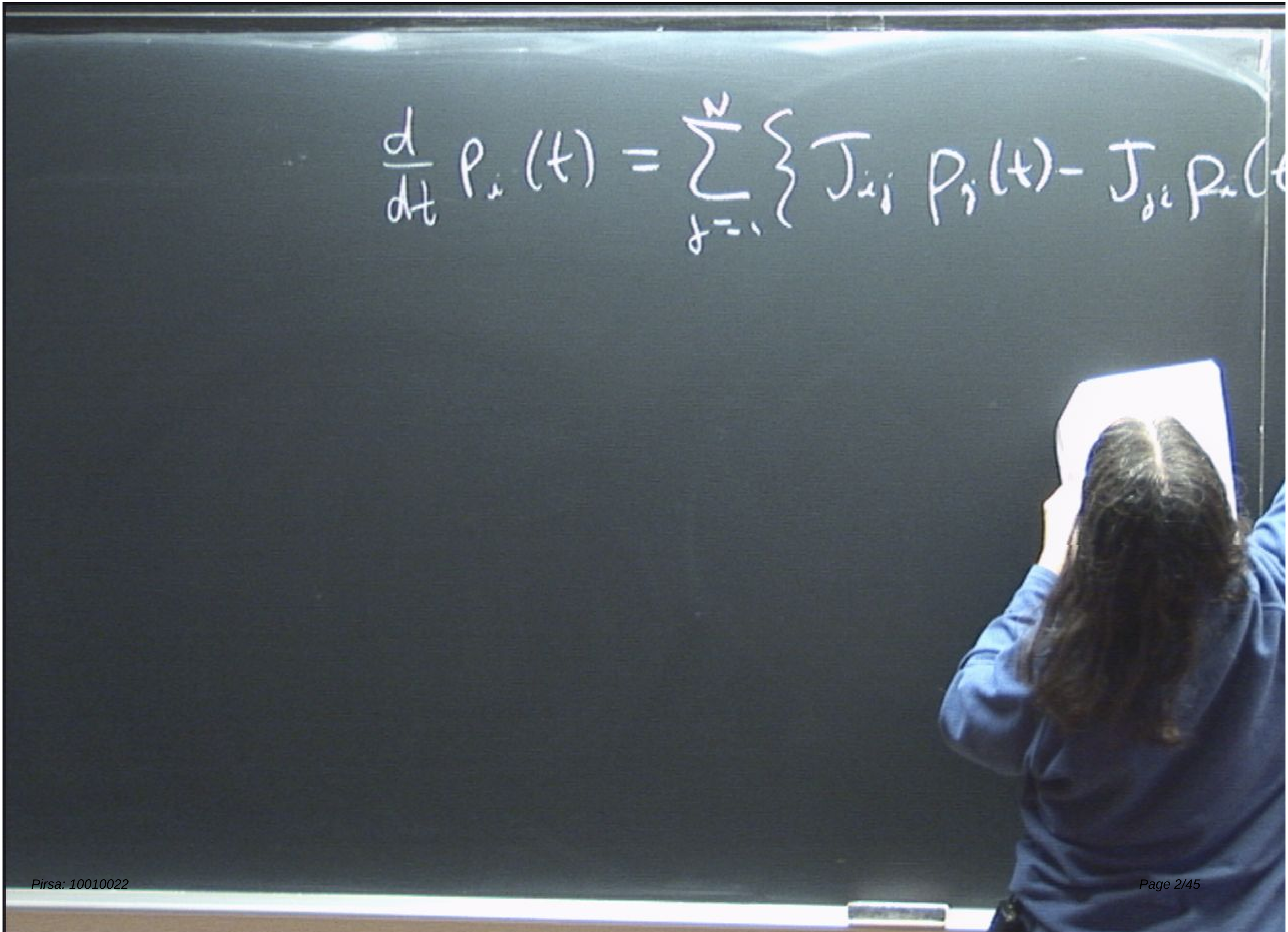
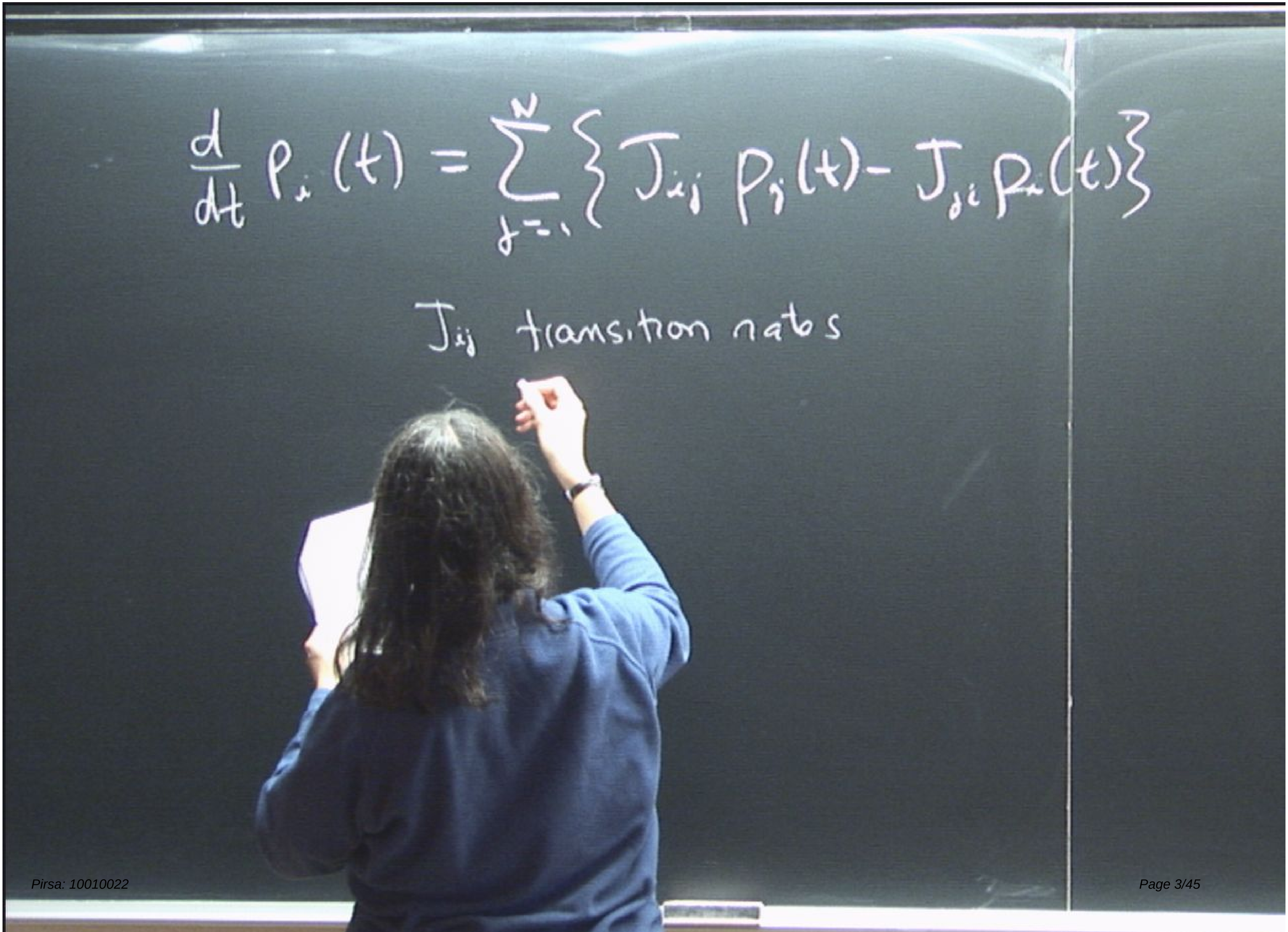
Title: Dynamical Systems - Review (PHYS 607) - Lecture 10

Date: Jan 15, 2010 09:00 AM

URL: http://pirsa.org/10010022

Abstract:


$\frac{d}{dt} P_i(t) = \sum_{j=1}^{N} \{ J_{ij} P_j(t) - J_{ji} P_i(t$


$\frac{d}{dt} P_i(t) = \sum_{j=1}^{N} \left\{ J_{ij} P_j(t) - J_{ji} P_i(t) \right\}$
J_{ij} transition rates

$$\frac{d}{dt} P_i(t) = \sum_{j=1}^{N} \left\{ J_{ij}\, P_j(t) - J_{ji}\, P_i(t) \right\}$$

