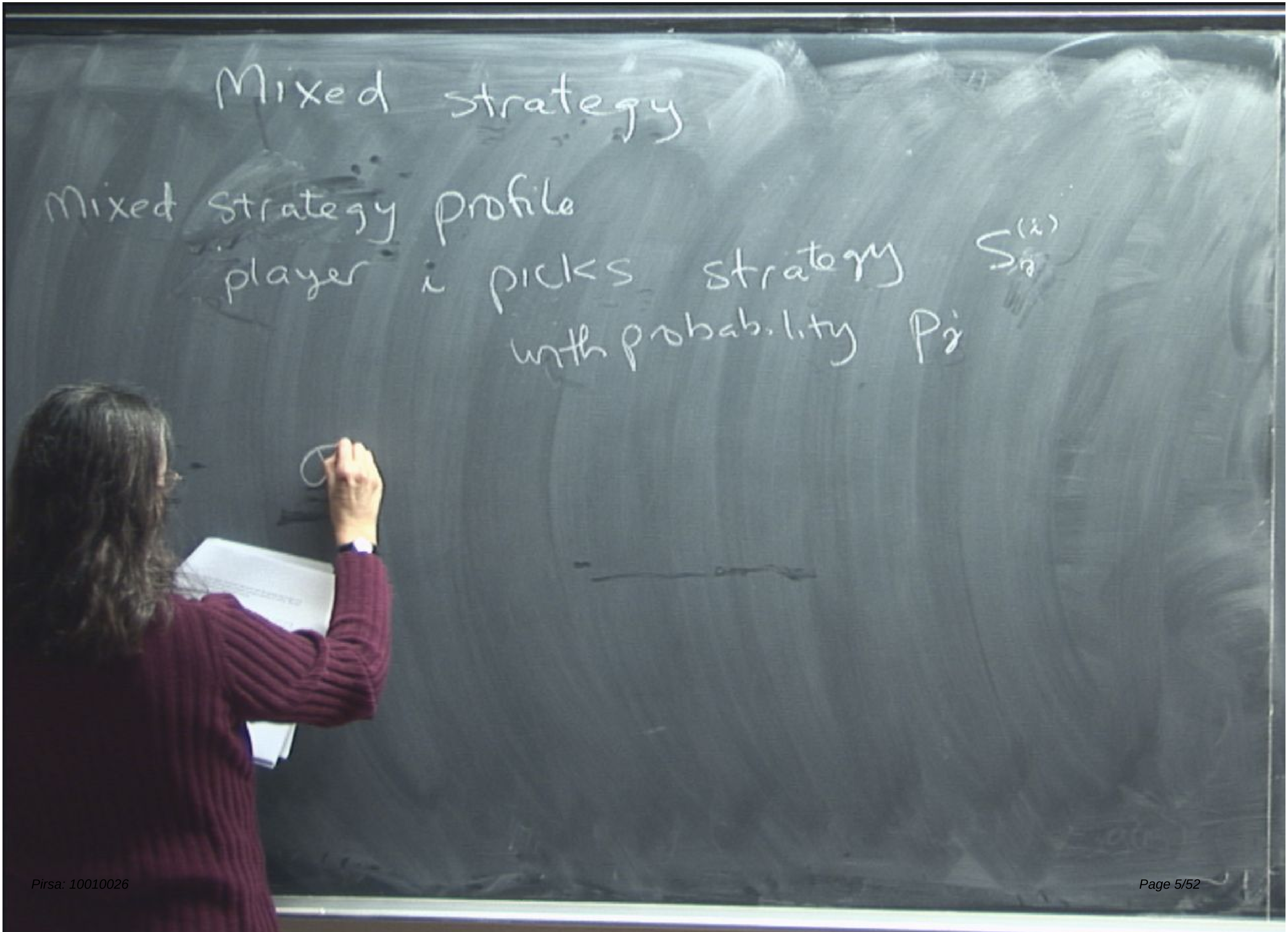
URL: http://pirsa.org/10010026

Abstract:


Mixed strategy


Mixed strategy
Mixed strategy profile


Mixed strategy
Mixed strategy profile
player i picks strategy s_k


Mixed strategy
Mixed strategy profile
player i picks strategy $S_j^{(i)}$
with probability p_j

Mixed strategy

Mixed strategy profile

player i picks strategy $S_j^{(i)}$

with probability p_j

for one player $\sigma = p_1 s_1 + p_2 s_2 + \dots + p_k s_k$

where $\sum_k p_k = 1$

all $p_k \geq 0$

Mixed strategy

Mixed strategy profile

player i picks strategy $S_j^{(i)}$

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for one player $\sigma = p_1 s_1 + p_2 s_2 + \dots + p_k s_k$

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payoff to player i

Mixed strategy

Mixed strategy profile

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for one player $\sigma = p_1 s_1 + p_2 s_2 + \dots + p_k s_k$

where $\sum_k p_k = 1$

all $p_k \geq 0$

payoff to player i

$\pi_i(\vec{\sigma})$

Mixed strategy

ixed strategy profile

player i picks strategy $S_j^{(i)}$

with probability p_j

or one player $\sigma = p_1 S_1 + p_2 S_2 + \dots + p_k S_k$

where $\sum_k p_k = 1$

all $p_k \geq 0$

payoff to player i

$$\pi_i(\vec{\sigma}) = \sum_{s_1 \in S_1} \dots \sum_{s_n \in S_n} p_{s_1} p_{s_2} \dots p_{s_n} \pi_i(s_1$$

Mixed strategy

ixed strategy profile

player i picks strategy $S_j^{(i)}$

with probability p_j..

or one
player $\sigma = p_1 S_1 + p_2 S_2 + \dots + p_K S_K$

where $\sum_k p_k = 1$

all $p_k \geq 0$

payoff to player i

$$\pi_i(\vec{\sigma}) = \sum_{s_1 \in S_1} \dots \sum_{s_n \in S_n} p_{s_1} p_{s_2} \cdots p_{s_n} \pi_i(s_1, \dots$$

d strategy

gy profile

i picks strategy $S_j^{(i)}$

with probability p_j

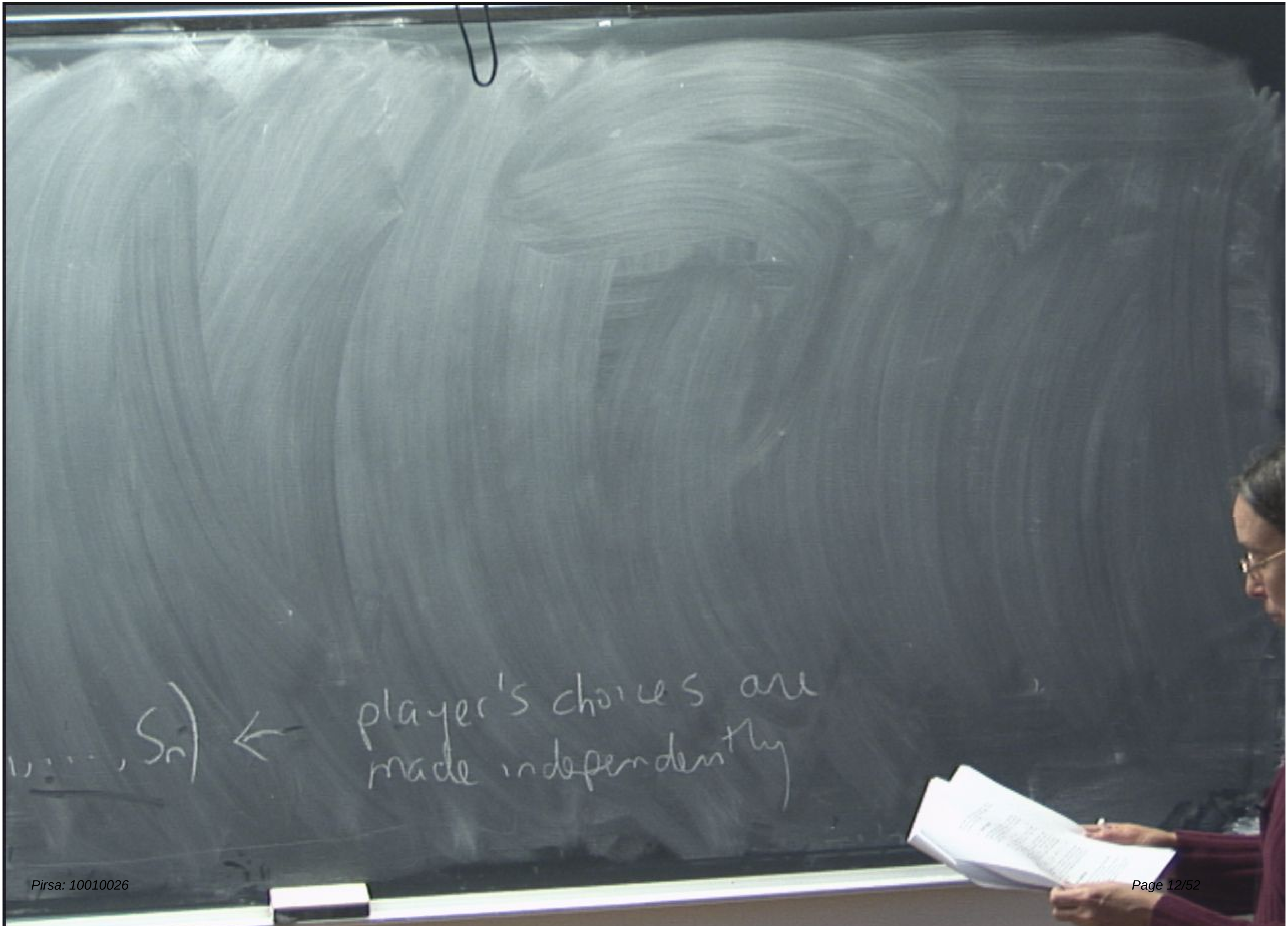
$= p_1 S_1 + p_2 S_2 + \cdots + p_k S_k$

