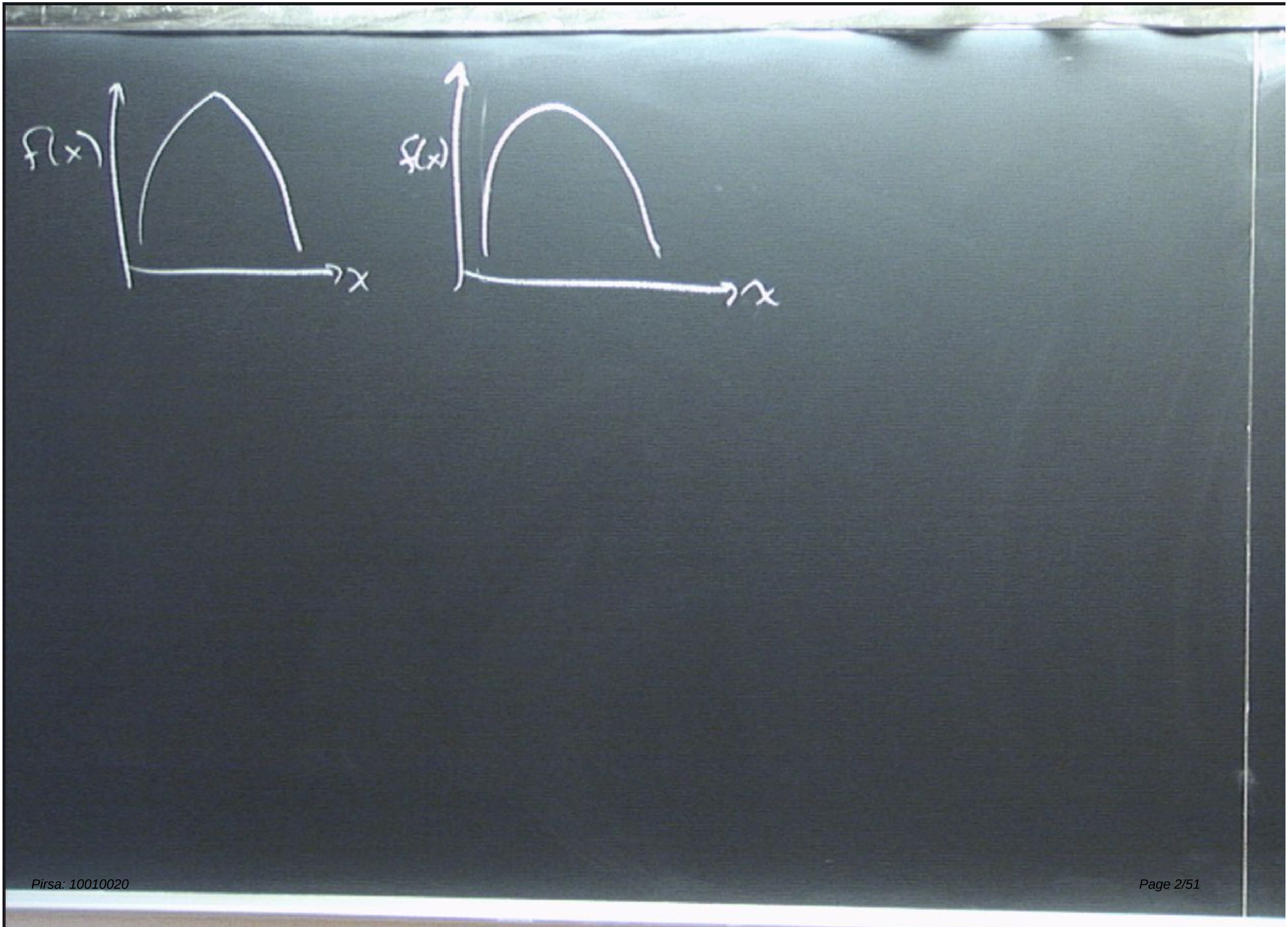
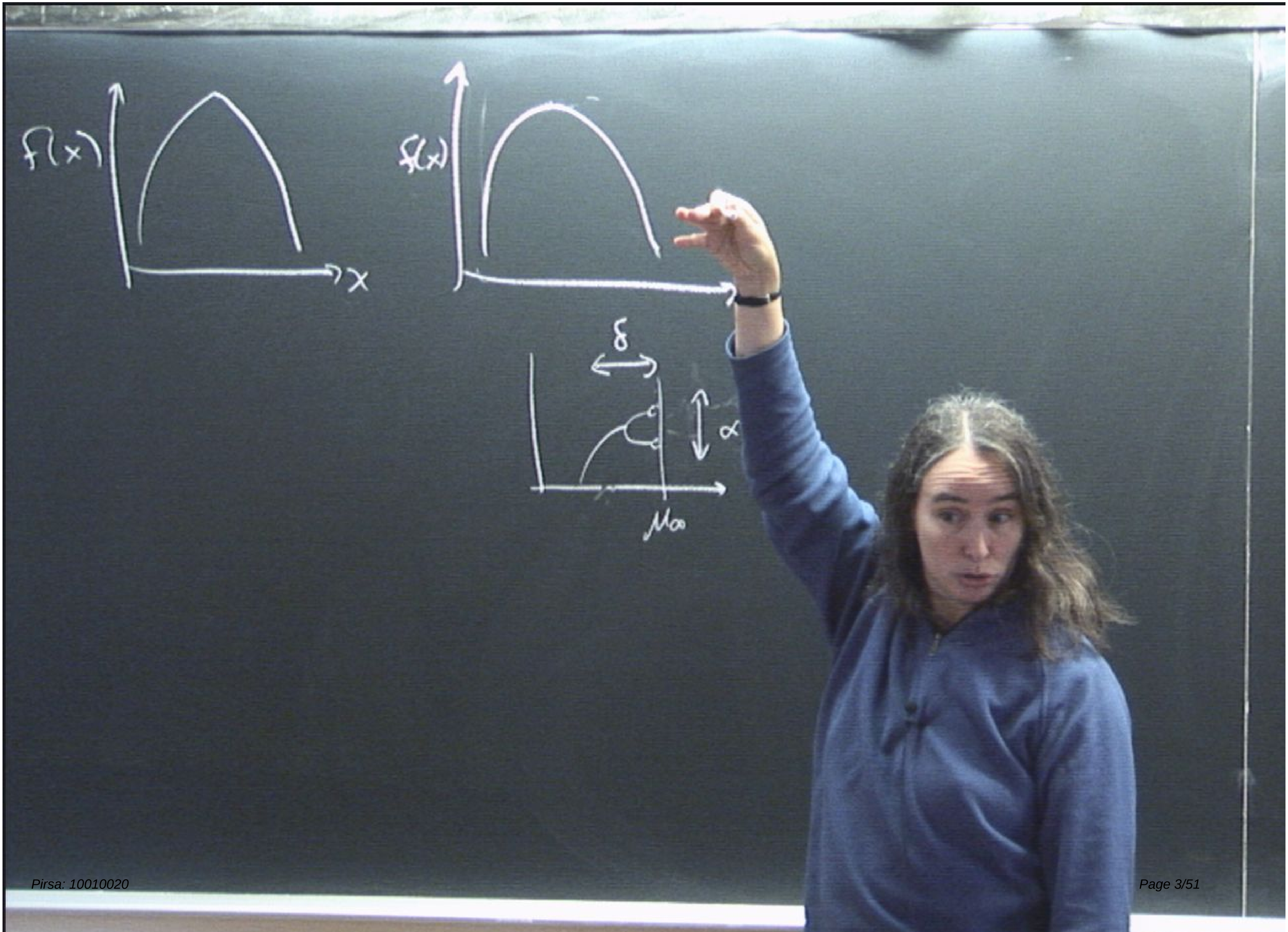
Title: Dynamical Systems - Review (PHYS 607) - Lecture 8

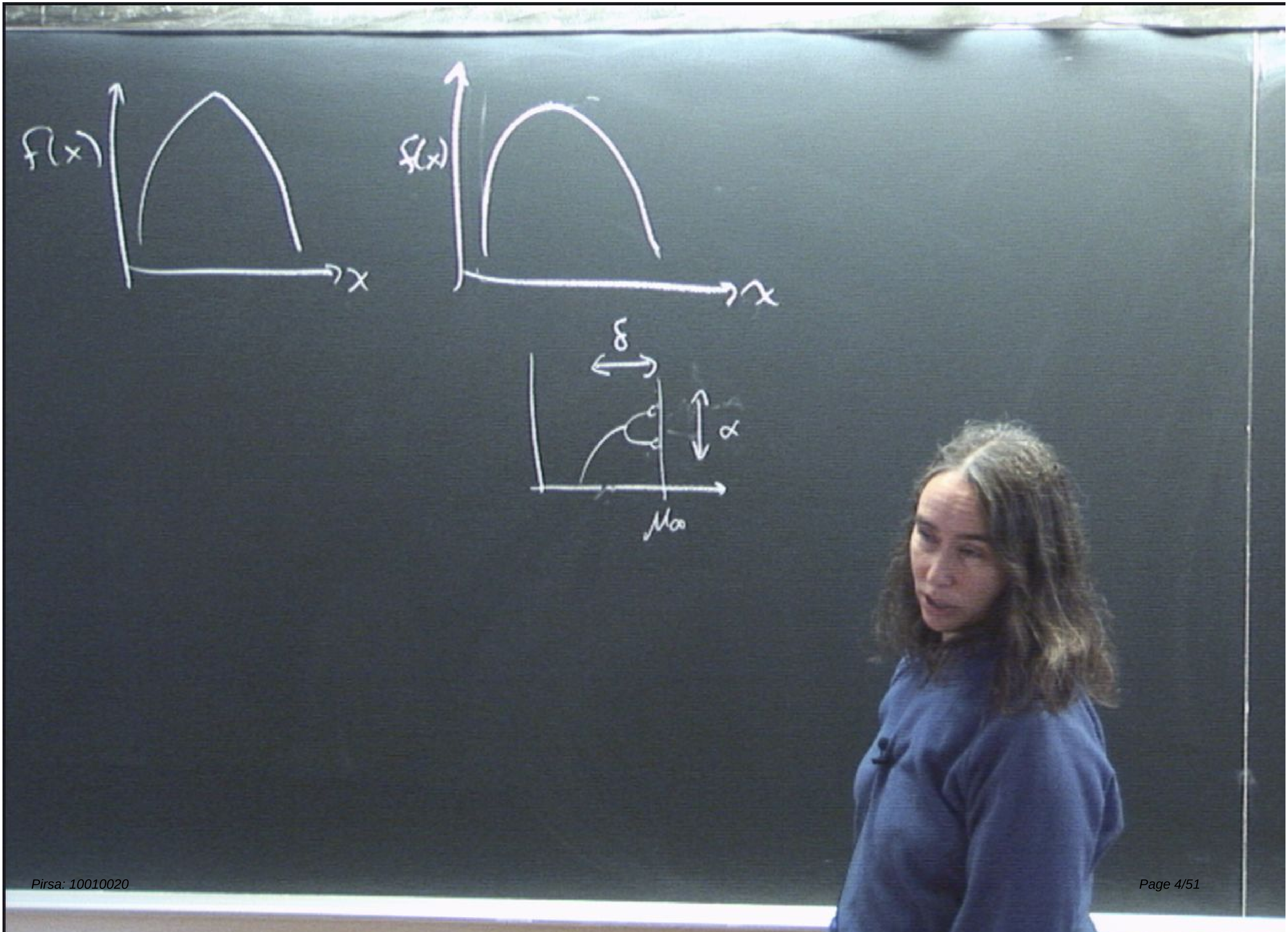
Date: Jan 13, 2010 09:00 AM

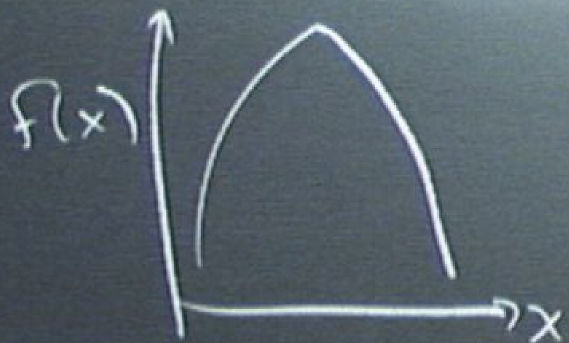
URL: http://pirsa.org/10010020

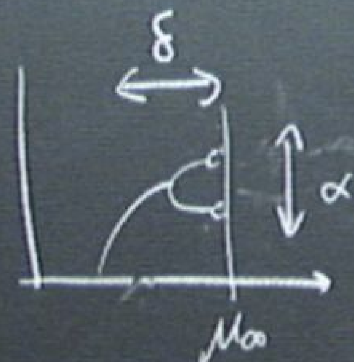
Abstract:


