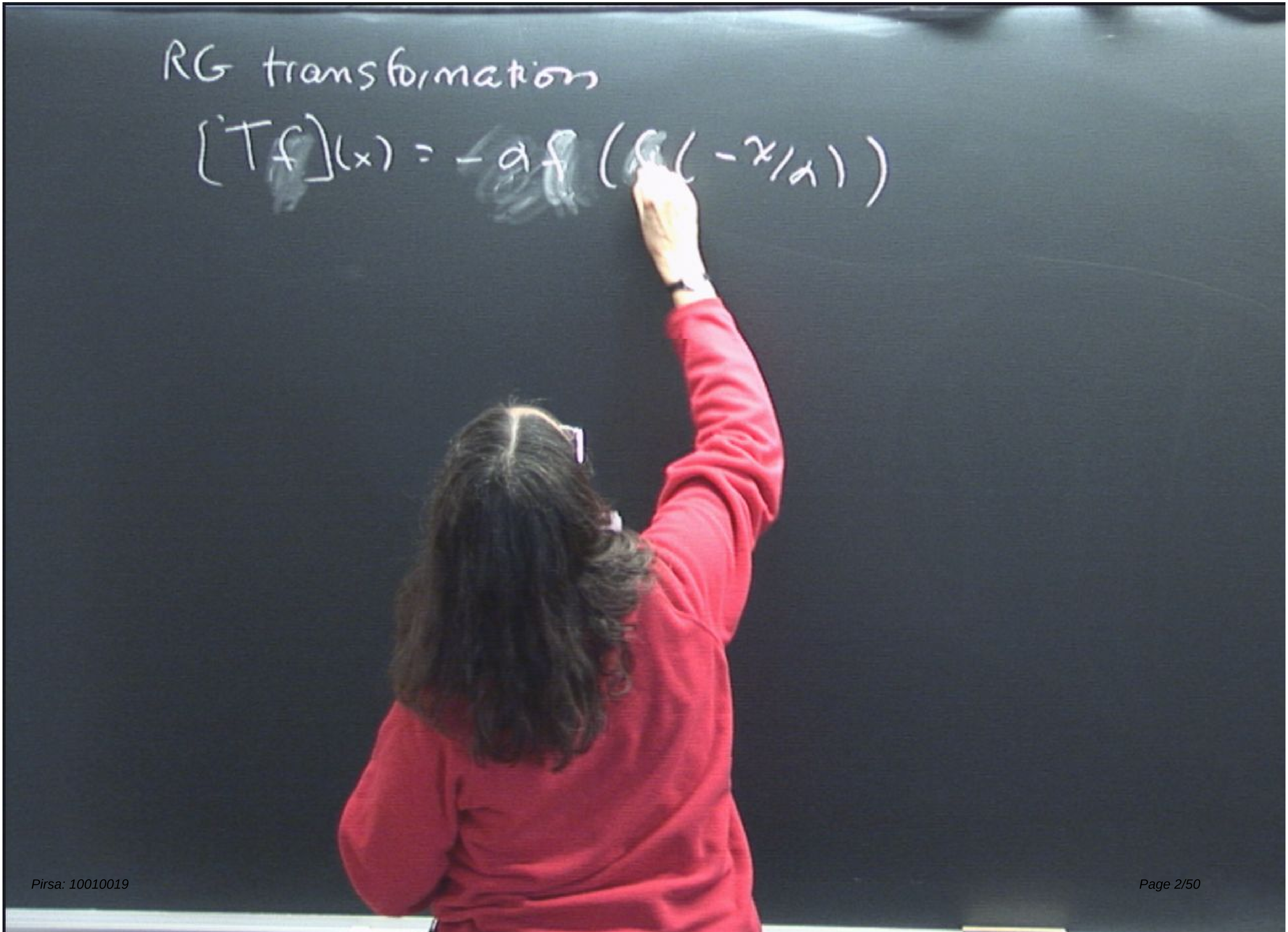
Title: Dynamical Systems - Review (PHYS 607) - Lecture 7

Date: Jan 12, 2010 09:00 AM

URL: http://pirsa.org/10010019

Abstract:


RG transformation
$[Tf](x) = -\alpha f(f(-x/\alpha))$

RG transformation

$$[Tf](x) = -\alpha f\left(f\left(-x/\alpha\right)\right)$$

fixed pt fn g satisfies

$$Tg = g$$

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To get δ

write $h(x) = g(x) + \epsilon\,\eta(x)$

$$[Th](x) \approx$$

RG transformation

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fixed pt fn g satisfies

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α fixed in this solution

To get δ

write $h(x) = g(x) + \epsilon\eta(x)$

$$[Th](x) \approx g(z) - \alpha\epsilon\eta\left(g\left(-\frac{z}{\alpha}\right)\right) - \alpha\epsilon\frac{dg}{dx}$$

tion

$\alpha f\,(f(-x/\alpha))$

g satisfies

g

in this solution

$(x) = g(x) + \epsilon\,\eta(x)$

$(x) \approx g(z) - \alpha\epsilon\,\eta\left(g\left(-\frac{z}{\alpha}\right)\right) - \alpha\epsilon \left.\frac{dg}{dx}\right|_{g(-z/\alpha)} \eta(-z/\alpha)$

RG transformation

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To get δ

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$$[Th](z) \approx g(z) - \alpha\epsilon\,\eta\left(g\left(-\frac{z}{\alpha}\right)\right)$$

$T:\quad \eta \to \eta' \quad : \quad T(g+\epsilon\eta) \approx g + \epsilon\,\mathcal{L}(\eta)$

RG transformation

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To get δ

write $h(z) = g(z) + \epsilon\,\eta(z)$

$$[Th](z) \approx g(z) - \alpha\epsilon\,\eta\left(g\left(-\tfrac{z}{\alpha}\right)\right) - \alpha\epsilon$$

T: $\eta \to \eta'$ $\quad T(g+\epsilon\eta) \simeq g + \epsilon\,\mathcal{L}(\eta)$

$\mathcal{L}(\eta) = -\alpha[\eta$

$(g(-x/\alpha))$

satisfies

n this solution

$= g(z) + \epsilon\eta(z)$

$\approx g(z) - \alpha\epsilon\eta(g(-\frac{z}{\alpha})) - \alpha\epsilon \left.\frac{dg}{dx}\right|_{g(-z/\alpha)} \eta(-z/\alpha)$

$T(g+\epsilon\eta) \simeq g + \epsilon\mathcal{L}(\eta)$

$$\mathcal{L}(\eta) = -\alpha\left[\eta(g(-z/\alpha)) + \frac{dg}{dx}\bigg|_{g(-z/\alpha)} \eta(-z/\alpha)\right]$$

$$\epsilon \frac{dg}{dx}\bigg|_{g(-z/\alpha)} \eta(-z/\alpha)$$

$$\mathcal{L}(\eta) = -\alpha\left[\eta(g(-z/\alpha)) + \frac{dg}{dx}\bigg|_{g(-z/\alpha)}\eta(-z/\alpha)\right]$$

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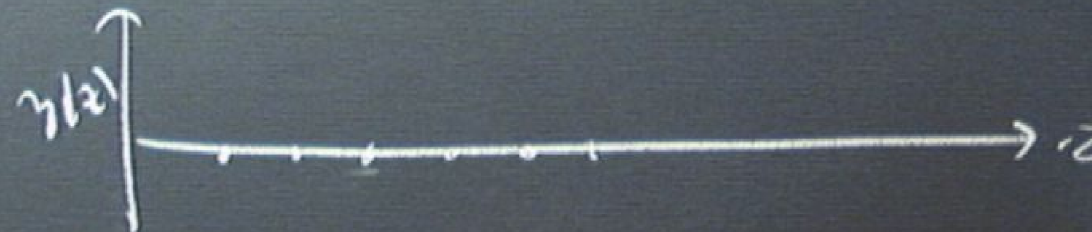

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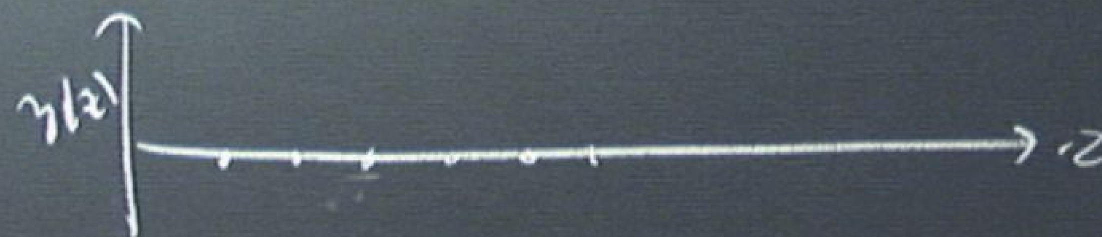

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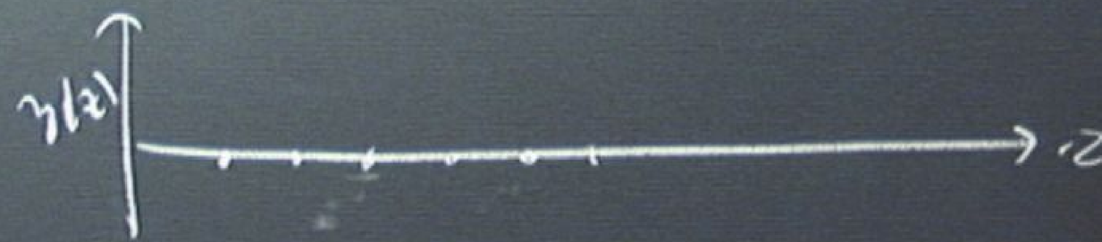