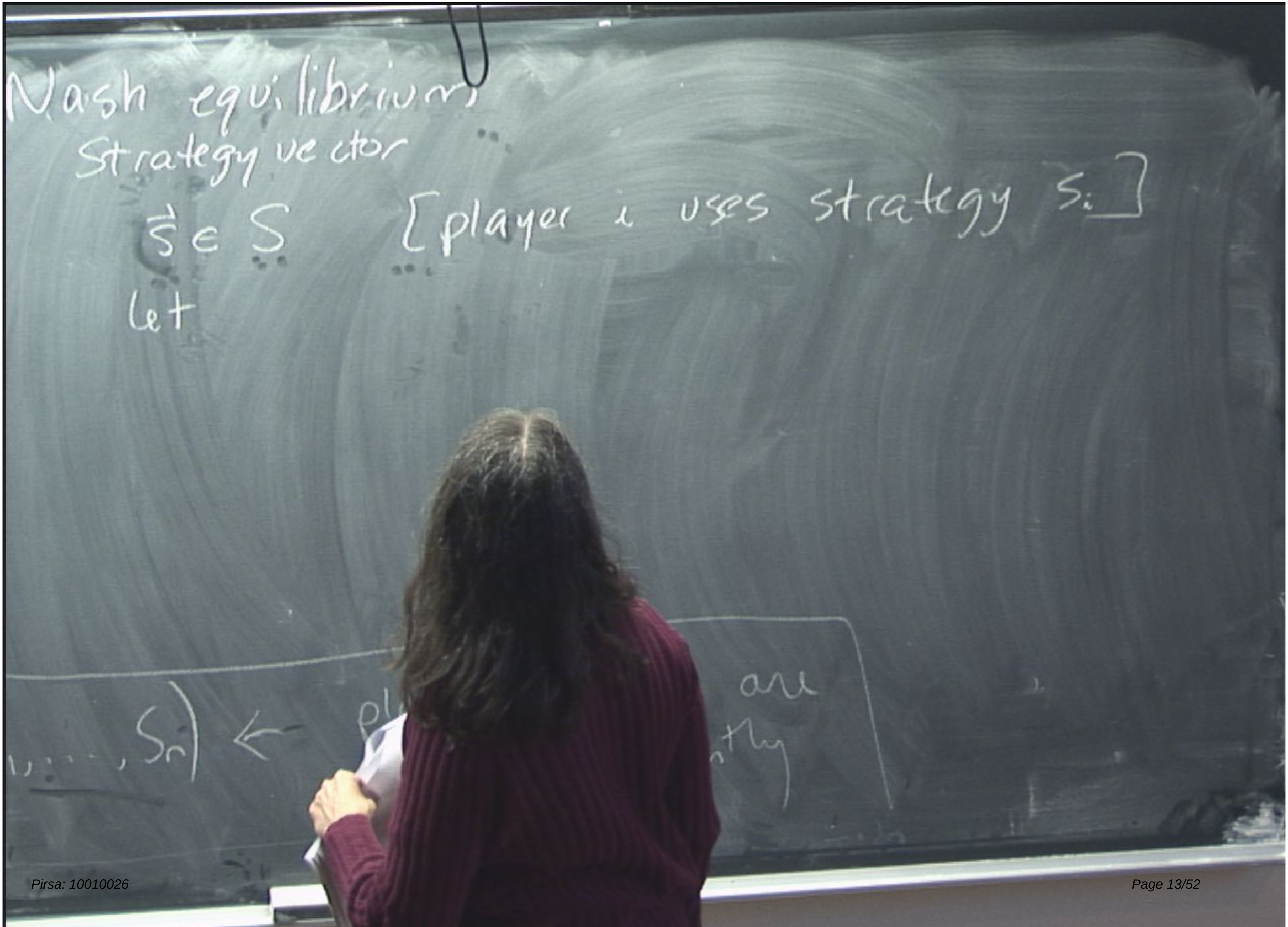
where $\sum_k p_k = 1$

all $p_k \geq 0$

to player i

$$(\vec{\sigma}) = \sum_{s_1 \in S_1} \cdots \sum_{s_n \in S_n} p_{s_1} p_{s_2} \cdots p_{s_n} \pi_i(s_1, \ldots, s_n)$$


Sn) ←
player's choices are
made independently


Nash equilibrium
strategy vector
$\vec{s} \in S$ [player i uses strategy s_i]
let
$, s_n) \leftarrow$
are

Nash equilibrium

Strategy vector

$\vec{s} \in S$ [player i uses strategy s_i]

let t_i be another strategy for player i

then $(s_{-i}, t_i) \leftarrow$ all players use $\vec{s}$ except for player i, who uses t_i

$\ldots, s_n) \leftarrow$ player's choices are made independently

Nash equilibrium

Strategy vector

$\vec{s} \in S$ [player i uses strategy s_i]

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Nash equilibrium

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A strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ ... ash equilibrium if for every i $\pi_i(s^*)$ ...

$(s_1, \ldots, s_n) \leftarrow$ player's choices are made independently

Nash equilibrium

Strategy vector

$\vec{s} \in S$ [player i uses strategy s_i]

let t_i be another strategy for player i

then $(s_{-i}, t_i) \leftarrow$ all players use $\vec{s}$ except for player i, who uses t_i

A strategy profile $s^* = (s_1^*, \ldots, s_r^*)$ is a Nash equilibrium if for every i $\pi_i(s^*) \geq \pi_i(s_{-i}^*, t_i)$ for all t_i

$\ldots, s_r) \leftarrow$ player's choices are made independently

Nash equilibrium

Strategy vector

$\vec{s} \in S$ [player i uses strategy s_i]

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$$\pi_i(s^*) \geq \pi_i(s_{-i}^*, t_i)$$

for all t_i

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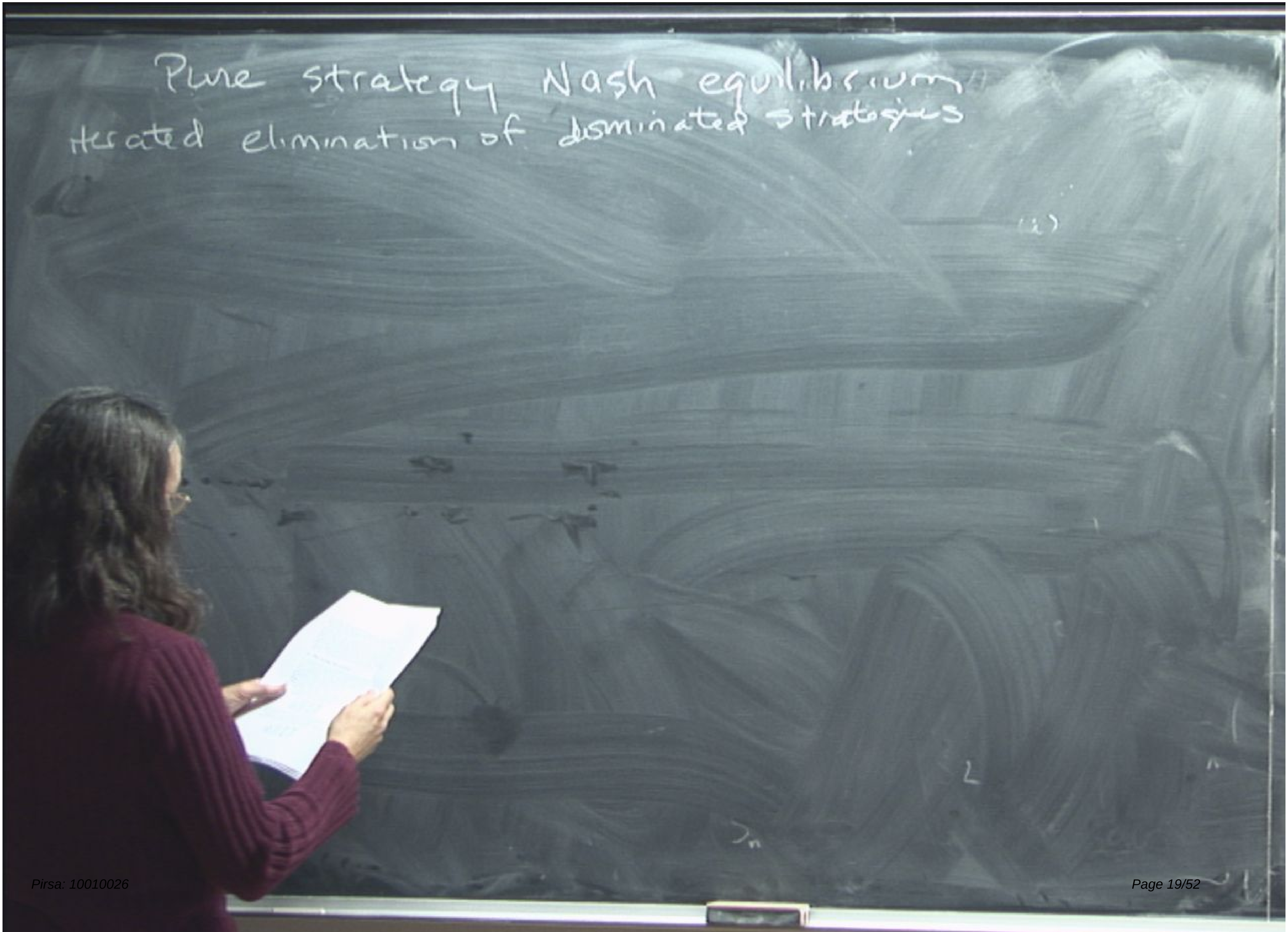
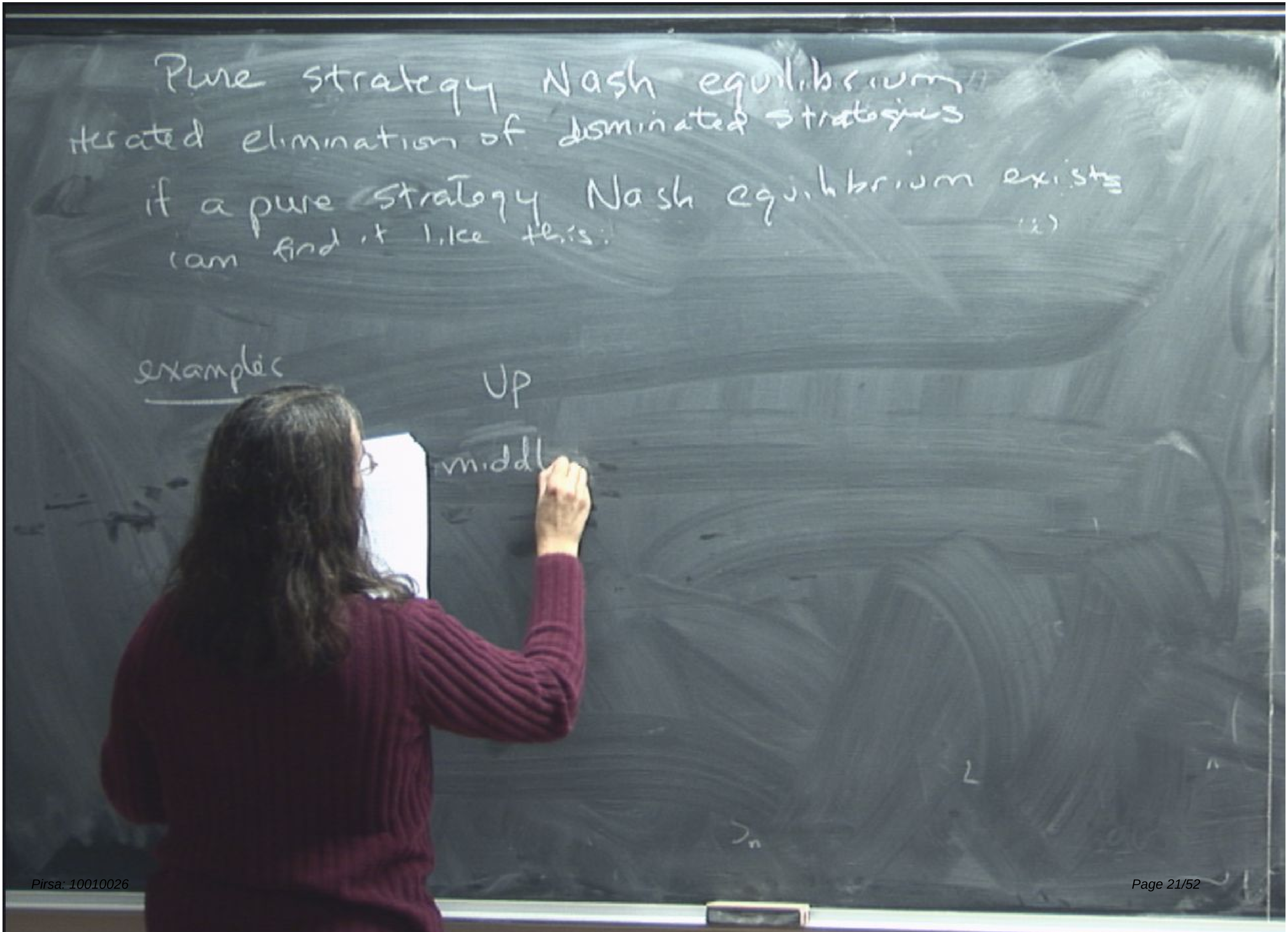