f(x)
x
f(x)
x

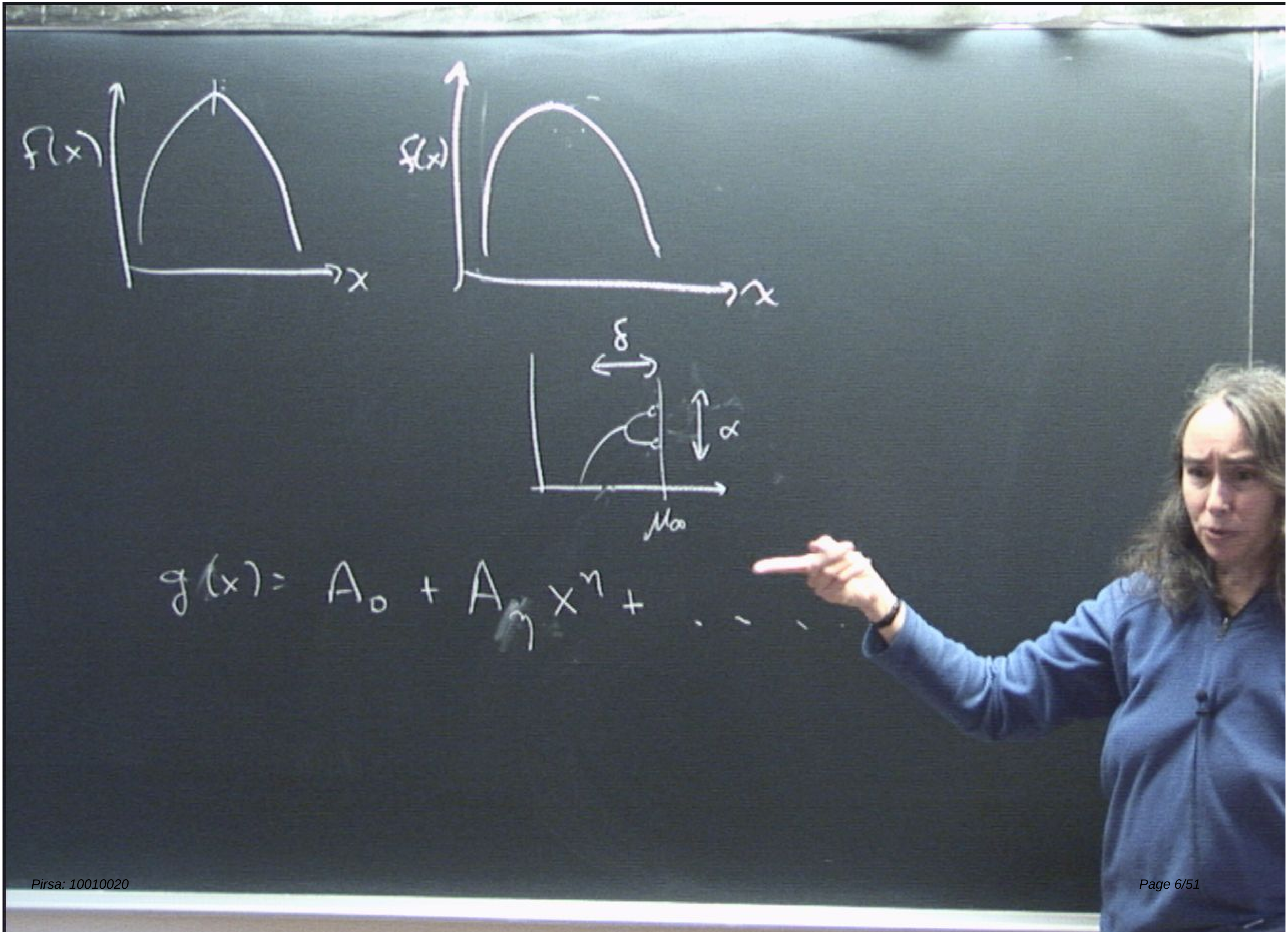

f(x)
x
f(x)
δ
α
μ∞

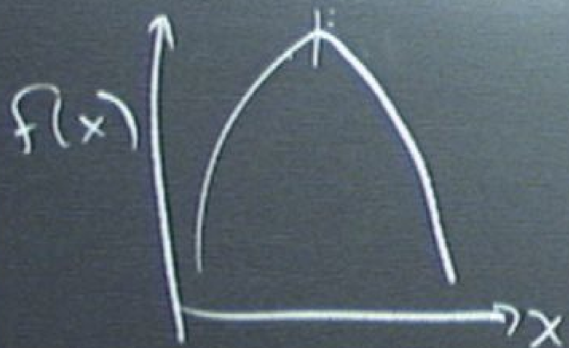

f(x)
x
f(x)
x
δ
α
μ∞

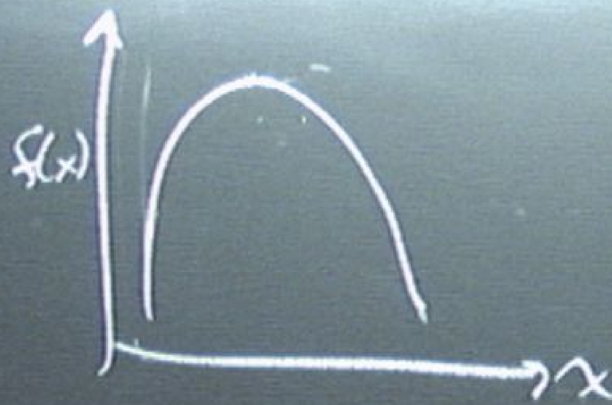


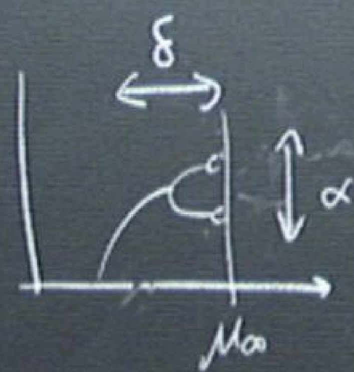


$$g(x) = A_0 + A_\eta x^\eta +$$

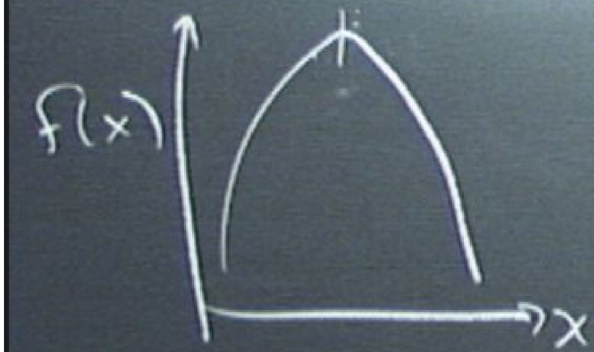

f(x)
x
f(x)
x
δ
α
μ∞
g(x) = A0 + Aη x^η + ...

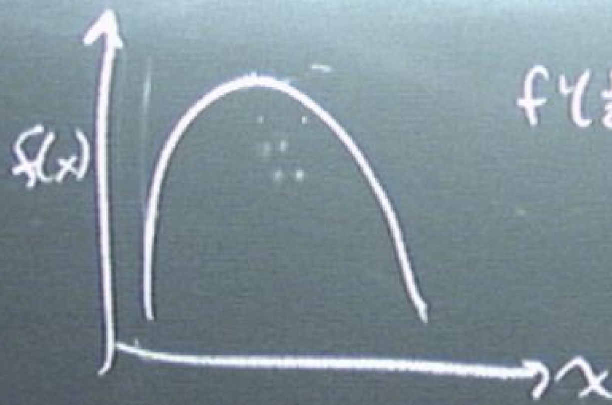






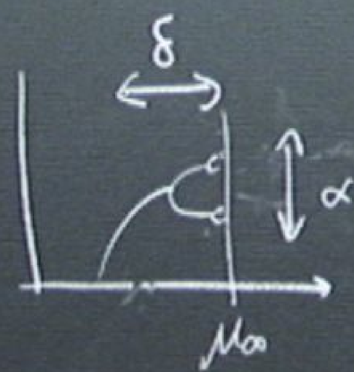
$$g(x) = A_0 + A_\eta x^\eta + \ldots$$



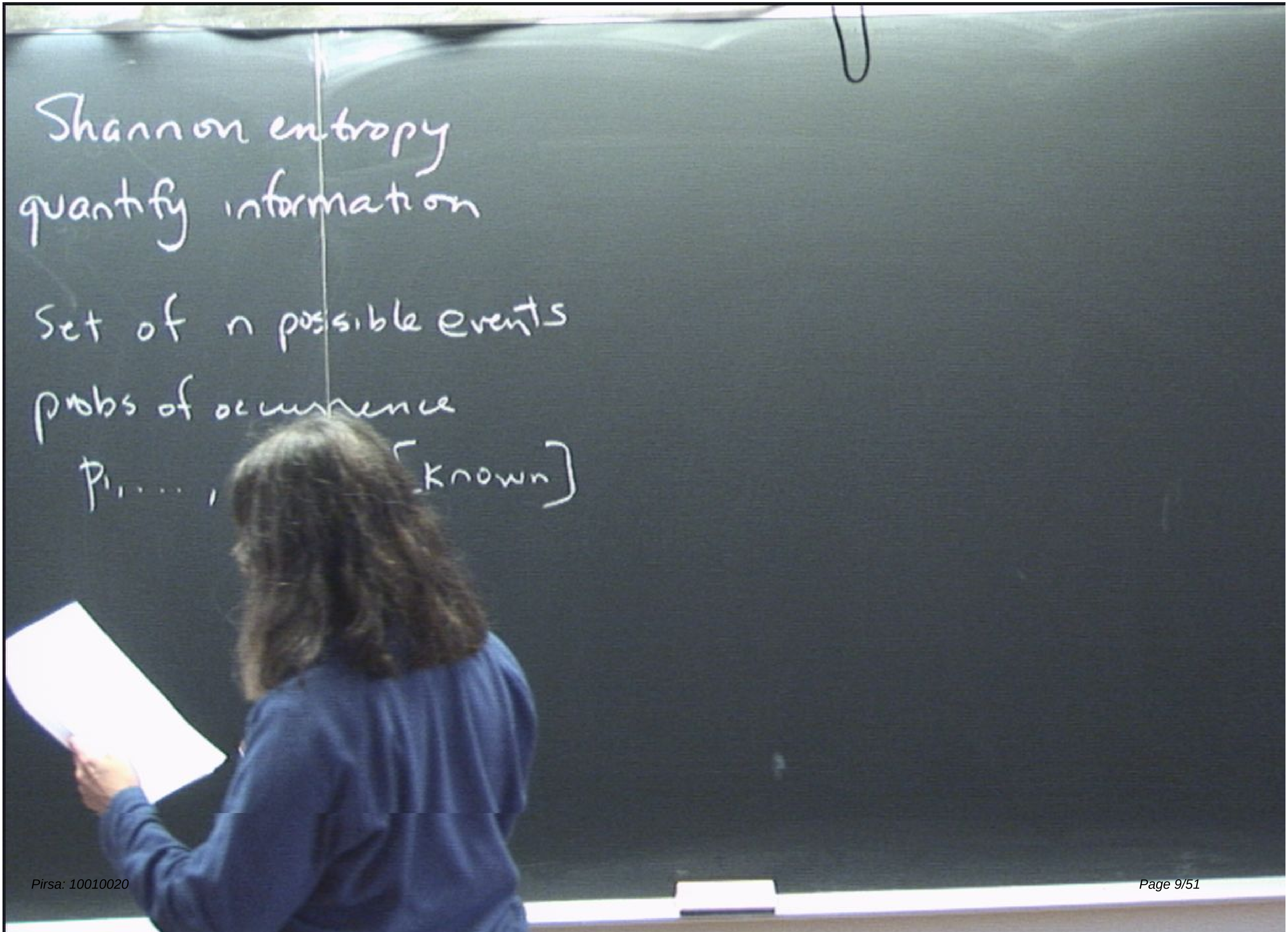


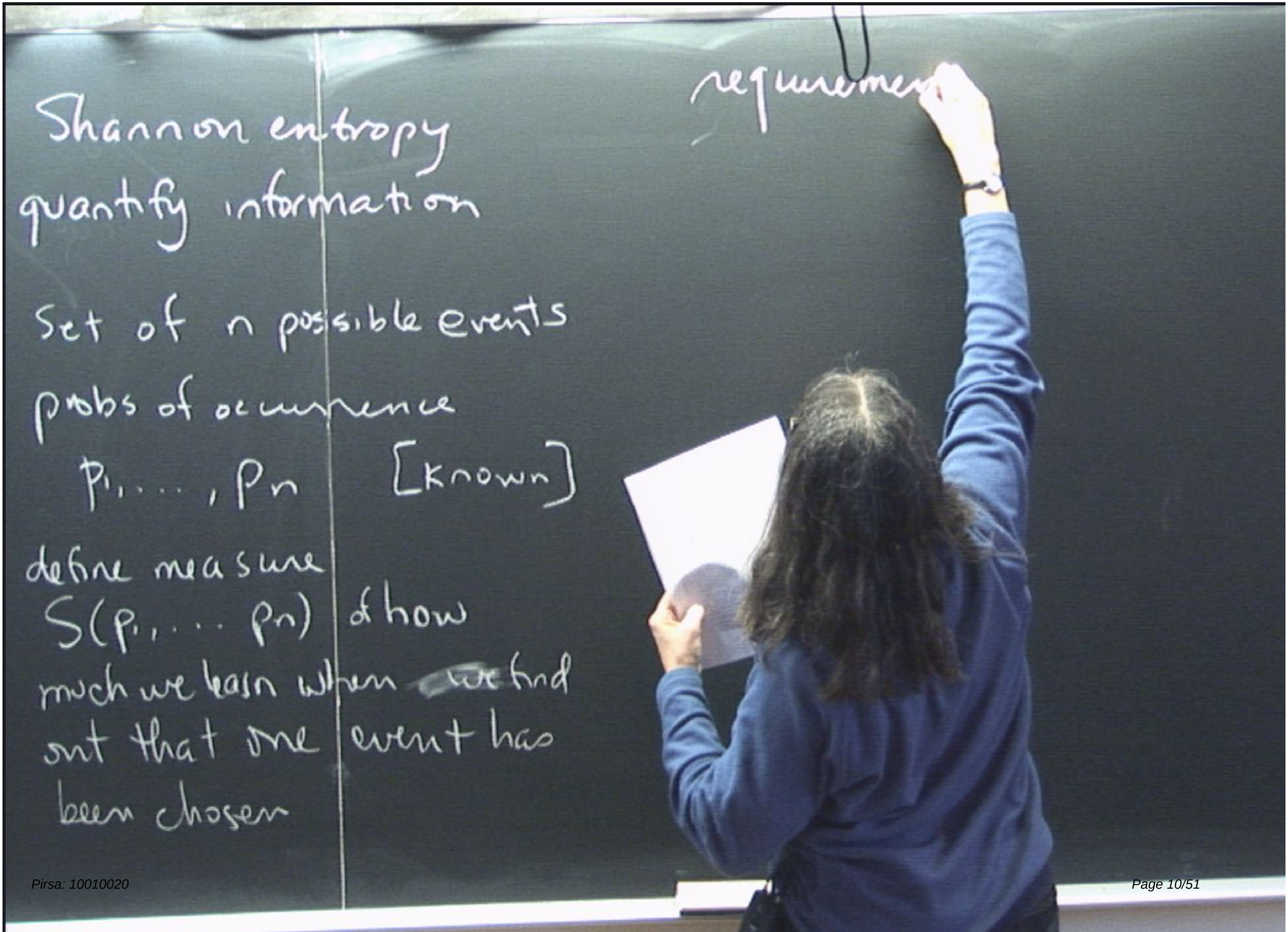
$f'(\tfrac{1}{2}) = 0$

Λ



$$g(x) = A_0 + A_\eta x^\eta + \ldots$$


Shannon entropy
quantify information
Set of n possible events
probs of occurrence
$p_1, \ldots,$ [known]