J_{ij} transition rates

$j \rightarrow i$

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$J_{ij} \geq 0$

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J_{ij} transition rates

from $j \longrightarrow i$ to

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from $j \to i$ to

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Steady state $\bar{p}_i$

$$\sum_j W_{ij} \bar{p}_j = 0$$

where $W_{ij} = J_{ij} - \delta_{ij} \sum_k J_{ki}$

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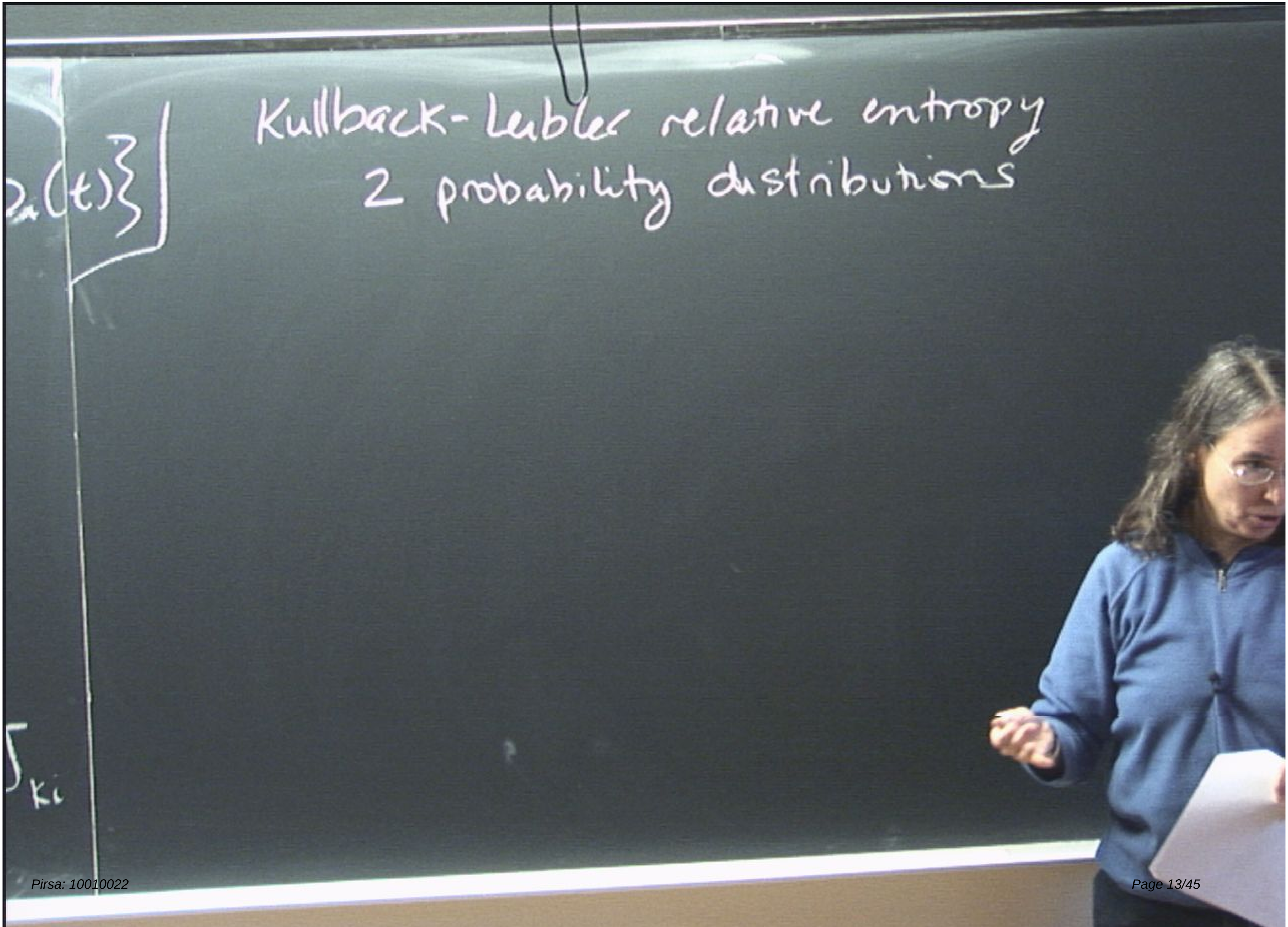
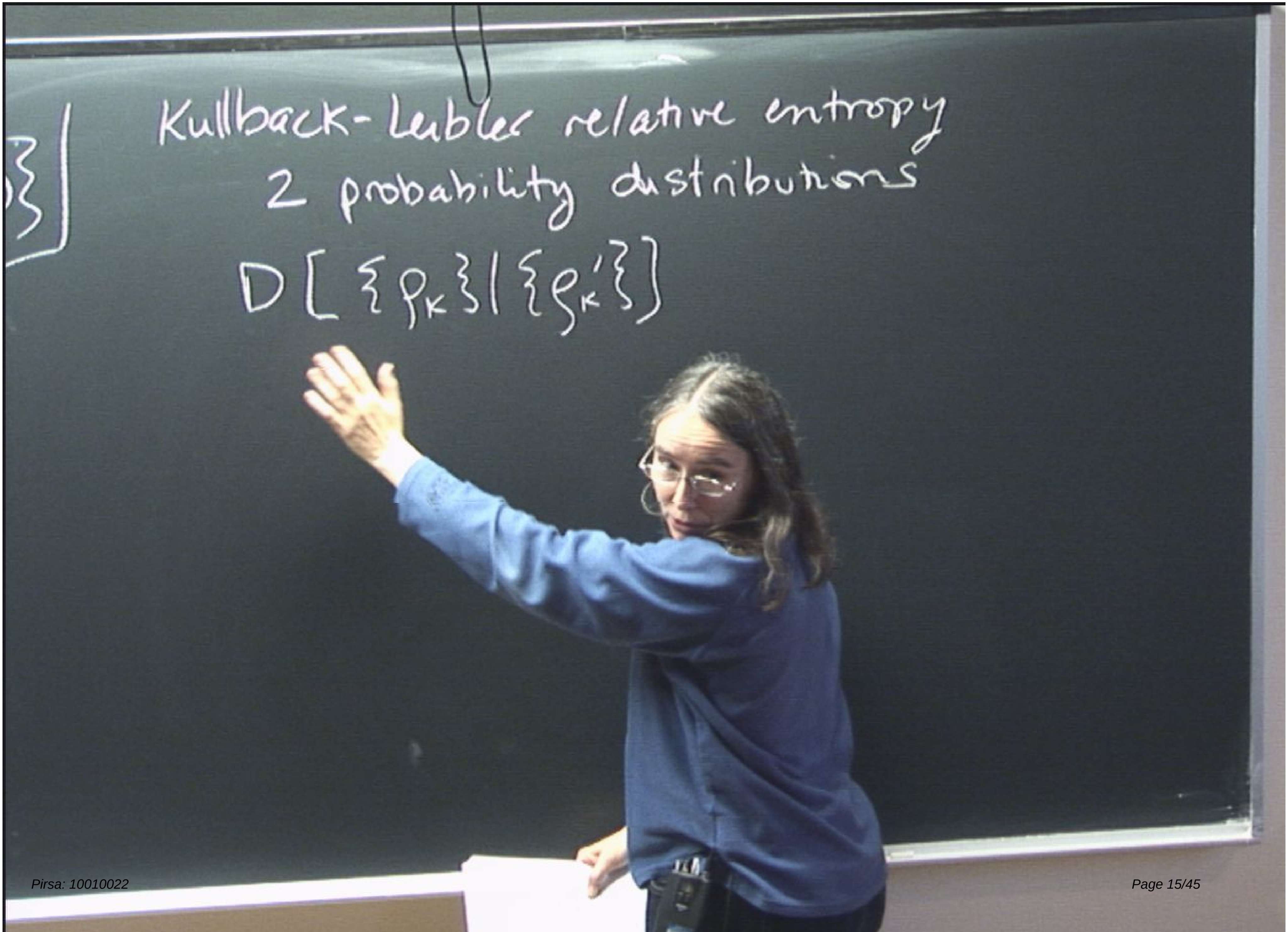

Kullback-Leibler relative entropy
2 probability distributions

Kullback-Leibler relative entropy

2 probability distributions

$p_a(t)\}$

$\int_{ki}$


Kullback-Leibler relative entropy
2 probability distributions
$D[\{p_k\} \| \{p'_k\}]$

Kullback-Leibler relative entropy

2 probability distributions

$$D[\{p_K\} \| \{p_K'\}] \equiv \sum_K p_K \ln\left(\frac{p_K}{p_K'}\right)$$

Kullback-Leibler relative entropy

2 probability distributions

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if I choose p'_K = uniform distribution (n possibilities)

$$\sum_K p_K \ln p_K - \sum_K p_K \ln\left(\frac{1}{N}\right)$$

$$= \log n - \sum_K p_K \ln p_K$$

Kullback-Leibler relative entropy

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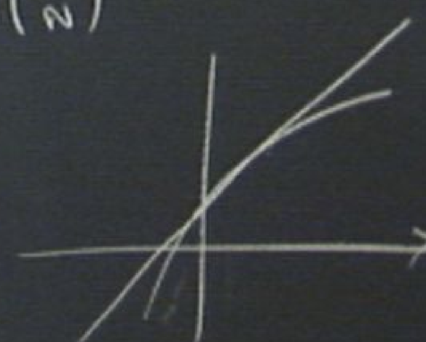

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don't dep... on p's

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if I

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$$\frac{d}{dt} \rho_i(t) = \sum_{j=1}^{N} \left\{ J_{ij}\, \rho_j(t) - J_{ji}\, \rho_i(t) \right\}$$

J_{ij} transition rates

from $j \longrightarrow i$ to

$J_{ij} \geq 0$

don't depend on p's

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Kullback-Leibler relative entropy

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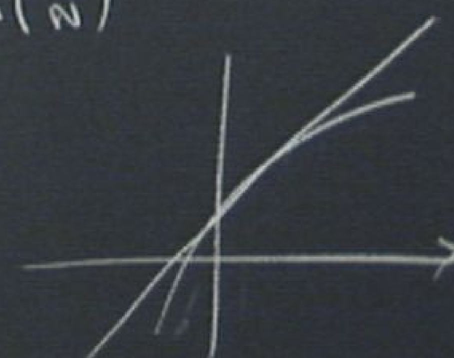