Pure strategy Nash equilibrium
Iterated elimination of dominated strategies

Pure strategy Nash equilibrium

Iterated elimination of dominated strategies

if a pure strategy Nash equilibrium exists (i)

can find it like this:


Pure strategy Nash equilibrium
iterated elimination of dominated strategies
if a pure strategy Nash equilibrium exists
can find it like this:
examples
Up
middl

Pure strategy Nash equilibrium

Iterated elimination of dominated strategies

if a pure strategy Nash equilibrium exists
can find it like this:

examples

	left	center	right
Up			
middle			
down			

Pure strategy Nash equilibrium

Iterated elimination of dominated strategies

if a pure strategy Nash equilibrium exists can find it like this:

examples

	left	center	right
Up	(12,4)	(2,6)	(6,7)
[illegible]	(5,6)	(1,5)	(4,9)
down	(11,2)	(4,3)	(7,1)

player 2 (column)

player 1 (row)

Pure strategy Nash equilibrium

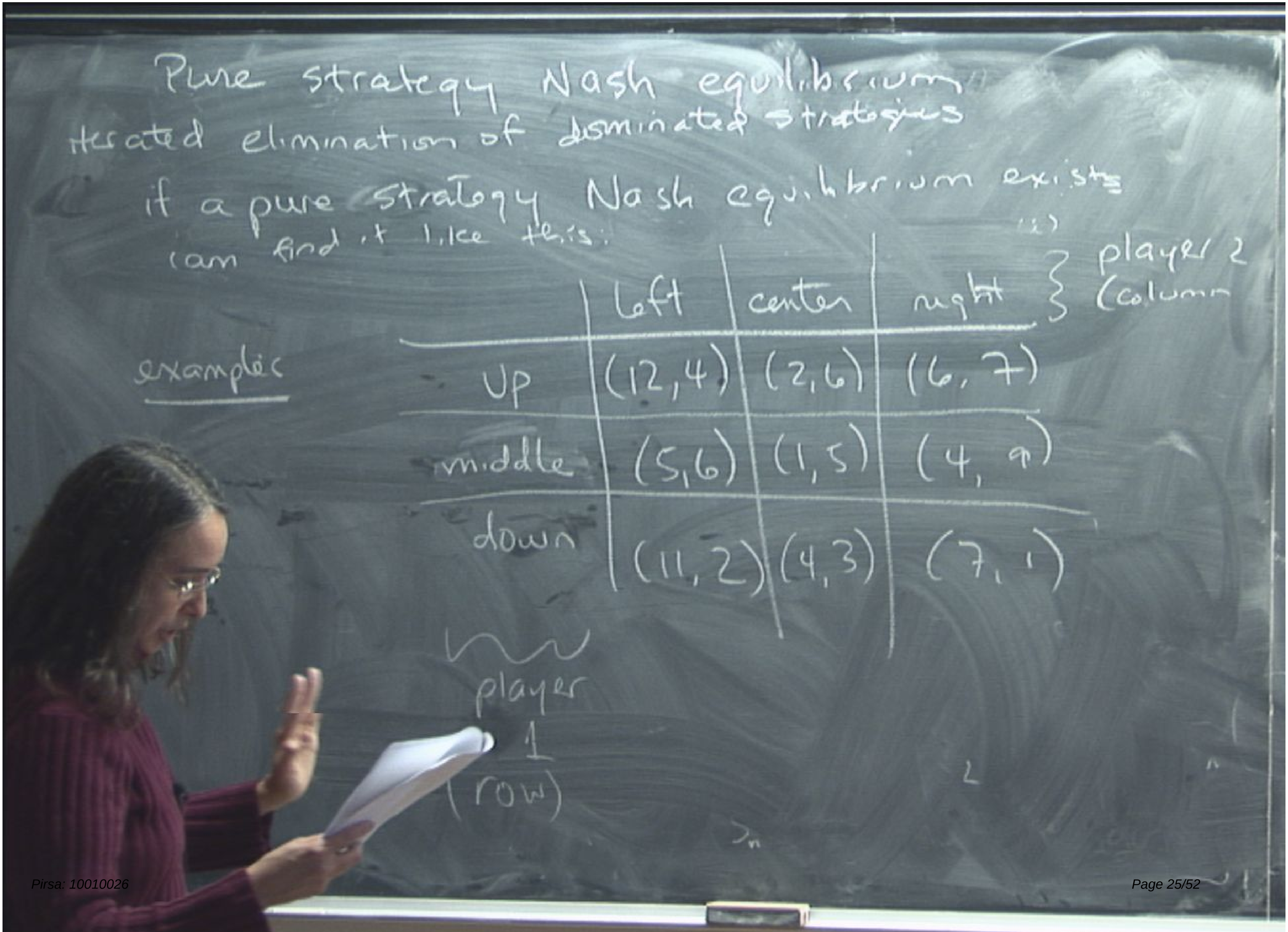
Iterated elimination of dominated strategies

if a pure strategy Nash equilibrium exists
can find it like this:

examples

player 2 (column

player 1 (row)	left	center	right
Up	(12,4)	(2,6)	(6,7)
middle	(5,6)	(1,5)	(4,9)
down	(11,2)	(4,3)	(7,1)