Shannon entropy
quantify information
Set of n possible events
probs of occurrence
$p_1, \dots, p_n$ [known]
define measure
$S(p_1, \dots p_n)$ of how
much we learn when we find
out that one event has
been chosen
requirements

requirements on S :

1) S continuous

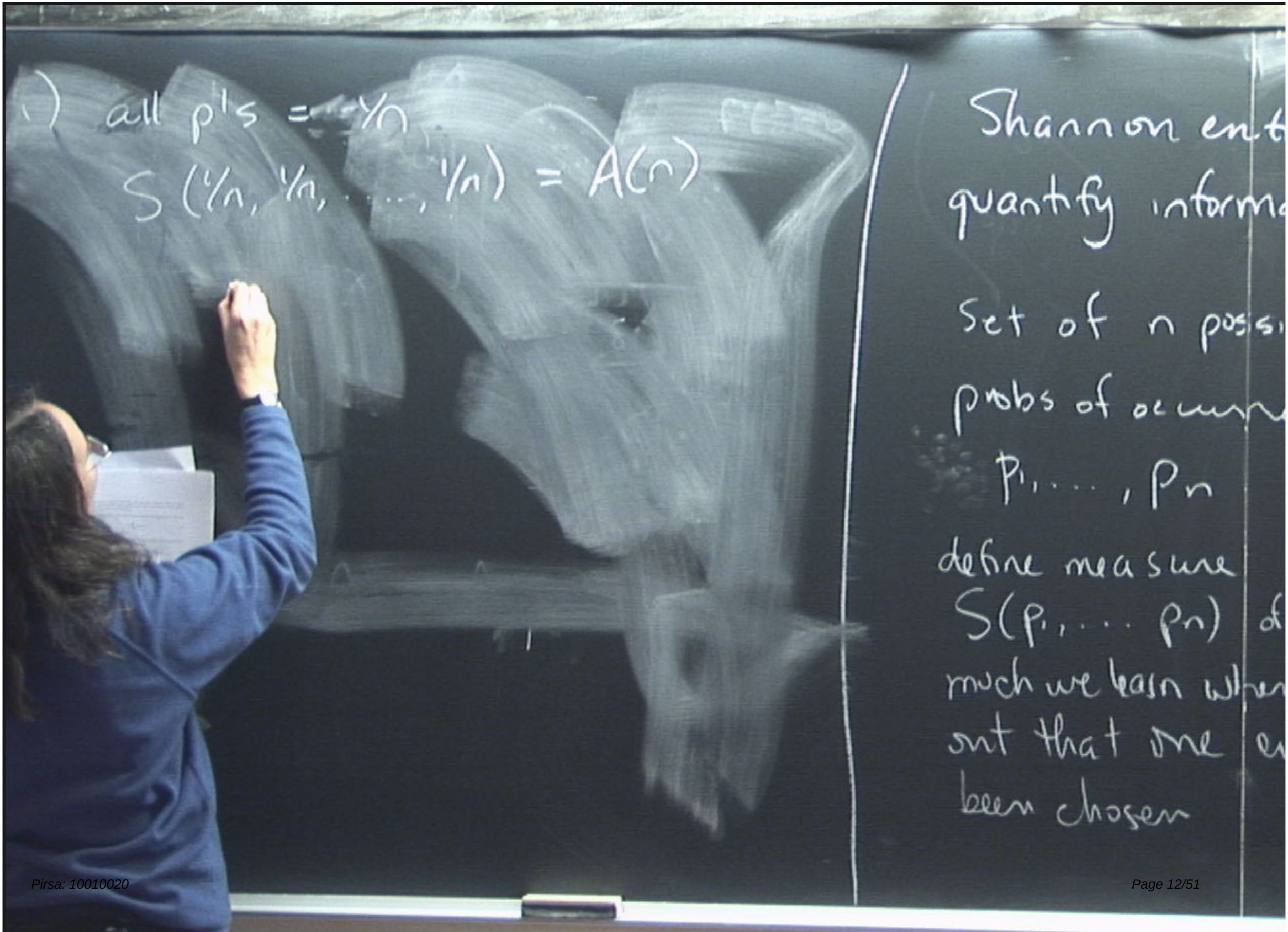
2) S monotonically increases with n if n choices are equally likely

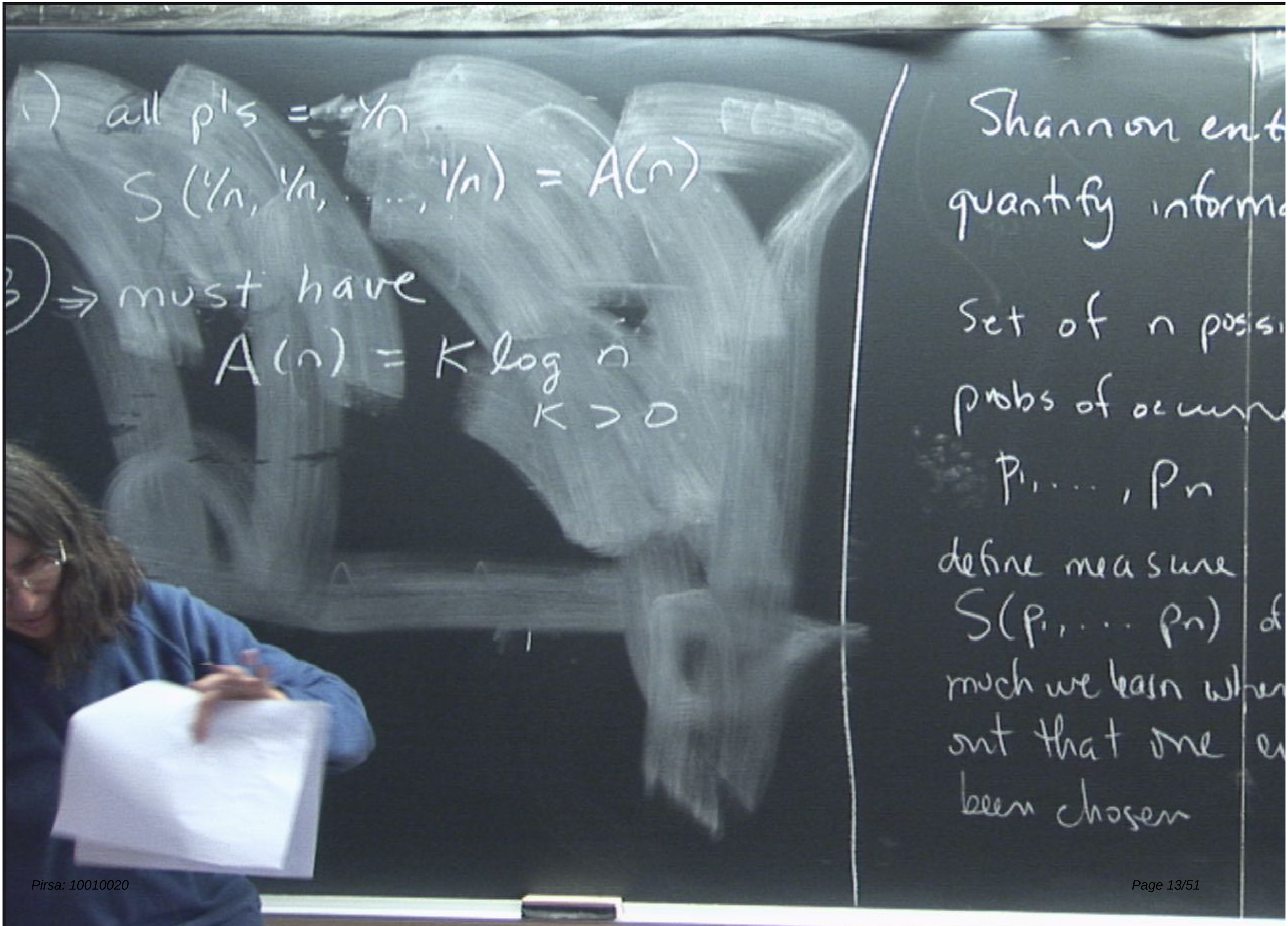
3) Breaking down a choice into steps doesn't change S.