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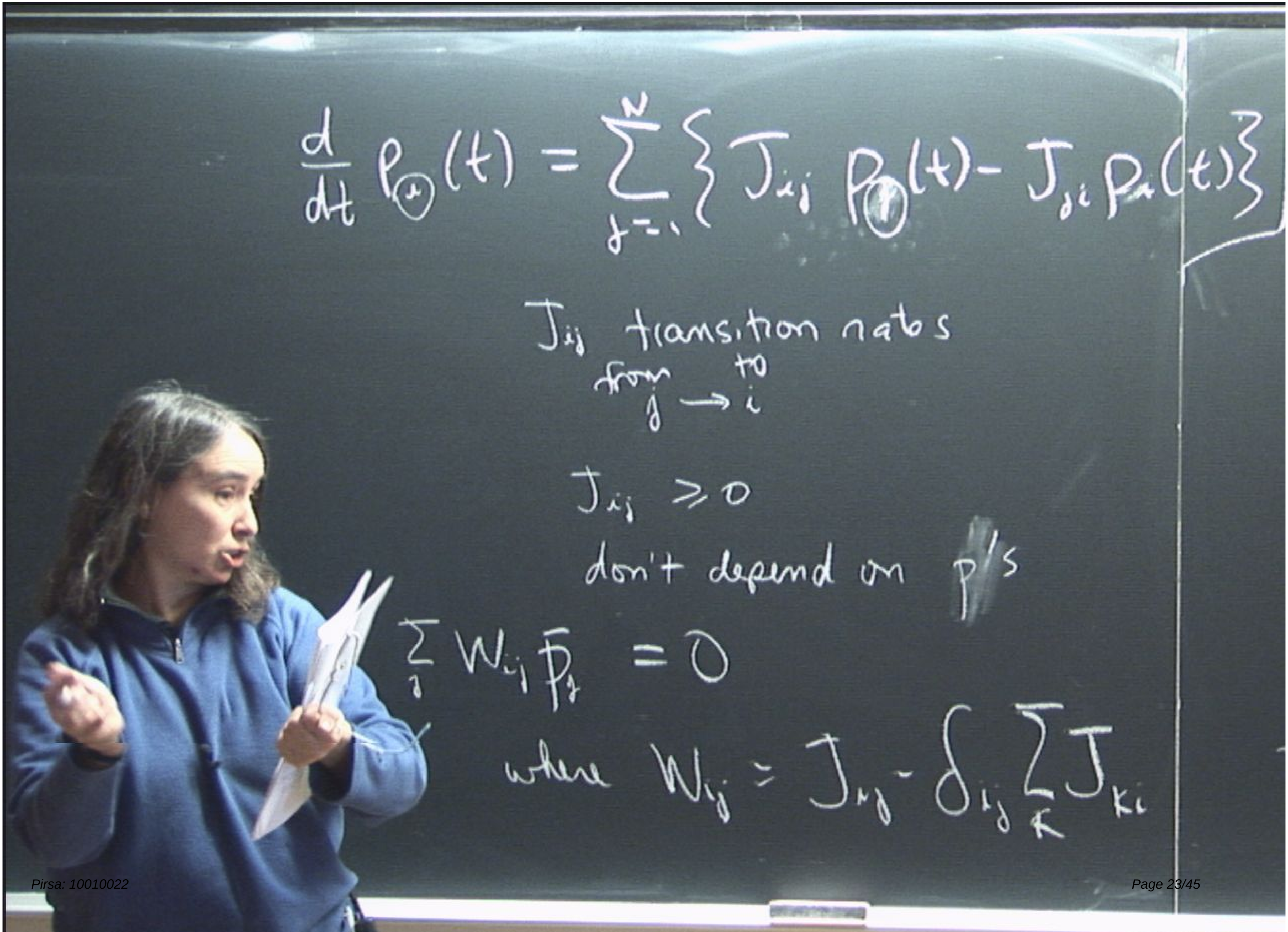
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Kullback-Leibler relative entropy

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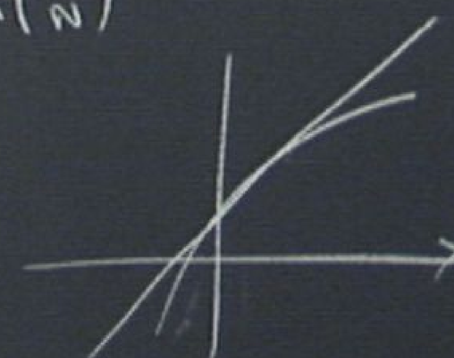
$$= \log N - \sum_k \rho_k \ln \rho_k$$