Pure strategy Nash equilibrium
iterated elimination of dominated strategies
if a pure strategy Nash equilibrium exists
can find it like this:
examples
player 2
(column
left
center
right
Up
(12,4)
(2,6)
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(1,5)
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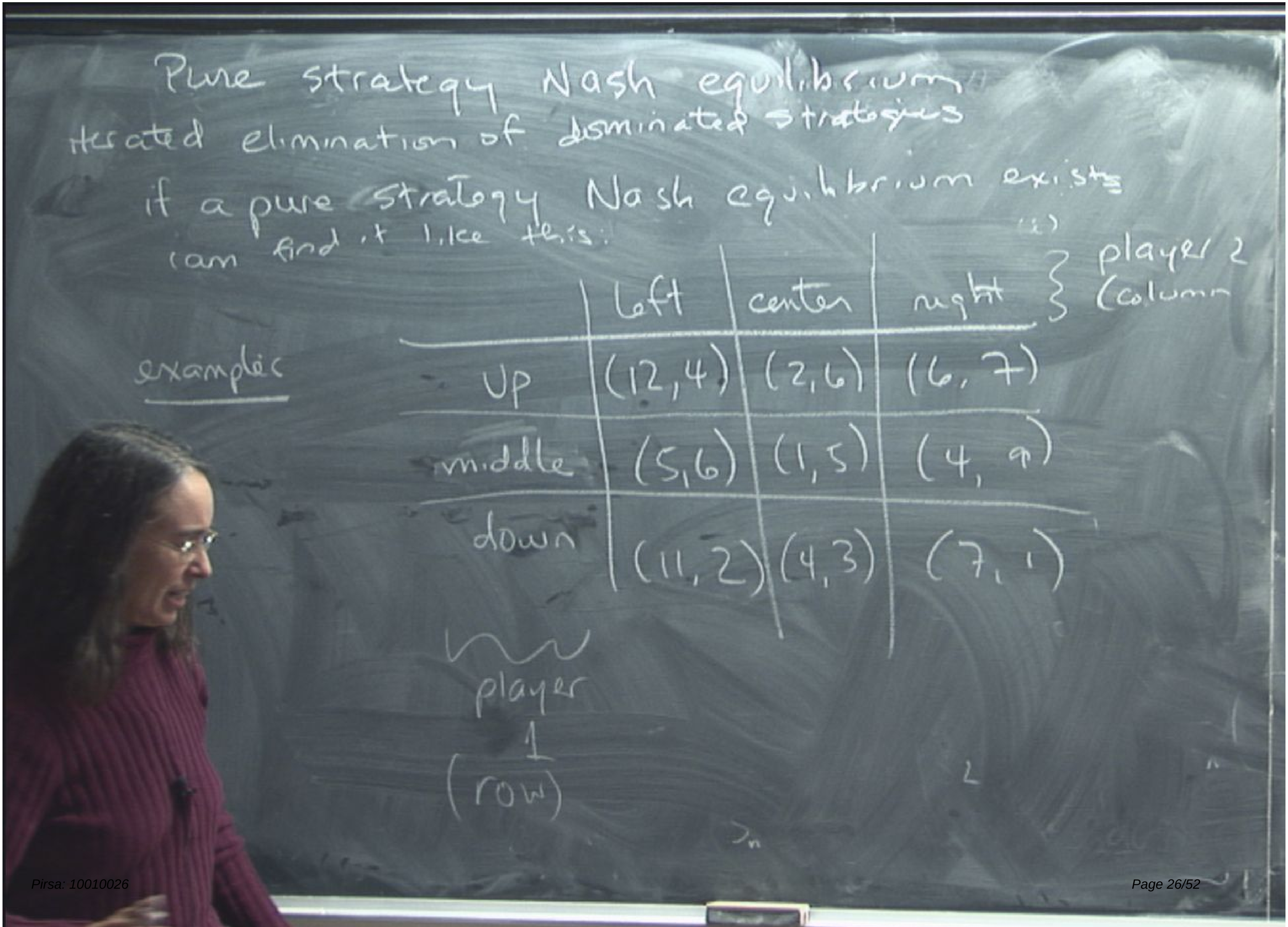
down
(11,2)
(4,3)
(7,1)
player
1
(row)


Pure strategy Nash equilibrium
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if a pure strategy Nash equilibrium exists
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player 2
(column
left
center
right
up
(12,4)
(2,6)
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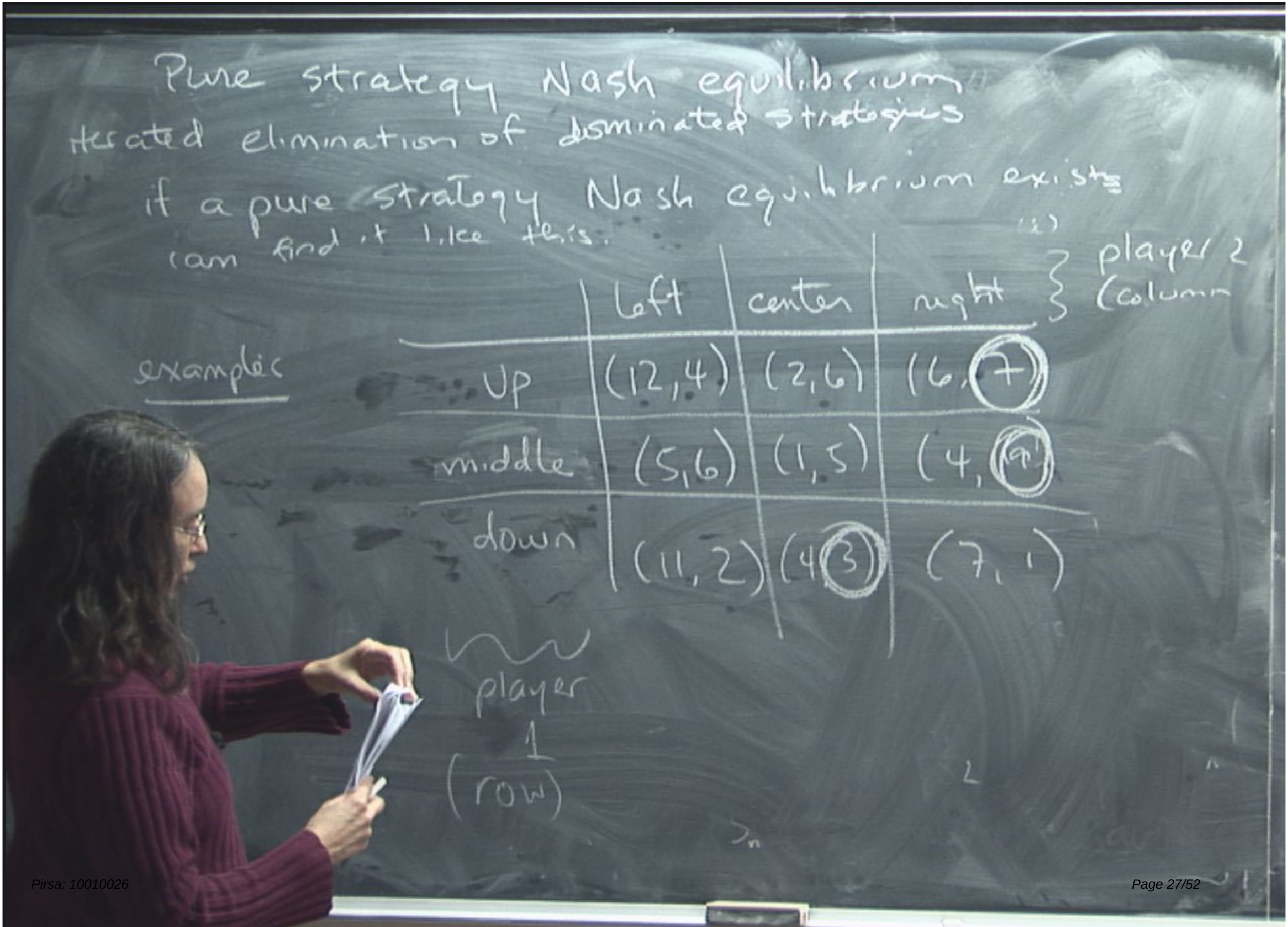
1
(row)


Pure strategy Nash equilibrium
iterated elimination of dominated strategies
if a pure strategy Nash equilibrium exists
can find it like this:
player 2 (column)
left
center
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examples
up
(12,4)
(2,6)
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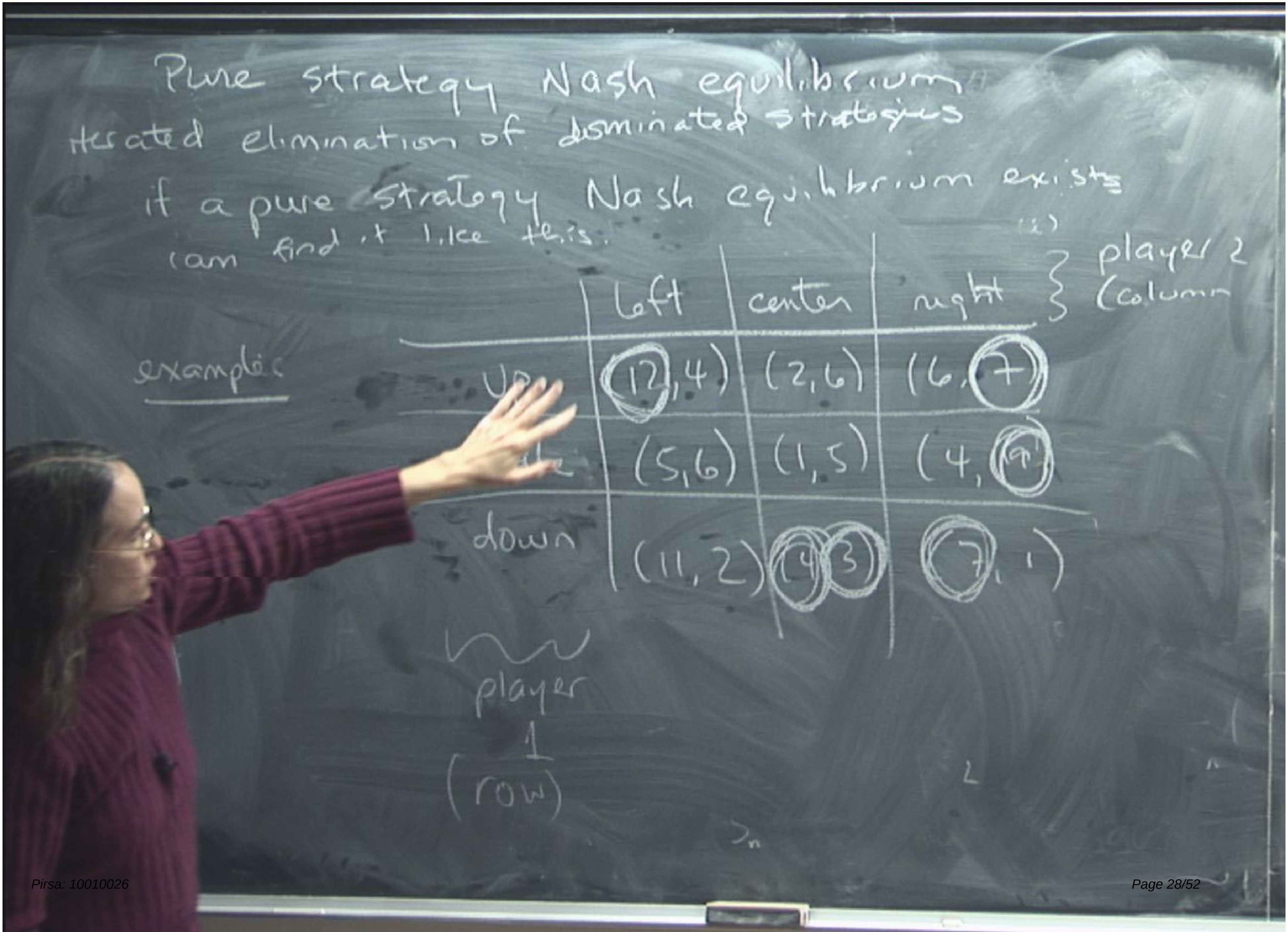
player 1 (row)