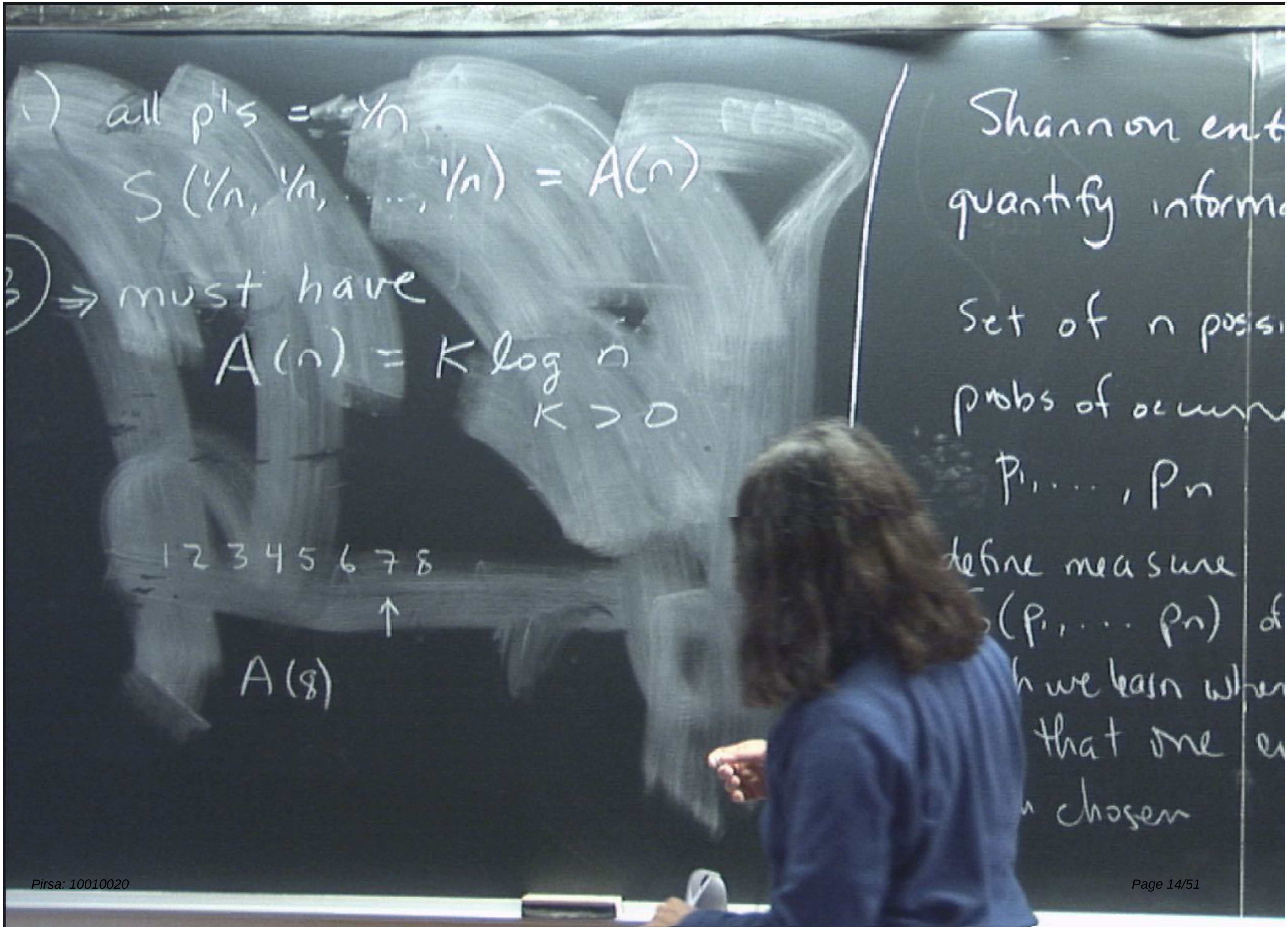
thm

$$S = -K \sum_{i=1}^{n} p_i \log p_i$$

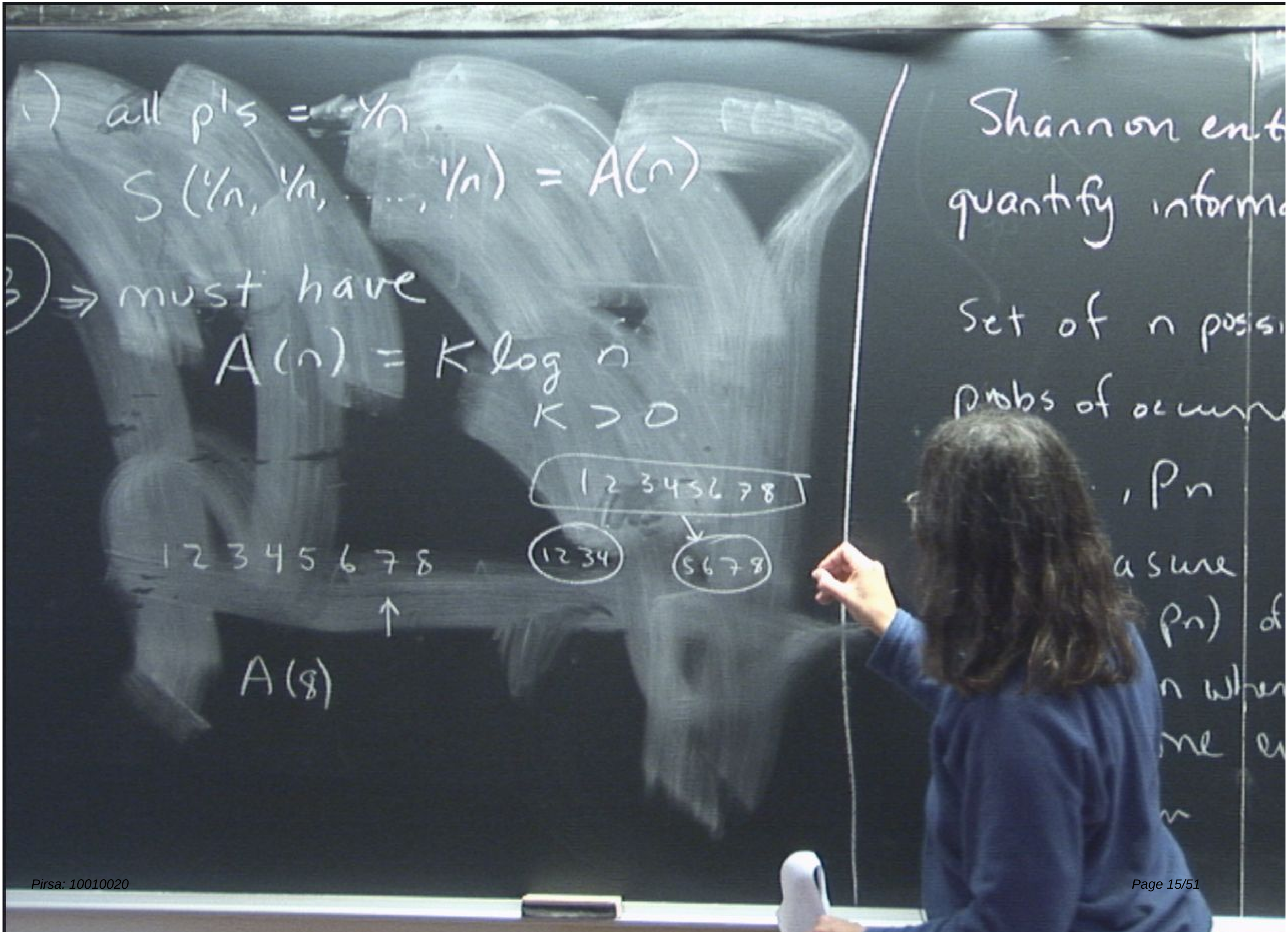
with $K > 0$


i) all p's = 1/n
S(1/n, 1/n, ..., 1/n) = A(n)
Shannon ent
quantify informa
Set of n possi
probs of occur
p1, ..., pn
define measure
S(p1, ... pn) of
much we learn whe
out that one e
been chosen


all p's = 1/n
S (1/n, 1/n, ..., 1/n) = A(n)
⇒ must have
A(n) = K log n
K > 0
Shannon ent
quantify informa
Set of n possi
probs of occurre
p1, ..., pn
define measure
S(p1, ... pn) of
much we learn whe
out that one e
been chosen



all p's = 1/n
S (1/n, 1/n, ..., 1/n) = A(n)
must have
A(n) = K log n
K > 0
1 2 3 4 5 6 7 8
A(8)
Shannon en
quantify informa
Set of n possi
probs of occurre
p1, ..., pn
define measure
S(p1, ... pn) of
we learn when
that one e
chosen


all p's = 1/n
S (1/n, 1/n, ..., 1/n) = A(n)
⇒ must have
A(n) = K log n
K > 0
1 2 3 4 5 6 7 8
1 2 3 4
5 6 7 8
1 2 3 4 5 6 7 8
A(8)
Shannon en
quantify inform
Set of n possi
probs of occur
, pn
asure
pn)
n when

1) all p's $= 1/n$

$S(1/n, 1/n, \ldots, 1/n) = A(n)$

3) $\Rightarrow$ must have

$A(n) = K \log n$

$K > 0$

1 2 3 4 5 6 7 8

1234 5678 $A(2)$

1 2 3 4 5 6 7 8

56 78

$A(8)$

Shannon ent

quantify informa

Set of n possi

probs of occurre

$p_1, \ldots, p_n$

define measure

$S(p_1, \ldots p_n)$ of

much we learn

out that one

been chosen

1) all p's $= 1/n$

$$S(1/n, 1/n, \ldots, 1/n) = A(n)$$

(3) $\Rightarrow$ must have

$$A(n) = K \log n$$

$$K > 0$$

1 2 3 4 5 6 7 8

A(8)

1 2 3 4 5 6 7 8

1 2 3 4   5 6 7 8   A(2)

5 6   7 8   A(2)

7   8   A(2)