δ is the only eigenvalue

$$\mathcal{L}\eta_i = \lambda\eta_i$$

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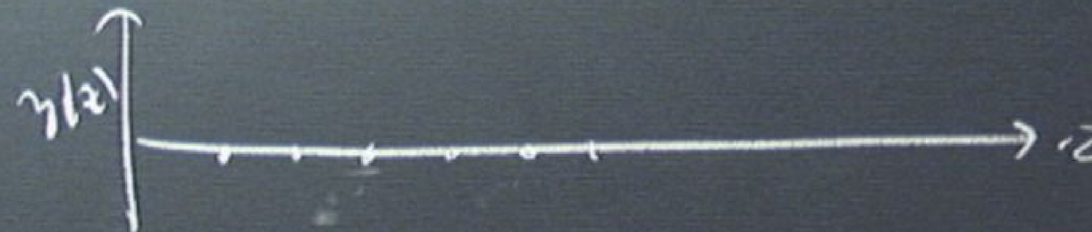

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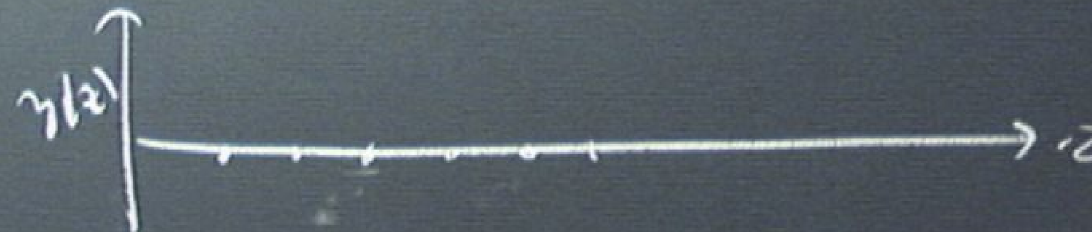

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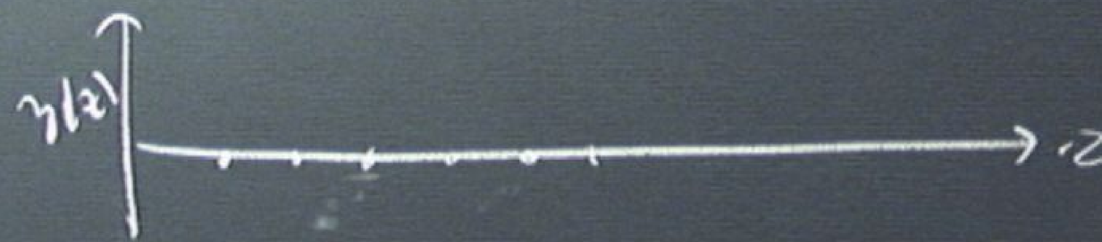

δ is the only eigenvalue > 1

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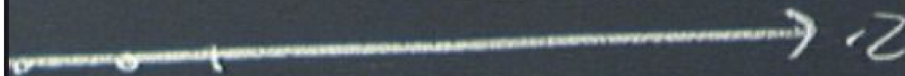

only

is the eigenvalue > 1

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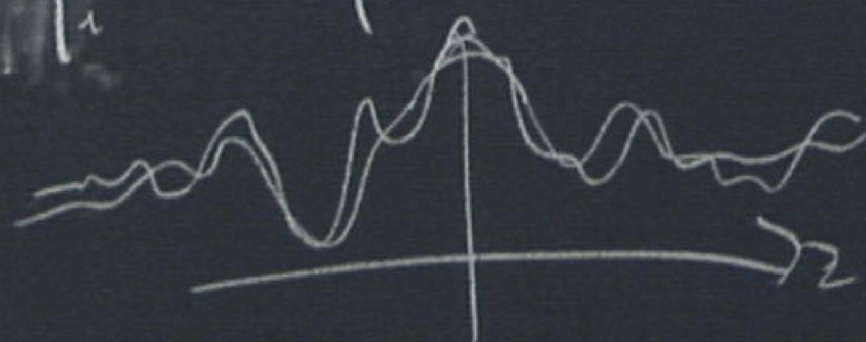
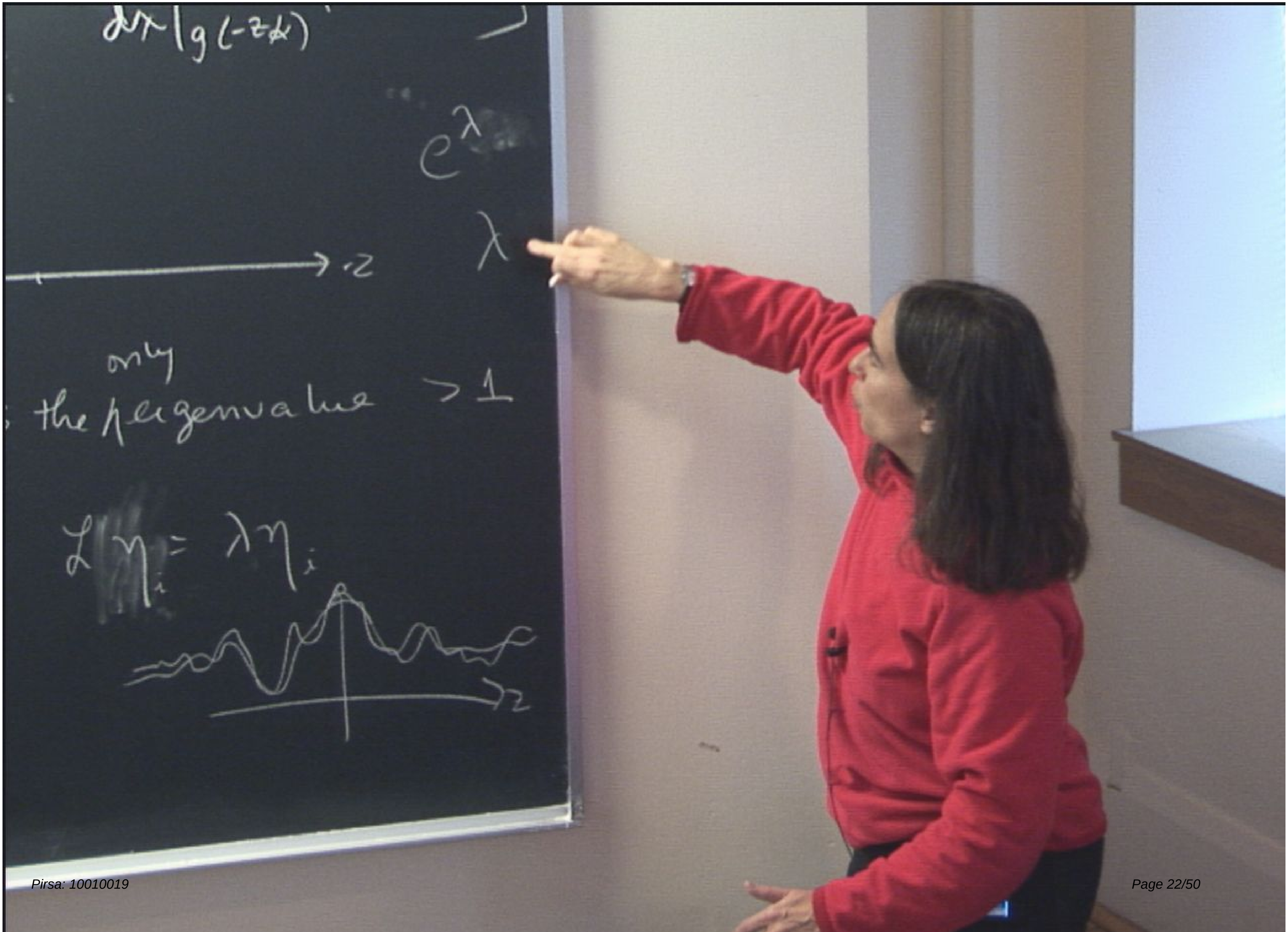
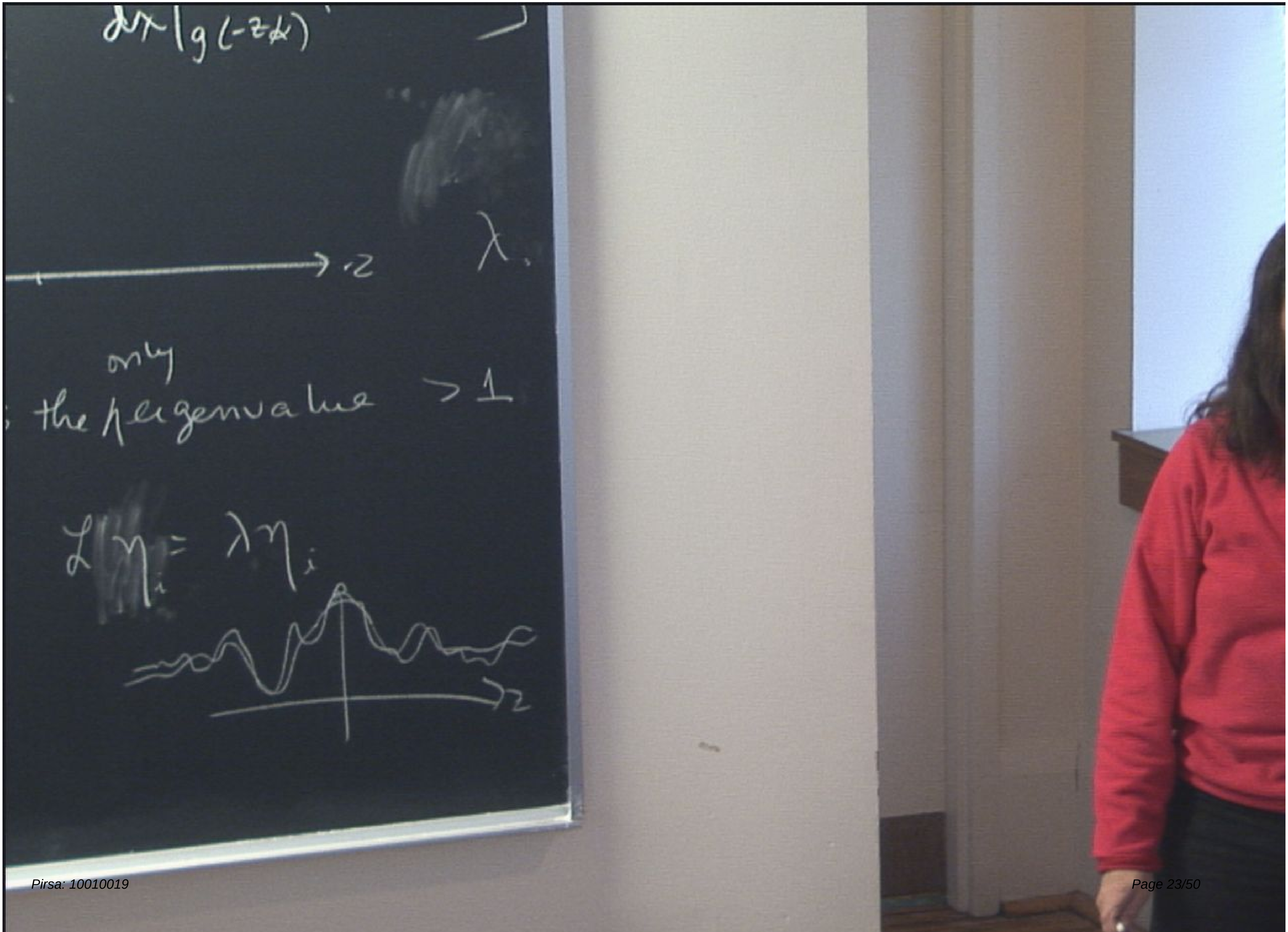