Pure strategy Nash equilibrium
iterated elimination of dominated strategies
if a pure strategy Nash equilibrium exists
can find it like this:
examples
left
center
right
player 2
(column
up
(12,4)
(2,6)
(6,7)
(5,6)
(1,5)
(4,9)
down
(11,2)
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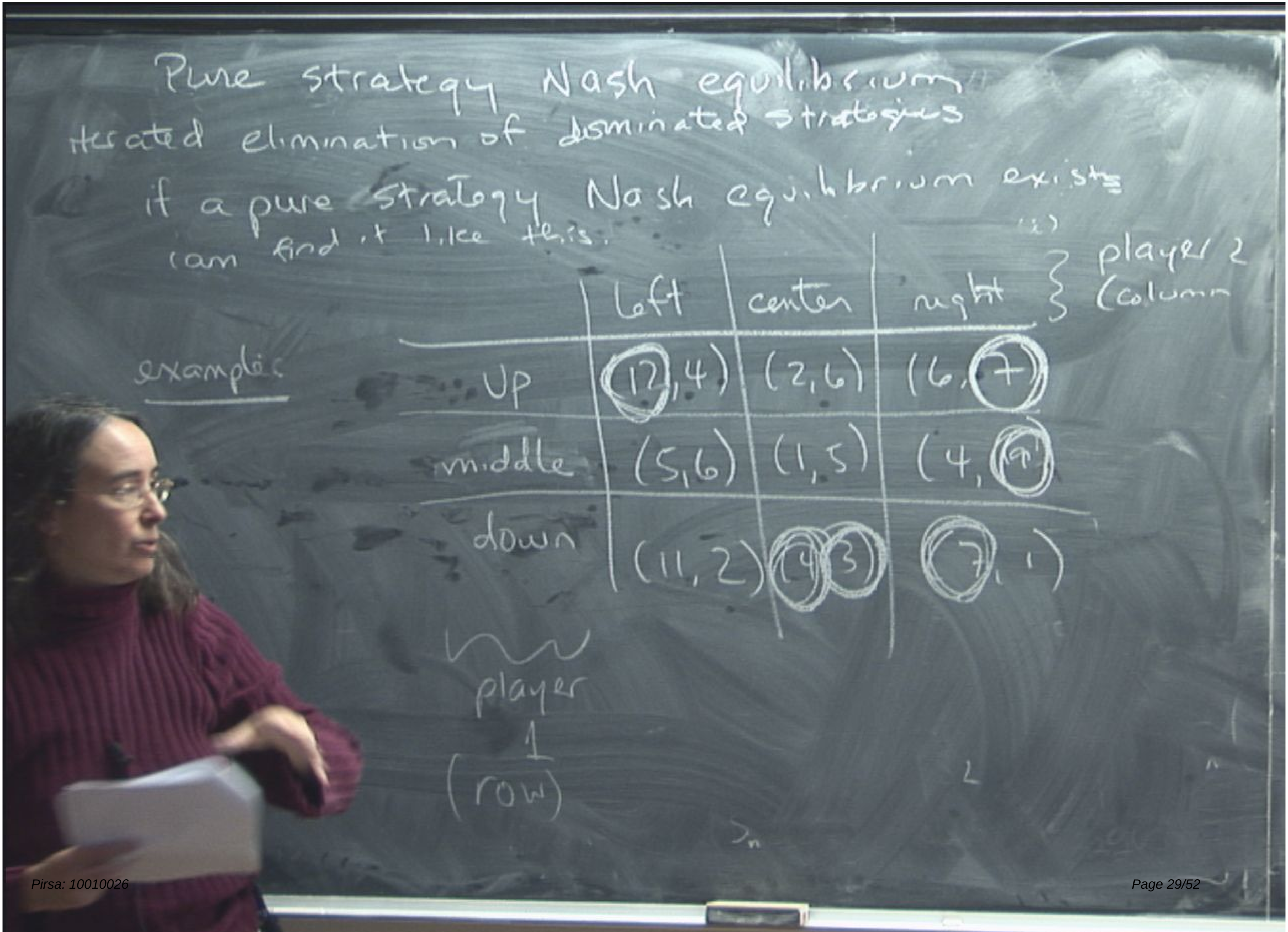
(7,1)
player
1
(row)


Pure strategy Nash equilibrium
iterated elimination of dominated strategies
if a pure strategy Nash equilibrium exists
can find it like this:
player 2 (column)
left
center
right
examples
up
(12,4) (2,6) (6,7)
middle
(5,6) (1,5) (4,9)
down
(11,2) (4,3) (7,1)
player 1 (row)

Pure strategy Nash equilibrium

iterated elimination of dominated strategies

if a pure strategy Nash equilibrium exists

can find it like this:

examples

player 1 (row)	left	center	right
up	$(\boxed{12},4)$	$(2,6)$	$(6,7)$
middle	$(5,6)$	$(1,5)$	$(4,9)$
down	$(11,2)$	$(\boxed{4},3)$	$(\boxed{7},1)$

} player 2 (column

Pure strategy Nash equilibrium

iterated elimination of dominated strategies

if a pure strategy Nash equilibrium exists

can find it like this:

player 2 (column)

examples

player 1 (row)	left	center	right
up	($\boxed{12}$,4)	(2,6)	(6,7)
middle	(5,6)	(1,5)	(4,9)
down	(11,2)	($\boxed{14}$,3)	($\boxed{7}$,1)

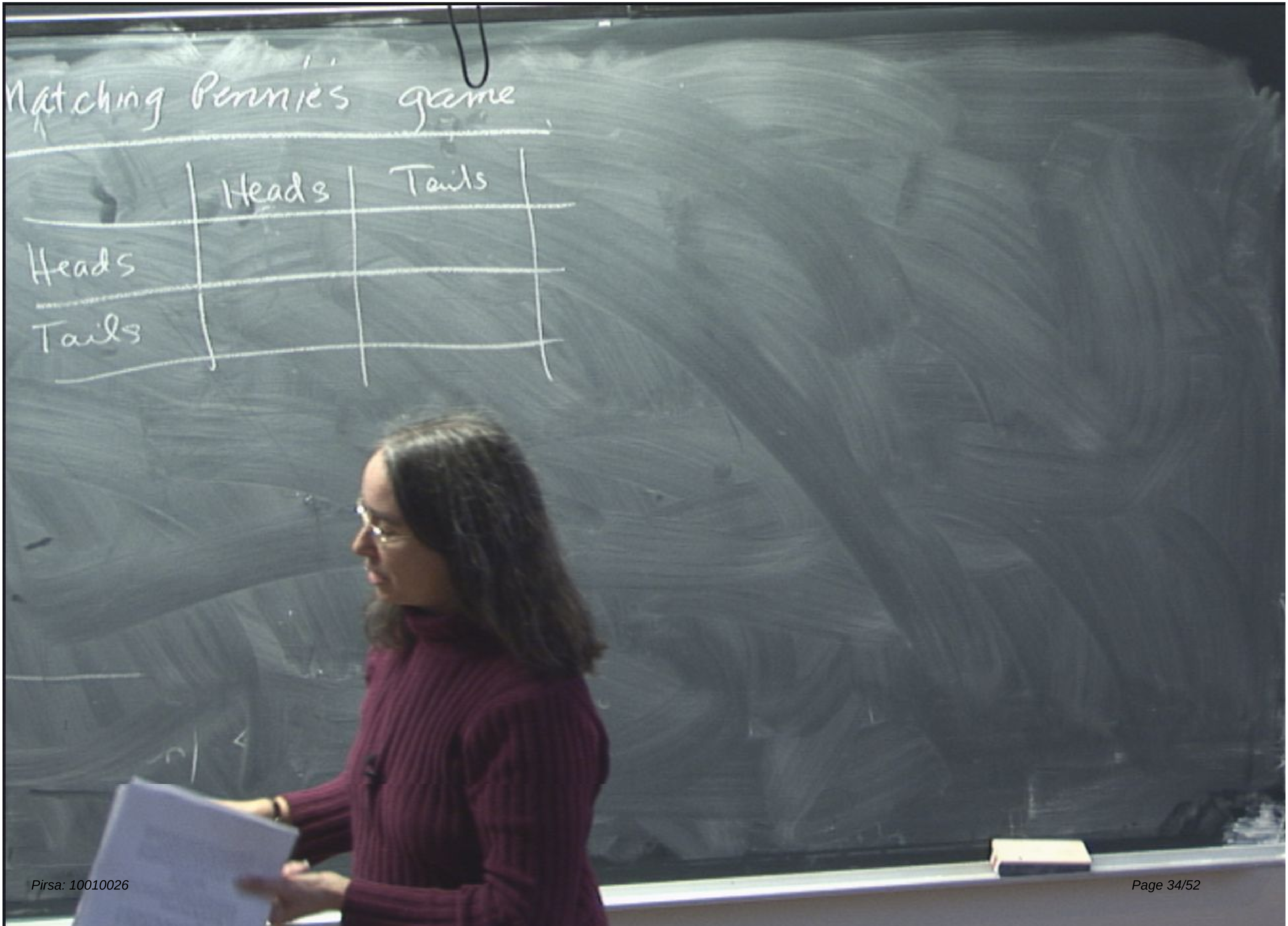
Pure strategy Nash equilibrium

if a pure strategy Nash equilibrium exists can find it like this:

examples

player 1 (row) \ player 2 (column)	left	center	right
up	(12, 4)	(2, 6)	(6, 7)
middle	(5, 6)	(1, 5)	(4, 9)
down	(11, 2)	(14, 3)	(7, 1)


Matching Pennies game


Matching Pennies game
Heads
Tails
Heads
Tails

Matching Pennies game

	Heads	Tails
Heads	(1, -1)	(-1, 1)
Tails	(-1, 1)	(1, -1)

Matching Pennies game

	Heads	Tails
Heads	($\boxed{1}$, −1)	(−1, 1)
Tails	(−1, 1)	($\boxed{1}$, −1)

Matching Pennies game

	Heads	Tails
Heads	(1, -1)	(-1, 1)
Tails	(-1, 1)	(1, -1)

looking for strategy profile

(σ_1^*, σ_2^*) for which

$$\pi_1(\sigma_1^*, \sigma_2^*) \geq \pi_1(\tilde{\sigma}_1, \sigma_2^*)$$

$$= \pi_1(\tilde{\sigma}_1, \sigma_{-1})$$

Matching Pennies game

	Heads	Tails
Heads	$(1, -1)$	$(-1, 1)$
Tails	$(-1, 1)$	$(1, -1)$

looking for strategy profile

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$$= \pi_1(\tilde{\sigma}_1, \sigma_{-1})$$

x_1 = prob that player 1 plays heads

x_2 = prob that player 2 plays heads

u

Matching Pennies game

	Heads	Tails
Heads	$(\boxed{1}, -1)$	$(-1, 1)$
Tails	$(-1, 1)$	$(\boxed{1}, -1)$

looking for strategy profile (σ_1^*, σ_2^*) for which

$$\pi_1(\sigma_1^*, \sigma_2^*) \geq \pi_1(\tilde{\sigma}_1, \sigma_2^*)$$
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x_1 = prob that player 1 plays heads

x_2 = prob that player 2 plays heads

$u_1(x_1, x_2) =$

$u_2(x_1, x_2) =$

Matching Pennies game

	Heads	Tails
Heads	$(\boxed{1}, -1)$	$(-1, 1)$
Tails	$(-1, 1)$	$(\boxed{1}, -1)$

looking for strategy profile (σ_1^*, σ_2^*) for which

$$\pi_1(\sigma_1^*, \sigma_2^*) \geq \pi_1(\tilde{\sigma}_1, \sigma_2^*)$$

$$= \pi_1(\tilde{\sigma}_1, \sigma_{-1})$$

x_1 = prob that player 1 plays heads

x_2 = prob that player 2 plays heads

$$u_1(x_1, x_2) = x_1 x_2 (1) + (1-x_1) x_2 (-1)$$

$$u_2(x_1, x_2) = x_1 x_2 (-1)$$

(M)atching Pennies game

	Heads	Tails
Heads	$(1, -1)$	$(-1, 1)$
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looking for strategy profile

(σ_1^*, σ_2^*) for which

$$\pi_i(\sigma_1^*, \sigma_2^*) \geq \pi_1(\tilde{\sigma}_i, \sigma_2^*)$$

$$= \pi_1(\tilde{\sigma}_i, \sigma_{-i})$$

X_1 = prob that player 1 plays heads

X_2 = prob that player 2 plays heads

$$u_1(X_1, X_2) = X_1 X_2 (1) + (1-X_1) X_2 (-1)$$

$$u_2(X_1, X_2) = X_1 X_2 (-1)$$

atching Pennies game

	Heads	Tails
Heads	$(1, -1)$	$(-1, 1)$
Tails	$(-1, 1)$	$(1, -1)$

looking for strategy profile

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$$\pi_i(\sigma_1^*, \sigma_2^*) \geq \pi_i(\tilde{\sigma}_i, \sigma_2^*)$$

$$= \pi_i(\tilde{\sigma}_i, \sigma_{-i})$$

X_1 = prob that player 1 plays heads

X_2 = prob that player 2 plays heads

$$u_1(X_1, X_2) = X_1 X_2 (1) + (1-X_1) X_2 (-1)$$
$$- X_1(1-X_2) + (1-X_1)(1-X_2)$$

$$u_2(X_1, X_2) = X_1 X_2 (-1) + (1-X_1) X_2$$
$$+ X_1(1-X_2) - (1-X_1)(1-X_2)$$

Matching Pennies game

	Heads	Tails
Heads	$(\boxed{1}, -1)$	$(-1, 1)$
Tails	$(-1, 1)$	$(\boxed{1}, -1)$

looking for strategy profile

(σ_1^*, σ_2^*) for which

$$\pi_i(\sigma_1^*, \sigma_2^*) \geq \pi_i(\tilde{\sigma}_i, \sigma_2^*)$$

$$= \pi_i(\tilde{\sigma}_i, \sigma_{-i})$$

x_1 = prob that player 1 plays heads

x_2 = prob that player 2 plays heads

$$u_1(x_1, x_2) = x_1 x_2 (1) + (1-x_1) x_2 (-1)$$
$$- x_1(1-x_2) + (1-x_1)(1-x_2)$$
$$= 4x_1x_2 - 2x_1 - 2x_2 + 1$$

$$u_2(x_1, x_2) = x_1 x_2(-1) + (1-x_1) x_2$$
$$+ x_1(1-x_2) - (1-x_1)(1-x_2)$$
$$= -4x_1x_2 + 2x_1 + 2x_2 - 1$$

atching Pennies game

	Heads	Tails
Heads	$(1,-1)$	$(-1,1)$
Tails	$(-1,1)$	$(1,-1)$

looking for strategy profile (σ_1^*, σ_2^*) for which

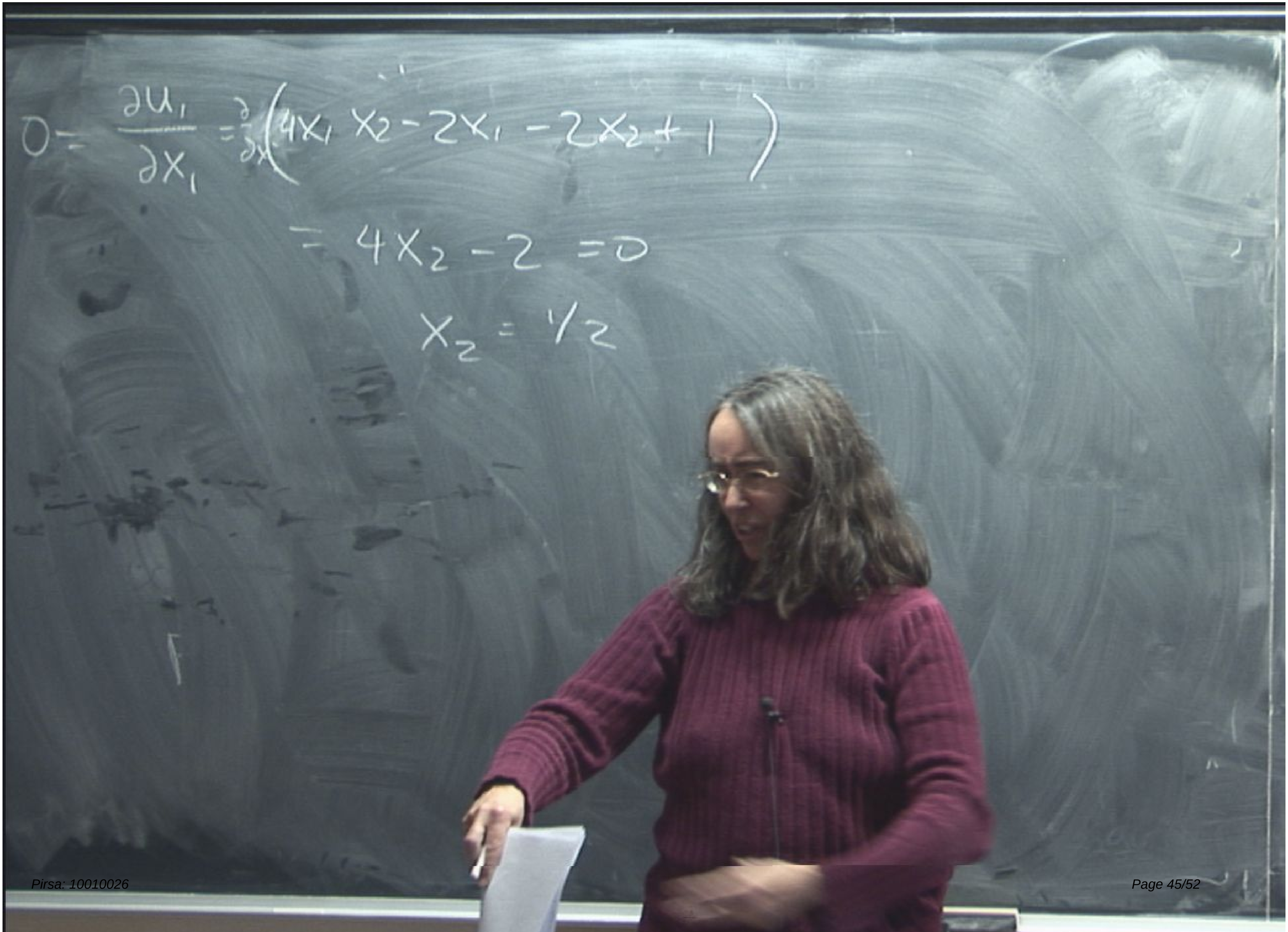
$$\pi_i(\sigma_i^*, \sigma_2^*) \geq \pi_i(\tilde{\sigma}_i, \sigma_2^*)$$
$$= \pi_i(\tilde{\sigma}_i, \sigma_{-i})$$