3 A(2)

Shannon ent

quantify informa

Set of n possi

probs of occurre

$p_1, \ldots, p_n$

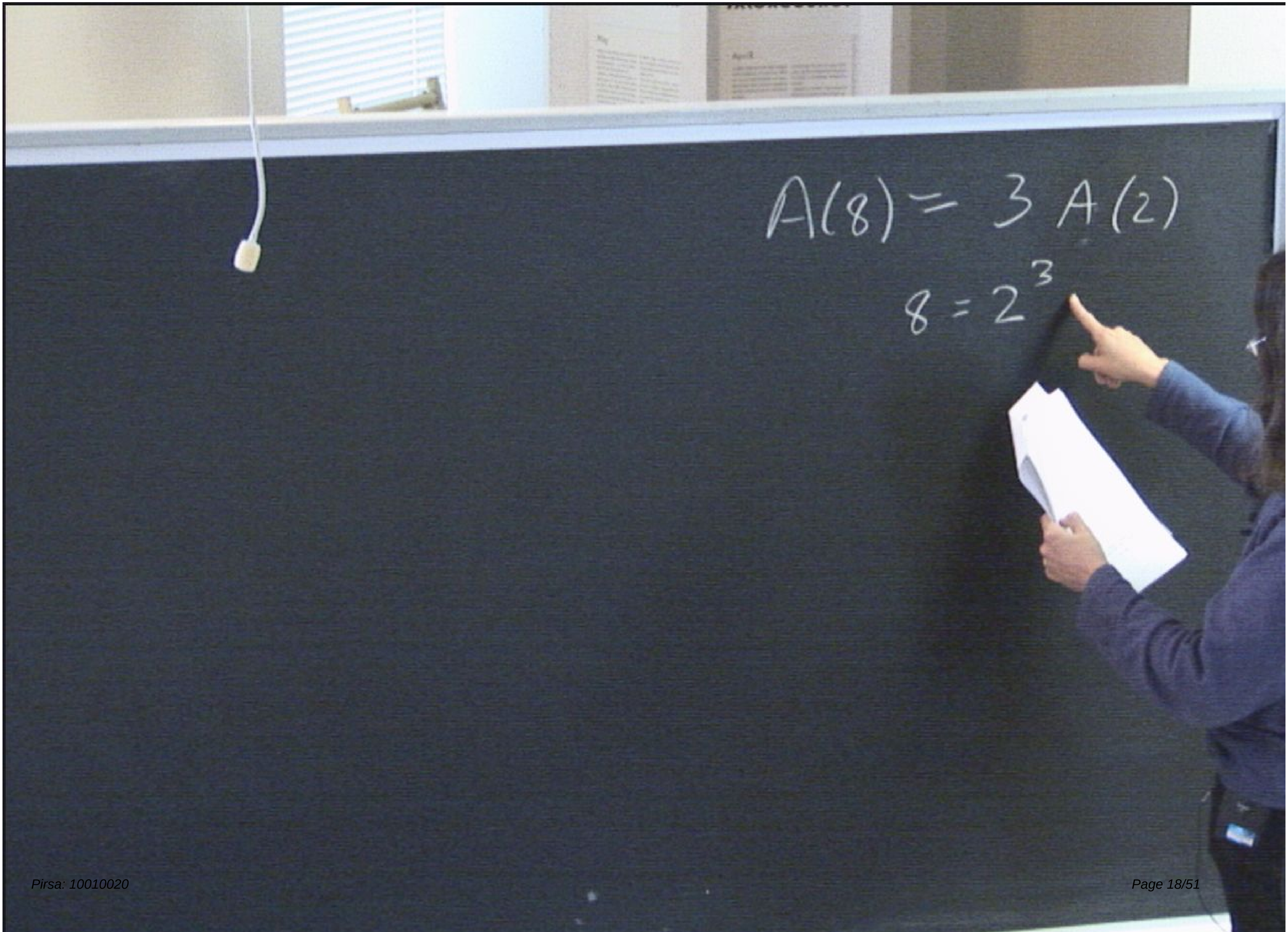
define measure

$S(p_1, \ldots p_n)$ of

much we learn when

out that one e

been chosen


$A(8) = 3\,A(2)$
$8 = 2^3$

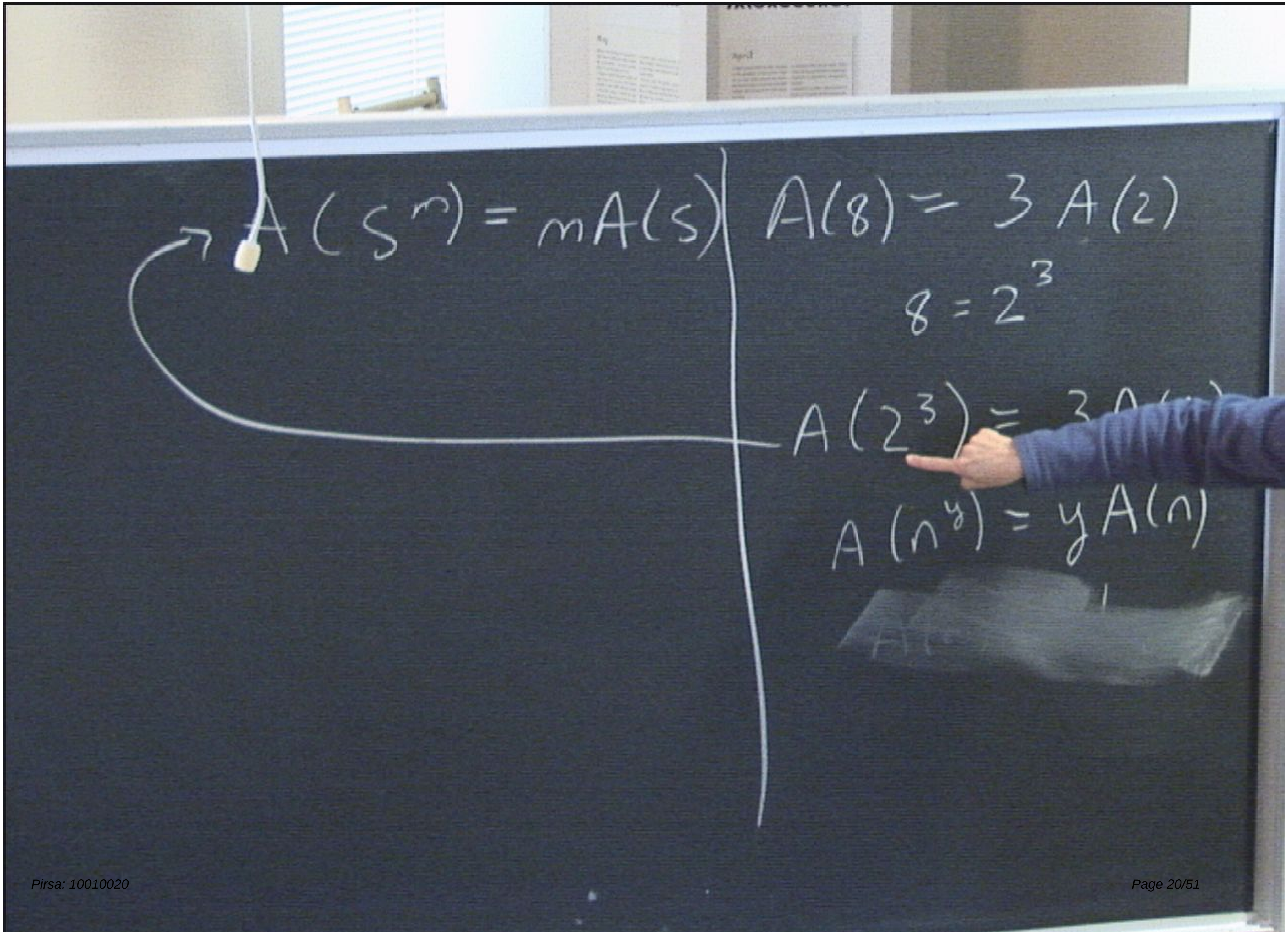
$$A(8) = 3\,A(2)$$

$$8 = 2^3$$

$$A(2^3) = 3\,A(2)$$

$$A(n^y) = y\,A(n)$$

$$A(m) = K \log m$$


$A(s^m) = mA(s)$
$A(8) = 3A(2)$
$8 = 2^3$
$A(2^3) =$
$A(n^y) = yA(n)$

$$A(S^m) = mA(S)$$

$$A(8) = 3A(2)$$

$$8 = 2^3$$

$$A(2^3) = 3A(2)$$

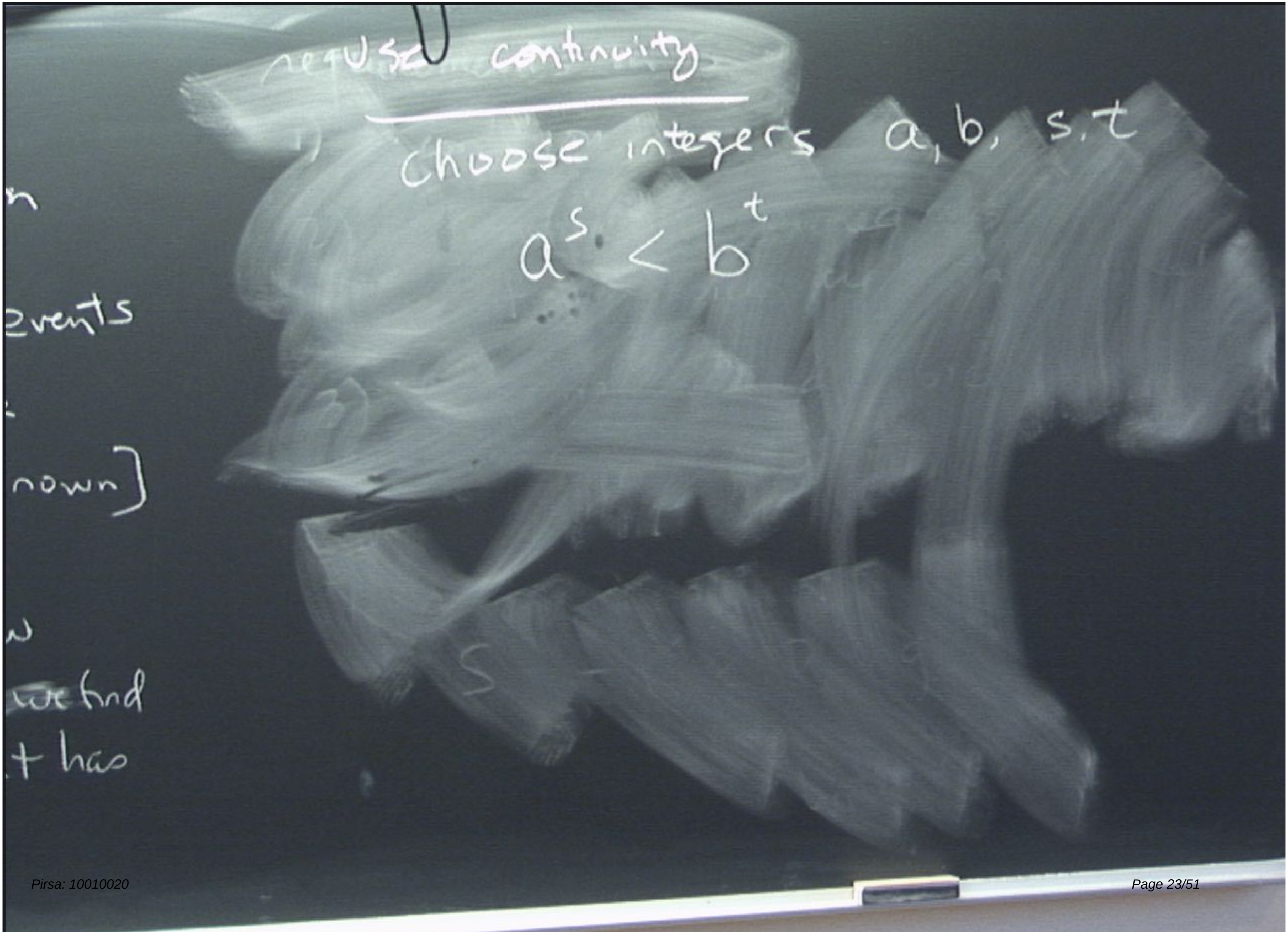
$$A(n^y) = yA(n)$$

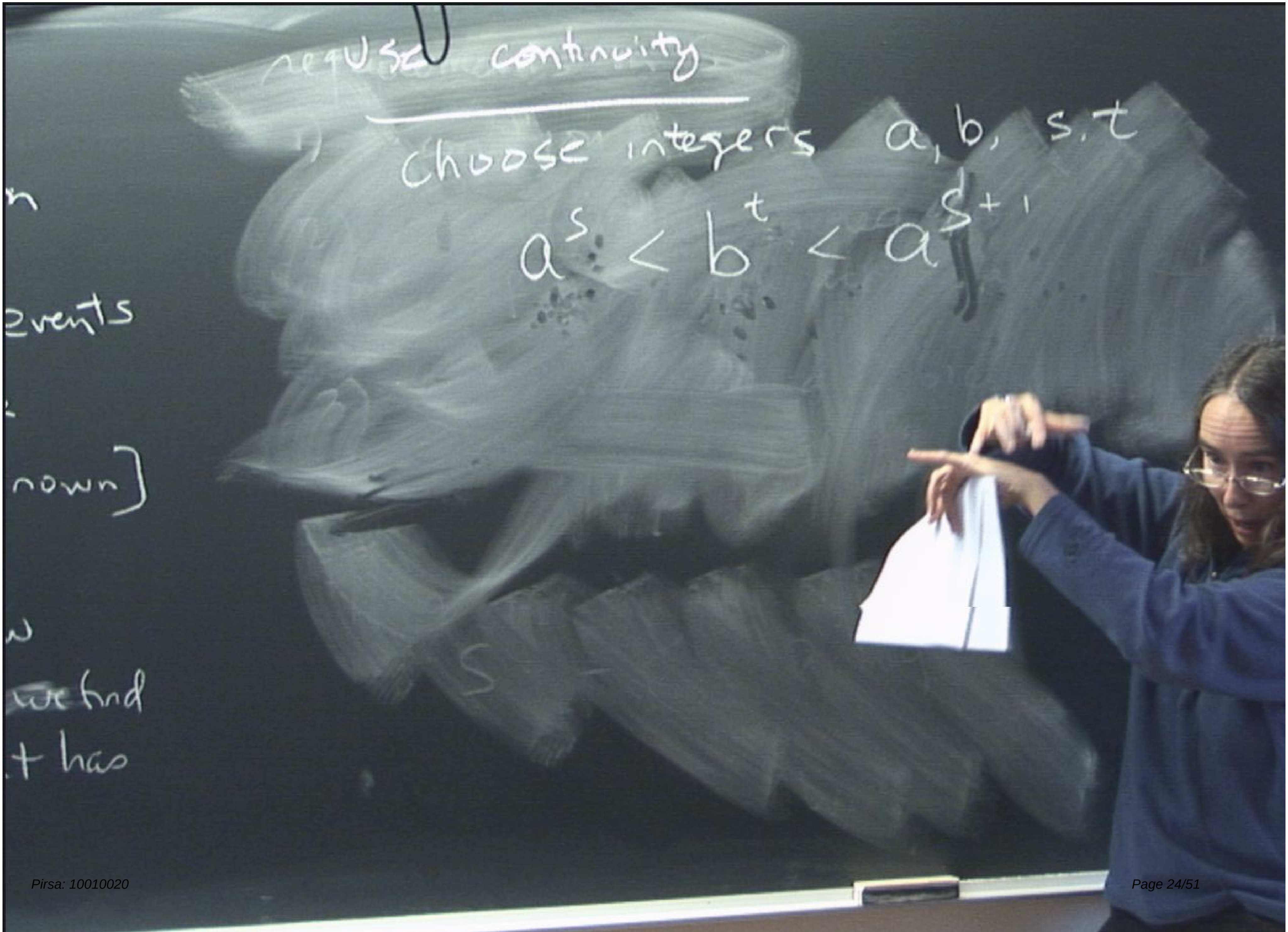
$$A(s^m) = mA(s)$$

$$A(8) = 3A(2)$$

$$8 = 2^3$$

$$A(2^3) = 3A(2)$$


continuity
Choose integers a, b, s.t
$a^s < b^t$
events
nown]
we find
t has


continuity
Choose integers a, b, s.t
a^s < b^t < a^{s+1}
events
nown]
we find
t has

continuity

Choose integers a, b, s, t

$$a^s < b^t < a^{s+1}$$

$$s \ln a < t \ln b < (s+1) \ln a$$

$$\frac{s}{t} \ln a < \ln b < \frac{(s+1)}{t} \ln a$$

$$\frac{\ln a}{\ln b} < \frac{t}{s} < \frac{s+1}{s} \frac{\ln a}{\ln b}$$

n

events

nown]

w

we find

t has

required continuity

Choose integers a, b, s, t

$$a^s < b^t < a^{s+1}$$

$$s \ln a < t \ln b < (s+1) \ln a$$

$$\frac{s}{t} \ln a < \ln b < \frac{(s+1)}{t} \ln a$$

$$\frac{\ln a}{\ln b} < \frac{t}{s} < \frac{s+1}{s} \frac{\ln a}{\ln b}$$

$$A(a^s) < A(b^t) < A(a^{s+1})$$

n

events

nown]

w

we find

t has

require continuity

Choose integers a, b, s, t

$$a^s < b^t < a^{s+1}$$

$$s \ln a < t \ln b < (s+1) \ln a$$

$$\frac{s}{t} \ln a < \ln b < \frac{(s+1)}{t} \ln a$$

$$\frac{\ln a}{\ln b} < \frac{t}{s} < \frac{s+1}{s} \frac{\ln a}{\ln b}$$

$$A(a^s) < A(b^t) < A(a^{s+1})$$

$$s A(a) < t A(b) < (s+1) A(a)$$

n

events

nown]

w

we find

t has

continuity

Choose integers a, b, s, t

$$a^s < b^t < a^{s+1}$$

$$s \ln a < t \ln b < (s+1) \ln a$$

$$\frac{s}{t} \ln a < \ln b < \frac{(s+1)}{t} \ln a$$

$$\frac{\ln a}{\ln b} < \frac{t}{s} < \frac{s+1}{s} \frac{\ln a}{\ln b}$$

$$A(a^s) < A(b^t) < A(a^{s+1})$$

$$sA(a) < tA(b) < (s+1) A(a)$$

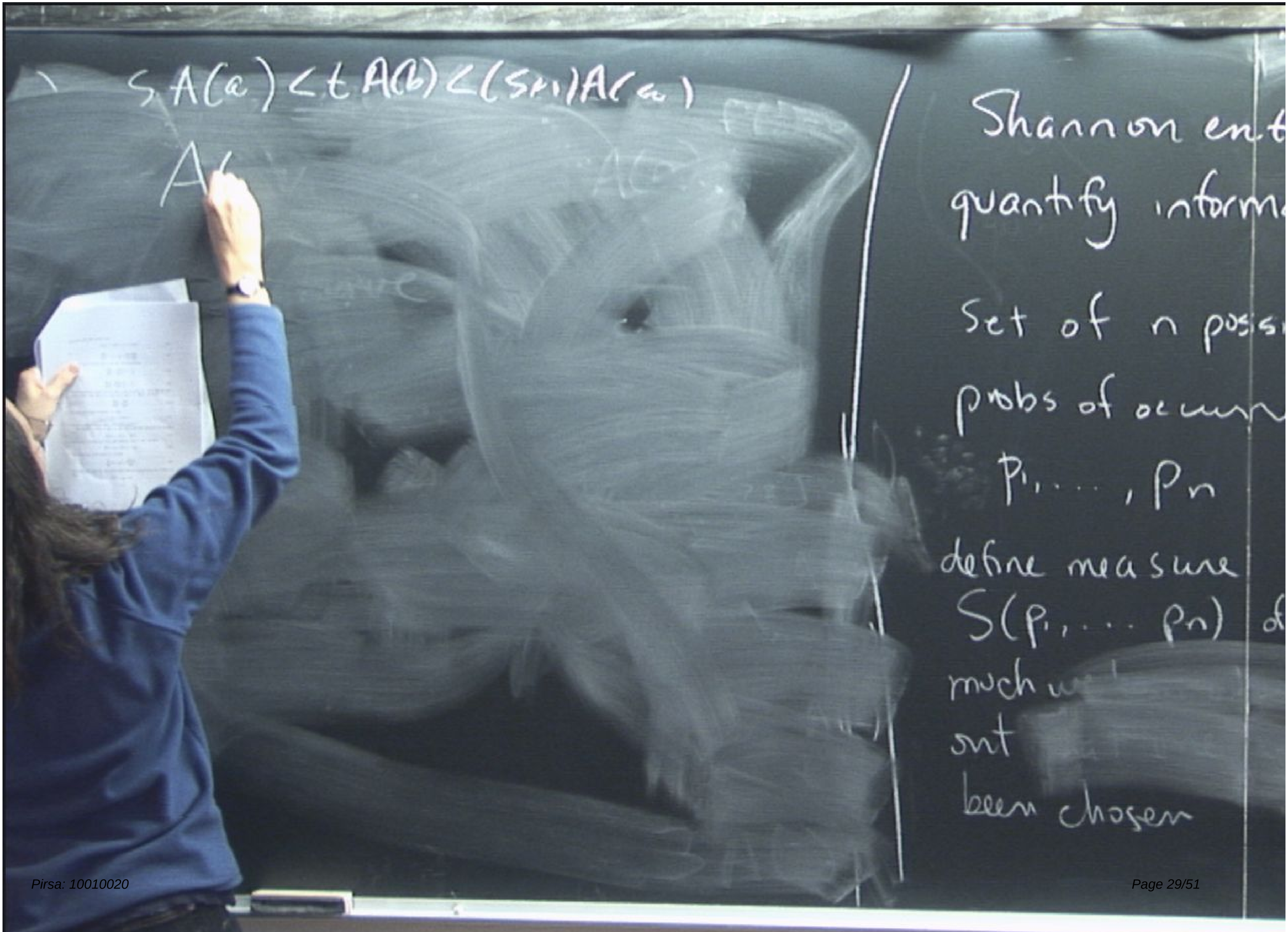
n

events

nown]