e^{λ}
λ
z
only
the eigenvalue > 1
$\mathcal{L}\eta_i = \lambda\eta_i$
z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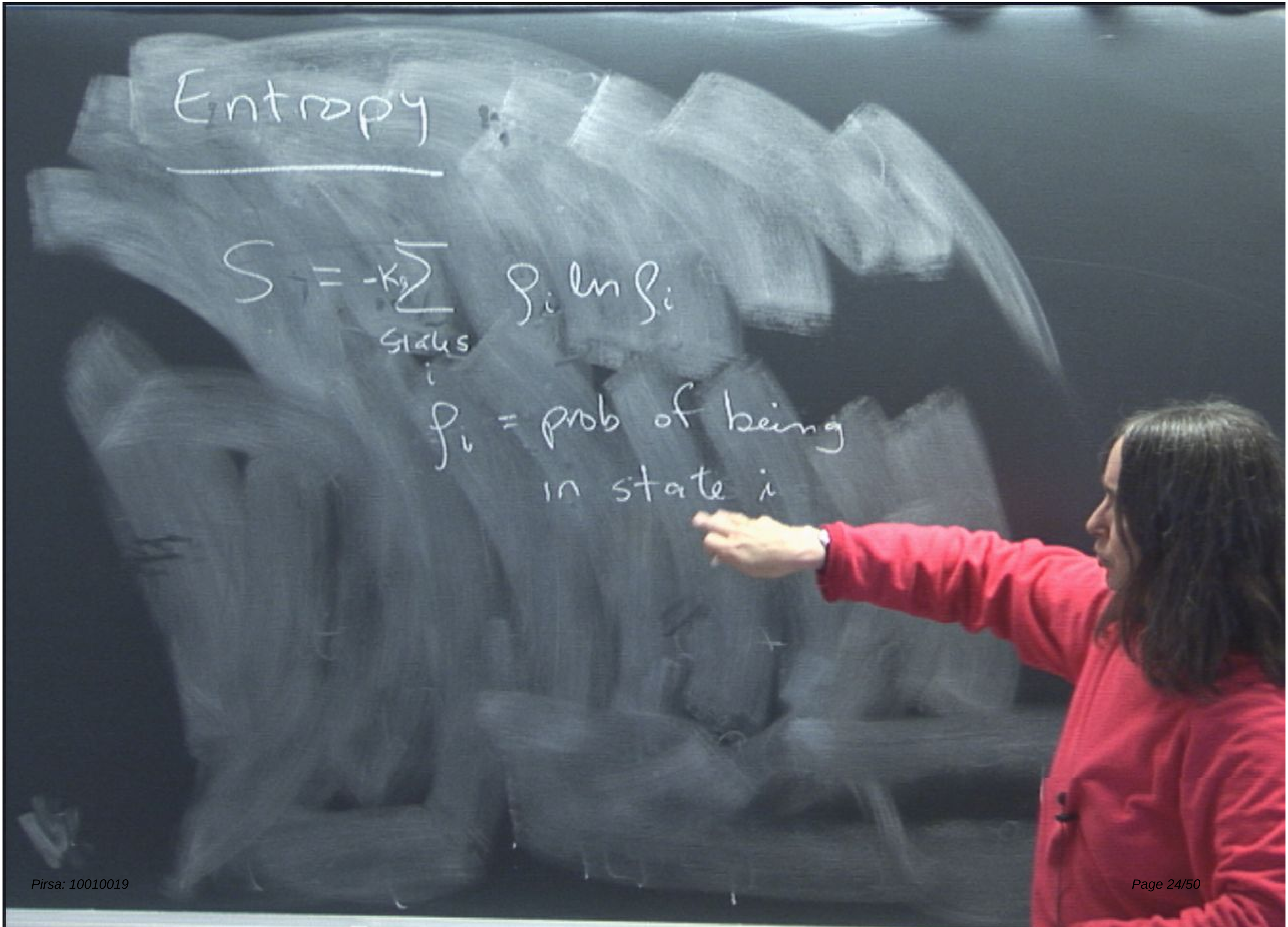
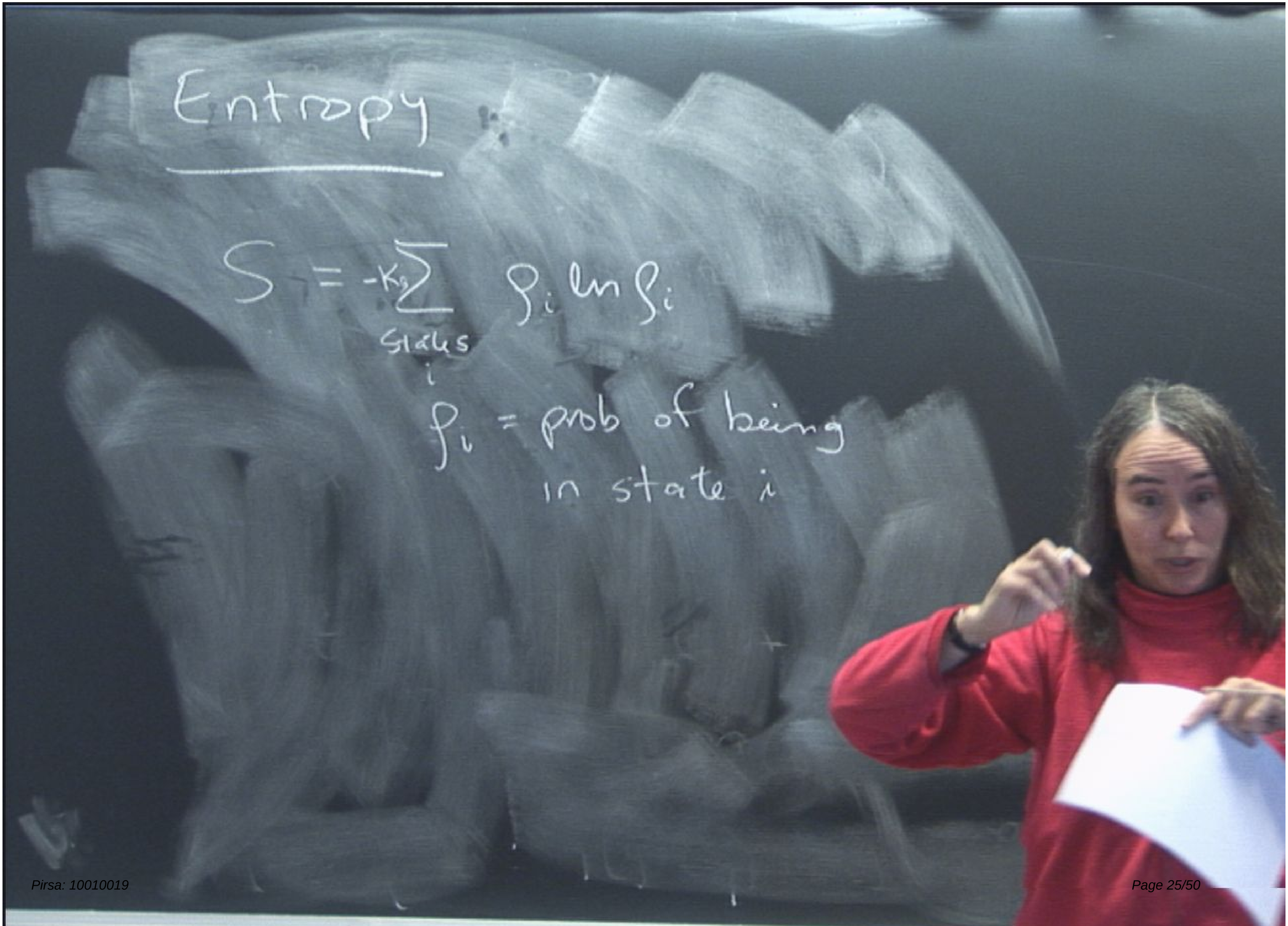