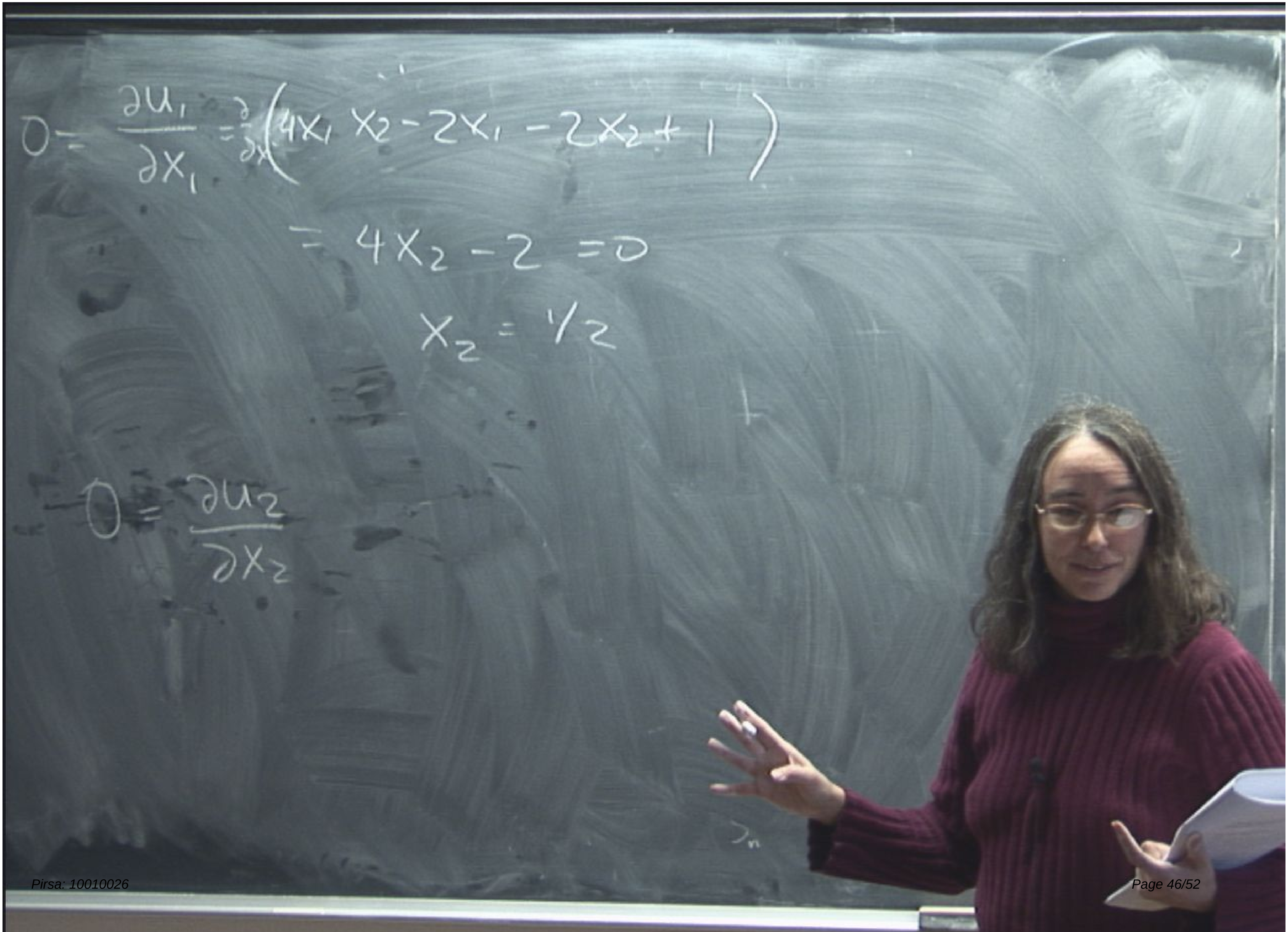
x_1 = prob that player 1 plays heads

x_2 = prob that player 2 plays heads

$$u_1(x_1, x_2) = x_1 x_2 (1) + (1-x_1) x_2 (-1)$$
$$- x_1(1-x_2) + (1-x_1)(1-x_2)$$
$$= 4x_1x_2 - 2x_1 - 2x_2 + 1$$

$$u_2(x_1, x_2) = x_1 x_2 (-1) + (1-x_1) x_2$$
$$+ x_1(1-x_2) - (1-x_1)(1-x_2)$$
$$= -4x_1x_2 + 2x_1 + 2x_2 - 1$$


$0 = \frac{\partial u_1}{\partial x_1} = \frac{\partial}{\partial x_1}(4x_1 x_2 - 2x_1 - 2x_2 + 1)$
$= 4x_2 - 2 = 0$
$x_2 = 1/2$


$0 = \frac{\partial u_1}{\partial x_1} = \frac{\partial}{\partial x_1}\left(4x_1 x_2 - 2x_1 - 2x_2 + 1\right)$
$= 4x_2 - 2 = 0$
$x_2 = 1/2$
$0 = \frac{\partial u_2}{\partial x_2}$

$$0 = \frac{\partial u_1}{\partial x_1} = \frac{\partial}{\partial x_1}\left(4x_1 x_2 - 2x_1 - 2x_2 + 1\right)$$