w

t has


s A(a) < t A(b) < (s+1) A(a)
A(
Shannon ent
quantify inform
Set of n possi
probs of occurr
p1, ..., pn
define measure
S(p1, ... pn) d
much w
ent
been chosen

$$s\,A(a) < t\,A(b) < (s+1)A(a)$$

$$\frac{A(a)}{A(b)} < \frac{t}{s} < \frac{s+1}{s}\,\frac{A(a)}{A(b)}$$

Shannon ent

quantify inform

Set of n possi

probs of occurr

$p_1, \ldots, p_n$

define measure

$S(p_1, \ldots p_n)$ of

much w

ont

been chosen

$s\,A(a) < t\,A(b) < (s+1)A(a)$

$$\frac{A(a)}{A(b)} < \frac{t}{s} < \frac{s+1}{s}\,\frac{A(a)}{A(b)}$$

① $$\frac{A(a)}{A(b)} < \frac{\ln a}{\ln b}\left(1 + 1/s\right)$$

② $$\frac{A(a)}{A(b)} > \frac{\ln a}{\ln b} \Big/ \left(1 + \frac{1}{s}\right)$$

Shannon ent

quantify inform

Set of n possi

probs of occurr

$p_1, \dots, p_n$

define measure

$S(p_1, \dots p_n)$ d

much w

snt

been chosen

$s\,A(a) < t\,A(b) < (s+1)A(a)$

$$\frac{A(a)}{A(b)} < \frac{t}{s} < \frac{s+1}{s}\,\frac{A(a)}{A(b)}$$

① $$\frac{A(a)}{A(b)} < \frac{\ln a}{\ln b}\left(1+1/s\right)$$

② $$\frac{A(a)}{A(b)} > \frac{\ln a}{\ln b}\Big/\left(1+\frac{1}{s}\right)$$

true for any s

$\Rightarrow$ true as $s \to \infty \Rightarrow \dfrac{A(a)}{A(b)} = \dfrac{\ln a}{\ln b}$

Shannon ent

quantify inform

Set of n possi

probs of occurr

$p_1, \dots, p_n$

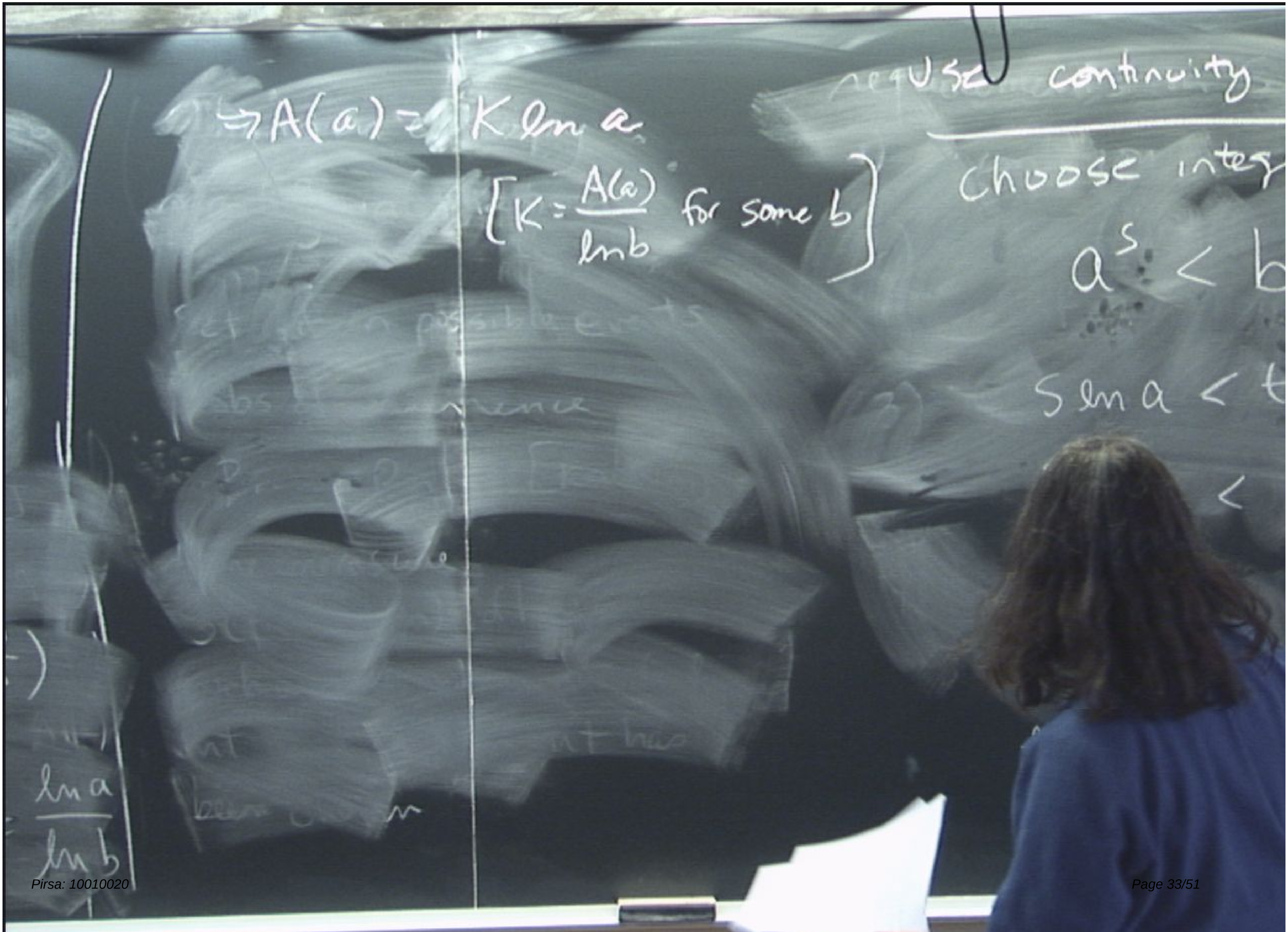
define measure

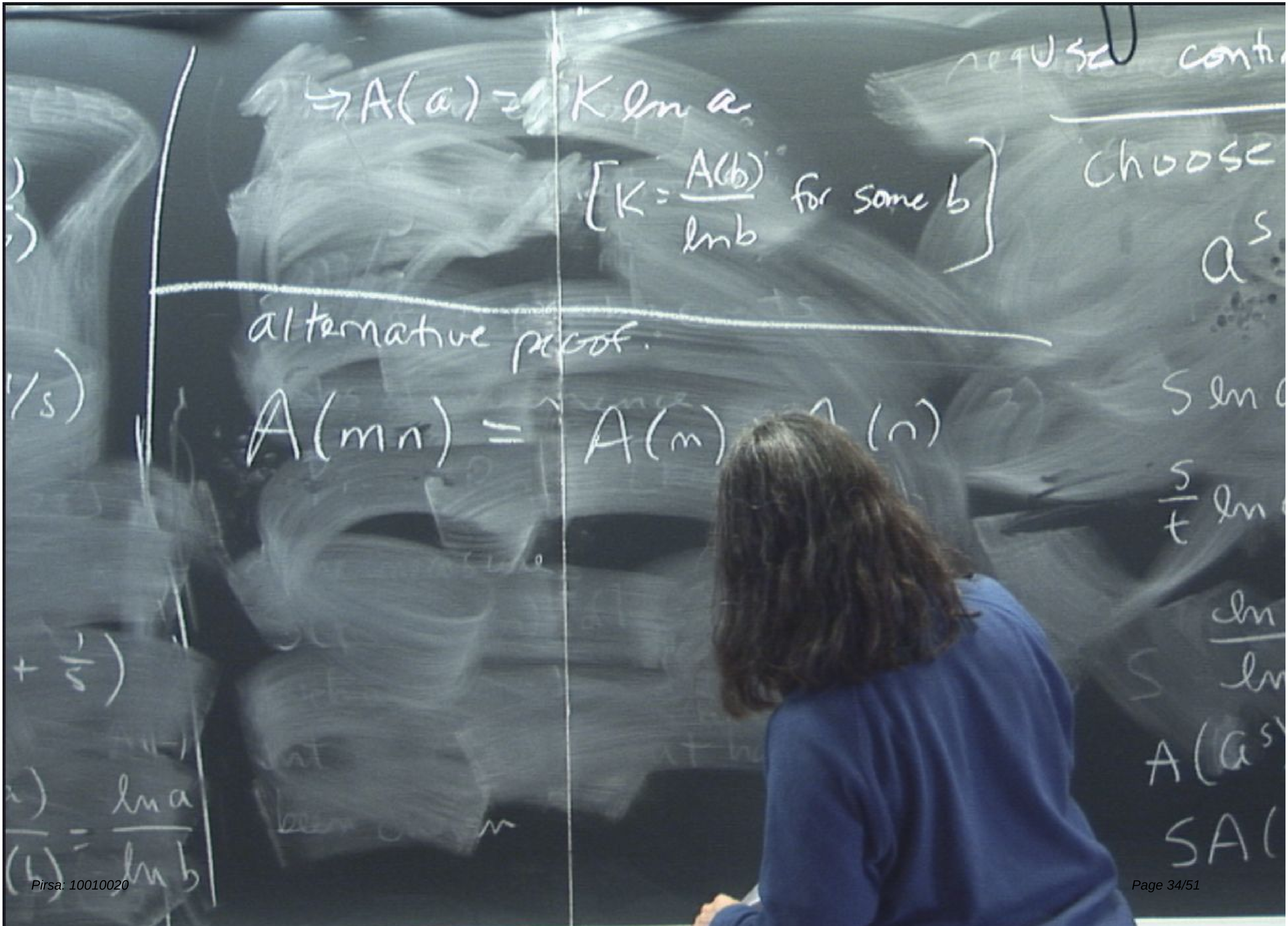
$S(p_1, \dots p_n)$ of

much w

unt

been chosen


⇒ A(a) = K ln a
[K = A(a)/ln b for some b]
use continuity
Choose integ
a^s < b
s ln a < t
ln a
ln b



⇒ A(a) = K ln a
[K = A(b)/ln b for some b]
choose
alternative proof.
A(mn) = A(m)

$\Rightarrow A(a) = K \ln a$

$\left[K = \frac{A(b)}{\ln b} \text{ for some } b \right]$

alternative proof:

$A(mn) = A(m) + A(n)$

1 2 3 4 5 6

$$\Rightarrow A(a) = K \ln a$$

$$\left[K = \frac{A(b)}{\ln b} \text{ for some } b \right]$$

alternative proof.

$$A(mn) = A(m) + A(n)$$

let $x = \ln m$

$y = \ln n$

$$\tilde{A}(x) = A(e^x)$$

$$\Rightarrow A(a) = K \ln a$$

$$\left[K = \frac{A(b)}{\ln b} \text{ for some } b \right]$$

alternative proof:

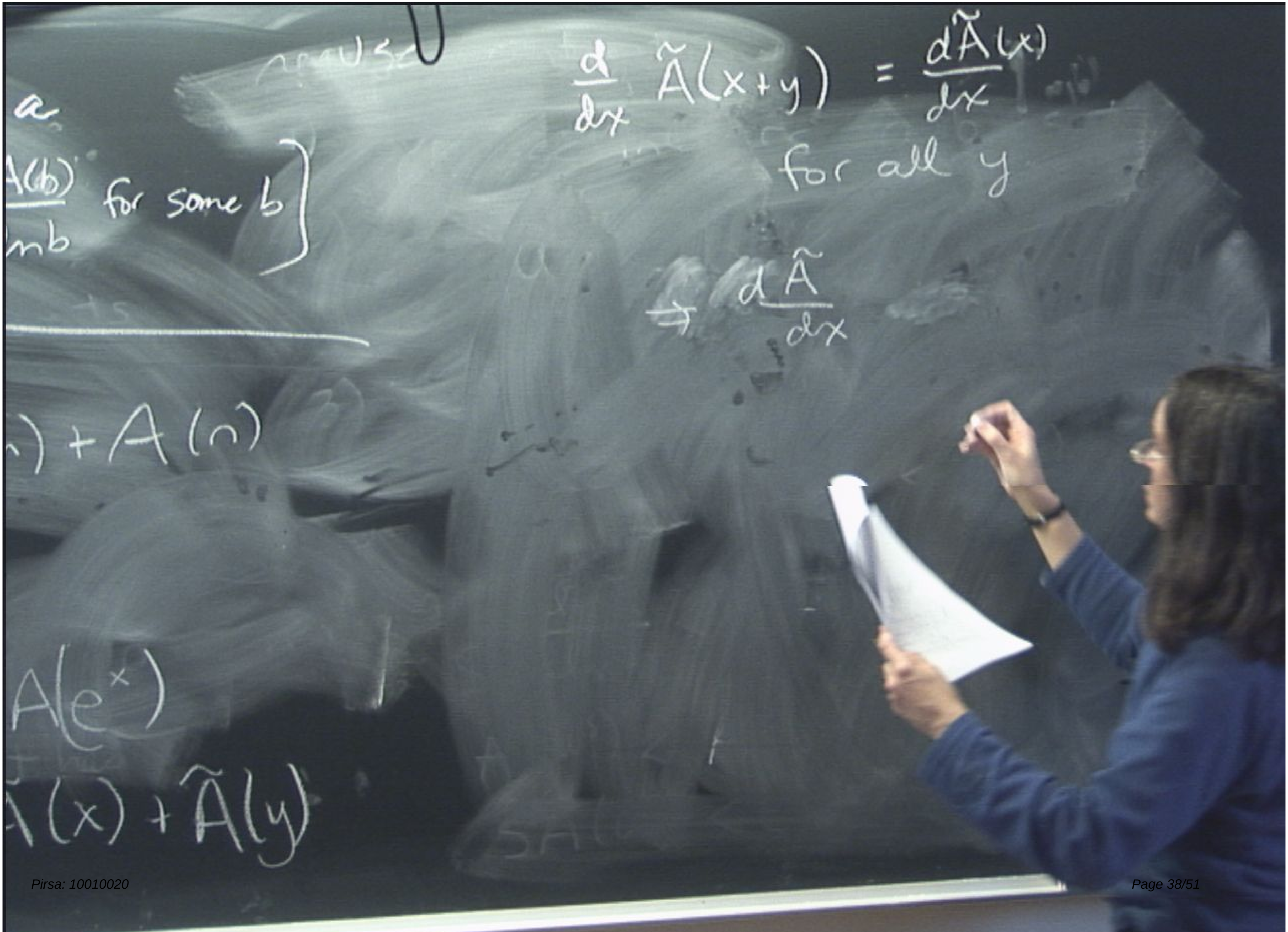
$$A(mn) = A(m) + A(n)$$

let $x = \ln m$

$y = \ln n$

$$\tilde{A}(x) = A(e^x)$$

$$\tilde{A}(x+y) = \tilde{A}(x) + \tilde{A}(y)$$


d/dx Ã(x+y) = dÃ(x)/dx
for all y
→ dÃ/dx
for some b
+ A(n)
A(e^x)
Ã(x) + Ã(y)

a

$\frac{A(b)}{\ \ mb}$ for some b

$) + A(n)$

$A(e^x)$

$\tilde{A}(x) + \tilde{A}(y)$

$$\frac{d}{dx}\tilde{A}(x+y) = \frac{d\tilde{A}(x)}{dx}$$

for all y

$\Rightarrow \frac{d\tilde{A}}{dx}$ is independent of x

$\Rightarrow \tilde{A}(x) = Kx + b$

$\Rightarrow A(m) = K \ln m + b$

$\Rightarrow A(a) = K \ln a$

$\left[K = \dfrac{A(b)}{\ln b} \text{ for some } b \right]$

ernative proof:

$(mn) = A(m) + A(n)$

let $x = \ln m$

$y = \ln n$

$\tilde{A}(x) = A(e^{x})$

$\tilde{A}(x+y) = \tilde{A}(x) + \tilde{A}(y)$

$\dfrac{d}{dx}\tilde{A}(x+y) =$

for a

$\Rightarrow \dfrac{d\tilde{A}}{dx}$

$\Rightarrow \tilde{A}(x) =$

$\Rightarrow A(m) =$

$K \ln(mn) + b = K$

$\Rightarrow b$

now consider

p_i that are not all equal

consider p_i rational

$\Rightarrow A(a) =$ ...

alternative pro...

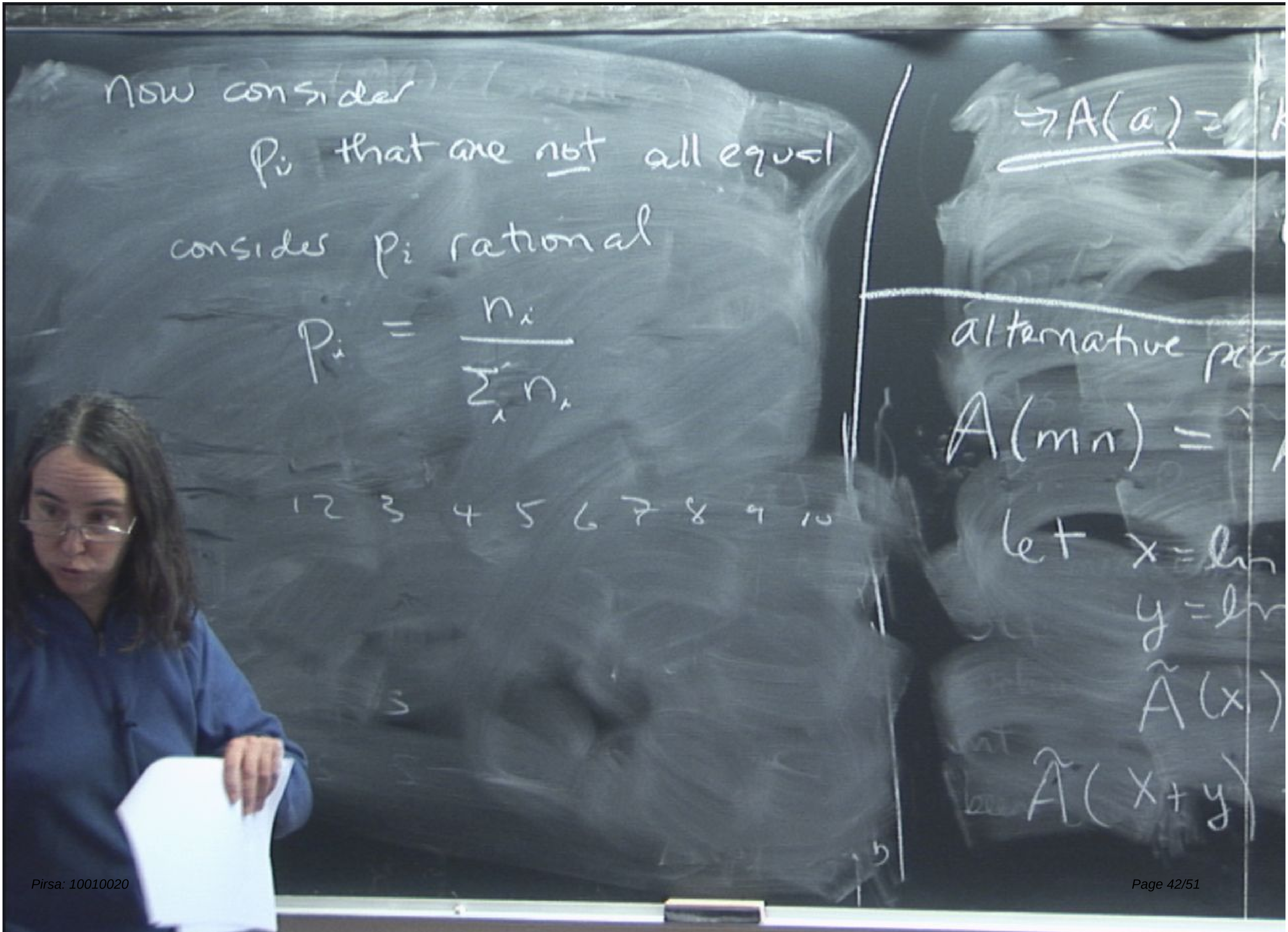
$A(mn) =$ ...

let $x = \ln$ ...

$y = \ln$ ...

$\tilde{A}(x)$ ...

$\tilde{A}(x+y)$ ...


now consider
p_i that are not all equal
consider p_i rational
$p_i = \frac{n_i}{\sum_i n_i}$
1 2 3 4 5 6 7 8 9 10
$\Rightarrow A(a) =$
alternative pr
$A(mn) =$
let $x = \ln$
$y = \ln$
$\tilde{A}(x)$
$\tilde{A}(x+y)$