Entropy
$S = -k_B \sum_{\substack{\text{states} \\ i}} \rho_i \ln \rho_i$
ρ_i = prob of being in state i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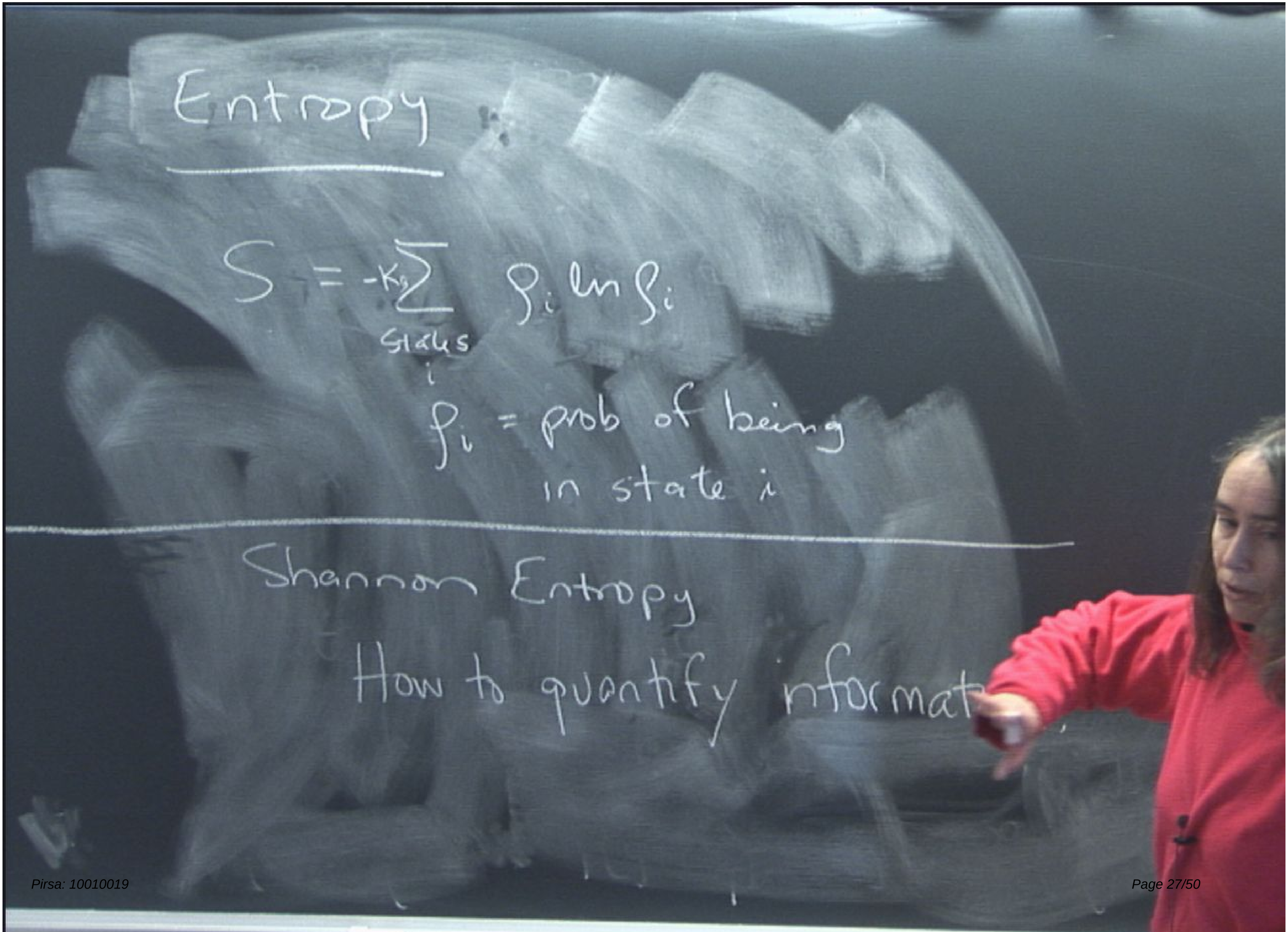
Shannon Entropy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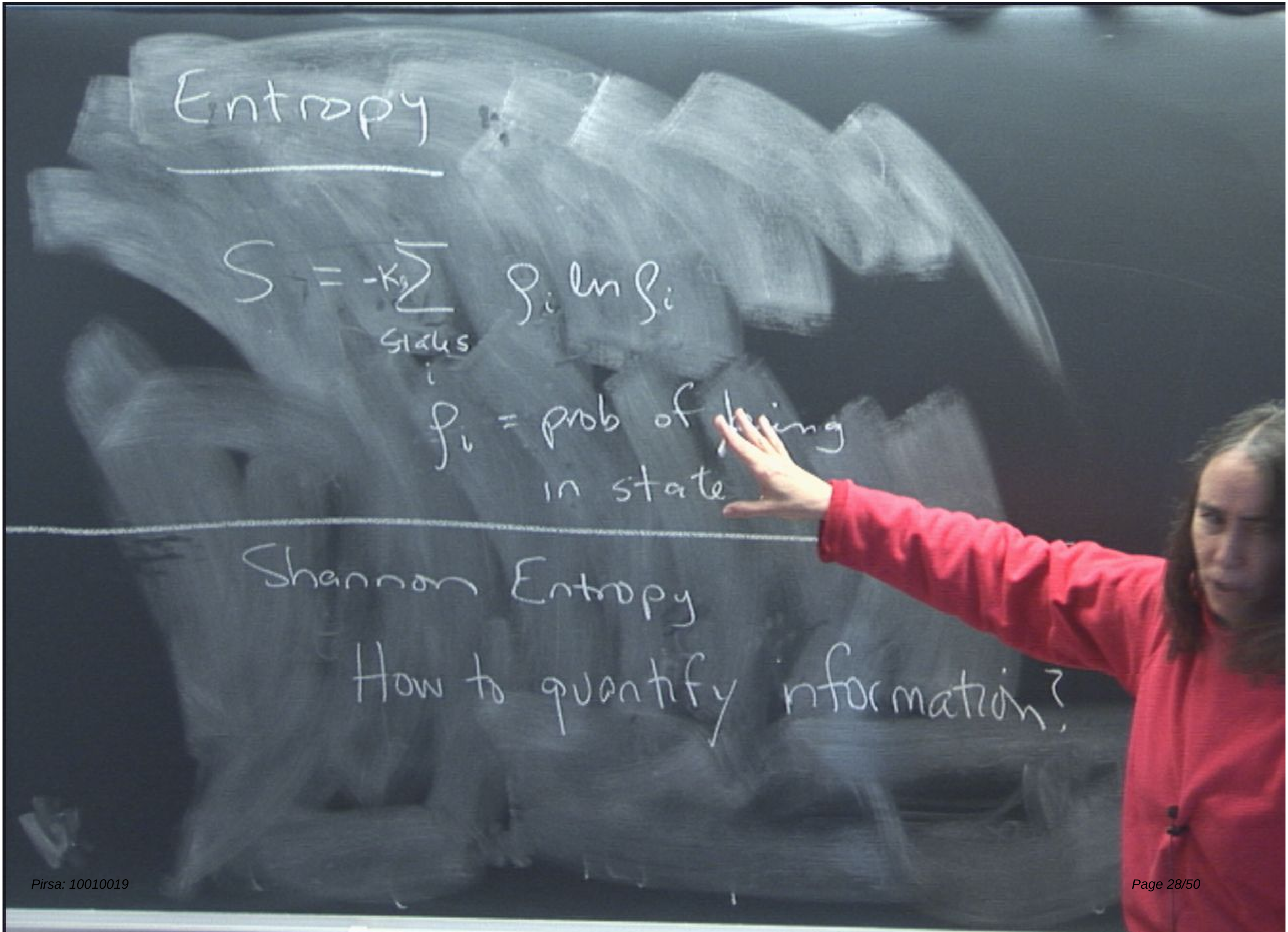
How to quantify informat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Shannon Entropy
How to quantify information?