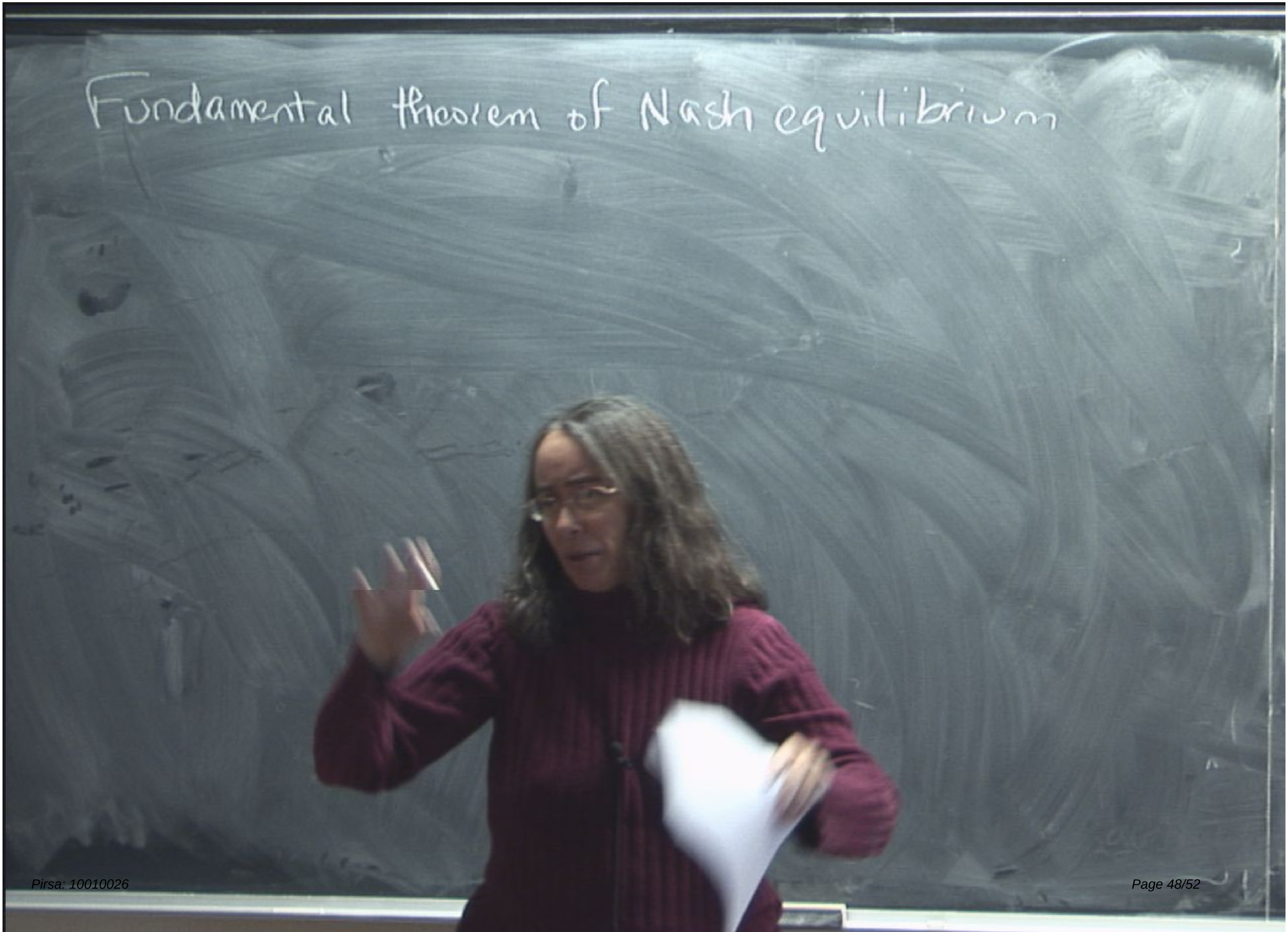
$$= 4x_2 - 2 = 0$$

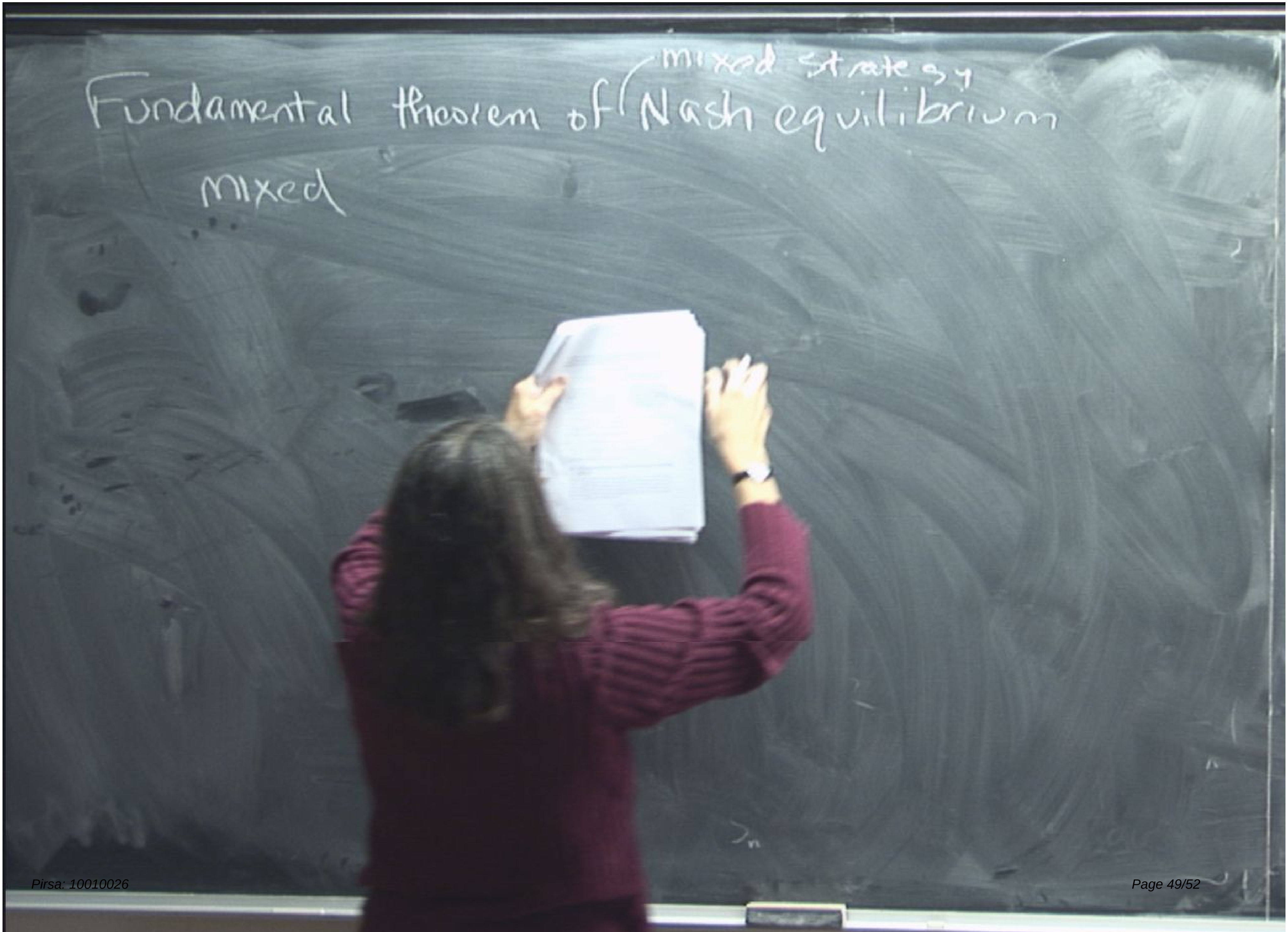
$$x_2 = 1/2$$

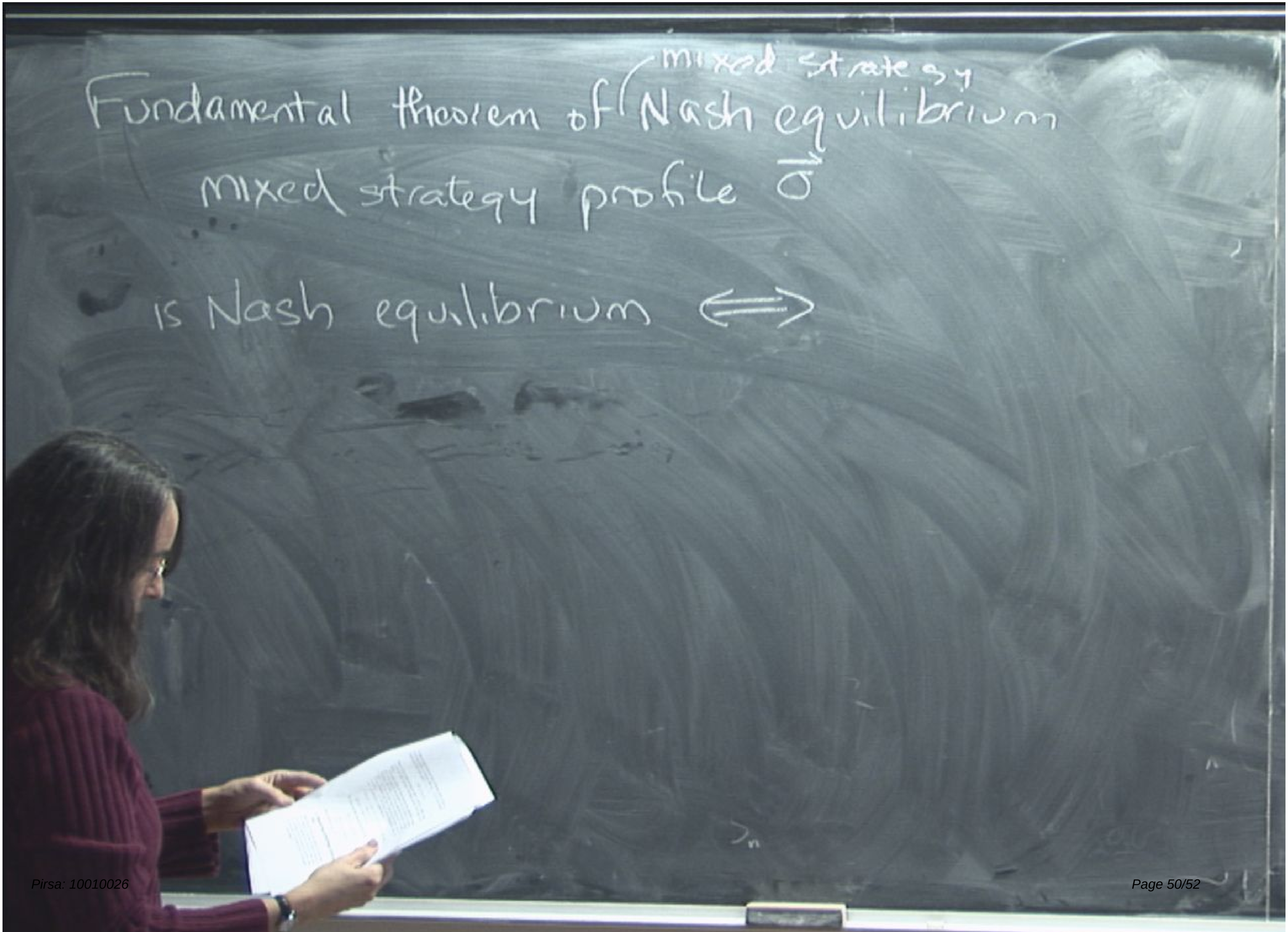
$$0 = \frac{\partial u_2}{\partial x_2} = \frac{\partial}{\partial x_2}\left(-4x_1 x_2 + 2x_1 + 2x_2 - 1\right) = 0$$

$$-4x_1 + 2 = 0$$

$$x_1 = 1/2$$


Fundamental theorem of Nash equilibrium


mixed strategy
Fundamental theorem of Nash equilibrium
mixed


mixed strategy
Fundamental theorem of Nash equilibrium
mixed strategy profile $\vec{\sigma}$
is Nash equilibrium $\Longleftrightarrow$

Fundamental theorem of mixed strategy Nash equilibrium

mixed strategy profile $\vec{\sigma}$

is Nash equilibrium $\iff$

for any player with pure strategy set S_i

if $s, s' \in S_i$ that occur with $p \geq 0$ in σ_i,

then $\pi_i(s, \sigma_{-i}) = \pi_i(s', \sigma_{-i})$

mixed strategy

Fundamental theorem of Nash equilibrium

mixed strategy profile $\vec{\sigma}$

is Nash equilibrium $\iff$

for any player with pure strategy set S_i

1) if $s, s' \in S_i$ that occur with $p \geq 0$ in σ_i

then $\pi_i(s, \sigma_{-i}) = \pi_i(s', \sigma_{-i})$

2) If s occurs with $p > 0$ in σ_i and

s' occurs w $p = 0$ in σ_i

then payoff for $s' \leq$ payoff for s