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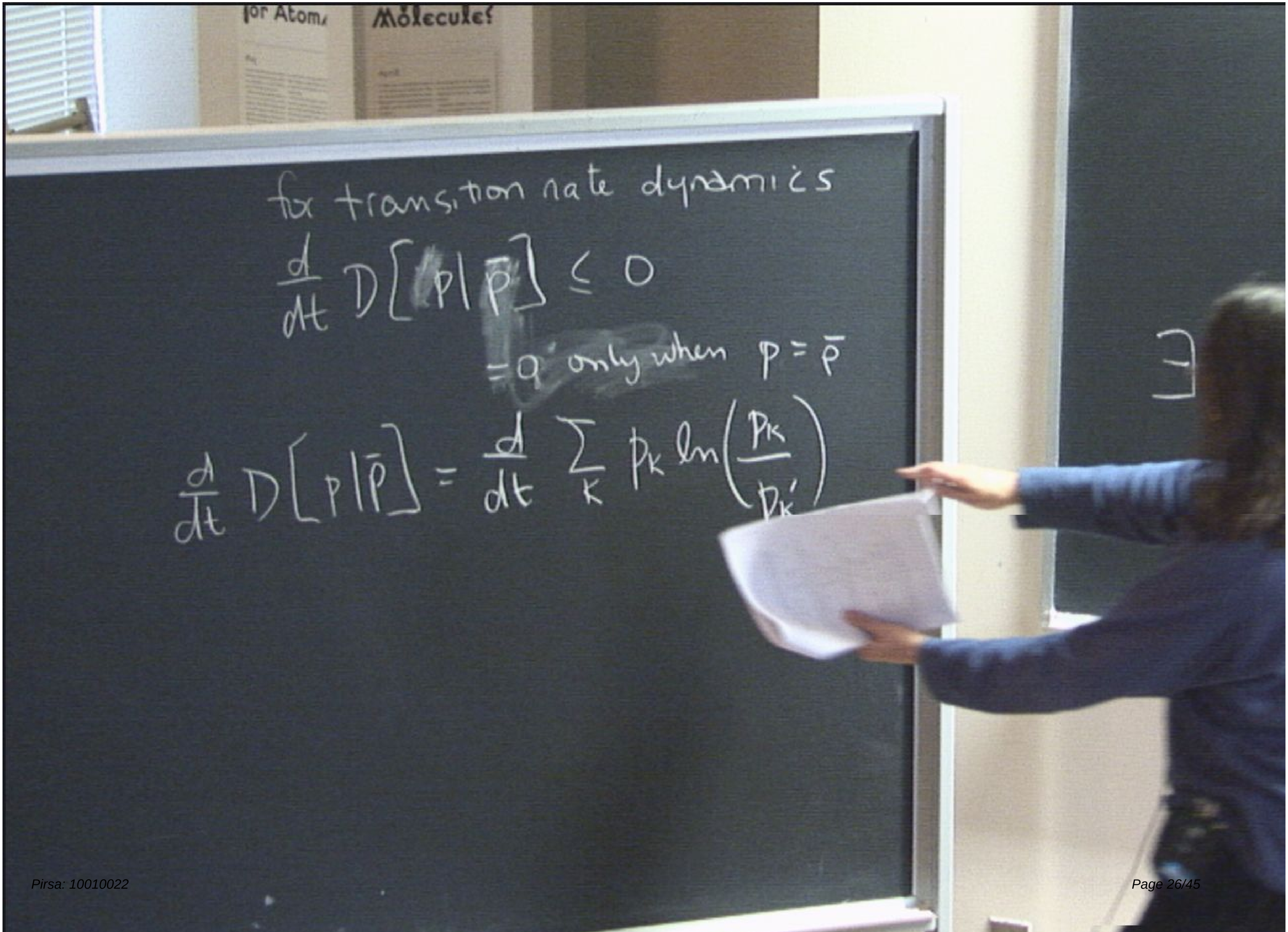
for transition rate dynamics
$\frac{d}{dt} D[p|\bar{p}] \leq 0$
$= 0$ only when $p = \bar{p}$
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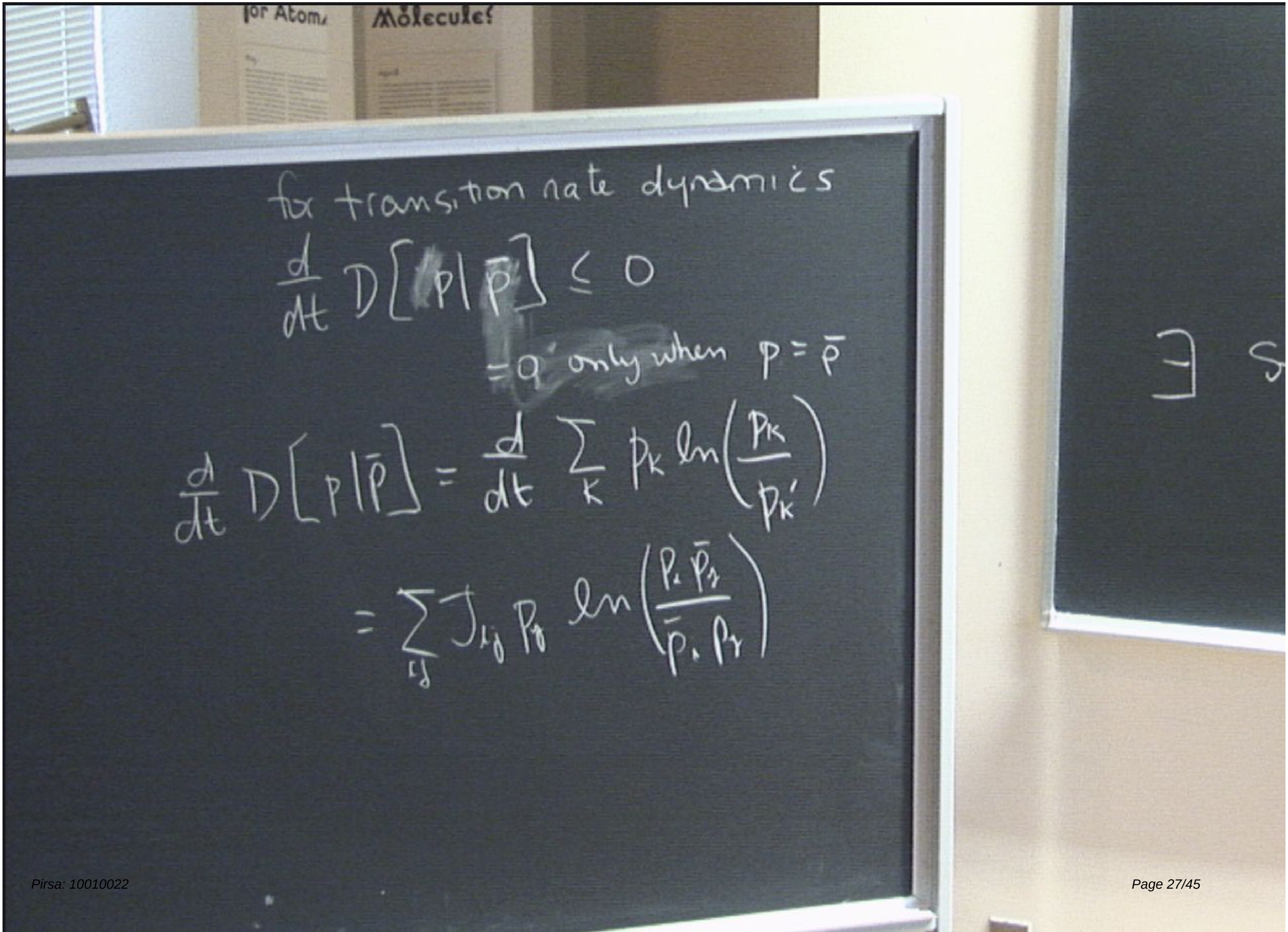
for Atoms
Molecules