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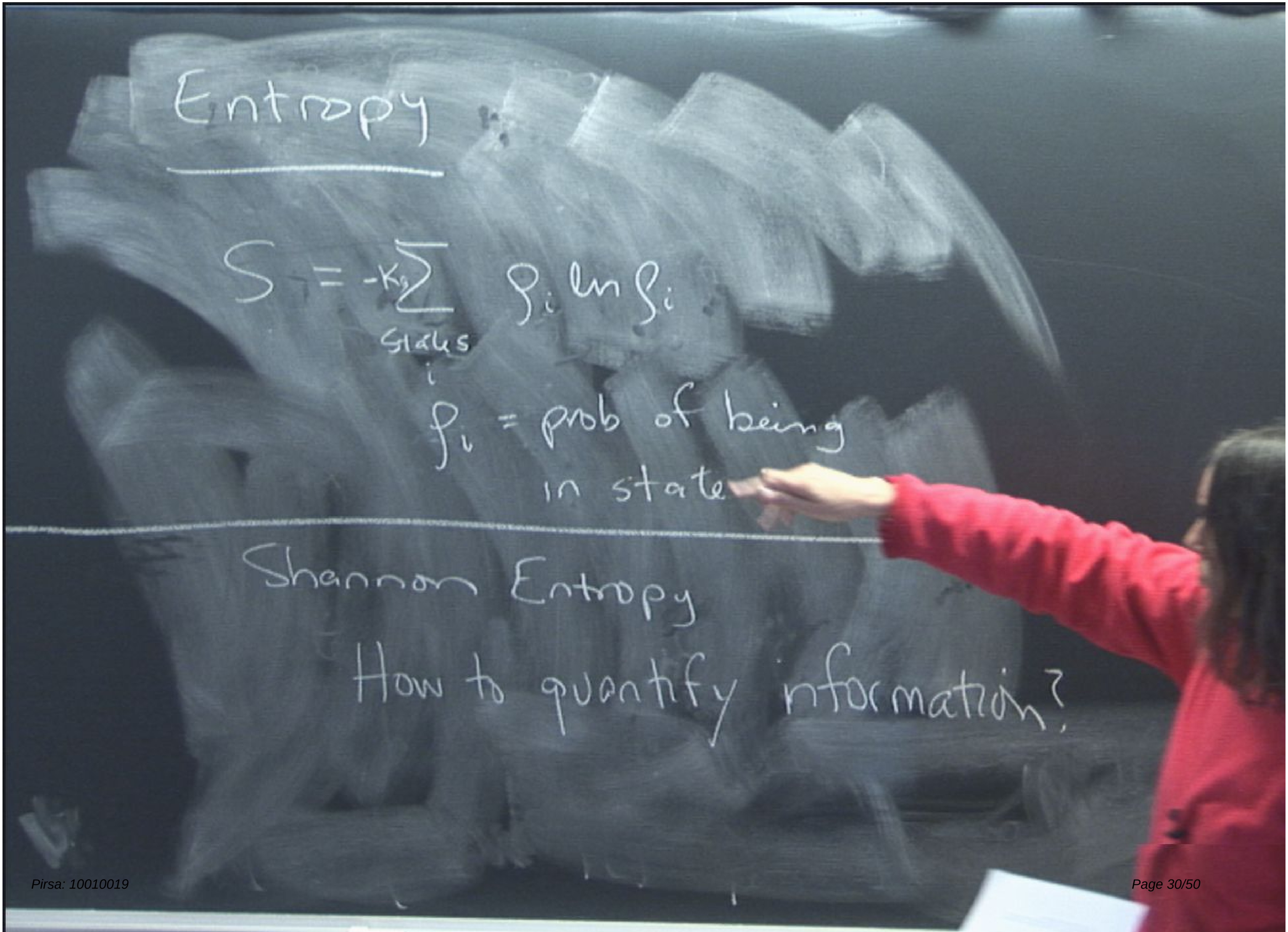
How to quantify information?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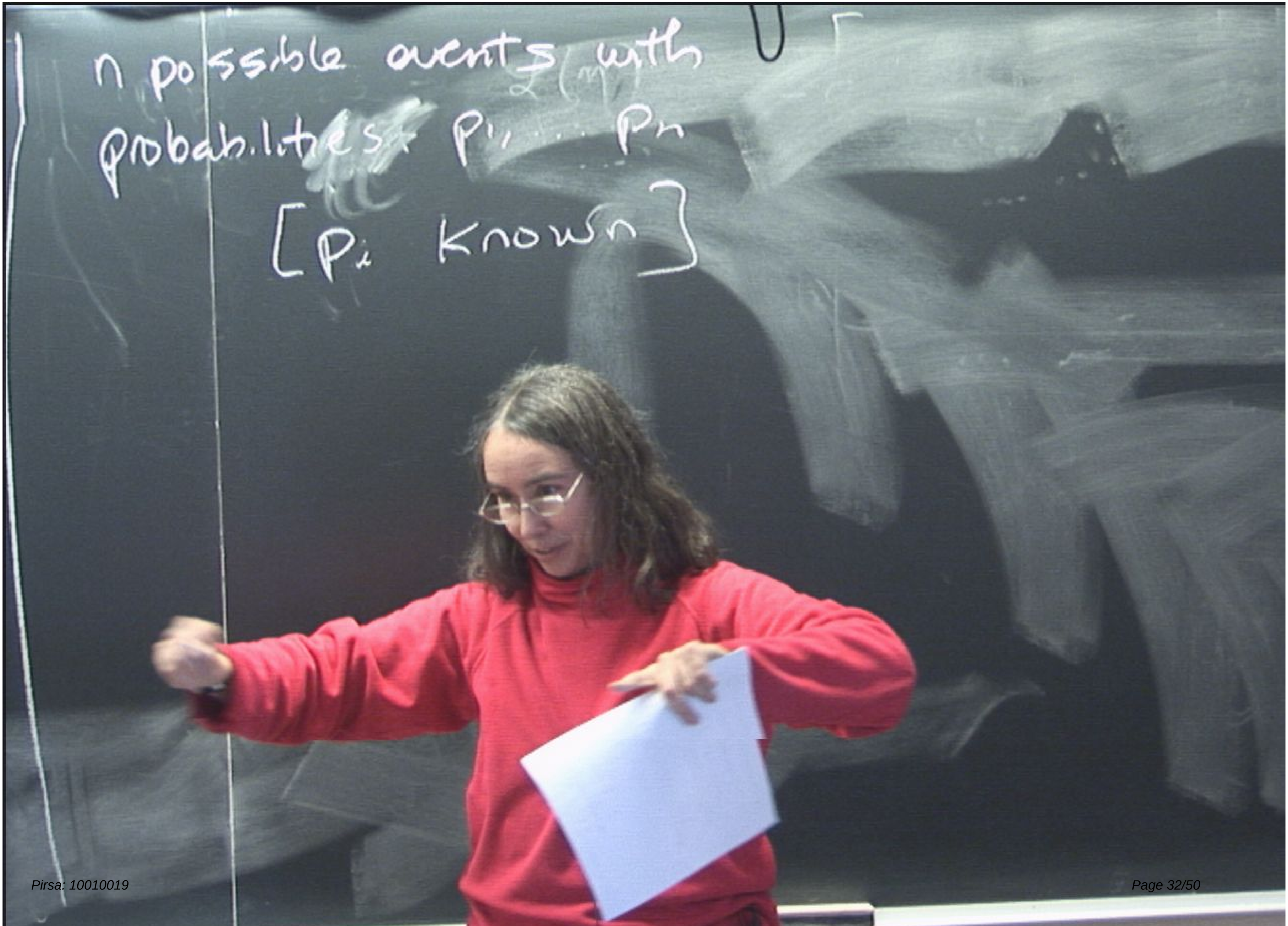
n possible events with
probabilities p1, ... pn
[pi known]

n possible events with

probabilities $p_1, \ldots p_n$

[p_i known]

How much do we learn when one event is chosen?

want to define a measure

$S(p_1, \ldots, p_n)$ of how much

choice is involved

n possible events with probabilities $p_1, \ldots p_n$

$[p_i$ known$]$

1) S is continuo

How much do we learn when one event is chosen?

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events with

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p_i known]

do we learn when
it is chosen?