now consider

p_i that are not all equal

consider p_i rational

$$p_i = \frac{n_i}{\sum_i n_i}$$

1 2 3 4 5 | 6 7 8 | 9 10

$p_1 = 1/2$ $p_2 = 3/10$ $p_3 = 1/5$

$\Rightarrow A(a) =$ K

alternative proo

$A(mn) =$

let $x = \ln$

$y = \ln$

$\tilde{A}(x)$

$\tilde{A}(x+y)$

now consider

p_i that are not all equal

consider p_i rational

$$p_i = \frac{n_i}{\sum_i n_i}$$

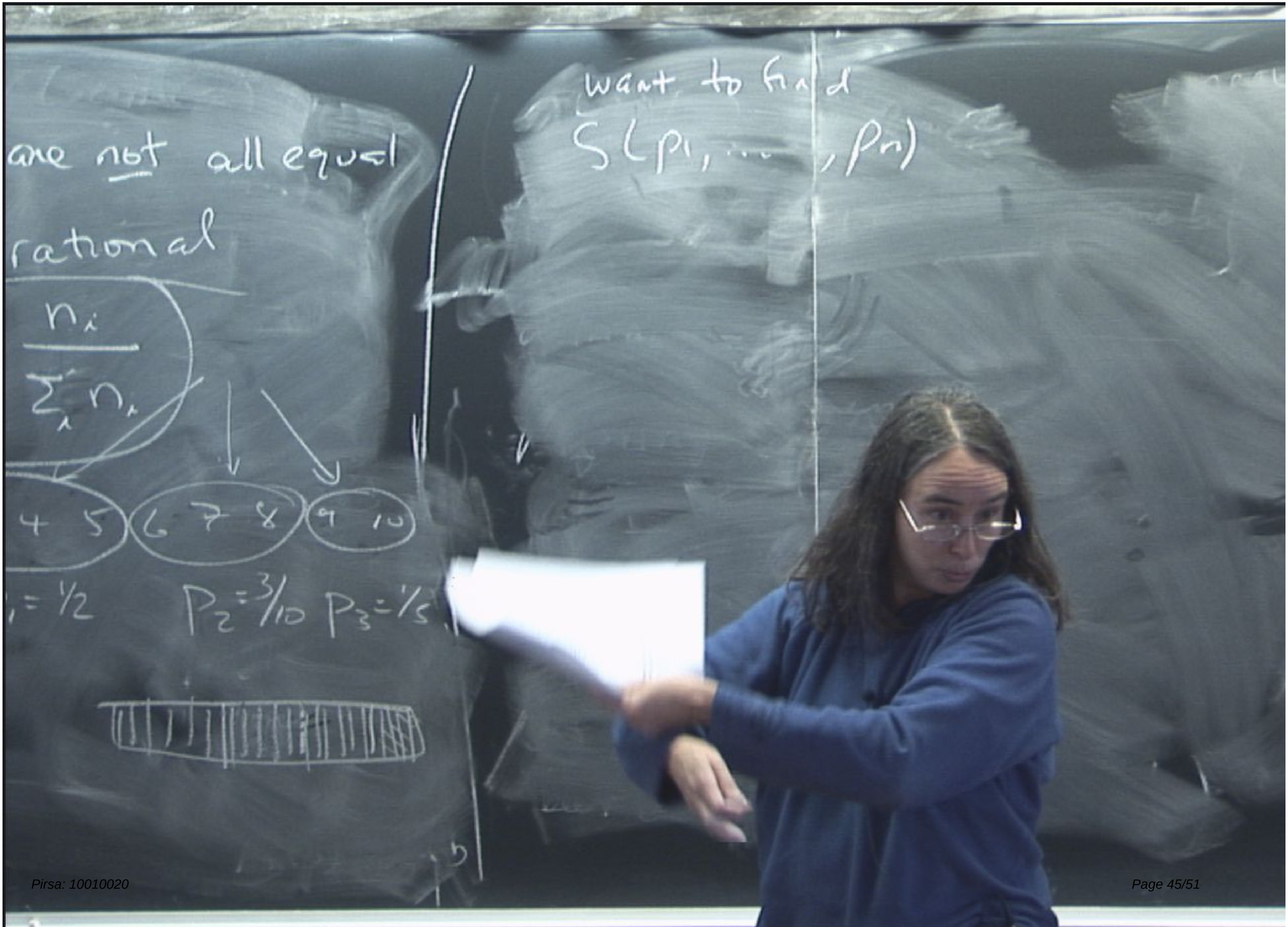
1 2 3 4 5   6 7 8   9 10

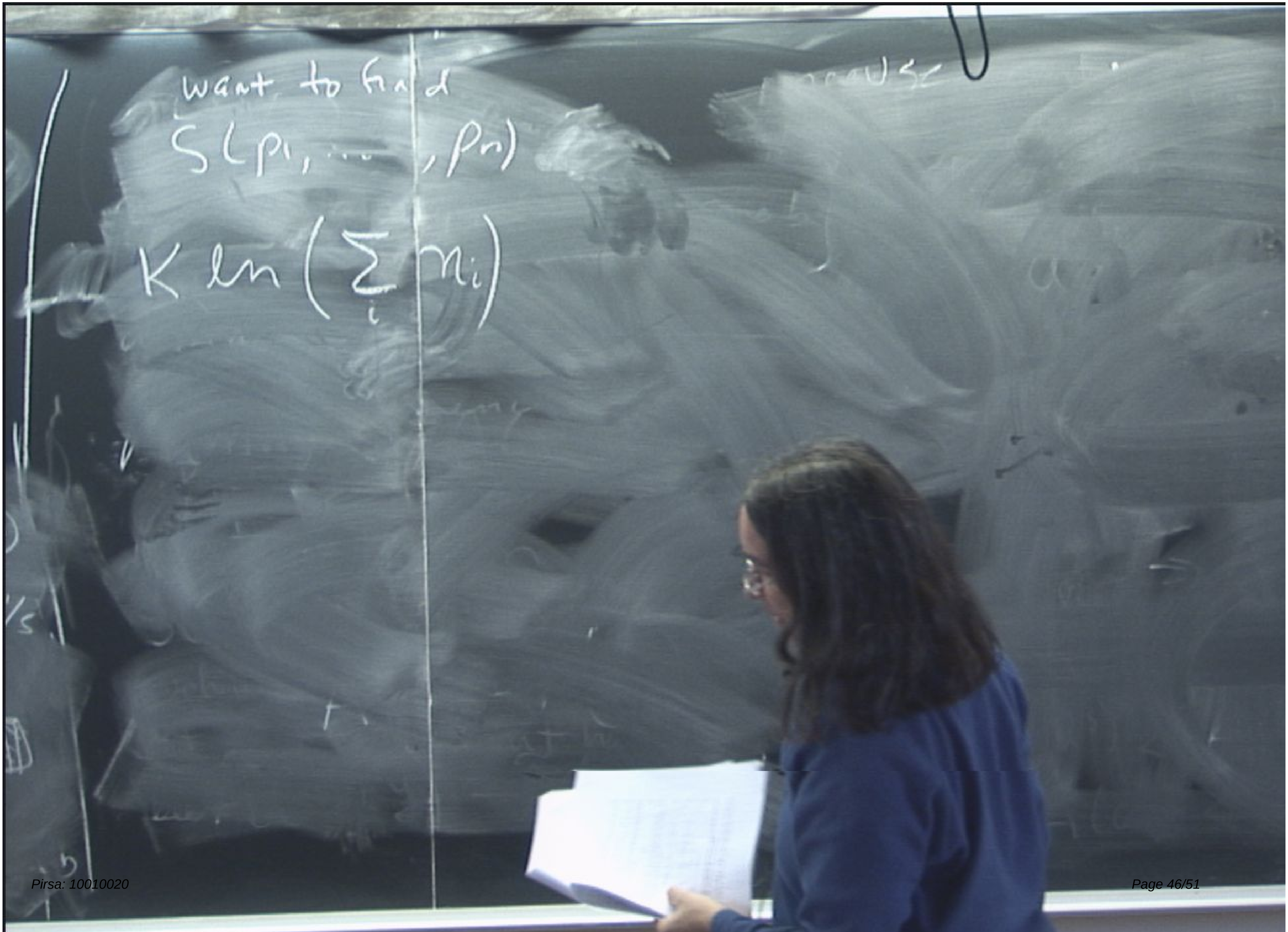
$p_1 = 1/2$ $p_2 = 3/10$ $p_3 = 1/5$

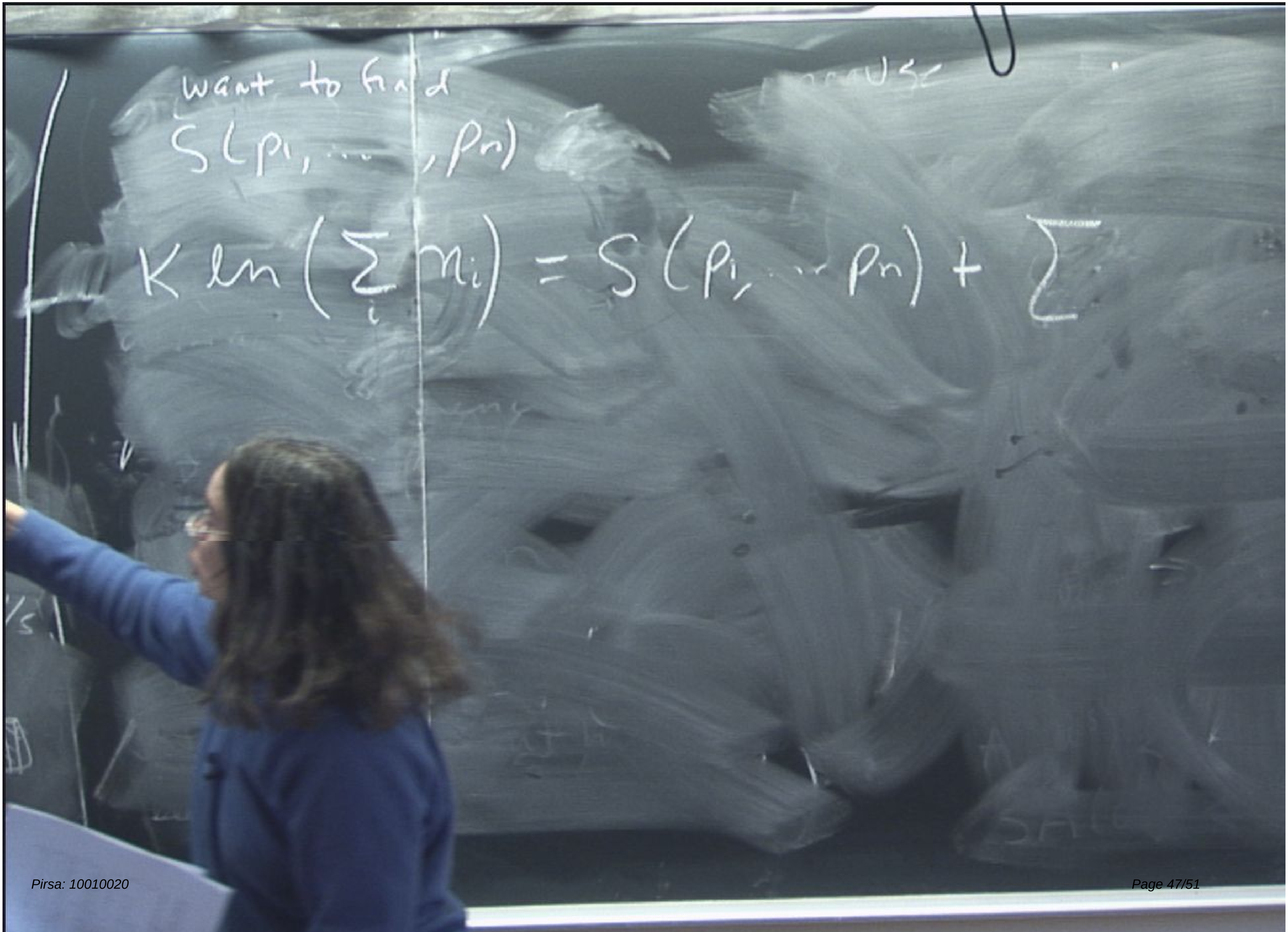
$\Rightarrow A(a) = K$

alternative pro

$A(mn) =$


want to find
S(p1, ..., pn)
are not all equal
rational
n_i / Σ_i n_i
4 5 6 7 8 9 10
= 1/2 P2 = 3/10 P3 = 1/5

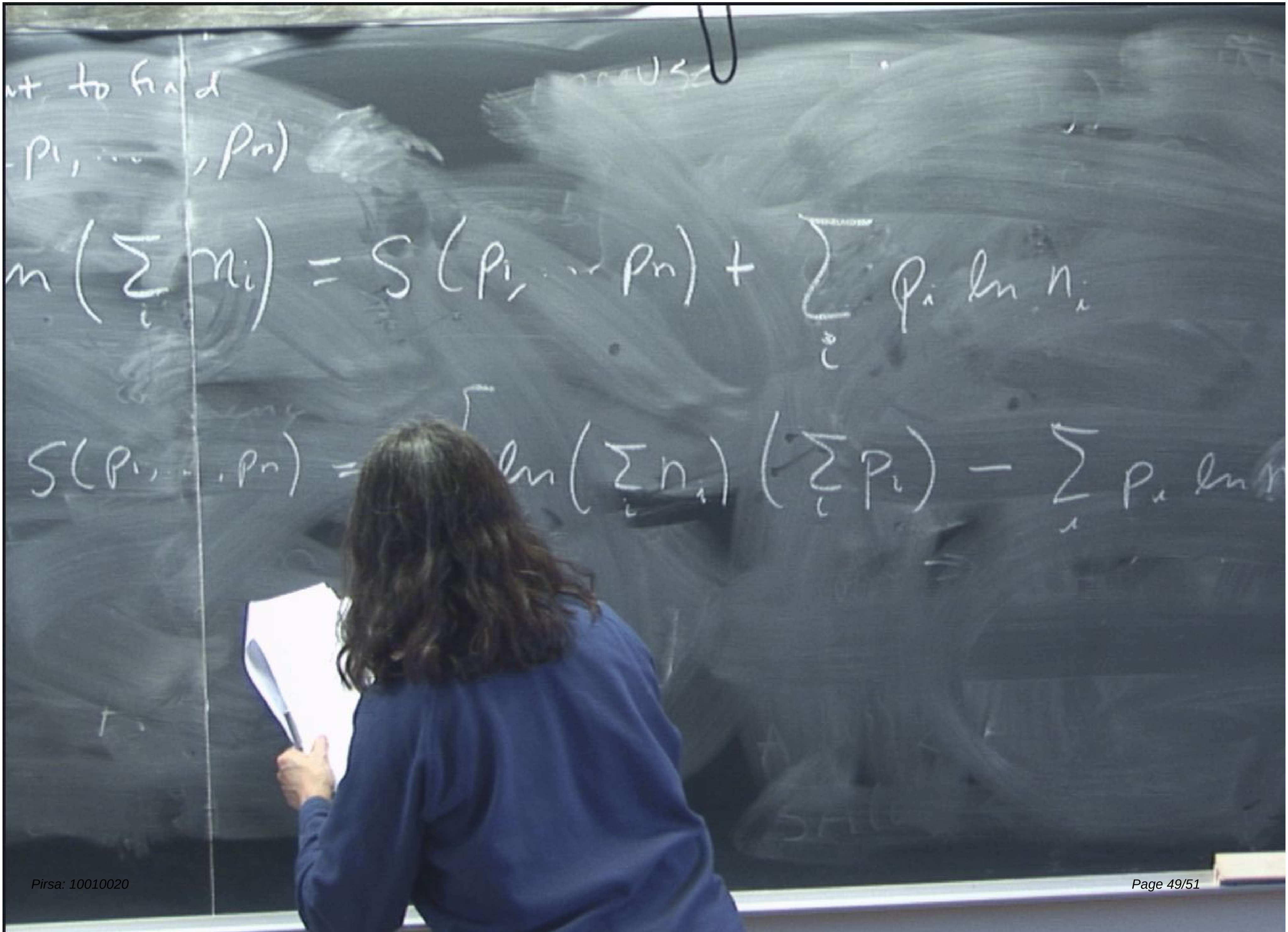

want to find
S(p1, ..., pn)
K ln(Σi ni)


want to find
S(p1, ..., pn)
K ln(Σi ni) = S(p1, ... pn) + Σ

want to find

$S(p_1, \ldots, p_n)$

$$K \ln\left(\sum_i n_i\right) = S(p_1, \ldots p_n) + \sum_i p_i \ln n_i$$



nt to find
$p_1, \ldots, p_n)$
$n(\sum_i n_i) = S(p_1, \ldots p_n) + \sum_i p_i \ln n_i$
$S(p_1, \ldots, p_n) = \ln(\sum_i n_i)(\sum_i p_i) - \sum_i p_i \ln n$

nt to find d

$(p_1, \ldots, p_n)$

$$\ldots n\left(\sum_i n_i\right) = S(p_1, \ldots p_n) + \sum_i p_i \ln n_i$$

$$S(p_1, \ldots, p_n) = K\left[\ln\left(\sum_i n_i\right)\left(\sum_i p_i\right) - \sum_i p_i \ln n_i\right]$$

$$= -K \sum_i p_i \ln\left(\frac{n_i}{\sum_i n_i}\right) = -K \sum_i p_i \, l\ldots$$

…nt to find d

$\ldots(p_1, \ldots, p_n)$

$$\ldots n\left(\sum_i n_i\right) = S(p_1, \ldots p_n) + K \sum_i p_i \ln n_i$$

$$S(p_1, \ldots, p_n) = K\left[\ln\left(\sum_i n_i\right)\left(\sum_i p_i\right) - \sum_i p_i \ln n_i\right]$$

$$= -K \sum_i p_i \ln\left(\frac{n_i}{\sum_i n_i}\right) = -K \sum_i p_i \, l\ldots$$