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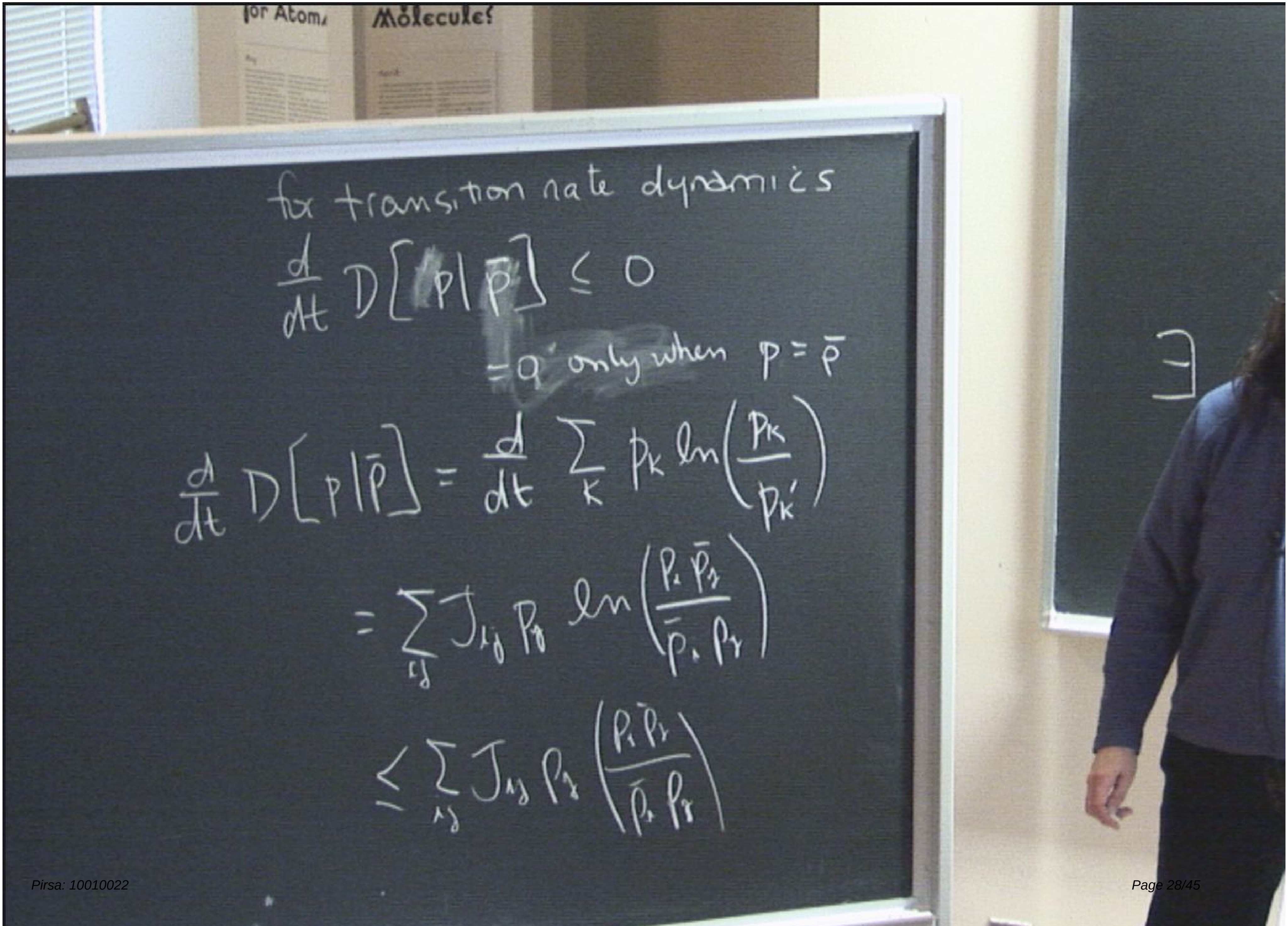
$\exists$ S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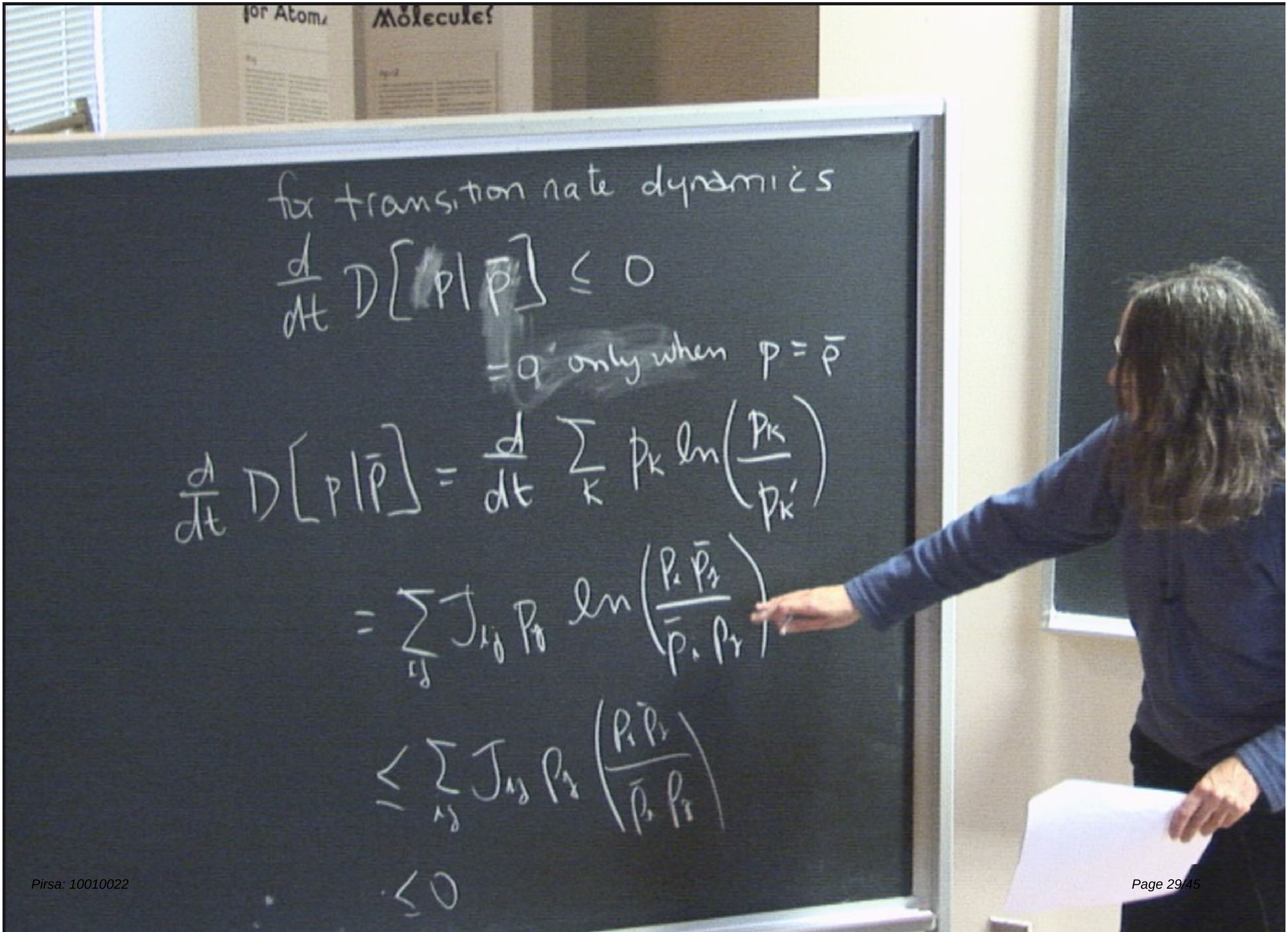
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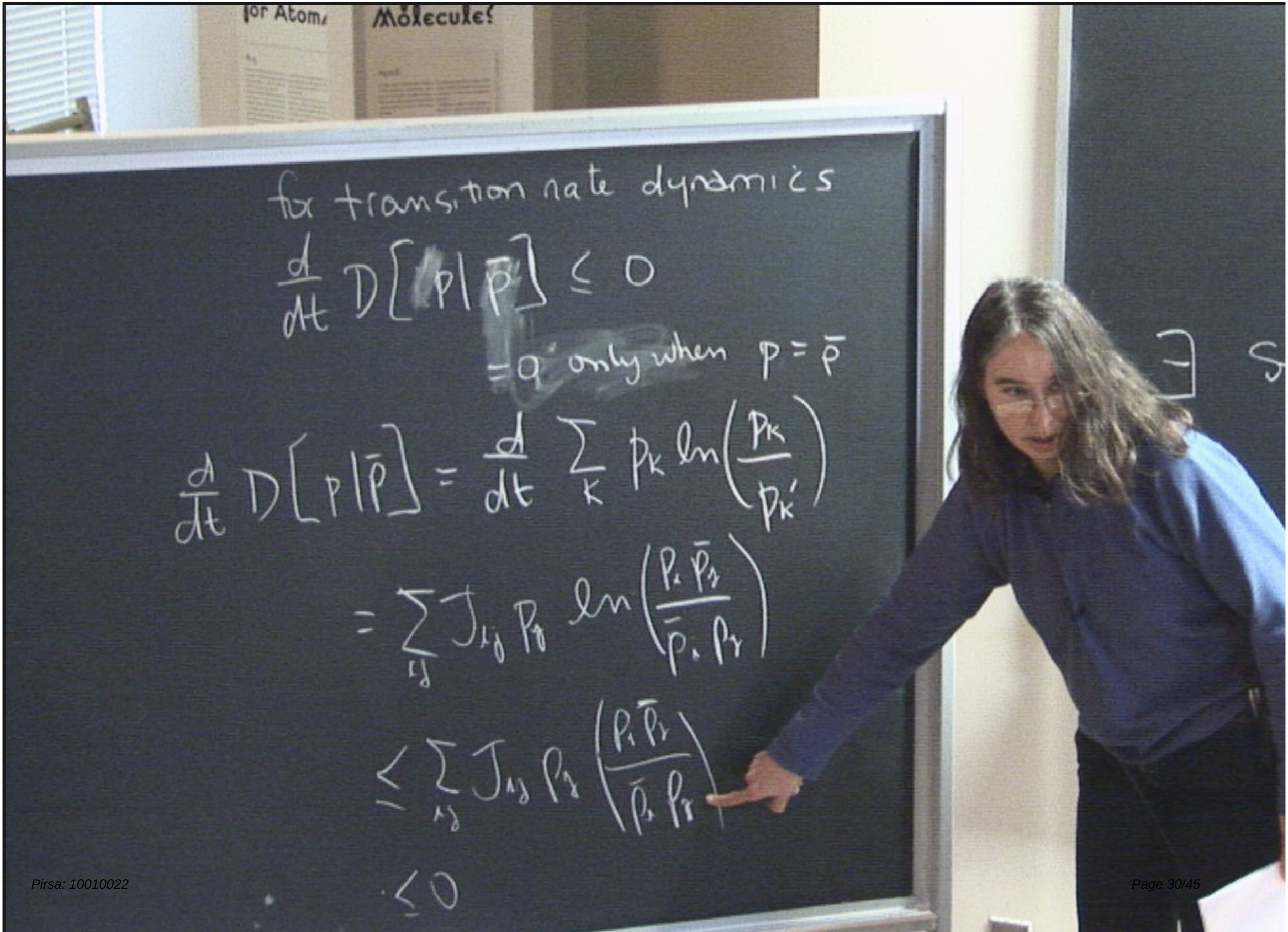
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for transition rate dynamics