define a measure

$\ldots, p_n)$ of how much
s involved

1) S is continuous in the p_i

events with

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p_i known

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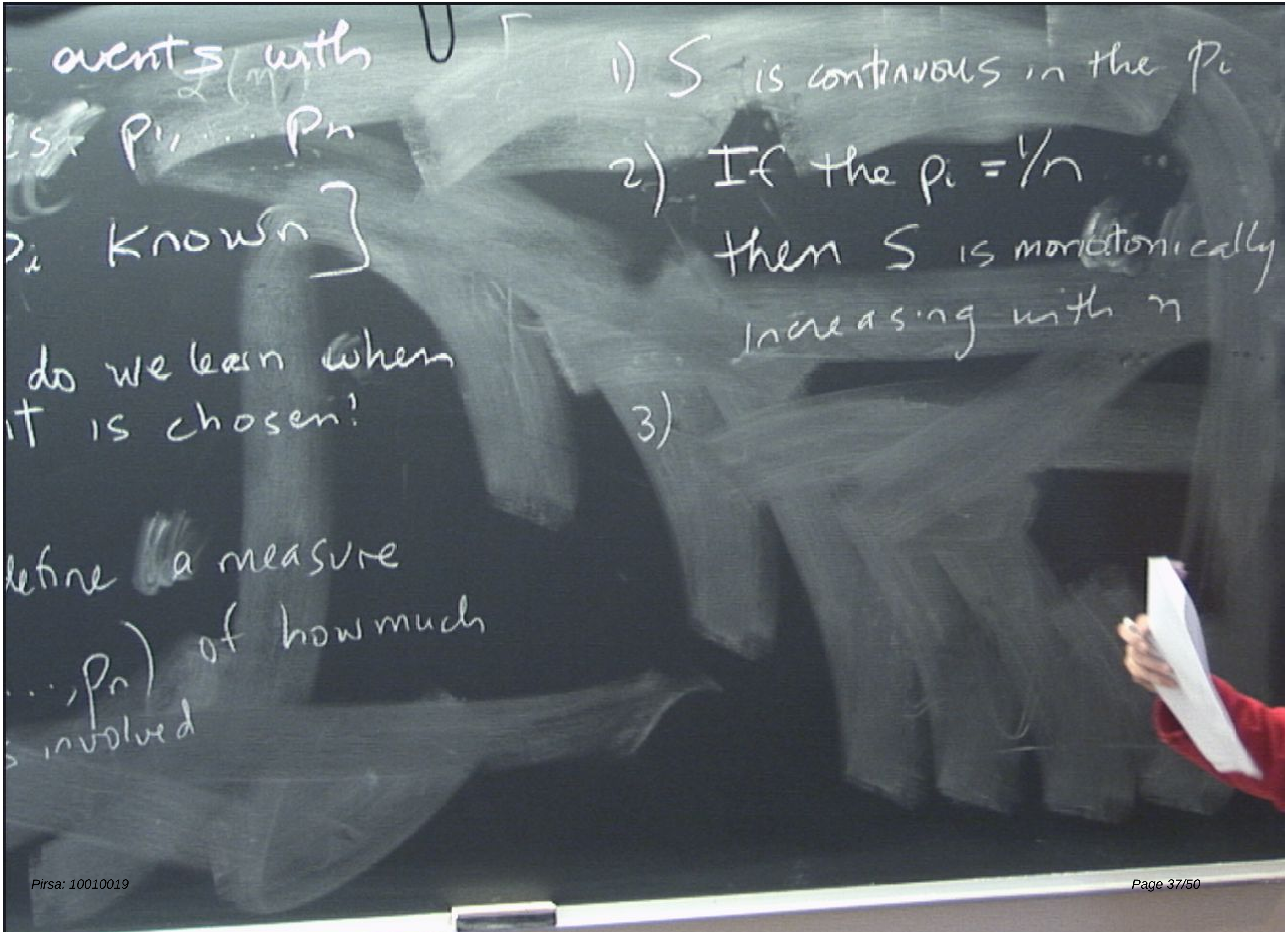
1) S is continuous in the p_i

2) If the $p_i = 1/n$

then S is monotonically

increasing with n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3) If you break down a choice into two steps, you learn the same amount as when you choose in 1 step

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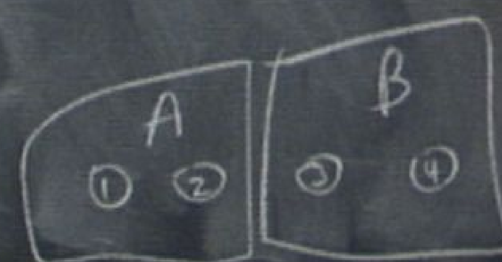

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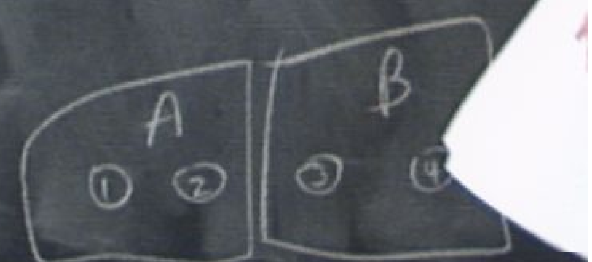

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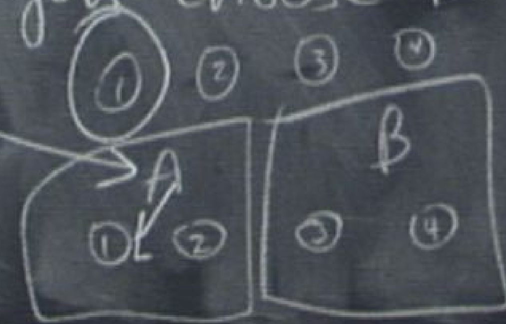

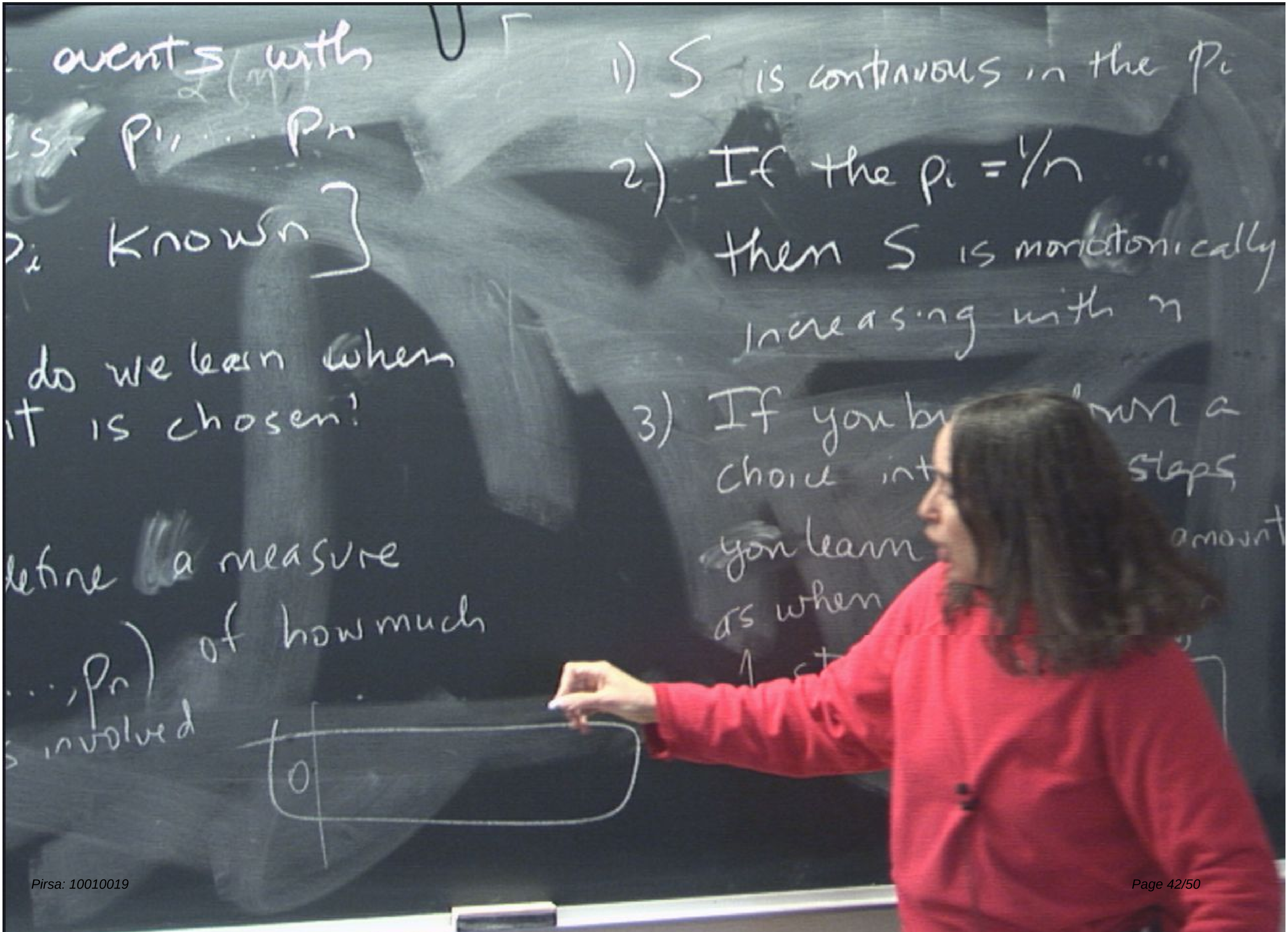
events with
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do we learn when
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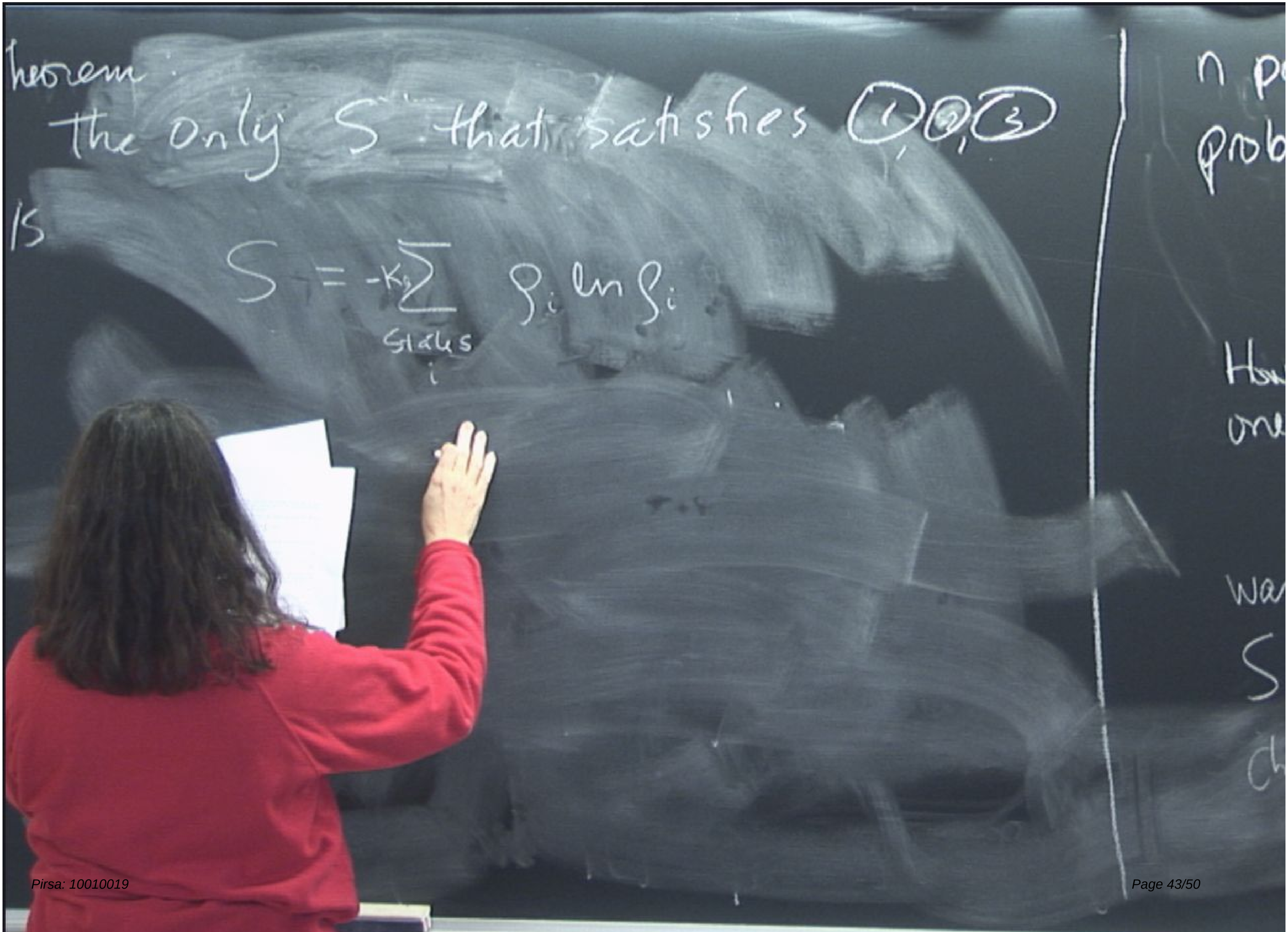
Theorem:
The only S that satisfies 1, 2, 3
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$S = -k_B \sum_{\text{states } i} \rho_i \ln \rho_i$