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J_{ij} transition rates

from $j \to i$ to

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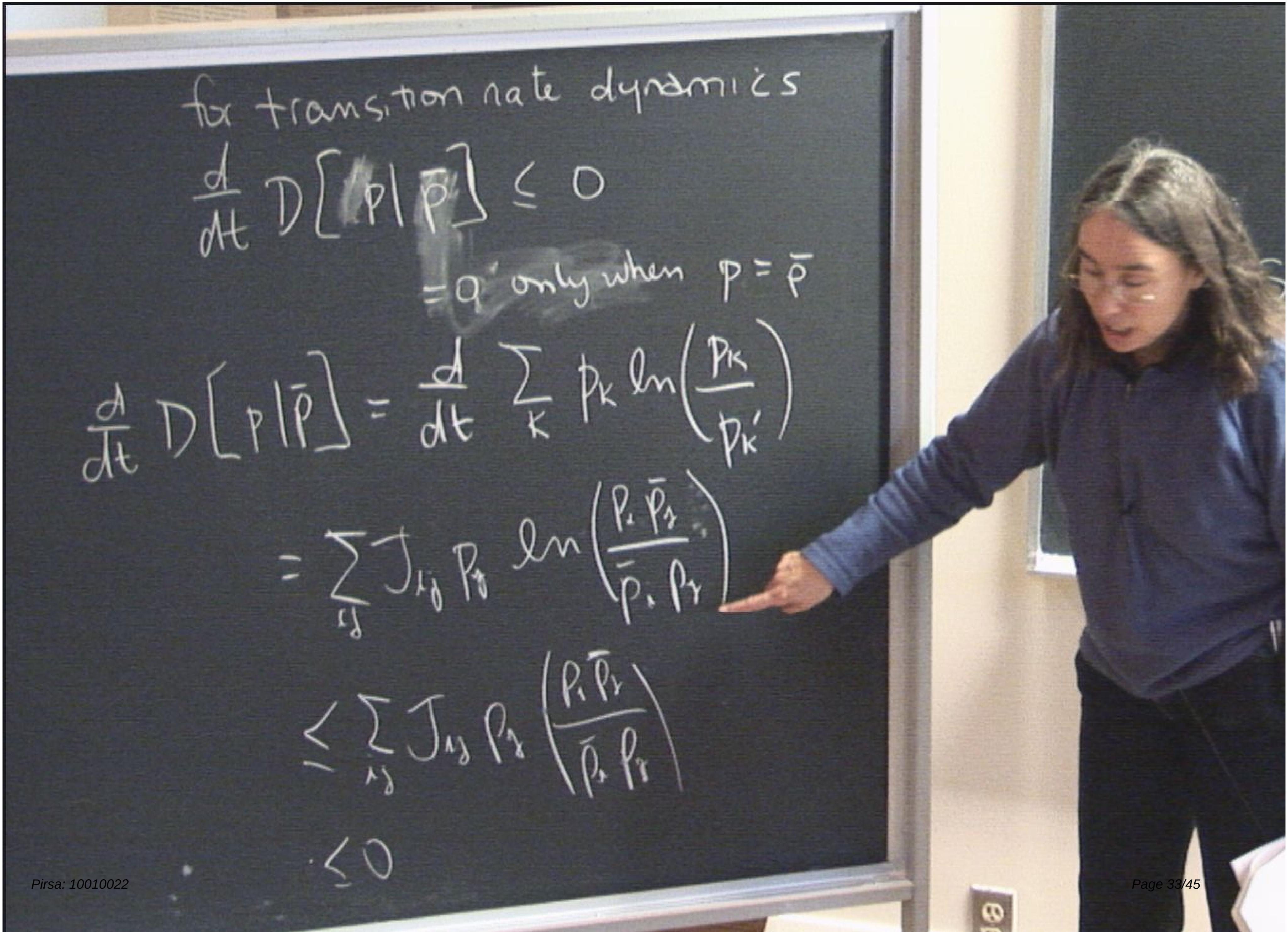
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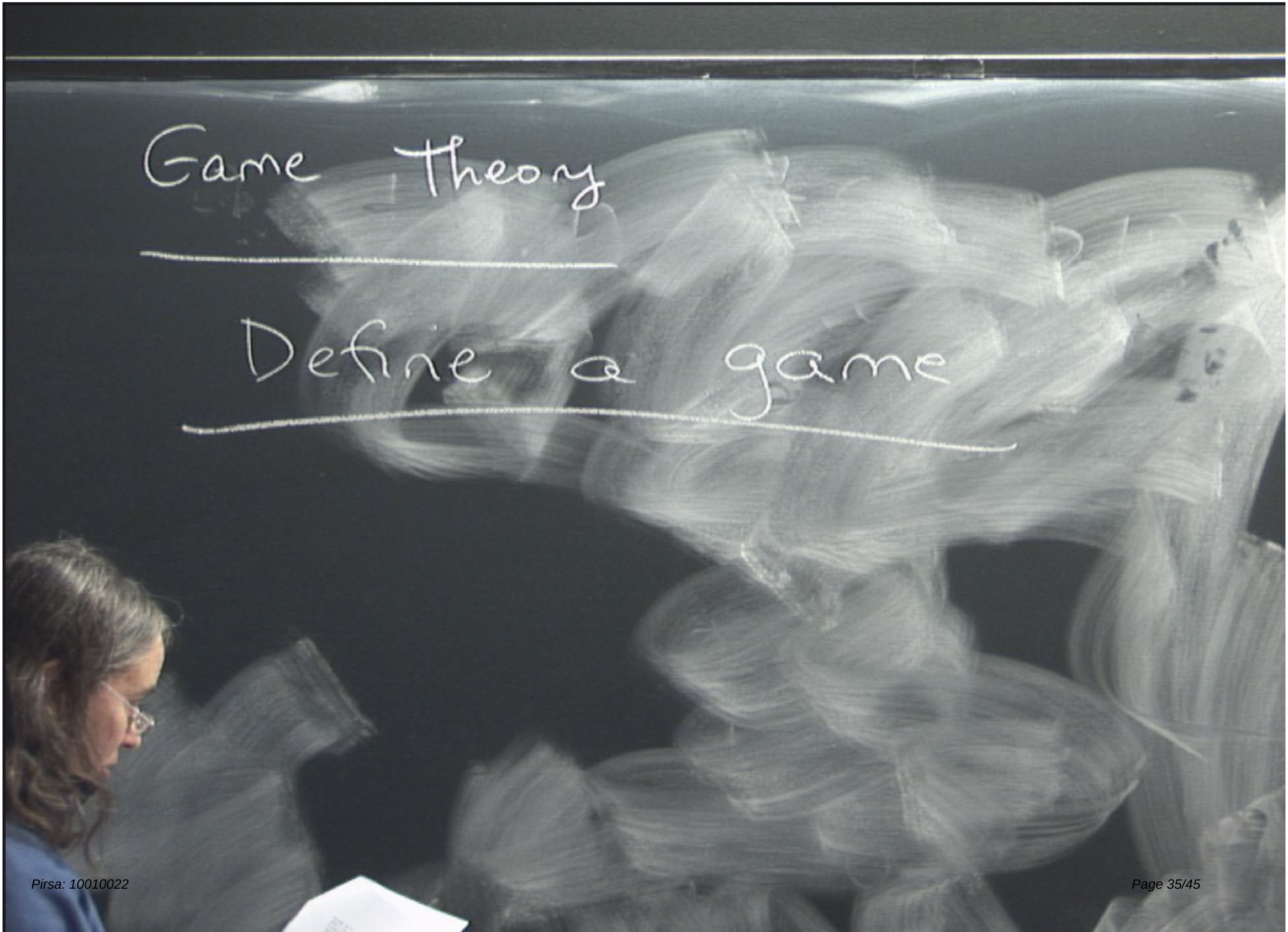
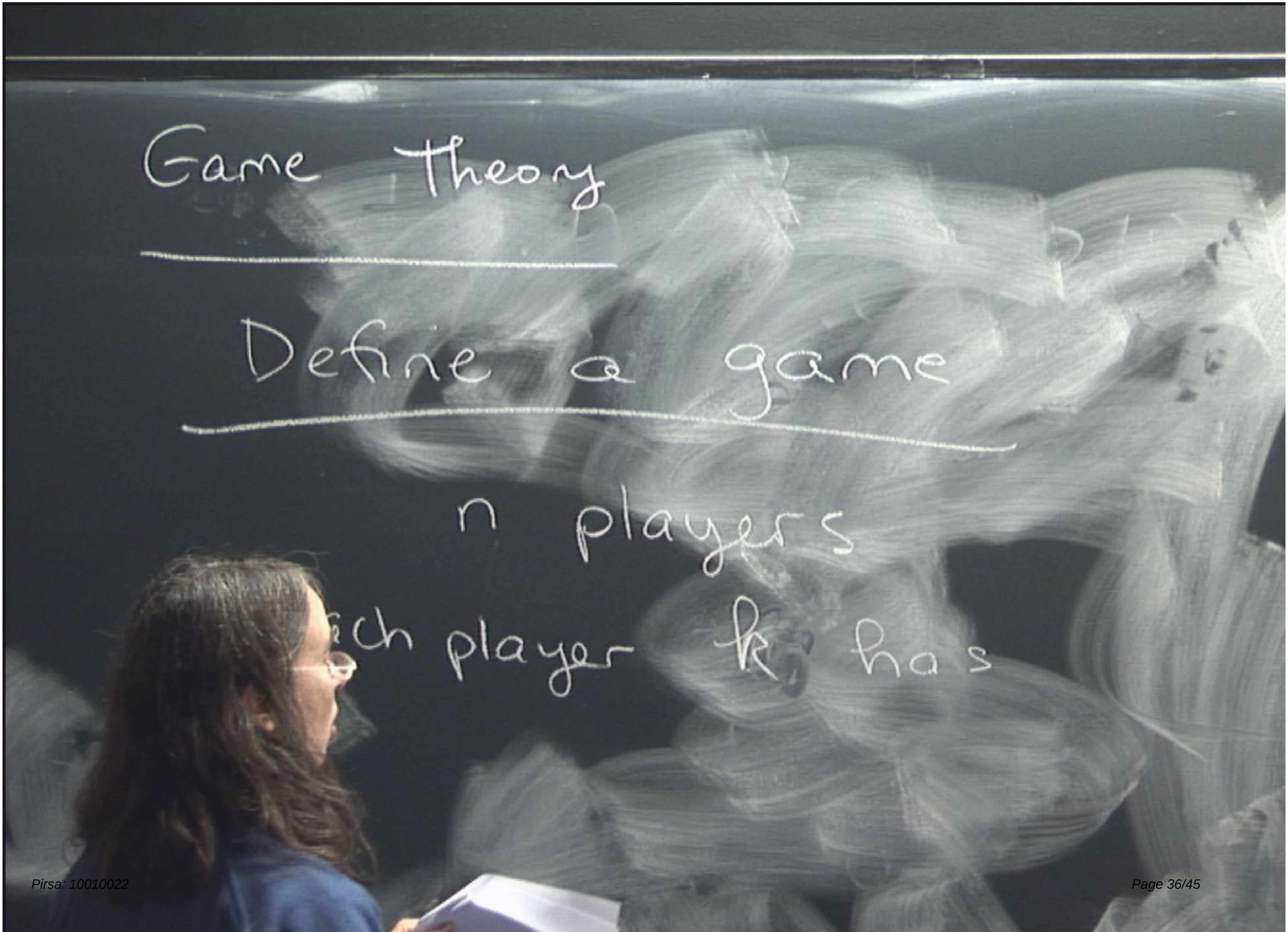
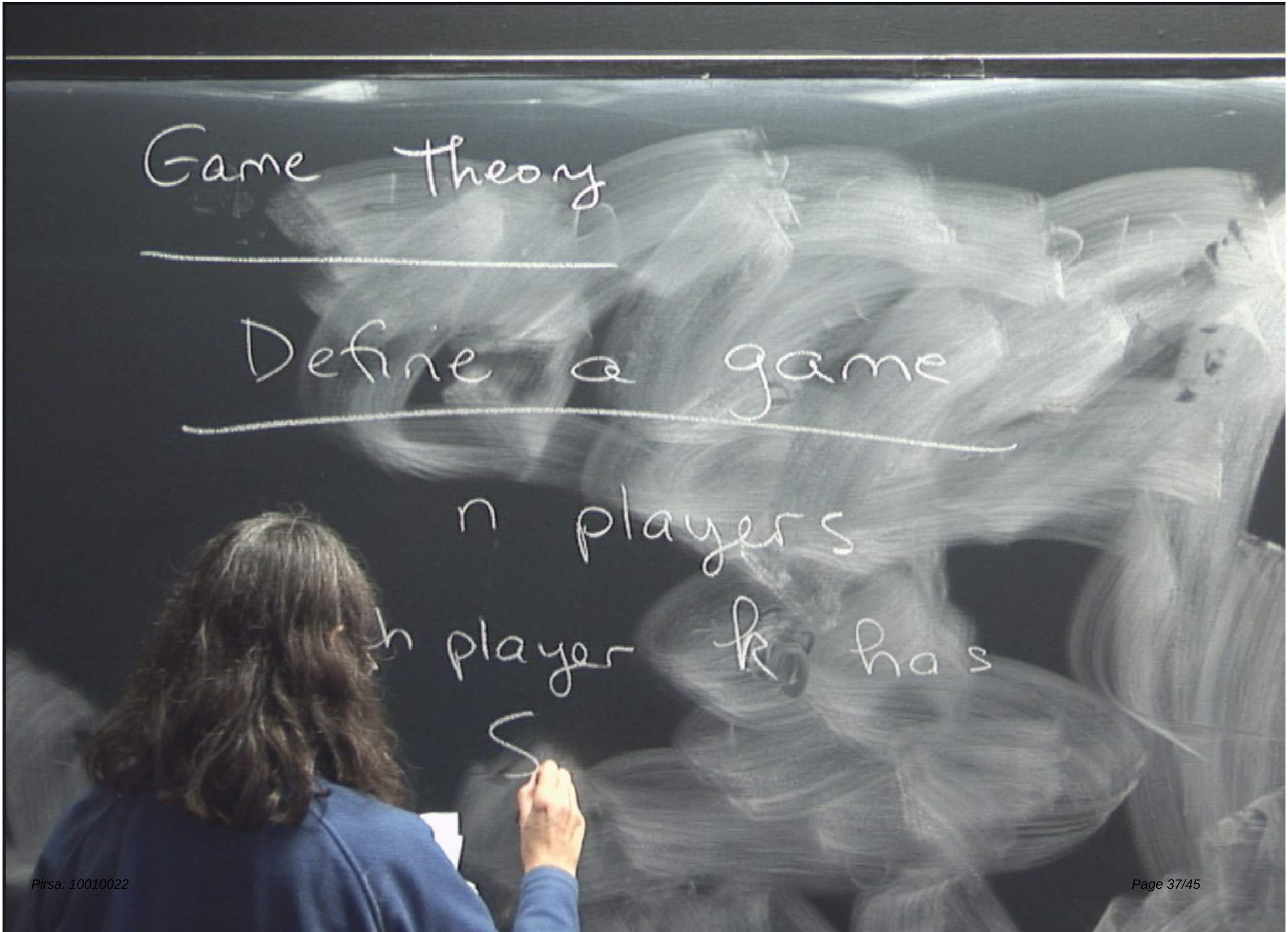

Game Theory
Define a game


Game Theory
Define a game
n players
ach player k has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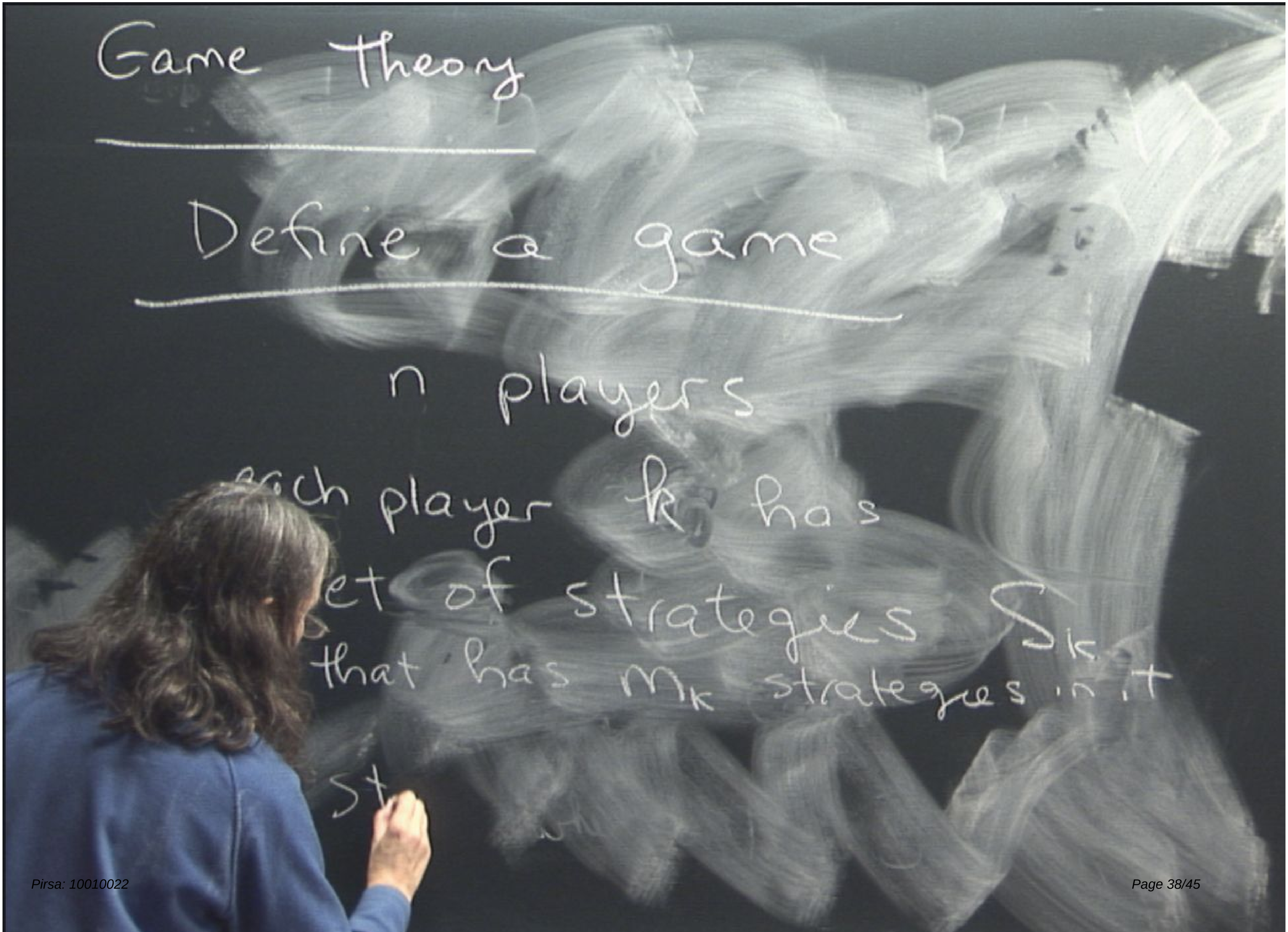
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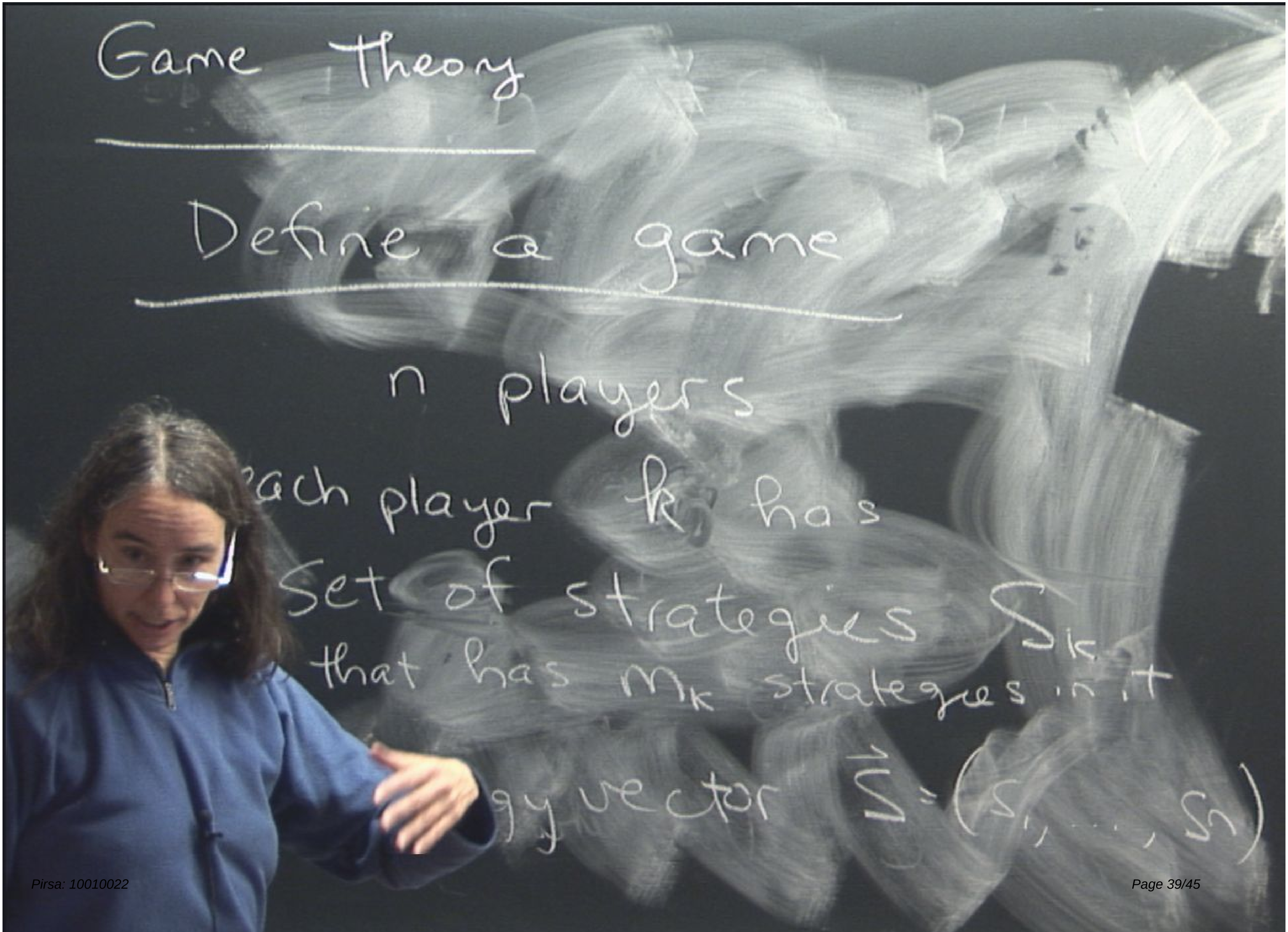