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with $k > 0$

n p
prob

How
one

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heorem:
the only S that satisfies ①, ②, ③

is

$$S = -k \sum_{\substack{\text{states}\\ i}} p_i \ln p_i$$

with $k > 0$

First consider the case when the choices are all equally likely [n choices]

$$S(1/n, 1/n, \ldots, 1/n) = A(n)$$

n p
prob

How
one

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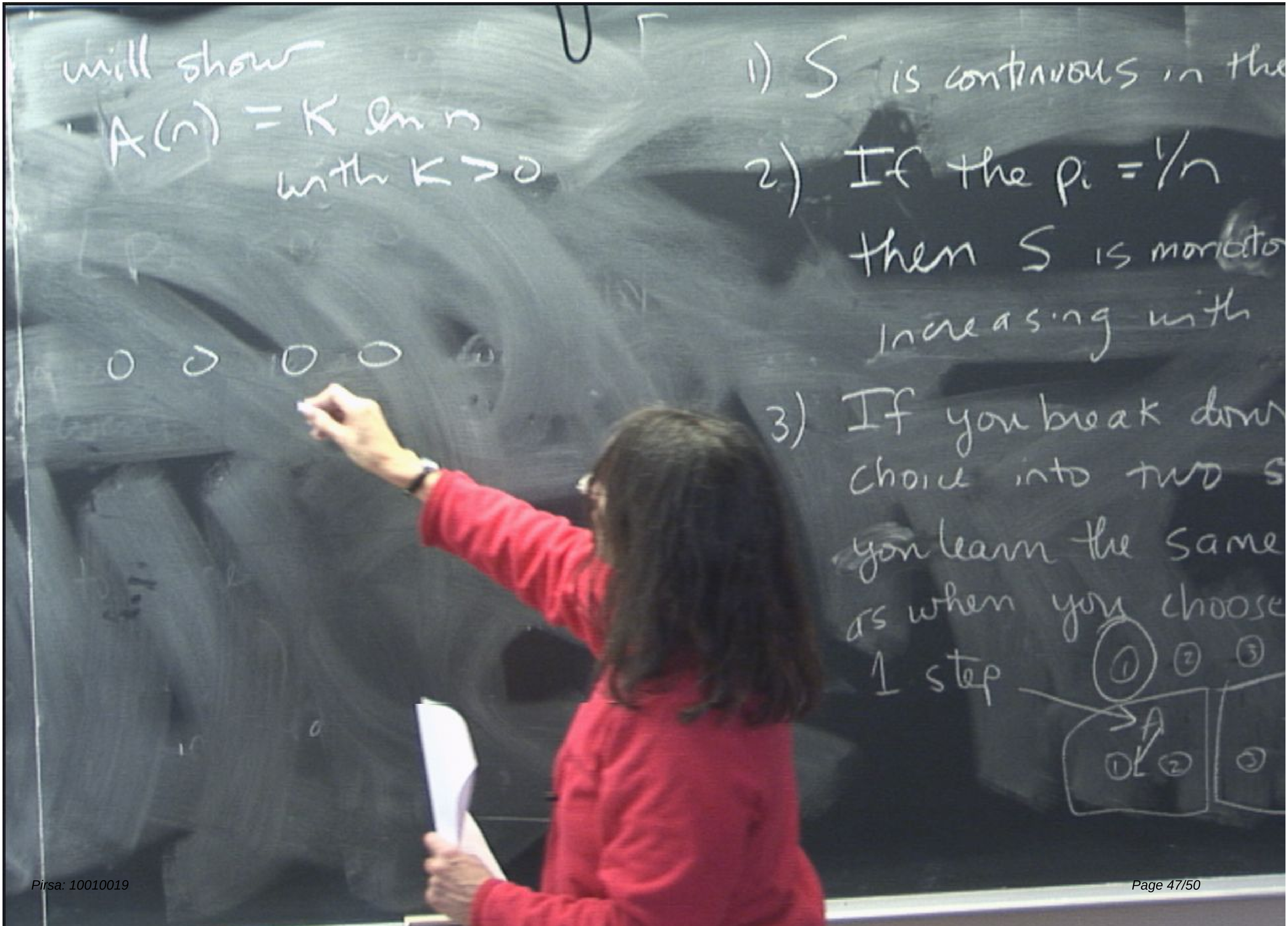
will show
A(n) = K ln n
with K > 0
1) S is continuous in the
2) If the pi = 1/n
then S is monotonic
increasing with
3) If you break down
choice into two s
you learn the same
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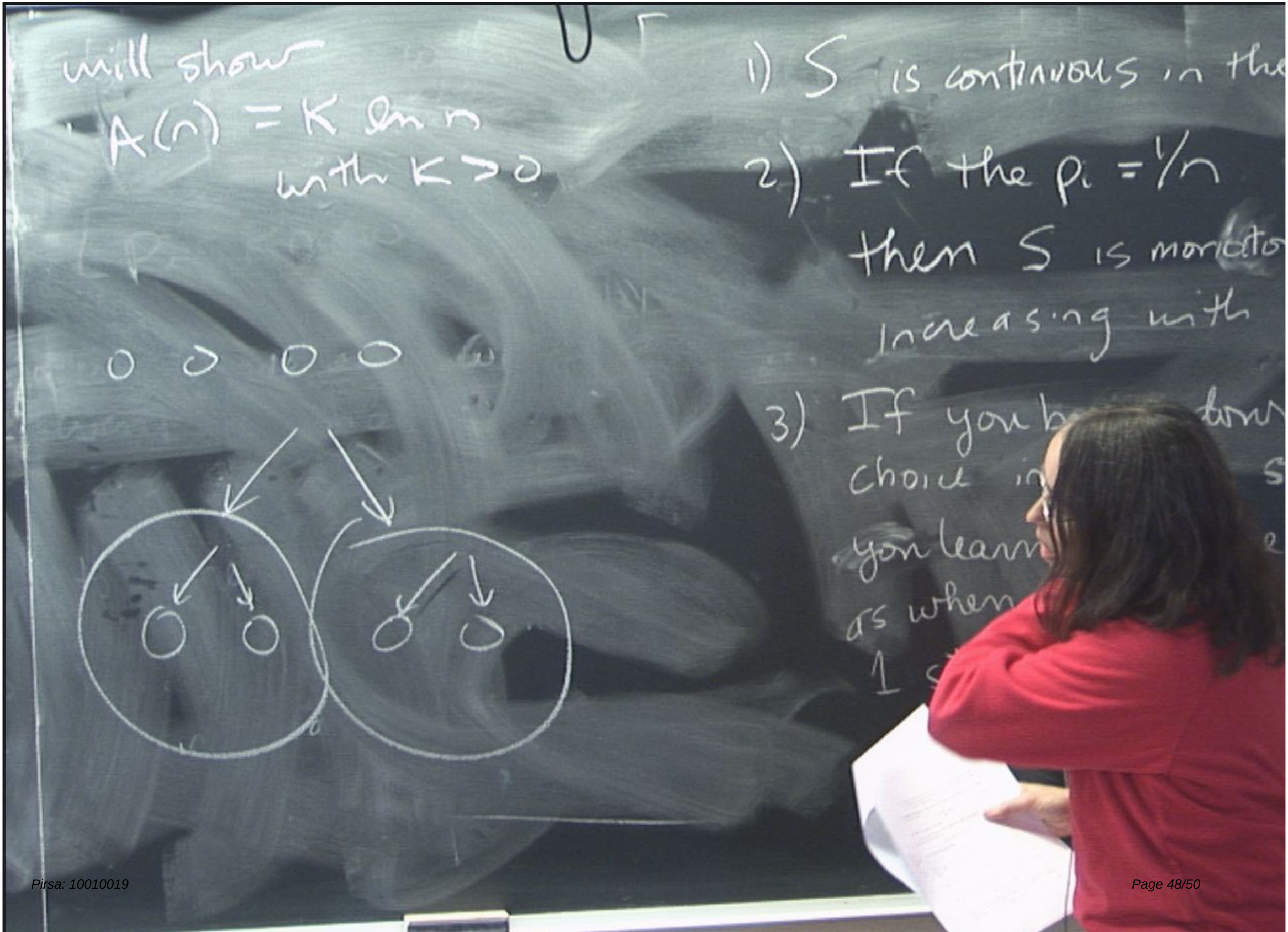