Game Theory
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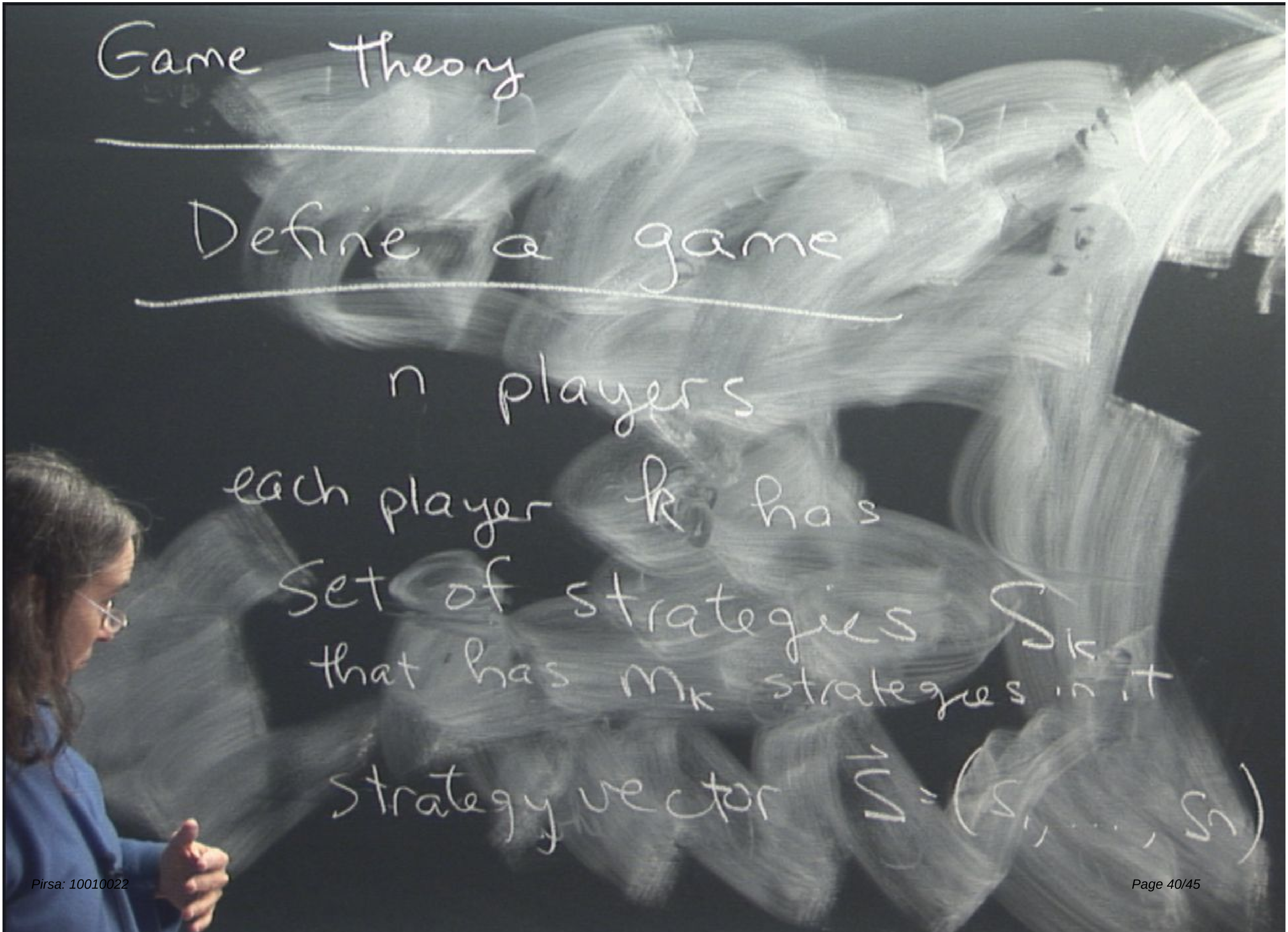
set of strategies S_k
that has m_k strategies in it
gy vector $\vec{S} = (s_1, \dots, s_n)$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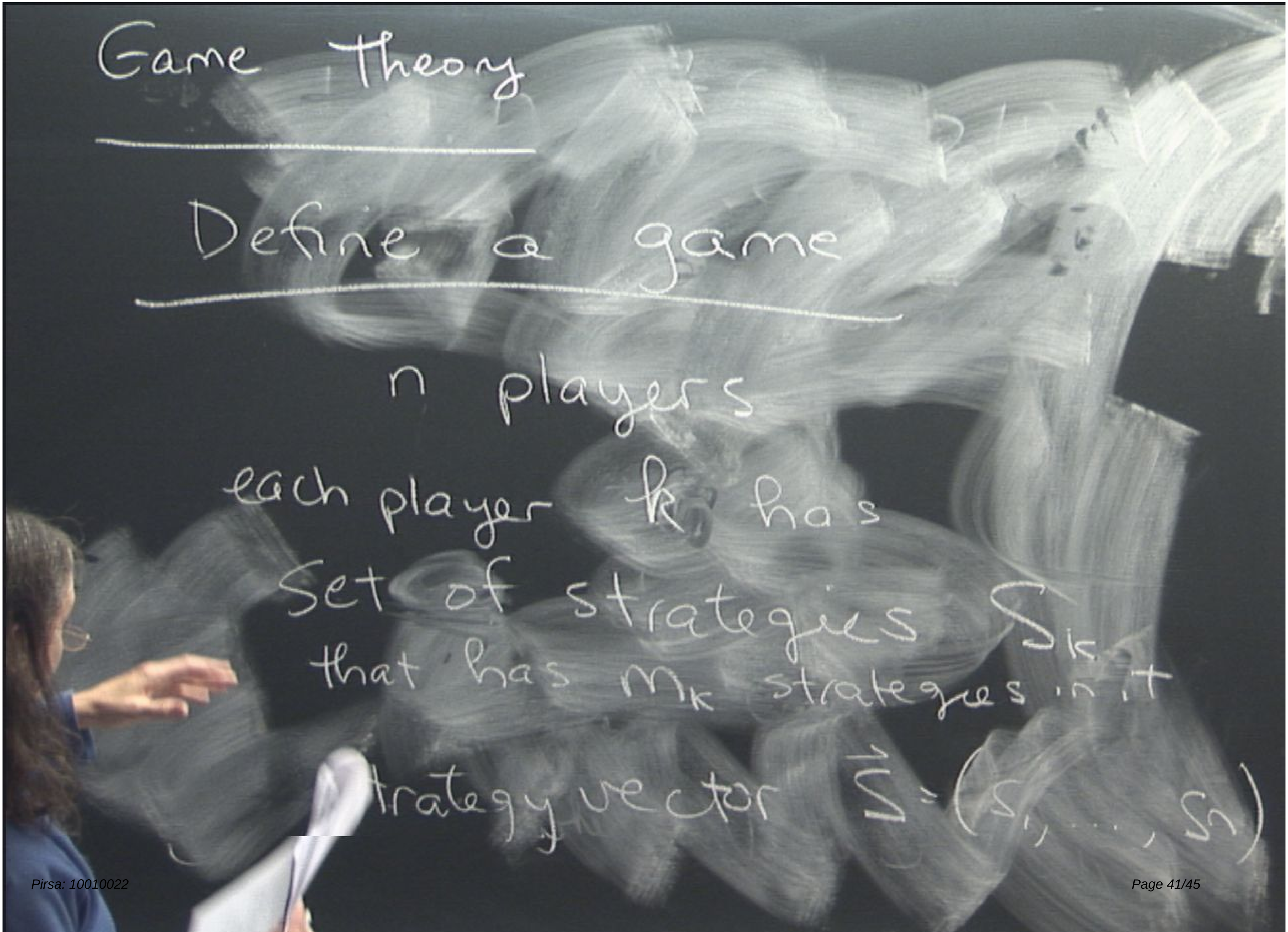
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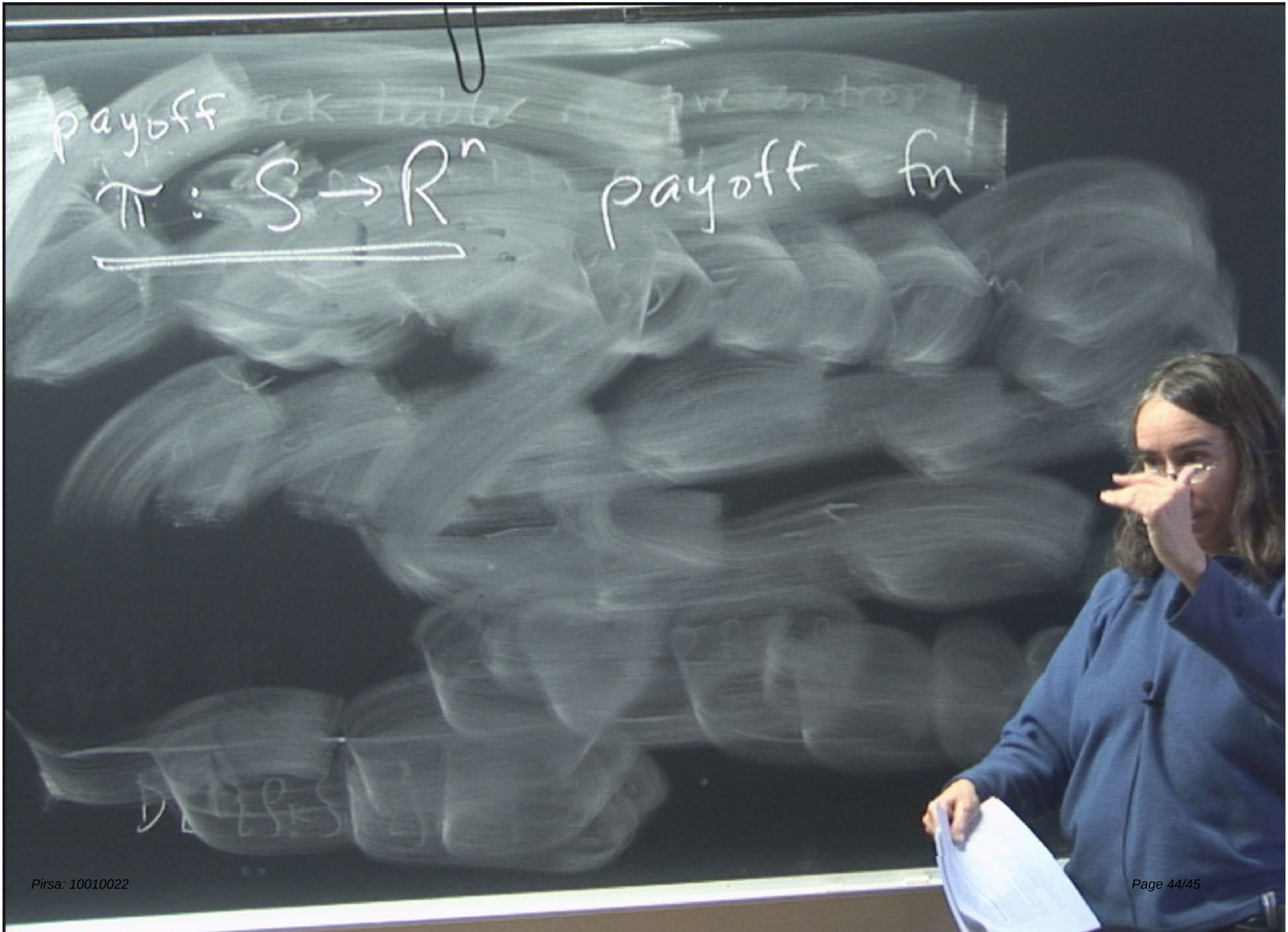
payoff
π : S → R^n


payoff
π : S → R^n payoff fn

payoff

$\pi : S \to R^n$ payoff fn