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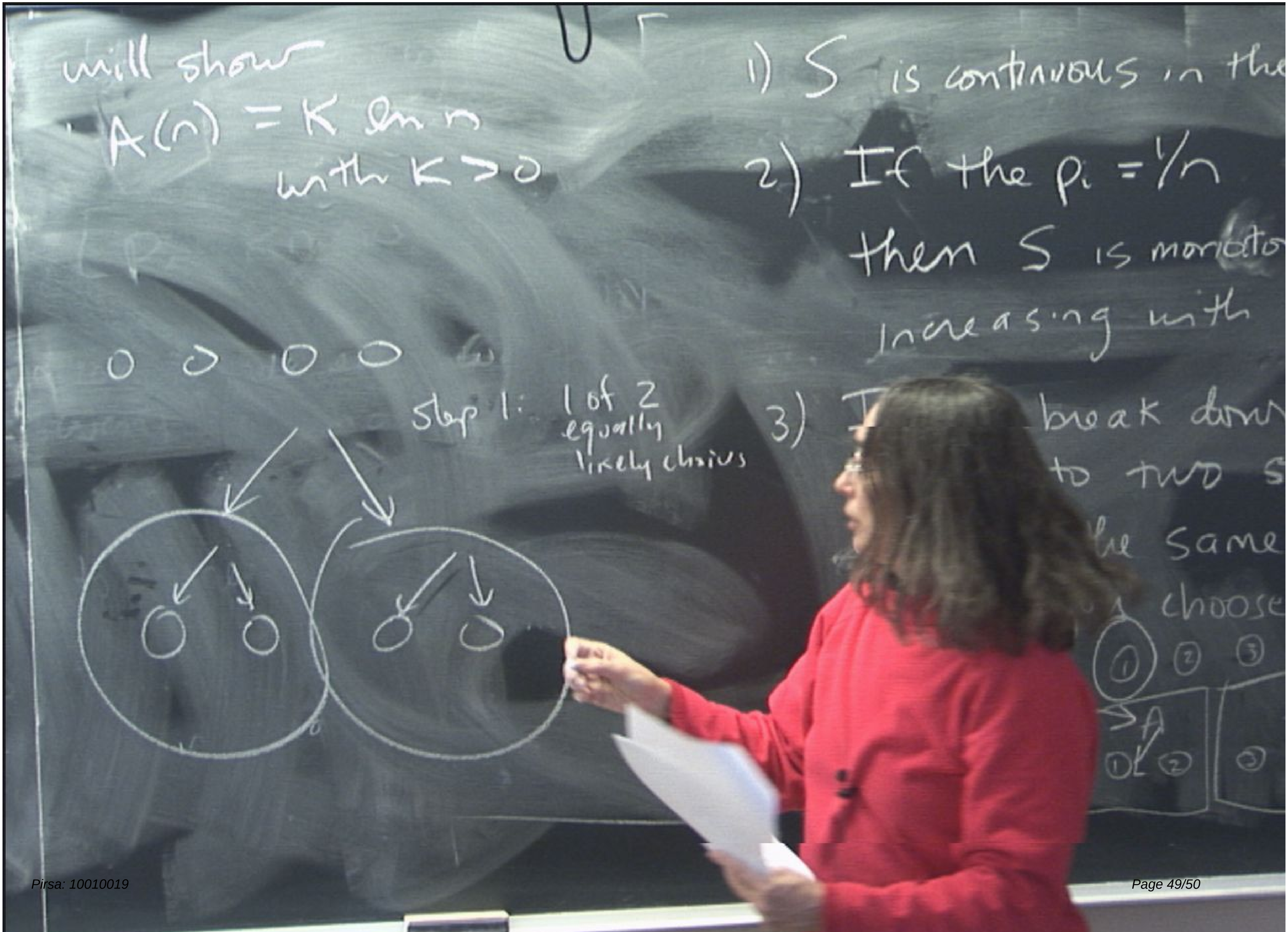
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A(n) = K ln n
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step 1: 1 of 2 equally likely choices
1) S is continuous in the
2) If the p_i = 1/n
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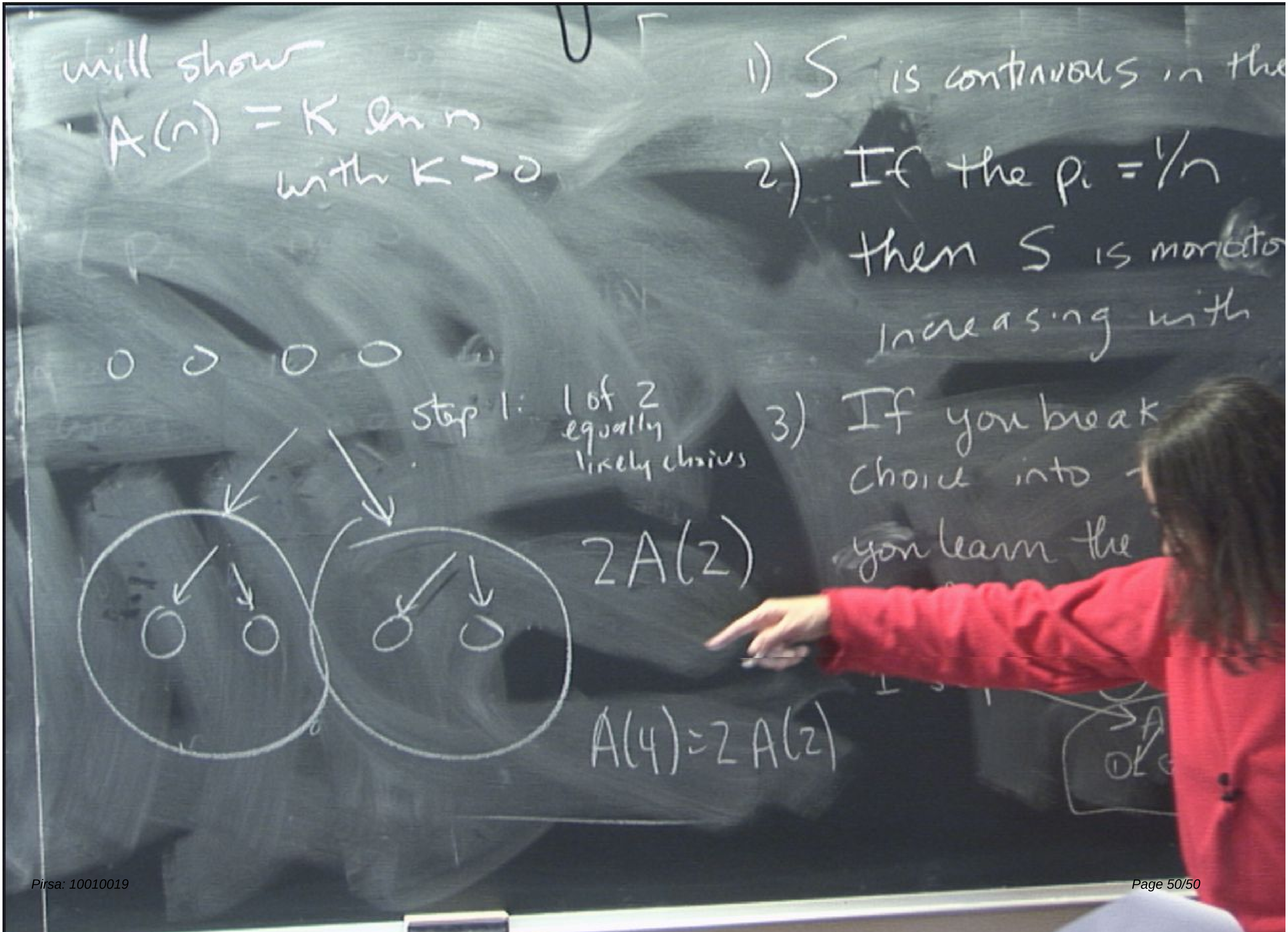
3)
break down
to two
the same
choose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A(n) = K ln n
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2A(2)
A(4) = 2 A(2)
1) S is continuous in the
2) If the pi = 1/n
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3) If you break
choice into
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